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## The Anisotropic and isotropic strength behavior of an illitic clay

Miles M. O'Brien  
*New Jersey Institute of Technology*

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THE ANISOTROPIC AND ISOTROPIC STRENGTH  
BEHAVIOR OF AN ILLITIC CLAY

BY

MILES M. O'BRIEN, JR., CAPTAIN USAF

A THESIS

PRESENTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE

OF

MASTER OF SCIENCE IN CIVIL ENGINEERING

AT

NEW JERSEY INSTITUTE OF TECHNOLOGY

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Newark, New Jersey

October, 1977

APPROVAL OF THESIS  
THE ANISOTROPIC AND ISOTROPIC STRENGTH  
BEHAVIOR OF AN ILLITIC CLAY  
BY  
MILES M. O'BRIEN, JR. CAPTAIN USAF

for  
DEPARTMENT OF CIVIL ENGINEERING  
NEW JERSEY INSTITUTE OF TECHNOLOGY  
BY  
FACULTY COMMITTEE

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## ABSTRACT

The cause and the nature of anisotropy in a cohesive soil was investigated by direct shear and triaxial compression tests. Laboratory techniques and procedures used to prepare isotropic clay samples are given where consolidation was accomplished under hydrostatic pressure. Direct shear tests were performed on these specimens trimmed at different inclinations to the physical horizontal from the block samples. The ratio of undrained shear strengths in any direction to shear strength in the vertical direction was found to be equal to one, proving isotropy existed. Similar tests were performed on specimens trimmed from the same clay consolidated one-dimensionally. Results from these tests showed the shear strength ratio to be maximum for specimens trimmed at  $90^{\circ}$  to the horizontal plane. In this case, the anisotropic characteristics were directly attributed to the sample stress history. These samples indicated preferred particle orientation. Undrained triaxial compression tests were performed on both hydrostatic and one-dimensionally consolidated samples. Triaxial test results confirmed the results of the direct shear tests and more accurately defined the stress/strength parameters. The angle between the failure plane and the test specimen's axis was essentially constant and the ratio between pore water pressure at failure and the mean consolidation stress remained a constant.

For each maximum consolidation stress, pore pressure was isotropic, but in all cases was higher for hydrostatically prepared samples. The hydrostatic method of preparing isotropic test specimens was effective and produced reliable results.

TABLE OF CONTENTS

Acknowledgement	i
List of Notations	iii
List of Figures	iv
List of Tables	iv
Chapter I Introduction	1
Chapter II Laboratory Procedures	10
Chapter III Direct Shear Test Procedures	22
Chapter IV Undrained Triaxial Compression Test Procedures and Results	31
Chapter V Summary and Conclusions	43
Bibliography	45



NOTATIONS

- $C_0$  Shear strength of a specimen oriented parallel to the horizontal plane.
- $C_\beta$  Shear strength of a specimen oriented at angle  $\beta$  to the horizontal plane.
- $C_{90}$  Shear strength of a specimen inclined at  $90^\circ$  to the horizontal plane.
- $K$  Ratio between the lateral effective stress and the vertical effective stress.
- $\alpha$  Angle between the plane of isotropy and the failure plane.
- $\alpha'$  Average of measured angles of inclination of the failure planes.
- $\alpha'_0$  Maximum measured angle of inclination of the failure plane.
- $\beta$  Angle between the horizontal plane and the specimen axis.
- $\epsilon_f$  Strain at failure.
- $u$  Pore water pressure.
- $u_f$  Pore pressure at failure.
- $\bar{\sigma}_{1c}$  Maximum effective consolidation stress.
- $\bar{\sigma}_1$  Major effective principal stress.
- $\bar{\sigma}_3$  Minor effective principal stress.
- $\bar{\sigma}_{mc}$  Mean effective consolidation stress =  $\bar{\sigma}_{1c}(1+2K)/2$
- $\tau_{CR}$  Maximum shear stress obtained from triaxial tests.

LIST OF FIGURES

		Page
Figure I	Specimen orientation relative to the plane of isotropy	3
Figure II	Grain size distribution of grundite clay	11
Figure III	One-dimensional consolidation chamber	13
Figure IV	Orientation of trimmed samples relative to the plane of isotropy	15
Figure V	Three-dimensional on hydrostatic consolidation chamber	17
Figure VI	Consolidation device	19
Figure VII	Orientation of trimmed isotropic test specimens	21
Figure VIIIa	Relative strength ratio from direct shear test	26
Figure VIIIb	Relative strength ratio from direct shear test	27
Figure IX	Comparison of pore pressure between H and F series test	37
Figure X	Comparison of pore pressure between H and F series test	38
Figure XI	Ratio of pore pressure at failure and the effective mean consolidation stress	41

LIST OF TABLES

Table I	Comparative strength data from direct shear tests	24
Table II	Summary of triaxial compression tests	35

## Chapter I

### INTRODUCTION

During the last decade there has been an increased interest in the investigation and analyses related to the directional variation of shear behavior of fine-grained soils. A study of the published works in soil mechanics indicates that many progressive soil engineers have begun to adapt their methods of analysis to take into account strength anisotropy.

Of the many types of material anisotropy described by Lekhnitskii<sup>1</sup>, the one most likely to be found in soils is cross anisotropy. A cross-anisotropic material contains an axis of rotational symmetry such that all orthogonal lines emanating from this axis are equivalent, that is, the material possesses a plane of isotropy normal to the axis of rotational symmetry. In a deposited soil which has not experienced any stress other than that from the overlying material, physical vertical and horizontal directions coincide, respectively, with the axis of rotational symmetry and the plane of material isotropy. To determine the directional variation of the shear strength of

1. S.G.Lekhnitskii. Theory of Elasticity of an Anisotropic Elastic Body. California, Holden-Day, Inc., 1963. p. 584.

soils, tests are generally performed on specimens trimmed in different directions as shown in Figure I.

Hvorslev <sup>1</sup> reported the variation in strength with direction for Vienna clay and Little Belt clay. Specimens for unconfined compression tests were trimmed such that  $\beta$ , the angle between the plane of material isotropy and the specimen axis, was 0, 45, and 90 degrees. Vienna clay exhibited maximum strength for vertical ( $\beta = 90$  degrees) specimens and minimum strength for horizontal specimens ( $\beta = 0$  degrees). Little Belt clay had the highest strength for  $\beta$  equal to 0 degrees and the lowest strength for  $\beta$  equal to 90 degrees. Lo<sup>2</sup> determined the anisotropy of shear strength in over-consolidated, undisturbed Welland clay. A large number of unconfined compression, and some triaxial and direct shear, tests indicated the maximum shear strength to exist in vertical specimens. Duncan and Seed <sup>3</sup> evaluated the strength anisotropy of San Francisco

1. M. J. Hvorslev. "Physical Components of the Shear Strength of Saturated Clays", Proceedings of the Research Conference of Shear Strength of Cohesive Soils. Colorado, American Society of Civil Engineering, 1960. pp. 169-274.

2. K. Y. Lo. "Stability of Slopes in Anisotropic Soils", Journal of the Soil Mechanics and Foundations Division, Proceedings. American Society of Civil Engineers, Vol 91:SM4. paper 4405, July 1965, p .85.

3. J. M. Duncan and H. B. Seed. "The Effect of Anisotropy and Reorientation of Principal Stress on the Shear Strength of Saturated Clay", Report ET-65-3, U.S. Army Engineers, Waterways Experiment Station, Vicksburg, Miss. 1965.

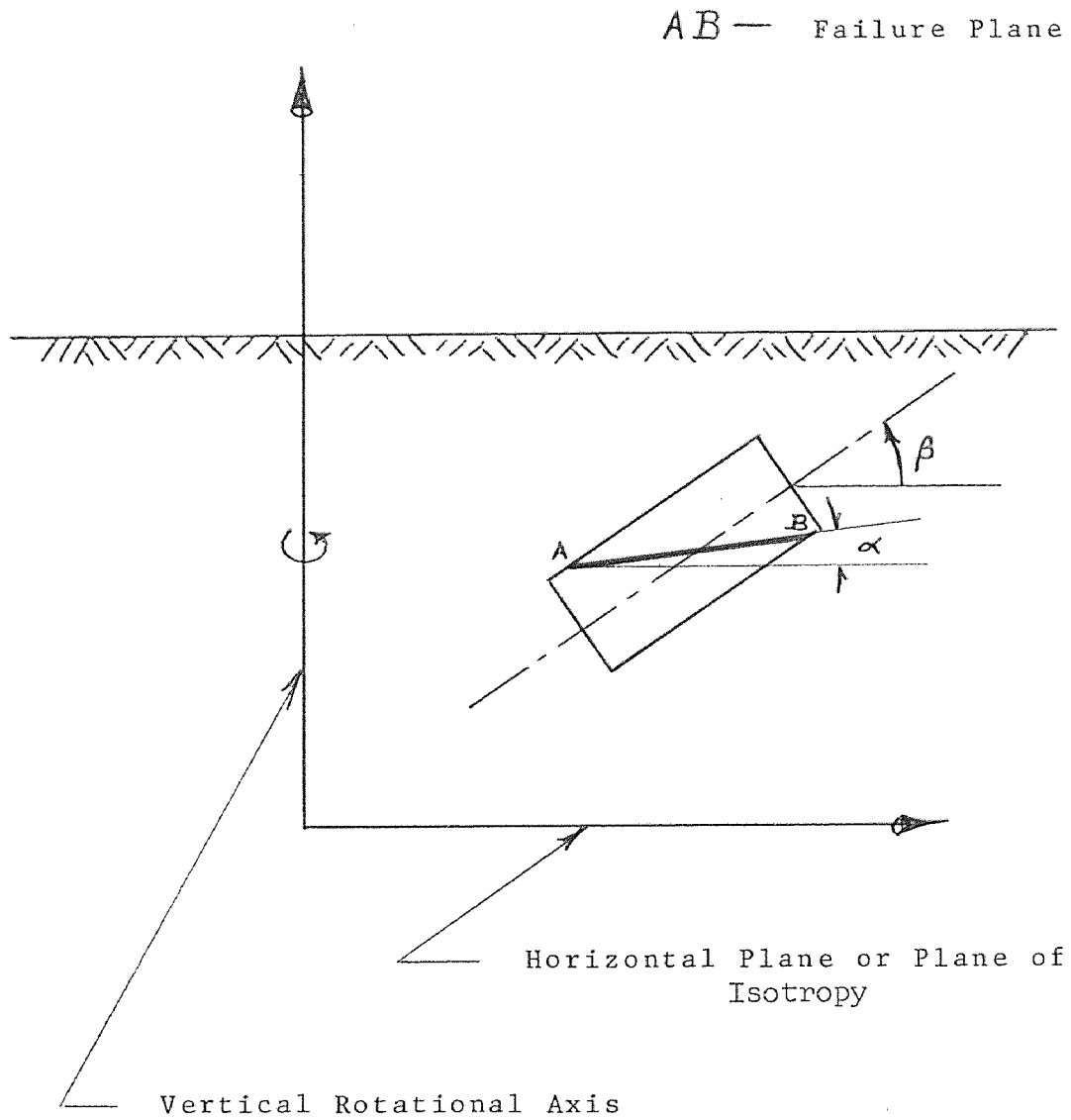


Figure I  
Specimen Orientation Relative to the Plane of Isotropy

Bay mud from anisotropically consolidated, undrained, plane-strain tests and found the maximum strength for  $\alpha$  equal to 60 degrees and the minimum strength for  $\alpha$  equal to 0 degrees,  $\alpha$  being the inclination of the failure plane of material isotropy. The variation in undrained shear strength was primarily attributed to two factors: anisotropy of pore-pressure parameter at failure and reorientation of principal stresses.

