Spring 5-31-1955

Viscosity of non-Newtonian suspensions

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VISCOSITY OF NON-NEWTONIAN SUSPENSIONS

A THESIS
SUBMITTED TO THE FACULTY OF
THE DEPARTMENT OF CHEMICAL ENGINEERING
OF
NEWARK COLLEGE OF ENGINEERING

BY

DAVID GULLETT, B.S.

IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
OF
MASTER OF SCIENCE
IN CHEMICAL ENGINEERING

AND

EDWARD CURTIS, B.S.

IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
OF
MASTER OF SCIENCE
WITH A MAJOR IN
THE FIELD OF CHEMICAL ENGINEERING

NEWARK, NEW JERSEY

1955

APPROVAL OF THESIS

FOR

DEPARTMENT OF CHEMICAL ENGINEERING

NEWARK COLLEGE OF ENGINEERING

BY

FACULTY COMMITTEE

APPROVED: ______________________

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NEWARK, NEW JERSEY

JUNE, 1955

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ACKNOWLEDGMENTS

The authors wish to express their appreciation to Dr. Jerome J. Salamone for his assistance and guidance in this project. It was his interest in this project that led to the attempt of developing expressions for the viscosity of non-Newtonian suspensions. It was also his confidence in the practicality of such a development that made possible the various steps in this work which carried it to a successful conclusion.

ABSTRACT

The object of this project was to obtain a reasonable correlation of the effect of velocity, concentration and particle size on apparent viscosity of non-Newtonian slurries.

Through the use of dimensional and graphical analysis an equation, $\mu/\mu_w = 1.02\ (Ak/GC)^{.105}$, was developed which filled these conditions. The average deviation of the apparent viscosity calculated from this equation compared to the experimental value was 14.4%.

The authors believe that this correlation should be tested under a greater variety of conditions of particle size and particle thermal conductivity and for suspending mediums other than water.

INTRODUCTION AND BACKGROUND

1. Objective of this Project

The object of this study was to develop an expression for the apparent viscosity of non-Newtonian suspensions in terms of variables concerning either the characteristics of the suspended material or the suspension itself. The interest in such an expression was prompted by the fact that at the present time workers in the field of design of heat transfer equipment using suspensions must empirically determine apparent viscosity for the specific material involved. These empirical determinations require pilot plant equipment usually utilizing pipeline viscometers and the measurement of the pressure drop in the line in order to calculate a viscosity value. It is, therefore, of considerable practical value to be able to express apparent viscosity in terms of readily known characteristics of a suspension such as concentration of the suspended material, the particle size of the solid, the rate of flow of the suspension and the like.

2. Viscosity - Basic Concepts[1,2]

In considering basic differences between the states of matter, solids and fluids, a major distinction can be made between the two in their ability to show resistance to motion. This distinction then becomes a fundamental property with which to distinguish solids and fluids.

The term viscosity is used to describe this property. In hydrodynamics which deals with the motion of fluids, viscosity is a unique and important property. In fact, it is the relative degree to which this property occurs in a material that enables it to be classed as a fluid or a solid. For example, the principal reason for the difference between the flow characteristics of water and asphalt is that asphalt has a much greater viscosity than water.

Gases and ordinary liquids may be considered as fluids which undergo continuous deformation when subjected to shearing stress. The resistance to such shearing stress is called the viscosity. When the viscosity is unchanged under conditions of constant temperature and static pressure, the fluid is referred to as a Newtonian liquid.

On the other hand, if the rate of shear does not remain constant under fixed conditions of temperature and static pressure, there is a resultant change in viscosity. Such materials are referred to as non-Newtonian liquids.

Where discrete particles of solid material are suspended or dispersed in a liquid, the viscosity of the continuous phase is critically affected in that the viscosity is non-Newtonian and is considered an apparent viscosity of the mixture. Although the size and shape of the discrete particles can generally be considered to be of minor import until high concentrations are reached so as to alter the continuous phase of the mix, there is nothing to indicate that they should not be taken into account even at low concentrations. As mentioned in the Objective of this Project, it was thought highly desirable to ascertain the effect of these characteristics.

The chief reason why the relative particle size should be of concern even at low concentrations is that with particles of small diameter, surface area would increase to the point of major significance. In the field of heat transfer equipment, such as heat exchangers, the effectiveness of heat transfer within a suspension will assume greater proportions as the surface area increases due to decrease of particle diameter. This will be especially true if an attempt is made to cover a wide variety of materials in thermal conductivity. This would normally be the case since in industrial operations slurries are often encountered running from carbonates and silicates of low thermal conductivities to metallic powders of high thermal conductivities.

3. Previous Investigators

Previous investigators[3] on the flow behavior of non-Newtonian fluids in conduits have developed apparent viscosities from pipeline viscometers using pressure drop data. The pipeline viscometer is first calibrated with water since its density and viscosity are known. A plot is then made of the friction factor versus the Reynolds Number. Then by calculating the friction factor, using the bulk density and pressure drop for the slurry when run through the pipeline, a corresponding Reynolds Number can be read from the plot and a bulk or apparent viscosity can be determined for the suspension from this Reynolds Number.

The only other method of expressing bulk or apparent viscosity of a slurry has been by using the volume fraction of solid in suspension. Bonilla[4], in work on heat transfer properties of chalk and water slurries, found a correlation between the viscosities of the slurry and the water using the Hatschek equation:

$$\mu_b = \mu_w \quad (1 - \phi)^{0.33}$$

where μ_b and μ_w are the viscosities of the slurry and the water respectively and ϕ is the volume fraction of the solid in suspension.

Again Orr and Dalla Valle[5], working with suspensions of solids in water and ethylene glycol, found that viscosity determinations with a Saybolt Type Viscosimeter gave results which agreed closely with viscosity calculated from the equation:

$$\mu_b = \mu_w \quad (1 - \phi/\phi')^{1.8}$$

The terms μ_b, μ_w and ϕ are as just described above and ϕ' is the fraction of the solid in a sedimented bed.

The term, volume fraction, is of interest as will be seen later in development of an expression for viscosity in terms of particle size and surface area of the solid in suspension. Volume of solid and diameter of the particle of solid can be conveniently used to express surface area of the solid present in the slurry.

4. Source of Project

The source of data for this project was obtained from work on heat transfer characteristics of non-Newtonian suspensions by Professor J. J. Salamone[6] of the Department of Chemical Engineering of Newark College of Engineering and some of his graduate students[7]. These investigators had been concerned with heat transfer data of various slurries when operating a laboratory counter-current heat exchanger. The system had included a pipeline viscometer with a manometer connected to it by means of pressure taps. A diagram of the equipment is shown in Figure 1.

The temperatures of the slurries at the heat exchange section were obtained from thermometers mounted in thermometer wells at the end of the calming section on each side. The temperature of the viscometer was read from a thermometer mounted in a well beyond the pressure drop section at the

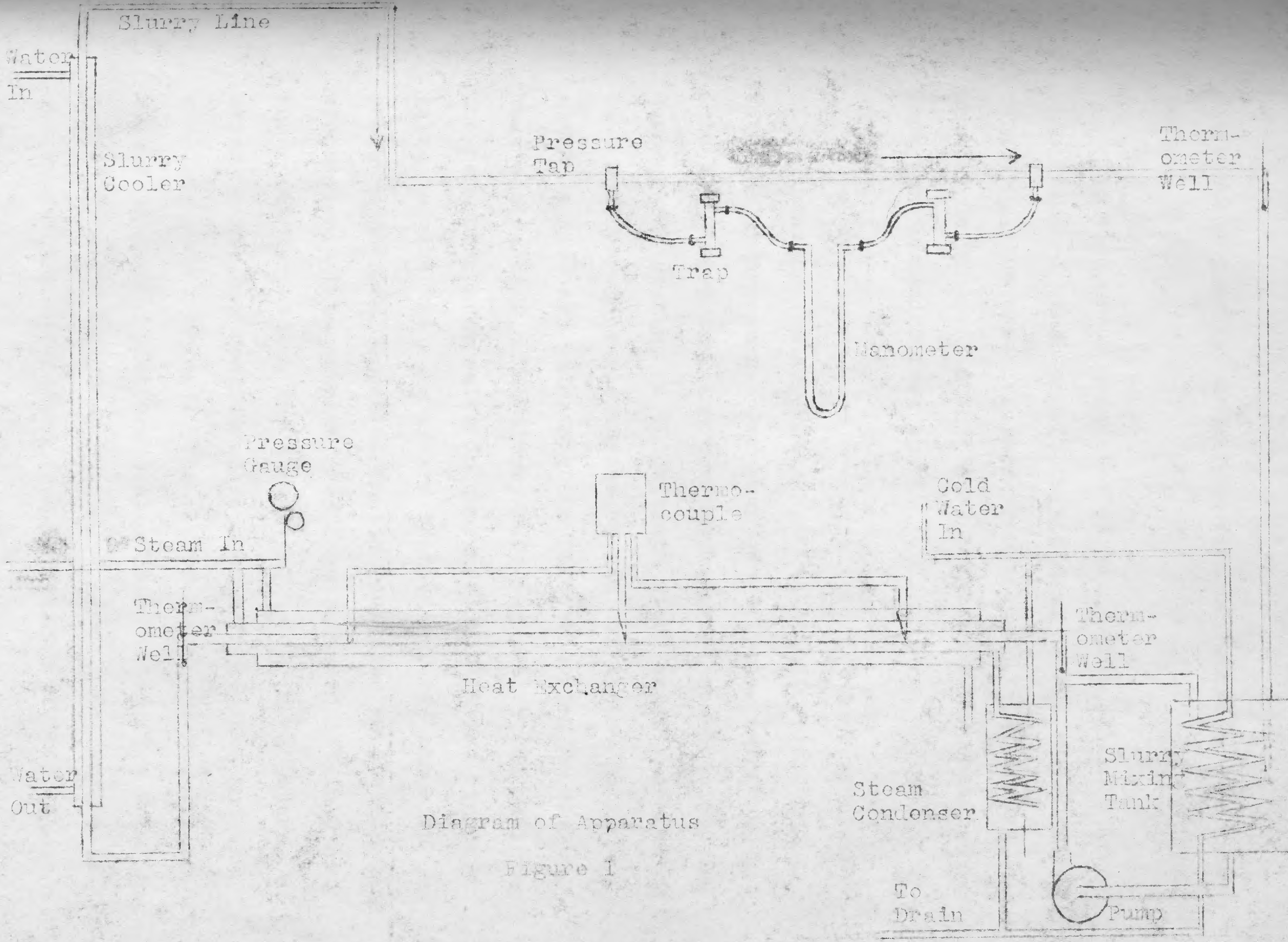


Diagram of Apparatus

Figure 1

point where the line drops downward to return to the slurry tank.

The viscometer consisted of an insulated ½ inch Iron Pipe Standard brass pipe. In the case of Professor Salamone's apparatus, the pressure taps were spaced 17-1/8 inches apart. In Binder and Pollara's apparatus, the pressure taps were spaced 6 feet apart.

The procedures used by these investigators consisted of calibrating the apparatus with water before proceeding with the slurry runs. For each set of runs, water was run into the slurry tank and the pump started to circulate it through the system. A "Lightning" Mixer which was mounted on the slurry tank was turned on and sufficient solid was added to give approximately the percent solid, be weight, that was desired. The slurry rate was set by manipulating the pump discharge valve in conjunction with the bypass valve to give the approximate desired rate as shown by the pressure drop differential on the manometer in the pipeline viscometer. When constant readings had been obtained on the manometer and on the outlet and inlet thermometers, a steady state was considered to have been reached and the readings were recorded. The slurry flow rate was determined by weighing on a platform scale the diverted flow from the slurry line over a known period of time. The density of the suspension was determined from the weight of four liters of the slurry using a flask in which the same volume of water had been previously weighed. Previously prepared curves of the weight fraction of solid versus density were then read off to provide a density value for each run. Tables I and II show the original basic data of these investigators which was used to derive the viscosity expressions appearing in this present thesis.

5. Original Plan of Project

This current study was originally started as separate projects with one author attempting to develop a correlation between velocity and viscosity, while the other author attempted to correlate viscosity with particle size and concentration. As will be seen in the several steps under Procedure and Theory it finally became apparent that all of these variables were necessary to define viscosity. From that point on the project was worked on as a joint problem. By means of dimensional and graphical analysis a relatively simple equation for the apparent viscosity of a slurry was found.

PROCEDURE AND THEORY

Part 1. Correlation of Slurry Velocity and Viscosity by Statistical Methods

a. Coefficient for Velocity and Viscosity at Constant Solid Content.

TABLE I

SOURCE OF MATERIALS AND OF PHYSICAL PROPERTIES USED

MATERIAL	SOURCE	DENSITY @ 20°C gm/cc	SPEC. HEAT @ 60°C BTU/ lb-°F	THERM. CONDUCTIVITY BTU hr-°F-ft	AVER. PARTICLE SIZE microns
Copper Powder	Charles Hardy, Inc. N.Y. City Electrolytic Copper Powder	8.92 Perry's Hdbk.	0.0932 Perry's Hdbk.	220 Perry's Hdbk.	Screened Fractions A-21 B-45 C-56 (measured) 30*
Carbon Powder	United Carbon Co. N.Y. City Uncompressed Carbon Black	2.0 Perry's Hdbk.	0.208 Perry's Hdbk.	3.0 Perry's Hdbk.	10 (measured)
Silica Powder	Exner Sand & Gravel Corp. N.Y. City Silica Flour	2.32 Perry's Hdbk.	0.194 Perry's Hdbk.	0.20 Perry's Hdbk.	1.5 (Company)
Chalk Powder	Thompson, Weinman & Co., Montclair, N.J. Atomite	2.71 (Company)	0.209 Perry's Hdbk.	0.40 Perry's Hdbk.	2.5 (Company
Snowflake White Powder	Thompson Weinman & Co., Montclair, N.J.	2.71 (Company)	0.209 Perry's Hdbk.	0.40 Perry's Hdbk.	6.0 (Company)
No. 1 White Powder	Thompson, Weinman & Co., Montclair, N.J.	2.71 (Company)	0.209 Perry's Hdbk.	0.40 Perry's Hdbk.	14.0 (Company)

* As calculated from size distribution data supply by manufacturer.

