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Design and analysis of an instrument system for rheological testing of whole blood samples

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DESIGN AND ANALYSIS OF AN INSTRUMENT SYSTEM

FOR

RHEOLOGICAL TESTING OF WHOLE BLOOD SAMPLES

BY

ROBERT RIMVYDAS SINUSAS

A DISSERTATION

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ABSTRACT

The dissertation covers the conceptual design of a rheometer for testing whole blood samples through to the clotting state. While the design aims at increased measurement accuracy, the design employs well known technologies and available components.

The main reason for preoccupation with blood testing is that recent investigations have indicated that physiological disorders can be correlated with rheological properties of blood. The pursuit of an improved means of correlation is initiated with a survey of the rheological properties of blood. The relevance of these blood properties is then discussed. From this discussion a broader blood model is developed as a guide to design of a special blood rheometer.

The contribution of this work is an engineering study of the exploitation of known technologies to produce a rheometer which permits more precise measurement of fluid properties by better quantization of instrument errors. The quantization of instrument errors inherently requires new and original analysis of sample holder geometry, mathematical treatment of sample holder geometry for Newtonian fluids, discussion of exact and end effect characteristics of solutions of sample holder geometry for Newtonian fluids. Further reduction in errors are achieved by the control of sample holder motions (through management of motions in discrete steps), a flexible digital control of required motions, and a method for obtaining transient responses and fast time constants for torque measurement using a conventional counter torque servo system.

APPROVAL OF DISSERTATION

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FACULTY COMMITTEE

APPROVED: ______________________ CHAIRMAN

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NEWARK, NEW JERSEY

DECEMBER, 1974

PREFACE

The design and analysis of the whole blood rheometer presented here is an outgrowth of the joint Newark College of Engineering - New Jersey College of Medicine and Dentistry (NJCMD) Blood Rheology Project. The Blood Rheology Project is directed by Dr. B. Rush, M.D., Chief of Surgery of NJCMD. The work reported in this dissertation was supported by the Foundation at NCE, Dr. A. Allentuch, Director. The support is gratefully acknowledged.

The early beginnings may be traced to Dr. N. Siskovic who from his own researches and published results using rheological tests on blood to diagnose disease perceived the need for a rheometer of greater accuracy and designed specifically for testing blood. Dr. Siskovic's arguments were persuasive and the task of designing a better rheometer was undertaken under the supervision of Dr. J.L. Martin who also has done extensive blood testing.

The design of an instrument is a difficult task because it is in an instrument that theory meets reality. An understanding of the theory and limitations of the state of the art plus the rather wide range of specialized knowledge must be acquired. This arduous apprenticeship was facilitated by the many hours of valuable discussion with Drs. Martin and Siskovic. Their contribution was more than verbal. When ideas crystalized on the design, quite a few were incorporated and checked for their viability and performance by Drs. Martin and Siskovic.

Many other persons have contributed. Dr. I. Cochin suggested the use of surplus inertial guidance gyroscopes to reduce the cost of the torque measuring system. Dr. C.R. Huang provided space in his laboratory and also contributed to the work by many valuable discussions.

Last, but really foremost, the contribution of my splendid wife Ellen must be gratefully acknowledged. She went to work so that I could go back to school. She acted as a purple prose filter and typed (and retyped) the manuscript. But, the greatest contribution was in the faith and encouragement that she constantly provided.

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I INTRODUCTION

Heading

The title in a telegraphic way presents the subject matter to be covered. By way of an introduction, the intended meaning of the words in the title "Design and Analysis of an Instrument System for Rheological Testing of Whole Blood Samples" are presented. The concern is with an instrument to measure the rheological properties of small samples of liquids with special consideration being given to testing the rheological properties of human blood. The concern is with more than just the instrument itself but with a means of absolute and relative calibration, automatic sequencing of the various tests and recording of the test data for processing by computer. The concern is with an instrument system rather than just with an instrument. The special concern is with blood, more specifically with whole blood. The terms "blood" and whole blood seem to have a range of meanings which are not equivalent from a rheological viewpoint. As used in this dissertation whole blood refers to the state of blood as it exists in the body unaltered in any way. Ideally the blood is instantenously withdrawn and injected into the sample holder and should have the properties of the blood in the donor's body. The sample holder in no way should change the properties of the blood. Outside the body blood clots in a few minutes via a rather complicated reaction chain[1]. Testing of whole blood includes testing the properties of the coagulum.

1. Macfarlane, R.G., "An Enzyme Cascade in the Blood Clotting Mechanism, and its Function as a Biochemical Amplifier", <u>Nature</u>, Vol. 202, May 2, 1964, pp. 498 - 499.

Blood has many properties. Our specific concern is with the rheological properties. Rheological refers to the flow properties from the Greek ρεω (to flow). The specification of the rheological properties would entail measurements over a wide range of flow conditions. Unfortunately this is not practical with blood. Blood is a living fluid and becomes damaged at high shear rates. This not only alters the fluid properties but speeds up the chemical reactions[2,3,4]. The measurement of rheological properties is thus restricted to rather slow and gentle flows.

The contribution of this dissertation lies in the design of a rheometer. Design means the systems design, that is, the conceptual design rather than the detailed execution at the nuts and bolts level. The design consists of proposing a certain way of meeting the technical requirements as well as providing an analysis of the parameters involved so as to aid the detailed design for someone "skilled in the art".

Why Test Blood?

Blood has very interesting rheological properties. Tests demon-

2. Leverett, L., Hellums, I., Alfrey, C., Lynch, E., "Red Blood Cell Damage by Shear Stress", Biophysical Journal, Vol. 12, #3, March 1972, pp. 257 - 273.

3. Williams, A.R., "Shear Induced Fragmentation of Human Erythrocytes", Biorheology, Vol. 10, #3, Sept. 1973, pp. 303 - 311.

4. Dintenfass, L., "Effect of Velocity Gradient on the Clotting Time of Blood and the Consistency (Rheology) of Clots (Thrombi) Formed in Vitro", (Proceedings of the XI Congress of the International Society on Blood Transfusion), Biblphie haemat, 29, Part 4, 1155, 1968.

strate that the rheological properties of blood are non-Newtonian[5], thizotropic[6,7], viscoelastic[8,9] and time dependent (changing from a fluid to a gel in a few minutes). In addition our interpretation of visual observations of blood suggests that sheared blood becomes anisotropic. If mountains are climbed because they are there, then blood should be tested for the challenge that it presents. Of course, instruments to test blood should be designed and built for the challenge that it poses to the instrument makers art.

There are more urgent reasons for testing blood than the challenge that blood testing poses. The need for testing blood becomes apparent if we look at the national death statistics (Figure 1). The deaths from circulation problems nearly equal those from all other causes. Perhaps not all the circulation problems are related to the

5. Copley, A.L., Huang, C.R., King, R.G., "Rheogoniometric Studies of Whole Human Blood at Shear Rates from 1000 to .0009 sec^{-1} Part I - Experimental Findings", Biorheology, Vol. 10, #1, March 1973, pp. 17 - 22.

6. Huang, R.C., Siskovic, N., Robertson, R.W.J., Wang, H.H., Orosz, P., Jr., "Thixotropy of Human Blood", Proceedings of the First International Congress of Biorheology, 1972, in press.

7. Dintenfass, L., "Blood Microrheology - Viscosity Factors in Blood Flow, Ischaemia and Thrombosis", London: Butterworth & Co., 1971, pp. 175 - 180.

8. Lessner, A., Zahavi, J., Silberberg, A., Frei, E.H., Dreyfus, F., "The Viscoelastic Properties of Whole Blood", In "Theoretical and Clinical Hemorheology", (Edited by H. Hartert and A.L. Copley), Berlin-Heidelberg, New York: Springer-Verlag, 1971.

9. Isogai, Y., Ilda, A., Chikatsu, I., Mochizuki, K., Abe, A., "Dynamic Viscoelasticity of Blood During Clotting in Health and Disease", Biorheology, Vol. 10, #3, Sept. 1973, pp. 411 - 424.

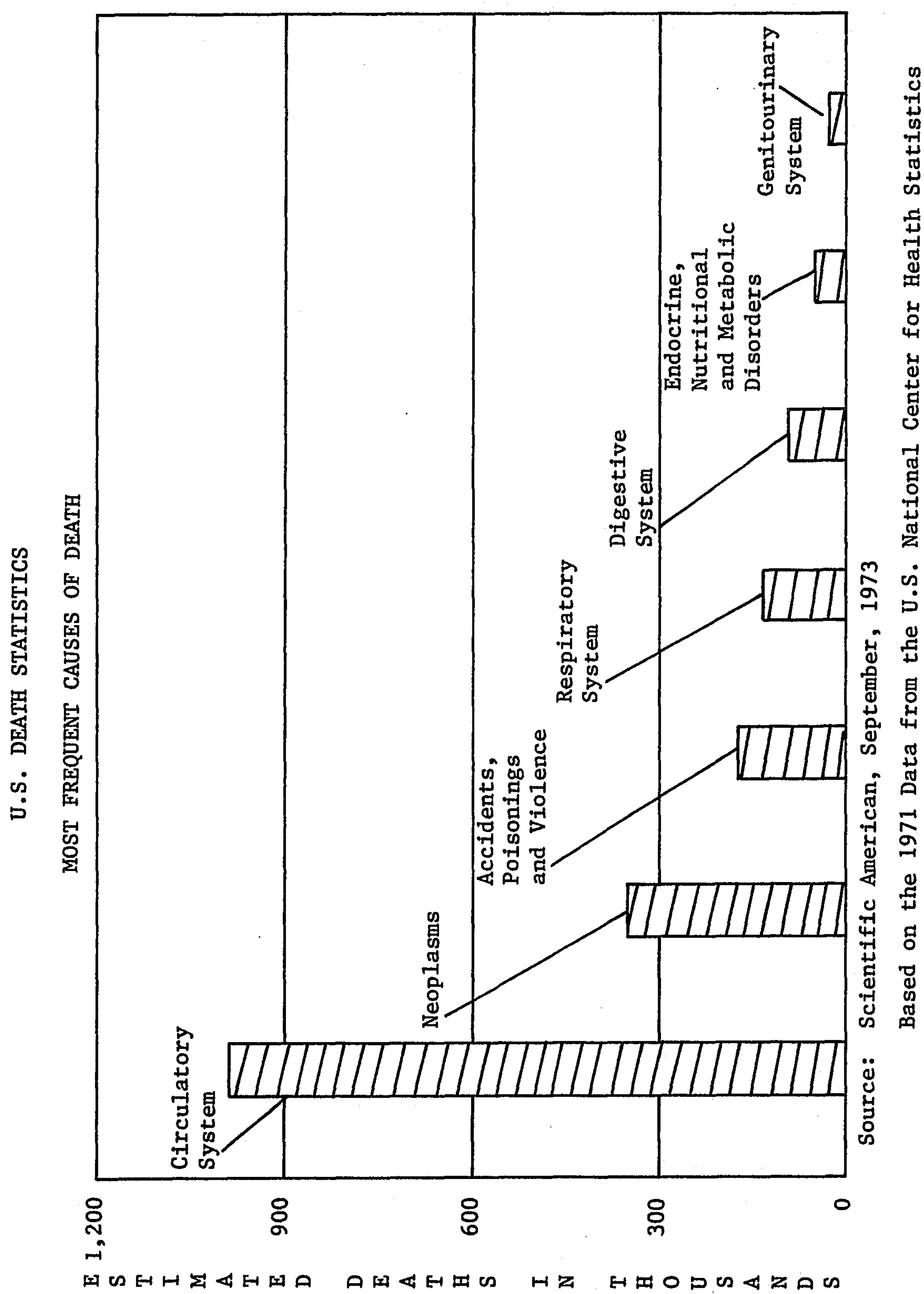


Figure 1 U.S. Causes of Death Statistics

rhelogical properties of blood, but it is reasonable that in some cases the rheological properties of blood are a contributing factor[10,11]. The gathering of large scale statistics of the relationship of rheological properties of blood and circulation problems is difficult and involves serious ethical considerations. The interpretation of such statistics is fraught with difficulties for blood properties are in a continuous state of flux. For instance, it has been reported that blood properties have a circadian[12,13,14] and seasonal cycle[15]. Furthermore, heart attack and clotting time have been correlated with solar activity[16,17].

10. Dintenfass, L., "Blood Microrheology Viscosity Factors in Blood Flow, Ischaemia and Thrombosis", London: Butterworth & Co., 1971.

11. Dintenfass, L., Stewart, J.H., Milton, C.W., Forbes, C.D., "Influence of ABO Blood Groups and Fibrinogen on Thrombus Formation and Aggregation of Red Cells in Cardiovascular and Malignant Diseases: New Aspects of Biorheological Characterization of Disease", Biorheology, Vol. 10, #4, Dec. 1973, pp. 585 - 594.

12. Bartter, F.D., Delea, C.S., Halberg, F., "A Map of Blood and Urinary Changes Related to Circadian Variations in Adrenal Cortical Function in Normal Subjects", Annals of the New York Academy of Sciences, 98: 1962, pp. 969 - 983.

13. Cranston, W.I., "Diurnal Variation in Plasma Volume in Normal and Hypertensive Subjects", American Heart Journal, 68: 1964, pp. 427-428.

14. Ehrly, A.M., Jung, G., "Circadian Rhythm of Human Blood Viscosity", Biorheology, Vol. 10, #4, Dec. 1973, pp. 577 - 583.

15. Yoshimuta, I., "Seasonal Changes in Human Body Fluids", Japanese Journal of Physiology, 8: 1958, pp. 165 - 179.

16. Chijevsky, A.L., Shishina, Y., "In the Rhythm of the Sun", Moscow: Nauka, 1969.

17. Gauquelin, M., "The Cosmic Clocks", New York: Avon, 1969.

In spite of such complications the reoccurence of a heart attack has been predicted twelve hours in advance by rheological tests on blood[18]. The pioneering work of Copley[19,20] and Dintenfass[10] has shown the importance of careful investigation of blood properties for the clarification of certain human physiological processes. There has been some indication that neoplasma (cancer) can also be detected from tests on blood[9,11]. The real problem is not so much to detect cancer but to detect cancer early. The keynote of our work is then not just an instrument for testing the rheological properties of blood but a very precise instrument for that purpose, so that the very small mechanical property changes can be sensed as they develop and can be evaluated.

Observations on Blood

To gain first hand experience with the complexities of blood, one of the Weissenberg rheogoniometers at NCE was modified by Dr. N. Siskovic for observing the blood cells via a closed circuit television[21]. The main features of this modification is shown in Figure 2. The results

18. Chmiel, H., "New Experimental Results in Hemorheology", Biorheology, Vol. 11, #1, Jan. 1974, pp. 87 - 96.

19. Copley, A.L., Krchma, L.C., Whitney, M.E., "Humoral Rheology, I., Viscosity Studies and Anomalous Flow Properties of Human Blood Systems with Heparin and Other Anticoagulanta", J. gen. Physiol., Vol. 26, 1942, pp. 49 - 64.

20. Copley, A.L., "On Biorheology", Biorheology, Vol. 10, #2, June 1973, pp. 87 - 105.

21. King, R.W., Copley, A.L., "The Karl Weissenberg 80th Birthday Celebration Essays", Edited by J. Harris, Kampala-Nairobi-Dar-es-Salam; The East African Literature Bureau, 1973.

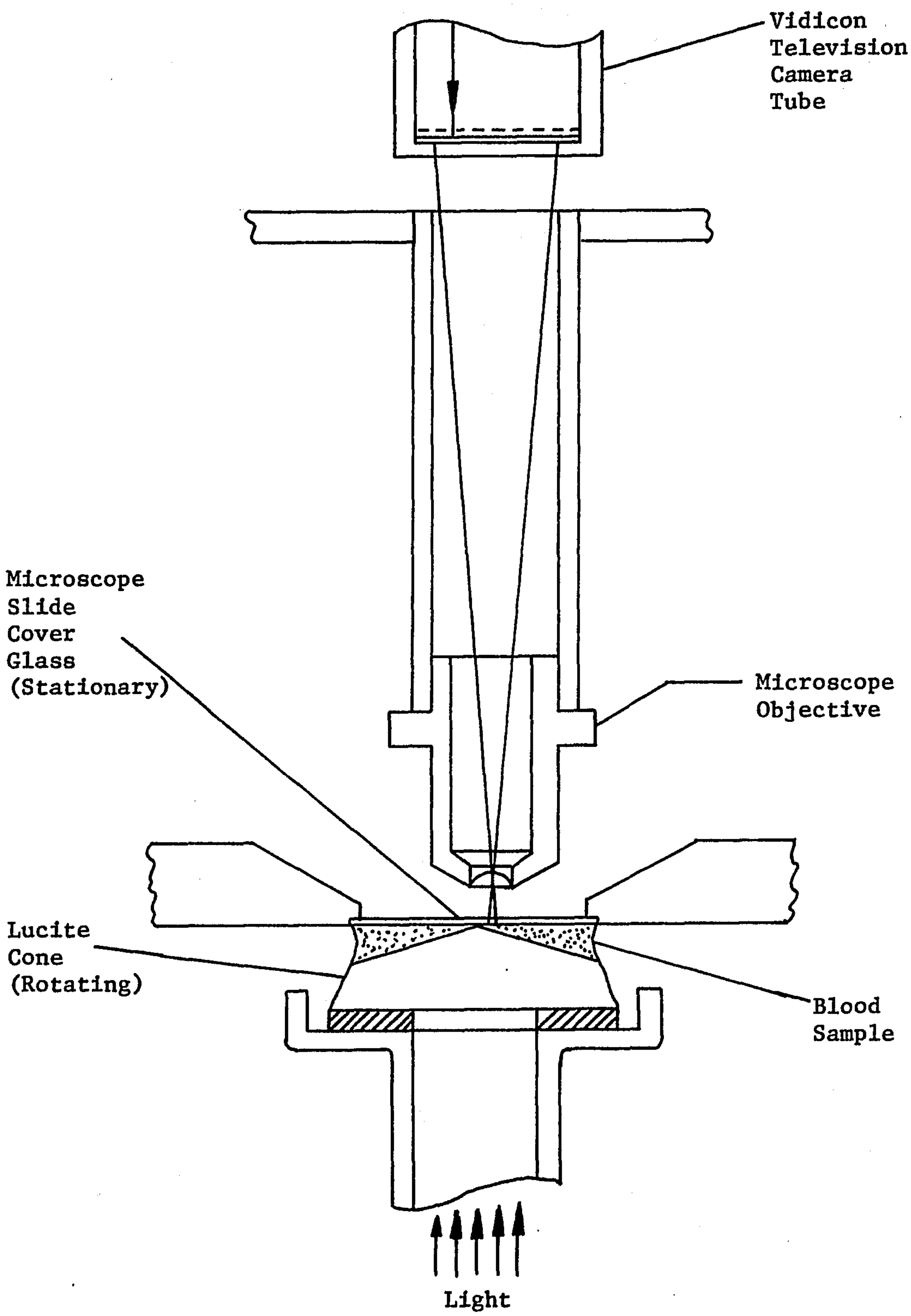


Figure 2 Television Pickup for Blood Cell Motion Patterns

obtained confirm those published[22,23]. The observed results can be interpreted based on an assumption that the red blood cells tend to stick together. These forces of adhesion are rather weak and the adhesion process is reversible. Starting at high shear rates and gradually reducing the shear rate one observes the following sequence of events that at the higher shear rates each blood cell moves independently. There were occasional visual suggestions that the cells themselves may be deformed but this observation could not be reliably confirmed with the apparatus. When the shear rate is reduced to around 40 sec^{-1} pairs of cells are seen moving together. As the shear rate is reduced lower and lower the number of cells in the clumps increase with decreasing shear rate. What is seen on the television screen is a dynamic process. Cells attach themselves to clumps. The clumps become too large and the hydrodynamic forces overcome the adhesive forces and the clump breaks up and one sees a continual forming, changing of shape and a breaking up of such clumps of cells. The statement that the number of cells in a clump increase with decreasing shear rate is valid only in a statistical sense.

The clumps also seemed to gradually change their character with time. When the shear rate is reduced the clumps that form have a ran-

22. Schmid-Schönbein, H., Gaehtgens, P., Hirsch, H., "On the Shear Rate Dependence of Red Cell Aggregation in Vitro", The Journal of Clinical Investigation, Vol. 47, 1968, pp. 1447 - 1454.

23. Schmid-Schönbein, H., Wells, R., Schildkraut, R., "Microscopy and Viscometry of Blood Flowing Under Uniform Shear Rate (Rheoscopy)", J. Appl. Physiol., Vol. 26, 1969, p. 674.

dom orientation of blood cells. As the shear rate is maintained the cells start to acquire some regularity. The red blood cells being disc shaped can now be seen as being stacked. Such clumps composed of stacks of discs (called rouleaux) appeared to be stronger than the clumps composed of random oriented cells, presumably because of the larger contact area between adjoining cells. Thus the explanation for the gradual change in clump character is that the roleaux are stronger and therefore survive better, but take longer to form because the cells must be precisely stacked. An example of such roleaux formation on our equipment is shown in Figure 3.

Rapid increases in shear rate do not instantaneously break up the clumps. There has to be a finite motion to separate the cells. Similarly a rapid decrease in shear rate does not instantaneously form larger clumps. There has to be finite motion for cells to join the clumps. Rapidly stopping the shear freezes the clumps. From observation the Brownian motion of cells is negligible (except for slow sedimentation) so that when motion stops the cells and clumps remain where they are. The apparent viscosity of the same sample of blood is nearly constant at the high shear rates. When cells start to form clumps the viscosity tends to increase with decreasing shear rate. The increase in viscosity with decreasing shear rate in part is due to the formation of cell clumps. However, as the description of experiments with blood which do not form clumps has shown[24], part

24. Schmid-Schönbein, H., Gosen, J.V., Klose, H.J., "Comparitive Microrheology of Blood; Effect of Desaggregation and Cell Fluidity on Shear Thining of Human and Bovine Blood", Biorheology, Vol. 10, #4, Dec. 1973, pp. 545 - 551.

Courtesy R.G. King

FIGURE 3 BLOOD ROULEAUX (.54 $sec.^{-1}$ for 3 min.)

of the viscosity change is due to fluidity and deformability of the cells.

Inferences

With the equipment available certain direct observations or measurements could not be made and confirmed. However, certain inferences could be made and these are presented. As may be expected from the model of clumps of cells that adhere to one another and are in equilibrium with the hydrodynamic shear forces, the shape of the cell clumps is not spherical. The implications of asymmetry in the shapes of the clumps suggests that blood becomes anistropic under shear. Incremental shear rates such as small amplitude oscillations in different directions should result in different incremental shear stresses. At higher shear rates where the cells do not form clumps the blood cells tended to orient themselves. Such orientation of cells could be detected because red blood cells are shaped in the form of a biconcave disc. This suggests that sheared blood should exhibit anisotropic fluid properties over a wide range of shear rates. Such orientation has been reported in literature[24], however no reports of anisotropic properties or treatment of blood anisotropy could be found.

Pursuing the idea to its logical conclusion that cell clumps are in equilibrium with the hydrodynamic forces would suggest that the clumps should assume different shapes in different sample holder geometries. Now if apparent viscosity at low shear rates is related in part to clump formation, then different geometries should give slightly different results on identical blood samples. The literature abounds in examples of different viscosities in different geometries. Unfortunately the data scatter even for the same geometry is quite large. It is not customary to publish error analysis. At this time it is not clear whether different geometries yield different results.

One observation of how different geometries can give different results came from observations near the apex of the cone (Figure 2). With decreasing shear rate the clump size increased, while the number of clumps increased. The clumps also now appeared to be surrounded by clear plasma. The clumps appeared to be large enough to bridge the gap between the cone and plate (Figure 2). Such bridging could not be observed by looking down on the clump. Whether such bridging did or did not occur is not important, what is important is that there could arise such a possibility. The hydrodynamic forces are small enough so that most of the cells are contained in large clumps surrounded by clear plasma. If bridgings do occur then the clumps are torn apart with the net result that the clumps (a network of clumps) float between the cone and the plate separated by a plasma layer. Such cell free layers have been reported for concentric-cylinder viscometers for shear rates below 1 sec^{-1} [25,26].

25. Cokelet, G.R., Merrill, E.W., Gilliland, E.R., Shin, J., Britten, A., Wells, R.E., Jr., Trans. Soc. Rheol., Vol. 7, 1963, p. 303.

26. Gregersen, M.I., Usami, S., Chien, S., Dellenback, R.J., Bibl. anat., Vol. 9, 1967, p. 276.

The real significance of the possibility of bridging is that we are no longer dealing with a homogenous fluid. "Rheological properties of whole blood" means that the properties are of a homogenous sample. If the network of clumps increases in size with decreasing shear rate, then the inference is that there is a lower shear rate limit for a given sample holder.

Clotting

Outside the body blood clots. The starting time of the clotting process is variable. The body manufactures anticoagulants which have a rather short life. Once the blood is outside the body and the anticoagulants are exhausted, clotting starts. The clotting can be speeded up considerably by one,bringing the blood into contact with foreign surfaces and two,by mechanical damage of the blood. Either method releases substances which activate the clotting process. Shearing of the blood not only damages the blood but also spreads the clotting initating substances so that the clotting starts sooner.

Clotting is a polymerization of fibrinogen into fibrin. The shearing of blood during clotting has a great effect on the clot that results. When clotting takes place at zero shear rate, the polymerization results in a three dimensional net of fibrin threads. The mesh size of this fibrin net is small enough so that the blood cells are entrapped in a random network of fibrin. When blood is sheared the structure of the clot changes as might be expected from the greater cross linking of the fibrinogen during polymerization. At high shear rates ($\gtrsim 400\ sec^{-1}$) the polymerization does not proceed throughout the whole blood sample but results in a suspension of discrete fibrin and platelet aggregates in serum. The detailed examination of the clotting process is not important to the design of the instrument and details of the clotting process may be found in Dintenfass' book[10] from which the preceeding description was abstracted.

Central to the testing of blood through the clotting stage is the effect of surfaces of the rheometer. As mentioned above the chemical nature of the surface can trigger the clotting process. Another way in which the surface properties enter into the testing of the blood clot is as follows: Assume that the bond between fibrinogen molecules is much stronger than the bond between the fibrinogen molecule and the rheometer surface. In that case either due to a sufficiently large displacement or a sufficiently large shear stress, the bond between the clot and the rheometer surface is broken. While the blood cells are trapped in the fibrin mesh, the plasma can seep out and so the result would be that the clot is separated from the rheometer surfaces by a layer of plasma. That this sort of process may indeed be occuring has been observed in our whole blood tests. The rheometer surfaces were clean when the clotting proceeded under steady shear.

If such separation of the clot from the rheometer surfaces does occur, then the measured properties no longer reflect the true properties of the clot. The problem is to have chemically inert surfaces so that the blood does not interact with the surfaces and start the clotting process and yet the surfaces must form strong bonds to the fibrinogen molecules. The resolution of this dilemma may lie in coating the surfaces with an organic substance such as fibrin. Copley and Scott Blair have extensively investigated the effects of fibrin coated surfaces[27,28].