Ladd and Lambe <sup>1</sup> and Seed and Noorany <sup>2</sup> noted that insitu anisotropic stresses are released during sampling and as the sample is extracted from the tube, it experiences a negative pore-water pressure, and the effective stress in the sample becomes isotropic. Because of the lack of stress anisotropy, Lo <sup>3</sup> concluded that in conventional triaxial tests, no reorientation of principal stresses is possible and, therefore, the measured direction strength variation is due to inherent

1.C.C.Ladd and T.W.Lambe. "The Strength of Undisturbed Clay Determined from Undrained Tests", Laboratory Shear Testing of Soils. Philadelphia, STP 361, American Society for Testing and Materials, 1964. pp.342-371.

2.H.S.Seed and L. Noorany. "In-Situ Strength Characteristics of Soft Clays", Journal of the Soil Mechanics and Foundation Division, Proceedings. American Society of Civil Engineers, March 1965, Vol 91:SM2, Paper 4274, Pp.48-80.

3.K.Y.Lo. Closure to "Stability of Slopes in Anisotropic Soils", Journal of the Soil Mechanics and Foundation Division, Proceedings, American Society of Civil Engineers, Vol 92:SM4, Paper 4405, July 1966, pp.72-82.

anisotropy. Loh and Holt <sup>1</sup> reported that, for Winnipeg Upper Brown clay, both shear strength and secant modulus had maximum values associated with horizontal specimens. In an account of in-situ vane shear tests, Aas <sup>2</sup> reported the ratio of shear strength in the horizontal and the vertical planes to be as high as two, which was confirmed by DiBiagio and Aas <sup>3</sup> through large-scale, in-situ, direct shear tests.

Many others have hypothesized that plate-like clay particles respond to shear strain by aligning themselves parallel to the direction of maximum shear strain. Studies of clay microstructure by Martin<sup>4</sup> and others indicate that, in one-dimensional consolidation under high pressures, clay particles acquire almost ideal orientation normal to the direction of the major principal stress, and the tendency toward parallel orientation is also observable

1. A.K. Loh and R.T. Holt. "Directional Variation in Undrained Shear Strength and Fabric of Winnipeg Upper Brown Clay", Canadian Geotechnical Journal. Vol 11:3, 1974, pp.430-437.

2. G. Aas. "Vane Tests for Investigation of Anisotropy of Undrained Shear Strength of Clays", Proceedings, Geotechnical Conference, Norwegian Geotechnical Institute, Oslo, Vol 1, 1967, pp.3-8.

3. E. DiBiagio and G. Aas. "The In-Situ Undrained Shear Strength Measured on a Horizontal Failure Plane by Large Scale Direct Shear Tests in Quick Clays", Proceedings, Geotechnical Conference, Norwegian Geotechnical Institute, Oslo, Vol 1, 1967, pp.19-26.

4. T.R. Martin. "Research on the Physical Properties of Marine Soils August 1961-July 1962", Research Report R62-42, Soil Engineering Division Pub. No. 127, 1962, Massachusetts Inst. of Technology, Cambridge, 1962.

even at very low values of major principal consolidation stress.

Interpretations of soil structure have also been made by indirect means. Seed et al <sup>1</sup> observed different strength behavior for specimens formed by kneading compaction and those formed by static compaction. It was suggested that the clay particles had a high degree of parallelism in kneaded specimens and a random orientation in specimens formed by static compaction.

It is evident from the preceding paragraphs that anisotropy of shear strength is of common occurrence in fine-grained soils. Maximum shear strength, which has been determined by various types of strength tests, may be found to occur in a horizontal plane, a vertical plane, or in some intermediate plane. Also, through direct observation and indirect interpretations it has been found that most clays, natural and remolded, have oriented structure even if they experienced very small one-dimensional consolidation stresses. In general, the observed anisotropy has been attributed to one or more of the following: (1) soil fabric, (2) reorientation of principal stresses or stress anisotropy, (3) directional variation of pore pressure, and (4) directional variation of effective stress

1. H.B. Seed et al. "Strength of Compacted Cohesive Soils", Proceedings of the Research Conference on Shear Strength of Cohesive Soils. American Society of Civil Engineers, Colorado, 1960, pp.877-964.



strength parameters. Items listed under (3) and (4) have been also referred to as inherent or intrinsic anisotropy.

It is well documented by Ladd and Lambe, and Seed and Nourany that the anisotropic stresses existing in the field are no longer operative on the specimens prior to shearing in triaxial apparatus, and therefore, according to Lo<sup>1</sup>, the measured directional variations in strength are due to intrinsic anisotropy and are not a consequence of stress anisotropy. It may then be asked whether intrinsic anisotropy would still exist if the soil did not have a history of anisotropic stresses.

Duncan and Seed<sup>2</sup> stated that most clays are anisotropic to some degree and the measured directional variation in strength is the combined effect of inherent anisotropy and reorientation of principal stresses. To understand what roles each one of these factors plays in the overall strength anisotropy, one would need to uncouple their effects. If the soil samples are prepared under truly hydrostatic stress conditions,

1. Lo. Op.Cit. p.4.

2. J.M. Duncan and H.B. Seed. "Strength Variation Along Failure Surfaces in Clays", Journal of the Soil Mechanics and Foundation Division, Proceedings, American Society of Civil Engineers, Vol 92:SM6, Nov 1966, pp. 81-104.

irrespective of the orientation of trimmed specimens, at no stage of testing would there be any reorientation of principal stresses, and any measurable directional variation in shear strength would indicate the effect of inherent anisotropy alone.

To obtain a better understanding of the causes and nature of strength anisotropy, the following experimental investigations were undertaken.

INVESTIGATIONS

## Chapter II

### LABORATORY PROCEDURES

Soil used in this study was an illite clay known by the trade name Grundite. It was mined in Grundy County, Illinois and purchased commercially from Illinois Clay Products Company. Initially, the soil had to be purified to a 95% pure clay. This procedure was accomplished by hydrometer tests on the initial soil composition. Figure II shows the grain size distribution by particle diameter in millimeters in accordance with U.S. Bureau of Soils Classification. Results found in Figure II show an initial composition of 3.0% fine sand, 40.5% silt, and 56.5% clay. Hydrometer tests on the illite clay proved a settlement time of 85 minutes was adequate to obtain pure clay suspension with the fine sand and silt settled out of the suspension. This 85 minute settlement was used as a standard throughout sample preparation. The clay suspension, with approximately 500% water content, was then transferred to a five gallon container, where it was allowed to stand. After continued settlement, excess distilled water was siphoned off. Vacuum was applied to a clay slurry to remove trapped air and establish a completely saturated soil-water slurry. This process was repeated until an adequate quantity of saturated clay slurry was formed for consolidation. Consolidation was accomplished both one-dimen-

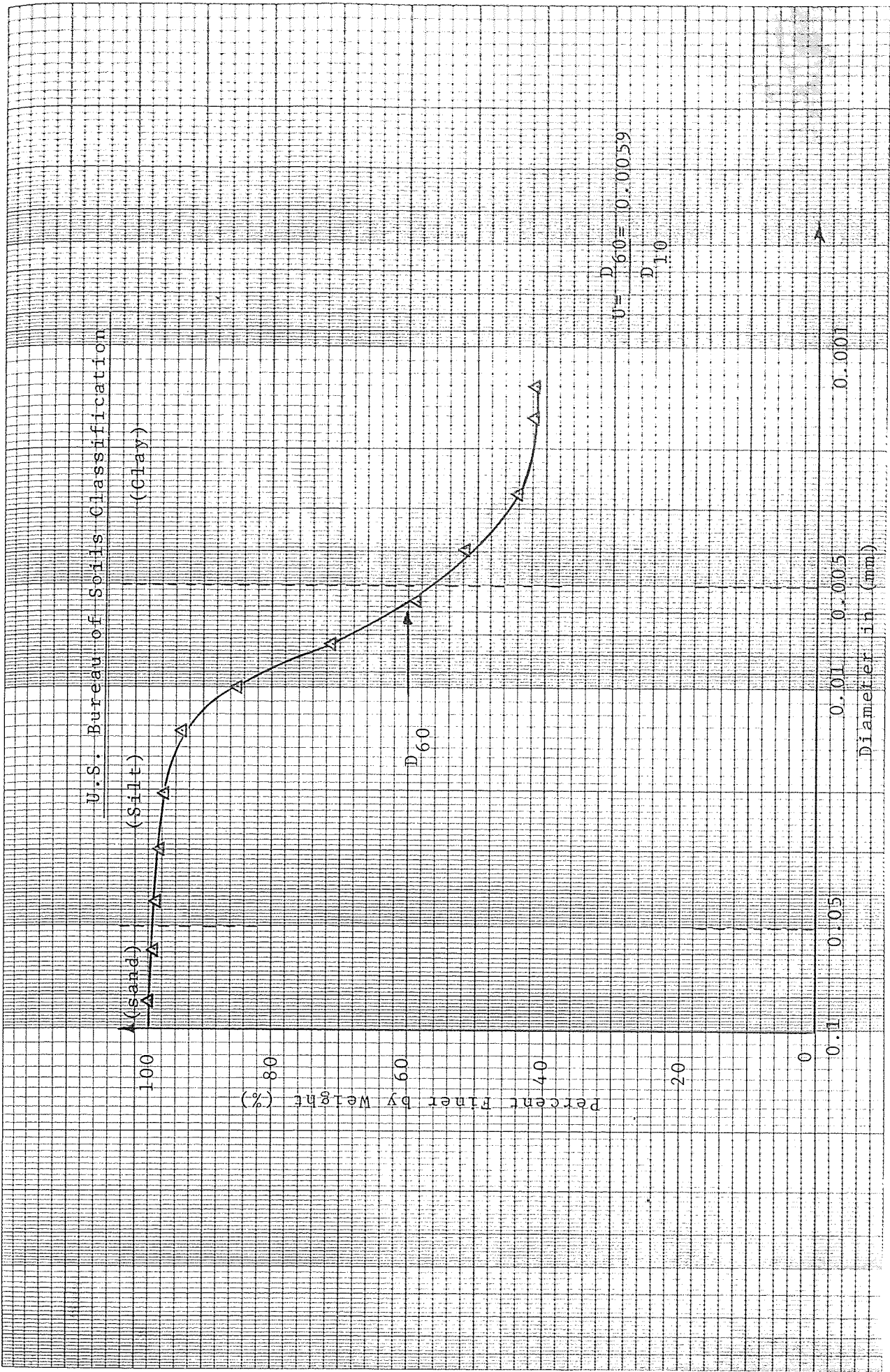


Figure II - Grain Size Distribution of Grundite

sionally and three-dimensionally to obtain block specimens with specific stress histories. The clay soil used in sample preparation had a liquid limit of 65% and a plastic limit of 32%.

