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One-dimensional compressible flow analysis : isentropic and normal shock

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ABSTRACT

Title of Thesis: One-Dimensional Compressible Flow
Analysis - Isentropic And Normal Shock.
Su-Bo Wong, Master of Science in Mechanical Engineering
1983.

Thesis Directed by: Rong-yaw Chen
Professor of Mechanical Engineering

A computer program to compute and analyse one-dimensional isentropic compressible flow through variable cross-sectional area with or without a normal shock is developed. The program is written in the "FORTRAN LANGUAGE".

In this work, the area change is the predominant cause of change of flow condition. One of the advantage of this program is set on general uses for isentropic flow. In common practice, the values of the isentropic flow property ratios were tabulated or graphically presented as function of the Mach number with a specified specific heat ratio, K (normally $K=1.40$ was presented). With todays technology, the most versatile method is by implementation of computer programming method.

The computer program presented can solve all the one-dimensional isentropic flow problems and to analyse the flow characteristic and the flow patterns in converging nozzle and

converging-diverging nozzle. The value of K can be assigned as any value as one's requirement. All the solutions are computed within 0.1 % error. For solving Mach number and location of normal shock inside the nozzle, ITERATION method is employed instead of numerical method. In most cases, a few iterations (less than ten) may arise a reasonable solutions.

ONE-DIMENSIONAL COMPRESSIBLE FLOW

ANALYSIS - ISENTROPIC AND NORMAL SHOCK

by

Su-Bo Wong

Thesis submitted to the Faculty of the Graduate School
of the New Jersey Institute of Technology in partial fulfillment
of the requirements for the degree of
Master of Science in Mechanical Engineering

1983

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Analysis - Isentropic and Normal Shock.

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NOTATION

a	Speed of sound
A	Cross-sectional area of duct
h	Specific enthalpy
K	Specific gas ratio
M	Mach number
\dot{M}	Mass flow rate
MF	Mass flux = \dot{M}/A
P	Static pressure
R	Specific heat constant
s	Specific entropy
T	Static temperature
V	Velocity of the fluid flow
v	Specific volume
X	Position coordinate
ρ	Density
max	Maximum
o	Stagnation condition
*	Condition at $M = 1$

1. INTRODUCTION

Fluid mechanics is that study of fluid motion involving a rational method of approach based on general physical laws and consistent with the results of modern experimental study. There is hardly a branch of engineering that is not concerned with fluids or does not make use of them. Real economy and value are achieved in studying at one time the same principles underlying the flow of different fluids. Such a study tends to develop a sound background and to make one versatile in approaching new problems.

A fluid may be considered as compressible or incompressible. As an example in the compressible flow, when the relative velocity of the fluid with respect to the immersed body became high, the results of the analysis for the pressure coefficient began to depart from that based on incompressible flow. Also, when the speed of the fluid relative to the immersed body approached the speed of sound in the flowing medium, owing to the compressibility of the fluid, the deviation from incompressible flow analysis became pronounced.

The fact that the flow is compressible indicates that the density, ρ of the flowing medium is a sensitive function of the pressure. The introduction of this new variable ρ into the equations of motion will necessitate the study of its

relationship to the other properties of the medium. This need then invokes the principles of thermodynamics which represent a separate and independent approach from the dynamic equations of motion. Since a new variable ρ has been added, this independent approach is necessary for the solution of the problem, and the concepts of thermodynamics will play an important role in the theory of compressible flow.

Friction, heat transfer, area change and electromagnetic fields have a great effect on compressible flow. In most physical situations, more than one of these effects occur simultaneously; for example, flow in a rocket nozzle involves area change, friction, and heat transfer. However, one of the effects is usually predominant; in the rocket nozzle, area change is the factor having greatest influence on the flow. The frequent predominance of one factor provides a justification for separating the effects, including them one at a time in the equations of motion, and studying the resultant property variations.

Whereas a certain loss of generality is incurred by treating each of the effects individually, this procedure does simplify the equations of motion so that the results of each of the effects can be easily appreciated. Further, this simplification enables approximate solutions to be derived for a wide range of problems in compressible flow; such

solutions are sufficiently accurate for many engineering applications. Attempts to include all the effects simultaneously in the equations of motion lead to mathematical complexities that mask the physical situation. In many cases exact solutions to these generalized equations of motion are impossible.

This study is concerned with compressible, isentropic flow through varying area ducts, such as nozzle passages. Friction and heat transfer are negligible for this isentropic flow; variation in properties are brought about by area change. One-dimensional steady flow of a perfect gas is assumed in order to reduce the equations to a workable form. Since this is a study of gas flows, changes in potential energy and gravitational forces are neglected.

The intention on this study has been to provide a good understanding of the physical behaviour of compressible fluid flow and an adequate appreciation of the principles behind the design of modern engines such as nozzles and their equivalent form a very important item of all turbine and jet devices. In this work, a computer program is implemented to solve most of the engineering problems arising from isentropic compressible flow.

In general, the changing variable properties are $T_0, P_0, T, P,$

$\rho_0, \rho, M, V, m/A$ (MF), K, R . Since K and R are the properties of the gas and usually listed in most of the science or engineering data booklets. The remainder nine properties at which three properties must be given in order to obtain the rest of the variable properties, this gives the initial cross-section (station 1) properties. The program may proceed further to consider any other sections of the duct with no discontinuities or shock wave occurs inbetween. since the stagnation properties are remains constant at any sections of the duct, by giving one properties from T, P, M, V, ρ, AR ($A1/A2$ - area ratio), the other properties can be obtained.

To shows the great effect of mach number and wisely needed in engineering applications; converging nozzle and converging-diverging nozzle were chosen to demonstrate on this study . The program is constructed to consider a fluid stored in a large reservoir and is discharged through a converging nozzle; by varying the back pressure to analyse the characteristics and compute the properties of the fluid flow at the exit plane. For converging-diverging nozzle, it is desired to consider the pressure distribution in the nozzle over a range of values of P_b/P_0 with P_0 maintained constant (except across the shock wave). The phenomenon of the choked or unchoked, shock wave occurs inside or outside the nozzle , overexpansion , underexpansion or designed condition; all those situations will be detected and printed

at the output of the solutions.

The main fundamental concepts are discussed in Chapter 2, which include the analysis, governing equations for isentropic flow and flow across the normal shock wave and the features of the fluid flows through the converging nozzle and converging-diverging nozzle.

Chapter 3 is the method of solutions which explain the techniques of writing this program and the procedures of inputting the data.

Conclusions are presented in Chapter 4 and suggested recommendations are in Chapter 5. Appendix - A, is the computer program and Appendix - B, is the sample of examples. Finally, the References.

2. ANALYSIS

2.1 Fundamental Concepts

Four basic laws can be readily applied in studying the flow of compressible fluids. These fundamental laws or principles upon which the analysis presented in this study depend directly or indirectly are :

(A) The law of conservation of mass

$$\rho VA = \text{constant}$$

(B) Newton's second law of motion

$$1/\rho dp + V dV = 0$$

(C) The first law of thermodynamics

$$h + v^2/2 = \text{constant}$$

(D) The second law of thermodynamics

$$T ds = dh - v dp$$

In any flow analysis some information about the properties of the fluid must be known. This information (such as equation of state of a perfect gas) is used in connection with the basic laws to provide maximum knowledge of the flow. Compressible flow can be treated at various levels of complexity. In this work, the most elementary level possible was taken and the results by virtue of their simplicity, are among the most instructive.

The basic approximations are that the flow is steady and one-dimensional or, more precisely, steady and quasi-one-dimensional. By this we mean that the flow can be treated according to a one-dimensional model even though the 'real' flow is in fact three-dimensional.

2.2 Classification of Compressible Flows

There are large differences in flow patterns with compressible flows. General behaviour of the flow depends on whether the fluid velocity is greater or less than the local velocity of sound. Thus, the compressible flows may be classified as follows :

- (1) $M < 1$ ---- Subsonic flow ;
- (2) $M = 1$ ---- Sonic flow ;
- (3) $M > 1$ ---- Supersonic flow.

A transonic flow is defined as a flow having regions in which the flow speed changes from subsonic to supersonic. For example, transonic flows can occur in converging-diverging nozzles and in flow over bodies.

A hypersonic flow is a supersonic flow at high mach number (often defined as a flow whose mach number is greater than 5) . Hypersonic flows are so called because they require treatment somewhat different from low mach number supersonic flows.

2.3 Assumptions on The Flow Through a Normal Shock Wave

The assumptions are made as follows:

- (1) The boundary surface forming the stream tube is far removed from the boundary layers adjacent to any solid surface. Since all friction forces may be assumed to be confined to the shearing stresses in the boundary layer, the configuration under discussion is a frictionless duct.
- (2) The shock process takes place at constant area; that is, the streamlines forming the boundary of the stream tube are parallel.
- (3) The shock wave is perpendicular to the streamlines.
- (4) The flow process, including the shock wave is adiabatic, no external work is performed, and the effects of body forces are negligible.

2.4 Governing Equations

(A) For Isentropic Flow:

The ratio of the speed of the fluid at a point to the local speed of sound at that point is a useful index for identifying the flow. this ratio is called the Mach number, M ;

$$M = \frac{V}{a} \quad (1)$$

For isentropic flow,

$$a^2 = \left(\frac{\partial P}{\partial \rho} \right)_s$$

Where subscript s denotes constant entropy.

For an ideal gas under isentropic process $P^{-K} = \text{const.}$

Thus,

$$a = \sqrt{\left(\frac{\partial P}{\partial \rho} \right)_s} = \sqrt{\frac{K P}{\rho}} = \sqrt{K R T} \quad (2)$$

$$\therefore M = \frac{V}{\sqrt{K R T}} \quad (3)$$

The stagnation temperature, T_0 is the temperature where the speed of the fluid is zero. With the energy equation for isentropic flow, we get

$$T_0 = T \left(1 + \frac{K-1}{2} M^2 \right) \quad (4)$$

The stagnation pressure P_0 and the stagnation temperature T_0 are related by

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{\frac{K}{K-1}} \quad (5a)$$

$$\text{or } \frac{P_0}{P} = \left(1 + \frac{K-1}{2} M^2 \right)^{\frac{K}{K-1}} \quad (5b)$$

The stagnation or total density, ρ_0 is the density corresponding to the stagnation temperature and pressure.

From the thermal equation of state,

$$P_0 = \rho_0 R T \quad (6a)$$

$$\text{or } P = \rho R T \quad (6b)$$

Substituting equations (4) & (5) into equation (6) yields

$$\frac{\rho_0}{\rho} = \left(1 + \frac{K-1}{2} M^2 \right)^{\frac{1}{K-1}} \quad (7)$$

The continuity equation may be transformed to read

$$\frac{\dot{M}}{A} = \rho V = \frac{PV}{RT} \quad (8a)$$

Substituting equation (3) into equation (8a) yields

$$\begin{aligned} \frac{\dot{M}}{A} &= \frac{PV}{RT} = \frac{PV}{(KRT)^{1/2}} \left(\frac{K}{RT}\right)^{1/2} \\ &= PM \left(\frac{K}{RT}\right)^{1/2} \end{aligned} \quad (8b)$$

From equation (4), we have

$$T = \frac{T_0}{\left(1 + \frac{K-1}{2} M^2\right)}$$

Hence, in terms of the stagnation temperature T_0 ; equation (8b) becomes

$$\frac{\dot{M}}{A} = PM \left[\frac{K}{RT_0} \left(1 + \frac{K-1}{2} M^2\right) \right]^{1/2} \quad (8c)$$

From equation (5b), it follows that

$$P = P_0 \left[1 + \frac{K-1}{2} M^2 \right]^{-\frac{K}{K-1}}$$

Hence, the continuity equation expressed in terms of the stagnation pressure P_0 for the actual flow becomes

$$\frac{\dot{M}}{A} = MP_0 \left(\frac{K}{RT_0}\right)^{1/2} \cdot \left[1 + \frac{K-1}{2} M^2 \right]^{\frac{-(K+1)}{2(K-1)}} \quad (8d)$$

or in terms of T, we have

$$\frac{\dot{M}}{A} = MP_0 \left(\frac{K}{RT} \right)^{1/2} \cdot \left(1 + \frac{K-1}{2} M^2 \right)^{\frac{K}{1-K}} \quad (8e)$$

(B) For Normal Shock Wave:

In below is the application of the equations developed in the above to normal shock wave.

