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Multiyear drought simulation with periodic-stochastic hydrologic processes

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MULTIYEAR DROUGHT SIMULATION

WITH

PERIODIC-STOCHASTIC HYDROLOGIC PROCESSES

by

Lankeswara Hemal Wijayaratne

Dissertation submitted to the Faculty of the Graduate School of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of
Doctor of Engineering Science
May 1988

APPROVAL OF DISSERTATION

MULTIYEAR DROUGHT SIMULATION
WITH
PERIODIC-STOCHASTIC HYDROLOGIC PROCESSES

BY

LANKESWARA HEMAL WIJAYARATNE

FOR

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

BY

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May 1988

ABSTRACT

Previous studies on multiyear droughts are often limited to the analysis of historic annual flow series. A major problem in these studies is the unavailability of long historic flow records, on which to perform the analysis. To overcome this difficulty, the present study has used synthetically generated mean annual flow series to suppliment the historic flows. For the purpose of generating flows, a general methodology was developed to propose a mathematical model based on the harmonic and stochastic analyses of the historic flow series. The main objective was to derive a large population of multiyear drought events from the generated flow series, and to utilize this population for the simualtion of statistical & stochastic behaviour of the drought parameters.

The methodology was applied to a study area which includes six watersheds in the northern part of New Jersey and one watershed in the central New Jersey area. Analyses reveal that the mean annual flow series recorded at each selected streamflow gaging station, represents a periodic-stochastic process. The best model was determined for each stream, and used to generate a long annual flow series. Multiyear point droughts were identified by analyzing both the historic and generated flow series at a fixed truncation level. Four important drought parameters, namely, the duration, severity, magnitude and the time of occurrence

were determined for each stream. The statistical properties of each of these parameters were then evaluated. It was found that the generated drought events closely follow the same statistical behaviour as the historic drought events. Based on the statistical properties, classical probability distributions such as gamma and log normal were fitted to the generated drought parameters. The applicability of these distribution functions to predict the extreme drought events have been illustrated with examples. In addition, the cross-correlation structure of the time of occurrence parameter of droughts has been identified with regard to spatial distribution of the point droughts over the study area.

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DEDICATION

The author wishes to dedicate this research effort to his parents, who brought him up, and to the motherland, Sri Lanka, which gave the free education to make him a man.

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LIST OF ABBREVIATIONS AND SYMBOLS

Autocorr.	:	Autocorrelation Coefficient
C	:	Centigrade
cfs	:	Cubic feet per second
cfs.yrs	:	Cubic feet per second years
Coef.	:	Coefficient
exp	:	Exponential
Fig.	:	Figure
ln	:	Natural logarithm
mi	:	Miles
mm	:	Milli meters
N. Br./No. Br.	:	North Branch
rad/yr	:	Radians per year
Skew.	:	Skewness
Std. Dev.	:	Standard Deviation
temp.	:	Temperature
USGS	:	United States Geological Survey
Vs.	:	Versus
yrs	:	Years

CHAPTER I. INTRODUCTION

1.1 General

The concept of a drought has been defined in different ways by researchers and scientists involved in different fields of study who analyze such phenomena. For example, the agriculturist views a drought as a period during which the soil moisture is insufficient to support crops. The geophysicist's view of drought may be climatical, general meteorological, hydrological or concerned with aspects of soil physics. The economist considers a drought as a period of low water supply which affects most socio-economic activities. The hydrologist however is concerned with drought in the context of a period of insufficient rainfall and low streamflow with resulting depleted reservoir storage.

The extent of the impact of a particular drought depends on the nature of the drought and the users conditions of operation. For example, from the view point of cultivation, the impact of a prolonged mild drought is worse than the impact due to a short-term severe drought. With regard to hydro power generation, the effect is the opposite; a prolonged less severe drought has little impact compared to the effect due to a short-term severe drought. The climate of a region is another important factor to be considered in drought definition. In some regions, a period of a few days without rain may cause a drought such as Bali; whereas in another region such as Libya, a drought may only be recognized after a number of years.

1.2 Hydrologic Drought

The term hydrologic drought is defined as a deficiency in water supply on the earths surface, or the deficiency in precipitation, effective precipitation, runoff or in accumulated water in various storage capacities (Yevjevich, 1967). Therefore, for an objective analysis of hydrologic droughts, one or more of the following hydrologic phenomena may be used.

(a) Precipitation at ground level,

(b) Evaporation from the ground, from bodies of water, through plants, etc.,

(c) Effective precipitation in the form of precipitation minus evaporation and infiltration,

(d) streamflow, or

(e) water stored in various natural or artificial storage spaces of all kinds.

Once a hydrologic phenomenon is selected, the variable or variables which describe the phenomenon must be determined, such as whether to use a point measurement or a total area value, whether intensities(or discharges) in the form of continuous series are used or discrete values in the form of daily, monthly, annual, or any other time interval values, or whether levels, stored water or similar variables are chosen. The selections must be made to meet the purpose of the study.

If point measurements such as data recorded at a precipitation gage or a streamflow gage are used, the droughts defined are known as **point droughts.** Usually, regional drought aspects are studied by generalizing the point drought characteristics at a number of locations in the study area.

1.3 Multiyear Hydrologic Drought

When developing or redeveloping a water resources system, considerations should be given to accommodate both the short-term and long-term impacts of the hydrologic cycle in the planning phase. Design parameters must be specified to withstand the extreme events such as floods and droughts. With regard to floods, the literature provides a large number of studies. Applicability of these studies is well documented. With regard to droughts, few studies are available. Among those, the majority deals with historic short-term droughts in specific regions. Therefore, these studies and the analytical methodologies presented have limitations when used in a sophisticated planning study. If one wants the operation of a water resources system to account for possible future water deficits, one needs a well established knowledge of the phenomenon of prolonged severe droughts which may last for a period longer than a year for that system. Such drought events can be recognized by examining historic streamflow sequences. If annual flow series are employed for this purpose, the term 'multiyear drought' is used to define a period of below normal streamflow lasting one year or more. This kind of event reflects the non-stationary nature of the hydrologic cycle. (The term 'non-stationary' is used to explain a time dependent process, in which the statistical properties are functions of time). Possible causes for this phenomenon are the changes of the weathers cyclic pattern accompanied by concomitant changes in watershed geomorphology.

1.4 Significance of the Present Study

According to the literature, few studies are available on the aspects of multiyear droughts. These studies are mostly limited to the analysis of historic streamflow series. As the mean annual flow series are used to identify and derive the mutiyear drought events, a basic disadvantage of such studies is the unavailability of extensive flow series, from which one may obtain samples of many droughts. The small number of drought events from a particular flow series, when subjected to statistical analyses in order to predict future occurrences, produces results which are not very reliable. This situation is mainly due to the large sampling errors of the estimated statistical moments.

The present study is undertaken to carry out a drought simulation, with the objective of analyzing large samples of drought events. This is an excellent method for obtaining reliable results for accurate prediction of drought behaviour. The advantage here is that when a large number of drought events are available, they can be subjected to statistical analyses by treating them as discrete time processes. What is important is to define the basic parameters which quantify the effect of a particular drought event.

If one can present a methodology which directly generates drought events or drought parameters, this methodology would be a useful tool. However, existing research studies related to this topic reveal that this is an impossible task, mainly because the drought events are independently distributed over time (Lee et.al., 1981). With this understanding, the drought simulation in the present study will be accomplished by analyzing a synthetically generated mean annual flow series. This approach believed to be more acceptable towards a detailed drought investigation, as a time dependent flow series can be generated by utilizing a pre-established mathematical model for that purpose.

Another advantage of a drought simulation study is that when a number of adjacent watersheds are used for the study, the determined point drought characteristics for the different watersheds can be generalized to investigate regional implications.

1.5 Proposed Approach

The present study proposes to use both historic and generated mean annual flow series for multiyear drought analysis. For the generation of flows, a general methodology will be developed which proposes a mathematical model which can simulate the pattern of historic flow series.

Since uncontrolled (natural) annual flow series are known to be periodically distributed over time, the proposed model for each watercourse is based on harmonic analysis and the theory of stochastic processes. The concept behind this modeling technique is that the hydrologic process is cyclic in nature and therefore, it is possible to break each annual flow event into a periodic component plus a residual component. This residual can be treated as the stochastic component and depending on the nature of its distribution, a suitable stochastic model can be suggested for its generation. Once the entire model is established for a particular flow series, it may be used to generate a flow series of desired length.

Identification and derivation of drought and surplus events will be accomplished by analyzing both historic and generated flow series by application of the techniques of time series analysis. For this purpose, low flow and high flow events are identified by defining a feasible truncation level. Flows greater than the truncation level are high flows while those smaller than the truncation level are low flows. Thus, a set of consecutive low flow events can be defined as a drought and a set of consecutive high flow events can be considered as a surplus. (In a precise mannar, they must be defined as a 'point drought' and a 'point surplus' respectively). Therefore, the two types of events resulting from a particular flow series form two other discrete time processes. Analysis of these derived random processes, especially the drought event series, is the most important part of the study. In the initial phase of such an analysis, the important parameters which quantify the effect of a particular drought (drought parameters) must first be defined. Then, the temporal distributions of these parameters can be investigated with application of statistical methods and probability theory. The final objective is to derive a theoretical distribution function which can explain the temporal variation pattern of each drought parameter. If this target can be achieved, it will provide a valuable tool to predict extreme drought events. For this purpose, therefore, a large population of droughts derivable from a generated flow series may be conveniently used. Moreover, the regional drought aspects may be studied by investigating the cross-correlation properties of the point drought characteristics over the study area.

1.5.1 Basic Considerations

In any time series analysis, the **Averaging Period** and the **Truncation Level** can be considered as the two fundamental parameters to be first determined. The averaging period is an indicator of the type of flow series to be chosen; the truncation level is used to distinguish the droughts from the surplus events and therefore, must be selected by the analyst depending upon the purpose of study.

Sample Averaging Period

This study deals with streamflows, which is known as a continuous hydrologic process. Usually, in time series analysis, a continuous process is treated as a discrete time sequence by means of a time averaging procedure.

Since the present study deals with multiyear droughts, it is appropriate to select annual flows with a sample averaging period of one year. Therefore, mean annual flows recorded at several gaging stations belonging to different basins in the study area will be used.

Truncation Level (X_C)

This is the level which distinguishes low flow events (droughts) from high flow events (surpluses). Selection of the truncation level is the initial step leading to a 'run' analysis. The term 'run' is used to define a sequence of observations of the same kind (low flow or high flow) preceeded and succeeded by one or more observations of a different kind (high flow or low flow). In general, the selection is not arbitrary, but rather it is a function of the type of water deficits being studied. It may be a constant, a stochastic variable, a deterministic function or any combination of the above.

In general, the truncation level is chosen to be some measure of the central tendancy. Statistics which commonly measure the central tendancy of a sample are the mean, the median and the mode. Each of these parameters measures the relative deviations of the observations in a specified fashion.

In fact, the selection of the truncation level must be based on the nature of the hydrologic process. If it represents a pure stochastic process, selecting the long-term mean as the truncation level is justifiable. When the process indicates a trend or a periodic pattern, as observed in an annual flow series, the selection of the truncation level should be based on some logic established by a sensitivity analysis of the process. In this regard, each mean annual flow series can be analyzed for several truncation levels, by revising the long-term mean (original truncation level) up and down at 5% intervals within a reasonable range. The results corresponding to different truncation levels also provides the analyst some useful insights into the process.

1.5.2 Drought and Surplus Parameters

Application of the theory of runs (Yevjevich, 1967) to a mean annual flow series by incorporating the truncation level introduces multiyear drought and surplus parameters. The fundamental parameters are shown in Fig.1.1. In mathematical terms, the events, droughts and surpluses, are referred to as negative runs and positive runs, respectively. Accordingly, the parameters, run length (time between successive intersections of X_c), run sum (cumulative deviation from X_c) and run intensity (average deviation from X_c) are defined. In more common drought terminology, the negative run length is termed **drought duration** (L_d), the negative run sum is termed **drought severity** (S_d) and the negative run intensity is called **drought magnitude** (M_d).

These three parameters are the fundamental descriptors of the drought events and are related by the expression, $S_d = M_d.L_d$; The duration and severity are usually considered to be the two most important primary parameters and therefore, a detailed analysis of the variation of the magnitude will not be conducted in the present study. Also, another important parameter, named the **time of occurrence of a drought** (τ_d) is defined. A brief description of the drought and surplus parameters is given below.

<u>Drought</u> <u>Duration</u> (L_d): the number of consecutive years, for which the mean annual flow is below the truncation level. It also measures the duration between a downcrossing and the successive upcrossing of the truncation level.

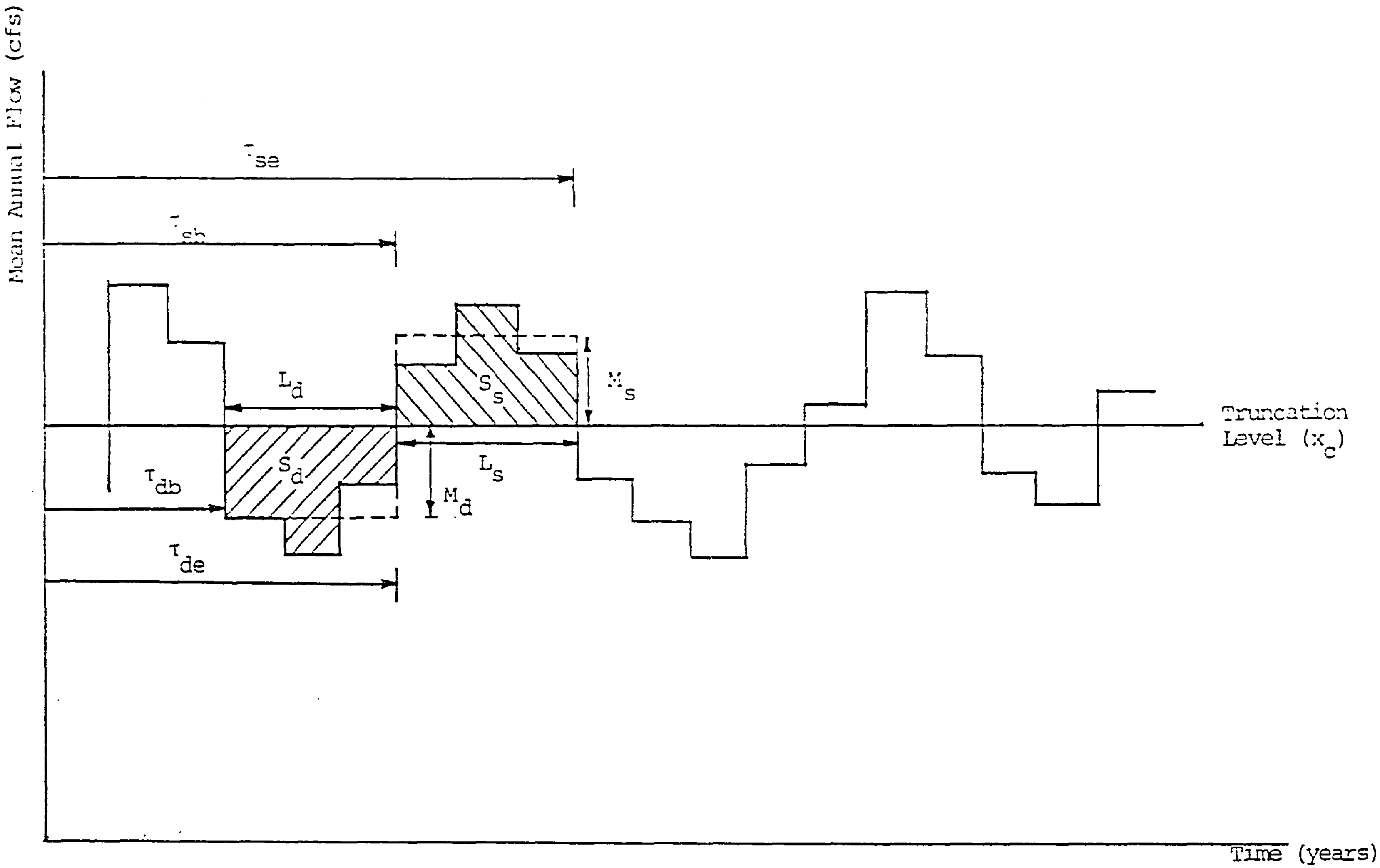


Fig. 1.1 Definition of Drought and Surplus Parameters Derived from a Mean Annual Flow Series

<u>Drought Severity</u> (S_d): the cumulative deficit (sum of the annual deficits) of stremflow for the drought duration considered. It is given by the area between the flow curve and the truncation level corresponding to the drought duration.

<u>Drought Magnitude</u> (M_d): the drought intensity expressed as the deficit per unit duration.

$$M_d = S_d/L_d$$

<u>Time of Occurrence of a Drought</u> (τ_d): Before defining this parameter, it is necessary to define two other parameters, namely, the time of the begining of the drought (τ_{db}) and the time of the ending of the drought (τ_{de}).

Then, τ_d is defined as:

$$\tau_d = 1/2(\tau_{db} + \tau_{de})$$

<u>Surplus Duration</u>(L_s): the number of consecutive years, during which the mean annual flow is above the truncation level. It also measures the duration between an upcrossing and the successive downcrossing of the truncation level.

<u>Surplus Severity</u> (S_s): the cumulative excess of streamflow for the surplus duration considered. This is given by the area between the flow curve and the truncation level corresponding to the surplus duration.

<u>Surplus Magnitude</u> (M_s): the intensity of a surplus event expressed as the surplus per unit duration.

$$M_s = S_s/L_s$$

<u>Time of Occurrence of a Surplus</u> (τ_s):

Denoting the time of the begining of the surplus by τ_{sb} and the time of the ending of the surplus by τ_{se},

τ_s is defined as:

$$\tau_s = 1/2(\tau_{sb} + \tau_{se})$$

CHAPTER II. OBJECTIVES AND STUDY AREA

2.1 Scope of the Study

In this thesis, a detailed study will be performed on point droughts in the study area selected, which will be identified on a multiyear basis by analyzing the mean annual flow series. These series must represent the uncontrolled natural flows of the streams involved. This means that the flows should be free from upstream regulation or diversion.

With regard to regionalization, no detailed study will be conducted other than evaluating the cross-correlation structure of the point drought parameters and presenting some drought parameters in the standardized form. In fact, a detailed study on regionalization has to be conducted under a different topic by considering both the basin geomorphology and the spatial variation of hydrology, which is beyond the scope of this thesis.

2.2 Objectives of the Study

The main objectives of the present study can be outlined as:

(1) Develop a methodology to determine the underlying mechanism of the hydrologic process of the historic mean annual flow series.

Based on the statistical properties, autocorrelation function and the spectral density function estimated for a particular historic flow series, the methodology would determine the mechanisms of the hydrologic process.

(2) To model each historic flow series.

An uncontrolled annual flow series will normally follow a periodic pattern due to the meteorologic cycle. Therefore, it would be possible to model the flow series by performing harmonic analyses and applying the theory of stochastic processes.

(3) To analyse the historic mean annual flow series for the multiyear drought and surplus events at selected truncation levels.

Then, to conduct statistical analyses on the derived drought and surplus parameters for different truncation levels. The purpose here is to carry out a sensitivity analysis of the drought parameters to fix one particular truncation level for each flow series. This truncation level must meet the criterion of balancing deficit and surplus durations in a feasible mannar.

(4) To generate a longer (than historic) flow series by application of the established model in step 2.

Having generated a sufficiently long synthetic annual flow series for each watershed, this series can be analyzed for its fixed truncation level (step 3) to obtain a large number of drought and surplus events. Before this analysis, it must be verified that the generated series statistically matches the historic series.

(5) To study the generated drought parameters in detail and to propose suitable distribution functions for their

probabilistic variations.

By use of these probability distributions, it will be possible to forecast extreme drought events for given return periods.

(6) To generalize the derived point drought characteristics with the objective of expressing their spatial distribution pattern.

The cross-correlation structure of the point drought parameters would be very useful for a study of regional droughts over the study area.

2.1 Study Area

The northern part of the state of New Jersey is chosen as the study area. This area is very densely populated and therefore, the existing water resources are of great economic importance. Among the variety of water demands, consumption, agriculture, recreation and navigation are assigned the highest priorities. Out of the total consumption in Northern New Jersey, almost 55% is being met by surface water regulation. However, as a result of continuous urbanization and increased land use, the hydrology in this area has been significantly changed during the past two decades. As the farms and woodlands are being replaced by streets, housing developments, apartment complexes and shopping centers, the the base flow and rainfall-runoff response of streams in the region has changed. The water demand has also increased with the growth of industries. Therefore, a detailed investigation of possible future droughts in the area is appropriate. An accurate knowledge of such events provides a useful tool to the water resources planner.

For the present study, seven streams were selected. Table 2.1 gives a listing of these streams, the number of available years of record and their drainage areas. Six of the streams are located in three adjacent river basins, namely, the Delaware, the Passaic and the Raritan. For example, the Pequest and the Musconetcong Rivers are in the Delaware basin; the Rockaway and the Passaic Rivers are in the Passaic basin; the Lamington and the North Branch Raritan Rivers are located in the Raritan basin. The other stream, the Manasquan River, belongs to the Manasquan river basin and is located in the Central part of New Jersey (see Fig. 2.1).

The reasons for this selection are twofold; (1) a study of adjacent river basins indicates how the hydrology is spatially distributed within the same time period. If the different basins indicate similar characteristics, it leads to a conclusion of hydrologic homogeneity within the region. On the other hand, if their behaviours are found to be different, that situation can be used to advantage in devising a water resources operational scheme to account for potential unfavorable impacts in one of the basins. (2)Since the Manasquan River is located in the central part of New

Jersey, it is possible to gain some insights as to how the hydrology varies from the northern part to the central part of the state.

Table 2.1 New Jersey Streams Used for the Study

Stream	Gage Location	USGS Station Number	Drainage Area(mi^2)	Period of Record
(1)Pequest River	Pequest	1445500	108.0	1926-1985
(2)Musconetcong River	Bloomsbury	1457000	143.0	1926-1985
(3)Rockaway River	Boonton	1380500	116.0	1938-1986
(4)Passaic River	Millington	1379000	55.4	1922-1986
(5)Lamington River	Pottersville	1399500	32.8	1922-1985
(6)N.Br. Raritan River	Raritan	1400000	190.0	1924-1985
(7)Manasquan River	Squankum	1408000	43.4	1932-1986

When one compares the sizes of the watersheds represented by the different gaging stations considered, the largest drainage area is represented by the gage near Raritan along the North Branch Raritan River. The smallest drainage area is represented by the gage at Pottersville along the Lamington River. In fact, the Lamington river basin is located within the North Branch Raritan river basin. Therefore, the Lamington River flows contribute to the flows observed at the Raritan gaging station along the North Branch Raritan River. The available longest record of

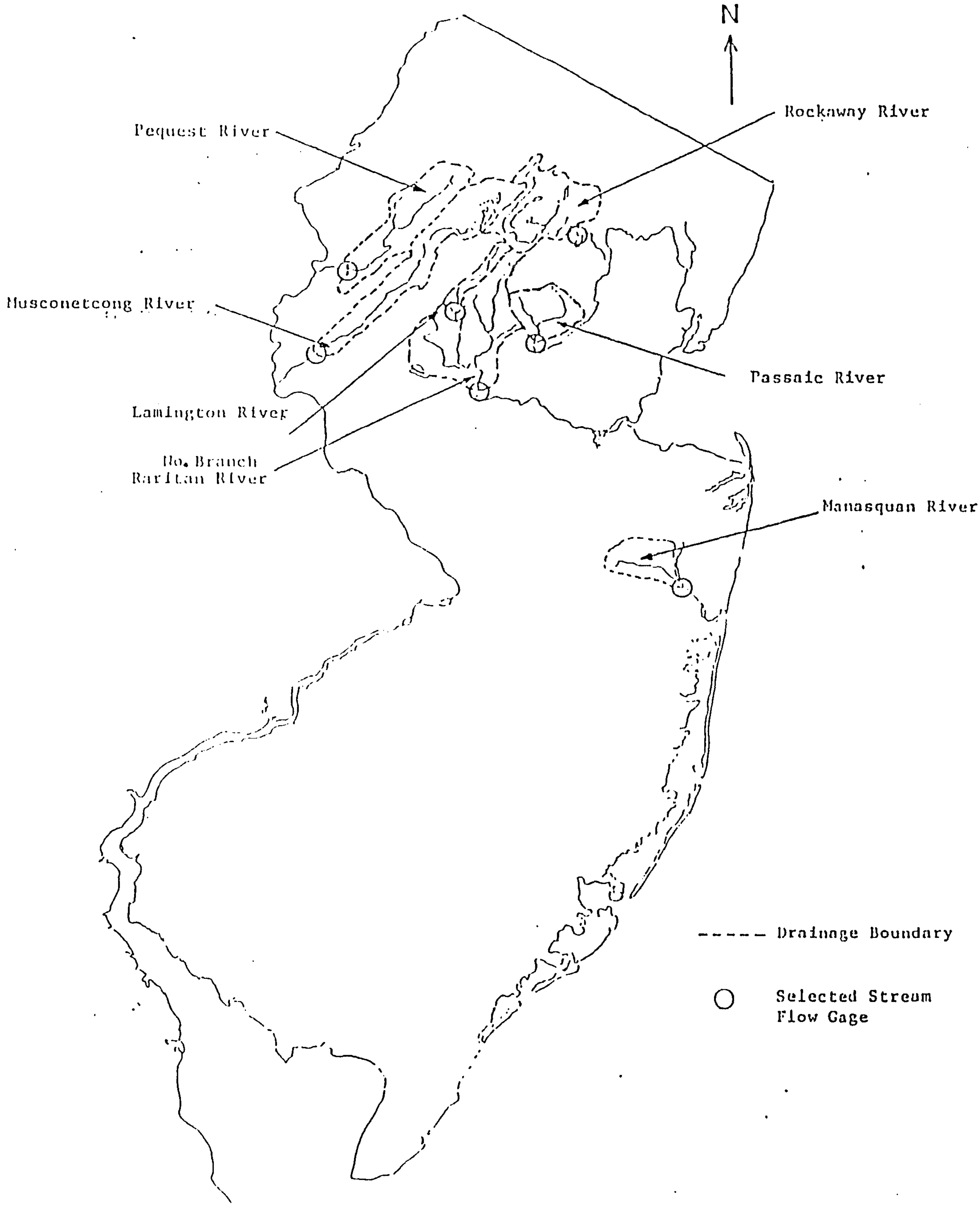


Fig. 2.1 Locations of Study Drainage Areas in New Jersey

historic mean annual flows in New Jersey has been recorded since the year 1898 and belongs to the gage at Little Falls along the Passaic River. Due to the highly non-stationary characteristic of this flow series, it was not considered in the drought study. The longest records used for the study have been recorded from 1922 and these are available for the gages at Millington and Pottersville.

The selected gaging stations measure uncontrolled streamflows and the records are available from the Division of Water Resources, U.S. Geological Survey (USGS) of New Jersey. Of the selected records, the longest is 64 years while the shortest is 49 years duration. This indicates the necessity of generating longer flow series for detailed analyses and therefore supports the main objective of this thesis.

CHAPTER III. LITERATURE REVIEW

3.1 General

Among the various aspects of hydrology, to date little attention has been given to the quantitative and qualitative analysis of droughts. Many of the existing drought studies refer to specific basins or particular historical droughts. Most of these works are limited to the statistical interpretation of the observed variation patterns of rainfall or streamflow series over time and space. Few attempts have been made to incorporate such studies with the operational and managerial aspects of water resources.

The earliest papers on droughts are concerned with drought indicators or indices which are variables and mostly measure the severity of the drought (A chronological list of drought indices is shown in Appendix A). Out of these indicators, the one usually known as the Palmer Index (Palmer, 1965) deserves special mention, because it allows the assessment of the begining and the ending of drought periods and at the same time, gives a formula for determining the severity of the drought. This index is usually a function of a number of variables, namely, the observed monthly precipitation, the monthly potential evaporation, the soil moisture recharge and the net loss of soil moisture during the month.

The second category of research is concerned with drought frequency analysis. One of the earliest papers belonging to this category has been presented by Whipple (1966) which gives a distribution function for the drought durations based on runoff deficiencies. He assumed that the drought event series of different streams are independently and identically distributed and has applied the station year method.

Historically, one of the principal obstacles to an effective investigation of droughts was the lack of an objective definition. Yevjevich, in 1967, clearly stated this problem, and was the first author to give a precise definition of a hydrologic point drought. In his paper, he presents various concepts of droughts and tries to characterize hydrologic droughts by means of the "run theory". Yevjevich, also considers the problem of prediction of large continental droughts. This can be thought of as a first work leading to a number of later studies which made use of run theory to study point and regional droughts. **Run theory** uses a truncation level to distinguish high flows and low flows. A set of cosequent low flows below this truncation level can be considered as a drought event. Then, the drought parameters such as **duration, severity** and **magnitude**(intensity) can be defined enabling the analyst to perform detailed analyses of the drought characteristics.

In the studies on stochastic point drought characterization succeeding Yevjevich, two different approaches have been followed: (1)the integration approach, which refers to runs of an infinite population, and (2)the combinatorial approach, which studies the runs in samples of a given size. Among those who follow the first approach, Downer, Siddiqui and Yevjevich (1967), Llamas and Siddiqui (1969) and, Saldarriaga and Yevjevich (1970) deserve mention. Others, such as Millan and Yevjevich (1971) and Millan (1972) followed the second approach. A third group of researchers, namely, Guerrero-Salazar and Yevjevich (1975), and Sen (1976,1980) studied point droughts from both view points.

Before giving a description of work done by each researcher, it is convenient to define the various drought parameters by mathematical expressions. The following notations are used in this chapter, otherwise they will be specified in each particular case.

(1) the magnitude of the hydrologic variable observed at time t, is denoted by X_t ,

(2)the j^{th} **negative run length** (drought duration) is denoted by L_j^- while the j^{th} **positive run length** (surplus duration) is denoted by L_j^+,

(3)the j^{th} **negative run sum** (drought severity) and the j^{th} **positive run sum** (surplus severity) are respectively denoted by S_j^- and S_j^+,

(4)the j^{th} **negative run intensity** (drought magnitude) and the j^{th} positive run intensity (surplus magnitude) are respectively denoted by I_j^- and I_j^+,

where, $I_j^- = S_j^-/L_j^-$ and, (3.1)

$I_j^+ = S_j^+/L_j^+$

Whenever these random variables are independent and identically distributed (i.i.d), the index j can be dropped. When the critical level (truncation level) is a constant, it will be denoted by X_c. If it is a quantile of order p of some distribution function (d.f.) F, it will be such that $p = F(X_c)$, and $q = 1-p = 1-F(X_c)$.

3.2 Studies on Short-term Droughts

Downner, Siddiqui and Yevjevich (1967) studied the distributions of positive and negative run lengths when X_t is a sequence of i.i.d. normal or log normal random variables. The authors have shown that, under these assumptions, the vectors (L_j^+, S_j^+) and (L_j^-, S_j^-) are also i.i.d. for all j. They obtained the parameters of L^+ and S^+ by means of the cumulant generating function of the vector (L^+, S^+). Expected values, variances and covariances for these parameters are defined as:

$E(L^+)=1/p$, $Var(L^+)=q/p$ (3.2)

$E(S^+) = 1/p(k_1{}^*-X_c)$, $Var(S^+) = q/p^2(k_1{}^*-X_c{}^2)+1/p.k_2{}^*$ (3.3)

$Cov(L^+, S^+)=q/p^2(k_1{}^*-X_c)$ (3.4)

where $k_1{}^*$ and $k_2{}^*$ are the first and second cumulants of the d.f. of the truncated variables $X^* = X|(X>X_c)$.

The corresponding parameters for L^- and S^- are easily

derived from the truncated variable $X^* = X|(X \leqslant X_c)$.
The subscript j has been dropped under the assumption of i.i.d.

Next, they generated standard normal and log normal pseudo-random numbers, estimated the above parameters and found that, in both cases, the relative errors in the estimates (comparing the analytical and empirical results) are less than ten percent.

Two years later, Llamas and Siddiqui (1969) published a paper on point drought characterization using the run theory. In this paper, they present the analytical moments and distributions of three random variables, namely, run length, run sum and run intensity under two alternative assumptions on the discrete stochastic process of X_t.
The assumptions are:
(i) X_t forms a sequence of independent and identical random variables with a common continuous distribution function,
(ii) when X_t represents a time discrete process, it is no longer independent, but its dependence is such that the sequence of random variable,

$$Z_t = \begin{cases} 1\ , & X_t > X_c \\ 0\ , & X_t \leqslant X_c \end{cases}$$

forms a two-state Markov chain with the following transition probability matrix.

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} \end{array} \qquad \text{for} \quad 0<\alpha<1 \text{ and, } 0<\beta<1$$

Under the first assumption, they found the following relationships for the three random variables considered.

$$E(L^-)=1/q, \qquad Var(L^-)=p/q^2 \tag{3.5}$$

$$E(S^-)=E(X_1^*)/q, \quad Var(S^-)=\{q.Var(X_1^*)+p.[E(X_1^*)]^2\}/q^2 \tag{3.6}$$

$$Cov(L^-,S^-)=p/q^2.E(X_1^*)$$

$$Corr(L^-,S^-)=p.E(X_1^*)/\sqrt{p.q.Var(X_1^*)+p^2.[E(X_1^*)]^2} \tag{3.7}$$

$$E(I^-)=E(X_1^*), \qquad Var(I^-)=q/p.Var(X_1^*).\ln(1/q) \tag{3.8}$$

where, X_1^* stands for the truncated random variable,

$$X_1^* = (X_c - X) \mid (X \leqslant X_c).$$

When compared with the study of Downer, Siddiqui and Yevjevich (1967), it can be concluded that equations (3.5) and (3.6) are equivalent to (3.2) and (3.3), except the difference of positive run and negative run parameters. Llamas and Siddiqui, also showed that under assumption (i), both L^- and L^+ are geometrically distributed as follw:

$$P(L^-=k)=q.p^{k-1} \tag{3.9}$$

$$P(L^+=k)=p.q^{k-1} \tag{3.10}$$

In addition, they derived approximations to the d.f. of the run sums and run intensities when X_t is normally distributed, and an approximation to the d.f. of S^- when the random variable is gamma distributed.

Under assumption (ii), the authors conclude that for $X_c = 0$, the model outlined above is equivalent to the independent sequence model except that $q = \alpha$ and $p = 1-\alpha$ for (L^-, S^-, I^-) and the same (p,q) will not apply for (L^+, S^+, I^+) unless $\beta = 1-\alpha$, which is the independent case.

Finally, the authors generalized the theory developed to the case of two mutually independent processes X_n and Y_n with a joint distribution function $F_{xy}(x,y)$ for n=1,2,... Given two critical levels c_1 and c_2 such that $0 < F_{xy}(c_1, c_2) < 1$, for possible events are considered.

$A_n = (X_n \leqslant c_1,\ Y_n \leqslant c_2)$; $B_n = (X_n \leqslant c_1,\ Y_n > c_2)$

$C_n = (X_n > c_1,\ Y_n \leqslant c_2)$; $D_n = (X_n > c_1,\ Y_n > c_2)$

Any sequence of **k** number of consecutive **A**'s followed and preceeded by any other event is said to be a **negative run** of length **k**. Similarly, any sequence of **k** number of consecutive **D**'s followed and preceeded by any other event is said to be a **positive run** of length **k**. Also, the **run sums** are defined as before. Denoting by L_1^-, S_1^- the negative run length and negative run sum corresponding to X_n and by L_2^- and S_2^- the negative run length and negative run sum corresponding to Y_n and taking,

$N_{11}^- = \{n \mid X_n \leqslant c_1, Y_n \leqslant c_2\}$, $S_{11}^- = \sum_{\text{over } N_{11}^-} (c_1 - X_n)$ and

$S_{12}^- = \sum_{\text{over } N_{11}^-} (c_2 - Y_n)$,

Llamas and Siddiqui (1969) showed that equations (3.5),(3.6) and (3.9) are still true while,

$$Cov(S_{11}^-, S_{12}^-) = p/q \,.E(X_1^*).E(Y_1^*) \qquad (3.11)$$

with $p = P(A_n)$ for n=1,2,....

Millan and Yevjevich (1971) used run theory in finite populations to determine either the probability of occurrence or return periods of historical droughts. In order to describe the historical droughts, the authors studied the longest negative run length and the largest negative run sum in a determinant record of size N as measures of the longest observed drought durations and the largest observed total deficits, respectively.

Studying the drought parameters from finite populations of size N, rather than from infinite populations makes the study more difficult. In fact, even for independent stochastic processes, the authors showed that it is only possible to obtain an approximation to the distribution of the longest negative run length. Thus, in this paper, the historical droughts are studied through the simulation of different samples of X_t, which is assumed to be a first order autoregressive stochastic process, under several and different assumptions on the value of:

(i)the probability, $p=P(X \leq X_c)$ for some truncation level X_c,

(ii)the sample size N,

(iii)the population first serial correlation coefficient ρ, and,

(iv)the population skewness coefficient β_1.

Denoting by L_m the longest run length for a given sample size N, D the deficit corresponding to L_m, D_m the largest run sum and L the corresponding run length, the joint cumulative frequencies of (L_m,D) and (D_m,L) are obtained from a large number of samples of **X** generated for each quadruplet of the parameters (i) to (iv) mentioned above.

From these two joint distributions, the authors obtained and analyzed a number of marginal and conditional distributions, such as the empirical distributions of L_m and D_m. The performed analysis led the authors to make the following conclusions:

(a)For a given probability **p**, the quantile l_0, such that $p(L \leqslant l_0)=p$, is always smaller than the quantile of L_m. However, for large values of L_m and L, their distribution functions converge. A similar conclusion can be drawn by comparing the empirical distributions of D_m and D,

(b)The non-normality of the determinant as measured by the skewness, hardly affects the distributions of the longest run length L_m. Moreover, if the random variable of the determinant is independent, the distribution of L_m does not depend on the underlying distribution. For D_m, however, the non-normality has a much greater effect than for L_m,

(c)The log normal probability function with two parameters fits very well both the empirical distributions of L_m and D_m,

(d)Expressing the characteristic parameters of L_m and D_m, namely, their means and standard deviations, as functions of the four independent parameters p,N,ρ and β_1, the two most significant variables found are the probability p associated with the critical level X_c and the sample size N. Next in importance comes the first serial correlation coefficient ρ, while the skewness β_1 has the least effect.

Finally, in order to study the return periods of historical droughts, the concepts of representative drought and representative sample size are introduced. They are defined as follows.

The representative sample size N_r, is the size of a sample which should have a historical drought with a duration equal to the average of the longest negative run length or with a total deficit equal to the mean of the largest negative run sum, for a very large number of generated samples of this sample size. This representative sample size N_r is evaluated from the empirical relationships between $E(L_m)$ or $E(D_m)$ and the sample size N mentioned in item (d) above.

As to the representative drought, it is defined as the longest run length median, that is the value l such that,

$$P(L_m \leq l) = P(L_m > l) = 0.5$$

Based on these two concepts, the authors conclude that when the historical drought is close to this representative drought duration, the length of the available record can be used for computing the return period. If, on the contrary, the probability of exeedence of an observed drought in a sample of size **N** is either very small or very large, that means that the longest observed drought does not behave according to the mean drought properties for the given structure and length of the record.

The method presented was applied with success, according to the authors, to selected annual runoff and precipitation records.

Millan (1972) analytically obtained the probability distribution function of the largest negative run length for simple determinants and truncation levels. An approximation to this distribution function is introduced for the cases when the critical level is of a linear trend type.

A model of the regional economy to determine the economical impacts of droughts is developed in the second part. However, drought impacts are outside the scope of this work.

For i.i.d. X_t , defining Z_t such that,

$$Z_t = \begin{cases} 0, & X_t > X_c \\ 1, & X_t \leqslant X_c \end{cases}$$

is Bernoulli distributed with $p=P(X_t<X_c)$, the probability of a success. If r_1 is the number of successes and r_2 is the number of failures in a sequence of **N** trials, it is shown that the d.f. of the longest negative run length (L_m) is equal to,

$$P(L_m \leqslant j) = \sum_{r_1=0}^{N} p^{r_1}(1-p)^{N-r_1} \cdot \left[\sum_{k=0}^{a} (-1)^k \binom{r_2+1}{k} \binom{N-k(j+1)}{r_2} \right]$$

where, $a = \min\{r_2+1, (N-r_2)/(j+1)\}$

and j satisfies the inequality,

$$N+1-r_2 \geq j+1 \geq (N+r_2+1)/(r_2+1)$$

If Z_t is not an independent sequence of Bernoulli random variable, but Z_t depends only on Z_{t-1} in such a way that it forms a two-state Markov chain with transition probability matrix,

$$T = \begin{array}{c} \\ 1 \\ 0 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 0 \end{array} \\ \begin{bmatrix} p_{11} & 1-p_{11} \\ p_{10} & 1-p_{10} \end{bmatrix} \end{array}$$

then, the d.f. of L_m is given by,

$$P(L_m \leq j) = [\sum_{s=1}^{N}\sum_{c=1}^{c_1} \frac{L(s,j,a)+L(s,j,a+1)}{\binom{s-1}{a-1}+\binom{s-1}{a}}\binom{s}{a}\binom{N-s-1}{b-1}$$

$$*\left(\frac{1-p_{11}}{1-p_{10}}\right)^b\left(\frac{p_{10}}{p_{11}}\right)^a p_{11}{}^s\ (1-p_{10})^{N-s}].P(X=1)$$

$$+\ [\sum_{s=0}^{N}\sum_{c=1}^{c_2} \frac{L(s,j,a)}{\binom{s-1}{a-1}}\binom{s-1}{b-1}\binom{N-s}{a}\left(\frac{1-p_{11}}{1-p_{10}}\right)^a\left(\frac{p_{10}}{p_{11}}\right)^b$$

$$*\ p_{11}{}^s(1-p_{10})^{N-s}].P(X=0) \qquad (3.12)$$

where, L(s,j,a) denotes the number of ways in which 's' elements can be arranged into a 'a' number of sets, each of which contains at least one element and the largest of which contains j or less number of elements.

L(s,j,a) is given by,

$$L(s,j,a) = \sum_{i=0}^{k}(-1)^i\binom{a}{i}\binom{s-ji-1}{a-1}$$

with $k = \min\{a, (s-a)/j\}$

and $s-a+1 \geqslant j \geqslant (s+a-1)/a$

'c' is the number of changes from one state to another. 'a' denotes the number of changes from failure to success and 'b' the number of changes from success to failure in a sequence of **N** trials with a total of 's' successes and 'c' changes.

For **c** even, $a=b=c/2$ and,

for **c** odd, $b=a+1$ if X_0 is a success,

$b=a-1$ if X_0 is a failure.

where X_0 is the observed value at t=0; **c** and **c** are the maximum number of changes from success to failure, and failure to success, respectively, in a sequence of **N** trials. They are determined by,

$$c_1 = \begin{cases} 0, & s=N \\ N+0.5-|2s-N-0.5|, & s \neq N \end{cases}$$

and

$$c_2 = \begin{cases} 0, & s=0 \\ N+0.5-|2s-N-0.5|, & s \neq 0 \end{cases}$$

$P(X=1)$ and $P(X=0)$ are the probabilities of success and failure at any trial.

Millan(1972), also claims that equation (3.12) which is exact for the case in which the Markov chain applies to Z_t, approximates fairly well the d.f. of L_m for other cases of dependence, namely, when the stochastic process of X_t can be represented by the first order linear autoregressive scheme. This approximation might be considered a good one if the first serial correlation coefficient is less than 0.4.

If the truncation level X_t is not a constant, but a linear function such that,

$$X_t = \alpha + \beta + t, \qquad t=1,2,\ldots$$

Millan states that it can be shown that for mild slope critical levels, the empirical distribution of L_m in a sample of size N can be obtained from (3.12) by replacing its trend truncation by a constant critical level equal to the trend's average value.

Gupta et. al.,(1973), using rainfall as the hydrologic variable, studied (i)the distribution of the length of the longest dry period in some fixed interval of time, and (ii)the length of time until a dry period that exeeds a pre assigned `critical' duration occurs. These distribution functions follow from considerations on extreme values of a random number of droughts.

Gupta and Duckstein(1975) present a generalization of the above mentioned paper of Gupta et. al.(1973). Here, the maximum dry period and the waiting time for the drought to exeed some critical duration for a point rainfall process are studied under the assumption that the number of rainfall events in the time interval [0,t] may be modeled by a non-homogeneous Poisson process. Analytical results are obtained only for two particular forms of the intensity function of the Poisson process.

Guerrero-Salazar and Yevjevich(1975) studied the properties of the runs either defined in an infinite population or a finite population of size **N**. Both the univariate case and the bivariate case, i.e., the cases when the determinant is a single discrete stochastic process X_t or a random vector of discrete stochastic process (X_t, Y_t) are considered. A special emphasis is given to the bivariate case.

The concept of a run in the univariate case is already introduced in this chapter. As to the bivariate case, given two critical levels, say c_1 and c_2, four types of events may be defined:

$A_t = (X_t \leqslant c_1,\ Y_t \leqslant c_2)$, $\quad B_t = (X_t \leqslant c_1,\ Y_t > c_2)$

$C_t = (X_t > c_1,\ Y_t \leqslant c_2)$, $\quad D_t = (X_t > c_1,\ Y_t > c_2)$

Let Z_t be the random variable such that,

$$Z_t = \begin{cases} 1 & \text{if } A_t \text{ occurs} \\ 0 & \text{if } B_t \text{ occurs} \\ -1 & \text{if } D_t \text{ occurs} \\ -2 & \text{if } C_t \text{ occurs} \end{cases}$$

A sequence of 1's followed and preceeded by elements of other kind is said to be a **negative-negative run**; a sequences of 0's followed and preceeded by elements of other kind are said to be a **negative-positive run. Positive-negative runs** and **positive-positive runs** are defined similarly.

The random variables studied by Guerrero-Salazar and Yevjevich are the longest run length of one kind, L_m, and the largest run sum, D_m. For a finite population and for the univariate case, the theoretical distribution of L_m is shown when the random variable of the determinant are independent or dependent. In the first case, the mean value of L_m is also given with an error term of order one. The problem of finding d.f. of the longest run length is also tackled for the bivariate case under four different assumptions:

(i) X_t and Y_t (t=1,2,...N) are mutually and serially independent,

(ii) X_t and Y_t are serially independent, but mutually dependent,

(iii) X_t and Y_t are serially dependent, but mutually independent,

(iv) X_t and Y_t are serially and mutually dependent

The first three cases can be handled easily, while the fourth one is more complex and only approximate solutions for the d.f. of the longest negative-negative run length and the longest negative-positive run length are obtained, when X_t and Y_t are normal bivariate random variables. The process of X_t and Y_t are assumed to be first-order linear autoregressive. The approximation seems to be satisfactory for the values of the first serial correlation coefficient up to 0.4.

The study of the largest run sum is much more difficult than the study of the longest run length of any kind and no analytical results were achieved even for the simplest case of a univariate independent Gaussian stochastic process.

Next, the paper analyses the run length distributions for infinite populations. As to the univariate case, for a determinant with independent random variables, nothing new is presented. For a dependent case, the distribution of the run length is obtained under two different assumptions. First, by assuming that the dependent series can be modeled by a two-state Markov chain or by a first-order linear autoregressive stochastic process, which can be approxinmated by a two-state Markov chain. Second, by assuming that the determinant is normal and each vector of size **n** $(X_{t+1}, X_{t+2}, \ldots X_{t+n})$, is multivariate normal and using a truncation level of the tetrachoric series exapnsion of the integral,

$$P(X_{t+1} \leq c, \ldots, X_{t+n} \leq c) = \int_{-\infty}^{c} \cdots\cdots \int_{-\infty}^{c} dF$$

where F is the multivariate normal distribution function.

If the two-state Markov chain is (exactly or approximately) the model for the underlying stochastic process, two different approaches are used to find the distribution of the run length. The mean and variance of the run length are also shown. One interesting thing to be noted is that the two approaches lead to different results, one of them being coincident with the result presented by Llamas and Siddiqui(1969).

Similarly, the bivariate case of the probability distribution of the longest run length for a finite population, the same four alternatives are investigated for the probability distribution and the first two moments of the negative-negative and negative-positive run lengths of infinite series for the bivariate case. Regarding run sums, again approximate results are shown, some of them already presented by other authors.

The results achieved are tested by generating a number of time series and comparing the behaviour of drought characteristics obtained both theoretically and from the experimental method.

After a review of some available techniques, the authors present a set of four alternatives to the study of drought durations and drought magnitudes when the time series is a periodic-stochastic process. A case study based on a particular monthly supply series and monthly demand series makes an attempt to study the drought parameters such as duration and magnitude.

After a review on the stochastic characterization of droughts, Tase(1976) developed a mathematical model of monthly precipitation over an area taking into account that the dependence among the observations in different observing stations over the area is a function of the coordinates of the station, the distance between two stations and the orientation of the line connecting two stations. In Chapter 5 of the paper, areal drought characteristics are defined.

They are, (1)the deficit area **A**, (2)the total areal deficit **D**, (3)the maximum deficit intensity **I** and are respectively defined by:

$$A = \sum_{k=1}^{m} I_k(k) \qquad (3.13)$$

$$D = \sum_{k=1}^{m} (X_c - X_k) \cdot I_k(k) \qquad (3.14)$$

$$I = X_c - \min(X_1, \ldots\ldots, X_m, X_c) \qquad (3.15)$$

where **m** is the number of observation stations (o.s) in the region under consideration; **k** is the set of o.s. such that at a given time unit, $X_k \leqslant X_c$; $\mathbf{I_k(k)}$ is the indicator function and $\mathbf{X_c}$ is the truncation level.

Using generated precipitation data from a new and regular rain-fall gage network, the basic parameters of the above three drought characteristics, namely, the mean, standard deviation, maximum value, minimum value, skewness and excess are estimated for different truncation levels. Concerning the distribution of the drought characteristics, a four parameter Beta distribution was fitted to the frequency distributions of the three random variables ascertained from the simulated data. A regression analysis was also carried out to find relationships among the deficit area, the total areal deficit and the maximum deficit intensity. The trivariate probability distribution function (p.d.f) of these three random variables is approximated by the product of the three marginal beta p.d.f. with the Jacoby polynomials. However, this distribution did not show a good fit.

Probabilities of areal coverages by droughts and probabilities of a specific region covered by a drought are also calculated. The author concludes that, (1)the probability of a whole region being completely drought affected is very low, (2)this probability is more dependent on the size than on the shape of the region, although for small regions, the shape may have a significant influence, (3)probability distributions of the deficit area are affected by the truncation level as well as by the size and the shape of the area.

In Chapter 6 of the paper, there is an endeavor to apply the rationale developed previousely to monthly precipitation data, which may be modeled by a periodic-stochastic process, such that each random variable, X_t, may be decomposed into a sum of two periodic functions with period equals to 12 months,

i.e., $X_t = m_t + s_t . Z_t$

where, m_t and s_t are the periodic functions and Z_t represents a second-order stochastic process. Moreover, it was assumed that both m_t and s_t are composed only of the 12-month harmonic in the Fourier series. Under these assumptions, run properties such as the expected value and variance of the negative run length and negative run sum are analytically derived.

Sen(1976) tackled the problem of finding the analytical d.f. and moments of the negative and positive run lengths

assuming that the stochastic process, X_t is first-order autoregressive. Both d.f. and moments of run lengths are found to be functions of the critical level X_c and the first-order serial correlation coefficient ρ.
He derived that,

$$P(L^+=k) = (1-r)r^{k-1} \tag{3.16}$$

where,

$$r = P(X_t>X_c | X_{t-1}>X_c)$$
$$= P(X_t>X_c, X_{t-1}>X_c)/P(X_{t-1}>X_c)$$

The joint probability indicated by the numerator was calculated in two different ways depending on whether $X_c = 0$ or $X_c \neq 0$ (He has assumed that X_t is standard normally distributed).

For $X_c = 0$, he used the following result from Sheppard(undated),

$$P(X_t>X_c, X_{t-1}>X_c) = (1/2\pi).\text{arc} \sin(\rho) + 1/4 \tag{3.17}$$

For $X_c \neq 0$, making use of the result from Cramer and Leabetter(1967), he gets

$$P(X_t> X_c, X_{t-1}> X_c) = (1/2\pi)\int_0^{\rho} \exp[-X_c^2/2(1+z)](1-z^2)^{-1/2}.dz \tag{3.18}$$

With these two results in mind and using (3.16), the mean, variance, skewness and coefficient of variation of the positive run length are derived as functions of r. The author completes his paper with an application for an annual runoff series in Turkey.

Sen(1980-b) theoretically studied the drought characteristics introduced by Tase(1976), namely, the deficit area(**A**), the total areal deficit(**D**) and the maximum deficit intensity(**I**).

In order to find the d.f. and the parameters of these random variables for a region with **m** observing stations, two assumptions are made as follows:

(i)the random vector {**X(t)**}, whose components $X_k(t)$ (k=1, 2,3,....,m) are, for each **t**, the model of the determinant factor at each station of the region under consideration is assumed to be constructed by spatially independent hydrologic variables,

(ii)at each observing station, the d.f. of the random variables are assumed to be identical and independent of time.

