# An analysis of the heating and densification process during rotational molding of a thermoplastic powder in a uniaxially rotating cylindrical cavity 

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AN ANALYSIS OF THE HEATING AND DENSIFICATION PROCESS DURING ROTATIONAL MOLDING OF A THERMOPLASTIC POWDER IN A UNIAXIALLY ROTATING CYLINDRICAL CAVITY

by<br>Floyd S. Ribe

Dissertation submitted to the Faculty of the Graduate School of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Engineering Science 1985
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# of Thesis: AN ANALYSIS OF THE HEATING AND DENSIFICATION PROCESS DURING ROTATIONAL MOLDING OF A THERMOPLASTIC POWDER IN A UNIAXIALLY ROTATING CYLINDRICAL CAVITY 

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## ABSTRACT

Title of Thesis: AN ANALYSIS OF THE HEATING AND
DENSIFICATION PROCESS DURING ROTATIONAL MOLDING OF A THERMOPLASTIC POWDER IN A UNIAXIALLY ROTATING CYLINDICAL CAVITY

Floyd Steven Ribe, Doctor of Engineering Science, 1985
Thesis directed by: Dr. Richard C. Progelhof Professor of Mechanical Engineering

This dissertation presents the results of an experimental and theoretical investigation of the heating and densification portion of the rotational molding process in a uniaxial, cylindrical mold.

A thorough literature survey is included which reviewed past analysis of the rotational molding process and other areas that assisted in understanding of the process.

The results presented in this dissertation included an analysis of the densification process by use of scanning Electronic Microscope (SEM) photography producing intermediate correlations between the physical properties of the densifying material and neck radius of adjacent coalescing spheres. In addition, a hybrid experimental procedure coupled with a small computer simulation was devised to determine the actual initial thermal conductivity and diffusivity of the powdered polymeric material.

Finally, a computer program was written to simulate the heating and densification process during the rotational molding process. Results showed good agreement with actual experimental findings.

## ACKNOWLEDGMENTS

The author would like to express appreciation to Dr. R. C. Progelhof for his_guidance and assistance throughout the preparation of this dissertation.

Also appreciated for their assistance are my many friends and co-workers at Picatinny Arsenal, Dover, NJ.

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## I. INTRODUCTION

This dissertation presents the results of an experimental and theoretical investigation of a portion of the rotational molding process of a thermoplastic material in a cylindical mold. In rotational molding or rotomolding of thermoplastic powders, the process consists of the following steps:

1 - Loading the mold with a fixed mass of resin
2 - Simultaineously heating and melting the thermoplastic powder while the mold rotates

3 - Cooling the mold
4 - Unloading the mold

During loading, a premeasured mass of thermoplastic powder is placed in a two or three part split mold of which one section is bolted to a platform. The mold is closed and locked. The mold platform is bolted to an arm which moves the mold from station to station and biaxially rotates the mold at a predetermined rate. Rotational speed and directional speed ratios are adjusted by gearing.

After loading, the arm moves the mold into an oven and the biaxially rotation is started. At this point the powdered material is still cool and due to gravity the
powder forms a pool at the bottom of the mold. Figure 1 shows a typical pool flow of a uniaxial clockwise rotating mold and is the type used in this investigation to experimentally verify the theoretical analysis. As the mold rotates, friction causes the material to follow and remains stationary to the motion of the mold surface. When the powder particles reach a point where friction is overcome by gravity, the individual powder granules begin to free fall over the top surface of the following adjacent material of the stationary pool. The stationary and free fall zones are shown in Figure l. Experiments have shown that the actual motion of the stationary and free falling powder is dependent upon particle geometry, air volume fraction and surface character of the powder.

This rotational process continues until the mold surface reaches a temperature that causes the material to begin to soften and adhere to the mold surface. This temperature is called the "stick temperature". With increasing mold temperature, more material adheres to the mold surface until the mold surface is covered. With a further increase in mold surface temperature, layers of powder will be built on the mold wall causing the pool to deplete. The material adhering to the surface of the mold is a porous powdered layer initially held together by point

Figure 1
TYPICAL POOL FLOW

contact of the individual powder particles.

During the period of time for which the particles are adhering to the wall of the mold, the material begins a densification process. During this process the material particles lose their individuality by joining themselves first at the contact points, then "melting" into a solid piece with minute air voids throughout the molten resin. The initial phase of this densification process is similar to the sintering, coalesence and fusion processes in the field of drop coalescense, paint technology, ceramics and glass.

When the heating process is completed, the mold is first air cooled allowing an even temperature distribution within the part and densification of the molded part to complete. Rapid cooing to room temperature is then usually accomplished by water spray on the exterior mold surface.

This investigation will analyze this rotational molding process excluding the cooling portion of the cycle. A literature survey will first be made to review past works. Finally two mathematical models will be developed to predict the material heating process which includes the time for complete densification.

## II. LITERATURE SURVEY

The literature available of the rotational molding process and the analysis thereof is very limited. Because of this, the literature survey will first examine the available published works on the subject and then at other related articles that also provide additional insight toward understanding the rotational molding process. These areas include a survey of articles concerning sintering which help describe the densification process in rotational molding, a survey of the thermal conductivity of porous material used to predict the thermal conductivity of powdered material similar to that encountered in the initial phase of the rotational molding system, and a rewiew of previously published works that analyzed the mass flow in a rotating cylinder.

Modeling of the Rotational Molding Process
One of the first to attempts to analytically model the rotational molding process was reported by Rao and Throne (31,32, and 14). They modeled the heat transfer to the mold and powder, the fluid flow of the powder, the sintering-melting and degradation during this process.

In their analysis, the authors' assume that an exponential internal mold surface temperature profile is resultant of a constant ambient oven temperature and
convective film coefficient . Assuming a polynomial temperature profile of the penetration thickness of the powdered material adjacent to the mold surface, they computed the amount of material adhering to the mold surface when the mold temperature reaches or exceeds the material stick temperature. By subtracting the predicted amount of powder that adhered to the mold surface, the new volume of the pool was determined. The analysis repeats until all material has left the pool.

Vanderbeck(30) modified Rao and Throne's models by improving some of the basic flow assumptions in the pool. As with Rao and Throne, Vanderbeck also assumes constant physical and thermal properties. Both authors disregard the fact of material that has traveled around the mold and has reentered the powdered pool and thus neglecting the insulating effect of the material adhering to the mold surface.

Throne(33) and Ahdout(34) modeled a rotational system which assumed the powdered material to be evenly distributed around the mold surface, neglecting the actual flow of the powder within the pool region and the mixing zone. Temperature dependent properties were calculated by using a linear interpolation between the known powdered state at the beginning and the final solid state at the end of the process.

The Throne, Rao, Vanderbeck and Adhout models all simulate in a very rough manner, a portion of specific phases of the rotational molding process. However, a complete simulation with temperature dependent properties to predict pool depletion and total densification has yet to be accomplished.

## Mass Flow

In the study of mass flow in a rotating cylinder, Lehmverg, Hehl and Schugerl(41) performed experiments using color tracers placed in the pool of powdered material of a rotating drum having transparent ends for visual
observations. As the drum rotated, the position of the tracers were recorded. Their results showed an area where most of the material remained stationary relative to the wall during mold rotation and a thin layer of material on top of the stationary pool that mixed as it rolled down the incline of the surface of the material. These results as well as results reported in references 53 and 54 , fully concur with the brief description of the typical mass flow in rotational molding made in the introduction.

## Sintering

The sintering, coalescence and fusion of particles have been studied in the field of drop coalescence, paint technology, ceramics, glass and polymers. References

1-28,35,51,53,54, and 56 were reviewed with respect to the sintering process of polymers. The following is a brief summary of the major applicable works.

Frenkel(1) analyzed the phenomena of "Cold Welding" of two amorphous spheres. Based on thermodynamic relationships, a relationship to predict the neck radius, $x$, of two coalesing spheres (Figure 2), is given by:

$$
x^{2}=3 a \gamma t / 2 \eta
$$

where $r$ is the surface tension; $\eta$, is the Newtonian viscosity; $a, ~ i s ~ t h e ~ r a d i u s ~ o f ~ t h e ~ s p h e r e ; ~ a n d ~ t ~ i s ~ t i m e . ~$

To illustrate the application of Frenkel's equation for isothermal sintering for polyethylene, the properties of which are listed in Table l, the following sintering equations are generated:

$$
\begin{aligned}
& \text { For } 105^{\circ} \mathrm{C} \\
& \qquad \frac{x^{2}}{a}=8.915 \times 10^{-6} \mathrm{t} \\
& \text { For } 150^{\circ} \mathrm{C} \\
& \frac{x^{2}}{a}=6.549 \times 10^{-5} \mathrm{t} \\
& \text { For } \frac{180}{}{ }^{\circ} \mathrm{C} \\
& \frac{x^{2}}{a}=1.959 \times 10^{-4} \mathrm{t}
\end{aligned}
$$

$$
\frac{x^{2}}{d}=8.915 \times 10^{-6} \mathrm{t} \quad \text { where } \mathrm{x} \text { and a are in }
$$

$$
\mathrm{cm} \text {. and } t \text { is seconds }
$$

The predicted results are plotted in Figure 3. Note the linear relationship between $\frac{x^{2}}{a}$ as a function of time that

Figure 2
SINTERING OF TWO ADJACENT SPHERES


## TABLE 1

## PROPERTIES OF POLYETHYLENE

Surface Tension(18)

$$
\gamma(T)=\gamma_{0}-(\partial \gamma / \partial T)\left(T-T_{0}\right)
$$

For Polyethylene

$$
\begin{aligned}
& \gamma(T)=31-(0.058)(T-105) \quad \text { Where: } \\
& Y \text { is in dynes } / \mathrm{cm} \\
& T \text { is in degrees } c .
\end{aligned}
$$

Viscosity (6)

$$
n=n_{0} \exp \left\{\frac{-E}{R} \frac{\left(T-T_{0}\right)}{(T) *\left(T_{0}\right)}\right\}
$$

For Polyethylene

$$
n=5.22 \times 10^{6} \exp \left\{-19.583+\frac{7402.5}{T}\right\}
$$

Where: $\eta$ is in Poise
T is in degrees $K$

## Figure 3

PLOTS OF FRENKEL'S EQUATIONS FOR POLYETHYLENE

intersects at the origin.

Frenkel also derived the theory of densification of glass for the second stage of sintering where the voids become individual bubbles that slowly reduce in diameter. The equation for the collapse of an individual bubble is given by:

$$
\begin{equation*}
r_{0}-r=\frac{\gamma}{2 \eta} t \tag{eq.2}
\end{equation*}
$$

where $r_{0}$ is the original pore radius at time zero and $r$ is the radius at time $t$. Using the polyethylene properties in Table l, the collapse of the voids for the three temperatures used are as follows:

For $105^{\circ} \mathrm{C}$

$$
r_{0}-r=2.97 \times 10^{-6} t
$$

For $150{ }^{\circ} \mathrm{C}$
$r_{0}-r=2.18 \times 10^{-5} t$
For $180^{\circ} \mathrm{C}$

$$
r_{0}-r=6.53 \times 10^{-4} t
$$

These equations are plotted in Figure 3.

Frenkel's equation(eq l) was experimentally confirmed by Kuczynski(2) where glass beads were heated on top of a glass plate having the same composition as the beads. After heating, the samples were rapidly cooled and mounted in Bakelite. The dimensions were measured and plotted in

Figure 4
KUCZYNSKI'S RESULTS OF SINTERING GLASS SPHERES


Figure 4. The experimental results confirmed the linear relationship of Frenkel's equation intersecting at the origin. Kuczynski along with Zaplatynskyj(13) confirmed Frenkel's colapsing theory, Equation 2, by heating capillary tubes and measuring the collapse of the internal diameter as a function of time.

Dillion, Matheson and Bradford(5) investigated the sintering of synthetic latex particles. Experimental results indicated that the coalescence process occurs by the same mechanism as was described by Frenkel equation.

Kuczynski, Neuville and Toner(3) performed the same type experiment as Kuczynski(2) using Poly(methy) Methacrylate (acrylic). Figure 5 presents the experimental results. They found for this polymer, Frenkel's equation(eq. l) is inadequate. The following empirical equation was then correlated:

$$
\left[\frac{x^{2}}{a^{1.02}}\right]^{P}=F(T) \quad t \quad \text { (Equation } 3 \text { ) }
$$

where $F$ is a function of temperature only and $P$ is the slope of the curve in Figure 5. Lontz(4) replots Figure 5 using $\frac{X}{a}$ for the $Y$ axis (as opposed to $\frac{x^{2}}{a}$ ) versus time(Fig. 6). It is obvious from this graph that the $y$ intersection of the linear curves tend to increase with temperature giving an indication of a time delay to the

Figure 5

$$
\frac{x^{2}}{a} \text { vs TIME }
$$



sintering process. This is not unexpected due to the transient temperature response of the spheres.

Shonhorn, Frisch and Kwei(10), studying the kinetics of wetting of surfaces by polymer melts, showed that polymers at high temperatures exhibited a shifted Frenkel curve, as is evident in the Kuczynski, Neuville and Toner's plots(Figure 5).

Lontz(4) developed a model of sintering between two viscoelastic spheres yielding the equation:

$$
\frac{x^{2}}{a}=\frac{3 t}{2 \eta} \frac{1}{1-\exp (-t / \lambda)}
$$

(Equation 4)
where the second right hand term is a correction factor to the Frenkel equation to account for viscoelastic effects. The term, $\lambda$, is a relaxation time constant. The relaxation time constant is determined experimentally for each resin and temperature. Its significance can be better understood by examining the four element Maxwell-Voigt model(Figure 7) for a viscoelastic body.

The spring $E_{\mid}$in Figure 7 act as the Hookean repsonse while the dashpot, ${ }^{7} 3$, corresponds to the Newtonian fluid response. The spring, $E_{2}$, and dashpot, ${ }^{7} 2$, represent the

```
Figure 7 FOUR ELEMENT MAXWELL - VOIGHT MODEL
```


retarded elastic response of the polymer molecules. When the stress, $\sigma$, is applied, E, and ${ }^{\eta} 3$ react instantly while because of the physical arrangement, $E_{2}$ and ${ }^{\eta} 2$ have a delayed response. Together the system has a viscoelastic effect. The ratio of ${ }^{\eta} 2$ to $E_{2}$ is the relaxation time constant, $\lambda$,or:

$$
\lambda=\eta_{2} / E_{2}
$$

(Equation 5)

Narkis(8) also studied the sintering of closely packed Poly(methyl) Methacrylate spheres in a circulated air oven. Experimental results showing neck radius verus time for four temperatures are shown in Figure 8. The shapes of the curves were very similar to the theoretical curves given by Lontz's equation(Eq. 4)

An examination of these curves, show a non-linear portion is followed by a linear portion. The non-linear portion is due to the viscoelastic effect (E2 and $\eta_{2}$ of our model) after which the relaxation term approaches unity reverting the Lontz equation to a shifted Frenkel equation (the linear portion). Note that the Kuczynski, Neuville and Toner(3) curves only showed the linear portion of the curves. They neglected to realize any relaxation effect but only stated that Frenkel's equation was inadequate. Also note, the viscoelastic effect is shortened as would be expected due to greater moleculer mobility as the sintering temperature is increasd.

| Figure 8 |
| :---: |
| SINTERING OF PMMA SPHERES BY NARKIS (8) |



Steiner, Manson and Nippert(9), using the basic differential Frenkel equation, numerically integrated the equation using the exact value for the $\sin \theta$ rather than Frenkel's assumption of $\sin \theta=\theta$. (Only valid for small values of $\theta$ ). Numerical results of $\frac{x^{2}}{a}$ versus time showed a better correlation to their experimental data for viscous sintering. A viscoelastic sintering equation which included a retardation time factor similar to that developed by Lontz was also derived.