Since T_0 is a constant, therefore equation (4) gives

$$\frac{T_2}{T_1} = \frac{1 + \frac{K-1}{2} M_1^2}{1 + \frac{K-1}{2} M_2^2} \quad (9)$$

From equation (8a), as $\dot{M}/A = \text{Const.}$, we can derived that

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{P_1 T_2}{P_2 T_1} \quad (10)$$

Substituting equation (3) into equation (10) gives

$$\frac{P_2}{P_1} = \frac{M_1}{M_2} \left(\frac{1 + \frac{K-1}{2} M_1^2}{1 + \frac{K-1}{2} M_2^2} \right)^{1/2} \quad (11)$$

Introducing the momentum equation,

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 = \text{constant.} \quad (12a)$$

$$\text{or } P_1 \left(1 + \frac{V_1^2}{P_1} \right) = P_2 \left(1 + \frac{\rho_2 V_2^2}{P_2} \right) \quad (12b)$$

From equation (2), noting that $a^2 = KP/\rho$, equation (12b)

yields

$$\frac{P_2}{P_1} = \frac{1 + KM_1^2}{1 + KM_2^2} \quad (13)$$

Equating the right-hand sides of equation (11) and (13),

We can obtained that

$$\frac{M_1 \left(1 + \frac{K-1}{2} M_1^2 \right)^{1/2}}{1 + KM_1^2} = \frac{M_2 \left(1 + \frac{K-1}{2} M_2^2 \right)^{1/2}}{1 + KM_2^2} \quad (14)$$

Squaring both sides of equation (14) and solving for M_2 yields

$$M_2 = M_1 \quad (15a)$$

$$\text{or } M_2^2 = \frac{M_1^2 + \frac{2}{K-1}}{\frac{2K}{K-1} M_1^2 - 1} \quad (15b)$$

Equation (15a) expresses the trivial result that no change occurs across the normal shock wave; that is, the latter has infinitesimal strength. Equation (15b) gives the solution for a normal shock wave of finite strength.

For a normal shock wave to occur, the mach number, M_1 must be greater than unity and equation (15b) shows that as M_1 in front of the normal shock wave is increased infinitely, the mach number, M_2 in back of the normal shock wave continually decreases but approaches the limiting value $\sqrt{(k-1)/2K}$. The limiting value for M_2 depends only on the specific heat ratio for the gas. For air and diatomic gases, $k=1.40$ and $\sqrt{(k-1)/2K} = 0.378$. (Note that the computer program presented is permitted to put any value of k as one's necessity).

The property ratios across a normal shock wave may be expressed in terms of the mach number ahead of the shock wave M_1 by substituting equation (15b) into equations (9), (10) and (13) to eliminate M_2 . Thus

$$\frac{T_2}{T_1} = \left(\frac{2K}{K+1} M_1^2 - \frac{K-1}{K+1} \right) \left[\frac{K-1}{K+1} + \frac{2}{(K+1) M_1^2} \right] \quad (16)$$

$$\frac{\rho_2}{\rho_1} = \frac{(K+1) M_1^2}{2+(K-1) M_1^2} \quad (17)$$

$$\frac{P_2}{P_1} = \frac{2}{K+1} M_1^2 - \frac{K-1}{K+1} \quad (18)$$

$$\frac{P_{o2}}{P_{o1}} = \left\{ \left(\frac{K-1}{K+1} + \frac{2}{(K+1) M_1^2} \right)^K \cdot \left[\frac{2K}{K+1} M_1^2 - \frac{K-1}{K+1} \right] \right\}^{-\frac{1}{K-1}} \quad (19)$$

It follows from equation (17) that as the mach number M_1 approaches infinity, the density ratio ρ/ρ_o approaches $(k+1)/(k-1)$ as a limiting value. (i.e. = 6, for $k=1.40$). On the other hand, the pressure ratio P_2/P_1 (see equation (18)) increases continuously with M_1 .

The flow conditions immediately in front of and in back of the normal shock wave are isentropic, thus all the governing equations derived for isentropic flow can be applied to those sections.

2.5 Performance of Converging Nozzle

Consider the configuration shown in fig.2.1., where a simple converging duct discharges into a region where the back pressure P_b is controlled by a valve. Let P_e be the pressure in the exit plane of the nozzle and P_o the reservoir pressure (stagnation pressure for isentropic flow).

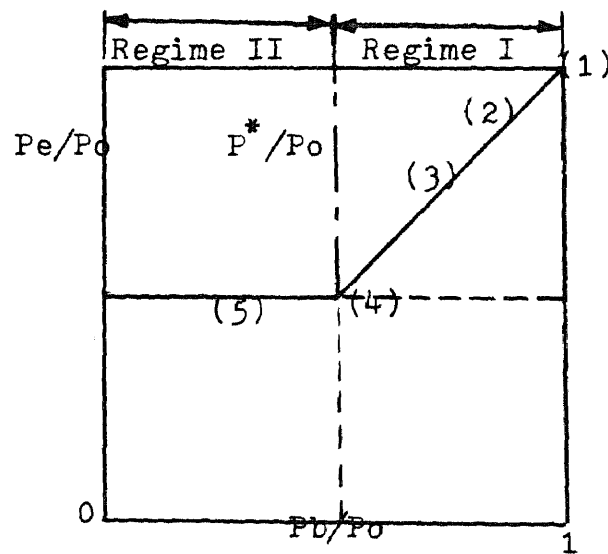
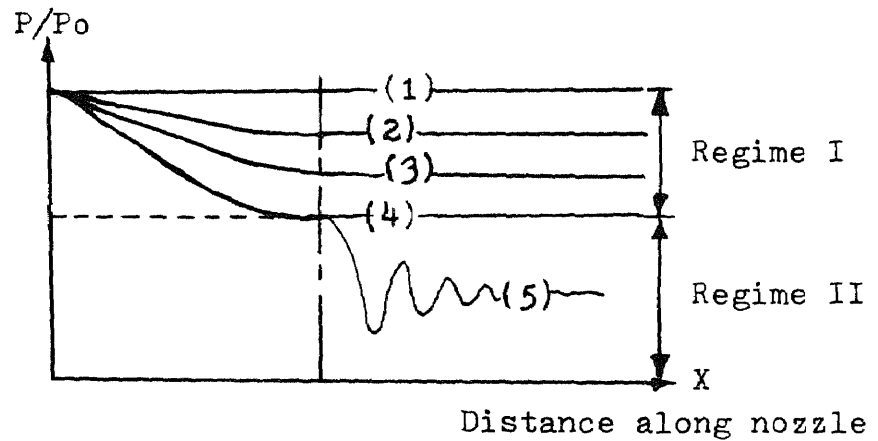
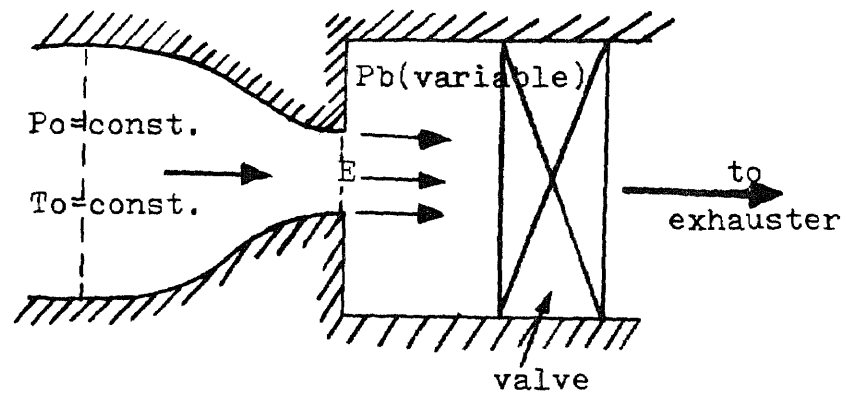


fig.2.1. Flow Through a Converging nozzle

If $P_b/P_o = 1$ (at condition 1, fig.2.1) then the pressure is constant throughout the nozzle and there is no flow. If P_b is slightly reduced from this value (condition 2, fig.2.1) a subsonic flow will be established with the exit pressure P_e equal to the back pressure P_b . If P_b is reduced further to condition 3, the flow remains subsonic with $P_b=P_e$, but the mass flow rate increases. This increase continues until P_b/P_o reaches the critical pressure ratio ($P_b/P_o = P^*/P_o = P_e/P_o$) where $Me = 1$, i.e. condition 4. If P_b is reduced further, (condition 5), the pressure P_e can not become less than P^* since Me stays at 1. Therefore the flow rate remains constant and pressure distribution inside the nozzle remains the same as for condition 4. The pressure distribution in the chamber outside the nozzle for P_b/P_e can not be predicted accurately by a one-dimensional model and is indicated by a wavy line.

The maximum mass flux, $(MF)_{max}$ can be found from equation (8c) with $Me = 1$, gives

$$\left(\frac{\dot{M}}{A} \right)_{max} = P_o \sqrt{\frac{K}{RT_o} \left(\frac{2}{K+1} \right)^{\frac{K+1}{K-1}}} \quad (20)$$

and from equation (5b),

$$\frac{P^*}{P_o} = \left(\frac{2}{K+1} \right)^{\frac{K}{K-1}} \quad (21)$$

For $k = 1.40$, $P^*/P_o = 0.5283$

2.6 Performance of Converging-Diverging Nozzle

Consider the case in which a fluid stored in a large reservoir is to be discharged through a converging-diverging nozzle. The experiment can be carried out similar to the above case (converging nozzle) by controlling the valve to vary the back pressure, P_b . It was pointed out that for certain ratios of back pressure to supply pressure, isentropic, one-dimensional solutions to the equations of motion are not possible. However, it is sufficient to analyse the normal shock occurred inside or at the exit plane and studied in details. It is desired to find the pressure distribution in the nozzle over a range of values of P_b/P_o , with P_o maintained constant. (see fig.2.3a)

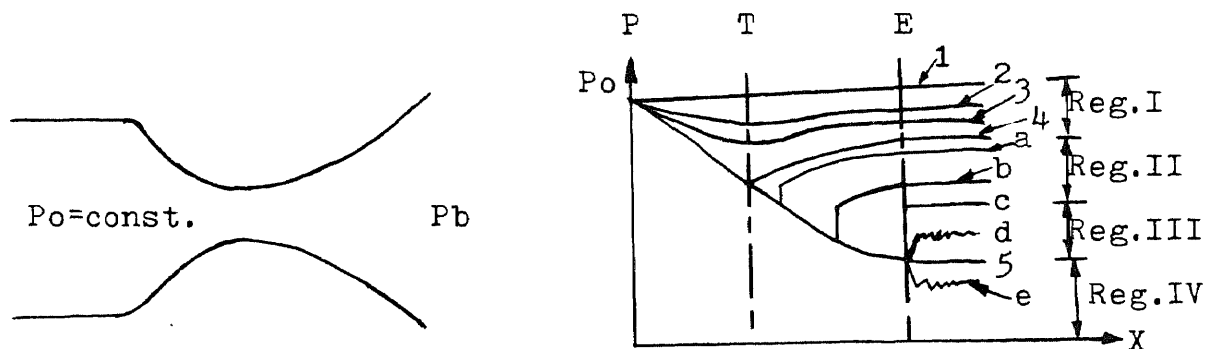


fig.2.3a

With $P_b = P_o$, there is no flow in the nozzle. As P_b is reduced below P_o , subsonic flow is induced through the nozzle with pressure decreasing to the throat then increasing in the diverging portion of the nozzle. When the back pressure is lowered to that of curve 4, sonic flow

occurs at the nozzle throat. Further reductions in back pressure can induced no more flow through the nozzle. As the back pressure is reduced below that of curve 4, a normal shock appears in the nozzle just downstream of the throat (curve a). Further reductions in back pressure cause the shock to move downstream (curve b), until for a low enough back pressure, the normal shock positions itself at the nozzle exit plane (curve c). Consider in detail a curve of P versus X with a shock in the nozzle (fig.2.3b). the static pressure decreases in the converging portion of the nozzle, with $M = 1$ at the throat. In the diverging portion, with the flow supersonic, the pressure continues to decrease up to the normal shock. After the shock, flow in the diverging part of the nozzle is subsonic, the static pressure increasing to the exit plane pressure. With subsonic flow at the exit, the exit plane pressure is equal to the back pressure.