One-dimensional consolidation chambers were manufactured using  $\frac{1}{2}$  inch lucite cylinders with 6 inch inside diameter, Figure III. Two inch head and base plates were used with a teflon piston and shaft. A hydrostatic pressure head was then applied for specific stress histories of 0.5, 1.0, 2.0, and 4.0 kg/cm<sup>2</sup> preloads. Consolidation of each sample was continued until measured pore pressure stabilized and no further vertical deformation was noted. At this point, the samples were left under the specified preload for a minimum of 24 hours to insure a uniform stress history throughout the prepared sample. Water content proved to be uniform, varying  $\pm 0.5\%$  from top to bottom of a 4 inch prototype. Upon completion of each one-dimensional consolidated sample, the sample was wrapped in plastic, sealed and stored in a 100% humidity locker.

Preparation procedures were followed carefully. Three block samples of each stress history were formed. This quantity proved adequate in supplying trimmed specimens for both triaxial and direct shear tests. Tests performed on specimens trimmed at

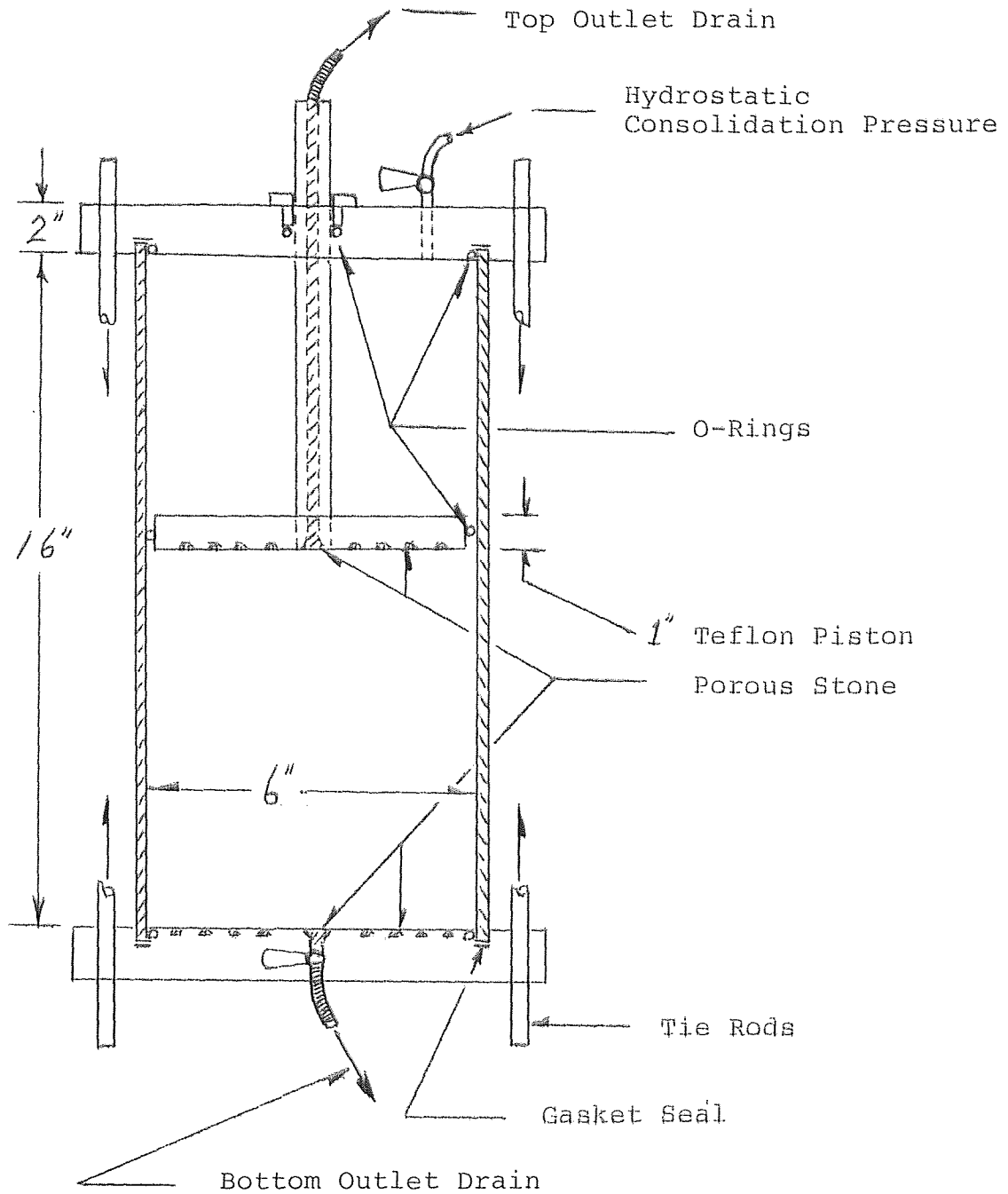


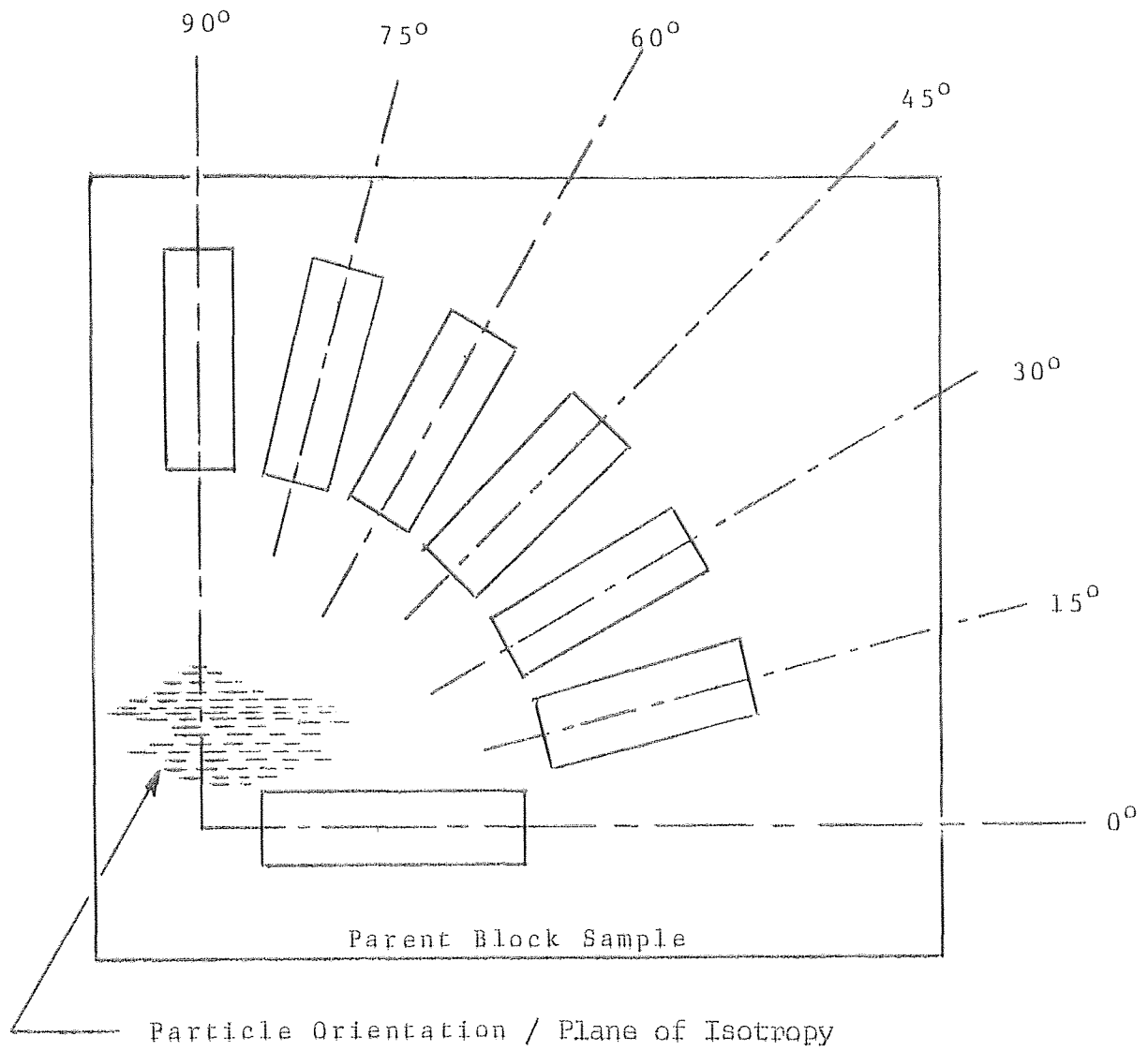
Figure III - One Dimensional Consolidation Chamber

specific angles from the horizontal of each sample. Standard angle planes of  $0^\circ$  (horizontal),  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ , and  $90^\circ$  were used on the one-dimensional samples (Figure IV). Chapter III will cover the results of the direct shear tests.

Triaxial test specimens were trimmed at similar angles. To conserve time and sample quantity, angle planes of  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$  were used for samples with stress histories of 0.5 and 2.0 kg/cm<sup>2</sup>. One-dimensional consolidated 4 kg/cm<sup>2</sup> triaxial specimens were trimmed at  $0^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ , and  $90^\circ$ , in order to obtain an accurate comparison with the direct shear tests. A complete discussion of triaxial test results are contained in Chapter IV.

Preparation of triaxial and direct shear specimens possessing isotropy presented a most intricate laboratory problem. The bulk sample had to be consolidated to a specific stress history with  $\bar{\sigma}_1 = \bar{\sigma}_2 = \bar{\sigma}_3$ , thus allowing consolidation to proceed equally along the X, Y, Z axis. To accomplish an equal, three-dimensional consolidation, the sample would have to be prepared in the shape of a sphere with a uniform hydrostatic pressure head applied over the entire surface of the sphere. Initial characteristics required for such consolidation would be: (a) a spherical porous core at the center of the sample to allow uniform drainage; (b) a rubber spherical membrane to enclose the sample which would contract evenly around





the sample as consolidation took place; (c) a water-tight pressure chamber capable of sustaining pressure to a maximum of  $6 \text{ kg/cm}^2$ , with an inlet valve for the pressure head and an outlet valve for pore water; (d) a means of suspending the sample inside the chamber to prevent anisotropy at the bottom of the sphere.

A search of local industries in Newark produced a satisfactory pressure chamber capable of holding a  $6 \text{ kg/cm}^2$  pressure head with minimum modification. It consisted of a stainless steel tank with a 16 inch diameter, 20 inches high. The tank had  $1/8$  inch wall thickness with horizontal reinforcement bonds around the circumference. The top of the tank was held in place with eight bolts. Shown in Figure V, the final consolidation chamber was modified by adding a  $1/4$  inch hard rubber gasket to seal the top, an inlet valve, and an outlet valve. A hydraulic jack and steel frame assembly were added to apply positive pressure to the top and bottom of the tank in order to prevent bulging. A stainless steel cradle was manufactured to hold a nylon net which suspended the sample near the center of the tank, thus preventing initial deformation of the sample due to gravity (Figure V). This would have occurred if the sample was allowed to lay on the tank bottom.