Under these assumptions, the author analytically determines the d.f. of the characteristics **A**, **D** and **I** and their means and variances.

Chander et.al.,(1981) presented a simplified analysis using a T-function and power transformation for the analysis of run length and expected run sum of a skewed correlated process. They have analytically derived the run sum distribution quantifying the extent of shortages (or excesses) during the deficit (or surplus) periods. A detailed methodology is also presented for the analytical estimation of the run sum distribution. The proposed methodology is explained with an application to some annual flow data.

Santos(1983), in her paper, presented a regional drought definition and an approach to the stochastic characterization of regional droughts. This characterization has been made by use of the probability theory through a set of random variables associated with regional droughts such as their duration, intensity and the area undergoing drought effects. The distribution, mean and variance of these random variables as well as of some other auxiliary variables are theoretically derived. Finally, she completes the paper with an application to some Portuguese data and making very useful conclusions.

3.3 Studies on Multiyear Droughts

Among the few researchers who have conducted studies on multiyear droughts, one must mention the works of Dracup et.al.(1980-a, 1980-b), Paulson et.al.(1985), Sadeghipour and Dracup(1985) and Lee et.al.(1986). These people deserve great credit for defining the basic concepts and investigating several analytical methods related to multiyear droughts. Dracup et.al.(1980-a) have discussed several considerations for developing a practical, analytical definition of droughts. These considerations include selection of the hydrologic variable, sampling averaging period and truncation level for a detailed drought analysis. Finally, a presentation is given on the application of the proposed drought definition procedure towards a frequency analysis of multiyear hydrologic droughts.

Dracup et.al.(1980-b) have performed several statistical tests on streamflow series for purposes of analysing multiyear hydrologic events. Mean annual flow series observed at different gages belonging to different watersheds have been used and statistical tests on both high flow and drought event parameters are conducted. The investigated drought parameters are namely, duration, severity and magnitude; the statistical tests include (1)stationarity in terms of linear trend; (2)randomness in terms of lag-1 serial correlation; (3)correlation and cross-correlation between the parameters. They conclude that their results can be considered as excellent indicators of the maximum response of a watershed in terms of drought duration and severity.

Lee et.al.(1986) developed a practical approach for frequency analysis of multiyear drought durations of annual streamflow series. The proposed approach employs a frequency curve smoothing technique to reduce the statistical uncertainities due to sample size limitations. Several hazard function models are tried out and the best model is selected to represent the duration-dependent termination rate of a drought data set. The advantage of this methodology is that once the form of the hazard model is determined and its parameters are estimated, the probability distribution and the exeedence probability of drought durations can be determined from their corresponding relationships with the hazard model.

Related to the regional frequency analysis of hydrologic multiyear droughts, it is worthwhile to mention the works of Paulson et.al.(1985) and Sadeghipour and Dracup(1985). In their paper, Paulson et.al.(1985) have used the multiple linear regression analysis to estimate the characteristics of drought duration, severity and magnitude from indices of watershed geomorphology and climate. Each of the three drought parameters has been estimated independently to identify its correlation to the selected geomorphological and climatic indices. Finally, they conclude that the proposed regression relations are primarily statistical models and are not necessarily reliable indicators of cause-and-effect physical relationships.

Sadeghipour and Dracup(1985) propose a method to standardize drought severities with a duration adjustment to enable comparison among drought events. For a regional study, the index drought method is selected and applied to standardized droughts to give a regional frequency curve. By taking advantage of random variations of droughts in both time and space, a multivariate simulation model named the regional extreme drought method is used to estimate the exeedence probabilities associated with regional drought maxima. They claim that this method is capable of generating a series of drought events which are more severe than historic events. Finally, they have combined the results of the index drought method and regional extreme drought analysis to obtain a single regional drought frequency curve.

Thus, at this point, it is clear that the available few studies on multiyear droughts are limited to the analysis of historic annual flow series. None of these studies provides a general methodology to conduct a thorough investigation of droughts with the objective of producing practically applicable results that would be beneficial for a water resources planning study. Some studies are concerned with the determination of the statistical properties of the observed droughts and the others attempt to derive probability distributions for the drought parameters with the application of general principals of probability. However, the conclusions are based on a small sample of drought events. As the sampling errors are totally neglected in these studies, the applicability of the results for the forecasting purpose would be a question. This situation in fact encouraged to develop the methodology of the present study to simulate the drought behaviour based on the analysis of large samples of droughts derived from generated flow series.

CHAPTER IV. THEORETICAL CONSIDERATIONS AND METHODOLOGY

4.1 Determination of the Underlying Mechanism of a Flow Series

In any time series analysis, as a first step, it is necessary to determine the underlying mechanism of the observed event series. This helps to understand the pattern of temporal variation and also provides sufficient clues for the analyst to model the process with mathematical techniques. Therefore, by estimating the basic statistical parameters and performing some statistical tests on the observed flow series, one can identify the basic behaviour of the process. Thereafter, by conducting detailed analyses in both the time domain and the frequency domain, the exact underlying mechanism can be determined.

4.1.1 Basic Statistical Parameters

(a) Mean (X_m)

The mean of a time series of variable X_i, is a measure of the central tendency and is defined by,

$$X_m = \sum_{i=1}^{n} X_i / n$$

where, n = total number of observations

(b) Standard Deviation (S_x)

The standard deviation of a sample measures the dispersion of the various values of the variable about the mean value, and is defined by,

$$S_x = \sqrt{\sum_{i=1}^{n} (X_i - X_m)^2 / n}$$

The above formula holds well for a large sample. When the sample size is small, the unbiased estimate is preferred in hydrology and is defined by,

$$\hat{S}_x = \sqrt{\sum_{i=1}^{n}(X_i - X_m)^2/(n-1)}$$

(c) Coefficient of Variation (C_v)

This parameter measures the dispersion of the variable about the mean as a ratio, and is defined by,

$$C_v = S_x/X_m$$

(d) Coefficient of Skewness (C_s)

This dimensionless ratio measures the symmetry of the distribution of an observed data set. The unbiased coefficient for a given sample can be estimated from,

$$C_s = n\sum_{i=1}^{n}(X_i - X_m)^3/[(n-1)(n-2).S_x^3]$$

(e) Autocorrealation Function

The autocorrelation function is frequently used to determine whether a given time series is independent or dependent. It is also employed to determine the spectral density function and the partial autocorrelation function, which are very important in attempting to model the time series.

Lag-k autocorrelation coefficient is defined by,

$$r(k) = \frac{\sum_{i=1}^{n-k} X_i.X_{i+1} - (\sum_{i=1}^{n-k} X_i).(\sum_{i=k+1}^{n} X_i)/(n-k)}{[\sum_{i=1}^{n-k} X_i^2 - (\sum_{i=1}^{n-k} X_i)^2/(n-k)]^{1/2}.[\sum_{i=k+1}^{n} X_i^2 - (\sum_{i=k+1}^{n} X_i)^2/(n-k)]^{1/2}}$$

4.1.2 Testing for Independent/Dependent Distribution

There are several statistical tests which can be used to determine whether a time series is independent or not. Among them, the following four tests have been selected for application to the present study. In each test, **n** denotes the total number of observations in the sample.

(a) Autocorrelation Test

This is considered to be a powerful test. It checks whether the calculated lag-one autocorrelation coefficient (r_1) is significantly different from the expected value. where,

Expected Value, $E(r_1) = -1/n$

Variance, $Var(r_1) = (n^3-2n^2+2)/(n^4-n^2)$

If the estimated value of r_1 lies within the 95% confidence limits, then the hypothesis that the sequence is independently distributed is accepted. Otherwise, the sequence is assumed to be dependent.

(b) Median Crossing Test (Fisz Test)

In this test, the observed values (X_i 's) are replaced by zeros if $X_i<m$ (median) and by 1's if $X_i>m$. If the sequence is independently distributed, then the number of times (q), zero is followed by 1 or, 1 is followed by zero, is approximately normally distributed with:

$$\text{Mean} = (n-1)/2$$

$$\text{Variance} = (n-1)/4$$

If **q** lies within the 95% confidence limits, then the sequence is assumed to be independent. Otherwise, it is dependent.

(c)<u>Turning Point Test</u> (Kendall's Test)

When the **i** th observation is denoted by X_i, in this test, X_i is assigned the value 1 for $X_{i-1} < X_i > X_{i+1}$ or $X_{i-1} > X_i < X_{i+1}$. Otherwise, X_i is assigned the value zero. Then, the total number of 1's (q) is approximately normally distributed with:

$$\text{Mean} = 2(n-2)/3$$

and,

$$\text{Variance} = (16n-29)/90$$

If **q** lies within the 95% confidence limits, the hypothesis of independent distribution is accepted. Otherwise, the series is considered to be dependent.

(d)<u>Rank Difference Test</u> (Meacham Test)

In this test, the variable X_i is replaced by its relative rank **R** , with the lowest being denoted by **1** (R_1). Then, the **U** statistic is calculated as:

$$U = \sum_{i=2}^{n} |R_i - R_{i-1}|$$

for large **n**, U is normally distributed with,

$$\text{Mean} = (n+1)(n-1)/3$$

and,

$$\text{Variance} = (n-2)(n+1)(4n-7)/90$$

If **U** lies within the 95% confidence limits, then the hypothesis that the sequence results from an independent process is accepted. Otherwise, it is a dependent process.

4.1.3 Testing for Stationarity/Non-stationarity of the Process

A stationary process is in a particular state of statistical equilibrium. Therefore, such a process must exhibit a constant moving average (mean) and a constant moving variance. For a strictly stationary process, the joint distribution of any set of observations must be unaffected by shifting all the times of observations forward or backward by any integer amount **k**, where **k** is less than the total number of observations **n** (Box and Jenkins, 1976). Therefore, by examining the autocorrelation function estimated for the observed time series, this property can be investigated.

Let σ_x = standard deviation (theoretical) of the process,

ρ_k = lag-k autocorrelation coefficient (theoretical),

and, γ_k = lag-k autocovariance,

Then, these parameters are related by,

$$\gamma_k = \rho_k \cdot \sigma_x^2$$

The autocovariance matrix is given by,

$$\Gamma_n = \begin{bmatrix} \gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_{n-1} \\ \gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_{n-2} \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_{n-3} \\ \vdots & \vdots & \vdots & & \vdots \\ \gamma_{n-1} & \gamma_{n-2} & \gamma_{n-3} & \cdots & \gamma_0 \end{bmatrix}$$

which is a symmetric matrix of order **n**.

where, n = total number of observations in the sample

The above matrix can be further reduced to:

$$\Gamma_n = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdot & \cdot & \rho_{n-1} \\ \rho_1 & 1 & \rho_1 & \cdot & \cdot & \rho_{n-2} \\ \rho_2 & \rho_1 & 1 & \cdot & \cdot & \rho_{n-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdot & \cdot & 1 \end{bmatrix}$$

P_n is referred to as the autocorrelation matrix.

In the analysis of an observed time series (a sample), the values of ρ_k must be replaced by r_k with σ_x replaced by s_x.

If the time series represents a stationary process, its autocorrelation matrix must be positive definite. Due to the difficulty of examining this property, an indirect approach is used. In this approach, the **Spectral Density Function** (Spectrum) is determined for the time series and its behaviour examined. The spectrum actually represents the frequency variation of the process. Therefore, the selected approach makes a frequency domain analysis instead of a time domain analysis. A major advantage of using the spectrum is that it indicates the hidden periodicities of the process(if they exist).

4.1.3.1 Sample Spectral Analysis

The theory of variance spectrum analysis postulates that a time series is a sample from a population characterized by a variation over a continuous spectrum of frequencies. The observed time series is a random sample of a process in time (or space) that is made up of contributions from all possible frequencies. A variance spectrum partitions the variance into a number of intervals or bands of frequency. The quantity usually shown is the spectral density, which is the amount of variance per interval of frequency (Haan, 1977).

Let **S(f)** = spectral density corresponding to frequency f,
For a complete random series, S(f) is a constant and is termed white noise.
The mathematical relationship of S(f) and the corresponding value of autocorrelation function, is given by,

$$S(f) = 2\int_{0}^{\infty} \rho(\tau).\cos(2\pi f \tau).d\tau$$

or $$\rho(\tau) = \int_{-\infty}^{\infty} S(f).\cos(2\pi f \tau).d\tau$$

also, $$\int_{-\infty}^{\infty} S(f).df = 1$$

Thus, S(f) represents the probability density function of the normalized variance of the time series.
For an observed discrete time series, the sample spectral density function, S(f) can be determined from the estimated autocorrelation function as follows:

$$\hat{S}(f) = \Delta t[r(0) + 2\sum_{k=1}^{m-1} r(k).\cos(2\pi kf.\Delta t) + r(m).\cos(2\pi\, mf.\Delta t)]$$

where, Δt = discrete time interval,

r(k) = lag-k autocorrelation coefficient,

m = maximum number of correlation lags. (not more than $n/4$)

Now, introducing the Nyquist frequency, $f_N = 1/[2.\Delta t]$, the frequency corresponding to lag-k can be written as:

$$f = k.f_N/m$$

Then, the final estimates for the sample spectral density function can be numerically evaluated by the following set of equations (Haan, 1977).

$$\hat{S}(0) = 0.5[\hat{S}'(0) + \hat{S}'(f_N/m)]$$

$$\hat{S}(k.f_N/m) = 0.25\hat{S}'[(k-1)f_N/m] + 0.5\hat{S}'(k.f_N/m) + 0.25\hat{S}'[(k+1)f_N/m), \quad \text{for } k=1,2,...(m-1)$$

$$\hat{S}(f_N) = 0.5\{\hat{S}'[(m-1)f_N/m] + \hat{S}'(f_N)\}$$

This new set of values smooth out the spectral density function derived from the discrete process.

If the function represents a smooth curve with one or more peaks, it can be mathematically proved that each peak corresponds to a dominant frequency of the observed time series.

4.1.3.2 Sufficient Conditions for Stationarity

With application of the former theory, one can test for the stationarity or non-stationarity characteristic of a given process as follows:

(1) Divide the observed time series into a number of samples, so that each subsample contains an equal number of observations. Then, estimate the mean and variance for each subsample and test whether they significantly

deviate from the overall mean and variance estimated for the entire time series. This test is usually performed at 95% significance levels.

(2) Determine the sample spectral density function and carefully observe its form of variation. If the function represents a smooth curve which is free from rapid fluctuations, the time series can be assumed to be stationary. Otherwise, the series is non-stationary.

Upon agreement of the conclusions determined from the above two tests, one can decide whether the hydrologic process represented by the observed time series is stationary or not. If non-stationarity is found, it may have been attributed by one or more of the following causes.

(a) periodicity of the process,

(b) presence of a trend pattern in the process,

(c) presence of a jump in the time series caused by a catastrophic change of the regional hyrology.

4.2 Selection of the Flow Series for Multiyear Drought Analysis

Based on the statistical tests mentioned earlier, it is possible to determine whether the mean annual flow series recorded at the selected gaging stationsare dependent, periodic, stochastic processes. This identification is important because the study proposes to use synthetic flows in addition to the historic flows. For generation of a longer flow series, the historic series must be capable of modeling by mathematical techniques. Therefore, they must be dependent at first. Also, since the annual flows are to be used, they must be of a periodic nature as well. Naturally, an annual flow series recorded at an uncontrolled gaging station must reflect these two properties, provided the flow series are not subject to upstream regulation.

4.3 Modeling the Flow Series

If an observed (historic) flow series is found to be a dependent, periodic, stochastic process, the following general model is suggested to be used for its representation.

$$X_t = X_m + \sum_{i=1}^{l} [A_i . \cos \omega_i (t-1) + B_i . \sin \omega_i (t-1)] + \varepsilon_t$$

$$\text{for } t = 1, 2, 3, \ldots$$

where, X_t = observed flow at time t,

X_m = long-term mean flow,

ω_i = frequency corresponding to i^{th} periodicity,

A_i = coefficient of cosine function,

B_i = coefficient of sine function,

and, ε_t = stochastic component of X_t.

The concept behind this modeling technique is that each flow event can be assumed to be a combination of a deterministic component and a stochastic component. The deterministic component is represented by the long-term mean X_m plus the attached harmonic function; the stochastic component is represented by the residual term ε_t. When applying this model, at the first step, the unknown parameters have to be determined. For this purpose a suitable parameter optimization method can be chosen. In the present study, the technique of minimizing the sum of the squares of residual terms (Least Square Method) will be employed in the form of the Cyclic Decent Method.

4.3.1 Cyclic Decent Method

The general idea of the Cyclic Decent method is to divide the parameters into subsets in such a way that the optimization with respect to parameters in any one subset, holding the remaining parameters fixed, can be done fairly easily (Bloomfield, 1976). The method then is to update the subsets successively by solving these manageable optimization problems in turn. The basic method cycles through the subsets in some fixed order, until a complete cycle results in an effective zero change in the function to be optimized. In sophisticated algorithms, the subsets may be chosen in a different sequence so as to accelerate the convergence of the method. When the function can not be reduced by varying any of the subsets of parameters, a minimum (local) has usually been reached.

The procedure can be described as below:

$$\text{Let} \quad R = \sum_{t=1}^{n} \varepsilon_t^2$$

$$= \sum_{t=1}^{n} \{ X_t - X_m - \sum_{\substack{i=1 \\ i \neq k}}^{\ell} (A_i . \cos \omega_i (t-1) + B_i . \sin \omega_i (t-1) - A_k . \cos \omega_k (t-1) - B_k . \sin \omega_k (t-1) \}^2$$

$$R = \sum_{t=1}^{n} [Y_t - A_k.\cos\omega_k(t-1) - B_k.\sin\omega_k(t-1)]^2$$

where,

$$Y_t = X_t - X_m - \sum_{\substack{i=1 \\ i \neq k}}^{l} [A_i.\cos\omega_i(t-1) + B_i.\sin\omega_i(t-1)]$$

Then, the optimization with respect to ω_k, A_k and B_k can be done as in the method of least squares. For this purpose, a Fortran program (Appendix E) can be wriiten. To avoid a long computation time, approximate values for ω_i $(i=1,2,\ldots,l)$ may be inputted as derived from thespectral density function. The output of the program will indicate the exact values of ω_i, A_i, B_i and the segregated stochastic component from each X_t.

4.3.2 <u>Modeling the Stochastic Component</u> (ε_t)

The method of least squares employed in the Cyclic Decent method assumes that the error terms (stochastic components) are uncorrelated and distributed with zero mean value. When the segregated components from an actual time series are carefully examined, they may be distrubuted with zero mean, but may not be uncorrelated. If the observed values follow a dependent process, then a suitable stochastic model can be proposed to represent this process. For this purpose, as a first step, it is necessary to determine the autocorrelation function and the partial autocorrelation of the series formed by ε_t terms. Then, based on the behaviour of these two functions, either an **Autoregressive(AR)** or **Moving Average(MA)** or a mixed **Autoregressive Moving**

Average(ARMA) model can be suggested. The best model, its order and parameters have to be decided to satisfy several conditions set out by the theory of stochastic processes (see Appendix B).

Once both the deterministic and stochastic components of a flow series are accurately modeled, it provides a tool to generate a flow series of desired length. As outlined under the objectives of this thesis, synthetic mean annual flow series will be used for detailed analyses of multiyear droughts.

4.4 Multiyear Drought Analysis of Historic Flow Series

Each selected historic mean annual flow series was subjected to drought analysis at the truncation levels of $0.8X_m$, $0.85X_m$, $0.9X_m$, $0.95X_m$, X_m, $1.05X_m$ and $1.1X_m$. The derived drought and surplus parameters resulting from each truncation level were then statistically analyzed as time series, and their basic statistical properties as well as the distribution characteristics were determined.

4.4.1 Fixation of a Truncation Level

The present study proposes synthetic annual flow series to be used for drought analysis. If it is desired to analyse these series for several truncation levels, in order to perform detailed analyses of resulting drought parameters, it would require a great effort. Even if it is done, the final results may have limited applications. Due to these reasons, it is necessary to select one technically feasible truncation level before analysing the generated flow series. The selection must be based on some meaningful criteria. For this study, the following criteria is setforth:

(a) As the state of New Jersey has rarely experienced critical droughts, the proper truncation level can be selected to result in a balance of the expected drought duration and the expected surplus duration.

i.e., $E(L_d) \simeq E(L_s)$

(b) For the fixed truncation level, both $E(L_d)$ and $E(L_s)$ shall not significantly differ from the semi-periodicity of the hydrologic process.

In the previous research papers on multiyear drought analysis, the long-term mean annual flow (X_m) has been used as the truncation level. Although this seems to be reasonable forstationary random processes, it may not be appropriate for periodic stochastic processes. This is due to the fact that periodic processes are dependent and therefore, the persistent properties may result in a balance of drought and surplus durations at a truncation level which is different from the long-term mean. But, if the degree of persistence is the same for both the low flow and high flow events, even for periodic processes the long-term mean would have been the most reasonable truncation level. However, when attempting to fix a truncation level by satisfying the above mentioned conditions (a) and (b), the long-term mean (X_m) can be set as the upper bound. In other words, if the truncation level meeting the two conditions is found to be greater than X_m, then X_m can be taken as the truncation level for synthetic flow series analysis.

4.4.2 Analysis of Drought Parameters

The derived point drought parameters such as duration (L_d), severity (S_d) and the time of occurrence (τ_d) can be subjected to detailed statistical analyses for the purpose of comparing the generated drought events with the historic events. If a reasonable agreement is observed, it is possible to propose suitable theoretical distributions to the drought parameters derived from the generated series.

For the above purpose, for each watershed, the following interesting relationships can be graphically obtained using the drought parameters derived from both the historic and generated flow series. The fixed truncation level based on the criteria outlined under 3.4.1 will be used.

(a) Relative Frequency Vs. Drought Duration

(b) Non-exeedence Probability Vs. Drought Duration

(c) Non-exeedence Probability Vs. Drought Severity

(d) Drought Duration-Severity-Frequency Curve

4.5 Use of Synthetic Flow Series for Drought Analysis

With application of the proposed model for each gaging station (located in a selected watershed), flow series of different length can be generated. Then, the basic statistical properties such as mean, standard deviation, skewness and autocorrelation function can be estimated for each series for the purpose of comparing with the correspon-ding parameters estimated for the historic series. In order to determine the most suitable series length, it is suggested to generate a series of one, two, three and four times as long as the historic series. In addition, the return period of interest also must be considered. The series length which more closely reproduces the historic statistical properties can be selected for detailed analysis. Once the series length is finalized, it can be used for the multiyear drought analysis at the fixed trunca-tion level. The statistical properties of the derived drought parameters can then be compared with those resulting from the historic flow series at the same truncation level. If a close agreement is observed, it provides an additional verification to confirm that the generated flow series resemblesthehistoric series.

4.6 Fitting Distributions for the Drought Parameters and Forecasting

Based on the estimated statistical properties and the performed statistical test results, a suitable probability distribution function can be proposed for each drought parameter of interest. If the statistical tests indicate that a particular drought parameter follows a random process, fitting a probability distribution is the only way of describing its magnitude variation. This situation is expected for drought duration and severity parameters, because they represent random variables.

Although, several types of probability distribution functions are available in the literature, the selection of the most appropriate one must be made to preserve the dominant statistical parameters. Moreover, the selected distribution must be confirmed by conducting a Goodness of Fit test such as Chi Squared test at acceptable confident levels.

Once the magnitude variation of a particular drought parameter is expressed by a probability distribution, it provides a basis for forecasting the future events within a given return period. What is important is an accurate estimation of the distribution parameters. As the droughts form a discrete time series, in making such forecast estimates, the periodicity of the hydrologic process must be considered as well. These forecast magnitudes can be treated as important considerations by the water resources engineer in future planning to minimize the unfavorable effects of prolonged drought events.

4.7 Determination of the Regional Variation of the Point Drought Characteristics

Having studied the point drought processes in several adjacent watersheds, the spatial variation of hydrology over the region can be understood. If different watersheds reflect different hydrologic status within the same time period, that behaviour may be favorable for devising a distribution scheme to minimize the unfavorable effects. More explicitly, water from a surplus region can be transported to a deficit region within the same time period. The decision will be made by performing an analysis over a common time period for which the historic flows have been recorded at each gaging station under study.

4.8 Application of the Techniques

In the application of the above outlined theory and mathematical techniques for the case study, many computer programs were written in Fortran IV language. Computations have been performed on a Vax-8800 computer located at the New Jersey Institute of Technology, Newark, New Jersy.

CHAPTER V. RESULTS AND DISCUSSION

The results of the study have been presented in the form of tables and figures. Values of different parameters, determined by analysing the flow series and the derived drought & surplus events have been tabulated in the best form to make necessary comparisons. Figures mainly indicate the stochastic behaviour of the flow series and the probability distributions fitted for various drought parameters.

When examining the basic statistical properties of the historic annual flow series belonging to each gaging station listed in Table 5.1, it is found that the mean and median are very close to each other for each watershed. Also, the coefficient of skewness is not that significant as it does not exeed 0.6. Therefore, based on these characteristics, it can be concluded that the selected flow series are approximately normally distributed. Also, it is interesting to note that the values for the Coefficient of Variation (CV) of the flow series recorded at the first six gages are almost equal and little higher than 0.3. On the other hand, the CV of the Squankum flow series considerably differs from the other CV values, as it is less than 0.3. This behaviour reveal two important facts; (1) the first six watersheds (belonging to the Delaware, the Passaic and the Raritan River basins) are more or less hydrologically homogeneous, and (2) the hydrology of the Manasquan River basin (located in Central

Table 5.1 Basic Statistical Properties of the Historic Mean Annual Flow Series

Gaging Station	Mean (cfs)	Median (cfs)	Std. Dev. (cfs)	Coef. Variation	Coef. Skew.	Lag-1 Autocorr.
Pequest	156.7	147.0	51.7	0.330	0.334	0.318
Bloomsbury	238.9	229.5	76.3	0.319	0.320	0.331
Boonton	231.3	228.0	72.5	0.313	0.247	0.392
Millington	89.7	86.4	30.0	0.334	0.521	0.274
Pottersville	56.1	54.4	17.4	0.310	0.474	0.280
Raritan	306.4	305.0	99.1	0.323	0.600	0.232
Squankum	75.2	75.9	20.1	0.267	0.539	0.253

Table 5.2 Characteristics of Historic Mean Annual Flow Series

Stream	Gaging Station	Description of the Behaviour
(1)Pequest R.	Pequest	Stationary, dependent and periodic
(2)Musconetcong R.	Bloomsbury	Stationary, dependent and periodic
(3)Rockaway R.	Boonton	Non-stationary, dependent and periodic
(4)Passaic R.	Millington	Non-stationary, dependent and periodic
(5)Lamington R.	Pottersvil.	Non-stationary, dependent and periodic
(6)N.Br. Raritan R.	Raritan	Non-stationary, dependent and periodic
(7)Manasquan R.	Squankum	Stationary, dependent and periodic

New Jersey) is different than the others and is subjected to smaller temporal variations.

The conclusions in Table 5.2 with regard to the nature of different annual flow series were made based on the variance analysis and the behaviour of Autocorrelation and Spectral Density functions (Appendix C). According to the results of the four statistical tests, namely, Autocorrelation test, Median Crossing test, Turning Point test and Rank Difference test, it was possible to conclude that each of the seven historic flow series selected represents a dependent process. Detailed analyses indicate that the non-stationarity nature of some of the series have been caused by the presence of periodicity in the hydrologic process.

Table 5.3 indicate the dominant frequencies of each historic annual flow series. The spectral density analysis was first used to estimate the approximate values for ω_j. Then, the exact values of ω_j and the coefficients A_j and B_j were determined by use of the Cyclic Decent method. In this procedure, the rough estimates of ω_j calculated from each peak of the spectrum were used to initialize the optimization scheme.

The presence of multi frequencies in a particular flow series is evidence that the periodicity of the hydrologic cycle has been changed over the time. On the other hand, a

Table 5.3 Parameters of the Periodic Component of the Historic Mean Annual Flow Series

Gaging Station	Number of Frequencies	Exact Values of Frequencies ω_j (rad/yr)	Cosine Coefficients A_j(cfs)	Sine Coefficients B_j(cfs)
Pequest	3	ω_1=0.228	14.96	-24.63
		ω_2=1.372	-16.97	-10.42
		ω_3=2.244	-12.80	-11.45
Bloomsbury	3	ω_1=0.224	13.85	-42.26
		ω_2=1.470	-7.80	7.00
		ω_3=2.247	-16.07	-17.22
Boonton	1	ω=1.400	25.12	15.74
Millington	3	ω_1=0.187	-8.96	-14.92
		ω_2=1.244	8.95	9.57
		ω_3=2.824	8.82	-2.67
Pottersville	3	ω_1=0.211	4.91	-7.15
		ω_2=1.124	5.57	4.53
		ω_3=2.111	4.79	-2.68
Raritan	1	ω=1.368	42.19	-1.52
Squankum	2	ω_1=1.244	3.28	9.06
		ω_2=2.222	-3.07	6.62

Table 5.4 Average Periodicities of Historic Mean Annual Flow Series

Stream	Gaging Station	Ave. Frequency (rad/yr)	Ave. Periodicity (years)
Pequest R.	Pequest	1.281	4.90
Musconetcong R.	Bloomsbury	1.314	4.80
Rockaway R.	Boonton	1.400	4.50
Passaic R.	Millington	1.418	4.40
Lamington R.	Pottersville	1.149	5.50
N.Br. Raritan R.	Raritan	1.368	4.60
Manasquan R.	Squankum	1.733	3.60

flow series with a single frequency can be assumed to be more stationary and is subjected to less temporal variations.

The average periodicity of each flow series listed in Table 5.4 was computed based on the average frequency calculated for different frequencies (ω_j's) tabulated in Table 5.3. The average periodicity can be treated as the period of the hydrologic cycle over the particular watershed.

For each flow series, the truncation level (X_c) listed in Table 5.5 was fixed by performing a sensitivity analysis on the flow series. In this procedure, each series was subjected to drought analysis at several selected truncation levels as explained in Chapter IV. The one given in Table 5.5 was chosen, because it results in a balance between the expected drought duration and the expected surplus duration, thus meeting the criteria. When considering the values in the fourth column of Table 5.5, it is interesting to note that the ratio of X_c/X_m is quite close to unity for the first six watersheds. For Pequest, Musconetcong and Rockaway watersheds, the chosen X_c values are rather close to the median values of the flow series. On the other hand, the fixed truncation level for the Manasquan river basin is well below the long-term mean. This situation may have been caused by the degree of persistence of high flows being greater than that of low flows. In other words, the tendancy

Table 5.5 Fixed Truncation Levels for the Analysis of Generated Annual Flow Series for Multiyear Droughts

Stream	Gaging Station	Truncation Level X_c(cfs)	X_c/X_m
Pequest R.	Pequest	149.00	0.95
Musconetcong R.	Bloomsbury	227.00	0.95
Rockaway R.	Boonton	227.00	0.98
Passaic R.	Millington	87.00	0.97
Lamington R.	Pottersville	56.00	1.00
N.Br. Raritan R.	Raritan	306.00	1.00
Manasquan R.	Squankum	64.00	0.85

Table 5.6 Basic Statistical Parameters of the Stochastic Component of the Historic Mean Annual Flows & Recommended Model

Gaging Station	Mean (cfs)	Std. Dev. (cfs)	Coef. Skew.	Lag-1 Autocorr.	Recommended Model
Pequest	0.0	43.91	0.037	0.269	ARMA(1,1)
Bloomsbury	0.0	67.26	0.184	0.252	AR(2)
Boonton	0.0	69.29	0.163	0.410	ARMA(1,1)
Millington	0.0	24.68	0.388	0.187	AR(1)
Pottersville	0.0	14.99	0.376	0.224	AR(2)
Raritan	0.0	94.41	0.410	0.245	AR(2)
Squankum	0.0	18.07	0.196	0.339	AR(2)

of the basin towards droughts is comparatively low.

Table 5.6 indicates the basic statistical properties of the series formed by the stochastic component (ε_t) segregated from each flow event of each historic sequence. As expected ε_t is found to be distributed with a zero mean for every watershed. According to the Autocorrelation test results, each of these ε_t series represents a dependent process. Furthermore, the variance analysis indicated that each process is stationary. Therefore, it was possible to model each ε_t series either by an **Autoregressive(AR)** or by a mixed **Autoregressive Moving Average(ARMA)** process. The behaviours of the autocorrelation and partial autocorrelation functions (see Appendix C) were very helpful to make the judgement. The suggested best model is listed in Table 5.6.

Having determined the parameters of the deterministic component and the model to represent the stochastic component, they were combined together to form the general model for representation of each historic annual flow series. Then, this model was used to generate synthetic flow series of different length for each watershed. The basic statistical properties of these series are given in Table 5.7(a) thru Table 5.7(g).

The statistical parameters of the generated flow series were compared with the corresponding parameters of the historic series, and the desired generated series length was

Table 5.7 Variation of the Statistical Parameters of the Generated Flow Series with the Series Length

(a) Pequest River at Pequest

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor. (r_1)
60	155.2	51.6	0.332	0.516	0.378
120	147.7	52.3	0.354	0.471	0.434
180	150.4	53.3	0.354	0.363	0.420
240	151.3	52.1	0.344	0.327	0.391

(b) Musconetcong River near Bloomsbury

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor. (r_1)
60	238.0	75.9	0.319	0.075	0.393
120	227.2	79.0	0.348	0.374	0.381
180	229.5	81.5	0.355	0.404	0.360
240	230.8	79.1	0.343	0.288	0.351

(c) Rockaway River at Boonton

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor. (r_1)
49	229.7	66.6	0.290	0.060	0.355
98	214.6	73.3	0.342	0.235	0.474
147	218.7	75.4	0.345	0.410	0.442
196	225.4	74.2	0.329	0.278	0.508

(d) Passaic River near Millington

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor. (r_1)
65	87.8	27.5	0.313	0.178	0.210
130	85.5	28.5	0.333	0.504	0.275
195	86.2	29.5	0.342	0.419	0.299
260	86.6	28.6	0.330	0.325	0.265

(e) Lamington River near Pottersville

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor. (r_1)
64	55.5	17.5	0.315	0.317	0.355
128	53.3	17.9	0.336	0.369	0.345
192	54.0	18.7	0.346	0.440	0.378
256	54.5	17.9	0.328	0.392	0.345

(f) No. Br. Raritan River near Raritan

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor. (r_1)
62	301.3	93.7	0.311	0.423	0.257
124	285.1	94.8	0.333	0.376	0.299
186	293.0	101.8	0.347	0.385	0.341
248	295.4	99.3	0.336	0.327	0.308

(g) Manasquan River at Squankum

Flow Series Length (N)	Statistical Parameters				
	Mean (cfs)	Std.Dev. (cfs)	Coef.Var. (CV)	Coef.Skew. (CS)	Lag-1 Autocor.(r_1)
55	74.0	19.3	0.261	0.322	0.318
110	70.6	19.8	0.280	0.644	0.290
165	72.0	20.3	0.282	0.579	0.285
220	73.2	20.5	0.280	0.533	0.306

Table 5.8 Comparison of the Series Lengths of Historic and Generated Mean Annual Flows

Gaging Station	Number of Flows in Historic Series(N_H)	Number of Flows in Generated Series(N_G)	N_G/N_H
Pequest	60	240	4
Bloomsbury	60	240	4
Boonton	49	196	4
Millington	65	195	3
Pottersville	64	192	3
Raritan	62	248	4
Squankum	55	220	4

determined so that the parameters resulting from two populations give the best tally. Table 5.8 indicates the selected generated series length and its ratio to the historic series length, for each watershed. Table 5.9 makes a comparison between the basic statistical parameters estimated from the historic and the selected length of generated mean annual flow series. When carefully observed these numbers, it is clear that the corresponding parameter values tally very closely. Therefore, the selected generated flow series may be used for detailed drought analyses as they resemble the historic series.

Table 5.10 provides some useful conclusions on the nature of the distribution of drought and surplus parameters derived from the historic flow series at $X_c = X_m$. These decisions were made based on the serial correlation test performed on the drought and surplus parameters. Table 5.11 indicates the correlation coefficients between duration and severity of drought as well as of surplus events. Based on these computed values, it can be concluded that there exists a strong correlation between the duration and severity of each type of event.

A comparison among the statistical properties of the drought durations and drought severities derived from historic and generated flow series can be made by examining different parameter values listed in Table 5.12 and Table 5.13 respectively. According to Table 5.12, the mean drought

Table 5.9

Comparison of the Basic Statistical Parameters of the Historic and Generated Flow Series

	Historic Flows				Generated Flows			
Gaging Station	Mean (cfs)	Std. Dev. (cfs)	Coef. Skew	Lag-1 Autocor.	Mean (cfs)	Std. Dev. (cfs)	Coef. Skew	Lag-1 Autocor.
Request	156.7	51.7	0.334	0.318	151.3	52.1	0.327	0.391
Bloomsburg	238.9	76.3	0.320	0.331	232.2	79.3	0.327	0.378
Boonton	231.3	72.5	0.247	0.392	225.4	74.2	0.278	0.508
Millington	89.7	30.0	0.521	0.274	86.2	29.5	0.419	0.299
Pottersville	56.1	17.4	0.474	0.280	54.0	18.9	0.440	0.378
Raritan	306.4	99.1	0.600	0.232	295.4	99.3	0.327	0.308
Squankum	75.2	20.1	0.539	0.253	73.2	20.5	0.533	0.306

Table 5.10 Nature of the Distribution of Drought and Surplus Parameters Based on the Serial Correlation Test ($X_c/X_m = 1$)

Gaging Station	Drought Parameters L_d	S_d	τ_d	Surplus Parameters L_s	S_s	τ_s
Pequest	-R	-R	+NR	+R	-R	+NR
Bloomsbury	-R	-R	+NR	-R	+R	+NR
Boonton	-R	-R	+NR	-R	-R	+NR
Millington	+R	+R	+NR	-R	+R	+NR
Pottersville	-R	-NR	+NR	-R	+R	+NR
Raritan	-R	-R	+NR	+R	+R	+NR
Squankum	-R	-R	+NR	+R	+R	+NR

Note: Sign refers to the sign of the lag-1 autocorrelation coefficient

R implies random and NR implies non-random

Table 5.11 Correlation Coefficients of Drought and Surplus Parameters ($X_c/X_m = 1$)

Gaging Station	Drought Severity Vs. Drought Duration	Surplus Severity Vs. Surplus Duration
Pequest	0.865	0.958
Bloomsbury	0.862	0.970
Boonton	0.962	0.986
Millington	0.983	0.927
Pottersville	0.966	0.803
Raritan	0.980	0.937
Squankum	0.962	0.969

Table 5.12
Comparison of the Statistical Properties of the Drought Durations Derived from Historic and Generated Annual Flow Series

Stream No.	X_c/X_m	Historic Drought Events					Generated Drought Events				
		Mean (yrs)	Std.Dev. (yrs)	Coef. Varia.	Coef. Skew.	Lag-1 Autocor.	Mean (yrs)	Std.Dev. (yrs)	Coef Varia.	Coef Skew.	Lag-1 Autocor.
1	0.95	2.00	1.30	0.650	1.223	0.008	2.84	1.94	0.683	0.919	0.006
2	0.95	2.00	1.30	0.650	1.223	0.008	2.53	2.00	0.791	1.619	0.022
3	0.98	2.09	1.51	0.722	1.930	-0.183	2.86	1.93	0.675	1.143	-0.058
4	0.97	2.07	2.34	1.130	3.171	-0.133	2.50	2.42	0.968	1.827	-0.112
5	1.00	2.20	2.34	1.064	3.019	-0.236	3.30	2.66	0.806	2.389	0.012
6	1.00	2.07	1.28	0.618	1.038	0.092	2.86	1.65	0.577	1.086	-0.084
7	0.85	1.80	1.03	0.572	1.241	-0.458	1.71	1.20	0.702	2.118	0.288

Table 5.13

Comparison of the Statistical Properties of the Drought Severities Derived from Historic and Generated Annual Flow Series

Stream No.	X_c/X_m	Historic Drought Events					Generated Drought Events				
		Mean (cfs-yrs)	Std.Dev. (cfs-yrs)	Coef. Varia.	Coef. Skew.	Lag-1 Autocor	Mean (cfs-yrs)	Std.Dev. (cfs-yrs)	Coef. Varia.	Coef. Skew.	Lag-1 Autocor.
1	0.95	66.52	95.74	1.439	2.725	-0.134	109.15	106.60	0.977	1.455	0.092
2	0.95	98.55	132.85	1.348	2.664	-0.102	143.95	152.38	1.059	1.491	0.103
3	0.98	116.32	135.56	1.165	2.909	-0.240	163.80	176.73	1.078	1.593	0.032
4	0.97	40.48	56.27	1.390	2.862	-0.120	57.33	65.58	1.144	1.737	-0.118
5	1.00	27.57	39.08	1.417	2.900	-0.173	50.08	50.95	1.017	2.566	0.011
6	1.00	150.43	166.22	1.105	2.376	-0.063	231.60	197.94	0.855	1.503	0.144
7	0.85	15.63	11.69	0.748	1.842	-0.433	20.83	20.08	0.964	1.742	0.275

durations resulting from two types of flow series are very close to each other. When compared the coefficients of variation, it can be concluded that the patterns of historic and generated drought duration variations are almost the same. As the skewness coefficient is quite high, the drought durations are non-normally distributed. The values of the lag-1 autocorrelation coefficients confirm the fact that durations are randomly distributed.

When compared the results listed in Table 5.13, it is apparent that there are significant differences between the corresponding properties of the drought severities resulting from historic and generated flow series. This may be due to the fact that drought severity is formed by the sum of annual deficits (i.e., $\sum_{i=1}^{L_d}(X_c - X_i)$) during a particular drought and therefore, it indicates a large variance. Due to this reason, the mean values are not comparable. However, the following conclusions can be noted.

(1) the temporal distribution pattern of the generated drought severity is almost similar to that of the historic drought severity (according to the values for coefficient of variation),

(2) drought severities are randomly distributed in each type of flow series; the coefficients of skewness indicate that they follow non-normal distributions.

Table 5.14 indicates a comparison between the longest drought durations derived from historic and generated flow series. As expected, it is found that the generated longest drought duration is always longer than that of the historic longest drought.

Table 5.14 Comparisonof the Longest Drought Durations Derived from Historic and Generated Flow Series

Gaging Station	Truncation Level (X_c/X_m)	Historic Longest (yrs)	Generated Longest (yrs)
Pequest	0.95	5	8
Bloomsbury	0.95	5	9
Boonton	0.98	6	8
Millington	0.97	10	11
Pottersville	1.00	10	14
Raritan	1.00	5	8
Squankum	0.85	4	6

Then, next step was to propose suitable probability distribution functions for derived drought durations and drought severities. For this purpose, the statistical moments of each drought parameter were computed. As the historic series result in a small number of drought events, the droughts derived from the generated flow series were used to compute the important statistical parameters. By testing various classical probability distribution functions, it was possible to conclude that the drought durations can be well fitted by a 3-Parameter Gamma distribution while the drought severities can be fitted by a

3-Parameter Log Normal distribution. These facts were confirmed by Chi Squared Goodness of Fit tests (see Appendix E). The distribution parameters estimated for different populations of drought events derived from generated flow series for different watersheds are given in Table 5.15 and Table 5.16. In estimating these parameters, the fitted probability density functions were considered in the following forms.

<u>3-Parameter Gamma Distribution</u>

Probability Density Function (PDF),

$$f(X;a,b,c) = [(X-c)/a]^{b-1}.\exp[-(X-c)/a]/[|a|.\Gamma(b)]$$

where, X = variable

a, b, c are distribution parameters.

$\Gamma(b)$ = Gamma Function of 'b'

<u>3-Parameter Log Normal(LN3) Distribution</u>

Probability Density Function (PDF),

$$f(x;a',b',c') = [\sqrt{2\pi}.c'(x-a')]^{-1}.\exp\{-1/2[1/c'.\ln(x/b'-a'/b')]^2\}$$

where, x = variable

a', b', c' are distribution parameters

The figures in Appendix D graphically compare the probability distributions of drought parameters derived from historic and generated flow series. For example, Fig. D.1.1 and Fig. D.1.2 respectively compare the distributions of historic and generated drought durations and drought severities for Pequest watershed. Fig. D.1.3 provides a

Table 5.15 Parameters of the 3-Parameter Gamma Distributions Fitted for Drought Durations Derived from Generated Flow Series

Gaging Station	X_c/X_m	Parameters		
		a	b	c
(1) Pequest	0.95	0.891	4.736	-1.380
(2) Bloomsbury	0.95	1.619	1.526	0.059
(3) Boonton	0.98	1.103	3.062	-0.517
(4) Millington	0.97	2.211	1.198	-0.149
(5) Pottersville	1.00	3.179	0.700	1.075
(6) Raritan	1.00	0.896	3.392	-0.179
(7) Squankum	0.85	1.271	0.892	0.576

Table 5.16 Parameters of the 3-Parameter Log Normal Distributions Fitted for Drought Severities Derived from Generated Flow Series

Gaging Station	X_c/X_m	Parameters		
		a'	b'	c'
(1) Pequest	0.95	-125.73	213.89	0.433
(2) Bloomsbury	0.95	-184.63	298.09	0.441
(3) Boonton	0.98	-195.82	322.75	0.465
(4) Millington	0.97	-66.52	109.45	0.497
(5) Pottersville	1.00	-19.99	56.67	0.652
(6) Raritan	1.00	-192.22	384.00	0.444
(7) Squankum	0.85	-17.00	33.41	0.498

comparison between the observed probability distribution and the fitted probability distribution for drought durations derived from generated flow series; Fig. D.1.4 indicates a comparison between the observed and fitted probability distributions for drought severities. In order to make a comparison of the probability distributions of the number of droughts of different durations resulting from two types of flow series, the relative frequencies observed from the historic flow series were compared with the corresponding relative frequencies resulting from the generated flow series. For example, Fig. D.1.5 indicates these relative frequency variations for Pequest watershed. Also, the Drought Frequency-Duration-Severity curves were constructed by use of the generated flow series for different watersheds. Such diagrams may be important for future planning purposes in the region. For example, Fig. D.1.6 shows the probability distributions of drought severities of 1-year, 2-year and 5-year durations for Pequest watershed. Similarly, the rest of the figures represent the corresponding variations for the other six watersheds. In addition, Fig. D.1.7 was constructed for Pequest watershed for the purpose of illustrating how the historic drought duration distribution is sensitive to the truncation level.

In order to make a comparison of the largest drought severities, the observed largest values were standardized by using the following relationship.

Table 5.17 Comparison of the Standardized Largest Drought Severities Derived from Historic & Generated Flow Series

Gaging Station	Truncation Level(X_c/X_m)	Historic Largest	Generated Largest
Pequest	0.95	2.134	2.312
Bloomsbury	0.95	2.126	2.296
Boonton	0.98	2.062	2.161
Millington	0.97	2.177	2.278
Pottersville	1.00	2.176	2.465
Raritan	1.00	2.074	2.287
Squankum	0.85	1.926	2.367

Table 5.18 Comparison of the Expected Drought Magnitudes derived from Historic & Generated Flow Series

Gaging Station	Truncation Level(X_c/X_m)	Historic Expected Magnitude (cfs)	Generated Expected Magnitude (cfs)
Pequest	0.95	26.00	33.60
Bloomsbury	0.95	41.00	48.50
Boonton	0.98	53.20	46.40
Millington	0.97	17.50	20.50
Pottersville	1.00	10.60	14.00
Raritan	1.00	65.40	75.20
Squankum	0.85	8.90	11.30

If the variable x can be fitted by a 3-Parameter Log Normal distribution, its standardized value(y) can be given by:

$$y = 1/c'[\ln(x/b'-a'/b']$$

where a',b' and c' are the distribution parameters as mentioned in a previous paragraph.

In other words, y belongs to the Standard Normal population. Based on this procedure, the standardized largest drought severities estimated from both the historic and generated flow series are tabulated in Table 5.17. As expected, the generated largest drought severity is found to be larger than the historic largest drought severity for every watershed.

Table 5.18 provides a comparison between the mean values of the Drought Magnitudes(M_d) computed from historic and generated flow series. When examining these values, it is found that except for the Rockaway Watershed (Boonton), the expected drought magnitudes resulting from generated flow series are greater than the corresponding magnitudes resulting from historic flow series. The differences must be basically due to the differences indicated by variance values of drought severities estimated from two types of flow series.

Table 5.19 and Table 5.20 respectively indicate the cross-correlation structures formed by and . These determinations were made by using the historic mean annual

TABLE 5.19

Cross-Correlation Structure of τ_d Determined from Historic Mean Annual Flow Series

	Pequest	Bloomsbury	Boonton	Millington	Pottersville	Raritan	Squankum
Pequest	1.000	0.991	0.999	0.964	0.999	0.991	0.974
Bloomsbury		1.000	0.991	0.963	0.991	0.999	0.973
Boonton			1.000	0.965	0.999	0.991	0.974
Millington				1.000	0.964	0.963	0.992
Pottersville					1.000	0.991	0.974
Raritan						1.000	0.973
Squankum							1.000

Note: τ_d = Time of Occurrence of a Drought
Period of Flow Series: 1938 - 1985
Truncation Level $(x_c) = x_m$

TABLE 5.20

Cross-Correlation Structure of τ_s Determined from Historic Mean Annual Flow Series

	Pequest	Bloomsbury	Boonton	Millington	Pottersville	Raritan	Squankum
Pequest	1.000	0.994	0.999	0.934	0.999	0.994	0.951
Bloomsbury		1.000	0.993	0.960	0.993	0.999	0.976
Boonton			1.000	0.929	0.999	0.993	0.947
Millington				1.000	0.929	0.960	0.993
Pottersville					1.000	0.993	0.947
Raritan						1.000	0.976
Squankum							1.000

Note: τ_s = Time of Occurrence of a Surplus
Period of Flow Series : 1938-1985
Truncation Level $(x_c) = x_m$

flow series chosen over the common period of (1938-1985), for which the records are available for every watershed under study. Analyses were performed by placing the truncation level at the long-term mean (i.e., $X_c=X_m$). A thorough investigation of the tabulated values in these tables, may reveal some important informations relevant for an inter-basin planning study.

Few applications of these results for solving practical problems have been explained by examples in Appendix E.

CHAPTER VI. CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and Conclusions

Based on the results of the study, the following conclusions have been drawn.

(1) The mean annual flow series for different watersheds used for the study are approximately normally distributed. Detailed analyses indicate that each flow series can be treated as a dependent periodic-stochastic time series. Some flow series were found to be non-stationary because of the presence of periodicity. The spectral analysis reveals how the periodicity of the hydrologic cycle has been changed over the time.

(2) Each historic flow series was subjected to harmonic and stochastic analyses for modeling the periodic and stochastic components, respectively. The methodology used consists of optimizing the parameters of the deterministic component such that the stochastic component is distributed with zero mean. Theoretically, this stochastic component seperated from each flow event in the sequence must be uncorrelated. However, the analytical results of the study indicate that each series formed by the stochastic components represents a dependent stationary process. Therefore, the flow series was modeled by preserving the autoregressive and moving average properties accordingly.

(3) Flow series generated by applying the proposed model for

each watershed was found to resemble the historic series. The length of the generated series to be used for drought analysis was selected, so that it preserves the historic statistical and stochastic properties with negligible deviations.

(4) The truncation level used for distinguishing the high flow and low flow events was selected to result in a balance of the expected drought and surplus durations derived from the historic annual flow series of each watershed.

(5) Drought and surplus parameters (such as duration and severity) derived from the historic flow series were found to be randomly distributed. Therefore, the two types of events resulting from a given flow series can be assumed to be forming two independent discrete time processes. However, strong correlations were observed between the drought duration and drought severity as well as between the surplus duration and surplus severity.

(6) When the statistical properties of the generated drought parameters were compared with those of the historic drought parameters, the drought duration characteristics were found to be quite similar. However, the generated drought severity characteristics indicate deviations from the historic drought severity characteristics. This is because of the larger variance accompanied by the severity compared with that of the duration, and is expected.

(7) When analyzed, the distribution properties of drought durations and drought severities derived from long generated flow series for different watersheds indicated the following:

(a) The distribution of the drought durations can be approximated by a 3-Parameter Gamma distribution,

(b) The distribution of the drought severities can be approximated by a 3-Parameter Log Normal distribution.

These two facts were confirmed by performing Chi Squared Goodness of Fit tests at 95% significant levels.

(8) The cross-correlation structures of the time of occurrence of a drought (τ_d), the time of occurrence of a surplus (τ_s) may provide some clues on how the water defficiencies and sufficiencies are spatially distributed over the study area.

(9) The established cumulative distribution functions and their parameters for different drought parameters can be directly used for forecasting the extreme drought events within a given return period.

(10) In every analytical phase, it was noted that the tendancy of the Manasquan river basin towards droughts is smaller than the other six watersheds. Therefore, compared with Central New Jersey, the high degree of urbanization in the Northern New Jersey has almost become a threat to the hydrology of this region.