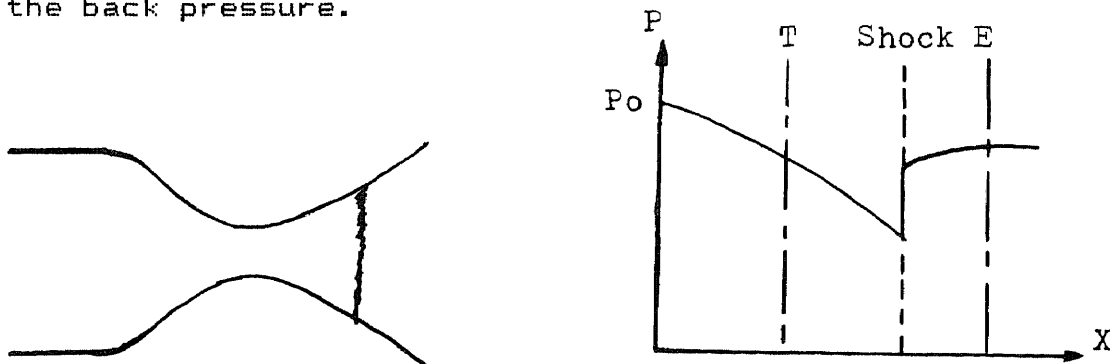


fig.2.3b

As the back pressure is lowered below that of curve c, a shock wave inclined at an angle to the flow appears at the exit plane of the nozzle (fig.2.3c). This shock wave,

weaker than a normal shock is called an oblique shock. Further reductions in back pressure cause the angle between the shock and the flow to decrease, thus decreasing the shock strength (fig.2.3d) until eventually the isentropic case, curve 5, is reached. Curve 5 corresponds to the design condition in which the flow is perfectly expanded in the nozzle to the back pressure.

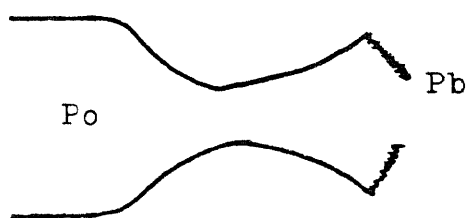


fig.2.3c

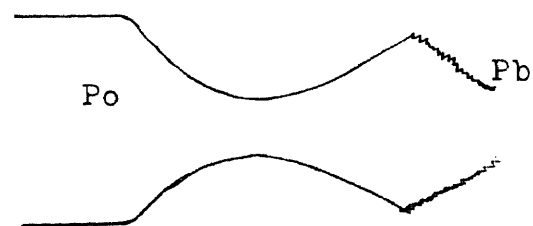


fig.2.3d

For back pressure below that of curve 5, exit plane pressure is greater than the back pressure. A pressure decrease occurs outside the nozzle in the form of expansion waves (fig.2.3e). Oblique shock waves and expansion waves represent flows which are not one-dimensional and can not be treated directly with the equations derived before.

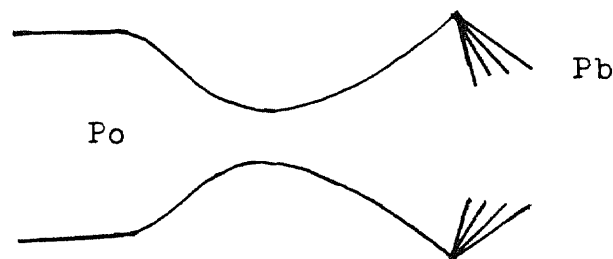


fig.2.3e

It is important to realize that for all back pressures below that of curve c the flow adjusts to the back pressure outside the nozzle. Over this range of back pressures

(below c), flow inside the nozzle remains unchanged as the back pressure is varied. For example, the exit plane pressure and exit velocity are the same for all back pressures below c . If a rocket nozzle is designed to operate isentropically at sea level, the rocket exhaust velocity and exit plane pressure do not change as the rocket moves upwards through the atmosphere (assuming constant chamber temperature and pressure).

Fig.2.3f depicts the variation of exit plane pressure with back pressure. For subsonic flow at the exit plane (curves 1,2,3,4, a,b,c), and for the design condition (curve 5), the exit plane pressure is equal to the back pressure. For supersonic flow at the exit plane (curve d,5,e) the exit plane pressure is equal to that for the design condition. For back pressure between c and 5, the exit plane pressure is less than the back pressure, so that the nozzle is termed overexpanded. For back pressure below 5, with the exit plane pressure greater than the back pressure, the nozzle is termed underexpanded.

Fig.2.3g shows the variation of pressure at the throat with back pressure, when back pressure is in between 1 and 4, the pressure at throat varies with back pressure (i.e. unchoked condition). Till back pressure equals or less than 4, choked condition, P_t always remain at P^* (i.e. critical stage).

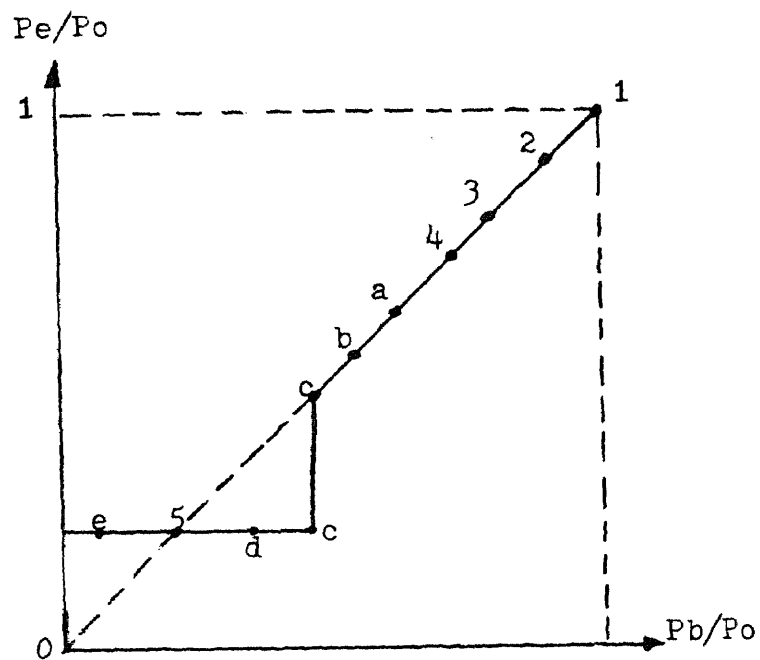


fig.2.3f

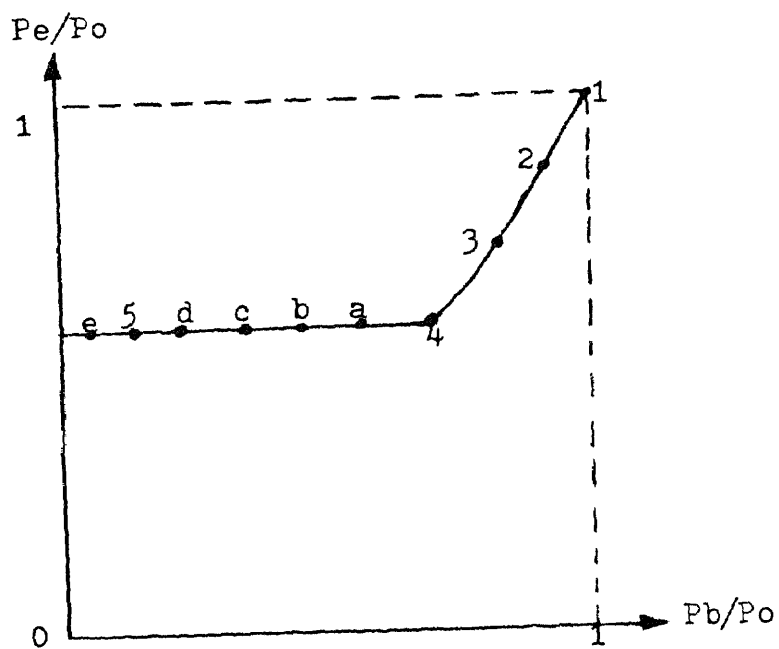


fig.2.3g

Summarising, there are four regimes of flow :

(I) Subsonic flow throughout the duct, maximum velocity is reached at the throat.

(II) Subsonic flow at the throat, then supersonic up to the normal shock, following by subsonic compression.

(III) Subsonic flow to the throat, followed by supersonic flow to the exit plane. Non-isentropic re-compression outside the duct through oblique shock waves.

(IV) Flow in the duct identical to (III), supersonic jet expanding out of the nozzle exit.

The nozzle is choked in regimes (II), (III) & (IV). The mass flow rate, \dot{M} is independent of the back pressure and is a maximum. Only in regime (I) can the mass flow rate be changed by variation in the back pressure.

3. METHOD OF SOLUTIONS

3.1 Introduction

By use of the governing equations developed in Chapter 2, we can express the flow property ratios (T/T_0 , P/P_0 , etc.) in a steady one-dimensional isentropic flow as a function of the local flow mach number, M for a given value of the specific heat ratio, k . In most of the engineering compressible text books, the values of the isentropic flow property ratios are tabulated or graphically presented as function of the mach number for $k = 1.40$. It has been a common practice of using table or graphs, whenever possible, in solving steady one-dimensional flow problems.

In most cases, the initial conditions at the inlet cross-section of the passage are specified. Using those conditions, the corresponding values of T/T_0 , P/P_0 , ρ/ρ_0 , etc. may be read directly from tables or graphs. The reference condition T_0 , P_0 , ρ_0 , A may then be calculated.

One condition at the exit cross-section of the flow passage must be known, so that tables or graphs may be employed for determine the property ratios, and hence the flow properties at the exit cross-section. Many problems of the practical

interest are not that straight-forward, and may be too complicated for a closed-form mathematical solution. A numerical solution is required and the most versatile method is by implementation of computer programming method. With present-days technology, computer is common and wisely used. It may solves the problem in shortest time with great efficient and high merit of accuracy. In this work, a computer program is developed to solve the isentropic flow problems instead of traditional method by using tables or graphs to compute and analyse the flow pattern. Also, it can be used for any different values of specific heat ratio, k .

One of the advantages of setting the program on general purpose is that many of the solution steps are common to all application area. For instant, the procedure for solving problems in converging or converging-diverging nozzle involve many of the same steps found in general program; i.e. for given three properties and compute the remainder properties.