Menges and associates(20) developed an isothermal growth correlation based on surface tension, neck curvatures and inner frictional forces. Theoretical results agreed with their experimental results.

Rosenzweig and Narkis(2l) performed a study of dimensional changes of sintering particles. Based on the Frenkel's sintering model( Figure 9), the authors derived the following relationships of two sintering spheres:

$$
\begin{gather*}
x=\left(2 a z-z^{2}\right)^{0.5}  \tag{Eq6}\\
L=2 a-2 z  \tag{Eq7}\\
4 a^{3}-3 z^{2} a+z^{3}-4 a_{0}^{3}=0 \tag{Eq8}
\end{gather*}
$$

Figure 9
GEOMETRY OF TWO SINTERING SPHERES
(FRENKEL'S MODEL)


Initially - Two spheres with length $L$, between centers
$a_{0}=$ initial radius of two spheres


During sintering neck radius x , is formed as lenth L, decreases


Final sintering results with sphere radius becoming equal to $2^{1 / 3} a_{0}$.

where: | $x$ | $=$ neck radius |
| ---: | :--- |
| $a_{0}$ | $=$ original sphere radius |
| $a$ | $=$ sphere radius during sintering |
| $2 * z$ | $=$ penetration depth of two sintering |
|  | $=$ spheres |
| $L$ |  |

These equations will be later used in this work to determine the density of the powdered material as a function of distance between centers of sintering spheres. The results were used to correlate the density of a group of spheres as a function of neck radius.

Thermal Conductivity
Progelhof, Throne, and Ruetsch(33) performed an in depth investigation of published articles studing the thermal conductivity of foamed, powdered or composite materials. In addition to the articles reviewed by Progelhof, Throne, and Ruetsch, references 34-46 were also reviewed. In most correlations evaluated, the values of $\mathrm{k}_{\mathrm{c}}$ and $k_{d}$ (where $k_{C}=$ thermal conductivity of the continuous material and $k_{d}=$ thermal conductivity of the discrete material) must be determined. In the case of rotomolding, until the material starts to stick to the mold, it is assumed that the polymeric material is the discrete phase that is surrounded by a continuous phase of air. Upon the
commencement of the fusing process, the polymeric material is assumed to be the continous phase surrounding the discrete phase, air.

Figure 10 illustrates the Maxwell Model of thermal conductivity as reported by Progelhof, Throne and Ruetsch (35) of polyethylene and air. The top curve represents the predicted thermal conductivity with air as the discrete phase while the bottom curve represents polyethylene as the descrete phase. The dotted lines labled $A^{\prime}, B^{\prime}$ and $C^{\prime}$ represents possible routes where air originially the continuous phase changes over to the descrete phase as is expected to occur in the rotational molding process.

The original intent of this investigation was to use one of these equations to predict the thermal conductivity of the powdered material while in the pool. Using this value, new values as a function of temperature and neck growth would be estimated. However, in performing the thermal conductivity calculations, a great variance was found. Figure 11 shows six thermal conductivity estimate equations and the results using air as the discrete material and then as the continuous material. The results show a variance of two magnitudes. To avoid justifying one value over another, this investigation will use experimental data to determine the actual value. The experimental procedures are discussed latter.

Figure 10
MAXWELL'S MODEL FOR THERMAL CONDUCTIVITY


Figure ll
COMPARISON OF VARIOUS
VARIOUS COMPOSITE THERMAL CONDUCTIVITY EQUATIONS

COMPUTED THERMAL CONDUCTIVITY
( J/cm-sec-K)

| Descrete Material | Polyethylene | Air |
| :--- | :--- | :--- |
| Continuous Material | Air | Polyethylene |
| Fraction of <br> Continuous Material | 0.58 | 0.42 |

EQUATION

| Yagi - Kunni | $9.90 \mathrm{E}-05$ | $3.24 \mathrm{E}-04$ |
| :--- | :--- | :--- |
| Maxwell | $4.98 \mathrm{E}-04$ | $2.52 \mathrm{E}-03$ |
| Lewis and Nielson | $2.76 \mathrm{E}-03$ | $1.10 \mathrm{E}-02$ |
| Russel | $1.08 \mathrm{E}-03$ | $2.66 \mathrm{E}-03$ |
| Geometric Mean | $1.39 \mathrm{E}-03$ | $1.39 \mathrm{E}-03$ |
| Series | $2.96 \mathrm{E}-03$ | $2.96 \mathrm{E}-03$ |

Thermal Conductivity of Polyethylene $=4.93 \mathrm{E}-03 \mathrm{~J} / \mathrm{cm}-\mathrm{s}-\mathrm{K}$ Thermal Conductivity of Air $=2.42 \mathrm{E}-04$

NOTE: Equations used in the computations of the thermal conductivity taken from Reference 35.

## III. STATEMENT OF THE PROBLEM

All previously published investigations of rotational molding were based upon models that had limited restrictions. One assumed constant physical and thermal properties, while the other assumes the material already distributed around the mold. Investigations dealing with sintering(densification) of particles all involved an isothermal process. In rotomolding, the "sintering" process starts with polymer powder being placed in a mold at ambient temperature. The mold is then heated as it is rotated in an oven. As the temperature of the polymer reaches the stick temperature, it begins to adhere to the mold wall. This process is continued until all powder has adhered and then begins to densify.

This dissertation is an investigation of the rotational molding process. It includes an analysis of the material flow in the pool and an investigation of the heating and densification process. It does not include the cooling portion of the cycle. A theoretical simulation model of the rotational molding process was then derived to predict pool depletion and densification times using temperature and time dependent properties. In lieu of justifying the use of one of the many composite thermal conductivity equations (reviewed earlier) over another, an
experimental procedure was developed in conjuction with a small computer program for determining the actual powder thermal conductivity. In the conculsions, the theoretical results predicted by this new model are compared to the actual measured rotational molding pool depletion times and with the other previous simulation results. The densification times between the simulations developed are then compared with the earlier simulation results of Throne and Adhout.

## IV. MASS FLOW IN ROTATIONAL MOLDING

In order to properly simulate the rotational molding process, an understanding of the mass flow within the mold cavity during that process is essential. Consider a simple cylindical mold cavity as shown in Figure 12 rotating in a clockwise direction. The angle theta $(\theta)$ is the included angle of the powdered material in the stationary zone of the pool. The angle phi $(\phi)$ is the angle that the stationary zone maintains as the mold rotates. This angle ( $\phi$ ) is a function of the material characteristics, the rotational speed, and the coeficent of friction of the mold surface. Based on Vanderbeck's experimental results, it has been shown that the angle ( $\phi$ ) remains constant during the depletion of the pool.

Consider a line segment $A-B$ which begins at the center of the cord formed by the surface of the stationary pool of powdered material and ends at the mold surface. If the mold is rotated for a time interval(DT), the line segment is also rotated to the postion $A^{\prime}-B^{\prime}$ shown in Figure 12. The area(volume) included by the two line segments and the mold surface is the amount of material that has entered the stationary pool leaving the free-falling zone during that time interval. This section is labled A-1.

Figure 12
ENTERING MASS FLOW IN ROTATIONAL MOLDING


Allowing the mold to rotate for another time increment will move Area $A-1$ to the location of Area $A-2$ as shown in Figure 12. Note that the shift of A-l has caused a portion of the area to protude past the surface line $B-C$ of the stationary region. This outside area represents the amount of material of Area A-l that has left the stationary pool and reentered the free-falling region during that time increment.

Rotating the mold by another time increment will move Area A-2 to the position of A-3. Again, a portion of the area, in the movement from $A-2$ to $A-3$, has exited Area $A-2$ of the stationary pool region and reentered the free fall zone during that time increment.

Continuing the rotation of the mold will cause the Area A-3 to move to A-4, then to A-5 and so on to A-8. After A-8 all material in the area that started at A-l has completely exited the stationary pool. The amount of material that exits the stationary pool at each time interval for each area is equal to the amount of area passing through the pool surface Line $B-C$ during that time increment.

From the preceding analysis, it is evident that the time length that the material remains in the pool depends on its location or position while entering the pool.

Material entering near the center surface of the pool (point A) remains in the pool for a short time duration while material entering near the mold surface remains longest in the pool.

The total mass leaving at each time interval is the amount that passes through line segment $B-C$ in Figure 13 to join the free-falling powder. Moving line segment $C-A$ in the opposite direction of the rotation for one time interval of rotation results in the postioning of that line labled $C^{\prime}-A^{\prime}$ in Figure 13. The area bounded by line segments $C-A, C^{\prime}-A '$ and the mold surface is the material exiting the stationary pool in the next time interval. Continuing back at one time increment movement intervals, the areas shown are the areas leaving in the succeding time intervals. Some areas have a portion of their areas above the pool surface line meaning that this portion of material has not yet entered the stationary pool. Notice that the area shown leaving the stationary pool in the Figure is the same amount of area that had entered in Figure 12.

Figure 14 combines Figures 12 and 13 into a complete illustration of the mass flow within the stationary pool zone. In this figure an arc time length of eight time increments(intervals) were used. It can be seen that the material entering in the center(area labled l) remains in

Figure 13
LEAVING MASS FLOW IN ROTATIONAL MOLDING


Figure 14
TOTAL MASS FLOW IN ROTATIONAL MOLDING

the pool for only one time increment while the areas labled 8 remain for all eight time increments. Those particles in between remain according to their relative position from the center.

It is assumed that after the material leaves the stationary pool, the material begins to fall down and mixes physically and thermally until it again rejoins the stationary pool and then repeats the cycle.

In addition to being rotated, the material is also being heated by the mold surface. When the mold reaches or exceeds the "stick temperature", the portion of material that is at or above that particular temperature will stick to mold wall, thus deminishing the amount of material in the pool. This process will continue until all material in the pool has left the pool by sticking to the mold surface.

## V. ANALYSIS OF THE HEAT TRANSFER FOR <br> SIMULATION ASSUMPTIONS

In the rotational molding process, the mass of material in the stationary pool is assumed to be heated by conduction from the mold surface. The time increment that an individual element is in contact with the mold surface is given by:

$$
t(\mathrm{~ms})=\quad \theta / \omega \quad \text { (Eq. 9) }
$$

where $\mathrm{t}(\mathrm{ms})=$ Contact time of an element with the mold surface. $\theta=$ angle theta(Figure 13) $\omega=$ angular velocity

Using typical values of an angle theta of 90 degrees, 7 RPM will result in a element contact time of 2.142 seconds.

During this contact time, the thermal penetration depth can be approximated by the results for a semi-infinite solid having a constant initial temperature and being subjected to sudden change in surface temperature:

$$
\begin{gathered}
T(x, t)=\operatorname{erf}\left\{\frac{x}{2 * S Q R(\alpha * t)}\right\} *(T i-T s)+T s \text { (Eq. 10) } \\
\text { where: } T(x, t)=\text { Temperature at time } t, \text { at a } \\
\text { distance } x \text { from the surface } \\
T i=\text { Original Initial temperature of the } \\
\text { solid }
\end{gathered}
$$

```
Ts = New surface temperature
x = distance from surface
\alpha = thermal diffusivity
t = time
```

Rearranging the above equation and solving for the distance, $x$ will result in the equation:

$$
x=2 * \operatorname{SQR}(\alpha \quad * t) * \operatorname{erfc}((T(x, t)-T s) /(T i-T s)) \quad \text { (Eq. ll) }
$$

The penetration depth will be at the position where $T(x, t)$
$=$ Ti. The inverse error function has the numeric value of
3.60. The equation reduces to :

$$
x=7.2 * \operatorname{SQR}(\alpha * t) \quad \text { (Eq. 12) }
$$

Using the numerical value of the thermal diffusivity of the High Density Polyethelyne powder as $1.7 \mathrm{E}-03 \mathrm{sq} \mathrm{cm} / \mathrm{sec}, \mathrm{a}$ contact time of 2.142 seconds will result in a penetration depth of 0.434 cm .

Since this penetration depth is very small compared to the depth of the pool, and that this penetration occurs only in the subareas adjacent to the mold surface, the computer simulation can use a rectangular system having the same number of columns as wedges in the actual system. This approximation will be in error when the pool is in the last phases of depletion. A comparison of the two systems is shown in Figure 15.

has a total area(or volume) equal to the area in the actual system and also a column width equal to the arc length of one wedge.

In this simulation, the material enters at position $A$ in Figure 16 at a temperature equal to that of the free-falling powder region. The temperature profile for one time increment is then calculated. The material is then shifted over one position (labled B) simulating rotational movement and a new temperature profile is calculated for the next time interval using the old profile as its initial temperature.

This process is continued for all postions(columns A-H). After the last position (H), the material is assumed to leave the stationary pool and join the free-falling powder region. It is assumed in this phase of the model that all of the particles are thermally mixed. The average or equilibrium temperature of the powder in the free fall zone is computed by using a mass weighted average of the average temperature of the mass entering the free-falling region with the remaining material in this region, or:

$$
\begin{gathered}
\bar{T}=\frac{\overline{T_{e}} * M_{e}+\overline{T_{F F}} *\left(M F F-M_{L}\right)}{M_{F F}} \\
\text { where: } \\
\\
\\
\\
\quad \bar{T}=\text { new ave temperature of free } \\
\text { falling region. }
\end{gathered}
$$



FIGURE 16
TEMPERATURE PROFILE DURING 8 TIME INTERVALS

$$
\begin{aligned}
\hline \mathrm{Te}_{\mathrm{e}}= & \text { new ave, temperature of mass } \\
& \text { entering } \\
M_{e}= & \text { mass entering } \\
\overline{T_{F F}=} & \text { previous average temperature } \\
& \text { of free falling region } \\
M_{F F}= & \text { previous amount of mass in free } \\
& \text { falling region } \\
M_{L}= & \text { amount of mass leaving which is } \\
& \text { equal to the mass entering, } M_{e} .
\end{aligned}
$$

Simplifing:

$$
\begin{equation*}
\bar{T}=\overline{T_{e}} * \frac{M_{e}}{M_{F F}}+T_{F F} *\left[1-\frac{M_{e}}{M_{F F}}\right] \tag{Eq.l4}
\end{equation*}
$$

The average temperature of the mass entering the free falling region, $\overline{T_{e}}$, is determined by the same mass weighted formula using the temperatures and mass within the penetration depth and that outside the penetration depth.

After the free falling region, the mass now re-enters the stationary pool at the present mixed mean free-falling temperature and the heating cycle described earlier is repeated.

