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A rheological equation of state for blood at low shear rates

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A RHEOLOGICAL EQUATION OF STATE FOR BLOOD AT LOW SHEAR RATES

BY

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

In this work a new rheological equation of state is presented and its application to blood is studied. The equation considered is a linear combination of Newtonian and non-Newtonian contributions to the apparent viscosity. The general form of the equation is

$$\tau_{ij} = -\left(A + B\,e^{-1/C\left|\frac{1}{2}\Delta_j^i : \Delta_i^j\right|^{\frac{1}{2}}}\right)\Delta_{ij}$$

Data from the literature were used to test the equation by generating curves of measured torque (T/L) versus applied shear rate (Ω) that would characterize the behavior of blood at all red cell concentrations and all shear rates in the Gilinson, Dauwalter and Merrill viscometer. The constants for the equation were obtained by appropriate curve-fitting using the "Least-Squares Estimation of Non-Linear Parameters" procedure of Marquardt and functions in terms of hematocrit were determined to represent this behavior.

Comparisons of the predicted results with those from the literature indicate considerable deviations at the lowest shear rates. However, it is believed that with more data in the very low shear region (less than 0.5 sec.$^{-1}$) to determine the correct values of the constants, better correlation with experimental data may be obtained. The results presented in this work were

found to correlate well for shear rates in the intermediate region and low region (greater than 5 $sec.^{-1}$ and less than 50 $sec.^{-1}$).

INTRODUCTION

With the increasing emphasis on applications of engineering methods to biological systems in both, the biomedical engineering field and the study of fundamental aspects of the human anatomy, the ability to predict quantitatively the rheological behavior of blood is increasing in importance. However, although the problem has been studied for more than a century, an examination of the literature indicates divergent opinions on even the most fundamental aspects of blood rheology. The existence of a yield stress, the strictly Newtonian behavior of plasma, the existence of rouleaux in healthy blood and the interpretation of shear stress versus shear rate behavior are examples of questions that are brought up in the literature and are inadequately answered.

To consider the problem from a strictly medical viewpoint, the study of the rheology of blood is necessary to obtain quantitative data of the energy in the system and the manner in which this energy is expended during flow. A corollary to this is the establishment of the magnitudes of the various components of the cardiovascular system that are responsible for altering the rheology of blood and perhaps their relation to diseases effecting the blood components.[1]

To consider the problem from an engineering viewpoint, prediction of the rheological behavior of blood is necessary in the design of blood circulating equipment, the realization of the limitations of this fluid, and the development of prosthetic replacements or aids for defective organs involved. A mathematical model and a resultant rheological equation of state that can predict the behavior of blood in the shear rates and the concentrations of interest would be useful in fulfilling these demands.

In this work such an equation of state is presented in the form suggested by Dr. C.R. Huang of Newark College of Engineering. The equation is tested by applying it to data from the literature by solving the equations of motion for the coaxial cylinder viscometer and presenting a method for evaluating the constants for the equation from viscometric data. Using these constants, shear stress versus shear rate behavior is predicted and compared to the data from the literature for varying red cell concentrations.

BACKGROUND

From a rheological viewpoint, blood may be described as a concentrated suspension of particles in a Newtonian suspending fluid. In man, the suspended cells occupy from 40 to 50 percent of the total volume of the suspension of which about 93 percent are red blood cells (erthrocytes) and the remainder are lycocytes and platelets. Plasma, the suspending medium, consists of an aqueous solution of 7 percent by weight of proteins and 1 percent of salts and miscellaneous organic compounds.[8] The behavior of the plasma may be approximately characterized as Newtonian and the non-Newtonian contributions of the suspended particles may be considered due almost entirely to the red cells. The negligible effects of the proteins and salts has been demonstrated experimentally by comparing the behavior of whole blood, defibrinated blood and red cell suspensions in Ringer solution.[6]

The problem then is one of characterizing the behavior of red cells suspended in plasma with respect to shear rate and red cell concentration (hematocrit). The dependence of shear stress on shear rate has been approached from two directions, the empirical curve fit and the approximate Casson model.[5]

The curve fit approach has primarily involved the use of the simple power law equation and the polynomial equation.[4] This approach lacks in several aspects. The inability to use the equations in predictive calculations, to change coordinate systems, and to fit the observed boundary conditions for blood without resorting to an unwieldly high order polynomial have been the major drawbacks.

The Casson model is somewhat more appealing due to its theoretical basis. In this model, derived to characterize the behavior of printing inks, mutually attractive particles suspended in a Newtonian medium at low shear rates are considered to form rigid, rod-like aggregates whose length varies inversely with the shear rate. With blood it is presumed that the aggregates are rouleaux whose mean length decreases with increasing shear rate. Using this model, a linear relationship between $\tau^{1/2}$ and $\dot{\gamma}^{1/2}$ exists and an equation of the form $\tau^{1/2} = \tau_y^{1/2} + b^{1/2}\dot{\gamma}^{1/2}$ is generally employed to analyze the data.[5,6,7,12]

Several problems exist with the use of this model. Essentially, it presupposes a yield stress which is a debatable point among workers in the field of blood rheology, and most recent papers have disputed its existence for blood. In addition, under just what conditions rouleaux formation takes place, and whether this is a characteristic of blood under normal circumstances has been questioned, thus throwing doubt on the very basis of the model. However, even with the knowledge of these drawbacks, several recent experimenters have used this model for lack of a better approach.

The other variable to be considered in the development of a rheological equation of state for blood is the dependence of the shear stress on hematocrit. The rheological effects of concentration on suspension and emulsion behavior has been the topic of considerable investigation and has been covered in sufficient detail in the references to which the interested reader is referred.[5,11,15]

In effect, most equations deal with either the very dilute range of concentrations or the highly concentrated, and no single equation is sufficiently broad to be used universally. The low concentration equations are essentially variations of the Einstein equation for the viscosity of suspensions of rigid spheres at infinite dilution. The high concentration equations are essentially variations of exponential behavior of apparent viscosity with increasing concentration of suspended material. Effects of hematocrit on blood rheology have been studied only from the basis of a polynomial curve fitting or an approximate exponential behavior. Such relationships as the Hatscheck, Mooney, or power series relationship are generally too complex for use in conjunction with a complex shear rate dependence in a rheological equation of state.[11]

DEVELOPMENT OF EQUATIONS

The general form of the equation used in this investigation to characterize blood at various shear rates is

$$\tau_{ij} = -\left(A + B\, e^{-1/C \left|\frac{1}{2} \Delta^{i}_{j} : \Delta^{j}_{i}\right|^{1/2}}\right) \Delta_{ij}$$

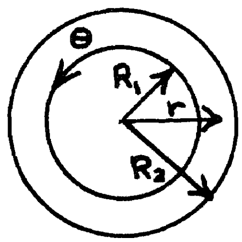
The form of this equation in somewhat more familiar notation may be represented as

$$\tau = A\dot{\gamma} + B\dot{\gamma}\, e^{-\dot{\gamma}/C}$$

This is essentially a three constant equation that assumes a linear combination of Newtonian and non-Newtonian effects of the fluid under shear. Resort to this form of equation was made primarily on an empirical basis where viscosity contributions to the system were assumed to come from the plasma and the red blood cells with the combination resulting in the effects shown by blood.