Test Signals

To conclude this brief description of blood properties the relation of blood properties to blood testing must be examined. To keep matters in their proper perspective only the angle of twist of a torsion bar (or the currents producing counter torques) and the angular velocity of the sample holder are the measurable quantities in rotational rheometers. The measured quantities include the characteristics of the instrument and do not represent the true blood response. All other quantities from viscosity to diagnosis of disease are inferred from models relating torsion bar angle and RPM to such quantities. One objective of such data reduction is the elimination (at least in theory) of the contribution of the instrument and to obtain an invariant representation of the properties of blood. Such representations assume that the blood is a homogenous continuum. More will be said about such representations in the next chapter.

Since some attempt is made to subtract the instrument contribu-

27. Copley, A.L., Scott Blair, G.W., "Comparative Observations on Adherence and Consistency of Various Blood Systems in Living and Artificial Capillaries", Rheologica Acta, Vol. 1, #2, 1958, pp. 170 - 176.

28. Copley, A.L., Scott Blair, G.W., "The Wettability of Glass and Fibrinized Surfaces in Relation to Intravascular Blood Coagulation", Fed. Prog., Vol. 19, 1960, p. 56.

tion, the question arises just what sort of motions or torques will simplify the subtraction of the instrument contribution and yet yield data on the blood properties. From a mathematical viewpoint the impulse, step, sinewave, exponential and white noise are suitable test signals. From a practical viewpoint the impulse must be discarded for it is not readily implementable. White noise is perhaps the best test signal, however, it involves lengthy computation and the interpretation of the results may not be easy, therefore white noise is not recommended as a test signal to be used initially with the instrument.

The step changes rapidly from equilibrium at one experimental condition to another constant experimental condition and allows observation of how the system (instrument and blood) achieves the new equilibrium condition. The attainment of equilibrium involves large motions and is not suitable for observing viscoelastic or anisotropic properties of blood or the properties of clots. Where large motions are not practical sine waves may be used. Now the experimental conditions are no longer constant, but the sinewaves have such good mathematical properties so that the loss of time invariant experimental conditions is not a problem. The exponential signal may be useful if the system has say two time constants one fast but of low amplitude and the other slow but of large amplitude. In tests with steps the slow time constant masks the fast time constant. If a slow exponential probing signal is applied, then the effects of the slow time constant are reduced and the fast time constant now predominates.

The reduction of the experimental data should match the test signals. This would mean a Fourier series representation for sine-wave signals and exponentials for the step and exponential signals. The representation in orthogonal exponentials may be preferable[29] for the step and exponential signals.

General Description of the Rheometer

Before going on to more detailed considerations a brief description of the proposed rheometer is in order. The simplified block diagram of the proposed rheometer is shown in Figure 4. The bottom left hand corner is the test sequencer. The need for an automatic test sequencer arises because there is about four minutes before clotting is complete in which to perform a score of tests on the blood. An operator setting switches and pushing buttons is out of the question. An automatic system is required which allows some operator interaction although it is fully automatic and capable of carrying out the full sequence of tests without operator interference. To achieve flexibility in testing, the test parameters are recorded on paper tape. The information about the test such as test duration, test type, amplitudes and so on are decoded, and the various timers and electronic circuits are initiated. The electronic circuits control electric currents which flow through the drive motor. In the electric

29. Armstrong, H.L., "On the Representation of Transients by Series of Orthogonal Functions", IRE Transactions on Circuit Theory, Vol. CT-6, Dec. 1959, pp. 351 - 354.

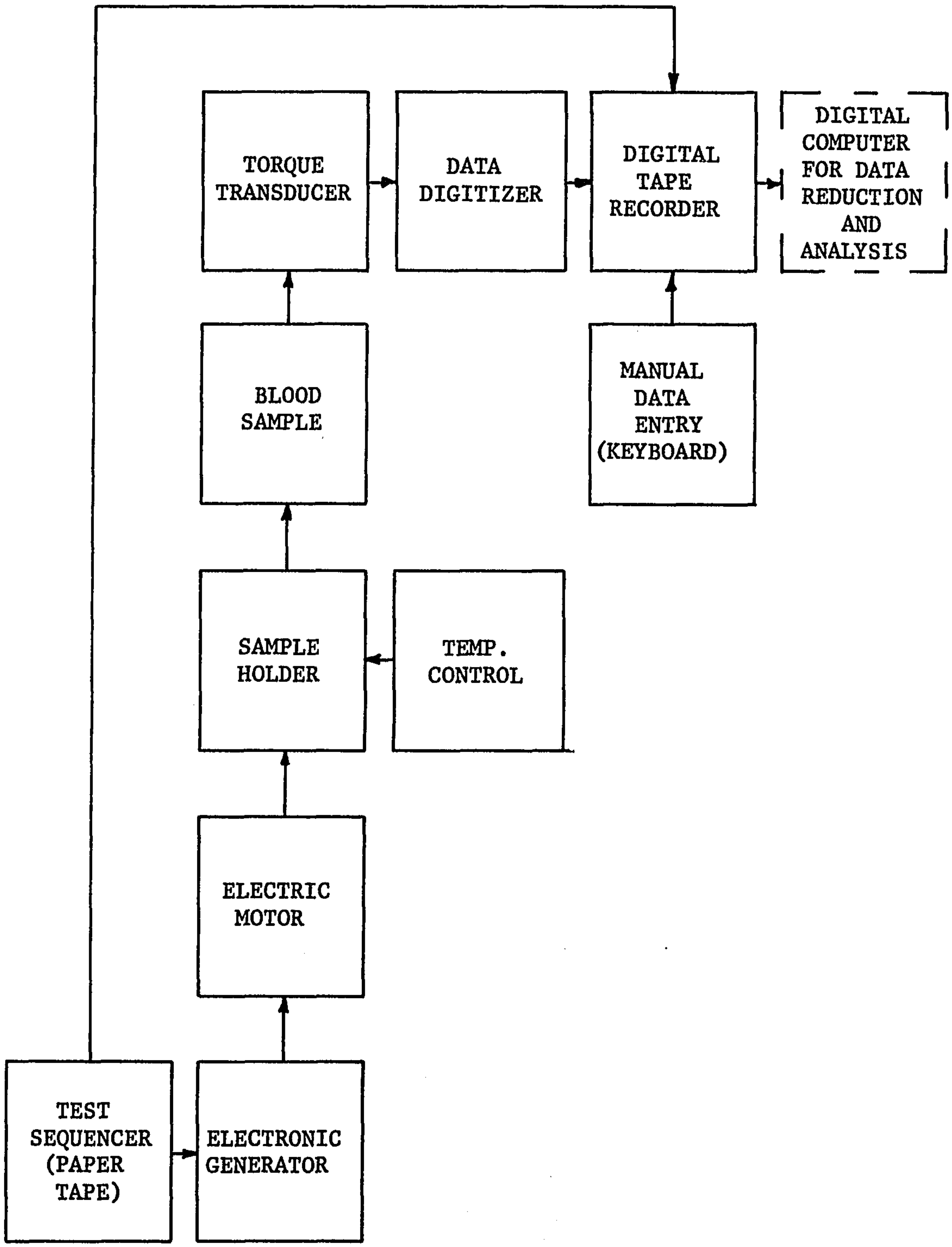


Figure 4 Block Diagram of Whole Blood Rheometer

motor these currents generate torques which result in the rotation of the motor and which in turn also rotate the blood sample holder. To maintain constant experimental conditions the sample is held at a constant temperature by a circulating water bath. In the sample holder the blood is sheared. As explained earlier the sheared blood produces a torque on the sample holder parts not driven by the motor. This resulting torque is related to the apparent viscosity of blood. The output torque is converted into an electric current proportional to the torque. This current is digitized in a digital current meter, and the resulting digital signal is recorded on magnetic tape for computer processing. A digital computer is the ideal tool for squeezing out the maximum of information from a minimum of data.

Additional information is recorded on the magnetic tape. Test information from the paper tape is recorded which tells the computer how to process the data. In addition some other information about the blood sample donor is available to the computer through keyboard entries.

System Design Philosophy

Having identified the major components of the rheometer, examination in greater detail of the system design philosophy will follow. Once testing begins, the design philosophy calls for all tests to be performed - except total test abortion or extension - to be initiated from a prerecorded paper tape. To begin with there is the paper tape. The sequence of tests to be performed is coded on the paper tape, and the duration and order of the tests is also coded into the paper tape before testing begins. The detailed information on the paper tape is:

1. test tape - steady rotation, sinusoidal oscillation and so on
2. speed - RPM in steady rotation, frequency in the case of oscillation
3. amplitude needed in the case of oscillation
4. test duration
5. sampling rate - time interval between recording data samples
6. time - at the start of the test

At the beginning of each new test the information on the paper tape is transferred to the magnetic tape. After the test begins the outputs from the actual blood tests are recorded on the magnetic tape following the information transferred from the paper tape. The entire magnetic tape is then processed on a digital computer.

Next is the testing. To infer fluid properties both the motion and the resulting torque must be recorded and, in a dynamic test, the time must be recorded. Furthermore, since the blood sample properties change with time, the time for the taking of the sample must be recorded. The easiest approach is to record all this information sequentially on magnetic tape. However, this slows data taking unacceptably. In addition, excessive input information takes a sizeable amount of the computer memory space unavailable for data processing. Our proposed approach is to make the motions and the timing very exact so that the timing and motion information need not be recorded along with each sample of data but may be reconstructed from one or two pieces of information recorded from the paper tape. The price to be paid is in the complexity of the devices and the timers that insure very precise timing and motion.

Next, consider the test sequence and the operator's interaction with the test sequence (Figure 5). Beyond starting and stopping the tests the operator's interaction with the test sequence is through extending any test or abandoning a test and skipping to the next test.

Turning on the electrical power energizes the electronics and starts heating the sample holder. No further operations may be performed until the sample holder warms up. When the sample holder reaches the operating temperature, the calibration tests may be started. The calibration tests use air as the test fluid. If the rotation of the test section is speeded up by a factor of thirty-three, then the torque obtained is the same as that obtained in tests with water, and more importantly the Reynold's number is about the same under these conditions. The calibration with air avoids contaminating the test cell. The calibration with air not only calibrates the torque transducer but also allows the estimation of the dynamics of the instrument, such as the resonant frequency and damping factor. The results of the calibration tests are recorded on magnetic tape. Since the digital computer can perform many complex calculations, it is our intent to eventually subtract the instrument system dynamics obtained in these calibration tests from the results obtained from the blood tests so as to obtain the more true behaviour of blood.

When the calibration tests are performed, the instrument is ready

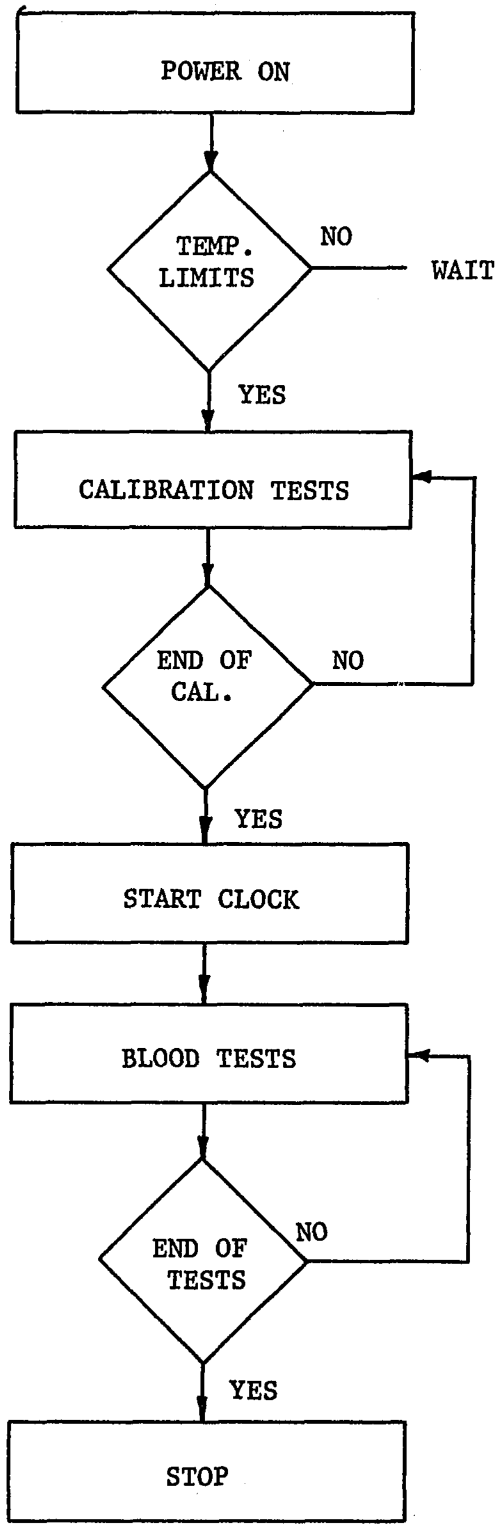


Figure 5 Test Sequence

to accept a blood sample. When a sample is taken a clock must be started. The function of the clock is to give the age of the sample as it proceeds through the various tests. Without starting the clock the test sequence can not be started. When the sample holder is filled and the test sequence started, the tests are performed sequentially with the operator only extending or skipping tests. At the end of the test sequence the instrument shuts off.

II FLUID DYNAMICS

Sample Holder Geometry

The sample holder is the heart of the instrument. It is impossible to make up for deficiencies of the sample holder in the rest of the instrument system. The following criteria were used in the selection of the sample holder geometry.

1) Except for the end effects present in any finite geometry, the flow conditions should be as uniform as possible throughout the sample. A large sample volume makes the construction of the sample holder easier to achieve greater accuracy and allows for more precise measurements because of the larger total forces obtainable from a larger sample volume. Non-uniform flow, which can not be discounted for a non-Newtonian fluid, creates problems without the saving grace of providing much more information.

2) Except for the end effects, the sample holder must have an exact analytical solution of the equation of motion for Newtonian fluids whatever the proportions of the sample holder or be reducible to a sample holder which is the difference of two sample holders and having an exact analytical solution for Newtonian fluids. If a solution for Newtonian fluids exists only as a approximation, then the testing of non-Newtonian fluids becomes difficult based on these two criteria. The number of possible sample holder geometries and motions is quite limited. Schlichting[1] lists some eleven exact solutions to

1. Schlichting, H., "Boundary Layer Theory", New York: McGraw-Hill, Fourth Edition, 1960, pp. 66 - 93.

the Navier-Stokes equation. Of these only the flow between two concentric rotating cylinders and the flow between moving parallel plates (Couette flow) have nearly uniform flow throughout the sample. The Couette flow would be the preferred geometry. Unfortunately the attainment of equilibrium of the clumps of red blood cells may involve considerable displacement of the plates. Large displacement of the plates creates difficulties in the design of the sample holder and so the choice falls on the concentric cylinder geometry.

Some other constraints on the sample holder must be considered.

1) The blood cell clump diameter increases with decreasing shear rate. Published results (120 - 150 μm diameter @ 5.8 sec^{-1})[2] and our observations suggest at least a ½ mm (500 μm) blood layer thickness is required to avoid bridging by a single clump of cells for a shear rate of 1 sec^{-1}. Of course the results are not interpretable by continuum theories if the clump of cells approaches the size of the blood layer thickness.

2) Medical opinion suggests the sample size of under 5 ml and preferrable 2 - 3 ml in volume.

3) The volume of the sample holder should be clearly defined. The blood surface exposed to air should be minimized and the end effects should also be minimized.

The selection of a sample holder geometry involves some compro-

2. Schmid-Schönbein, H., Wells, R., Schildkraut, R., "Microscopy and Viscometry of Blood Flowing under Uniform Shear Rate (Rheoscopy)", J. Appl. Physiol., Vol. 26, 1969, p. 674.

mises between the conflicting requirements. The proposed sample holder geometry involves a rotating cylinder with a cylindrical slot and a tube (used for torque sensing) inserted into this slot with the sample being sheared between the rotating cylinder and the inserted tube. A lip on the torque sensing tube confines the sample to be sheared. This sample holder geometry is shown in Figure 6.

Flow Stability

The proposed sample holder geometry is subject to flow instabilities when the Reynolds number exceeds a certain value. According to Rayleigh[3] the motion of an inviscid fluid between two concentric cylinders rotating in opposite directions is unstable at any speed. The motion of the fluid is stable if the outer cylinder rotates and the inner cylinder is stationary. If the inner cylinder rotates and the outer cylinder is stationary then the motion is unstable.

To understand the reason of this asymmetry and the nature of the instability consider the case of the inner cylinder rotating and the outer cylinder being stationary. If the viscous forces are negligible compared to the inertial forces, then a fluid particle proceeds from the inner cylinder to the outer cylinder due to centrifugal effects. Not all fluid particles can be moving outward at the same time and hence the nature of the instability is to set up circulation cells (Taylor[4] vortices) perpendicular to the main motion. This sort of

3. Lord Rayleigh, "On the Dynamics of Revolving Fluids", Scientific Papers, Cambridge, Mass., Cambridge University Press, 1916.

4. Taylor, G.I., "Stability of a Viscous Liquid between two Rotating Cylinders", Phil. Trans. A, Vol. 223, 1923, pp. 289 - 343.

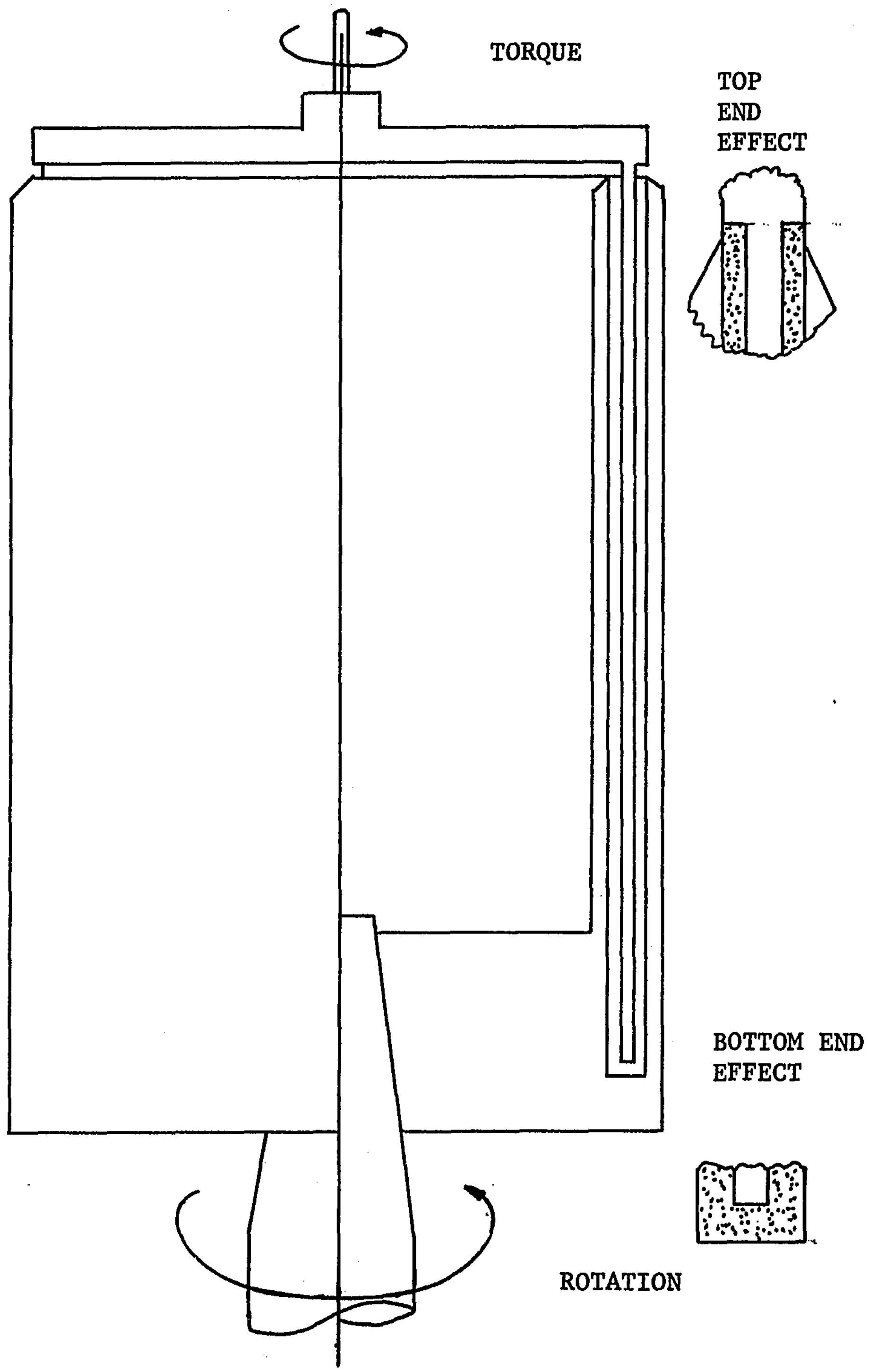


Figure 6 Sample Holder Geometry

instability can not occur if the outer cylinder is moving and the inner cylinder is stationary.

In the case of viscous fluids the radial motion of the fluid particles is opposed by viscous forces and the formation of circulation cells does not occur until a certain Reynolds number (ratio of inertial to viscous forces on a fluid particle) is reached. The instability investigations for viscous fluids with the ratio of the outer to inner radii near unity has been extensively investigated due to the importance of the results in the hydrodynamic lubrication of rotating shafts.

According to Prandtl[5] the condition for the start of Taylor vortex formation is:

$$\frac{U_i h}{\nu} \sqrt{\frac{h}{R_i}} > 41.3 \qquad 2.1$$

U_i - peripheral velocity of inner cylinder

R_i - radius of inner cylinder

h - gap between inner and outer cylinder

ν - Kinematic viscosity

For the ratio of the radii not near unity, the criteria for stability becomes more complicated and the solution more difficult[6,7]. For the

5. Prandtl, L., "Abriß der Strömungslehre", Braunschweig, 1935, Second Edition, p. 100.

6. Chrandrasekhar, S., "The Stability of Viscous Flow between Rotating Cylinders", Proc. Roy. Soc. Lond. A, Vol. 246, 1958, pp. 301-311.

7. Yu, Y., Sun, D., "Stability of a Viscous Flow between two Rotating Coaxial Cylinders", J. Franklin Inst., Vol. 277, #2, Feb. 1964, pp. 140 - 149.

flow to be nearly uniform throughout the sample, the gap between the cylinders must be small compared to the radius, and hence Prandtl's formula is satisfactory.

Fluid Properties

The fluid mechanical properties of a liquid are known when, given the applied stresses, the motion can be calculated (at least in theory). Such fluid properties must be independent of the particular method or geometry used to obtain such properties. To investigate the mechanical parameters characteristic of a fluid one ideally selects a mathematical model which contains (through the constitutive equations in the model) sufficient parameters to characterize observed or publicly reported fluid motion patterns. As mentioned in the introduction blood is a very complicated fluid and the mathematical model reflects such complexities. Rather than attack the whole problem, the steady flow case should be investigated first. Then, the information gained from the steady rotation tests may be used to characterize some of the viscoelastic and time dependent aspects of blood flow. The following discussion deals strictly with steady flow conditions. For the selection of a mathematical model for blood under steady flow conditions a survey of the published results as well as of our own observations showed the following characteristics of blood to be significant.

1) The published results[8,9] led to the conclusion that blood is

8. Martin, J.L., Jacobs, R.M., Copley, A.L., Final Report on Apparent Viscosity of Whole Blood Systems at Moderate Pressures, Office of Naval Research Contract N-00014-71A-0124, Department of Mechanical Engineering, Newark College of Engineering, Newark, N.J.

9. Ahuja, A.S., Bugliarello, G., "A Note on the Compressibility of Blood", Biorheology, Vol. 7, #4, May 1971, pp. 199 - 203.

not very compressible. Compared to the pressures used by Martin and Jacobs the gravitational and centrifugal effects are quite small. Thus blood may be taken as being incompressible.

2) The question of temperature dependence of blood viscosity has not been settled as yet. Some investigators find that there is no change in blood viscosity relative to plasma from 20^{o} to 38^{o}C. Others report no change relative to water but increases at low shear rate. The latest result available states that different results are obtained with different viscometer types[10]. In light of the above results the only reasonable conclusion is that if the sample holder is held at a constant temperature, then slight temperature variations will have negligible effects on the measured viscosity.

3) Blood is viscoelastic and may or may not have a yield stress. However, once a large enough displacement has occured then blood behaves as a true fluid. No reports to the contrary have been reported.

4) Sheared blood becomes anisotropic. This is inferred from our microscopic observations of blood. Blood cells and clumps tend to orient themselves in the sheared blood which suggests that sheared blood becomes anisotropic. The blood cells being disc shaped oriented themselves with the plane of the disc in what would be the constant shear rate plane of a Newtonian fluid. As reported[11] at high shear rates the blood cells deform into ellipsoid shaped discs and the

10. Barbee, J.H., "The Effect of Temperature on the Relative Viscosity of Human Blood", Biorheology, Vol. #10, #1, March 1973, pp. 1-5.

11. Schmid-Schönbein, H., Gosen, J.V., Klose, H.J., "Comparative Microrheology of Blood: Effect of Desaggregation and Cell Fluidity on the Shear Thining of Human and Bovine Blood", Biorheology, Vol. #10, Dec. 1973, pp. 545 - 551.

membrane moves in a tank-tread rotation. The clumps had a more irregular shape but in general formed parallel as well as perpendicular to the direction of flow.

5) Suggestions have been made that blood may be a polar fluid[12,13,14]. However, Cowin has argued against blood being a polar fluid[15]. The search of literature has not uncovered any experimental verification that blood is a polar fluid. From our observations we can report that neither the blood cells nor the clumps had any visible angular velocity independent of the velocity of the fluid. While a cell would tumble occasionally, such tumbling was random and not part of a regular tumbling motion. However, while the cells do not have rigid body angular velocity, the tank-tread motion of the cell membrane leads to the conclusion that blood must be considered a polar fluid over a certain shear range.

The Casson Equation

The stress-shear rate relations for blood follow the Casson[16] equation.

$$\sqrt{\sigma} = k_0 + k_1\sqrt{\dot{\gamma}} \qquad 2.2$$

12. Kline, K.A., Allen, S.J., DeSilva, C.N., Biorheology, Vol. 5, 1968, p. 111.

13. Valantis, K.C., Sun, C.T., Biorheology, Vol. 6, 1969, p. 85

14. Kline, K.A., Allen, S.J., Keshavarzi, M., "Dissipative Effects Due to Hydrodynamic Interactions between Red Cells in a Theory of Pulse Transmission and Oscillatory Flow in Arteries", Biorheology, Vol. #9, #1, March 1972, pp. 1 - 22.

15. Cowin, S.C., "On the Polar Fluid as a Model for Blood Flow in Tubes", Biorheology, Vol. #9, March 1972, pp. 23 - 25.

16. Casson, N., "A Flow Equation for Pigment-Oil Suspensions of the Printing Ink Type", In Rheology of Disperse Systems, G.E. Mills, Editor, New York: Pergamon Press, 1959.

σ - shear stress

$\dot{\gamma}$ - rate of shear

k_0, k_1 - constants involving viscosity, axial ratio and volume fraction of fluids

Casson's equation holds for blood (bovine) which does not form clumps[11]. Casson's equation is also applicable to clumping blood[17]. Furthermore, Casson's equation seems to hold when the blood clumps are lubricated by a layer of plasma at the sample holder walls[18], but at very low shear rates there is a departure from Casson's equation due to artifacts introduced by the sample holder[17].