TABLE II

BASIC ORIGINAL DATA FROM INVESTIGATIONS BY SALAMONE AND BINDER & POLLARA

RUN NO.	VISCO-METER TEMP °C	SLURRY TIME min/75# slurry	SLURRY TEMP. °C Inlet	Outlet	DENSITY #/cu ft	% SOLID	APPAR-ENT VISCOS-ITY (ht sect) cps.
COPPER A							
19	60.5	0.532	50.65	68.35	67.86	10.0	0.85
20	58.2	0.600	46.75	65.35	68.23	10.6	0.86
21	61.0	0.658	48.80	70.25	68.28	10.7	0.88
22	59.5	0.773	46.00	69.47	68.23	10.6	0.97
23	59.0	0.983	43.10	70.49	68.23	10.6	1.19
24	58.5	1.289	40.10	72.00	68.48	11.0	1.52
25	60.0	0.517	49.90	67.10	67.44	9.40	0.81
26	60.5	0.559	50.90	69.12	67.44	9.40	0.81
27	57.0	0.600	47.95	66.25	67.44	9.40	0.83
28	59.5	0.657	47.10	68.20	67.44	9.40	0.86
29	59.2	0.757	45.35	68.95	67.65	9.60	0.93
30	58.5	1.000	41.95	69.85	67.65	9.60	1.14
31	61.4	0.527	50.30	67.20	65.25	6.20	0.76
32	61.0	0.587	48.70	67.05	65.56	6.45	0.78
33	58.0	0.632	46.10	64.50	65.56	6.45	0.79
34	60.0	0.715	46.60	67.95	65.56	6.45	0.82
35	60.0	0.802	45.10	68.45	65.56	6.45	0.85
36	59.0	1.072	41.80	69.70	65.56	6.45	1.11
37	56.0	0.560	46.20	62.65	64.52	4.80	0.74
38	60.5	0.598	48.35	66.70	64.52	4.80	0.74
39	59.5	0.656	47.15	66.90	64.52	4.80	0.75
40	59.0	0.720	46.35	67.64	64.52	4.80	0.79
41	60.0	0.800	45.25	68.05	64.73	5.10	0.85
42	59.0	1.024	42.55	68.90	64.73	5.10	1.03
43	60.0	0.555	50.50	67.35	64.11	4.15	0.70
44	57.0	0.589	47.20	64.80	64.11	4.15	0.71
45	60.0	0.647	48.25	68.05	64.11	4.15	0.72
46	59.3	0.723	46.85	68.15	64.21	4.30	0.76
47	59.0	0.837	45.15	68.60	64.21	4.30	0.82
48	58.0	1.280	40.20	70.50	64.21	4.30	1.05

TABLE II (Continued)

BASIC ORIGINAL DATA FROM INVESTIGATIONS BY SALAMONE AND BINDER & POLLARA

RUN NO.	VISCO-METER TEMP°C	SLURRY TIME min/75# slurry	SLURRY TEMP. °C Inlet	Outlet	DENSITY #/cu ft	% SOLID	APPAR-ENT VISCOS-ITY (ht sect) cps.
49	58.0	0.560	48.30	63.80	63.27	2.80	0.64
50	57.0	0.597	46.95	64.65	63.07	2.50	0.64
51	56.5	0.667	45.65	64.55	63.27	2.80	0.66
52	57.0	0.765	44.60	66.05	63.27	2.80	0.68
53	58.0	0.968	43.20	68.95	63.27	2.80	0.73
54	57.0	1.458	38.50	70.80	63.27	2.80	0.90
COPPER B							
55	64.0	0.577	54.76	70.85	63.48	4.30	0.65
56	63.5	0.614	54.10	70.90	63.69	4.84	0.70
57	63.0	0.686	52.89	71.21	63.89	5.20	0.75
58	63.0	0.810	51.30	71.89	63.89	5.20	0.82
59	63.0	1.010	49.50	73.20	63.89	5.20	0.89
60	64.0	1.720	44.75	77.05	63.69	4.84	2.00
61	64.4	0.601	55.12	71.21	62.65	2.75	0.56
62	64.0	0.643	54.20	71.30	62.65	2.75	0.58
63	63.7	0.704	53.09	71.35	62.65	2.75	0.61
64	63.4	0.846	51.40	71.93	62.65	2.75	0.65
65	63.0	1.179	48.48	73.84	62.75	3.00	0.72
COPPER C							
66	65.4	0.590	55.00	70.98	62.96	2.90	0.58
67	65.0	0.710	53.56	71.45	63.17	3.20	0.64
68	65.0	0.800	52.18	71.91	63.06	3.10	0.68
69	65.0	0.936	50.98	72.75	63.06	3.10	0.72
70	65.0	1.238	48.49	74.35	63.37	3.60	0.78
71	65.0	0.602	54.80	70.70	62.54	2.15	0.56
72	65.0	0.683	53.70	70.96	62.54	2.15	0.61
73	64.5	0.810	51.80	71.45	62.65	2.34	0.66
74	64.0	1.040	49.58	72.66	62.54	2.15	0.72
75	65.0	1.546	45.92	75.06	62.65	2.34	0.78

TABLE II (Continued)

RUN NO.	VISCO-METER TEMP°C	SLURRY TIME min/75# slurry	SLURRY TEMP. °C Inlet	Outlet	DENSITY #/cu ft	% SOLID	APPARENT VISCOSITY (ht sect) cps.
SILICA							
76	65.5	0.596	55.18	71.12	62.44	3.40	0.66
77	65.5	0.650	54.28	71.28	62.44	3.40	0.66
78	65.4	0.729	53.25	71.78	62.44	3.40	0.66
79	65.0	0.875	51.70	72.51	62.44	3.40	0.66
80	65.0	1.258	47.82	73.95	62.44	3.40	0.66
81	65.5	0.597	55.08	71.33	63.69	7.06	0.68
82	65.5	0.656	53.88	71.51	63.58	6.75	0.70
110	65.5	0.579	55.05	71.14	64.40	9.10	0.68
111	65.2	0.645	53.70	71.25	64.36	8.94	0.69
112	64.8	0.728	52.30	71.65	64.32	8.94	0.73
113	64.2	0.937	50.25	72.55	64.32	8.94	0.84
CARBON							
83	65.5	0.570	57.05	71.59	61.81	4.35	0.64
84	66.0	0.620	56.55	71.89	61.85	4.60	0.65
85	65.0	0.702	55.72	72.09	61.85	4.60	0.69
86	65.0	0.814	54.88	72.85	61.85	4.60	0.75
87	65.0	1.064	52.74	73.72	61.89	5.00	0.91
88	69.5	0.609	60.20	73.84	62.02	6.00	0.94
ATOMITE							
89	66.5	0.589	55.90	71.35	62.81	4.00	0.54
90	66.5	0.683	55.30	71.58	62.81	4.00	0.56
91	66.5	0.722	54.25	72.05	62.81	4.00	0.58
92	66.0	0.871	52.55	72.68	62.81	4.00	0.63
93	65.8	1.012	50.80	73.21	62.81	4.00	0.68
94	67.0	0.583	56.38	72.00	63.98	7.05	0.67
95	67.0	0.639	55.45	72.08	64.02	7.15	0.68
96	66.5	0.738	54.12	72.47	64.02	7.15	0.71
97	66.5	0.877	52.35	72.89	64.02	7.15	0.74
98	66.5	1.067	50.25	73.60	64.06	7.25	0.80
99	67.2	0.572	56.28	71.98	65.25	10.4	0.76
100	66.8	0.632	55.25	72.11	65.28	10.4	0.78
101	66.2	0.701	54.20	72.20	65.25	10.4	0.79
102	66.0	0.806	52.60	72.55	65.32	10.5	0.82
103	65.5	0.977	50.60	73.15	65.32	10.5	0.88
104	65.0	1.415	46.90	74.48	65.40	10.7	1.04

TABLE II (Continued)

BASIC ORIGINAL DATA FROM INVESTIGATIONS BY SALAMONE AND BINDER & POLLARA

RUN NO.	VISCO-METER TEMP°C	SLURRY TIME mass rate lbs/min	DENSITY #/cu ft	% SOLID	APPARENT VISCOSITY (visco-meter) #/min-ft
ATOMITE					
1	52.0	47.75	64.6	6.4	.0303
2	54.8	57.60	63.9	5.0	.0184
3	56.0	30.60	63.6	4.0	.0706
4	53.5	23.10	63.6	4.0	.0348
5	53.8	38.60	63.5	4.0	.0275
6	55.8	45.62	63.3	3.3	.0206
7	57.0	51.75	65.1	7.8	.0268
8	57.2	41.25	65.2	7.8	.0262
9	57.6	37.60	65.2	7.8	.0256
10	58.2	31.30	65.3	8.0	.0313
11	57.4	21.25	65.4	8.3	.0350
SNOWFLAKE					
1	58.5	56.7	64.0	4.8	.0184
2	59.1	48.5	64.0	4.8	.0255
3	59.1	43.5	64.0	4.8	.0215
4	60.0	35.7	64.0	4.8	.0225
5	58.1	28.6	64.0	4.8	.0224
6	57.0	17.2	64.0	4.8	.0436
7	57.2	20.3	66.3	10.4	.0493
8	59.0	27.7	66.3	10.4	.0308
9	62.0	35.1	66.3	10.4	.0256
10	62.0	42.4	66.3	10.4	.0238
11	62.7	50.6	66.3	10.4	.0216
12	64.2	56.1	66.3	10.4	.0211

TABLE II (Continued)

BASIC ORIGINAL DATA FROM INVESTIGATIONS BY SALAMONE AND BINDER & POLLARA

RUN NO.	VISCO-METER TEMP°C	SLURRY TIME mass rate lbs/min	DENSITY #/cu ft	% SOLID	APPARENT VISCOSITY (visco-meter) #/min-ft
NO. 1 WHITE					
1	57.6	47.6	64.5	5.6	.0339
2	58.0	42.1	64.5	5.6	.0314
3	58.1	39.6	64.5	5.6	.0322
4	57.5	34.0	64.5	5.6	.0406
5	56.2	25.1	64.5	5.6	.0386
6	59.0	54.0	66.4	11.5	.0284
7	56.5	44.3	66.4	11.5	.0316
8	59.5	35.4	66.4	11.5	.0223
9	58.3	26.6	66.4	11.5	.0307
10	57.0	22.8	66.4	11.5	.0272
COPPER					
1	62.6	59.2	63.6	3.0	.0233
2	64.3	44.8	63.6	3.0	.0339
3	61.9	39.3	63.6	3.0	.0206
4	58.4	29.4	63.6	3.0	.0214
5	61.1	32.2	63.6	3.0	.0354
6	57.0	25.6	63.6	3.0	.0266
7	54.6	19.4	63.6	3.0	.0401

As an initial step it was decided to determine the correlation coefficient for the apparent viscosity and the linear velocity of each slurry. Isothermal conditions were assumed for the viscometer, although there was a deviation in average temperature of 10% for 130 runs. However, in the case of each material which constituted the slurry, the concentration of the solid material in the suspension was scattered over a number of values from about 2% solids to 10% solids. For this reason each individual slurry was broken down into a number of groups of nominal solid content wherever possible. In other words, those values which were nearest to a whole figure such as 5, 6, 7, 8 or 10% were grouped together. For each group correlation coefficients for the linear velocity and viscosity were determined by a standard statistical method[8].

The value of the coefficient of correlation, r, is calculated from the equation:

$$r = \frac{\overline{\mu\, v} - \bar{\mu}\,\bar{v}}{\left(\frac{\Sigma \mu^2}{n} - \bar{\mu}^2\right)^{\frac{1}{2}} \left(\frac{\Sigma v^2}{n} - \bar{v}^2\right)^{\frac{1}{2}}}$$

where μ is the apparent viscosity of the slurry, v is the linear velocity of the slurry, $\bar{\mu}$ and $\bar{v}$ are the means of their respective terms,

$\bar{\mu}\,\bar{v}$ is the product of the means

$\overline{\mu\, v}$ is the mean of the products of μ and v and n is the number of runs involved.

$\Sigma\mu^2$ and Σv^2 are the summation of the squares of the viscosity and velocity respectively.

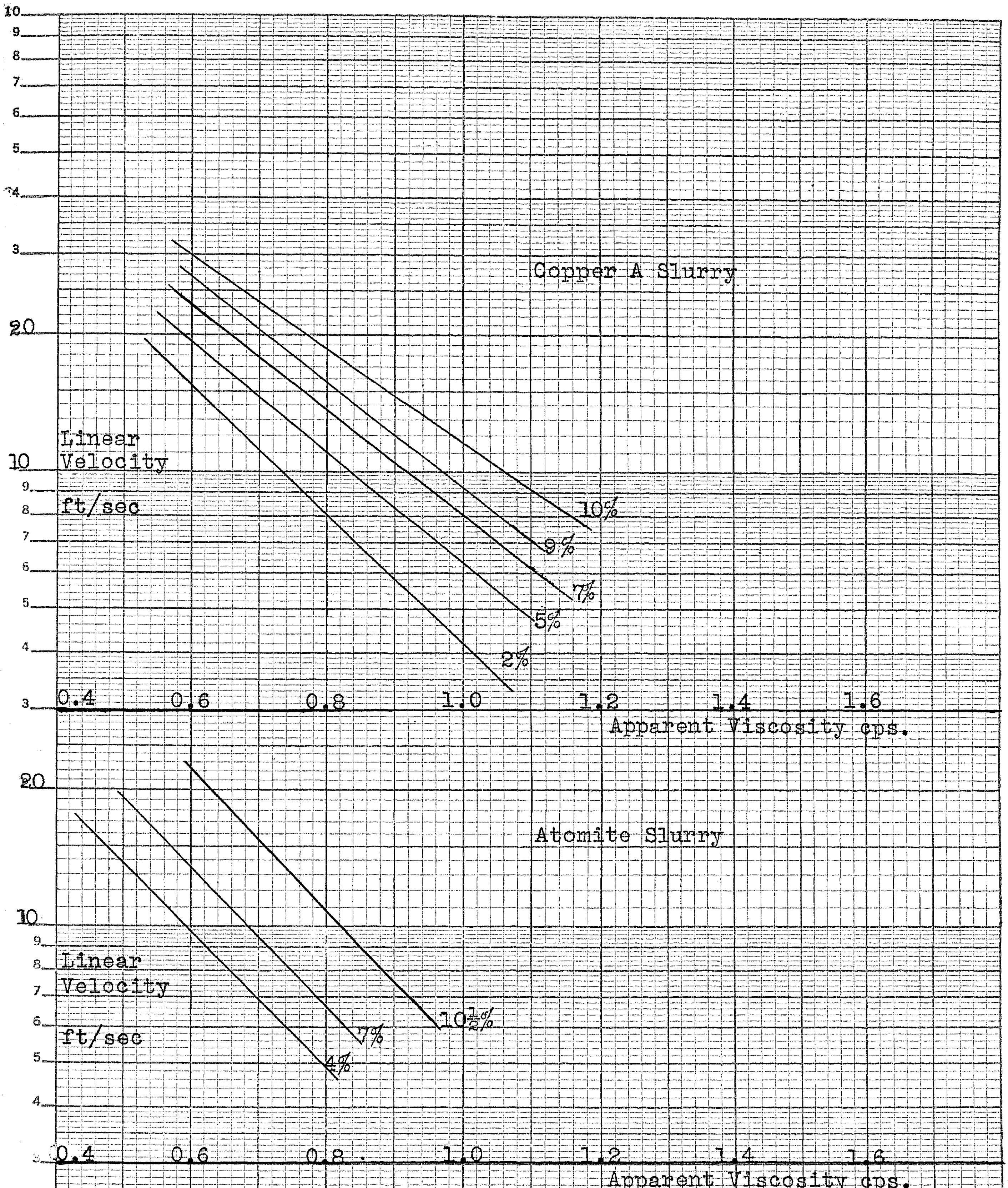
A tabulation is presented in Table III for the correlation coefficient of viscosity and velocity at constant solid content. Since most of the values lie between 0.6 and 1.0 a satisfactory correlation is indicated. The negative value of the correlation coefficient indicates that viscosity decreases at higher flow rates. On the other hand, if a plot is made of velocity versus viscosity for these groups of nominal solid contents (Figure 2) it will be seen that there is a family of curves with viscosity increasing as the solid content increases. The two items, Copper A and Atomite, were selected because there were enough runs to provide an adequate range of concentrations of solid content to illustrate this point.

b. Coefficient for Viscosity and Solid Content.