When the polymer powder in contact with the mold surface attains the stick temperature, material above this temperature will adhere to the mold wall causing the powdered pool to become smaller in size. The simulation
will still use the same amount of wedges (columns), however, the time increment (interval) will become smaller to compensate for the smaller contact time.

The cycle is repeated until all material has left the powdered pool and adheres to the mold.

## VI. THEORETICAL MODEL USING NODAL ANALYSIS

In the theoretical analysis presented in this dissertation, a numerical method will be used. The geometry of the specimen is subdivided into small but finite subvolumes of thickness of $\Delta X$. For each subvolume there is located a center nodal point which has been assigned a reference number, Figure l7. Note that the exterior subvolumes has a thickness of $1 / 2 \Delta X$ with its node located at the surface.

In this actual transient heat transfer process, the temperature profile within a subvolume varies with position and time. However for this simplified model it was assumed that the temperature of each subvolume can be denoted by a nodal "mixed mean" or "equilibration" temperature. Thus the temperature within a node is assumed to be only a function of time. The mixed mean or equilibration temperature of the element is defined as the temperature the element will attain if all the internal energy of the element was distributed evenly throughout that element.

The temperature of each subvolume is assumed to be represented by the temperature of the node. It is further assumed that the rate of energy transfer between adjacent nodal points is approximated by the steady-state conduction

Figure 17
NODAL CONVENTION

equation using the nodal temperature values as the descriptive temperature between the nodes. Thus in this model, a discontinuous temperature profile is being used to approximate the actual temperature profile (Figure 18). It is obvious that as the subvolumes become smaller, the approximate temperature profile will approach the actual temperature profile.

Writing an energy balance on node $i$ results in the following equation:

$$
q(i n)-q(o u t)+q(\text { generated })=q(\text { stored }) \quad(e q \quad 15)
$$

Or:

$$
\begin{equation*}
\sum q_{i}+q(\text { generated })=m_{i} c_{i} \frac{d T}{d t} \tag{eq16}
\end{equation*}
$$

where: $\quad q_{i}=$ heat flow entering node $i$
$m_{i}=$ mass of subvolume $i$ $\frac{d T}{d t}=$ the time derivative of temperature of subvolume i

The initial temperatures of all nodes were constant room temperature and the mold surface was subjected to a changing temperature. Because the temperature change will only occur within the penetration depth, nodal equations must then be made to include this depth. Temperatures of nodal volumes outside the penitration depth will not change, thus performing computations of these nodes are needless and a waste of valuable computer time.

Figure 18
APPROXIMATION OF THE TEMPERATURE PROFILE


Figure 19 shows a typical stationary pool that is divided into eight wedges with each wedge divided into $N$ nodal volumes. The nodal volumes have a fixed thickness such that the total distance extend slightly past the final penetration depth.

For this geometry, three types of nodal equations must be used. These equations are for: the surface nodes, the interior nodes and the final interior nodes outside the penetration depth.

The energy balance for the interior nodes of the column, Figure 19, is given by:

$$
q(i-1: i)+q(i+l: i)=m_{i} c_{i} \frac{\partial T}{\partial t}
$$

where: $q(i-1: i)=$ heat flow from node $i-1$ to node $i$

$$
=k(i-1: i) \frac{A}{\Delta X}\left(T_{i-1}-T i\right) \quad(e q 18)
$$

$k(i-1: i)=$ the average thermal conductivity between node i-l and node i
$=1 / 2[k(i-1)+k(i)]$
(eq 19)
$q(i+1: i)=$ heat flow from node $i+1$ to node $i$
$=k(i+1: i) \frac{A}{\Delta X}\left(T_{i+1}-T_{i}\right) \quad(e q 20)$
$k(i+1: i)=$ the average thermal conductivity between node $i+1$ and node $i$
$=1 / 2[k(i+1)+k(i)]$
(eq 21)
$m_{i} \quad=$ the mass of node $i$
$=\rho A \Delta X$
(eq 22)


FIGURE 19
NODAL ASIGNMENTS FOR COMPUTER SIMULATION
which results in the relationship:
$k(i-1 ; i) \frac{A}{\Delta X}\left(T_{i-1}-T_{i}\right)+k(i+1 ; i) \frac{A}{\Delta X}\left(T_{i+1}-T_{i}\right)=\rho A \Delta \times C \frac{d T}{d t}$
In this slab geometry, the cross-sectional area A is constant. Solving for the time rate of change of temperature:

$$
\begin{equation*}
\frac{d T}{d t}=\frac{k(i-1: i)}{\rho C \Delta X^{2}}\left(T_{i-1}-T_{i}\right)+\frac{k(i+1: i)}{\rho C \Delta X^{2}}\left(T_{i+1}-T_{i}\right) \tag{eq24}
\end{equation*}
$$

The surface node will be assigned the same temperature as the mold surface. Since the mold temperature changes as a function of time, then:

$$
\begin{equation*}
T(\text { surface node })=T(\text { mold })=T(t) \tag{eq25}
\end{equation*}
$$

The last interior node, which being outside the penetration depth, is always at a temperature of the pool temperature for that particular wedge, or:

$$
T_{j, \cap}=T(\text { pool })_{j} \quad(\text { eq 26) }
$$

Where: $T j, n=$ temperature of node $n$ in wedge $j$
$T($ pool $) j=$ pool temperature outside the penetration depth for wedge $j$

$$
=\bar{T}
$$

There are three basic methods of approximating the time derivative, the Pure Explicit or Euler's Method, the Pure Implicit Method, and the Implicit Crank-Nicolison Method.

Euler's Method estimates the temperature T' at the nodal point one time increment, $\Delta \theta$, later by computing the time derivative of the present temperature $T$, multiplying it by the time increment between $T$ and $T$ ', and then adding this to the present temperature $T$, or:

$$
T^{\prime}=T+\left.\frac{d T}{d t}\right|_{t} \Delta \theta
$$

(eq 27)

This is shown graphically in Figure 20.

The Pure Implicit Method estimates the future nodal temperature $\mathrm{T}^{\prime}$ in a similar fashion as the Euler's Method. But instead of using the time derivative of the present temperature $T$, it uses the derivative of the future temperature $\mathrm{T}^{\prime}$, or:

$$
\begin{equation*}
T^{\prime}=T+\left.\frac{d T}{d t}\right|_{t^{\prime}}, \quad \Delta \theta \tag{eq28}
\end{equation*}
$$

This is shown in Figure 21.

In the Implicit Crank-Nicolson Method, the arithmetic mean value of the derivatives at the beginning and at the end of the time interval is used to determine the future tmeperature. Or:

$$
\begin{equation*}
T^{\prime}=T+\left[\frac{\left.\frac{d T}{d t}\right|_{t}+\left.\frac{d T}{d t}\right|_{t^{\prime}}}{2}\right] \Delta \theta \tag{eq29}
\end{equation*}
$$

This is shown in Figure 22.

Figure 20
EULER'S METHOD OF APPROXIMATION


Figure 21
THE PURE IMPLICIT METHOD OF APPROXIMATION


Figure 22
THE CRANK-NICOLSON METHOD OF APPROXIMATION


If the Pure Implicit or the Implicit Crank-Nicolson Method is used to approximate the time derivative, the result will be a set of $N$ simultaneous equations with $N$ unknowns. This would require the computer to solve a tri-diagonal matrix for each time increment for each column.

Euler's Method, however, results in a set of $N$ equations with one unknown per nodal energy balance equation. Consequenty, the future temperature $T$ ' can be found directly by solving a simple algebraic equation. Therefore, in this dissertation, Euler's Method of approximation was used.

Substituting for the time derivative for an interior nodal equation results in:

$$
\begin{equation*}
\frac{T_{i}^{\prime}-T_{i}}{\Delta \theta}=\frac{k(i-1: i)}{\rho C \Delta X^{2}}\left(T_{i-1}-T_{i}\right)+\frac{k(i+1: i)}{\rho C \Delta X^{2}}\left(T_{i+1}-T_{i}\right) \tag{eq30}
\end{equation*}
$$

Solving for the future temperature $\mathrm{T}^{\prime}$ :

$$
\begin{aligned}
T_{i}^{\prime}=T_{i} & \left(1-\frac{\Delta \theta}{\rho C \Delta X^{2}}\{k(i-1: i)+k(i+1: i)\}\right) \\
& +\frac{\Delta \theta}{\rho C \Delta X^{2}}\left\{k(i-1: i) T_{i-1}+k(i+1: i) T_{i+1}\right\}
\end{aligned}
$$

(eq 31)

In the use of the Euler's Method of approximation, it must be noted that the equations become unstable when the expression $\frac{1-K \Delta \theta}{\rho C \Delta X^{2}} \Sigma k \quad$ becomes negative ( $K$ is some constant).

Often the k's, $k, \rho, C$ and $\Delta X$ are predetermined. Therefore it is necessary to compute the value of $\Delta \theta$ such
that the value of the expression is greater than or equal to zero. Or:

$$
\begin{equation*}
1-\frac{K \Delta \theta \Sigma k}{O C \Delta X^{2}} \geq 0.0 \tag{eq32}
\end{equation*}
$$

Or:

$$
\frac{\mathrm{K} \Delta \theta \Sigma \mathrm{k}}{\rho \mathrm{C} \Delta \mathrm{X}^{2}} \leq 1.0
$$

(eq 33)

Solving for $\Delta \theta$ :

$$
\Delta \theta \leq \frac{\rho C \Delta X^{2}}{K \Sigma k}
$$

(eq 34)
Usually the maximum allowable value of

$$
\begin{equation*}
\Delta \theta_{\operatorname{MAX}}=\frac{\rho C \Delta X^{2}}{K \Sigma k} \tag{eq35}
\end{equation*}
$$

will be used, or:

If the values of $\rho, C$, and $k$ are not constant, the minimum values of $\rho$ and $C$ and the maximum value of $k$ must be used to determine the stability constants.

Since the nodal thickness, $\Delta X$ in the simulation developed, becomes smaller as the material densifies, the maximum time increment ( $\Delta \theta$ ) will be recalculated at each iteration assuring that all nodal equations remain stable.

The second simulation model developed includes the effect of convection heating by oven air. The exterior surface nodal equation of the mold is given by:

$$
\begin{align*}
\mathrm{T}_{\mathrm{eM}}{ }^{\prime}=\mathrm{T}_{\mathrm{eM}}+ & {\left[1-\frac{2 \Delta \theta}{\rho_{\mathrm{M}} \mathrm{C}_{\mathrm{M}} \Delta \mathrm{X}_{\mathrm{M}}}\left(\frac{k_{M}}{\Delta \mathrm{X}_{\mathrm{M}}}+\mathrm{H}_{A}\right)\right] } \\
& +\frac{2 \Delta \theta}{\rho_{M} C_{M} \Delta \mathrm{X}_{M}}\left[\frac{k_{M}}{\Delta \mathrm{X}_{M}} * T_{I M}+H_{A} * T_{\text {OVEN }}\right] \tag{eq36}
\end{align*}
$$

where: $T^{\prime}{ }_{e M}=$ Future temperature of the external mold surface one time increment ( $\Delta \theta$ ) latter.
$T_{e M} \quad=$ Present temperature of the external mold surface.
$\rho_{\mathrm{M}} \quad=$ Density of the mold material
$K_{M} \quad=$ Thermal Conductivity of mold material
$C_{M} \quad=$ Specific heat of mold material
$\Delta X_{M}=$ Mold thickness
$H_{A} \quad=$ Coefficient of Convection of the air
in the oven
Toven $=$ Oven temperature
$\mathrm{T}_{\mathrm{IM}}=$ Temperature of mold's internal surface

The maximum allowable time increment for this nodal equation is:

$$
\begin{equation*}
\Delta \theta_{M A X}=\frac{\rho_{M} C_{M} \Delta X_{M}}{2\left(\frac{K_{M}}{\Delta \mathrm{X}_{M}}+H_{A}\right)} \tag{eq37}
\end{equation*}
$$

The mold's internal surface nodal temperature equation is:

$$
\begin{aligned}
& T_{I M}^{\prime}=\left[1-\frac{2 \Delta \theta k_{M}}{\rho_{M} C_{M} \Delta X_{M}^{2}}\right]^{T} I M+\frac{2 \Delta \theta K_{M}}{\rho_{M} C_{M} \Delta X_{M}^{2}} * T_{e M} \quad \text { (eq 38) } \\
& \text { where: } T^{\prime \prime}= \\
& \text { Future temperature of the internal } \\
& \text { mold surface one time increment }
\end{aligned}
$$

$$
(\Delta \theta \text { ) later. }
$$

$\mathrm{T}_{\mathrm{IM}} \quad=$ Present temperature of the internal
mold surface.

The maximum allowable time increment for this nodal equation is:

$$
\begin{equation*}
\Delta \theta_{M A X}=\frac{\rho_{M} C_{M} \Delta X_{M}^{2}}{2 K_{M}} \tag{eq39}
\end{equation*}
$$

When using more than one type of nodal equation, the minimum $\Delta \theta_{\text {MAX }}$ must be the largest time increment used in the numerical analysis. This will assure stability for all equations.

The temperature of the first node of the powdered material now becomes equal of $T_{I M}$ •

The equations developed in this chapter were used to predict the powder temperatures during the rotational molding process.

## VII. INVESTIGATION INTO THE DENSIFICATION PROCESS

To accurately simulate the rotational molding process from powder rotation to the formation of molten mass, the powder densification process must be understood. Experimental studies were performed to determine the basic densification phenomena and the results were used to describe the densification in the simulation model.

Because the individual powder particles used in rotational molding have such irregular shape (see Figure 23), it becomes very difficult to obtain experimental observational data that can be used to describe the densification process . Thus, it becomes necessary to experiment with a much simpler particle geometry and then hypothesize how the process occurs with much more complex geometry. However, problems still occur in attempting to freeze the densification process at a particular time frame while the material is in an actual rotational mold. The solution was to model the rotational mold in an environment with a flat plate apparatus, Figure 24. The powdered material was heated through the base plate and at a particular instant of time the heating process was stopped by removing the plate from the heater and placing the plate on dry ice.

TYPICAL ROTATIONAL MOLDING MATERIAL DURING DENSIFICATION


Figure 24
FLAT PLATE APPARATUS USED IN DENSIFICATION STUDY


Figure 25 compares a photo of a rotational molded part of Dupont acrylic microspheres made in the uniaxial rotational mold in the NJIT laboratory and a photo of a part molded on the flat plate apparatus. Both molding processes were conducted under similar temperature time histories. (material data listed in Appendix C) Notice the similarities; both have a totaly densified region, a region with slight necking, and a region where spheres are attached but no appreciable or noticeable radius at the points of contact. Hence, the flat plate apparatus is a viable means of analyzing and understanding the densification process in rotational molding.

An analysis of the densification process was performed after a review of a flat plate experiment using significantly larger spheres illustrating a non-necking densification process. This experiment involved using one-eighth diameter High Density Polyethylene(HDPE) spheres (Appendix $B$ for data) packed in a body centered cubic array using an apparatus shown in Figure 24. This apparatus was heated through its base, simulating the non-isothermal heating that occurs in the rotational molding process.