To test this equation against data from the literature, the equations of motion are first solved for the system on which the data were taken to enable the use of instrument readings to eliminate some of the assumptions and approximations made by the original experimenters. Then methods of determining the constants for the equation are determined using the solved equations of motion.

The data were obtained from a type of coaxial cylinder viscometer, therefore, the following derivation assumes cylindrical coordinates. The inner cylinder is rotating at Ω_1 revolutions per second and the torque (T/L) is measured on the outside cylinder.



The equations of motion may be written in cylindrical coordinates as follows [2]

r-component

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial\theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r$$

θ-component

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta} + \frac{\partial\tau_{\theta z}}{\partial z}\right) + \rho g_\theta$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z$$

From physical grounds the following statements can be made to simplify these equations. In steady state laminar flow the fluid moves in a circular pattern and the velocity components v_r and v_z are zero. There is no pressure gradient in the θ direction and all terms of the equation of continuity are zero.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)$$

$$+ \frac{\partial}{\partial z}(\rho v_z) = 0$$

The equations of motion can, therefore, be reduced to

$$0 = \frac{1}{r^2}\frac{d}{dr}(r^2\tau_{r\theta}) \qquad (1)$$

and by definition

$$\dot{\gamma} = r\frac{d}{dr}\left(\frac{v_\theta}{r}\right) \qquad (2)$$

Substitution into the proposed equation of state gives

$$\tau_{r\theta} = -\left[r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]\left[A + Be^{-\frac{1}{c}\left[r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]}\right] \qquad (3)$$

Integrating equation (1)

$$\tau_{r\theta} = \frac{c_1}{r^2} \qquad (4)$$

From the boundary condition at $r = R_2$, the torque is known and the integration constant c_1 can be found.

$$T = -\tau_{r\theta}\Big|_{r=R_2} \cdot 2\pi R_2 L \cdot R_2 \qquad (5)$$

Combine (4) and (5)

$$T = -\left(\frac{c_1}{r^2}\right)\Big|_{r=R_2} \cdot 2\pi R_2 L \cdot R_2 \qquad (6)$$

$$c_1 = -\frac{T}{2\pi L}$$

$$\tau_{r\theta} = -\frac{T}{2\pi L}\frac{1}{r^2} \qquad (7)$$

Eliminating $\tau_{r\theta}$ by combining equations (3) and (7)

$$-\frac{T}{2\pi L}\frac{1}{r^2} = -\left[r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]\left[A + Be^{-\frac{1}{c}\left[r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]}\right] \qquad (8)$$

Let $\frac{v_\theta}{r} = \omega$

$$\frac{d\omega}{dr}\left[A + Be^{-\frac{r}{c}\frac{d\omega}{dr}}\right] - \frac{1}{r}\frac{T}{2\pi L} = 0 \qquad (9)$$

Rearrange

$$\frac{d\omega}{d\ln r}\left[A + Be^{-\frac{1}{c}\frac{d\omega}{d\ln r}}\right] - \frac{T}{2\pi L} = 0 \qquad (10)$$

Let $\frac{d\omega}{d\ln r} = y$

$$y\left[A + Be^{-y/C}\right] - \frac{T}{2\pi L} = 0 \qquad (11)$$

This is an algebraic equation in terms of y. For any given run (hematocrit and R_1 and R_2 set), the solution to this equation should yield a constant. This equation is then used in determining A, B and C from viscometric data.

PROCEDURE

To test the proposed equation in its application to blood, suitable viscometric data were obtained from the literature.[6] The data consisted of polynomial equations describing the apparent viscosity of blood at various concentrations and shear rates. In order to use the data, however, it was necessary to convert the results presented in the article back to what would have been the rotational speed of the inner cylinder and the torque measured on the outer cylinder.

The data were presented in the literature in terms of hematocrit concentration with the following polynomial:

$$\ln \eta = a_0 + a_1 H + a_2 H^2 + a_3 H^3 + a_4 H^4 + a_5 H^5$$

The regression coefficients for the various shear rates are given in Table I.

Since the apparent viscosity η is defined as the shear stress divided by the shear rate and the shear rate for the viscometer used was related to the angular velocity of the rotating bob by a proportionality constant, it was a simple matter to back out Ω_1, τ, and subsequently T/L. The equations for this procedure follow.

$$\Omega_1 = \dot{\gamma}/1.04$$

$$\tau_{r\theta}\Big|_{r=R_2} = \eta\dot{\gamma}$$

$$T/L = -\tau_{r\theta}\Big|_{r=R_2} \cdot 2R_2 \cdot R_2$$

The data were plotted to determine the type of relationship existing between the variables. The plot of T/L versus Ω_1 for plasma yielded a straight line as expected for a Newtonian liquid. The plots for various percentages of red cells yielded curves concave downward, somewhat similar to pseudoplastic behavior. Subsequent examination of these data on log-log coordinates ruled out a simple power law relationship, so the $B\dot{\gamma}e^{-\dot{\gamma}/C}$ type of relationship was used.

The literature data were plotted for hematocrit values of .1 through .9 and the curves were smoothened out by inspection. Each curve was then used to approximate a set of constants (A, B and C) for a given concentration using the "Least Squares Estimation of Non-Linear Parameters" procedure of Donald W. Marquardt. An IBM 360 computer was used to perform the calculations.

The values of A, B and C were subsequently cross-plotted to obtain their dependence on concentration of red cells. The CEIR Time-Sharing system using an IBM 420 computer was used to fit appropriate functions to represent the constants by least squares criterion. The functions for the coefficients are found on page of this report.

These coefficients were then used with equation (11) to generate the values of T/L using the proposed model. Comparison and discussion of these results are found in subsequent sections of this paper.