3.2 Techniques of Programming the Program

A general computer flow diagram is presented in fig.3.1. This flow diagram is specifically for the one-dimensional isentropic flow with variable cross-sectional area and is

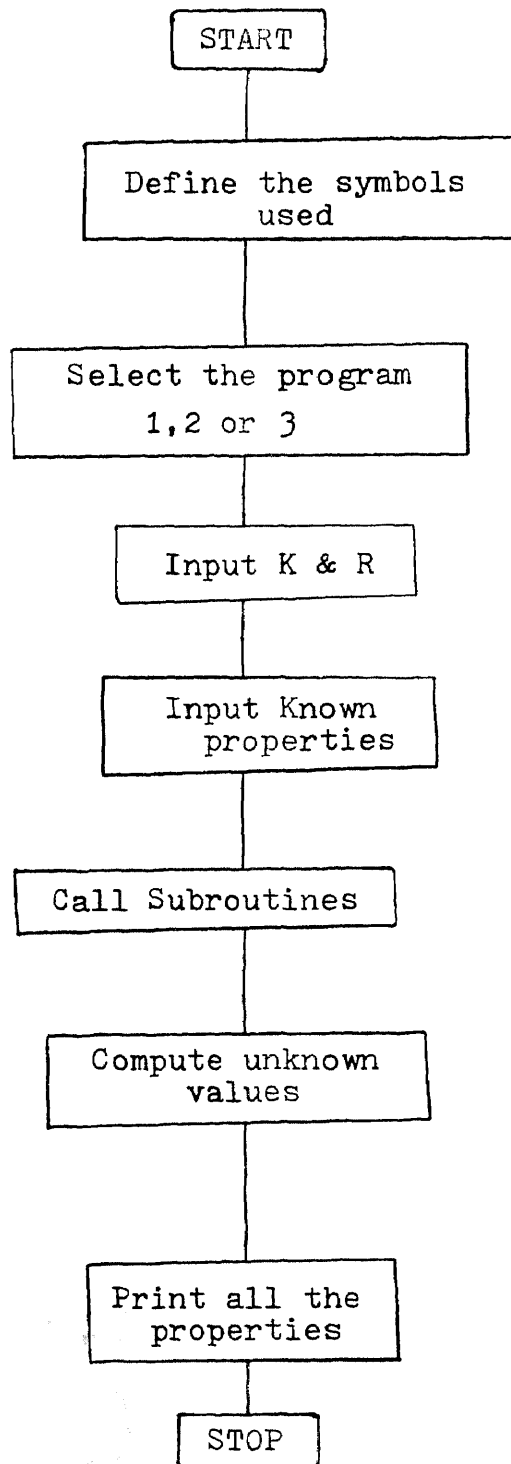


fig.3.1. A General Flow Diagram

valid for all of the application areas discussed in Chapter 2. Each major step of the flow chart will be discussed in general terms rather than with respect to a specific example.

Firstly, the general program was established. In general, the changing variable flow properties are $T_0, P_0, T, P, \rho, \rho_0, M, V, MF, K$ and R . Since K and R are the properties of the gas and usually listed in most of the text books. The remainder nine properties at which three must be given to make the solution possible.

From the knowledge of mathematical statistics, if one is interested only in what particular objects are selected when r objects are chosen from n objects (where r and n are any arbitrary numbers), without regard to their arrangement in a line, then the unloaded selection is called a "COMBINATION".

Employing the formula with $r = 3$ and $n = 9$;

Combination formula :

$$\binom{n}{r} = \frac{n}{r! (n-r)!}$$

i.e. $\binom{9}{3} = \frac{9!}{3! (9-3)!} = \underline{84}.$

From the above, it is understood that there are 84 ways of setting the problems.

Mass flux, MF is a most common property that either known or unknown value. This property is taken as a reference and divide the program into two parts. i.e. (1) Given two properties with mass flux, MF ; or (2) Given three properties without MF.

Using the same combination knowledge, we get in case (I) is 28 ways and 56 ways in case (II). Due to certain properties gives in the combination is exactly the terms of the governing equation, it is unable or should say not enough information to solve the problem. Such as occurred in case (I) has two cases and eight cases in case (II). When this situation appears, the output will print " NOT ENOUGH INFORMATION".

The logical If statement is used as a transfer control for the given condition. A pattern of entering the code number is fixed by entering the code number in ascending order.

For example, The coding as follows:

To - 1	T - 3	DN - 5	M - 7
Po - 2	P - 4	DNO- 6	V - 8

For given To,Po,P then enter 124 without any punctuation in between the numbers.

One of the most tedious part in the program is to solve for

M by trial and error. Here, ITERATION method is used instead of other numerical method, like Newton-Raphson method, because numerical method may easily diverged and make the solution impossible.

For instant, given MF, Po and To from equation (8c), we have

$$MF = M Po \left(\frac{K}{RT_0} \right)^{1/2} \left(1 + \frac{K-1}{2} M^2 \right)^{-\frac{(K+1)}{2(K-1)}}$$

Solving for M,

$$M = \frac{MF \left(1 + \frac{K-1}{2} M^2 \right)^{\frac{(K-1)}{2(K-1)}}}{Po \left(\frac{k}{RT_0} \right)^{1/2}}$$

By iteration, the DO-LOOP being employed. The Fortran statement may be written as follows :

```

DUMMY=PO*SQRT(K/R/TO)
M=MF*(1+C2)**PWS/DUMMY
DO 111 I=1,30
MP=MF*(1+C2*M*M)**PWS/DUMMY
EP=ABS(1-MP/M)
IF (EP.LT.0.00001) GO TO 1000
M=MP
111 CONTINUE
1000 -----
-----

```

In most cases, only a few iterations (normally less than 10) may gives the reasonable solution within 0.001 % error.

Due to many solution steps are common in most of the cases, in preparing the program, one must solve all the 84 cases at one time to search where the solution steps are common. By grouping those steps together to form the SUBROUTINE. In this program, eleven subroutines were employed.

3.3 Comments on the Computer Program

When the program get started, it prints the NOTATION of the symbols used then follows by select program:

- (1) Given three properties of a flow in a duct, find the other properties of the flow;
- (2) Converging nozzle flow from reservoir;
- (3) Converging-Diverging nozzle.

3.3.1 Program (1)

1. Select either (a) Given mass flux with two properties
or (b) Given three properties without mass flux.
2. Input K and R.
3. Input the known properties with the coding specified in ascending order.
4. Input the values of the known properties.

5. The computer may compute the unknown properties at this initial section and print out the solutions.
6. It may proceed further to compute any section of the duct by entering one known properties.
7. The program may keep on repeated until all the section of the required properties are obtained

3.3.2 Program (2)

1. Input K and R
2. Input T_0, P_0 and P_b (Back pressure)
3. The computer may detect the characteristic of the flow pattern through the nozzle. Refer to Chapter two, fig.2.1.
 - (i) $P_0 = P_b$
 - (ii) Regime I ($P_0 < P_b < P(iv)$)
 - (iii) Regime II ($P_b < P(iv)$)
 - (iv) $P_b = P(iv)$
4. Print the properties at the exit plane.

3.3.3 Program (3)

1. Input K and R
2. Input T_0, P_0 and P_b (Back pressure)
3. Input area ratio between exit and throat (A_e/A_t)
4. The program is able to detect the charecteristic of the flow pattern. Refer to fig.3.2 in below :

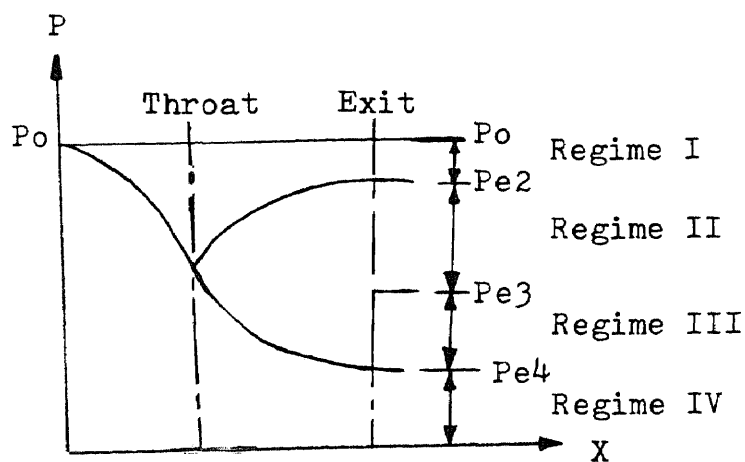


fig.3.2

- (i) Find Me_2 and Me_4
- (ii) Determine Pe_2 , Pe_3 and Pe_4
- (iii) Using Logical IF to classify the flow pattern. Generally, it consists of the following cases. (Note : All the details had been included in Chapter 2).

- (a) $P_o = P_b$
- (b) Regime I ($P_o > P_b > P_{e2}$)
- (c) $P_b = P_{e2}$
- (d) Regime II ($P_{e2} > P_b > P_{e3}$)
- (e) $P_b = P_{e3}$
- (f) Regime III ($P_{e3} > P_b > P_{e4}$)
- (g) $P_b = P_{e4}$
- (h) Regime IV ($P_{e4} > P_b$)

In the above eight cases, the most difficult part of the program is in Regime II (i.e. to locate normal shock inside

the nozzle). There are many ways of solving this problem. Here, the iteration method is employed to find the area ratio AR , where the normal shock occurs. The program is constructed as follows:

(i) By averaging the area ratios AR at throat and exit to assume the normal shock wave occurs half way between the throat and the exit plane.

(ii) From the throat condition with $M = 1$ and the known P_0, T_0 . Using subroutine with estimated AR to compute the properties before the normal shock occurs.

(iii) Compute the properties after normal shock wave occurs.

(iv) Using subroutine to determine the properties at the exit plane.

(v) To compare is the computed exit pressure $P_e = P_b$?

(vi) (a) If $P_e > P_b$; take AR as the average of the estimated AR and area ratio at the throat.

(b) If $P_e < P_b$; take AR as the average of the estimated AR and the area ratio at the exit.

(vii) Repeating the procedures by taking the average until P_e within 0.1 % error equals to P_b . In most cases, within ten iterations the solution may arise.

(5) Print all the necessary solutions. i.e

(i) Throat properties;

(ii) exit plane properties;

(iii) normal shock, before and after.

The program constructed in the above are three programs

that are commonly needed for studying one-dimensional isentropic compressible flow with variable cross-sectional area. The program may be modified or extended to fit others requirement.

4. CONCLUSION

A computer program to aid the analysis of one-dimensional isentropic compressible flow through variable cross-sectional area, and discharges of gas from a reservoir through a converging or converging-diverging nozzle has been developed. The program was written in FORTRAN LANGUAGE.

In this work, the area change is the predominant cause of change of flow condition. One of the advantage of this program is set on general uses for isentropic cases. Three application solutions on fluid flow through a variable cross-sectional area duct are described and had demonstrated how those problems being solved. The three applications are :

(1) Three properties of the initial conditions at the inlet cross-section or any particular section of the flow passage are specified, the corresponding properties can be computed and proceed further with one condition given at any section or at the exit cross-section of the flow passage, all other required properties are computed.

(2) A fluid stored in a large reservoir with given P_0 and T_0 is discharged through a converging nozzle and by varying the back pressure to analyse the features and the patterns of the flow, choked or unchoked and compute the properties at the exit plane.

(3) A fluid stored in a large reservoir with given P_0 and T_0 is discharged through a converging-diverging nozzle. The fluid flow has been carefully considered in all flow patterns, i.e. subsonic or supersonic. It can detect the shock occurred inside, outside or at the exit plane. Furthermore it analyses the flow at design condition, overexpansion or underexpansion, and choked or unchoked conditions. It also computes all the necessary properties at throat, exit plane, before and after the normal shock wave. Especially, it can locate the position of the normal shock wave occurred inside the nozzle.

The computer program presented in Appendix A is not limited to solving these three types of applications, instead it can be easily extended to fit all other applications or any section of the isentropic compressible flow. In general, graphs or tables for $K = 1.40$ is attached at the textbook for solving isentropic flow. In this program, value of K can be assigned as any value as one's requirement.

5. RECOMMENDATIONS

(1) Nozzle design and operation have been studied up to this point by means of a one-dimensional flow analysis. Although this method of analysis is adequate for the solution of many engineering problems, certain limitations become apparent. For example, in the design of a supersonic nozzle, area ratios can be determined for a given supersonic Mach number. But the length of the nozzle or the rate of change of area with axial distance cannot be prescribed from one-dimensional flow considerations. Further, due to presence of boundary layers on the nozzle walls, the area available to the main flow is somewhat reduced; the areas calculated from a one-dimensional flow analysis may have to be enlarged to account for boundary layers. For an advanced and complete analysis of the operation and design of a converging or converging-diverging nozzle, a study of two- and three-dimensional analysis is required. A good engineering approximations may then be obtained for the solution of a wide range of compressible flow problems.

(2) Compressible flow in ducts was analyzed for the case in which changes in flow properties were brought about solely by area change. In a real flow situation, however, frictional forces are present and may have a decisive effect on the resultant flow characteristic. Naturally, the inclusion of friction terms in the equations of motion makes

the resultant analysis more complex. For this reason, to study the effect of friction on compressible flow in ducts, certain restrictions may be placed on the flow. (a) By considering compressible flow with friction in constant-area insulated ducts, which eliminate the effects of area change and heat addition. In a practical sense, these restrictions limit the applicability of the resultant analysis; however, certain problems such as flow in short ducts can be handled, and furthermore an insight is provided into the general effects of friction on a compressible flow. (b) Dealing with the flow with friction in constant-area ducts, in which the fluid temperature is assumed constant, this approximates the flow of a gas through a long uninsulated pipe line. Thus these two cases cover a wide range of frictional flows and are consequently of great significance.