The actual consolidation device needed to allow pore water drainage was designed and manufactured at the college. It con-

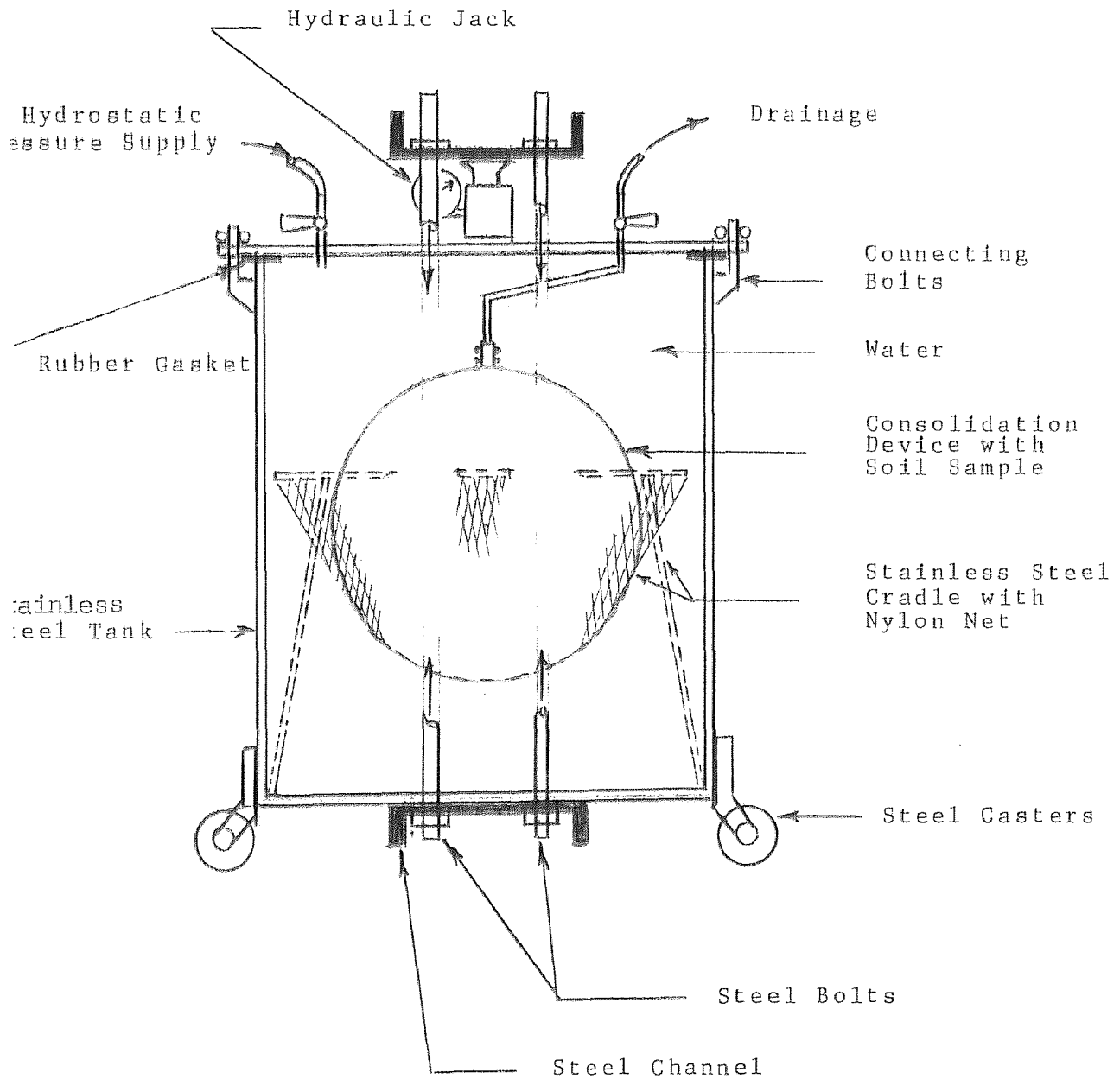


Figure V - Three-Dimensional Hydrostatic Consolidation Chamber

sisted of a 8.5 inch long stainless steel tube,  $\frac{1}{4}$  inch in diameter. A porous ball was formed at one end of the rod by using a fine sand-epoxy mixture shaped inside a ping pong ball, with a tube fitting located at the center of the ping pong ball. After 48 hours of drying time for the epoxy, the ping pong ball was cut away (Figure VI). A brass slide collar was designed to seal the neck of the rubber membrane and still allow the membrane to slide down the drainage tube as the sample consolidated. Finally, a flairless tube fitting was attached to the top of the drainage tube and then to the drainage fitting on the tank lid.

The procedures for the actual sample preparation were as follows: A soil slurry was prepared in the same manner as for the one-dimensional sample. Once the air was removed from the slurry and all excess distilled water was removed, the rubber balloon-shaped membrane was attached to the end of a 3 inch diameter lucite injector tube. The balloon was then de-aired under a vacuum and the tube filled with saturated clay slurry. A vacuum was reapplied to remove as much trapped air as possible. With the balloon sitting in the nylon net within the partially filled cylindrical chamber, the soil slurry was forced into the balloon with a hand-operated piston. Soil slurry was forced into the membrane until a 12 to 14 inch diameter was obtained. At this point, the porous drainage ball assembly was placed in the neck of the membrane with the ball located at the center

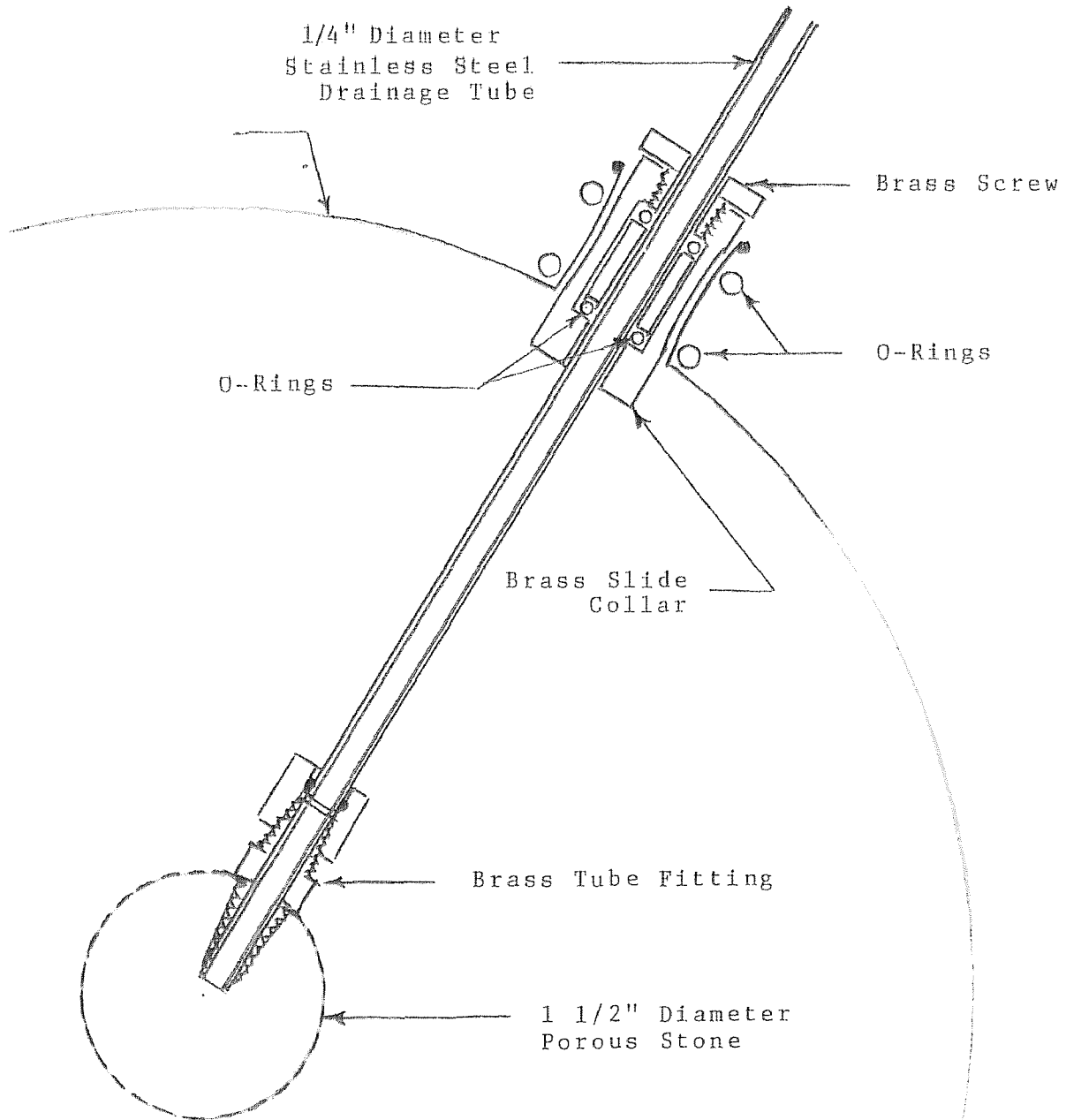


Figure VI - Consolidation Device

of the spherical membrane. The slide collar was installed with lubricated o-rings compressed around the drainage tube. The membrane neck was stretched over the slide collar and sealed with o-rings. At this point, the remainder of the tank was filled with distilled water and the drainage tube attached to the tank lid. With the clay slurry sealed within the membrane, the tank was sealed with a lubricated hard rubber gasket and the lid bolted in place. A 3/4 inch by 3 inch steel cross bar was placed on top of the tank and the entire tank assembly was compressed between a steel channel frame by means of a hydraulic jack. A constant pressure head was applied to the hydrostatic system by a self-compensating mercury control. The pressure head was then regulated to the consolidation pressure. Consolidation was allowed to proceed until pore water drainage stabilized. At this point, the consolidation pressure was left on the sample for 48 hours to insure complete and uniform consolidation.

After complete consolidation, the apparatus was disassembled and the sample removed, sealed and stored in a humidity locker for testing. Test specimens were trimmed from the spherical, three-dimensionally consolidated samples in the same manner as the one-dimensional tests.  $0^\circ$  represented the horizontal plane of the spherical sample. From the horizontal plane, samples were trimmed at standard angle planes of  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$ . Figure VII indicates the random orientation of each spec-

imen trimmed with its axis along one of the angular planes. The comparative test results of the spherical samples with respect to one-dimensional samples at the same angle planes are discussed in Chapter III for direct shear tests and Chapter IV for the triaxial tests.