The remarkable success of Casson's equation may be better understood if both sides of the equation 2.2 are squared

$$\sigma = k_0^2 + 2k_0k_1\sqrt{\dot{\gamma}} + k_1^2\dot{\gamma} \qquad 2.3$$

At high shear rates the contribution from $\sqrt{\dot{\gamma}}$ is small and thus the fluid behaves as a Newtonian fluid. At intermediate shear rates both $\dot{\gamma}$ and $\sqrt{\dot{\gamma}}$ contribute. At low shear rates $\sqrt{\dot{\gamma}}$ is the predominant term. Finally at zero shear rate a yield stress is to be expected as predicted by many investigators. Such a yield stress is not likely to be found if a lubricating plasma layer forms between the network of clumps and the sample holder walls. It would be expected that Casson's equation should not describe a two phase flow where the network of

17. Schmid-Schönbein, H., Wells, R., Schildkraut, R., "Microscopy and Viscometry of Blood Flowing Under Uniform Shear Rate (Rheoscopy)", J. Appl. Physiol., Vol. 26, 1969, p. 674.

18. Cokelet, G.R., Merrill, E.Q., Gilliland, E.R., Shin, H., Britten, A., Wells, R.E., "Rheology of Human Blood Measurement Near and at Zero Shear Rate", Trans. Soc. Rheol., Vol. #7, 1963, pp. 303-317.

clumps is lubricated by plasma and moderate increases in shear rate do not break up the network of clumps but merely squeeze out the plasma to form a thicker lubricating layer. That Casson's equation even approximately describes such two phase flow is remarkable.

Casson developed his equation for a fluid with suspended particles. Such particles could adhere to one another and form rods. Inherent in Casson's formulation is that the axial ratio (ratio of rod length to diameter) is high. At a given shear rate there is a maximum axial ratio that can exist. Rods of greater axial ratio will break down into smaller fragments due to hydrodynamic tension exerted on the rod.

Casson's theory is applicable to blood in the region where clumps of high axial ratio are formed prior to the formation of lubricating plasma layers. In ordinary viscometers the range of shear rates for which Casson's theory holds is quite limited. That Casson's equation describes blood flow properties outside the region of Casson's theory is a phenomenological fact not to be confused with Casson's theory.

Tensorial Invariance

Casson's equation fits the data but is not a tensorially invariant formulation of fluid flow properties. Tensorial invariance is a requirement that blood properties be independent of the coordinate system used in the description, the motion of the reference coordinate frame and the particular sample holder geometry. As mentioned in the introduction there may be dependence on sample holder geometry when the clump size is of the same order of magnitude as the smallest dimension of the sample holder. In that case the liquid is no longer homogenous and the tensor description will fail.

There is no lack of blood flow equations. The more recent theories extend Casson's theories[19] and treat blood cells as drops of liquid[20,21] and consider blood as a polar fluid[22] or approach the problem from thermodynamics[23]. As might be expected the claim to validity rests on the agreement with experimental data. These theories treat special flows and a fully invariant blood flow equation for steady shear incorporating the known blood flow properties is yet to be developed.

The simplest continuum model that is tensorially invariant and fits the properties of blood is that put forward by Ericksen[24,25]. Ericksen considers fluids with a single preferred direction at each

19. Aroesty, J., Gross, J.F., "Pulsatile Flow in Small Blood Vessels I - Casson Theory", Biorheology, Vol. 9, #1, March 1972, pp. 33-43.

20. Kline, K.A., "On a Liquid Drop Model of Blood Rheology", Biorheology, Vol. 9, #4, Dec. 1974, pp. 287 - 299.

21. Gauthier, F.J., Goldsmith, H.L., Mason, S.G., "Flow of Suspensions through Tubes - X Liquid Drops as Models of Erothrocytes". Biorheology, Vol. 9, #4, Dec. 1972, pp. 205 - 224.

22. Kline, K.A., Allen, S.J., Keshavrzi, M., "Dissipative Effects Due to Hydrodynamic Interactions between Red Cells in a Theory of Pulse Transmission and Oscillatory Flow in Arteries", Biorheology, Vol. 9, #1, March 1972, pp. 1 - 22.

23. Mahalingam, R., Poon, T.K., "Qüasi-Thermodynamic Interpretation of the Behavior of Formed Elements in Blood Flow", Biorheology, Vol. 10, #3, Sept. 1973, pp. 329 - 341.

24. Ericksen, J.L., "Theory of Anisotropic Fluids", Trans. Soc. Rheol., Vol. 4, 1960, pp. 29 - 39.

25. Ericksen, J.L., "Anisotropic Fluids", Arch. Rational Mech. Anal., Vol. 4, 1960, pp. 231 - 237.

point. This theory is applicable to particles which are figures of revolution. The preferred direction being the axis of revolution. Examples of such particles would be rods, dumbells, ellipsoids of revolution and so on. The preferred direction is represented by a vector $(\bar{n})$ where the magnitude of the vector may be associated with the degree of asymmetry. For example, the magnitude of the vector is proportional to the separation of the two masses of the dumbell shaped particle. The magnitude of this direction vector is treated as a variable which implies that the dumbell particle can stretch. Ericksen's formulation includes provisions for polar fluids but does not include two preferred directions as required by the stretched disk shape that blood cells assume under shear. Ericksen's model is suitable in the shear rate range where blood cells form clumps.

An assumption is made that the structure represented by the direction vector $\bar{n}$ is symmetric with respect to reflections in planes parallel and perpendicular to $\bar{n}$. The result in Cartesian coordinates for the isothermal flow of a non polar and incompressible liquid with negligible clump inertia as obtained by Ericksen is

$$t_{ij} = t_{ji} = -p\delta_{ij} + (\lambda_1 + \lambda_2 d_{km} n_k n_m) n_i n_j + \lambda_3 d_{ij} + \lambda_4 (d_{ik} n_k n_j + d_{jk} n_k n_i) \qquad 2.4$$

$$d_{ii} = 0 \qquad 2.5$$

$$t_{ij,j} + f_i = \rho a_i \qquad 2.6$$

$$n^2 = n_i n_i \qquad 2.7$$

$$d_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) \qquad 2.8$$

$$a_i = \frac{\partial u_i}{\partial t} + u_j u_{i,j} \qquad 2.9$$

t_{ji} -- stress tensor on j surface in i direction

p - pressure

λ - fluid parameters (functions of n^2)

δ_{ij} - Kronecker delta

n_i - preferred direction vector of the particles

d_{ij} - rate of shear tensor

f_i - body force per unit volume

ρ - volume density of total mass

a_i - acceleration vector

u_i - velocity vector

, - (comma) denotes partial differentiation

To compare Ericksen's result with Casson's equation, impose the conditions of Casson's theory

$$u_1 = Kx_2, \qquad u_2 = u_3 = 0 \qquad K = \text{const.} \qquad 2.10$$

$$\dot{\gamma} = d_{21} \qquad 2.11$$

$$\sigma = t_{21} \qquad 2.12$$

Casson's equation 2.3

$$\sigma = k_0^2 + 2k_0 k_1 \sqrt{\dot{\gamma}} + k_1^2 \dot{\gamma} \qquad 2.13$$

is compared to Ericksen's

$$\sigma = \lambda_1 n_1 n_2 + 2\lambda_2 n_1^2 n_2^2 + \lambda_3 \dot{\gamma} + \lambda_4 (n_1^2 + n_2^2)\dot{\gamma} \qquad 2.14$$

One may argue that since the λ's are functions of n^2, then $\lambda_1 n_1 n_2$ may be associated with k_0^2. Treating λ_3 as a constant associates λ_3 with k_1^2. Further arguing that since the magnitude of the direction vector

$\bar{n}$ is related to shear rate, then n, λ_2 and λ_4 may have the proper form to yield any dependence on $\sqrt{\dot{\gamma}}$.

On examining closer the Casson and Ericksen theories it is noted that the contribution to the energy density due to oriented elements in Ericksen's theory involves n^2. In Casson's theory the energy contribution involves $\dot{\gamma}^2 J$, where J is the axial ratio (length to radius). Furthermore the mean axial ratio depends as $1/\sqrt{\dot{\gamma}}$ on $\dot{\gamma}$ and thus the contribution to the energy by the rods in Casson's theory involves J^{-3}. Thus the necessary condition for compatibility of Ericksen's and Casson's theories requires the representation of the magnitude of the vector $\bar{n}$ to be proportional to $J^{-3/2}$. Even if Ericksen's equation can be made compatible with Casson's equation, this is not sufficient proof that Ericksen's equation describes blood in the clump formation stage. Only careful experiments can establish the applicability of Ericksen's theories to blood. While time dependence of flow has not been emphasized, it must be pointed out that both the Casson and Ericksen theories admit both thixotropy and viscoelasticity via gradual growth of clumps and the stretching of the clumps respectively.

Newtonian Liquid Motion

If the proposed rheometer is to be an aid in untangling the various blood models, then the motion of a Newtonian liquid in the sample holder geometry must be known so that the instrument accuracy may be assessed. To this end the equations governing the motion of a Newtonian liquid are derived. The starting point is Cauchy's first law of motion.

$$m a^i = \sigma^{ji}{}_{;j} + m f^i \qquad 2.15$$

$$\frac{dx}{dt} = u^j \qquad 2.16$$

$$a^i = \frac{\partial u^i}{\partial t} + u^j u^i{}_{;j} \qquad 2.17$$

Note: Coordinates are chosen such that they are equal to distance along a coordinate line in the proper system of units

m - mass density

a^i - acceleration vector

f^i - body force vector

u^i - velocity vector

x^i - coordinates of the path of motion ($x^i = x^i(t)$)

σ^{ji} - stress tensor on j surface in the i direction

; - (semicolon) denotes covariant differentiation

, - (comma) denotes partial differentiation

Constitutive equation for Newtonian liquid

$$\sigma^{ij} = \mu g^{in} d^j_n - p g^{in} \delta^j_n \qquad 2.18$$

g^{kn} - contravariant metric tensor

d^i_j - symmetric part of the velocity strain tensor

δ^i_j - Kronecker delta

μ - dynamic viscosity

p - pressure

$$d^i_j = \tfrac{1}{2}(u^i_{;j} + u^j_{;i}) \qquad 2.19$$

In a cylindrical coordinate system (ρ,θ,ζ) with velocity only in the θ direction but varying in the ρ and ζ directions the components of the equation of motion are:

$$\rho)\quad -m\rho u^{\theta}u^{\theta} = -p,_{\rho} + mf^{\rho} \qquad 2.20$$

$$\theta)\quad m\frac{\partial u^{\theta}}{\partial t} = \mu(u^{\theta},_{\rho\rho} + \frac{1}{\rho}u^{\theta},_{\rho} - \frac{1}{\rho^{2}}u^{\theta} + u^{\theta},_{\zeta\zeta}) + mf^{\theta} \qquad 2.21$$

$$\zeta)\quad 0 = -p,_{\zeta} + mf^{\zeta} \qquad 2.22$$

For constant velocity and no body forces in the θ direction the velocity distribution is given by

$$u^{\theta},_{\rho\rho} + \frac{1}{\rho}u^{\theta},_{\rho} - \frac{1}{\rho^{2}}u^{\theta} + u^{\theta},_{\zeta\zeta} = \nabla^{2}u^{\theta} - \frac{u^{\theta}}{\rho^{2}} = 0 \qquad 2.23$$

For the liquid sheared between two infinitely long coaxial cylinders with no velocity variation in the ζ direction, the equation of motion is in the form of Euler's equation. The complete solution for the velocity profile is

$$u^{\theta} = c_{1}\rho + \frac{c_{2}}{\rho} \qquad 2.24$$

where c_1 and c_2 are constants determined by the radii and angular velocities of the cylinders.

III END EFFECTS

Imperfections

It has been argued previously that the sample holder geometry should be such that the flow pattern of a Newtonian liquid can be found. Otherwise there is no hope of adequately treating non-Newtonian liquids. Except for end effects the chosen geometry does have an exact solution, so what remains is to calculate the end and other secondary effects for Newtonian liquids. The main concern of this chapter is the evaluation of end effects under steady shear rate conditions.

The problem is a classical boundary value problem. Given the value or the normal derivative of velocity on the boundaries, find the velocity in the interior. Having found the velocity in the interior, calculate the shear stresses via the constitutive equation. From the shear stresses calculate the torque on the sample holder surfaces. Comparison of the calculated torque to the measured torque for a Newtonian liquid with known viscosity, allows the assessment of accuracy of the instrument.

Before proceeding with the end effect solution the other secondary effects should be considered. These fall into two categories namely misalignment errors and machining errors.

While the theory calls for concentric cylinders in reality this is impossible to achieve in practice even if the cylinders were perfect. The misalignment can consist of an offset and/or a skew. The exact solution of motion of a Newtonian fluid between rotating cylinders that are non concentric is known[1]. The solution for skew misalignment is not exact. For now one must deal with finite cylinders at arbitrary angles of skew. An estimate of the skew error for small skews may be approached by assuming that the offset solution applies locally and then integrate over the cylinder to obtain the total contribution. Since skew is treated as an offset the errors due to skew and offset are additive.

The machining errors must be small if they are to be treated at all. Since the type of machining error is not known nothing can be said about their effects. The most reasonable way to treat machining errors would be via some sort of perturbation scheme. It is realized that the end effects are affected by misalignment and machining errors. However, since the contribution of end effects is small, the contribution of misalignment and machining errors to the end effects is assumed to be negligible.

Potential Solution

To get a feel for the end effects we note that the design of the sample holder calls for high shear rates and low fluid velocities so as to minimize the inertial effects of the liquid. This suggests that the potential flow solution

$$\nabla^2 u = 0 \qquad 3.1$$

is a good approximation to the viscous flow solution

1. Heyda, J.F., "A Green's Function Solution for the Case of Laminar Incompressible Flow between Non-concentric Circular Cylinders", J. Franklin Institute, Vol. 267, #1, Jan. 1959, pp. 25 - 34.

$$\nabla^2 u - \frac{u}{\rho^2} = \frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} - \frac{u}{\rho^2} + \frac{\partial^2 u}{\partial \zeta^2} = 0 \qquad 3.2$$

if u/ρ^2 is small compared to the derivative terms. Select a gap of ½ mm and a radius of 15 mm. The magnitude of the shear terms ($\partial u^2/\partial \rho^2$ and $\partial u/(\rho \partial \rho)$) as compared to the velocity term (u/ρ^2) is in the order of 25:1. Thus, a first approximation the end effects can be examined by substituting Laplace's equation for equation 3.2 in the sample holder problem.

Since the potential flow solution is an approximation there is no need to expend a large amount of effort on solving an approximation. The shape of a tightly stretched membrane is a solution to Laplace's equation and thus an insight into the end effects will be obtained by examining such a membrane solution. Figure 7 is a forshortened model of the sample holder geometry of Figure 6. The lines represent equal height (velocity) contours.

Examination of the model reveals that the end effects decay very rapidly so that one gap width away the end effects are not noticeable. This observation justifies treating the end effects (top and bottom) separately.

Preliminaries

The equation of viscous flow

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} - \frac{u}{\rho^2} + \frac{\partial^2 u}{\partial \zeta^2} = 0 \qquad 3.3$$

$$u = u(\rho, \zeta)$$

is a second-order linear homogeneous elliptic partial differential equation with variable coefficients. A change of variables

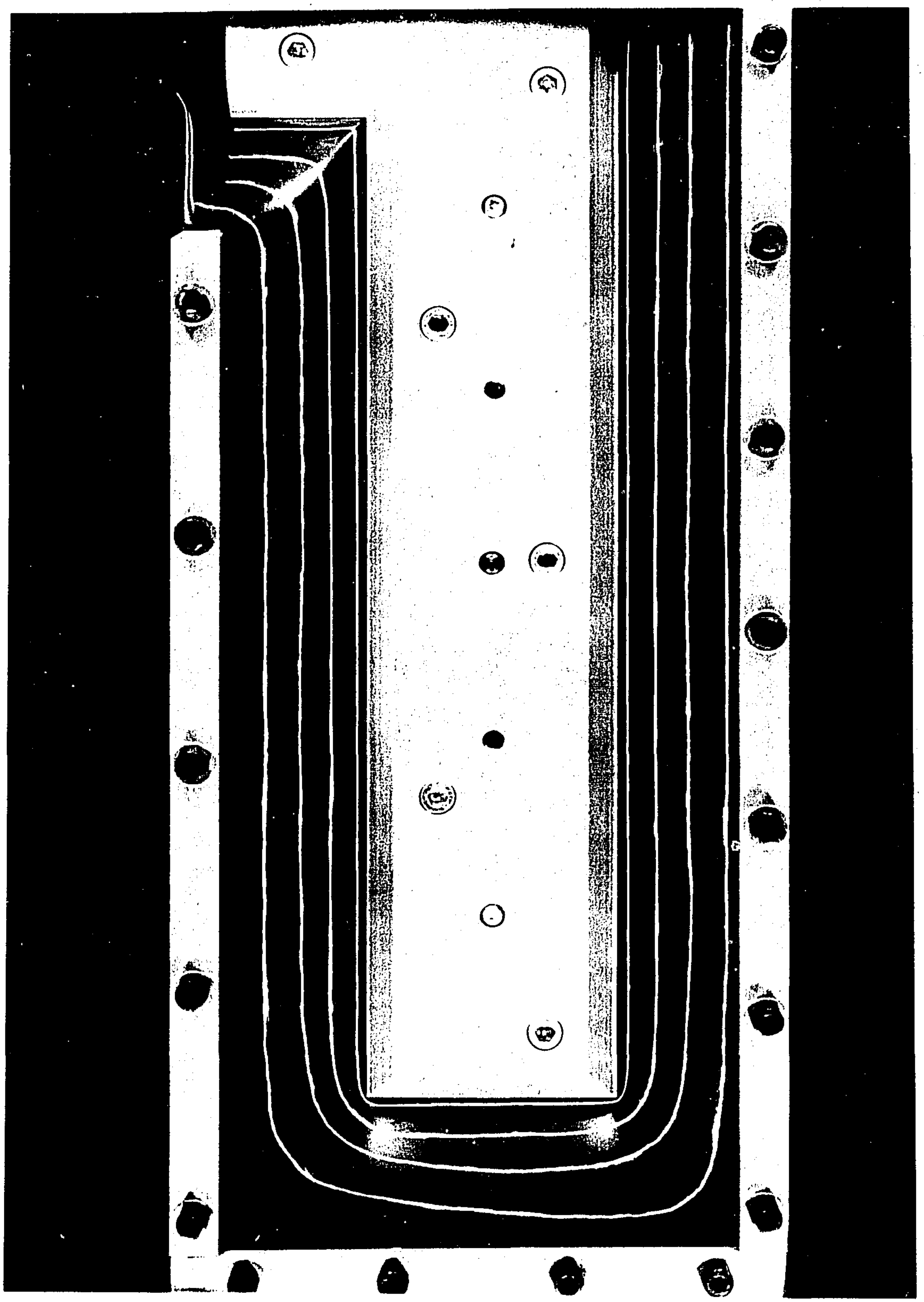


FIGURE 7 MEMBRANE SOLUTION OF FLOW PATTERN

$$u(\rho,\zeta) = \rho^{-\frac{1}{2}}v(\rho,\zeta) \qquad 3.4$$

reduces the equation to the normal form

$$Lv = \frac{\partial^2 v}{\partial\rho^2} + \frac{\partial^2 v}{\partial\zeta^2} - \frac{3v}{4\rho^2} = 0 \qquad 3.5$$

The linear operator L is self-adjoint.

A solution to the problem must satisfy the differential equation and satisfy the imposed boundary conditions. Specific details will be presented as needed, but for the moment the boundary conditions are those for a closed, simply connected domain in two dimensions with smooth boundary curves and continuous values of velocity on the boundary. In addition the solution must be unique, depend in a continuous manner on the boundary data and of course exist. What do the above criteria require in order for the solution to correspond to acceptable viscometric flows? Let this question be investigated by examining the differential equation 3.5.

Uniqueness

A solution if it exists, must be unique. To demonstrate uniqueness consider the problem as being three dimensional. The liquid volume ($\maltese$) being in the shape of an anulus bounded by the surfaces (S) of the sample holder. Multiply the differential equation 3.5 by v and integrate over the volume.

$$\iiint_{\maltese} v\nabla^2 v\, d\maltese = \frac{3}{4}\iiint_{\maltese} \frac{v^2}{\rho^2} d\maltese \qquad 3.6$$

Begin by assuming that v is bounded and continuous and has continuous first and second derivatives throughout the volume (v is of class C^2 in $\maltese$) and that on the boundary v is continuous (v is of class C^0 on S).

Then by Green's theorem

$$\iint_S v\frac{\partial v}{\partial n}dS = \iiint_{V} (\nabla v)^2 dV + \frac{3}{4}\iiint_{V} \frac{v^2}{\rho^2}dV \qquad 3.7$$

where $\partial v/\partial n$ is the normal derivative.

Consider that v is the difference of two solutions to the problem satisfying the same boundary conditions. Since the boundary conditions are the same v or $\partial v/\partial n$ are zero on the boundaries and the integral on the left is equal to zero. Now $(\nabla v)^2$ and v^2/ρ^2 are positive because of the squaring. The only way that the sum of the integrals on the right of the equal sign in equation 3.7 is equal zero is for both v and ∇v to be identically zero throughout the volume. Therefore the difference between two solutions meeting the same boundary conditions is zero on the boundaries and throughout the volume. This demonstrates the uniqueness of the solution.

Dependence on Boundary Conditions

The solution must depend on the boundary conditions. This dependence must be continuous which means that if the boundary conditions are changed slightly there is a slight change and not a great change in the values of velocity throughout the sample. To investigate the dependence on boundary data without actually solving the problem, investigate the interior maxima and minima and their relation to the boundary values.

For this investigation the problem is considered as being two dimensional $v(\rho,\zeta)$. Suppose that an interior maximum exists. In that case at the maximum

$$\frac{\partial v}{\partial \rho} = \frac{\partial v}{\partial \zeta} = 0 \qquad 3.8$$

then at the the minimum

$$\frac{\partial^2 v}{\partial \rho^2} \leqslant 0 \qquad 3.9$$

$$\frac{\partial^2 v}{\partial \zeta^2} \leqslant 0 \qquad 3.10$$

Comparison with the differential equation 3.5 leads to the result that v must be non positive at an interior maximum. The conclusion is that a positive maximum can not exist in the interior. By a similar argument a negative minimum can not exist in the interior. This implies that if the maximum and minimum occurs in the interior then the function v must be identically zero.

Once more consider that v is the difference of two solutions v_1 and v_2 whose boundary values are slightly different. On the boundary let the absolute value of the difference between the two solutions be less than ε, that is $-\varepsilon \leqslant v \leqslant \varepsilon$ on the boundary. We now show that the difference of the solutions is bounded by $\pm\varepsilon$, that is, we show that everywhere in the region $-\varepsilon \leqslant v \leqslant \varepsilon$.

Proof: Consider the following four cases.

1) Largest maximum (M) and smallest minimum (m) of v occur both in the interior. The conclusion from the preceeding arguments is that v is zero and therefore trivially between $-\varepsilon$ and ε.

2) Maximum (M) and minimum (m) occur on the boundary. It follows that in the interior v is between $-\varepsilon$ and ε.

3) Maximum (M) occurs in the interior and minimum (m) is on the

boundary. The maximum must be non positive $M \leqslant 0$, but greater than the minimum hence v is between $-\varepsilon$ and 0 and trivially between $-\varepsilon$ and ε.

4) Maximum (M) on the boundary and minimum (m) occurs in the interior. The minimum must be non negative $m \geqslant 0$ but, of course, less than the maximum and therefore v is between 0 and $+\varepsilon$ and trivially between $-\varepsilon$ and $+\varepsilon$. This establishes continuous dependence on the boundary data.

An interesting conclusion emerges. Even though the solution has continuous dependence on the boundary conditions, the range of the solution in the interior need not lie in the range of boundary values. Consider the case where the boundary values are positive in the range $\alpha > \beta > 0$. Nothing may be said about the value of an non negative minimum. It is possible that the range of some of the values in the interior is less than β and that the interior minimum lies in the range between 0 and α. A physical interpretation is that conditions for the formation of vortices may exist (see Chapter II).

Existence

Existence means the absence of contradictions between the imposed boundary conditions and the fundamental nature of the problem, and a demonstration of how a solution to the problem can be constructed. The main concern for the present is the avoidance of imposing conditions both on the function and the geometry that are contradictory to the problem. The physical nature of the velocity field is as follows

1) There is no slip at a solid-liquid interface. On a fixed wall both the normal and tangential components of velocity must vanish.

2) The velocity is single valued and continuous, possessing continuous partial derivatives of as many orders as needed. This assumption excludes motions of physical interest such as shocks and cavitation but is necessary for viscometric measurements. For uniqueness it is required that the velocity be of class C^2 and the boundaries be of at least class C^0. Thus the requirements of uniqueness are compatible with the nature of the velocity field.

The problem now centers on whether the imposed boundary conditions especially the sharp corners violate the requirements of the velocity field. General existence proofs are quite difficult and will not be attempted. Discussing the existence of solutions for the general elliptic equation with variable coefficients

$$\sum_{i,j=1}^{n} a_{ij}(x_1,\ldots,x_n)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i(x_1,\ldots,x_n)\frac{\partial u}{\partial x} + a(x_1,\ldots,x_n)u = f(x_1,\ldots,x_n) \qquad 3.11$$

Petrovsky[2] cites the results of Oleynik[3].

2. Petrovsky, I.G., "Lectures on Partial Differential Equations", Interscience Publishers Inc., New York: 1954, pp. 231 - 233.

3. Oleynick, O.A., "On the Dirichlet Problem for Equations of the Elliptic Type", Matematichesky Sbornik, Vol. XXIV, 1, 1949, pp. 1 - 14 (in Russian).

> O.A. Oleynik considered the Dirichlet problem (a) for arbitrary regions G in which $a \leqslant 0$, and (b) for sufficiently small regions in other cases. She showed that, regardless of what continuous function is prescribed on the boundary of the region G, the conditions which must be imposed on the boundary of that region to insure the solvability of the problem do not depend on whether the Dirichlet problem is being solved for the Laplace equation or for equation (3.11).

Further Petrovsky states that

> If a homogeneous ($f \equiv 0$) elliptic equation (3.11) with sufficiently smooth coefficients has in some bounded region G one (and, consequently, only one) solution of the Dirichlet problem for every continuous function prescribed on the boundary, then Harnack's first theorem holds for the solution functions, namely, if a sequence of such functions converges in the interior of G and the limit function satisfies equation (3.11).

While the above statements do not prove existence for our particular problem, we note that square corners and certain discontinuities are admissible for Laplace's equation in two dimensions, as in the case of the elastic torsion of a rectangular bar. Thus, a solution for our particular problem does exist.

General Representations for the Solution

Existence requires that a method for solving the differential equation for our boundary conditions be presented. One such method is due to Vekua[4,5]. Vekua does not prove the existence of a solution to our differential equation (3.5) but gives a method for obtaining a solution if the boundary curve is smooth (corners must be rounded) and the value of the function on the boundaries is continuous.