In a similar manner, if correlation coefficients are calculated for the viscosity and the percent solid of each slurry, a positive value of r is obtained (refer to Table V) which also indicates that viscosity increases with increasing amount of concentration.

Part 2. Correlation of Viscosity and Particle Size

From a practical standpoint it is quite easy to see how an increasing amount of solid matter would increase the



Apparent Viscosity and Linear Velocity at
Equal Concentrations of Solid Material

Figure 2

TABLE III

CORRELATION COEFFICIENT FOR VISCOSITY AND VELOCITY AT CONSTANT SOLID CONTENT

Material		r
Copper A		
Solid Content	10%	-1.010
	9%	-0.814
	7%	-0.860
	5%	-0.863
	2%	-0.218
Copper B		
Solid Content	5%	-0.872
	2%	-0.641
Copper C		
Solid Content	3%	-0.832
	2%	-0.340
Silica Flour		
Solid Content	9%	-0.631
	3½%	-0.720
Atomite		
Solid Content	10½%	-0.740
	7 %	-0.124
	4 %	-0.608
Carbon Black		
Solid Content	5%	-0.896

TABLE IV

APPARENT VISCOSITY AND LINEAR VELOCITY AT EQUAL CONCENTRATIONS OF SOLID MATERIAL

RUN NO.	APPARENT VISCOSITY cps	LINEAR VELOCITY ft/sec
Copper A		
	10% Solids	
19	0.84	16.4
20	0.83	14.4
21	0.86	13.3
22	0.95	11.2
23	1.12	8.9
24	1.44	6.7
	9% Solids	
25	0.78	17.0
26	0.81	15.8
27	0.80	14.7
28	0.83	13.4
29	0.88	11.6
30	1.08	8.8
	7% Solids	
31	0.73	17.2
32	0.74	15.4
33	0.73	14.2
34	0.79	12.6
35	0.79	11.3
36	0.99	8.4
	5% Solids	
37	0.71	16.3
38	0.68	15.3
39	0.71	13.9
40	0.77	12.7
41	0.79	11.4
42	0.97	8.9
43	0.68	16.6
44	0.70	15.7

TABLE IV (Continued)

APPARENT VISCOSITY AND LINEAR VELOCITY AT EQUAL CONCENTRATIONS OF SOLID MATERIAL

RUN NO.		APPARENT VISCOSITY cps		LINEAR VELOCITY ft/sec
	Copper A			
			5% Solids	
45		0.69		14.3
46		0.74		12.7
47		0.78		11.0
48		0.97		7.2
			2% Solids	
49		0.61		16.7
50		0.63		15.7
51		0.64		14.0
52		0.67		12.2
53		0.69		9.7
54		0.86		6.4
	Atomite			
			10½% Solids	
99		0.70		15.8
100		0.72		14.3
101		0.71		12.9
102		0.74		11.3
103		0.80		9.3
104		0.97		6.4
			7% Solids	
94		0.62		15.9
95		0.63		14.5
96		0.64		12.5
97		0.67		10.5
98		0.72		8.7
			4% Solids	
89		0.50		16.0
90		0.50		13.8
91		0.52		13.0
92		0.57		10.8
93		0.63		9.3

TABLE V

CORRELATION COEFFICIENT FOR VISCOSITY AND % SOLID

Material	r
Copper A	0.845
Copper B	0.980
Copper C	0.234
Silica Flour	0.407
Atomite	0.984
Carbon Black	0.817

apparent viscosity. On the other hand, the fact that the apparent viscosity decreases as the flow rate becomes greater would indicate that the movement of the fluid over the solid is a greater factor in the apparent viscosity. Therefore, particle size of the solid material which will affect the surface area over which the fluid flows must be taken into consideration. However, the wide variations in the flow rate and mass fraction did not allow a correlation between apparent viscosity and particle size under conditions of constant flow rate and concentration.

Part 3. Combined Effect of Velocity and Concentration on Viscosity

Because of the difficulties mentioned in Part 2, the effect of particle size upon viscosity was temporarily neglected. The next phase was to determine the combined effect of velocity and concentration on viscosity.

The effect of these variables upon viscosity was shown by dimensional analysis[9] to be of the form $\mu = f(Dv\rho/\phi)$, where f represents some function. The addition of D (pipe diameter) is necessary to make the equation dimensionally sound. Concentration is represented by the volume fraction (ϕ) which being dimensionless has no effect upon the validity of the initial relationship $\mu = f(Dv\rho)$. Velocity (v) and density (ρ) are listed as separate terms in these first relationships. However, the product of these terms, the mass velocity (G), is used in the remainder of the text including the tables, graphs and sample calculations.

A graphical plot of the viscosity versus DG/Ø is shown in Figure 3. Although there is a considerable amount of scattering of data in this plot, there is strong indication that a series of definite relationships exists. At least two distinct groups are present. One group, copper particles varying in size from 21 to 56 microns, shows good correlation. The second group contains the non-metallic materials; silica, carbon and three sizes of chalk. Particle size in this group ranges from 1.5 microns for the silica to 14 microns for the largest chalk particle, No. 1 White. Scattering is much more pronounced in this second group.

At this stage of the investigation it was not possible to tell whether the formation of two separate groups was traceable to the difference in particle size or to the fact that the two groups were of a different nature.

Part 4. Combined Effect of Velocity, Concentration and Particle Size on Viscosity

It was decided to neglect the second possibility at this time and to determine what effect the introduction of particle size would have.

At this point it became necessary to make two very

Apparent Viscosity and Combined Effect of Concentration and Velocity

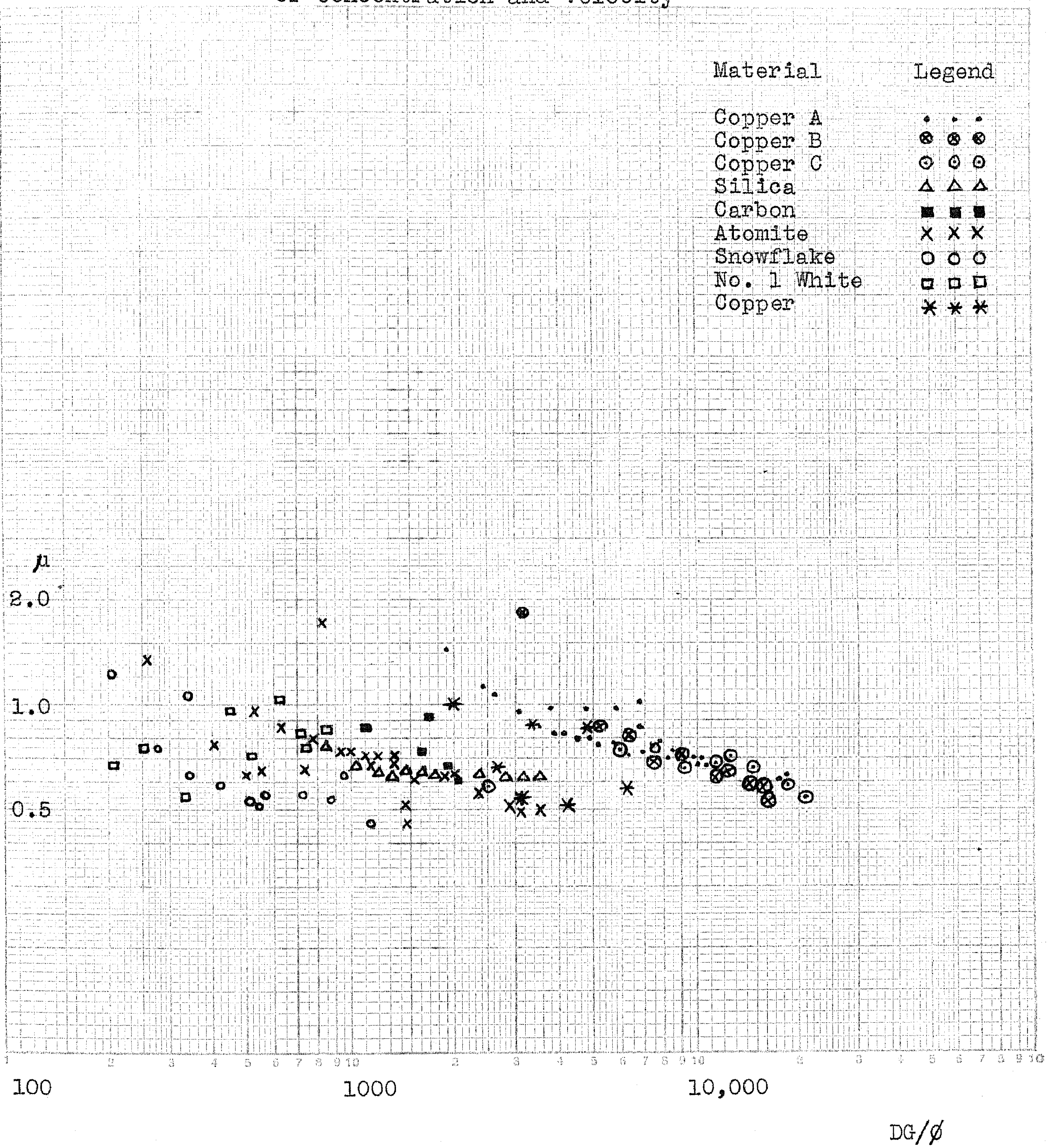


Figure 3

TABLE VI

COMBINED EFFECT OF CONCENTRATION AND VELOCITY ON APPARENT VISCOSITY

RUN NO.	G	DG	Ø	DG/Ø	μ
			Copper A		
19	1120	58.0	0.01225	4,740	0.84
20	990	51.3	0.01310	3,910	0.83
21	904	46.8	0.01315	3,570	0.86
22	775	40.0	0.01310	3,060	0.95
23	607	31.6	0.01310	2,410	1.12
24	462	24.9	0.01360	1,890	1.44
25	1155	59.8	0.01140	5,250	0.78
26	1060	54.8	0.01140	4,810	0.81
27	990	51.3	0.01140	4,490	0.80
28	904	46.8	0.01140	4,120	0.83
29	787	40.7	0.01170	3,480	0.88
30	596	30.9	0.01170	2,640	1.08
31	1130	58.6	0.00738	7,940	0.73
32	1020	52.8	0.00762	6,950	0.74
33	945	49.0	0.00762	6,440	0.73
34	834	43.2	0.00762	5,770	0.79
35	742	38.4	0.00762	5,040	0.79
36	556	28.8	0.00762	3,780	0.99
37	1060	55.0	0.00558	9,860	0.71
38	990	51.3	0.00558	9,200	0.68
39	904	46.8	0.00558	8,410	0.71
40	824	42.6	0.00558	7,640	0.77
41	745	38.0	0.00596	6,380	0.79
42	580	30.0	0.00596	5,800	0.97
43	1070	55.5	0.00480	10,700	0.68
44	1010	52.4	0.00480	10,100	0.70
45	920	47.6	0.00480	9,200	0.69
46	825	42.7	0.00499	8,560	0.74
47	714	37.0	0.00499	7,420	0.78
48	466	24.1	0.00499	4,840	0.97

TABLE VI (Continued)

COMBINED EFFECT OF CONCENTRATION AND VELOCITY ON APPARENT VISCOSITY

RUN NO.	G	DG	Ø	DG/Ø	μ
			Copper A		
49	1065	55.2	0.00319	17,300	0.61
50	1000	51.8	0.00284	18,300	0.63
51	888	46.0	0.00319	14,400	0.64
52	778	40.4	0.00319	12,650	0.67
53	615	31.8	0.00319	10,000	0.69
54	408	21.2	0.00319	6,740	0.86
			Copper B		
55	1030	53.4	0.00484	11,020	0.62
56	968	50.4	0.00555	9,080	0.70
57	868	45.2	0.00598	7,540	0.74
58	735	38.2	0.00598	6,380	0.80
59	588	30.6	0.00598	5,110	0.88
60	346	17.9	0.00555	3,130	1.84
61	990	51.4	0.00310	16,550	0.54
62	923	48.4	0.00310	15,600	0.58
63	845	44.8	0.00310	14,500	0.60
64	702	36.4	0.00310	11,750	0.64
65	504	26.2	0.00350	7,500	0.70
			Copper C		
66	1010	52.4	0.00329	15,950	0.57
67	837	43.4	0.00364	11,950	0.63
68	744	38.5	0.00365	10,530	0.65
69	635	32.9	0.00365	9,030	0.69
70	480	24.8	0.00411	6,040	0.75
71	998	51.2	0.00242	21,100	0.55
72	864	44.7	0.00242	18,450	0.60
73	735	38.1	0.00264	14,480	0.65
74	572	29.6	0.00242	12,250	0.70
75	386	20.0	0.00264	7,580	0.76

TABLE VI (Continued)