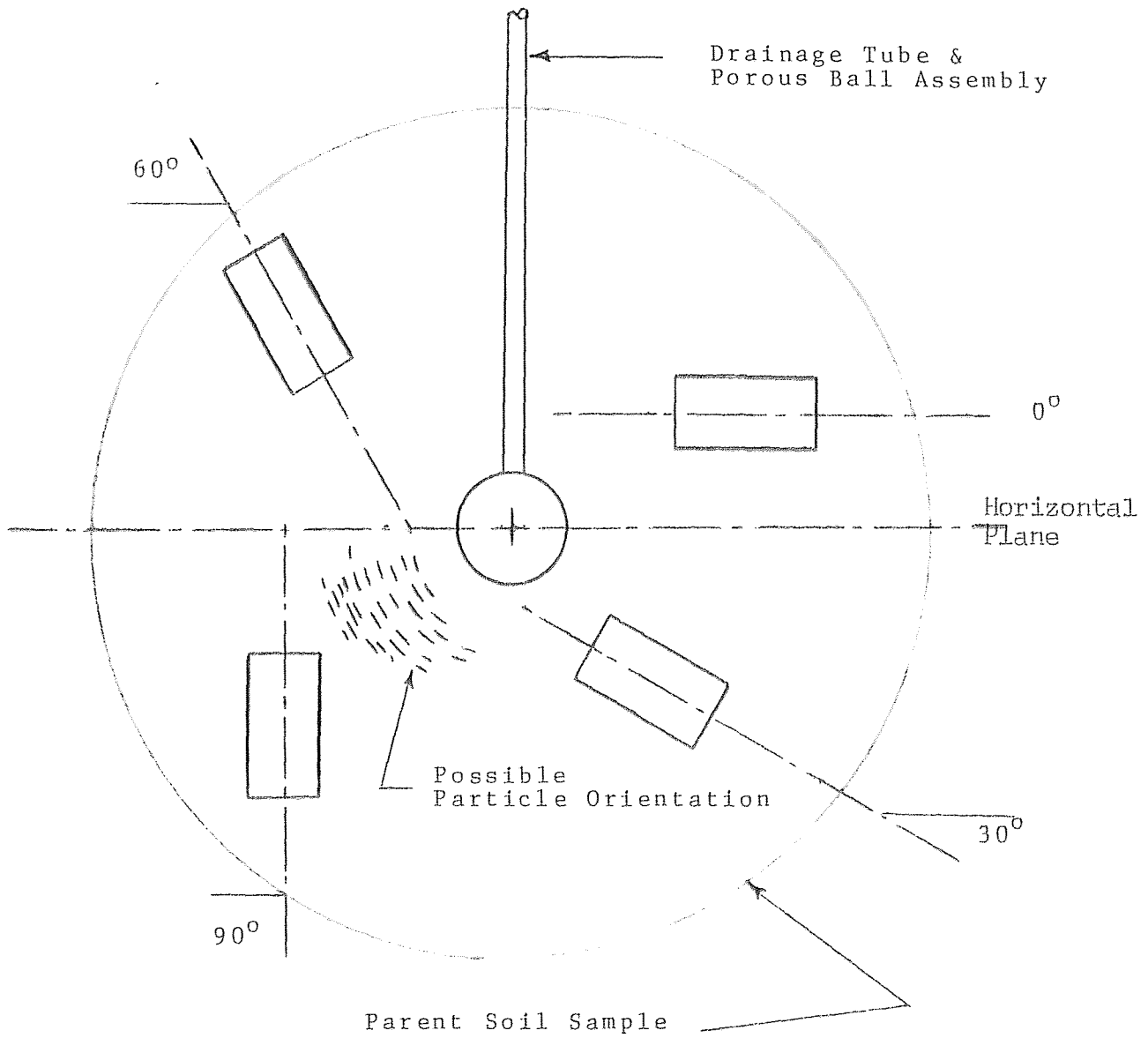


Figure VII - Orientation of Trimmed Isotropic Test Specimens



## Chapter III

### DIRECT SHEAR TESTS AND RESULTS

The direct shear test was used to evaluate the variation of shear strength in prepared soil samples. This method of testing proved effective, since the testing procedures are relatively simple and can be performed in minimum time. There were 59 direct shear tests performed on both one-dimensional samples and three-dimensional samples. Tests were performed on the standard manual direct shear apparatus using a 2.0 inch by 2.0 inch shear box, dead-weight load comprising the normal force, and a manual crank drive. Midway through the testing, an electric motor-operated gear drive was installed to produce a constant strain rate of 0.01 inches/second. This modification eliminated the possibility of manual error, but did present a problem of recording shear force and strain readings. In an effort to obtain accurate readings at a constant strain rate, a super-8 movie camera and flood lights were installed above the apparatus. A film history of tests 23 through 59 was recorded in this manner. Unfortunately, the photographic process was not successful. It was determined after the film was processed that the super-8 film was not adequate to record strain and proving ring readings with sufficient clarity. Readings could not be read from the film when shown on a screen, a film editor, or a microfilm

reader. As a result, shear force computations and graphic comparisons are based on peak proving ring readings obtained during each test.

Through the use of rapid strain rates, the pore pressure which built up within the sample during shearing did not have sufficient time to dissipate. Thus, the tests were considered undrained shear tests and their results could be compared with the undrained triaxial tests discussed in Chapter IV.

Procedures stated in Chapter II were followed in trimming direct shear samples from the parent block. In each case, the test sample was trimmed at an angle  $\beta$  from the horizontal. The sample was placed under a normal load equal to the lateral consolidation stress experienced by the parent block sample. A rapid shear force was then induced on the sample, forcing the sample to fail along a plane parallel to the angle  $\beta$ .

One-dimensional consolidated samples were tested with  $\beta$  ranging from  $0^\circ$  to  $180^\circ$ . Samples were taken at  $15^\circ$  intervals. Table I contains strength data for representative samples along with the strength ratio  $C_\beta / C_0$ . This ratio tends to be the simplest means of comparison. Here  $C_\beta$  represents the shear strength of a sample oriented at  $\beta$  degrees from the horizontal and  $C_0$  is the shear strength of a sample taken along the hori-

TABLE I COMPARATIVE STRENGTH DATA FROM DIRECT SHEAR TESTS

<u>TYPE OF CONSOLIDATION</u>	<u>ONE-DIMENSIONAL</u>			<u>THREE-DIMENSIONAL</u>				
<u>Maximum Consolidation Stress = 0.5 kg/cm<sup>2</sup> *</u>								
Angle $\beta$ from Horizontal Degrees	0	30	60	90	0	30	60	90
Shear Stress at Failure (PSI)	300	274	315	300	455	480	458	458
Strength Ratio $C_{\beta} / C_0$	1.00	0.91	1.05	1.00	1.00	1.05	1.01	1.01
<u>Maximum Consolidation Stress = 1.0 kg/cm<sup>2</sup> *</u>								
Angle $\beta$ from Horizontal Degrees	0	30	60	90	0	30	60	90
Shear Strength at Failure (PSI)	650	677	614	587	931	776	733	708
Strength Ratio $C_{\beta} / C_0$	1.00	0.92	0.92	0.90	1.00	1.06	1.00	0.97
<u>Maximum Consolidation Stress = 4.0 kg/cm<sup>2</sup> *</u>								
Angle $\beta$ from Horizontal Degrees	0	30	60	90	0	30	60	90
Shear Stress at Failure (PSI)	2374	2228	2035	1800	2527	2560	2491	2458
Strength Ratio $C_{\beta} / C_0$	1.00	0.94	0.86	0.76	1.00	1.01	0.99	0.97

\*Laboratory apparatus was designed and calibrated under the metric system. For simplification, the type of test are listed using the metric units.

zontal plane from the same parent block sample. Figure VIII presents a graphic illustration of the stress ratio plotted against the angle  $\beta$ . In each case for one-dimensional consolidated samples of illite clay, the degree of anisotropy increases as the normal consolidation stress increases. For example, a  $4 \text{ kg/cm}^2$  sample showed the greatest degree of anisotropy. The stress ratio  $C_\beta / C_0$  was equal to one when  $\beta$  equaled  $0^\circ$ . The shear strength decreased to the minimum value at  $\beta = 90^\circ$ . An interpretation of this finding would indicate a distinct variation of strength within the soil relative to the horizontal plane of symmetry. Samples oriented along the horizontal, being subjected to the greatest normal consolidation stress, exhibited the highest shear strength. As sample orientation reached  $90^\circ$  from the horizontal, the normal stress approached the lateral consolidation stress, resulting in a lower shear strength. From these observations, we may conclude that the shear strength of a cohesive soil is directly related to the soil stress history.

The isotropic or anisotropic soil properties are also directly related to the soil stress history. As in the case of the  $0.5 \text{ kg/cm}^2$  sample shown in Figure VIII a, the strength ratio  $C_\beta / C_0$  is equal to one and is represented by a straight line. In this case, a one-dimensional sample consolidated to  $0.5 \text{ kg/cm}^2$  possesses isotropic characteristics; that is, identical shear

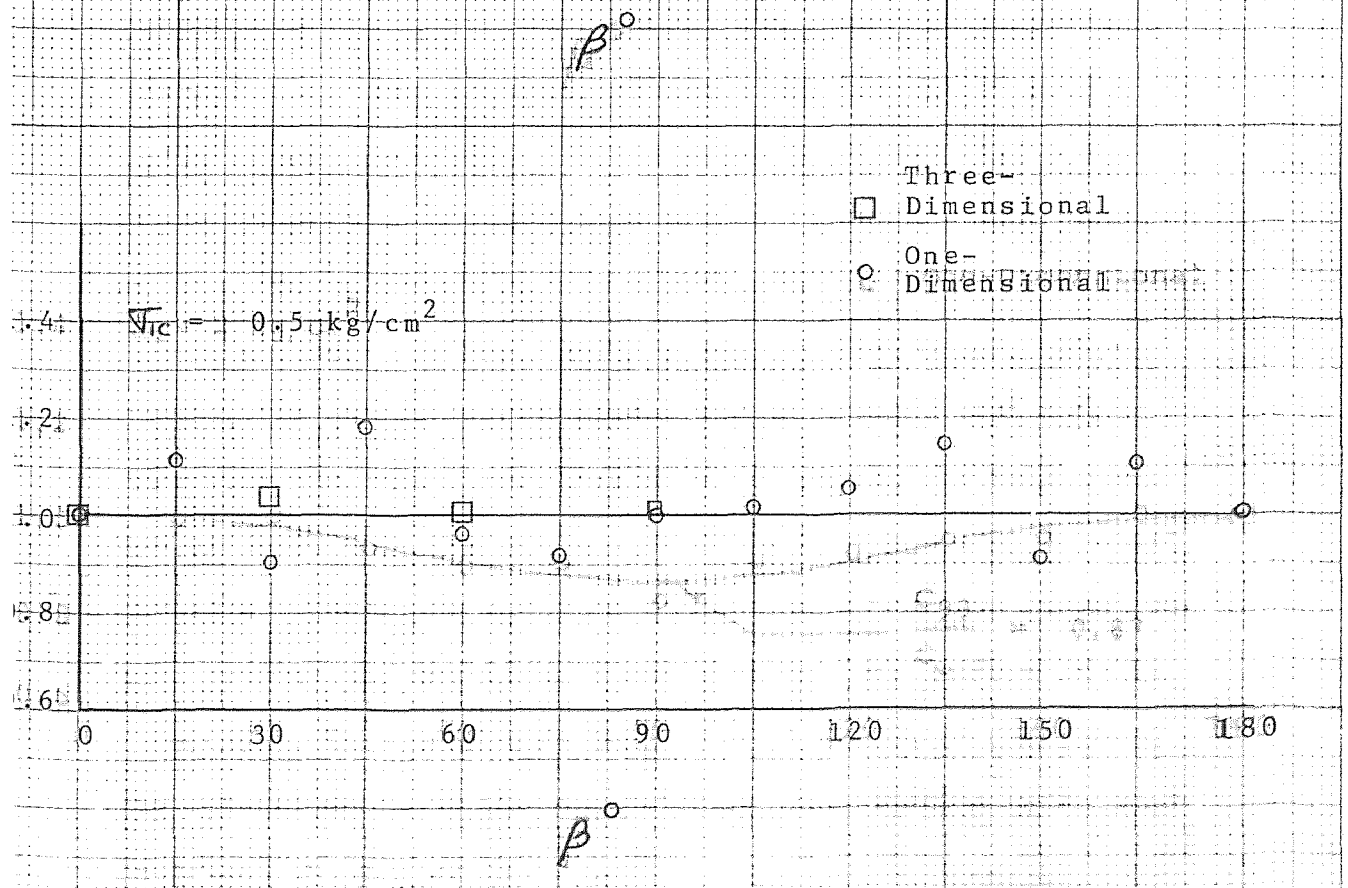
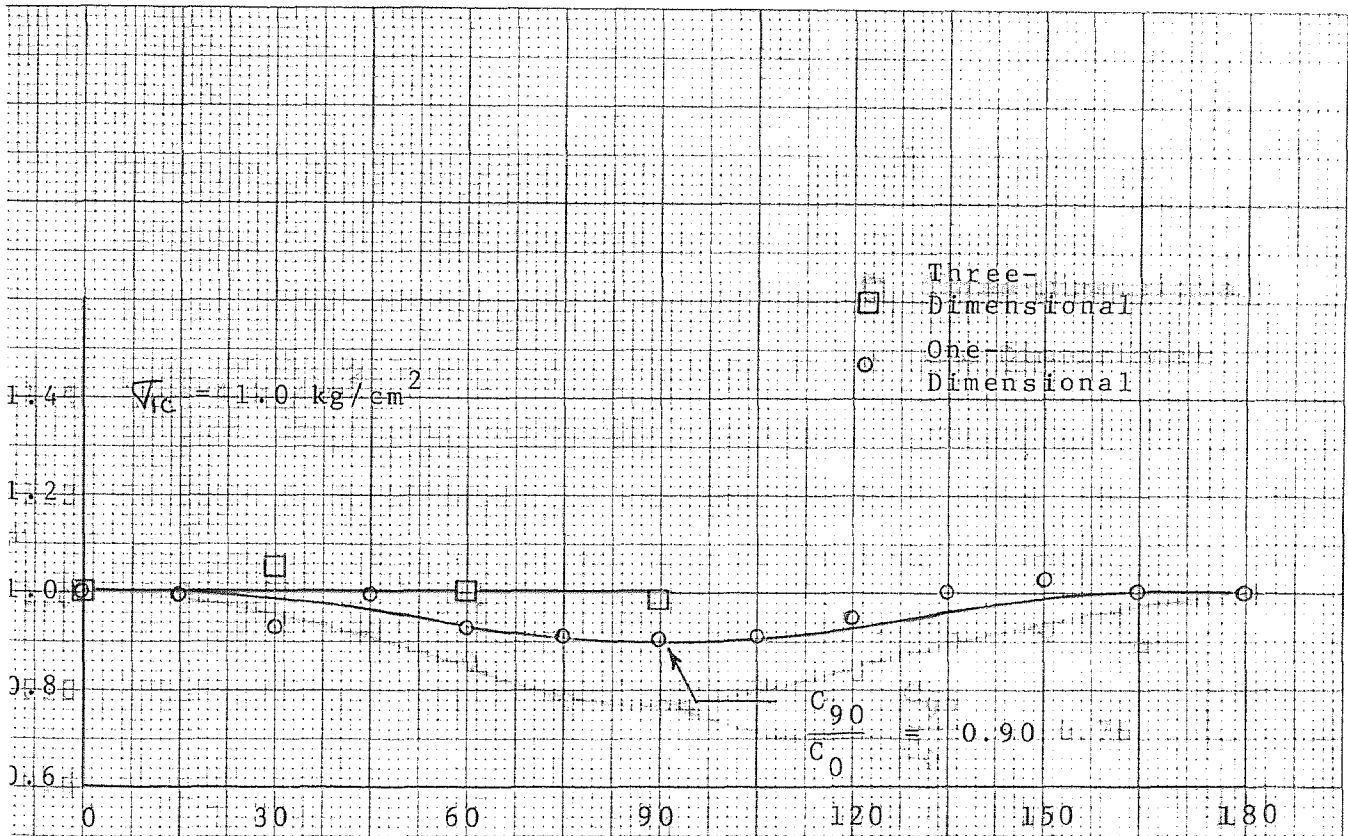