6.2 Recommendations

The present study was carried out to investigate the statistical and stochastic properties of multiyear drought parameters by analyzing the mean annual flow series. As the available historic flow records are not long enough to yield a considerable number of drought events, it was proposed to use synthetic flow series in addition to the historic flow sequences. In fact, a generated flow series of a desired length can be used to derive a large sample of drought events and thus, allows the analyst to perform a simulation study. Although such a study can reveal many important features of practical significance, it also involves significant calculations. Intime, one may be able to develop deterministic relationships between the drought parameters and the statistical and stochastic properties of the flow sequence. For this purpose, the present study can be used as a base study to recognize the possible ways to build up such relationships. The selection of the right variables to be correlated is very important. For example, the parameters of the hydrologic cycle can be thought of as making a direct contribution to the drought duration. Therefore, the dominant frequencies of the historic flow series must be able to be related to the drought duration characteristics. If this relationship can be satisfactorily presented, the distributions of the other drought parameters such as severity can be easily predicted by use of the observed strong correlation between duration and severity. It is also necessary to include a technique of preserving the persistant characteristics of the flow series, for a complete study.

Further analyses of the statistics of drought parameters must be able to detect how the drought characteristics are related to the non-stationarity of the flow series. Such a study would be very interesting.

To establish the conclusions of this study it is recommended that a similar study based on the precipitation analyses be conducted. However, in such a study, the applicability of a generation technique is doubtful and therefore, simulation has to be carried out by using other possible methods.

APPENDICES

APPENDIX A DROUGHT INDICES

Author(year)	Index	Description
Ivanov(1948)	$k = P/E$ $0 \leq k \leq 0.12$ -deserts $0.13 \leq k \leq 0.29$ -semi deserts $0.3 \leq k \leq 0.59$ -steppes $0.6 \leq k \leq 0.99$ -forested steppes $1.0 \leq k \leq 1.49$ -sufficient moisture $1.5 \leq k$ -excess moisture	P = annual precipitation (mm) E = annual evapotranspiration derived from, $E = 0.0018(25+T)^2(100-a)$ where, T = mean monthly temp. a = mean monthly relative humidity
Popov(1948)	$I = P_e/[2.4(T-T')r]$	P_e = annual effective precipitation(mm) T-T' = annual mean wet bulb depression(C) r = factor depending on day length I = index of aridity
Capot-Rey(1951)	$I = 0.5\left[\frac{P}{(T+10)} + \frac{12p}{(t+10)}\right]$	P = mean annual precipitation(mm) T=mean annual temp. (C°) p = monthly precipitation(mm) t = monthly temp.(C°) I = improved aridity index
Emberger(1955)	$I = 100p/(M^2-m^2)$	p = monthly precipitation(mm) M = mean maximum temp. in the hottest month(C°) m = mean minimum temp. in the coldest month (C°)

APPENDIX A DROUGHT INDICES (Continued)

Author(year)	Index	Description
Palmer(1965)	$X_i = \sum_{t=1}^{i} \frac{(P-\hat{P}).(\overline{PE}+\overline{R})}{(0.31t+2.691)(\overline{P}+\overline{L})}$ $4 \leqslant X_i$ -extremely wet $3 \leqslant X_i \leqslant 3.99$ -very wet $2 \leqslant X_i \leqslant 2.99$ -moderately wet $1 \leqslant X_i \leqslant 1.99$ -slightly wet $0.5 \leqslant X_i \leqslant 0.99$ -incipient wet spell $-0.49 \leqslant X_i \leqslant 0.49$ -near normal $-0.99 \leqslant X_i \leqslant -0.5$ -incipient $-1.99 \leqslant X_i \leqslant -1$ -mild drought $-2.99 \leqslant X_i \leqslant -2$ -moderate drought $-3.99 \leqslant X_i \leqslant -3$ -severe drought $X_i \leqslant -4$ -extreme drought	$\overline{PE}$ = mean monthly potential evapotranspiration (mm) $\overline{R}$ = mean monthly soil moisture recharge (mm) $\overline{P}$ = mean monthly precipitation(mm) $\overline{L}$ = mean monthly soil moisture loss(mm) P = areal monthly precipitation(mm) $\hat{P}$ = climatically appropriate water balance for existing conditions(mm) X_i= drought severity index
Sly(1970)	I = 100P/(P+SM+IR)	P = growing season precipitation(mm) SM= soil moisture available to crops at beginning of growing season IR = calculated growing season irrigation requirement(mm) I = climatic moisture index
Budyko (1970)	$k = P/(0.18\sum\theta)$	$\sum\theta$= annual sum of dailymean temperatures higher than 10C $0.18\sum\theta$ = potential evapotranspiration(mm) P = annual precipitation(mm) k = hydrothermal coefficient

APPENDIX B THEORY OF STOCHASTIC PROCESSES (Modeling)

B.1 Linear Stationary Processes

The general linear process can be written as follows:

$$z_t = \mu + a_t + \psi_1 \cdot a_{t-1} + \psi_2 \cdot a_{t-2} + \cdots\cdots\cdots$$
$$= \mu + a_t + \sum_{j=1}^{\infty} \psi_j \cdot a_{t-j}$$

where, z_t = value of the variable at time t,

μ = mean value of all z_t's,

a_t = white noise of z_t,

ψ_j = operational function of a_{t-j}

By denoting $\bar{z}_t = z_t - \mu$

$$\bar{z}_t = a_t + \sum_{j=1}^{\infty} \psi_j \cdot a_{t-j} \qquad \text{(B.1)}$$

The white noise process a_t consists of a sequence of uncorrelated random variables with zero mean and constant variance.

Thus, $E[a_t] = 0$; $Var[a_t] = \sigma_a^2$

Autocovariance function, $\gamma_k = E[a_t \cdot a_{t+k}] = \begin{cases} \sigma_a^2, & k = 0 \\ 0, & k \neq 0 \end{cases}$

Therefore, the autocorrelation function of white noise has the particularly simple form:

$$\rho_k = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

The general linear process in the form of (B.1) would not be very useful in practice, because it contains an infinite number of parameters. Therefore, it is necessary to introduce finite, parsimony models as derivations of the model given by (B.1). The literature provides three types of such models. They are namely, **Autoregressive(AR)**, **Moving Average(MA)** and mixed **Autoregressive Moving Average(ARMA)** processes.

B.1.1 Autoregressive(AR) Processes

The autoregressive process of order p, denoted by AR(p) can be written as:

$$\bar{z}_t = \phi_1.\bar{z}_{t-1}+\phi_2.\bar{z}_{t-2}+.............+\phi_p.\bar{z}_{t-p}+a_t$$

The terms $\phi_1,\phi_2,......,\phi_p$ are known as autoregressive parameters of order 1, 2,......p respectively.

By introducing the backward operator B, the above expression can be simplified as follow:

$$(1-\phi_1.B-\phi_2.B^2-.......-\phi_p.B^p)\bar{z}_t = a_t$$

or,
$$\phi(B)\bar{z}_t = a_t \qquad (B.2)$$

This process is invertible for all values of ϕ. For stationarity, the roots of $\phi(B) = 0$ must lie outside the unit circle.

The autoregressive parameters $\phi_1,\phi_2,.....,\phi_p$ are related to the autocorrelation coefficients $\rho_1,\rho_2,....,\rho_p$ by the following set of Yule-Walker equations.

$$\rho_1 = \phi_1 + \phi_2\rho_1 + ...+\phi_p\rho_{p-1}$$
$$\rho_2 = \phi_1\rho_1 + \phi_2 + ...+\phi_p\rho_{p-2}$$
$$\vdots$$
$$\rho_p = \phi_1\rho_{p-1}+\phi_2\rho_{p-2}+..........+\phi_p$$

For a real study, these theoretical autocorrelation coefficients must be replaced by the sample autocorrelation coefficients $r_1, r_2, \ldots\ldots, r_p$.

Also, $\text{Var}[a_t] = (1 - \rho_1\phi_1 - \rho_2\phi_2 - \ldots\ldots\ldots - \rho_p\phi_p).\text{Var}[\bar{z}_t]$

AR(1) Model

By taking into account only the first order autoregressive parameter ϕ_1, AR(1) model can be written as:

$$(1-\phi_1 B)\bar{z}_t = a_t$$

For stationarity, $-1 < \phi_1 < 1$

From Yule-Walker equations: $\phi_1 = \rho_1$

and, $\text{Var}[a_t] = (1-\phi_1^2).\text{Var}[z_t]$

AR(2) Model

This model can be written as:

$$(1-\phi_1 B - \phi_2 B^2)\bar{z}_t = a_t$$

For stationarity, the roots of $(1 - \phi_1 B - \phi_2 B^2) = 0$ must lie outside the unit circle.

i.e., $\phi_1 + \phi_2 < 1$

$\phi_2 - \phi_1 < 1$

$-1 < \phi_2 < 1$

The application of the Yule-Walker equations results in:

$$\phi_1 = \rho_1(1-\rho_2)/(1-\rho_1^2)$$

and, $\phi_2 = (\rho_2 - \rho_1^2)/(1-\rho_1^2)$

Also, $\text{Var}[a_t] = (1-\rho_1\phi_1 - \rho_2\phi_2).\text{Var}[z_t]$

Similarly, higher order Autoregressive models can be derived.

B.1.2 Moving Average(MA) Processes

The moving average process of order **q**, denoted by MA(q) can be written as:

$$\bar{z}_t = (1 - \theta_1 . B - \theta_2 . B^2 - \ldots\ldots\ldots\ldots - \theta_q . B^q) a_t$$

or, $$\bar{z}_t = \theta(B) . a_t \qquad (B.3)$$

where, $\theta_1, \theta_2, \ldots\ldots, \theta_q$ are known as moving average parameters of order 1,2,.....,q respectively.

The process is stationary for all values of . For invertibility, the roots of $(1 - \theta_1 B - \theta_2 B^2 - \ldots\ldots - \theta_q B^q) = 0$ must lie outside the unit circle.

$$Var[a_t] = Var[\bar{z}_t]/(1 + \theta_1^2 + \theta_2^2 + \ldots\ldots + \theta_q^2),$$ and

the autocorrelation coefficients are related to the moving average parameters as follows.

$$\rho_k = \begin{cases} \dfrac{-\theta_k + \theta_1 . \theta_{k+1} + \ldots\ldots\ldots\ldots + \theta_{q-k} . \theta_q}{1 + \theta_1^2 + \theta_2^2 + \ldots\ldots\ldots + \theta_q^2}, & k = 1,2,\ldots\ldots,q \\ 0, & k > q \end{cases}$$

MA(1) Model

This model can be written as:

$$\bar{z}_t = (1 - \theta_1 B) a_t$$

For invertibility, the roots of $(1 - \theta_1 B) = 0$ must lie outside the unit circle.

This implies, $-1 < \theta_1 < 1$

and, $$\rho_k = \begin{cases} -\theta_1/(1+\theta_1^2), & k=1 \\ 0, & k \geq 2 \end{cases}$$

$$Var[a_t] = Var[\bar{z}_t]/(1+\theta_1^2)$$

MA(2) Model

By taking into account the first two moving average parameters, this model can be written as:

$$\bar{z}_t = (1 - \theta_1 B - \theta_2 B^2) a_t$$

For invertibility, $\theta_2 + \theta_1 < 1$

$$\theta_2 - \theta_1 < 1$$

$$-1 < \theta_2 < 1$$

Also, the following relationships hold:

$$\rho_1 = -\theta_1(1-\theta_2)/(1+\theta_1^2+\theta_2^2)$$

$$\rho_2 = -\theta_2/(1+\theta_1^2+\theta_2^2)$$

$$\rho_k = 0 \quad \text{for } k \geqslant 3$$

and, $\mathrm{Var}[a_t] = \mathrm{Var}[z_t]/(1+\theta_1^2+\theta_2^2)$

Similarly, higher order moving average models can be derived.

B.1.3 Autoregressive Moving Average (ARMA) Processes

The mixed autoregressive moving average model of order (p,q), denoted by ARMA(p,q) can be written as:

$$(1-\phi_1 B-\phi_2 B^2- \ldots -\phi_p B^p)\bar{z}_t = (1-\theta_1 B-\theta_2 B^2- \ldots -\theta_q B^q)a_t$$

or, $\phi(B)\bar{z}_t = \theta(B)a_t$

For stationarity, the roots of $\phi(B) = 0$ must lie outside the unit circle.

For invertibility, the roots of $\theta(B) = 0$ must lie outside the unit circle.

ARMA(p,q) process accounts **q** number of autocorrelation coefficients $\rho_1, \rho_2, \ldots, \rho_q$ whose values depend directly on the choise of the **q** moving average parameters. Also, the **p** values $\rho_p, \rho_{p-1}, \ldots, \rho_{q-p+1}$ provide the necessary starting values for the difference equation $\phi(B)\rho_k = 0$, where $K \geq q+1$, which then entirely determines the autocorrelations at higher lags. If $q-p<0$, the whole autocorrelation function ρ_j, for $j = 0,1,2,\ldots$ will consist of a mixture of damped exponentials and/or damped sine waves, whose nature is dictated by the polynomial $\phi(B)$ and the starting values. If however, $q-p>0$ there will be $q-p+1$ initial values $\rho_0, \rho_1, \ldots\ldots, \rho_{q-p}$ which do not follow this general pattern.

It can be shown that the autoregressive parameters are related to the autocorrelation coefficients by,

$$\rho_k = \phi_1.\rho_{k-1} + \phi_2.\rho_{k-2} + \ldots\ldots\ldots\ldots + \phi_p.\rho_{k-p} \qquad \text{for } k>q+1$$

ARMA(1,1) Model

This model can be written as:

$$(1 - \phi_1 B)\bar{z}_t = (1 - \theta_1 B)a_t$$

For stationarity, $-1<\phi_1<1$

For invertibility, $-1<\theta_1<1$

Also, it can be shown that,

$$\rho_1 = (1-\phi_1\theta_1)(\phi_1 - \theta_1)/(1+\theta_1^2-2\phi_1\theta_1)$$

$$\rho_2 = \phi_1\rho_1$$

and, $Var[a_t] = (1 - \phi_1^2).Var[\bar{z}_t]/(1 + \theta_1^2 - 2\phi_1\theta_1)$

Similarly, higher order models can be derived.

B.2 Model Identification

The identification of a model for a stochastic process is necessarily inexact. This is because of the difficulty of recognizing the exact mechanism of the process by examining few properties and also the inability of a simple mathematical model to represent a complicated natural process.

Some useful clues for identification of a suitable model for a given process are given below.

Stationary Processes: for large lags, the autocorrelation function of a stationary process quickly dies out.

Non-stationary Processes: if the process is non-stationary its autocorrelation function falls off slowly and very nearly linearly. When the degree of differencing is **d**, the autocorrelation function of $\nabla^d z_t$ must die out quickly.

Autoregressive Processes

For an autoregressive process, the following properties are dominant:

(i) autocorrelation function tails off,

(ii) if it is an AR(p) process, the partial autocorrelation function indicates a cutoff after lag **p**.

Moving Average Processes

For a moving average process, the following characteristics are dominant:

(i) if the process is MA(q), its autocorrelation function indicates a cutoff after lag **p**,

(ii) partial autocorrelation function tails off.

Autoregressive Moving Average Processes

If the process is ARMA(p,q), it should emphasise the following characteristics:

(i) autocorrelation function is a mixure of exponential and damped sine waves after first (q-p) lags,

(ii) partial autocorrelation function is a mixture of exponential and damped sine waves after first (p-q) lags.

B.3 Partial Autocorrelation Function

The partial autocorrelation function is a device which exploits the fact that whereas an AR(p) process has an autocorrelation function which is infinite in extent, it can by its very nature be described in terms of **p** non-zero functions of the autocorrelations.

Let ϕ_{kj} = j th partial autocorrelation coefficient in an autoregressive process of order k.

Then,

$$\rho_j = \phi_{k1} \cdot \rho_{j-1} + \cdots\cdots + \phi_{k,k-1} \cdot \rho_{j-k+1} + \phi_{kk} \cdot \rho_{j-k} \quad \text{for } j=1,2,\ldots k$$

where, terms represent the autocorrelation coefficients.

The general relationship between the autocorrelation coefficients can be given by the following set of Yule-Walker equations.

$$\begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots\cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots\cdots & \rho_{k-2} \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots\cdots & 1 \end{bmatrix} \begin{bmatrix} \phi_{k1} \\ \phi_{k2} \\ \cdot \\ \cdot \\ \phi_{kk} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdot \\ \cdot \\ \rho_k \end{bmatrix}$$

For example, $\phi_{11} = \rho_1$

$$\phi_{22} = (\rho_2 - \rho_1)/(1-\rho_1^2)$$

By replacing the theoretical autocorrelation coefficients (ρ_j), by the sample estimates (r_j), in general, the following recursive formulae can be used.

$$\phi_{p+1,j} = \phi_{pj} - \phi_{p+1,p+1} \cdot \phi_{p,p-j+1}; \quad j = 1,2,\ldots,p$$

$$\phi_{p+1,p+1} = r_{p+1} - \sum_{j=1}^{p} \phi_{pj} \cdot r_{p+1-j} / (1 - \sum_{j=1}^{p} \phi_{pj} \cdot r_j)$$

It must be noted that for an autoregressive process of order p, p', ϕ_{kk} will be non-zero for k<p and will be zero for k≥p.

B.4 Linear Non-stationary Processes

Many time series behave like they have no fixed mean, thus indicating non-stationary character. However, a new series formed by the **d** th differences (d=1,2,..) of the original observations may be stationary. Thereafter, a suitable Autoregressive(AR), Moving Average(MA) or a mixed Autoregressive Moving Average(ARMA) model can be proposed for this stochastic process. Such a model is referred as an **Integrated Autoregressive Moving Average**(ARIMA) model. In this case, the model must be designated by three variables, namely, the difference(d), the number of autoregressive parameters(p) and the number of moving average parameters(q). The general model, therefore is denoted by ARIMA(p,d,q). The orders **p,q** and the values of the corresponding parameters have to be determined for the stationary process formed by the **d** th differences.

General ARIMA Model

Consider the model given by,

$$\psi(B).\bar{z}_t = \theta(B).a_t$$

where, $\psi(B)$ is a non-stationary autoregressive operator, such that d of the roots of $\psi(B)=0$ are unity and the remainder lie outside the unit circle.

Then, the above model can be rewritten as:

$$\psi(B).\bar{z}_t = \phi(B).(1-B)^d.\bar{z}_t = \theta(B).a_t$$

where, $\phi(B)$ is a stationary autoregressive operator.

Since $\quad \nabla^d \bar{z}_t = \nabla^d z_t \quad$ for $d \geq 1$

$$\phi(B).\nabla^d z_t = \theta(B).a_t$$

i.e., the d th difference of the original observations form a stationary process.

ARIMA(0,1,1) Model

This model can be written as:

$$\nabla z_t = a_t - \theta_1.a_{t-1}$$
$$= (1-\theta_1 B)a_t$$

where, θ_1 = moving average parameter of order 1.

ARIMA(0,2,2) Model

This can be written as:

$$\nabla^2 z_t = a_t - \theta_1.a_{t-1} - \theta_2.a_{t-2}$$
$$= (1-\theta_1 B - \theta_2 B^2)a_t$$

corresponding to p=0, d=2 and q=2.

θ_1, θ_2 are the first and second order moving average parameters.

<u>ARIMA(1,1,1) Model</u>

This model can be written as:

$$(1-\phi_1 B).\nabla z_t = (1-\theta_1 B)a_t$$

where, ϕ_1= first order autoregressive parameter of ∇z_t,

θ_1= first order moving average parameter of ∇z_t.

APPENDIX C

STATISTICAL & STOCHASTIC BEHAVIOUR OF HISTORIC FLOW SERIES

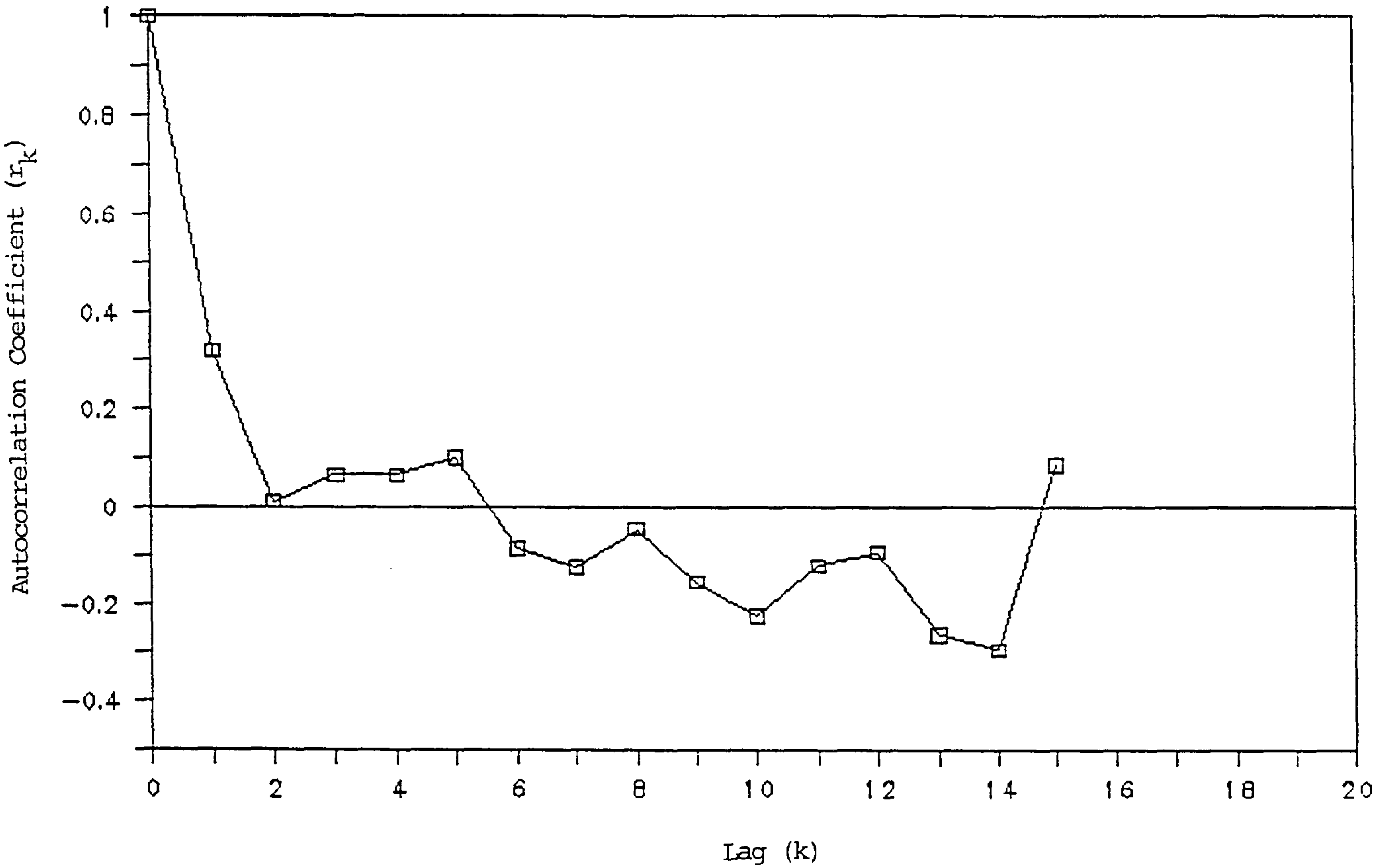


Fig. C.1.1 Autocorrelation Function of the Historic Mean Annual Flow Series - Pequest River at Pequest

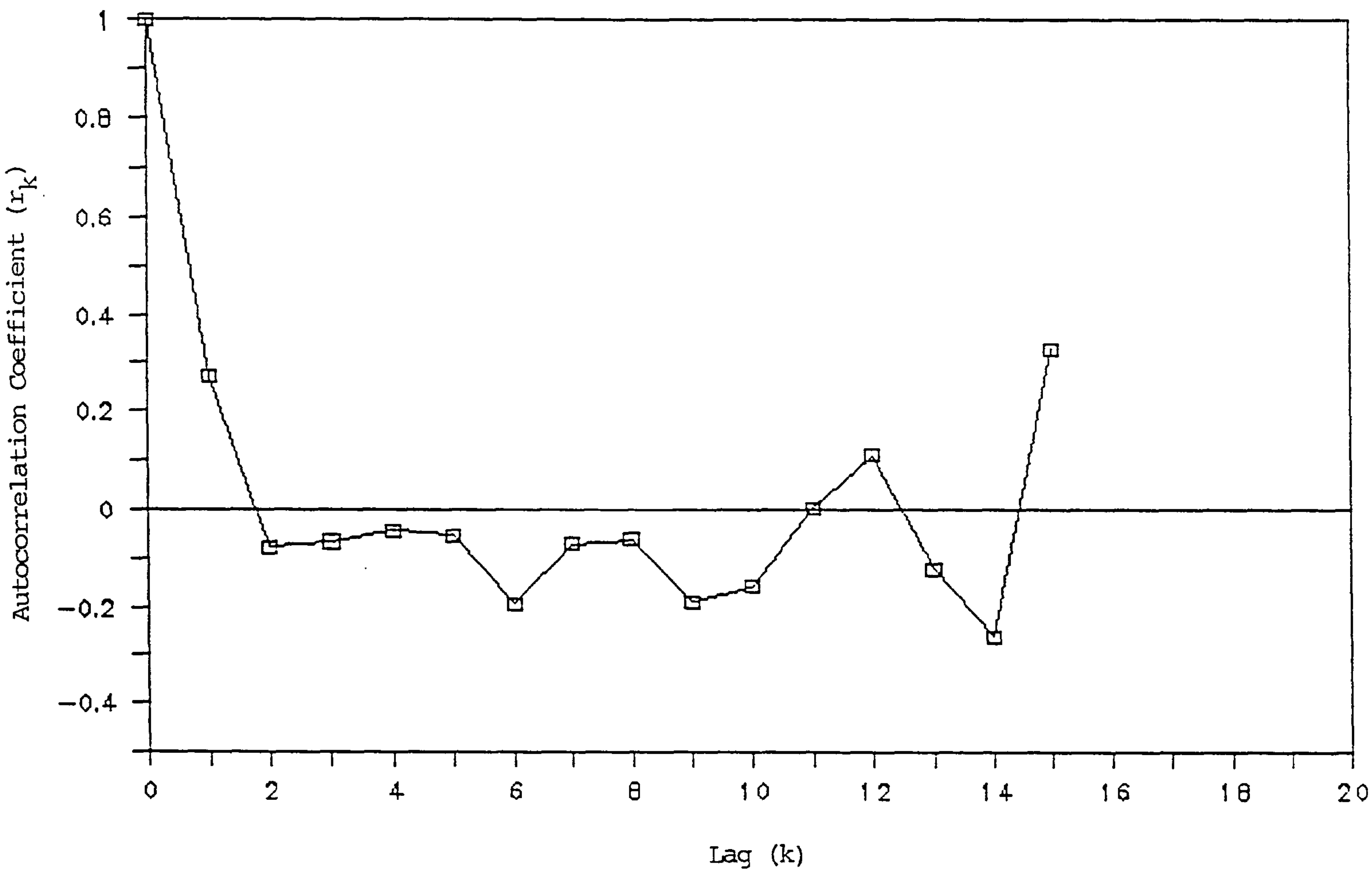


Fig. C.1.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Pequest River at Pequest

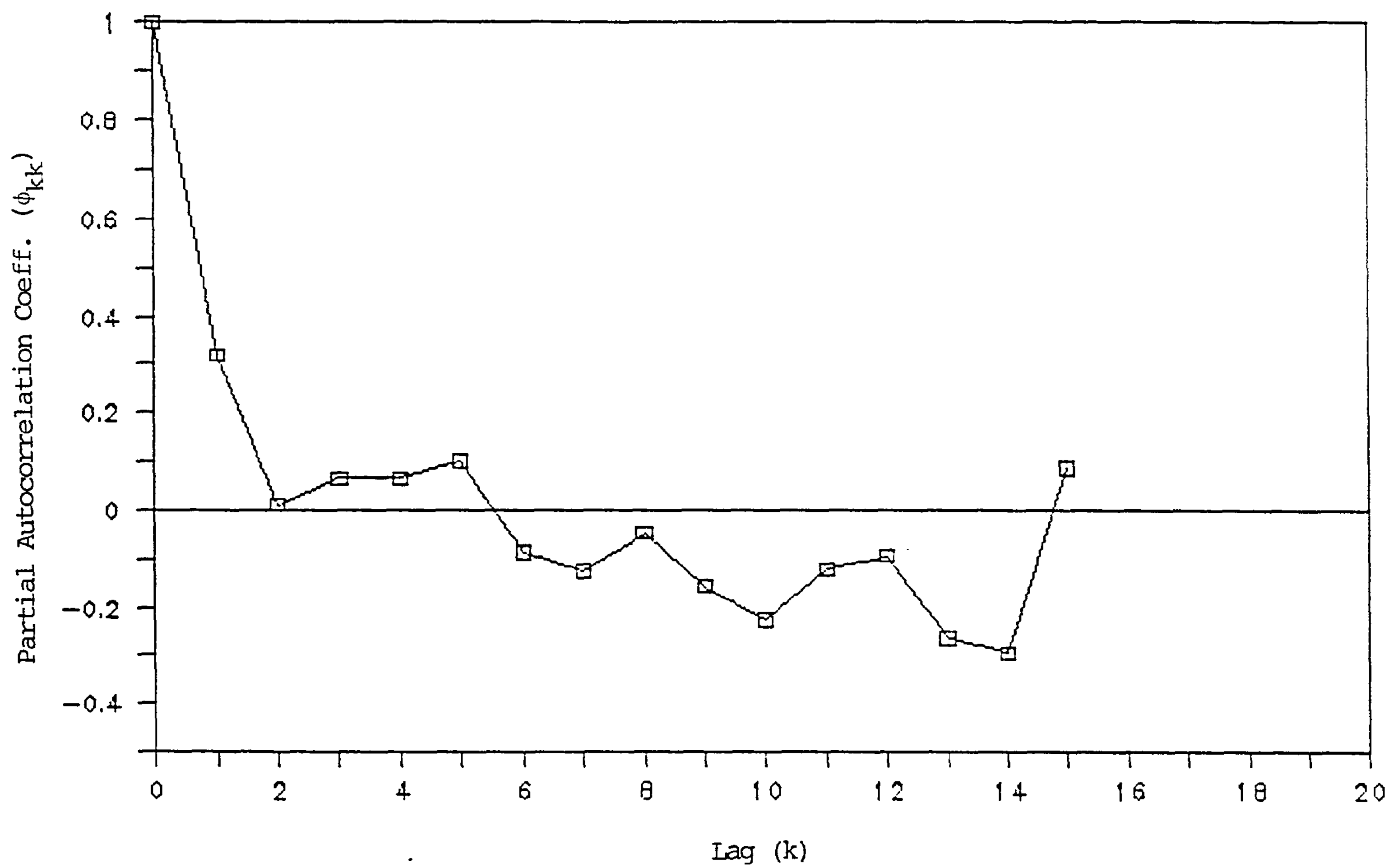


Fig. C.1.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Pequest River at Pequest

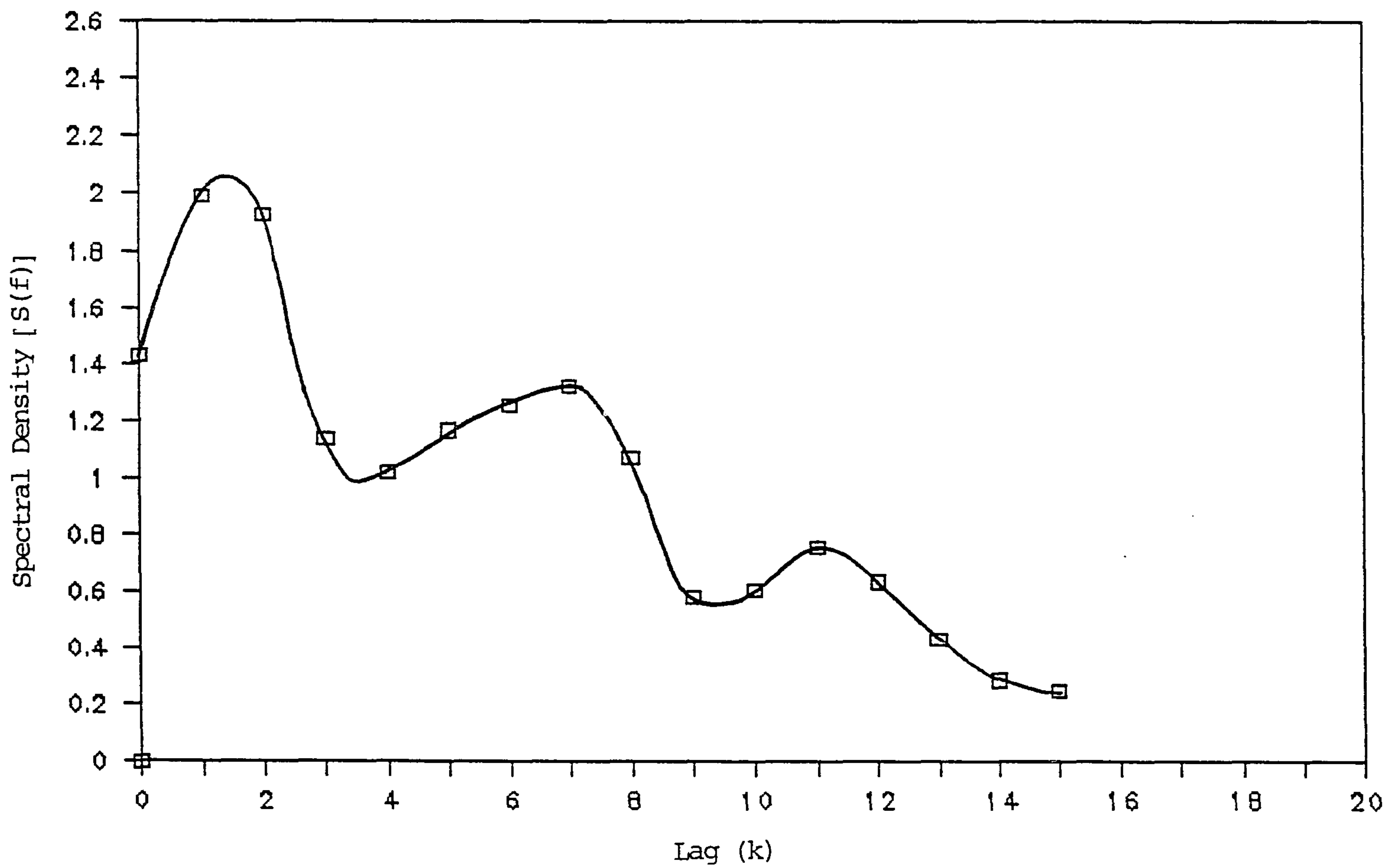


Fig. C.1.4 Spectral Density Function of the Historic Mean Annual Flow Series - Pequest River at Pequest

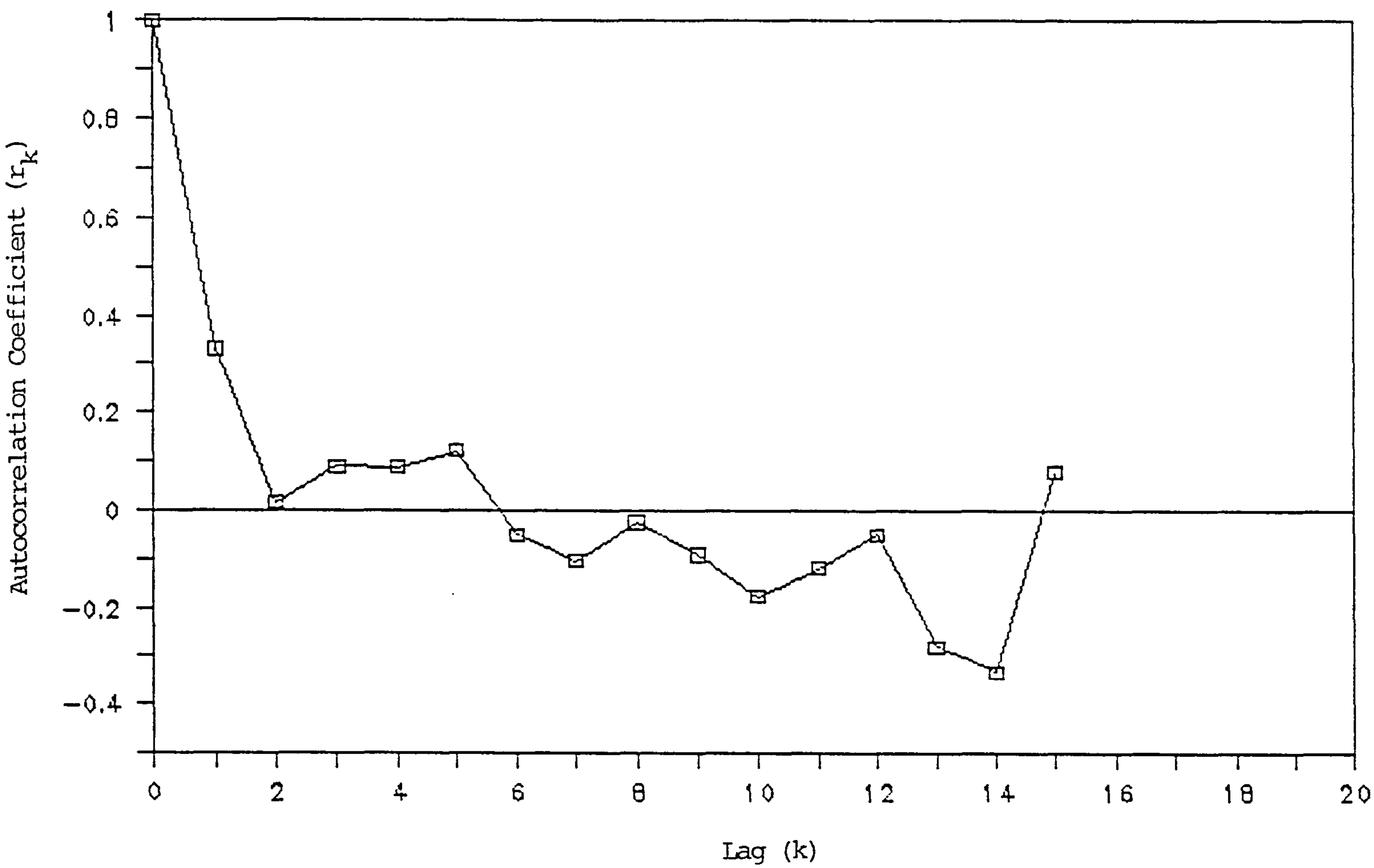


Fig. C.2.1 Autocorrelation Function of the Historic Mean Annual Flow Series - Musconetcong River near Bloomsbury

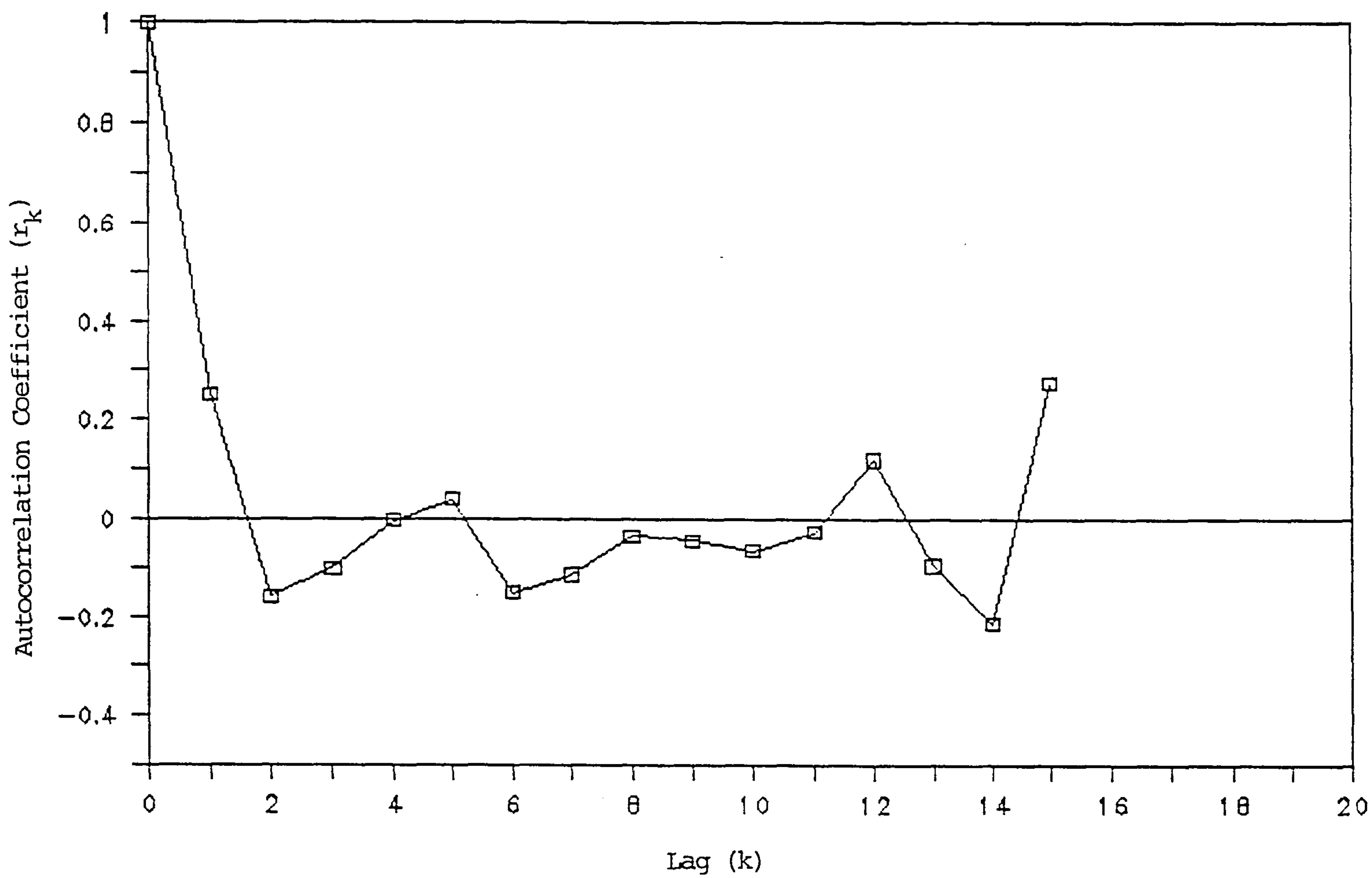


Fig. C.2.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Musconetcong River near Bloomsbury

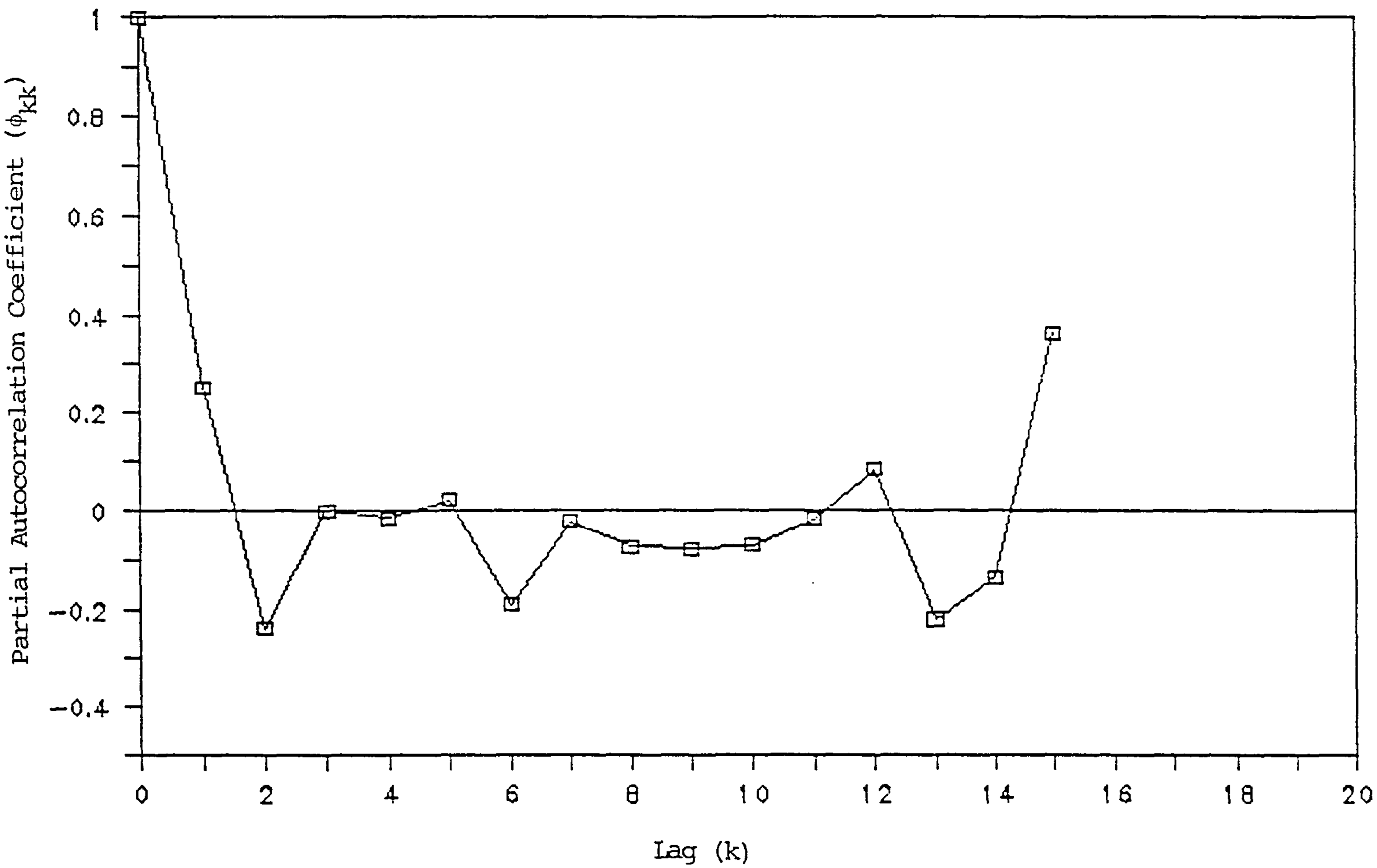


Fig. C.2.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Musconetcong River at Bloomsbury

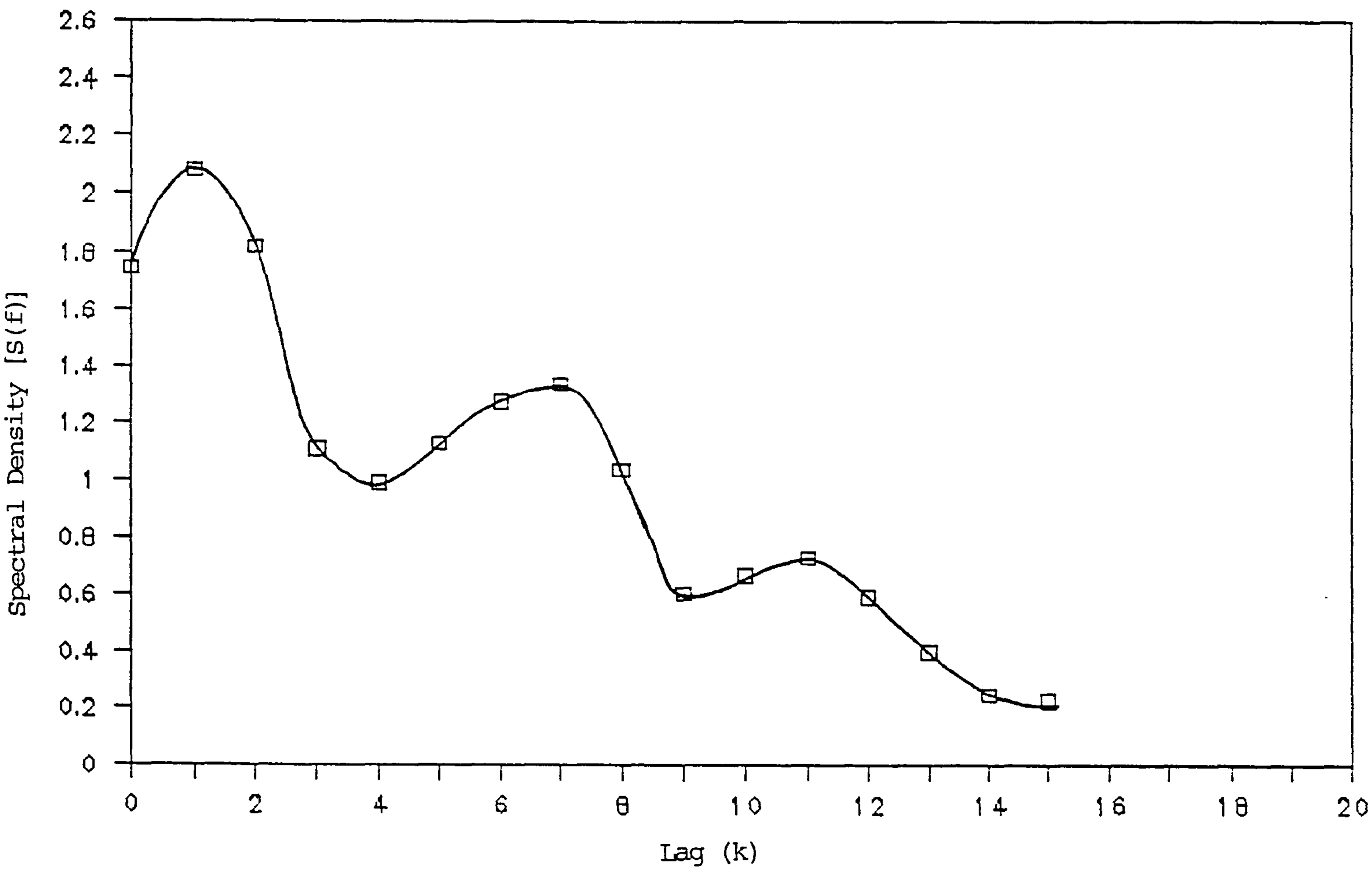


Fig. C.2.4 Spectral Density Function of the Historic Mean Annual Flow Series - Musconetcong River near Bloomsbury

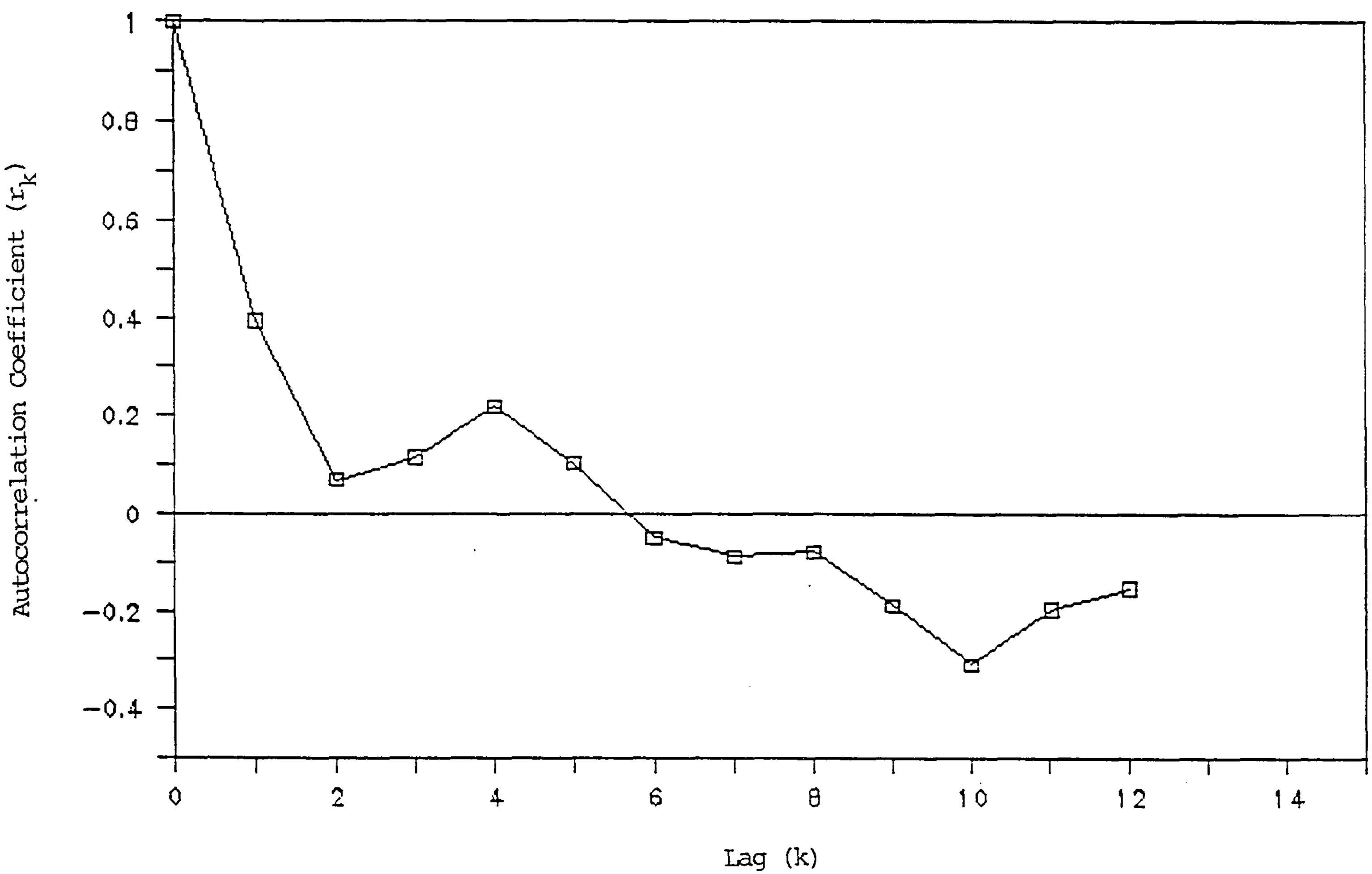


Fig. C. 3.1 Autocorrelation Function of the Historic Mean Annual Flow Series-Rockaway River at Boonton