COMBINED EFFECT OF CONCENTRATION AND VELOCITY ON APPARENT VISCOSITY

RUN NO.	G	DG	Ø	DG/Ø	μ
			Silica		
76	996	51.6	0.0147	3,520	0.62
77	915	47.4	0.0147	3,215	0.61
78	815	42.2	0.0147	2,870	0.61
79	695	36.0	0.0147	2,450	0.63
80	468	24.3	0.0147	1,650	0.61
81	995	51.5	0.0293	1,760	0.63
82	828	42.8	0.0296	1,450	0.65
110	1030	53.4	0.0406	1,315	0.63
111	924	47.8	0.0396	1,210	0.64
112	816	42.4	0.0396	1,070	0.68
113	635	32.9	0.0396	832	0.78
			Carbon		
83	1045	54.2	0.0216	2,510	0.60
84	960	49.6	0.0228	2,190	0.61
85	845	43.8	0.0228	1,920	0.68
86	730	37.8	0.0228	1,660	0.74
87	558	28.9	0.0249	1,160	0.88
88	975	50.5	0.0298	1,700	0.91
			Atomite		
89	1010	52.4	0.0150	3,490	0.50
90	870	45.1	0.0150	3,010	0.50
91	825	42.6	0.0150	2,840	0.52
92	683	35.4	0.0150	2,360	0.57
93	587	30.4	0.0150	2,015	0.63
94	1020	52.8	0.0268	1,975	0.62
95	930	48.2	0.0268	1,800	0.63
96	807	41.8	0.0268	1,565	0.64
97	675	35.0	0.0268	1,315	0.67
98	558	28.9	0.0274	1,055	0.72

TABLE VI (Continued)

COMBINED EFFECT OF CONCENTRATION AND VELOCITY ON APPARENT VISCOSITY

RUN NO.	G	DG	Ø	DG/Ø	μ
99	1040	54.0	0.0403	1,340	0.70
100	940	48.6	0.0403	1,205	0.72
101	846	43.8	0.0403	1,090	0.71
102	736	38.1	0.0406	936	0.74
103	608	31.5	0.0406	775	0.80
104	420	21.8	0.0414	528	0.97
			Atomite		
1	377	19.5	0.0215	907	0.75
2	455	23.6	0.0170	1,490	0.46
3	242	12.3	0.0148	831	1.75
4	182	9.45	0.0148	639	0.86
5	304	15.8	0.0135	1,170	0.68
6	360	18.7	0.0128	1,460	0.51
7	408	21.2	0.0297	715	0.66
8	326	16.9	0.0308	548	0.65
9	296	15.4	0.0308	500	0.64
10	251	13.0	0.0321	405	0.78
11	168	8.51	0.0329	259	1.37
			Snowflake		
1	448	23.4	0.0205	1,140	0.46
2	383	19.9	0.0205	972	0.63
3	344	17.9	0.0205	875	0.53
4	282	14.6	0.0205	714	0.56
5	226	11.7	0.0205	571	0.56
6	136	7.05	0.0205	342	1.08
7	160	8.3	0.0408	203	1.22
8	219	11.4	0.0408	279	0.75
9	277	14.4	0.0408	353	0.64
10	334	17.3	0.0408	422	0.59
11	400	20.8	0.0408	508	0.54
12	444	23.0	0.0408	564	0.52

TABLE VI (Continued)

COMBINED EFFECT OF CONCENTRATION AND VELOCITY ON APPARENT VISCOSITY

RUN NO.	G	DG	∅	DG/∅	μ
			No. 1 White		
1	376	19.5	0.0230	850	0.84
2	333	17.3	0.0230	754	0.78
3	312	16.2	0.0230	704	0.82
4	268	13.9	0.0230	604	1.01
5	198	10.3	0.0230	446	0.96
6	426	22.2	0.0437	507	0.70
7	350	18.2	0.0437	416	0.78
8	280	14.6	0.0437	334	0.55
9	210	10.9	0.0437	249	0,76
10	180	9.35	0.0437	214	0.68
			Copper		
1	468	24.3	0.00384	6,340	0.58
2	354	18.4	0.00384	4,790	0.84
3	311	16.2	0.00384	4,220	0.51
4	232	12.0	0.00384	3,110	0.53
5	254	13.2	0.00384	3,440	0.88
6	202	10.5	0.00384	2,730	0.66
7	153	7.94	0.00384	2,060	1.00

important assumptions. First, the particles were assumed to be of uniform size and shape (spherical). Secondly, the size measurement as listed by the manufacturers was accepted at face value.

Particle size of the suspended material was introduced by means of the expression A, which is actually the surface area of solid particle per unit volume of slurry. This expression and the relation $6\phi/D_p$ (diameter of particle) are interchangeable.

The relationship between the variables used in Figure 3, the expression A and viscosity is represented by the form $\mu = f(DG/A)$. Dimensional analysis shows the correct form to be actually $\mu = f(G/A)$ or its counterpart $\mu = f(D_pG/6\phi)$. The first form is used as a matter of convenience in calculation and tabulation.

Figure 4 graphically shows the relationship between the apparent or bulk viscosity of the slurry and these variables. It can be seen that the grouping mentioned in Part 3 has not been eliminated or reduced, and has actually become more noticeable. The first group still contains only the copper particles. However, the second group has split into two separate categories. Carbon particles (10 micron size) and the largest chalk particles (14 microns) make up the first category, while silica and the other chalk particles comprise the second. Particle size of this last

Apparent Viscosity and Combined Effect of Concentration, Particle Size and Velocity

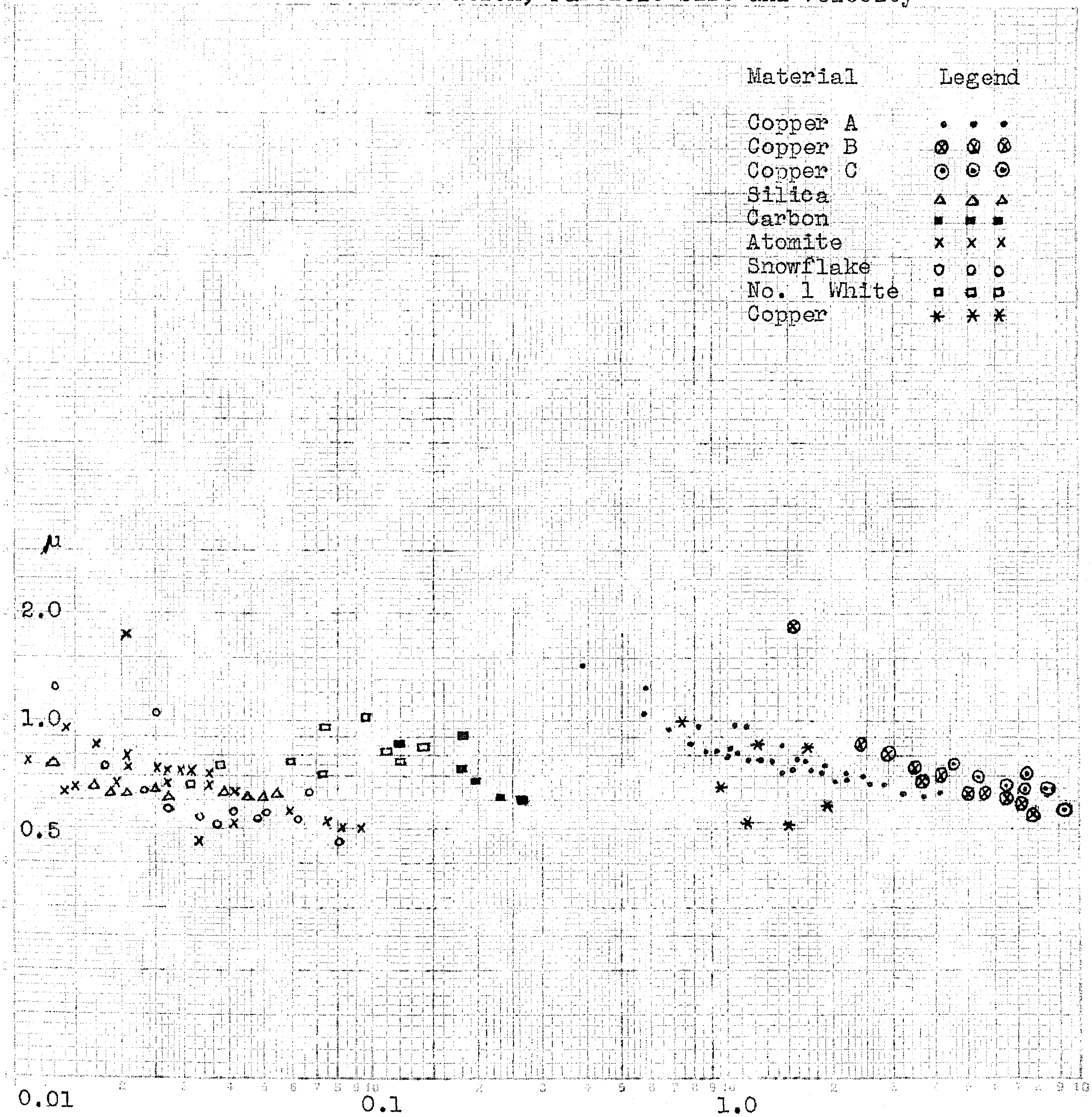


Figure 4

TABLE VII

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE ON APPARENT VISCOSITY

RUN NO.	A x 10^2	G/A	μ
	Copper A		
19	10.6	1.06	0.84
20	11.3	0.87	0.83
21	11.4	0.79	0.86
22	11.3	0.68	0.95
23	11.3	0.58	1.12
24	11.8	0.39	1.44
25	9.9	1.16	0.78
26	9.9	1.08	0.81
27	9.9	1.00	0.80
28	9.9	0.91	0.83
29	10.2	0.77	0.88
30	10.2	0.58	1.08
31	6.3	1.78	0.73
32	6.6	1.53	0.74
33	6.6	1.41	0.73
34	6.6	1.25	0.79
35	6.6	1.12	0.79
36	6.6	0.83	0.99
37	4.9	2.18	0.71
38	4.9	2.05	0.68
39	4.9	1.87	0.71
40	4.9	1.70	0.77
41	5.2	1.43	0.79
42	5.2	1.12	0.97
43	4.2	2.57	0.68
44	4.2	2.42	0.70
45	4.2	2.20	0.69
46	4.3	1.90	0.74
47	4.3	1.64	0.78
48	4.3	1.08	0.97

RUN NO.	A x 10^2	G/A	μ
	Copper A		
49	2.8	3.73	0.61
50	2.5	4.01	0.63
51	2.8	3.25	0.64
52	2.8	2.80	0.67
53	2.8	2.22	0.69
54	2.8	1.47	0.86
	Copper B		
55	2.0	5.04	0.62
56	2.3	4.28	0.70
57	2.4	3.57	0.74
58	2.4	2.98	0.80
59	2.4	2.44	0.88
60	2.3	1.52	1.84
61	1.3	7.80	0.54
62	1.3	7.30	0.58
63	1.3	6.68	0.60
64	1.3	5.53	0.64
65	1.4	3.57	0.70
	Copper C		
66	1.1	9.42	0.57
67	1.2	7.01	0.63
68	1.2	6.48	0.65
69	1.2	5.49	0.69
70	1.4	3.50	0.75
71	0.8	12.4	0.55
72	0.8	11.0	0.60
73	0.9	8.50	0.65
74	0.8	7.22	0.70
75	0.9	4.46	0.76

TABLE VII (Continued)

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE ON APPARENT VISCOSITY

RUN NO.	A x 10^2	G/A	μ
	Silica		
76	179	0.056	0.62
77	179	0.050	0.61
78	179	0.046	0.61
79	179	0.039	0.63
80	179	0.027	0.61
81	379	0.027	0.63
82	364	0.025	0.65
110	494	0.021	0.63
111	485	0.019	0.64
112	485	0.017	0.68
113	485	0.013	0.78
	Carbon		
83	39.5	0.27	0.60
84	41.7	0.23	0.61
85	41.7	0.20	0.68
86	41.7	0.18	0.74
87	45.4	0.12	0.88
88	54.6	0.18	0.91
	Atomite		
89	109	0.093	0.50
90	109	0.081	0.50
91	109	0.075	0.52
92	109	0.059	0.57
93	109	0.054	0.63
94	196	0.052	0.62
95	199	0.046	0.63
96	199	0.041	0.64
97	199	0.035	0.67
98	202	0.027	0.72

RUN NO.	A x 10^2	G/A	μ
	Atomite		
99	295	0.035	0.70
100	295	0.031	0.72
101	295	0.029	0.71
102	298	0.025	0.74
103	298	0.021	0.80
104	304	0.014	0.97
	Atomite		
1	178	0.021	0.75
2	138	0.033	0.46
3	111	0.021	1.75
4	111	0.017	0.86
5	111	0.027	0.68
6	91.3	0.041	0.51
7	220	0.019	0.66
8	220	0.015	0.65
9	220	0.014	0.64
10	226	0.011	0.78
11	234	0.007	1.37
	Snowflake		
1	55.2	0.081	0.46
2	55.2	0.068	0.63
3	55.2	0.062	0.53
4	55.2	0.050	0.56
5	55.2	0.041	0.56
6	55.2	0.025	1.08
7	125	0.013	1.22
8	125	0.018	0.75
9	125	0.023	0.64
10	125	0.027	0.59
11	125	0.033	0.54
12	125	0.037	0.52

TABLE VII (Continued)

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE ON APPARENT VISCOSITY

RUN NO.	A x 10^2	G/A	μ
	No. 1 White		
1	27.8	0.14	0.84
2	27.8	0.12	0.78
3	27.8	0.11	0.82
4	27.8	0.097	1.01
5	27.8	0.071	0.96
6	59.0	0.073	0.70
7	59.0	0.060	0.78
8	59.0	0.048	0.55
9	59.0	0.037	0.76
10	59.0	0.031	0.68

RUN NO.	A x 10^2	G/A	μ
	Copper		
1	2.1	1.95	0.58
2	2.1	1.70	0.84
3	2.1	1.49	0.51
4	2.1	1.12	0.53
5	2.1	1.23	0.88
6	2.1	0.98	0.66
7	2.1	0.73	1.00

group ranges from 1.5 to 6 microns. There is some spillage of the 14 micron size chalk data into this last grouping. It seems likely that additional data for chalk of this size would place this material in the same category as the other chalk particles.

Although the overall effect of the introduction of particle size has been to further separate the groups shown in Figure 3, there has been an improvement in correlation within the individual groups.

In an effort to improve the overall correlation, the G/A group was raised to a fractional power and plotted against the viscosity in Figure 5. This method of approach was quickly abandoned when it became obvious that to obtain a curve of a satisfactory nature, the group G/A would have to be raised to an exponent so small that it would be impossible to accurately write an equation to fit the curve.