4. Vekua, I.N., "New Methods for Solving Elliptic Equations", New York: John Wiley & Sons, Inc., 1967, Chapter 1.

5. Bitsadze, A.V., "Boundary Value Problems for Second Order Elliptic Equations", New York: John Wiley & Sons, Inc.- 1968, Chapters 1 & 2.

We discuss Vekua's results without a complete presentation of the method. The restricted Riemann-Vekua function for our problem on which the method hinges will be presented. Vekua's method is cited to show that a solution method exists and not that it is a practical method of solving problems. In fact for our boundary conditions Vekua's method is too complicated to be of practical value and for this reason is presented only in outline form as an aid in reading the original text. Vekua's method is a complex variable method and the first step is to assign complex values to the arguments of an analytic function of real variables.

> Let $f(x_1,\ldots x_n)$ be an analytic function of the real variables $x_1,\ldots,x_n$ in some domain D of the space of n dimensions. Then there exists a unique function $f^*(z_1,\ldots,z_n)$ of the complex variables $z_1 = x_1 + iy_1,\ldots, z_n = x_n + iy_n$, analytic in some domain D^* of the space of $2n$ dimensions, which coincides with $f(x_1,\ldots,x_n)$ when $y_1 = y_2 = \ldots = y_n = 0$, where obviously, $D \subset D^*$. The function $f^*(z_1,\ldots z_n)$ is called the analytic continuation of the function $f(x_1,\ldots,x_n)$ from the domain of real values of the arguments $x_1,\ldots,x_n$ into the domain of complex values.

For our specific case introduce the complex variables

$$r = \rho + i\zeta \qquad 3.12$$

$$z = \overline{r} = \rho - i\zeta \qquad 3.13$$

Since the original problem is in cylindrical coordinates, the domain $\mathcal{R}$ for the real variables ρ,ζ of equation (3.5) is $\rho \geqslant 0$ and ζ unrestricted. Since ρ and ζ are real, then by the way the variables r and z are defined the four dimensional domain $\mathcal{D}^*$ is split into two two-dimensional domains $\mathcal{D}$ and $\overline{\mathcal{D}}$ such that the domains $\mathcal{D}$ and $\overline{\mathcal{D}}$ lie in the complex plane of the variable $r = \rho + i\zeta$ where $\bar{\mathcal{D}}$ is situated symmetrically to $\mathcal{D}$ relative to the real axis with $r \in \mathcal{D}$ and $z \in \bar{\mathcal{D}}$. For our purposes all the domains are simply-connected domains. Change the the variables of equation (3.5)

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \zeta^2} - \frac{3}{4}\frac{v}{\rho^2} = 0 \qquad 3.5$$

To the complex variables of equations (3.12) and (3.13) then equation (3.5) leads to

$$Lv = \frac{\partial^2 v}{\partial r \partial z} - \frac{3v}{4(r + z)^2} = 0 \qquad 3.14$$

It should be pointed out that $\partial v/\partial r$ can not in general be regarded as partial derivatives since $v(\rho,\zeta)$ is not a function of r. The proper notation for an analytic function $v(\rho,\zeta)$ continued analytically into the domain of complex values of ρ,ζ via the equations (3.12) and (3.13) is

$$V(r,z) = v\left(\frac{r + z}{2}, \frac{r - z}{2i}\right) \qquad 3.15$$

If the variables were real, then equation (3.14) would be a linear hyperbolic differential equation, and the boundary value problem is reduced to an initial value problem. Such a hyperbolic equation could be solved by the Riemann-Volterra method by finding the Riemann-Green function for equation (3.14). The Riemann function is used to solve an arbitrary initial value problem for the hyperbolic differential equation on an arbitrary free initial curve. Since the Riemann-Volterra method is extensively treated in literature the details will not be presented. While the Riemann-Volterra method is widely known it is seldom used because a simple expression for the Riemann function is difficult to obtain. In our case the Riemann function was found from Riemann's original work[6]. If the variables of equation (3.14) were real then the Riemann function $R(r,z;\eta,\xi)$ for the equation would be

$$R(r,z;\eta,\xi) = F\left(\frac{3}{2}, -\frac{1}{2}; 1; -\frac{(r-\eta)(z-\xi)}{(r+z)(\eta+\xi)}\right) \qquad 3.16$$

$$= P_{\frac{1}{2}}\left(\frac{(r-z)(\eta-\xi) + 2rz + 2\xi\eta}{(r+z)(\eta+\xi)}\right) \qquad 3.17$$

Where $P_{\frac{1}{2}}$ is the Legendre function of one-half degree and of the first kind. F is the hypergeometric function of Gauss, and ξ,η are dummy variables associated with r and z respectively. It can be shown that

$$\frac{\partial P_{\frac{1}{2}}(r,z;\eta,\xi)}{\partial r}\bigg|_{z=\xi} = \frac{\partial P_{\frac{1}{2}}(\eta,\xi;r,z)}{\partial \eta}\bigg|_{\xi=z} = 0 \qquad 3.18$$

$$\frac{\partial P_{\frac{1}{2}}(r,z;\eta,\xi)}{\partial z}\bigg|_{r=\eta} = \frac{\partial P_{\frac{1}{2}}(\eta,\xi;r,z)}{\partial \xi}\bigg|_{\eta=r} = 0 \qquad 3.19$$

$$P_{\frac{1}{2}}(r,z;r,z) = P_{\frac{1}{2}}(\eta,\xi;\eta,\xi) = 1 \qquad 3.20$$

Furthermore $P_{\frac{1}{2}}(r,z;\eta,\xi)$ satisfies equation (3.14 with respect to its first two arguments r,z, and hence is the Riemann function for the differential equation. It can be shown that

$$P\ (r,z,\eta,\xi) = \left(\frac{r+z}{\eta+\xi}\right)^{\frac{1}{2}} P_{\frac{1}{2}}\left(\frac{(r-z)(\eta-\xi) + 2rz + 2\eta\xi}{(r+z)(\eta+\xi)}\right) \qquad 3.21$$

is the Riemann function for the complex version of equation (3.3).

In converting the results to complex variables certain precautions must be observed. There is no problem with the analytic

6. Riemann, B., "Ueber Die Fortpflanzung Ebener Luftwellen Von Endlicher Schwingunsweite" found in "Collected Works of Bernhard Riemann" Edited by H. Weber, New York, N.Y., Dover Publications Inc., 1953, (in German).

continuation into the complex domain and complex arguments may be assigned to the arguments of Legendre's function. However, $P_{\frac{1}{2}}(w)$ may be multiple valued unless there is a cut made in the complex w plane along the real axis from $w = -1$ to $w = -\infty$. For a plane with such a cut, the Lengendre function $P_{\frac{1}{2}}(w)$ is analytic throughout the cut plane[7]. This condition is imposed on

$$w = \frac{(r - z)(\eta - \xi) + 2rz + 2\eta\xi}{(r + z)(\eta + \xi)} \qquad \begin{array}{l} r,\eta \in \mathcal{D} \\ z,\xi \in \overline{\mathcal{D}} \end{array} \tag{3.22}$$

Since ξ and η are of the same domain as z and r respectively and since the domains $\mathcal{D}$ and $\overline{\mathcal{D}}$ are reflections about the real axis, the condition for single-valueness and avoidance of singularities both in equation (2.2) and in equation (3.5) requires that $\rho > 0$. The Legendre function with complex arguments of equation (3.17) satisfies the requirements for the Riemann function of the complex equation (3.14).

The Riemann function is so defined as to make the solution simpler. Starting with the identity $\partial^2 PS/\partial r\partial z = \partial^2 PS/\partial r\partial z$ it can be shown that for S(r,z) an analytic function and the Riemann function $P_{\frac{1}{2}}(r,z;\eta,\xi)$, the following identity holds

$$\frac{\partial^2 SP}{\partial r\partial z} - P\left[\frac{\partial^2 S}{\partial r\partial z} - \frac{3}{4}\,\frac{S}{(r + z)^2}\right] = \frac{\partial}{\partial r}\left[S\,\frac{\partial P}{\partial z}\right] + \frac{\partial}{\partial z}\left[S\,\frac{\partial P}{\partial r}\right] \tag{3.23}$$

because the Riemann function satisfies equation (3.14). Interchanging the variables r, with η and z with ξ and integrating with respect to η and ξ between the limits r_0 to r and z_0 to z respectively where r_0 and z_0 are fixed points, $r_0 \in \mathcal{D}$, $z_0 \in \overline{\mathcal{D}}$, yield the results

7. Whittaker, E.T., Watson, G.N., "A Course of Modern Analysis", New York: Macmillan Co., 1948, Chapter 15.

$$S(r,z)P_{\frac{1}{2}}(r,z;r,z) - S(r,z_0)P_{\frac{1}{2}}(r,z_0;r,z) - S(r_0,z)P_{\frac{1}{2}}(r_0,z;r,z)$$

$$+ S(r_0,z_0)P_{\frac{1}{2}}(r_0,z_0;r,z) - \int_{r_0}^{r}\int_{z_0}^{z} P_{\frac{1}{2}}(\eta,\xi;r,z)\left[\frac{\partial^2 S(\eta,\xi)}{\partial\eta\partial\xi} - \frac{3S(\eta,\xi)}{4(\eta+\xi)^2}\right] d\xi d\eta$$

$$= \int_{r_0}^{r} S(\eta,z)\frac{\partial P_{\frac{1}{2}}(\eta,z;r,z)}{\partial\eta}\, d\eta - \int_{r_0}^{r} S(\eta,z_0)\frac{\partial P_{\frac{1}{2}}(\eta,z_0;r,z)}{\partial\eta}\, d\eta$$

$$+ \int_{z_0}^{z} S(r,\xi)\frac{\partial P_{\frac{1}{2}}(r,\xi;r,z)}{\partial\xi}\, d\xi - \int_{z_0}^{z} S(r_0,\xi)\frac{\partial P_{\frac{1}{2}}(r_0,\xi;r,z)}{\partial\xi}\, d\xi \qquad 3.24$$

For any analytic function that is a solution of the differential equation 3.14, the integrand of the double integral is zero. On imposing the requirements of the Riemann function, any analytic solution of the differential equation 3.14 may be expressed as

$$V(r,z) = V(r,z_0) + V(r_0,z) - V(r_0,z_0)P_{\frac{1}{2}}(r_0,z_0;r,z)$$

$$+ \int_{r_0}^{r} V(\eta,z_0)\frac{\partial P_{\frac{1}{2}}(\eta,z_0;r,z)}{\partial\eta} d\eta - \int_{z_0}^{z} V(r_0,\xi)\frac{\partial P_{\frac{1}{2}}(r_0,\xi;r,z)}{\partial\xi} d\xi$$

$$3.25$$

where $V(r,z_0)$, $V(r_0,z)$ are arbitrary analytic functions in $\mathcal{D}$, $\overline{\mathcal{D}}$ and $V(r_0,z_0)$ is an arbitrary constant. If the variables were real, then the interpretation of equation (3.25) would be that the the solution is found by prescribing values on the characteristics.

The next step is to reduce the variables to the variables of a single domain. The points r_0 and z_0 in $\mathcal{D}$, $\overline{\mathcal{D}}$ are arbitrary and thus without loss of generality z_0 can be set equal to the conjugate of r_0 $V_0(z_0 = \overline{r}_0)$. The dummy variables ξ and η may also be taken as complex conjugates ($\xi = \overline{\eta}$). The variables r and z are complex conjugates by equations (3.12) and (3.13) ($z = \overline{r}$). All the variables are now in terms of the variables of the domain $\mathcal{D}$, where $\mathcal{D}$ is defined to be the portion of the complex ρ, ζ plane where $\rho > 0$. Using the $\mathcal{D}$ domain variables does not mean that the conjugate domain $\overline{\mathcal{D}}$ has disappeared. Not at all, what happens of course, is that as the variable r varies in $\mathcal{D}$, the conjugate $\overline{r}$ varies in $\overline{\mathcal{D}}$. If f(r) is an analytic function in $\mathcal{D}$, the conjugate function $\overline{f(r)}$ will obviously be an analytic function of $\overline{r}$ in $\overline{\mathcal{D}}$.

By substitution in equation (3.17) it can be shown that the Riemann function $P_{\frac{1}{2}}(r,\overline{r};\eta,\overline{\eta})$ takes on real values. The arguments of the Riemann function $(\eta,\overline{r}_0;r,\overline{r})$ and $(r_0,\overline{\eta};r,\overline{r})$ take on complex conjugate values and by Schwarz's reflection principle the corresponding Riemann functions take on complex conjugate values. Since

$$V(r,z) = v\left(\frac{r+z}{2}, \frac{r-z}{2i}\right) \qquad 3.15$$

Then the Taylor series expansion of $V(r_0,\overline{r})$ and $V(r,\overline{r}_0)$ about the points r_0 and $\overline{r}_0$ respectively leads to the result that $V(r_0,\overline{r})$ and $V(r,\overline{r}_0)$ take on the complex conjugate values. The constant $V(r_0,\overline{r}_0)$ is a real constant. Because the functions involved are complex conjugates, equation (3.25) may be written as

$$v(\rho,\zeta) = \mathrm{Re}\left[\Phi(r) - \int_{r_0}^{r} \Phi(\eta)H(r,\eta)d\eta\right] \qquad 3.26$$

where

$$\Phi(r) = 2V(r,\overline{r}_0) - V(r_0,\overline{r}_0)P_{\frac{1}{2}}(r_0,\overline{r}_0;r,\overline{r}) \qquad 3.27$$

$$H(r,\eta) = \frac{\partial}{\partial\eta} P_{\frac{1}{2}}(\eta,\overline{r}_0;r,\overline{r}) \qquad 3.28$$

Any real analytic solution to equation (3.5) is given by equation (3.26) with $\Phi(r)$ an arbitrary analytic function. Clearly $\Phi(r)$ is not just any arbitrary function but is uniquely defined by $v(\rho,\zeta)$. Our problem is not to represent $v(\rho,\zeta)$ but to find it from prescribed boundary conditions. If $\Phi(r)$ could be found from the boundary conditions the solution for $v(\rho,\zeta)$ is obtained. By boundary condition it is meant that $v(\rho,\zeta)$ is known as in the limit it approaches the boundary with a value of $v(\tau)$ where point τ is on the boundary. Equation (3.26) may be written as the following boundary value problem on replacing r by τ (see Vekua Appendix I).

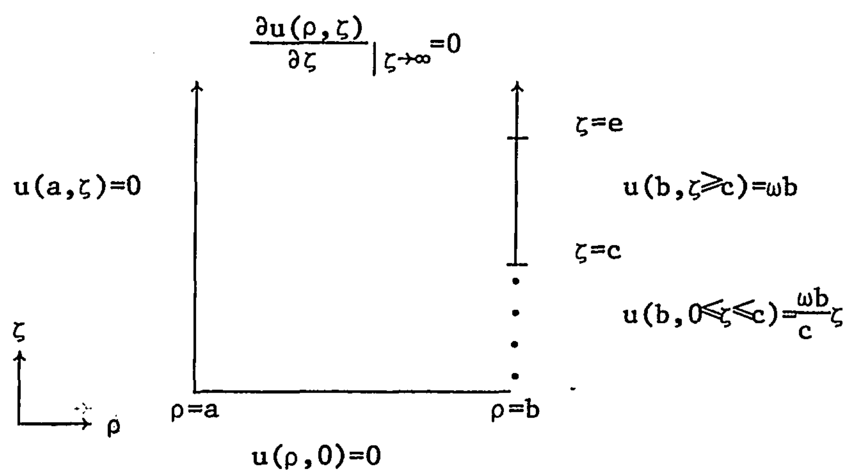
$$v(\tau) = \mathrm{Re}[\,\Phi(\tau) - \int_{r_0}^{\tau} \Phi(\eta)H(\tau,\eta)d\eta] \qquad 3.29$$

There are other ways of obtaining $\Phi(r)$ but all involve integral equations in one form or another (see Vekua Chapter III). The intention is not to solve the problem by this approach, but to present a general method by which the problem may be solved and results from an easier to use approximate solution may be checked. The general solution is presented in the spirit of an existence proof rather than a working tool. If an exact solution is required, here is one way to go about it. From an engineering viewpoint the preferred approach to the end effects is by numerical methods. The use of a computer is unavoidable. Even an exact solution involving sharp corners requires the evaluation of an infinite series. The objection to a numerical solution is on the grounds of elegance and accuracy. Elegance is not an engineering consideration but accuracy is. Our concern is with torque and hence the spacing of the first contour of Figure 7 (indicative of the magnitude of the velocity gradient) from the torque sensing part of the sample holder. In a full scale sample holder the area where the torque is significantly different may be in the order of one to two percent of the total area. Even a ten percent error in the numerical solution of the end effects represents acceptable accuracy levels. This argument begs the question as far as accuracy is concerned and an exact solution is required for at least one end effect as a check on the numerical method. A fair estimate is needed for the velocity distribution of the end effects so that the convergence of the numerical solution will be rapid. To this end the analytical solution of the top end effect is investigated.

Top End Effect

Actually there are two top end effects but since the two are separated by a solid wall only one effect needs to be considered. In this treatment the top and bottom end effects are considered as being non-interacting. The justification for this assumption is the potential flow solution to the problem. The geometry is thus a semi-infinite one with uniform flow conditions at infinity. At the top end blood is exposed to air. The exact shape that the blood surface will take depends on the filling, the surface tension and to a certain extent on the angular velocity. The problem is further complicated. Exposure of the blood surface to air forms a tough black skin on the exposed surface. The formation of this skin contributes significantly to the measured torque. The formation of this black skin apparently is in part due to a drying effect, for when the surrounding air is approximately at the temperature of the sample and at nearly one hundred percent relative humidity the black film does not form. Under such ambient conditions the contribution by the surface effects to the measured torque could not in actual measurements be identified due to the limitations on the precision of our measurements.

An exact treatment of the air-blood interface is not possible at this time due to uncertainties in the experimential conditions. As an engineering approximation the blood-air interface is assumed to be a flat surface with no surface effects. For easier treatment the geometry is inverted so that the blood-air interface $(b, 0 \leqslant \zeta \leqslant c)$ is at the bottom. The boundary conditions for the top end effect are as follows:



$$u(\rho,0) = 0 \qquad 3.30$$

$$u(a,\zeta) = 0 \qquad 3.31$$

$$\frac{\partial u(\rho,\zeta)}{\partial \zeta}\Big|_{\zeta\to\infty} = 0 \qquad 3.32$$

$$u(b,\zeta\geqslant c) = \omega b \qquad 3.33$$

Inspection of the membrane model (Figure 7) suggests that the velocity variation in the gap $(b, 0\leqslant\zeta\leqslant c)$ is nearly linear. The boundary condition for the gap is

$$u(b,0\leqslant\zeta\leqslant c) = \frac{\omega b}{c}\zeta \qquad 3.34$$

These boundary conditions are separable and thus a solution via the separation of variables technique is considered. The governing partial differential equation (3.5)

$$\frac{\partial^2 u}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial\rho} - \frac{u}{\rho^2} + \frac{\partial^2 u}{\partial\zeta^2} = 0 \qquad 3.5$$

admits two separation of variable solutions

$$u(\rho,\zeta) = \sum_{n=1}^{\infty} [c_{1_n}J_1(\lambda n\rho) + c_{2_n}Y_1(\lambda_n\rho)][c_{3_n}\sinh(\lambda_n\zeta) + c_{4_n}\cosh(\lambda_n\zeta)] \qquad 3.35$$

$$u(\rho,\zeta) = \sum_{n=1}^{\infty} [c_{5_n}I_1(\lambda_n\rho) + c_{6_n}K_1(\lambda_n\rho)][c_{7_n}\sin(\lambda_n\zeta) + c_{8_n}\cos(\lambda_n\xi)] \qquad 3.36$$

where J_1 and Y_1 are the Bessel functions of the first and second kind of order unity and I_1 and K_1 are the modified Bessel functions. The eigenvalues (λ_n) are obtained by orthogonalizing either the Bessel or the circular functions. The orthogonal expansion is most simple when the boundary conditions have either zero value or zero derivative on the boundaries.

The ζ boundary conditions have zero value at $\zeta = 0$ and zero derivative for large values of ζ and thus the ξ functions should be orthogonalized. The preferred expansion would be in terms of sines but this would mean a finite ζ solution. Inspection of Figure 7 reveals that full flow condition is very nearly attained by $\zeta = c$. Therefore a finite solution $\zeta = e$ would be a very good approximation to the infinite ζ solution. The solution of equation (3.36) may be written as

$$u(\rho,\zeta) = \sum_{n=1}^{\infty} [c_{5_n} I_1(\lambda_n \rho) + c_{6_n} K_1(\lambda_n \rho)] \sin\lambda_n \zeta \qquad 3.37$$

$0 \leqslant \zeta \leqslant e$

which satisfies the boundary condition at $\zeta = o$ (equation 3.30). To satisfy the boundary condition at $\rho = a$ (equation 3.31) it is required that

$$c_{5_n} = -c_{6_n} \frac{K_1(\lambda_n a)}{I_1(\lambda_n a)}$$

and the solution now is

$$u(\rho,\zeta) = \sum_{n=1}^{\infty} c_{6_n} [-K_1(\lambda_n a) \frac{I_1(\lambda_n \rho)}{I_1(\lambda_n a)} + K_1(\lambda_n \rho)] \sin\lambda_n \zeta \qquad 3.39$$

The Fourier series expansion of the odd function $u(b, 0 \leqslant \zeta \leqslant e)$ where the interval from $\zeta = 0$ to $\zeta = e$ represents a quarter period $(\pi/2)$ is

$$u(b, 0 \leqslant \zeta \leqslant e) = \sum_{n=1}^{\infty} \frac{2\omega bc}{e} \frac{\sin(\frac{n\pi c}{2e})}{(\frac{n\pi c}{2e})^2} \sin\frac{n\pi\zeta}{2e} \qquad 3.40$$

$n = 1,3,5.....$

The two expressions (3.39)and (3.40) must be equal at $\rho = b$ and thus λ_n and c_{6_n} are found to be

$$\lambda_n = \frac{n\pi}{2e} \qquad 3.41$$

$$c_{6_n} = \frac{\frac{2\omega bc}{e} \frac{\sin(\frac{n\pi c}{2e})}{(\frac{n\pi c}{2e})^2}}{-K_1(\lambda_n a)\frac{I_1(\lambda_n b)}{I_1(\lambda_n a)} + K_1(\lambda_n b)} \qquad 3.42$$

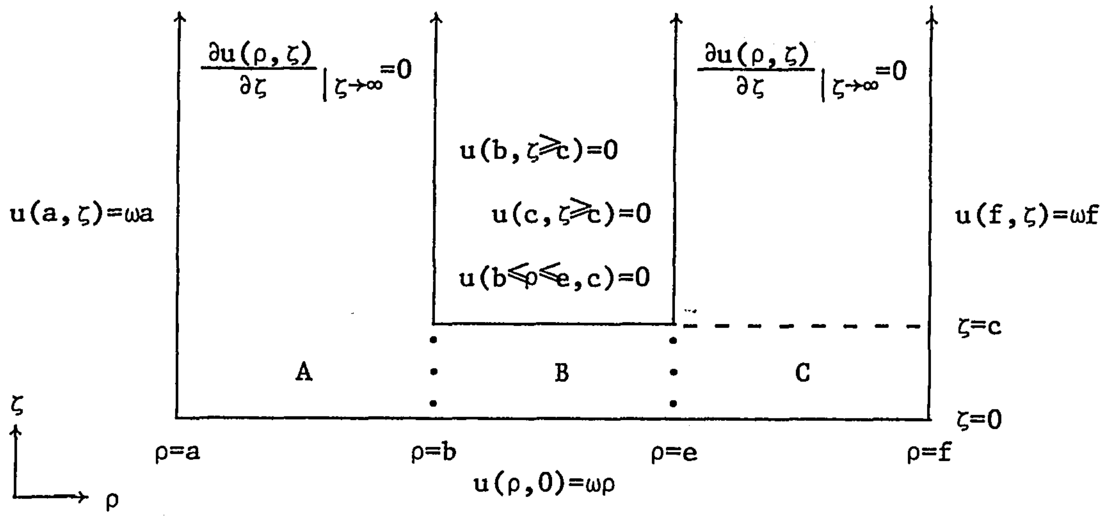
$n = 1,3,5...$

With these values equation (3.39) is the desired solution. Note that at $\zeta = e$ the derivative $\partial u/\partial\zeta$ is zero, satisfying the last boundary condition.

Certain objections may be raised. For one, the solution is finite instead of infinite and there is no information on how to pick the "flow at infinity" distance e. First of all the infinite boundary condition is a mathematical idealization of the assumption that the end effects decay rapidly which allows the top end and bottom end effects to be treated separately and does not represent a physical boundary condition. Thus a finite solution is as valid as an infinite solution for if the full flow is not rapidly attained, then the end effects are not separable and the infinite solution does not reflect the problem at hand. Since the full flow solution is known (equation 2.24) how fast the full flow solution is achieved may be determined from the trial values for e. The assumption of a linear velocity profile in the gap ($0\leqslant\zeta\leqslant c$) may be questioned. Spraying of liquids and powders through a narrow slit onto the sheared fluid in the gap and observing the pattern leads to the conclusion that the assumption of a linear velocity profile is certainly a good approximation. If more refined measurements show that the velocity profile is not linear, then still there are no conceptual difficulties in incorporating the exact velocity profile in the Fourier series expansion and obtaining a solution.

Bottom End Effect

The boundary conditions for the bottom end effect are as follows:



A solution by separation of variables would involve matching the solutions of three different regions. The solutions of course are interacting and the separation of the variables solution becomes too involved for engineering purposes. The separation of the variables solution can provide a first estimate of the velocity distribution which may be useful. Consider the transformation

$$u(\rho,\zeta) = v(\rho,\zeta) + \omega\rho \qquad 3.43$$

The form of the governing differential equation (3.3) does not change

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial v}{\partial \rho} - \frac{v}{\rho^2} + \frac{\partial^2 v}{\partial \zeta^2} = 0 \qquad 3.44$$

But the boundary conditions do change. The boundaries that were zero acquire an $-\omega\rho$ velocity and the boundaries that had an $\omega\rho$ velocity have a zero velocity after the transformation from u to v. The boundary conditions for the "A" region $(a\leqslant\rho\leqslant b,\zeta)$ now very closely resemble the boundary conditions of the top end effect. The only difference of course is in equations (3.33) and (3.34) which acquire negative signs when the velocity in the gap is assumed to vary linearly with ζ. The solution for the "B" region is obtained by inspection

$$v_B(\rho,\zeta) = -\frac{\omega\rho\zeta}{c} \qquad 3.45$$

Of course, the solution is only an approximate solution because a linear velocity variation is assumed at $\rho = b$ and $\rho = e$, but an inspection of the membrane solution (Figure 7) suggests that the approximation may be a good one. This estimate is based on the observation that the contours in the "B" region have a small curvature and equation (3.45) requires zero curvature. It may be argued that Figure 7 is a solution to a different problem, but still it does provide a reasonable picture of what might be occuring. From the first order estimate of velocity distribution, more refined estimates may now be obtained by numerical methods. From the exact and numerical solutions to the top end effect an estimate on the accuracy of the numerical solution may be made.

