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Stress analysis of a stepped shaft with a keyway

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2) STRESS ANALYSIS OF A STEPPED SHAFT WITH A KEYWAY //

1) BY
IRFAN HUSSAIN SABIR //

Thesis submitted to the Faculty of the Graduate School of
the New Jersey Institute of Technology in partial fulfillment of
the requirements for the degree of
Master of Science in Mechanical Engineering
1987

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ABSTRACT

Title of thesis : Stress Analysis of a Stepped Shaft with a Keyway.

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A stress concentration case which is encountered in practice more than any other is the shoulder fillet. Shafts, axles, spindles, rotors etc., usually involve, in each case, a number of diameters connected by shoulders with fillet radii and very often it is necessary for a keyway to be near a shoulder. The objective in this study is to analyse the stresses in a shouldered shaft with a keyway subjected to an axial tensile load and to study the dependence of the keyway-end stresses on the distance between the shoulder fillet and the keyway end. This is done by performing a finite element analysis of the model. Two three-dimensional models with a grid of 3-D isoparametric solid elements are generated using the GE CALMA software. In Model 1 the keyway cuts into the shoulder fillet whereas in Model 2 the keyway end terminates just before the fillet. The stress determination is done by ANSYS, a powerful finite element analysis package. A comparative study of the two models shows that stress reductions occur at the keyway end when the keyway runs into the shoulder fillet. The results of this analysis also serve as a guide for the prediction of stresses for a stepped shaft with a keyway subjected to a bending moment.

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TABLE OF CONTENTS

	Page
List of Figures	
Chapter	
1. INTRODUCTION	
1.1 Introductory Comments	1
1.2 Literature Survey	2
2. FINITE ELEMENT ANALYSIS TECHNIQUE	
2.1 Model Analysis	11
2.2 General Method	12
2.3 Advantages and Disadvantages	15
3. MESH GENERATION, BOUNDARY CONDITIONS AND SOLUTION OF THE MODEL	17
4. MODEL OF STEPPED SHAFT WITH KEYWAY	
4.1 Model 1 : Keyway cutting into shoulder fillet ...	40
4.2 Model 2 : Keyway ending before shoulder fillet ..	45
5. DISCUSSION OF RESULTS	50
6. CONCLUSION	63
APPENDIX 1 Ansys Input Listing of Model 1	64
Ansys Input Listing of Model 2	75
APPENDIX 2 Nodal Displacement and Stress Listing of Model 1	85
Nodal Displacement and Stress Listing of Model 2	93
Bibliography	101

LIST OF FIGURES

Figure		Page
1	Plot of Model 1	22
2	Model 1 : Points for generating curves	23
3	Model 1 : Curves for defining the sub-areas	24
4	Model 1 : Sub-areas which make up the sub-volumes	25
5	Model 1 : Sub-volumes which constitute the entire model	26
6	Model 1 : Nodal Plot	27
7	Model 1 : Elemental Plot	28
8	Model 1 : Restraints and loads	29
9	Plot of Model 2	30
10	Model 2 : Points for generating curves	31
11	Model 2 : Curves for defining the sub-areas	32
12	Model 2 : Sub-areas which make up the sub-volumes	33
13	Model 2 : Sub-volumes which constitute the entire model	34
14	Model 2 : Nodal Plot	35
15	Model 2 : Elemental Plot	36
16	Model 2 : Restraints and loads	37
17	Model 1 : Element faces at end of small shaft	42
18	Model 2 : Element faces at end of small shaft	46
19	Model 1 : Deformed geometry	51
20	Model 2 : Deformed geometry	52
21	Model 1 : Stress contours (x-component)	54
22	Model 1 : Stress contours (Von Mises)	55
23	Model 1 : Stress contours (max. shear)	56

24 Model 1 : Stress contours (max. principal) 57
25 Model 2 : Stress contours (x-component) 58
26 Model 2 : Stress contours (Von Mises) 59
27 Model 2 : Stress contours (max. shear) 60
28 Model 2 : Stress contours (max. principal) 61

1. INTRODUCTION.

1.1 INTRODUCTORY COMMENTS.

Shafts containing keyways are frequently-used components and it is often necessary for a keyway to be near a shoulder. These components are almost always subjected to varying loads, which makes them liable to fatigue failure. An elastic analysis of the stresses is therefore of value for design purposes.

Keyways and shoulders cause discontinuities in profile and hence stress concentrations. They have been studied separately and a detailed study of the interactions of the stress fields caused by these disturbances has also been carried out[10]. The frozen-stress photoelastic technique has been used in most of the work performed in this area. Little effort has been made to obtain theoretical solutions, probably due to the complicated boundary.

In the present study an attempt is made to analyse the stresses in a shouldered shaft with keyway subjected to axial load using the finite element analysis technique. The GE CALMA software is used to generate the model and then ANSYS, a powerful finite element analysis package, is used for determining the stresses. However, the scope of the investigation is limited due to severe limitations caused by the use of three dimensional solid elements. The university version of ANSYS could not support the high number of elements needed to generate satisfactorily the curved boundary of the model.

1.2 LITERATURE SURVEY

Notation used

- a Distance between shoulder-radius run out and nearest point of keyway end, measured positive along the shank.
- D Shoulder diameter.
- d Shank diameter.
- h Shoulder depth.
- k Keyway depth.
- r Fillet radius.

(a) Stress Analysis of Shouldered Shafts and Plates.

Tension: Photoelastic stress analysis has been applied to the study of shouldered plates in tension since one of the founders of the subject obtained values in 1921[6]. After Coker[6] Timoshenko and Dietz[39] carried out an extensive investigation, also using celluloid as the model material. The work of Maris[23] was limited to plates without full fillets. In 1934 and 1935 further photoelastic results were published by Weibel[41] and Frocht[12], both using phenolformaldehyde materials. In 1951, after better materials had become available, Frocht and Landsberg[13] carried out further tests using a Bakelite resin and Allison[1] used epoxy resin models. Some results were published by Ibrahim and McCallion[19] as examples of the application of finite-difference calculations to two dimensional elastic-stress analysis problems.

Probably because of the greater experimental difficulties involved in testing them, shouldered shafts have received comparatively little attention. Peterson[31] obtained values for stress concentration factor by using an empirical equation. In 1962 I. M. Allison[4] carried out a photoelastic investigation on the elastic stress concentration factors associated with shouldered shafts subjected to axial load. He found that under axial load the peak stress in the shouldered shaft is a principal stress lying in the axial plane and located on the surface of the shaft. The stress concentration factor was always higher than the corresponding factor for bending or torsion. For constant values of shoulder-fillet ratio (r/d) the stress concentration factor increased initially and then decreased as the ratio of shank diameter to shoulder diameter (d/D) was decreased below a value of 0.55. Allison concluded that for stepped shafts with no keyways for values of d/D greater than 0.8 and all values of r/d the stress concentration factor for axial load is about the same for bending. The results of Fessler, Rogers, and Stanley[8] were published in 1969 who carried out room-temperature and frozen stress photoelastic tests to obtain the peak stresses and stress distributions in shouldered plates in tension and shouldered shafts in pure tension and pure torsion. They derived an empirical equation from the flat-plate tests which is also applicable to shafts in tension. According to Fessler et al. the analysis of the stress distribution in flat shouldered plates subjected to tension showed that the extent of disturbance in the shank is independent of the fillet ratio and the width ratio over a defined range of shapes. The same applied for the fillet ratio in shafts. The stress

concentrations in shouldered shafts subjected to torsion were much smaller than those due to tension for the same shapes.

Torsion: Stress concentration factors for shouldered shafts in torsion were obtained with an electrical analogue by Jacobsen[20] and Thum and Bautz[38]. Weigand[42] used mechanical extensometers in his study of actual shafts. He used a strain gauge of 1.3 mm gauge length. Allison[2] carried out frozen stress photoelastic tests and Rylander et al.[34] used birefringent coatings on aluminium shafts to obtain further results. Fatigue-strength reduction factors have also been published by Peterson[29] for shouldered shafts in torsion. Sonntag[37] obtained an empirical expression for the peak stresses in shafts in torsion and Heywood[18], who collected all the results available before 1951, also derived empirical expressions. Allison employed the photoelastic "stress freezing" technique to evaluate the elastic stress concentration factors associated with the fillet blend radius in a number of shouldered shafts. The full range of practical sizes of blend radius and depth of shoulder were examined and comprehensive results for the shaft subjected to torsion were published. He found that for given values of r and d , the stress concentration factor increases initially, reaches a maximum value and then decreases as D is increased. According to Allison this behaviour also occurs in pure bending and to a lesser extent in the case of axial load. He also showed that the stress concentration factor increases continuously as r/d is decreased. Fessler, Rogers, and Stanley[8] also obtained results for shouldered shafts in torsion using photoelastic methods.

Bending: Peterson and Wahl[30] made measurements with a mechanical strain gauge on a large steel shaft having a D/d ratio of 1.5 and obtained results for three values of r/d . Thum and Bautz[38] employed a similar method and obtained a curve for a steel shaft also having a D/d ratio of 1.5. Using the photoelastic stress freezing method, Leven and Hartman[22] have obtained results with three dimensional models for D/d ratios of 1.25 and 2.0. In 1961 Allison[3] published a paper which describes a photoelastic determination of the elastic stress concentration factors associated with shouldered shafts subjected to pure bending. He found that the general shape of the families of curves for the bending tests is similar in form to that of the torsion and axial load tests. He pointed out four points of similarity to the torsion test results: the stress concentration factor becomes very large as r/d tends to 0, for all values of r/d the stress concentration factor tends to 1 as d/D tends to 1, for all values of h/D the stress concentration factor tends to 1 as h/r tends to 1, and as the value of d/D is decreased from 1 the stress concentration factor increases initially, reaches a maximum and then decreases. Allison tried to explain the reason for the last behaviour. According to him it is unlikely that the stress concentration factor is a simple function of d/D and r/d only, and a combination of two separate effects is likely to determine the value of stress concentration factor. He described the two effects qualitatively as : (i) The diffusion of the axial stress around the blend radius. (ii) The rate at which the general level of stress decreases in passing from the the smaller diameter shaft to

the larger in the axial direction. According to Allison for the larger values of d/D , i.e. small shoulder depths, the first effect will be predominant, since the the general level of stress in the small and large diameter shafts does not change excessively, and the axial stress will diffuse almost completely around the blend radius. The axial stress on the surface of the large diameter shaft increases from zero, at the shoulder, to its maximum value in a fairly short length of the shaft and the value of the stress concentration factor will be almost entirely a function of the blend radius alone. As d/D is decreased the second effect becomes increasingly important especially as the decrease in stress level at a particular radius, for bending or torsion is $(d/D)^4$. Allison explained that in the presence of large shoulders the second factor inhibits the diffusion of stress around the blend radius to such an extent that a decrease in the stress concentration factor is produced. Under these conditions the stress concentration factor, for a particular value of r/d , would increase initially as d/D is decreased, but would ultimately reach a limit and then decrease at a continuously increasing rate as the "inhibiting" factor becomes dominant.

(b) Stress Analysis of Plain Shafts with Keyways.

Using St. Venant's torsion theory, Filon[11] obtained an approximate theoretical solution for a shaft in torsion containing one or two keyways in the form of hyperbolic notches. With a similar approach Gronwall[17] and Shepherd[36] have studied further notch-

like keyways and Okubo[27] has investigated the practically important rectangular-sectioned keyway. Griffith and Taylor[15][16] used the soap-film technique for evaluating the St. Venant stress function for a hollow shaft with a rectangular-sectioned keyway and Leven[21] used a numerical method for a series of rectangular-sectioned keyways in solid shafts. Nisida and Hondo[26] employed their photoelastic-wedge method[25] in the evaluation of shear stresses in the prismatic section of a number of keyways. Moore[24], Gough[14], and Peterson[28] have investigated prototype shafts with keyways under a variety of loading conditions and Peterson concluded that 'the side-milled keyway was preferable to the end-milled type from the standpoint of fatigue. Timoshenko[40] used the brittle-coating method to investigate the cause of a fatigue crack at the tip of a keyway in a motor-generator shaft. On the basis of comparative tests, he concluded that the fatigue was due to excessive bending stresses, probably caused by bearing misalignment. Dorey[7] has tested two large shouldered shafts with keyways in torsional fatigue. Fessler, Rogers, and Stanley[9] used the frozen-stress photoelastic technique to determine the complete surface stress field in empty keyways of British Standard proportions for rectangular keyways. They carried out a detailed stress analysis on a series of plain solid shafts containing keyways and loaded in direct tension or compression, bending (with the keyway in the plane of bending) or torsion. They found that the position of the maximum stress for tension did not coincide with that for torsion. According to Fessler et al. the stresses due to bending can be obtained from those due to direct loading because the ratio of the measured

stresses to the stresses in the same fibre in the plain shafts is the same for both forms of loading. They showed that there is a close correspondence between the two different loadings if an allowance is made for the stress gradient in the bending case. This they did by scaling down the stress indices for the tension model by the factor $(d-2k)/d$. Thus the bending stress indices can be derived from the direct loading ones by multiplying the latter by the factor $(d-2k)/d$. This modification produced an excellent agreement between the high-stress indices and also the peak-stress indices. The authors were therefore satisfied that the bending-stress indices, particularly the more important higher ones, could be safely predicted from the direct-loading data. They also concluded that for torsion as well as direct loading the maximum stress index at the upper edge of the keyway wall was independent of r/d over the range they covered. In direct loading the peak stress index was always in the fillet at the end of the keyway.

(c) Stress Analysis of Shouldered Shafts with Keyways.

Limited work has been done on the stress analysis of a shaft having a keyway end near a shoulder. Timoshenko[40] used the brittle-coating technique to determine the position and direction of fatigue cracks at the end of such a keyway, when the shaft was subjected to bending or torsion. He compared these cracks with a crack from an actual service failure of a motor-generator shaft, and concluded that the service failure was probably due to bearing misalignment. Fessler, Rogers, and Stanley[10] performed

photoelastic stress analysis on a series of shouldered shafts with keyways in the shank subjected to direct loading as well as torsion. For all models two identical diametrically opposite keyways were cut in the shank. They varied the shoulder-fillet ratio (shoulder-fillet radius/shank diameter) and the distance between the keyway end and shoulder but both the ratio of the shoulder diameter to the shank diameter and the ratio of the keyway-fillet radius to the shank diameter were held constant. It was shown for both forms of loading that variations in the shoulder-fillet ratio had no significant effect on the greater principal stresses in the keyway fillet when the keyway end touched the shoulder-fillet toe. The shoulder-fillet ratio had very little effect on the stresses at the top of the wall in direct loading, but in torsion changes in this quantity cause appreciable stress increases. Fessler et al. also studied the dependence of the keyway stresses on the distance between shoulder fillet and keyway end; important stress reductions occurred when the keyway end was continued through the shoulder fillet and into the shoulder proper. For direct loading the fillet stresses were the more important and they were greatest when the keyway end was positioned in the shoulder fillet. The presence of the keyway nowhere increased the shoulder-fillet stress indices above those which would be expected in a shouldered shaft without a keyway.

The important conclusions that resulted from the investigations of Fessler, Rogers, and Stanley are listed below:

- (i) Variation of the shoulder-fillet ratio r/d had no significant effect on the stresses in the keyway fillet.

- (ii) Variation of r/d had no significant effect on the maximum top-of-the-keyway-wall stresses in direct loading but increases those due to torsion by up to 20 percent above the plain-shaft value.
- (iii) When the shoulder-fillet toe is more than one third of the shaft diameter from the keyway end the shoulder does not effect the keyway-end stresses.
- (iv) The shoulder-fillet stresses are never increased by the keyway; they are reduced in line with the keyway for small values of a/d .
- (v) The greatest stresses in the keyway end always occur in the fillet for direct loading. In torsion the maximum stress at the top of the wall is greater than the maximum fillet stress except when the keyway is cut well into the shoulder.
- (vi) The stresses at the ends of the keyways cut well into the shoulder are less than those present when the keyway terminates in the shank near the shoulder fillet or just inside the shoulder fillet.

2. FINITE ELEMENT ANALYSIS TECHNIQUE.

2.1 MODEL ANALYSIS.

The model analysis of a stepped shaft with keyway is complex in nature because of the three dimensional geometry and the the curved and discontinuous boundary of the model. Analytical solutions are possible for problems having simple geometry and simplified external loading configurations. The stress analysis of a solid body involves the solution of a set of mathematical equations which govern the behaviour of the material subjected to mechanical loading. However, geometric irregularities and three dimensional nature of the problem make these solutions impractical. Simplified assumptions can be made but these may effect the reality of the model.

An alternate to the classical methods is to utilise approximate techniques involving computer and numerical analysis. The most widely used among these methods is the finite element analysis using high speed computers to perform a very large number of matrix operations. It is a numerical technique in which a structural part is divided into small but finite elements. These elements are then superimposed onto a grid system, where identifiable points of the elements, called nodes, are referenced with respect to a coordinate system. The finite element method provides a systematic technique for deriving the approximation functions for simple subregions by which a geometrically complex region can be represented.

2.2 GENERAL METHOD.

During the past two decades, the finite element method of analysis has become a very popular technique for the solution of complex problems in engineering. The success of this method is based largely on the basic finite element procedures used: the formulation of the problem in variational or weighted residual form, the finite element discretization of this formulation, and the effective solution of the resulting finite element equations. These basic steps are the same whichever problem is considered and provide a general framework and - in conjunction with the use of the digital computer - a quite natural approach to engineering analysis. The final result is a complete numerical process implemented on the digital computer: the formulation of the finite element matrices, the numerical integration to evaluate the matrices, the assemblage of the element matrices into the matrices that correspond to the complete finite element system, and the numerical solution of the system equilibrium equations.

The finite element procedure is endowed with two basic features which account for its superiority over other competing methods. First, a geometrically complex domain of the problem is represented as a collection of geometrically simple subdomains, called finite elements. Second, over each finite element the approximation functions are derived using the basic idea that any continuous function can be represented by a linear combination of algebraic polynomials. The approximation functions are derived using concepts

from interpolation theory, and are therefore called interpolation functions. Thus, the finite element method can be interpreted as a piecewise application of the variational methods, in which the approximation functions are algebraic polynomials and the undetermined parameters represent the value of the solution at a finite number of preselected points, called nodes, on the boundary and the interior of the element. The order of the interpolation functions depends on the number of nodes in the element.

The development of the finite element method as an analysis tool essentially began with the advent of the electronic digital computer. It was initially developed on a physical basis for the analysis of problems in structural mechanics; however it was soon recognized that the method can be applied equally well to the solution of many other classes of problems. In 1941 Hrenikoff introduced the so-called framework method, in which a plane elastic medium was represented as a collection of bars and beams. The use of piecewise continuous functions defined over a subdomain to approximate the unknown function dates back to the work of Courant (1943), who used an assemblage of triangular elements and the principle of minimum potential energy to study the St. Venant torsion problem. Although certain key features of the finite element method can be found in the works of Hrenikoff (1941) and Courant (1943), the formal presentation of the finite element method is attributed to Argyris and Kelsey (1960) and to Turner, Clough, Martin, and Topp (1956). However the term "finite element" was first used by Clough in 1960. Since its inception, the literature on finite element applications has grown exponentially, and today there

are numerous journals which are primarily devoted to the theory and development of the finite element method.

Steps Involved In The Finite Element Analysis Of A Typical Problem.

Listed below are the basic steps involved in the finite element analysis of a problem.

a. Discretization of the given domain into a collection of pre-selected finite elements. These elements are connected at common nodal points and collectively approximate the shape of the domain.

(i) Construct the finite element mesh of preselected elements.

(ii) Number the nodes and elements.

(iii) Generate the geometric properties needed for the problem.

b. Derivation of element equations for all typical elements in the mesh.

(i) Construct the variational formulation of the given differential equation over the typical element.

(ii) Assume that a typical variable u is of the form

$$u = \sum_{i=1}^n u_i U_i$$

and substitute it into b.(i) to obtain element equations in the form

$$[K(e)] \{u(e)\} = \{F(e)\}$$

Ψ_i are the interpolation functions, K is called the stiffness matrix and F is called the force vector.

(iii) Derive or select, if already available in the literature, element interpolation functions Ψ_i and compute the element

matrices.

- c. Assembly of element equations to obtain the equations of the whole problem.
 - (i) Identify the interelement continuity conditions among the primary variables by relating element nodes to global nodes.
 - (ii) Identify the "equilibrium" conditions among the secondary variables.
 - (iii) Assemble element equations using steps c.(i) and c.(ii) and the property of superposition.
- d. Imposition of the boundary conditions of the problem.
 - (i) Identify the specified global primary degrees of freedom.
 - (ii) Identify the specified global secondary degrees of freedom (if not already done in step c.(ii)).
- e. Solution of the assembled equations.
- f. Postprocessing of the solution.
 - (i) Compute the gradient of the solution or other desired quantities from the primary degrees of freedom computed in step e.
 - (ii) Represent the results in tabular and/or graphic form.

2.3 ADVANTAGES AND DISADVANTAGES.

The finite element method has the following advantages:

- a. The material properties in adjacent elements do not have to be the same. This allows the method to be applied to bodies composed of several materials.

- b. Irregularly shaped boundaries can be approximated using elements with straight sides or matched exactly with curved boundaries. The method, therefore, is not limited to 'nice' shapes with easily defined boundaries.
- c. The size of the elements can be varied. This property allows the element grid to be expanded (in a region of non-interest) or refined (in a region of importance) as the need exists. Usually the grid has to be refined at the point of application of the loading.
- d. Boundary conditions such as discontinuous surface loadings present no difficulties for the method. Mixed boundary conditions can be easily handled.

The primary disadvantage of the finite element method analysis technique is the need for computer programs and computer facilities. The computations involved in the finite element analysis are too numerous for hand calculations even when solving very small problems. The availability of computer memory and the computations costs are the limiting factors when solving a problem.

3. MESH GENERATION, BOUNDARY CONDITIONS AND SOLUTION OF THE MODEL.

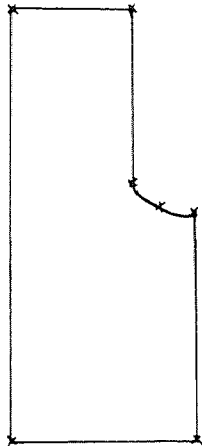
The GE CALMA software and ANSYS, a general purpose finite element analysis package, were utilised for the stress analysis of the three dimensional model of a shouldered shaft with keyway subjected to axial load. The model creation, mesh generation, application of boundary conditions and post processing was performed by using the GE CALMA package and the solution was carried out by means of ANSYS. A very large number of elements are required due to the following reasons:

- (a) The three dimensional nature of the model.
- (b) To closely approximate the circular shape of the shank and shoulder, the radial fillet, and the complicated keyway.
- (c) A refined mesh is needed at regions of interest and at the point of load application. To accomplish this a fine grid pattern has to be generated near the fillet and the keyway and also at the end of the shank where the load is applied.

The university version of ANSYS imposed a severe limitation on the investigation because it could not support the high number of elements needed to simulate the complicated shape of the model and give realistic results.

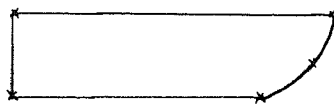
Described below is the method in which the model was created, mesh was generated, boundary conditions were applied, solution was carried out and post processing was performed.

Creating the Stepped Shaft: The GE CALMA package consists of GEOMOD, SUPERTAB and SUPERB. To run this package the letters IDEAS have to be entered. At any level of the program the menu can be seen by typing in **M** (**Menu**). A branch is made from **System Design (SD)** to **Object Modeling (O)** inside which is a module called **Task (TA)**. Task consists of **Profile 2-D (P)** using which a profile is generated by first creating points and then joining them by means of lines and curves. The dimensions of the model to be produced can be controlled by the shape of the profile which in turn depends on the coordinates of the points used to create the profile. The figure below shows the profile created.



Task has another module **Object 3-D (O)** in which the 2-D profile is revolved 360 degrees about the Y-axis to get the 3-D model of the stepped shaft without keyway.

Creating the Key: Using **Profile 2-D** another profile is created as shown.



In Object 3-D this profile is extruded in the Z direction to give a 3-D model of the key. The extrusion distance becomes the thickness of the key.

Creating Stepped Shaft with Keyway: In Object Modeling a Boolean (B) operation is performed which consists of cutting the stepped shaft with the key. Before that the shaft and key have to be aligned properly so that the Boolean cut gives the keyway at the desired position.

Cutting the Model in Half: Since the 3-D model of the stepped shaft with keyway is symmetrical about the X-Y plane, only half of it need be analysed. To cut the model in half another Boolean operation is performed using the Plane Cut command. Plane specification method used is XY Plane and the Positive Side is chosen as the desired output.

Geometry Transfer: An exit is made from the System Design family and at this point an entry is made into the Finite Element Method (FEM) family. Using the Transfer Geometry command the geometry of the cut stepped shaft is transferred from Object Modeler to Enhanced Mesh Generation (EMG).

Creating Subareas, Subvolumes: In the EMG module the necessary points, curves, subareas and subvolumes are created. When creating subareas the angle subtended by any curve should not exceed 120 degrees (limitation imposed by SUPERTAB). In order to overcome this some curves have to be broken down into smaller segments. The manner in which the model is going to be divided into subvolumes is

decided. Then additional points and curves are created that are needed to define subareas and subvolumes. A subarea is defined by curves, a subvolume by subareas and all the subvolumes together constitute the model. 30 points and 59 curves had to be created for Model 1 whereas Model 2 required 25 points and 45 curves. When creating each subarea the curves that define it have to be entered. Similarly the subareas that make up each subvolume have to be entered when creating the subvolume.

Mesh Generation: The model is now ready for mesh generation. The mesh density at any particular region in the model can be controlled. This can be done by various ways,

- (a) By specifying the number of elements on a particular curve.
- (b) By giving a length multiplier.
- (c) By specifying local element lengths at particular points.

Method (c) is used to generate a mesh that has a fine grid pattern at regions of interest and a coarse grid (less elements) at other regions that are not very crucial for analysis.