The G/A group was then plotted against the viscosity raised to a fractional exponent ($\mu^{.333}$) in Figure 6. Better correlation was obtained and an equation was written for the curve drawn. Bulk viscosities calculated from this equation had a mean deviation[10] of 25% when compared to the experimentally determined viscosities. This method was abandoned at this point since further reduction of the fractional exponent would lead to the same problem that arose in Figure 5.

Apparent Viscosity and Combined Effect of Concentration, Particle Size and Velocity Raised to a Fractional Power

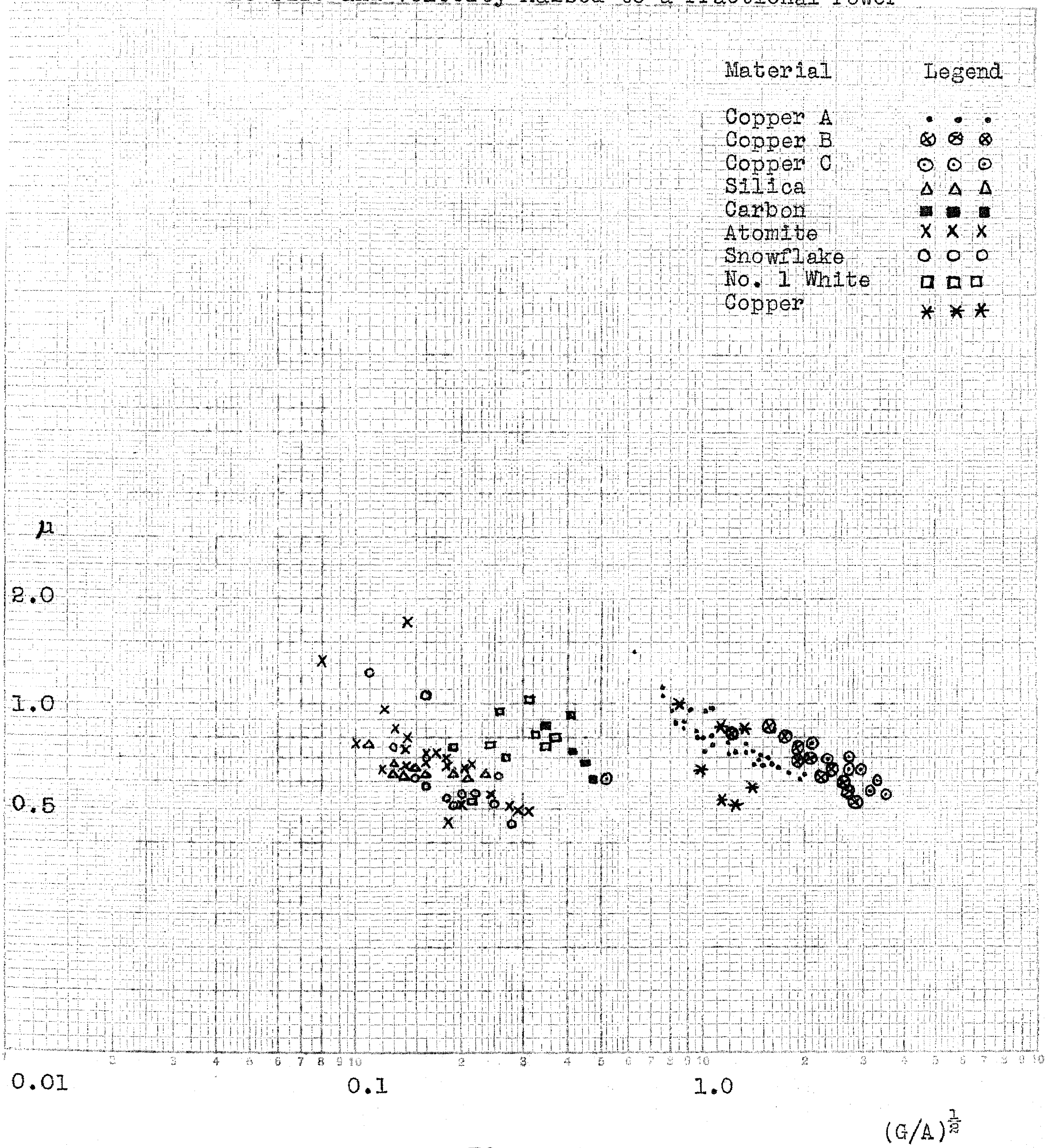


Figure 5

TABLE VIII

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE, RAISED TO A FRACTIONAL POWER, ON APPARENT VISCOSITY

RUN NO.	$(G/A)^{\frac{1}{2}}$	μ
Copper A		
19	1.02	0.84
20	0.93	0.83
21	0.88	0.86
22	0.82	0.95
23	0.76	1.12
24	0.62	1.44
25	1.08	0.78
26	1.04	0.81
27	1.00	0.80
28	0.95	0.83
29	0.87	0.88
30	0.76	1.08
31	1.33	0.73
32	1.24	0.74
33	1.19	0.73
34	1.12	0.79
35	1.06	0.79
36	0.91	0.99
37	1.48	0.71
38	1.43	0.68
39	1.37	0.71
40	1.31	0.77
41	1.20	0.79
42	1.06	0.97
43	1.60	0.68
44	1.56	0.70
45	1.48	0.69
46	1.38	0.74
47	1.28	0.78
48	1.04	0.97

RUN NO.	$(G/A)^{\frac{1}{2}}$	μ
Copper A		
49	1.93	0.61
50	2.00	0.63
51	1.80	0.64
52	1.67	0.67
53	1.49	0.69
54	1.21	0.86
Copper B		
55	2.25	0.62
56	2.07	0.70
57	1.89	0.74
58	1.73	0.80
59	1.56	0.88
60	1.23	1.84
61	2.79	0.54
62	2.70	0.58
63	2.59	0.60
64	2.35	0.64
65	1.89	0.70
Copper C		
66	3.07	0.57
67	2.65	0.63
68	2.55	0.65
69	2.34	0.69
70	1.87	0.75
71	3.52	0.55
72	3.32	0.60
73	2.92	0.65
74	2.69	0.70
75	2.11	0.76

RUN NO.	$(G/A)^{\frac{1}{2}}$	μ
Silica		
76	0.24	0.62
77	0.22	0.61
78	0.21	0.61
79	0.19	0.63
80	0.16	0.61
81	0.16	0.63
82	0.15	0.65
110	0.14	0.63
111	0.13	0.64
112	0.13	0.68
113	0.11	0.78
Carbon		
83	0.52	0.60
84	0.48	0.61
85	0.45	0.68
86	0.42	0.74
87	0.35	0.88
88	0.42	0.91
Atomite		
89	0.31	0.50
90	0.28	0.50
91	0.27	0.52
92	0.24	0.57
93	0.23	0.63
94	0.23	0.62
95	0.21	0.63
96	0.20	0.64
97	0.18	0.67
98	0.16	0.72

TABLE VIII (Continued)

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE, RAISED TO A FRACTIONAL POWER, ON APPARENT VISCOSITY

RUN NO.	$(G/A)^{\frac{1}{2}}$	μ
	Atomite	
99	0.18	0.70
100	0.17	0.72
101	0.17	0.71
102	0.16	0.74
103	0.14	0.80
104	0.12	0.97
	Atomite	
1	0.14	0.75
2	0.18	0.46
3	0.14	1.75
4	0.13	0.86
5	0.16	0.68
6	0.20	0.51
7	0.14	0.66
8	0.12	0.65
9	0.12	0.64
10	0.10	0.78
11	0.08	1.37
	Snowflake	
1	0.28	0.46
2	0.26	0.63
3	0.25	0.53
4	0.22	0.56
5	0.20	0.56

RUN NO.	$(G/A)^{\frac{1}{2}}$	μ
	Snowflake	
6	0.16	1.08
7	0.11	1.22
8	0.13	0.75
9	0.15	0.64
10	0.16	0.59
11	0.18	0.54
12	0.19	0.52
	No. 1 White	
1	0.37	0.84
2	0.35	0.78
3	0.33	0.82
4	0.31	1.01
5	0.26	0.96
6	0.27	0.70
7	0.24	0.78
8	0.22	0.55
9	0.19	0.76
10	0.18	0.68
	Copper	
1	1.40	0.58
2	1.30	0.84
3	1.22	0.51
4	1.06	0.53
5	1.11	0.88
6	0.99	0.66
7	0.85	1.00

Apparent Viscosity Raised to a Fractional Power and the Combined Effect of Concentration, Particle Size and Velocity

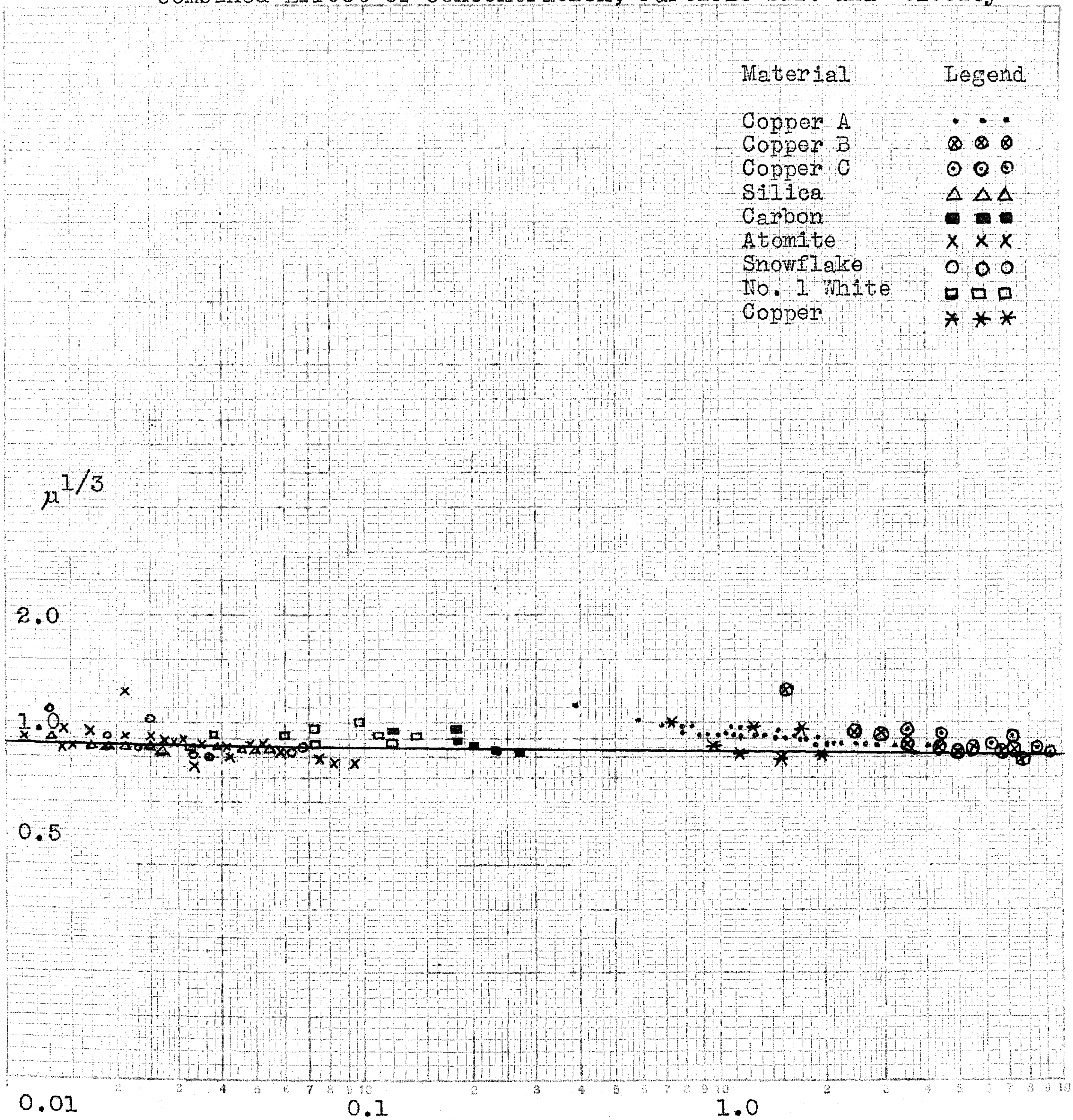


Figure 6

TABLE IX

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE ON APPARENT VISCOSITY RAISED TO A FRACTIONAL POWER

RUN NO.	G/A	$\mu^{1/3}$
Copper A		
19	1.06	0.94
20	0.87	0.94
21	0.79	0.95
22	0.68	0.98
23	0.58	1.03
24	0.39	1.12
25	1.16	0.96
26	1.08	0.93
27	1.00	0.93
28	0.91	0.94
29	0.77	0.96
30	0.58	1.03
31	1.78	0.90
32	1.53	0.91
33	1.41	0.90
34	1.25	0.92
35	1.12	0.92
36	0.83	0.99
37	2.18	0.89
38	2.05	0.88
39	1.87	0.89
40	1.70	0.91
41	1.43	0.92
42	1.12	0.99
43	2.57	0.88
44	2.42	0.89
45	2.20	0.88
46	1.90	0.91
47	1.64	0.92
48	1.08	0.99

RUN NO.	G/A	$\mu^{1/3}$
Copper A		
49	3.73	0.84
50	4.01	0.86
51	3.25	0.86
52	2.80	0.88
53	2.22	0.88
54	1.47	0.95
Copper B		
55	5.04	0.85
56	4.28	0.89
57	3.57	0.91
58	2.98	0.93
59	2.44	0.96
60	1.52	1.22
61	7.80	0.81
62	7.30	0.83
63	6.68	0.84
64	5.53	0.86
65	3.57	0.89
Copper C		
66	9.42	0.83
67	7.01	0.86
68	6.48	0.87
69	5.49	0.88
70	3.50	0.91
71	12.4	0.82
72	11.0	0.84
73	8.50	0.87
74	7.22	0.89
75	4.46	0.91

RUN NO.	G/A	$\mu^{1/3}$
Silica		
76	0.056	0.85
77	0.050	0.85
78	0.046	0.85
79	0.039	0.86
80	0.027	0.85
81	0.027	0.86
82	0.025	0.87
110	0.021	0.86
111	0.019	0.86
112	0.017	0.88
113	0.013	0.92
Carbon		
83	0.27	0.84
84	0.23	0.85
85	0.20	0.88
86	0.18	0.90
87	0.12	0.96
88	0.18	0.97
Atomite		
89	0.093	0.79
90	0.081	0.79
91	0.075	0.80
92	0.059	0.83
93	0.054	0.86
94	0.052	0.85
95	0.046	0.86
96	0.041	0.86
97	0.035	0.88
98	0.027	0.90