(3) Flow assumed to be adiabatic and study the effects on a gas flow of area change and friction. Here, the effect of heat addition or loss on a gas flow may be investigated. Flows with heat transfer occur in a wide variety of situations, for example, combustion chambers, in which the heat addition is supplied internally by a chemical reaction, or heat exchangers, in which heat flow occurs the system boundaries.

(4) Furthermore, the investigation may be ventured into the case of flow with applied electric and magnetic fields. Under certain conditions, however, a gas can be made into an electrically conducting fluid, possessing electrical

properties similar to a solid conductor. If such a conducting gas stream is allowed to pass through a magnetic field aligned perpendicular to the flow, an electrical field is induced normal to the flow direction and the magnetic field. Since the gas is able to conduct electrically, the electric field can be used to generate a current between electrodes placed in the fluid, and this current is able to produce work through an external load.

APPENDIX A

FORTRAN IV COMPUTER PROGRAM

```

C-----
C      MAIN PROGRAM
C-----
C
C      * PROGRAM FOR SOLVING ONE-DIMENSIONAL ISENTROPIC *
C      * COMPRESSIBLE FLOW, INCLUDING FLOW THROUGH *
C      * CONVERGING NOZZLE AND CONVERGING-DIVERGING *
C      * NOZZLE *
C-----
COMMON V,DN,DNO,K,R,M,MF,TO,T,P,PO,C2,PW2,PW3,PW5
REAL*4 MF,M,K,MP,ME2,ME3,ME4,ME5,MB1,MB2,MFB1,MFB2
WRITE(2,1)
1  FORMAT(1X,'TO DEFINE THE SYMBOLS USED :')
   WRITE(2,2)
2  FORMAT(//,7X,'MF = MASS FLUX',//,
$6X,' R = SPEC.HEAT CONST.',//,
$6X,' K = SPEC.GAS RATIO.',//,
$6X,' M = MACH NO.',//,
$6X,' T = STATIC TEMPERATURE',//,
$6X,' P = STATIC PRESSURE',//,
$6X,' DN = STATIC DENSITY',//,
$6X,' V = LOCAL FLUID VELOCITY',//,
$6X,' TO = STAGNATION TEMPERATURE',//,
$6X,' PO = STAGNATION PRESSURE',//,
$6X,' DNO= STAGNATION DENSITY')
   WRITE(2,31)
31  FORMAT(//,2X,'SELECT PROGRAM :',
$//,7X,'1 - GIVEN THREE PROPERTIES OF A FLOW IN A DUCT',
$,15X,'FIND THE OTHER PROPERTIES OF THE FLOW',
$//,7X,'2 - CONVERGING NOZZLE FLOW FROM RESERVOIR',
$//,7X,'3 - CONVERGING-DIVERGING NOZZLE',)
   READ ,J1
9999 IF (J1.EQ.1) WRITE (2,10)
10  FORMAT(//,1X,'SELECT EITHER :',//,
$6X,'1 - GIVEN MASS FLUX WITH TWO PROPERTIES',//,
$2X,'OR 2 - GIVEN THREE PROPERTIES WITHOUT MASS FLUX',//)
   JJ=0
   IF (J1.EQ.1) READ ,JJ
   WRITE(2,6)
6  FORMAT(//,3X,'ENTER K,R')
   READ ,K,R
   WRITE(2,17) K,R
17  FORMAT(//,3X,'SPECIFIC HEAT RATIO=',F12.4,//,
$3X,'SPECIFIC GAS CONSTANT=',F12.4,//)
   IF (JJ.EQ.1) WRITE (2,5)

```

```

5   FORMAT(//,3X,'ENTER MF')
    IF (JJ.EQ.1) READ ,MF
    C1=2/(K-1)
    C2=(K-1)/2
    C3=(K-1)/(K+1)
    C4=2/(K+1)
    PW1=K-1
    PW2=1/(K-1)
    PW3=K/(K-1)
    PW4=(K-1)/K
    PW5=(K+1)/(2*(K-1))
    KNOR=0
    IF ((J1.EQ.2).OR.(J1.EQ.3)) GO TO 2001
    WRITE(2,3)
3   FORMAT(//,1X,'SELECT THE CONDITIONS :',//,
$3X,'ENTERING THE CODE OF THE GIVEN CONDITIONS IN ASCENDI
NG ORDER',
$//,12X,'###  CODES  ###',//,
$11X,'TO - 1',6X,'DN - 5',//,
$11X,'PO - 2',6X,'INO - 6',//,
$11X,'T - 3',6X,'M - 7',//,
$11X,'P - 4',6X,'V - 8',//)
    READ ,N
    WRITE(2,4)N
4   FORMAT(//,1X,'THE CONDITIONS GIVEN ARE CODES ##',1X,I4,1
X,'##',//)
    KK=1
    IF ((JJ.EQ.1).AND.(N.GT.100)) GO TO 9996
    IF ((JJ.EQ.2).AND.(N.GT.100)) GO TO 9998
    IF ((JJ.EQ.2).AND.(N.LT.100)) GO TO 9996

C   -----
C   PROGRAM FOR GIVEN MF & TWO PROPERTIES
C   -----
    IF (N.EQ.12) GO TO 100
    IF (N.EQ.14) GO TO 150
    IF (N.EQ.23) GO TO 200
    IF (N.EQ.34) GO TO 250
    IF (N.EQ.35) GO TO 300
    IF (N.EQ.45) GO TO 350
    IF (N.EQ.38) GO TO 400
    IF (N.EQ.48) GO TO 450
    IF (N.EQ.13) GO TO 500
    IF (N.EQ.15) GO TO 520
    IF (N.EQ.18) GO TO 540
    IF (N.EQ.78) GO TO 560
    IF (N.EQ.57) GO TO 580
    IF (N.EQ.17) GO TO 600
    IF (N.EQ.37) GO TO 620
    IF (N.EQ.28) GO TO 640
    IF (N.EQ.25) GO TO 660
    IF (N.EQ.24) GO TO 680
    IF (N.EQ.27) GO TO 700

```

```

IF (N.EQ.47) GO TO 720
IF (N.EQ.67) GO TO 740
IF (N.EQ.56) GO TO 760
IF (N.EQ.68) GO TO 780
IF (N.EQ.16) GO TO 782
IF (N.EQ.26) GO TO 785
IF (N.EQ.36) GO TO 787
IF (N.EQ.46) GO TO 1111
IF (N.EQ.58) GO TO 1111
100 CONTINUE
WRITE(2,8)
8 FORMAT(1X,'INPUT TO,PO')
READ , TO,PO
CALL ISEN7
GO TO 2222
150 WRITE(2,15)
15 FORMAT(1X,'INPUT TO,P')
READ , TO,P
M=MF/SQRT(1+C2)/(P*SQRT(K/R/TO))
DO 155 I=1,30
MP=MF/SQRT(1+C2*M*M)/(P*SQRT(K/R/TO))
EP=ABS(1-MP/M)
IF (EP.LT.0.00001) GO TO 1500
M=MP
155 CONTINUE
1500 T=TO/(1+C2*M*M)
CALL ISENT1
PO=P*(1+C2*M*M)**PW3
GOTO 2222
200 WRITE(2,20)
20 FORMAT(1X,'INPUT PO,T')
READ , PO,T
M=MF*(1+C2)**PW3/(PO*SQRT(K/R/T))
DO 205 I=1,30
MP=MF*(1+C2*M*M)**PW3/(PO*SQRT(K/R/T))
EP=ABS(1-MP/M)
IF (EP.LT.0.00001) GO TO 2000
M=MP
205 CONTINUE
2000 TO=T*(1+C2*M*M)
CALL ISENT1
P=PO/(1+C2*M*M)**PW3
GO TO 2222
250 WRITE(2,25)
25 FORMAT(1X,'INPUT T,P')
READ , T,P
M=MF/(P*SQRT(K/R/T))
CALL ISENT1
PO=P*(1+C2*M*M)**PW3
TO=T*(1+C2*M*M)

```

```

GO TO 2222
300 CONTINUE
WRITE(2,30)
30  FORMAT(1X,'INPUT T, DN')
    READ ,T, DN
    P=DN*R*T
36  M=MF/(P*SQRT(K/R/T))
    V=M*SQRT(K*R*T)
    DNO=DN*(1+C2*M*M)**PW2
    PO=P*(1+C2*M*M)**PW3
    TO=T*(1+C2*M*M)
    GO TO 2222
350 WRITE(2,35)
35  FORMAT(1X,'INPUT P, DN')
    READ ,P, DN
    T=P/(DN*R)
    GO TO 36
400 WRITE(2,40)
40  FORMAT(1X,'INPUT T, V')
    READ , T, V
    DN=MF/V
    P=DN*R*T
46  M=V/(K*R*T)**0.5
    PO=P*(1+C2*M*M)**PW3
    TO=T*(1+C2*M*M)
    DNO=DN*(1+C2*M*M)**PW2
    GO TO 2222
450 WRITE(2,45)
45  FORMAT(1X,'INPUT P, V')
    READ ,P, V
    DN=MF/V
    T=P/(DN*R)
    GO TO 46
500 WRITE(2,50)
50  FORMAT(1X,'INPUT TO, T')
    READ ,TO, T
    TOT=TO/T
    M=((2*(TOT-1))/(K-1))**0.5
    V=M*(K*R*T)**0.5
    DN=MF/V
    CALL ISENT2
    GO TO 2222
520 WRITE(2,52)
52  FORMAT(1X,'INPUT TO, DN')
    READ ,TO, DN
    V=MF/DN
55  M=SQRT(V*V/((TO*K*R)-(V*V*C2)))
    T=TO/(1+C2*M*M)
    CALL ISENT2
    GO TO 2222

```

```

540 WRITE(2,54)
54  FORMAT(1X,'INPUT TO,V')
    READ ,TO,V
    DN=MF/V
    GO TO 55
560 WRITE(2,56)
56  FORMAT(1X,'INPUT M,V')
    READ ,M,V
    DN=MF/V
59  T=V*V/(M**K**R)
    TO=T*(1+C2**M**M)
    CALL ISENT2
    GO TO 2222
580 WRITE(2,58)
58  FORMAT(1X,'INPUT DN,M')
    READ ,DN,M
    V=MF/DN
    GO TO 59
600 WRITE(2,60)
60  FORMAT(1X,'INPUT TO,M')
    READ ,TO,M
    CALL ISEN8
    GO TO 2222
620 WRITE(2,62)
62  FORMAT(1X,'INPUT T,M')
    READ ,T,M
790 TO=T*(1+C2**M**M)
    CALL ISEN8
    GO TO 2222
640 WRITE(2,641)
641  FORMAT(1X,'INPUT PO,V')
    READ ,PO,V
    DN=MF/V
662 M=V*(1+C2)**(0.5*PW3)*SQRT(DN/PO/K)
    DO 644 I=1,30
    MP=V*(1+C2**M**M)**(0.5*PW3)*SQRT(DN/PO/K)
    EP=ABS(1-MP/M)
    IF (EP.LT.0.00001) GO TO 645
    M=MP
644 CONTINUE
645 P=PO/(1+C2**M**M)**PW3
    T=P/(DN**R)
    TO=T*(1+C2**M**M)
    DNO=DN*(1+C2**M**M)**PW2
    GO TO 2222
660 WRITE(2,661)
661  FORMAT(1X,'INPUT PO,DN')
    READ ,PO,DN
    V=MF/DN
    GO TO 662