All too often the mathematical solution and the performance of equipment built in accordance to the required specifications differ significantly. Therefore, it is necessary to very accurately test the performance of the sample holder. If the performance differs from the predicted performance, the cause must be found and corrected. A brief description of how the sample holder may be calibrated is given in Chapter VI.

IV SAMPLE HOLDER DRIVE

Stepping Motor

The idea behind the sample holder motion is to generate a function Θ (of time) which is proportional to the desired angular position of the sample holder. This function Θ is operated on by several "black boxes", such as the power amplifier and the motor. This results in a close approximation of the position function Θ by the sample holder. While it would seem that the logical starting point for a discussion of the drive system would be the generation of the function Θ, the practical starting point is the motor. The other black boxes offer wider design options than the motor and thus the motor to a large extent determines the nature of the other "black boxes".

As discussed in the introduction, the required rotary motions of the sample holder are quite complicated and therefore, a flexible drive system is required. The preferred choice is an electric motor and an electronic control system for the motor. For this application there is nothing superior in an electric drive system over say a hydraulic of a pneumatic system, except that the needed hydraulic or pneumatic components are not available at a reasonable price. The electric motors commonly encountered are not suitable for our application. A brief review of the required motor characteristics is in order.

The stationary part of the motor (called the stator) is in the form of a hollow iron cylinder. On the inside of this cylinder there are two very similar coils wound at right angles to each other. When current is passed through a coil a magnetic field is generated. When currents flow in both coils two magnetic fields are generated at right angles to each other with the net magnetic field being the vector sum. Very specifically the windings of the two coils are so distributed that when the current in one coil varies as the sine of the variable

$$I_1 = I_0 \sin\Theta \qquad 4.1$$

and the current in the other coil varies as

$$I_2 = I_0 \cos\Theta \qquad 4.2$$

Then the net magnetic field of the stator is of a constant magnitude whose axis is at an angle Θ. If the variable Θ is a function of time, then the axis of the stator's magnetic field rotates in space. Insert into the stator a bar magnet (called the rotor) which is free to rotate about the axis of the stator and about its own axis. The interaction of the magnetic field of the bar magnet with the magnetic field of the stator results in a torque on the bar magnet which in turn causes the bar magnet to rotate so as to align the axis of the rotor with the magnetic field of the stator. As the stator magnetic field rotates in space due to the sinusoidaly varying currents, the rotor follows this rotating stator field and we have a crude motor with the desired properties. While the motor may be improved, the principles of operation remain the same and thus the explanation is easier to understand in terms of the simple bar magnet motor.

We note that the torque is zero once the stator and rotor fields become aligned. The rotor experiences a torque only when the rotor and stator fields are not aligned. Because of the way the stator is wound the torque varies in a sinusoidal way with the angular difference between the rotor and stator fields. The torque is

$$T = K_1 I_0 \sin(\Theta - \zeta) \qquad 4.3$$

T - torque

K_1 - motor constant

I_0 - peak current

Θ - stator field angle

ζ - rotor field angle

For small angular displacements

$$\sin(\Theta - \zeta) \approx \Theta - \zeta \qquad 4.4$$

and thus the rotor and stator may be visualized as being connected by a spring. The spring constant being a property of the motor and the stator currents. Since the rotor and whatever is connected to the rotor has a moment of inertia, the effect is that of a spring-mass system connected to the stator. Frictional effects on the rotor will constitute damping. Since the rotor is a permanent magnet, therefore the motor will function as a generator. The generated sinusoidal voltage magnitude and the frequency are both proportional to the angular velocity of the rotor.

$$e_g = K_2 \frac{d}{dt} \sin \zeta \qquad 4.5$$

e_g - generated voltage in one winding

K_2 - motor constant

ζ - rotor angle

t - time

Thus the motor is a bilateral electromechanical transducer. Still further motor properties are electrical in nature. The stator windings have inductance and resistance and there is also mutual inductance between the stator windings. Furthermore, if the rotor is in the shape of a bar, then the inductance of a stator coil varies with rotor position because the reluctance of the magnetic path for the coil varies with the rotor position because the reluctance of the magnetic path for the coil varies with rotor angle. A still further complication is due to the winding capacitances which in effect make the stator coils parallel tuned circuits.

It is not the intent here to get too deeply involved with motor models[1] and theory, rather the intent is merely to provide background information so as to clarify the design choices. Based on the preceeding discussion the ideal motor would have a stiff spring constant and a low inertia so as to respond quickly and minimize the positioning error. The winding inductances should be low so that the current through the coils could be changed rapidly without developing high voltages. A further benefit of low inductance would be to make the resonant frequency of the coil-winding capacity tuned circuit very high where the magnetic losses become appreciable and the energy storage effects of such a tuned circuit are small compared to the dissipative effects, which makes this resonant circuit heavily damped.

1. Sinha, N.K., Elliot, A.R., Wong, R.C.S., "A Realistic Model for Permanent Magnet Stepping Motors". IEEE Transactions on Industrial Electronics and Control Instrumentation, Vol. IECI-21, #3, Aug. 1974.

There is one more consideration in motor selection. The bar magnet motor is of course assumed to be a precision motor, still a small percentage of 360° is a significant error. One can see that accurate small angle motions would be difficult to achieve or are achieved at high cost. The thing to do would be to gear down the motor. Gearing is especially attractive since the top speed of the sample holder is in the order of one revolution per second. Mechanical gears have their problems. While it is possible to cut the gears oversize and then run in the geartrain, it is best to avoid mechanical gearing. This leads to consideration of electrical gearing and mounting the sample holder directly on the rotor shaft. Motors with electrical gearing are commonly called stepping motors. The name stepping motor arose out of the primary use of such motors in computer controlled positioning applications. Since the angle of twist per cycle of stator current is small such a motor could be commanded to step an integer number of cycles and stop thus avoiding the complications and costs of a servo system and yet achieve fine resolution and acceptable accuracy.

The stepping motor is characterized by numerous toothed projections on both the rotor and stator. The stator coils are the two phase windings discussed at the beginning of this section. The stator may typically have fourty-eight teeth and the rotor may have fifty teeth. Assume that the stator field is aligned with a stator tooth, then due to the attraction a rotor tooth aligns with this stator tooth. Because of the different number of rotor and stator teeth all other rotor teeth (except of course the diametrically opposite tooth) are misaligned with the stator teeth. The misalignment of course is smallest for the teeth adjacent to the aligned rotor-stator teeth. When the stator field shifts in space due to sinusoidally varying current in the stator coils, the rotor tooth that was aligned with the stator tooth does not follow the motion of the stator magnetic field as in the case of the bar magnet rotor. Rather when the stator field moves to the adjacent stator tooth the adjacent rotor tooth (which initially was slightly misaligned) now becomes fully aligned because of the greater attractive force. Thus while the stator field moves through one tooth spacing the rotor turns through the misalignment angle of the adjacent teeth which is a much smaller angle than the angle between the stator teeth. As the stator field rotates in space, successive teeth become aligned.

The recommendation is to use a stepping motor with the sample holder mounted directly on the rotor shaft. As with any other motor, the accuracy of the stepping motor depends on the accuracy of machining and winding of the stator coils. The remaining problem with the accuracy of the motor lies in the bearings. The speed is slow and the load is light and therefore no difficulties are envisioned in the performance and life of precision bearings that should be installed.

Power Amplifier

The axis of the stator magnetic field and hence of the rotor position is determined by the currents in the stator coils. Therefore it is desirable that the currents in these coils not be influenced by the inductive effects of the coils or the generated voltage due to the angular velocity of the rotor. What is wanted is a power amplifier whose output current is proportional to the input voltage (or current) with the output current not being affected by the voltage across the output terminals. A simple embodiment of such an amplifier is shown in Figure 8. This amplifier is for one phase of the motor. Another such amplifier is required for the other stator coil. The applied voltage V_{in}, is split into two output voltages of opposite polarity namely kV_{in} and $- kV_{in}$ where k represents the gain of the amplifier A_1. These two voltages are applied to the bases B_1 and B_2 of the NPN silicon transistors Q_1 and Q_2. If the peak to peak value of the voltages applied to the bases (kV_{in}) is much greater than the conduction window of the transistor, then to a first approximation the base-emitter potential difference may be assumed to be constant the the potential difference across R_3 is $2kV_{in}$. Transistor conduction is nearly zero when base-emitter voltage is + .5 volts and the conducting at nearly maximum rates when base-emitter voltage is + .7 volts and thus the transistor conduction window is approximately .2V. If the magnitude of the potential difference across the resistors R_1 and R_2 is large compared to the voltage swings at the emitters E_1 and E_2 then to a first approximation the currents flowing through the equal value resistors R_1 and R_2 is constant. Now the current that flows in the resistor R_3 is by Ohm's law $2kV_{in}/R_3$. Since the total current that flows in R_1 and R_2 is constant then the current through transistor Q_1 is $I_1 + I_3$ and the current through transistor Q_2 is $I_2 - I_3$. The current I_2 must be greater or equal to I_3 because a negative current will not flow through a transistor. To a

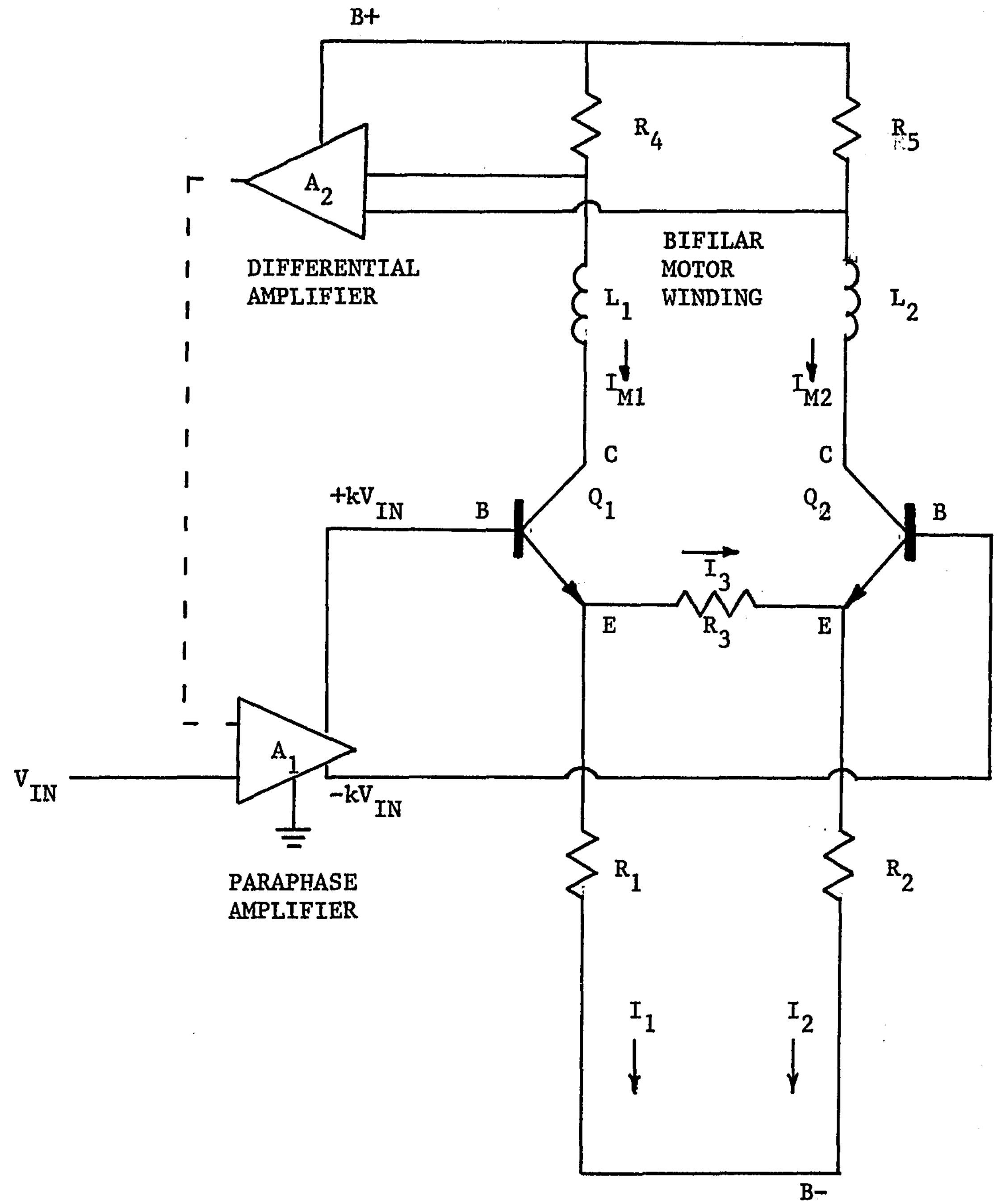


FIGURE 8

POWER AMPLIFIER

first approximation the collector voltage has little effect on the current that flows through the transistor, thus the collector currents may be taken as equal to the emitter currents independent of the collector voltage. Of course the collector voltage can not be greater than the breakdown voltage of the transistor nor should the transistor be in saturation (voltage of collector just slightly greater than the voltage of the emitter). The inductances L_1 and L_2 represent the windings of the same motor coil, but the windings are so wound (bifilar) that their magnetic fields tend to cancel. If the currents I_{m1} and I_{m2} are equal then the net magnetic flux is zero. From the preceeding discussion it follows that the net magnetic field is proportional to the current that flows in resistor R_3 and in turn the net field is proportional to V_{in}.

The amplifier meets the design objective in that the currents are independent of the voltage across the motor and the magnetic field is proportional to the input voltage. Certain improvements may be added to insure better performance. All amplifiers have distortion which may be minimized but not eliminated by negative feedback. The resistor R_3 introduces some feedback but it may not be sufficient to keep the distortion down to an acceptable level. Overall feedback may be introduced by inserting two small equal value resistors R_4 and R_5. The voltage difference across these two resistors is proportional to the current difference and in turn proportional to the net magnetic field of the stator coil. This difference voltage is amplified by the differential amplifier A_2 and the input of the paraphase amplifier.

Another improvement in performance may be obtained by switching the B+ supply voltage. At low speeds the voltage across the motor is low, however at highest speed the voltage can be quite high due to inductive and generator effects. To avoid saturating the transistors the B+ voltage must be set higher than the maximum expected voltage but at low speeds this results in needlessly high power dissipation in the transistors which shortens the life of the transistors. Circuitry may be added which senses the voltage at the collectors and if needed increases or decreases the B+ voltage. Since the collector voltage does not influence the current that flows through the motor, raising or lowering the B+ voltage has no effect on the performance of the amplifier. For motion of the highest accuracy it is necessary to take the feedback from the rotor itself. There are several ways that such feedback may be obtained with perhaps the optical encoders being the most practical. As usual cost and complexity increase sharply with accuracy.

There is a disadvantage to the described amplifier in that the amplifier does not contribute to the damping. As mentioned, the motor acts also as a generator and thus if the output impedance of the amplifier has a finite resistive component then the motor acting as a generator would dissipate some energy and hence provide damping. As it stands, the amplifier acts as a current source and hence the motor acting as a generator does not dissipate energy. The required damping must now be provided by viscous damping of the rotor or by additional circuitry in the amplifier.

The importance of damping is best illustrated by considering the motor performance in terms of energy instead of voltages, currents or torques. An electric motor is quite efficient with the major sources of energy dissipation being the Ohmic losses in the windings and the friction in the bearings. For the rotor to slow down from one speed to another steady speed a certain amount of energy must be dissipated. If this change of speed must occur in a given time period then energy must change at a certain rate. Fortunately the motor speeds are low and very precise bearings may have appreciable friction, still the problem of damping requires very close attention.

Sine/Cosine Generator

As a "black box" the power amplifier-motor combination take the inverse (arc) of the sine/cosine of the position function Θ and with the output being the sample holder angular position Θ. The requirement is then to generate voltages proportional to the sine/cosine of Θ which constitute the input to the power amplifier.

To accurately take the sine or cosine of a function by analog means becomes quite difficult. The preferred way is to generate the sine or cosine by digital techniques. Digital implies discrete steps which of course are highly undesirable, but if the steps are small enough then the digital approach may be acceptable. Consider dividing one revolution of the sample holder into a million steps ($\approx$ 1.3 arc seconds per step). At the highest speed which is in the order of one revolution per second this would require a million steps per second which is well within the capabilities of digital circuits. The step size is small and is further reduced by the low pass filtering action of the rotor (as mentioned previously the rotor inertia-magnetic torque forms a spring-mass system which has a resonant frequency). For forcing functions above the resonant frequency, the amplitude of the rotor response decreases by a factor of one hundred for each factor of ten increase in the forcing function frequency above the rotor resonant frequency. Thus for a resonant frequency of the rotor of ten Hz and a forcing function of 1 MHz the rotor step size is reduced by a factor of 1×10^{-10} at the highest motor speeds planned for the unit. There is a problem at the low speeds. If the number of steps per second is equal to the resonant frequency and if the rotor is underdamped, then resonance may occur which would magnify the step size. However, for a rotor resonant frequency of 10 Hz and 1×10^{6} steps per revolution this would mean a sample holder revolution every 10^{5} seconds (38 hours) which is not very useful for testing blood. Of course it is desirable to make the rotor resonant frequency as high as possible, but this would still leave a sufficient margin for steps (attenuation). Furthermore one million pulses per second is quite slow compared to what is possible at reasonably low cost.

The stepping motor as described has electrical gearing and typically two hundred electrical cycles are required for one revolution of the shaft. At one million steps per revolution this would require that each cycle of the sine wave contain five thousand steps. Such voltage steps may be generated by a digital to analog converter. The digital to analog converter (DAC) operation may be understood by considering the converter as being comprised of many switchable current sources say 1 ma, 2 ma, 4 ma, 8 ma and so on. Thus the output current may have any value within the range of the device in one milliampere steps, with the value of the current being determined by the binary signal applied to its input. The current may be converted to a voltage with the digital to analog converter having a voltage rather than a current output. The widespread use of computers and the need to interface the computer outputs with analog devices such as video terminals have made precision digital to analog converters of say 2^{14} (16,384 discrete levels) available as components at reasonable prices.

The problem is to generate binary coded signals which when applied to the digital to analog converter would result in an output voltage. Actually two such codes must be generated and two digital to analog converters are needed - one for generating sine and one for generating cosine. Since the construction for the cosine generator is very similar to the sine generator only the sine generator need be considered. Consider a digital counter and a binary code converter. The digital counter totalizes the number of pulses that have been applied to the counter. This count of pulses represents a binary number which is applied to the code converter. The code converter converts the binary number of the counter to another binary number by the rules of transformation. Very specifically the code converter is a sine look up table. Thus the output of the counter is a binary number proportional to the angle Θ and the output of the look up table is a binary number proportional to the sine of Θ which is converted by the digital to analog converter into a voltage proportional to the sine of Θ. The counter must be able to count up or count down and of course there is a need for a detector which will reverse the count direction when the maximum count (peak of the sine wave) is reached. The frequency of the resulting voltage and therefore the angular velocity of the sample holder is proportional to the pulse rate applied to the counter. This scheme of generating sine waves is attractive from a conceptual and a technical viewpoint but not from a fiscal viewpoint. The code converter is really a computer memory with the counter number representing the memory cell location. For the system under discussion the requirement is to store five thousand numbers (locations) with each stored number capable of representing the number 5000. Memory technology is rapidly changing so that in a few years the memory approach may prove very attractive and if money is no object the memory approach is the preferred approach. But, as of today the memory approach is more expensive than alternative ways of generating sine numbers. Of course the memory size can be reduced drastically by various techniques. The whole period of the sine need not be stored. Only half a period or with some circuit complexity one quarter period need be stored. By presetting the counter and adding a binary adder/subtractor only the increments of the sine need be stored with the true sine value obtained by adding the increments. Even these approaches which drastically reduce the memory capacity, the memory approach is not quite competitive.

An alternate way to obtain binary numbers is to solve the differential equation

$$\frac{d^2\Theta}{dt^2} + \Theta = 0 \qquad 4.6$$

by digital techniques[2]. This technique is very slow and thus seldom used, but is suitable for our purposes since the rotor speeds are quite slow. The digital counter is a digital integrator because it totalizes the number of pulses. If pulses are applied at a certain rate then the count is the time integral of the pulse rate.

$$C(t) = \int_0^t R(t)dt \qquad 4.7$$

Consider now a digital rate multiplier (DRM) which is a programmable divider. The pulse rate that appears at the output is related to the pulse rate at the input by

$$R_{out} = \frac{R_{in} \cdot N}{M} \qquad 4.8$$

$$0 \leqslant N \leqslant M$$

where the integer N is the programmed division and M is the highest division capability of the digital rate multiplier (M is an integer constant). Consider now two counters and two digital rate multipliers. The input rate to the rate multipliers is the same say a constant rate R_c. The output of the rate multipliers is connected to the input of the counters. One counter is initially preset to zero and counts up, the other counter is initially preset to M and counts down for each applied pulse. The output (i.e. the count) of each counter is cross connected to the other rate multiplier and represents

2. Schmid, H., "An Operational Hybrid Computing System Provides Analog Computation with Digital Elements", <u>IEEE Transactions on Electronic Computers</u>, VEC-12, #6, Dec. 1963, pp. 715 - 732.

the division number N. This connection is shown in Figure 9. By going through the various substitutions it is found that

$$N_u = R_c t - \left(\frac{R_c}{M}\right)^2 \int_0^t \left[\int_0^t N_u dt\right] dt \qquad 4.9$$

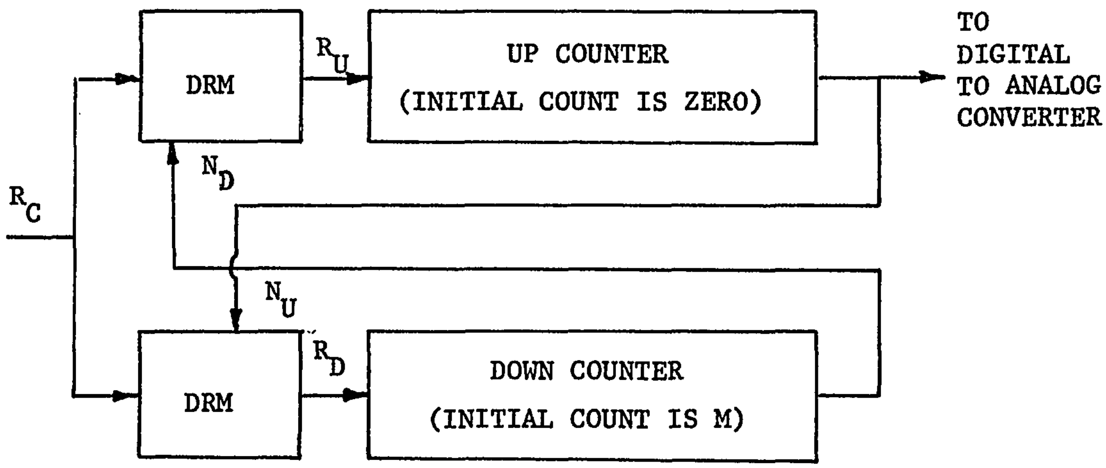
The solution of the above equation is

$$N_u = M \, SIN\left(\frac{R_c}{M} t\right) \qquad 4.10$$

Connecting the up counter to the digital to analog converter yields a sinusoidal output voltage. The down counter yields the cosine function.

The above results are incorrect because integration was performed on integer variables. The result is that there is an error in the generation of the sine. The error of course decreases as M increases. For M = 1027(2^{10}) the error is about 1% (8 steps). The error may be decreased by using a larger value of M and not using the least significant digits. The error is also somewhat attenuated by the filter action of the rotor, for if the rotor does not respond to a single step it also does not respond to the full extent to eight steps (out of one million steps per revolution). It should also be pointed out that the pulses coming out of the digital rate multiplier are not evenly spaced, but the integrator action of the counters provides some smoothing.

The sine/cosine generator circuit as it now stands generates only the first quadrant (0 - 90°). To generate the other quadrants, the up counter must switch to a down counter and vice versa. Both counters should be preset with the proper values so that the error does not



DRM - DIGITAL RATE MULTIPLIER

$$R_C = \frac{R_C N_D}{M}$$

$$R_D = \frac{R_C N_U}{M}$$

$$N_U = 0 + \int_o^t R_U dt$$

$$N_D = M - \int_o^t R_D dt$$

FIGURE 9

SINE/COSINE GENERATOR

accumulate. The polarity of the digital to analog converters must be switched at appropriate times so that rotation rather than an oscillation results.

Position Function Generators

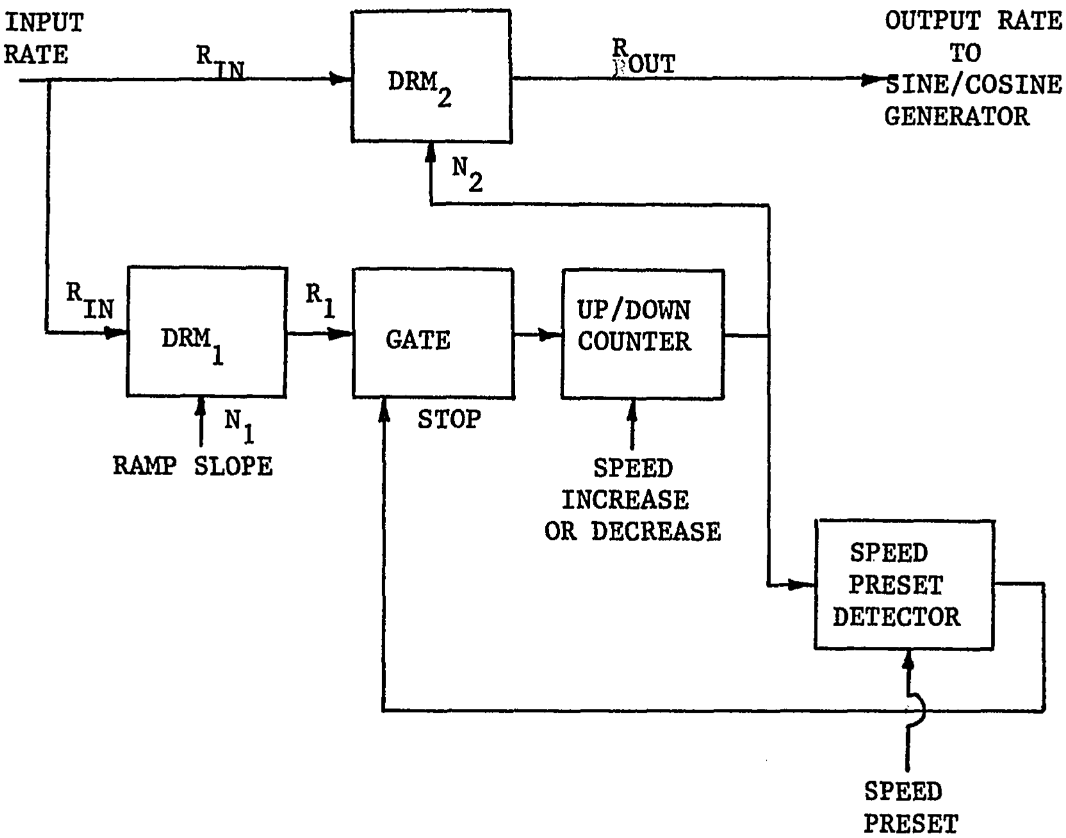
Whether the sine/cosine generation is by a look up table or integration, the angular velocity of the sample holder is proportional to the pulse rate applied to the sine/cosine generator. A steady pulse rate results in a constant angular velocity. For determining the fluid properties other motions must be considered. Step, sinewave, exponential and random motions may be of interest. Although the modulation of the pulse rate may be achieved in many ways, the technique presented here uses the digital rate multiplier and counters as the basic building blocks. Again it should be pointed out that the pulse rate from the digital rate multiplier has the correct frequency on the average but the instantaneous frequency may vary. Further processing and the filtering action of the rotor reduce these instantaneous frequency variations to acceptable limits. If need be a binary divider may be added which will reduce the instantaneous frequency variations. The greater the division rate the smaller the frequency fluctuations.