As energy was applied to the base plate, the lower portion of the bottom spheres became less viscous and was forced down by the weight of the spheres above. The less

COMPARISON BETWEEN ROTATIONAL MOLDING AND
FLAT PLATE MOLDING OF ACRYLIC MICROSPHERES

TOP


FLAT Plate molded

TOP


HOT PLATE
viscous material flowed between the spheres, filling the voids(Figure 26a-c).

The upper portion of the bottom layer next became soft allowing the pressure of the harder spheres above to deform the lower level spheres(Figure 26d). The final shape of a bottom layer spheres appeared to be flat on the bottom due to the flat plate and dimpled on the top due to the more viscous upper level spheres. With continued heating of the plate, the interior spheres deformed next. As seen in the bottom layer analysis, the lower portion of the more viscous upper layer retained its spherical shape while the lower layer was deformed (Figure 27-a).

As the upper portion of the interior sphere increases in temperature, the harder, more viscous next upper layer spheres deformed the interior sphere below(Fig. 27a-d). The final shape is spherical in the lower region and dimpled on top (Figure 27e). Duplication of the experiments using one-eighth diameter acrylic spheres produced the same results as those obtained with the HDPE spheres.

The experimental results indicated a different densification process that was described in the literature survey. Unlike sintering, the one-eighth diameter spheres

## Figure 26

THE MELTING OF BOTTOM LAYER SPHERES
(a)


Original set-up
(b)

(c)

(d)

(e)


As heat in applied, the lower portion of the bottom spheres become less viscous (melts) and is forced down by the weight above.

The displaced material begins to fill the voids

The heat continues and causes the upper portion of the bottom layer to become soft allowing the pressure of the more viscous, harder spheres to deform the lower level spheres.

The final shape is flat on the bottom(caused by the plate) and dimpled on top (caused by the harder upper level spheres)

Figure 27
THE MELTING OF MIDDLE LAYER SPHERES
(a)

(b)

(c)

(d)
(e)


As was seen in the Bottom Layer Analysis, the lower portion of the middle layer retains its spherical shape during melting because it is more viscous (harder) than the layer below causing the lower level to be deformed by the spheres above it.

As the upper portion of the sphere increases in temperature, it allows the harder, more viscous upper layer spheresto deform it.

The final shape is spherical on the bottom and dimpled on top.
showed no necking but rather filled the voids between the individual spheres by viscous flow.

The spheres used is this experiment were approximately one hundred times greater in size than normal rotational material. The only useful conclusions that were drawn from this study was that material flow caused by lower viscosity may also be a significant factor in the mechanism that occurs in the densification process of rotational molding.

The next set of experiments were performed with the Dupont acrylic microspheres using the Flat Plate apparatus. Because the size distribution of the microspheres was very close to that of a rotational molding grade polyethylene powder, the results obtained using the microspheres gave considerable insight into the actual densification mechanism.

Several experiments were conducted terminating the densification process at different time intervals. The specimens were then mounted and photographed with a Scanning Electronic Microscope. The temperature - time profiles used in these experiments were identical to those used by Vanderbeck(30) in his experiments. A direct comparison between results of the computer simulation developed in a latter section in this dissertation using a model of the densification process observed here and the
actual experiments results of Vanderbeck.

Figure 28 shows the photographs of six simulated rotomolding experimental runs of the acrylic microspheres that were frozen at various time frames. Though difficult to see in the photographs, all the spheres have adhered together at the points of contact and the densification process was initiated.

As the temperature rises, the spheres nearer to the heat source began the necking process by sintering. With increasing time and mold temperature, neck radius of the spheres near the high temperature surface continue to increase. In addition, the necking process starts to occur with the interior spheres.

The latter photographs show that as the neck growth reaches a maximum of between $10 \%$ and $20 \%$, there appears to be a dramatic change occuring. Necking is overcome by viscous flow from melting. This process continues until all the spheres have lost their individuality and have melted into one homogenous part. This drastic change is a characteristic of a material where by the temperature causes the material to flow faster than the slow viscous flow of the sintering process. This temperature at which material flow overcomes the sintering process will be referred to as the melt interface temperature (Tm).


HOT PLATE

FIGURE 28

SCANNING ELECTRONIC
MICROSCOPE PHOTOGRAPHS OF ACRYLIC MICROSPHERES FOR VARIOUS TIME INTERVALS

8 MINUTES


10 MIINUTES





16 MINUTES (CONTINUED)


HOT PLATE





## VIII. DETERMINATION OF THE PHYSICAL PROPERTIES

As discussed previously, the simulation model will use nodal analysis in which the average neck size for a representative particle will be computed at each node as a function of time. The value of the thermal conductivity of each nodal subvolume at any particular time will be a direct function of the neck size within the nodal volume at that instant of time. The equation used to estimate the instantaneous thermal conductivity showing the effect of necking due to sintering is:

$$
\begin{equation*}
K_{E i}=K_{P}+\left[\frac{X_{i}}{a}\right]^{N}\left(K_{S}-K_{P}\right) \tag{eq40}
\end{equation*}
$$

where: $K_{E i}=$ thermal conductivity at node $i$
$K_{P}=$ thermal conductivity of initial powder
$K_{S}=$ thermal conductivity of the solid polymer
$X_{i}=$ neck radius of node $i$
a = original sphere radius
$\mathrm{N}=$ power law exponent (value = l)
Even though the actual polyethylene powdered material is not spherical, the value of the original sphere radius, a, is the average sift size radius.

The thermal conductivity of the powdered material ( $K_{p}$ ) was to be initially be determined using the equations
mentioned earlier in the literature survey for predicting the thermal conductivity of composite system. Figure ll compared the results of using various composite material equations for determining thermal conductivity based on the material data in Appendix A. The calculated values of equivalent thermal conductivities ( KEi ) can be seen to vary greatly. Therefore it becomes necessary to experimentally determine, with use of a simple computer simulation, the actual thermal conductivity of the powder. The experiment and the subsequent determination of the powder thermal conductivity will be described in the next chapter.

The neck size for each node is calculated in the computer simulation by adding the calculated change in neck size during that time increment to the previous neck size of the node. Using Frenkel's Equation:

$$
\begin{equation*}
x=\left[\frac{3 a t \gamma}{2 \eta}\right]^{0.5} \tag{eqI}
\end{equation*}
$$

and differentiating eq. l with respect to time will give the change of neck radius as a function of time, or:

$$
\begin{equation*}
\frac{\partial x}{\partial t}=\left[\frac{3 a \gamma}{8 \eta t}\right]^{0.5} \tag{eq41}
\end{equation*}
$$

Converting the equation to a small finite change in time ( $\Delta t$ ), the change in neck radius for a time $\Delta t$ is :

$$
\begin{equation*}
\Delta x=\left[\frac{3 a r}{8 n t}\right]^{0.5} \Delta t \tag{array}
\end{equation*}
$$

When the neck radius exceeds $50 \%$ of the sphere radius, Frenkel states that the collapse of the bubble formed is:

$$
\begin{equation*}
r_{0}-r=\frac{\gamma}{2 \eta} t \tag{eq2}
\end{equation*}
$$

differenting with respect to time:

$$
\begin{equation*}
\frac{\partial r}{\partial t}=-\frac{\gamma}{2 \eta} \tag{eq43}
\end{equation*}
$$

for a small finite change in time:

$$
\begin{equation*}
\Delta r=-\frac{\gamma}{2 \eta} \Delta t \tag{eq44}
\end{equation*}
$$

since the negative change in bubble radius ( $\Delta r$ ) is equal to the increase of neck radius, the change in sphere radius above 50\% necking is:

$$
\begin{equation*}
\Delta X=\frac{r}{2 \eta} \Delta t \tag{eq45}
\end{equation*}
$$

As the temperature of a node exceeds the melt temperature, material flow overcomes necking in the densification process. Hence, the neck radius for that particular node instentaniously becomes equal to the radius of the spheres. In other words, the material has completely densified into a homogenious part for that nodal subvolume.

Density is computed in the similar manner as to that of thermal conductivity. The origial powder density was found by experimentally by weighing a volume of the
polyethylene material and comparing it to the same volume of water. The density measured for the polyethelyne powder is listed in Appendix $A$. The density at the complete melt is that of the solid polymer. Density and other physical data of the base resin was supplied by the manufacturer.

The intermediate density is a assumed to be linearly proportional to the distance between the centers of two sintering spheres as was shown in Figure 9. As length L decreases, the density increases proportionaly. Using Frenkel's model, the length $L$ can be correlated to the neck radius, $x$, thus allowing a direct coorelation of density to neck radius. Figure 29 is a plot of length $L$ vs neck radius using equations 6 through 8 and solving them simutaneously with $a=0.0125 \mathrm{~cm}$. The curve between $0 \%$ and $40 \%$ neck radius/ sphere radius ratio can be approximated with a straight line fit with only a $2.5 \%$ variance. Therefore, it is possible to state that density is proportional to neck radius, or:

$$
\begin{equation*}
\rho_{E i}=\rho_{P}+\frac{x_{i}}{a}\left(\rho_{s}-\rho_{p}\right) \tag{eq46}
\end{equation*}
$$

where: $\quad \rho_{E i}=$ Equivalent density at node $i$
$\rho_{\mathrm{P}}=$ initial powder density
$\rho_{s} \quad=$ density of solid polymer
$x_{i}=$ neck radius at node $i$
a $=$ sphere radius

Figure 29


Because of the density change, the distance between nodes in the simulation also changes. The distance between node $i$ and node $i-l$ is one half the thickness of node $i$ added to the half the thickness of node i-l. These distances are a function of their relative nodal densities. Therefore the nodal distance between node $i$ and node $i-1$ is proportional to the ratio of the original density to the new average density between node $i$ and node i-l, or:

$$
\begin{equation*}
D x_{i, i-1}=\frac{\rho_{p}}{\left(\rho_{E_{i}}+\rho_{E_{i-1}}\right) * \frac{1 / 2}{2}} * D x \tag{eq47}
\end{equation*}
$$

where: $D x_{i, i-1}=$ Nodal distance between node $i$ and node i-l

$$
\begin{array}{ll}
\rho_{\mathrm{p}} & =\text { Density of original powdered material } \\
\rho_{\mathrm{E}_{\mathrm{i}}} & =\text { Equivalent density at node } \mathrm{i} \\
\rho_{\mathrm{E}_{\mathrm{i}}-1} & =\text { Equivalent density at node } \mathrm{i}-1 \\
\mathrm{Dx} & =\text { original nodal distance }
\end{array}
$$

Since specific heat is energy per amount of mass to raise the temperature of the mass one degree, the specific heat of the powder is assumed equal to that of the solid since the mass of air in the powder is neglible. Hence the specific heat remains constant in the simulation developed.

Since the consistency of the thermal conductivity equations of composite materials vary too greatly, it becomes necessary to determine the conductivity of the initial state of the polymeric powder by a combination of an experiment and a computer simulation. The experiment will entail heating the powder and measuring temperature-time histories at specific locations while the computer program will simulate the experiment in an attempt to duplicate the temperature - time histories and thus determining the thermal diffusivity which includes the thermal conductivity.

The experimental set-up used in shown in Figure 30. The apparatus consists of a cardboard tube 3.25 inches long, 3.375 inches inner diameter with a 0.25 inch wall thickness. The tube is beveled at the bottom to minimize heat transfer up the tube wall. Aluminum foil was then epoxied to the beveled edge, bottom of the tube, to contain the powdered material.

Six thermocouples were positioned as shown in Figure 30. To maintain their correct height level, sewing thread was positioned through the tube at the different levels.

Figure 30
THERMAL CONDUCTIVITY EXPERIMENTAL SET-UP


The thermocouple was then wrapped along the taught thread. The experimental proceedure used in these experiments were as follows:

1. The tube with the powdered material was placed on a hot plate and is heated slowly.
2. The temperature at each thermocouple location was then recorded every two minutes for seventy minutes.
3. This data was then used in a computer simulation to determine the thermal conductivity of the powdered material.

Since the experiment was conducted with six thermocouples, the computer simulation can estimate six average thermal conductivities (actually diffusivity) for each experimental run. They are the diffusivities from position 2 to 3,2 to 4,3 to 4,5 to 6,5 to 7 and 6 to 7 . Using two consecutive runs at the same heating rate, twelve estimates of the diffusivity were made.

The computer program used was a simple flat plate simulation having a nodal geometry as is shown in Figure 31. The simulation allowed input of an actual measured temperature - time history at node 1 and predicted the temperature - time history of one of the other two thermocouples located at node i in the simulation. In chosing the correct diffusivity in the simulation, the predicted temperature - time history duplicated the actual.

Figure 31
NODAL GEOMETRY


$$
\begin{gathered}
A^{*}=\text { DISTANCE BETWEEN THE TWO THERMOCOUFLES } \\
\text { USED IN THE SIMULATION }
\end{gathered}
$$

Six predictions of diffusivities per experimental run were obtained in using the simulation. They included predicted diffusivities between positions stated in the previous paragraph. The method of selecting the correct diffusivity was through trial and error. If the diffusivity used was too low, the predicted temperature time history was lower than the actual. If the diffusivity used was too high, the predicted history was higher.

The extra nodes from $i+1$ to $n$ were added to negate end effects. A technique to check that this assumption was valid was to perform the simulation with more nodes. If the predicted temperature time history at node $i$ was unchanged then node $i$ was uneffected by the end effects. In the actual experiment, end effects were negated by adding an additional quantity of powder, approximately one and a half inches above the last thermocouple.

There is essentially one nodal equation used in the flat plate simulation. For the internal nodes( node 2 to node $n-l)$, the equation is:

$$
\begin{array}{r}
T_{j}^{\prime}=T_{j} *\left[1-2 \frac{D T * A L}{D X^{2}}\right]+\frac{D T * A L}{D X^{2}} *\left[T_{j-1}+T_{j+1}\right] \quad \text { (eq. 48) } \\
\text { where } T_{j}^{\prime}=\text { the temperature of node } j \text { one time } \\
\quad \text { increment ( } D T \text { ) latter. } \\
T_{j}=\text { the present temperature of node } j
\end{array}
$$

```
T
T j+1 = the present temperature of node j+1
DT = time increment
AL = thermal diffusivity = k/\rhoC
k = thermal conductivity
C
\rho = density
```

The last node $n$ is assumed adiabatic. The equation for this node is the same as equation 41 except $T_{n+1}$ will have the same value as $T_{n-1}$, or:

$$
\begin{equation*}
T_{n}^{\prime}=T_{n} *\left[1-2 \frac{D T * A L}{D X^{2}}\right]+2 \frac{D T * A L}{D X^{2}} * T_{n-1} \tag{Eq}
\end{equation*}
$$

The computer simulation performs the following steps:

1 - Sets values for nodal distance( DX ) and diffusivity ( $A L$ ) and then determines the maximum time increment.

2 - Reduces the time increment to a division of a one minute interval so that temperatures will be calculated for each minute.

3 - Sets all nodes to the initial temperatures.
4 - Increments time by one time increment and sets
node 1 to the temperature of the lower thermocouple at that time.

5 - Calculates the new nodal temperature for node 2 through node $n$.

6 - If the time is at a minute interval, then print the predicted temperature of the other thermocouple.

7 - Change the old nodal temperature values to the new predicted values.