```

```

680 WRITE(2,681)
681 FORMAT(1X,'INPUT PO,P')
   READ ,PO,P
   M=SQRT(C1*((PO/P)**PW4-1))
688 T=M*M*P*P*K/(MF*MF*R)
   V=M*SQRT(K*R*T)
   TO=T*(1+C2*M*M)
   DN=MF/V
   DNO=DN*(1+C2*M*M)**PW2
   GO TO 2222
700 WRITE(2,701)
701 FORMAT(1X,'INPUT PO,M')
   READ ,PO,M
   P=PO/(1+C2*M*M)**PW3
   GO TO 688
720 WRITE(2,721)
721 FORMAT(1X,'INPUT P,M')
   READ ,P,M
   PO=P*(1+C2*M*M)**PW3
   GO TO 688
740 WRITE(2,741)
741 FORMAT(1X,'INPUT DNO,M')
   READ ,DNO,M
   DN=DNO/(1+C2*M*M)**PW2
766 V=MF/DN
   T=V*V/(M*M*K*R)
   TO=T*(1+C2*M*M)
   P=DN*R*T
   PO=P*(1+C2*M*M)**PW3
   GO TO 2222
760 WRITE(2,761)
761 FORMAT(1X,'INPUT DN,DNO')
   READ ,DN,DNO
767 DU=(DNO/DN)**(K-1)-1
   M=SQRT(2*DU/(K-1))
   GO TO 766
780 WRITE(2,781)
781 FORMAT(1X,'INPUT DNO,V')
   READ ,DNO,V
   DN=MF/V
   GO TO 767
782 WRITE(2,783)
783 FORMAT(1X,'INPUT TO,DNO')
   READ ,TO,DNO
   PO=DNO*R*TO
   CALL ISEN7
   GO TO 2222
785 WRITE(2,786)
786 FORMAT(1X,'INPUT PO,DNO')
   READ ,PO,DNO

```

```

      TO=PO/R/DNO
      CALL ISEN7
      GO TO 2222
787  WRITE(2,788)
788  FORMAT(1X,'INPUT T,DNO')
      READ ,T,DNO
      M=MF*(1+C2)**FW2/(DNO*SQRT(K*R*T))
      DO 789 I=1,30
      MP=MF*(1+C2*M*M)**FW2/(DNO*SQRT(K*R*T))
      EP=ABS(1-MF/M)
      IF (EP.LT.0.00001) GO TO 790
      M=MP
789  CONTINUE

```

```

C -----
C PROGRAM FOR GIVEN THREE PROPERTIES WITHOUT MASS FLUX
C -----

```

```

9998 CONTINUE
      IF (N.EQ.123) GO TO 800
      IF (N.EQ.134) GO TO 802
      IF (N.EQ.135) GO TO 804
      IF (N.EQ.136) GO TO 806
      IF (N.EQ.137) GO TO 1111
      IF (N.EQ.138) GO TO 1111
      IF (N.EQ.246) GO TO 808
      IF (N.EQ.248) GO TO 810
      IF (N.EQ.247) GO TO 1111
      IF (N.EQ.234) GO TO 812
      IF (N.EQ.245) GO TO 814
      IF (N.EQ.456) GO TO 816
      IF (N.EQ.256) GO TO 818
      IF (N.EQ.356) GO TO 820
      IF (N.EQ.156) GO TO 822
      IF (N.EQ.568) GO TO 824
      IF (N.EQ.567) GO TO 1111
      IF (N.EQ.125) GO TO 826
      IF (N.EQ.126) GO TO 1111
      IF (N.EQ.128) GO TO 828
      IF (N.EQ.127) GO TO 832
      IF (N.EQ.124) GO TO 834
      IF (N.EQ.345) GO TO 1111
      IF (N.EQ.346) GO TO 836
      IF (N.EQ.348) GO TO 838
      IF (N.EQ.347) GO TO 842
      IF (N.EQ.146) GO TO 844
      IF (N.EQ.168) GO TO 846
      IF (N.EQ.167) GO TO 848
      IF (N.EQ.145) GO TO 850
      IF (N.EQ.148) GO TO 852
      IF (N.EQ.147) GO TO 855
      IF (N.EQ.236) GO TO 857

```



```
IF (N.EQ.235) GO TO 860
IF (N.EQ.238) GO TO 863
IF (N.EQ.237) GO TO 866
IF (N.EQ.158) GO TO 868
IF (N.EQ.157) GO TO 871
IF (N.EQ.258) GO TO 873
IF (N.EQ.257) GO TO 878
IF (N.EQ.268) GO TO 880
IF (N.EQ.267) GO TO 882
IF (N.EQ.358) GO TO 885
IF (N.EQ.357) GO TO 887
IF (N.EQ.368) GO TO 889
IF (N.EQ.237) GO TO 892
IF (N.EQ.458) GO TO 894
IF (N.EQ.457) GO TO 896
IF (N.EQ.468) GO TO 898
IF (N.EQ.467) GO TO 903
IF (N.EQ.178) GO TO 1111
IF (N.EQ.378) GO TO 1111
IF (N.EQ.478) GO TO 905
IF (N.EQ.278) GO TO 907
IF (N.EQ.578) GO TO 913
IF (N.EQ.678) GO TO 910
800 WRITE(2,801)
801 FORMAT(1X,'INPUT TO,PO,T')
   READ ,TO,PO,T
   M=SQRT(C1*(TO/T-1))
   P=PO/(1+C2*M*M)**PW3
   CALL ISEN1
   GO TO 2222
802 WRITE(2,803)
803 FORMAT(1X,'INPUT TO,T,P')
   READ ,TO,T,P
   M=SQRT(C1*(TO/T-1))
   PO=P*(1+C2*M*M)**PW3
   CALL ISEN1
   GO TO 2222
804 WRITE(2,805)
805 FORMAT(1X,'INPUT TO,T,DN')
   READ ,TO,T,DN
920 M=SQRT(C1*(TO/T-1))
   P=DN*R*T
   PO=P*(1+C2*M*M)**PW3
   CALL ISEN1
   GO TO 2222
806 WRITE(2,807)
807 FORMAT(1X,'INPUT TO,T,DNO')
   READ ,TO,T,DNO
   M=SQRT(C1*(TO/T-1))
   DN=DNO/(1+C2*M*M)**PW2
```

```

      P=DN*R*T
      CALL ISEN2
      GO TO 2222
808  WRITE(2,809)
809  FORMAT(1X,'INPUT PO,P,DNO')
      READ ,PO,P,DNO
      M=SQRT(C1*((PO/P)**PW4-1))
      DN=DNO/(1+C2*M*M)**PW2
      CALL ISEN5
      GO TO 2222
810  WRITE(2,811)
811  FORMAT(1X,'INPUT PO,P,V')
      READ ,PO,P,V
      M=SQRT(C1*((PO/P)**PW4-1))
      T=V*V/(M*M*K*R)
      TO=T*(1+C2*M*M)
      CALL ISEN1
      GO TO 2222
812  WRITE(2,813)
813  FORMAT(1X,'INPUT PO,T,P')
      READ ,PO,T,P
      M=SQRT(C1*((PO/P)**PW4-1))
      TO=T*(1+C2*M*M)
      CALL ISEN1
      GO TO 2222
814  WRITE(2,815)
815  FORMAT(1X,'INPUT PO,P,DN')
      READ ,PO,P,DN
      M=SQRT(C1*((PO/P)**PW4-1))
972  DNO=DN*(1+C2*M*M)**PW2
      CALL ISEN5
      GO TO 2222
816  WRITE(2,817)
817  FORMAT(1X,'INPUT P,DN,DNO')
      READ ,P,DN,DNO
      M=SQRT(C1*((DNO/DN)**PW1-1))
      PO=P*(1+C2*M*M)**PW3
      CALL ISEN5
      GO TO 2222
818  WRITE(2,819)
819  FORMAT(1X,'INPUT PO,DN,DNO')
      READ ,PO,DN,DNO
      M=SQRT(C1*((DNO/DN)**PW1-1))
      P=PO/(1+C2*M*M)**PW3
      CALL ISEN5
      GO TO 2222
820  WRITE(2,821)
821  FORMAT(1X,'INPUT T,DN,DNO')
      READ ,T,DN,DNO
      M=SQRT(C1*((DNO/DN)**PW1-1))

```

```

      TO=T*(1+C2*M*M)
      CALL ISEN2
      GO TO 2222
822  WRITE(2,823)
823  FORMAT(1X,'INPUT TO,DN,DNO')
      READ ,TO,DN,DNO
922  M=SQRT(C1*((DNO/DN)**PW1-1))
      T=TO/(1+C2*M*M)
      CALL ISEN2
      GO TO 2222
824  WRITE(2,825)
825  FORMAT(1X,'INPUT DN,DNO,V')
      READ ,DN,DNO,V
      M=SQRT(C1*((DNO/DN)**PW1-1))
      T=V*V/(M*M*K*K*R)
      TO=T*(1+C2*M*M)
      CALL ISEN2
      GO TO 2222
826  WRITE(2,827)
827  FORMAT(1X,'INPUT TO,PO,DN')
      READ ,TO,PO,DN
      DNO=PO/(R*TO)
      M=SQRT(C1*((DNO/DN)**PW1-1))
      CALL ISEN3
      GO TO 2222
828  WRITE(2,829)
829  FORMAT(1X,'INPUT TO,PO,V')
      READ ,TO,PO,V
      DNO=PO/R/TO
841  CALL ISENM1
831  DN=DNO/(1+C2*M*M)**PW2
      CALL ISEN3
      GO TO 2222
832  WRITE(2,833)
833  FORMAT(1X,'INPUT TO,PO,M')
      READ ,TO,PO,M
      CALL ISEN6
      GO TO 2222
834  WRITE(2,835)
835  FORMAT(1X,'INPUT TO,PO,P')
      READ ,TO,PO,P
2004 CONTINUE
      M=SQRT(C1*((PO/P)**PW4-1))
      T=TO/(1+C2*M*M)
      CALL ISEN1
      IF (J1,EQ,2) GO TO 2005
      IF (J1,EQ,3) GO TO 2042
      GO TO 2222
836  WRITE(2,837)
837  FORMAT(1X,'INPUT T,P,DNO')

```

```

      READ ,T,P,DNO
      DN=P/R/T
      M=SQRT(C1*((DNO/DN)**PW1-1))
      CALL ISEN4
      GO TO 2222
838  WRITE(2,839)
839  FORMAT(1X,'INPUT T,P,U')
      READ ,T,P,U
      DN=P/R/T
      M=U/SQRT(K*R*T)
840  DNO=DN*(1+C2*M*M)**PW2
      CALL ISEN4
      GO TO 2222
842  WRITE(2,843)
843  FORMAT(1X,'INPUT T,P,M')
      READ ,T,P,M
      DN=P/R/T
      GO TO 840
844  WRITE(2,845)
845  FORMAT(1X,'INPUT TO,P,DNO')
      READ ,TO,P,DNO
921  PO=DNO*R*TO
      M=SQRT(C1*((PO/P)**PW4-1))
      T=TO/(1+C2*M*M)
      CALL ISEN1
      GO TO 2222
846  WRITE(2,847)
847  FORMAT(1X,'INPUT TO,DNO,U')
      READ ,TO,DNO,U
      PO=DNO*R*TO
      GO TO 841
848  WRITE(2,849)
849  FORMAT(1X,'INPUT TO,DNO,M')
      READ ,TO,DNO,M
923  PO=DNO*R*TO
      DN=DNO/(1+C2*M*M)**PW2
      CALL ISEN3
      GO TO 2222
850  WRITE(2,851)
851  FORMAT(1X,'INPUT TO,P,DN')
      READ ,TO,P,DN
      T=P/(R*DN)
      M=SQRT(C1*((TO/T)-1))
      PO=P*(1+C2*M*M)**PW3
      CALL ISEN1
      GO TO 2222
852  WRITE(2,853)
853  FORMAT(1X,'INPUT TO,P,U')
      READ ,TO,P,U
      CALL ISENM1