Before generating the mesh the geometry is checked by checking the nodes on boundaries and the subarea curvature. The geometry is also checked for free boundaries.

For both the models the element chosen is a solid linear tetrahedron. The physical constants and the material properties associated with the element are entered (otherwise the default values are used).

The command for generating the mesh is then given. The package first generates a 2-D mesh on the outer surface of the model. With the help of a module called the Mesh Previewer the number of solid

elements that are going to be generated is estimated. The Mesh Previewer also allows the 2-D mesh to be examined in various ways. If after previewing the mesh it is decided to go ahead then the entire mesh is generated. Even at this point there is still the option of discarding or retaining the mesh. After experimenting with a number of meshes, the mesh which best satisfied the required conditions was retained. Model 1 consists of 125 nodes and 323 elements; Model 2 comprises of 118 nodes and 279 elements.

Boundary Conditions: A switch is made from the **EMG** module to the **Model Creation (MC)** module. Here the desired loads are applied to the model and nodes that have to be fixed in a particular direction are restrained. Loading can be given in the form of elemental pressure or nodal forces. Displacements of nodes can be restrained in the X, Y or Z directions and their X, Y or Z rotations can also be restrained.

Solution and Post processing: Using the **File Translator (FT)** in **EMG** it is possible to Write an **ANSYS** input file. This file contains the data in a form compatible with the **ANSYS** package and also the necessary commands to run the program using **ANSYS**. An exit is made from the **GE CALMA** package and the **ANSYS** input file is edited by adding the necessary starting and ending **JCL**. The input file is submitted to **ANSYS** for analysis by the command **@PUB\$ACS:IANSYS**. **ANSYS** solves the problem and creates a number of files one of which is **FILE12.DAT** that contains the data of the stresses and displacements.

SDRC I-DEAS 3.0: OBJECT MODELING 15-APR-87 01:20:44
DATABASE: VIEW: NO STORED VIEW UNITS = MM
TASK: OBJECT DISPLAY: NO STORED OPTION
OBJECT: 2-CUT_SHAFT BIN: 1-MAIN

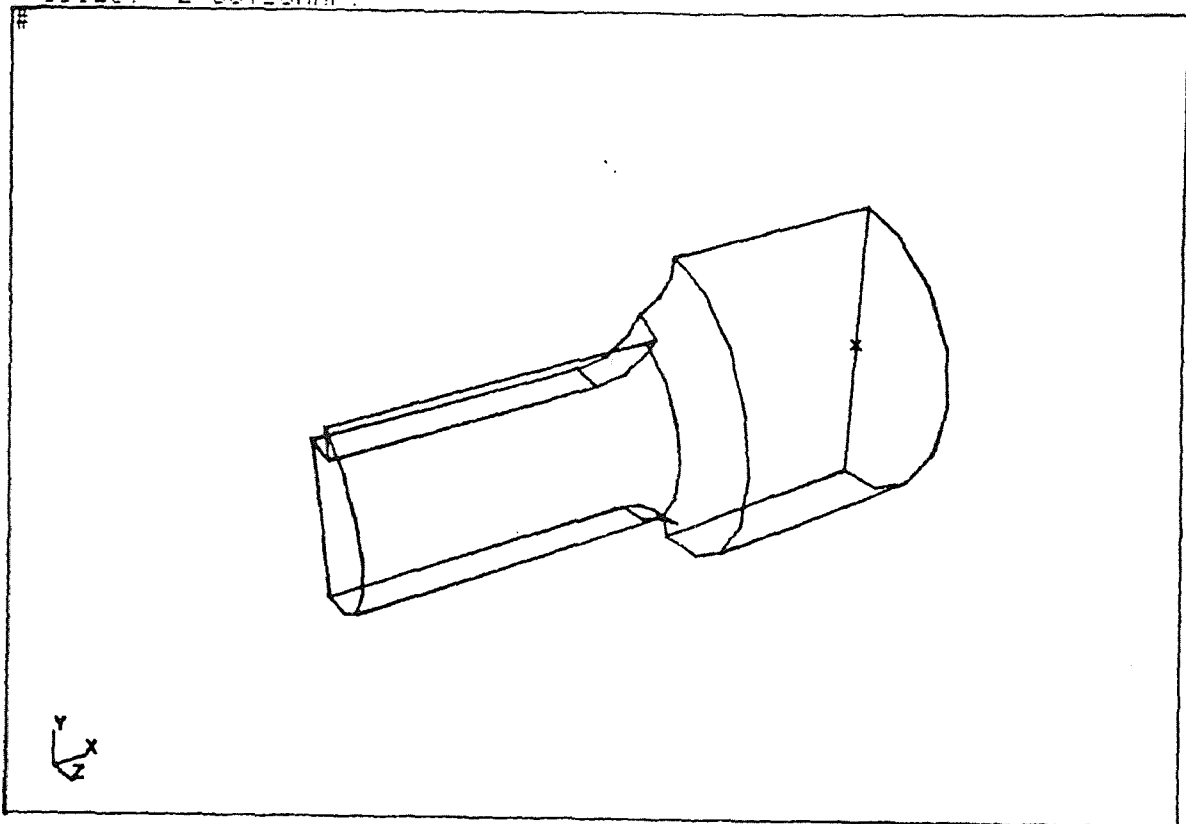


Fig.1 Plot of Model 1

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 01:32:17
UNITS = MM
DISPLAY: NO STORED OPTION

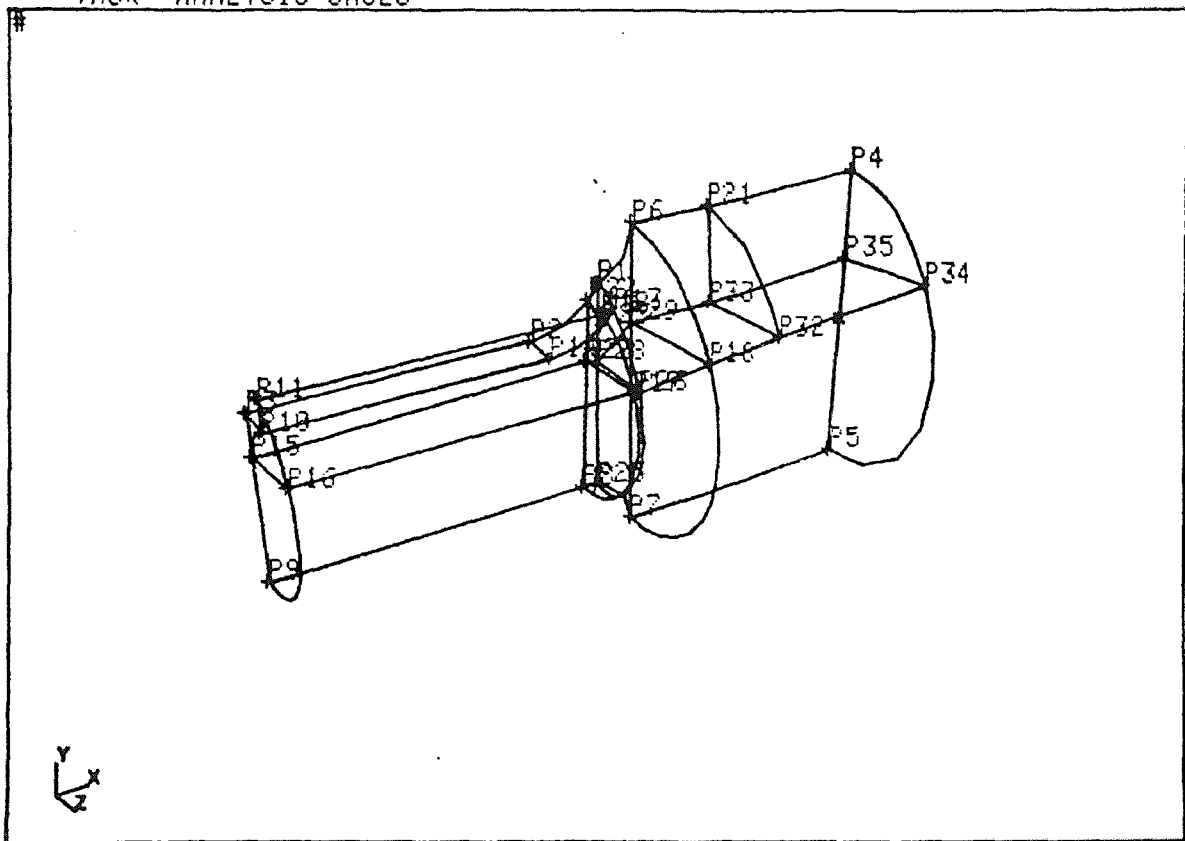


Fig.2 Model 1 : Points for generating curves

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 01:34:22
UNITS = MM
DISPLAY: NO STORED OPTION

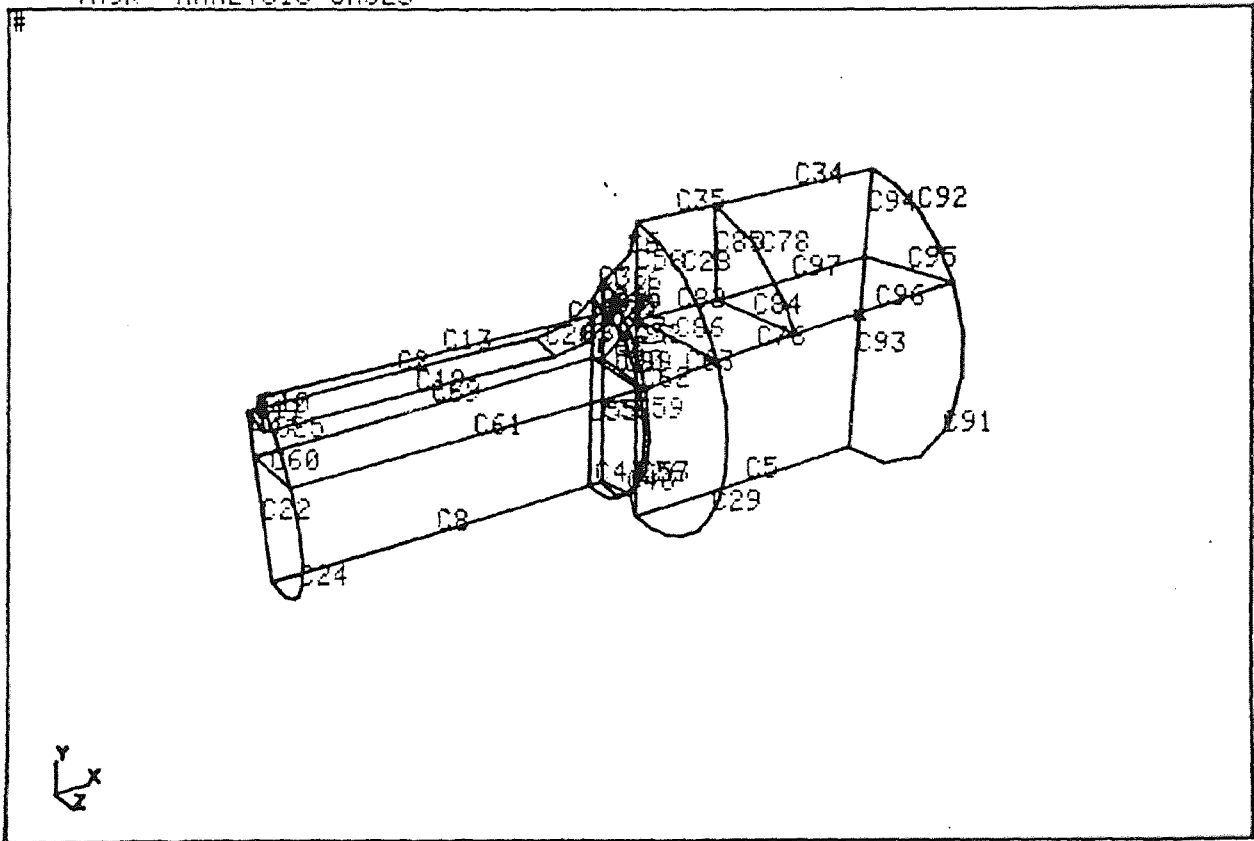


Fig.3 Model 1 : Curves for defining the sub-areas

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: FREE MESH GEOMETRY

15-APR-87 01:45:49
UNITS = MM
DISPLAY: NO STORED OPTION

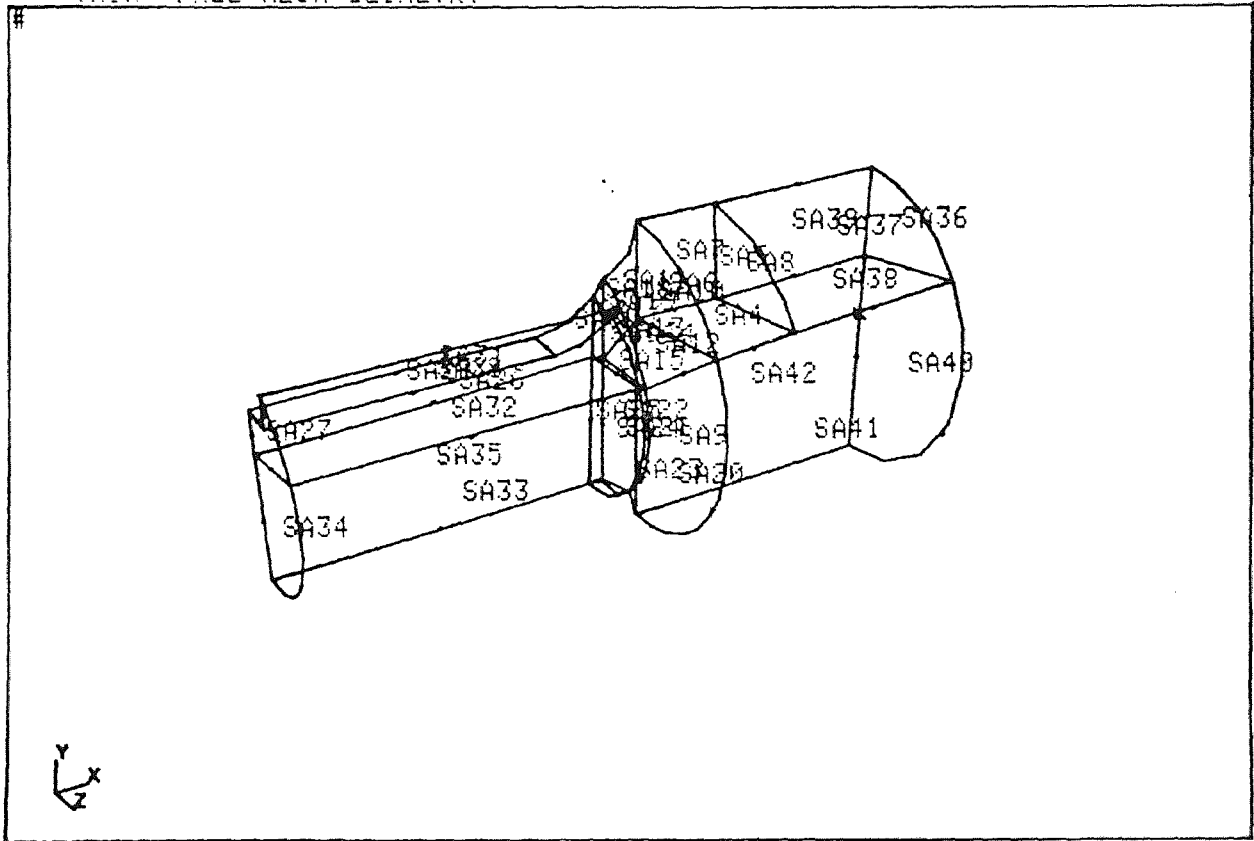


Fig.4 Model 1 : Sub-areas which make up the sub-volumes

SORC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: FREE MESH GEOMETRY

15-APR-87 01:46:56
UNITS = MM
DISPLAY: NO STORED OPTION

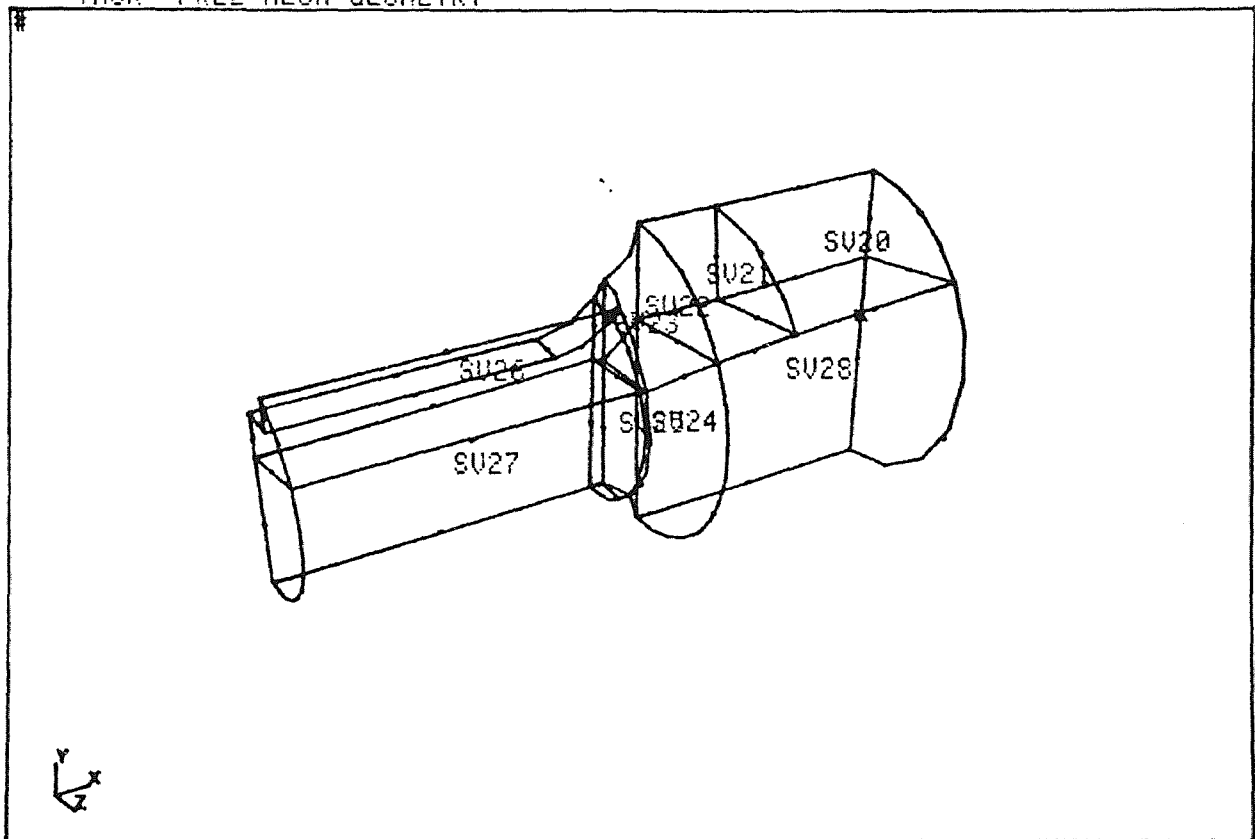


Fig.5 Model 1 : Sub-volumes which constitute the entire model

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: FREE MESH GEOMETRY

15-APR-87 01:49:52
UNITS = MM
DISPLAY: NO STORED OPTION

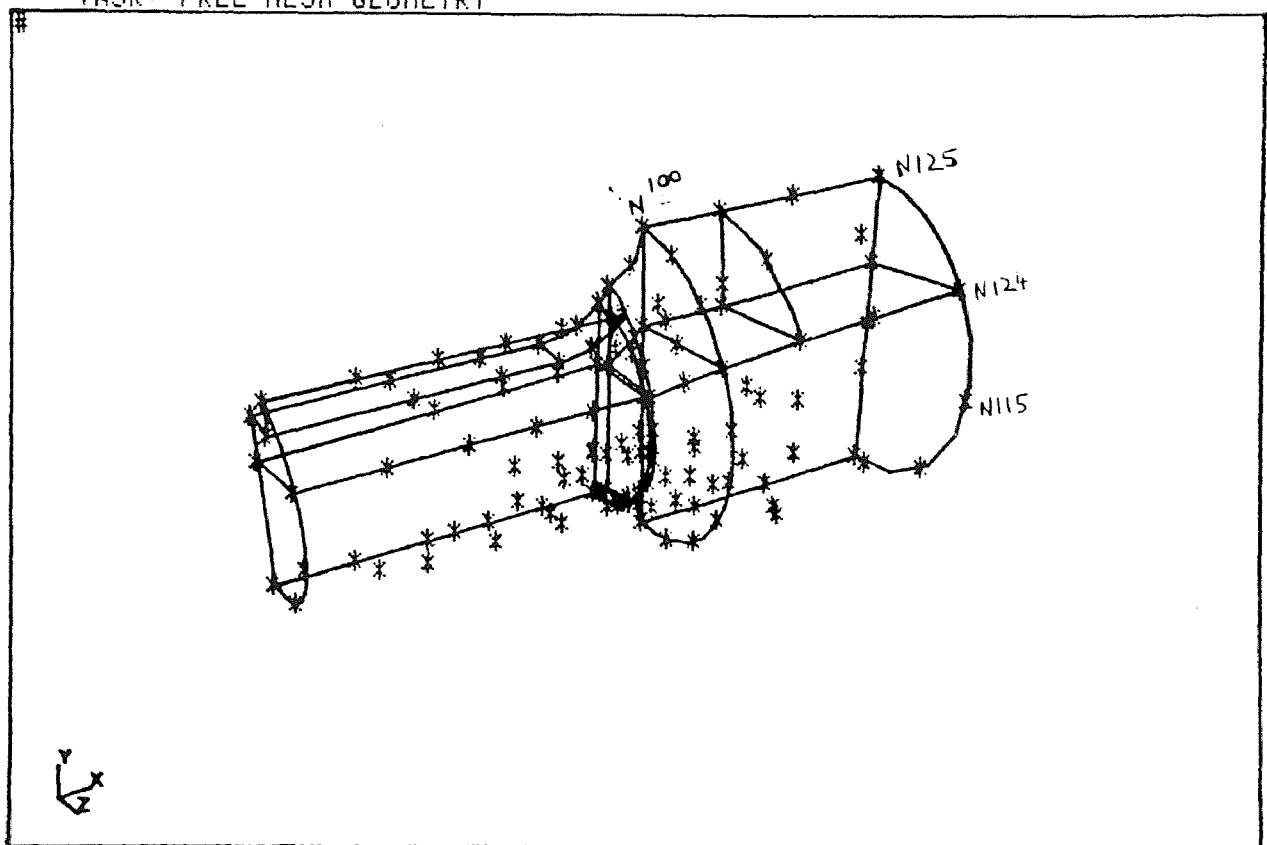


Fig.6 Model 1 : Nodal Plot (selected nodes numbered)