TABLE IX (Continued)

COMBINED EFFECT OF CONCENTRATION, VELOCITY AND PARTICLE SIZE ON APPARENT VISCOSITY RAISED TO A FRACTIONAL POWER

RUN NO.	G/A	$\mu^{1/3}$
	Atomite	
99	0.035	0.89
100	0.031	0.90
101	0.029	0.89
102	0.025	0.91
103	0.021	0.93
104	0.014	0.99
	Atomite	
1	0.021	0.91
2	0.033	0.77
3	0.021	1.22
4	0.017	0.95
5	0.027	0.88
6	0.041	0.80
7	0.019	0.87
8	0.015	0.87
9	0.014	0.86
10	0.011	0.92
11	0.007	1.12
	Snowflake	
1	0.081	0.77
2	0.068	0.86
3	0.062	0.81
4	0.050	0.82
5	0.041	0.82

RUN NO.	G/A	$\mu^{1/3}$
	Snowflake	
6	0.025	1.03
7	0.013	1.06
8	0.018	0.91
9	0.023	0.86
10	0.027	0.84
11	0.033	0.81
12	0.037	0.80
	No. 1 White	
1	0.14	0.94
2	0.12	0.92
3	0.11	0.94
4	0.097	1.00
5	0.071	0.98
6	0.073	0.88
7	0.060	0.92
8	0.048	0.82
9	0.037	0.91
10	0.031	0.88
	Copper	
1	1.95	0.83
2	1.70	0.94
3	1.49	0.80
4	1.12	0.81
5	1.23	0.96
6	0.98	0.87
7	0.73	1.00

TABLE X

DEVIATION OF RESULTS OF EQUATION, DERIVED AS FRACTIONAL POWER OF VISCOSITY FROM OBSERVED RESULTS

RUN NO.	DE-RIVED μ	OB-SERVED μ	DEVI-ATION %
	Copper A		
19	0.53	0.84	37
20	0.55	0.83	34
21	0.57	0.86	34
22	0.57	0.95	40
23	0.59	1.12	47
24	0.61	1.44	58
25	0.53	0.78	32
26	0.53	0.81	35
27	0.53	0.80	34
28	0.55	0.83	34
29	0.57	0.88	35
30	0.59	1.08	45
31	0.51	0.73	30
32	0.51	0.74	31
33	0.52	0.73	29
34	0.52	0.79	34
35	0.53	0.79	33
36	0.55	0.99	44
37	0.51	0.71	28
38	0.51	0.68	25
39	0.51	0.71	28
40	0.51	0.77	34
41	0.52	0.79	34
42	0.53	0.97	45
43	0.51	0.68	25
44	0.51	0.70	27
45	0.51	0.69	26
46	0.51	0.74	31
47	0.51	0.78	34
48	0.53	0.97	45
49	0.49	0.61	20
50	0.49	0.63	22
51	0.50	0.64	22
52	0.51	0.67	24
53	0.51	0.69	26
54	0.51	0.86	41
	Copper B		
55	0.48	0.62	29
56	0.48	0.70	31
57	0.50	0.74	32
58	0.51	0.80	36
59	0.51	0.88	42
60	0.51	1.84	72
61	0.47	0.54	13
62	0.48	0.58	17
63	0.47	0.60	22
64	0.49	0.64	23
65	0.50	0.70	29
	Copper C		
66	0.47	0.57	18
67	0.47	0.63	25
68	0.49	0.65	25
69	0.49	0.69	29
70	0.51	0.75	32
71	0.46	0.55	16
72	0.44	0.60	27
73	0.47	0.65	28
74	0.49	0.70	30
75	0.50	0.76	34

TABLE X (con.)

DEVIATION OF RESULTS OF EQUATION,
DERIVED AS FRACTIONAL POWER OF VISCOSITY,
FROM OBSERVED RESULTS

RUN NO.	DERIVED μ	OBSERVED μ	DEVIATION %
Silica			
76	0.72	0.62	16
77	0.73	0.61	16
78	0.73	0.61	16
79	0.73	0.63	16
80	0.74	0.61	21
81	0.74	0.63	17
82	0.74	0.65	14
110	0.75	0.63	19
111	0.75	0.64	17
112	0.79	0.68	16
113	0.80	0.78	3
Carbon			
83	0.61	0.60	2
84	0.61	0.61	0
85	0.62	0.68	9
86	0.67	0.74	9
87	0.64	0.88	27
88	0.64	0.91	30
Atomite			
89	0.66	0.50	32
90	0.68	0.50	36
91	0.68	0.52	31
92	0.69	0.57	21
93	0.70	0.63	11
94	0.70	0.62	13
95	0.72	0.63	14
96	0.72	0.64	12
97	0.73	0.67	9
98	0.75	0.72	4
Atomite			
99	0.73	0.70	4
100	0.73	0.72	1
101	0.73	0.71	3
102	0.74	0.74	0
103	0.75	0.80	6
104	0.83	0.97	14
Atomite			
1	0.75	0.75	0
2	0.73	0.46	59
3	0.75	1.75	57
4	0.80	0.86	7
5	0.74	0.68	9
6	0.73	0.51	43
7	0.77	0.66	16
8	0.83	0.65	28
9	0.83	0.64	30
10	0.84	0.78	8
11	0.86	1.37	37
Snowflake			
1	0.68	0.46	48
2	0.70	0.63	11
3	0.70	0.53	32
4	0.72	0.56	28
5	0.72	0.56	29

TABLE X (con.)

DEVIATION OF RESULTS OF EQUATION, DERIVED AS FRACTIONAL POWER OF VISCOSITY, FROM OBSERVED RESULTS

RUN NO.	DE-RIVED μ	OB-SERVED μ	DEVI-ATION %
	Snowflake		
6	0.75	1.08	30
7	0.83	1.22	31
8	0.78	0.75	4
9	0.75	0.64	14
10	0.74	0.59	25
11	0.73	0.54	35
12	0.73	0.52	40
	No. 1 White		
1	0.64	0.84	24
2	0.64	0.78	18
3	0.64	0.82	22
4	0.66	1.01	36
5	0.71	0.96	26
6	0.71	0.70	1
7	0.72	0.78	8
8	0.73	0.55	33
9	0.74	0.76	3
10	0.75	0.68	10

RUN NO.	DE-RIVED μ	OB-SERVED μ	DEVI-ATION %
	Copper		
1	0.51	0.58	12
2	0.51	0.84	39
3	0.52	0.51	2
4	0.53	0.53	0
5	0.53	0.88	40
6	0.52	0.66	20
7	0.57	1.00	43

Mean of Deviation 25%

Part 5. Combined Effect of Velocity, Concentration, Particle Size and Thermal Conductivity of Particle on Viscosity

Investigation of the second possibility that there is something in the basic nature of the particle in addition to its size that affects the viscosity of the slurried particles was the next phase.

With the exception of the largest chalk particles (No. 1 white - 14 microns) which are in both the second and third groups, there is a definite grouping by material. From right to left in Figure 4 this grouping is in order of descending thermal conductivity.

Although the relationship was not justified dimensionally, a plot of the combined effect of the variables used in Figure 4 plus the particle thermal conductivity on viscosity is shown in Figure 7.

Prior to the introduction of thermal conductivity as a variable, conditions in the viscometer had been considered isothermal even though there had been a deviation for 130 test runs of 10% from the average value. With the introduction of this new variable, a heat term, it was deemed necessary to apply a temperature correction to the apparent viscosity. This was done by introduction of the viscosity of water (μ_W). μ_W was determined from the average vis-

Ratio of Apparent Viscosity to Viscosity of Suspending Fluid and the Combined Effect of Concentration, Particle Size, Velocity and Thermal Conductivity of the Particle

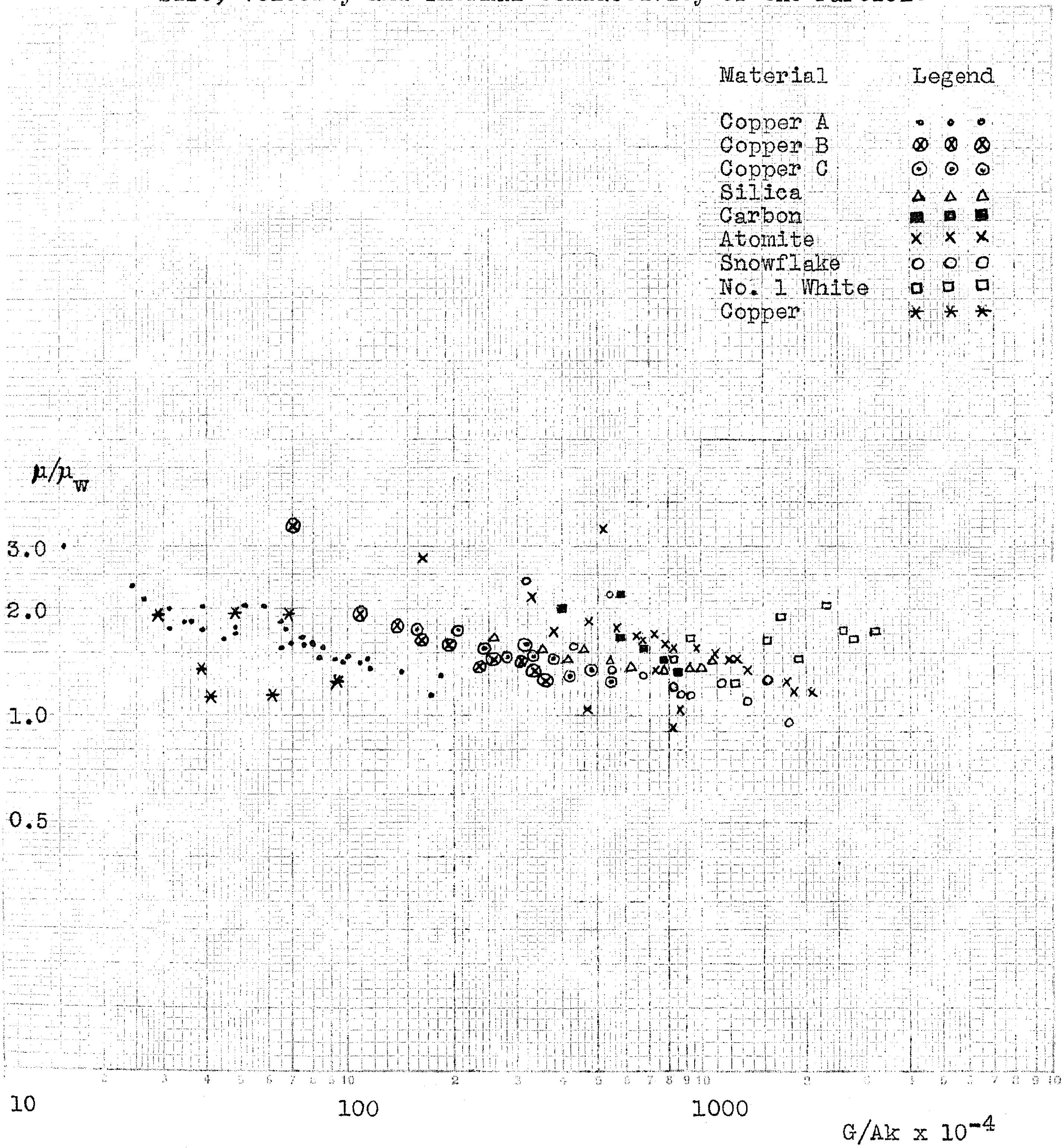


Figure 7

TABLE XI

COMBINED EFFECT OF CONCENTRATION, VELOCITY, PARTICLE SIZE, THERMAL CONDUCTIVITY OF PARTICLE AND VISCOSITY OF SUSPENDING FLUID ON APPARENT VISCOSITY

RUN NO.	μ	μ_w	μ/μ_w	G/Ak x10^{-4}
		Copper A		
19	0.84	0.47	1.79	48.2
20	0.83	0.48	1.77	39.5
21	0.86	0.46	1.87	35.9
22	0.95	0.47	2.02	30.9
23	1.12	0.48	2.34	24.2
24	1.44	0.48	3.00	15.4
25	0.78	0.47	1.66	52.8
26	0.81	0.46	1.72	48.4
27	0.80	0.49	1.66	45.4
28	0.83	0.47	1.77	31.3
29	0.88	0.48	1.84	35.0
30	1.08	0.48	2.25	26.6
31	0.73	0.46	1.59	80.4
32	0.74	0.46	1.61	70.0
33	0.73	0.48	1.52	64.8
34	0.79	0.47	1.68	57.2
35	0.79	0.47	1.68	50.8
36	0.99	0.48	2.06	38.9
37	0.71	0.50	1.42	98.0
38	0.68	0.47	1.45	92.6
39	0.71	0.48	1.48	84.5
40	0.77	0.48	1.60	77.8
41	0.79	0.47	1.84	65.4
42	0.97	0.48	2.02	50.9
43	0.68	0.47	1.45	116
44	0.70	0.49	1.43	110
45	0.69	0.47	1.47	100
46	0.74	0.48	1.54	86
47	0.78	0.48	1.63	74.5
48	0.97	0.48	2.02	48.6
		Copper A		
49	0.61	0.48	1.27	175
50	0.63	0.49	1.29	184
51	0.64	0.49	1.31	146
52	0.67	0.49	1.37	128
53	0.69	0.48	1.44	101
54	0.86	0.49	1.76	67
		Copper B		
55	0.62	0.44	1.41	238
56	0.70	0.44	1.59	194
57	0.74	0.45	1.65	161
58	0.80	0.45	1.78	136
59	0.88	0.45	1.96	109
60	1.84	0.44	3.50	69.5
61	0.54	0.44	1.23	357
62	0.58	0.44	1.32	333
63	0.60	0.44	1.37	305
64	0.64	0.45	1.42	254
65	0.70	0.45	1.56	160
		Copper C		
66	0.57	0.44	1.30	425
67	0.63	0.43	1.47	318
68	0.65	0.44	1.48	283
69	0.69	0.44	1.57	242
70	0.75	0.44	1.71	160
71	0.55	0.44	1.25	568
72	0.60	0.44	1.37	494
73	0.65	0.44	1.48	381
74	0.70	0.44	1.59	324
75	0.76	0.44	1.73	201

TABLE XI (con.)