Steady Rotation/Step/Staircase Motions

The steady rotation is rarely a step function because to achieve steady rotation at a given speed, the speed must have been changed from some other speed (most frequently standstill). The staircase is of course programmed step functions of a specified duration. For testing blood step functions, with the implied infinite accelerations, may not be used because blood may become damaged by high shear stresses. The step portion is actually a ramp of some specified slope. The ramp terminates when the desired speed is achieved. Of course both speed increasing steps and speed decreasing steps are required. If the duration of the ramp is long and the steady rotation is short the step generator becomes a ramp generator of sorts. The step rate generator is shown in Figure 10. Two digital rate multipliers are used with the same input rate applied to both digital rate multipliers. The rate of one rate multiplier (DRM_1) is present and is used to program the slope of the ramp in the following way. The pulses of this rate multiplier (DRM_1) pass through a gate to an up/down counter which counts these pulses. The accumulated count controls the rate of the output pulses of the second rate multiplier (DRM_2) whose output goes to the sine/cosine generator. The accumulated count of course increases or decreases linearly with time depending whether the counter counts up or down. The output pulse rate of the second rate multiplier therefore also varies linearly with time. When the count reaches a certain preset limit the gate is closed and the count no longer changes which means that the output rate to the sine/cosine generator is now constant resulting in a steady rotation. Presetting a new limit and resetting the gate results in a ramp transition to another constant speed.

Sinewave Oscillation

The sinewave oscillator is very similar to the sine/cosine generator described previously. Instead of connecting the sine number



DRM - DIGITAL RATE MULTIPLIER

$$R_1 = \frac{N_1 R_{IN}}{M_1}$$

$$N_2 = N_0 + \int_o^t R_1 dt$$

$$R_{OUT} = \frac{N_2 R_{IN}}{M_2}$$

FIGURE 10

STEP GENERATOR

(N_u) to the digital to the analog converter, the connection is to a digital rate multiplier which will result in a sinusoidally varying output pulse rate. Of course the additional rate multiplier may be omitted with the pulse rate taken directly from the input of the up counter (R_u) see Figure 9.

A varying pulse rate results in a varying sample holder velocity but the direction of rotation remains the same. To reverse the direction the direction of the up/down counters of the sine/cosine generator must be reversed. Thus at the 180^o and 360^o points of the sinewave the direction of rotation must be reversed.

The question of oscillatory testing of blood is as yet open. Perhaps instead of reversing direction the superposition of small sinusoidal oscillation on a steady shear rate may be more significant. To achieve such motion not all the digits of the rate multiplier would be controlled by the sine oscillator. The most significant digits would be preset to select the steady rotation speed. Such a generator would require an additional rate multiplier as described at the beginning of this section.

<u>Exponential Motion</u>

One problem with a growing exponential motion is that the exponential can not last too long or else the velocity becomes too high. The major consideration is what to do when the speed reaches a certain value. Of the several choices the design shown holds the speed constant. Since no reports of testing blood with exponential motions have been found, just what sort of tests will be physiologically significant is not known.

The circuit to generate exponential motions uses two digital rate multipliers and one counter as shown in Figure 11. The output rate is given as

$$R_{out} = \frac{R_{in}N_0}{M_1} + \frac{R_{in}N_2}{M_2} \int_0^t R_{out}dt \qquad 4.11$$

with the solution

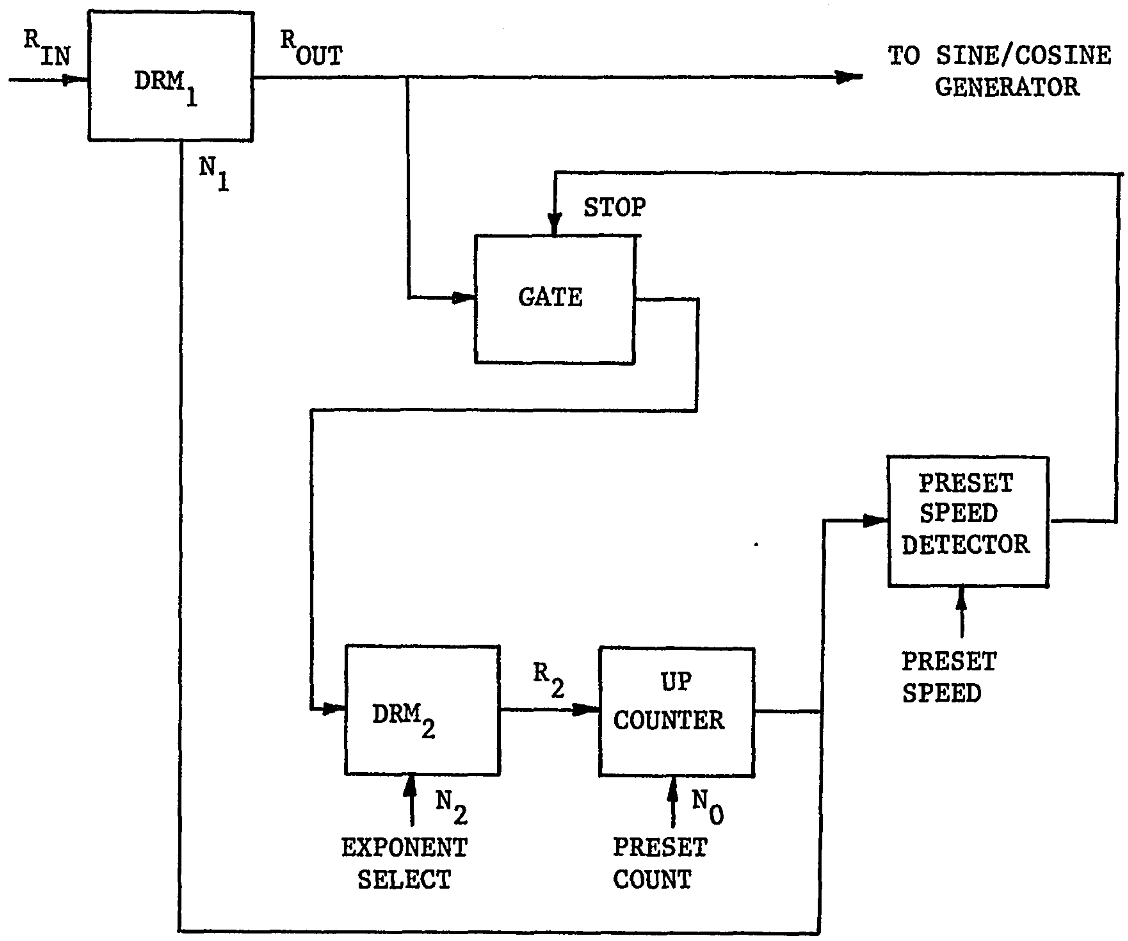
$$R_{out} = \frac{R_{in}N_0}{M_1} e^{\frac{R_{in}N_2}{M_2}t} \qquad 4.12$$

The output of the rate multiplier (DRM_1) is integrated by the counter. The output of the counter in turn controls the output rate of the digital rate multiplier (DRM_1). The integration process is modified by the second rate multiplier, which in effect varies with the exponent. When a preset speed is reached the count stops and the speed remains constant.

Random Motion

Testing with noise is a well established practice[3] in communication systems, process control and in analysis of structures for traffic and earthquake resistance. The description of blood behavior subjected to random shear rates was not found in the literature, and hence it is not known what the test parameters should be. Instead of concentrating on a particular motion generator, the emphasis will be on the

3. Korn, G.A., "Random-Process Simulation and Measurements", McGraw-Hill, New York, N.Y., 1966.



DRM - DIGITAL RATE MULTIPLIER

$$R_2 = \frac{N_2 \ R_{OUT}}{M_2}$$

$$N_1 = N_0 + \int_o^t R_2 dt$$

$$R_{OUT} = \frac{N_1 \ R_{IN}}{M_1}$$

FIGURE 11

EXPONENTIAL MOTION

generation of random sequences with the implementation of a particular motion being deducible from the examples of motion generators presented in this chapter. A truly random noise generator will not have the expected statistical properties if the observation time is finite. Each repetition of the experiment will yield a different result because for example the mean value may be different. Pseudo-random noise generators constructed from shift registers[4,5] have nearly the ideal statistical properties over a finite time interval. Such a generator looks and acts like a truly random noise over a short time interval, but it is in fact periodic. The longer the period between repetitions, the longer the shift register.

A shift register is an electronic "bucket brigade". On command (clock pulse) the contents (binary "0" or "1") shift by one register right (or left). Except during transfer it is possible to change the state of a register (fill or empty the bucket). The random sequence of "0" and "1" is generated by feedback from two or more register stages to the start of the shift register. Such a generator produces the maximum number of "0" and "1" sequences possible with n registers which is

$$N = 2^n - 1 \qquad 4.13$$

where N is the number of clock pulses for one period (before repetition) and n is the number of registers. The reason for the minus one in the

4. Elspas, B., "The Theory of Autonomous Linear Sequential Networks", <u>IRE Transactions on Circuit Theory</u>, March 1959.

5. Golomb, S.W., "<u>Shift Register Sequences</u>", Holden-Day Inc., San Francisco, California, 1967.

above equation is that the sequence of n zeros is not generated. Thus for twenty registers ($n = 20$) the sequence will recur after 1,048,575 shifts. The feedback arrangement varies with n, but is reasonably simple. For instance the feedback is taken from the seventeenth and twentieth register and summed by an exclusive "or" gate and feedback to the input (register 1). An exclusive "or" gate has an "0" output if the two inputs are the same ("00" or "11") and an "1" output if the two inputs differ.

If the contents of the registers could be viewed, then except for the all zero each sequence would be generated once. The output of each register is the same but delayed by some integer number of clock pulses depending on where the register is in the sequence. The statistics of the output are such that "0" and "1" occur equally often. After a run of ones or zeros there is a fifty percent chance that the run will end with the next shift; and thus it is not possible to predict an entire sequence from any partial sequence. The difference between the shift register sequence and some "natural" random occurrence such as tossing a coin is that there is a finite probability that the whole sequence of N tosses of the coin may turn up heads, whereas the maximum run of "1's" from a shift register is n and this occurs only once per period. The probability density function of the shift register output of course is discrete because the sequence is finite, but the envelope is nearly Gaussian and approaches the Gaussian distribution as $n \to \infty$. For a sequence length 8191 or more the probability density of the output, deviates less than ten percent from the true Gaussian envelope.

The natural tendency is to use as long a sequence as possible so as to better approximate the Gaussian distribution. This tendency should be resisted. The period of the sequence should be nearly the same as the test duration. The reason for this is the same as the reason for not using a "natural" noise source. A short segment from a long sequence may not have the desired statistical properties. The power density spectrum of the output is discrete because the output is periodic. The envelope of the power density spectrum is given by

$$P = \left[\frac{\sin(\pi f/fc)}{(\pi f/fc)}\right]^2 \qquad 4.14$$

where fc is the clock frequency. The difficulty with a long sequence and a high frequency clock so as to obtain a long sequence for a short test time is that the output signal contains high frequencies.

The advantages of the shift register generator are several. The sequence period can be adjusted to match the test duration period. The statistical properties, i.e., the probability density function or the power density spectrum envelope are not influenced by the clock frequency. Finally by presetting the same initial sequence, the same sequence is generated each time thus in effect producing the same "random" sequence.

V TORQUE PICKUP AND RECORDING

Torsion Bar

A low hysteresis torsion bar such as a quartz rod in the linear elastic region is an ideal device for measuring torque. Unfortunately for testing blood a torsion bar poses some problems: As blood clots it changes from a liquid to a gel. The change in torque is appreciable. The result is that a torsion bar which remains in the elastic region for torques which are required to break up the clot has very small deflections at low shear rates before clotting starts. The accurate measurement of small angles is quite difficult, but here in addition the requirement of measuring small angles is coupled with the requirement of measuring a wide range in order to handle the peak torques. One way to overcome this problem is to measure the blood properties before clotting starts, then change the torsion bar, draw another sample and test through the clotting stage. Since each blood sample must be freshly drawn this method is not suitable. Considerable thought went into various schemes for changing the torsion bar while a test is in progress. However, all proposed schemes tended to misalign the test section and hence were impractical.

The ability to handle a wide range of torques is not the only output problem that must be managed. The requirement is also to perform dynamic testing. Thus, the nature of the spring mass system formed by a torsion bar and sample holder must be carefully considered. Because of the small volume of blood available for testing (~ 3 cc) the absolute torque levels are quite small, and various fluid mechanics considerations constrain the sample holder size to ~ 3 cc. The result is that the resonant frequency

$$f_R = \frac{1}{2\pi}\sqrt{\frac{k}{I}} \qquad 5.1$$

f_R - torsional resonant frequency

k - torsional spring constant

I - polar moment of inertia

must be of the order of a few Hertz in order to obtain adequate sensitivity. It is desirable to have a much higher resonant frequency. An examination of the frequency response curves of Figure 12 for the torsion bar torque measuring systems indicates that the highest applied frequency should be less than 1/10 of the resonant frequency of .1 Hz in our case. The clump breakup process under step changes in shear rate is expected to be a relatively fast process which would involve higher frequencies than .1 Hz. Therefore the resonant frequency should be much greater than the 1 Hz level dictated by the sample volume considerations. It is difficult to raise the resonant frequency. For example, to raise the resonant frequency by a factor of ten while maintaining the same sensitivity requires that the moment of inertia be decreased by a factor of one hundred. On the other hand, the sensitivity is reduced by a factor of one hundred for a tenfold resonant frequency increase. Since neither a hundredfold decrease in sensitivity is acceptable nor a hundredfold decrease in moment of inertia can be achieved, the resonant frequency remains at a few cycles per second.

A further difficulty is noted by examining the equation of motion

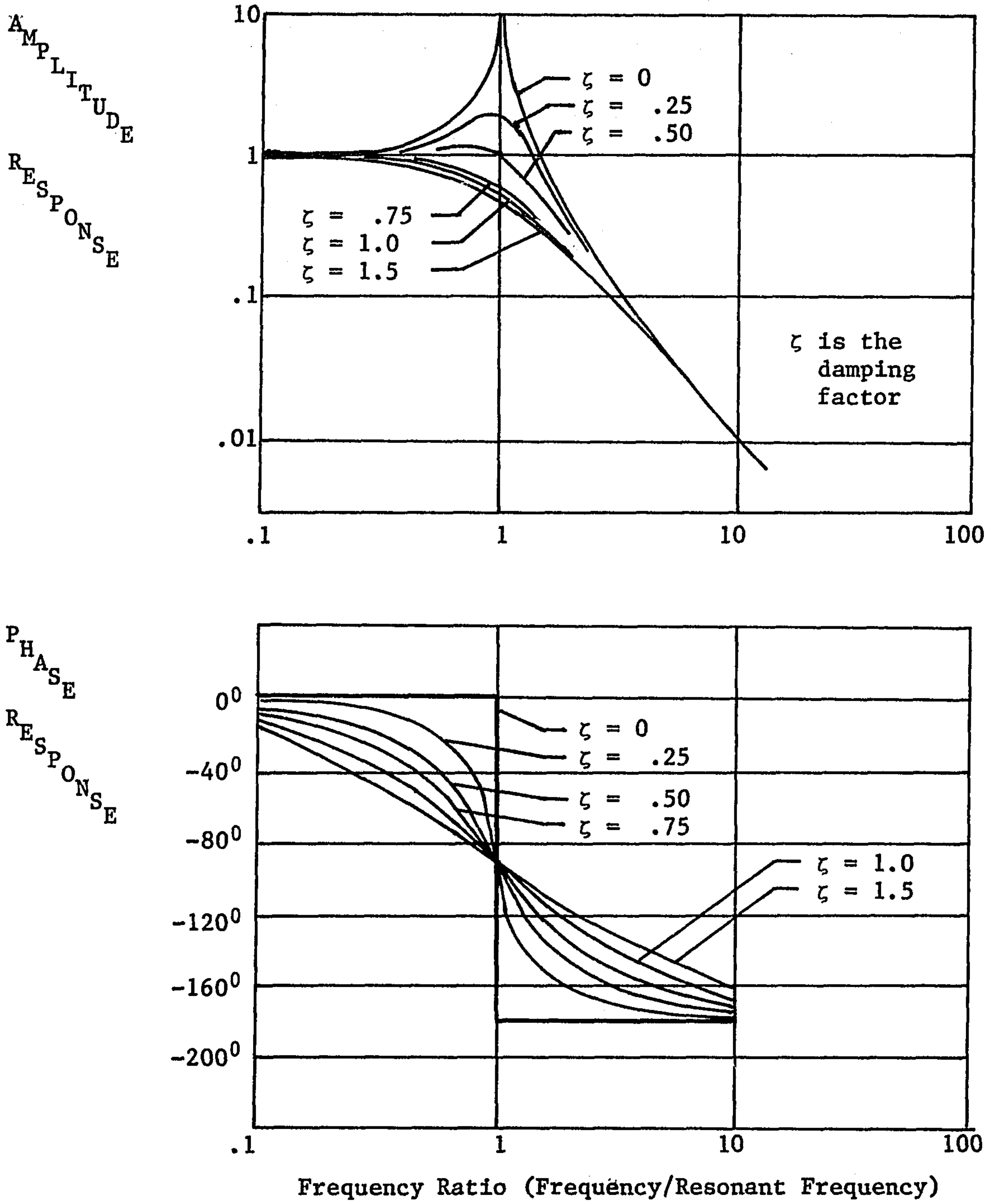


FIGURE 12

FREQUENCY CHARACTERISTIC OF TORQUE MEASUREMENT of the torsion bar and sample holder

$$I\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + k\theta = \tau \qquad 5.2$$

θ - angle of deflection

I - polar moment of inertia

D - damping

k - torsional spring constant

τ - applied torque

t - time

The applied torque is due to the shearing of the fluid. For a Newtonian fluid this is given by

$$\tau = \mu G\frac{d}{dt}(\alpha - \theta) \qquad 5.3$$

G - geometric factor

μ - viscosity of the fluid

$\alpha - \theta$ - angular difference between the driven and the torsion sensing parts of the sample holder

$d\alpha/dt$ - apparent shear rate

Thus the equation of motion for a Newtonian fluid is

$$I\frac{d^2\theta}{dt^2} + (D + \mu G)\frac{d\theta}{dt} + k\theta = \mu G\frac{d\alpha}{dt} \qquad 5.4$$

As might be expected the test fluid contributes to the damping. For a Newtonian fluid the problem is not too serious because μG is a constant but for blood the apparent viscosity is a function of shear rate and to infer the time dependent properties of blood involves considerable data processing. The simplest solution is to increase the external damping such that in comparison to D the damping of the fluid (μG) is negligible and need not be considered. Heavy external damping makes the system response sluggish (see Figure 12) and thus once more major data processing is needed to find the time dependent properties of blood.

Counter Torque Servo

To overcome the problems associated with a torsion bar the use of a counter torque servo system is considered. The twist angle θ is detected and a torque proportional to the twist angle is applied so as to nearly cancel the torque due to the fluid with the result that the twist angle is now very small. That the angle of twist is nearly stationary is of great theoretical importance because one of the boundary conditions in the solution of the partial differential equation no longer involves a moving boundary dependent on fluid motion and the system dynamics. The applied torque of equation 5.3 now is

$$\tau = \mu G \frac{d}{dt}(\alpha - \theta) - A\theta \qquad 5.5$$

where A is the servo system transfer function. The equation of motion 5.2 now reads

$$I\frac{d^2\theta}{dt^2} + (D + \mu G)\frac{d\theta}{dt} + (k + A)\theta = \mu G\frac{d\alpha}{dt} \qquad 5.6$$

In effect the spring constant is increased. Practically all of the spring constant is designed to be due to the servo system with the torsion bar serving a suspension and positioning function.

The spring constant of the counter torque system is now the servo system transfer function (A), which can be changed and in effect changes the torsion bar stiffness while a test is in progress. The resonant frequency may be increased. Finally with a higher resonant frequency the damping may be increased. Damping does decrease the response speed, but since the resonant frequency is high the response is still adequate.

At first sight it appears that the servo approach is technically superior to the torsion bar. On closer inspection this is not the case. The real advantage of the servo system arises from state of the art and cost factors. A quartz torsion bar is an excellent torque to angle transducer. The main problem is in measuring angles accurately. If the counter torque generator is an electric motor, then the counter torque is proportional to the current. Electric current can be measured very accurately. The inaccuracies of the servo system are due to the conversion of electric current into torque by the motor. Thus the tradeoff is an accurate motor versus an accurate instrument to measure angles. This very same problem faced the designers of gyroscopes for the missile and aircraft inertial navigation systems. Their decision was to counter torque the gyroscopes rather than measure angles. Huge sums of money were spent on the development of counter torque servo systems and thus we are the beneficiaries of the government's efforts in this area. Furthermore, progress is rather rapid in the area of inertial navigation systems and now counter torque gyroscopes are available as surplus equipment at a few cents on the dollar. Until a large effort is expended on angle measuring technology and such equipment is available as surplus there really is not much choice but to use the counter torque servo system. Indeed a viscometer employing counter torque principles has been built by Gilinson, Dauwalter and Merrill of MIT[1].

Counter Torque Generator

The counter torque generator (torquer) is basically an electric motor but comes in many varieties and trade names. Each motor type has its advantages and disadvantages. Which motor is used depends on who designed the gyroscope. Primarily the motors fall into two categories depending on whether the rotor is made of a "hard" magnetic material (permanent magnet) or a "soft" magnetic material (does not remain magnetized once an external field is removed). The theory of the permanent magnet rotor type motor has been discussed previously and need not be further elaborated upon.

A permanent bar magnet and a "soft" magnetic material bar rotor will tend to align themselves with the magnetic field of the stator. However, the principle of operation is different. Whereas action of the permanent magnet rotor may be explained on the basis of mutual attraction of the magnetic poles, the torque on the "soft" magnetic rotor is due to a minimization of the reluctance of the magnetic path. The generic name for such "soft" magnetic material rotor type motors is not surprisingly a variable reluctance motor. A closer examination of the characteristics of variable reluctance motors reveals that the torque is proportional to the square of the stator current

1. Gilinson, P.J. Jr., Dauwalter, C.R., Merrill, E.W., "A Rotational Viscometer Using an A.C. Torque to Balance Loop and Air Bearing", Transaction of the Society of Rheology, Vol. 7, 1963, pp. 319-331.

with the torque varying with angular displacement from the "aligned" position. How the torque varies with displacement depends in part on the motor design. The important difference as compared to the permanent magnet motor is that the torque varies as the current squared (see equation 4.3). The disadvantage of the variable reluctance motor is that it is a nonlinear transducer from current to torque which complicates the design of the servo system.

The way to identify a variable reluctance motor is to look at the torque-current relationship. If the torque varies as the current squared then the motor is a variable reluctance motor. For a tyroscope the torque is specified indirectly. Both the maximum torque and the linearity of the torque are usually specified as so many radians per second. To precess the spin axis a torque is applied to the gyroscope. The magnitude of the applied torque and the angular momentum of the gyroscope wheel determine the precession rate $\dot{\phi}$. Since the angular momentum of the gyroscope wheel is given, the magnitude of the torque can be calculated from the equation

$$\tau = H\dot{\phi} \qquad 5.7$$

τ - torque

H - angular momentum

$\dot{\phi}$ - turning rate

Since the torque is proportional to the square of the current, the variable reluctance motor may be operated either on alternating or direct current. For various traditional reasons quite a large percentage of the torquers operate on alternating current. Alternating current operations have their problems. Not only must the magnetic fields be well controlled, but phase shifts of the alternating current due to hysteresis, eddy currents, winding resistance and capacitance of the windings must be controlled. There are several types of A.C. variable reluctance torquers of which the Microsyn[2] which was developed at MIT is the most commonly encountered. The Microsyn was used by Gilinson, Dauwalter and Merrill in their viscometer.

The basic Microsyn has a dumbbell shaped rotor and a stator that is arranged to carry two coils on each of the equally spaced four pole pieces. One of the coils of each pole is connected in series to form a primary winding and the other coil of each pole is also connected in series to form a secondary winding. The primary coils of each pole are alike and the secondary coils of each pole are alike. Since the current through the primary coil is the same and since the coils are alike the magnitude of the magnetomotive potential (ampere turns) due to the primary current is the same for each pole. The same is true for the magnetomotive potential due to the secondary current. The coils are so arranged on the poles that there is no torque on the rotor if only the primary or secondary winding is excited. The connection of the primary and secondary coils is not identical for each pole. The coils are so connected that the magnetomotive potentials of the primary and secondary coils add at the 0° - 180° poles and subtract at the 90° - 270° poles, when the primary and secondary currents are in phase. When the primary and secondary currents are 180°

2. Mueller, R.K., "Dynamo Transformer", U.S. Patent 2,488,734, Nov. 1949.

out of phase the magnetomotive potentials add at the 90^o - 270^o poles and subtract at the 0^o - 180^o poles. Excitation of both the primary and secondary windings results in a torque on the rotor. The direction of the torque depends on whether the primary and secondary currents are in phase or 180^o out of phase. The magnitude of the torque is proportional to the primary-secondary current product. If the current flowing in the primary is held constant then the resulting torque is proportional to the magnitude of the current flowing in the secondary. The result is a linear transducer with the magnitude and direction of the torque governed by the magnitude and phase relation of the current flowing in the secondary winding.

Angle Detector

The main motivation for using a counter torque servo system with all its complexities is to avoid measuring small angles. However, the counter torque servo system does not completely avoid measuring angles in fact it requires the measurement of even smaller angles than the torsion bar approach. The great distinction between the torsion bar and counter torque angular measurement requirements is that the counter torque system does not require accurate measurement over a wide dynamic range. All that is required is to sense that an angular rotation has occured and the direction of the rotation because the counter torque feedback system acts to restore the original angular position. The main requirement on the angle detector is high sensitivity to angular motions, low sensitivity to displacements and low reaction torque. Linearity of the angle detector is not that important since over a small range any detector is linear.