```

```

854  PO=P*(1+C2*M*M)**PW3
      T=TO/(1+C2*M*M)
      CALL ISEN1
      GO TO 2222
855  WRITE(2,856)
856  FORMAT(1X,'INPUT TO,P,M')
      READ ,TO,P,M
2010 CONTINUE
      T=TO/(1+C2*M*M)
      GO TO 854
857  WRITE(2,858)
858  FORMAT(1X,'INPUT PO,T,DNO')
      READ ,PO,T,DNO
      TO=PO/R/DNO
      M=SQRT(C1*(TO/T-1))
859  P=PO/(1+C2*M*M)**PW3
      CALL ISEN1
      GO TO 2222
860  WRITE(2,861)
861  FORMAT(1X,'INPUT PO,T,IN')
      READ ,PO,T,IN
      P=R*IN*T
      M=SQRT(C1*((PO/P)**PW4-1))
862  TO=T*(1+C2*M*M)
      CALL ISEN1
      GO TO 2222
863  WRITE(2,864)
864  FORMAT(1X,'INPUT PO,T,V')
      READ ,PO,T,V
      M=V/SQRT(K*R*T)
865  TO=T*(1+C2*M*M)
      GO TO 859
866  WRITE(2,867)
867  FORMAT(1X,'INPUT PO,T,M')
      READ ,PO,T,M
      GO TO 865
868  WRITE(2,869)
869  FORMAT(1X,'INPUT TO,IN,V')
      READ ,TO,IN,V
      CALL ISENM1
      T=V*V/(K*R*M*M)
870  DNO=DN*(1+C2*M*M)**PW2
      CALL ISEN2
      GO TO 2222
871  WRITE(2,872)
872  FORMAT(1X,'INPUT TO,IN,M')
      READ ,TO,IN,M
      T=TO/(1+C2*M*M)
      GO TO 870
873  WRITE(2,874)

```

```

874  FORMAT(1X,'INPUT PO, DN, V')
      READ ,PO, DN, V
      M=V*(1+C2)**(0.5*PW3)*SQRT(DN/K/PO)
      DO 875 I=1,30
      MP=V*(1+C2**M**M)**(0.5*PW3)*SQRT(DN/K/PO)
      EP=ABS(1-MP/M)
      IF (EP.LT.0.00001) GO TO 876
      M=MP
875  CONTINUE
876  P=PO/(1+C2**M**M)**PW3
877  DNO=DN*(1+C2**M**M)**PW2
      CALL ISEN5
      GO TO 2222
878  WRITE(2,879)
879  FORMAT(1X,'INPUT PO, DN, M')
      READ ,PO, DN, M
      GO TO 876
880  WRITE(2,881)
881  FORMAT(1X,'INPUT PO, DNO, V')
      READ ,PO, DNO, V
      TO=PO/R/DNO
      CALL ISENM1
      P=PO/(1+C2**M**M)**PW3
      DN=DNO/(1+C2**M**M)**PW2
      CALL ISEN3
      GO TO 2222
882  WRITE(2,883)
883  FORMAT(1X,'INPUT PO, DNO, M')
      READ ,PO, DNO, M
      P=PO/(1+C2**M**M)**PW3
884  DN=DNO/(1+C2**M**M)**PW2
      CALL ISEN5
      GO TO 2222
885  WRITE(2,886)
886  FORMAT(1X,'INPUT T, DN, V')
      READ ,T, DN, V
      M=V/SQRT(K*R*T)
      TO=T*(1+C2**M**M)
      GO TO 870
887  WRITE(2,888)
888  FORMAT(1X,'INPUT T, DN, M')
      READ ,T, DN, M
      TO=T*(1+C2**M**M)
      GO TO 870
889  WRITE(2,890)
890  FORMAT(1X,'INPUT T, DNO, V')
      READ ,T, DNO, V
      M=V/SQRT(K*R*T)
891  TO=V*V*(1+C2**M**M)/(K*R**M**M)
      DN=DNO/(1+C2**M**M)**PW2

```

```

      CALL ISEN2
      GO TO 2222
892  WRITE(2,893)
893  FORMAT(1X,'INPUT PO,T,M')
      READ ,T,PO,M

      GO TO 891
894  WRITE(2,895)
895  FORMAT(1X,'INPUT P,DN,V')
      READ ,P,DN,V
      T=P/R/DN
      M=V/SQRT(K*R*T)
      DNO=DN*(1+C2*M*M)**PW2
      CALL ISEN4
      GO TO 2222
896  WRITE(2,897)
897  FORMAT(1X,'INPUT P,DN,M')
      READ ,P,DN,M
      PO=P*(1+C2*M*M)**PW3
      GO TO 972
898  WRITE(2,899)
899  FORMAT(1X,'INPUT P,DNO,V')
      READ ,P,DNO,V
      M=SQRT(DNO*V*V/(K*P*(1+C2)**PW2))
      DO 900 I=1,30
      MP=SQRT(DNO*V*V/(K*P*(1+C2*M*M)**PW2))
      EP=ABS(1-MP/M)
      IF (EP.EQ.0.00001) GO TO 901
      M=MP
900  CONTINUE
901  T=V*V/(M*M*R*K)
      TO=T*(1+C2*M*M)
902  PO=P*(1+C2*M*M)**PW3
      CALL ISEN1
      GO TO 2222
903  WRITE(2,904)
904  FORMAT(1X,'INPUT P,DNO,M')
      READ ,P,DNO,M
      PO=P*(1+C2*M*M)**PW3
      GO TO 884
905  WRITE(2,906)
906  FORMAT(1X,'INPUT P,M,V')
      READ ,P,M,V
      T=V*V/(M*M*R*K)
      TO=T*(1+C2*M*M)
      GO TO 854
907  WRITE(2,908)
908  FORMAT(1X,'INPUT PO,M,V')
      READ ,PO,M,V
      T=V*V/(M*M*R*K)
      TO=T*(1+C2*M*M)

```

```

P=PO/(1+C2*M*M)**PW3
CALL ISEN1
GO TO 2222
913 WRITE(2,909)
909 FORMAT(1X,'INPUT DN,M,U')
READ ,DN,M,U
T=U*U/(M*M*R*K)
TD=T*(1+C2*M*M)
GO TO 870
910 WRITE(2,911)
911 FORMAT(1X,'INPUT DNO,M,U')
READ ,DNO,M,U
T=U*U/(M*M*R*K)
GO TO 891
C -----
C PROGRAM FOR CONVERGING NOZZLE AND
C CONVERGING-DIVERGING NOZZLE
C -----
2001 KK=0
WRITE (2,2003)
2003 FORMAT(1X,'INPUT TD,PO,PB(BACK PRESSURE)')
READ ,TD,PO,PB
IF (PO.EQ.PB) WRITE (2,2002)
2002 FORMAT(//,4X,'### FLUID IS REMAIN STATIONARY ###',//)
2007 FORMAT(//,3X,'## UNCHOKED CONDITION (SUBSONIC FLOW) #
#')
2008 FORMAT(//,3X,'## CRITICAL CONDITION (SONIC FLOW) ##
#')
2009 FORMAT(//,3X,'## CHOKED CONDITION ##')
IF (PO.EQ.PB) GO TO 9997
IF (PO.LT.PB) GO TO 9996
IF (J1.EQ.3) GO TO 2020
M=1
CALL ISEN6
WRITE (2,2006)
2006 FORMAT(//,3X,'## PROPERTIES AT THE EXIT PLANE ##',//)
IF (PB.EQ.P) WRITE (2,2008)
IF (PB.LT.P) WRITE (2,2009)
IF (PB.LE.P) GO TO 2222
P=PB
GO TO 2004
2005 CONTINUE
IF ((M.GT.0.9995).AND.(M.LT.1.0005)) M=1
IF (M.NE.1) GO TO 2040
CALL ISEN6
WRITE (2,2008)
P=PB
GO TO 2222
2040 WRITE (2,2007)
GO TO 2222
2020 WRITE(2,2021)
2021 FORMAT(//,1X,'INPUT AREA RATIO BETWEEN EXIT & THROAT (AE
/AT)',//)

```



```

      READ ,AR10
      AR=AR10
C -----
C TO FIND MACH NUMBERS,ME2 & ME4 AT THE EXIT PLANE
C -----
      ME2=((C4+C3)**PW5)/AR
      DO 2030 I=1,30
      MP=((C4+ME2*ME2*C3)**PW5)/AR
      EP=ABS(1-MP/ME2)
      IF (EP.LT.0.00001) GO TO 2031
      ME2=MP
2030 CONTINUE
2031 ME4=SQRT((AR**(1/PW5)-C4)*(K+1)/(K-1))
      DO 2032 I=1,30
      MP=SQRT(((ME4*AR)**(1/PW5)-C4)*(K+1)/(K-1))
      EP=ABS(1-MP/ME4)
      IF (EP.LT.0.00001) GO TO 2033
      ME4=MP
2032 CONTINUE
2033 CONTINUE
      IF (ME2.GT.1) ME4=ME2
      IF (ME4.LT.1) ME2=ME4
      M=ME2
C -----
C TO DETERMINE PE2,PE3,PE4
C -----
      CALL ISEN6
      PE2=P
      ME3=ME4
      M=ME4
      CALL ISEN6
      PE4=P
      ME5=SQRT((ME3*ME3+C1)/(2*PW3*ME3*ME3-1))
      PE3=PE4*((2*K*ME3*ME3)/(K+1)-C3)
      IF ((PE3.GT.PB).AND.(PB.GT.PE4)) WRITE (2,2035)
      IF (PB.LT.PE4) WRITE (2,2035)
      IF (PB.GT.PE2) WRITE(2,2007)
      IF (PB.EQ.PE4) WRITE (2,2027)
      IF (PB.EQ.PE3) WRITE (2,2036)
      IF (PB.LT.PE4) WRITE (2,2045)
2045 FORMAT(8X,'##### UNDEREXPANDED #####',//)
      IF ((PE3.GT.PB).AND.(PB.GT.PE4)) WRITE (2,2048)
2035 FORMAT(//,3X,'## SHOCK WAVE OCCURED OUTSIDE THE NOZZLE
      ##',//)
2048 FORMAT(8X,'##### OVEREXPANDED #####',//)
2036 FORMAT(3X,'## SHOCK AT EXIT PLANE ##',//)
2023 FORMAT(1X,'## THE CRITICAL PROPERTIES AT THE THROAT ARE
      :',//)
2024 FORMAT (1X,'## THE PROPERTIES AT THE THROAT ARE : ##',
      //)
2027 FORMAT(//,3X,'## DESIGN CONDITION ##',//)
2037 FORMAT(//,3X,'## SHOCK INSIDE ##')
      IF (PB.LT.PE3) PB=PE4

```

```

P=PB
GO TO 2004
2042 CONTINUE
IF (PB,NE,PE3) GO TO 2047
WRITE (2,2036)
WRITE (2,2043)
2043 FORMAT(//,3X,'## PROPERTIES BEFORE THE NORMAL SHOCK WAV
E ##',//)
WRITE (2,7) MF,TO,PO,T,P,IN,DNO,M,V
M=SQRT((M*M+C1)/(K*C1*M*M-1))
PO=PO/((C3+C4/M/M)**K*(K*C4*M*M-C3))**PW2
P=P1*C4*K*M*M-C3
DUM=(K+1)*M*M/(2+(K-1)*M*M)
V=V/DUM
DN=DNB1*DUM
T=T*(K*C4*M*M-C3)*(C3+C4/M/M)
WRITE (2,2044)
2044 FORMAT(//,3X,'## PROPERTIES AFTER THE NORMAL SHOCK WAVE
##',//)
GO TO 2222
2047 IF ((PE2,GT,PB),AND,(PB,GT,PE3)) GO TO 2049
WRITE (2,7) MF,TO,PO,T,P,IN,DNO,M,V
IF (PB,LE,PE2) WRITE (2,2023)
IF (PB,LE,PE2) M=1
IF (PB,LE,PE2) CALL ISEN6
IF (PB,LE,PE2) GO TO 2222
WRITE (2,2024)
MF=MF*AR
CALL ISEN7
GO TO 2222
2049 WRITE (2,2037)
PF=PO
ARR=ARID
ARL=1.0
DO 2050 I=1,30
M=1
PO=PF
CALL ISEN6
AR=(ARR+ARL)/2
M=SQRT((AR**((1/PW5)-C4)*(K+1)/(K-1))
DO 2056 I1=1,30
MP=SQRT(((M*AR)**((1/PW5)-C4)*(K+1)/(K-1))
EP=ABS(1-MP/M)
IF (EP,LT,0.00001) GO TO 2057
M=MP
2056 CONTINUE
2057 CONTINUE
CALL ISEN6
MB1=M
DNOB1=DNO
TOB1=TO