COMBINED EFFECT OF CONCENTRATION, VELOCITY, PARTICLE SIZE, THERMAL CONDUCTIVITY OF PARTICLE AND VISCOSITY OF SUSPENDING FLUID ON APPARENT VISCOSITY

RUN NO.	μ	μ_w	μ/μ_w	G/Ak x10^{-4}
		Silica		
76	0.62	0.43	1.44	1115
77	0.61	0.43	1.42	1025
78	0.61	0.43	1.42	914
79	0.63	0.46	1.37	788
80	0.61	0.44	1.39	525
81	0.63	0.43	1.47	558
82	0.65	0.43	1.52	460
110	0.63	0.43	1.47	417
111	0.64	0.43	1.49	382
112	0.68	0.44	1.55	346
113	0.78	0.46	1.70	262
		Carbon		
83	0.60	0.44	1.37	886
84	0.61	0.43	1.42	763
85	0.68	0.44	1.55	670
86	0.74	0.44	1.68	580
87	0.88	0.44	2.00	407
88	0.91	0.41	2.22	595
		Atomite		
89	0.50	0.42	1.19	2120
90	0.50	0.42	1.19	1830
91	0.52	0.42	1.24	1730
92	0.57	0.42	1.36	1435
93	0.63	0.43	1.47	1230
94	0.62	0.42	1.48	1200
95	0.63	0.42	1.50	1090
96	0.64	0.42	1.53	950
97	0.67	0.42	1.60	795
98	0.72	0.43	1.68	642

RUN NO.	μ	μ_w	μ/μ_w	G/Ak x10^{-4}
		Atomite		
99	0.70	0.44	1.59	812
100	0.72	0.42	1.72	735
101	0.71	0.43	1.66	660
102	0.74	0.43	1.72	570
103	0.80	0.43	1.86	472
104	0.97	0.44	2.20	326
		Atomite		
1	0.75	0.52	1.44	552
2	0.46	0.51	0.91	842
3	1.75	0.50	3.44	515
4	0.86	0.52	1.72	388
5	0.68	0.52	1.33	710
6	0.51	0.50	1.02	885
7	0.66	0.49	1.06	438
8	0.65	0.49	1.37	334
9	0.64	0.48	1.34	304
10	0.78	0.48	1.58	248
11	1.37	0.50	2.82	161
		Snowflake		
1	0.46	0.48	0.96	1790
2	0.63	0.48	1.29	1530
3	0.53	0.48	1.10	1375
4	0.56	0.47	1.22	1128
5	0.56	0.48	1.17	905

TABLE XI (con.)

COMBINED EFFECT OF CONCENTRATION, VELOCITY, PARTICLE SIZE, THERMAL CONDUCTIVITY OF PARTICLE AND VISCOSITY OF SUSPENDING FLUID ON APPARENT VISCOSITY

RUN NO.	μ	μ_w	μ/μ_w	G/A k x10^{-4}
Snowflake				
6	1.08	0.49	2.20	545
7	1.22	0.49	2.47	320
8	0.75	0.48	1.59	438
9	0.64	0.46	1.37	555
10	0.59	0.46	1.30	668
11	0.54	0.45	1.20	802
12	0.52	0.44	1.16	896
No. 1 White				
1	0.84	0.48	1.73	3220
2	0.78	0.48	1.65	2845
3	0.82	0.48	1.77	2660
4	1.01	0.49	2.10	2290
5	0.96	0.50	1.92	1690
6	0.70	0.48	1.42	1900
7	0.78	0.49	1.68	1565
8	0.55	0.47	1.21	1250
9	0.76	0.48	1.65	938
10	0.68	0.49	1.45	805

RUN NO.	μ	μ_w	μ/μ_w	G/Ak x10^{-4}
Copper				
1	0.58	0.45	1.27	91.0
2	0.84	0.44	1.94	68.8
3	0.51	0.46	1.13	60.4
4	0.53	0.48	1.11	41.5
5	0.88	0.46	1.92	49.4
6	0.66	0.49	1.35	39.3
7	1.00	0.51	1.98	29.7

cometer temperature for each individual run.

The graphical representation, Figure 7, of the form $\mu/\mu_w = f(G/Ak)$ shows little of the grouping by either material or particle size that was present in Figure 4. Since this function is dimensionally unstable and is presented only as an intermediate step to a final form, no attempt was made to write an equation or determine the mean deviation. However the correlation shown by the plot appears to be of a satisfactory nature.

The one remaining problem was to modify the relation $\mu/\mu_w = f(G/Ak)$ so that it would be dimensionally sound and yet not affect the correlation which had been obtained. These qualifications were met by introduction of the heat capacity (C) of the suspending medium, in this case water. This resulted in the development of the dimensionally valid form $\mu/\mu_w = f(GC/Ak)$.

From a graphical plot (Figure 8) of this form the following equation was obtained: $\mu/\mu_w = 1.02(Ak/GC)^{.105}$.

Substitution of the correct values in the above equation gives values of μ/μ_w which compare favorably with those obtained from the observed data. The average deviation of the results calculated from the derived equation compared to those based upon the observed data is 14.4%.

Ratio of Apparent Viscosity to Viscosity of Suspending Fluid and Combined Effect of Concentration, Particle Size, Velocity, Thermal Conductivity of Particle and Heat Capacity of the Suspending Fluid

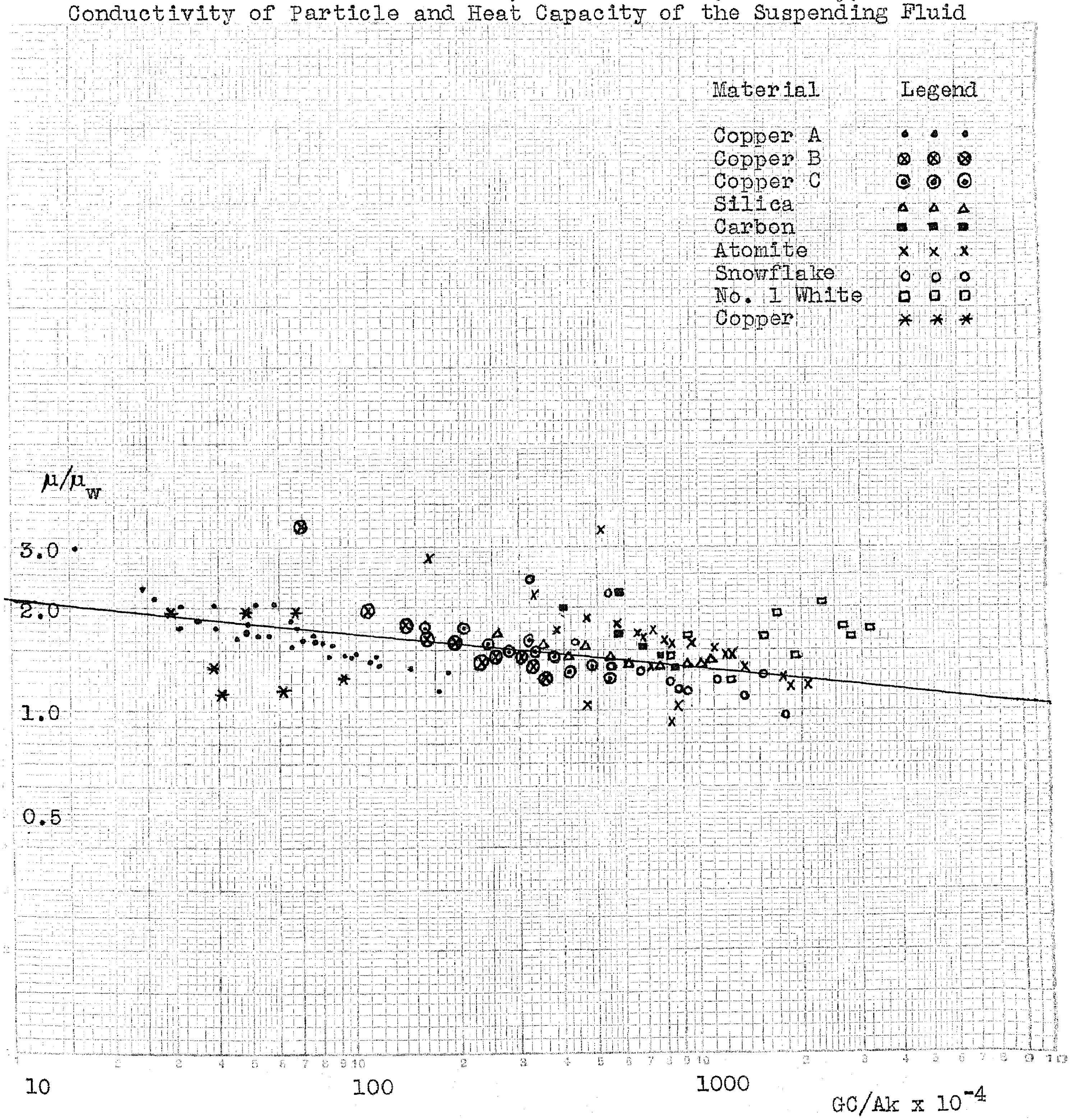


Figure 8

TABLE XII

COMBINED EFFECT OF CONCENTRATION, VELOCITY, PARTICLE SIZE, THERMAL CONDUCTIVITY OF PARTICLE, VISCOSITY OF SUSPENDING FLUID AND HEAT CAPACITY OF SUSPENDING FLUID ON APPARENT VISCOSITY

RUN NO.	μ/μ_w	GC/Ak $\times 10^{-4}$
Copper A		
19	1.79	48.2
20	1.77	39.5
21	1.87	35.9
22	2.02	30.9
23	2.34	24.2
24	3.00	15.4
25	1.66	52.8
26	1.72	48.4
27	1.66	45.4
28	1.77	31.3
29	1.84	35.0
30	2.25	26.6
31	1.59	80.4
32	1.61	70.0
33	1.52	64.8
34	1.68	57.2
35	1.68	50.8
36	2.06	38.9
37	1.42	98.0
38	1.45	92.6
39	1.48	84.5
40	1.60	77.8
41	1.84	65.4
42	2.02	50.9
43	1.45	116
44	1.43	110
45	1.47	100
46	1.54	86
47	1.63	74.5
48	2.02	48.6

RUN NO.	μ/μ_w	GC/Ak $\times 10^{-4}$
Copper A		
49	1.27	175
50	1.29	184
51	1.31	146
52	1.37	128
53	1.44	101
54	1.76	67
Copper B		
55	1.41	238
56	1.59	194
57	1.65	161
58	1.78	136
59	1.96	109
60	3.50	69.5
61	1.23	357
62	1.32	333
63	1.37	305
64	1.42	254
65	1.56	160
Copper C		
66	1.30	425
67	1.47	318
68	1.48	283
69	1.57	242
70	1.71	160
71	1.25	568
72	1.37	494
73	1.48	381
74	1.59	324
75	1.73	201

RUN NO.	μ/μ_w	GC/Ak $\times 10^{-4}$
Silica		
76	1.44	1115
77	1.42	1025
78	1.42	914
79	1.37	788
80	1.39	525
81	1.47	558
82	1.52	460
110	1.47	417
111	1.49	382
112	1.55	346
Carbon		
83	1.37	886
84	1.42	763
85	1.55	670
86	1.68	580
87	2.00	407
88	2.22	595
Atomite		
89	1.19	2120
90	1.19	1830
91	1.24	1730
92	1.36	1435
93	1.47	1230
94	1.48	1200
95	1.50	1090
96	1.53	950
97	1.60	795
98	1.68	642

TABLE XII (con.)

COMBINED EFFECT OF CONCENTRATION, VELOCITY, PARTICLE SIZE, THERMAL CONDUCTIVITY OF PARTICLE, VISCOSITY OF SUSPENDING FLUID AND HEAT CAPACITY OF SUSPENDING FLUID ON APPARENT VISCOSITY

Atomite

RUN NO.	μ/μ_w	GC/Ak $x10^{-4}$
99	1.59	812
100	1.72	735
101	1.66	660
102	1.72	570
103	1.86	472
104	2.20	326

Atomite

RUN NO.	μ/μ_w	GC/Ak $x10^{-4}$
1	1.44	552
2	0.91	842
3	3.44	515
4	1.72	388
5	1.33	710
6	1.02	885
7	1.06	438
8	1.37	334
9	1.34	304
10	1.58	248
11	2.82	161

Snowflake

RUN NO.	μ/μ_w	GC/Ak $x10^{-4}$
1	0.96	1790
2	1.29	1530
3	1.10	1375
4	1.22	1128
5	1.17	905

Snowflake

RUN NO.	μ/μ_w	GC/Ak $x10^{-4}$
6	2.20	545
7	2.47	320
8	1.59	438
9	1.37	555
10	1.30	668
11	1.20	802
12	1.16	896

No. 1 White

RUN NO.	μ/μ_w	GC/Ak $x10^{-4}$
1	1.73	3220
2	1.65	2845
3	1.77	2660
4	2.10	2290
5	1.92	1690
6	1.42	1900
7	1.68	1565
8	1.21	1250
9	1.65	938
10	1.45	805

Copper

RUN NO.	μ/μ_w	GC/Ak $x10^{-4}$
1	1.27	91.0
2	1.94	68.8
3	1.13	60.4
4	1.11	41.5
5	1.92	49.4
6	1.35	39.3
7	1.98	29.7

TABLE XIII

DEVIATION OF DERIVED EQUATION RESULTS FROM OBSERVED RESULTS

RUN NO.	DE-RIVED μ/μ_w	OB-SERVED μ/μ_w	DEVI-ATION %
	Copper A		
19	1.78	1.79	0.6
20	1.82	1.77	2.8
21	1.84	1.87	1.6
22	1.88	2.02	7.0
23	1.94	2.34	16.4
24	2.01	3.00	33.0
25	1.77	1.66	6.6
26	1.78	1.72	3.4
27	1.79	1.66	7.8
28	1.82	1.77	2.8
29	1.84	1.84	0
30	1.91	2.25	15.1
31	1.69	1.59	6.3
32	1.71	1.61	6.2
33	1.72	1.52	13.2
34	1.75	1.68	4.2
35	1.77	1.68	5.4
36	1.82	2.06	11.6
37	1.65	1.42	16.2
38	1.66	1.45	14.4
39	1.68	1.48	13.5
40	1.69	1.60	5.6
41	1.73	1.84	6.0
42	1.78	2.02	11.9
43	1.63	1.45	12.4
44	1.63	1.43	14.0
45	1.65	1.47	12.2
46	1.68	1.54	9.1
47	1.70	1.63	4.3
48	1.78	2.02	11.9

RUN NO.	DE-RIVED μ/μ_w	OB-SERVED μ/μ_w	DEVI-ATION %
	Copper A		
49	1.56	1.27	22.8
50	1.56	1.29	21.0
51	1.59	1.31	21.4
52	1.61	1.37	17.5
53	1.65	1.44	14.6
54	1.73	1.76	1.7
	Copper B		
55	1.51	1.41	7.1
56	1.54	1.59	3.2
57	1.57	1.65	4.8
58	1.60	1.78	10.1
59	1.64	1.96	16.3
60	1.72	3.50	50.6
61	1.45	1.23	17.9
62	1.46	1.32	10.6
63	1.47	1.37	7.3
64	1.50	1.42	5.6
65	1.58	1.56	1.2
	Copper C		
66	1.43	1.30	10.0
67	1.46	1.47	0.7
68	1.49	1.48	0.7
69	1.51	1.57	3.8
70	1.57	1.71	8.2
71	1.38	1.25	10.4
72	1.40	1.37	2.2
73	1.44	1.48	2.7
74	1.47	1.59	5.0
75	1.54	1.73	11.0

TABLE XIII (con.)