Various angle detectors may be used such as pheumatic, capacitive, inductive, optical and so on. Gyroscopes that employ inductive torquers will typically employ inductive angle sensors with the Microsyn being most prevalent of the various inductive angle sensor types. The structure of the Microsyn angle sensor is very similar to the Microsyn torquer, but the theory of operation differs. The primary winding is connected as in the case of the torquer and no net torque is exerted on the rotor due to current flowing in the primary windings. The secondary coil on each pole may be viewed as forming the secondary winding of a transformer with an induced voltage that is proportional to the magnitude of the voltage across the primary coil. The Microsyn is a variable reluctance motor which means that as the rotor is turned the inductance of the coils varies. Because of the symmetry of the Microsyn the inductances of the opposite poles will increase (or decrease) due to the turning of the rotor. The total voltage across the primary winding is distributed unevenly across the four coils with the coils with the higher inductance having a greater voltage difference. The secondary coils are connected in series so that the voltages induced in each two pairs of coils on opposite poles are in opposition. The result is that when the voltages across the primary coils are equal the voltage difference across the secondary winding is zero. As the rotor is moved from the null position a net voltage appears across the secondary with the phase of this secondary voltage changing by 180^{o} as the rotor is turned through the null. Since the magnitude of the primary winding inductance changes is a function of the angular displacement from the null position, the magnitude of the induced voltage of the secondary winding is a function of the angular displacement from the null position and of course proportional to the voltage across the primary winding.

The use of a Microsyn torquer and sensor together on the same shaft of the counter torque servo system is now obvious. The primary windings of both the torquer and sensor are excited from the same source. The output of the secondary winding of the sensor is electronically amplified and applied to the secondary winding of the torquer. The result is of course a servo system. Either the gain of the amplifier or the magnitude of the primary excitation or both may be used to vary the gain of the servo system (A of equation 5.6) and hence the effective spring constant.

It must be pointed out that the resulting torque of a Microsyn servo system is periodic because alternating current is used. Since the frequency of the alternating current is above the torsional resonant frequency, the system does not respond fully to the instantaneous torque value but only to the mean value. While the motion due to the instantaneous value of torque may be very small, it is not zero and thus the question arises as to the magnitude of motion below which the small motions have no effect on the blood. No mention of such a lower limit of motion magnitude was found in the literature which could mean that the problem has not been investigated or that no lower limit could be found. If there is no lower limit this would mean that different results would be obtained by investigators using viscometers having different noise levels. Here noise is used in the sense of small undersized motion arising from any source. In the absence of information prudent design would require a low noise design. This means paying special attention to balancing all the components including the whole rheometer so that the lateral motions of the floor are not converted into rotational motions due to mass unbalance.

There is yet another reason for reducing the seismic noise level of the instrument. The reliability of an electronic component is in part predictable from the noise such a component generates. Ball bearings are inspected by measuring the noise generated. The formation and the breakup of clumps is a noise process and thus the intriguing question arises as to whether the noise level and spectrum of sheared blood has any physiological signifance? To make tests that would settle the question, the noise level of the viscometer system must be below the noise level of the blood.

Bearings

The various torquers not only generate tangential forces which contribute to the torque but radial forces as well. By making the torquers symmetrical, the radial forces may be made to cancel. Unfortunately the cancellation is never perfect and thus there is a need for a bearing to restrict radial motion which tends to misalign the sample holder. The ideal bearing would restrict motion in the radial direction and have no effect on motion in the tangential direction. Unfortunately real bearings have cross coupling with the result being that strong radial support means greater interaction in the tangential direction, all other things being equal. Thus the secret of a good bearing design is to minimize the radial forces.

The bearing problem is also encountered in gyroscopes because the gyroscope must turn freely and yet support is needed to overcome gravitational attraction and accelerations. To cancel the gravitational effects the gyroscope is floated in a dense liquid. The flotation is never quite perfect but the effects of gravity are greatly reduced. Various schemes for bearings have been suggested such as electrostatic, magnetic, superconductive, hydrostatic and so on. The most commonly encountered bearing in high quality gyroscopes is magnetic[3] with the Magnesyn type of bearing being the most common.

The construction of the Magnesyn bearing is as follows. The rotor is a cylinder made of a "soft" magnetic material. The stator has eight symmetrically spaced poles. Each pole has a very similar coil. The coils of two adjacent poles are so connected that the resulting magnetic field due to current flowing in the coils of two adjacent poles goes up one pole, through the pole-rotor air gap, through the rotor, through the pole-rotor air gap of the other pole, down the other pole and completing the circuit through the stator casing. In series with two coils is inserted a capacitor and this series inductor capacitor circuit is connected to a sinusoidal voltage source. The inductor-

3. Gilinson, P.J., Jr., Denhard, W.G., Frazier, R.H., "A Magnetic Support for Floated Inertial Instruments", Proc. Natl. Specialists Meeting on Guidance of Aerospace Vehicles, Institute of the Aeronautical Sciences, New York, N.Y., 1960, p. 222.

capacitor circuit forms a series resonant circuit in which the maximum current flows at resonance. Beyond the resonant frequency the current rapidly decreases with increasing applied frequency. The value of the capacitor is adjusted such that the frequency of the voltage generator is slightly higher than the resonant frequency. The current in the coils attracts the rotor, but the rotor is in equilibrium because the pair of poles on the other side of the stator also attract the rotor. Consider now what happens when the rotor is displaced toward one pair of poles. The air gap becomes smaller and therefore the reluctance of the magnetic path decreases. A decrease in reluctance increases the inductance with the result that the resonant frequency of the series inductor-capacitor circuit decreases. Since the frequency of the voltage source remains the same the result is that the current decreases because the difference between resonant and applied frequency is now greater. Now the force between the rotor and the pair of poles is given by

$$F = \frac{I^2}{2} \frac{dL}{dx} \tag{5.8}$$

F - attractive force

I - electric current

dL/dx - change in inductance with displacement

If the current decreases more rapidly than the inductance change then the attractive force on the rotor diminishes. Just the reverse action occurs at the opposite pair of poles: air gap increases, inductance decreases and thus current increases which in turn means that the force increases attempting to restore the rotor to its original position. Again because of the alternating current excitation the force is periodic and care must be taken to avoid structural resonances or motions which may affect blood properties.

The use of an air bearing was also considered specifically the Weissenberg Rheogoniometer air bearing was investigated. The air bearing turned out to be very noisy even after the rotor was carefully balanced. The source of the noise could not be identified. The noise seemed to be partially correlated with floor noise. Perhaps the non-linearity of the air coupled with some turbulence were responsible for the noise, because the noise tended to decrease with decreases in the air pressure. Unfortunately the rotor made contact with the stator before the noise reached acceptable levels. Due to the high noise level the autorotational torque versus angular position could not be determined. The autorotational torque is due to machining imperfections of the rotor and stator. A flat spot on the rotor which aligned on a flat spot on the stator contributed to the autorotational torque. From the investigation of one air bearing the air bearing as a class can not be rejected as being unsuitable, but since the air bearing investigated is a high quality bearing it does suggest that the design of an air bearing requires great care and effort.

Data Recording

The current applied to the torquer may also be sent through a chart recorder and except for the torquer scale factor the torque may be recorded on paper. Recording the torque on a paper chart and then trying to analyze the data, to a large extent negates the laboriously achieved accuracy. The other types of recorders commonly encountered also have dynamic range problems. As far as information is concerned the requirements of the rheometer are quite modest. While high resolution is required the rate changes are quite slow. This suggests coding the test data, and for example, trading dynamic range for bandwidth.

Recording of the test data is just a small step leading from the testing of a blood sample to diagnosis of the disease. The coding of the test data must be made compatible with the data reduction system. Nowadays the most economical way to process data is by digital computer. While it is possible to perform data analysis such as Fourier spectrum analysis by analog techniques, the equipment is quite specilized and required large capital expenditures for the equivalent accuracy of a digital computer. While a digital computer is also expensive, its availability, speed and flexibility allow it to be used for many other purposes. Thus, the cost of a Fourier spectrum analysis of test data is quite low compared to the amortization costs of a very accurate analog spectrum analyzer that is not in constant use.

The decision to use a digital computer for test data reduction means that the data coding must be binary. However, there are many binary codes. The question now arises whether the test data should be coded to fit the computer format or coded in the easiest to generate format, and let the computer translate this code to the required format for the data. Either decision can be implemented but since the computer is a data processor it is just as simple to let the computer rearrange the test data to the required format and not burden the rheometer with this task. Of course, a program must be written to tell the computer how to do the conversion.

Quantization Errors

For computer processing the test data is no longer continuous but is sampled. Of course, a continuous function is, for engineering purposes determined by the instantaneously sampled data if the sampling frequency is high enough. Specifically if the analog test data contains no frequencies higher than B Hertz, then it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart, that is, the sampling frequency (Nyquist frequency) must be twice the signal frequency. In practice the requirements of the theory can only be approximated. The sampling duration is non-zero and it is impossible to require that a signal contain no frequencies beyond B Hertz.

The effect of the non-zero sampling duration is that the signal may change while the sample is taken which of course leads to error. The exact amount of error depends on the nature of the signal and the sampling electronics. The order of magnitude of the error may be computed by calculating the greatest amount that a sinewave of B Hertz changes during the time it requires to take a sample. Of course, the use of a sample and hold circuit which reduces the sampling duration and maintains the sampled signal level for some time to allow for a more accurate quantization of the sampled signal reduces the problem of sampling duration but introduces errors of its own.

The presence of higher signal frequencies beyond the Nyquist frequency results in an error know as "aliasing". An aliasing error results because it is impossible to decide from the ordinates of the samples whether the applied frequency (f) is above or below the sampling frequency (fs). For example, a sinewave sampled at a rate eight times its frequency (fs = 8f) yields the same sample values (except for phase shifts) as a sinewave of the same amplitude but sampled at eight-sevenths of its frequency (fs = (8/7)f). To estimate the root mean square of the aliasing error the following formula may be used.

$$\varepsilon_a = \left[\frac{\int_B^\infty S(f)df}{\int_0^\infty S(f)df}\right]^{\frac{1}{2}} \qquad 5.9$$

ε_a - RMS error

$S(f)$ - signal power spectral density

B - Nyquist frequency

As may be seen the aliasing error is the square root of the ratio of the signal power beyond the Nyquist frequency to the total signal power applied to the sampler[4]. The above result is valid if the sampling duration is zero. This can not be implemented in practice, but the result is still valid because usually a filter is interposed which rapidly attenuates all high frequencies. The sample holder torsion bar (whether material or servo) forms a low pass filter which rapidly attenuates the signal beyond the resonant frequency (see Figure 12). Recall that the resonant frequency was increased so that all the information may be measured and thus the resonant frequency forms a limit to the highest frequency of interest and frequencies beyond that should be attenuated to reduce aliasing error. The low

4. Downing, J.J., "Modulation Systems and Noise", Englewood Cliffs, N.J., Prentice-Hall, 1964, pp. 140 - 143.

limit for the Nyquist frequency thus is twice the resonant frequency. For some motions the frequency content is much lower than the resonant frequency. In this case so as to avoid sampling at a high rate which results in many data points and little information a low pass filter may be interposed to limit the bandwidth of signals applied to the sampler and the sampling rate may be reduced.

A filter (including the sample holder-torsion bar) has a phase response as well as an amplitude response. As it may be seen from Figure 12. The changes in phase may start at a much lower frequency than any changes in the amplitude. A phase shift between the input of a sinewave to a filter and a phase shift at the output means that there is a time delay in passing through the filter. An ideal filter would delay all frequencies equally up to the frequency where the amplitude response has decreased significantly. A constant delay for all frequencies means that the phase shift varies linearly with frequency. An unequal time delay versus frequency means that a signal containing many frequencies simultaneously will have a different waveform shape at the output because the different frequencies were delayed by different amounts. One solution is to use filters with nearly ideal phase response. Unfortunately such filters have an amplitude response that decreases very slowly with increasing frequency and in effect one trades aliasing error for phase error. Now aliasing error can not be removed by processing the data, but phase error can be minimized at least conceptually by processing the data. By testing a fluid with known properties such as air or water the rheometer transfer function may be determined. If the inverse transfer function is simulated in the computer and the test data is passed through the inverse transfer function then to a large extent the instrument contribution may be subtracted. In practice such a procedure can not be performed exactly nor is such a procedure as simple as it sounds.

Another kind of error is the quantization error, that is, the resolution error. A decision must be made of how many discrete levels the full range of torque is to be divided. The countertorque current is measured by a digital current meter and the result is displayed and recorded. Digital current meters are available instruments with conversion time, resolution and money closely intertwined. The greater the resolution and the shorter the time to obtain an accurate reading, the greater is the cost. An alternative is to use a up-down counter and a digital to analog current converter. The error angle opens up a gate and the counter counts up or down depending on the direction of the error. The total count is recorded and also applied to the digital to analog converter. The output current of course now is the countertorque current.

Digital Data Recording

The preferred recording device is an incremental digital tape recorder. Punched paper tape or cards are too slow and cumbersome. The incremental tape recorder differs from the ordinarily encountered tape recorders in that on command a binary number is recorded, an advance is made to the next position and motion stops until the next record command. While the recording formats are standarized to allow interchange of data, there are several systems of data recording so that the recording of data must be compatible with the tape reading capability of the computer. With the recording format fixed, the main concern is what data to record.

The counter torque current must be recorded but much more data must also be recorded such as shear rate, elapsed time from the taking of the sample, elapsed time from the starting of the test, temperature, donor and so on. Such information could be recorded each time the torque is recorded but some of this information is redundant.and the fast memory used for computation is quite limited. Another limitation is the speed of the tape recorder. Though a tape recorder may accept a few hundred record commands per second (when the information per sample is large) the sample recording rate (sampling rate) is comparitively slow. Thus, there is a need to record the minimum information per sample. Three different recording modes will be considered.

First, there is keyboard entry. Data may be recorded from a manual keyboard. Information that should be recorded in this way is information that is valid for the whole series of tests such as information about the donor and so on. The important consideration is to insure that this data will not be confused with test data but will be printed exactly as recorded. This means that some sort of coding at the beginning or throughout each block of data is required so that the keyboard data may be unambiguously identified by the computer.

Second, there is the test information. If the tests were performed manually, then the operation would have to set switches selecting the type of motion, motion parameters and so on. Because the time for testing whole blood is so short, the tests must be automatically sequenced. Instead of manually setting the switches, the switches may be set electronically prior to each test by information recorded on a paper tape. When one test is finished, the next one starts. The information on the paper tape describes exactly the test to be performed and if this information is transferred to the magnetic tape all the parameters of the test would be recorded. A program reading this information could branch to the desired data processing routine. Additional information beyond the information of the paper tape such as elapsed time, temperature and so on could also be recorded at this time. To transfer this information onto magnetic tape is a fast process (less than a second) and after this fixed length information block the test data starts.

Third, there is the test data. The two main variables from which the blood properties are to be inferred are of course the torque and shear rate. For data of the highest accuracy both must be recorded. If the motion is very accurate, then the shear rate information may be omitted. This involves a tradeoff. Either the funds are spent to make the motion accurate or if the motion is not that accurate then the shear rate must be recorded and processed and the price must be paid for the extra data processing. Recording of the shear rate may also require a tape recorder that has greater recording rate capabilities to accomodate the faster data rates.

Neither torque nor shear rate are available as such - a conversion factor is needed. For shear rate a number proportional to angular velocity may be recorded and the conversion factor obtained from the sample holder size may be stored in the program. For torque this problem is more acute because the very high accuracy torque sensing system is subject to long term drifts and shifts in sensitivity. It is suggested that a calibration test with air as the working fluid be performed by incorporating it in the test sequence. The torque pickup conversion factor and the rheometer dynamics could in this way be obtained for each sample to be tested.

VI ANCILLARY EQUIPMENT

Temperature and Humidity Control

The shear rate and resulting torque are of course of prime importance. Most of the effort should be spent on attaining great accuracy and precision in their measurement. However, there are also some other pieces of equipment needed to realize measurements of the highest accuracy.

Temperature control of the sample holder is an example of such ancillary equipment. The blood viscosity is not very sensitive to temperature but the clotting process being a chemical reaction is sensitive to temperature. The time at which clotting in unsheared blood starts is typically

$4^{o}C$ - 60 - 75 minutes
$14^{o}C$ - 20 - 30 minutes
$37^{o}C$ - 4 - 7 minutes

As may be seen the clotting time follows the chemical reaction rule of thumb that is, doubling for every ten degrees centigrade drop in temperature. The question of the effect of a papid change in temperature on the rheological properties of blood has not been thoroughly investigated. In the absence of concrete information it is best to avoid problems by keeping the sample holder and the blood transfer equipment at body temperature ($37^{o}C$). The sample holder must be preheated and kept at this temperature during the test period.

Several schemes were investigated for temperature control of the sample holder. The heat loss from the sample holder is large enough so that preheating the sample holder and then turning off the heat during testing is not feasible which means that the heating process must continue throughout the test. Electrical heating seems an obvious choice but it turns out that accurate absolute temperature control is quite complicated. Furthermore the requirement that the sample holder be demountable for cleaning makes electrical heating too complicated. The recommended technique is to use circulating water. A design showing the circulating water constant temperature bath is shown in Figure 13.

This design avoids the stiction introduced by slip rings but does require a hollow motor shaft. For small and medium size motors the rotor is pressed to fit onto the motor shaft and for this reason motors with hollow shafts are not available. Drilling a long concentric hole through the shaft turns out to be a problem. Electrochemical machining which removes the metal atom by atom was investigated for this purpose. Since in electrochemical machining there is no physical contact between the tool and the workpiece, a straight concentric hole may be drilled through the rotating motor shaft. To check the feasibility of electrochemical machining some holes were drilled in steel blocks with satisfactory results. For a simple description of electromachining see the article by Hoare and LaBoda[1].

1. Hoare, J.P., LaBoda, M.A., "Electrochemical Machining", <u>Scientific American</u>, Vol. 230, #1, Jan. 1974, pp. 30 - 37.

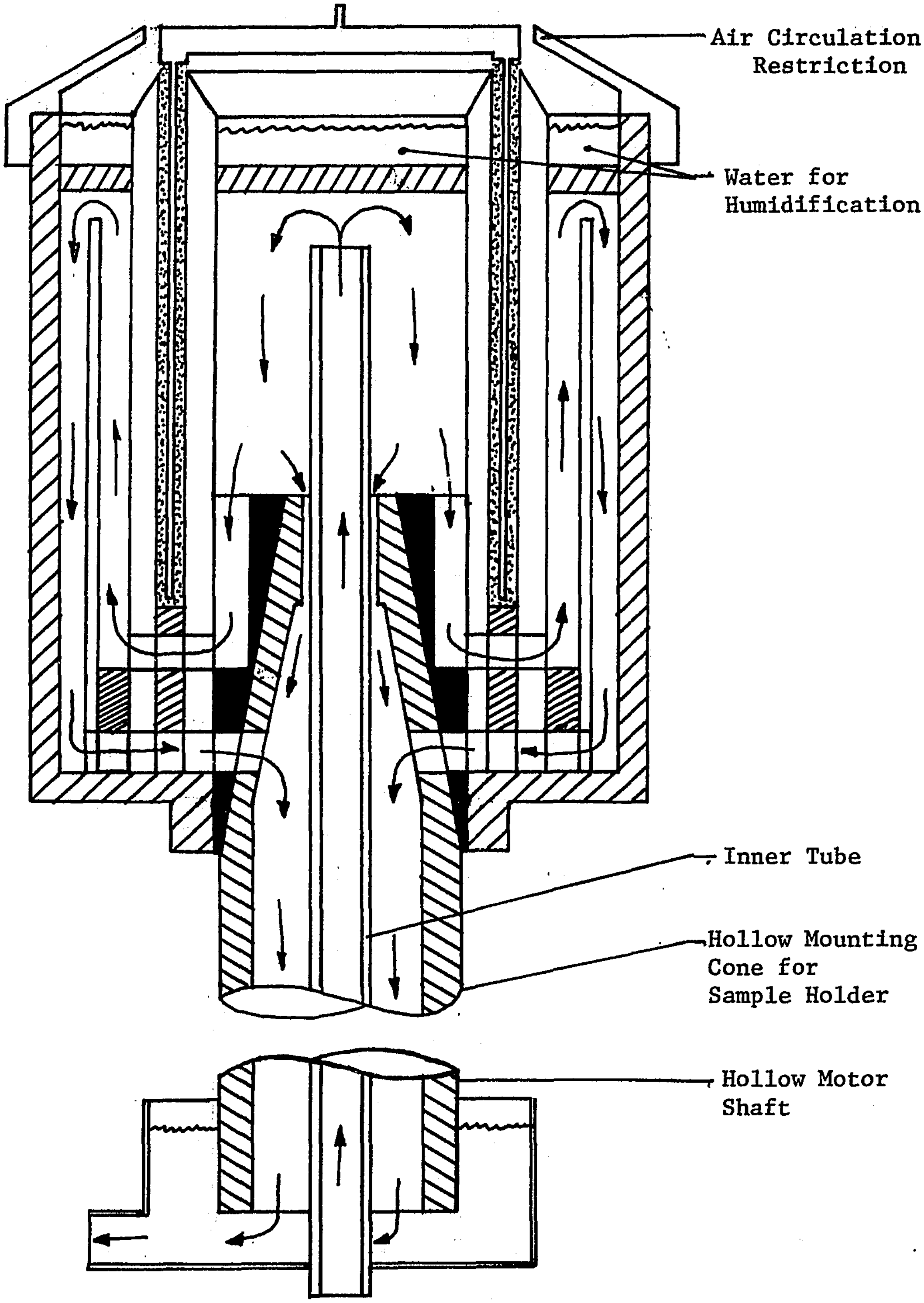


FIGURE 13 WATER JACKET AND HUMIDIFIER

One source of annoyance that may be encountered is the water bath. To monitor the temperature of the sample holder, thermistors should be inserted in the supply and return water lines. It may turn out that the temperature of the supply line could vary appreciably more than the required temperature tolerance. The reason for this is that typically the water bath temperature is controlled by a mercury thermometer which switches electric power to a heating coil. The thermal inertia of the thermometer coupled with the insufficient mixing of the heated water produces these temperature fluctuations while the mean temperature is very stable. Except for redesign there is not very much that can be done and therefore an averaging circuit must be included in the temperature monitoring system.

Under ambient conditions which the operator finds comfortable the exposed surfaces of the blood sample tend to form a tough black surface layer which influences the torque measurements. When the relative humidity and temperature of the air around the exposed blood surfaces is near one hundred percent and thirty-seven degrees centigrade then the effects of such surface film could not be detected, but, the operator is uncomfortable. Therefore, it is necessary to achieve the desired humidity and temperature conditions locally around the exposed surfaces of the blood. This may be achieved by restricting air circulation and having exposed water surfaces from which evaporation can take place. The water of course is heated by the sample holder and thus the required conditions of temperature and humidity are achieved. This arrangement is shown in Figure 13.

Alignment

Alignment of the sample holder is a procedure that is crucial to the measurement of the rheological properties. While the results mainly depend on the technique and effort expended, the results also partially depend on the measuring instruments. The concentricity alignment requires the measurement of distances. The most sensitive instrument for measuring small distances is the ultra-micrometer[2] which is capable of measuring atomic distances ($\sim 10^{-10}$m). The ultra-micrometer is a capacitive transducer with the distance variable capacitor forming the capacitor part of an inductive capacitive resonant circuit which determines the frequency of an oscillator. Changes in capacitance due to displacement change of the frequency of the oscillation. The output of this oscillator is mixed with the output of another oscillator of nearly the same frequency as the variable oscillator and the variation of the difference frequency indicates displacement of the capacitive transducer. The ultra-micrometer has not attained widespread use for several reasons. The first being that it is nonlinear since the resonant frequency varies as the square root of capacitance. The second reason is that it is a relative displacement indicator rather than an absolute distance measuring tool because of the fringe fields and the slight misalignments of the capacitor. The last reason is that it is subject to long term drifts. While

2. Whiddington, R., "The Ultra-micrometer; an Application of the Thermionic Valve to the Measurement of very Small Distances", Phil. Mag., Vol. #40, Nov. 1920, pp. 634 - 639.

stability has been improved by circuit modification[3,4] still there are objectionable long term drifts. Of course none of these objections are valid for the ultra-micrometer as an alignment tool, and thus their use is recommended.

Several different ultra-micrometers are required. First, one to align the concentricity of the motor shaft rotation with respect to the rheometer frame. It is not recommended that the sample holder be mounted directly on the tapered motor shaft because of the difficulties involved in machining the motor shaft. But that an adjustable mounting fixture be attached to the motor shaft and the sample holder mounted on this fixture. Another ultra-micrometer is needed to check that the mounted sample holder is indeed concentric. A third ultra-micrometer should be mounted directly on the motor shaft to check the concentricity of the torque sensing part of the sample holder. To avoid dangling wires this ultra-micrometer should be battery powered and the difference frequency coupled through an air gap to the indicators.

The rheometer must also be aligned with the local gravitational gradient so that the bearings of the torque sensor do not have to support part of the weight of the torque sensor. The vertical alignment may be performed by an autocollimator and a pool of mercury.

3. Alexander, W., "An Electronic Ultra-micrometer", Electron. Eng., Vol. 23, Dec. 1951, pp. 479 - 480.

4. Zuckerwar, A.J., Shope, W.W., "A Solid State Converter for Measurement of Aircraft Noise and Sonic Boom", IEEE Transactions on Instrumentation and Measurement, Vol. IM-23, #1, March 1974, pp. 23 - 27.

An autocollimator is a telescope with an illuminated graticule. The illumination and the position of the graticule is such that a collimated image of the graticule is projected if a mirror is placed so that it is perpendicular to the optical axis of the telescope then the reflected image of the graticule is superimposed on the graticule. If the mirror is tilted slightly off the optical axis of the telescope then the reflected image of the graticule is shifted. The mirror in this instance is a pool of mercury which, except at the edges where the surface tension effects predominate is perpendicular to the local gravitational gradient. Thus the autocollimator placed on the guides which adjust the height of the torque transducer may be used to align the rheometer to the local gravity gradient.

Torsion Bar Calibration

A counter torque servo measures relative torque and must be periodically calibrated by standard torsion bars. Even the standard torsion bars must be periodically checked for consistency and recalibrated by a standards laboratory. The consistency of the various torsion bars may be checked with a sine-bar and a microbalance. A sine-bar consists of two cylinders spaced at a known distance apart. By placing gage blocks under one cylinder the sine-bar is tilted with the angle of tilt given by the sine of the gage block height and the separation between the bars - hence the name sine-bar. The sine-bar is used to impose a known angle of twist on the torsion bar. The other end of the torsion bar is attached to a micro-balance. From the length of the balance bar and the weights required to counter balance the applied torque the consistency among the various torsion bars may be checked. The absolute check on the calibration of the torsion bars in this manner requires that the local acceleration of gravity be accurately known.

Sample Holder Calibration

The sample holder itself must be calibrated. It is very important to calibrate the end effects. The calibration of the end effects is performed by varying the length of the sample holder. In this way the magnitude of the end effects remains constant but the portion of the torque contribution from the uniform flow region is varied. Of course several different sample holders may be manufactured for this purpose, but this turns out to be an expensive proposition (about $1,000.00 per sample holder). An alternative is to use rings which may be accurately positioned . In effect this allows different insertion depths. The use of rings also allows the depth of the top and bottom end effect to be varied.

For the calibration of the end effects the test fluid is not important. Once the end effects have been calibrated, the sample holder must be calibrated by using fluids of known viscosity. Air and water are of course ideal test fluids of known viscosity. For higher viscosities standard fluids are available. Part of the calibration procedure is to make an error analysis.