Figure VIIIa - Relative Strength Ratio From Direct Shear Test

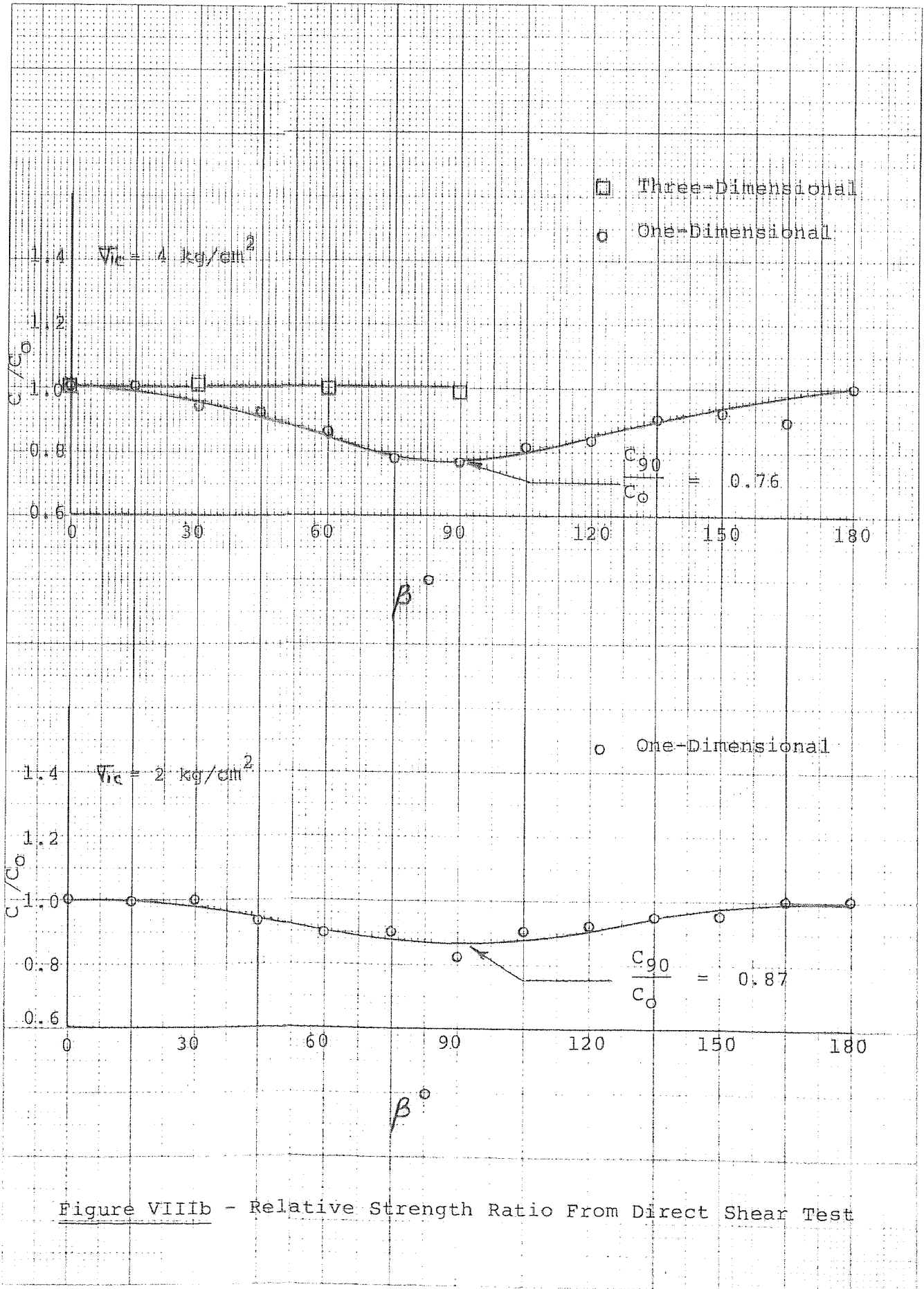


Figure VIIIb - Relative Strength Ratio From Direct Shear Test

strength in all directions. Rationale for the existence of isotropy in this case would be due to the relatively low consolidation stress. With a low consolidation stress, the soil sample holds pore water as shown by the high water content. The normal consolidation pressure is not great enough to force a rearrangement of the clay particles into a uniform pattern as it did in the case of the 4 kg/cm<sup>2</sup> sample. As a result, the clay particles maintained a random orientation and, thus, isotropic properties.

In comparison, three-dimensional, consolidated samples prepared by the hydrostatic method had isotropic properties through all stages of consolidation from 0.5 kg/cm<sup>2</sup> to 4.0 kg/cm<sup>2</sup>. The hydrostatic results shown in Figure VIII b are represented by a straight line with a strength ratios equal to one. The rationale for this phenomenon is drawn from the sample's stress history. Under a hydrostatic consolidation stress, the sample experienced a stress history of  $\bar{\sigma}_1 = \bar{\sigma}_2 = \bar{\sigma}_3$ . With the three principal stresses equal, the sample was consolidated with the absence of shearing forces. As a result, particle orientation remained random throughout the consolidation process and the final shear strength was uniform in all directions. This principle held true in all cases and proved the effectiveness of the hydrostatic consolidation techniques described in Chapter II.

The results of the direct shear tests proved that this testing technique was an efficient means of evaluating the variation in shear strength as the angle  $\beta$  ranged from  $0^\circ$  to  $90^\circ$ . Two factors should be mentioned that limit the accuracy of the direct shear tests. The first disadvantage was the lack of control over pore water pressure during the test. The only technique available to control pore water pressure was the control of the strain rate. By using a slow strain rate, the pore pressure was allowed to dissipate, resulting in a drained shear test. As discussed in this section, the use of a rapid strain rate resulted in an undrained test, since the pore water pressure did not have sufficient time to dissipate. The overall effect of this type of test was a build-up of pore pressure as the shearing force increased. Exact control over the pore pressure was not possible, however, at best, we are approximating the undrained condition. The second disadvantage to direct shear tests was the control over the insitu forces which were present during the sample's initial consolidation. In effect, the only force or stress which could be simulated during the test was the normal force on the sample. The stress history would have vertical and horizontal shear forces present along with lateral and normal consolidation forces. In the direct shear apparatus, vertical shear forces and lateral loading could not be reconstructed. Therefore, the results of this type of testing provided an



approximation of the true shear strength for that sample along an induced failure plane.

## Chapter IV

TRIAXIAL COMPRESSION TESTS

Undrained triaxial tests were determined to be the most accurate and effective means of obtaining the effective stress strength parameters for the illite clay samples. This method of testing would furnish a complete picture of both the isotropic and anisotropic properties experienced by the parent block samples resulting from their stress history. An evaluation of existing laboratory equipment revealed the need for a more modern, machine drive apparatus which would limit human error to a minimum and attain consistent results. To satisfy this requirement, a variable speed, constant drive triaxial test assembly was procured from Wykeham Farrance of England. The constant speed drive along with rotary bushing triaxial test cell heads proved effective in reducing chamber piston friction and interference on the samples. At the time, this apparatus was the most economical means of performing triaxial tests with " frictionless " piston characteristics.

Two triaxial chambers, both with rotary bushings, allowed continuous testing with minimum delay. As one sample was being tested, the second sample was being consolidated in preparation for testing. A self-compensating mercury control system furnished a constant hydrostatic pressure head

for chamber pressure and back pressure. Sample volume change was measured by a double-walled, burette apparatus using a water paraffin surface. To obtain accurate readings in change of pore water pressure without disturbing the test sample, a pressure transducer - PA. 208TC - was mounted in a locally-designed and manufactured brass housing with watertight connections to the chamber base assembly. Pore water pressure readings were obtained through electrical digital readout in milli-volts and converted to pounds per square inch.