DEVIATION OF DERIVED EQUATION RESULTS FROM OBSERVED RESULTS

RUN NO.	DERIVED μ/μ_w	OBSERVED μ/μ_w	DEVIATION %
	Silica		
76	1.28	1.44	11.1
77	1.30	1.42	8.4
78	1.32	1.42	7.0
79	1.34	1.37	2.2
80	1.40	1.39	0.7
81	1.38	1.47	4.8
82	1.40	1.52	7.9
110	1.43	1.47	2.0
111	1.44	1.49	3.4
112	1.45	1.55	6.5
113	1.49	1.70	12.4
	Carbon		
83	1.31	1.37	4.4
84	1.34	1.42	5.6
85	1.36	1.55	12.3
86	1.38	1.68	16.7
87	1.43	2.00	28.5
88	1.38	2.22	37.8
	Atomite		
89	1.20	1.19	0.8
90	1.21	1.19	1.7
91	1.22	1.24	1.6
92	1.25	1.36	8.1
93	1.28	1.47	12.9
94	1.28	1.48	11.5
95	1.28	1.50	14.7
96	1.30	1.53	15.0
97	1.34	1.60	16.2
98	1.37	1.68	18.4
	Atomite		
99	1.33	1.59	16.3
100	1.34	1.72	1.34
101	1.37	1.66	1.37
102	1.38	1.72	1.38
103	1.41	1.86	1.41
104	1.48	2.20	1.48
	Atomite		
1	1.39	1.44	3.5
2	1.33	0.91	48.0
3	1.39	3.44	59.8
4	1.44	1.72	16.3
5	1.35	1.33	2.1
6	1.31	1.02	28.4
7	1.42	1.06	34.0
8	1.46	1.37	6.6
9	1.47	1.34	9.7
10	1.50	1.58	5.1
11	1.57	2.82	44.4
	Snowflake		
1	1.21	0.96	26.0
2	1.24	1.29	3.8
3	1.25	1.10	13.7
4	1.28	1.22	4.9
5	1.30	1.17	11.1

TABLE XIII (con.)

DEVIATION OF DERIVED EQUATION RESULTS FROM OBSERVED RESULTS

RUN NO.	DERIVED μ/μ_W	OBSERVED μ/μ_W	DEVIATION %
Snowflake			
6	1.39	2.20	36.8
7	1.46	2.47	40.8
8	1.43	1.59	10.0
9	1.39	1.37	1.5
10	1.36	1.30	4.6
11	1.33	1.20	9.7
12	1.30	1.16	12.1
No. 1 White			
1	1.15	1.73	33.5
2	1.16	1.65	29.8
3	1.17	1.77	33.9
4	1.18	2.10	43.8
5	1.23	1.92	35.9
6	1.21	1.42	14.8
7	1.23	1.68	26.8
8	1.26	1.21	4.1
9	1.31	1.65	20.6
10	1.33	1.45	8.3

RUN NO.	DERIVED μ/μ_W	OBSERVED μ/μ_W	DEVIATION %
Copper			
1	1.67	1.27	31.5
2	1.74	1.94	10.3
3	1.74	1.13	53.6
4	1.81	1.11	63.2
5	1.78	1.92	7.3
6	1.81	1.35	34.2
7	1.88	1.98	3.0

SUMMARY AND CONCLUSION

A reasonable correlation of the effects of velocity, concentration, particle size, particle thermal conductivity and heat capacity of the suspending medium upon the apparent or bulk viscosity of certain non-Newtonian slurries was developed. The equation, $\mu/\mu_W = 1.02(Ak/GC)^{.105}$, which represents the effect of these variables, was obtained through dimensional and graphical analysis. For 130 runs the average deviation of the apparent viscosity calculated from this equation compared to the experimentally obtained values was 14.4%.

Although a satisfactory correlation was obtained, certain assumptions were necessary, which require confirmation through collection and interpretation of additional data.

The particle size values obtained from the manufacturer were accepted at face value. The accuracy of these values and the uniformity of different batches of the same material should be the subject of additional investigation.

The effect of particle size is somewhat masked by the fact that the different materials are not represented over the same particle size range. The non-metallic materials range in size from 1.5 to 14 microns while the only metal, copper, ranges between 21 and 56 microns in size. The non-metallic materials category should be expanded to include particles in the 21 to 56 micron range. Copper particles smaller than 21 microns should also be included in the materials evaluated.

The shape of the particles investigated was assumed to be spherical. The particle shape should actually be determined and data obtained for both spherical and non-spherical particles.

The overall effect of particle thermal conductivity on apparent viscosity of the slurry is to vary transfer of heat through the suspension so that extremes in thermal conductivity would cause extremes in the effect of temperature on viscosity of the suspending medium. It is recommended that the selection of materials be arranged to include a wider range of thermal conductivity values.

Additional information on the net effect of particle thermal conductivity and the heat capacity of the suspending medium should be obtained by the substitution of other fluids for water.

SAMPLE CALCULATIONS

1. Conversion of Apparent Viscosity in Heat Section to Apparent Viscosity in Viscometer

Copper A Run 19

Apparent Viscosity in heat section 0.85

Temperature in heat section is

$$\tfrac{1}{2}(50.65 + 68.35) = 59.5^{\circ}C$$

Viscometer Temperature 60.5°C

Viscosity of water at 60.5°C, 0.467 cps

Viscosity of water at 59.5°C, 0.473 cps

$$(0.85)\ \frac{0.467}{0.473} = 0.84 \text{ cps apparent viscosity in viscometer at } 60.5^{\circ}C$$

2. Conversion of Apparent Viscosity in #/min-ft to cps

Snowflake White Powder Run 1

Apparent Viscosity 0.0184 #/min-ft

Factor #/min-ft to cps 24.8

$$(24.8)\ (0.0184) = 0.46 \text{ cps apparent viscosity}$$

3. Linear Velocity of Slurry

Copper A Run 19

Observed rate minutes per 75# slurry

$$\frac{75}{0.532} = 140.8 \text{ \#/min of slurry}$$

Density of slurry 67.86 $\#/ft^3$

$$\frac{140.8}{67.86} = 2.08 \ ft^3/\text{min of slurry}$$

$$\frac{2.08}{60} = 0.0346 \ ft^3/\text{sec of slurry}$$

Viscometer is ½" Std. Pipe

Cross sectional area is 0.00211 ft^2

$$\frac{0.0346}{0.00211} = 16.4 \ \text{ft/sec linear velocity of}$$

Copper A Run 19

4. <u>Correlation Coefficient for Copper A, 10% Solids</u>

Runs

Correlation Coefficient r is expressed by the equation:

$$r = \frac{\overline{\mu v} - \bar{\mu}\,\bar{v}}{\left(\frac{\Sigma \mu^2}{n} - \bar{\mu}^2\right)^{\frac{1}{2}} \left(\frac{\Sigma v^2}{n} - \bar{v}^2\right)^{\frac{1}{2}}}$$

where μ is the apparent viscosity of the slurry

v is the linear velocity of the slurry and

$\bar{\mu}$ and $\bar{v}$ are the means of their respective terms,

$\bar{\mu}\,\bar{v}$ is the product of the means

$\overline{\mu v}$ is the mean of the products of μ and v

and n is the number of runs involved

$\Sigma \mu^2$ and Σv^2 are the summation of the squares of the viscosity and velocity respectively.

4. (continued)

For Copper A, 10% solids runs

$$\bar{\mu} = 1.01$$

$$\bar{v} = 11.8$$

$$\mu^2 = 6.3646$$

$$v^2 = 902.8$$

$$n = 6$$

$$\bar{\mu}^2 = 1.02$$

$$\bar{v}^2 = 139.2$$

$$\overline{\mu\ v} = 11.22$$

$$\bar{\mu}\ \bar{v} = 11.9$$

$$\frac{\Sigma \mu^2}{n} = 1.06$$

$$\frac{\Sigma v^2}{n} = 150.5$$

substituting in the above equation

$$r = \frac{11.22 - 11.90}{(1.06 - 1.02)^{\frac{1}{2}}\ (150.5 - 139.2)^{\frac{1}{2}}}$$

$$r = 1.01$$

5. Mass Velocity of Slurry

Copper A Run 19

$$\text{Mass velocity (G)} = 140.8 \ \frac{\#}{\text{min.}} \times \frac{\text{min.}}{60 \text{ sec}} \times \frac{1}{.00211 \text{ ft}^2}$$

$$= 1115 \ \frac{\#}{\text{sec.} - \text{ft}^2}$$

Determination of $\frac{\#}{\text{minute}}$ and ft^2 pipe cross sectional area values shown in Sample Calculation No. 3

6. Volume Fraction of Solid in Slurry

Copper A Run 19

Weight % copper in slurry = 10.7%

Slurry density = $68.28\#/ft^3$

Solid particle density = $8.92 \times 62.4\#/ft^3$

$$\text{Volume Fraction } \emptyset = \frac{(0.107)\ (68.28)}{(8.92)\ (62.4)}$$

$$= 0.01313 \ \frac{\text{ft}^3 \text{ solid}}{\text{ft}^3 \text{ slurry}}$$

7. Surface Area of Particles

Spherical particles of the same diameter equal to the average diameter are assumed.

$$\text{Volume per solid particle} = \frac{\pi D^3}{6} \text{ ft}^3$$

$$N = \frac{\text{number of solid particles}}{\text{ft}^3 \text{ slurry}} = \frac{6\emptyset}{\pi D^3}$$

7. (Continued)

$$A = \frac{\text{surface area of solid particles}}{\text{ft}^3 \text{ slurry}}$$

$$= (N) \ \frac{\text{surface area}}{\text{particle}}$$

$$= \frac{6\phi}{\pi D^3} \ \pi D^2 \ = \frac{6\phi}{D}$$

Copper A Run 19

$$A = \frac{(6)\ (.01313\ \frac{\text{ft}^3 \text{ solid}}{\text{ft}^3 \text{ slurry}}}{0.0000679 \text{ ft}} =$$

$$= \frac{1162 \text{ ft}^2 \text{ solid surface}}{\text{ft}^3 \text{ slurry}}$$

8. Dimensional Analysis of the Effect of Velocity and Concentration on Apparent Viscosity of Non-Newtonian Slurries

A system using the net dimensions of mass (M), length (L) and time (Θ) is utilized.

Letting f = any function, the effect of the variables upon viscosity is shown by the following:

(1) $\mu = f(D, v, \rho, \phi)$

where D = pipe diameter, v = linear velocity, ρ = bulk density and ϕ = volume fraction.

This is replaced by an infinite series.

8. (Continued)

(2) $\mu = \alpha D^a v^b \rho^c \phi^d + \alpha'$ etc. ----

Substitution of the dimensions gives

(3) $\frac{M}{L\theta} = (L)^a \left(\frac{L}{\theta}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{L^3}{L^3}\right)^d$

Summation of the exponents of like dimensions gives the condition equations:

$\Sigma M \quad 1 = c$

$\Sigma L \quad -1 = a + b - 3c + 3d - 3d$

$\Sigma \theta \quad -1 = -b$

Simultaneous solution gives c = 1, b = 1, a = 1 and d = 0.

Substitution in equation 2 gives

$\mu = f(Dv\rho)$

ϕ is dimensionless and does not affect validity of equation.

UNITS

A	=	Surface area of particle, ft^2/ft^3 of suspension
C	=	Specific heat of fluid, Btu/(lb) (^oF)
D	=	Pipe diameter, ft.
D_p	=	Particle diameter, ft.
G	=	Mass velocity, lb/sec. - ft^2
k	=	Thermal conductivity of suspended solid, Btu/(hr)(ft^2) (^oF/ft)
v	=	Linear velocity of suspension, ft/sec.
ϕ	=	Volume fraction of solid in suspension
ρ	=	Density of slurry or bulk density, lb/ft^3
μ	=	Apparent or bulk viscosity of suspension
μ_w	=	Viscosity of water

REFERENCES

(1) a. Perry, Chemical Engineers' Handbook, McGraw Hill Book Company, New York, (1950)

b. Brown, Unit Operations, John Wiley & Sons, Inc., New York, (1951)

(2) Richardson, E.G., Dynamics of Real Fluids, Edmond Arnold & Co., London, (1950)

(3) a. Wilhelm, R.H., Wroughten, D.M., and Loeffel, W.F., Ind. Eng. Chem., 31, 622, (1939)

b. Binder, R.C., and Busher, J.E., (Tr. A.S.M.E.), J. of App. Mech., 13, No. 2, A 101, (1946)

c. Babbit, H.E., and Caldwell, D.H., Ind. Eng. Chem., 33, 249, (1941)

(4) Bonilla, C.F., et al., "Preprints of Symposium on Heat Transfer", 44th Annual Meeting A.E.Ch.E., Dec., (1951)

(5) Orr, C., Jr., and Dalla Valle, J.M., Preprint No. 13, Heat Transfer Symposium, Annual Meeting, A.E.Ch.E., Dec., (1953)

(6) Salamone, J.J., Doctor of Engineering Science Thesis, New York University, (April 1954)

(7) Binder, R.C. and Pollara, F., Master of Science in Chemical Engineering Thesis, Newark College of Engineering, (June 1954)

(8) Simon, Leslie E., An Engineer's Manual of Statistical Methods, John Wiley & Sons, New York, (1941)

(9) McAdams, W.H., "Heat Transmission", 3rd Edition, Chapter V, McGraw Hill Book Co., New York, (1954)

(10) Youden, W.J., "Statistical Methods for Chemists", John Wiley & Sons, Inc., New York, (1951)