It is necessary to periodically check the dimensions of the sample holder to verify that no significant changes have occurred due to handling. For the most part the checking of the sample holder dimensions is a check of roundness. It is not expected that the sample holder will shrink or expand but it may distort. Calibration of the ultra-micrometer by imposing known displacements should be sufficient to check the sample holder.

While on the subject of the sample holder a few words should be said about the manufacture of the sample holder. The accuracy requirements place the sample holder in the class of gage blocks, not the very best grade of gage blocks but nonetheless in the gage block class. Discussions with a local gage manufacturer* have established the feasibility of assembling the sample holder from cylindrical gage blocks.

Sample holders made from gage blocks are quite expensive and some thought was given to reducing the cost. One way by which a cost reduction may be achieved is by electroforming[5]. Electroforming is an electroplating process. The difference between plating and electroforming is that in plating a good bond between the base material and the plating is desired whereas in electroforming a poor bond is desired so that the form may be removed. Ideally a very thin conductive layer that acts as a lubricant would be deposited on the tool. The layer has to be conductive so that current may pass for plating and the lubricating properties are needed to separate the electrodeposited

* Blanchette Tool & Gage Mfg. Co., 845 Bloomfield Ave., Clifton, N.J. 07012

5. Symposium on Electroforming - Applications, Uses and Properties of Electroformed Metals, Dassas, Texas, Feb. 6,7, 1962. American Society for Testing and Materials, Phila., Pa., ASTM Special Technical Publication #318.

metal from the form. The parting agent must be very thin if close dimensional tolerances are required. Electroforming is capable of reproducing great surface detail. For example, surface roughness comparitors, phonograph record masters and printing plates are manufactured by electroforming. However, electroforms tend to have poor dimensional tolerances because many metals are deposited in a state of internal stress. There are processess[6] (mostly proprietary) which produce deposits with low internal stresses, and thus are capable of greater dimensional accuracies. To investigate the feasibility of electroforming the sample holder on gage block forms, several nickel electroforms were made by Mr. R. Schmanske, an NCE undergraduate student, under the direction of Dr. J.L. Martin. The parting agent was formed by dipping the gage block for a few minutes in a sodium dichromate bath. The results were encouraging and it is recommended that this method of manufacturing sample holders be investigated further.

Rheometer Structure

Some form of the structural skeleton is needed to hold the various components of the rheometer. The important requirement is that the structure be rigid so that the relative position between the torque sensing and the rotating part of the sample holder is constant. There are two ways to achieve rigidity. One way is to use very stiff (heavy) structural members and the other way is to use lighter members

6. McCutchen, H.L., "Electroforming and Electrodeposition of Stress Free Nickel from the Sulfamate Bath". U.S. Patent 3,374,154, March 19, 1968.

and obtain the stiffness out of the geometrical shape of the structure.

How this rigid structure is achieved is not that important. An engineer being concerned with conservation of resources would be based in favor of stiffness by geometry. The quest for minimum weight could lead to visually unappealing structures and it is recommended that some thought be given to aesthetic considerations. An example of a rheometer structure based on the golden triangle[7] is shown in Figure 14. The stepping motor mounts below the platform and the sample holder are on top of the platform.

For cleaning the sample holder must be removable. Furthermore, to avoid a long alignment procedure the mount of the sample holder must be self aligning. This suggests that the sample holder mount be conical for both the rotating and torque sensing parts of the sample holder. Mounting the sample holder on cones requires that the torque sensing unit be lifted so as to allow the removal of the sample holder. This means that the torque sensing unit must also be mounted on a cone at the apex of the pyramid. A provision has to be made for positioning the torque sensing part of the sample holder for alignment purposes. Some of the adjustment may be made with the bearing, but there is a need for a mechanical adjustment in order not to load the bearings with these position adjustments. Another adjustment that is needed is an adjustment for vertical height which governs the depth to which the torque sensing part of the sample holder is inserted into

7. Huntley, H.E., "The Divine Proportion", Dover Publication, Inc., New York, N.Y., 1970, pp. 40 - 42.

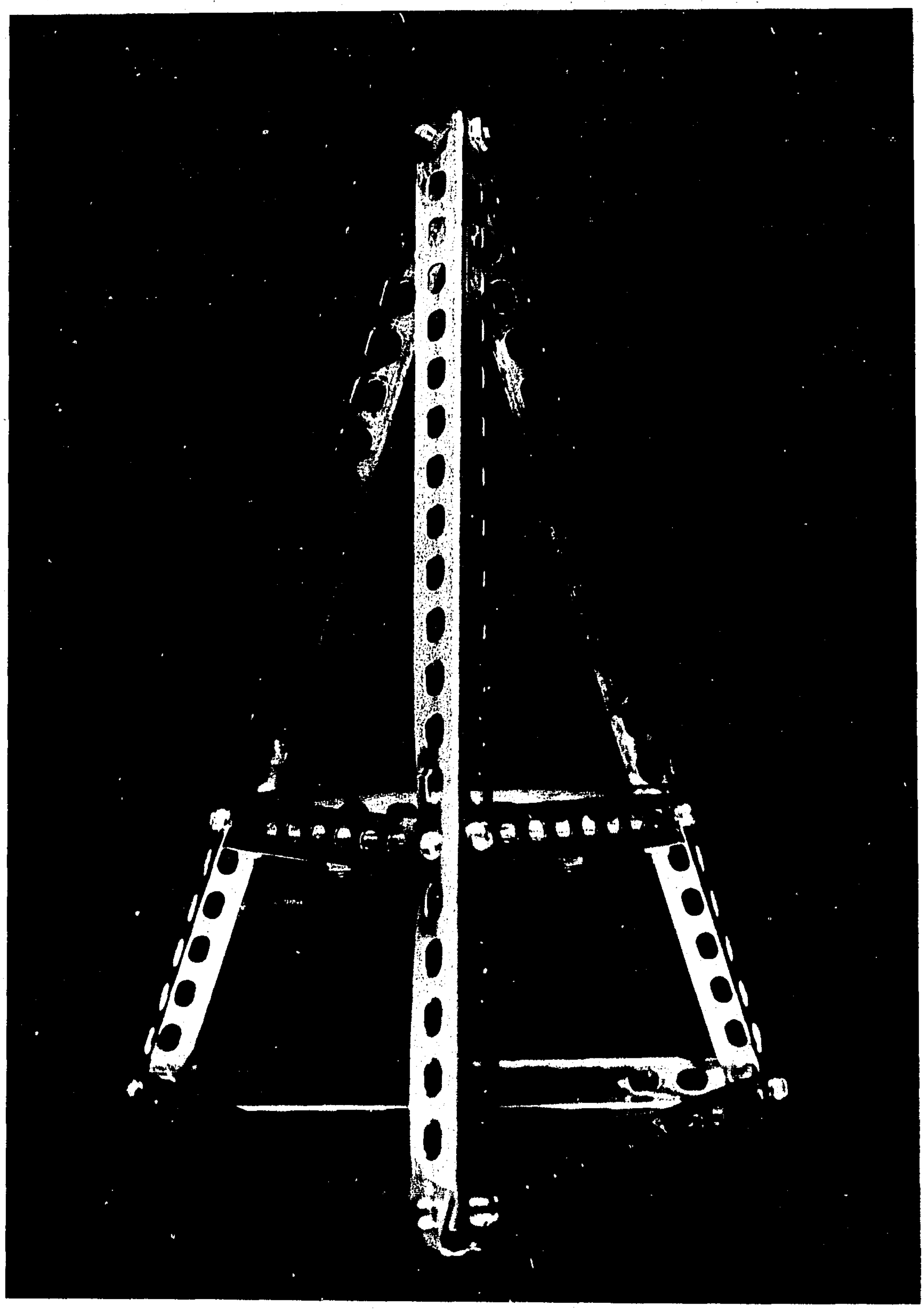


FIGURE 14 RHEOMETER STRUCTURE

the rotating part of the sample holder. These adjusting mechanisms may be mounted on the apex cone.

Rheometer Table

The rheometer is typically placed on a table for the convenience of the operator. The design of the table and the placement of the rheometer are important to the performance of the unit. The table legs and joints form springs and the rheometer forms the mass and thus we have a spring mass system with the forcing function consisting of the floor motions. Typically the spectrum of floor motions show decreasing amplitude with increasing frequency. Therefore, it is desirable to have a high resonant frequency for the rheometer table. A low mass rheometer structure is helpful in realizing this goal. Very specifically, a resonant frequency that is higher than the resonant frequency of the torque sensor is desirable. Resonant frequencies near 60 and 30 Hz should be avoided, because unbalanced 3600 and 1800 RPM electric motors generate vibrations at these frequencies.

It is important to properly position the rheometer on the table. The lateral motions of the floor may result in angular motion of the rheometer structure because the rheometer is eccentrically placed on the table. Unless the equivalent springs of the table legs are equal, the best place for the rheometer is not at the geometric center of the table. The best position may be found by using the rheometer itself as a torsional seismometer. If the resonant frequency of the torque sensor is low, then for frequencies above the resonant frequency, the torque sensor does not respond to the rheometer structure motions and angular motions of the rheometer structure appear as noise. The rheometer is positioned so as to minimize the noise. The torsional seismometer approach is also one way to check on the balance of the torsion pickup.

Sample Holder Filling

Filling the sample holder with blood is more a matter of technique rather than equipment. But, since the problem is important it should be mentioned. The problem is important because if the sample holder is not filled properly erroneous readings will result. In addition, the filling must be rapid because the time before clotting sets in is short. The blood sample may be injected with a syringe through a rubber stopper in the side of the sample holder and the blood rises on either side of the torque sensing part of the sample holder. Blood has a surface tension and the gap is narrow with the result that the filling is uneven. One technique that is of some help is to oscillate the sample holder. The amplitude is made small enough so that the oscillation does not interfere with the filling, but the motion tends to reduce the surface tension effects so that the filling is more even.

CLOSURE

The design of this rheometer was in response to the reported results that correlated the rheological properties of blood with various diseases. Much has been claimed for diagnosis of disease by testing blood, unfortunately the usefulness is in doubt because the data scatter for the published results is quite large. None of the publications surveyed contained a full and correct error analysis so there is the hope that a more accurate instrument could pinpoint disease. If on the other hand the measurement errors are small compared to the encountered differences in blood viscosities, the blood testing as now practiced may be of a very limited diagnostic value. Perhaps by special techniques, say the rheological equivalent of staining in microscopy, blood testing may become useful, but such ideas are pure speculation. The work presented here rests on the hope that increased measurement accuracy will lead to a more definite and correct diagnosis.

A discussion of error analysis is unpleasant because error implies fault and one is required not only to confess the faults but to discuss and assign numerical values to them. Uncertainty does not imply fault, but is equally unpopular because it implies ignorance. Assuming that a rheometer as outlined in the dissertation is built would greater accuracy be realized in blood testing? The honest answer is - maybe. Just because the instrument is capable of greater accuracy does not mean that the research results are automatically more accurate.

Experimental results are subjected to two somewhat correlated sources of error, namely measurement errors and non-measurement errors. Measurement errors are associated with uncertainties that exist in the measured quantities. Non-measurement errors are associated with deviations of the actual experimental conditions as they exist during the experiment from the boundary conditions assumed in the theoretical model.

In blood testing non-measurement errors may predominate. For instance, the torque and the angular velocity may be known exactly, but if the blood has formed clumps so that a plasmal layer forms, then the non-measurement error predominates because it is assumed that no such plasma layer occurs when the apparent viscosity is calculated. Thus, we can see that improvements in measurement accuracy may have little effect on the final results. As for measurement errors an awareness must be developed that random uncertainties do contain varying components of systematic errors and that systematic errors can also have a random component.

Several contributions to aid the reduction of both measurement and non-measurement errors, are claimed. The Chapter I survey of blood properties presents a more lucid account of blood properties at the cell level and their influence on the rheological properties. The effect of the sample holder geometry and the test conditions on the formation of a plasma layer both before and after clotting has been discussed and should result in the reduction of non-measurement error.

There should be an insistence that the sample holder geometry have an exact solution for Newtonian fluids and the problem of vortex formation. While this is obvious to mechanical engineers, this seems to have been ignored in blood testing. Also, the requirement that the constitutive equations be tensorially invariant is also obvious. A discussion of how the theory that best fits the experimental results could be related to a tensorially invariant theory is an important consideration as well.

The exact solution for the velocity distribution has not been pursued because the concern is more with an instrument of sufficient accuracy rather than a perfect instrument. Still, the first important steps have been taken. The Riemann function for the problem has been found and an approximate solution to one end effect has been obtained.

The design of the rheometer was undertaken in response to an important health problem and not as an end in itself. The temptation to do extensive inventing and development was firmly resisted. Only well known technologies with readily available components were used in the design. Granted, that the result is not the best rheometer that could be built given enough development effort, but the described rheometer does represent a notable advance in the state of the art.

That a technology is well known does not mean that it has been applied previously to rheometers. The art of the design is to bring together various technologies so as to create a superior instrument. The splitting of one revolution into a million parts and generating the motions digitally is novel. The ultra-micrometer is used to advantage as an alignment tool, using a long neglected instrument for its great inherent sensitivity in an application where its poor long term stability is not important. Perhaps the best example of novel exploitation of existing technology is the use of a well proportioned truss structure as the instrument stand. It is not the use of any one technology but the harmonious synthesis of different technologies that results in a superior instrument and forms the major contribution to knowledge of this work.

REFERENCES

Ahuja, A.S., Bugliarello, G., "A Note on the Compressibility of Blood", Biorheology, Vol. 7, #4, May 1971, pp. 199 - 203.

Alexander, W., "An Electronic Ultra-micrometer", Electron. Eng., Vol. 23, Dec. 1951, pp. 479 - 480.

Aroesty, J., Gross, J.F., "Pulsatile Flow in Small Blood Vessels I - Casson Theory", Biorheology, Vol. 9, #1, March 1972, pp. 33 - 43.

Armstrong, H.L., "On the Representation of Transients by Series of Orthogonal Functions", IRE Transactions on Circuit Theory, Vol. CT-6, Dec. 1959, pp. 351 - 354.

Barbee, J.H., "The Effect of Temperature on the Relative Viscosity of Human Blood", Biorheology, Vol. #10, #1, March 1973, pp. 1 - 5.

Bartter, F.D., Delea, C.S., Halberg, F., "A Map of Blood and Urinary Changes Related to Circadian Variations in Adrenal Cortical Function in Normal Subjects", Annals of the New York Academy of Sciences, 98: 1962, pp. 969 - 983.

Bitsadze, A.V., "Boundary Value Problems for Second Order Elliptic Equations", New York: John Wiley & Sons, Inc., 1968, Chapters 1 & 2.

Casson, N., "A Flow Equation for Pigment-Oil Suspensions of the Printing Ink Type", In Rheology of Disperse Systems, G.E. Mills, Editor, New York: Pergamon Press, 1959.

Chijevsky, A.L., Shishina, Y., "In the Rhythm of the Sun", Moscow: Nauka, 1969.

Chmiel, H., "New Experimental Results in Hemorheology", Biorheology, Vol. 11, #1, Jan. 1974, pp. 87 - 96.

Chrandrasekhar, S., "The Stability of Viscous Flow between Rotating Cylinders", Proc. Roy. Soc. Lond. A, Vol. 246, 1958, pp. 301 - 311.

Cokelet, G.R., Merrill, E.W., Gilliland, E.R., Shin, H., Britten, A., Wells, R.E., "Rheology of Human Blood Measurement Near and at Zero Shear Rate", Trans. Soc. Rheol., Vol. #7, 1963, pp. 303 - 317.

Copley, A.L., "On Biorheology", Biorheology, Vol. 10, #2, June 1973, pp. 87 - 105.

Copley, A.L., Huang, C.R., King, R.G., "Rheogoniometric Studies of Whole Human Blood at Shear Rates from 1000 to .0009 sec^{-1} Part I - Experimental Findings", Biorheology, Vol. 10, #1, March 1973, pp. 17 - 22.

Copley, A.L., Krchma, L.C., Whitney, M.E., "Humoral Rheology, I. Viscosity Studies and Anomalous Flow Properties of Human Blood Systems with Heparin and Other Anticoagulanta", J. gen. Physiol., Vol. 26, 1942, pp. 49 - 64.

Copley, A.L., Scott Blair, G.W., "Comparative Observations on Adherence and Consistency of Various Blood Systems in Living and Artificial Capillaries", Rheologica Acta, Vol. 1, #2, 1958, pp. 170 - 176.

Copley, A.L., Scott Blair, G.W., "The Wettability of Glass and Fibrinized Surfaces in Relation to Intravascular Blood Coagulation", Fed. Prog., Vol. 19, 1960, p. 56.

Cowin, S.C., "On the Polar Fluid as a Model for Blood Flow in Tubes", Biorheology, Vol. #9, March 1972, pp. 23 - 25.

Cranston, W.I., "Diurnal Viaiation in Plasma Volume in Normal and Hypertensive Subjects", American Heart Journal, 68: 1964, pp. 427-428.

Dintenfass, L., "Effect of Velocity Gradient on the Clotting Time of Blood and the Consistency (Rheology) of Clots (Thrombi) Formed in Vitro", (Proceedings of the XI Congress of the International Society on Blood Transfusion), Biblphie haemat, 29, Part 4, 1155, 1968.

Dintenfass, L., "Blood Microrheology - Viscosity Factors in Blood Flow, Ischaemia and Thrombosis", London: Butterworth & Co., 1971.

Dintenfass, L., Stewart, J.H., Milton, C.W., Forbes, C.D., "Influence of ABO Blood Groups and Fibrinogen on Thrombus Formation and Aggregation of Red Cells in Cardiovascular and Malignant Diseases: New Aspects of Biorheological Characterization of Disease", Biorheology, Vol. 10, #4, Dec. 1973, pp. 585 - 594.

Downing, J.J., "Modulation Systems and Noise", Englewood Cliffs, N.J., Prentice-Hall, 1964, pp. 140 - 143.

Ehrly, A.M., Jung, G., "Circadian Rhythm of Human Blood Viscosity", Biorheology, Vol. 10, #4, Dec. 1973, pp. 577 - 583.

Elspas, B., "The Theory of Autonomous Linear Sequential Networks", IRE Transactions on Circuit Theory, March 1959.

Ericksen, J.L., "Theory of Anisotropic Fluids", Trans. Soc. Rheol., Vol. 4, 1960, pp. 29 - 39.

Ericksen, J.L., "Anisotropic Fluids", Arch. Rational Mech. Anal., Vol. 4, 1960, pp. 231 - 237.

Gauquelin, M., "The Cosmic Clocks", New York: Avon, 1969.

Gauthier, F.J., Goldsmith, H.L., Mason, S.G., "Flow of Suspensions through Tubes - X Liquid Drops as Models of Erothrocytes", Biorheology, Vol. 9, #4, Dec. 1972, pp. 205 - 224.

Gilinson, P.J., Jr., Denhard, W.G., Frazier, R.H., "A Magnetic Support for Floated Inertial Instruments", Proc. Natl. Specialists Meeting on Guidance of Aerospace Vehicles, Institute of the Aeronautical Sciences, New York, N.Y., 1960, p. 222.

Gilinson, P.J., Jr., Dauwalter, C.R., Merrill, E.W., "A Rotational Viscometer Using an A.C. Torque to Balance Loop and Air Bearing", Transaction of the Society of Rheology, Vol. 7, 1963, pp. 319 - 331.

Golomb, S.W., "Shift Register Sequences", Holden-Day Inc., San Francisco, California, 1967.

Gregersen, M.I., Usami, S., Chien, S., Dellenback, R.J., Bibl. Anta., Vol. 9, 1967, p. 276.

Heyda, J.F., "A Green's Function Solution for the Case of Laminar Incompressible Flow between Non-concentric Circular Cylinders", J. Franklin Institute, Vol. 267, #1, Jan. 1959, pp. 25 - 34.

Hoare, J.P., LaBoda, M.A., "Electrochemical Machining", Scientific American, Vol. 230, #1, Jan. 1974, pp. 30 - 37.

Huang, C.R., Siskovic, N., Robertson, R.W.J., Wang, H.H., Orosz, P., Jr., "Thixotropy of Human Blood", Proceedings of the First International Congress of Biorheology, 1972, in press.

Huntley, H.E., "The Divine Proportion", Dover Publication, Inc., New York, N.Y., 1970, pp. 40 - 42.

Isogai, Y., Ilda, A., Chikatsu, I., Mochizuki, K., Abe, A., "Dynamic Viscoelasticity of Blood During Clotting in Health and Disease", Biorheology, Vol. 10, #3, Sept. 1973, pp. 411 - 424.

King, R.W., Copley, A.L., "The Karl Weissenberg 80th Birthday Celebration Essays", Edited by J. Harris, Kampala-Nairobi-Dar-es-Salam; The East African Literature Bureau, 1973.

Kline, K.A., "On a Liquid Drop Model of Blood Rheology", Biorheology, Vol. 9, #4, Dec. 1974, pp. 287 - 299.

Kline, K.A., Allen, S.J., DeSilva, C.N., Biorheology, Vol. 5, 1968, p. 111.

Kline, K.A., Allen, S.J., Keshavrzi, M., "Dissipative Effects Due to Hydrodynamic Interactions between Red Cells in a Theory of Pulse Trans-Mission and Oscillatory Flow in Arteries", Biorheology, Vol. 9, #1, March 1972, pp. 1 - 22.

Korn, G.A., "Random-Process Simulation and Measurements", McGraw-Hill, New York, N.Y., 1966.

Lessner, A., Zahavi, J., Silberberg, A., Frei, E.H., Dreyfus, F., "The Viocoelastic Properties of Whole Blood", In "Theoretical and Clinical Hemorheology", (Edited by H. Hartert and A.L. Copley), Berlin-Heidelberg, New York: Springer-Verlay, 1971.

Leverett, L., Hellums, I., Alfrey, C., Lynch, E., "Red Blood Cell Damage by Shear Stress", Biophysical Journal, Vol. 12, #3, March 1972, pp. 257 - 273.

Macfarlane, R.G., "An Enzyme Cascade in the Blood Clotting Mechanism, and its Function as a Biochemical Amplifier", Nature, Vol. 202, May 2, 1964, pp. 498 - 499.

Mahalingam, R., Poon, T.K., "Quasi-Thermodynamic Interpretation of the Behavior of Formed Elements in Blood Flow", Biorheology, Vol. 10, #3, Sept. 1973, pp. 329 - 341.

Martin, J.L., Jacobs, R.M., Copley, A.L., Final Report on Apparent Viscosity of Whole Blood Systems at Moderate Pressures, Office of Naval Research Contract N-00014-71A-0124, Department of Mechanical Engineering, Newark College of Engineering, Newark, N.J.

McCutchen, H.L., "Electroforming and Electrodeposition of Stress Free Nickel from the Sulfamate Bath", U.S. Patent 3,374,154, March 19, 1968.

Mueller, R.K., "Dynamo Transformer", U.S. Patent 2,488,734, Nov. 1949.

Oleynick, O.A., "On the Dirichlet Problem for Equations of the Elliptic Type", Matematichesky Sbornik, Vol. XXIV, 1, 1949, pp. 1 - 14 (in Russian).

Petrovsky, I.G., "Lectures on Partial Differential Equations", Interscience Publishers Inc., New York: 1954, pp. 231 - 233.

Prandtl, L.,"Abriß der Strömungslehre", Braunschweig, 1935, Second Edition, p. 100.

Lord Rayleigh, "On the Dynamics of Revolving Fluids", Scientific Papers, Cambridge, Mass., Cambridge University Press, 1916.

Riemann, B., "Ueber Die Fortpflanzung Ebener Luftwellen Von Endlicher Schwingunsweite" found in "Collected Works of Bernhard Riemann" Edited by H. Weber, New York, N.Y., Dover Publications Inc., 1953, (in German)

Schlichting, H., "Boundary Layer Theory", New York: McGraw-Hill, Fourth Edition, 1960, pp. 66 - 93.

Schmid, H., "An Operational Hybrid Computing System Provides Analog Computation with Digital Elements", IEEE Transactions on Electronic Computers, VEC-12, #6, Dec. 1963, pp. 715 - 732.

Schmid-Schönbein, H., Gaehtgens, P., Hirsch, H., "On the Shear Rate Dependence of Red Cell Aggregation in Vitro", The Journal of Clinical Investigation, Vol. 47, 1968, pp. 1447 - 1454.

Schmid-Schönbein, H., Wells, R., Schildkraut, R., "Microscopy and Viscometry of Blood Flowing Under Uniform Shear Rate (Rheoscopy)", J. Appl. Physiol., Vol. 26, 1969, p. 674.

Schmid-Schönbein, H., Gosen, J.V., Klose, H.J., "Comparitive Microrheology of Blood; Effect of Desaggregation and Cell Fluidity on Shear Thining of Human and Bovine Blood", Biorheology, Vol. 10, #4, Dec. 1973, pp. 545 - 551.

Sinha, N.K., Elliot, A.R., Wong, R.C.S., "A Realistic Model for Permanent Magnet Stepping Motors", IEEE Transactions on Industrial Electronics and Control Instrumentation, Vol. IECI-21, #3, Aug. 1974.

Symposium on Electroforming - Applications, Uses and Properties of Electroformed Metals, Dassa, Texas, Feb. 6,7, 1962. American Society for Testing and Materials, Phila., Pa., ASTM Special Technical Publication #318.

Taylor, G.I., "Stability of a Viscous Liquid between two Rotating Cylinders", Phil. Trans. A, Vol. 223, 1923, pp. 289 - 343.

Valantis, K.C., Sun, C.T., Biorheology, Vol. 6, 1969, p. 85.

Vekua, I.N., "New Methods for Solving Elliptic Equations", New York: John Wiley & Sons, Inc., 1967, Cahpter I.

Whiddington, R., "The Ultra-micrometer; an Application of the Thermionic Valve to the Measurement of very Small Distances", Phil. Mag., Vol. #40, Nov. 1920, pp. 634 - 639.

Whittaker, E.T., Watson, G.N., "A Course of Modern Analysis", New York: Macmillan Co., 1948, Chapter 15.

Williams, A.R., "Shear Induced Fragmentation of Human Erythrocytes", Biorheology, Vol. 10, #3, Sept. 1973, pp. 303 - 311.

Yoshimuta, I., "Seasonal Changes in Human Body Fluids", Japanese Journal of Physiology, 8: 1958, pp. 165 - 179.

Yu, Y., Sun, D., "Stability of a Viscous Flow between two Rotating Coaxial Cylinders", J. Franklin Inst., Vol. 277, #2, Feb. 1964, pp. 140 - 149.

Zuckerwar, A.J., Shope, W.W., "A Solid State Converter for Measurement of Aircraft Noise and Sonic Boom", IEEE Transactions on Instrumentation and Measurement, Vol. IM-23, #1, March 1974, pp. 23 - 27.

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- 1966 - 1971 Monsanto Co., W. Caldwell, N.J., Technical specialist. Digital frequency synthesizer development. Assistant to engineering manager.
- 1964 - 1966 Fairchild, Clifton, N.J., Senior engineer. Development of 3 GHz sampling oscilloscope.
- 1962 - 1964 General Atronics, Philadelphia, Pa., Staff engineer, Design and development of special purpose oscilllo-scopes.
- 1960 - 1962 Waterman Products, Philadelphia, Pa., Design engineer. Oscilloscope design.