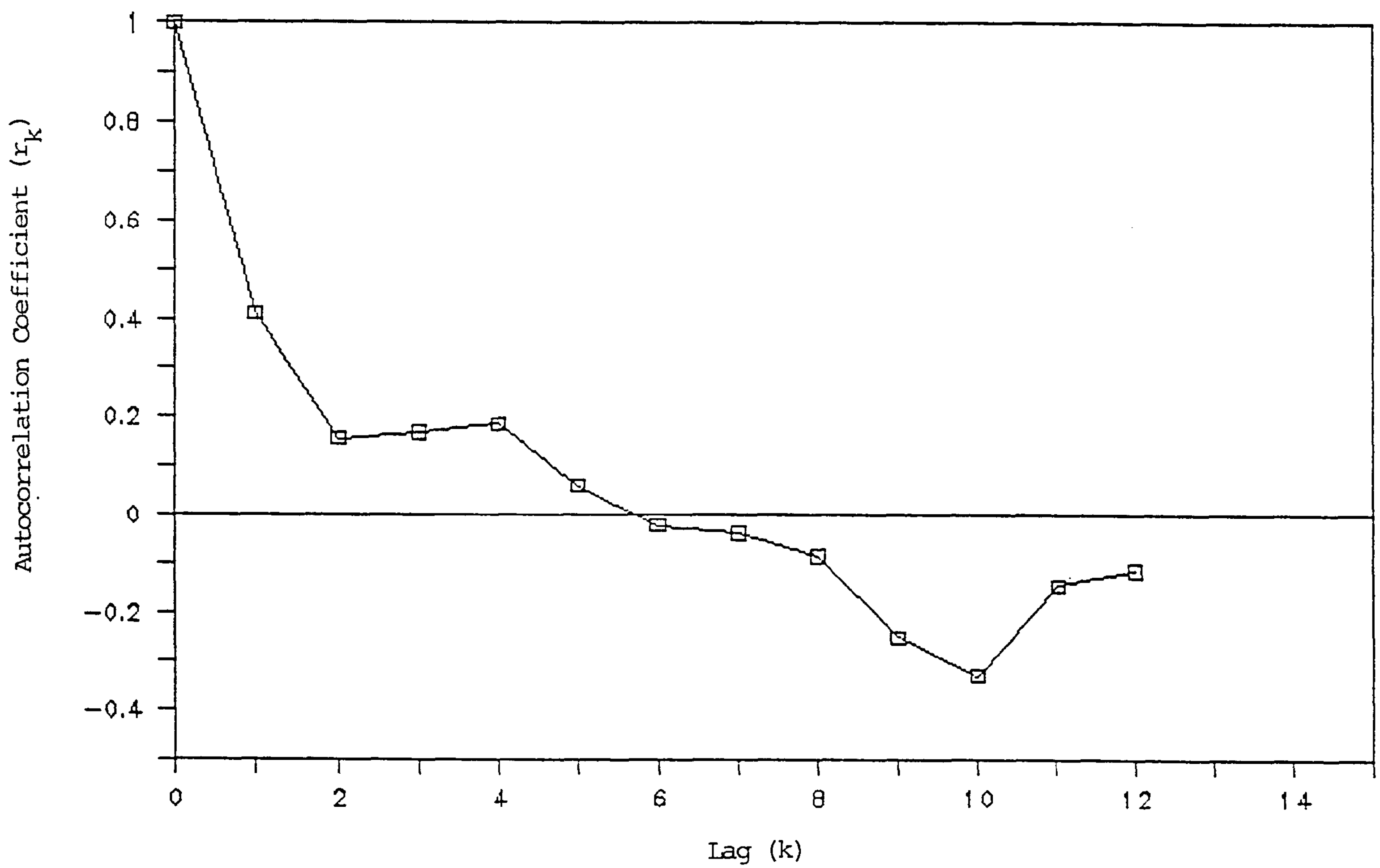


Fig. C. 3.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Rockaway River at Boonton

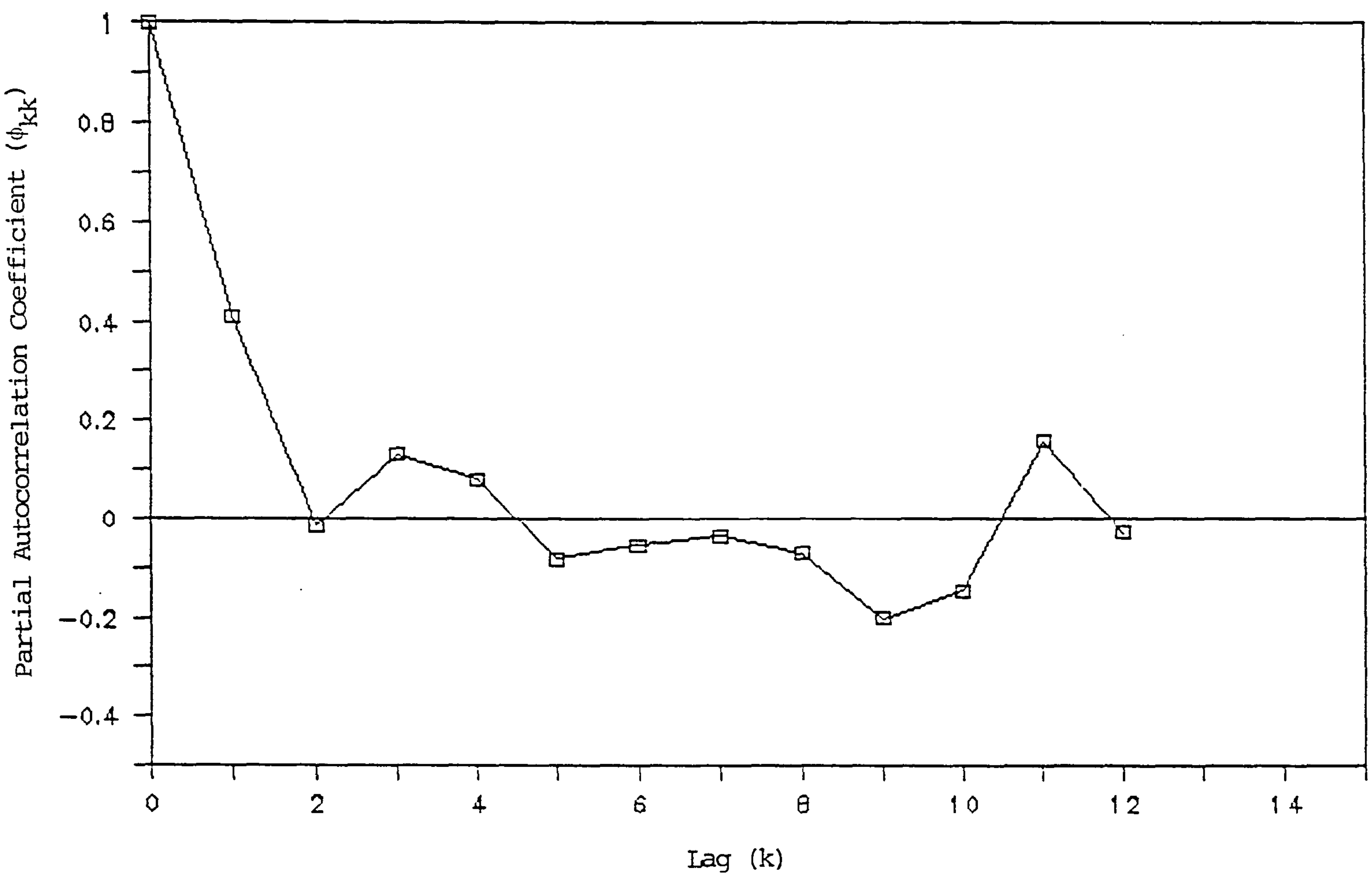


Fig. C. 3.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Rockaway River at Boonton

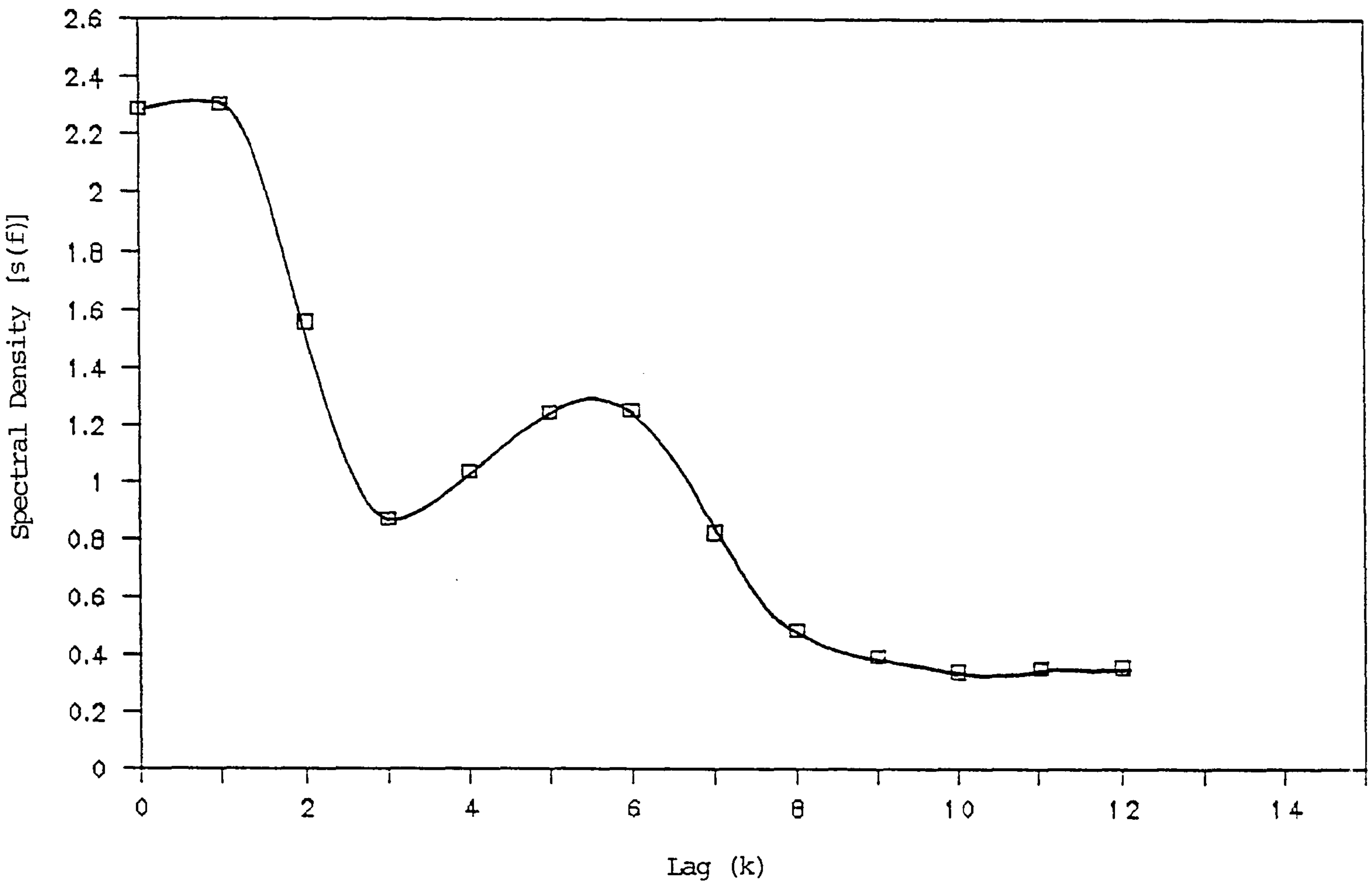


Fig. C. 3.4 Spectral Density Function of the Historic Mean Annual Flow Series - Rockaway River at Boonton

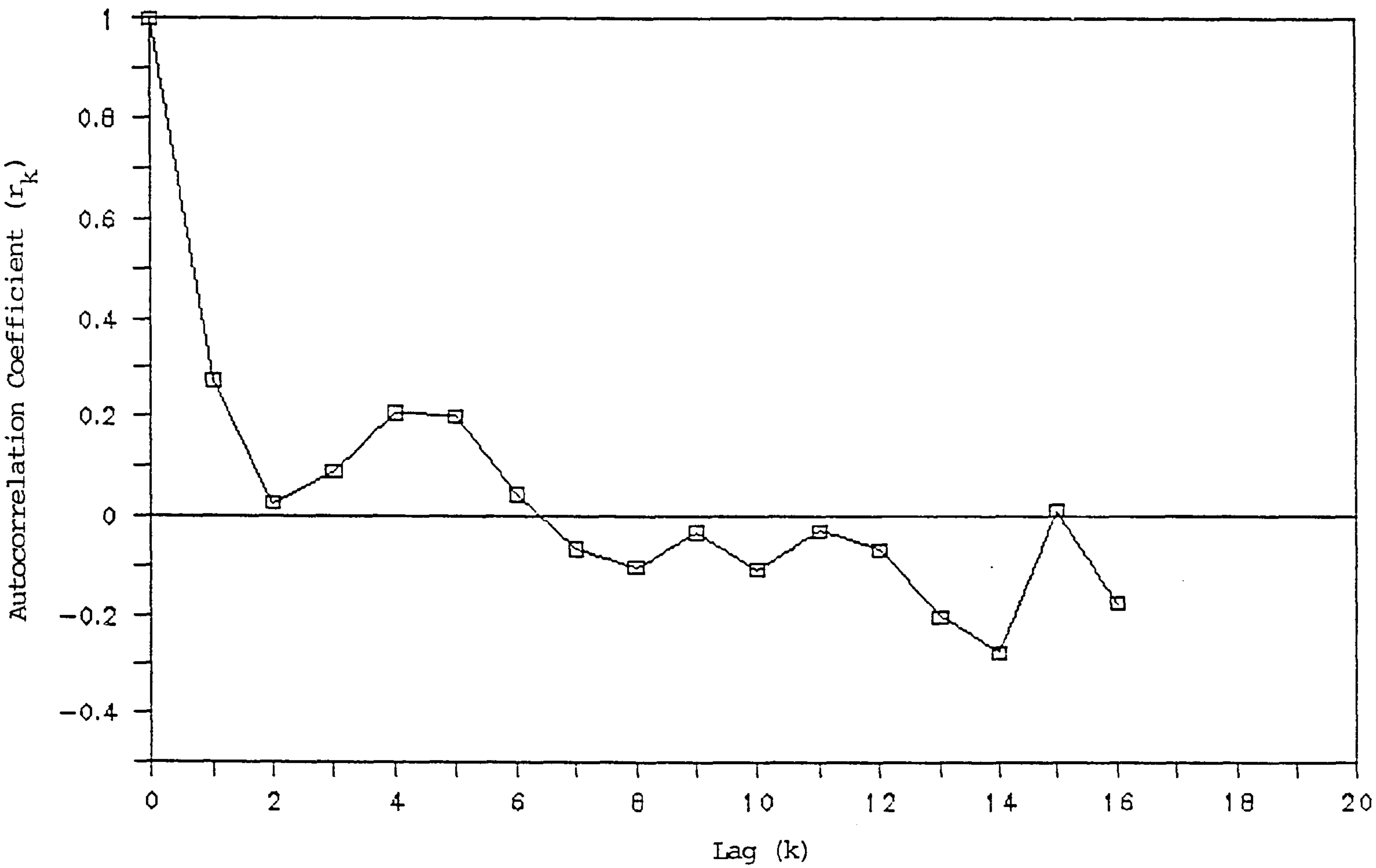


Fig. C.4.1 Autocorrelation Function of the Historic Mean Annual Flow Series - Passaic River near Milltinton

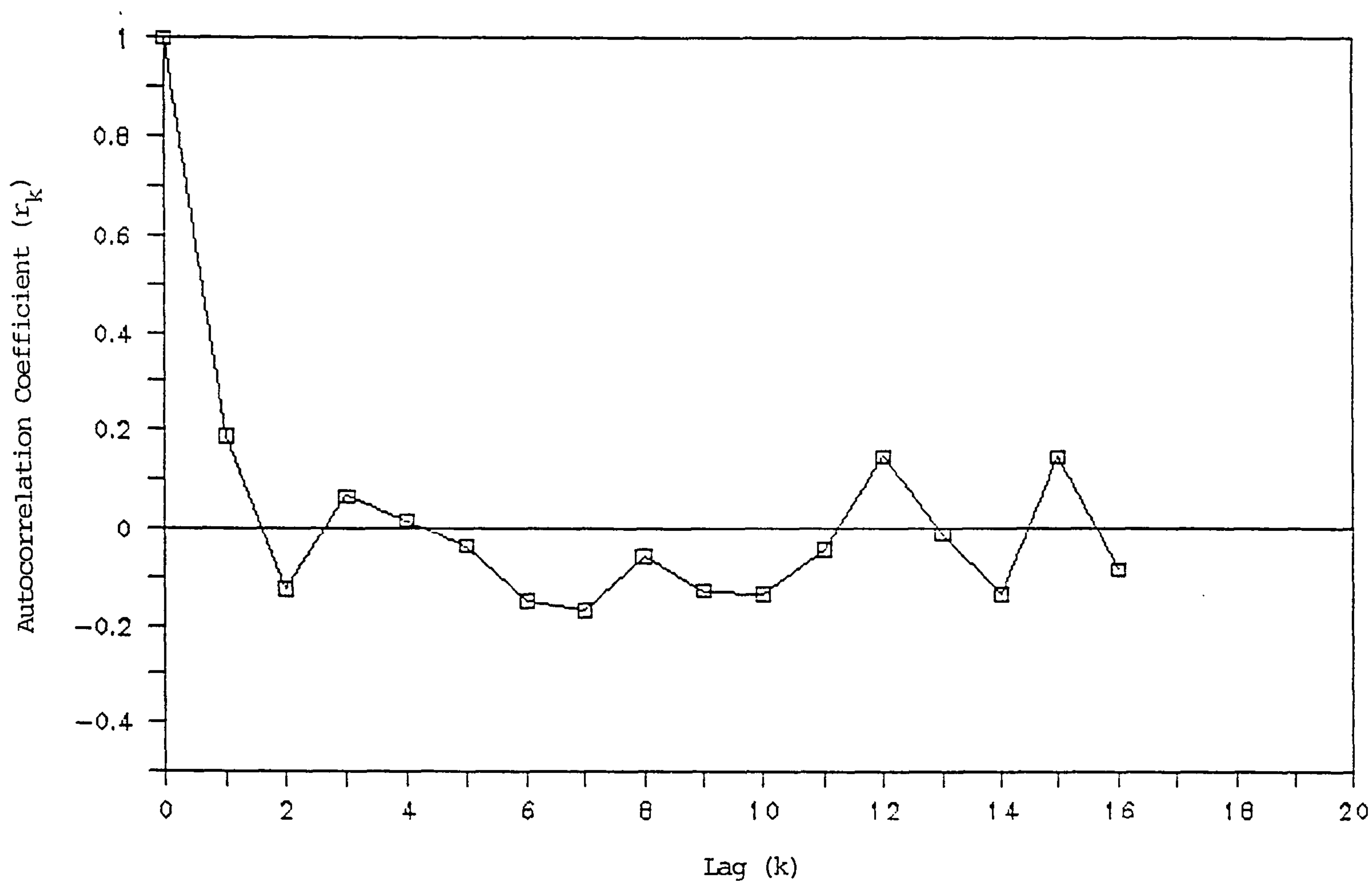


Fig. C.4.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Passaic River near Millington

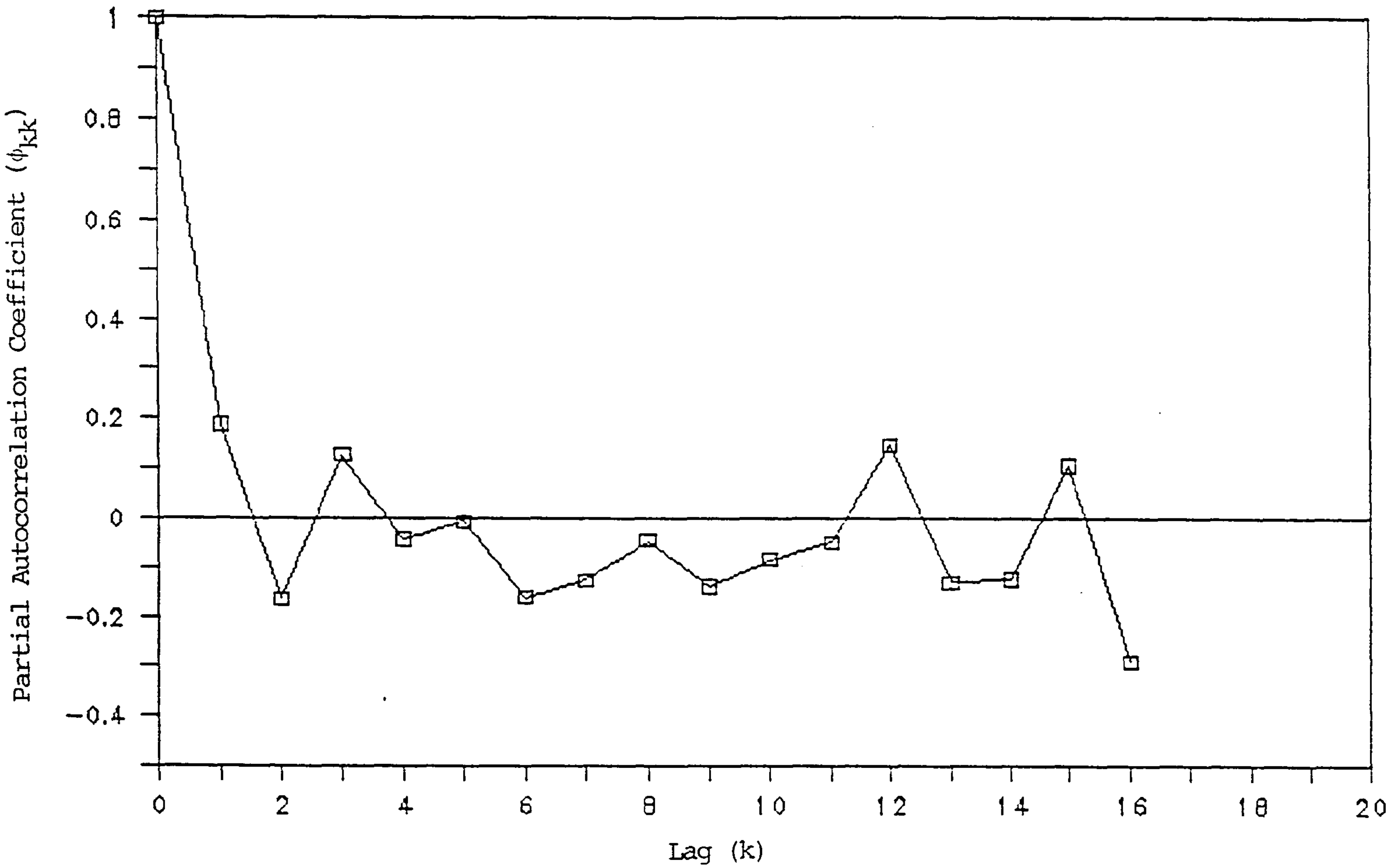


Fig. C.4.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Passaic River near Millington

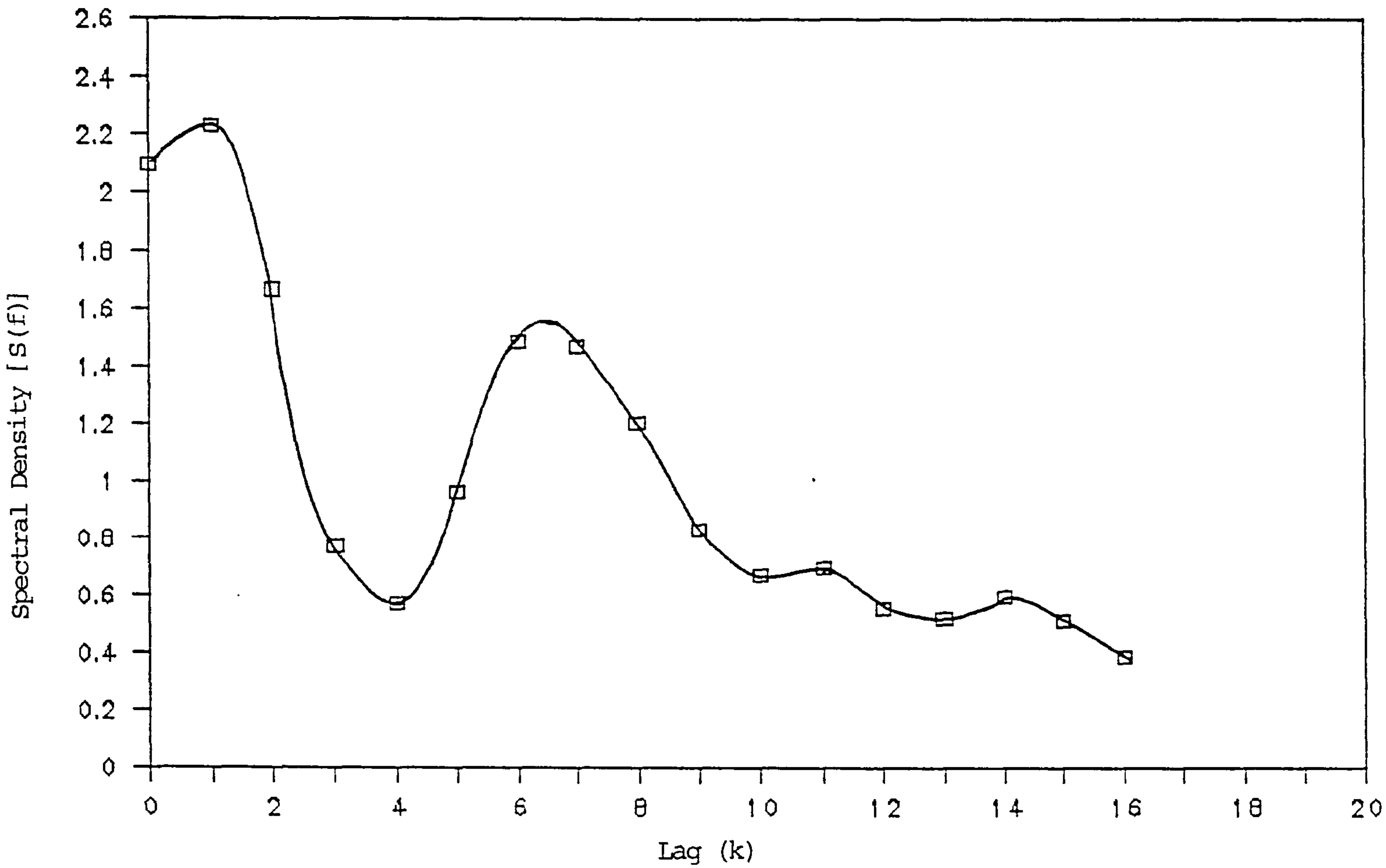


Fig. C.4.4 Spectral Density Function of the Historic Mean Annual Flow Series - Passaic River near Millington

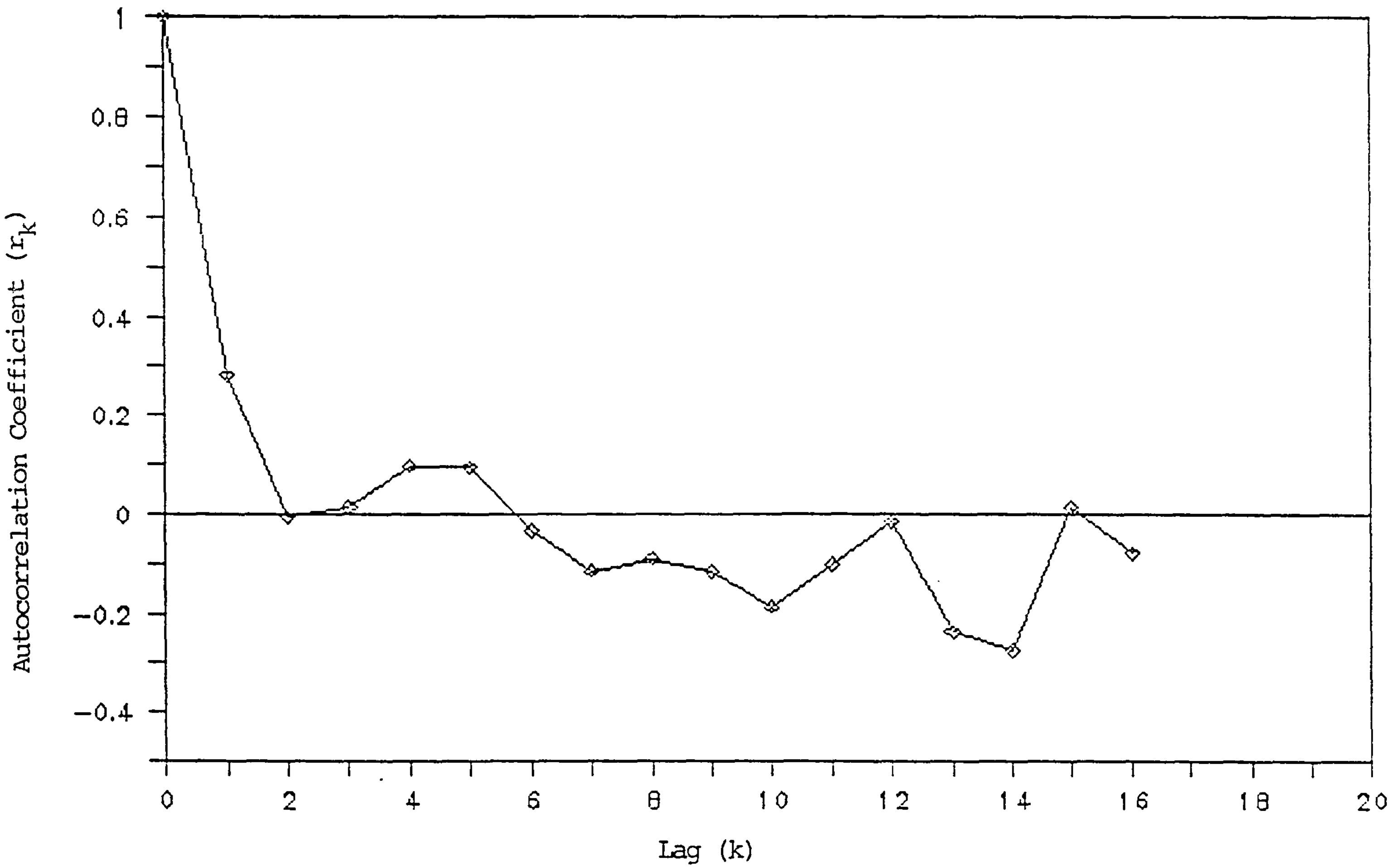


Fig. C.5.1 Autocorrelation Function of the Historic Mean Annual Flow Series - Lamington River near Pottersville

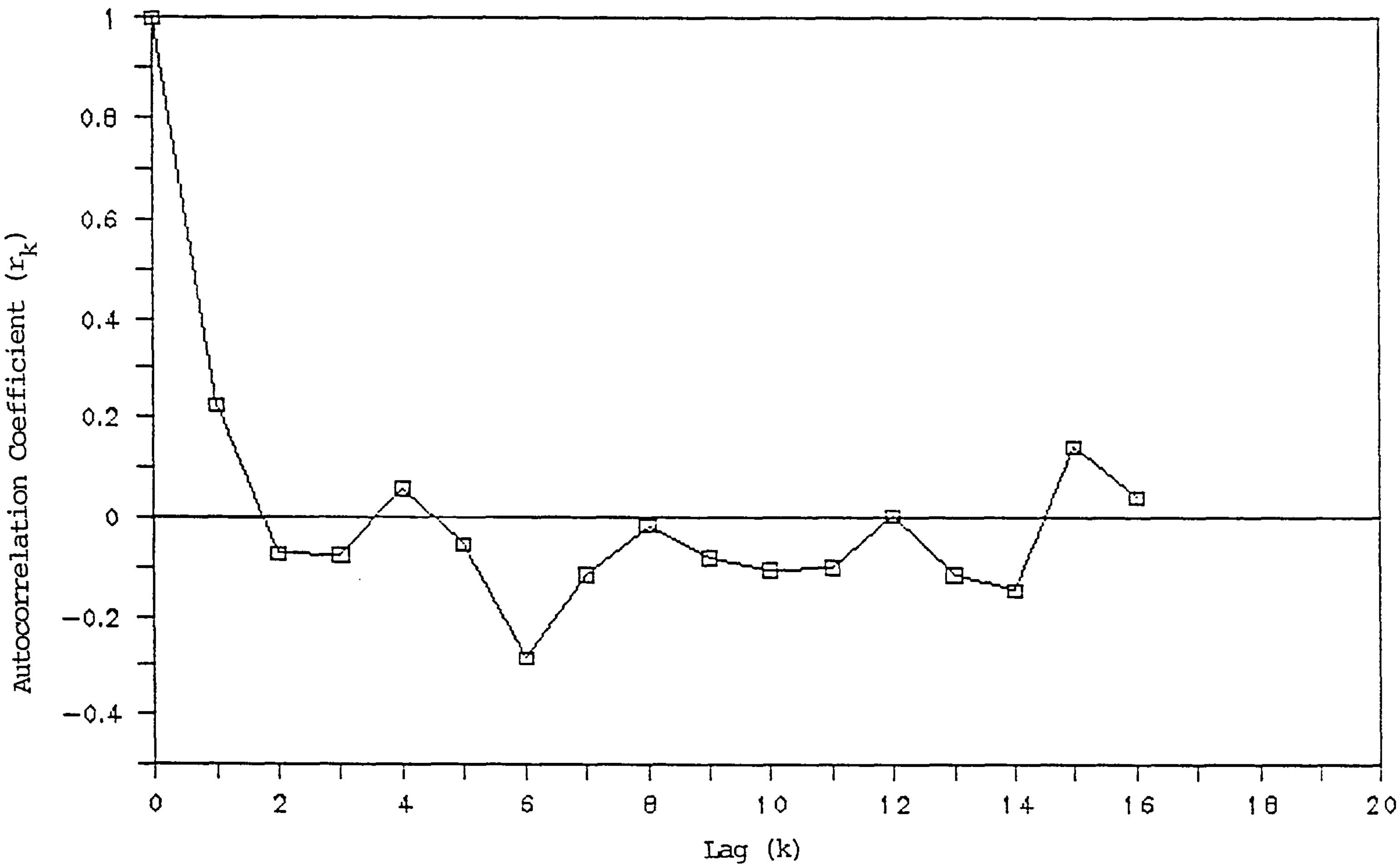


Fig. C.5.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Lamington River near Pottersville

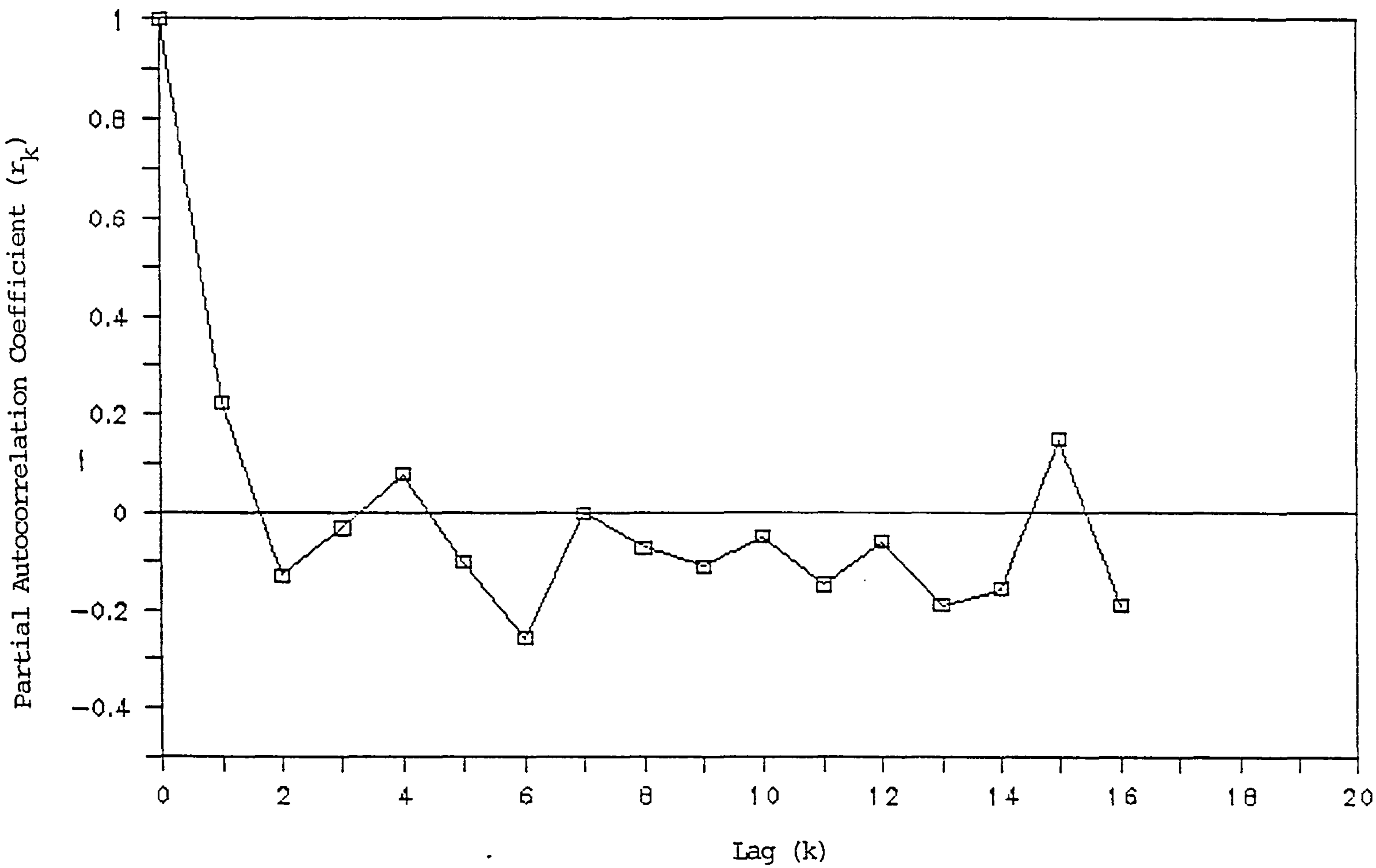


Fig. C.5.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Lamington River near Pottersville

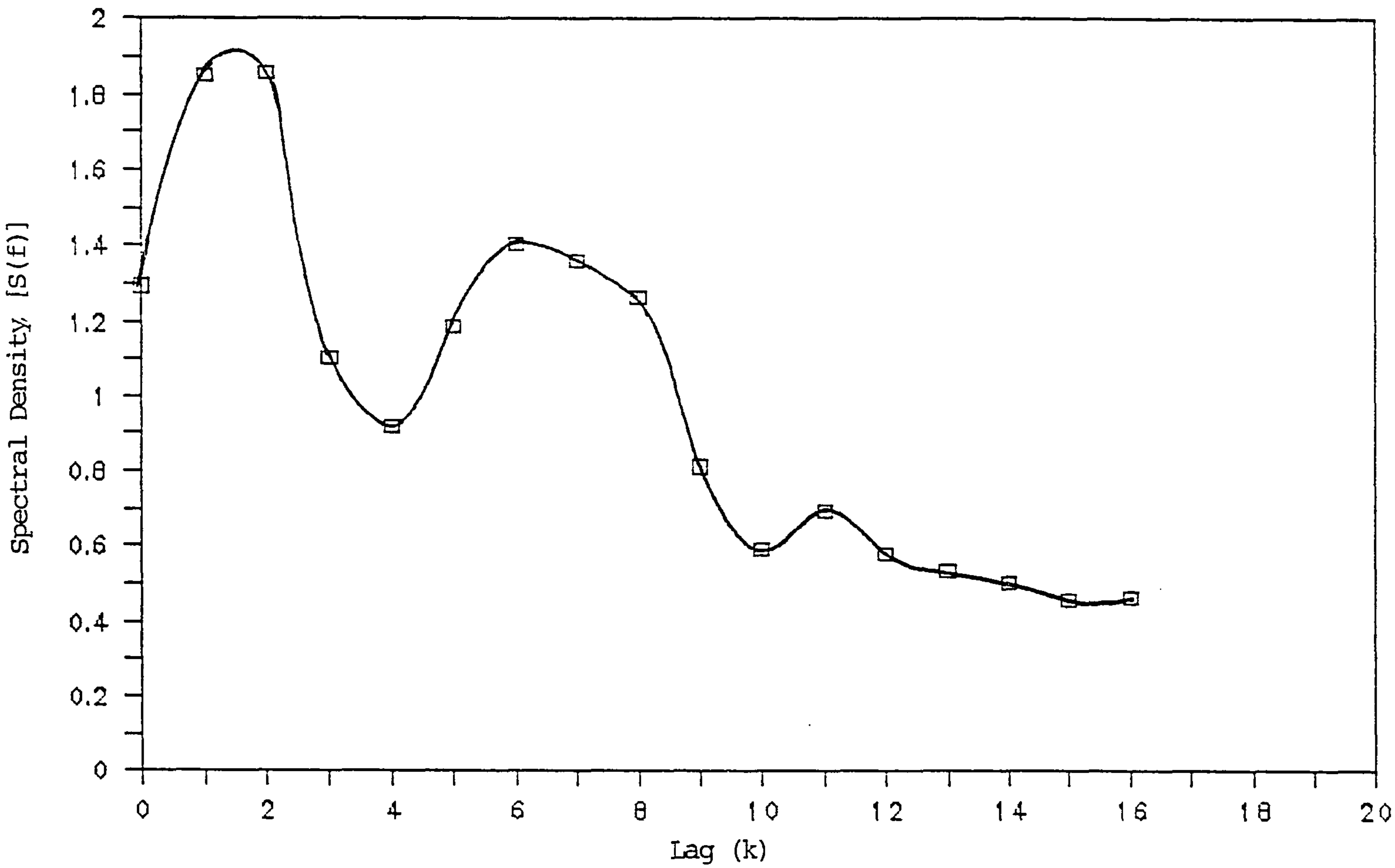


Fig. C.5.4 Spectral Density Function of the Historic Mean Annual Flows Series - Lamington River near Pottersville

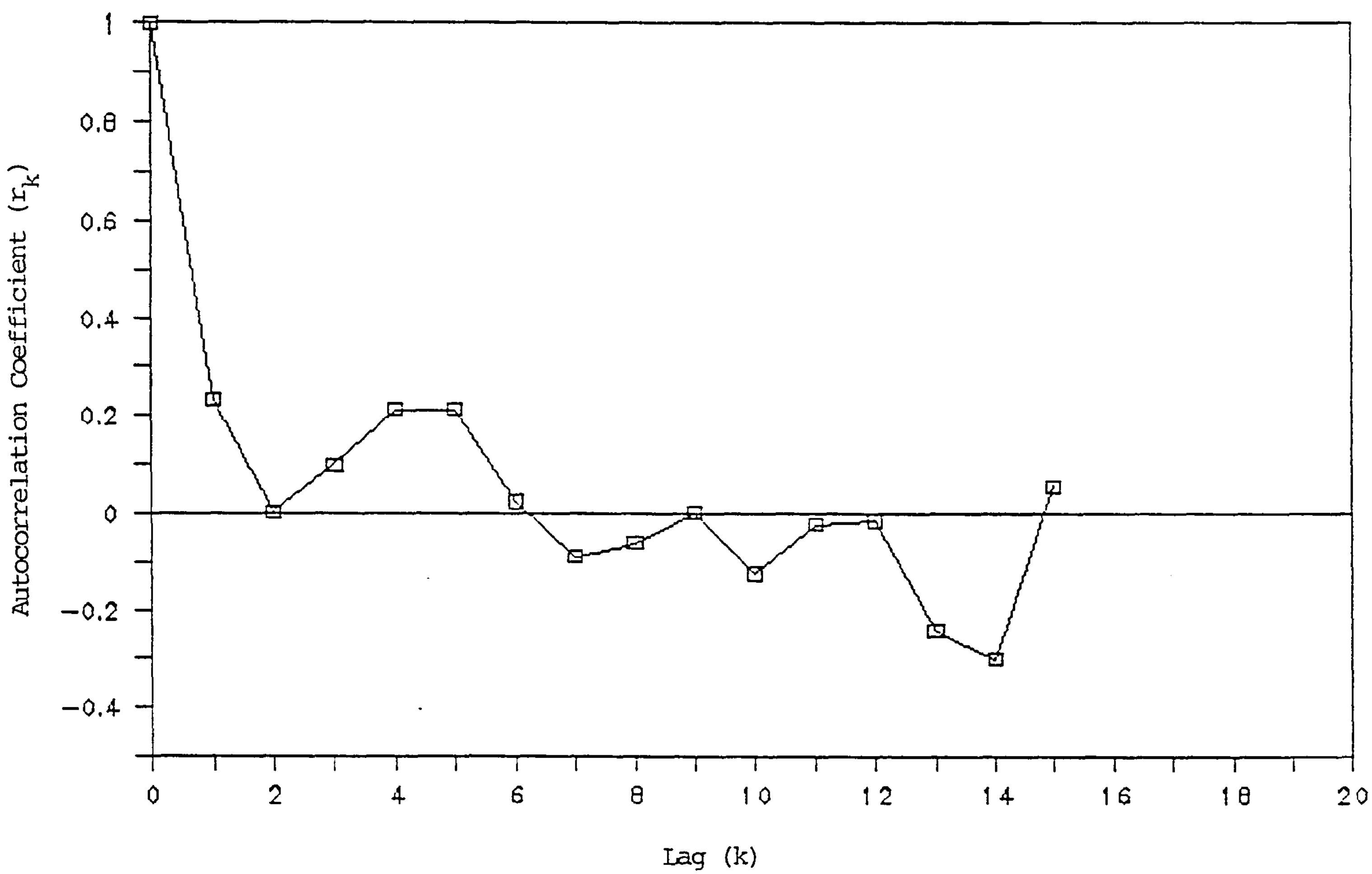


Fig. C. 6.1 Autocorrelation Function of the Historic Mean Annual Flow Series - North Branch Raritan River near Raritan

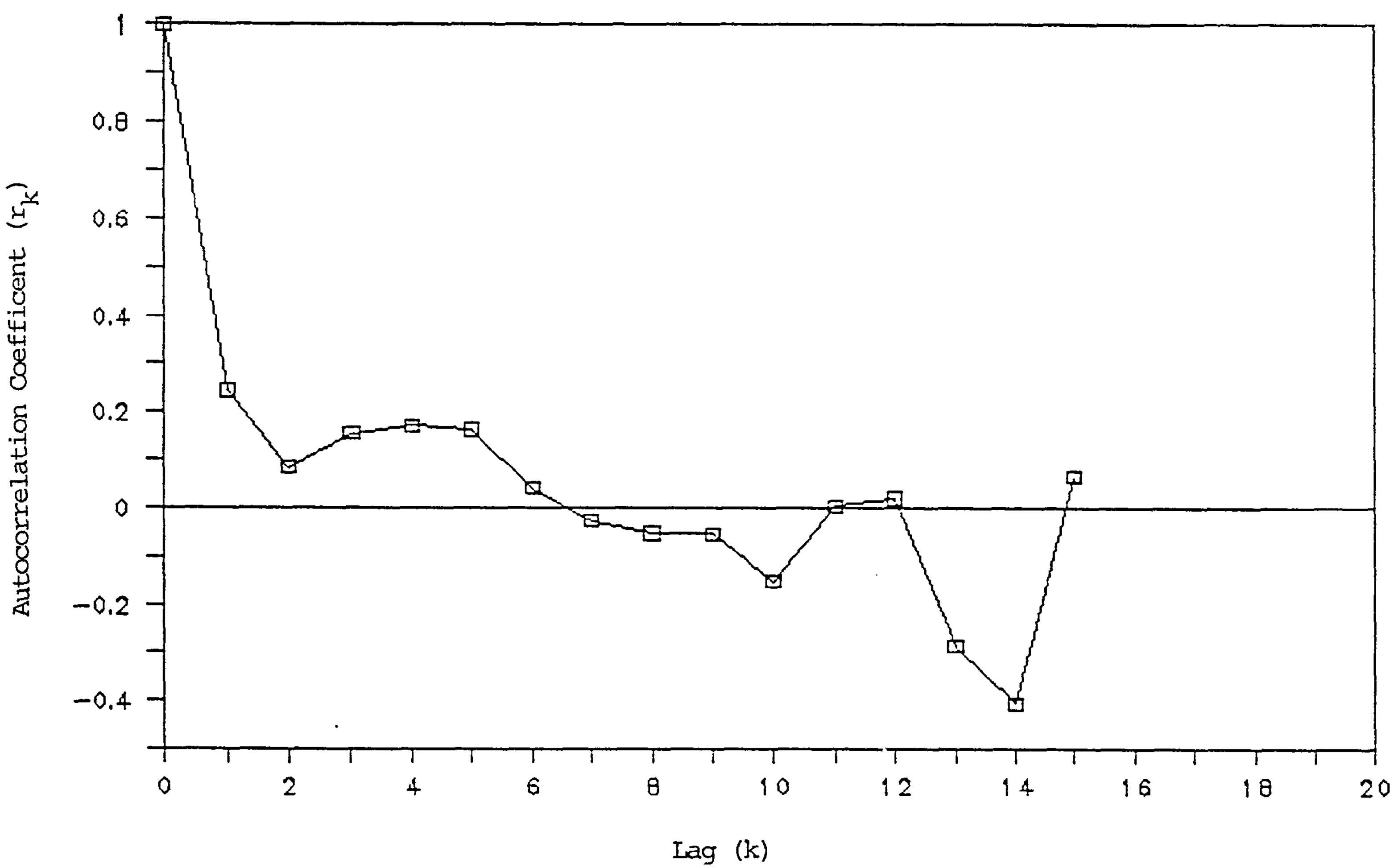


Fig. C. 6.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - North Branch Raritan River near Raritan

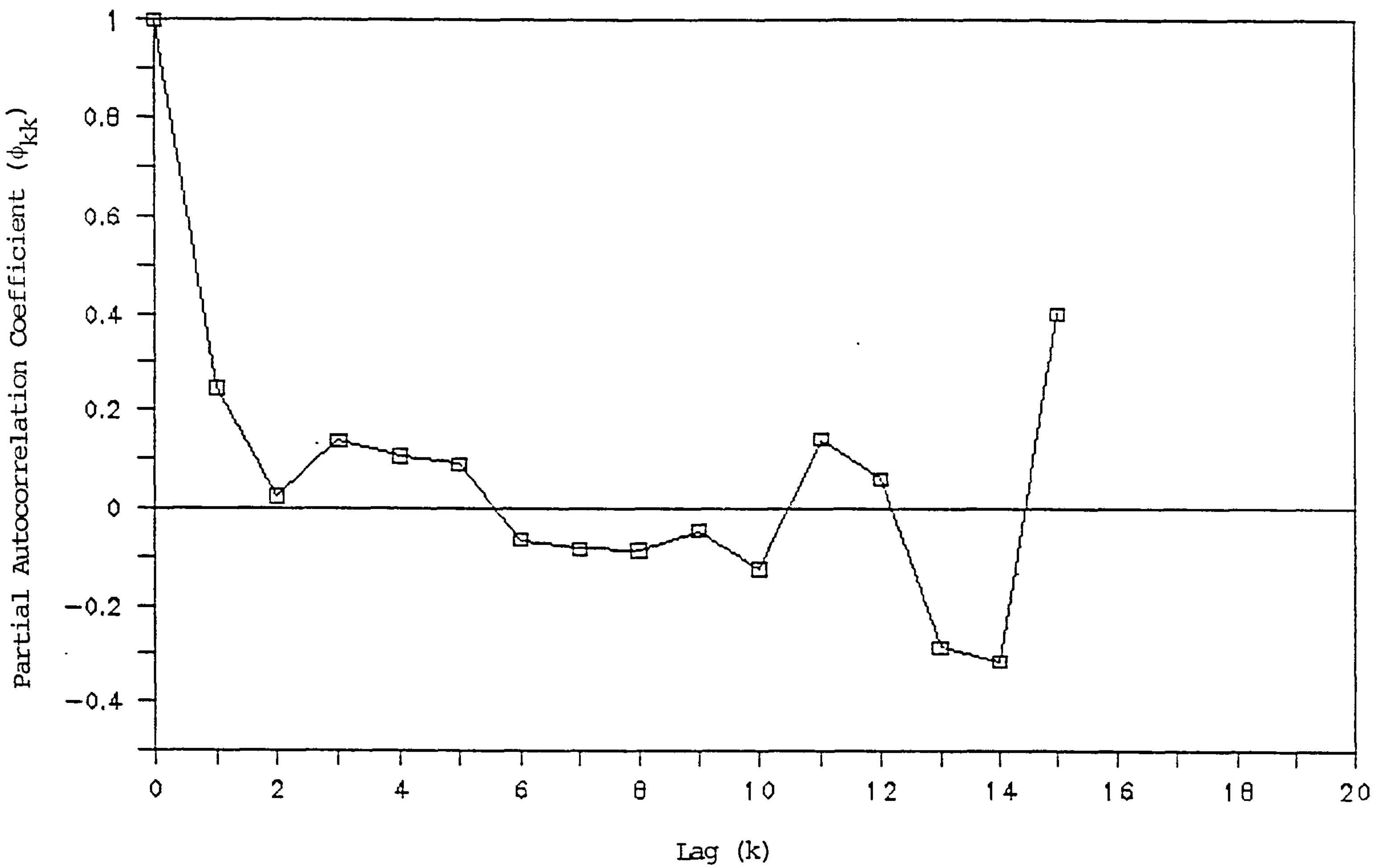


Fig. C. 6.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - North Branch Raritan River near Raritan