DISCUSSION OF EQUATION

To apply the proposed rheological equation of state to blood, it was necessary to make the following general assumptions regarding the nature of the terms in the equation and the procedures in solving for the coefficients.

1. The Newtonian component and the non-Newtonian component are represented by the first and second terms of the equation respectively.

2. The gross effect of the components on the viscosity of the system is accomplished by a linear combination of Newtonian and non-Newtonian effects.

3. The coefficients A and B are dependent on the concentration of red cells only.

4. The constant C is a characteristic of the red cells and has a single value.

The general behavior of the equation, however, is generally consistent with the boundary conditions observed in the data from the literature. It was suggested that there is no yield stress as the shear rate goes to zero and that a Newtonian region exists at very low shear rates. A second Newtonian plateau was observed at very high shear rates with a highly non-Newtonian region between.[6] All of these conditions are qualitatively observed in the behavior of the terms of the equation. As the shear rate goes to zero and H is fixed, the second term approaches one and a Newtonian region is attained for very low shear rates. As the shear rate becomes very large, the exponential term approaches $-\infty$ very rapidly and the second term goes to zero, providing the second Newtonian plateau. The intermediate shear rates result in a combined effect of the two terms in the equation to give non-Newtonian behavior.

RESULTS

The regression coefficients for the polynomial from the literature relating apparent viscosity to hematocrit for blood are found in Table I and the apparent viscosities determined directly from the equation are in Table II. These apparent viscosities were used to calculate the measured values of the torque per unit depth of the outer bob of the viscometer (T/L) for varying hematocrits and rotational speeds of the inner bob of the viscometer (Ω). These torques represent the experimental data and are summarized in Table III. The values for 90 percent red cells were at the experimental limit as indicated in the original article.

Table IV contains a summary of the compared values of experimental torques with those predicted by the equation. The deviations tend to increase as the rate of shear is decreased with extremely poor correlation at the lowest shear rate studied. These torque values represent a summary of results achieved in this investigation.

Figure 1 is a plot of the dependence of T/L on Ω for plasma, indicating that for all practical purposes, a straight line characteristic of Newtonian behavior is obtained. The same plot is shown for whole blood in Figure 2 with both, the experimental and the calculated curves exhibiting a pseudoplastic type of behavior.

Figures 3,4, and 5 graphically illustrate the results for various shear rates. The tendency to obtain a greater deviation from experimental results may be observed here. The plot for the lowest shear rate was omitted due to a complete break down of the model and deviations of 100 percent or more appeared. It is believed that sedimentation effects become very important for the 0.05 $sec.^{-1}$ shear rate, contributing significantly to the errors.

Figures 6 and 7 show values of the coefficients for various hematocrit levels as obtained from the "Least-Squares Estimation of Non-Linear Parameters" procedure of Marquardt. The fitted equations for these curves are also presented. These values are related to the coefficients A and B in the equation of state by a factor of 2π. The values of the coefficients in terms of hematocrit (H) were determined to be

$$A = 0.635 \; e^{3.858 \, H}$$

$$B = 54.7 \; H^{2.388}$$

$$C = -0.15$$

DISCUSSION AND RECOMMENDATIONS

That the qualitative behavior of the equation corresponds to the qualitative behavior of blood was discussed previously in this paper. The comparison of the experimental to the predicted values of T/L, however, indicates considerable deviations in the lowest shear rates studied. There are several points at the procedure that may have contributed to this error.

The most outstanding problem in this investigation was the lack of adequate experimental data to accurately test the proposed equation. Although the proposed model allows for a separation of contributing effects of the Newtonian and non-Newtonian components, at present there is still a considerable difficulty in finding the coefficients. The least-squares estimation used in this work provides a relatively simple method using a computer, however, the experimental data is a limiting factor.

Since there were only four actual points to work with for each curve, it became necessary to interpolate between the data to obtain meaningful results for the coefficients. This may have effected the results adversely by improperly weighing the data for the curve fitting procedure. The slope of the curve of T/L versus Ω changes very rapidly at the lowest shear rates studied, and the coefficients determined from the data failed to describe this region accurately. A larger amount of low shear data would help alleviate this problem.

Finally, the problem of sedimentation of the red cells while working at the very low shear rates may have a considerable influence on the validity of the very low shear experimental data. Distribution changes of the suspended medium may significantly effect the viscosity, and the assumption of a homogeneous suspension in the model may preclude its application to shear rates where sedimentation is observed.

On the basis of the findings in this investigation, the following conclusions are made regarding the proposed rheological equation of state.

1. The equation may be applied to blood in shear regions where sedimentation effects are negligible.

2. The equation satisfactorily satisfies the necessary boundary conditions for blood.

3. A relatively straight-forward procedure is

available for determining the parameters of the equation.

The following improvements are recommended to further validate the proposed equation's application to blood.

1. Obtain more data in the very low shear region placing particular emphasis on the 0.01 to 0.05 $sec.^{-1}$ range to more accurately describe the behavior of the curve.

2. The effects of components other than red cells should be studied in greater detail to determine their effects more accurately.

3. The shear curve should be studied more carefully to investigate the possibility of different regions existing where different mechanisms may be controlling. Essentially, three regions may be expected: a very low shear Newtonian region, a low shear non-Newtonian region, and a high shear Newtonian region. The characterization procedure may be simplified by studying the regions separately.

4. The equation should be tested against data obtained from other types of viscometers to check the broad applicability to other coordinate systems.

5. A more detailed concentration dependence of the coefficients may be investigated.

The large deviations from the experimental data, therefore, may not be as much a shortcoming in the equation as in the insufficiency of the data to test it accurately. From a qualitative viewpoint, however, there is a reasonable similarity in the predicted and experimental curves, and the application of this form of rheological equation of state to blood may be acceptable.