```

```

MFB1=MF
PP=PO
P1=P
VB1=V
DNB1=DN
TB1=T
IF (M.LT.1) GO TO 2070
M=SQRT((M*M+C1)/(K*C1*M*M-1))
PO=PP/((C3+C4/MB1/MB1)**K*(K*C4*MB1*MB1-C3)**PW2
F=P1*C4*K*MB1*MB1-C3
DUM=(K+1)*MB1*MB1/(2+(K-1)*MB1*MB1)
V=VB1/DUM
DN=DNB1*DUM
T=TB1*(K*C4*MB1*MB1-C3)*(C3+C4/MB1/MB1)
PO2=PO
MB2=M
MFB2=MF
F2=P
VB2=V
DNB2=DN
TB2=T
MF=MF*AR/ARID
CALL ISEN7
EP=ABS(1-P/PB)
IF (EP.LT.0.001) GO TO 2051
2070 IF (P.GT.PB) ARL=AR
IF (P.LT.PB) ARR=AR
M=1
PO=PP
2050 CONTINUE
P=PB
2051 WRITE(2,2071) AR
2071 FORMAT(//,3X,'NORMAL SHOCK WAVE OCCURS AT AREA RATIO =',
F6.3,//)
WRITE (2,2043)
WRITE (2,7)MFB1,TO,PP,TB1,P1,DNB1,DNO,MB1,VB1
WRITE (2,2044)
WRITE (2,7) MFB2,TO,PO2,TB2,P2,DNB2,DNO,MB2,VB2
WRITE (2,2006)
P=PB
WRITE(2,7) MF,TO,PO,T,P,DN,DNO,M,V
WRITE (2,2023)
M=1
PO=PP
CALL ISEN6
2222 CONTINUE
IF (KK.EQ.1) WRITE(2,1200)
IF (KK.GE.2) WRITE(2,1201) KK
1200 FORMAT(//,3X,'THE RESULTS FOR INITIAL SECTION ## STATION
1 ##',//)
1201 FORMAT(//,3X,'RESULTS FOR OTHER SECTION. ##STATION',I2,'
##')

```

```

WRITE(2,7) MF,TD,PD,T,P,IN,DND,M,V
7  FORMAT(//,6X,'MASS FLUX =',F12.3,/,/,
$6X,'STAGNATION TEMPERATURE =',F12.3,/,/,
$6X,'STAGNATION PRESSURE=',F12.3,/,/,
$6X,'STATIC TEMPERATURE=',F12.3,/,/,
$6X,'STATIC PRESSURE=',F12.3,/,/,
$6X,'STATIC DENSITY=',F12.3,/,/,
$6X,'STAGNATION DENSITY=',F12.3,/,/,
$6X,'MACH NUMBER=',F12.3,/,/,
$6X,'LOCAL FLUID VELOCITY=',F12.3,/,/)
IF ((J1.EQ.2).OR.(J1.EQ.3)) GO TO 9997
GO TO 998
1111 WRITE(2,1112)
1112 FORMAT(//,2X,'NOT ENOUGH INFORMATION TO GET ALL THE PROPE
RTIES')
GO TO 9997
998 WRITE(2,999)
999 FORMAT(//,2X,'CONTINUE ? YES-- ENTER 1 ; NO-- ENTER 0
')
READ ,J
IF (J.EQ.0) GO TO 9997
C -----
C GIVEN ONE PROPERTY FROM ANY CROSS-SECTION TO
C COMPUTE THE OTHER UNKNOWN PROPERTIES
C -----
WRITE (2,1919)
1919 FORMAT(//,2X,'FIND PROPERTIES AT OTHER SECTION',/,/)
KK=KK+1
WRITE(2,51)
51 FORMAT(1X,'THE GIVEN PROPERTY IS',/,/,
$6X,'1 - T2',6X,'4 - M2',/,/
$6X,'2 - P2',6X,'5 - V2',/,/
$6X,'3 - DN2',5X,'6 - AR(A1/A2)',/,/)
READ ,J2
IF (J2.EQ.1) GO TO 1203
IF (J2.EQ.2) GO TO 1205
IF (J2.EQ.3) GO TO 1207
IF (J2.EQ.4) GO TO 1209
IF (J2.EQ.5) GO TO 1225
IF (J2.EQ.6) GO TO 1212
1203 WRITE (2,53)
53 FORMAT(1X,'INPUT T2')
READ ,T
GO TO 920
1205 WRITE (2,1206)
1206 FORMAT(1X,'INPUT P2')
READ ,P
GO TO 921
1207 WRITE (2,57)
57 FORMAT(1X,'INPUT DN2')
READ ,DN
GO TO 922

```

```

1209 WRITE (2,1210)
1210 FORMAT(1X,'INPUT M2')
      READ ,M
      GO TO 923
1225 WRITE(2,61)
61    FORMAT(1X,'INPUT V2')
      READ ,V
      GO TO 841
1212 WRITE(2,1213)
1213 FORMAT(1X,'INPUT AR(A1/A2)')
      READ ,AR
      MF=MF*AR
      CALL ISEN7
      GO TO 2222
9996 WRITE (2,9995)
9995 FORMAT (//,3X,'WRONG INFORMATION, CHECK THE INPUT DATA A
      GAIN.')
```

```

9997 CONTINUE
      STOP
      END
```

```

C-----
C   SUBROUTINE PROGRAMS
C-----
```

```

SUBROUTINE ISENT1
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 MF, M, K, MP
V=M*SQRT(K*R*T)
DN=MF/V
DNO=DN*(1+C2*M*M)**PW2
RETURN
END
```

```

SUBROUTINE ISENT2
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 MF, M, K, MP
P=DN*R*T
PO=P*(1+C2*M*M)**PW3
DNO=DN*(1+C2*M*M)**PW2
RETURN
END
```

```

SUBROUTINE ISENM1
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 MF, M, K, MP
M=V*SQRT((1+C2)/(K*R*TO))
DO 830 I=1,30
MP=V*SQRT((1+C2*M*M)/(K*R*TO))
EP=ABS(1-MP/M)
IF (EP.LT.0.00001) GO TO 912
M=MP
```

```
830 CONTINUE
```

```
912 CONTINUE
```

```
RETURN
```

```

END
SUBROUTINE ISEN1
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 M, K, MF
DN=P/(R*T)
DNO=PO/(R*TO)
V=M*SQRT(K*R*T)
MF=DN*V
RETURN
END
SUBROUTINE ISEN2
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 M, K, MF
P=DN*R*T
PO=DNO*R*TO
V=M*SQRT(K*R*T)
MF=DN*V
RETURN
END
SUBROUTINE ISEN3
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 M, K, MF
T=TO/(1+C2*M*M)
P=DN*R*T
V=M*SQRT(K*R*T)
MF=DN*V
RETURN
END
SUBROUTINE ISEN4
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 M, K, MF
TO=T*(1+C2*M*M)
PO=DNO*R*TO
V=M*SQRT(K*R*T)
MF=DN*V
RETURN
END
SUBROUTINE ISEN5
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 M, K, MF
T=P/(R*DN)
TO=PO/(R*DNO)
V=M*SQRT(K*R*T)
MF=DN*V
RETURN
END
SUBROUTINE ISEN6
COMMON V, DN, DNO, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 MF, M, K, MF
DNO=PO/R/TO

```

```

DN=DND/(1+C2*M*M)**PW2
CALL ISEN3
RETURN
END
SUBROUTINE ISEN7
COMMON V, DN, DND, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 MF, M, K, MF
DUMMY=PO*SQRT(K/R/TO)
M=MF*(1+C2)**PW5/DUMMY
DO 111 I=1,30
MP=MF*(1+C2*M*M)**PW5/DUMMY
EP=ABS(1-MP/M)
IF(EP.LT.0.00001) GO TO 1000
M=MP
111 CONTINUE
1000 T=TO/(1+C2*M*M)
CALL ISENT1
P=PO/(1+C2*M*M)**PW3
RETURN
END
SUBROUTINE ISEN8
COMMON V, DN, DND, K, R, M, MF, TO, T, P, PO, C2, PW2, PW3, PW5
REAL*4 MF, M, K, MF
T=TO/(1+C2*M*M)
63 V=M*SQRT(K*R*T)
DN=MF/V
CALL ISENT2
RETURN
END

```

APPENDIX B

Sample of Example

Problem : Air flows through a frictionless adiabatic converging-diverging nozzle. The air stagnation temperature and pressure are 500K and $7.0 \times 10^5 \text{ N/M}^2$ respectively. The diverging portion of the nozzle has an area ratio between the exit plane and the throat (A_e / A_t) of 11.91. The back pressure at the exit plane is controlled to be at $2.2623 \times 10^5 \text{ N/M}^2$. Analyse the flow characteristic and calculate all the properties of the flow. Assume $K = 1.40$ and $R = 287.04 \text{ J/KG-K}$.

Results obtained from computer: (All in SI UNIT)

FASTFOR (CONVERSATIONAL, VER 9)

TO DEFINE THE SYMBOLS USED :

MF = MASS FLUX
 R = SPEC.HEAT CONST.
 K = SPEC.GAS RATIO.
 M = MACH NO.
 T = STATIC TEMPERATURE
 P = STATIC PRESSURE
 DN = STATIC DENSITY
 V = LOCAL FLUID VELOCITY
 TO = STAGNATION TEMPERATURE
 PO = STAGNATION PRESSURE
 DNO= STAGNATION DENSITY

SELECT PROGRAM :

- 1 - GIVEN THREE PROPERTIES OF A FLOW IN A DUCT,
FIND THE OTHER PROPERTIES OF THE FLOW
- 2 - CONVERGING NOZZLE FLOW FROM RESERVOIR
- 3 - CONVERGING-DIVERGING NOZZLE

*3

ENTER K,R

*1.4,287.04

SPECIFIC HEAT RATIO= 1.4000

SPECIFIC GAS CONSTANT= 287.0400

INPUT T0,P0,PB(BACK PRESSURE)

*500,700000,226230

INPUT AREA RATIO BETWEEN EXIT & THROAT (AE/AT)

*11.91

SHOCK INSIDE

NORMAL SHOCK WAVE OCCURS AT AREA RATIO = 4.239

PROPERTIES BEFORE THE NORMAL SHOCK WAVE

MASS FLUX = 298.482

STAGNATION TEMPERATURE = 500.000

STAGNATION PRESSURE= 700000.000

STATIC TEMPERATURE= 178.491

STATIC PRESSURE= 19026.570

STATIC DENSITY= 0.371

STAGNATION DENSITY= 1.600

MACH NUMBER= 3.001
LOCAL FLUID VELOCITY= 803.741

PROPERTIES AFTER THE NORMAL SHOCK WAVE

MASS FLUX = 298.482
STAGNATION TEMPERATURE = 500.000
STAGNATION PRESSURE= 229633.900
STATIC TEMPERATURE= 478.400
STATIC PRESSURE= 199918.500
STATIC DENSITY= 1.433
STAGNATION DENSITY= 1.600
MACH NUMBER= 0.475
LOCAL FLUID VELOCITY= 208.325

PROPERTIES AT THE EXIT PLANE

MASS FLUX = 106.233
STAGNATION TEMPERATURE = 500.000
STAGNATION PRESSURE= 229633.900
STATIC TEMPERATURE= 497.756
STATIC PRESSURE= 226230.000
STATIC DENSITY= 1.582
STAGNATION DENSITY= 1.600
MACH NUMBER= 0.150
LOCAL FLUID VELOCITY= 67.146

THE CRITICAL PROPERTIES AT THE THROAT ARE :

MASS FLUX = 1265.208
STAGNATION TEMPERATURE = 500.000
STAGNATION PRESSURE= 700000.000
STATIC TEMPERATURE= 416.667
STATIC PRESSURE= 369797.000
STATIC DENSITY= 3.092
STAGNATION DENSITY= 4.877
MACH NUMBER= 1.000
LOCAL FLUID VELOCITY= 409.194

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