SORC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: FREE MESH GEOMETRY

15-APR-87 01:52:27
UNITS = MM
DISPLAY: NO STORED OPTION

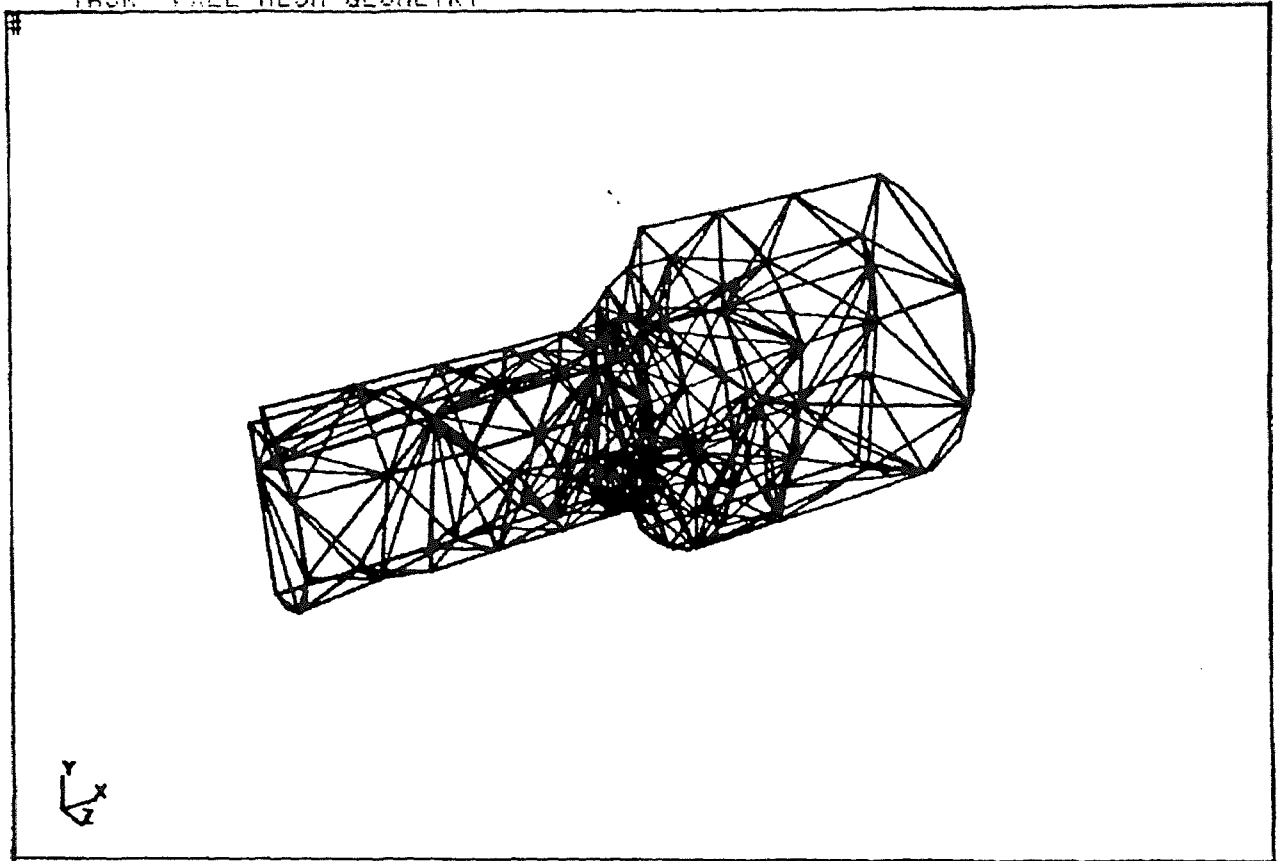


Fig.7 Model 1 : Elemental Plot

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: FREE MESH GEOMETRY

15-APR-87 01:56:55
UNITS = MM
DISPLAY: NO STORED OPTION

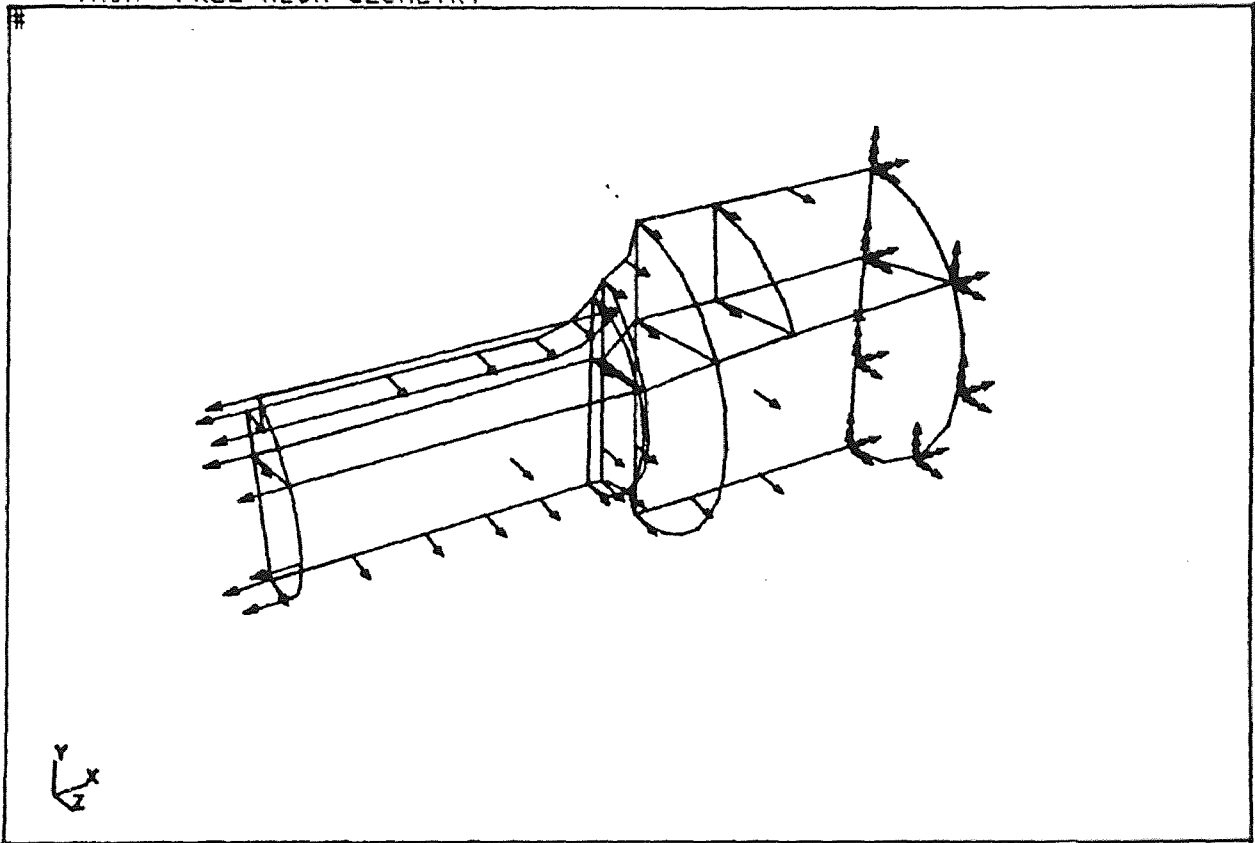


Fig.8 Model 1 : Restraints and loads

SORC I-DEAS 3.0: OBJECT MODELING 15-APR-87 02:08:46
DATABASE: AFT WITH KEY UNITS = MM
VIEW: NO STORED VIEW DISPLAY: NO STORED OPTION
TASK: OBJECT BIN: 1-MAIN
OBJECT: 3-SHAFT WITH KEY

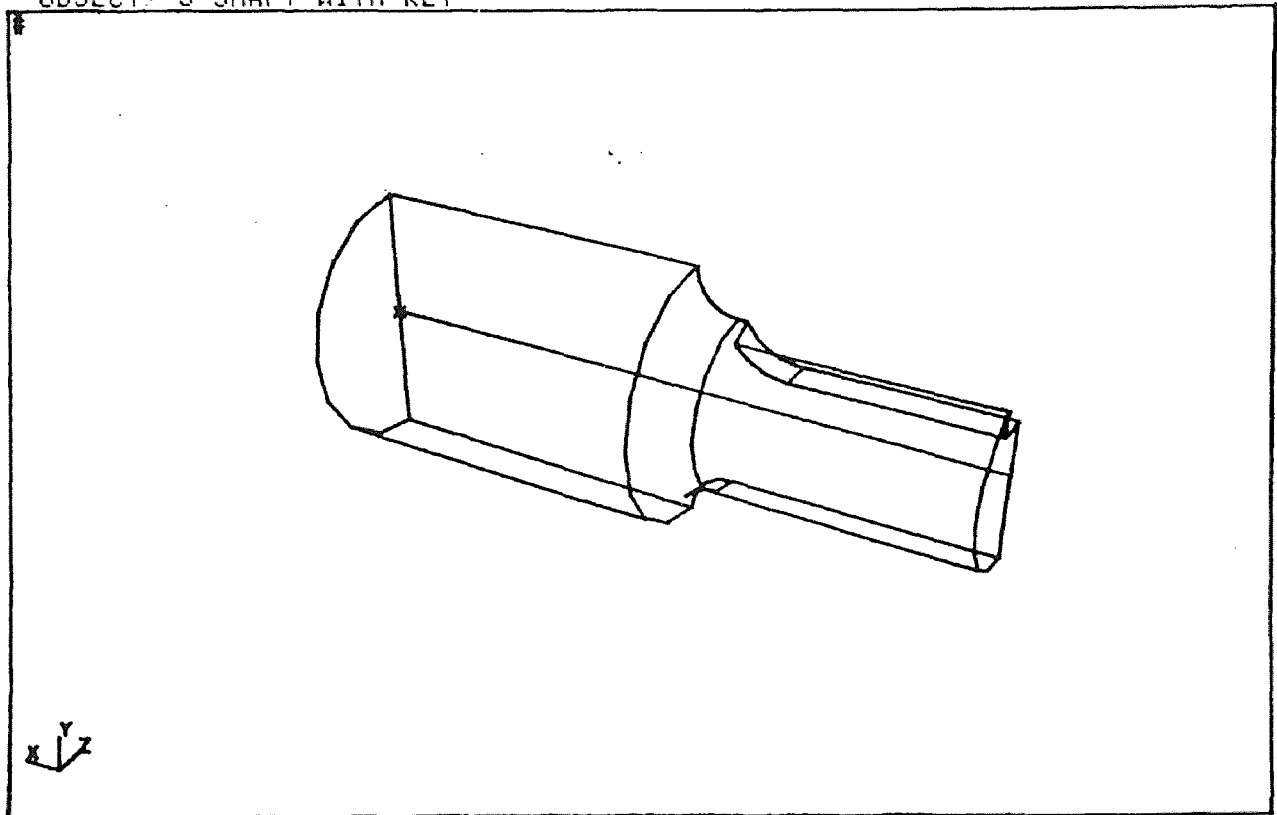


Fig.9 Plot of Model 2

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: APT WITH KEY
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 02:14:12
UNITS = MM
DISPLAY: NO STORED OPTION

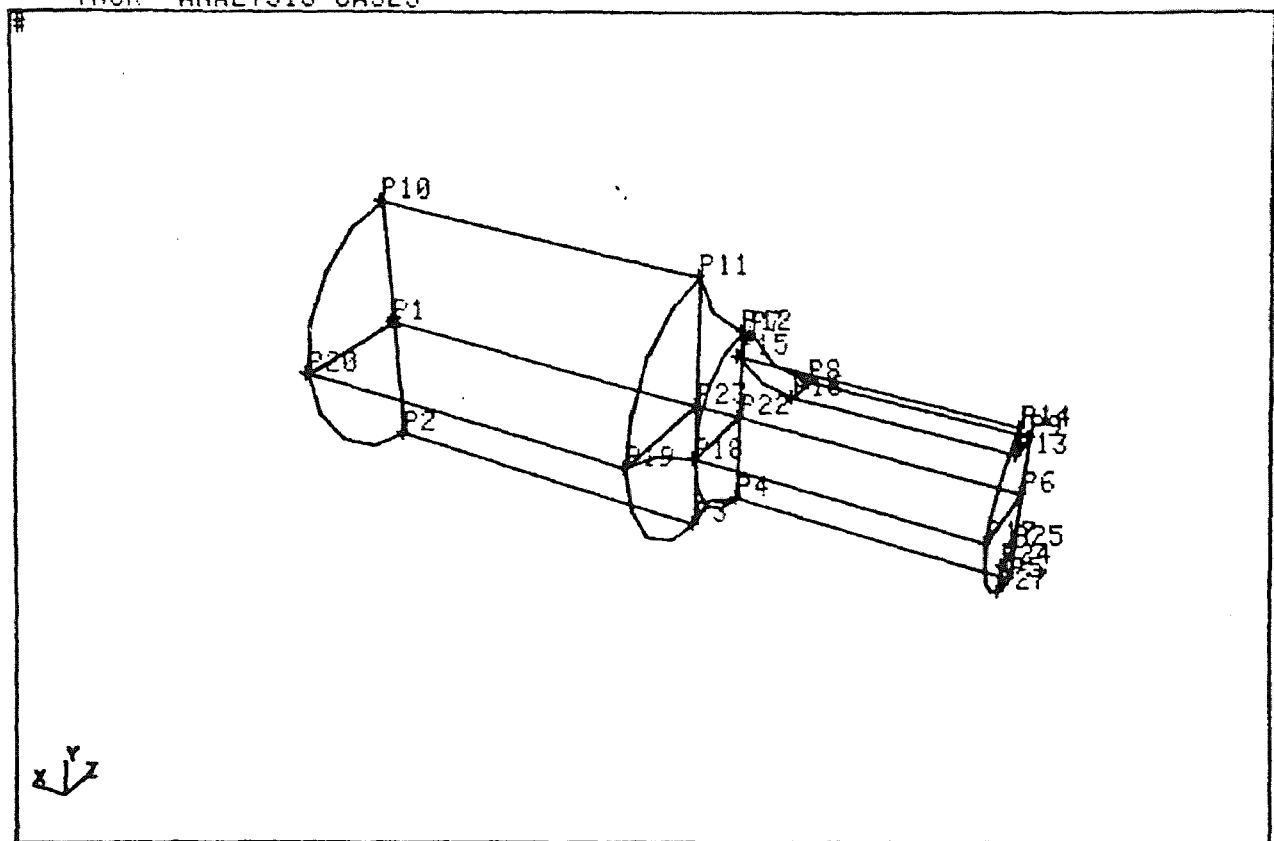


Fig.10 Model 2 : Points for generating curves

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: AFT WITH KEY
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 02:15:07
UNITS = MM
DISPLAY: NO STORED OPTION

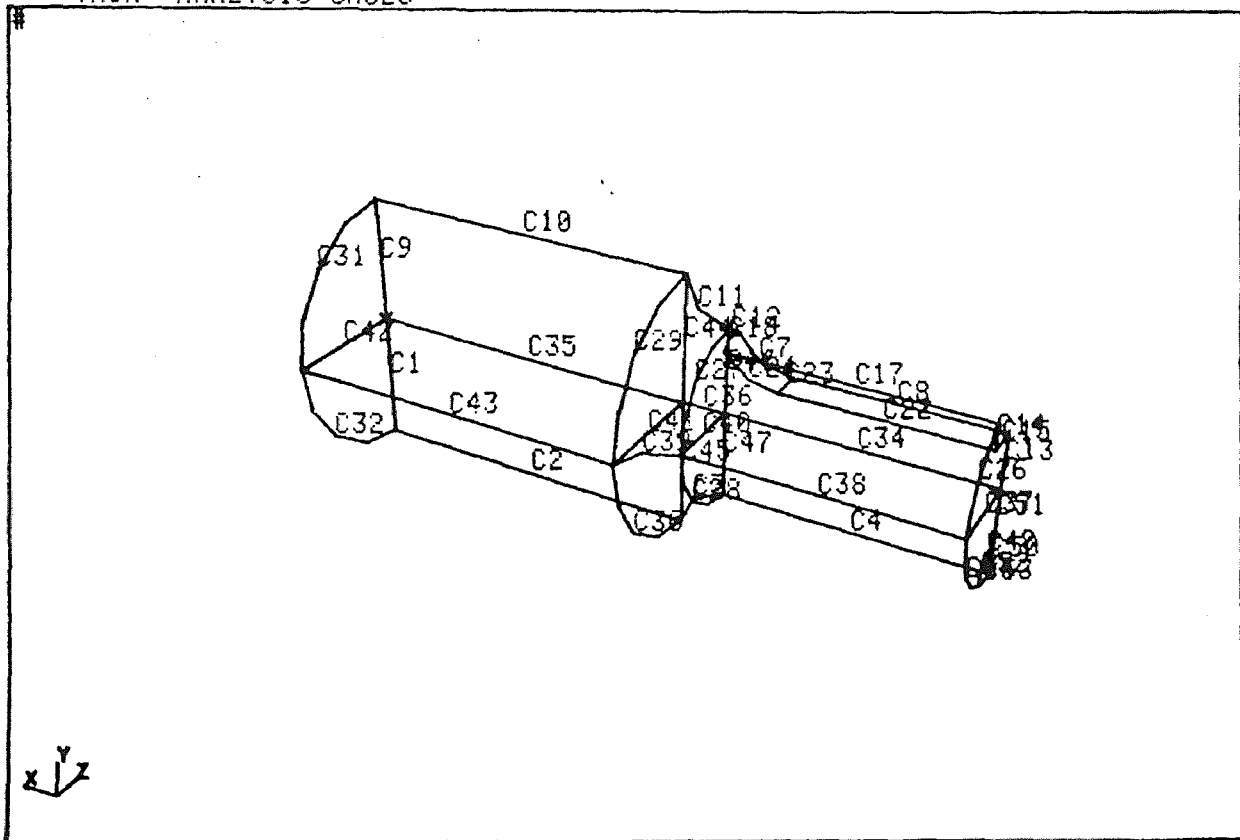


Fig.11 Model 2 : Curves for defining the sub-areas

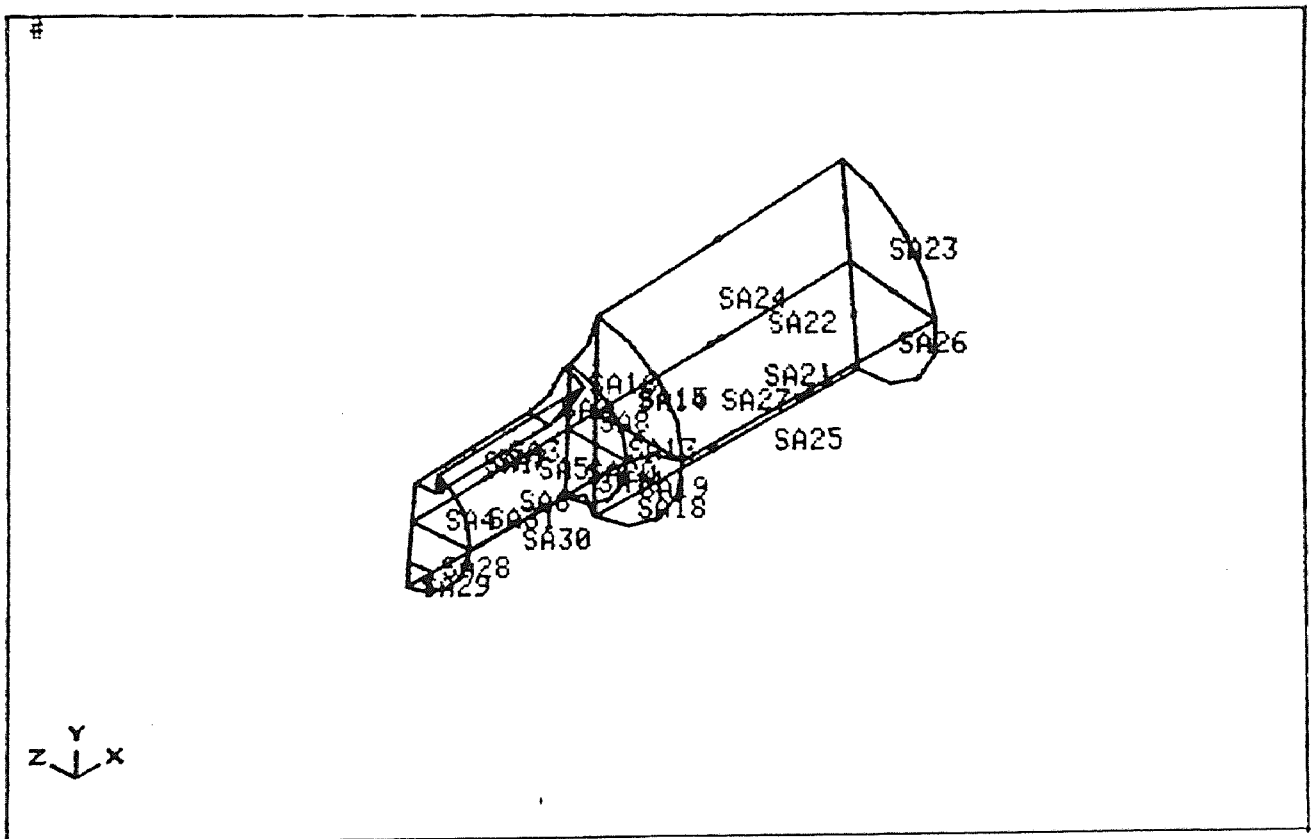


Fig.12 Model 2 : Sub-areas which make up the sub-volumes

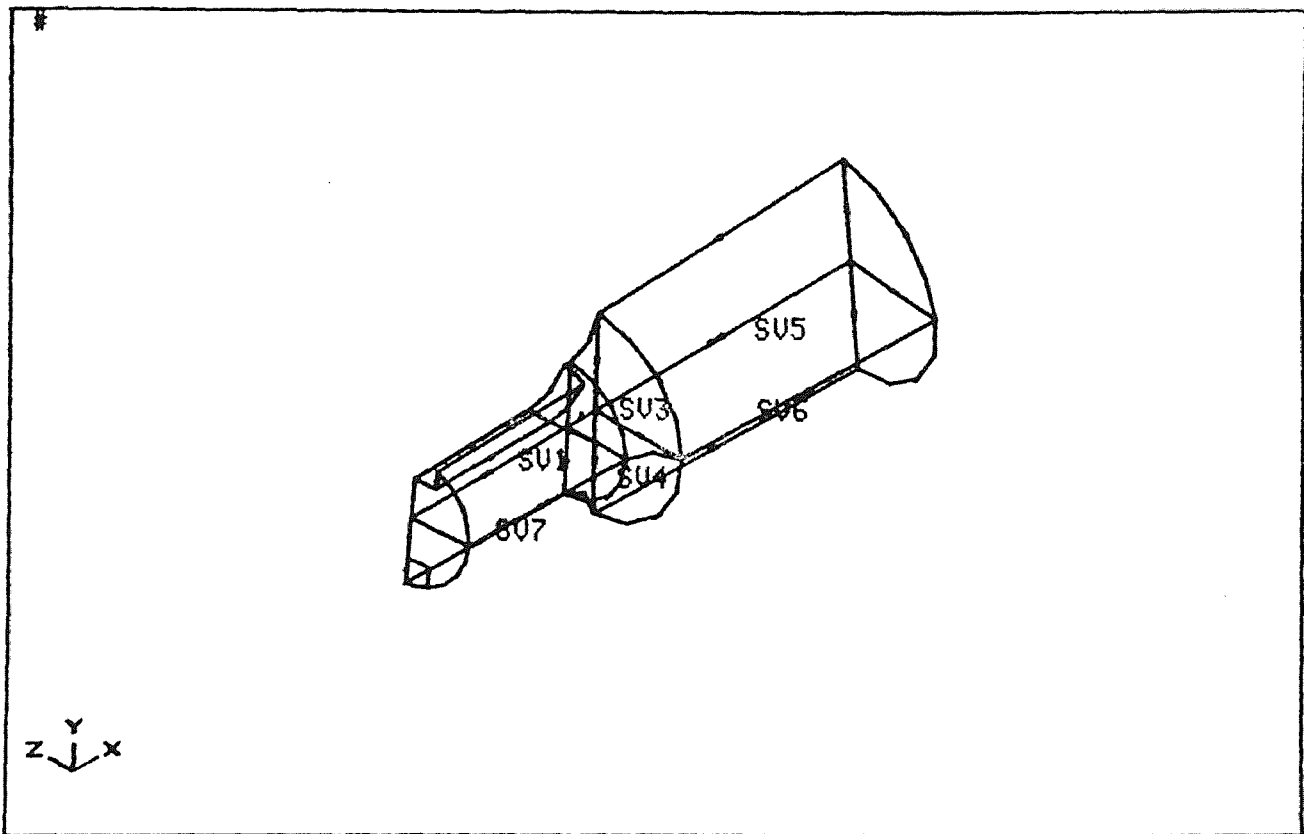


Fig.13 Model 2 : Sub-volumes which constitute the entire model

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: AFT WITH KEY
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 02:18:45
UNITS = MM
DISPLAY: NO STORED OPTION

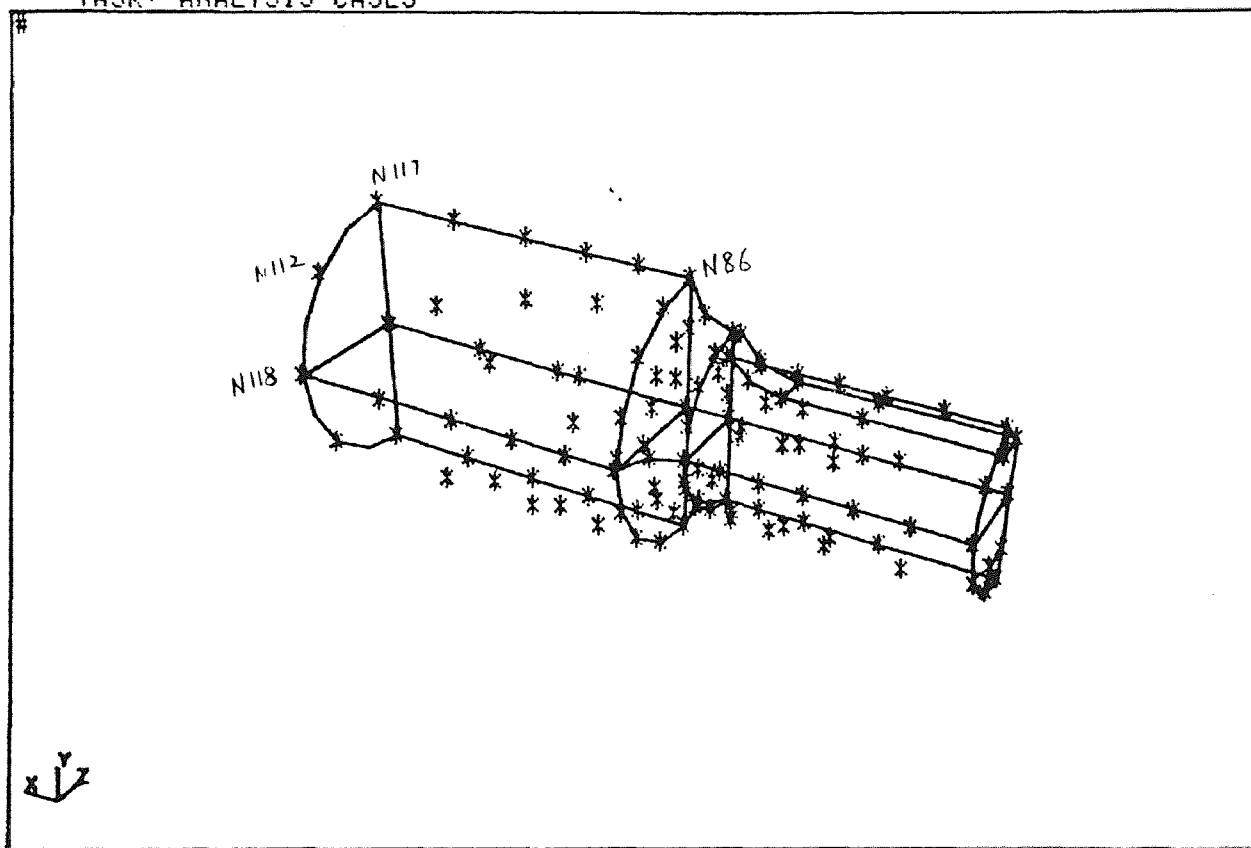


Fig.14 Model 2 : Nodal Plot (selected nodes numbered)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: AFT WITH KEY
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 02:21:56
UNITS = MM
DISPLAY: NO STORED OPTION