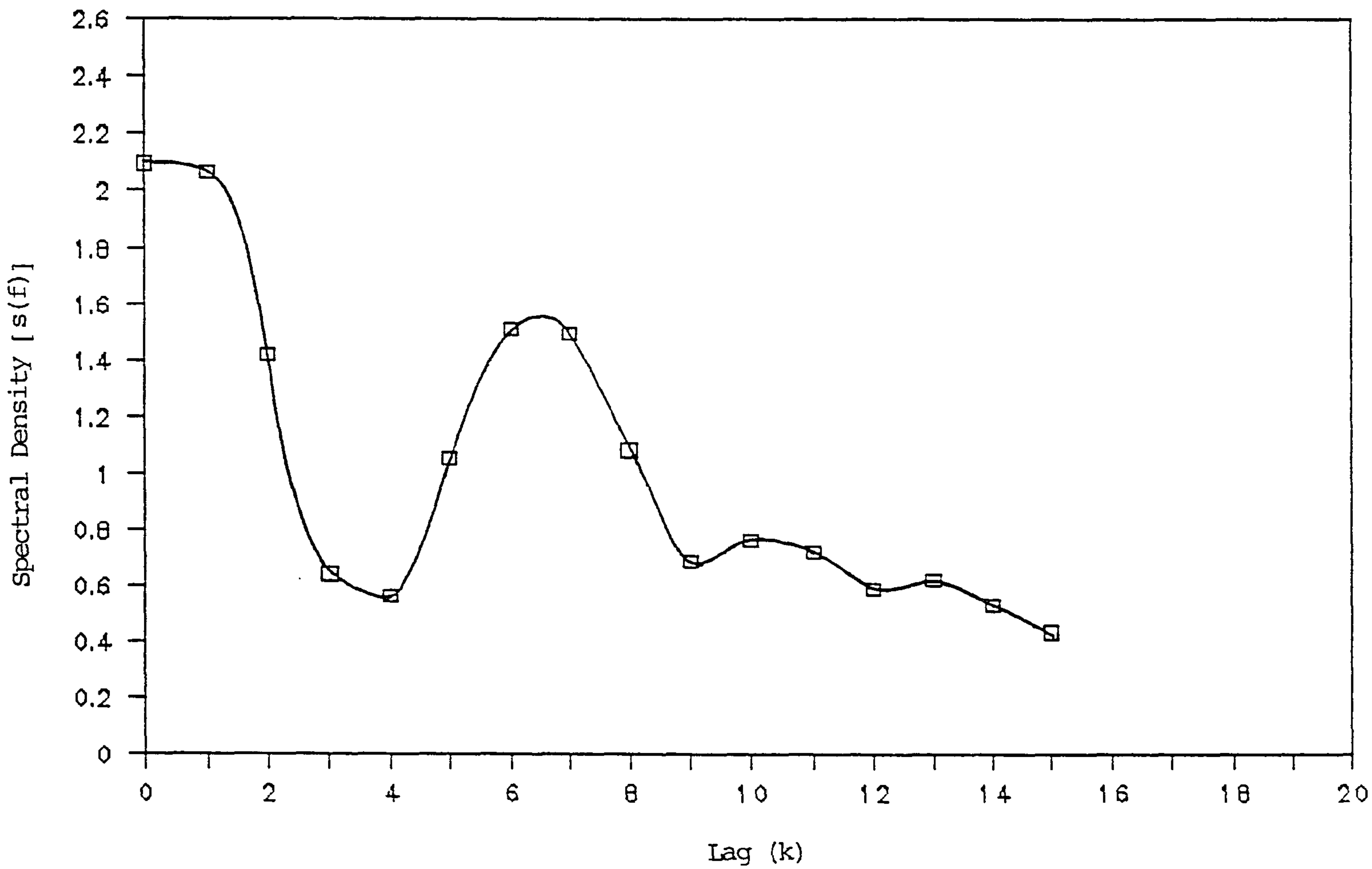


Fig. C. 6.4 Spectral Density Function of the Historic Mean Annual Flow Series - North Branch Raritan River near Raritan

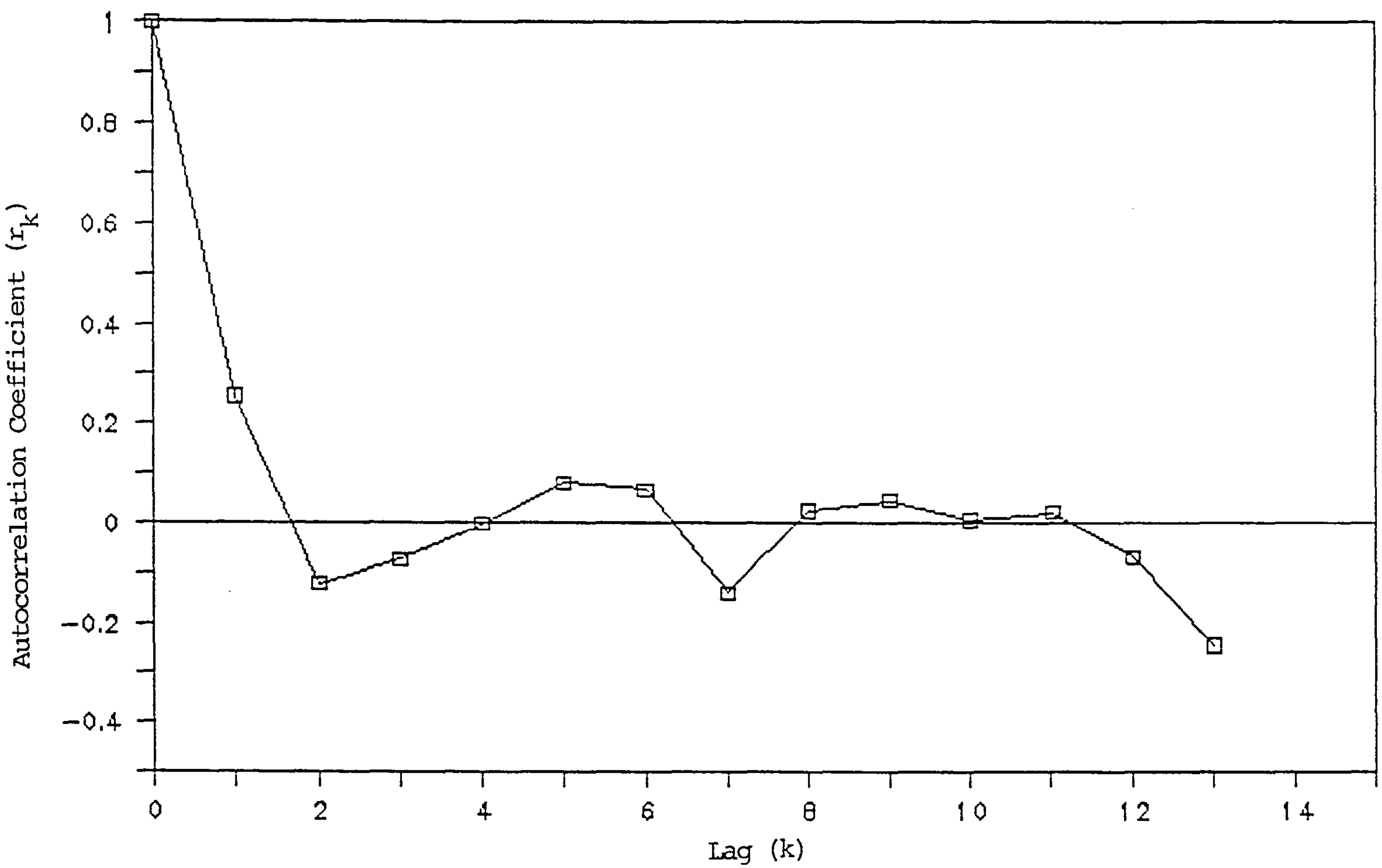


Fig. C.7.1 Autocorrelation Function of the Historic Mean Annual Flow Series - Manasquan River at Squankum

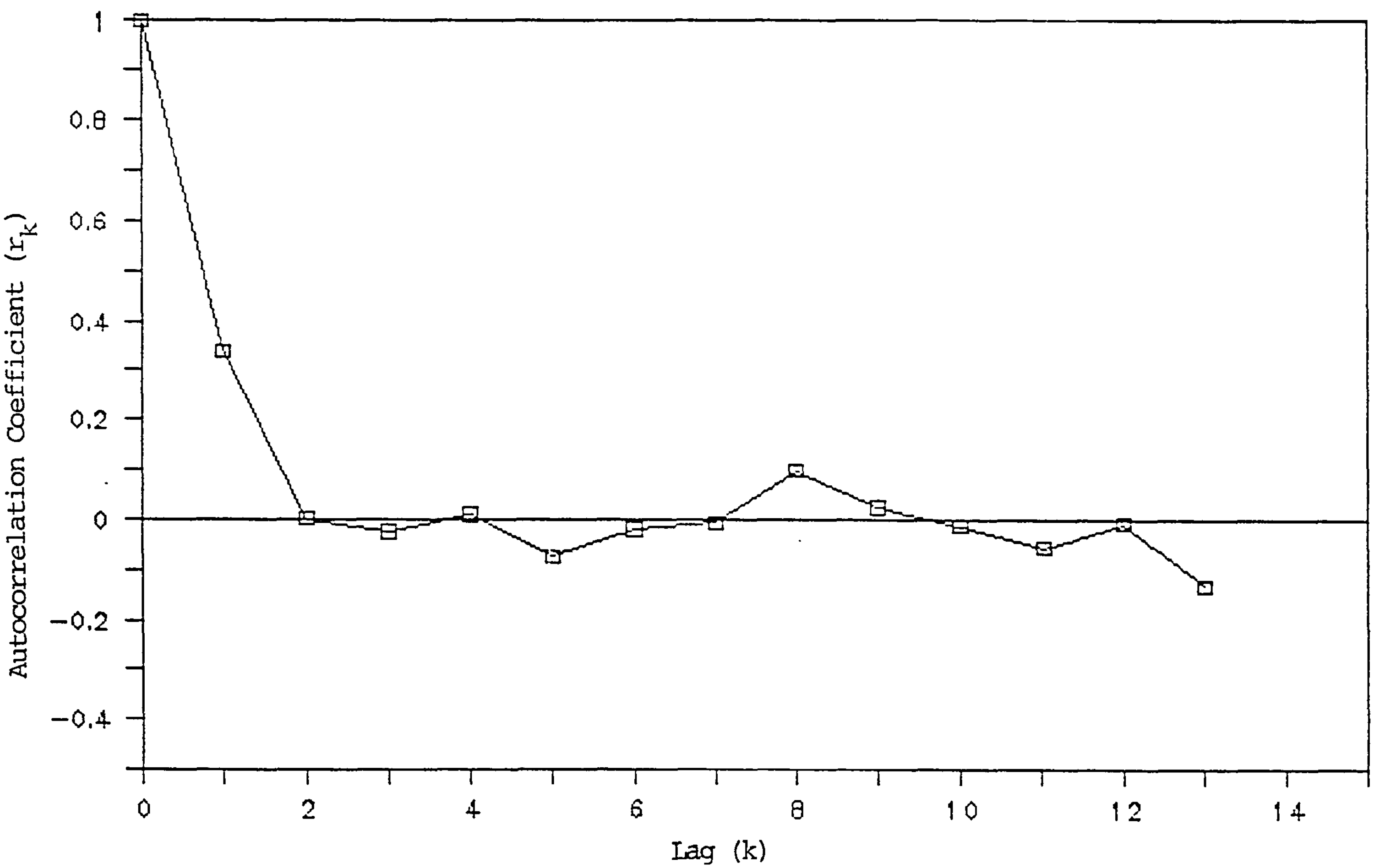


Fig. C.7.2 Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Manasquan River at Squankum

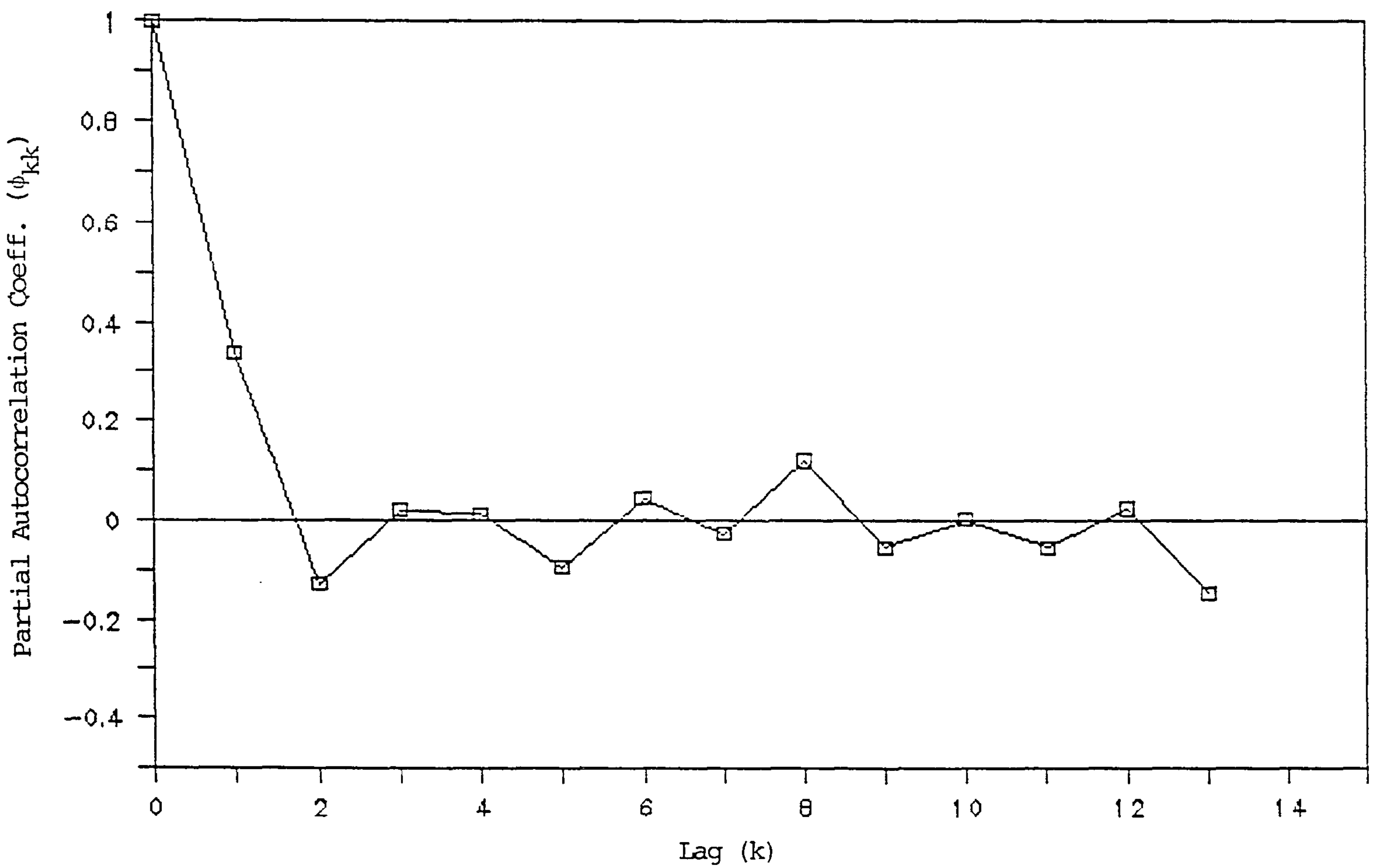


Fig. C.7.3 Partial Autocorrelation Function of the Stochastic Component (ε_t) of Historic Mean Annual Flows - Manasquan River near Squankum

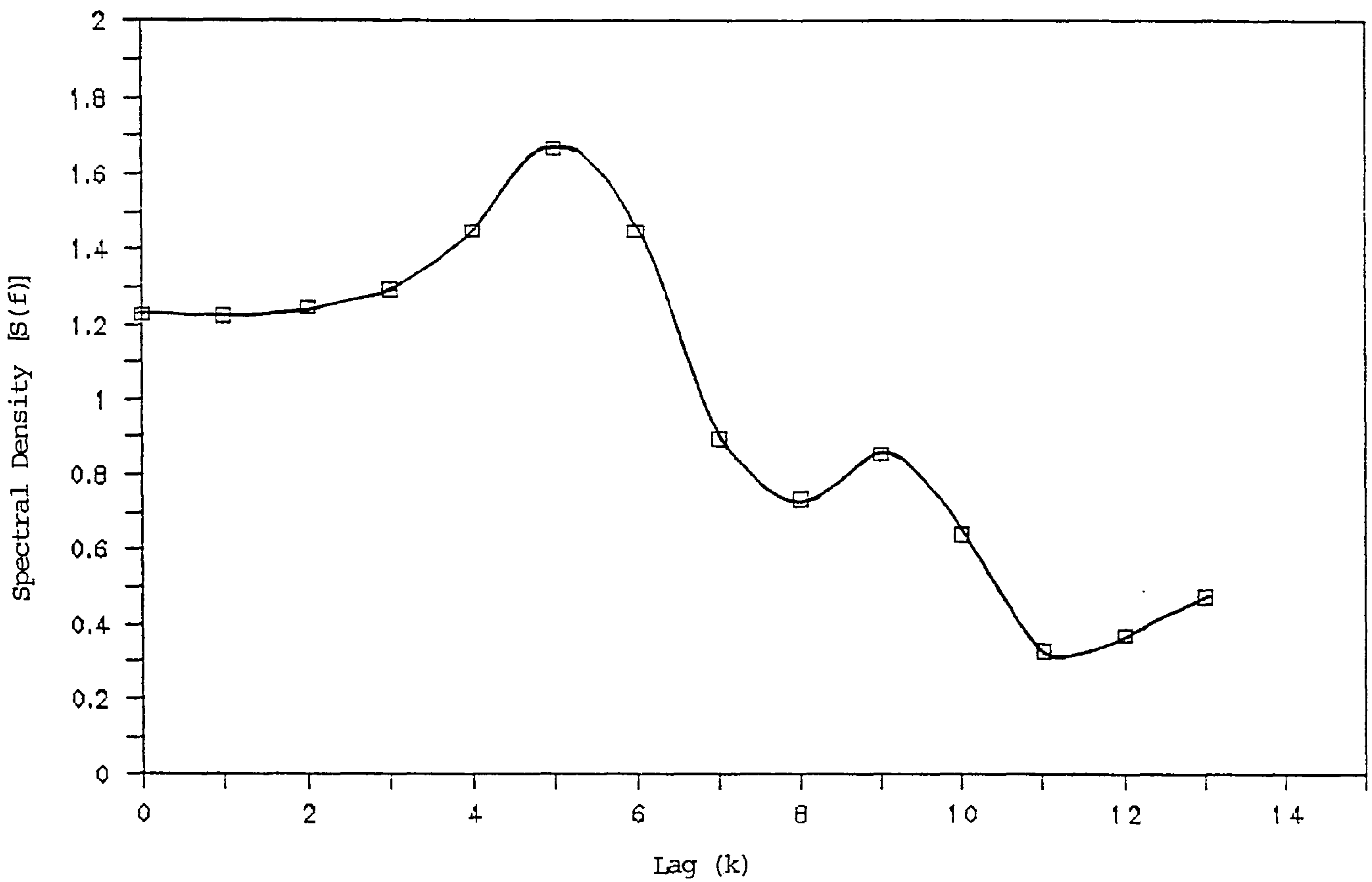


Fig. C.7.4 Spectral Density Function of the Historic Mean Annual Flow Series - Manasquan River at Squankum

APPENDIX D

PROBABILITY DISTRIBUTIONS OF DROUGHT PARAMETERS

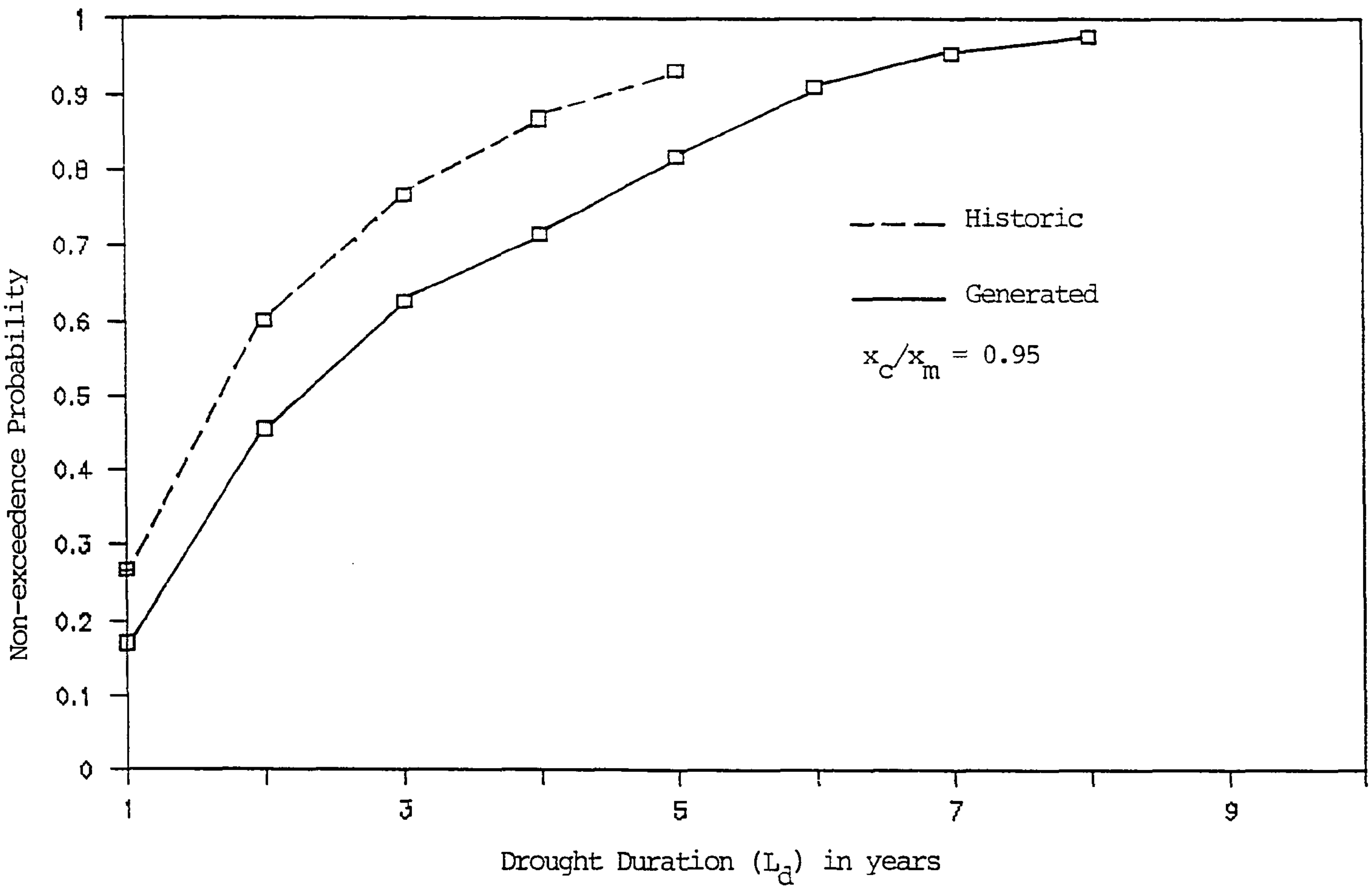


Fig. D.1.1 Probability Distributions of Drought Durations Derived from Historic and Generated Flow Series - Pequest River at Pequest

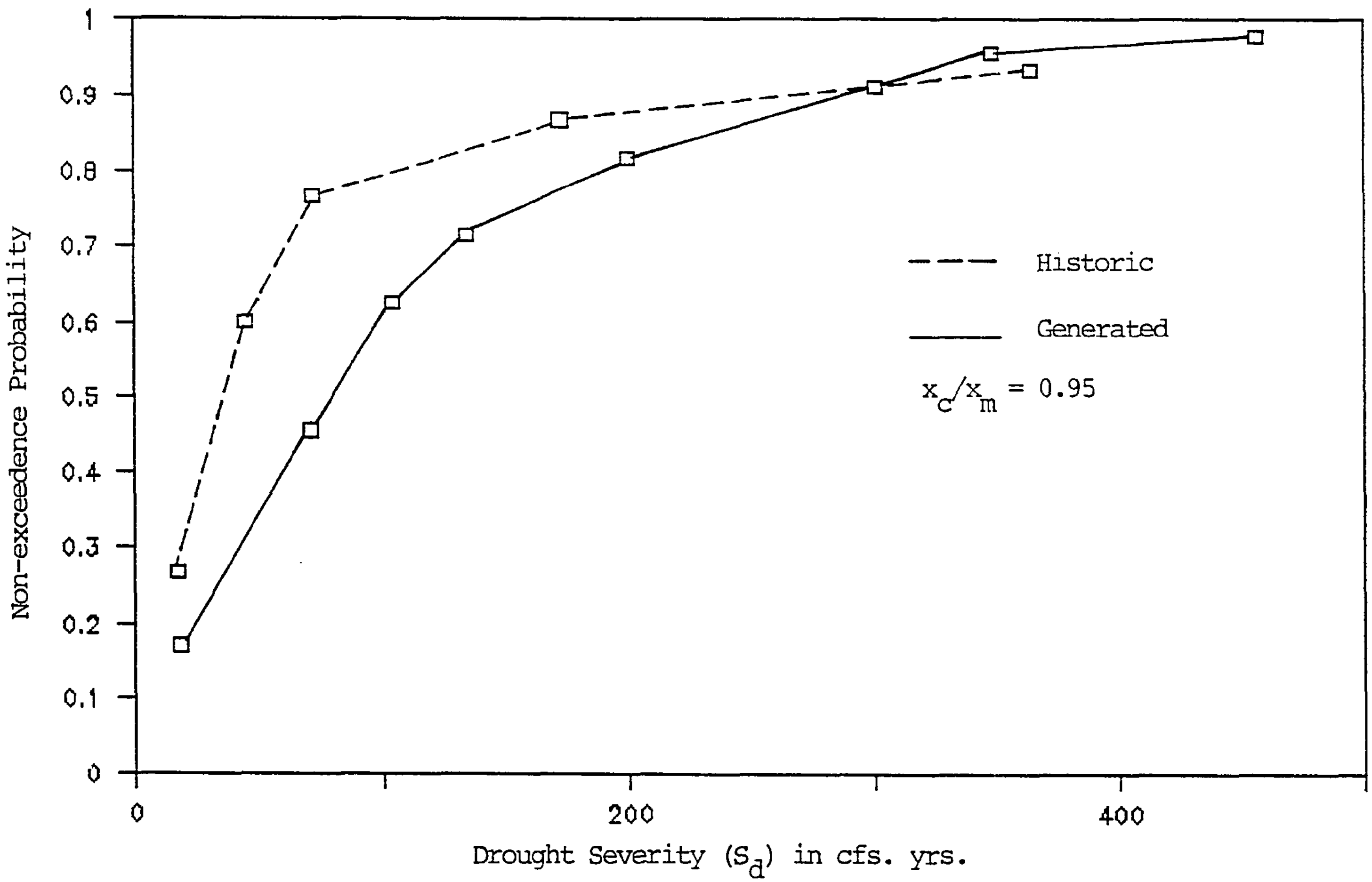


Fig. D.1.2 Probability Distributions of Drought Severities Derived from Historic and Generated Flow Series - Pequest River at Pequest

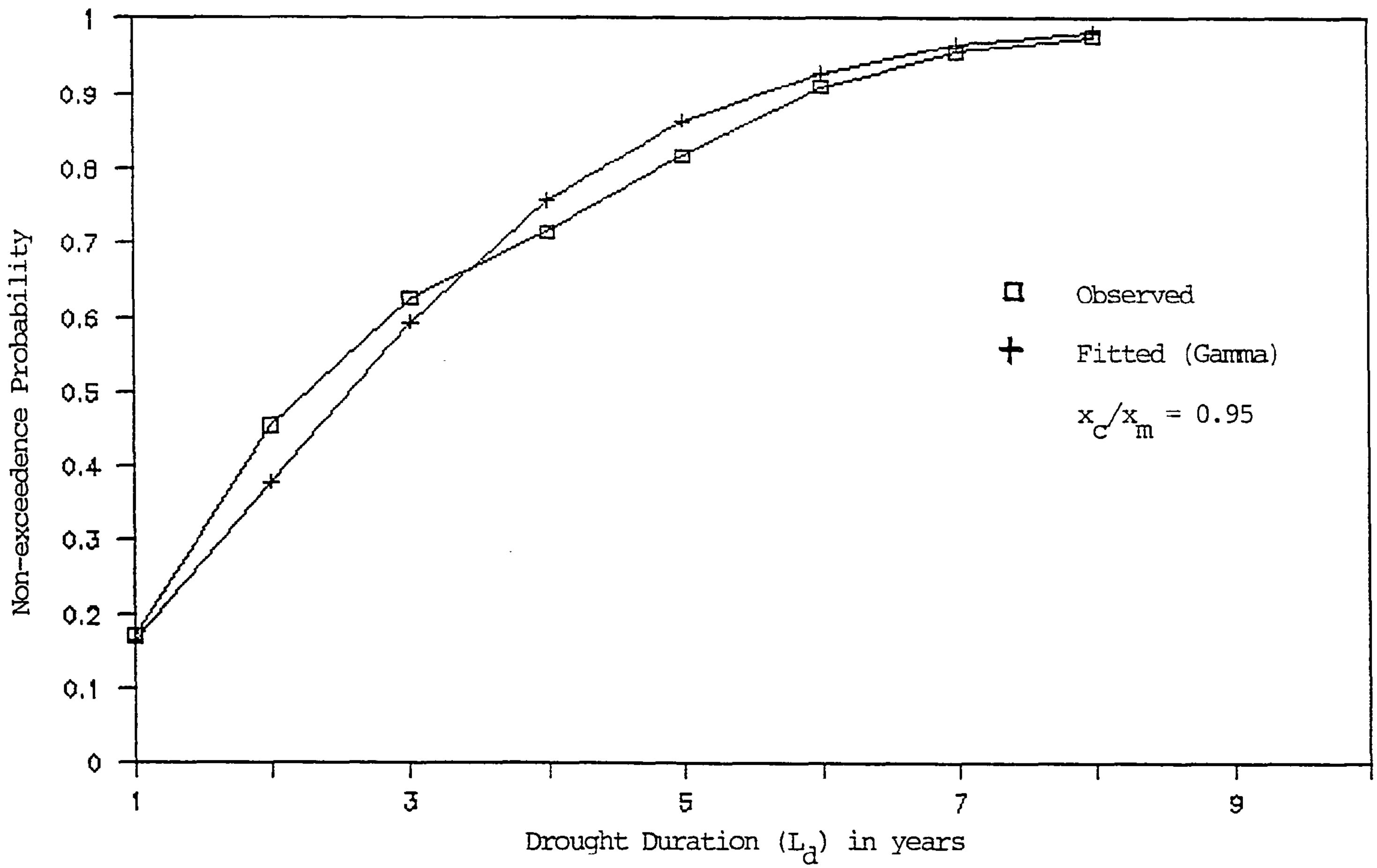


Fig. D.1.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - Pequest River at Pequest

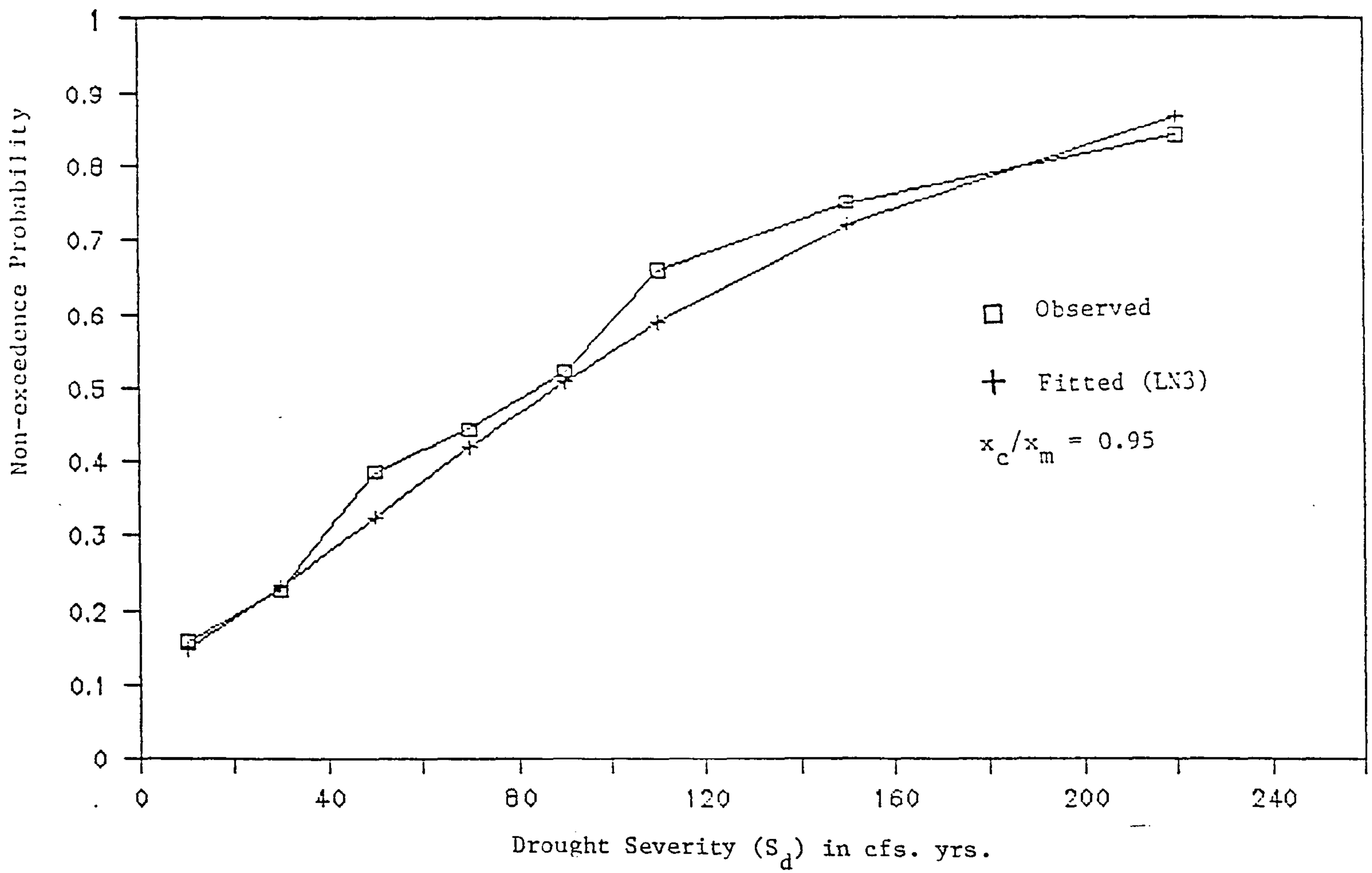


Fig. D.1.4 Comparison of Observed and Fitted Probability Distributions for Drought Severities Derived from Generated Flow Series - Pequest River at Pequest

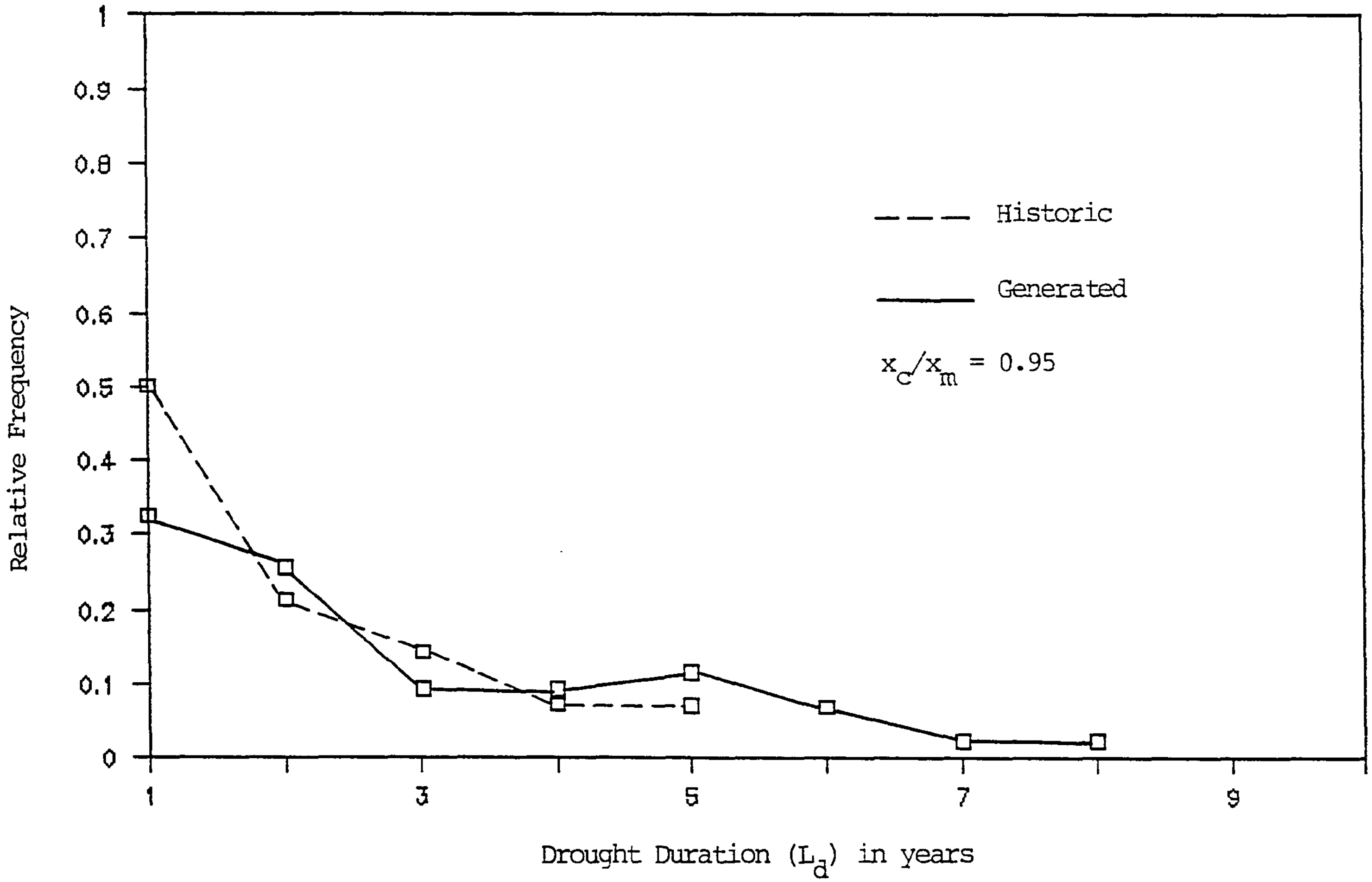


Fig. D.1.5 Probability Density Curves of Drought Durations - Pequest River at Pequest

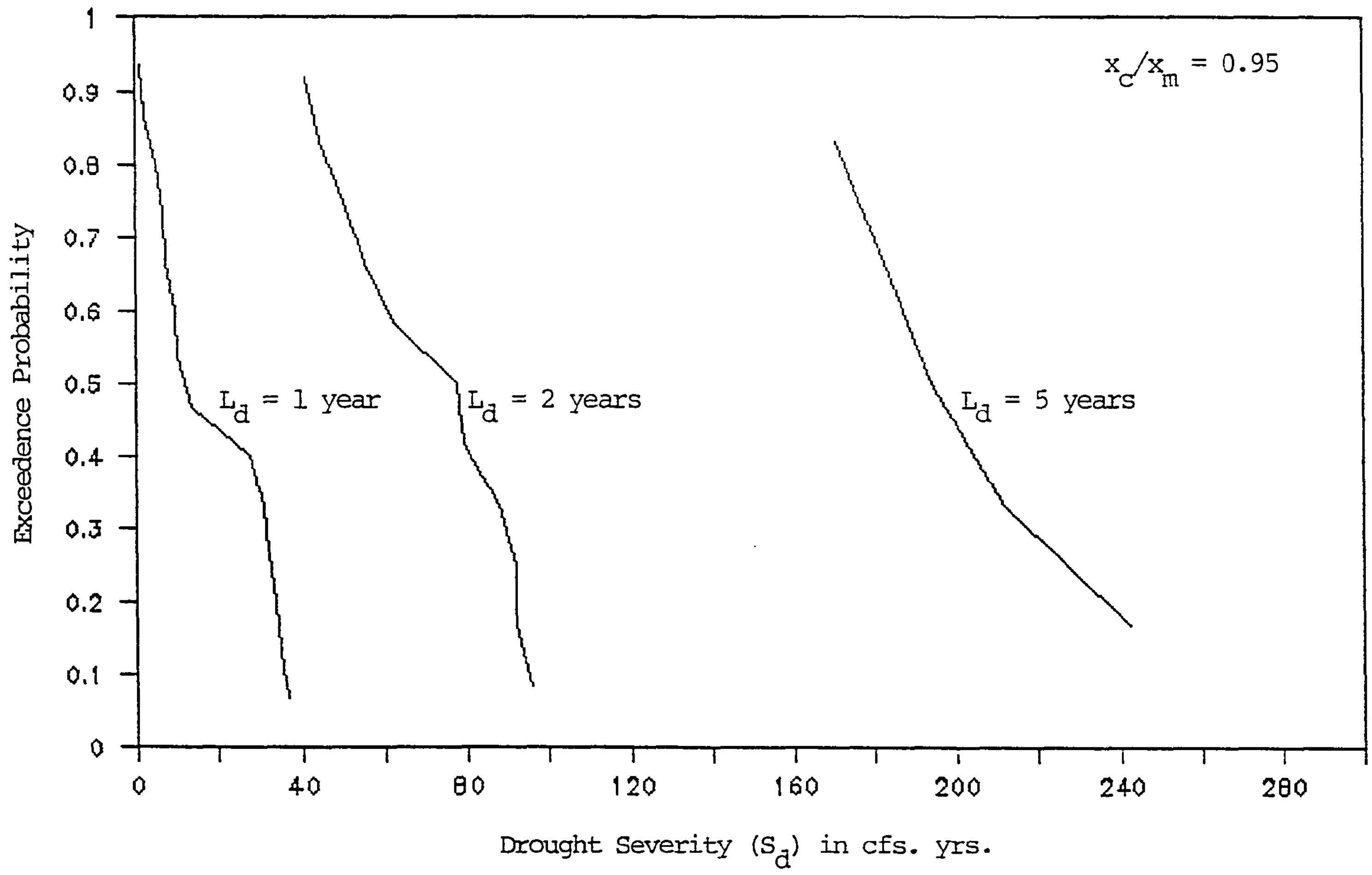


Fig. D.1.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - Pequest River at Pequest

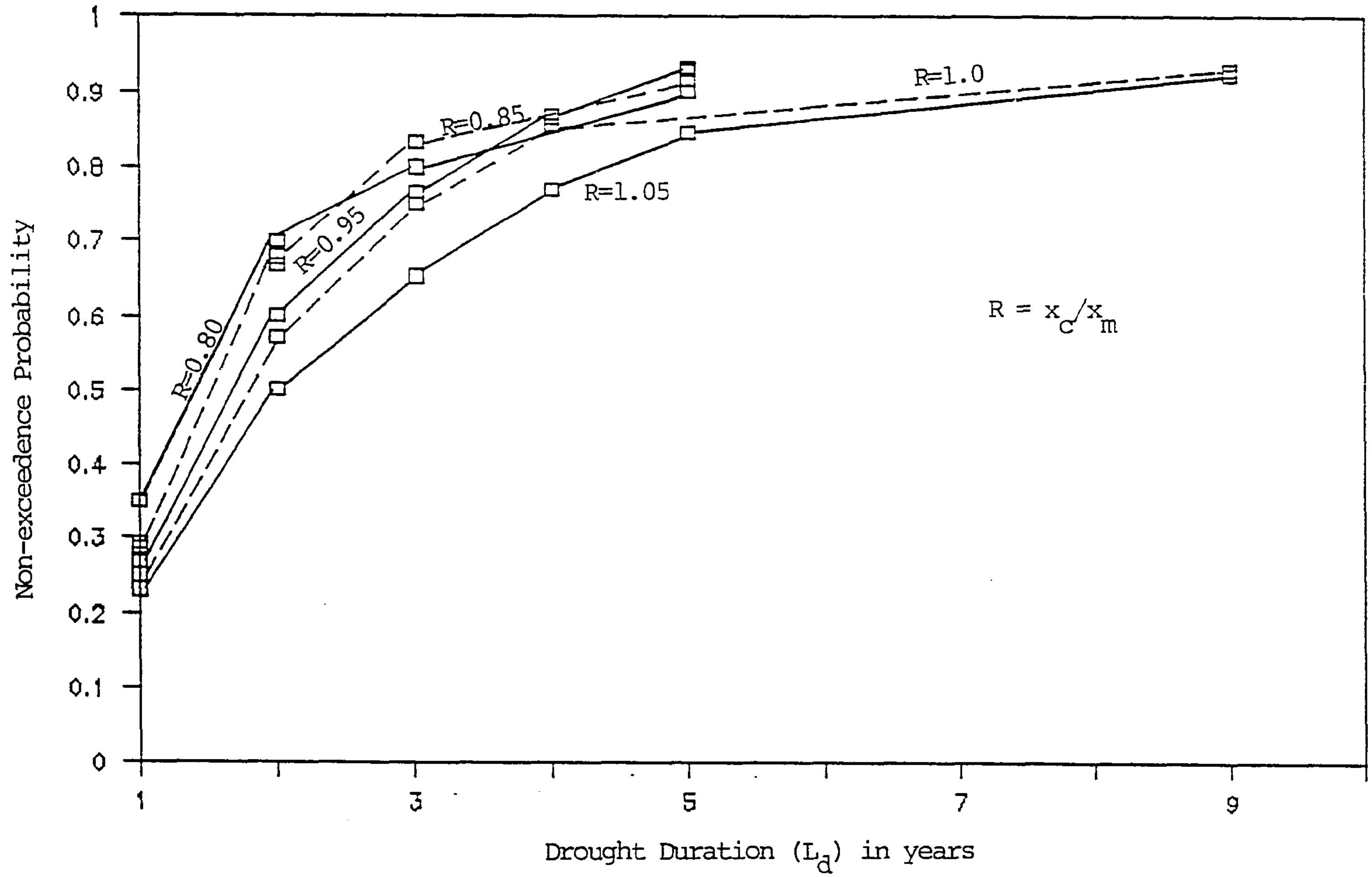


Fig. D .1.7 Probability Distributions of Drought Durations Derived from Historic Flow Series at Different Truncation Levels - Pequest River at Pequest

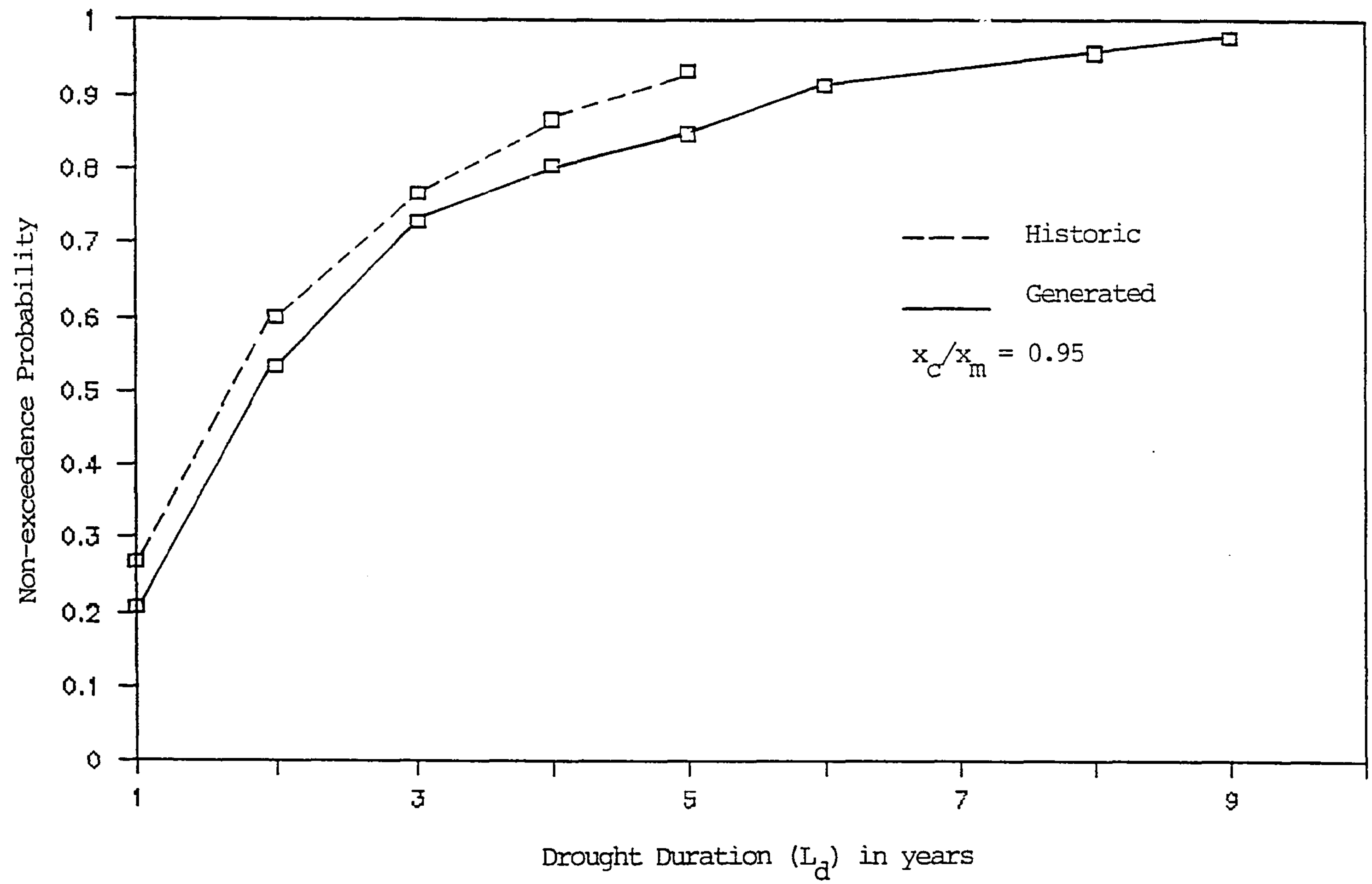


Fig. D.2.1 Probability Distributions of Drought Durations Derived from Historic and Generated Flow Series - Musconetcong River near Bloomsbury