APPENDIX

TABLE I

CONSTANT	SHEAR RATE (SEC.$^{-1}$)			
	52	5.2	.52	.052
$a_0 x10^{-1}$	2.028	1.817	1.499	1.342
$a_1 x10^{-2}$	2.928	2.423	2.599	4.250
$a_2 x10^{-3}$	-.157	.985	3.598	5.820
$a_3 x10^{-5}$	1.385	-1.372	-9.467	-16.425
$a_4 x10^{-7}$	-2.813	.299	9.524	16.958
$a_5 x10^{-9}$	1.788	.353	-3.335	-6.035

TABLE II

HEMATOCRIT	APPARENT VISCOSITY (CPS.) AT SHEAR RATES			
	52	5.2	.52	.052
.1	1.634	1.663	1.982	2.700
.2	2.219	2.602	4.451	9.489
.3	3.093	4.295	9.995	31.168
.4	4.358	7.110	19.767	78.058
.5	6.087	11.448	33.817	147.049
.6	8.420	17.813	52.624	231.123
.7	11.997	27.372	80.606	351.631
.8	19.462	43.854	129.987	584.455
.9	43.155	80.253	224.190	1083.90

TABLE III

HEMATOCRIT	MEASURED TORQUE (T/L)			
	50	5.0	.50	.05
.1	334.061	34.012	4.051	.551
.2	453.648	53.201	9.100	1.939
.3	632.483	87.801	20.433	6.371
.4	891.024	145.349	40.410	15.957
.5	1244.370	234.044	69.130	30.060
.6	1721.43	364.163	107.579	47.247
.7	2452.68	559.559	164.781	71.882
.8	3978.57	896.50	265.728	119.478
.9	8822.2	1640.59	458.303	221.577

TABLE IV

HEMATOCRIT	T/L EXPERIMENTAL	T/L PREDICTED	% DEVIATION
FOR SHEAR RATE = 50 sec^{-1}			
.1	334.1	293.5	12.1
.2	453.6	431.7	4.8
.3	632.5	635.3	-.5
.4	891.0	934.6	-4.8
.5	1244.4	1374.9	-10.4
.6	1721.4	2022.3	-17.5
.7	2452.7	2974.3	-21.2
.8	3978.6	4374.2	-10.0
.9	8822.2	6432.8	27.2
FOR SHEAR RATE = 5 sec^{-1}			
.1	34.0	32.7	3.8
.2	53.2	60.5	-13.8
.3	87.8	109.2	-24.0
.4	145.3	184.2	-26.8
.5	234.0	292.1	-24.8
.6	364.2	441.2	-21.2
.7	559.6	642.7	-14.7
.8	896.5	912.3	-1.7
.9	1640.6	1272.4	22.4

TABLE IV (CONT'D)

HEMATOCRIT	T/L EXPERIMENTAL	T/L PREDICTED	% DEVIATION
FOR SHEAR RATE = .5 $sec.^{-1}$			
.1	4.05	3.58	11.7
.2	9.10	7.72	14.1
.3	20.43	15.32	25.0
.4	40.41	27.17	32.5
.5	69.13	44.13	36.3
.6	107.60	67.18	37.8
.7	164.78	97.59	40.9
.8	265.73	137.07	48.2
.9	458.30	187.97	57.7
FOR SHEAR RATE = .05 $sec.^{-1}$			
.1	.55	.36	34.1
.2	1.94	.80	59.0
.3	6.37	1.60	74.6
.4	15.96	2.84	82.0
.5	30.06	4.63	84.8
.6	47.25	7.05	85.0
.7	71.88	10.23	85.7
.8	119.49	14.36	88.2
.9	221.58	19.66	91.0