As in the direct shear testing, soil samples were trimmed from the parent block using angle  $\phi$  from the horizontal of  $0^{\circ}$ ,  $30^{\circ}$ ,  $60^{\circ}$ , and  $90^{\circ}$ , for each consolidation stress. Sample dimensions were 1.4 inches in diameter and 3.0 inches high. Procedures used in mounting the samples in the test chambers were identical to those of Bishop and Henkle<sup>1</sup>. Special attention was given to the end effects induced on the sample by the end caps. As described by Khera and Krizek<sup>2</sup>, stainless steel end caps measuring 1.4 inches in diameter and 0.25 inches thick were coated with silicone grease and positioned at the

1. A.W. Bishop and D.J.Henkel. The Measurement of Soil Properties in the Triaxial Test. London, Edward Arnold, Ltd. 1964, 2nd. ed. p.45.

2. R.P.Khera and R.J.Krizek. "Measurement of Control of Radial Deformation in the Triaxial Test of Soils", Material Research and Standards. Vol 7:9 Sept. 1967. P. 394.

top and bottom of the test specimen. Each cap contained a small porous stone at its center which intersected 1/16 inch radial drainage holes emanating from the center to the circumference. Slotted filter paper was then carefully wrapped around the test specimen, joining the drainage holes in the upper and lower end caps. The use of slotted filter paper, which does not soften in water ( i.e. Whatman's # 54), was determined to provide the most even and quickest drainage technique. This technique proved effective in minimizing end restraints on the test samples. In each case, failed samples showed uniform deformation with no noticeable distortion at its ends.

Test samples were then consolidated in the triaxial cell, taking the specimen back to its original stress condition experienced by the parent block. An effective consolidation stress equal to the mean consolidation pressure was used for consolidation. A constant back pressure of  $2\text{kg/cm}^2$  was used to insure complete saturation during consolidation. Over a period of six hours, the change in volume of the samples stabilized and remained constant thereafter. The volume parameter was used to show the completion of consolidation. All samples remained under consolidation for a full twelve hours to insure uniformity, saturation, and stabilization.

Once consolidation was completed, the sample was ready for testing. With the sample/chamber assembly mounted in the

triaxial machine with constant normal stress, lateral stress, pore pressure, and change in volume readings, a constant 3 millimeter per hour deformation rate was applied to the sample. Axial deformation was obtained from dial readings and a proving ring provided axial load readings. Dial readings and pore water pressure were recorded approximately every 0.01 inches of deformation until clear shear failure had occurred.

Table II contains test results obtained from twenty-four triaxial tests. Eight tests were performed on samples trimmed from the 4 kg/cm<sup>2</sup>, one-dimensionally consolidated block. In this case, specimens were treated with the orientation of angle  $\beta$ , varying in 15° increments. The intent was to obtain a true picture of strength variation as angle  $\beta$  increases from 0°. These results were then to be compared to direct shear results. After completion of the first set of eight tests, it was decided that in order to conserve time and reduce the amount of parent samples required, the remaining tests would be limited to angle  $\beta$  equal to 0°, 30°, 60°, and 90°. This variation provided identical information and did not sacrifice accuracy in lieu of quantity of test specimens.

For each failed triaxial specimen, the value of angle  $(\beta - \alpha)$  is shown in Table II. This angle is defined as the inclination between the failure plane of the test sample and the axis of

TABLE II - SUMMARY OF TRIAXIAL COMPRESSION TESTS

Test No.	Moisture Content (%)		Failure Strain $\epsilon_f$ %	$(\bar{\sigma}_1 - \bar{\sigma}_3)$ PSI	$\frac{M}{PSI}$	$\beta$ Deg.	Consolidation Stress $\bar{\sigma}_{1c}$ (kg/cm <sup>2</sup> )		$(\beta - \alpha)$ Deg.	$\tau_{cr}$ PSI	$C_p/C_o$
	Initial	Final									
1	37.2	34.5	7.60	35.10	17.80	90°	4.0	b	16.65	1.21	
2	35.7	33.6	7.20	33.20	17.80	75°	4.0	33	14.45	1.04	
3	36.3	33.8	7.30	35.70	15.90	60°	4.0	30	15.16	1.10	
4	36.2	35.2	9.60	30.30	21.00	0°	4.0	30	13.78	1.00	
5	35.9	33.5	10.00	36.30	16.30	45°	4.0	28	15.48	1.12	
6	36.1	33.9	6.70	37.04	17.40	90°	4.0	30	15.96	1.16	
7	36.7	35.2	8.57	38.00	16.60	30°	4.0	31	15.93	1.16	
8	36.3	34.8	9.64	36.80	15.79	15°	4.0	29	15.55	1.13	
9*	48.2	46.6	0.89	2.25	1.82	60°	0.5(H)	30	1.03	0.26	
10	47.7	45.0	10.18	10.50	3.36	30°	0.5(H)	b	4.17	1.04	
11	47.8	45.5	7.68	10.00	3.35	0°	0.5(H)	b	4.00	1.00	
12	48.8	47.0	8.39	8.49	3.98	90°	0.5(H)	b	3.56	0.89	
13	43.8	41.4	10.00	17.97	8.81	0°	2.0	30	7.68	1.00	
14	43.5	41.9	8.93	18.56	9.20	30°	2.0	29	7.98	1.04	
15	43.5	40.7	9.71	16.35	8.37	60°	2.0	30	7.10	0.92	
16	43.5	41.8	9.89	16.51	8.81	90°	2.0	26	7.14	0.93	
17	35.1	33.7	9.46	45.47	25.03	90°	4.0(H)	30	19.99	0.99	
18	35.5	34.4	10.79	45.50	25.50	60°	4.0(H)	31	20.06	0.99	
19	35.2	34.0	9.29	44.25	27.92	30°	4.0(H)	32	19.85	0.98	
20	35.7	34.6	10.79	45.18	29.44	0°	4.0(H)	32	20.29	1.00	
21	59.2	54.4	10.00	6.74	2.60	0°	0.5	35	2.78	1.00	
22	61.6	55.0	10.89	7.29	2.52	30°	0.5	b	2.95	1.06	
23**	61.5	56.4	4.15	4.60	1.75	60°	0.5	b	1.87	0.67	
24	60.8	55.6	11.07	7.14	2.33	90°	0.5	b	2.85	1.03	

NOTES:

\*Results of this test are negated by the use of an inappropriate proving ring to measure deviator stress.

\*\*Results of this test are negated due to test sample requiring reconsolidation (poor testing techniques).

b - Sample failed by bulging.

major principal stress during shear. The results of all tests gave  $(\beta-\alpha)$  values ranging from  $26^\circ$  to a maximum of  $35^\circ$  or an average value of  $30.4^\circ$ . Lo<sup>1</sup> found a much greater variation in values of  $(\beta-\alpha)$  for an undisturbed soil, yet assumed  $(\beta-\alpha)$  to be equal to a constant. Hvorslev's<sup>2</sup> evaluation of  $(\beta-\alpha)$  showed the variation to be not more than  $3^\circ$  for remolded soils. A conclusion drawn by Hvorslev<sup>3</sup> was that failure of a sample may occur in a plane with its inclination  $\alpha \approx \alpha_0 \pm 4^\circ$  when the corresponding shear strength in this plane was slightly less than the shear strength at  $\alpha_0$  by 1.0 to 1.5 per cent. He deduced that this variation was due to anisotropy, or irregularities within the test specimen. Hvorslev<sup>4</sup> did not consider changes in pore water pressure during his study. This is one important factor which must be evaluated to prove isotropy or anisotropy.

Pore water pressure values obtained from triaxial tests are shown graphically in Figure IX and Figure X. It is interesting to note that the comparison of the pore water pressure values ( $u$ ) are nearly identical for normally consolidated samples de-

1. K.Y. Lo. "Stability of Slopes in Anisotropic Soils", Journal of the Soils Mechanics and Foundation Division, Proceedings, American Society of Civil Engineers, Vol 91:SM4, Paper 4405, July 1965. P.90.