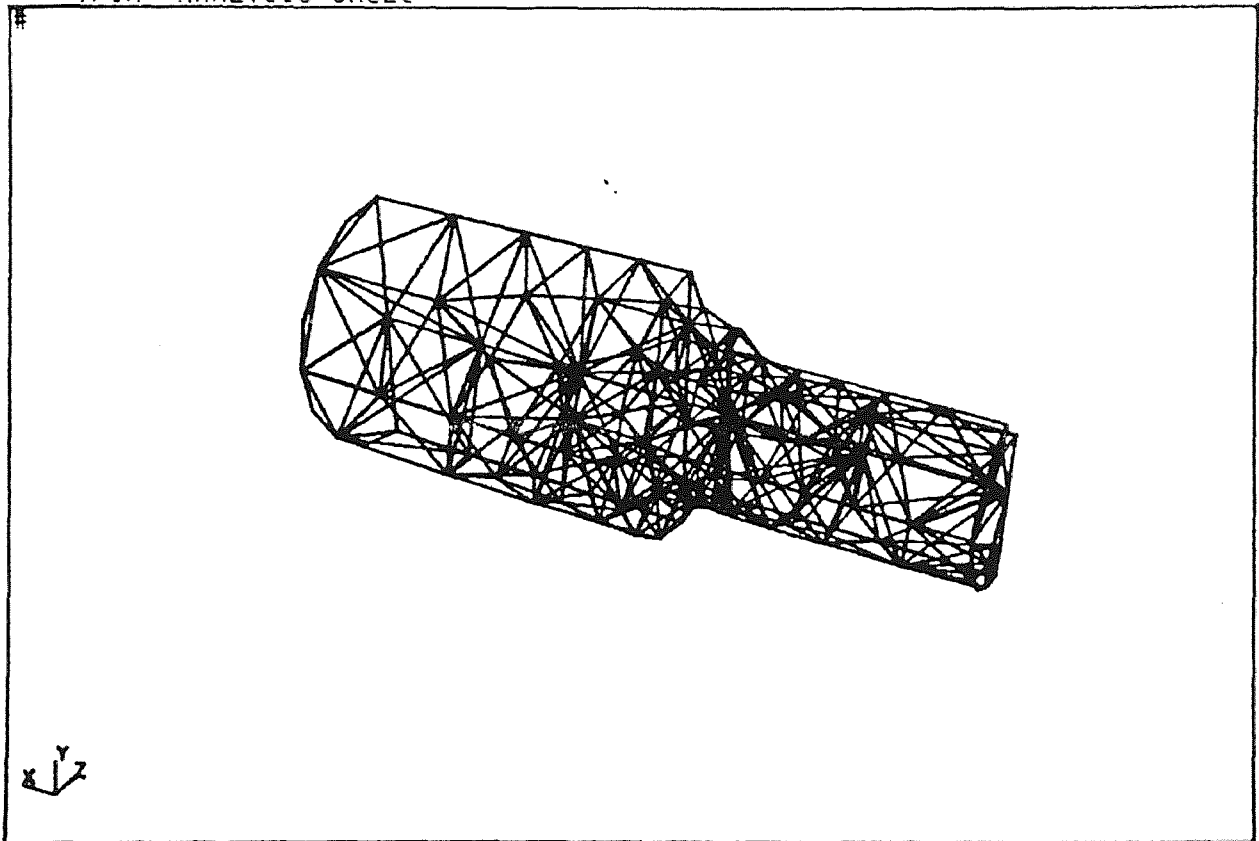


Fig.15 Model 2 : Elemental Plot

SORC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: AFT WITH KEY
VIEW: NO STORED VIEW
TASK: ANALYSIS CASES

15-APR-87 02:11:18
UNITS = MM
DISPLAY: NO STORED OPTION

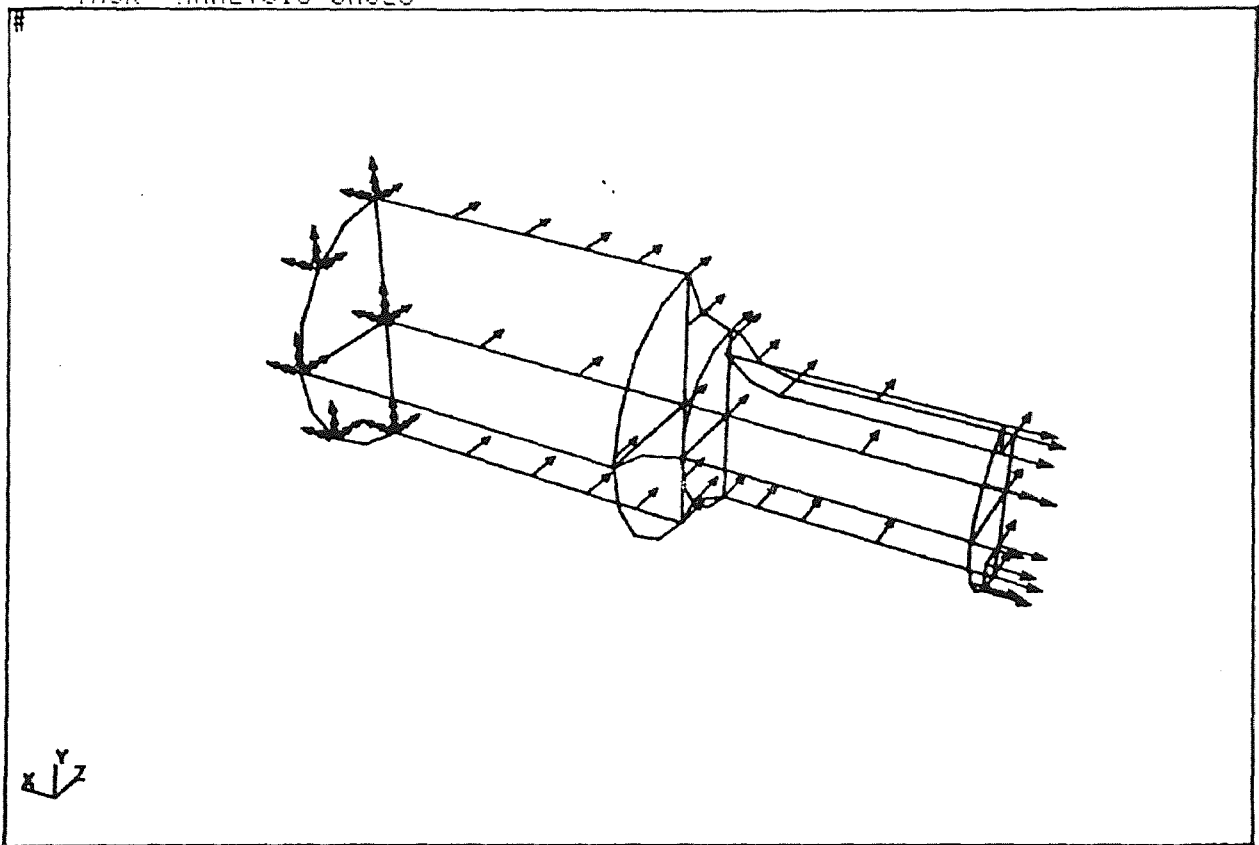


Fig.16 Model 2 : Restraints and loads

The GE CALMA package is entered again and a Universal File of the ANSYS output file (FILE12.DAT) is created by a module called Data Loader (DL) in the FEM family. This Universal File is Read into the SUPERTAB file by means of the File Translator. Now the Output Display (OD) module of FEM is utilized. Here the displacement plots and the stress plots are viewed. The deformed shape of the model after the loading has been applied is observed and the stress contours for Von Mises stresses, maximum shear stresses, maximum principal stresses etc. are seen.

The results can also be examined by means of the ANSYS post processing facilities.

4. MODEL OF STEPPED SHAFT WITH KEYWAY.

Notation

a	Distance between shoulder-radius run out and nearest point of keyway end, measured positive along the shank.
b	Keyway width.
D	Shoulder diameter.
d	Shank diameter.
I	Stress index, i.e. measured stress/nominal stress.
k	Keyway depth.
L	Length of the shoulder.
l	Length of the shank.
P	Direct load.
r	Fillet radius.
Diameter ratio	D/d
Fillet ratio	r/d
Shank	The smaller section of shaft.
Shoulder	The bigger section of shaft.

<u>Quantity</u>	<u>Units</u>
Distance	mm
Area	mm ²
Load	milliNewton
Pressure	milliNewton/mm ²

4.1 MODEL 1 : KEYWAY CUTTING INTO SHOULDER FILLET.

Model 1 consists of a stepped shaft with a single keyway, the keyway end cutting into the fillet. The axial position of the keyway end relative to the shoulder was defined by 'a', the length of the cylindrical portion between the run-out of the shoulder radius and the nearest point of the keyway end. A negative value of 'a' was chosen for Model 1 which indicates that the keyway end extends into the fillet. The shoulder-diameter ratio (D/d) was fixed as 1.5 because larger ratios are rarely used for highly stressed components. Most of the results for stress concentration factors in shouldered shafts without keyways had been obtained for this or for smaller shoulder-diameter ratios. The shank and shoulder were made large enough to avoid end effects in the region of the shoulder fillet and the keyway end. The details of Model 1 are given below.

$$a = -3.2662 \text{ mm}$$

$$b = 15 \text{ mm}$$

$$D = 60 \text{ mm}$$

$$d = 40 \text{ mm}$$

$$k = 6.0405 \text{ mm}$$

$$L = 50 \text{ mm}$$

$$l = 60 \text{ mm}$$

$$r = 10 \text{ mm}$$

$$D/d = 1.5$$

$$r/d = 0.25$$

The model is symmetrical about the X-Y plane, therefore it was cut and only half of it was analysed. The displacements of all the nodes at the end of the big shaft were fixed in the X, Y, and Z directions and an axial tensile load of $P = 1 \times 10^8$ milliNewtons was applied to the end of the shank. All nodes on the cut surface were restrained to move in the Z direction. Shown below are the calculations for the nodal forces in order to produce a load of 1×10^8 mN at the end of the shank. These calculations are based on the location of the nodes and the number of elements connected to it. Fig.17 shows the nodes and element faces at the end of the shank.

Cross-sectional area of shank = 585.21089

Pressure acting on shank end = $1 \times 10^8 / 585.21089$
 $= 170878.57$

Face A

Area = 59.53545

Distance of node from center of area

Node	Distance
2	9.650
4	4.266
7	9.460

Load on face A = $PA = (170878.57) \times (59.53545)$
 $= 10173333$

The load on face A is shared by nodes 2, 4 and 7. The magnitudes of these nodal forces depend on the distance of the node from the center of area and can be determined as follows :

SDRC I-DEAS 3.0: OBJECT MODELING
DATABASE:
VIEW: NO STORED VIEW
TASK: PROFILE
PROFILE: 1-PROFILE1

23-APR-87 23:03:00
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DISPLAY: NO STORED OPTION
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SKIN: NO CURRENT SKIN GROUP EXIST

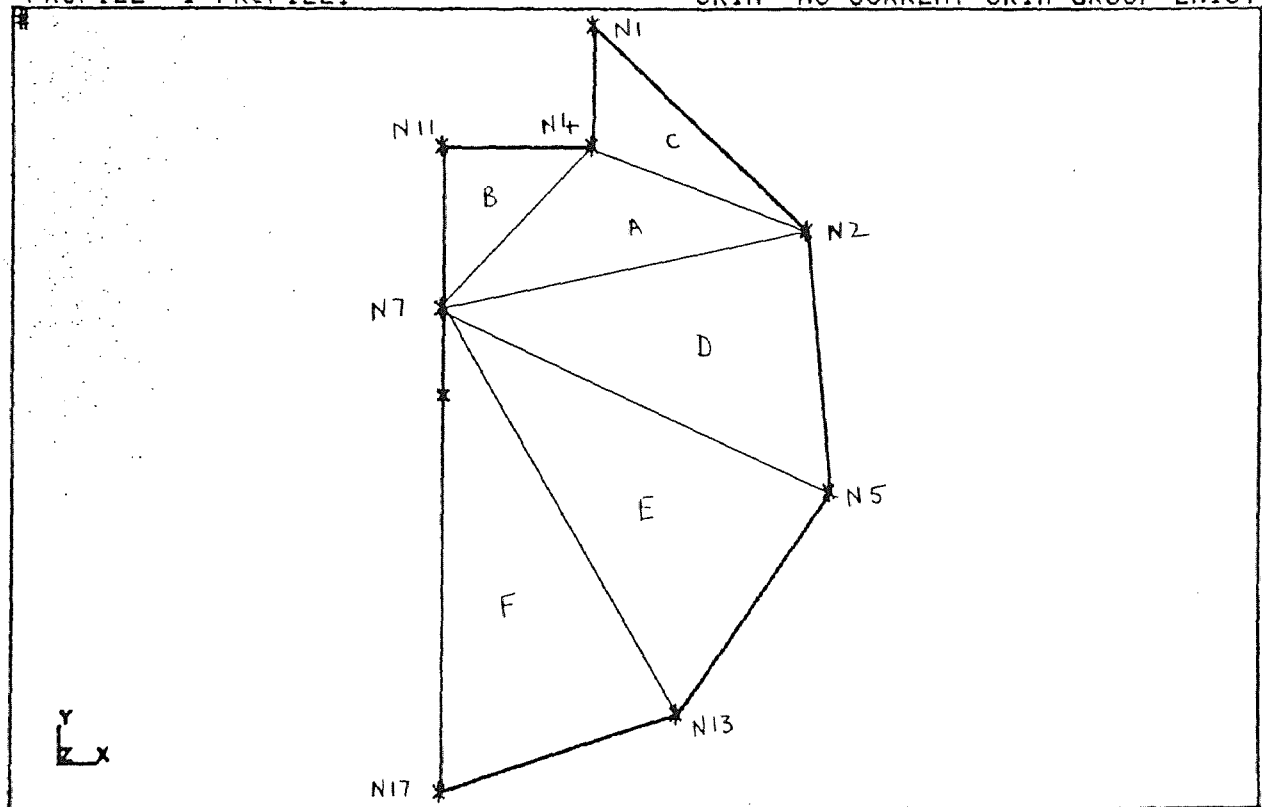


Fig.17 Model 1 : Element faces at end of small shaft

$$(9.65)PA_2 = (4.266)PA_4 = (9.46)PA_7$$

$$\text{Therefore } PA_4 = (2.262)PA_2$$

$$PA_7 = (1.02)PA_2$$

$$\text{Also } PA_2 + PA_4 + PA_7 = PA$$

$$(1 + 2.262 + 1.02)PA_2 = 10173333$$

$$\text{Hence } PA_2 = 2375836.7$$

$$PA_4 = 5374142.6$$

$$PA_7 = 2424638.5$$

Similar calculations were done for all the faces and the following values of nodal forces were obtained.

Face B

$$PB_4 = 1489261.1$$

$$PB_7 = 1419265.8$$

$$PB_{11} = 4762074$$

Face C

$$PC_1 = 4141607.9$$

$$PC_2 = 1174782.6$$

$$PC_4 = 2801404.7$$

Face D

$$PD_2 = 8613610.6$$

$$PD_5 = 6821979.6$$

$$PD_7 = 5469642.7$$

Face E

$$PE_5 = 9984432.1$$

$$PE_7 = 6260238.9$$

$$PE_{13} = 8426860.7$$

Face F

$$PF7 = 5931402.1$$

$$PF13 = 9543626$$

$$PF17 = 9015731.2$$

Total Nodal Forces

$$P1 = PC1$$

$$= 4141607.9$$

$$P2 = PA2+PC2+PD2$$

$$= 12164230$$

$$P4 = PA4+PB4+PC4$$

$$= 11370276$$

$$P5 = PD5+PE5$$

$$= 16806412$$

$$P7 = PA7+PB7+PD7+PE7+PF7$$

$$= 21505188$$

$$P11 = PB11$$

$$= 4762074$$

$$P13 = PE13+PF13$$

$$= 17970487$$

$$P17 = PF17$$

$$= 9015731.2$$

4.2 MODEL 2 : KEYWAY ENDING BEFORE SHOULDER FILLET.

In Model 2 the value of 'a' was chosen to be positive so that the keyway end terminated before the fillet. The overall dimensions of the model are so chosen that the end effects of the loading system should not disturb the fringe pattern in the vicinity of the fillet and the keyway end. The details of Model 2 are given below.

$$a = 1.9413 \text{ mm}$$

$$b = 15 \text{ mm}$$

$$D = 60 \text{ mm}$$

$$d = 40 \text{ mm}$$

$$k = 6.0405 \text{ mm}$$

$$L = 80 \text{ mm}$$

$$l = 60 \text{ mm}$$

$$r = 10 \text{ mm}$$

$$D/d = 1.5$$

$$r/d = 0.25$$

The same restraints are applied to Model 2 as were applied to Model 1 and again a tensile load of $P=1 \times 10^8$ mN is applied at the end of the small shaft. Fig.18 shows the nodes and element faces at the end of the shank. The methodology used to calculate the nodal forces to produce a load of 1×10^8 mN at the end of the shank was the same as used in Model 1. Summarized below are the forces on nodes due to each face load and the total nodal forces.

SDRC I-DEAS 3.0: OBJECT MODELING
DATABASE:
VIEW: NO STORED VIEW
TASK: PROFILE
PROFILE: 2-PROFILE2

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DISPLAY: NO STORED OPTION
WORKSET: NO STORED WORKING_SET
SKIN: NO CURRENT SKIN GROUP EXIST

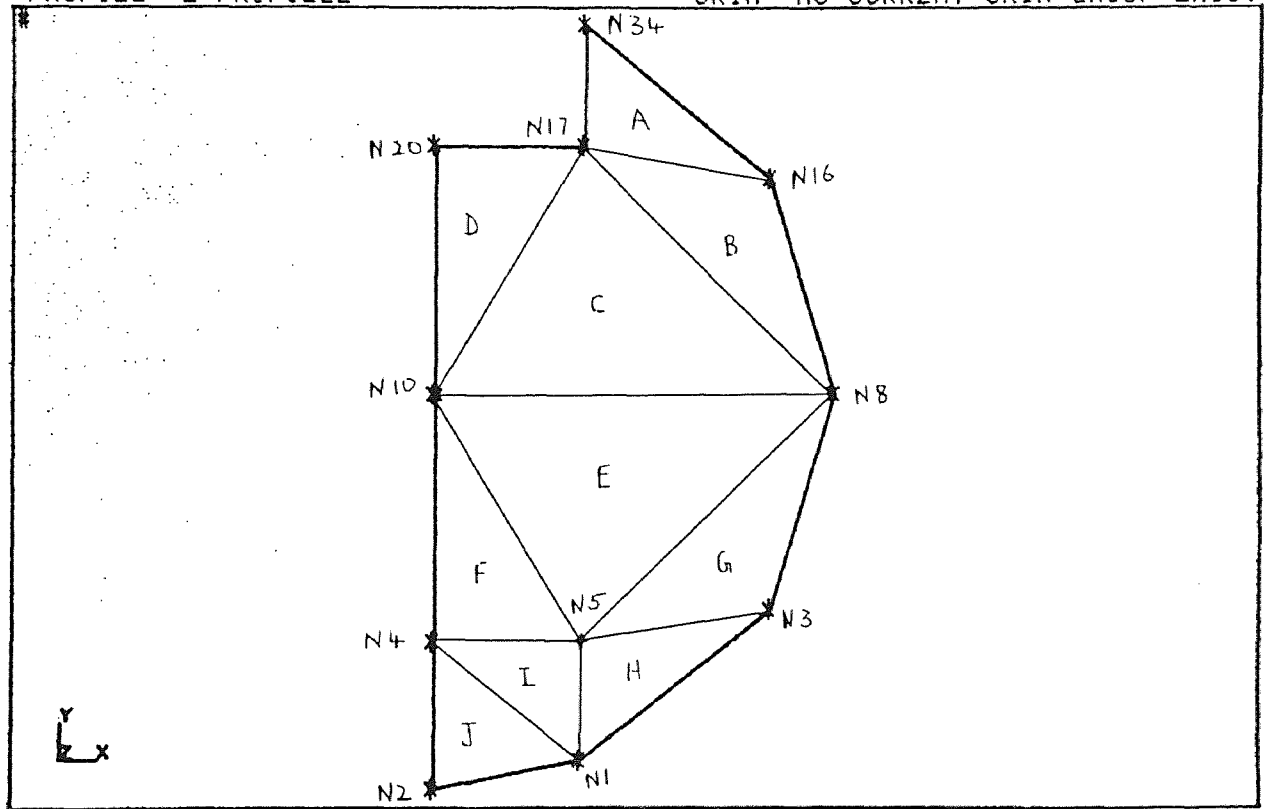


Fig.18 Model 2 : Element faces at end of small shaft

Face A

PA16 = 1201095.5

PA17 = 2425011.7

PA34 = 1506173.7

Face B

PB8 = 1893061.1

PB16 = 4819733.6

PB17 = 2053971.3

Face C

PC8 = 6503252.7

PC10 = 7498250.4

PC17 = 8883443.2

Face D

PD10 = 2079441.6

PD17 = 2780213.4

PD20 = 3722200.5

Face E

PE5 = 8883907.7

PE8 = 6503020.4

PE10 = 7498018.1

Face F

PF4 = 3721533.2

PF5 = 2779985.3

PF10 = 2080337.1

Face G

PG3 = 4819585.2

$$PG5 = 2067602.1$$

$$PG8 = 1898916.6$$

Face H

$$PH1 = 1489966.4$$

$$PH3 = 1199422.9$$

$$PH5 = 2452484.6$$

Face I

$$PI1 = 1256021.2$$

$$PI4 = 1107810.7$$

$$PI5 = 1821230.8$$

Face J

$$PJ1 = 1514640.8$$

$$PJ2 = 2026589.4$$

$$PJ4 = 1513126.2$$

Total Nodal Forces

$$P1 = PH1+PI1+PJ1$$

$$= 4260628.4$$

$$P2 = PJ2$$

$$= 2026589.4$$

$$P3 = PG3+PH3$$

$$= 6019008.1$$

$$P4 = PF4+PI4+PJ4$$

$$= 6342470.1$$

$$P5 = PI5+PH5+PG5+PE5+PF5$$

$$= 18005211$$

$$\begin{aligned} P8 &= PB8+PC8+PE8+PG8 \\ &= 16798251 \end{aligned}$$

$$\begin{aligned} P10 &= PD10+PC10+PE10+PF10 \\ &= 19156047 \end{aligned}$$

$$\begin{aligned} P16 &= PA16+PB16 \\ &= 6020829.2 \end{aligned}$$

$$\begin{aligned} P17 &= PA17+PB17+PC17+PD17 \\ &= 16142640 \end{aligned}$$

$$\begin{aligned} P20 &= PD20 \\ &= 3722200.5 \end{aligned}$$

$$\begin{aligned} P34 &= PA34 \\ &= 1506173.7 \end{aligned}$$

5. DISCUSSION OF RESULTS.

The displacement and stress listings of both the models are included in Appendix 2. In Model 1 the keyway extends into the fillet whereas in Model 2 the keyway end terminates before the fillet. UX, UY, UZ are the nodal displacements in the X, Y, and Z directions respectively after the axial tensile load had been applied. SX, SY, SZ, SXY, SYZ, SXZ are the component stresses and SIG1, SIG2, SIG3 are the principal stresses. SI is the stress intensity (maximum shear stress) and is the maximum difference of the principal stresses, i.e. maximum value of (SIG1-SIG2) or (SIG2-SIG3) or (SIG3-SIG1). SIGE is the equivalent stress (Von Mises stress).

$$SIGE = (SIG1^2 + SIG2^2 + SIG3^2 - SIG1 * SIG2 - SIG2 * SIG3 - SIG3 * SIG1)^{1/2}$$

Fig.19 shows the deformed geometry of Model 1 and the deformed geometry of Model 2 is shown in Fig.20.