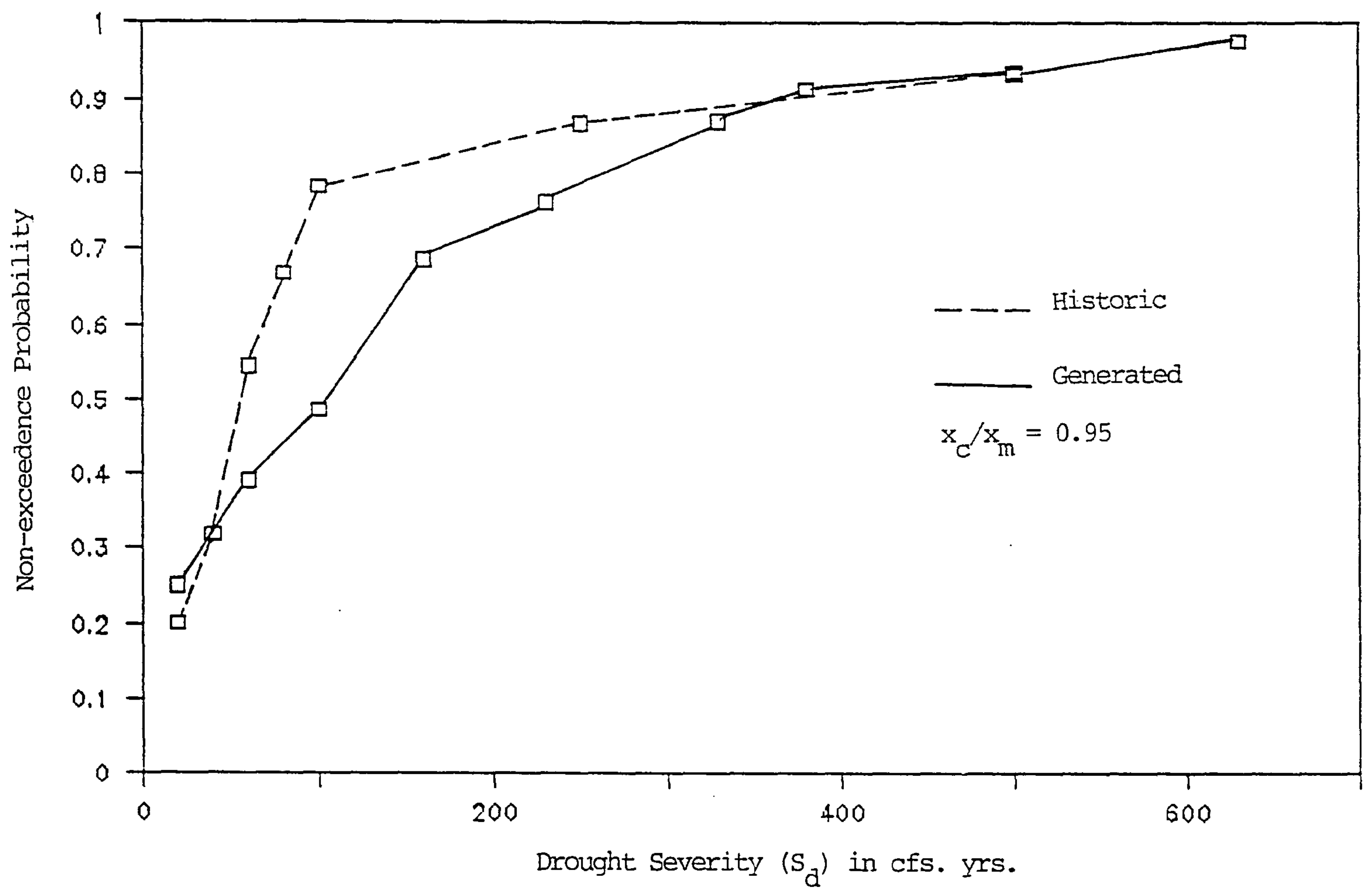


Fig. D.2.2 Probability Distributions of Drought Severities Derived from Historic and Generated Flow Series - Musconetcong River near Bloomsbury

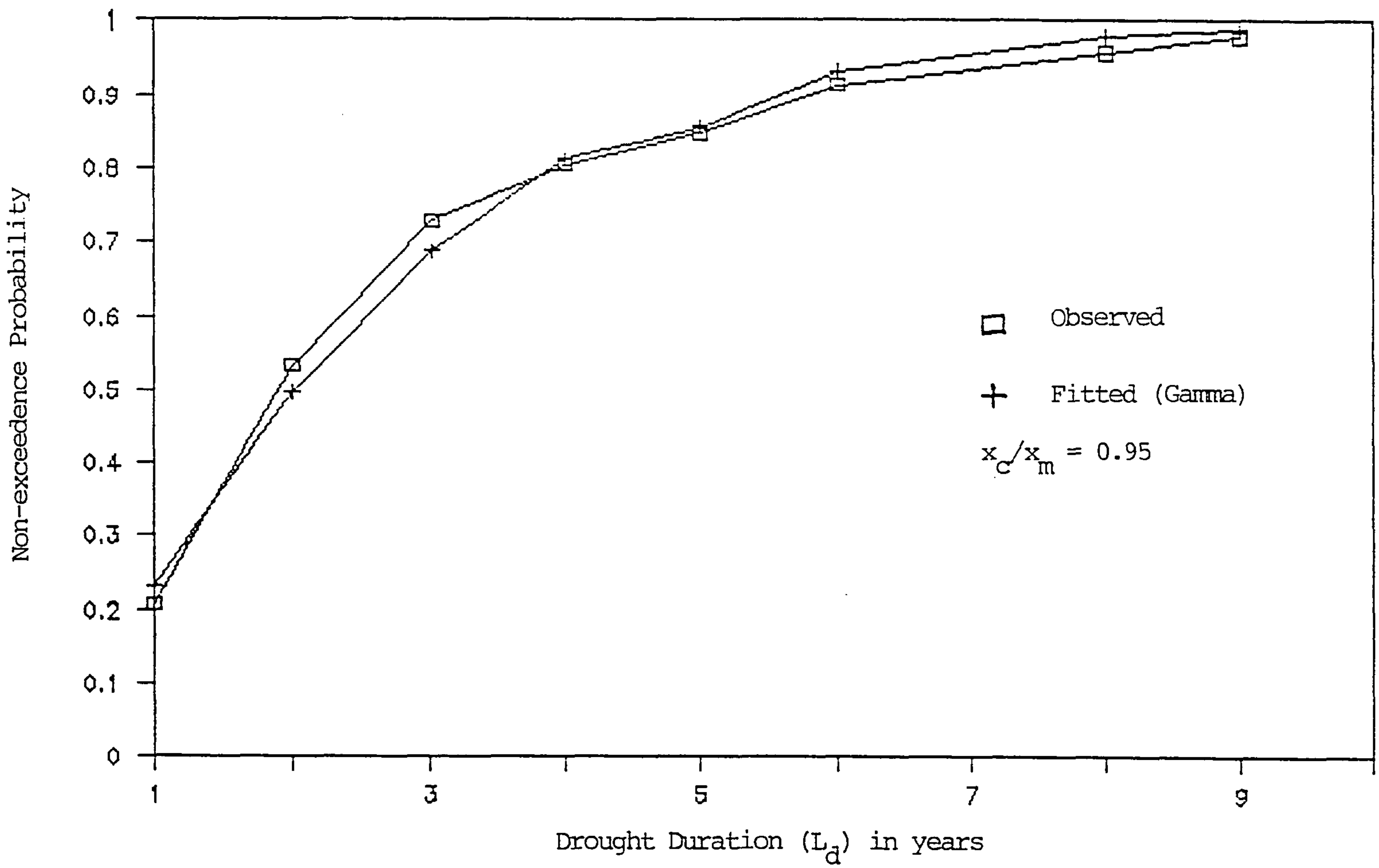


Fig. D.2.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - Musconetcong River near Bloomsbury

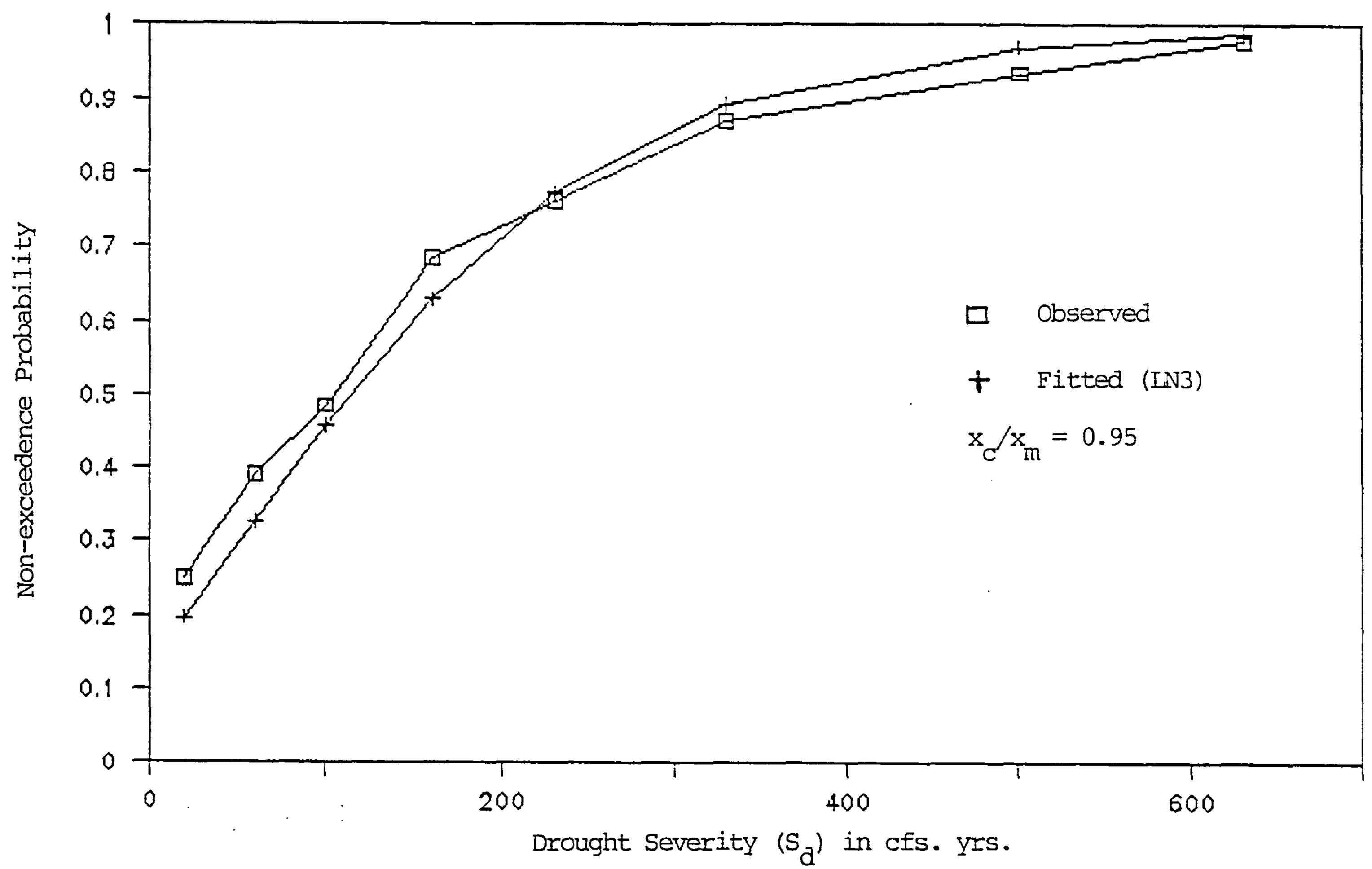


Fig. D.2.4 Comparison of Observed and Fitted Probability Distributions for Drought Severities Derived from Generated Flow Series - Musconetcong River near Bloomsbury

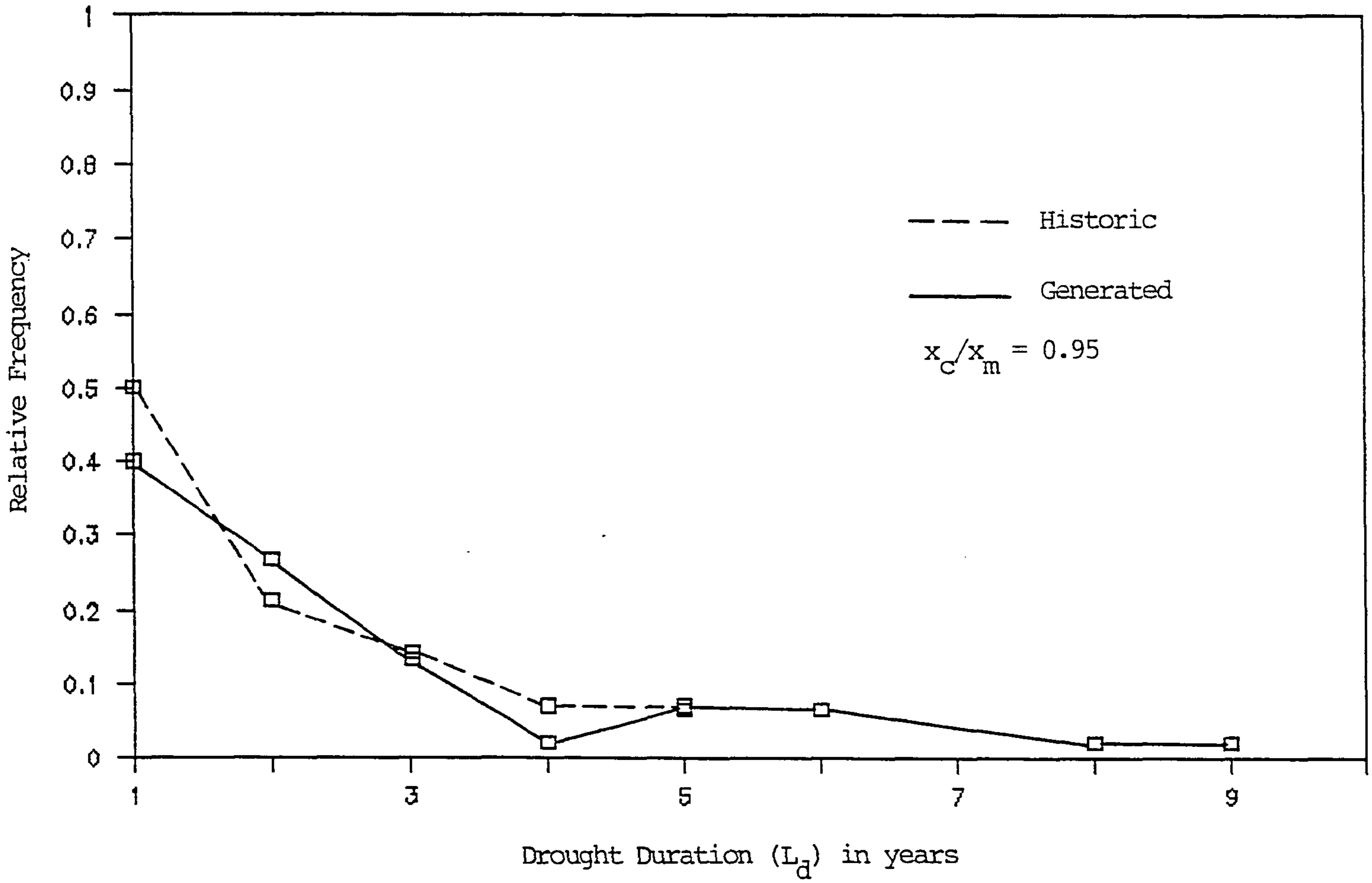


Fig. D.2.5 Probability Density Curves of Drought Durations - Musconetcong River near Bloomsbury

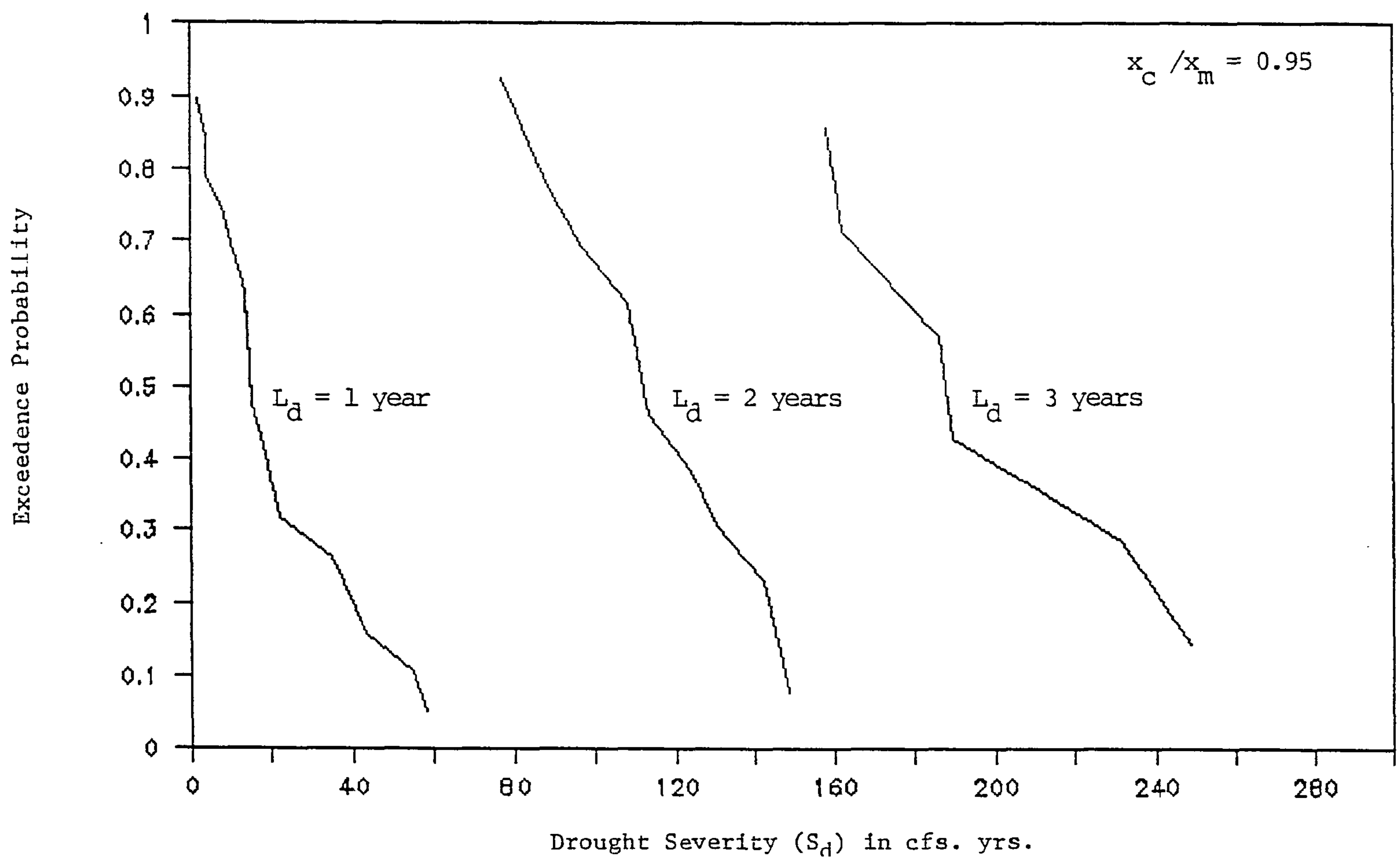


Fig. D.2.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - Musconetcong River near Bloomsbury

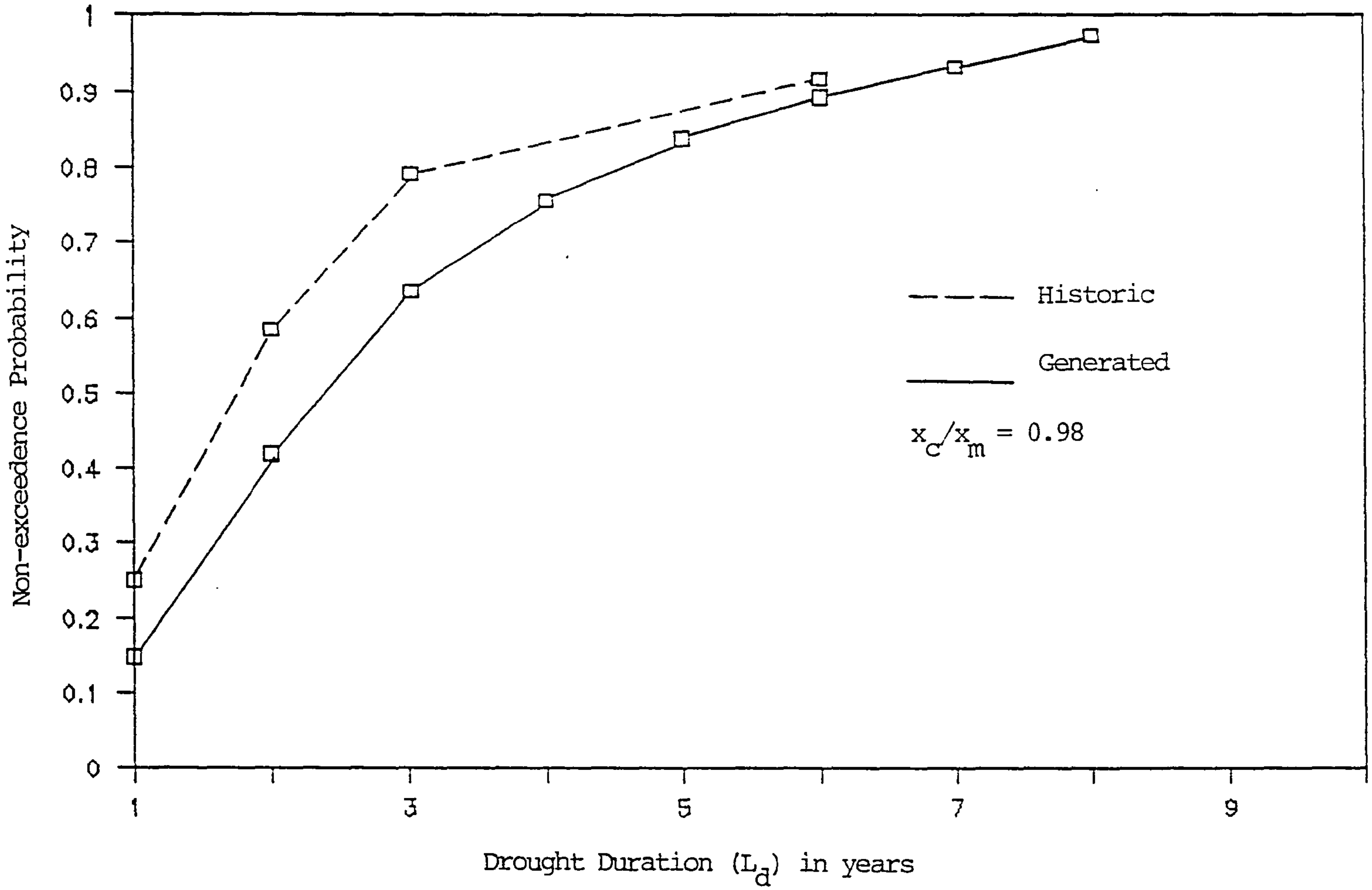


Fig. D.3.1 Probability Distributions of Drought Durations Derived from Historic and Generated Flow Series - Rockaway River at Boonton

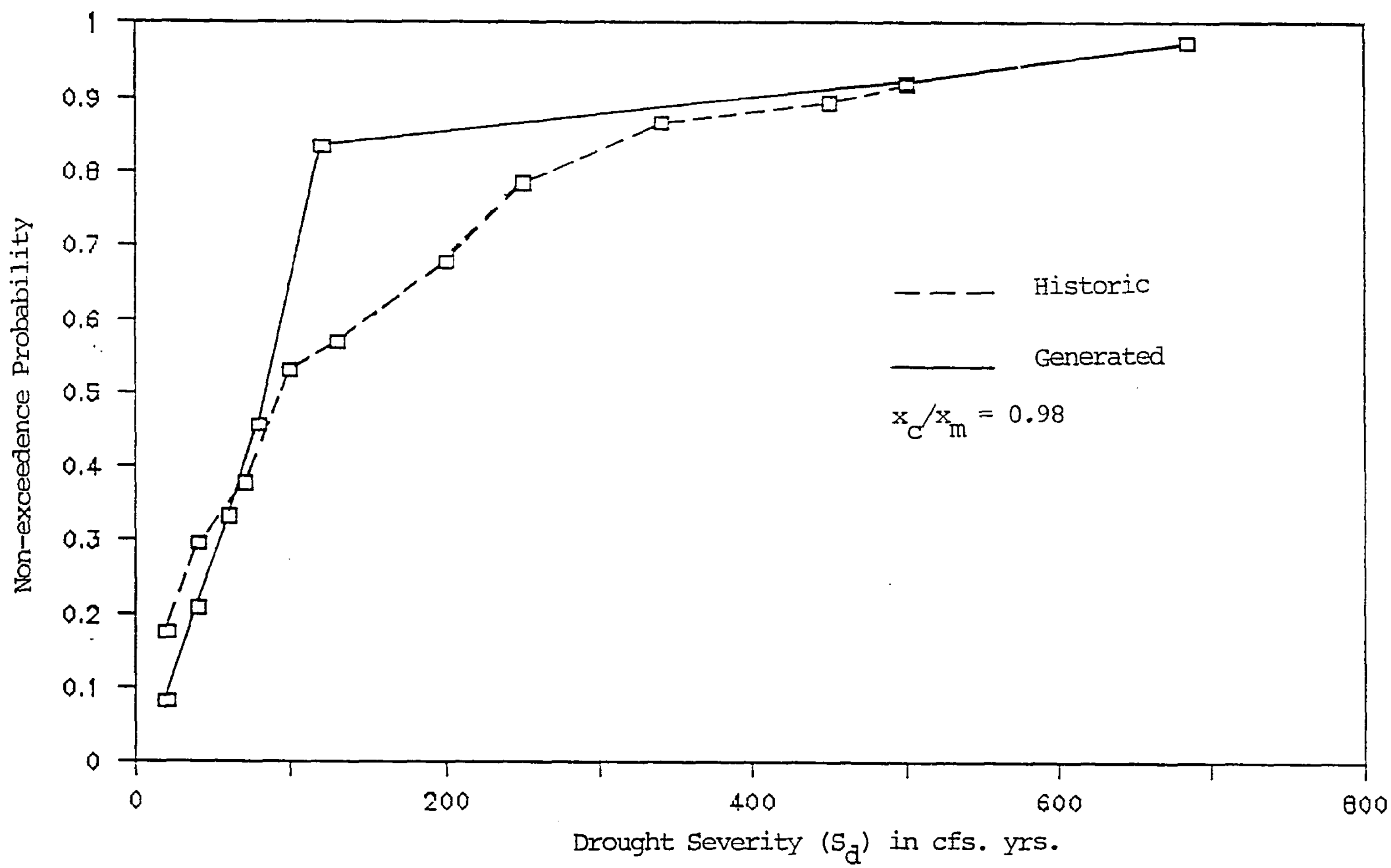


Fig. D.3.2 Probability Distributions of Drought Severities Derived from Historic and Generated Flow Series - Rockaway River at Boonton

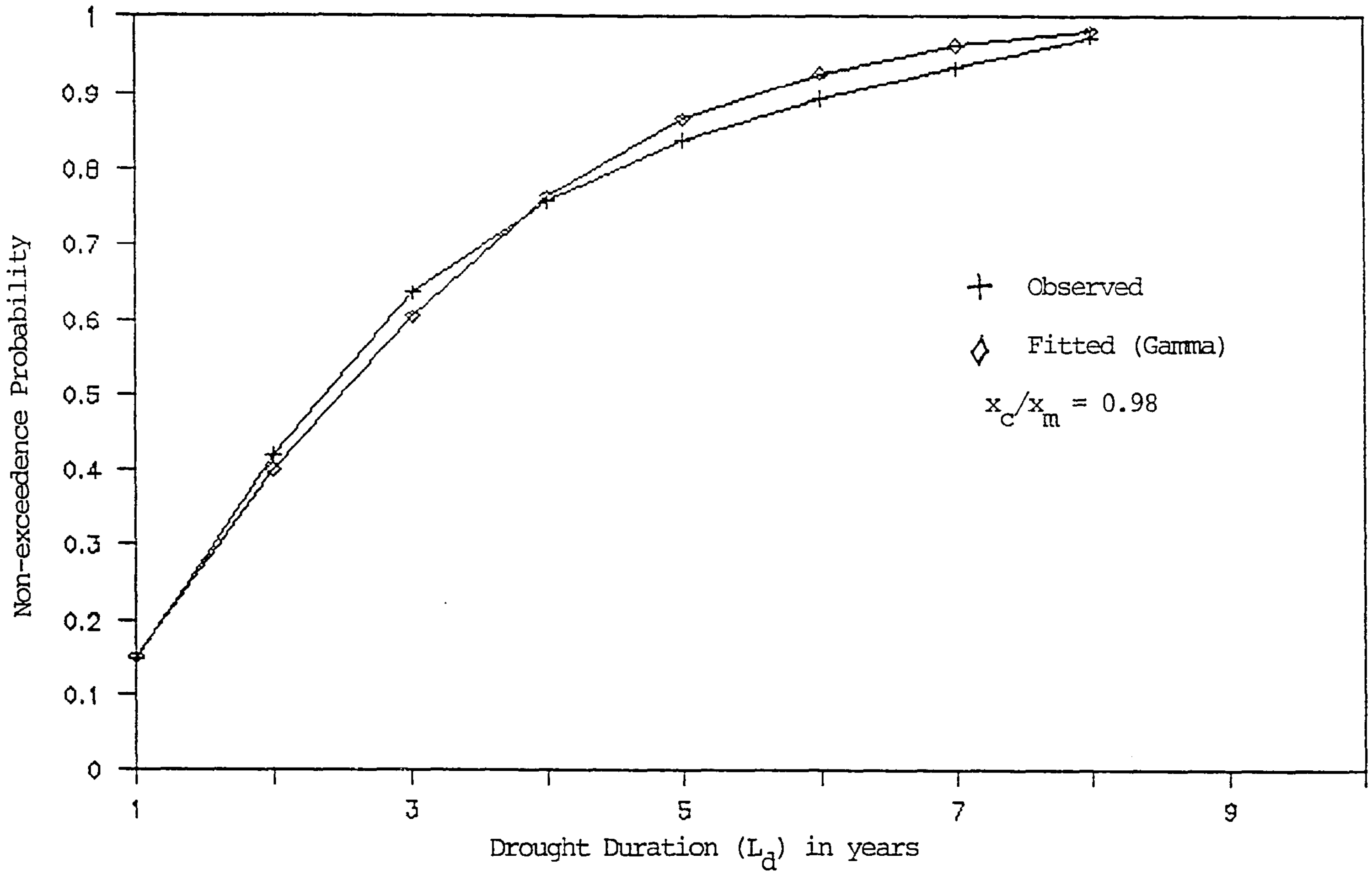


Fig. D.3.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - Rockaway River at Boonton

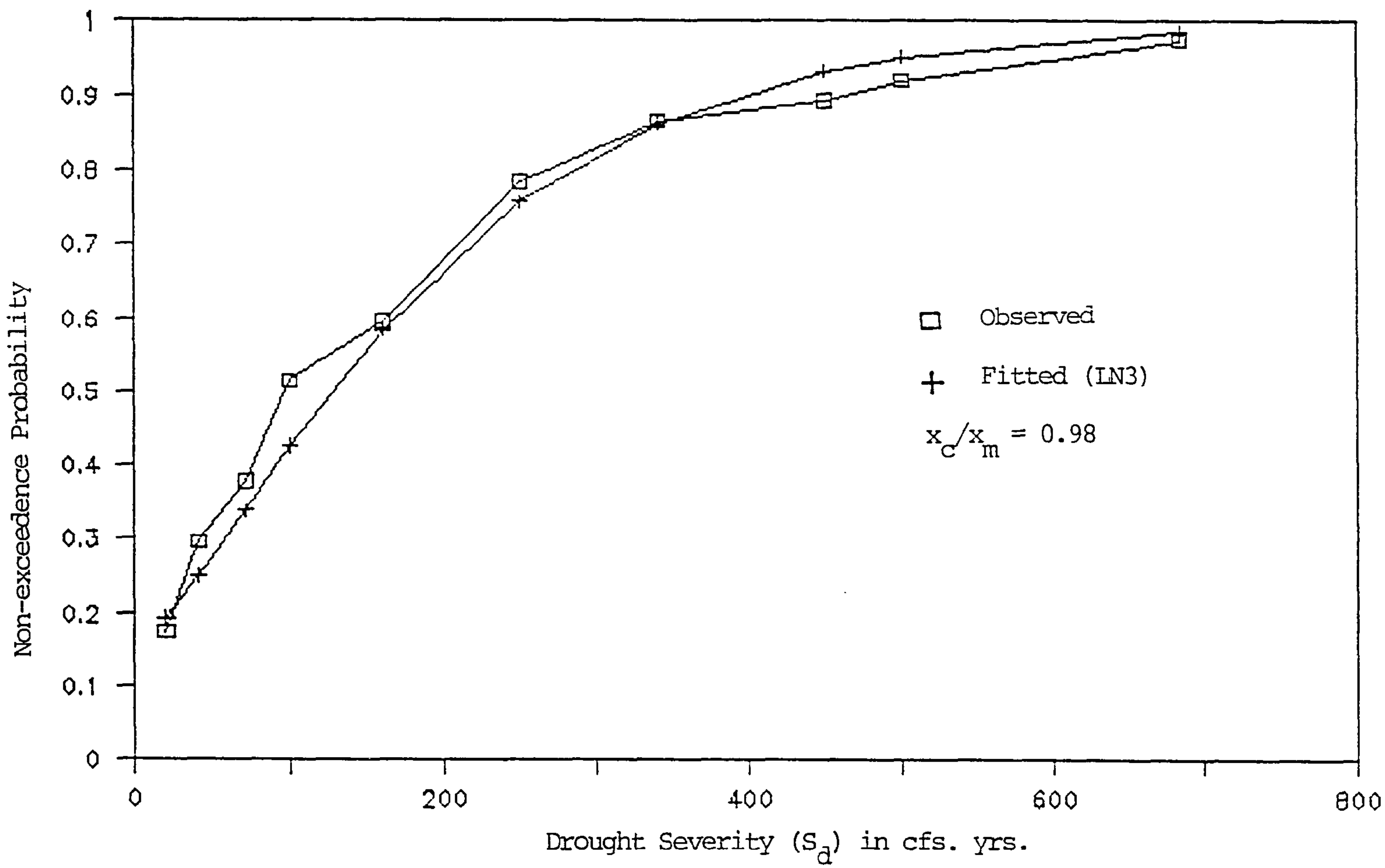


Fig. D.3.4 Comparison of Observed and Fitted Probability Distributions for Drought Severities Derived from Generated Flow Series - Rockaway River at Boonton

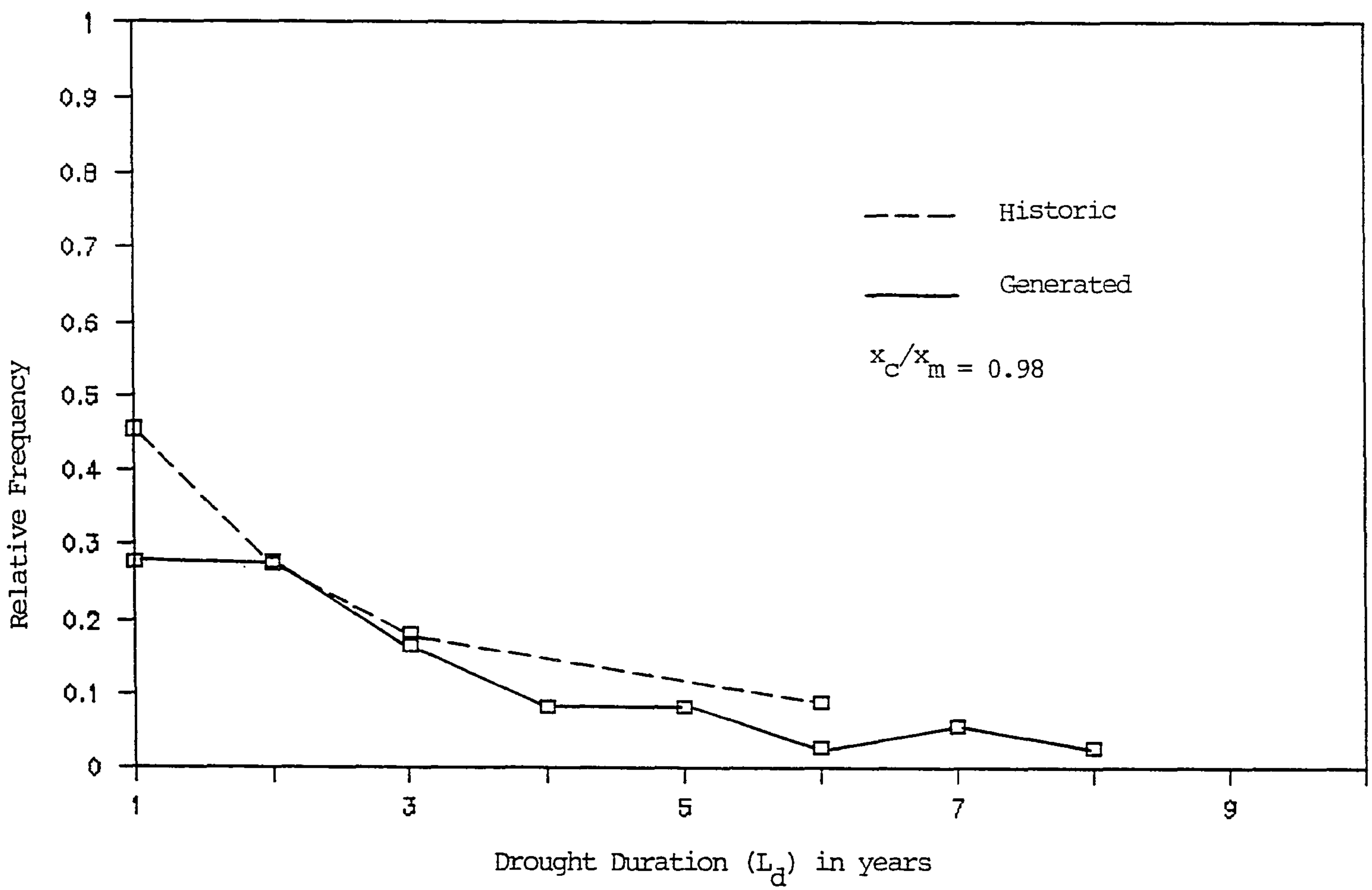


Fig. D.3.5 Probability Density Curves of Drought Durations - Rockaway River at Boonton

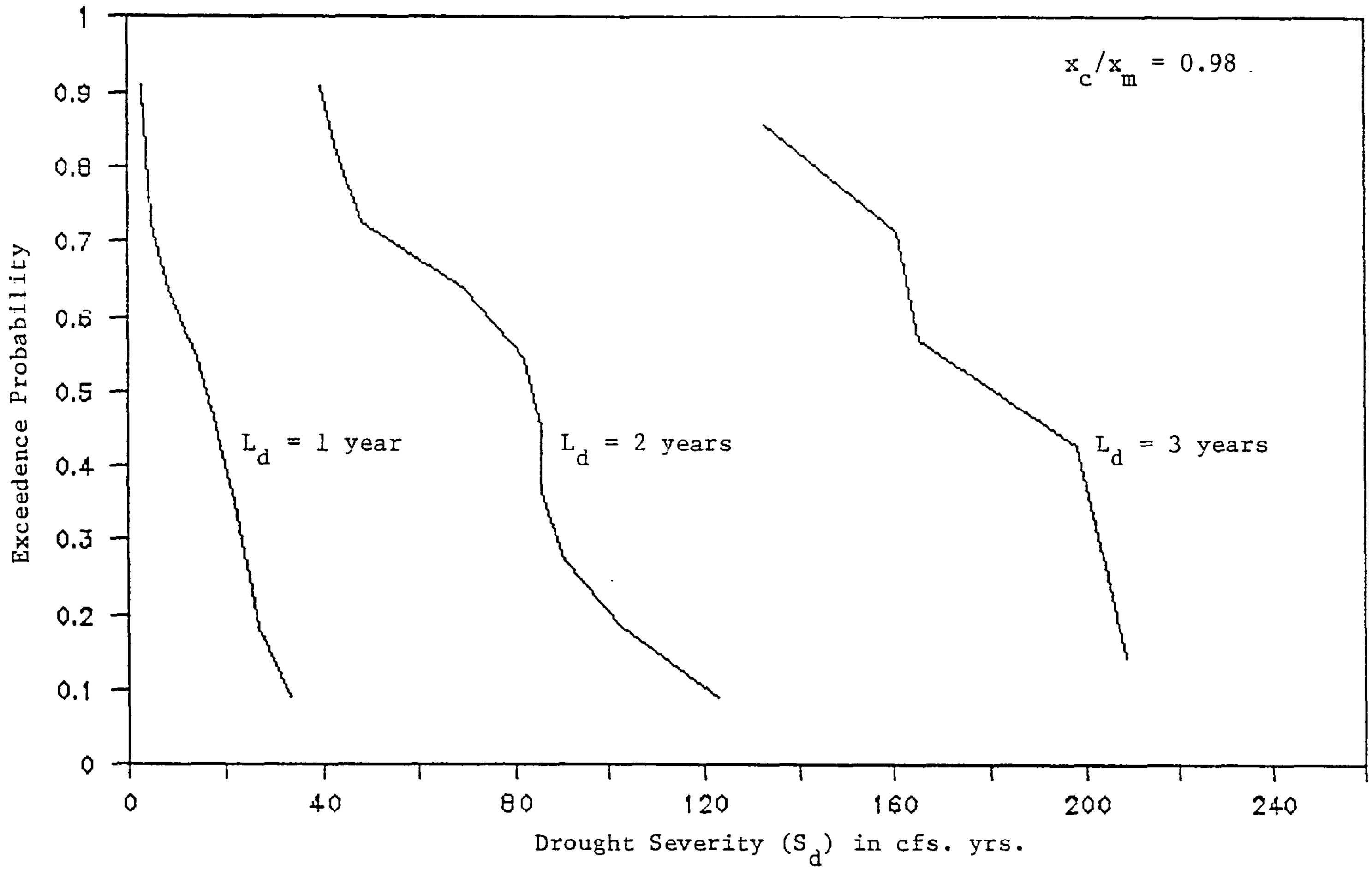


Fig. D.3.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - Rockaway River at Boonton

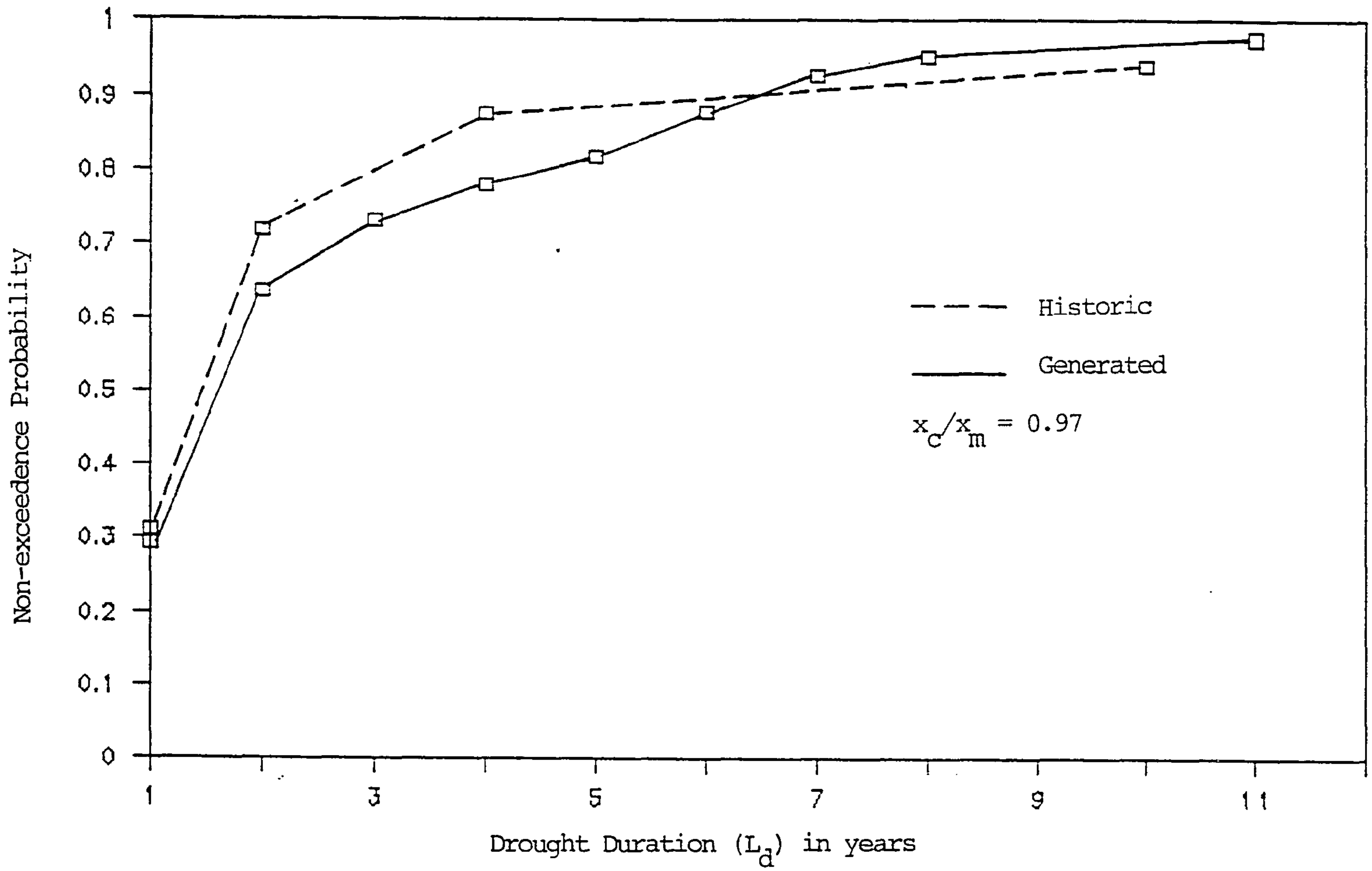


Fig. D.4.1 Probability Distributions of Drought Durations Derived from Historic and Generated Flow Series - Passaic River near Millington

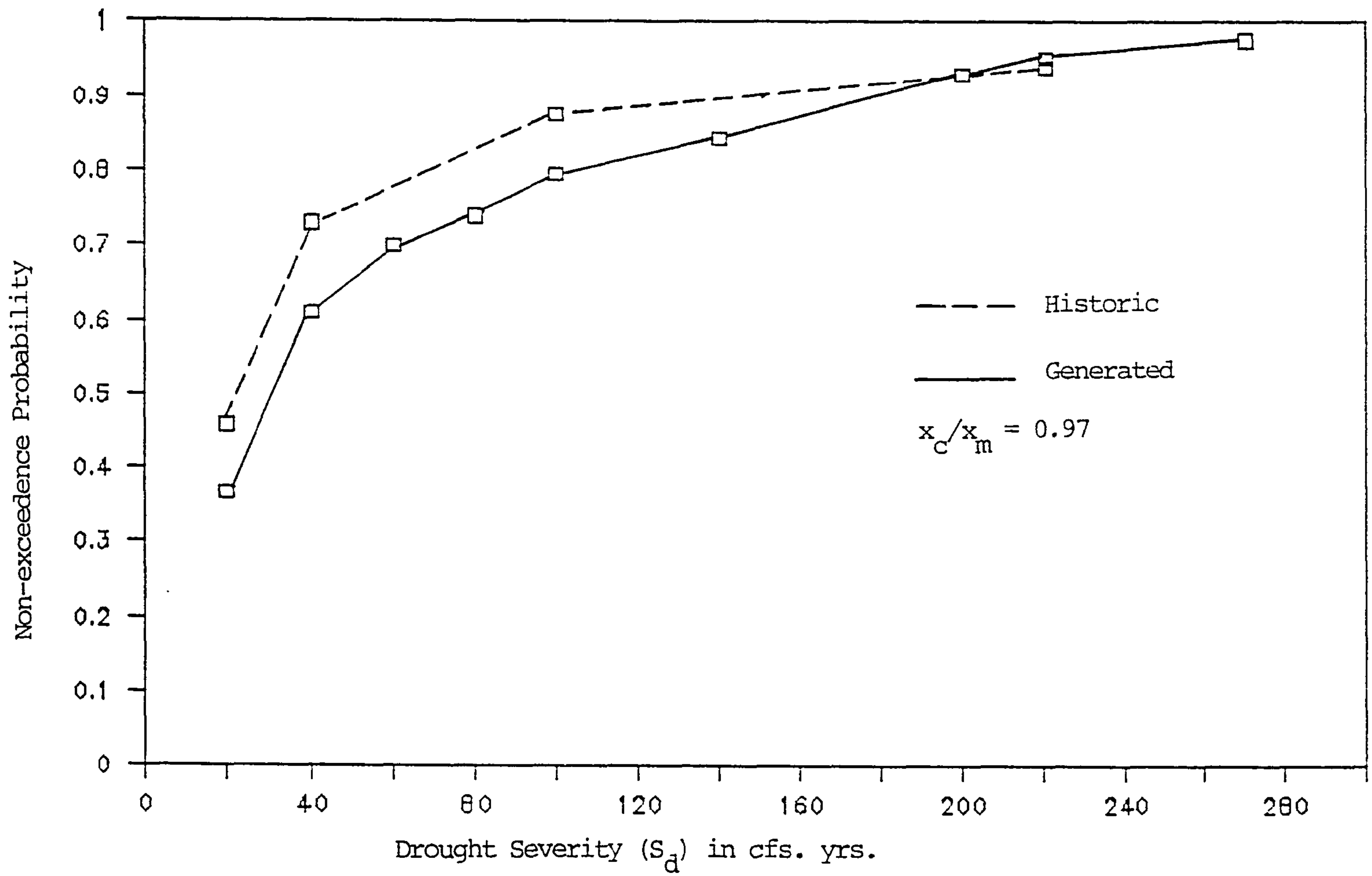


Fig. D.4.2 Probability Distributions of Drought Severities Derived from Historic and Generated Flow Series - Passaic River near Millington

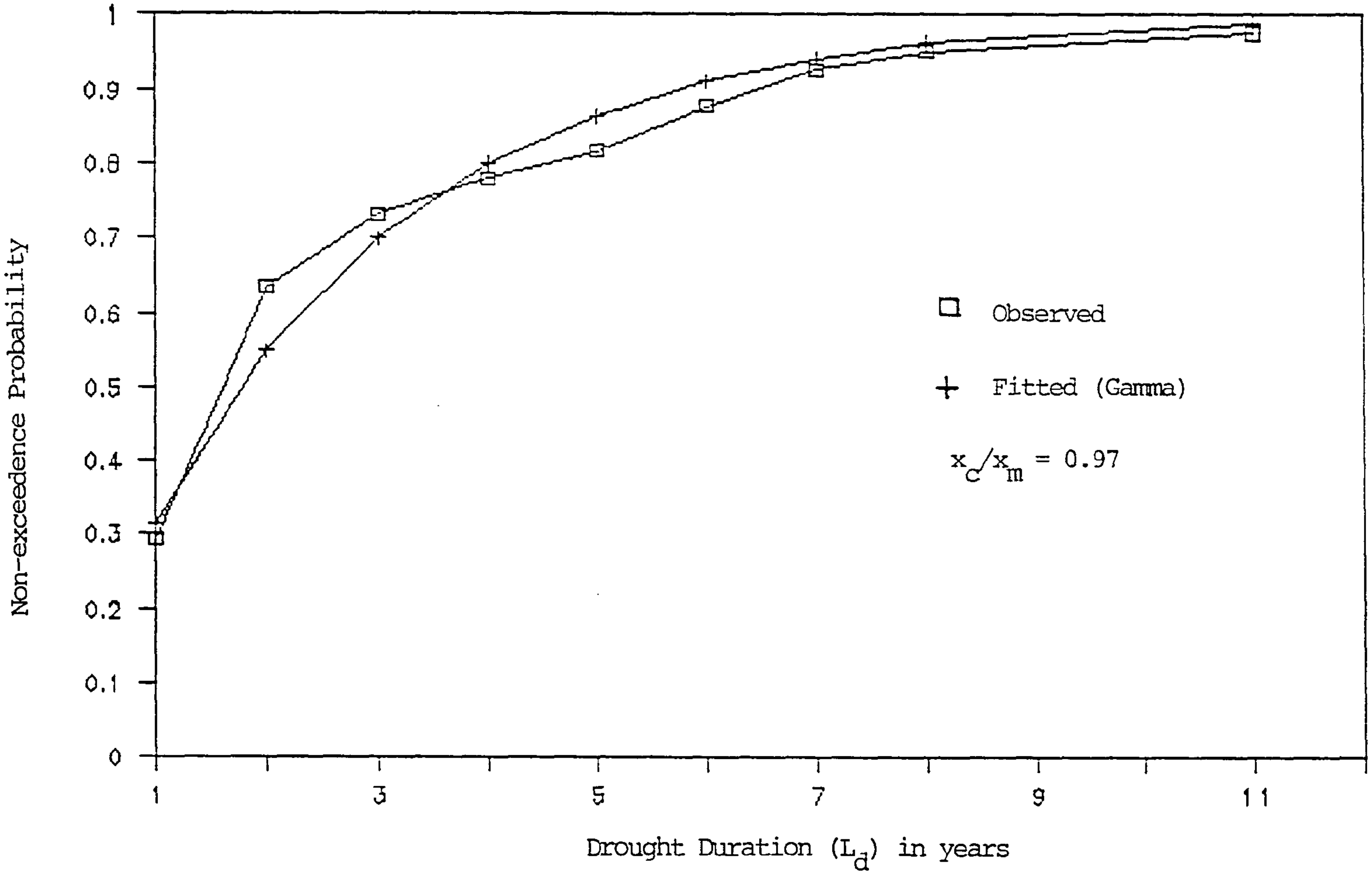


Fig. D.4.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - Passaic River near Millington