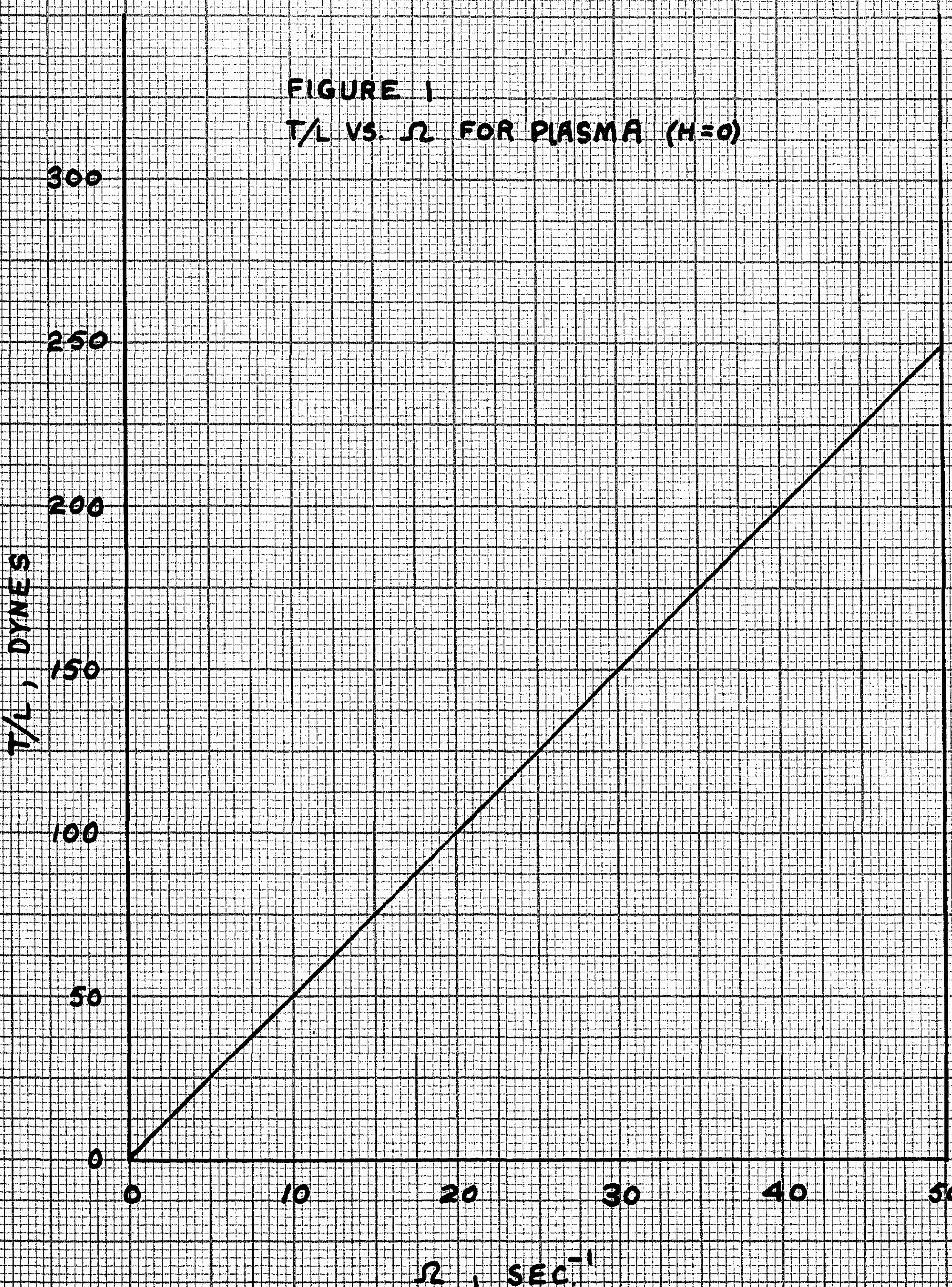


FIGURE 1
T/L VS. Ω FOR PLASMA (H=0)
300
250
200
150
100
50
0
T/L, DYNES
0
10
20
30
40
50
Ω, SEC.⁻¹

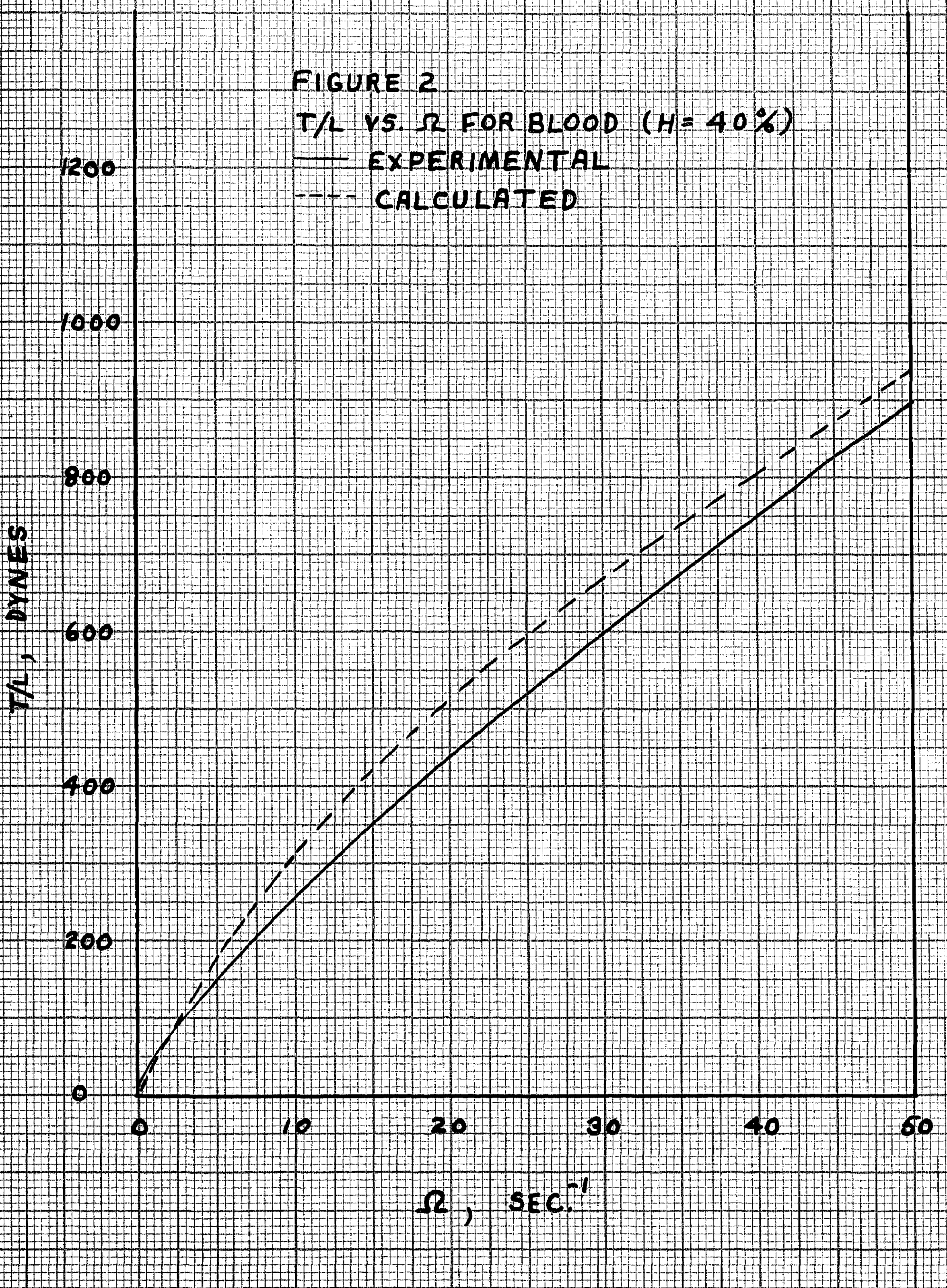


FIGURE 2
T/L VS. Ω FOR BLOOD (H = 40%)