The stress concentration factor is based on the nominal stress in the smaller diameter of the shouldered shaft without the keyway. It is the ratio of the peak stress in the component to the maximum stress in a shaft of the smaller diameter without the keyway under the same loading conditions. In this analysis the major region of interest was the stresses at the keyway end and the stresses in the fillet. High stresses were produced near the end of the small shaft but this was expected because of the load application at the shank end. These high stresses are insignificant according to the

SORC I-DEAS 3.0 PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: POST PROCESSING

1-MAY-87 20:16:13
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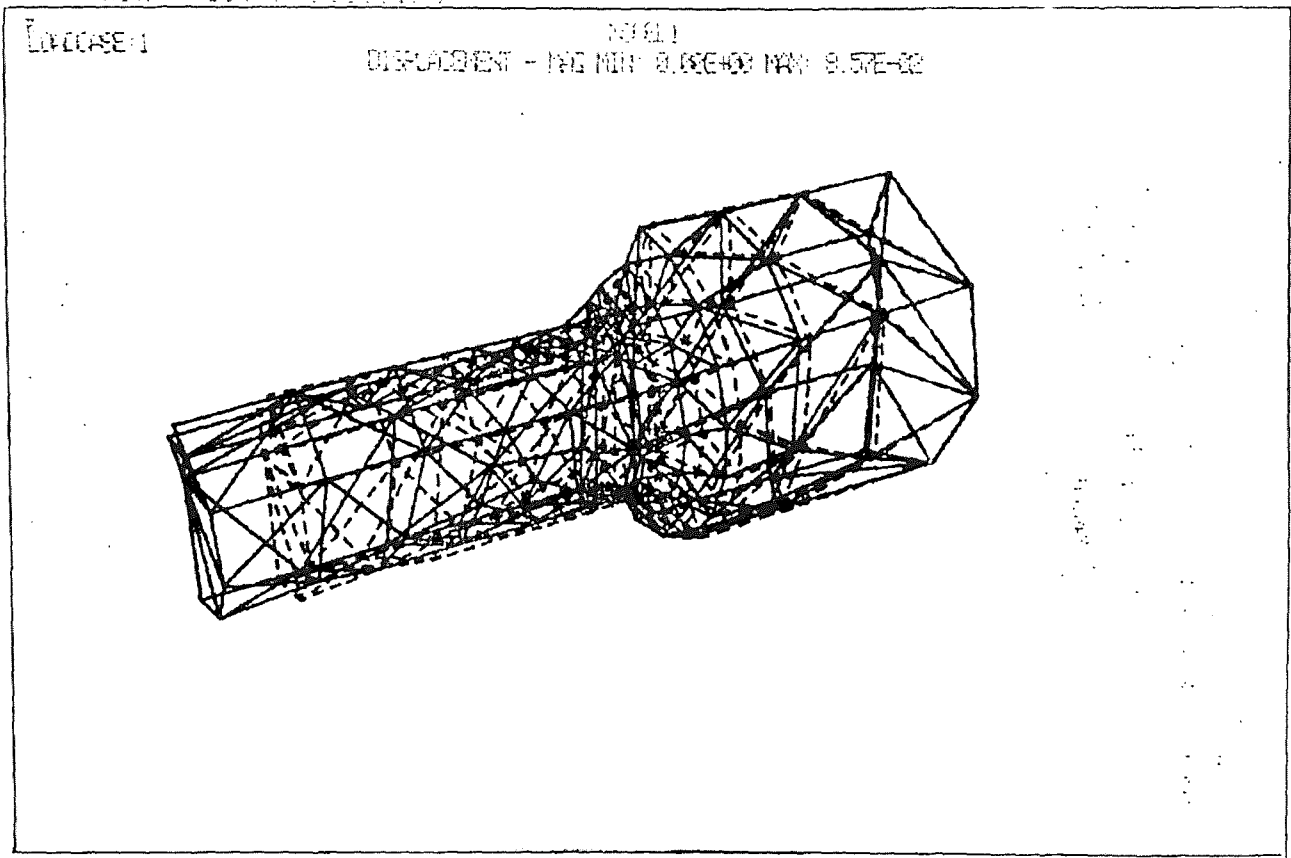


Fig.19 Model 1 : Deformed geometry

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: APT WITH KEY
VIEW: NO STRESS/ANNO
TASK: POST PROCESSING

1-MAY-87 20:24:53
UNITS = MM
DISPLAY: NO STORED OPTION

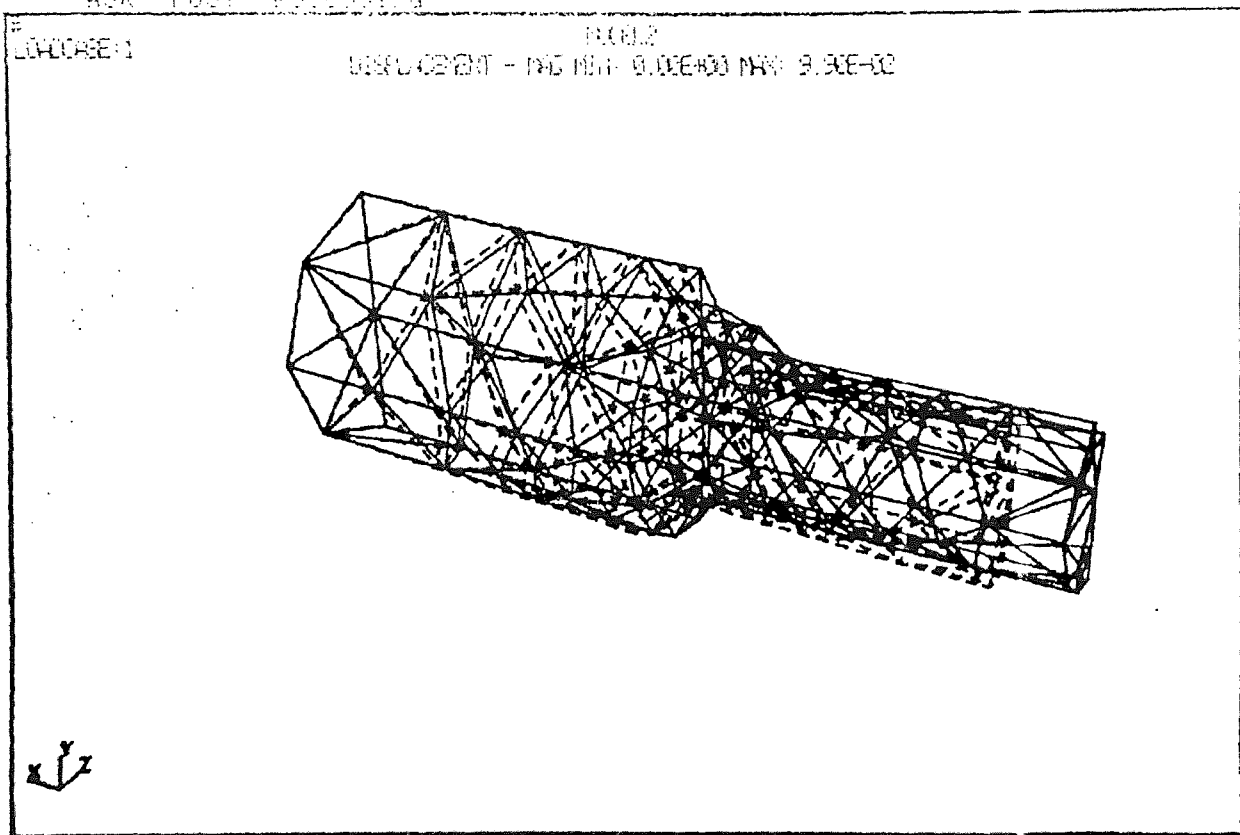


Fig.20 Model 2 : Deformed geometry

principle of St. Venant. The phenomena that the stress system tends to become regular at distances removed from the disturbance is known as the principle of St. Venant. It states that if the forces acting on a small region of the body are replaced by a different, but statically equivalent system, no change in the stress or deformation will be experienced at points sufficiently far removed from the loads. It is valid for both normal and shear stress.

Figs.21-24 show the various stress contours for Model 1 and Figs.25-28 show the stress contours for Model 2. From the contours it is clearly seen that the keyway and fillet are highly stressed whereas the stresses in the shaft of the bigger diameter are low particularly at the end.

Neglecting the stresses near the end of the shank, where the loading is applied, the stress concentration factor in Model 2, where the keyway end terminated in the shank just before the shoulder fillet, was found to occur in the keyway near the end and was at the top of the wall. The keyway was cut into the shoulder fillet in Model 1. By doing so the stress concentration factor increased and in this case it was at the point where the keyway cut into the shoulder fillet. The stresses at the keyway end for both the models were studied. It was observed that important stress reductions occur at the keyway end when the keyway runs into the fillet. The stresses at the keyway end in Model 1 decreased by almost 20% when compared to the keyway-end-stresses of Model 2. These results compare favorably with those of Fessler, Rogers, and Stanley[10] who concluded that stresses at the ends of keyways cut well into the shoulder are less than those present when the keyway

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
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TASK: POST PROCESSING

29-APR-87 00:15:04
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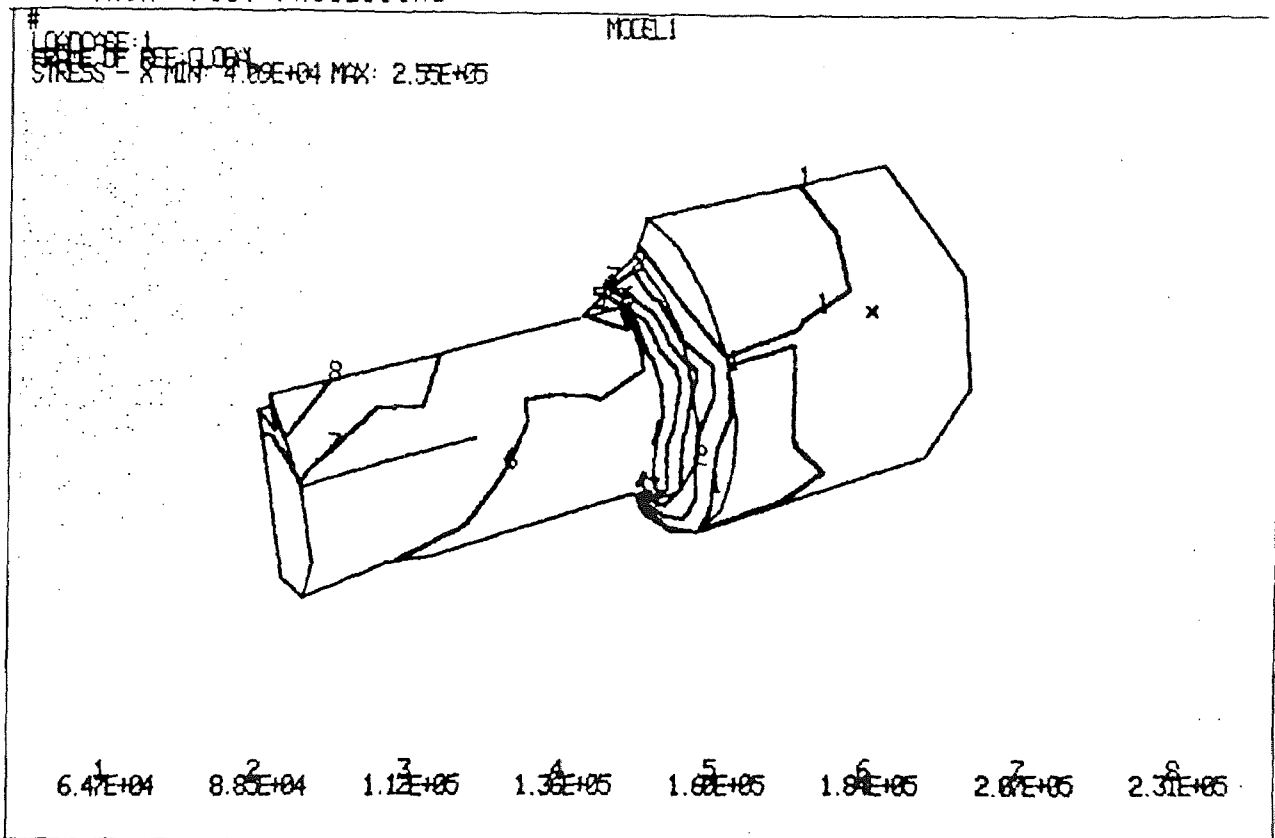


Fig.21 Model 1 : Stress contours (x-component)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: POST PROCESSING

29-APR-87 00:11:51
UNITS = MM
DISPLAY: NO STORED OPTION

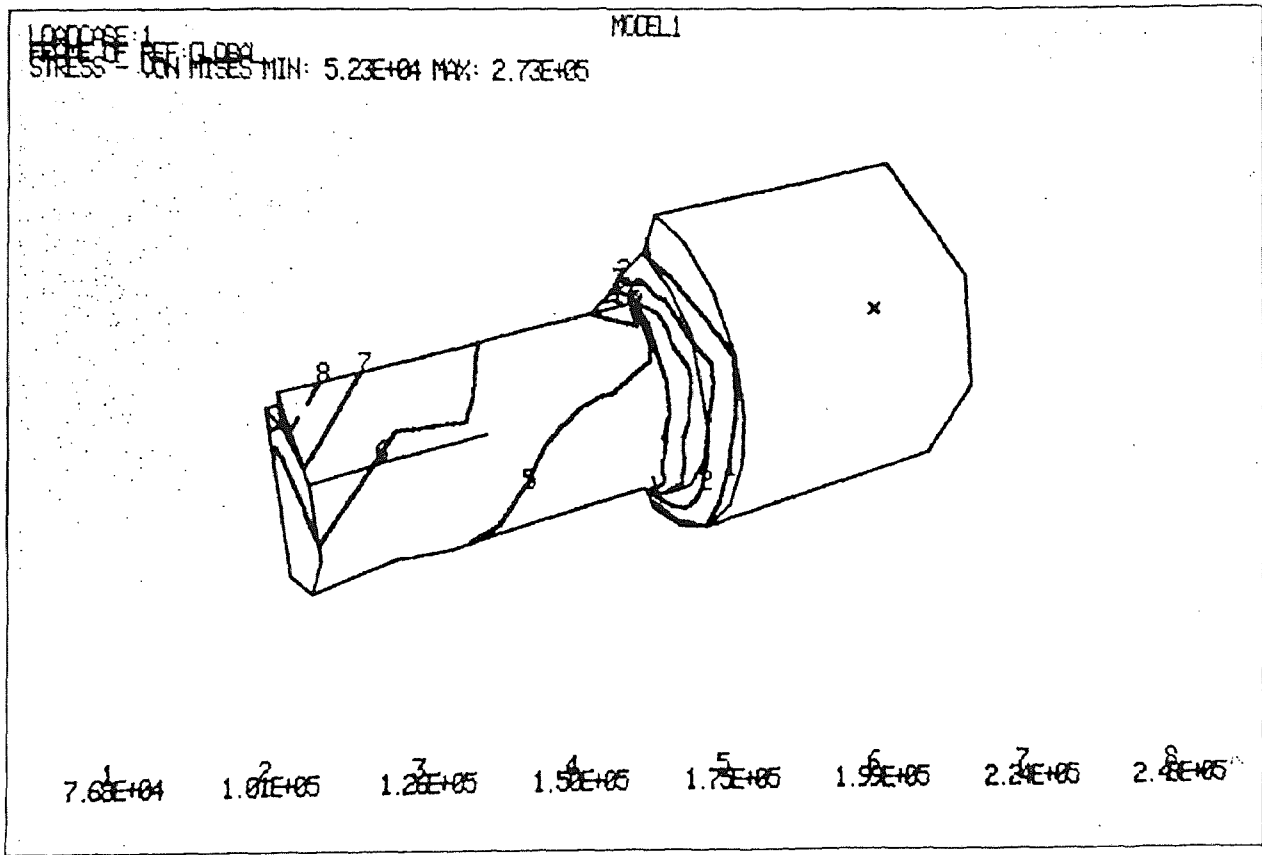


Fig.22 Model 1 : Stress contours (Von Mises)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: POST PROCESSING

29-APR-87 00:09:11
UNITS = MM
DISPLAY: NO STORED OPTION

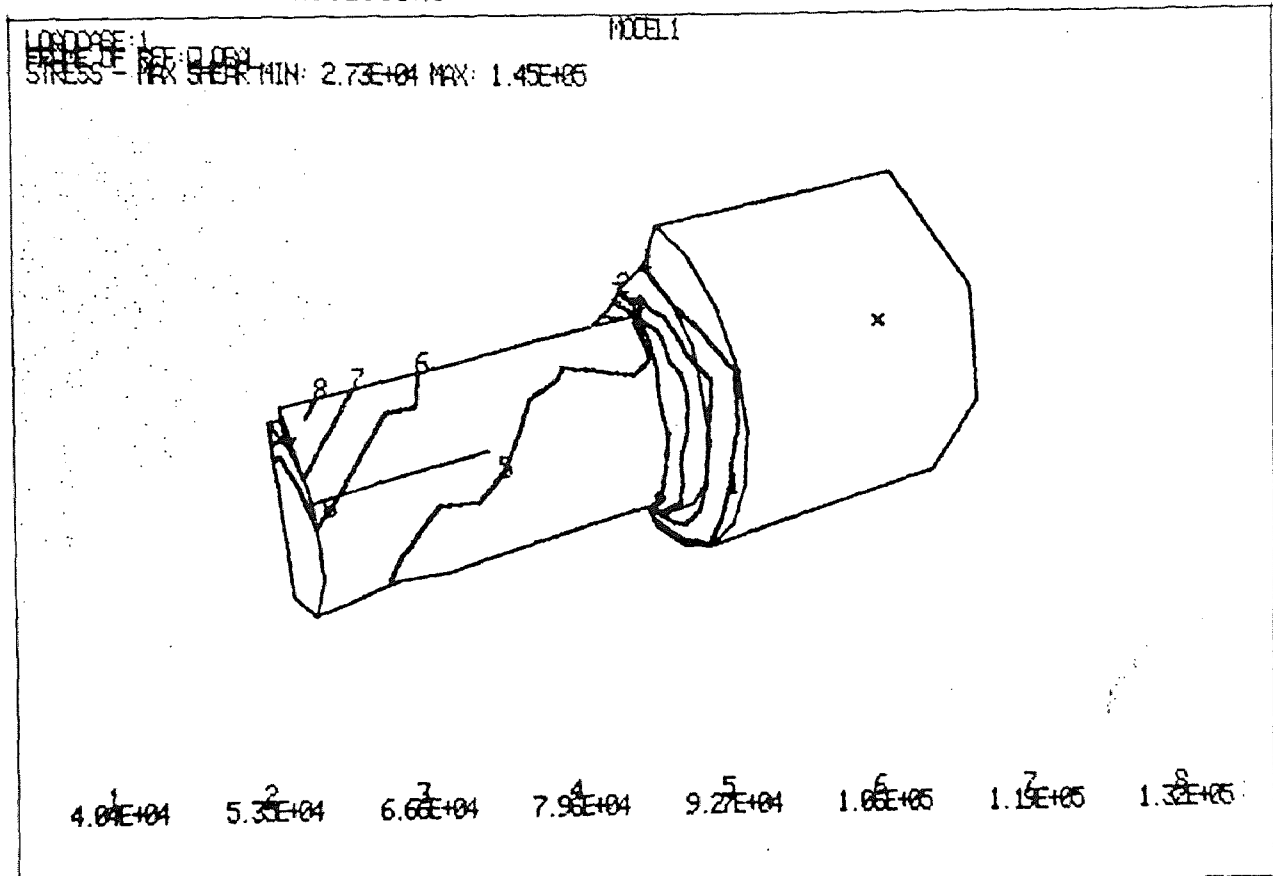


Fig.23 Model 1 : Stress contours (max. shear)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE:
VIEW: NO STORED VIEW
TASK: POST PROCESSING

29-APR-87 00:18:13
UNITS = MM
DISPLAY: NO STORED OPTION

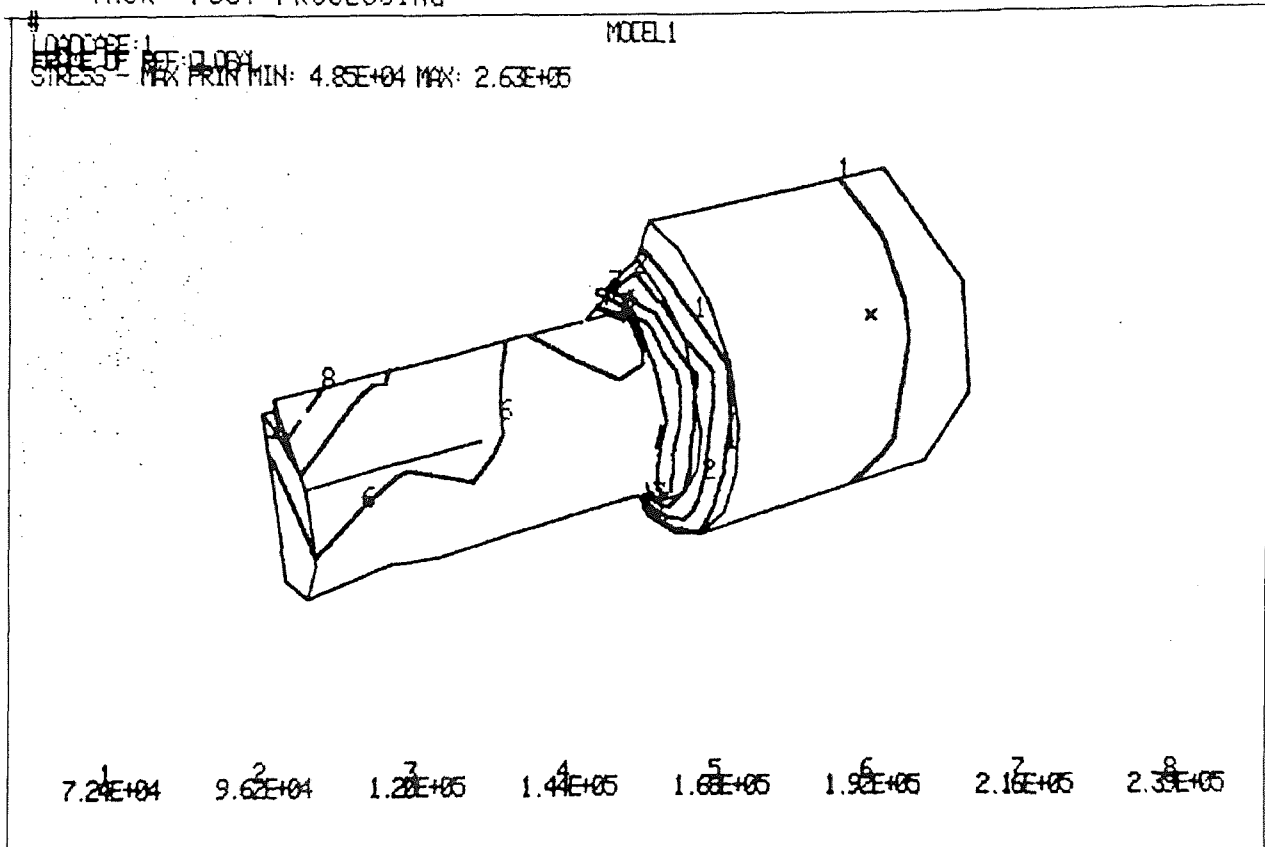


Fig.24 Model 1 : Stress contours (max. principal)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
ADDRESS: RPT WITH KEY
VIEW: NO STORED VIEW
TYPE: POST PROCESSING

29-APR-87 00:23:55
UNITS = MM
DISPLAY: NO STORED OPTION

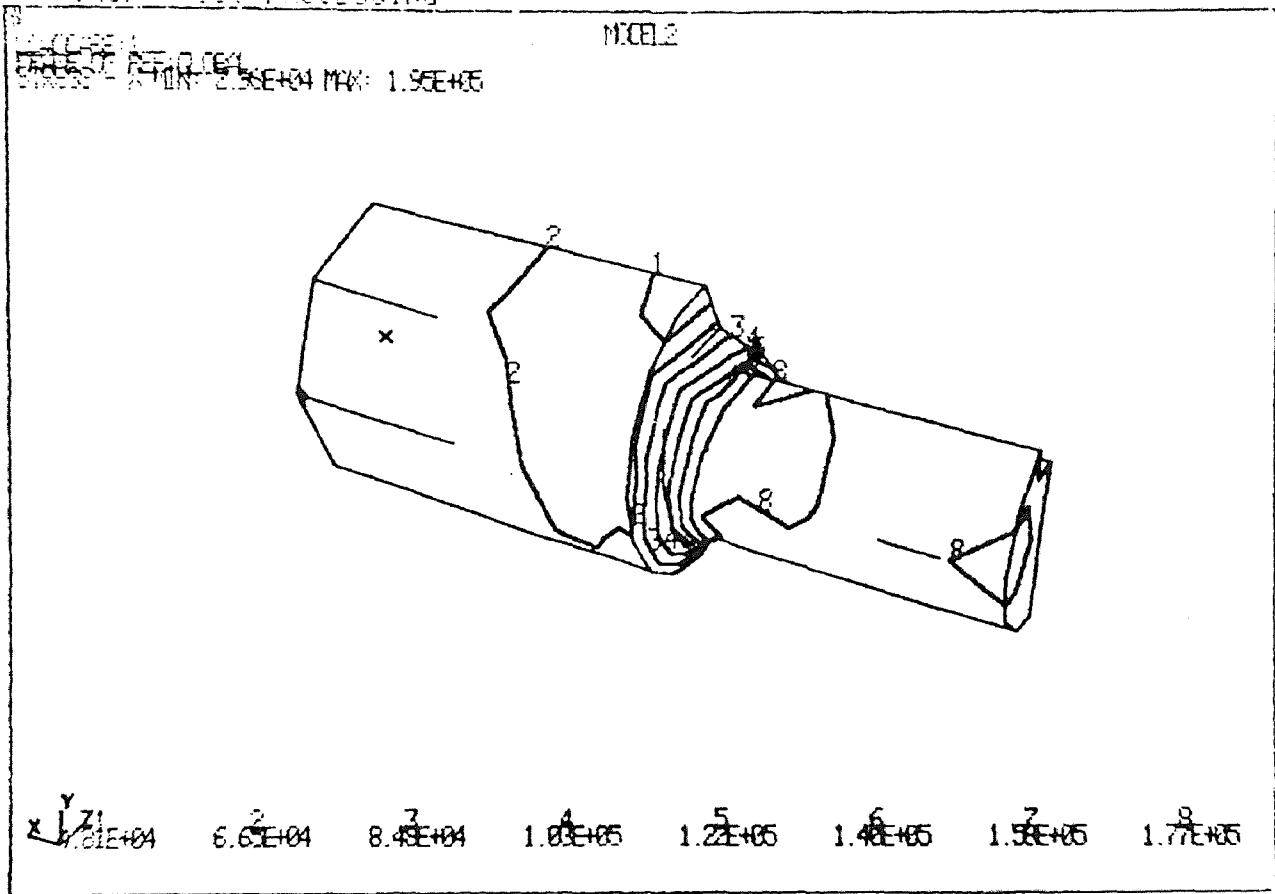


Fig.25 Model 2 : Stress contours (x-component)