8 - Go back to step 4 until the simulation is over.

To check the validity of this simulation, results of a trail simulation are compared to the Heisler Charts in Figure 32. Note the exact fit proving the simulation.

In the performance of this simulation, if the temperature time history of node $i$ is identical to the experiment, the correct diffusivity must be ajusted accordingly and the simulation repeated. An example of the simulation - experiment matching for the Phillips Petroleum polyethylene(Appendix A) is shown in Figure 33.

Figure 34 shows the values of the diffusivity found with the corresponding thermal conductivity. This spread in values will be used in the rotational molding simulation to determine the effects in molding time. Appendix D lists the computer program used in this flat plate simulation.

Figure 32


Figure 33
COMPARISON BETWEEN MEASURED AND COMPUTER
PREDICTED TEMPERATURES


## FIGURE 34

EXPERIMENTALLY AND COMPUTER SIMULATED DETERMINATION
OF THERMAL CONDUCTIVITY

| $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | Thermocouples | $\begin{gathered} \text { Diffusi } \\ (\mathrm{F} t 2 / \mathrm{Hr}) \end{gathered}$ | ity <br> (Cm2/sec) | Thermal Cond <br> ( $B / h r-f t-F)$ | uctivity <br> ( $\mathrm{J} / \mathrm{cm}-\mathrm{s}-\mathrm{K}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2-3 | 6.0E-03 | 1.55E-03 | 0.105 | 1.82E-03 |
|  | 2-4 | 6.0E-03 | 1.55E-03 | 0.105 | 1.82E-03 |
|  | 3-4 | 6.0E-03 | 1.55E-03 | 0.105 | 1.82E-03 |
|  | 5-6 | 7.5E-03 | $1.94 \mathrm{E}-03$ | 0.131 | 2.27E-03 |
|  | 5-7 | $6.5 \mathrm{E}-03$ | 1.68E-03 | 0.113 | $1.96 \mathrm{E}-03$ |
|  | 6-7 | 7.5E-03 | 1.94E-03 | 0.131 | $2.27 \mathrm{E}-03$ |
| 2 | $2-3$ | $6.0 \mathrm{E}-03$ | 1.55E-03 | 0.105 | $1.82 \mathrm{E}-03$ |
|  | 2-4 | 6.0E-03 | 1.55E-03 | 0.105 | $1.82 \mathrm{E}-03$ |
|  | 3-4 | 6.0E-03 | $1.55 \mathrm{E}-03$ | 0.105 | 1.82E-03 |
|  | 5-6 | 5.0E-03 | 1.29E-03 | 0.087 | $1.51 \mathrm{E}-03$ |
|  | 5-7 | 7.0E-03 | 1.81E-03 | 0.122 | 2.11E-03 |
|  | $6-7$ | 9.5E-03 | $2.45 \mathrm{E}-03$ | 0.160 | 2.77E-03 |

Average Thermal Conductivity $=.155 \mathrm{~B} / \mathrm{Hr}-\mathrm{ft}-\mathrm{F}$ Standard Deviation $=0.020 \mathrm{~B} / \mathrm{Hr}-\mathrm{ft}-\mathrm{F}$

NOTE: Thermal Conductivity calculated computed diffusivity using a density of $31.8 \mathrm{lb} / \mathrm{ft3}$ and a specific heat of $0.55 \mathrm{~B} / \mathrm{lb}-\mathrm{F}$ (Appendix A).

## X. THE COMPUTER SIMULATION FOR ROTATIONAL MOLDING

The rotational molding computer simulation (Appendix F) consists of the main program named ROTOTP, supported by the following subroutines: PRINT and CONST, and functions: VIS, SU, RHO, RADX, COND, and CP. The main program ROTOTP performs the simulation, PRINT is the printing routine and CONST computes the constants used in the nodal temperature equations. In addition CONST compute the neck growth for each node during every time increment. The functions VIS, SU, RHO, COND, and CP computes the viscosity, surface tension, density, thermal conductivity and specific heat respectively for each node. Function RADX computes the distance between adjacent nodes based on the density at those nodes.

The main program is divided into ten parts as is labeled in the program listing. Preceding Section I is a glossary of all variables used in the main programs. Section I initializes all the variables and sets all program flags. The variables include: the mold stick temperature(SS), the complete melt temperature(SM), the printing interval(PT), the number of wedges(NC), the material radius(RAD), the initial temperature(IT), the mold radius(RO), the radius from the center of the mold to the surface of the stationary pool material(RI), acceleration
due to gravity(G), rotational speed of the mold(RPM), the angle of response(BA), and the coefficients used in the mold temperature-time history equation(A, $B$, and $Y Y$ ).

Section II performs initial computations using the initial values in Section $I$. They include computation of the pool angle theta(TH), cross-sectional area of stationary pool(AA), radial mold velocity(W), maximum penetration thickness(AD), maximum time increment( $D T$ ), the nodal distance(DX), and the number of nodes per column(N). This section then sets each node to the initial temperature and each nodal neck radius to zero. Then all major values are printed.

Section III begins the iteration process. The mold temperature(MT) based on time is determined. For each iteration the maximum time increment(DT) and time increment used(TS) is determined using the maximum diffusivity and the minimum nodal distance( all usually located at the surface node). Cross sectional areas(AT and AS), penetration thickness(AD) and free-fall time(TF) is also calculated for each iteration.

Section IV computes the nodal temperatures and nodal neck size at each nodal subvolume. The values Cl, C2, C3, $D B$, and $D A$ in the temperature equation and the new neck size is determined in the subroutine CONST. The program
also checks each node to determine if its temperature is equal to or has exceeded the complete melt temperature(SM). When reached, the nodal neck radius becomes equal to the orginal radius of the material simulating full densification of the material.

Section $V$ controls the printing of nodal temperatures and neck sizes. The printing time interval(PT) was selected in Section I. In addition, the temperature and neck size are printed at the point in time when there is no material remaining in the stationary pool and when all material has completely densified. At the end of the simulation, the neck size of each node location is printed showing the amount of neck growth that occured before complete melt had overcome the sintering process.

If mold sticking has not yet occured in the simulation, the program enters Section VI where it calculates the new free-fall section's average temperature for the next time interval. It calculates the temperature energy of the last column of the stationary pool that enters the free-fall zone during the next time interval and adds it to that remaining in the pool after a portion of it re-enters the stationary pool.

The program then rotates the stationary pool nodes
over one column distance simulating one time increment of travel. For example, the values of the nodes in the next to the last column move over to the last column. The nodal values in the second to the last column are shifted to the next to the last column and so on.

Next, the nodes in the first column are then initialized to the temperature of the material that had just exited from the free-fall zone and re-entered the stationary pool. The computer simulation then returns to Section IV and begins another iteration process for the next time interval.

If there is material sticking to the mold, the program by-passes Section VI and goes on to Section VII where it determines the amount of material in the last column that will stick to the mold wall and the amount that enters the free-fall zone during the next time interval.

This routine then sets up new nodal columns to simulate the stuck material. Since the time interval(TS) decreases as the cross-section of the stationary pool decreases, the simulation maintains, for each stuck column, its width(DC), position in the mold(PS), the number of nodes(N3), its height(H2), the distance between the last two nodes of the column(H3), and the nodal neck sizes (ANECK). It must be noted that the stuck height(H2) is
very seldom an integer of the nodal distance(DX), therefore all nodal distances are the length $D X$ except between the last two nodes which has a distance larger than DX making up the extra distance.

This section then calculates the energy of the material in the stationary pool column not sticking but entering the free-fall zone.

Section VIII simulates the rotation of the mold when sticking has occured. The stationary pool and then the stuck material is rotated one column width in the same manner described in the discussion of Section VI. The values of the stuck column's width(DC), number of nodes(N3), height(H2), and the distance between the last two nodes(H3) is also shifted along with the column. The position(PS) is updated to reflect its new position in the mold.

Section IX checks if the rotation causes any of the stuck material to re-enter the stationary pool. If no re-entering occurs, the computer simulation goes to Section X. If re-entering occurs, the simulation determines the portion of column width(EN) that re-enters the pool as well as the portion remaining outside(OT).

The portion of the column remaining outside (OT) is combined to the adjacent column to make one equivalent column having a combined average nodal temperatures with a width equal to OT plus the width of the adjacent column.

The nodal temperatures and neck size of the stationary pool's first column is assigned the same values as the re-entering material(EN). Since the nodal distance of the last two nodes is usually larger than the normal nodal distance (DX), the last node is replaced by two nodes. The next to the last node will have the same neck size and temperature as the last node of the stuck material re-entering the pool. The remaining material is to be combined with the re-entering powder from the free fall zone to form an equivalent node. Additional nodal subvolumes are added representing the re-entering powder from the free fall zone adding an additional height equal to the penetration thickness.

If there is not enough mass in the stationary pool to allow adding the height of a penetration thickness, then fewer subvolumes are used having an equivalent cross sectional area equal to that of the remaining pool material. The last node will then be modeled as an adiabatic edge. The temperatures and neck radius nodal values of these added subvolumes are equal to that of the re-entering material from the free-fall zone.

Section $X$ calculates the new angle theta(TH) based on the amount of material in the stationary pool zone. The program then returns to Section III and starts the iteration again for the next time increment.

When the amount of material in the stationary pool becomes zero, angle theta (TH) becomes zero. Therefore, the rotational simulation portion of this program is no longer needed making this simulation a simple onedimensional heat transfer simulation having the number of columns equal to that of the number of columns of stuck material. The simulation then consists of the iteration process in Sections III and IV until the temperature of the last nodes reaches or exceeeds the complete melt temperature(SM). At that point, the simulation calls SUBROUTINE PRINT and then terminates the simulation by printing the neck size of each node height level at the point when complete melt had overcome the sintering process.

To simulate heating by oven convection in rotational molding, a few significant modifications were made to the original nodal model and to the computer program earlier developed (as listed in Appendix F.).

The original model assumed a temperature-time relationship which was described by a polynomial equation for the mold's internal surface, NODE l, that is in direct contact with the powdered material. The temperature values of the other nodes are then calculated during each iteration based on the temperature time history of NODE 1 .

With oven convection, two additional nodes are used in the model accounting of the effect of the mold itself and the resistance between the heated air and the mold's surface. Figure 35 shows the position of these extra nodes. The node labled $A$ in the figure represents the oven air and has a temperature value equal to the oven temperature, or:

$$
T(A)=T(\text { OVEN })
$$

NODE B represents the exterior surface of the mold. Its nodal temperature equation (eq 36) is listed in Chapter VI. NODE 1 represents the interior surface of the mold having equation 38 as its nodal temperature equation. The

Figure 35 OVEN CONVECTION SURFACE NODAL CONVENTION

distance between NODE $B$ and NODE 1 is equal to the thickness of the mold.

With the additional nodal equations, there are two additional stability equations, each having a maximum allowable time increment. The computer simulation calculates for each iteration the three maximum time increments and then allows the time increment used to be no greater than the minimum value of the three.

The future temperature values of NODE $B$ and NODEl as well as the remaining nodes are computed for each iteration using the nodal temperature equations developed. Appendix H is the listing of the Rotational Molding Simulation having oven convection.

## XII. DISCUSSION OF RESULTS

Using the computer simulation program earlier developed, a number of runs were performed in order to predict pool depletion and total densification times with variations in temperature-time histories, rotational speeds, material amount, and thermal properties of the powdered material. The results of the rotomolding simulations as well as results of the Vanderbeck and Throne/Ahdout simulations are compared in Table 2 to the actual experimental results performed by Vanderbeck(30). The simulations reported in Table 2 used the polynominal temperature-time equations discussed earilier.

Section I of Table 2 compares each simulation at various power input levels having 50 grams of powdered material in the mold cavity. The different power inputs correspond to a particular mold temperature-time history as described by Vanderbeck(30). Results show a 40\% to 55\% error in the Vanderbeck simulation, a $-3.6 \%$ to $-10.2 \%$ error in the Throne/Ahdout simulation and a $-2.1 \%$ to $-11.1 \%$ error in the Rotational Mold Simulation developed here.

In Section II and III, the thermal diffusivity of the powder used in Section I was decreased by $20 \%$ and increased by $20 \%$ respectively. Simulations using the 500 watt and
TABLE 2
POOL DEPLETION TIMES


800 watt temperature-time histories resulted in the predicted pool depletion times shown for both the Throne/Ahdout and the Rotational Mold Simulations. For the 20\% change, the Rotomolding Simulation showed a change between $0.8 \%$ and $1.6 \%$ while the Throne/Ahdout Flat Plate simulation resulted in a $1.6 \%$ to $3.3 \%$ change.

Section IV and $V$ in Table 2 show the results of varying the rotational speed of the mold. The Vanderbeck simulation yielded a difference of $27 \%$ to $42 \%$ between the predicted and actual experimental results. Because the Throne/Ahdout simulation is a flat plate analysis with no rotational parameter, the pool depletion times are constant for power input reguardless of the actual rotational speed. The Throne/Ahdout simulation produced a $4.8 \%$ to $11.2 \%$ difference between actual and experimental results. The Rotational Mold Simulation had a $11.2 \%$ to $16.7 \%$ difference.

Note that the Rotational Mold Simulation developed here predicts a time difference of 10 seconds between the 10 RPM and 25 RPM pool depletion times of the 800 watt power input while the actual experimental difference was 9 seconds. For the 700 watt power input, the predicted and actual experimental time differences were identical with 10 seconds.

Section VI is the actual and predicted pool depletion
time results of a mold containing 84.5 grams of powder. The Throne/Ahdout has a difference ranging from 45.1\% to 49.1\%. The Rotomold Simulation produced a $0.0 \%$ to $7.1 \%$ difference.

Table 3 is a comparison of pool depletion and total densification times with the Rotomold and Throne/Ahdout simulation containing a 50 gram charge using oven convection coefficients in lieu of the polynomial temperature equations. Using a typical oven temperature of 371 degrees Centigrade, an aluminum mold thickness of 0.655 cm yielded results showing a close agreement between the two simulation for typical air convection coefficients.

The previous comparison of the 50 gram charge simulations show that the Rotomold Simulation developed here and the Throne/Ahdout Simulation will predict pool depletion and total densification times within a satisfactory tolerance. The Vanderbeck Simulation however, showed between $27 \%$ and $57 \%$ error. It also showed at those power inputs an insensitivity to changes in thermal conductivity.