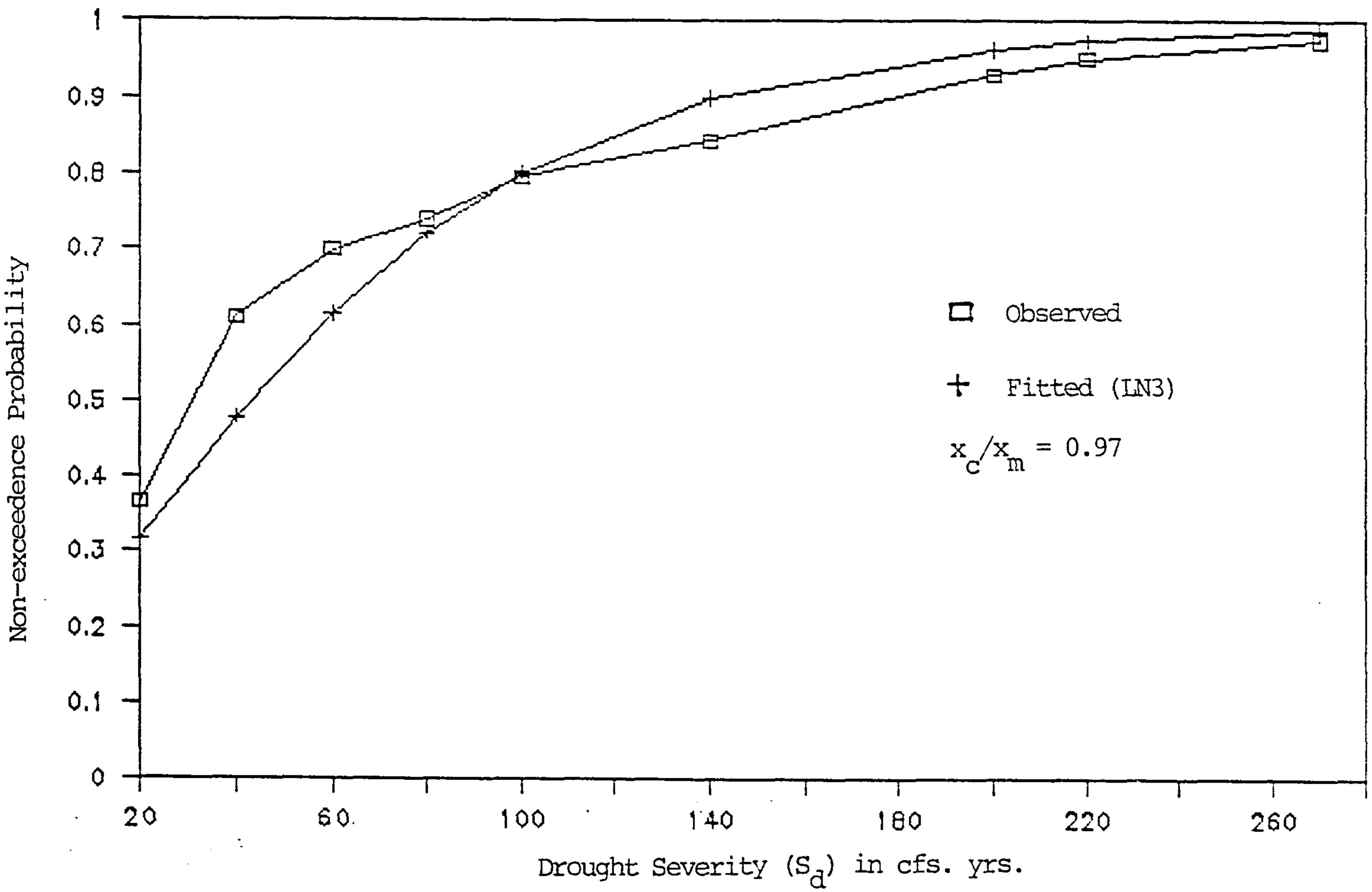


Fig. D.4.4 Comparison of Observed and Fitted Probability Distributions for Drought Severities Derived from Generated Flow Series - Passaic River near Millington

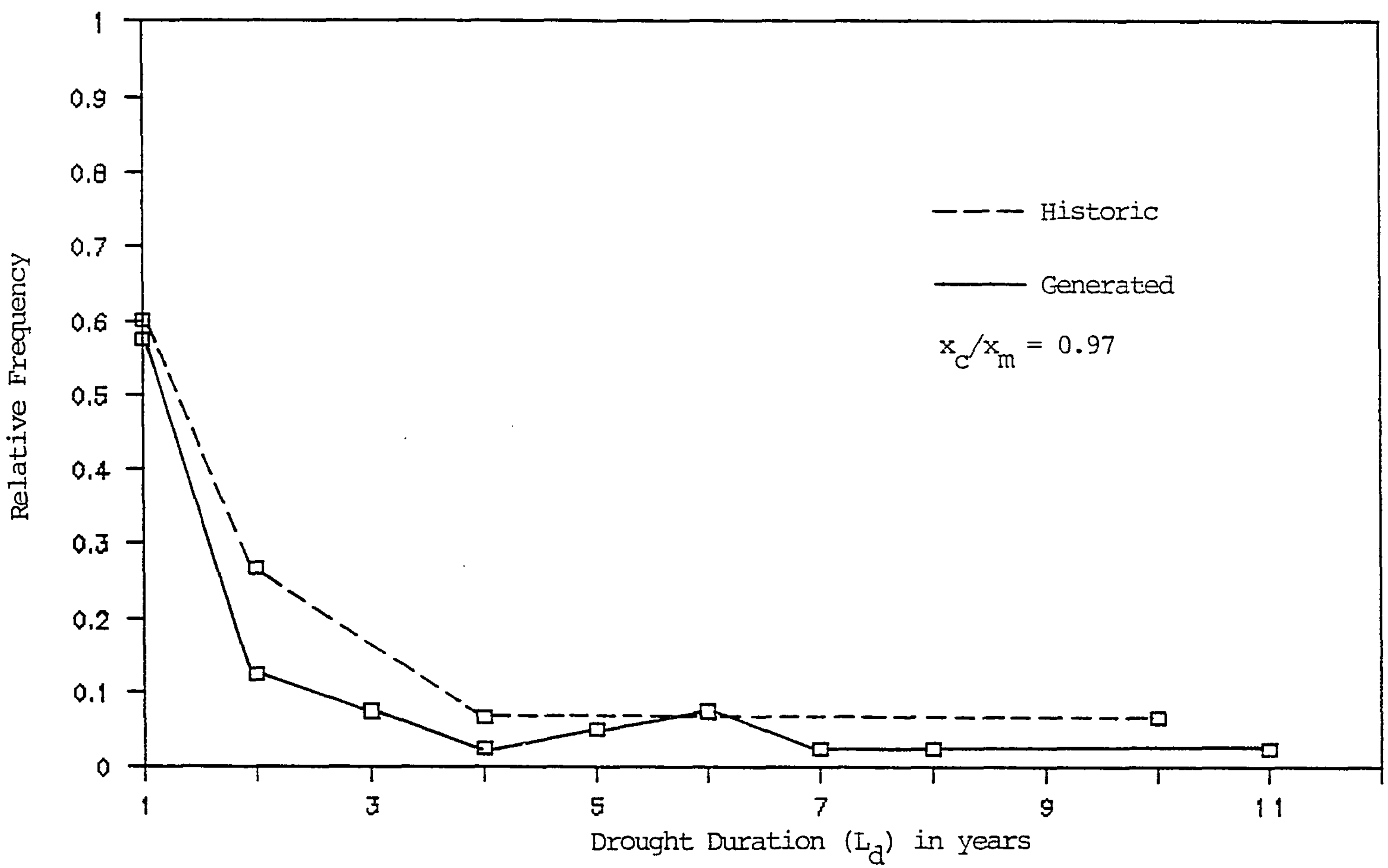


Fig. D.4.5 Probability Density Curves of Drought Durations - Passaic River near Millington

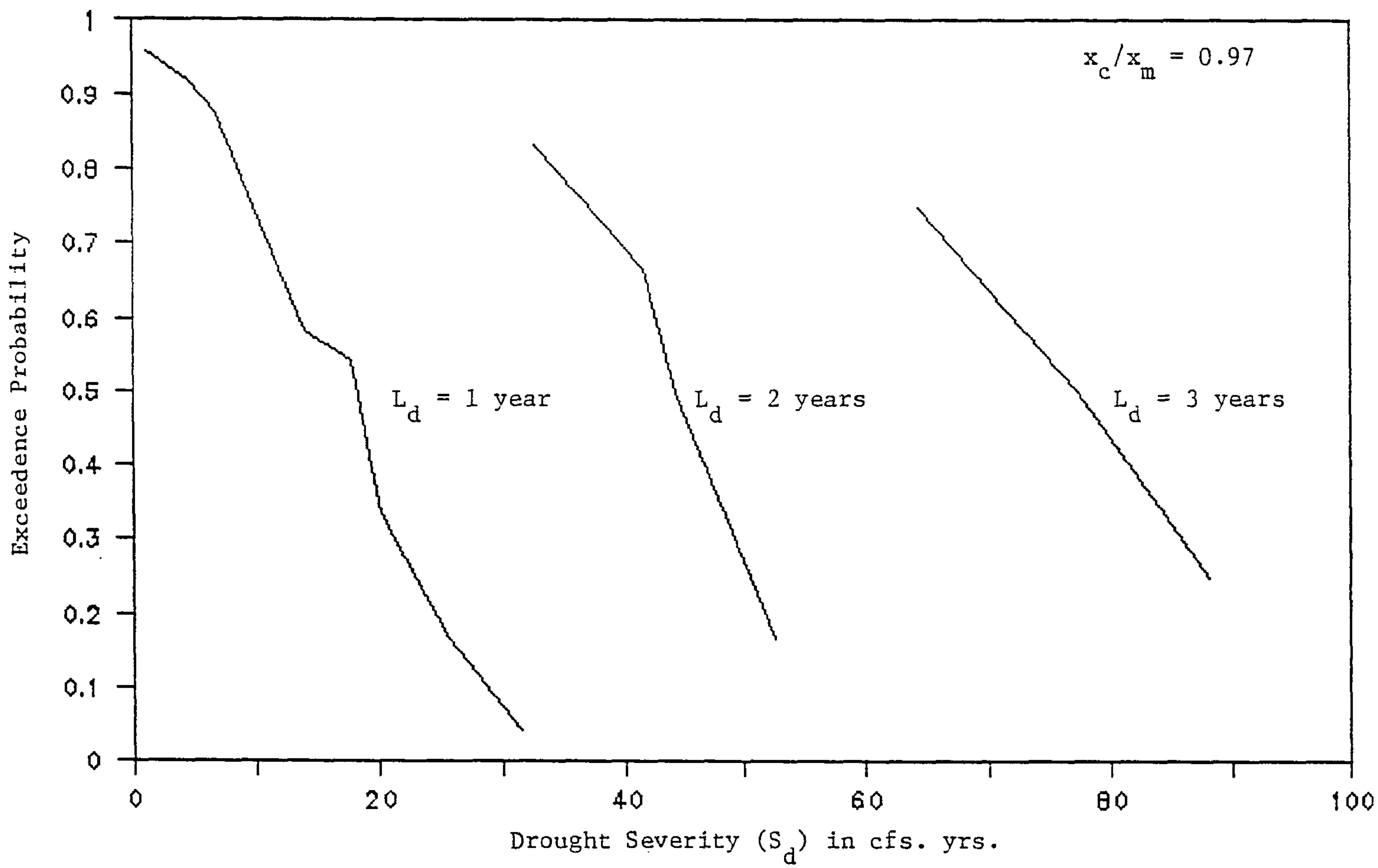


Fig. D.4.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - Passaic River near Millington

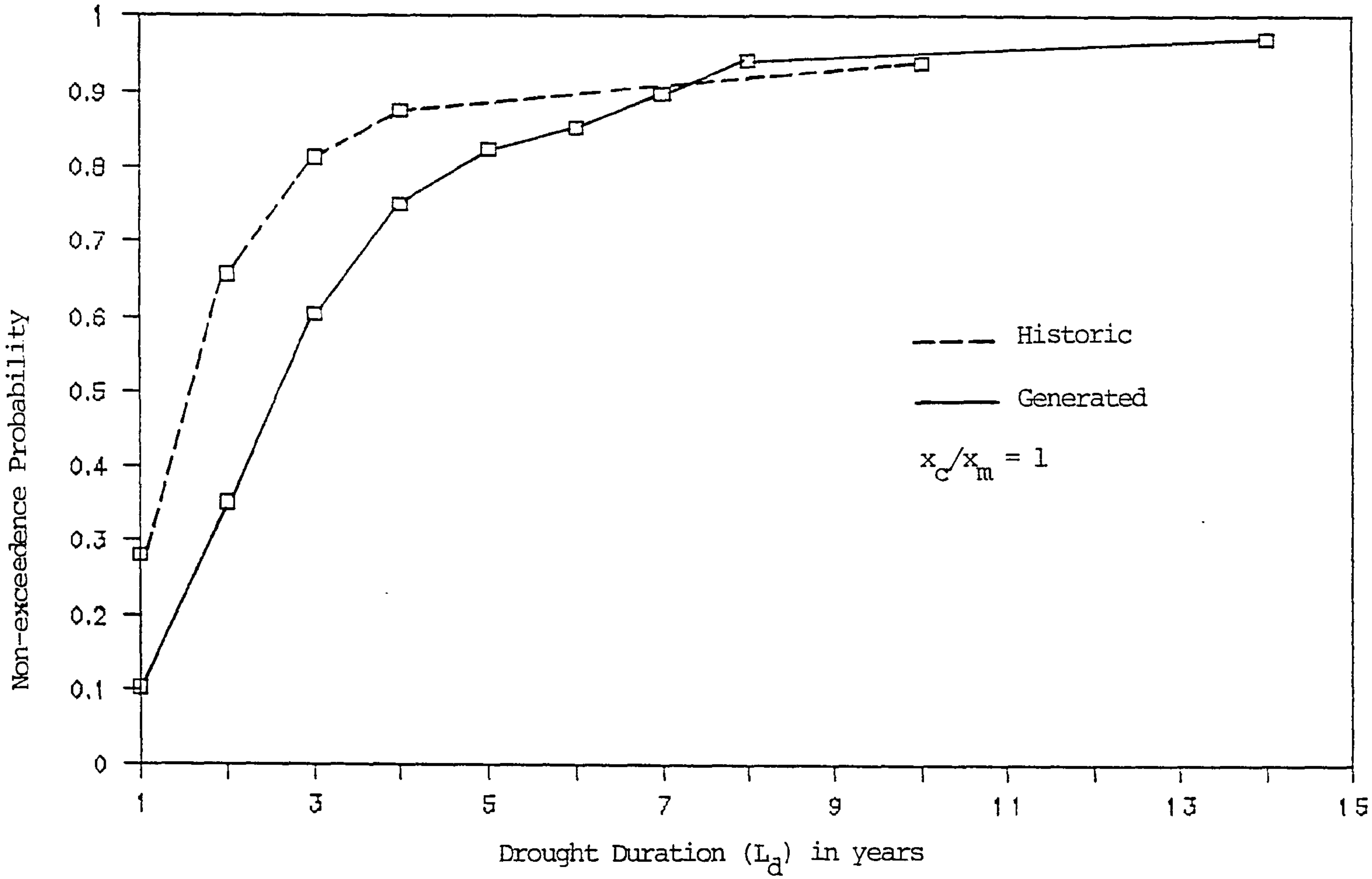
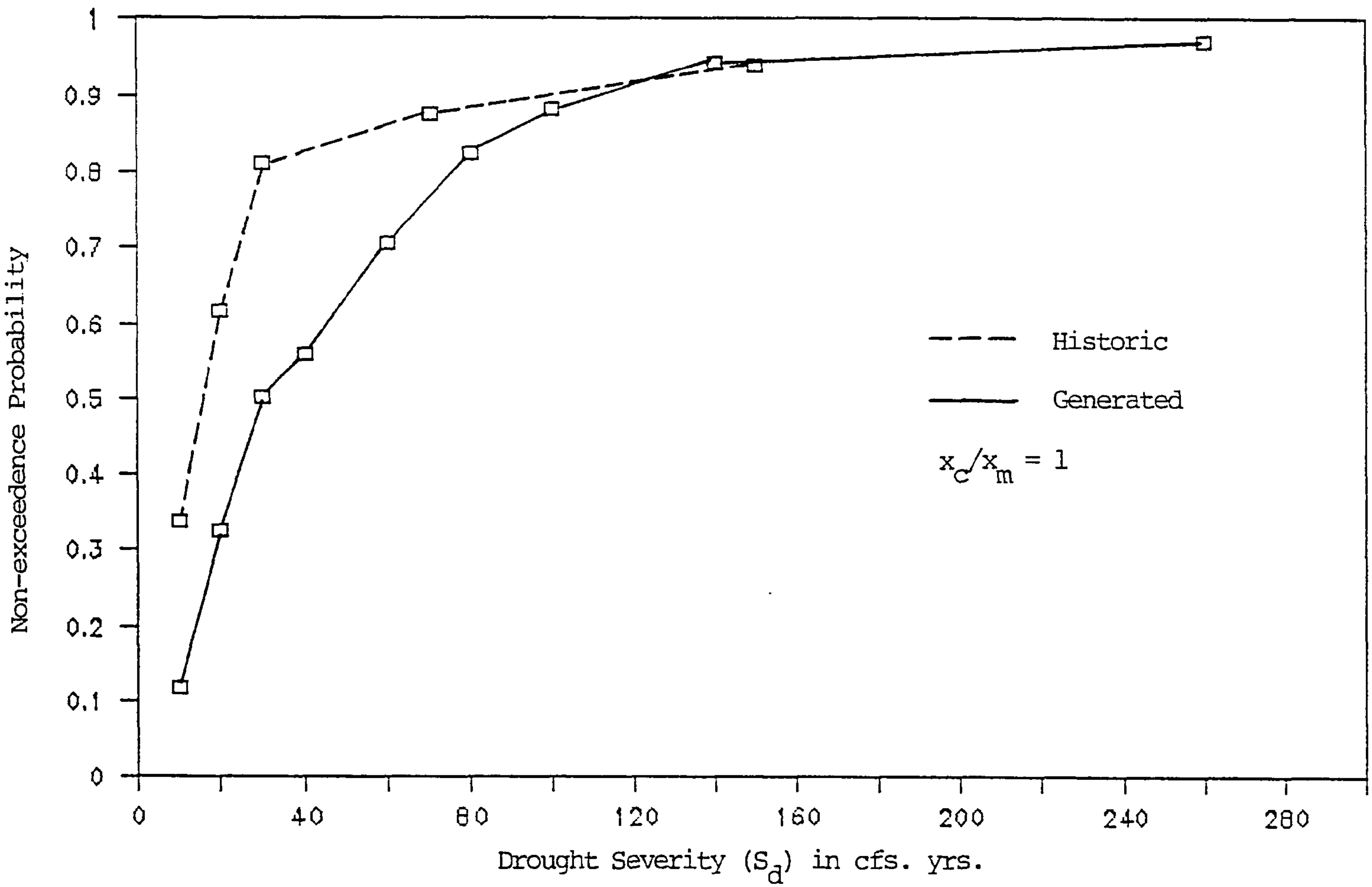


Fig. D.5.1 Probability Distributions of Drought Durations Derived from Historic and Generated Flow Series - Lamington River near Pottersville



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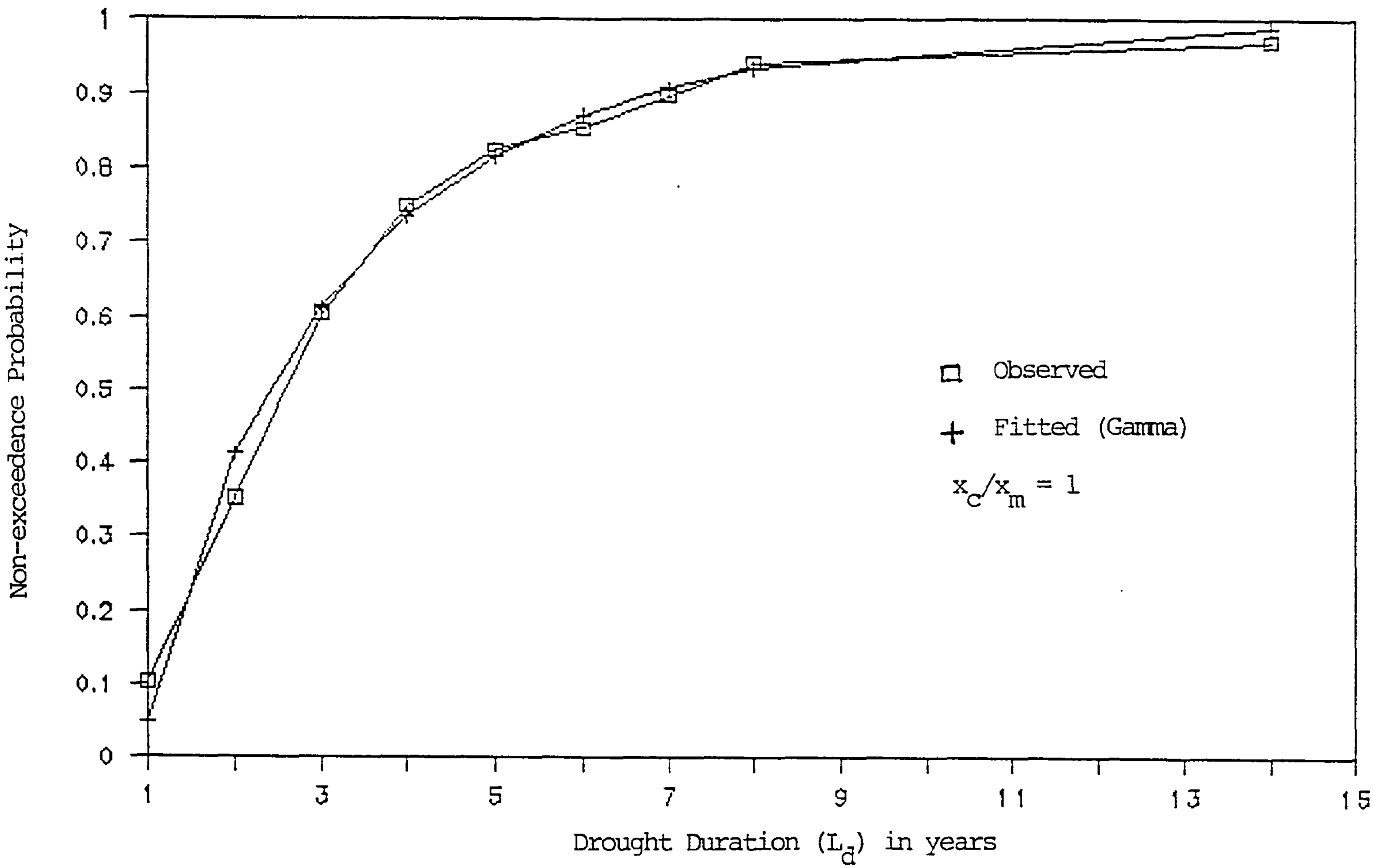
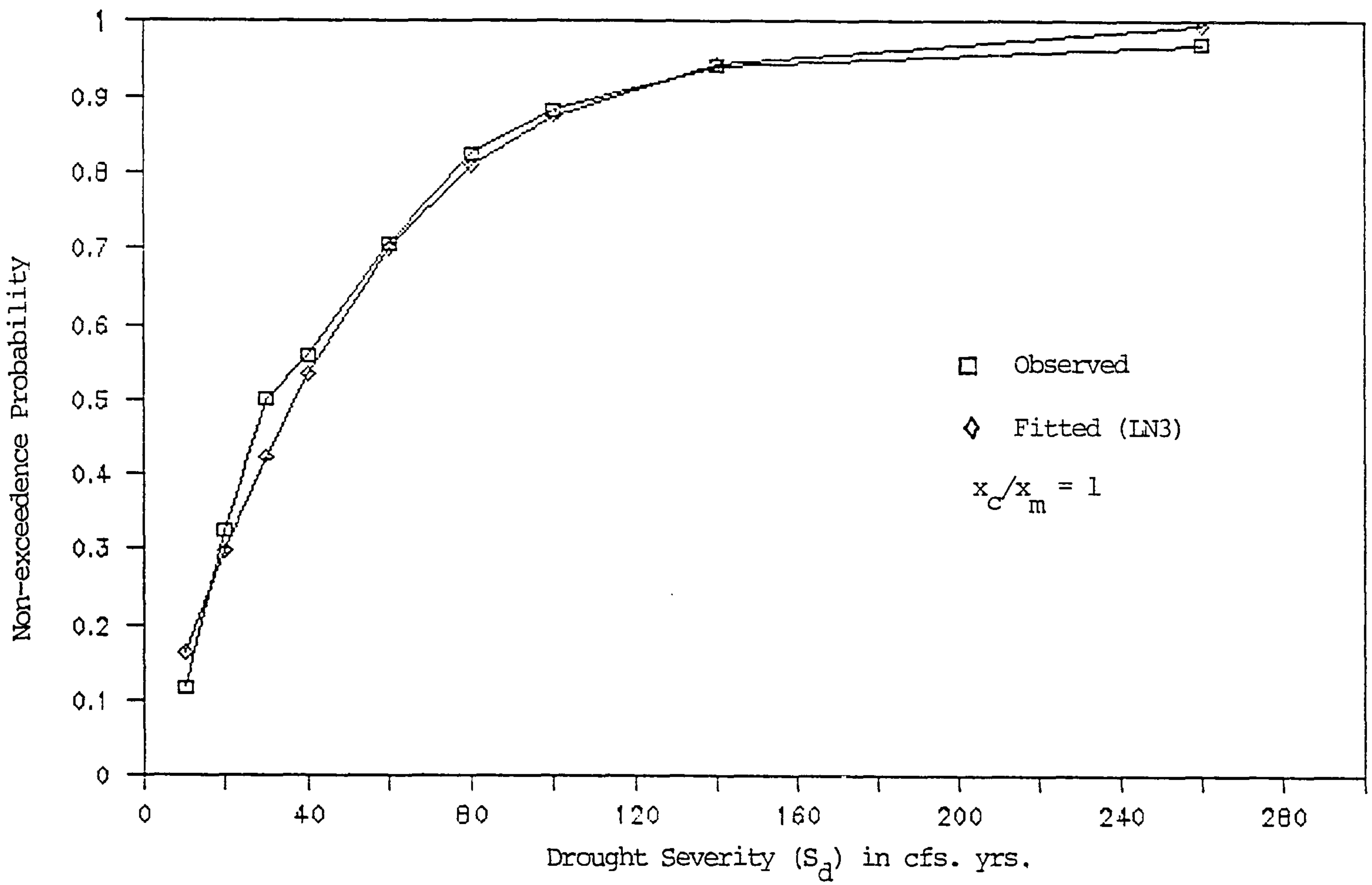


Fig. D.5.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - Lamington River near Pottersville



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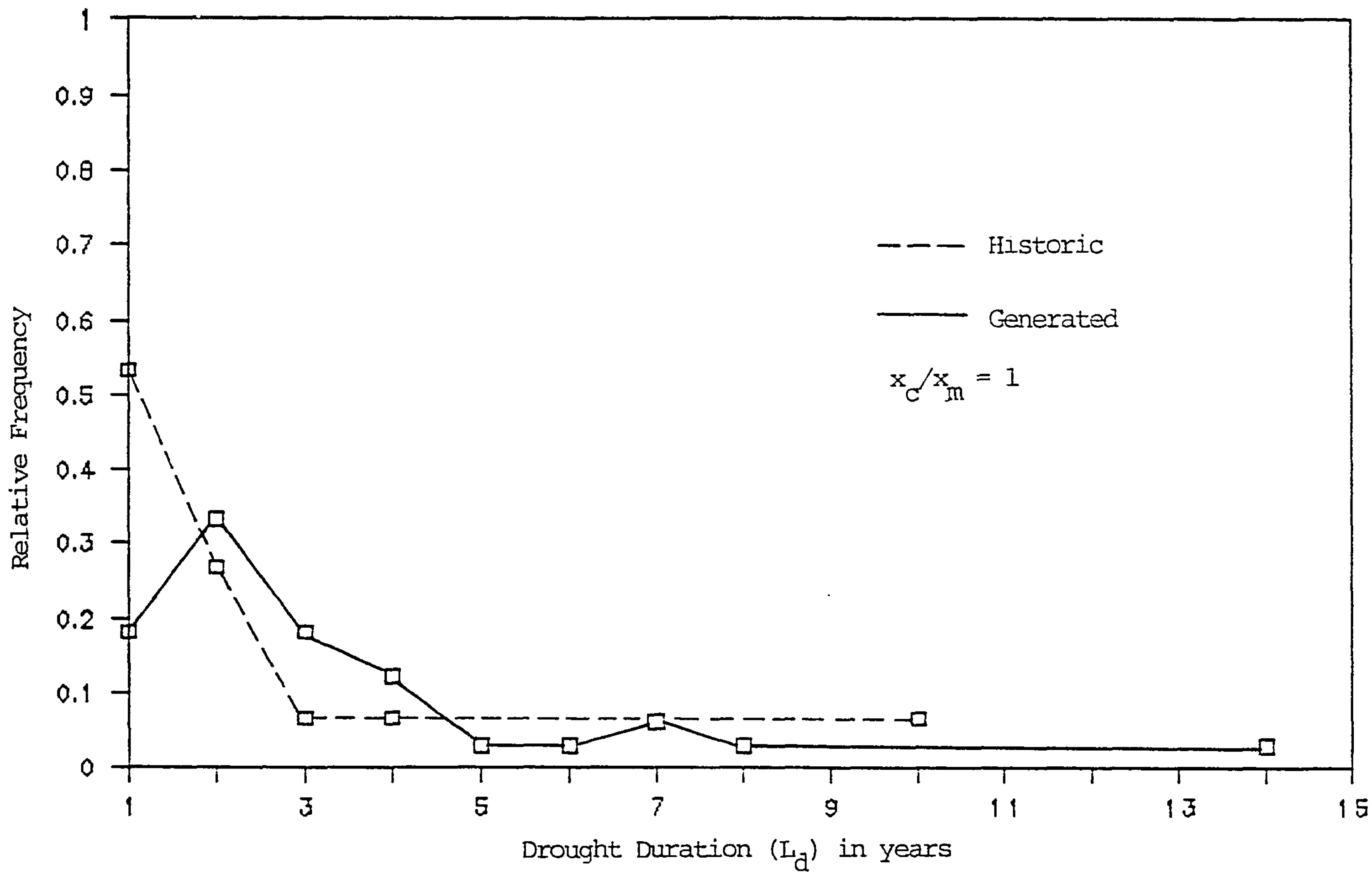

Fig. D.5.5 Probability Density Curves of Drought Durations - Lamington River near Pottersville

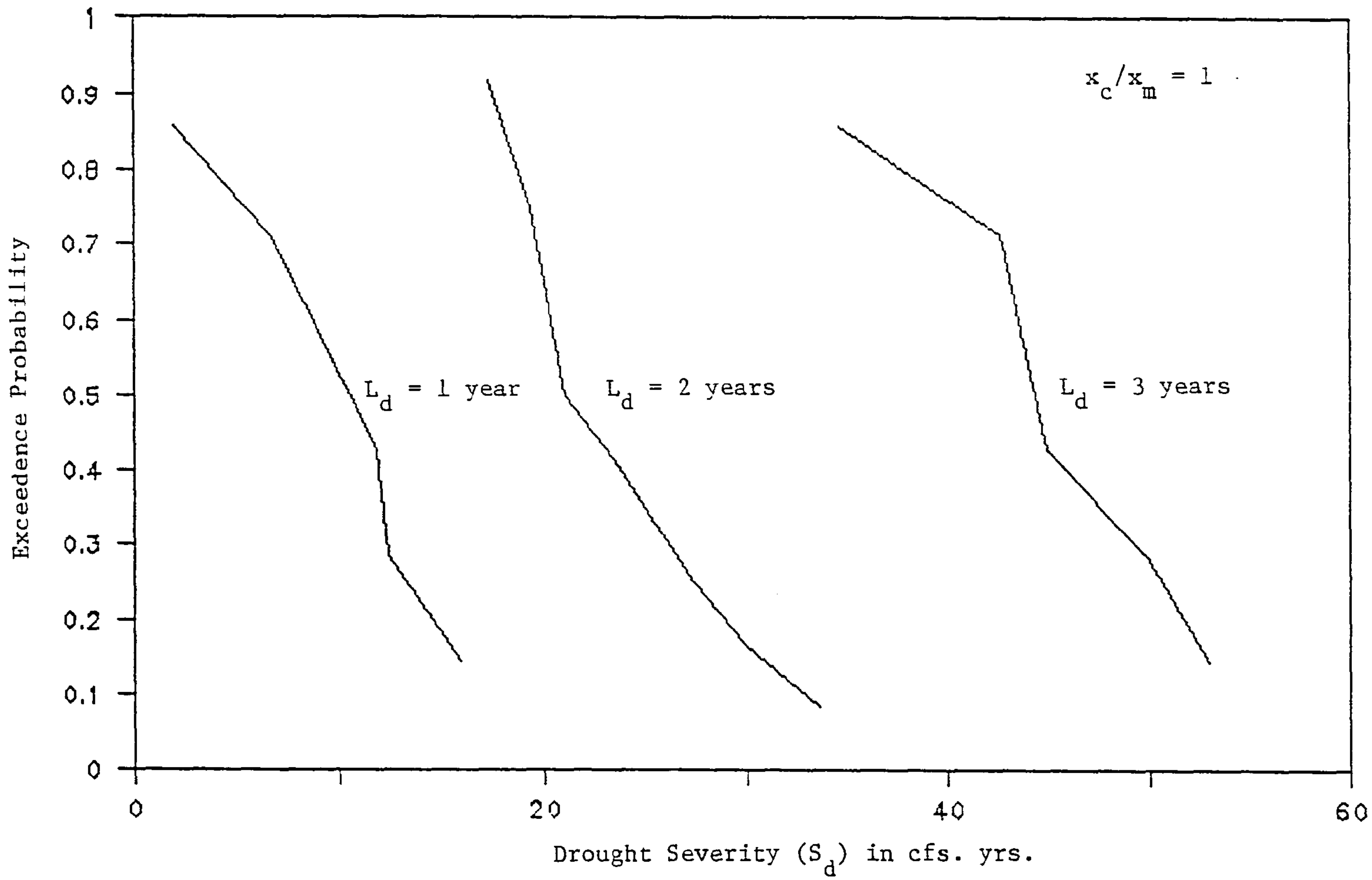

Fig. D.5.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - Lamington River near Pottersville

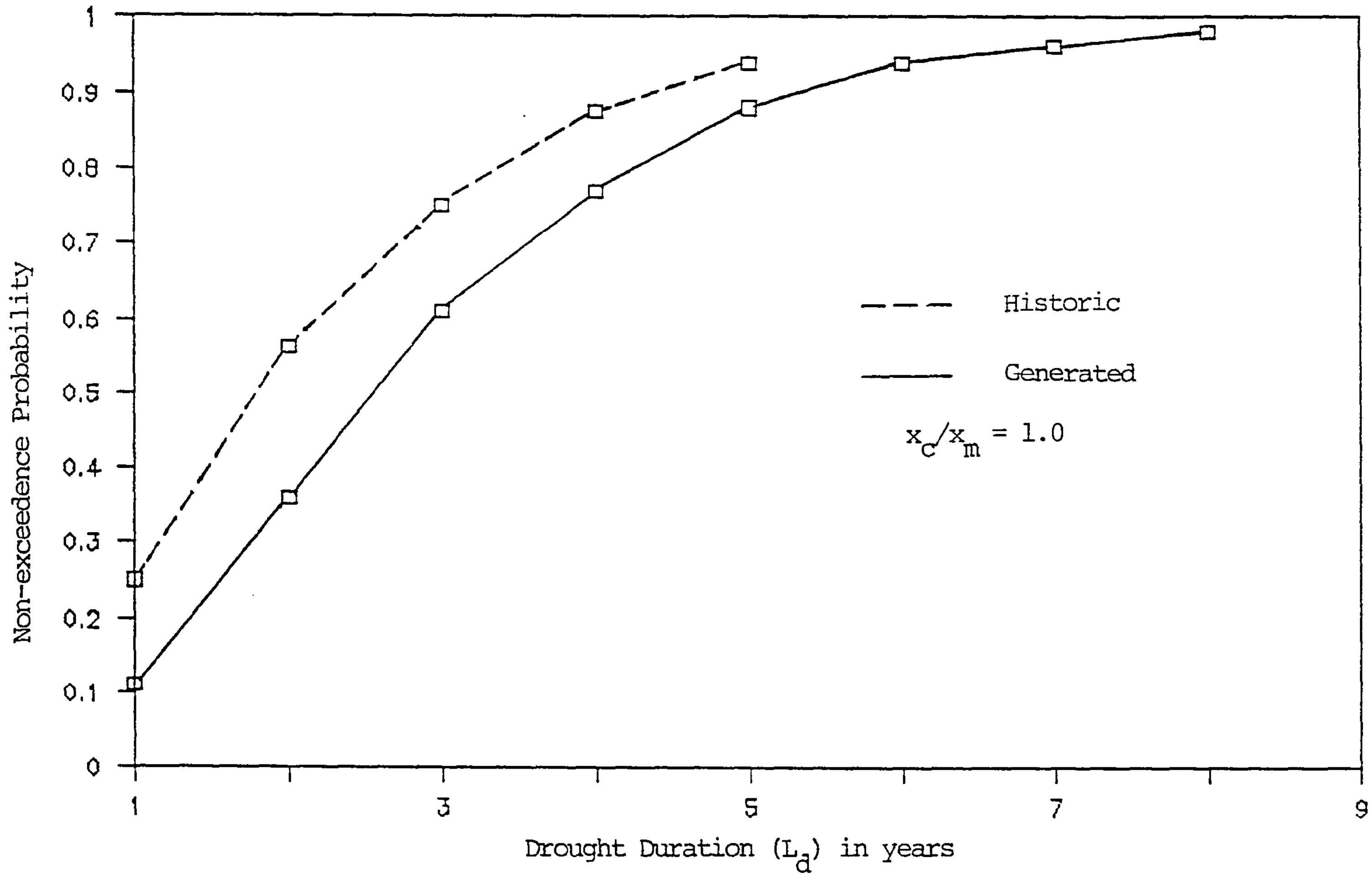

Fig. D.6.1 Probability Distributions of Drought Durations Derived from Historic and Generated Flow Series - North Branch Raritan River near Raritan

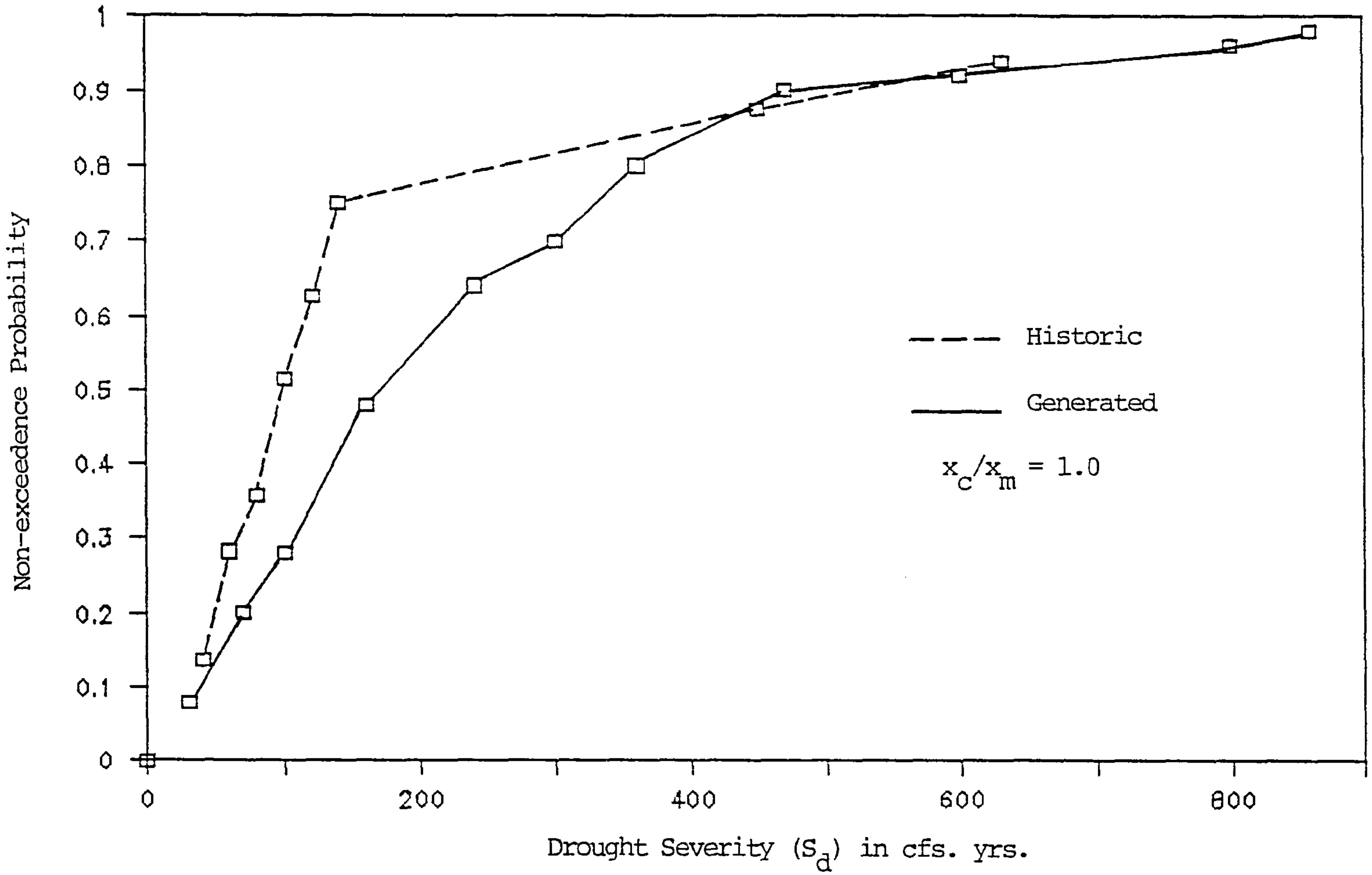

Fig. D.6.2 Probability Distributions of Drought Severities Derived from Historic and Generated Flow Series - North Branch Raritan River near Raritan

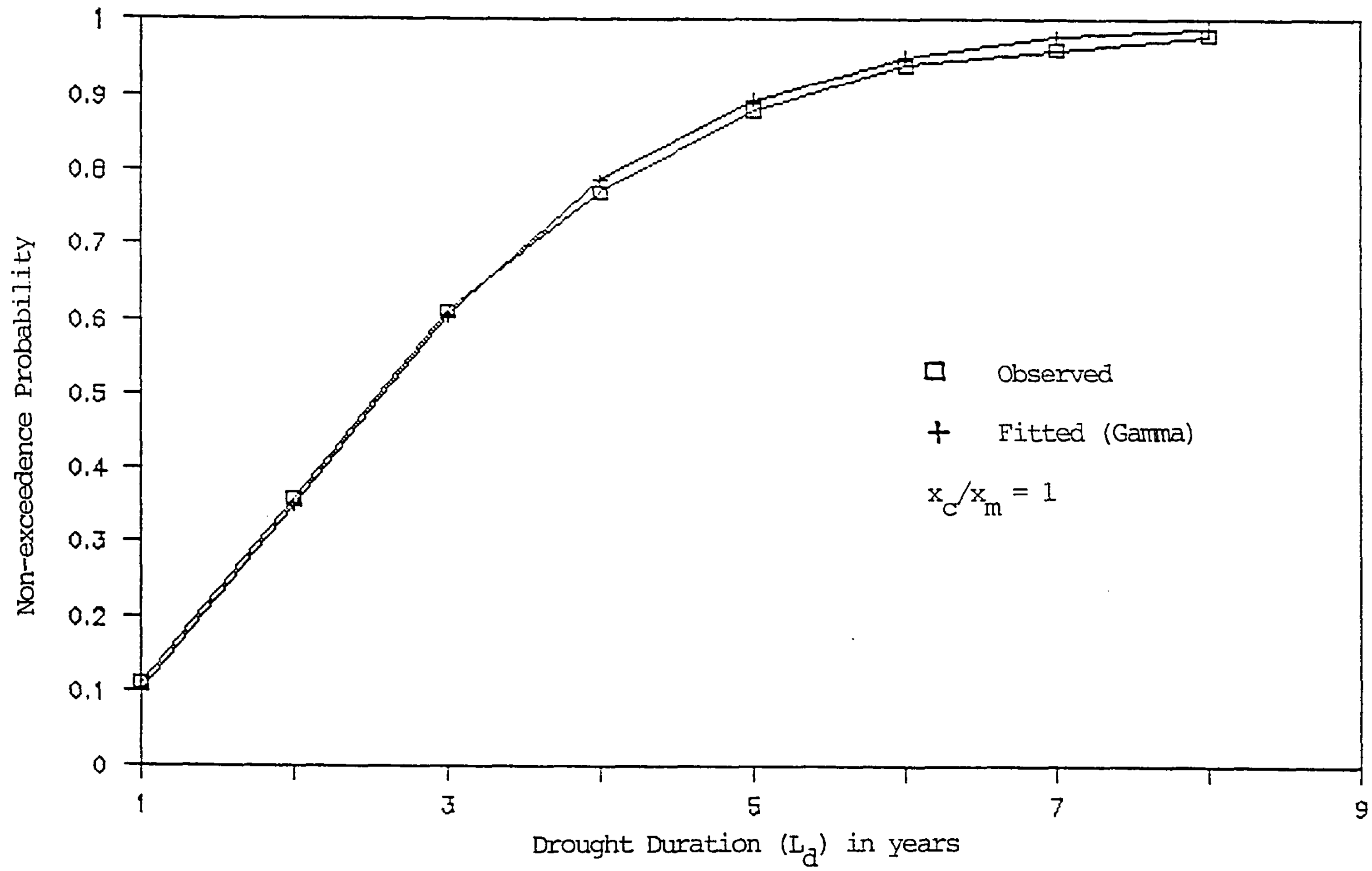

Fig. D.6.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - North Branch Raritan River near Raritan

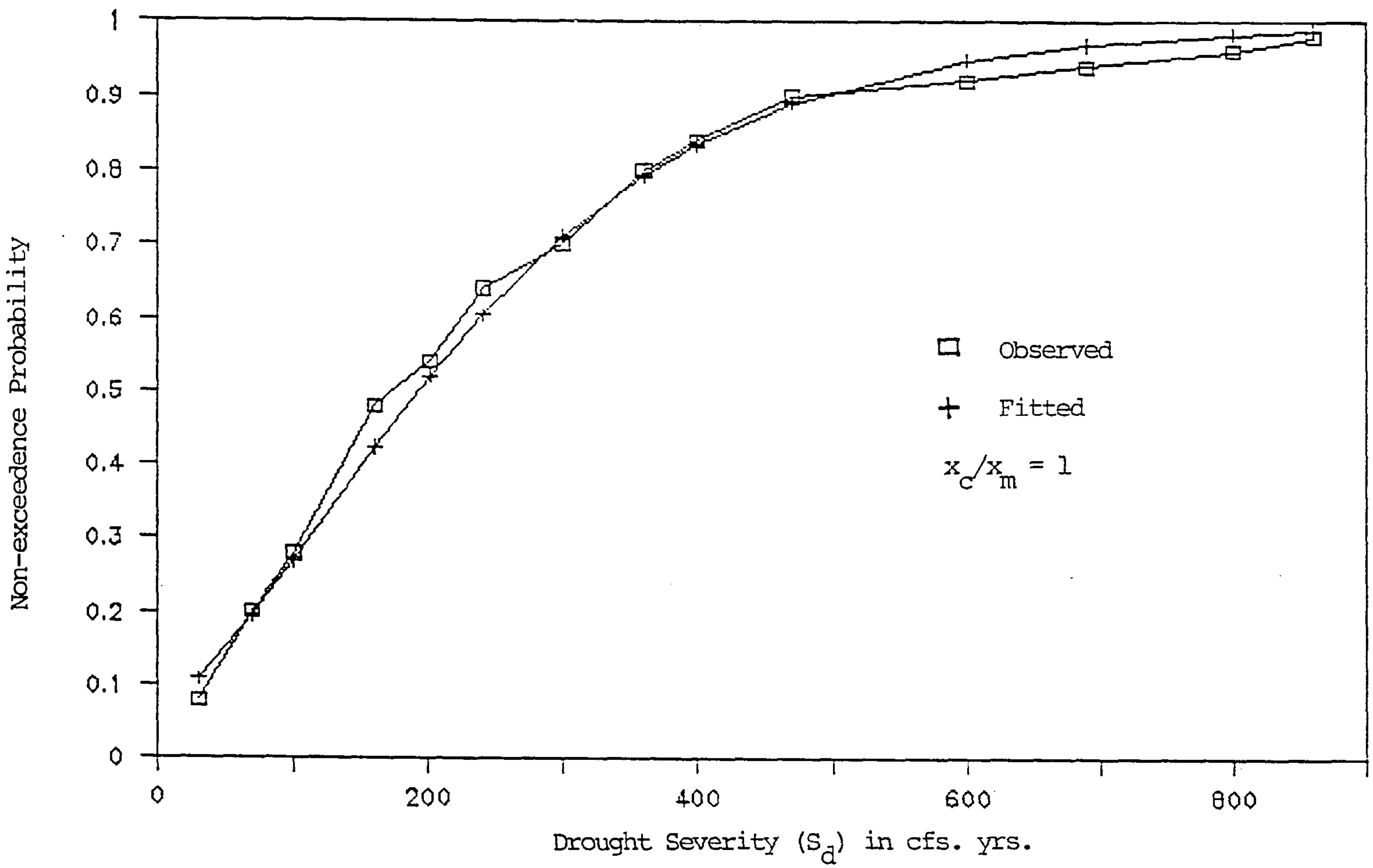

Fig. D.6.4 Comparison of Observed and Fitted Probability Distributions for Drought Severities Derived from Generated Flow Series - North Branch Raritan River near Raritan

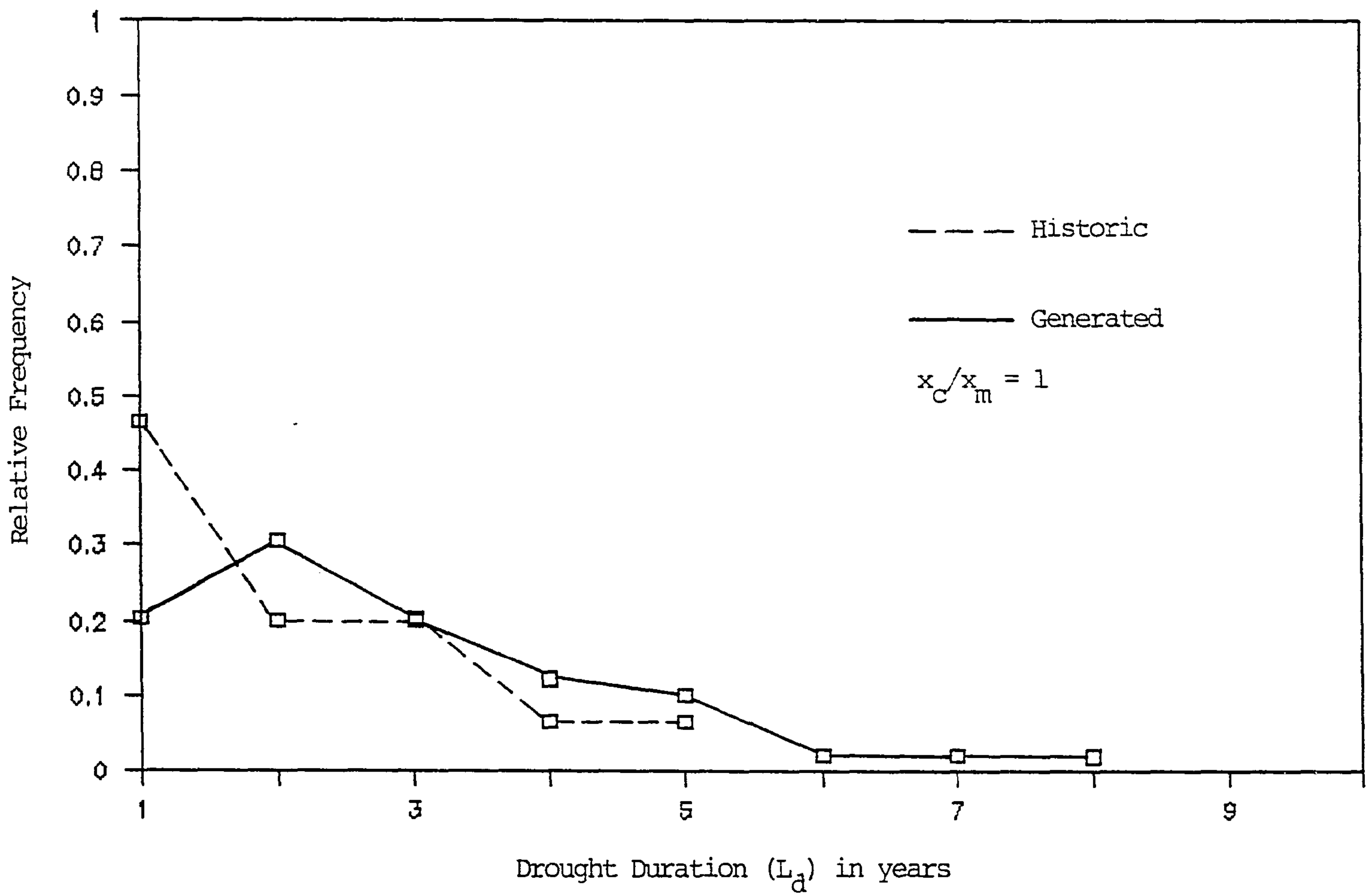

Fig. D.6.5 Probability Density Curves of Drought Durations - N. Branch Raritan River near Raritan