SORC I-DEAS 3.0: PRE/POST PROCESSING
 DATABASE: APT WITH KEY
 VIEW: NO STORED VIEW
 TASK: POST PROCESSING

29-APR-87 00:33:45
 UNITS = MM
 DISPLAY: NO STORED OPTION

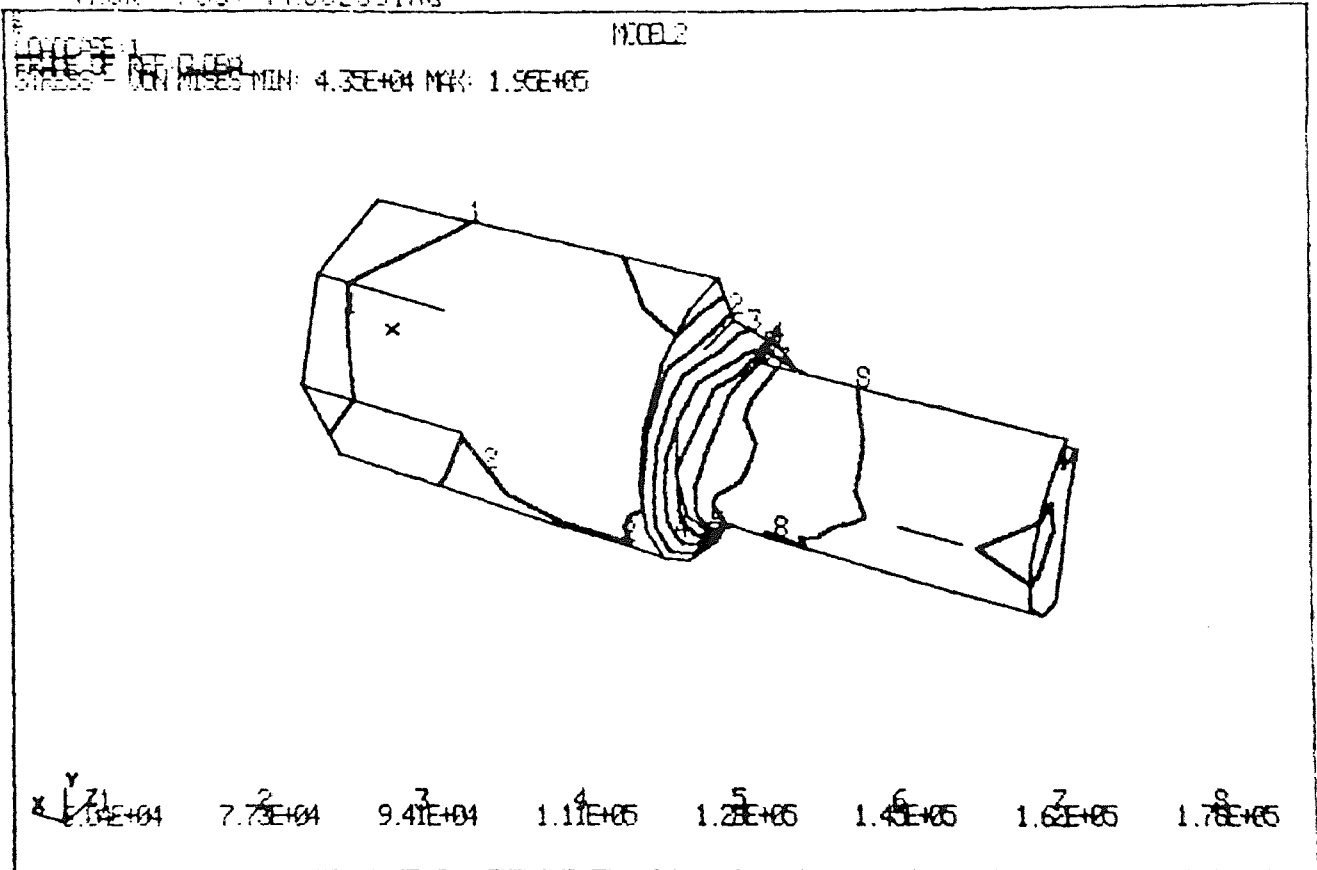


Fig.26 Model 2 : Stress contours (Von Mises)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: AFT WITH KEY
VIEW: NO STORED VIEW
TASK: POST PROCESSING

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DISPLAY: NO STORED OPTION

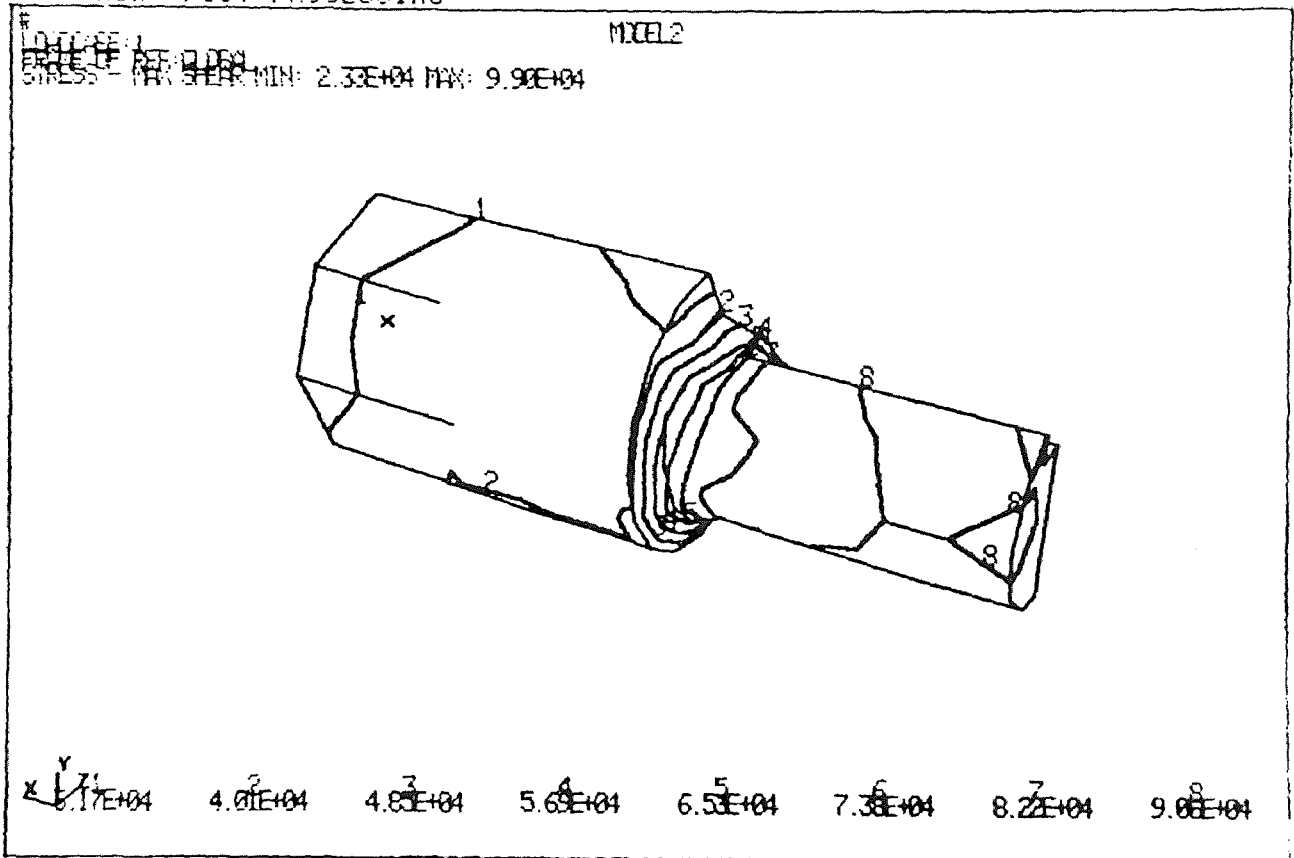


Fig.27 Model 2 : Stress contours (max. shear)

SDRC I-DEAS 3.0: PRE/POST PROCESSING
DATABASE: AFT WITH KEY
VIEWING STORED VIEW
TASK: POST PROCESSING

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UNITS = MM
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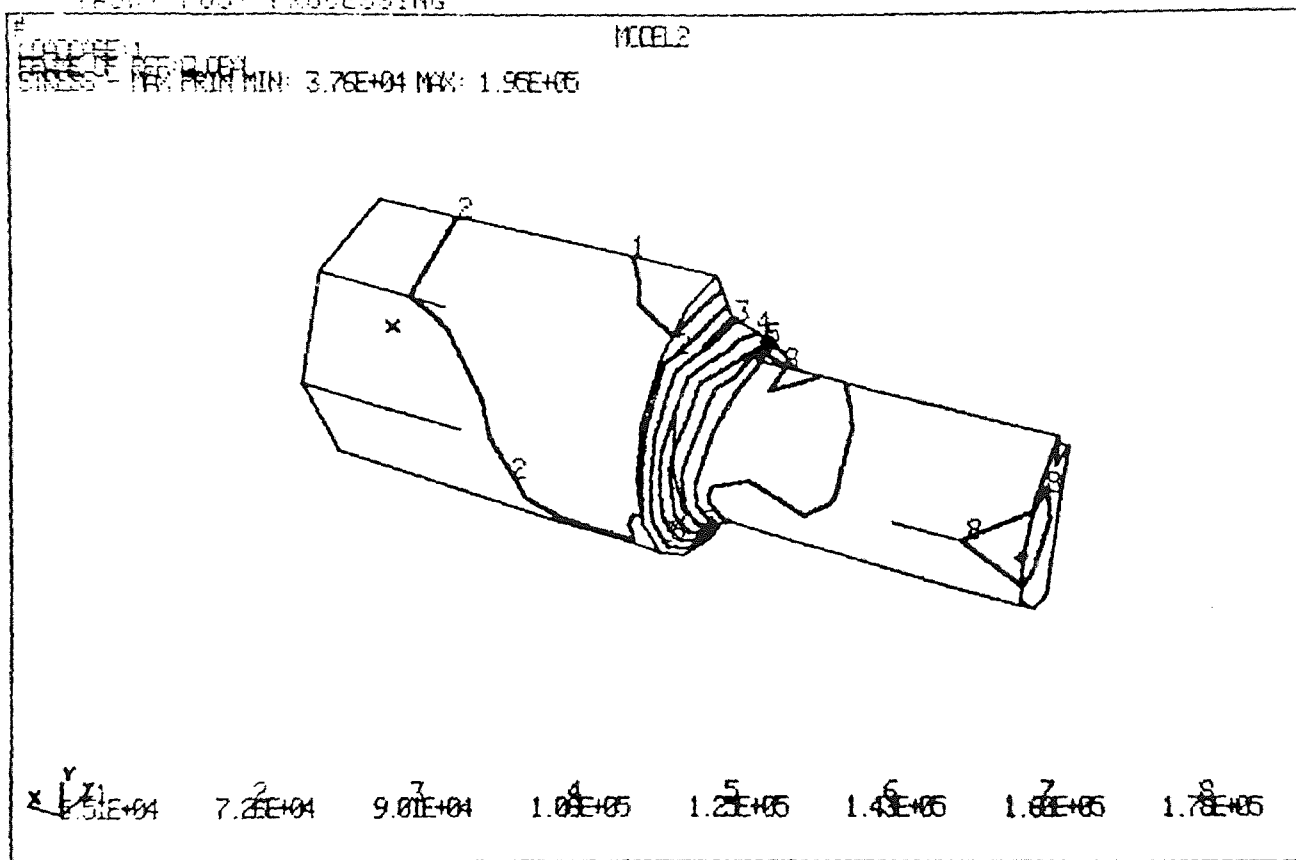


Fig.28 Model 2 : Stress contours (max. principal)

terminates in the shank, near the shoulder fillet or just inside the shoulder fillet. Varying the value of 'a' did not affect the stresses at points on the surface of the shaft which are diametrically opposite the keyway. The stresses in these regions for Model 1 as well as Model 2 remained almost the same.

It was shown in [9] that the stress distribution obtained from a bending load can be predicted from the direct-loading analysis. Hence using the results of this analysis the stresses can be determined for a stepped shaft with a keyway subjected to a bending moment instead of an axial load. This can be done by scaling down the stress indices for the tension model by the factor $(d-2k)/d$. In other words the bending stress indices can be derived from the direct loading ones by multiplying the latter by the factor $(d-2k)/d$.

6. CONCLUSION.

Based on this study it is concluded that the keyway end stresses decrease as the value of 'a' is decreased. The complete investigation of the stresses at the ends of keyways as well as in the fillet would require a study of the full permitted range of each shape parameter, that is

shoulder-fillet-radius ratio (r/d)

shoulder-diameter ratio (D/d)

proximity of the shoulder fillet to the keyway end (a/d).

The present investigation was limited to only one loading type, namely, tension. The shoulder-fillet ratio as well as the shoulder-diameter ratio were fixed. Two values of a/d were used; one positive and one negative. The stresses obtained were low when compared to the results of other investigations. However, useful observations were made and using the results of this analysis the stresses in a shouldered shaft with a keyway subjected to bending can be predicted. A lot of further research can be performed in this area. The two ratios D/d and r/d can be varied and different values of 'a' can be tried to study the effect of running the keyway well past the fillet into the shoulder. Instead of tension the analysis can be repeated for torsion or bending.

The university version of ANSYS imposed severe limitations. It could not support the large number of solid elements required to satisfactorily generate the curved and discontinuous boundary of the model. Also, the solid element used has only three degrees of freedom restricting the application of a bending moment to it.

APPENDIX 1.

ANSYS INPUT LISTING OF MODEL 1.

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E, 121, 115, 124, 124, 117, 117, 117, 117
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ANSYS INPUT LISTING OF MODEL 2.

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E, 55, 61, 32, 32, 56, 56, 56, 56
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E, 42, 43, 56, 56, 68, 68, 68, 68
E, 43, 66, 64, 64, 38, 38, 38, 38
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E, 66, 50, 57, 57, 49, 49, 49, 49
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E, 68, 43, 56, 56, 55, 55, 55, 55
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E, 71, 44, 48, 48, 62, 62, 62, 62
E, 53, 58, 46, 46, 72, 72, 72, 72
E, 52, 42, 69, 69, 67, 67, 67, 67
E, 38, 65, 64, 64, 63, 63, 63, 63
E, 66, 43, 68, 68, 55, 55, 55, 55
E, 38, 66, 64, 64, 65, 65, 65, 65
E, 68, 42, 69, 69, 56, 56, 56, 56
E, 62, 46, 58, 58, 70, 70, 70, 70
E, 73, 59, 51, 51, 53, 53, 53, 53
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E, 62, 57, 48, 48, 71, 71, 71, 71
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E, 67, 42, 69, 69, 68, 68, 68, 68
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E, 54, 59, 77, 77, 58, 58, 58, 58
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E, 56, 78, 55, 55, 61, 61, 61, 61
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E, 57, 61, 55, 55, 78, 78, 78, 78
E, 61, 77, 62, 62, 54, 54, 54, 54
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E, 56, 60, 78, 78, 61, 61, 61, 61
E, 56, 78, 69, 69, 55, 55, 55, 55
E, 53, 72, 73, 73, 59, 59, 59, 59

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/COM
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D, 14, UZ, 0., , ,
D, 15, UZ, 0., , ,
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D, 21, UZ, 0., , ,
D, 27, UZ, 0., , ,
D, 32, UZ, 0., , ,
D, 33, UZ, 0., , ,
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D, 75, UZ, 0., , ,
D, 76, UZ, 0., , ,
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D, 85, UZ, 0., , ,
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D, 87, UZ, 0., , ,
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D, 96, UZ, 0., , ,
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D, 99, UZ, 0., , ,
D, 107, UZ, 0., , ,
D, 110, UZ, 0., , ,
D, 111, UZ, 0., , ,
D, 112, UX, 0., , , , UY, UZ, ROTX, ROTY, ROTZ
D, 114, UX, 0., , , , UY, UZ, ROTX, ROTY, ROTZ
D, 115, UX, 0., , , , UY, UZ, ROTX, ROTY, ROTZ
D, 116, UX, 0., , , , UY, UZ, ROTX, ROTY, ROTZ
D, 117, UX, 0., , , , UY, UZ, ROTX, ROTY, ROTZ
D, 118, UX, 0., , , , UY, UZ, ROTX, ROTY, ROTZ
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EPDELE, 1,          279
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F, 2, FX, -2026589.4
F, 3, FX, -6019008.1
F, 4, FX, -6342470.1

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F, 8, FX, -16798251.0
F, 10, FX, -19156047.0
F, 16, FX, -6020829.2
F, 17, FX, -16142640.0
F, 20, FX, -3722200.5
F, 34, FX, -1506173.7
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/POST1
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PRDISP
PRNSTR
FINISH
/EOF
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APPENDIX 2.

NODAL DISPLACEMENT AND STRESS LISTING OF MODEL1.

THE FOLLOWING X,Y,Z DISPLACEMENTS ARE IN NODAL COORDINATES

NODE	UX	UY	UZ
1	-0.85362130E-01	-0.78161055E-02	-0.22252665E-02
2	-0.77945924E-01	-0.38196244E-02	-0.49214835E-02
3	-0.66640583E-01	-0.40569531E-02	-0.18617366E-02
4	-0.80224990E-01	-0.50932597E-02	-0.20276887E-02
5	-0.76930749E-01	0.30025452E-03	-0.54627452E-02
6	-0.63861289E-01	-0.12567958E-02	-0.44596993E-02
7	-0.74765023E-01	-0.22954694E-02	0.00000000E+00
8	-0.51477836E-01	-0.28281639E-02	-0.15944172E-02
9	-0.50911541E-01	0.18145878E-03	-0.45406241E-02
10	-0.56347776E-01	-0.12165116E-02	-0.16628685E-02
11	-0.82298614E-01	-0.55634216E-02	0.00000000E+00
12	-0.56233826E-01	-0.12833064E-02	0.00000000E+00
13	-0.75988605E-01	0.38248910E-02	-0.32193887E-02
14	-0.64372492E-01	0.31011644E-02	-0.38027029E-02
15	-0.53663022E-01	0.33713472E-02	-0.45373414E-02
16	-0.49526357E-01	0.10117346E-02	0.44061547E-03
17	-0.75808784E-01	0.50362342E-02	0.00000000E+00
18	-0.62599209E-01	0.54643525E-02	0.00000000E+00
19	-0.43306889E-01	-0.13268810E-02	-0.34328702E-02
20	-0.40576336E-01	-0.23543035E-02	-0.18230279E-02
21	-0.41341761E-01	-0.79278239E-03	-0.17580059E-02
22	-0.44347871E-01	0.30302906E-02	-0.45128037E-02
23	-0.40320656E-01	0.33494333E-03	-0.43578086E-02
24	-0.41732879E-01	-0.96040120E-03	0.00000000E+00
25	-0.55357404E-01	0.53670787E-02	-0.28528364E-02
26	-0.45717448E-01	0.54767455E-02	-0.32745159E-02
27	-0.51509293E-01	0.67000203E-02	0.00000000E+00
28	-0.31630342E-01	-0.74195566E-03	0.00000000E+00
29	-0.38810658E-01	0.41256530E-02	-0.41270509E-02
30	-0.31873542E-01	-0.69497790E-03	-0.17846523E-02
31	-0.37323506E-01	0.43464692E-02	0.00000000E+00
32	-0.24240989E-01	0.47692667E-03	0.00000000E+00
33	-0.34969906E-01	-0.12792675E-02	-0.34734589E-02
34	-0.30941949E-01	-0.24200195E-02	-0.18835783E-02
35	-0.37598981E-01	0.24494905E-02	-0.45766831E-02
36	-0.37131124E-01	0.20578078E-02	-0.39359432E-02
37	-0.31743556E-01	0.12536165E-03	-0.43041832E-02
38	-0.29915440E-01	0.26312754E-02	-0.25402162E-02
39	-0.41576033E-01	0.67896110E-02	0.00000000E+00
40	-0.34947874E-01	0.55444994E-02	-0.28283053E-02
41	-0.23675762E-01	-0.12264444E-02	0.00000000E+00
42	-0.26020348E-01	-0.10403347E-02	-0.16422261E-02

43	-0.31551226E-01	0.35419463E-02	-0.42409203E-02
44	-0.32874625E-01	0.62011094E-02	0.00000000E+00
45	-0.25518426E-01	0.33076357E-02	0.33558935E-03
46	-0.29614576E-01	0.38488846E-02	-0.12542085E-02
47	-0.19587743E-01	-0.11112992E-02	0.00000000E+00
48	-0.22768524E-01	-0.78869803E-03	-0.13281150E-02
49	-0.23853577E-01	0.41737972E-04	-0.33583522E-02
50	-0.25141841E-01	0.22253685E-02	-0.17945647E-02
51	-0.23007766E-01	0.35706992E-03	0.00000000E+00
52	-0.22828602E-01	-0.12562914E-02	-0.27994527E-02
53	-0.21560442E-01	-0.20696788E-02	-0.96187483E-03
54	-0.23794402E-01	0.16786654E-02	-0.37071412E-02
55	-0.28964789E-01	0.45354849E-02	-0.31895310E-02
56	-0.24277158E-01	0.33001782E-02	-0.31072378E-02
57	-0.24408342E-01	0.45890077E-02	-0.16546767E-02
58	-0.21561109E-01	-0.12909797E-02	-0.11109774E-02
59	-0.24485547E-01	0.52166630E-02	0.00000000E+00
60	-0.23688875E-01	0.33221953E-02	0.00000000E+00
61	-0.23915577E-01	0.26329893E-02	-0.15475760E-02
62	-0.17902500E-01	-0.12595943E-02	0.00000000E+00
63	-0.19376733E-01	-0.13124267E-02	-0.78074365E-03
64	-0.21840614E-01	-0.75267578E-03	-0.25779721E-02
65	-0.20777213E-01	-0.12031442E-02	-0.18193341E-02
66	-0.22954624E-01	-0.10651733E-04	-0.32010248E-02
67	-0.22670529E-01	0.10546916E-02	-0.33305647E-02
68	-0.23743452E-01	0.15936645E-02	-0.24515607E-02
69	-0.23131745E-01	0.26021474E-02	-0.30506850E-02
70	-0.20601994E-01	-0.26278731E-03	-0.12344960E-02
71	-0.19337426E-01	0.32677043E-02	0.00000000E+00
72	-0.18545081E-01	-0.86330659E-03	0.00000000E+00
73	-0.18382742E-01	0.15820130E-02	-0.26683301E-02
74	-0.22954788E-01	0.18683985E-02	-0.33078181E-02
75	-0.22988219E-01	0.36660319E-02	-0.26202502E-02
76	-0.23019156E-01	0.41406372E-02	-0.18520669E-02
77	-0.22207324E-01	0.47165590E-02	-0.97976498E-03
78	-0.21813609E-01	0.45978562E-02	0.00000000E+00
79	-0.17642181E-01	0.37601513E-02	0.00000000E+00
80	-0.18844457E-01	0.29259134E-02	-0.17023727E-02
81	-0.18616098E-01	0.38534110E-02	-0.14639498E-02
82	-0.18101931E-01	0.29372762E-02	-0.24891323E-02
83	-0.15456695E-01	-0.13248349E-02	0.00000000E+00
84	-0.15456695E-01	-0.12425599E-02	-0.10125017E-02
85	-0.16764251E-01	-0.11889217E-02	-0.10703041E-02
86	-0.16669027E-01	-0.87558064E-03	-0.19064724E-02
87	-0.18090416E-01	-0.30677130E-03	-0.26517803E-02
88	-0.17831835E-01	0.96465407E-03	-0.30513186E-02
89	-0.17886977E-01	0.20302121E-02	-0.29214655E-02
90	-0.15294019E-01	0.39871321E-02	0.00000000E+00
91	-0.12720936E-01	0.46340206E-02	0.00000000E+00
92	-0.13687767E-01	0.35923224E-02	-0.13688331E-02
93	-0.15287886E-01	0.41675653E-02	-0.95150167E-03
94	-0.13121104E-01	0.36974400E-04	-0.22199550E-02
95	-0.15644361E-01	0.37098788E-02	-0.23768407E-02

96	-0.13527900E-01	0.22769763E-02	-0.22906959E-02
97	-0.71754493E-02	0.12459561E-02	0.00000000E+00
98	-0.11238940E-01	-0.11629628E-02	0.00000000E+00
99	-0.79428145E-02	0.59503132E-03	-0.15398189E-02
100	-0.13062649E-01	-0.16771255E-02	0.00000000E+00
101	-0.12603435E-01	-0.21865062E-02	-0.12647505E-02
102	-0.10042930E-01	-0.31116152E-02	0.00000000E+00
103	-0.14137658E-01	-0.15152496E-02	-0.17867754E-02
104	-0.13105982E-01	-0.17083076E-02	-0.78918958E-03
105	-0.15228279E-01	-0.89553954E-03	-0.28730565E-02
106	-0.10656480E-01	-0.13989934E-02	-0.29350213E-02
107	-0.14860869E-01	0.14356518E-02	-0.27804632E-02
108	-0.15530909E-01	0.26482958E-02	-0.31633795E-02
109	-0.15288265E-01	0.55125584E-03	-0.33328997E-02
110	-0.12441386E-01	0.93075003E-03	-0.36486901E-02
111	-0.69917118E-02	0.30978545E-02	0.00000000E+00
112	-0.11038402E-01	0.33156717E-02	-0.20888071E-02
113	-0.13021662E-01	0.25107959E-02	-0.30310195E-02
114	0.00000000E+00	0.00000000E+00	0.00000000E+00
115	0.00000000E+00	0.00000000E+00	0.00000000E+00
116	0.00000000E+00	0.00000000E+00	0.00000000E+00
117	0.00000000E+00	0.00000000E+00	0.00000000E+00
118	-0.56720153E-02	-0.25147944E-02	0.00000000E+00
119	-0.43290957E-02	-0.20984862E-02	-0.12982841E-02
120	-0.10583830E-01	-0.25716563E-02	-0.14472173E-02
121	-0.54458780E-02	-0.13199539E-02	-0.21824160E-02
122	-0.73955566E-02	0.11844029E-02	-0.25066811E-02
123	0.00000000E+00	0.00000000E+00	0.00000000E+00
124	0.00000000E+00	0.00000000E+00	0.00000000E+00
125	0.00000000E+00	0.00000000E+00	0.00000000E+00

MAXIMUMS

NODE	1	1	5
VALUE	-0.85362130E-01	-0.78161055E-02	-0.54627452E-02

THE FOLLOWING X,Y,Z STRESSES ARE IN GLOBAL COORDINATES

NODE	SX SIG1	SY SIG2	SZ SIG3	SXY SI	SYZ SIGE	SXZ
1	0.2548E+06	-0.1634E+05	0.1061E+05	-0.4827E+05	-7460.	-8119.
2	0.2633E+06	0.1248E+05	-0.2672E+05	0.2901E+06	0.2726E+06	-501.1
3	0.2054E+06	-0.1066E+05	-1279.	-0.1808E+05	-2749.	2021.
4	0.2098E+06	775.6	-0.1711E+05	0.2269E+06	0.2186E+06	2792.
5	0.2178E+06	-978.9	5518.	-0.1227E+05	-3632.	-7034.
	0.2201E+06	9233.	-7042.	0.2272E+06	0.2197E+06	
	0.2128E+06	-6183.	5446.	-0.3015E+05	-1328.	
	0.2190E+06	7570.	-0.1448E+05	0.2335E+06	0.2234E+06	
	0.1900E+06	-5493.	-5000.	4713.	-409.9	