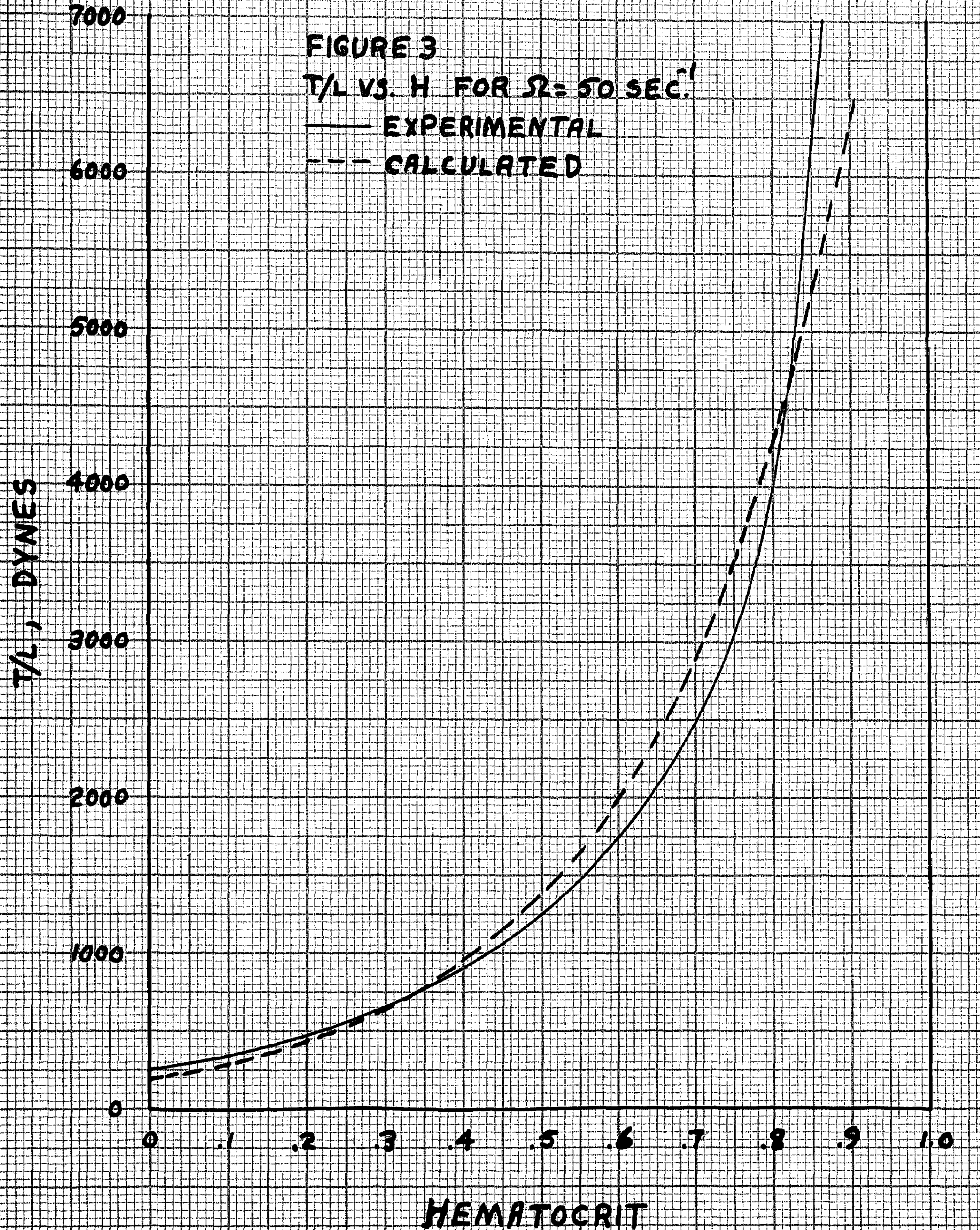


FIGURE 3
T/L VS. H FOR Ω = 50 SEC.⁻¹
EXPERIMENTAL
CALCULATED
T/L, DYNES
7000
6000
5000
4000
3000
2000
1000
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
HEMATOCRIT

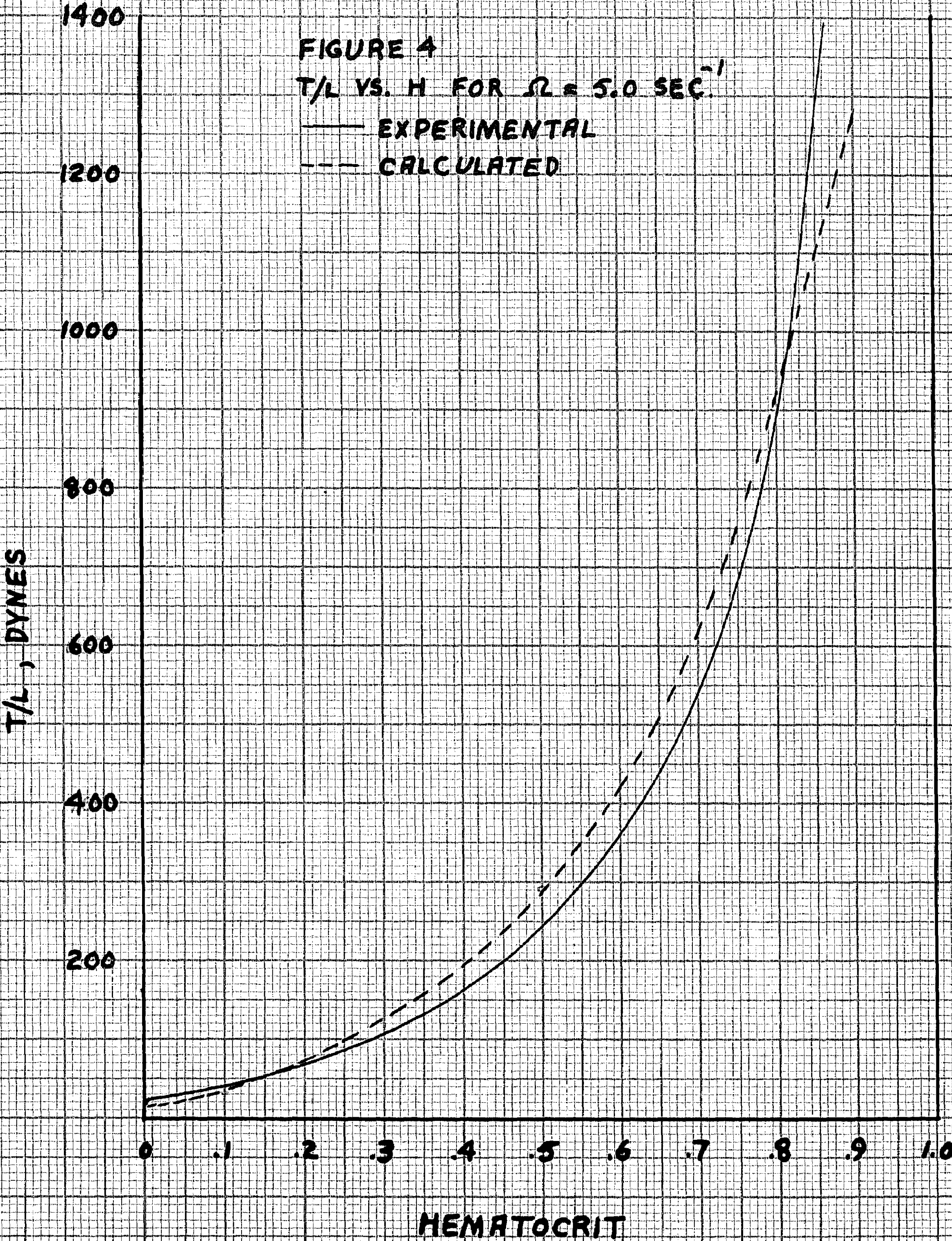


FIGURE 4
T/L VS. H FOR Ω = 5.0 SEC.⁻¹
EXPERIMENTAL
CALCULATED
1400
1200
1000
800
600
400
200
T/L , DYNES
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
HEMATOCRIT