2. Hvorslev. Op. Cit. p. 265.

3. Ibid. p.270.

4. Ibid. p.271.

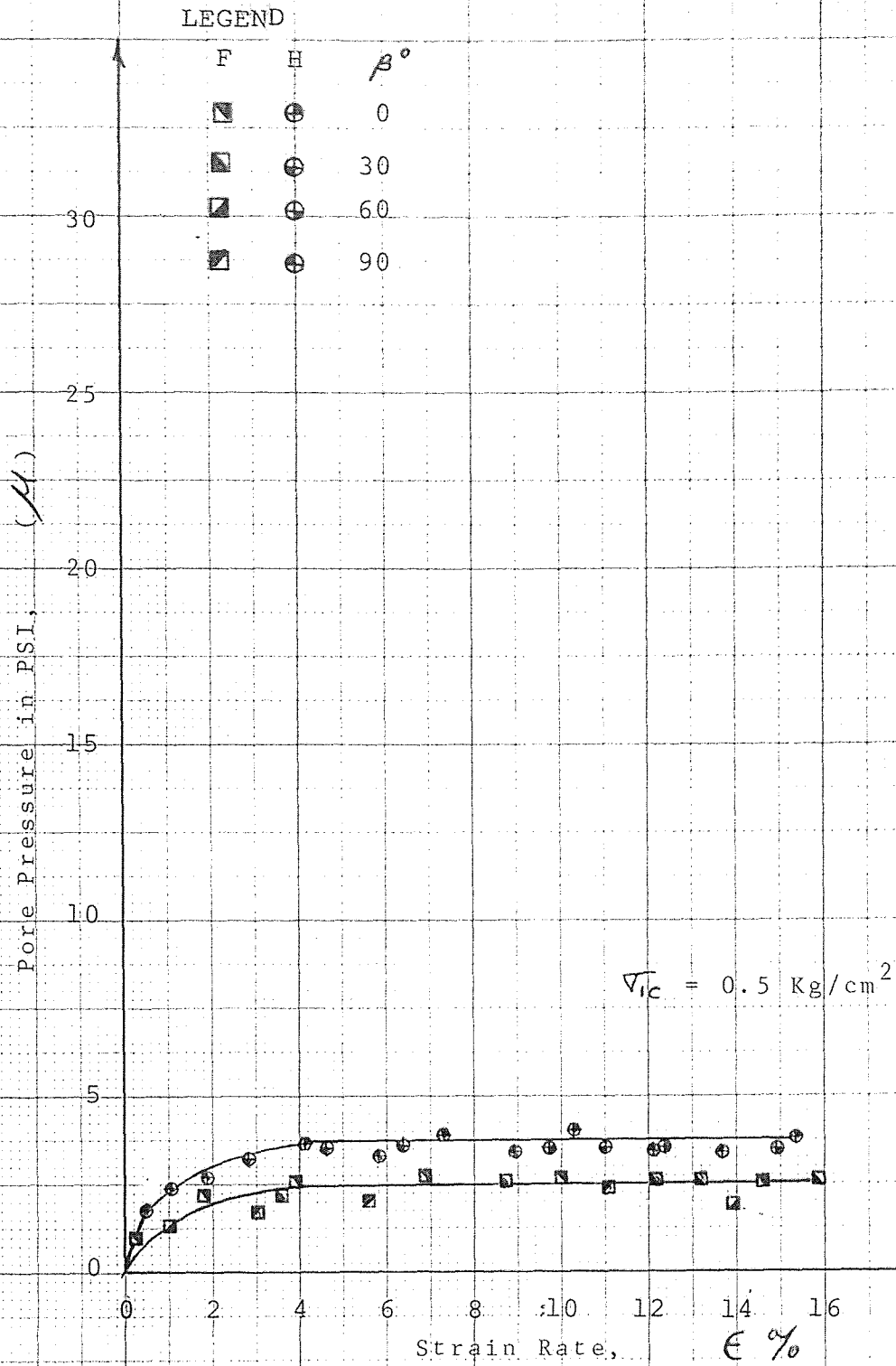


Figure IX - Comparison of Pore Pressure Between H and F Series Test



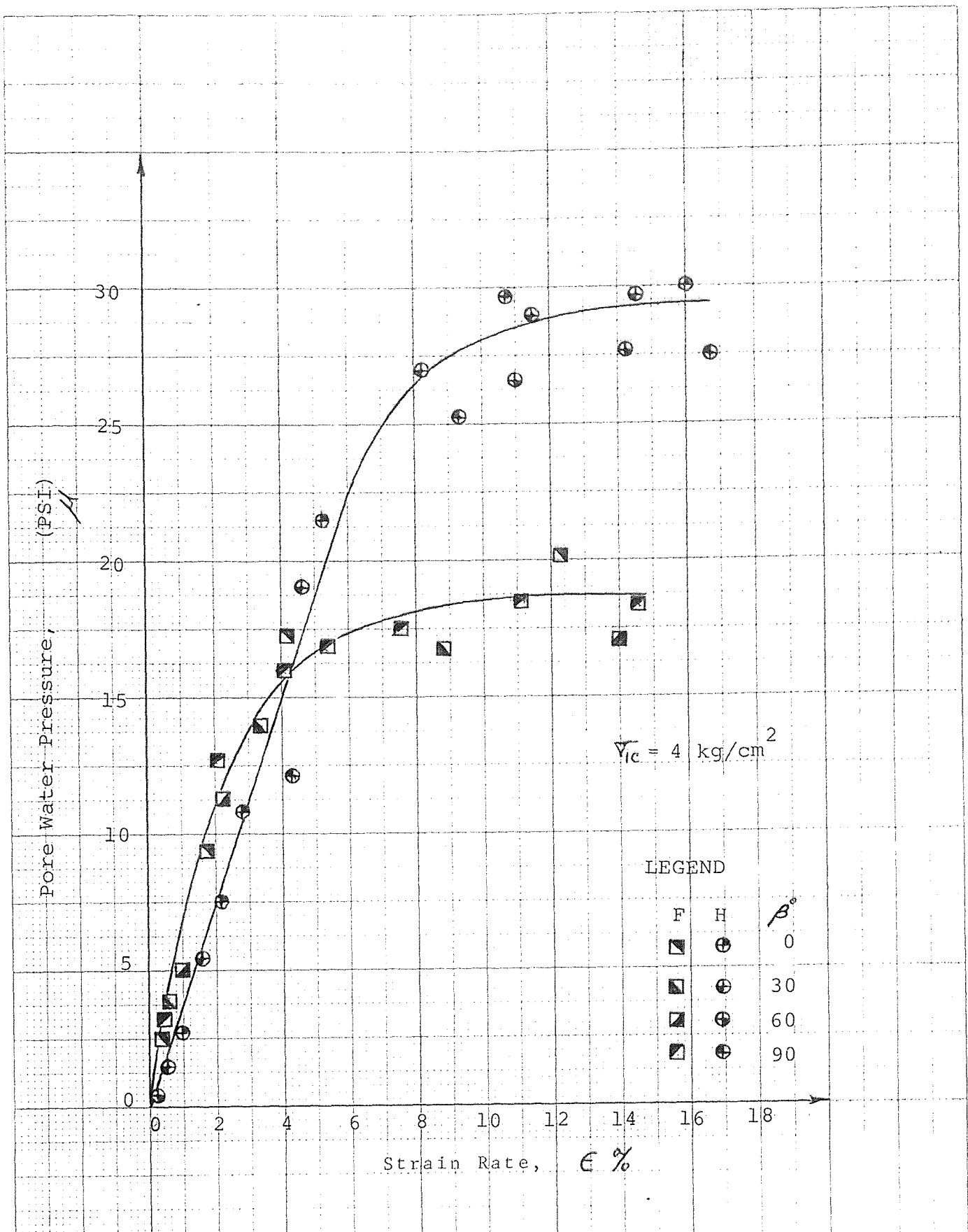


Figure X - Comparison of Pore Pressure Between H and F Series Test

picted in Figure X. Pore pressure values for all F-series tests (normally consolidated) may be represented by one curve. Similarly, the H-series tests, hydrostatically consolidated samples, were also well represented by a single curve. One observation which can be readily made from Figures IX and X is the marked difference in pore pressure values between the H-series and the F-series tests. In the F-series tests, values fall on one curve despite sample orientation. This phenomenon would indicate isotropy of pore pressure which may seem to be a contradiction to the anisotropic characteristics presented in Figure VIII, discussed previously in Chapter III. In the  $4 \text{ kg/cm}^2$  normally consolidated sample, the strength ratio  $C_{\beta} / C_0$  discussed in Chapter III showed a distinct anisotropic characteristic. However, the pore pressure shown in Figure X tends to be isotropic. This phenomena would lead to the conclusion that for a specific soil sample, the specimen orientation would have negligible effect on pore water pressure while the shear strength varied as the specimen orientation varied by angle  $\beta$  from the horizontal. The second observation is that the pore pressure for three-dimensionally consolidated, hydrostatic samples is consistently higher than the pore pressure for F-series tests. This characteristic is true, despite the fact that the maximum consolidation stress was the same for both series. The final observation is the difference in slope of the pore pressure graphs shown in Figure X. The pore pressure builds slower for the  $4 \text{ kg/cm}^2$  H-series test specimens. The

F-series tests show the pore pressure reaching its maximum faster than the H-series, where it then levels off well below the maximum pore pressure attained by the H-series specimens.

The ratio of pore pressure at failure and the effective mean consolidation stress is plotted against the consolidation stress in Figure XI. In each case, the data points form a single straight line. This was true regardless of the magnitude of consolidation stress. This phenomenon was reported by Khera<sup>1</sup>, where additional data from triaxial tests on screw-extruded Grundite specimens were plotted on the same graph. Despite the great difference in stress history and sample formation, all points fell on the same straight line.

Based on the comparisons and data presented here, it may be concluded that the difference in shear strength is due to sample stress history. The H and F-Series tests were consolidated under identical consolidation stresses and tested in the same apparatus using the identical technique. The only difference between the two series was that the H-series were consolidated three-dimensionally and, therefore, exhibited isotropic characteristics regardless of sample orientation.

1. R.P.Khera. "Remolding Stresses and Directional Strength Behavior of the Illitic Clay", Journal of Testing and Evaluation. January 1976, Vol 4:6.P.106.

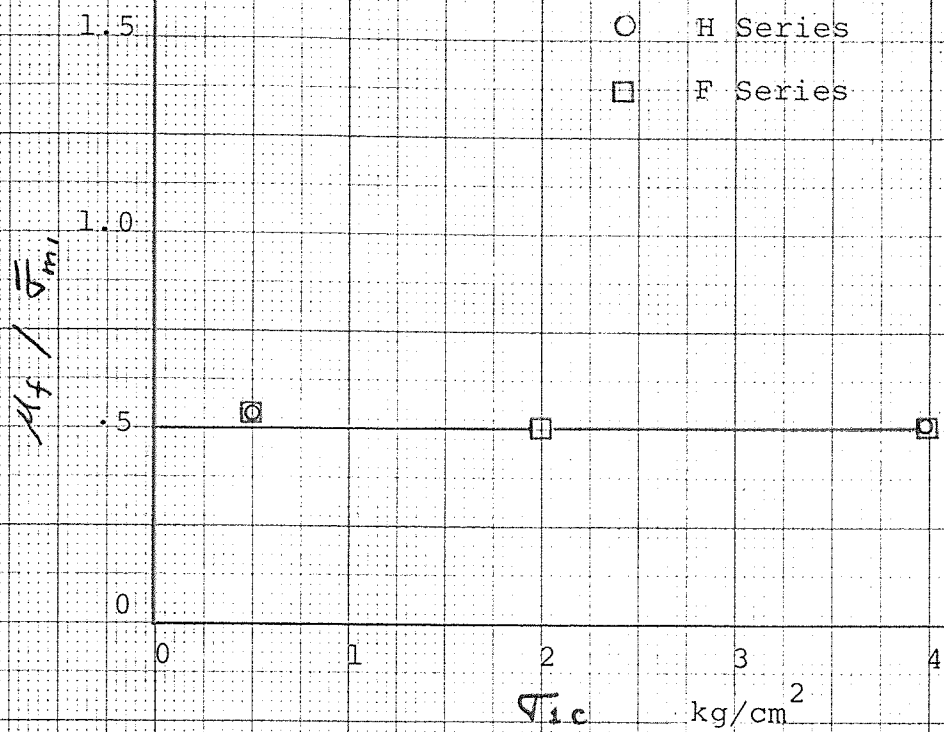


Figure XI - Ratio of Pore Pressure at Failure and the Effective Mean Consolidation Stress

Similarly, the F-series tests exhibited anisotropic properties. The shear strength decreased as the specimen orientation reached an angle  $\beta$  equal to  $90^\circ$  from horizontal. The strength, therefore, was greatest for  $\beta$  equal to  $0^\circ$ , indicating a preferred orientation of soil particles, parallel to the horizontal plane.

Slack test results on samples prepared for testing were reported by Khera<sup>1</sup>. These tests showed expansion and development of fissures in planes parallel to the horizontal plane. Identical tests on H-series specimens did not indicate any bias in particle arrangement. As a result, it may be assumed that particle orientation in the H-series is completely random.

1., Ibid. P.107.

## Chapter V

SUMMARY AND CONCLUSIONS

Laboratory samples of Grundite were prepared by two methods, one-dimensional consolidation and three-dimensional hydrostatic consolidation. The test specimens were trimmed from the parent samples varying the specimen orientation from the horizontal plane between  $0^{\circ}$  and  $90^{\circ}$ . Stress/strength parameters were determined by undrained triaxial compression tests and direct shear tests. For triaxial specimens, consolidation stress was equal to the mean effective normal stress experienced by the parent block sample. Based on the results obtained, the following conclusions are drawn:

1. Test specimens prepared using the three-dimensional, hydrostatic consolidation method had isotropic undrained shear strength. These samples showed no bias in soil structure, which is a unique phenomena and not normally found in natural deposits.

2. The same soil prepared using the one-dimensional consolidation method exhibited anisotropic characteristics, which were directly attributed to its stress history. These samples did indicate preferred particle orientation and anisotropic, undrained shear strength. Previous research studies reporting anisotropy, were made on soils with similar anisotropic stress history.

3. The maximum shear strengths were exhibited by specimens orientated parallel to the horizontal plane. Vertically orienta-

ted specimens had the minimum strength. Strength ratios between the  
these planes varied from 0.90 for 1 kg/cm<sup>2</sup>, 0.87 for 2 kg/cm<sup>2</sup>,  
and 0.76 for 4 kg/cm<sup>2</sup> samples.

4. The angle between the failure plane and the specimen  
axis was essentially constant.

5. For a given maximum consolidation stress, hydrostatically  
prepared specimens exhibited a higher pore water pressure than  
samples with anisotropic stress history. In both cases, the pore  
pressure was isotropic.

6. The ratio between pore pressure at failure and the mean  
consolidation stress tends to be constant.

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