	0.1904E+06	-2282.	-8617.	0.1991E+06	0.1960E+06	
6	0.1925E+06	1315.	445.8	-5793.	-944.2	-1036.
	0.1939E+06	4765.	-4392.	0.1983E+06	0.1939E+06	
7	0.1897E+06	-4775.	431.6	-9670.	-307.0	-2523.
	0.1924E+06	2452.	-9515.	0.2020E+06	0.1964E+06	
8	0.2066E+06	706.4	4350.	3680.	-98.31	4344.
	0.2068E+06	5107.	-256.6	0.2070E+06	0.2044E+06	
9	0.1978E+06	2795.	2557.	-348.7	-1797.	2587.
	0.1981E+06	6181.	-1080.	0.1991E+06	0.1956E+06	
10	0.2007E+06	2307.	6685.	-7748.	-2095.	-1613.
	0.2018E+06	9632.	-1771.	0.2036E+06	0.1982E+06	
11	0.2284E+06	-0.1533E+05	5831.	-0.5968E+05	5030.	22180.
	0.2440E+06	6782.	-0.3187E+05	0.2759E+06	0.2587E+06	
12	0.2019E+06	-2754.	9197.	-0.2300E+05	-1715.	330.9
	0.2072E+06	0.1133E+05	-0.1023E+05	0.2175E+06	0.2077E+06	
13	0.1897E+06	-7336.	-2058.	5788.	303.2	-2423.
	0.1899E+06	-1611.	-7996.	0.1979E+06	0.1948E+06	
14	0.1848E+06	508.3	216.8	5026.	698.2	-2243.
	0.1850E+06	2850.	-2341.	0.1873E+06	0.1848E+06	
15	0.1852E+06	5658.	991.9	2386.	154.5	1547.
	0.1852E+06	5900.	671.2	0.1846E+06	0.1820E+06	
16	0.1825E+06	4083.	2568.	1793.	-632.4	-5.268
	0.1827E+06	6156.	283.9	0.1824E+06	0.1795E+06	
17	0.1877E+06	-9260.	-4923.	5845.	-419.9	-2344.
	0.1879E+06	-4925.	-9461.	0.1974E+06	0.1951E+06	
18	0.1814E+06	-1923.	1347.	6270.	1308.	-644.0
	0.1817E+06	2150.	-2958.	0.1846E+06	0.1821E+06	
19	0.1877E+06	-626.5	62.66	3065.	-728.7	3900.
	0.1879E+06	1246.	-1960.	0.1898E+06	0.1882E+06	
20	0.1891E+06	-523.7	-205.2	1794.	-1065.	2424.
	0.1892E+06	1061.	-1864.	0.1910E+06	0.1896E+06	
21	0.1911E+06	-311.4	2293.	1055.	-1688.	2161.
	0.1912E+06	3749.	-1891.	0.1931E+06	0.1903E+06	
22	0.1748E+06	4674.	1180.	-118.9	-265.7	2052.
	0.1749E+06	5113.	692.9	0.1742E+06	0.1720E+06	
23	0.1795E+06	3954.	2411.	2677.	-1962.	2615.
	0.1797E+06	5895.	238.7	0.1795E+06	0.1767E+06	
24	0.1962E+06	-1631.	8795.	-840.5	-3319.	-3192.
	0.1963E+06	9678.	-2646.	0.1989E+06	0.1931E+06	
25	0.1788E+06	3367.	1470.	3970.	1201.	726.2
	0.1789E+06	4286.	432.3	0.1785E+06	0.1766E+06	
26	0.1736E+06	3102.	1476.	-888.4	712.2	1589.
	0.1736E+06	3914.	622.2	0.1730E+06	0.1714E+06	
27	0.1742E+06	2236.	1592.	881.4	1191.	1032.
	0.1742E+06	3430.	326.1	0.1739E+06	0.1724E+06	
28	0.1929E+06	0.1021E+05	0.1057E+05	0.1790E+05	-981.2	-3553.
	0.1958E+06	0.1255E+05	5284.	0.1905E+06	0.1870E+06	
29	0.1712E+06	3974.	2939.	-1904.	269.8	2217.
	0.1713E+06	5061.	1775.	0.1695E+06	0.1679E+06	
30	0.1878E+06	5600.	4928.	8993.	-1592.	1009.
	0.1887E+06	7721.	1841.	0.1869E+06	0.1840E+06	
31	0.1697E+06	5438.	3608.	-4084.	242.9	2754.
	0.1699E+06	6635.	2181.	0.1677E+06	0.1656E+06	

32	0.1704E+06	0.1167E+05	9387.	8828.	-733.9	2878.
	0.1722E+06	0.1240E+05	6796.	0.1654E+06	0.1627E+06	
33	0.1885E+06	764.0	3690.	2371.	-2053.	1251.
	0.1886E+06	5350.	-988.8	0.1896E+06	0.1865E+06	
34	0.2000E+06	3319.	8575.	5237.	-343.8	499.8
	0.2002E+06	9537.	2099.	0.1981E+06	0.1945E+06	
35	0.1728E+06	4695.	2082.	347.5	284.0	1637.
	0.1729E+06	5016.	1679.	0.1712E+06	0.1696E+06	
36	0.1724E+06	5246.	4547.	157.1	-702.0	2320.
	0.1725E+06	6623.	3079.	0.1694E+06	0.1677E+06	
37	0.1824E+06	0.1057E+05	9632.	7393.	-1356.	6396.
	0.1834E+06	0.1242E+05	6703.	0.1767E+06	0.1740E+06	
38	0.1723E+06	9933.	6673.	-3839.	-721.3	7562.
	0.1732E+06	0.1078E+05	4841.	0.1684E+06	0.1655E+06	
39	0.1725E+06	2793.	2931.	-3632.	207.4	947.2
	0.1726E+06	3943.	1679.	0.1709E+06	0.1698E+06	
40	0.1718E+06	6754.	3331.	-6395.	940.1	3940.
	0.1723E+06	7201.	2424.	0.1699E+06	0.1676E+06	
41	0.1820E+06	0.2601E+05	0.2176E+05	0.3846E+05	1700.	-5905.
	0.1915E+06	0.2407E+05	0.1420E+05	0.1773E+06	0.1726E+06	
42	0.1981E+06	0.1521E+05	0.1507E+05	0.1947E+05	648.5	55.84
	0.2013E+06	0.1778E+05	9311.	0.1920E+06	0.1879E+06	
43	0.1754E+06	0.1084E+05	6706.	-6229.	432.1	8830.
	0.1771E+09	0.1025E+05	5522.	0.1716E+06	0.1693E+06	
44	0.1709E+06	7506.	7748.	-0.1238E+05	1275.	4681.
	0.1721E+06	9175.	4859.	0.1672E+06	0.1651E+06	
45	0.1514E+06	7867.	6166.	-0.1134E+05	562.3	3285.
	0.1529E+06	8970.	3631.	0.1492E+06	0.1466E+06	
46	0.1698E+06	0.1207E+05	7286.	-9484.	755.2	7312.
	0.1709E+06	0.1284E+05	5482.	0.1654E+06	0.1619E+06	
47	0.1369E+06	0.1954E+05	0.1651E+05	0.3540E+05	-808.7	-2845.
	0.1474E+06	0.1746E+05	8017.	0.1394E+06	0.1350E+06	
48	0.1695E+06	0.2156E+05	0.1636E+05	0.2888E+05	352.2	6273.
	0.1766E+06	0.1984E+05	0.1092E+05	0.1657E+06	0.1615E+06	
49	0.1714E+06	0.1614E+05	0.1694E+05	6703.	-648.7	16540.
	0.1744E+06	0.1805E+05	0.1201E+05	0.1624E+06	0.1595E+06	
50	0.1571E+06	0.1333E+05	8957.	-8038.	-618.5	11450.
	0.1595E+06	0.1326E+05	6695.	0.1528E+06	0.1496E+06	
51	0.1417E+06	0.1314E+05	0.1166E+05	0.1062E+05	464.6	5525.
	0.1445E+06	0.1314E+05	8834.	0.1357E+06	0.1336E+06	
52	0.1997E+06	0.1449E+05	0.1450E+05	0.1899E+05	-1619.	7412.
	0.2037E+06	0.1865E+05	6304.	0.1974E+06	0.1916E+06	
53	0.2186E+06	718.3	0.1553E+05	0.1942E+05	-2023.	-4528.
	0.2208E+06	0.1621E+05	-2182.	0.2230E+06	0.2146E+06	
54	0.1812E+06	0.2336E+05	0.2083E+05	-3719.	-844.7	27270.
	0.1870E+06	0.2339E+05	0.1502E+05	0.1720E+06	0.1680E+06	
55	0.1806E+06	0.1265E+05	7182.	-9955.	2081.	8824.
	0.1817E+06	0.1363E+05	5113.	0.1766E+06	0.1725E+06	
56	0.1842E+06	0.1994E+05	0.1656E+05	-0.1675E+05	470.0	15900.
	0.1882E+06	0.2000E+05	0.1244E+05	0.1758E+06	0.1722E+06	
57	0.1724E+06	0.1496E+05	0.1347E+05	-0.1965E+05	1246.	7766.
	0.1756E+06	0.1623E+05	8988.	0.1666E+06	0.1632E+06	
58	0.1775E+06	0.1403E+05	0.1319E+05	0.3350E+05	341.0	-539.0

	0.1860E+06	0.1652E+05	2234.	0.1838E+06	0.1771E+06	
59	0.1793E+06	0.1644E+05	0.1932E+05	-0.2917E+05	1020.	3538.
	0.1848E+06	0.1964E+05	0.1060E+05	0.1742E+06	0.1699E+06	
60	0.1589E+06	0.1802E+05	0.1728E+05	-0.2645E+05	293.3	6450.
	0.1650E+06	0.1878E+05	0.1049E+05	0.1545E+06	0.1505E+06	
61	0.1556E+06	0.1590E+05	0.1515E+05	-0.1407E+05	-233.2	12490.
	0.1589E+06	0.1676E+05	0.1090E+05	0.1481E+06	0.1452E+06	
62	0.7956E+05	7089.	3580.	0.2614E+05	-2083.	-5511.
	0.8862E+05	4146.	-2530.	0.9115E+05	0.8803E+05	
63	0.1220E+06	0.1703E+05	0.1038E+05	0.2964E+05	361.3	2082.
	0.1309E+06	0.1377E+05	4828.	0.1260E+06	0.1219E+06	
64	0.1642E+06	0.2424E+05	0.2132E+05	0.2323E+05	2640.	18380.
	0.1712E+06	0.2265E+05	0.1591E+05	0.1553E+06	0.1521E+06	
65	0.1569E+06	0.2102E+05	0.1323E+05	0.3094E+05	1639.	10200.
	0.1662E+06	0.1816E+05	6792.	0.1594E+06	0.1541E+06	
66	0.1683E+06	0.1807E+05	0.2229E+05	8348.	1328.	21310.
	0.1726E+06	0.1947E+05	0.1659E+05	0.1560E+06	0.1546E+06	
67	0.1671E+06	0.2447E+05	0.3086E+05	2069.	-342.8	36150.
	0.1768E+06	0.2529E+05	0.2031E+05	0.1565E+06	0.1541E+06	
68	0.1570E+06	0.1750E+05	0.1776E+05	404.4	-129.7	21830.
	0.1613E+06	0.1851E+05	0.1247E+05	0.1488E+06	0.1459E+06	
69	0.1627E+06	0.2270E+05	0.2279E+05	-0.1518E+05	-2176.	28060.
	0.1711E+06	0.2293E+05	0.1413E+05	0.1570E+06	0.1528E+06	
70	0.1344E+06	0.1312E+05	0.1069E+05	0.1476E+05	2217.	10810.
	0.1378E+06	0.1298E+05	7427.	0.1304E+06	0.1277E+06	
71	0.9374E+05	3029.	261.7	-0.1188E+05	-366.4	6817.
	0.9725E+05	2255.	-2476.	0.9973E+05	0.9750E+05	
72	0.8166E+05	3673.	1159.	0.1272E+05	1474.	5515.
	0.8569E+05	2845.	-2041.	0.8773E+05	0.8541E+05	
73	0.9910E+05	3683.	8445.	-4177.	-774.7	19020.
	0.1061E+06	5217.	-90.58	0.1062E+06	0.1037E+06	
74	0.1496E+06	0.2413E+05	0.3021E+05	-5232.	-2396.	40490.
	0.1636E+06	0.2407E+05	0.1628E+05	0.1473E+06	0.1436E+06	
75	0.1808E+06	0.2564E+05	0.2766E+05	-0.2300E+05	-974.0	23440.
	0.1884E+06	0.2687E+05	0.1888E+05	0.1695E+06	0.1657E+06	
76	0.1721E+06	0.2298E+05	0.2159E+05	-0.2801E+05	-449.9	14170.
	0.1790E+06	0.2329E+05	0.1434E+05	0.1647E+06	0.1604E+06	
77	0.1866E+06	0.2262E+05	0.2620E+05	-0.3887E+05	278.7	8374.
	0.1959E+06	0.2667E+05	0.1278E+05	0.1832E+06	0.1767E+06	
78	0.1807E+06	0.2398E+05	0.2292E+05	-0.4179E+05	1018.	7560.
	0.1916E+06	0.2346E+05	0.1257E+05	0.1790E+06	0.1738E+06	
79	0.1112E+06	0.1552E+05	8912.	-0.3231E+05	2014.	7263.
	0.1218E+06	0.1246E+05	1407.	0.1204E+06	0.1153E+06	
80	0.1263E+06	0.1339E+05	0.1165E+05	-0.2060E+05	-2378.	17070.
	0.1331E+06	0.1147E+05	6774.	0.1263E+06	0.1241E+06	
81	0.1237E+06	0.1651E+05	0.1164E+05	-0.2850E+05	-1849.	14000.
	0.1330E+06	0.1277E+05	6049.	0.1270E+06	0.1238E+06	
82	0.1118E+06	7594.	0.1049E+05	-0.1903E+05	-3051.	22230.
	0.1208E+06	7225.	1920.	0.1189E+06	0.1164E+06	
83	0.7179E+05	0.1541E+05	5849.	0.2547E+05	-45.75	3806.
	0.8219E+05	7416.	3447.	0.7875E+05	0.7686E+05	
84	0.7291E+05	0.1598E+05	1255.	0.2737E+05	3211.	10780.
	0.8618E+05	5716.	-1760.	0.8794E+05	0.8455E+05	

85	0.8873E+05	0.1322E+05	4181.	0.2326E+05	3419.	11360.
	0.9832E+05	7317.	488.2	0.9783E+05	0.9466E+05	
86	0.8273E+05	0.1197E+05	5250.	0.2594E+05	6922.	20190.
	0.9672E+05	5122.	-1881.	0.9860E+05	0.9539E+05	
87	0.1220E+06	0.1115E+05	0.1313E+05	0.1167E+05	3366.	20050.
	0.1283E+06	0.1137E+05	6611.	0.1217E+06	0.1194E+06	
88	0.9061E+05	5098.	0.1935E+05	-657.5	-2079.	37570.
	0.1077E+06	5979.	1385.	0.1063E+06	0.1042E+06	
89	0.9087E+05	6708.	0.1975E+05	-0.1236E+05	-3021.	38430.
	0.1099E+06	6796.	630.0	0.1093E+06	0.1064E+06	
90	0.4593E+05	1824.	-6235.	-0.2037E+05	2107.	3597.
	0.5405E+05	-2853.	-9682.	0.6374E+05	0.6061E+05	
91	0.6516E+05	-1542.	-6908.	-9735.	-1828.	2332.
	0.6750E+05	-1630.	-9159.	0.7666E+05	0.7337E+05	
92	0.6882E+05	-1737.	-5978.	-9237.	-2687.	3675.
	0.7072E+05	-836.9	-8779.	0.7950E+05	0.7607E+05	
93	0.5658E+05	2247.	-7264.	-0.1759E+05	-2591.	5588.
	0.6273E+05	-1025.	-0.1014E+05	0.7287E+05	0.6898E+05	
94	0.7960E+05	-6899.	-2908.	1331.	1536.	6261.
	0.8047E+05	-2640.	-8035.	0.8851E+05	0.8597E+05	
95	0.6610E+05	-2494.	-5725.	-0.1361E+05	-3702.	9699.
	0.7067E+05	-3879.	-8915.	0.7959E+05	0.7734E+05	
96	0.7411E+05	-4929.	-3418.	-5667.	-1485.	5121.
	0.7531E+05	-1870.	-7686.	0.8300E+05	0.8032E+05	
97	0.8132E+05	7117.	6644.	-4443.	-444.8	1726.
	0.8187E+05	0.1028E+05	2926.	0.7895E+05	0.7589E+05	
98	0.7449E+05	-1952.	-1221.	3323.	588.2	2883.
	0.7500E+05	-117.2	-3565.	0.7856E+05	0.7690E+05	
99	0.7760E+05	1610.	1785.	-1116.	-58.41	3671.
	0.7803E+05	5070.	-2100.	0.8013E+05	0.7706E+05	
100	0.4131E+05	1789.	-4158.	0.1856E+05	-840.4	3589.
	0.4887E+05	-2668.	-7259.	0.5613E+05	0.5398E+05	
101	0.5686E+05	-64.68	-4508.	9311.	2680.	3043.
	0.5916E+05	-23.55	-6852.	0.6601E+05	0.6313E+05	
102	0.5397E+05	-397.4	-4924.	0.1040E+05	1336.	2589.
	0.5676E+05	-1381.	-6733.	0.6350E+05	0.6119E+05	
103	0.4389E+05	-1532.	-9012.	0.1428E+05	3212.	6765.
	0.4990E+05	-4703.	-0.1185E+05	0.6175E+05	0.5866E+05	
104	0.4167E+05	2287.	-6502.	0.1728E+05	1324.	3942.
	0.4874E+05	-2594.	-8695.	0.5744E+05	0.5478E+05	
105	0.6557E+05	-4973.	-4335.	9218.	3947.	12690.
	0.7025E+05	-3730.	-0.1026E+05	0.8052E+05	0.7749E+05	
106	0.6608E+05	-5006.	-4718.	3888.	2873.	4960.
	0.6710E+05	-2377.	-8374.	0.7548E+05	0.7275E+05	
107	0.4094E+05	-8645.	7055.	-4963.	-2668.	25700.
	0.5540E+05	-4094.	-0.1196E+05	0.6736E+05	0.6391E+05	
108	0.5056E+05	-9261.	-2993.	-8811.	-2368.	14620.
	0.5718E+05	-6179.	-0.1269E+05	0.6988E+05	0.6695E+05	
109	0.5736E+05	-8765.	-137.3	2984.	637.1	16450.
	0.6379E+05	-4313.	-0.1102E+05	0.7481E+05	0.7174E+05	
110	0.6534E+05	-6016.	-905.0	-945.9	-753.7	6649.
	0.6770E+05	-1412.	-7858.	0.7555E+05	0.7260E+05	
111	0.7656E+05	0.1062E+05	0.1476E+05	-6184.	-1368.	-214.5

	0.7746E+05	0.1572E+05	8759.	0.6871E+05	0.6584E+05	
112	0.6610E+05	-1791.	-2486.	-4771.	-2396.	1059.
	0.6676E+05	757.6	-5693.	0.7245E+05	0.6965E+05	
113	0.5868E+05	-6780.	-4976.	-1842.	-2888.	2304.
	0.5898E+05	-2636.	-9417.	0.6840E+05	0.6530E+05	
114	0.8050E+05	0.2729E+05	0.3126E+05	-5545.	-1244.	292.0
	0.8132E+05	0.3160E+05	0.2613E+05	0.5519E+05	0.5303E+05	
115	0.7737E+05	0.2241E+05	0.1847E+05	-168.2	-1170.	5263.
	0.7824E+05	0.2545E+05	0.1457E+05	0.6367E+05	0.5945E+05	
116	0.7816E+05	0.1658E+05	0.1488E+05	-4003.	346.3	2594.
	0.7886E+05	0.2005E+05	0.1071E+05	0.6815E+05	0.6450E+05	
117	0.7343E+05	0.1399E+05	0.1507E+05	3604.	-1345.	4143.
	0.7468E+05	0.1750E+05	0.1031E+05	0.6437E+05	0.6135E+05	
118	0.6485E+05	3715.	569.5	8197.	-380.3	-141.4
	0.6607E+05	3931.	-874.9	0.6695E+05	0.6482E+05	
119	0.7011E+05	8954.	9513.	6596.	-102.3	3510.
	0.7155E+05	0.1137E+05	5658.	0.6589E+05	0.6338E+05	
120	0.5372E+05	-1731.	-4863.	6358.	2541.	2210.
	0.5498E+05	-642.4	-7207.	0.6218E+05	0.5945E+05	
121	0.6910E+05	3201.	5232.	2810.	305.2	5065.
	0.6987E+05	6744.	916.6	0.6896E+05	0.6638E+05	
122	0.7040E+05	1922.	1052.	-216.6	1124.	2029.
	0.7059E+05	4564.	-1779.	0.7237E+05	0.6959E+05	
123	0.8758E+05	0.3577E+05	0.3577E+05	-0.1148E+05	0.0000E+00	0.000
	0.9001E+05	0.3577E+05	0.3334E+05	0.5666E+05	0.5549E+05	
124	0.7614E+05	0.2405E+05	0.2869E+05	7332.	1235.	8670.
	0.7891E+05	0.2828E+05	0.2168E+05	0.5723E+05	0.5438E+05	
125	0.7691E+05	0.2844E+05	0.2117E+05	0.1125E+05	-540.1	3560.
	0.7988E+05	0.2735E+05	0.1929E+05	0.6059E+05	0.5710E+05	

NODAL DISPLACEMENT AND STRESS LISTING OF MODEL 2.

THE FOLLOWING X,Y,Z DISPLACEMENTS ARE IN NODAL COORDINATES

NODE	UX	UY	UZ
1	-0.93890238E-01	0.26886320E-01	0.18045247E-02
2	-0.95165977E-01	0.27332610E-01	0.00000000E+00
3	-0.91913989E-01	0.24494094E-01	0.43636567E-02
4	-0.92275127E-01	0.25207000E-01	0.00000000E+00
5	-0.92205888E-01	0.25090368E-01	0.19550797E-02
6	-0.78266276E-01	0.21038741E-01	0.36685932E-02
7	-0.70890435E-01	0.19819977E-01	0.00000000E+00
8	-0.85699774E-01	0.21616245E-01	0.44931865E-02
9	-0.76119073E-01	0.18225560E-01	0.51220037E-02
10	-0.87782498E-01	0.21563370E-01	0.00000000E+00
11	-0.64464741E-01	0.16059768E-01	0.46702316E-02
12	-0.63135221E-01	0.16999071E-01	0.30960468E-02
13	-0.66301182E-01	0.15168763E-01	0.51900240E-02
14	-0.61232295E-01	0.13482626E-01	0.00000000E+00
15	-0.55427178E-01	0.15089339E-01	0.00000000E+00
16	-0.85620518E-01	0.18697051E-01	0.41839756E-02
17	-0.85190936E-01	0.18164005E-01	0.18092154E-02
18	-0.69790486E-01	0.14115296E-01	0.42356279E-02
19	-0.59880338E-01	0.10964579E-01	0.19522857E-02
20	-0.85640063E-01	0.18106812E-01	0.00000000E+00
21	-0.60030083E-01	0.10996443E-01	0.00000000E+00
22	-0.55903505E-01	0.13592924E-01	0.43527929E-02
23	-0.57468272E-01	0.12492468E-01	0.50294124E-02
24	-0.52002564E-01	0.13540864E-01	0.24742857E-02
25	-0.58922136E-01	0.12132548E-01	0.33521654E-02
26	-0.58194664E-01	0.10719630E-01	0.42379690E-02
27	-0.44378214E-01	0.72643319E-02	0.00000000E+00
28	-0.46135945E-01	0.10743312E-01	0.39578952E-02
29	-0.40044767E-01	0.82809779E-02	0.28140369E-02
30	-0.49436101E-01	0.10156673E-01	0.46829735E-02
31	-0.51984183E-01	0.10013226E-01	0.30586238E-02
32	-0.37202751E-01	0.61228978E-02	0.00000000E+00
33	-0.46180841E-01	0.12846676E-01	0.00000000E+00
34	-0.82771725E-01	0.16766945E-01	0.18557707E-02
35	-0.72332472E-01	0.13319646E-01	0.18977627E-02
36	-0.62295029E-01	0.10516483E-01	0.19691093E-02
37	-0.53297871E-01	0.81461171E-02	0.19164135E-02
38	-0.44440138E-01	0.70629850E-02	0.17581783E-02
39	-0.50308260E-01	0.89940314E-02	0.43925715E-02
40	-0.44212015E-01	0.11503084E-01	0.24195368E-02
41	-0.50129037E-01	0.80034175E-02	0.33428423E-02
42	-0.35578793E-01	0.47580476E-02	0.00000000E+00
43	-0.37687182E-01	0.51250498E-02	0.15447596E-02
44	-0.36607602E-01	0.76241479E-02	0.34254324E-02
45	-0.42340200E-01	0.79839989E-02	0.44436174E-02
46	-0.37990133E-01	0.89267837E-02	0.26017351E-02