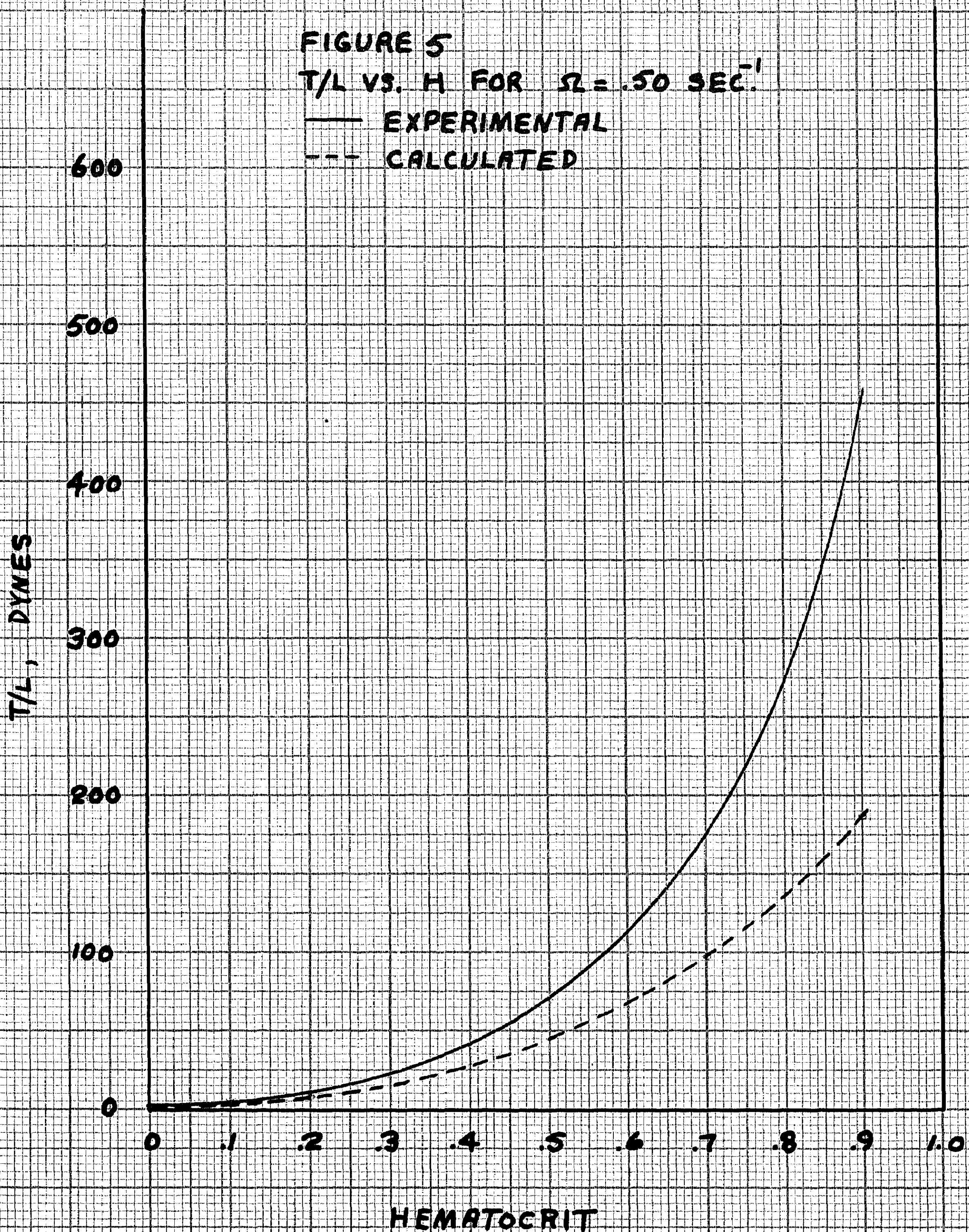


FIGURE 5
T/L VS. H FOR Ω = .50 SEC.⁻¹
EXPERIMENTAL
CALCULATED
600
500
400
300
200
100
0
T/L, DYNES
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
HEMATOCRIT