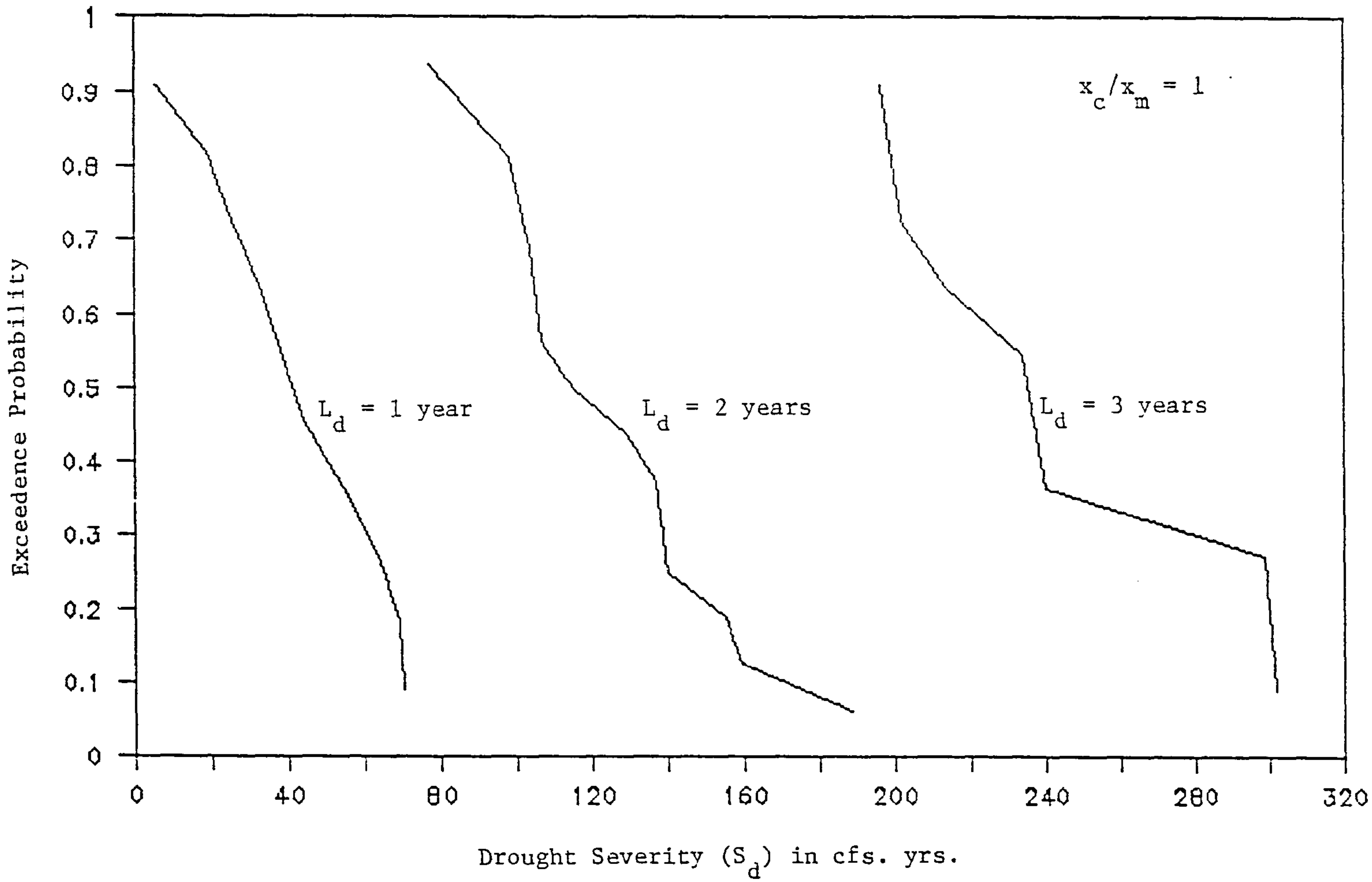

Fig. D.6.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - N. Branch Raritan River near Raritan

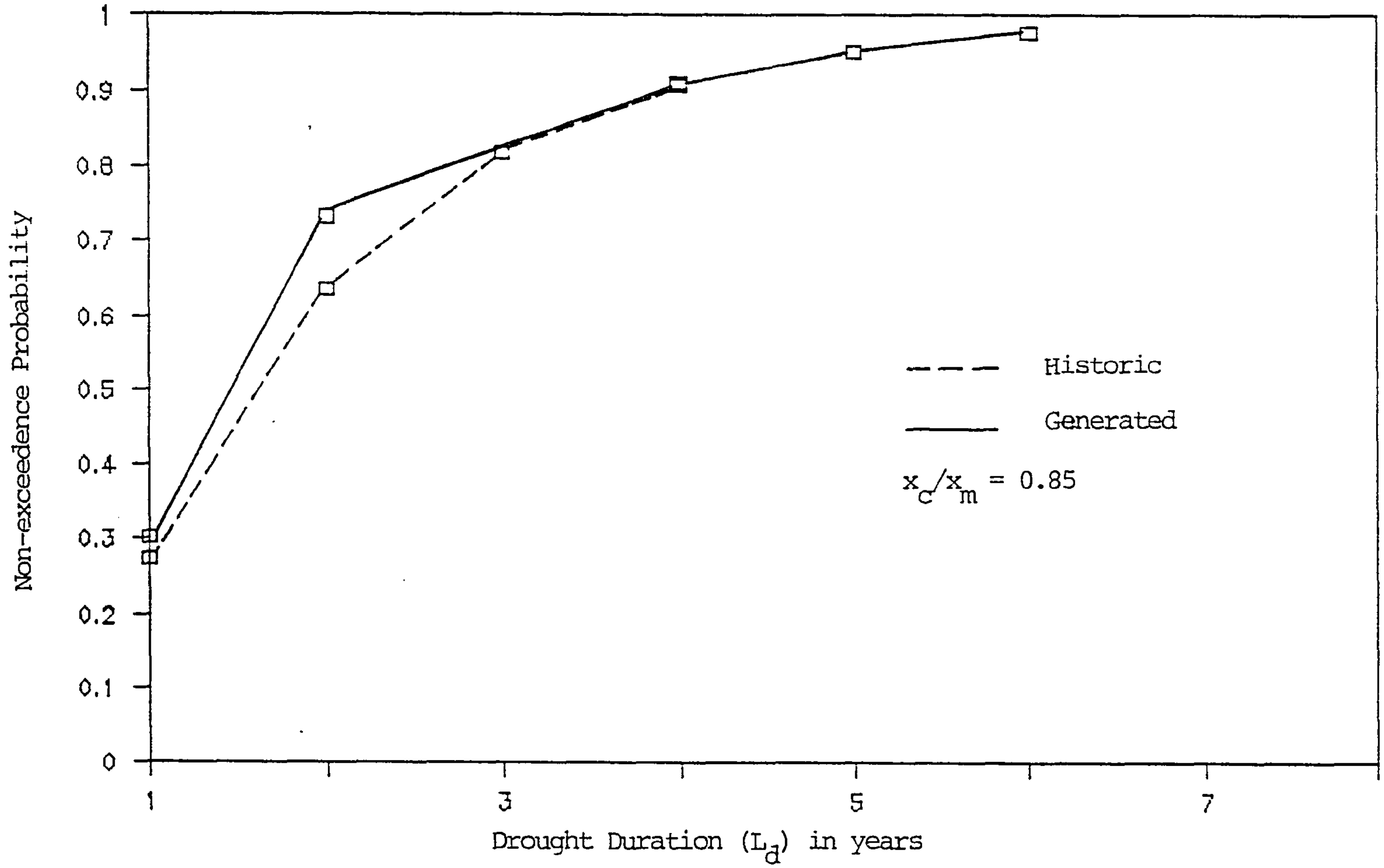

Fig. D.7.1 Proability Distributions of Drought Durations Derived from Historic and Generated Flow Series - Manasquan River near Squankum

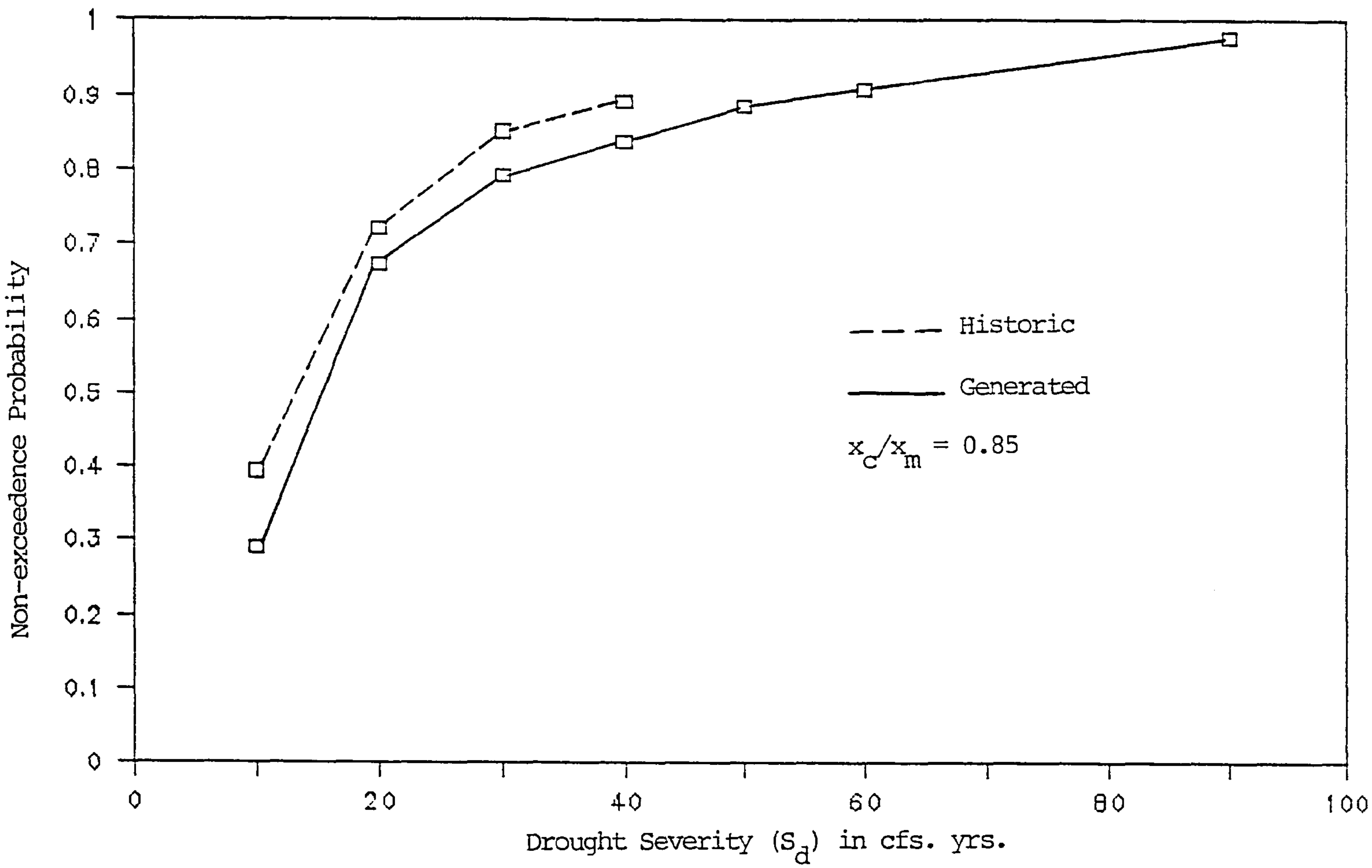

Fig. D.7.2 Probability Distributions of Drought Severities Derived from Historic and Generated Flow Series - Manasquan River at Squankum

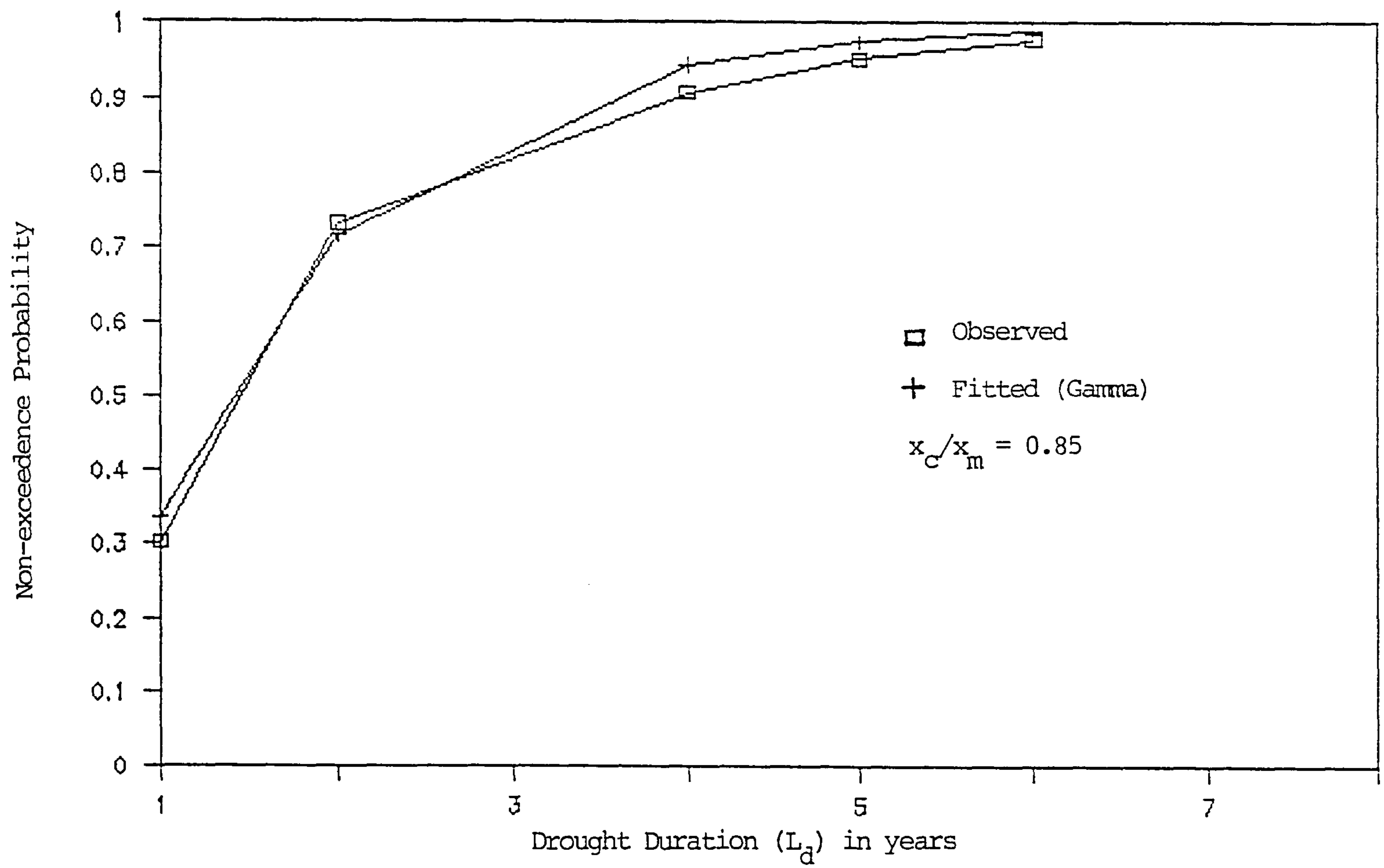

Fig. D.7.3 Comparison of Observed and Fitted Probability Distributions for Drought Durations Derived from Generated Flow Series - Manasquan River at Squankum

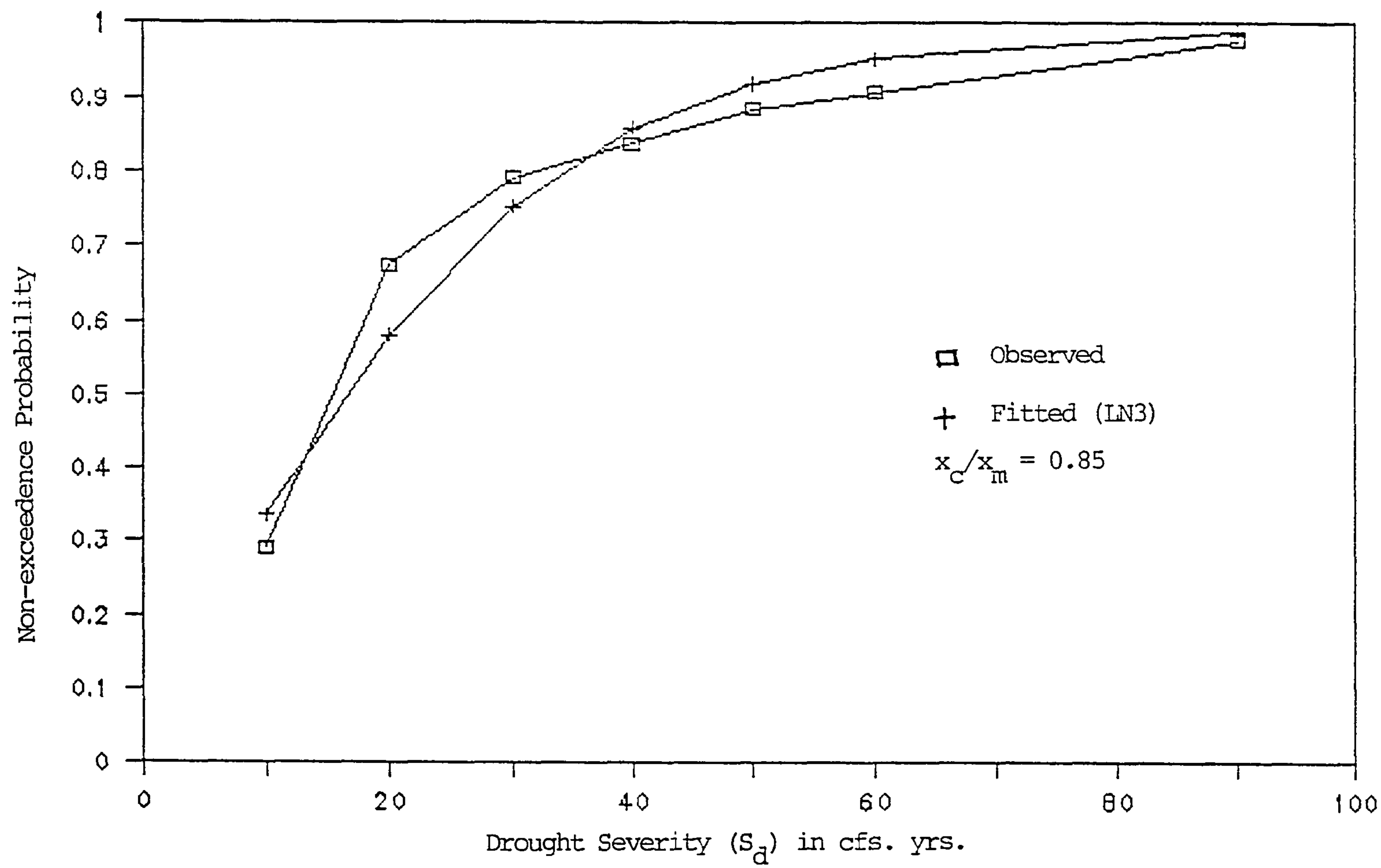

Fig. D.7.4 Comparison of Observed and Fitted Probability Distributions for Drought Severities Derived from Generated Flow Series - Manasquan River at Squankum

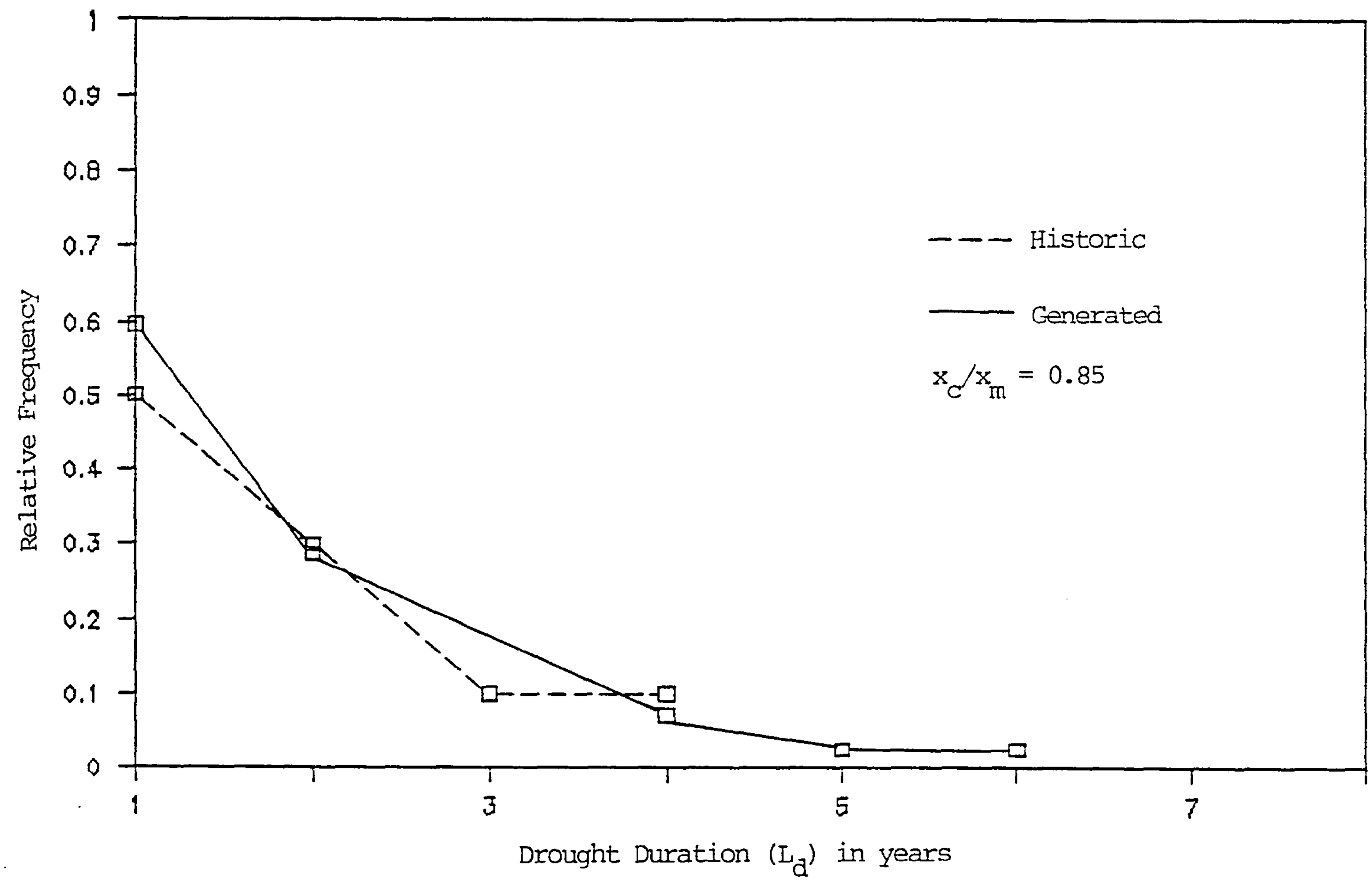

Fig. D.7.5 Probability Density Curves of Drought Durations - Manasquan River at Squankum

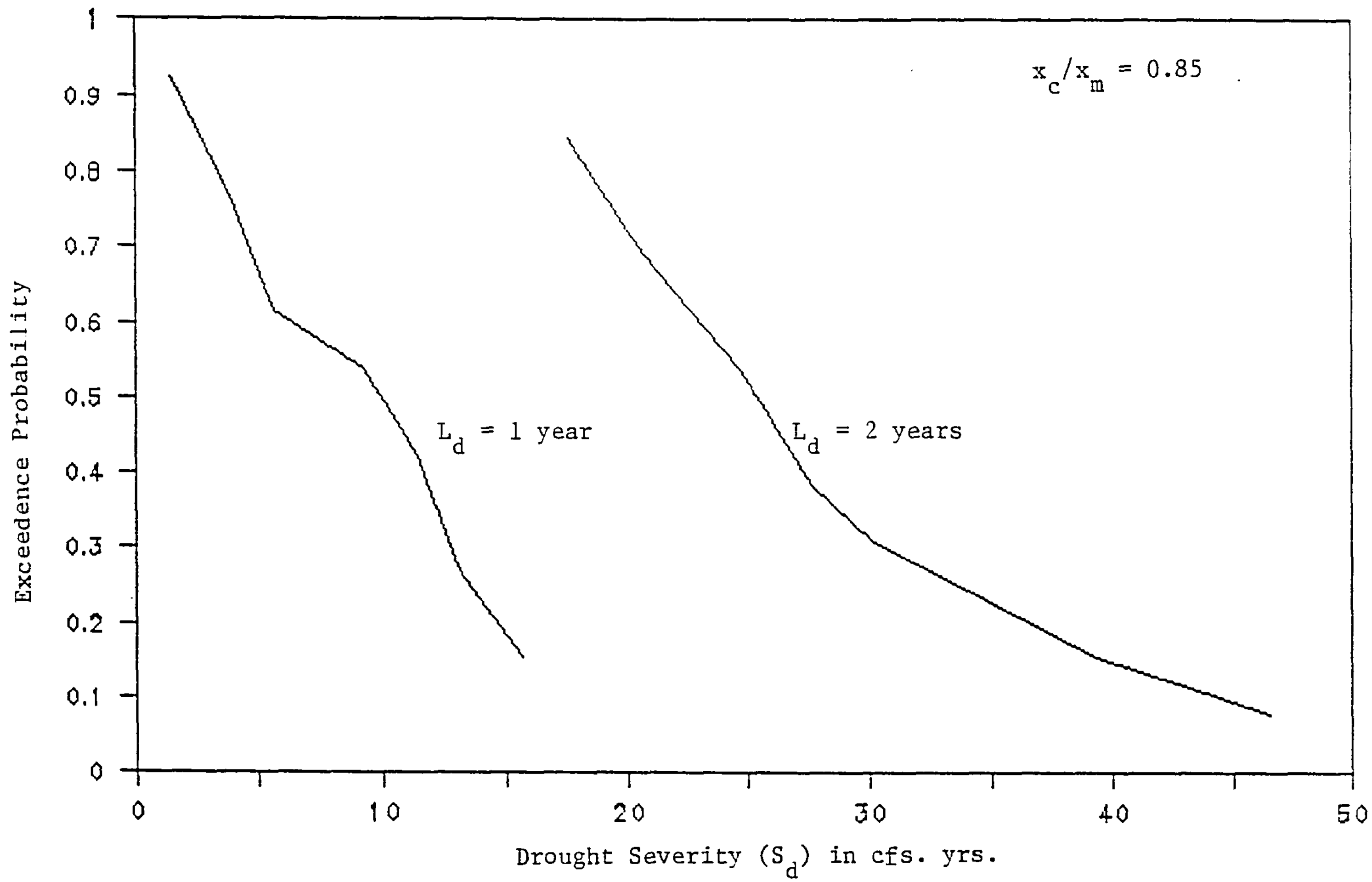

Fig. D.7.6 Drought Frequency - Duration - Severity Curves Derived from Generated Flow Series - Manasquan River at Squankum

APPENDIX E

MISCELLANEOUS RESULTS AND APPLICATIONS

Table E.1.1 Variation of the Statistical Properties of the Multiyear Drought & Surplus Durations Derived from Historic Flow Series of Different Watersheds.

(a) Pequest River at Pequest - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.78	1.39	0.781	1.925	-0.165
0.85	1.82	1.25	0.687	1.912	-0.366
0.90	1.85	1.28	0.692	1.740	-0.140
0.95	2.00	1.30	0.650	1.223	0.008
1.00	2.39	2.22	0.929	2.507	-0.279
1.05	2.75	2.38	0.865	1.863	-0.393
1.10	3.18	2.60	0.818	2.008	-0.286

(b) Pequest River at Pequest - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	4.20	3.29	0.783	1.250	0.010
0.85	3.17	2.89	0.912	2.087	-0.168
0.90	2.43	1.74	0.716	1.669	-0.084
0.95	2.00	1.36	0.680	1.954	0.039
1.00	1.86	1.35	0.726	2.484	0.036
1.05	1.85	1.41	0.762	2.449	0.079
1.10	1.83	1.47	0.803	2.416	-0.094

(c) Musconetcong River near Bloomsbury - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.67	1.41	0.844	2.121	-0.204
0.85	1.80	1.32	0.733	1.913	-0.300
0.90	1.92	1.26	0.656	1.660	0.000
0.95	2.00	1.30	0.650	1.223	0.008
1.00	2.07	1.27	0.614	1.165	-0.049
1.05	2.67	2.57	0.963	2.410	-0.327
1.10	3.09	2.74	0.887	1.923	0.136

(d) Musconetcong River near Bloomsbury - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	4.30	3.40	0.791	1.141	0.032
0.85	3.64	2.98	0.819	1.740	-0.056
0.90	2.36	1.78	0.754	1.657	-0.184
0.95	2.00	1.36	0.680	1.954	0.039
1.00	1.93	1.34	0.694	2.219	-0.003
1.05	1.92	1.44	0.750	2.131	0.041
1.10	1.92	1.51	0.786	2.088	0.082

(e) Rockaway River at Boonton - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.44	1.33	0.924	3.000	-0.143
0.85	1.78	1.30	0.730	2.269	-0.322
0.90	1.73	1.19	0.688	2.376	-0.332
0.95	1.82	1.25	0.687	1.912	-0.366
1.00	2.46	2.62	1.065	2.810	-0.303
1.05	3.50	2.98	0.851	1.734	-0.547
1.10	4.29	3.15	0.734	0.890	-0.546

(f) Rockaway River at Boonton - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	4.00	4.21	1.053	2.422	-0.159
0.85	3.63	4.34	1.196	2.464	-0.094
0.90	2.60	3.03	1.165	2.885	-0.070
0.95	2.60	3.03	1.165	2.885	-0.070
1.00	1.90	1.60	0.842	2.254	-0.089
1.05	2.13	1.73	0.812	1.966	-0.213
1.10	2.29	1.80	0.786	1.809	-0.326

(g) Passaic River near Millington - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.55	1.29	0.832	2.420	-0.156
0.85	1.46	1.20	0.822	2.682	-0.176
0.90	1.85	1.41	0.762	2.449	-0.007
0.95	2.00	2.10	1.050	3.024	-0.130
1.00	2.69	2.98	1.108	2.913	0.030
1.05	3.00	3.05	1.017	2.689	0.062
1.10	3.80	3.05	0.803	2.533	-0.039

(h) Passaic River near Millington - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	4.09	3.65	0.892	2.333	-0.160
0.85	3.31	2.02	0.610	1.161	0.153
0.90	2.92	1.98	0.678	0.966	0.091
0.95	2.13	1.36	0.638	1.709	0.154
1.00	2.08	1.38	0.663	2.078	-0.052
1.05	1.92	1.17	0.609	1.847	-0.207
1.10	1.73	1.19	0.688	2.376	-0.043

(i) Lamington River near Pottersville - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.67	1.41	0.844	2.121	-0.290
0.85	1.58	1.24	0.785	2.346	-0.148
0.90	1.77	1.30	0.734	1.844	0.085
0.95	2.00	1.85	0.925	2.804	-0.232
1.00	2.20	2.34	1.064	3.019	-0.236
1.05	2.64	2.44	0.924	2.420	-0.083
1.10	2.92	2.43	0.832	2.326	0.059

(j) Lamington River near Pottersville - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	5.22	3.19	0.611	0.719	-0.134
0.85	3.58	3.03	0.846	1.751	-0.162
0.90	3.00	2.31	0.770	1.295	-0.307
0.95	2.13	1.55	0.728	2.390	0.078
1.00	1.93	1.34	0.694	2.219	0.000
1.05	1.79	1.19	0.665	1.762	0.085
1.10	1.85	1.21	0.654	1.662	0.048

(k) North Branch Raritan River near Raritan - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.67	1.41	0.844	2.121	-0.204
0.85	1.69	1.32	0.781	1.970	0.000
0.90	1.73	1.22	0.705	1.940	0.104
0.95	1.87	1.25	0.668	1.565	0.134
1.00	2.07	1.28	0.618	1.038	0.092
1.05	2.43	2.38	0.979	2.811	-0.120
1.10	3.27	3.69	1.128	2.917	-0.139

(l) North Branch Raritan River near Raritan - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	4.44	3.58	0.806	0.988	0.096
0.85	2.92	1.75	0.599	1.232	0.027
0.90	2.27	1.22	0.537	2.117	0.250
0.95	2.13	1.30	0.610	1.959	0.037
1.00	1.93	1.39	0.720	1.989	0.077
1.05	1.86	1.23	0.661	1.465	0.149
1.10	1.75	1.29	0.737	1.780	0.193

(m) Manasquan River at Squankum - Drought Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	1.36	0.51	0.375	0.661	-0.535
0.85	1.80	1.03	0.572	1.241	-0.458
0.90	2.33	1.32	0.566	0.463	-0.658
0.95	2.44	1.51	0.619	0.697	-0.667
1.00	2.50	1.60	0.640	0.554	-0.674
1.05	3.00	1.87	0.623	1.178	-0.474
1.10	3.33	2.24	0.673	1.102	-0.504

(n) Manasquan River at Squankum - Surplus Durations

X_c/X_m	Mean (yrs)	Std. Dev. (yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	3.46	2.73	0.789	1.496	-0.430
0.85	3.50	2.68	0.766	1.759	0.022
0.90	3.10	2.60	0.839	2.420	0.006
0.95	3.00	2.63	0.877	2.489	-0.071
1.00	3.22	2.68	0.832	2.445	-0.042
1.05	2.10	1.10	0.524	0.388	0.205
1.10	1.70	0.95	0.559	1.718	-0.086

Table E.1.2 Variation of the Statistical Properties of the Multiyear Drought & Surplus Severities Derived from Historic Flow Series of Different Watersheds.

(a) Pequest River at Pequest - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	46.42	79.75	1.718	2.445	-0.248
0.85	50.36	84.06	1.669	2.637	-0.218
0.90	55.83	88.86	1.592	2.736	-0.615
0.95	66.52	95.74	1.439	2.725	-0.134
1.00	89.44	124.56	1.393	2.726	-0.219
1.05	117.39	145.23	1.237	2.583	-0.274
1.10	152.09	166.01	1.092	2.450	-0.252

(b) Pequest River at Pequest - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	232.59	214.23	0.921	1.845	-0.088
0.85	167.30	178.72	1.068	2.369	-0.113
0.90	123.21	117.10	0.950	1.821	-0.194
0.95	98.40	102.05	1.037	2.087	-0.145
1.00	90.06	92.69	1.029	2.000	-0.108
1.05	81.55	83.35	1.022	1.856	-0.100
1.10	73.16	73.35	1.003	1.683	-0.207

(c) Musconetcong River near Bloomsbury - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	68.42	104.96	1.534	2.353	-0.279
0.85	81.42	116.88	1.436	2.456	-0.238
0.90	82.17	121.70	1.481	2.684	-0.135
0.95	98.55	132.85	1.348	2.664	-0.102
1.00	122.86	146.26	1.190	2.571	-0.093
1.05	172.84	204.83	1.185	2.419	-0.231
1.10	224.81	247.31	1.100	2.090	0.075

(d) Musconetcong River near Bloomsbury - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	352.38	348.15	0.988	2.099	-0.064
0.85	275.40	304.64	1.106	2.355	-0.064
0.90	185.05	185.57	1.003	1.895	-0.044
0.95	147.29	162.47	1.103	2.078	0.020
1.00	123.79	146.94	1.187	2.053	0.018
1.05	117.30	135.03	1.151	1.876	-0.011
1.10	103.57	120.22	1.161	1.716	-0.070

(e) Rockaway River at Boonton - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	61.72	91.67	1.485	2.730	-0.071
0.85	79.19	106.46	1.344	2.800	-0.091
0.90	82.48	114.34	1.386	2.925	-0.221
0.95	103.16	127.17	1.233	2.916	-0.237
1.00	127.01	157.43	1.240	2.870	-0.287
1.05	198.78	216.43	1.089	2.143	-0.443
1.10	275.41	256.62	0.932	1.864	-0.551

(f) Rockaway River at Boonton - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	323.59	411.31	1.271	2.326	-0.063
0.85	278.18	364.32	1.310	2.269	-0.056
0.90	189.96	282.46	1.487	2.461	-0.035
0.95	159.89	248.63	1.555	2.372	-0.029
1.00	132.93	173.03	1.302	1.828	-0.062
1.05	139.29	162.02	1.163	1.545	-0.213
1.10	132.02	144.42	1.094	1.366	-0.352

(g) Passaic River near Millington - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	23.30	32.51	1.395	2.269	-0.283
0.85	25.77	36.49	1.416	2.482	-0.268
0.90	32.98	41.66	1.263	2.547	-0.248
0.95	36.85	52.44	1.423	2.829	-0.118
1.00	53.53	73.83	1.379	2.735	0.008
1.05	68.56	90.21	1.316	2.651	0.043
1.10	95.25	109.82	1.153	2.457	-0.005

(h) Passaic River near Millington - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	129.54	114.86	0.887	1.041	0.018
0.85	94.27	86.19	0.914	1.327	0.174
0.90	80.09	79.55	0.993	1.414	0.183
0.95	59.14	66.45	1.124	2.074	0.205
1.00	57.86	61.77	1.068	1.985	0.201
1.05	52.84	56.84	1.076	1.890	0.231
1.10	49.24	52.36	1.063	1.790	0.213

(i) Lamington River near Pottersville - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	15.88	21.49	1.353	2.226	-0.346
0.85	15.83	23.30	1.472	2.525	-0.179
0.90	19.19	26.01	1.355	2.616	-0.142
0.95	21.68	33.28	1.535	2.869	-0.163
1.00	36.54	46.84	1.282	2.734	-0.173
1.05	36.54	46.84	1.282	2.734	-0.155
1.10	47.35	54.50	1.151	2.612	-0.096

(j) Lamington River near Pottersville - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	95.47	78.71	0.824	1.837	-0.204
0.85	61.03	68.06	1.115	2.212	-0.120
0.90	47.54	43.62	0.918	1.363	-0.087
0.95	34.65	35.81	1.033	2.014	0.046
1.00	28.95	32.24	1.114	1.940	0.043
1.05	25.60	28.90	1.129	1.830	0.089
1.10	22.18	26.44	1.192	1.674	0.037

(k) North Branch Raritan River near Raritan - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	85.42	111.30	1.303	1.657	-0.324
0.85	80.59	119.04	1.477	2.193	-0.224
0.90	93.65	131.99	1.409	2.403	-0.116
0.95	121.08	149.08	1.231	2.412	-0.095
1.00	150.43	166.22	1.105	2.376	-0.063
1.05	196.96	233.73	1.187	2.597	-0.159
1.10	281.86	363.28	1.289	2.577	-0.120

(l) North Branch Raritan River near Raritan - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	471.24	480.51	1.020	2.023	0.033
0.85	301.41	260.80	0.865	1.779	0.124
0.90	223.74	234.01	1.046	2.010	0.229
0.95	189.90	215.95	1.137	1.965	0.234
1.00	157.96	197.23	1.249	1.909	0.252
1.05	139.38	179.62	1.286	1.790	0.261
1.10	132.35	167.06	1.262	1.603	0.227

(m) Manasquan River at Squankum - Drought Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	8.66	7.76	0.896	1.231	-0.294
0.85	15.63	11.69	0.748	1.842	-0.433
0.90	23.60	17.49	0.741	1.076	-0.665
0.95	32.59	22.30	0.684	0.920	-0.702
1.00	43.11	29.34	0.681	0.698	-0.701
1.05	47.86	37.48	0.783	0.703	-0.585
1.10	59.36	44.43	0.748	0.762	-0.574

(n) Manasquan River at Squankum - Surplus Severities

X_c/X_m	Mean (cfs.yrs)	Std. Dev. (cfs.yrs)	Coef. Var.	Coef. Skew.	Lag-1 Autocorr.
0.80	84.87	76.66	0.903	1.386	-0.141
0.85	79.52	65.96	0.829	1.397	0.232
0.90	67.18	55.03	0.819	1.567	0.190
0.95	55.72	46.26	0.830	1.379	0.233
1.00	49.62	36.21	0.730	1.346	0.323
1.05	35.07	27.39	0.781	1.393	0.400
1.10	27.47	24.82	0.904	1.412	0.413

Table E.2.1

Chi Squared Test for Drought Durations Derived from Generated Flow Series for Pequest River at Pequest; $X_c/X_m=0.95$

Assumed Distribution: 3-parameter Gamma

Drought Duration (L_d) yrs	Observed Frequency (f_{ob})	Theoretical Probability (p_{th})	Theoretical Frequency (f_{th})	$(f_{ob}-f_{th})^2/f_{th}$
1	14	0.165	7.10	6.706
2	11	0.214	9.20	0.353
3	4	0.214	9.20	2.939
4	4	0.165	7.10	1.354
5	5	0.107	4.61	0.033
6	3	0.063	2.71	0.031
7	1	0.038	1.63	0.243
8	1	0.017	0.73	0.100

$\sum f_{ob}=43$ $\sum (f_{ob}-f_{th})^2/f_{th}=11.759$

No. of Degrees of Freedom = 8-3-1
= 4

$\chi^2_{0.01,4} = 13.277 > 11.759$

Therefore, the hypothesis that the drought duration is gamma distributed is acceptable within 0.01 chance of error.

Table E.2.2

Chi Squared Test for Drought Severities Derived from Generated Flow Series for Pequest River at Pequest; X_c/x_m=0.95

Assumed Theretical Distribution: 3-parameter Log Normal(LN3)

Drought Severity (cfs.yrs)	Observed Frequency (f_{ob})	Theoretical Probability (p_{th})	Theoretical Frequency (f_{th})	$(f_{ob}-f_{th})^2/f_{th}$
0-20	8	0.147	6.32	0.447
20-40	6	0.085	3.67	1.479
40-60	4	0.093	4.00	0.000
60-80	3	0.094	4.04	0.268
80-100	4	0.089	3.83	0.008
100-120	4	0.081	3.48	0.078
120-180	5	0.132	5.68	0.081
180-260	5	0.145	6.24	0.246
>260	4	0.134	5.76	0.538

$\sum f_{ob}=43$ $\sum (f_{ob}-f_{th})^2/f_{th})=3.145$

No. of Degrees of Freedom = 9-1-3
= 5

$\chi^2_{0.05,5}$ = 11.07 >> 3.145

Therefore, the hypothesis that the drought severities are log normally distributed is acceptable within the 0.05 chance of error.

E.3 Application of the Results

The following simple examples illustrate some applications of the results obtained in the present study. However, the results may have more applications to a real water resources planning study over the region concerned.

Example 1

Problem:

Determine (a)the longest drought duration, its severity and (b)the largest drought severity & its duration expected within a period of 50 years for the Pequest Watershed.

Solution:

As the return period (50 years) is included in both the historic flow record and the generated flow series length, for the determination of the required estimates, the behaviour of the drought parameters established either from historic flow series or from the generated flow series can be used. If both are used, a comparison between the estimates resulting from two types of flow series can be made.

(i) Determination of the Non-exeedence Probability

From Table 5.4,
Average periodicity of the hydrologic cycle over the Pequest Watershed = 4.9 years

Therefore, the expected number of hydrologic cycles within a period of 50 years = 50/4.9
= 10

∴ Non-exeedence Probability (p) = 1-1/10
= 0.90

(ii) Longest Drought Duration & Its Severity

From Fig. D.1.1,
Corresponding to a non-exeedence probability of 0.90,

the longest drought duration resulting from historic flow series = 5 years

the longest drought duration resulting from generated flow series = 6 years

The latter result must be more reliable as it is obtained from a large population of flows.

From Table 5.18,
The expected drought magnitude resulting from historic flow series of Pequest River = 26.0 cfs

the expected drought magnitude resulting from generated flow series = 33.6 cfs

Thus,
the expected drought severity during the longest drought duration from historic series = (5)x(26.0)
= 130 cfs.yrs

Similarly,
the expected drought severity during the longest drought duration derived from generated flow series = (6)x(33.6)
= 201.6 cfs.yrs

Compared with 130 cfs.yrs, the value of 201.6 cfs.yrs must be more reliable as this estimate resulted from a large population of drought events.

(iii) Largest Drought Severity and Its Duration

From Fig. D.1.2,

Corresponding to a non-exeedence probability of 0.90,

the largest drought severity resulting from historic flow series = 300 cfs.yrs

the largest drought severity resulting from generated flow series = 300 cfs.yrs

Therefore, the <u>expected</u> drought duration corresponding to a severity of 300 cfs.yrs resulting from historic flow series

$$= 300/26.0$$
$$= 12 \text{ years}$$

Similarly, the <u>expected</u> drought duration corresponding to a severity of 300 cfs.yrs resulting from generated flow series

$$= 300/33.6$$
$$= 9 \text{ years}$$

The latter result can be considered to be more reliable.

Example 2

Problem:

Determine the probable longest drought duration and the largest drought severity within a period of 300 years for the Pequest Watershed.

Solution:

These estimates can be made by using the drought simulation results derived for the Pequest River in the present study. As a first step, it is necessary to determine the non-exeedence probability corresponding to the given return period. Then, making use of the observed probabilistic relationships or the fitted theoretical probability density functions for the interested drought parameters derived from generated flow series, the required estimates can be forecast.

As the return period is 300 years which exeeds the period for which the flows were generated (240 years), it is appropriate to use the fitted probability distribution functions to solve this problem.

(i) Determination of the Non-exeedence Probability

From Table 5.4,
Average periodicity of the hydrologic cycle over the Pequest Watershed = 4.9 years

Therefore, the expected number of hydrologic cycles within a period of 300 years = 300/4.9
= 61

∴ Non-exeedence probability = 1-1/61
= 0.984

(ii) Determination of the Longest Drought Duration

From Table 5.15,
The distribution parameters of the fitted Gamma density function for the drought durations derived for the Pequest Watershed are:

$a = 0.891$, $b = 4.736$ and $c = -1.380$

Corresponding to a non-exeedence probability of 0.984, and a shape parameter(b) of 4.736, the standard gamma quantile (see Table E.1),

$$w = 10.70$$

Therefore, the probable longest drought duration (L_d),

$$L_d = c + a.w$$
$$= -1.38 + 0.891(10.7)$$
$$= 9 \text{ years}$$

(iii) Determination of the Largest Drought Severity

From Table 5.16,
The parameters of the fitted Log Normal distribution function for the drought severities derived for the Pequest Watershed are:

$a' = -125.73$, $b' = 213.89$ and $c' = 0.433$

The standard normal quantile corresponding to a non-exeedence probability of 0.984 is:

$$z = 2.145$$
(from the standard normal probability chart)

Therefore, the anticipated largest deficit,

$$\begin{aligned} S_d &= a'+b'.\exp(c'.z) \\ &= -125.73 + 213.89.\exp[(0.433)(2.145)] \\ &= 416.0 \text{ cfs.yrs} \end{aligned}$$

As the Chi Squared test, performed for the fitted Log Normal distribution for drought severities, indicates the goodness of fit within a 0.05 chance of error, the above result must be reliable at 95% significant levels.

Example 3

Problem:

Determine the largest 2-year drought severity within a period of 80 years for the Pequest Watershed.

Solution:

For this purpose, the Relative Frequency Vs. Drought Duration curve established for the drought events derived from generated flow series (Fig. D.1.5), can be used.

Expected total number of hydrologic cycles = 80/4.9
= 16

As the hydrologic cycle consists of a drought period followed by a surplus period,

Expected total number of drought events = 2(16)
= 32

From Fig. D.1.5,
Relative frequency of a 2-year drought = 0.25
(i.e. Among every four drought events, there is one 2-year drought event)

Therefore, the expected total number of 2-year droughts
= (0.25)x(32)
= 8

∴ Exeedence parbability of the largest 2-year drought
= 1/8
= 0.125

Now from Fig. D.1.6,
the largest 2-year drought severity = 95.0 cfs.yrs

Table E.3 Standard gamma variates and probabilities

Probability of non-exceedance	*Standard gamma variate for shape parameter* (b)														
	0.1	0.5	1	2	3	4	5	6	8	10	12	14	16	18	22
0.001	0.0000	0.0000	0.0010	0.0454	0.1905	0.4286	0.7394	1.1071	1.9708	2.9605	4.0424	5.1954	6.4053	7.6621	10.2881
0.010	0.0000	0.0001	0.0101	0.1486	0.4360	0.8232	1.2791	1.7853	2.9061	4.1302	5.4282	6.7824	8.1811	9.6163	12.5740
0.020	0.0000	0.0003	0.0202	0.2147	0.5672	1.0162	1.5295	2.0891	3.3071	4.6183	5.9959	7.4237	8.8914	10.3915	13.4693
0.050	0.0000	0.0020	0.0513	0.3554	0.8177	1.3663	1.9701	2.6130	3.9808	5.4254	6.9242	8.4639	10.0360	11.6343	14.8937
0.100	0.0000	0.0079	0.1054	0.5318	1.1021	1.7448	2.4326	3.1519	4.6561	6.2213	7.8293	9.4696	11.1353	12.8216	16.2436
0.200	0.0000	0.0321	0.2231	0.8244	1.5350	2.2968	3.0895	3.9037	5.5761	7.2892	9.0309	10.7940	12.5739	14.3675	17.9872
0.300	0.0000	0.0742	0.3567	1.0973	1.9138	2.7637	3.6336	4.5171	6.3122	8.1329	9.9716	11.8237	13.6864	15.5576	19.3204
0.400	0.0001	0.1375	0.5108	1.3764	2.2851	3.2113	4.1477	5.0910	6.9914	8.9044	10.8262	12.7546	14.6881	16.6258	20.5111
0.500	0.0006	0.2275	0.6931	1.6783	2.6741	3.6721	4.6709	5.6702	7.6692	9.6687	11.6684	13.6681	15.6679	17.6678	21.6676
0.600	0.0037	0.3542	0.9163	2.0223	3.1054	4.1753	5.2366	6.2919	8.3898	10.4757	12.5532	14.6243	16.6904	18.7525	22.8668
0.700	0.0174	0.5371	1.2040	2.4392	3.6156	4.7622	5.8904	7.0056	9.2089	11.3873	13.5480	15.6954	17.8325	19.9610	24.1978
0.800	0.0694	0.8212	1.6094	2.9943	4.2790	5.5150	6.7210	7.9060	10.2325	12.5188	14.7767	17.0133	19.2338	21.4394	25.8195
0.900	0.2662	1.3528	2.3026	3.8897	5.3223	6.6808	7.9936	9.2747	11.7709	14.2060	16.5981	18.9580	21.2924	23.6061	28.1843
0.950	0.5804	1.9207	2.9957	4.7439	6.2958	7.7537	9.1535	10.5130	13.1481	15.7052	18.2075	20.6686	23.0971	25.4992	30.2404
0.980	1.1190	2.7059	3.9120	5.8339	7.5166	9.0841	10.5804	12.0270	14.8166	17.5098	20.1352	22.7094	25.2434	27.7444	32.6683
0.990	1.5885	3.3174	4.6052	6.6384	8.4059	10.0451	11.6046	13.1085	16.0000	18.7831	21.4899	24.1391	26.7429	29.3096	34.3548
0.999	3.3637	5.4138	6.9078	9.2334	11.2289	13.0622	14.7941	16.4547	19.6262	22.6574	25.5893	28.4461	31.2436	33.9926	39.3748

Note:

[These results were generated by use of a computer program]

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