It is confusing however, that for the 50 gram charge simulations, the Throne/Ahdout Simulation which is a flat plate simulation with no rotation or thermal mixing has as

TABLE 3

## DEPLETION AND DENSIFICATION TIMES

| $\begin{array}{r} \text { Convection } \\ (\mathrm{B} / \mathrm{Hr}-\mathrm{Ft} 2-\mathrm{F}) \end{array}$ | n Coefficient <br> (J/sec-cm2-K) | Simul | ```e on (sec) Densifi- cation``` | Throne/Ah Simulation Depletion | ```dout (sec) Densifi- cation``` |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $1.1356 \mathrm{E}-03$ | 442 | 577 | 431 | 572 |
| 4 | 2.2712E-03 | 241 | 300 | 232 | 295 |
| 6 | $3.4068 \mathrm{E}-03$ | 174 | 208 | 167 | 204 |
| 8 | $4.5424 \mathrm{E}-03$ | 141 | 162 | 134 | 158 |
| 10 | 5.6780E-03 | 126 | 135 | 125 | 135 |
| Material Stick Temperature (deg C) $=110$ <br> Material Melt Temperature (deg C) $=138$ <br> Oven Air Temperature (deg C) $=371.0$ <br> Amount of material in mold (grams) $=50.0$ |  |  |  |  |  |
| Properties of Aluminum Mold |  |  |  |  |  |
|  | Thermal Cond Density (Kg/ Specific Hea Thermal Diff | ductivi <br> /Cm3) <br> at ( $\mathrm{J} / \mathrm{K}$ <br> fusivit | $\begin{aligned} & (\mathrm{J} / \mathrm{cm}-\mathrm{sec} \\ & 2.707 \mathrm{E}-03 \\ & \mathrm{~K})=8.70 \\ & (\mathrm{Cm} 2 / \mathrm{sec}) \end{aligned}$ | $\begin{aligned} & -K)=2.02 \\ & 85 \mathrm{E}-02 \\ & =8.588 \end{aligned}$ |  |

good as or even better results then the Rotational Mold Simulation developed. It would seem logical that the flat plate simulation temperature profile would show a maximum temperature at the mold's surface dropping rapidly to a minimun temperature at the inner surface. The difference between the minimum and maximum temperature would then increase with time causing a dramatic increase in predicted pool depletion and densification time for the powdered material. The flat plate simulation did not predict this. In fact, its predictions for the temperature profiles were very close to the Rotomold Simulation.

This seeming discrepency can be explained. In the Vanderbeck experiments, the pool depletion times for 500 through 800 watts are shown(Table 3) equivalent to an oven convection coefficient of approximently l.137E-03 to $2.271 \mathrm{E}-03 \mathrm{~J} / \mathrm{Sec}-\mathrm{cm} 2-\mathrm{K}(2.0$ to $4.0 \mathrm{~B} / \mathrm{hr}-\mathrm{ft} 2-\mathrm{F})$. The ratio of the convection coefficient to the thermal conductivity of the powder divided by the thickness ( $h /(k / L)$ ) determines the temperature profile. Krieth (55) states that if the ratio, $h /(k / L)$ is less than 0.1 then the temperature throughout the material can be assumed constant for transcient thermal analysis. In other words, the heat input is so slow that it allows the material to distribute its internal energy evenly.

Pre-distributing 50 grams of powdered material around the mold will result in a material height of 0.4 cm , the ratio, $h /(k / L)$ varies between 0.22 and 0.46 . Although these values are not below 0.l, they are close enough to produce similar results. In addition, because these ratios are low, a $20 \%$ change in either direction of the thermal conductivity (as was reported in Section II and III of Table 2) would produce very little change in the pool depletion and total densification times.

With 80 grams of material charge, the material height (the value L) becomes 1.67 cm increasing the ratio four fold. Section VI of Table 2 shows the dramatic change in prediction times of the Throne/Ahdout simulation as compared to the actual results. The error now increases to a range of $45 \%$ to $49 \%$.

By increasing the thermal conductivity, Table 4 shows again the limitation of the Throne/Ahdout simulation. Notice that as the conductivity, $k$, becomes smaller (making $h /(k / l)$ larger), the deviation between the Rotational Molding Simulation and the Throne/Ahdout Simulation becomes extremely large.

Similar results are found if the value of the coefficient of convection becomes large as may occur in

## DEPLETION AND DENSIFICATION TIMES

AS A FUNCTION OF THERMAL CONDUCTIVITY

| Thermal | Thermal | Ribe |  | Throne/Ahdou |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Conduc- | Diffu- |  |  | lation) |  |
| tivity | sivity | (sec) |  |  |  |
| (J/Cm-sec-K) | (cm2/sec) | Deplet | Densif. | Deple | Dens |
| $1.99 \mathrm{E}-02$ | 1.7E-02 | 95 | 84 | 116 | 113 |
| $1.99 \mathrm{E}-03$ | 1.7E-03 | 126 | 126 | 135 | 135 |
| $1.99 \mathrm{E}-04$ | I. $7 \mathrm{E}-04$ | 154 | 245 | 200 | 258 |
| 1.99E-05 | 1.7E-05 | 215 | 873 | 418 | 919 |
| Material Stick Temperature (deg C) = 110 |  |  |  |  |  |
| Material Melt Temperature (deg C) $=138$ |  |  |  |  |  |
| Oven Air Temperature ( $\operatorname{deg} \mathrm{C})=371.0$ |  |  |  |  |  |
| Amount of material in mold (grams) $=50.0$ |  |  |  |  |  |
| Oven Convection Coefficient (J/cm2-sec-K) $=5.678 \mathrm{E}-03$ |  |  |  |  |  |
| Properties of Aluminum Mold |  |  |  |  |  |
| Thermal Conductivity (J/cm-sec-K) $=2.025$ |  |  |  |  |  |
| Density ( $\mathrm{Kg} / \mathrm{Cm3}$ ) $=2.707 \mathrm{E}-03$ |  |  |  |  |  |
| Specific Heat (J/Kg-K) $=8.7085 \mathrm{E}-02$ |  |  |  |  |  |
| Thermal Diffusivity ( $\mathrm{Cm} 2 / \mathrm{sec}$ ) $=8.588$ |  |  |  |  |  |

radiant and forced Fluid heating of the mold. Table 5 illustrates the effect of changing convection coefficients by factors of magnitude. Again, high errors (19\% to $34 \%$ ) occur in the prediction of the pool depletion and densification times.

In summary, the Throne/Ahdout Simulation is accurate only when using certain combinations of the simulation parameters, otherwize large errors will occur in pool depletion and total densification predictions. The Rotational Molding Simulation developed in this dissertation does predict accurate times for the pool depletion and densification as was verified by the experimental results of Van der Beck(30). It models closely the material flow, mold heating of the powdered material, rotation of the stationary pool and material melted to the mold wall, and the thermal and physical mixing during the free-fall zone.

TABLE 5

DEPLETION AND DENSIFICATION TIMES
AS A FUNCTION OF CONVECTION COEFFICIENT

| Convection | Depletion Times (sec) |  |  | Densification |  | Times |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Coefficient | Ribe | T/A | \% | Ribe | T/A |  |
| (J/Cm2-S-K) | Sim. | Sim. | Diff. | Sim. | Sim. | Diff. |
| $3.407 \mathrm{E}-03$ | 331 | 445 | 34 | 355 | 468 | 32 |
| $3.407 \mathrm{E}-02$ | 122 | 151 | 24 | 137 | 178 | 30 |
| $3.407 \mathrm{E}-01$ | 101 | 121 | 20 | 109 | 130 | 19 |
| 3.407 | 99 | 118 | 19 | 105 | 125 | 19 |

```
Material Stick Temperature (deg C) = 110
Material Melt Temperature (deg C) = 138
Oven Air Temperature (deg C) = 371.0
Amount of material in mold (grams) = 80.0
Thermal Diffusivity (Cm2/sec) = 1.7E-03
Properties of Aluminum Mold
Thermal Conductivity (J/cm-sec-K) = 2.025
Density (Kg/Cm3) = 2.707E-03
Specific Heat (J/Kg-K) = 8.7085E-02
Thermal Diffusivity (Cm2/sec) = 8.588
NOTE: \(T / A=\) Throne/Adhout
```


## XIII. CONCLUSIONS AND REMARKS

This dissertation has presented an investigation of the densification process in rotational molding of a thermoplastic powder in a cylindrical cavity.

First, a thorough literature survey was performed to review past works of rotational molding analysis, as well as the study of sintering, the effects of temperature on viscosity and surface tension for polymeric material, a review of thermal conductivity equations of composite materials and a review of other areas that would assist in the analysis of the densification process in rotational molding.

Next, an in depth study of the mass flow in a rotating cylinder together with an analysis of the heat transfer during rotational molding were performed to attain the understanding necessary to model the densification process.

The densification process in rotational molding was then modeled mathematically using Nodal Analysis. Nodal temperature equations were derived for typical oven convection heating and a special case where the mold's temperature-time history is known. The special case was modeled to provide a comparison and validation of the
simulation developed with previous experimental research.

The research for this dissertation included an exhaustive investigation and analysis into the đensification process (neck formation) by use of Scanning Electronic Microscope(SEM) photography. Because of the large depth-of-field of the SEM, photographs which were almost impossible to obtain before, allow the analysis to be performed. Based on this analysis, the intermediate physical property correlations needed in the nodal temperature equations were derived.

Lacking an agreement between results of published composite thermal conductivity equations, a hybrid experimental procedure coupled with a computer simulation was devised to determine the actual initial thermal conductivity of the powdered polymeric material.

Finally, a computer program was written to simulate the heating and densification during the rotational molding process. Results showed agreement with actual rotational molding experimental findings. In addition, results of other simulations were compared showing their shortcomings.

It is important to include in this summary a discussion of one logistical drawback in the use of this simulation; that being the amount of computer time required.

Because of the intricate modeling, the simulation will require from 800 to 6000 seconds of computer time. This equates to a maximum of $\$ 650.00$ per simulation when used on a Control Data Corp. mainframe system(CDC 6500).

If, because of cost, the use of the Rotational Molding Simulation becomes prohibitive, modifications to the Throne/Ahdout Simulation will improve its accuracy. A complete examination and modification to the Throne/Ahdout Simulation is beyond the scope of work for this dissertation. However let it be noted that the major modification would encompass a thermal mixing routine to simulate the free fall thermal/physical mixing zone for the powdered material that has a temperature below the stick temperature.

Since miminal computer cost was not an original requirement, this dissertation has performed and completed all its objectives. It has investigated the heating and densification portion of the rotational molding process. Included, was an analysis of the heat transfer, mass flow and neck formation colmenating into a valid and proven simulation.

NOMINAL PHYSICAL PROPERTIES OF MARLEX LX470 (POLYETHYLENE)

| PROPERTY ${ }^{1}$ | ASTM | Units | Value |
| :---: | :---: | :---: | :---: |
| Density | D1505 | $\mathrm{g} / \mathrm{cm} 3$ | 0.943 |
| Melt Index | D1238 | $\mathrm{g} / 10 \mathrm{~min}$ | 3.0 |
| ```Flow Index, CIL, 190C 10.4 MPa}``` |  | $\mathrm{g} / 10 \mathrm{~min}$ | 3.0 |
| Brittleness Temperature | D746 | Deg. C | -118 |
| Specific Heat @ 90C(ref 58) |  | cal/g-C | 0.55 |
| ROTATIONAL MOLDED PROPERTIES 3 |  |  |  |
| ESCR, Condition A, F50 | D1693 | h | 200 |
| Tensile Strength at Yield 2" (50.8 mm) per min. | $\begin{aligned} & \text { D638 } \\ & \text { Type IV } \\ & \text { Spec } \end{aligned}$ | MPa | 22.1 |
| Elongation <br> 2" (50.8 mm) per min. | $\begin{aligned} & \text { D638 } \\ & \text { TYPE IV } \\ & \text { Spec } \end{aligned}$ | \% | 350 |
| Flexural Modulus | D790 | MPa | 965 |
| Impact ARM Standard ${ }^{4}$ at -28.9 deg C |  | J | 68 |

1 physical properties reported herein were determined on compression molded speciments prepared in accordance with Procedure $C$ of ASTM Dl928 (Ref 57).

2 Data obtained using a gas extrusion plastometer based on design by Canadian Industries, Ltd., with a die having an orifice diameter of 0.49 mm and a land length of 4.48 mm .

3 Physical properties are based on parts molded at optimum conditions (Ref 57).

4 Ten pound dart with 0.5 inch point in center of 0.125 inch thick unsupported 3.5 inch diameter area.

## APPENDIX A (continued)

U. S. STANDARD SIEVE. SIZE DISTRIBUTION OF MARLEX LX470

Measured Specific Gravity of Powder $=0.5096$ Specific Gravity of Resin $\quad=0.9430$

Void Fraction of Air $=45.96 \%$

## APPENDIX B

PHYSICAL PLASTIC PROPERTIES OF ONE-EIGHT DIAMETER
SPHERES MADE OF POLYETHYLENE AND ACRYLIC

| PROPERTY | UNITS | POLY- <br> ETHYLENE | ACRYLIC |
| :---: | :---: | :---: | :---: |
| Impact Strength, Notched Izod | Ft-lb/in | $1-10$ | 0.4-0.6 |
| Tensile Strength | PSI x 1000 | 2.5-5 | 7.9 |
| Tensile Modulus | PSI x 1000 | $85-160$ | $350-450$ |
| Thermal Conductivity | Cal/cm2/sec/ <br> C/cm x 10000 | 8 | 4-6 |
| Specific Gravity |  | 0.94-0.96 | 1.18-1.19 |
| Elongation | \% | 5-10 | 2-10 |
| Flexural Strength | PSI x 2000 | $2-3$ | $14-16$ |
| Flexural Modulus | PSI x 1000 | 90-150 | 350-450 |

## APPENDIX C

PHYSICAL PROPERTIES OF MICROSPHERES COMMERICALLY NAMED
"ELVACITE" ACRYLIC RESINS 2021 by DuPont

```
Density(resin) 1.196 Kg/M3
Glass Transition Temperature }100\textrm{Deg C
Tukon Hardness, Knoop No. }2
Tensile Strength (23 Deg C., 50% RH) 106 MPa l5kPsi
Elongation at Break (23 Deg C., 50% RH) 4%
```

U. S. STANDARD SIEVE SIZE DISTRIBUTION

Sieve Percent of material
Size Inches mm $\quad$--
35
40
45
50
60
80
100
120
170
200
230
270
PAN

| 0.0232 | 0.590 |
| :---: | :---: |
| 0.0197 | 0.500 |
| 0.0164 | 0.420 |
| 0.0138 | 0.350 |
| 0.0117 | 0.297 |
| 0.0088 | 0.250 |
| 0.0070 | 0.177 |
| 0.0059 | 0.149 |
| 0.0049 | 0.125 |
| 0.0035 | 0.088 |
| 0.0029 | 0.074 |
| 0.0025 | 0.063 |
| 0.0021 | 0.055 |


| Percent of material |
| :---: |
| stopped by sieve |
| 0.48 |
| 0.16 |
| 0.18 |
| 0.27 |
| 3.26 |
| 18.87 |
| 65.58 |
| 4.83 |
| 3.28 |
| 1.92 |
| 0.92 |
| 0.10 |
| 0.08 |
| 0.07 |
| -100.00 |