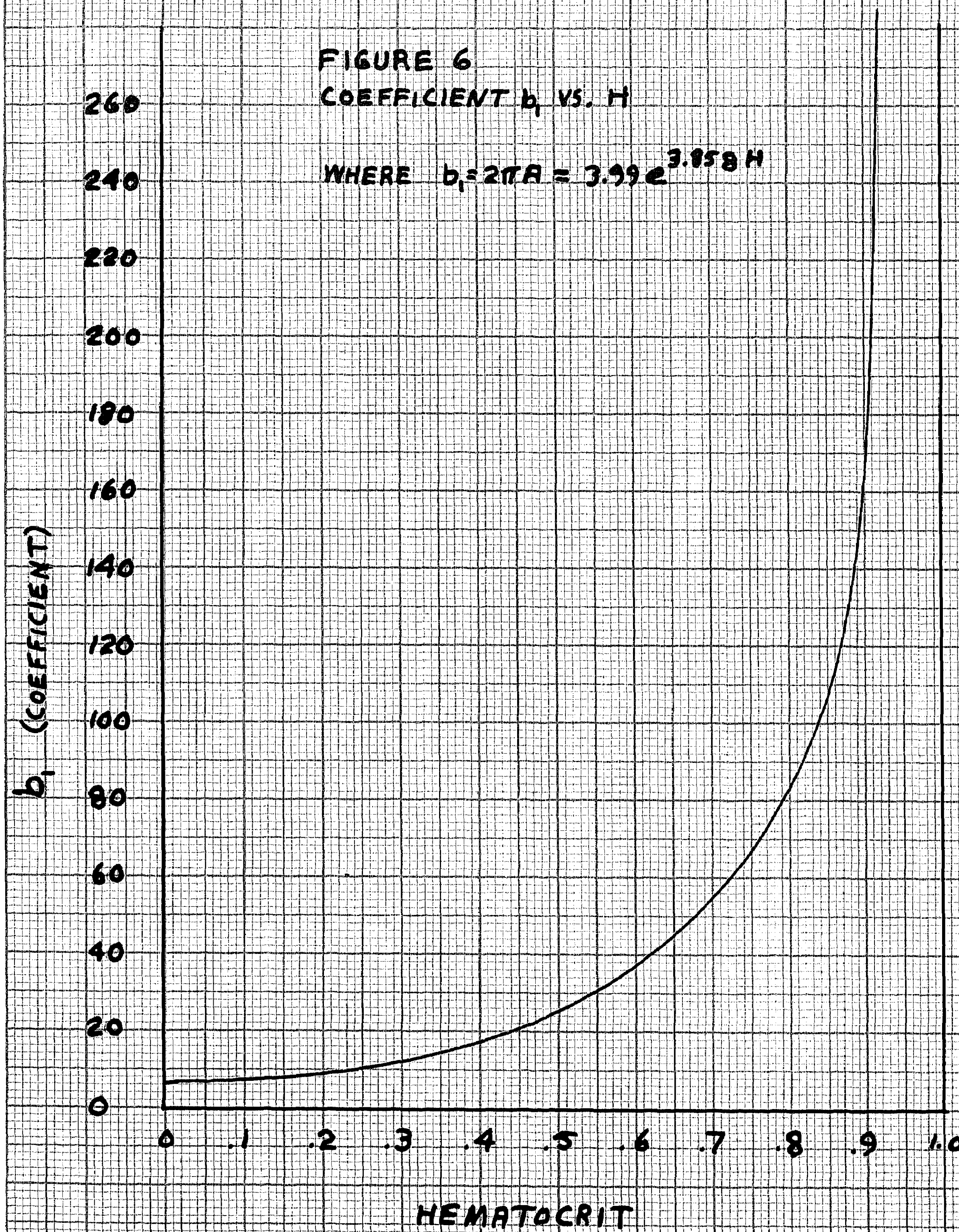


FIGURE 6
COEFFICIENT b₁ VS. H
WHERE b₁ = 2πA = 3.99e^(3.858 H)
260
240
220
200
180
160
140
120
100
80
60
40
20
0
b₁ (COEFFICIENT)
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
HEMATOCRIT

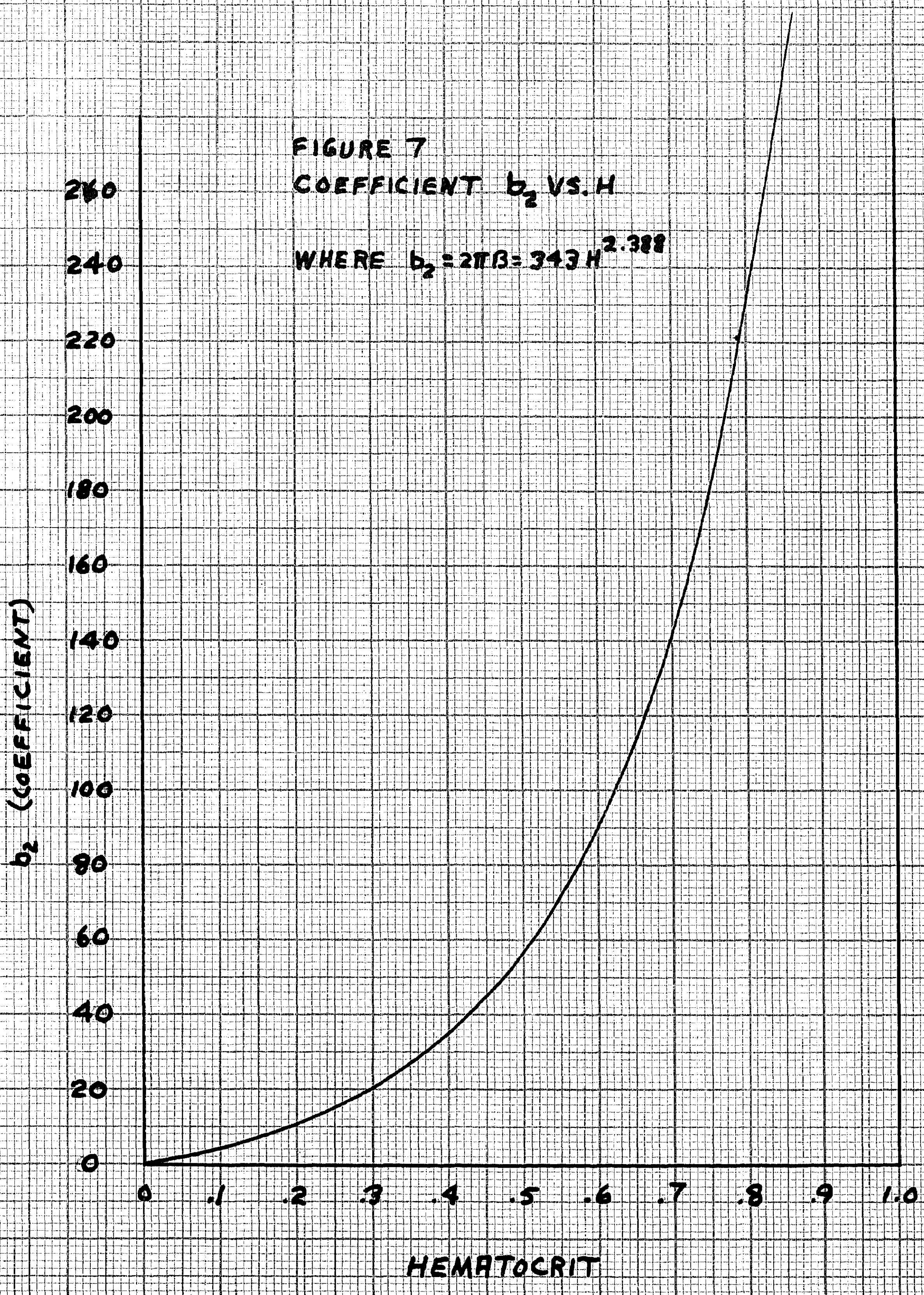


FIGURE 7
COEFFICIENT b_2 VS. H
WHERE $b_2 = 2\pi\beta = 343\,H^{2.388}$
260
240
220
200
180
160
140
120
100
80
60
40
20
0
b_2 (COEFFICIENT)
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
HEMATOCRIT

NOMENCLATURE

a_i = constants in apparent viscosity generating equation

A, B, C = coefficients in the proposed rheological equation of state

b = constant in the Casson model

H = hematocrit, red cell concentration by volume

L = depth of liquid in the viscometer cup

r = radius

R_1, R_2 = radius of inner, outer culinder of coaxial cylinder viscometer

T = torque

y = $d\omega/d \ln r$

$\tau = \tau_{ij}$ = shear stress tensor

$\dot{\gamma} = \dot{\gamma}_{ij}$ = rate of shear tensor

η = apparent viscosity

Ω, Ω_1 = rotational rate of inner bob in coaxial cylinder viscometer

v_θ = angular velocity

$\omega = v_\theta/r$

Δ_{ij} = rate of deformation tensor

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