47	-0.37294109E-01	0.72355238E-02	0.24559551E-02
48	-0.35766432E-01	0.62933743E-02	0.33689796E-02
49	-0.43048495E-01	0.68894106E-02	0.42466823E-02
50	-0.41977787E-01	0.67266368E-02	0.32232556E-02
51	-0.39169111E-01	0.10029916E-01	0.00000000E+00
52	-0.30083286E-01	0.32784178E-02	0.00000000E+00
53	-0.38524308E-01	0.96767164E-02	0.13715978E-02
54	-0.31369991E-01	0.48855693E-02	0.00000000E+00
55	-0.32636018E-01	0.38117370E-02	0.21293209E-02
56	-0.32929765E-01	0.40709270E-02	0.63774945E-03
57	-0.34088104E-01	0.48945872E-02	0.29675324E-02
58	-0.31117116E-01	0.68413574E-02	0.19448844E-02
59	-0.31891366E-01	0.77171835E-02	0.00000000E+00
60	-0.26054800E-01	0.18830537E-02	0.00000000E+00
61	-0.27293901E-01	0.26226656E-02	0.17394421E-02
62	-0.29226081E-01	0.46681872E-02	0.24734684E-02
63	-0.45842036E-01	0.62681844E-02	0.18239848E-02
64	-0.39100892E-01	0.45471034E-02	0.16560692E-02
65	-0.44282189E-01	0.64936793E-02	0.32420369E-02
66	-0.37908477E-01	0.47692352E-02	0.29770733E-02
67	-0.30662495E-01	0.37252306E-02	0.00000000E+00
68	-0.31999365E-01	0.36507406E-02	0.10091642E-02
69	-0.30292743E-01	0.33597757E-02	0.87186000E-03
70	-0.31206040E-01	0.72624803E-02	0.29672975E-02
71	-0.29138896E-01	0.51213563E-02	0.30372775E-02
72	-0.32289084E-01	0.84681973E-02	0.19336710E-02
73	-0.32333636E-01	0.88380947E-02	0.00000000E+00
74	-0.25359654E-01	0.21896696E-02	0.00000000E+00
75	-0.24211985E-01	0.55200028E-02	0.00000000E+00
76	-0.18688991E-01	0.18692092E-02	0.00000000E+00
77	-0.22384938E-01	0.27470077E-02	0.25325723E-02
78	-0.28775748E-01	0.31342063E-02	0.20402701E-02
79	-0.27461655E-01	0.34082603E-02	0.28500289E-02
80	-0.29459736E-01	0.74917294E-02	0.28325142E-02
81	-0.29860826E-01	0.79487480E-02	0.12990140E-02
82	-0.27444814E-01	0.59943773E-02	0.33922309E-02
83	-0.25787333E-01	0.67755851E-02	0.19482279E-02
84	-0.29873064E-01	0.81715518E-02	0.00000000E+00
85	-0.27058717E-01	0.77175341E-02	0.00000000E+00
86	-0.22278642E-01	0.18601192E-02	0.00000000E+00
87	-0.20662331E-01	-0.21282030E-03	0.00000000E+00
88	-0.25732458E-01	0.23877748E-02	0.12828208E-02
89	-0.22713031E-01	0.19904012E-02	0.10243990E-02
90	-0.20265938E-01	0.11462822E-03	0.15221868E-02
91	-0.24537243E-01	0.19654321E-02	0.24216046E-02
92	-0.25024701E-01	0.31487850E-02	0.31849093E-02
93	-0.25925429E-01	0.45647584E-02	0.29235458E-02
94	-0.23128506E-01	0.32290109E-02	0.35932165E-02
95	-0.22108530E-01	0.60539410E-02	0.00000000E+00
96	-0.15594271E-01	0.43619720E-02	0.00000000E+00
97	-0.19794400E-01	0.47044385E-02	0.20758852E-02
98	-0.16804232E-01	-0.75640541E-03	0.00000000E+00
99	-0.12184475E-01	-0.20939514E-02	0.00000000E+00

100	-0.18053646E-01	0.32011909E-02	0.30999840E-02
101	-0.15865406E-01	-0.63970497E-03	0.14903742E-02
102	-0.15996682E-01	0.79485718E-04	0.26010124E-02
103	-0.10133408E-01	-0.16870535E-02	0.21999757E-02
104	-0.21003948E-01	0.12651237E-02	0.27858099E-02
105	-0.13937169E-01	0.10027764E-02	0.32627434E-02
106	-0.12114445E-01	0.29417788E-02	0.25273613E-02
107	-0.83628859E-02	0.33826649E-03	0.00000000E+00
108	-0.19337459E-01	0.20466864E-02	0.35438693E-02
109	-0.23750627E-01	0.45637136E-02	0.31709058E-02
110	-0.74609458E-02	0.30722155E-02	0.00000000E+00
111	-0.59890933E-02	-0.22403708E-02	0.00000000E+00
112	0.00000000E+00	0.00000000E+00	0.00000000E+00
113	-0.67538734E-02	0.14583898E-03	0.26612706E-02
114	0.00000000E+00	0.00000000E+00	0.00000000E+00
115	0.00000000E+00	0.00000000E+00	0.00000000E+00
116	0.00000000E+00	0.00000000E+00	0.00000000E+00
117	0.00000000E+00	0.00000000E+00	0.00000000E+00
118	0.00000000E+00	0.00000000E+00	0.00000000E+00

MAXIMUMS

NODE	2	2	13
VALUE	-0.95165977E-01	0.27332610E-01	0.51900240E-02

THE FOLLOWING X,Y,Z STRESSES ARE IN GLOBAL COORDINATES

NODE	SX	SY	SZ	SXY	SYZ	SXZ
	SIG1	SIG2	SIG3	SI	SIGE	
1	0.1908E+06	-4432.	1399.	1267.	1985.	-1861.
	0.1910E+06	2071.	-5281.	0.1962E+06	0.1927E+06	
2	0.1908E+06	-2724.	2066.	6784.	576.5	-3238.
	0.1912E+06	2038.	-3059.	0.1943E+06	0.1918E+06	
3	0.1876E+06	-1067.	-501.1	0.1016E+05	-33.68	4630.
	0.1890E+06	1999.	-5016.	0.1940E+06	0.1906E+06	
4	0.1949E+06	-2571.	2358.	4376.	1131.	-1544.
	0.1951E+06	2647.	-3056.	0.1982E+06	0.1954E+06	
5	0.1881E+06	-2569.	830.8	7027.	25.06	1217.
	0.1888E+06	2255.	-4762.	0.1936E+06	0.1902E+06	
6	0.1926E+06	767.6	2456.	2707.	546.8	-1010.
	0.1928E+06	3424.	-325.2	0.1931E+06	0.1913E+06	
7	0.1945E+06	2318.	2083.	1407.	-28.77	107.7
	0.1945E+06	4017.	305.3	0.1942E+06	0.1924E+06	
8	0.1579E+06	-0.1246E+05	-8840.	424.9	-533.5	2324.
	0.1602E+06	-7063.	-0.1649E+05	0.1767E+06	0.1722E+06	
9	0.1794E+06	-2373.	-947.1	-579.5	-255.4	179.2
	0.1803E+06	126.1	-4334.	0.1846E+06	0.1825E+06	
10	0.1845E+06	-1385.	961.2	-180.0	-55.35	-2331.
	0.1847E+06	1563.	-2221.	0.1870E+06	0.1851E+06	

11	0.1828E+06	1512.	254.8	386.7	-583.7	-2640.
	0.1828E+06	1901.	-187.3	0.1830E+06	0.1820E+06	
12	0.1894E+06	3459.	1816.	-2468.	-1014.	-1261.
	0.1894E+06	4202.	1017.	0.1884E+06	0.1868E+06	
13	0.1802E+06	355.6	-854.4	-225.8	139.9	-2440.
	0.1803E+06	585.8	-1164.	0.1814E+06	0.1806E+06	
14	0.1803E+06	3723.	1030.	-392.2	-151.2	-2743.
	0.1804E+06	4237.	368.5	0.1800E+06	0.1781E+06	
15	0.1856E+06	5111.	4243.	-5679.	-496.9	-2044.
	0.1858E+06	6468.	2633.	0.1832E+06	0.1813E+06	
16	0.1832E+06	701.4	-128.6	-5877.	60.59	2245.
	0.1843E+06	1918.	-2416.	0.1867E+06	0.1846E+06	
17	0.1835E+06	-1114.	717.2	-5437.	165.9	-922.2
	0.1841E+06	1629.	-2688.	0.1868E+06	0.1847E+06	
18	0.1882E+06	1008.	1602.	-2345.	-12.10	-690.7
	0.1883E+06	2167.	321.2	0.1880E+06	0.1871E+06	
19	0.1847E+06	507.8	386.0	-698.2	413.6	-2250.
	0.1848E+06	1554.	-782.6	0.1856E+06	0.1844E+06	
20	0.1907E+06	-1030.	3103.	-6958.	-135.5	-2996.
	0.1910E+06	3110.	-1347.	0.1924E+06	0.1902E+06	
21	0.1846E+06	2080.	1267.	222.4	459.2	-1953.
	0.1847E+06	3010.	248.7	0.1845E+06	0.1831E+06	
22	0.1811E+06	3141.	36.37	-1459.	-1375.	-2790.
	0.1813E+06	3744.	-673.8	0.1819E+06	0.1798E+06	
23	0.1784E+06	2697.	-114.6	730.3	523.4	-3035.
	0.1785E+06	3265.	-755.1	0.1792E+06	0.1772E+06	
24	0.1834E+06	5677.	5234.	-7089.	-562.9	-3117.
	0.1839E+06	7775.	2729.	0.1811E+06	0.1787E+06	
25	0.1784E+06	750.1	-1313.	579.6	518.9	-2843.
	0.1785E+06	965.1	-1585.	0.1801E+06	0.1788E+06	
26	0.1813E+06	899.0	209.7	1359.	846.9	-1280.
	0.1814E+06	1789.	-736.4	0.1821E+06	0.1809E+06	
27	0.1808E+06	9895.	7044.	0.1029E+05	644.1	1302.
	0.1817E+06	0.1027E+05	5755.	0.1760E+06	0.1738E+06	
28	0.1855E+06	9571.	9358.	-5409.	-850.2	-6173.
	0.1861E+06	0.1242E+05	5881.	0.1802E+06	0.1771E+06	
29	0.1807E+06	0.1251E+05	0.1406E+05	-7454.	-437.5	-.1073E+5
	0.1821E+06	0.1573E+05	9464.	0.1726E+06	0.1696E+06	
30	0.1740E+06	4597.	2194.	-1214.	957.6	-4319.
	0.1742E+06	5776.	790.0	0.1734E+06	0.1710E+06	
31	0.1776E+06	3754.	1596.	3313.	1190.	-2113.
	0.1778E+06	4561.	598.5	0.1772E+06	0.1753E+06	
32	0.1591E+06	0.1003E+05	0.1053E+05	1246.	-202.9	-5693.
	0.1604E+06	0.1193E+05	7371.	0.1530E+06	0.1508E+06	
33	0.1932E+06	0.1459E+05	0.1521E+05	-0.1563E+05	1716.	-4572.
	0.1948E+06	0.1625E+05	0.1192E+05	0.1829E+06	0.1808E+06	
34	0.1802E+06	4490.	330.7	9243.	-750.4	-1617.
	0.1807E+06	4121.	199.5	0.1805E+06	0.1786E+06	
35	0.1877E+06	1122.	1336.	2992.	-132.9	-1655.
	0.1879E+06	2465.	-176.4	0.1881E+06	0.1868E+06	
36	0.1867E+06	1080.	972.0	1076.	222.1	-1548.
	0.1867E+06	1497.	530.0	0.1862E+06	0.1857E+06	
37	0.1832E+06	441.3	345.1	2275.	432.5	-672.1

	0.1832E+06	1193.	-464.5	0.1837E+06	0.1829E+06	
38	0.1762E+06	3840.	3028.	3763.	942.8	1624.
	0.1765E+06	5286.	1265.	0.1753E+06	0.1733E+06	
39	0.1733E+06	1941.	417.0	271.5	1830.	780.3
	0.1733E+06	3288.	-992.5	0.1743E+06	0.1722E+06	
40	0.1935E+06	0.1466E+05	0.1621E+05	-0.1408E+05	1241.	-6314.
	0.1949E+06	0.1783E+05	0.1158E+05	0.1834E+06	0.1803E+06	
41	0.1749E+06	-656.8	287.8	343.7	597.0	1905.
	0.1749E+06	851.6	-1263.	0.1762E+06	0.1751E+06	
42	0.1296E+06	0.1396E+05	9477.	0.2735E+05	565.4	1124.
	0.1374E+06	0.1187E+05	3739.	0.1337E+06	0.1299E+06	
43	0.1694E+06	0.1269E+05	9661.	0.1559E+05	39.83	2590.
	0.1719E+06	0.1322E+05	6594.	0.1653E+06	0.1621E+06	
44	0.1769E+06	0.1970E+05	0.2481E+05	-0.1102E+05	-419.1	-.3055E+05
	0.1857E+06	0.2214E+05	0.1356E+05	0.1721E+06	0.1680E+06	
45	0.1769E+06	0.1181E+05	0.1326E+05	-1712.	-1049.	-.1138E+05
	0.1781E+06	0.1479E+05	9087.	0.1690E+06	0.1663E+06	
46	0.1738E+06	0.1732E+05	0.1840E+05	-0.1737E+05	207.2	-.2056E+05
	0.1806E+06	0.1874E+05	0.1012E+05	0.1705E+06	0.1664E+06	
47	0.1603E+06	0.1140E+05	0.1349E+05	-6376.	-1933.	-.1550E+05
	0.1624E+06	0.1406E+05	8697.	0.1537E+06	0.1511E+06	
48	0.1657E+06	0.1573E+05	0.2136E+05	-2119.	-2460.	-.2559E+05
	0.1728E+06	0.1930E+05	0.1076E+05	0.1620E+06	0.1579E+06	
49	0.1706E+06	5281.	1770.	-878.1	1557.	-1999.
	0.1707E+06	6471.	446.4	0.1703E+06	0.1674E+06	
50	0.1707E+06	7506.	5370.	2945.	1221.	-2782.
	0.1712E+06	9243.	3140.	0.1681E+06	0.1652E+06	
51	0.1797E+06	0.2511E+05	0.2088E+05	-0.3281E+05	1398.	-9355.
	0.1906E+06	0.2076E+05	0.1436E+05	0.1763E+06	0.1732E+06	
52	0.1025E+06	0.1177E+05	8637.	0.2754E+05	1073.	-2262.
	0.1122E+06	9193.	1539.	0.1106E+06	0.1070E+06	
53	0.1785E+06	0.2367E+05	0.2103E+05	-0.3179E+05	264.9	-.1201E+05
	0.1885E+06	0.2197E+05	0.1263E+05	0.1759E+06	0.1715E+06	
54	0.1025E+06	-2750.	377.5	1883.	431.1	-5596.
	0.1036E+06	174.4	-3608.	0.1072E+06	0.1054E+06	
55	0.1505E+06	0.1342E+05	9437.	0.2298E+05	-120.6	-6424.
	0.1578E+06	0.1167E+05	3827.	0.1540E+06	0.1504E+06	
56	0.1263E+06	0.1048E+05	7612.	0.2728E+05	-511.4	-931.8
	0.1338E+06	8938.	1668.	0.1321E+06	0.1287E+06	
57	0.1574E+06	0.1363E+05	0.1661E+05	9908.	-469.3	-.2322E+05
	0.1647E+06	0.1525E+05	7642.	0.1571E+06	0.1535E+06	
58	0.1104E+06	5150.	4929.	-0.1730E+05	1171.	-.1710E+05
	0.1182E+06	4806.	-2554.	0.1208E+06	0.1174E+06	
59	0.1081E+06	8952.	3413.	-0.2306E+05	1670.	-7533.
	0.1164E+06	5289.	-1205.	0.1176E+06	0.1145E+06	
60	0.8097E+05	6328.	302.6	0.1899E+05	-1479.	-3336.
	0.8701E+05	3822.	-3234.	0.9024E+05	0.8705E+05	
61	0.9456E+05	3478.	1980.	0.1573E+05	-1928.	-.1041E+05
	0.1010E+06	2904.	-3883.	0.1049E+06	0.1017E+06	
62	0.1131E+06	2615.	0.1010E+05	-1551.	-110.7	-.2514E+05
	0.1216E+06	5689.	-1531.	0.1231E+06	0.1197E+06	
63	0.1754E+06	224.1	1351.	-728.5	480.8	2780.
	0.1754E+06	1891.	-379.3	0.1758E+06	0.1747E+06	

64	0.1824E+06	4358.	7276.	4919.	1675.	6462.
	0.1829E+06	7770.	3317.	0.1796E+06	0.1774E+06	
65	0.1683E+06	2021.	469.1	421.5	771.5	2753.
	0.1683E+06	3295.	-881.2	0.1692E+06	0.1672E+06	
66	0.1748E+06	6789.	4976.	5521.	3253.	2215.
	0.1756E+06	9734.	1234.	0.1743E+06	0.1703E+06	
67	0.9893E+05	0.1920E+05	7853.	0.3416E+05	2911.	-5470.
	0.1140E+06	0.1150E+05	522.6	0.1134E+06	0.1084E+06	
68	0.1518E+06	0.1440E+05	9832.	0.2816E+05	1073.	608.6
	0.1582E+06	0.1243E+05	5499.	0.1527E+06	0.1494E+06	
69	0.1152E+06	0.1700E+05	9904.	0.3074E+05	302.9	-4451.
	0.1251E+06	0.1165E+05	5367.	0.1198E+06	0.1168E+06	
70	0.1236E+06	8496.	0.1335E+05	-0.2142E+05	2285.	-.3930E+05
	0.1405E+06	7784.	-2924.	0.1435E+06	0.1385E+06	
71	0.1130E+06	5656.	0.2017E+05	-2107.	698.6	-.4593E+05
	0.1327E+06	7671.	-1587.	0.1343E+06	0.1300E+06	
72	0.1283E+06	0.1759E+05	0.1099E+05	-0.4219E+05	2148.	-.2279E+05
	0.1472E+06	0.1100E+05	-1361.	0.1486E+06	0.1429E+06	
73	0.1280E+06	0.2602E+05	0.1273E+05	-0.5179E+05	2559.	-.1166E+05
	0.1509E+06	0.1298E+05	2837.	0.1481E+06	0.1433E+06	
74	0.7173E+05	0.1504E+05	1621.	0.2910E+05	-1650.	-4112.
	0.8447E+05	4881.	-965.4	0.8544E+05	0.8275E+05	
75	0.8473E+05	-4048.	-4555.	-4621.	1272.	-2444.
	0.8510E+05	-3002.	-5971.	0.9107E+05	0.8963E+05	
76	0.7526E+05	-2355.	-2662.	1529.	-697.0	-608.6
	0.7551E+05	-1084.	-4180.	0.7969E+05	0.7821E+05	
77	0.7426E+05	-4893.	-2737.	-472.4	-29.47	-6362.
	0.7545E+05	-1998.	-6826.	0.8228E+05	0.8001E+05	
78	0.1186E+06	0.1588E+05	0.1065E+05	0.2921E+05	-3296.	-.1220E+05
	0.1305E+06	0.1179E+05	2838.	0.1277E+06	0.1235E+06	
79	0.9962E+05	2455.	7494.	0.1037E+05	-1939.	-.3047E+05
	0.1107E+06	3339.	-4432.	0.1151E+06	0.1114E+06	
80	0.7119E+05	-3453.	-3464.	-0.1284E+05	3073.	-.1541E+05
	0.7783E+05	-3978.	-9578.	0.8740E+05	0.8479E+05	
81	0.6491E+05	3910.	-7636.	-0.2441E+05	3130.	-7850.
	0.7501E+05	-3728.	-0.1010E+05	0.8512E+05	0.8218E+05	
82	0.6869E+05	-7296.	912.5	-7446.	1201.	-.1996E+05
	0.7594E+05	-3865.	-9774.	0.8571E+05	0.8297E+05	
83	0.7406E+05	-4004.	-6512.	-7499.	2723.	-5224.
	0.7578E+05	-3205.	-9028.	0.8481E+05	0.8212E+05	
84	0.5758E+05	6894.	-5653.	-0.2647E+05	1015.	-2770.
	0.6928E+05	-2324.	-8132.	0.7741E+05	0.7470E+05	
85	0.6805E+05	-1429.	-8062.	-0.1126E+05	2191.	-2938.
	0.7085E+05	-3303.	-8988.	0.7984E+05	0.7721E+05	
86	0.2977E+05	5628.	-8939.	0.1566E+05	-916.1	-1860.
	0.3763E+05	-2139.	-9033.	0.4666E+05	0.4372E+05	
87	0.4886E+05	204.6	-7223.	7045.	-1478.	-1989.
	0.5106E+05	-1532.	-7683.	0.5874E+05	0.5615E+05	
88	0.7565E+05	0.1174E+05	245.3	0.2417E+05	-3260.	-4609.
	0.8473E+05	4545.	-1635.	0.8637E+05	0.8351E+05	
89	0.3675E+05	1605.	-0.1169E+05	0.1664E+05	-2702.	-3267.
	0.4399E+05	-4461.	-0.1286E+05	0.5685E+05	0.5325E+05	
90	0.5940E+05	-2601.	-6075.	5976.	-2277.	-2531.

	0.6086E+05	-2093.	-8042.	0.6890E+05	0.6625E+05	
91	0.6315E+05	-1231.	-3606.	8690.	-2485.	-9061.
	0.6740E+05	-1398.	-7689.	0.7509E+05	0.7218E+05	
92	0.6034E+05	-8968.	-1553.	5078.	-1063.	-.1700E+05
	0.6637E+05	-5320.	-0.1123E+05	0.7761E+05	0.7487E+05	
93	0.5299E+05	-0.1004E+05	4369.	-1309.	-32.81	-.2535E+05
	0.6400E+05	-3681.	-0.1300E+05	0.7701E+05	0.7285E+05	
94	0.5918E+05	-7371.	-796.2	-614.9	-256.2	-9554.
	0.6186E+05	-2142.	-8713.	0.7058E+05	0.6758E+05	
95	0.7968E+05	-3078.	-5034.	-3960.	1103.	-1546.
	0.7995E+05	-2788.	-5594.	0.8554E+05	0.8418E+05	
96	0.8550E+05	941.5	-10.38	-1247.	264.3	956.7
	0.8556E+05	1750.	-877.1	0.8644E+05	0.8516E+05	
97	0.7846E+05	-3802.	-3330.	-2047.	1061.	-1499.
	0.7861E+05	-1933.	-5346.	0.8396E+05	0.8233E+05	
98	0.5991E+05	-2689.	-3584.	1723.	-109.9	-905.2
	0.6001E+05	-2428.	-3950.	0.6396E+05	0.6322E+05	
99	0.6893E+05	102.6	-149.4	45.79	66.58	1290.
	0.6898E+05	863.2	-959.8	0.6994E+05	0.6906E+05	
100	0.7506E+05	-2556.	-1311.	-84.79	163.6	-2052.
	0.7516E+05	-655.8	-3310.	0.7847E+05	0.7718E+05	
101	0.6220E+05	-2428.	-3109.	1088.	-535.4	-254.6
	0.6226E+05	-1782.	-3813.	0.6608E+05	0.6509E+05	
102	0.6835E+05	-1941.	-3089.	564.2	-1777.	-1776.
	0.6851E+05	-434.6	-4750.	0.7326E+05	0.7122E+05	
103	0.7158E+05	1158.	792.9	672.2	-104.7	-307.6
	0.7178E+05	2143.	-392.9	0.7217E+05	0.7095E+05	
104	0.5860E+05	-5542.	-4684.	1412.	-2488.	-3023.
	0.5894E+05	-2497.	-8078.	0.6702E+05	0.6444E+05	
105	0.7666E+05	-672.1	105.4	-207.8	-826.7	-1372.
	0.7675E+05	1089.	-1752.	0.7851E+05	0.7714E+05	
106	0.8165E+05	1413.	1721.	-308.6	-662.7	-348.0
	0.8176E+05	2712.	310.3	0.8145E+05	0.8028E+05	
107	0.7814E+05	6675.	6660.	96.08	-56.92	-527.1
	0.7833E+05	0.1010E+05	3048.	0.7528E+05	0.7243E+05	
108	0.6693E+05	-3044.	-669.3	71.17	-461.2	-5105.
	0.6734E+05	-183.2	-3941.	0.7128E+05	0.6948E+05	
109	0.6405E+05	-6663.	-2639.	-1656.	2335.	-4886.
	0.6460E+05	-1955.	-7892.	0.7249E+05	0.6974E+05	
110	0.8621E+05	0.1471E+05	0.1876E+05	-4255.	1616.	-534.3
	0.8691E+05	0.2012E+05	0.1265E+05	0.7426E+05	0.7127E+05	
111	0.7211E+05	0.1165E+05	0.1514E+05	4166.	-1391.	224.7
	0.7271E+05	0.1659E+05	9602.	0.6311E+05	0.6024E+05	
112	0.7473E+05	0.1850E+05	0.1794E+05	2994.	-2228.	-2721.
	0.7555E+05	0.2214E+05	0.1349E+05	0.6206E+05	0.5861E+05	
113	0.7933E+05	0.1698E+05	0.1321E+05	-75.68	-158.0	-4174.
	0.7996E+05	0.1843E+05	0.1113E+05	0.6883E+05	0.6587E+05	
114	0.8228E+05	0.1941E+05	0.1994E+05	-3065.	2295.	-3252.
	0.8312E+05	0.2358E+05	0.1493E+05	0.6818E+05	0.6479E+05	
115	0.8025E+05	0.2632E+05	0.2632E+05	-542.3	180.6	-2470.
	0.8122E+05	0.3133E+05	0.2033E+05	0.6090E+05	0.5670E+05	
116	0.9438E+05	0.3855E+05	0.3855E+05	-0.1149E+05	0.0000	0.0000
	0.9665E+05	0.3855E+05	0.3627E+05	0.6038E+05	0.5927E+05	

117	0.7576E+05	0.3094E+05	0.3094E+05	8382.	0.0000	0.0000
	0.7728E+05	0.3094E+05	0.2943E+05	0.4785E+05	0.4711E+05	
118	0.8543E+05	0.3490E+05	0.3490E+05	-545.6	0.0000E+00	-9957.
	0.8733E+05	0.3490E+05	0.3300E+05	0.5433E+05	0.5341E+05	

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