Measured Specific Gravity of Powder $=0.730$ Specific Gravity of Resin $=1.196$

Void Fraction of Air $=38.96 \%$

## APPENDIX D

## FLAT PLATE SIMULATION

```
90 REM SLAB TEMPERATURE TIME HISTORY SIMULATION
95 DIM T(30),TN(30),TT(15),TP(15)
97 OPENl,4,2
100 AL=6.200E-03:REM FT2/HR
105 PRINT#1,"AL = ";AL
110 AL=AL*0.04 :REM IN2/SEC
120 DX=0.09375 :N=17 :REM DX IN INCHES
125 TM=0.0 :NN=0
130 TS=DX@ 2/(2*AL)
140 M=INT(60/TS)+1
150 DT=60/M
160 FORJJ=1TOll:READ TT(JJ):NEXTJJ
170 FORJJ=1TO11: READ TP(JJ):NEXTJJ
180 FORI=1TON:T(I)=TP(I):NEXTI
200 TM=TM+DT:NN=NN+1
210 GOSUB 1000
220 T(N+1)=T(N-1)
230 FOR I=2TON
240 TN(I)=T(I)*(1-2*DT*AL/DX\odot2)+DT*AL/DX@2*(T(I-1)+T(I+1))
245 IFTN(I)<T(I)THEN STOP
250 NEXTI
260 FORI=2TON:T(I)=TN(I):NEXTI
265 PRINTTM;T(9)
270 IF INT(NN/M)=NN/M THEN PRINT#l,TM;TM/60,T(1),T(9),T(16)
280 IF TM/60>60 THEN PRINT#l:CLOSEl:STOP
290 GOTO200
1000 REM SUBROUTINE TO DETERMINE IST NODE TEMPERATURE
l010 FOR JJ=2TOlI
1020 IF(TM/60 <TT(JJ)) THEN 1030
1025 NEXTJJ
1030 RA=(TM/60 -TT(JJ-1))/(TT(JJ)-TT(JJ-1))
1035 IFRA=OTHEN T(l)=TP(JJ-1):RETURN
1040 T(l)=TP(JJ-1)+RA*(TP(JJ)-TP(JJ-l))
1050 RETURN
2000 DATA 0,2,5,10,15,20,30,40,50,60,100
2010 DATA 64,67,86,127,153,171,195,210,220,227,227
```

APPENDIX E
FLOW DIAGRAM FOR THE ROTATIONAL MOLDING SIMULATION



## APPENDIX F

ROTATIONAL MOLDING SIMULATION




$52=80 * \mathrm{CJS+74+z}=0+* F A N(H * 75+2 \cdot 6)$
$A T=0.5 *(S 1+52) *(S 1-S 2) * S I N\left(W^{*} T S\right)+0.5 * R C * R O(W * T S-S I N(* * T S))$
$A S=(A-0.5+* 0 x+0 t$
IF（AS．GT．AT）AS＝AT

$N=N M A X$
$I F L A G=1$
540 CONTINUE
OO～55－T $\mathrm{I}=\mathrm{I}$ ，NS
$T(I, N+1)=T(I, Y-1)$
ANECKけI，N＋IT＝ANECKII，N－I）
551 cONTINUE
56－CONTINげ
NNAX $=N+1$
N
c
C－＊＊＊＊SECTIONIV
COMPUFE－NOOAL FEMPERATUFES ANA NEEK RAOIUS
58 S $4 T=A * T M / 60.0+3 *(T M / 63.0)+* 2+Y Y$
एण ठटण $\Gamma=1$ ，NE
G NOOAL TEMPERATURE EQUATICNS FOR STATIONAFY POOL
$T(1,1)=4 T$
$T N(1)=4 T$

1 BNECK（1））
मNECK（1，1）＝3NECKけI
$00630 \mathrm{~J}=2, \mathrm{~V}$

1 ANECK（I，J＋1），BNECK（J））

10A／C2＊T（I，J＋1）
Oु．CONTINUE
$00540 \mathrm{JJ}=\mathrm{i}, \mathrm{N}$
$T(I, J J)=T N+5 J$
IF（ $(J J, E Q \cdot N E=$ ）．AND．（TN（JJ）．GT．SM：）ANEK（NEC）$=$ GNECK（JJ）
IFt JJ．EQ．NECT．ANO．TTNT与J）．GF：SMM NEC＝NECFI
IF（TN（JJ），GT．SM）BVECK（JJ）＝RAD

54：CONTINUE
6ても CONTFNUE
IF（CT．LT．N2） 50 TO 890
C—— VOOAL FEMPERATURE EQUATIENS FOR－SFUEK NATERIAL
$00880 \mathrm{I}=\mathrm{NZ}, \mathrm{CT}$
「だけ二小
CALL CONST（C1，C2，CZ，DB，DA，MT，MT，MT，ANECK II，1），ANEGK（I，2），
1 BNECK（1）
$\operatorname{ANECK}(I, 1)=3$ NECK（1）

IF（N3（I）．EQ． 11 50 TO 780
－JVAL $=$ N3－I +
$00770 \mathrm{~J}=2$ ，JTVAL

1 ANECK（I，J＋1），BNECK（J））
 1DA／C2＊T（I，J＋1））
TTE CONTINUE

| ogram | R485 | $74 / 825$ | DPT=O, ROUNL= $4 / \mathrm{S} / \mathrm{M} /-\mathrm{D},-\mathrm{OS}$ | FTN 5.1+501 | 129 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CALL CONST(C1,C2,C3, DB, DA, T(I, J), T(I,J-1), T(I,J+1), ANEGK(I,J), |  |  |  |  |  |
|  |  |  |  |  |  |
| 32 J CONTINUE |  |  |  |  |  |
| CALL CONST(C1, [2, C3, OB, DA, T(I,J),T(I,J-1),T(I,J), ANECK (I, J), |  |  |  |  |  |
| $B B=O B * T S /(H 3(I) *(G 3 / O X) * * 2 *(H 3(I)-D X / 2))$ |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| $00870 \mathrm{JJ=1.J}$ |  |  |  |  |  |
| IF((JJ.EQ.NEC).AND. (TN(JJ).GT.SM)) ANEK(NEC) = SNECK(JJ) |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| 97: CONTINUE |  |  |  |  |  |
|  |  |  |  |  |  |
| CON |  |  |  |  |  |
| IF (IT (NC, N).JE.SS).AND. (IREM.EQ.1).AND.(N.EQ.NMAX)) GO TO 9 G |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| ***** SECTIONV ***** |  |  |  |  |  |
| PRINT RJUTIVE |  |  |  |  |  |
| 991 I3 $=0$ |  |  |  |  |  |
| GALE PRFATF,L-K,F3$I 3=1$ |  |  |  |  |  |
| IZ-0 |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  | -- | . NMAX ${ }^{\text {a }}$ | GOTO992 - - - . - - - - - - - |  |  |
| $\operatorname{IREM}=0$ |  |  |  |  |  |
| $\begin{aligned} & A F=0.0 \\ & A A\end{aligned}=0.0$ |  |  |  |  |  |
| FH $=0.0$$L K=L K-1$ |  |  |  |  |  |
|  |  |  |  |  |  |
| 29 CONTINUE |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| 3: FORMA T(10X,I5,5X,F10.6) <br> 3こ GONFINUE |  |  |  |  |  |
| 3- CONFINUE---- |  |  |  |  |  |
| GONTINUEITM I TMI |  |  |  |  |  |
|  |  | 4. EQ-Ol |  |  |  |

QGPAM R485 $\quad 7+1825 \quad 0 P T=0, R O U N D=4 / S / M /-D,-D S \quad F T N 5.1+601 \quad 130$



| ROGRAM R485 | 74/325 ODT $=0$, ROUND $=4 / 5 / 4 /-3,-D S$ | FTN 5.1+6.1 | 132 |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| $J K=0$$J K=N 3(J I-1)+1$ |  |  |  |
|  |  |  |  |
| 185] $\begin{aligned} & \text { IF } \\ & J K=J K-1\end{aligned}$ |  |  |  |
|  |  |  |  |
| G0 To 1550 |  |  |  |
|  |  |  |  |
|  |  |  |  |
| IF ${ }_{\text {It }}$ |  |  |  |
| D0 $1670 \mathrm{I}=2$, JK |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| AK $=J K+1$$A 3=O C(J I-1) *(H 3(J I-1)-0 \times / 2)$ |  |  |  |
|  |  |  |  |
|  |  |  |  |
| IF (JL.EQ. 1) $A 4=0$ T* ( +3 (JI) $-1.5 * D \times$ ) |  |  |  |
| IF (JL.EQ.1) QYECK(JI, JK $)=A N E C K(J I, N 3(J I)+2)$ |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| Ste |  |  |  |
| PS(JI-1) $=$ PS(JI) |  |  |  |
| $-1=-J I-1$$60 ¢ 01550$ |  |  |  |
| 176t PSTJT=1177 CONTINUE |  |  |  |
|  |  |  |  |
| $\mathrm{F}(1,1)=47$$\mathrm{~V} 4=\mathrm{NS}(\mathrm{JI})+1$ |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| $\operatorname{ANECK}(1, J)=\operatorname{ANECK}(J I, J)$ |  |  |  |
| 1. $\mathrm{RM}=\mathrm{H2} 2(J I)-\left(\mathrm{N}_{4}-0.5\right) * 0 \mathrm{X}$ |  |  |  |
|  |  |  |  |
| $\mathrm{N} 4=\mathrm{N} 4+1$ |  |  |  |
|  |  |  |  |
| IF(IFLAJ.EG.1) 30TJ 1870 |  |  |  |
|  |  |  |  |
|  |  |  |  |
| NNTP $\mathrm{NG}+\mathrm{NL}+1$ |  |  |  |
| $001860 \mathrm{I}=2, \mathrm{~N}$ |  |  |  |
|  |  |  |  |






```
WUINE PRINT
                74/825 OPT=3,ROUND= A/ S/M/-D,-OS
                FTN 5.1+601
```



```
    NTN=N3(KJ)+2
    90-20-4-J=士口HFY
    00 203 II=KJ,KJ1
    NFMP=N3+IET+2
    IF(NTMP. EQ.NTN) GO TO 203
    IOIFF=NFN-NFMP
    DO 202 LL=1,IJI=F
    FHEIqNFMP+t5+=0.0
        202 CONTINUE
        2O2 GONTINUE
            IF(K.EQ.0) WRITE(6,102)J,(T(I,J),I=KJ,KJ1)
```



```
        204 CONTINUE
        20%-CONFINUE
        30% RETURN
            ENG
```



```
        DIMENSION N3(200)
    G
    C
    C
        COMMON SS,SM,AL,AM,DX,N,IM,AA,AT,AD,TH,DL,N2,CT,NC,N3,RAD,TS
        A1=0.5*+A+TB+A*A+
        C1=RAOX(ANE)*DX
        G2=RAOX+ANAJ*OX
        C3=RAOX(A1)*DX
```



```
        IF(ANB/2AD.GE.0.5) DELB=0.5*SU(T1)*TS/(VIS(T1))
        OS=CONOtANEH+ARHO&A1)*EPHATH+
        DA=CONO(ANA)/(RHO(A1)*CP(A1))
        BNK=ANB+0ELG
        IF(BNK.GT.RAD) 3NK=RAD
        IF&日EL马.GF:G:F) FEFHRN
        WRITE (6,1O)TM,ANB,ANA,BNK,RAD,TS,SU(T1),VIS(T1),DELB
```



```
        २ETURN
        ENQ-
```

```
CIIN CONL TH/825 OTT=3,ROUND= A/ S/ M/-D,-OS FTN 5.1+601 I36
```



FUNGFION GONOTM
DIMENSION N3（200）
C CONDUCTIVITY FUNCTION（JCULE／（GM－SEC－DEG K）

```
C CI = THERMAL COVDUCTIVITY AT TEMPERATURE SS
```

C TE THERMAL CONOUCTIVITY AT TEMPERAFURE SM

$C_{1}=1.99 E-03$
$C 2=4.9325 E-03$
$X=Y / R A D$
IFTX.GT. J.OJ~OTO 10
$C O N D=C 1$
GO 701
$10 \operatorname{IF}(X . G T \cdot 1.0) 30 T 020$
COMO=CIFX* (Cこ-で)
GO TO 1
2TーCONO=TZ
1 CONTINUE
RETURN
END
FUNEFチOY R+4Ot+
DIMENSION N3 (200)
C
C DENSITY FUNGTI ON ( $\angle G / C M * * 3$ )
C
C $\quad$ R1=DENSITY AT SS
C- RC=TENSITYAT SH
C

R1=5.0937E-04
ママニタ.805 8E-04
$x=Y / R \Delta D$

$2 \mathrm{HO}=\mathrm{Q} 1$
GOTOI
1. IF (X.GT. 1.0) $30 T 020$
$R H 0=R 1+\left(R-Z-\right.$ R1 $^{2} * x$
GO TO 1
2T-RHOR 2
1 continue
RETURN
END














TEMPERATURE DF STUCK MATERIAL


```
NECX RADIUS DF STUCK MATERIAL
```

    \(\begin{array}{clll}\text { AREA } & 10 & 11 & 12 \\ 1 & 00003 & 00003 & 00004\end{array}\)
    

TEHPERATURE OF STUCK MATERIAL

| AREA | 10 | 11 | 12 | 13 | 14 | 15 | 16. | 17 | 18 | 19 | 20 | 21 | 22 | 21 | 24 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 124.010 | 124.018 | 124.018 | 124.018 | 124.018 | 124.010 | 124.018 | 124.018 | 124.018 |  |  |  |  |  | 124.018 |
| 2 | 122.109 | 122.089 | 122.052 | 122.059 | 122.037 | 122.02 | 122.020 | 122.020 | 124.018 | $\begin{aligned} & 124.018 \\ & 121.987 \end{aligned}$ | $\begin{array}{r} 124.018 \\ 121.976 \end{array}$ | 124.018 121.968 | 124.018 121.969 | $\begin{aligned} & 124,018 \\ & 121.950 \end{aligned}$ | $\begin{aligned} & 124.018 \\ & 121.941 \end{aligned}$ |
| 3 | 120.352 | 120.303 | 120.213 | 120.219 | 120.274 | 120.154 | 120.135 | 120.134 | 120.093 | 120.074 | 120.054 | 120.040 | 120.045 | 120.013 | 1219.94 |
| 5 | 118.725 117.133 | 118.595 116.886 | 118.434 116.665 | 118.432 116.859 | 118.368 116.593 | 118.336 116.364 | 118.308 116.833 | 116.304 116.53 | 110.261. | 118.230 | 118.217 | 118.204 | 118.217 | 118.163 | 110.170 |
| 6 | 118.416 | 128.073 | 114.8.44 | 116.859 114.853 | 116.593 | 216.364 $114.80 n$ | 1164533 | 116.533 114.819 | 116.519 116.813 | 114.018 | 114.894 | 116.491 116.916 | 116.516 116.968 | 116.495 | 116.487 114.900 |
| 7 | 113.607 | 113.268 | 113.105 | 113.154 | 113.176 | 113.219 | 113.259 | 113.329 | 113.468 | 113.513 | 113.559 | 113.615 | 123.697 | 113.739 | 113.791 |
| 8 | 111.921 | 111.699 | 112.660 | 111.777 | 111.812 | 111.990 | 112.117 | 112.250 | 111.400 | 111.552 | 111.706 | 111.859 | 112.009 | 112.136 | 113.791 112.305 |
| AREA | ${ }_{110} 10.197$ | 110.433 26 | 110.715 27 | 110.945 | 111.153 | 111.378 | 111.612 | 111.852 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $\rightarrow{ }_{\text {AREA }}$ | 124.018 | -124.01 | 27.010 124.010 | 28 124.01 | 24 | 124.018 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 3.9 |
| ,-."2 | 121.934 | 1421.929 | 121.927 | 121.914 | IR1i908 | 1821.909 | 12 |  |  |  |  | 18 | 124.018 | 124.024 |  |
| 3 | 119.987 | 119.974 | 119.980 | 119.960 | 119.953 | 129.941 | 119.930 | 119.923 | 119.938 | 119.932 | 119.930 | 119.924 | 119.912 | 119.868 | 119.879 |
| 4 | 116.162 | 128.153 | 118.169 | 116.152 | 116.151 | 128.143 | 118.137 | 118.136 | 118.160 | 118.167 | 118.175 | 118.180 | 118.181 | 118.156 | 118.180 |
| 5 | 116.472 | 116.478 | 118.511 | 116.509 | 116.525 | 116.532 | 116.540 | 116.356 | 116.604 | 116.620 | 116.645 | 116.686 | 116.686 | 116.685 | 116.726 |
| 6 | 115.002 | 115.034 | 115.093 | 115.115 | 115.158 | 115.188 | 115.221 | 115.261 | 115.332 | 115.369 | 115.419 | 115.467 | 115.518 | 115.566 | 115.627 |
| 7 | 113.042 | 113.908 | 113.998 | 114.050 | 114.105 | 114.166 | 114.229 | 114.299 | 114.393 | 114.456 | 114.534: | 114.615 | 114.708 | 114.828 | 114.909 |
| 8 | 112.482 | 112.647 | 112.814 | 112.920 | 113.054 | 113.182 | 113.315 | 113.452 | 113.8as | 113.695. | 113.835 | 113.997 | 11.493 | 114.486 | 114.909 |
| AREA | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 1s.193 | 11Samaa | 11s. 2 |
| 1 | 124.010 | 124.018 | 124.018 | 124.01: | 124.020 | 124.018 | 124.01勫 | 124.016 | 124.018 | 124.014 | 124.018 | 124.018 |  |  |  |
| 2 | 121.860 | 121.873 | 121.884 | 121.898 | 121.906 | 121.925 | 121.033 | 121.946 | 121.959 | 121.973 | 221.989 | 122.017 |  |  |  |
| 3 | 119.901 | 119.925 | 119.930 | 119.944 | 119.967 | 120.002 | 120.020 | 120.046 | 120.074 | 120.104 | 120.138 | 120.194 |  |  |  |
| 4 | 116.218 | 118.256 | 118.269 | 118.275 | 118.313 | 118.366 | 118.396 | 118.438 | 110.463 | 118.331 | 118.594 | 118.66 |  |  |  |
| 5 | 116.747 | 116.西52 | 116. 162 | 116.893 | 116.950 | 117.023 | 127.067 | 117.126 | 117.190 | 117.25 | 117.336 | 117.451 |  |  |  |
| 6 | 115.713 | 115.807 | 115.808 | 115.867 | 115.946 | 116.039 | 116*098 | 116.174 | 116.259 | 116.352 | 116.453 | 116.582 |  |  |  |
|  |  | 418 14A | 114.474 | 114.627 | 114.774 | $\begin{array}{r} 114.913 \\ n \operatorname{nnn} \end{array}$ | $\begin{array}{r} 113.011 \\ n .000 \end{array}$ | $\begin{array}{r} 115.131 \\ 0.000 \end{array}$ | $\begin{array}{r} 115.276 \\ 0.000 \end{array}$ | $\begin{array}{r} 115.437 \\ 0.000 \end{array}$ | $\begin{array}{r} 125.610 \\ 0.000 \end{array}$ | $\begin{array}{r} 115.806 \\ 0.000 \end{array}$ |  |  |  |




TOTAL AREAICZIC
HEDGE AREAm
.0294
.909
ORIGINAL PEMITRATION DEPTH=
ANGLE(DEGIO 20.060
ARC LEMGTHITIME INCREMEMTE -291A0
NO. OF HEDGES IN POOL 6

## TEMPERATURE HISTORY OF POUOER

$122^{2} \quad 3$
$126.195 \quad 126.195 \quad 126.195 \quad 126.195 \quad 126.195 \quad 126.195126 .195126 .195$ $123.981 \quad 123.981123 .981 \quad 123.993123 .993123 .094123 .944123 .994$ $121.986 \quad 121.986121 .986122 .009122 .009122 .012122 .012122 .012$ 120.209120 .209120 .209120 .245120 .245120 .247120 .247120 .246 $\begin{array}{lllllllll}118.709 & 118.709 & 118.709 & 118.760 & 118.760 & 118.754 & 118.749 & 118.743 \\ 117.483 & 117.483 & 117.482 & 117.545 & 117.537 & 117.486 & 117.444 & 117.440\end{array}$ $117.483117 .483 \quad 117.482117 .545$ 117.537 117.486 117.464 117.440 $\begin{array}{llllllllll}116.580 & 116.564 & 116.536 & 116.571 & 116.521 & 116.311 & 116.248 & 116.187 \\ 115.811 & 115.637 & 115.563 & 115.365 & 115.234 & 114.785 & 114.703 & 114.630\end{array}$ $113.274113 .222113 .181113 .019113 .006 \quad 112.704112 .709112 .708$ 110.264110 .460110 .666110 .765110 .884110 .045110 .939111 .012 109.999110 .039110 .092110 .136110 .182110 .158110 .218110 .266

TEAPERATURE OF STUCK MATERIAL

| AREA | 10 | 11 | 12 | 13 | 14 | 15 | 6 | 7 | 10 | - | 20 | 21 | 22 | 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 126.195 | 126.19 | 126. | 126.195 | 126.195 | 126.195 | 126. | 126.193 | 126.195 | 26.1 | 126.195 | 126.195 | 126.195 | 26.108 | 126.195 |
| 2 | 124.001 | 124.001 | 124.008 | 124.008 | 124.017 | 124.017 | 124.027 | 124.027 | 124.032 | 124.032 | 124.042 | 124.041 | 124,051 | 124.050 | 124.058 |
| 3 | 122.025 | 122.025 | 122.039 | 122.039 | 122.056 | 122.055 | 122.076 | 122.074 | 122.084 | 122.082 | 122.101 | 122.097 | 122.116 | 122.111 | 122.126 |
| 4 | 120.266 | 120.264 | 120.286 | 120.283 | 120.307 | 120.302 | 120.330 | 120.323 | 120.336 | 120.327 | 120.352 | 120.341 | 120.367 | 120.354 | 120.371 |
| 5 | 118.767 | 118.758 | 118.782 | 118.769 | 118.793 | 118.777 | 118.801 | 118.782 | 114.749 | 118.768 | 118.783 | 118.760 | 118.768 | 118.744 | 118.755 |
| 6 | 117.453 | 117.423 | 117.435 | 117.401 | 117.412 | 117.376 | 117.386 | 117.349 | 117.343 | 117.305 | 117.311 | 117.273 | 117.270 | 117.234 | 117.236 |
| 7 | 116.169 | 116.109 | 116.093 | 116.036 | 116.025 | 115.970 | 115.959 | 115.910 | 115.092 | 115.845 | 115.838 | 115.795 | 115.780 | 115.743 | 115.736 |
| 8 | 114.590 | 114.532 | 114. 502 | 114.453 | 114.435 | 114.392 | 114.373 | 114.339 | 114.324 | 114.293 | 114.286 | 114.258 | 114.242 | 114.224 | 114.223 |
| 9 | 112.710 | 112.721 | 112.727 | 112.739 | 112.755 | 114.761 | 112.755 | 112.766 | 112.775 | 112.783. | 112.794 | 112.800 | 112.803 | 112.821 | 112.866 |
| 10 | 111.101 | 111.193 | 111.259 | 111.331 | 111.388 | 111.437 | 111.431 | 111.483 | 111.515 | 111.560 | 111.597 | 111.641 | 111.683 | 111.740 | 111.815 |
| 11 | 110.028 | 110.056 | 110.089 | 110.122 | 110.161 | 110.210 | 110.363 | 110.421 | 110.551 | 110.638 | 110.738 | 110.675 | 111.059 | 111.160 | 111.370 |
| area | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 1 | 126.195 | 126.195 | 126. 195 | 126. 195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126. 195 | 126.195 | 126.195 | 126.195 | 126. 295 | 126. 195 |
| 2 | 124.056 | 124.063 | 124.067 | 124.064 | 124.071 | 124.067 | 124.071 | 124.067 | 124.068 | 124.073 | 124.068 | 124.071 | 124.074 | 124.068 | 124.072 |
| 3 | 122.120 | 122.131 | 122.138 | 122.129 | 122.142 | 122.132 | 122.138 | 122.128 | 122.129 | 122.131 | 122.125 | 122.129 | 122.135 | 122.123 | $122.128$ |
| 4 | 120.356 | 120.367 | 120.373 | 120.357 | 120.371 | 120.354 | 120.358 | 120.340 | 120.338 | 120.348 | 120.328 | 120.3321 | 120.340 | 120.321 | 120.331 |
| 5 | 118.730 | 118.736 | 118.738 | 118.711 | 118.725 | 110.698. | 118.699 | -118.673 | 118.666 | 118.67.2 | 118.667 | 110.650 | 118.657 | 118.635 | 118.645 |
| 6 | 117.200 | 117.206 | 117.202 | 117.170 | 117.182 | 117.150 | 117.147 | 117.120 | 117.108 | 117.113 | 117.087 | 117.091 | 117.100 | 117.078 | 117.091 |
| 7 | 115.702 | 115.126 | 115.718 | 115.689 | 115.683 | 115.655 | 115.655 | 115.635 | 115.625 | 115.633 | 115.615 | 115.626 | 115.641 | 115.629 | 115.649 |
| 8 | 114.208 | 114.286 | 114. 290 | 114.270 | 114.285 | 114.274 | 114.285 | 114.281 | 114.283 | 114.301 | 114.301 | 114.324 1 | 114.352 | 114.357 | 114.392 |
| 9 | 112.868 | 113.027 | 113.051 | 113.064 | 113.090 | 113.103 | 113.133 | 113.153 | 113.175 | 113.211 | 113.233 | 113.274 | 113.321 | 113.345 | 113.387 |
| 10 | 111.082 | 111.165 | 111.272 | 111.349 | 111.428 | 111.515 | 111.602 | 111.666 | 111.759 | 111.861 | 111.931 | 112.0081 | 112.121 | 112.170 | $112.313$ |
| 11 | 111.485 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 10.000 | 1. 0.000 | 112.000 | $0.000$ | $\begin{array}{r} 0.000 \\ \hline \end{array}$ |
| AREA | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | $48$ | 49 | 50 | 51 | 52 | 53 | 54 |
| 1 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126. 195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126. 295 | 126.195 |
| 2 | 124.065 | 124.062 | 124.060 | 124.053 | 124.037 | 124.03\% | 124.030 | 124.027 | 124.021 | 124.013 | 123.995 | 123.9971 | 124.002 | 123.995 | 123.997 |
| 3 | 122.115 | 122.108 | 122.102 | 122.087 | 122.051 | 122.053 | 122.039 | 122.030 | 122.009 | 121.996 | 121.952 | 121.939 | 121.948 | 121.937 | 121.941 |
| 4 | 120.312 | 120.303 | 120.302 | 120.283 | 120.237 | 120.239 | 120.221 | 120.206 | 120.174 | 120.158 | 120.092 | 120.0671 | 120.083 | 120.071 | 120.079 |
| 5. | 118.623 | 118.607 | 118.600 | 118.579 | 118.512 | 118.516 | 118.498 | 118.484 | 118.548 | 118.434 | 118.361 | 118.344 | 118.369 | 118.360 | 118.375 |
| 6 | 117.071 | 117.053 | 117.042 | 117.025 | 116.944 | 116.953 | 116.941 | 116.931 | 116.899 | 116.892 | 116.823 | 116.820 | 116.857 | $116.857$ | $116.083$ |
| 7 | 115.638 | 115.625 | 115.617 | 115.611 | 115.528 | 115.547 | 115.547 | 115.546 | 115.531 | 115.537 | 115.486 | 115.304 1 | 115.557 | 115,371 | 115.611 |
|  | 11S.63 | -.: $10 \times$ | 114.402 | 114.412 | 114.349 | 114.382 | 114.396 | 114.413 | 114.427 | 114.450 | 114.434 | 114.478 | 114.551 | 114.580 | 114.836 |
|  |  |  |  |  |  | - | 112.543 | 113.582 | 113.641 | 113.681 | 113.713 | 113.7891 | 113.884 | 113.926 | 114.000 |
|  |  |  |  |  |  |  |  |  |  |  |  | 1.1442 | 112.592 | 113.651 | 113.742 |


| AREA | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |  | 6. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 |
| 2 | 123.999 | 123.994 | 123.998 | 123. 997 | 124.003 | 121.999 | 123.999 | 124.007 | 124.003 | 126.003 | 124.005 | 124.011 | 124.009 | 124.011 | 124.010 |
| 3 | 121.947 | 121.938 | 121.948 | 121.947 | 121.960 | 121.952 | 121.954 | 121.970 | 121.964 | 121.966 | 121.971 | 121.984 | 121.980 | 121.987 | 121.987 |
| 4 | 120.090 | 120.080 | 120.097 | 120.100 | 120.119 | 120.111 | 120.116 | 120.139 | 120.133 | 120.139 | 120.144 | 120. 153 | 120.150 | 120.151 | 120.155 |
| 5 | 118.394 | 116.388 | 118.415 | 118.422 | 118.451 | 118.446 | 118.459 | 118.493 | 118.491 | 116.503 | 118.516 | 118.536 | 118.538 | 118.550 | 118.560 |
| 6 | 116.912 | 116.915 | 116.949 | 116.966 | 117.004 | 117.007 | 117.027 | 117.052 | 117.058 | 117.079 | 117.103 | 117.138 | 117.148 | 117.175 | 117.196 |
| 7 | 115.653 | 115.668 | 115.706 | 115.735 | 115.787 | 115.002 | 115.836 | 115.876 | 115.892 | 115.925 | 115.963 | 116.015 | 116.036 | 116.083 | 116.116 |
| 8 | 114.692 | 114.722 | 114.753 | 114.797 | 114.866 | 114.893 | 114.947 | 115.002 | 115.029 | 115.075 | 115.128 | 115.200 | 115.231 | 119,287 | 115.335 |
| 9 | 114.067 | 114.109 | 113.497 | 113.593 | 113.715 | 113.774 | 113.892 | 113.990 | 114.040 | 114.118 | 114.213 | 114.343 | 114.398 | 114.526 | 114.614 |
| 10 | 113.820 | 113.874 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0. 000 | 0.000 | 0.000 | - 0.000 | 0.000 | -0.000. | 0.000 | 0.000 |
| AREA | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 |  |  |  |  |  |  |  |
| 1 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 | 126.195 |  |  |  |  |  |  |  |
| 2 | 124.009 | 124.008 | 124.000 | 123.960 | 123.947 | 123.958 | 123.961 | 123.971 |  |  |  |  |  |  |  |
| 3 | 121.99.2 | 121.991 | 121.985 | 121.936 | 121.913 | 121.936. | 121.943 | 121.963. |  |  |  |  |  |  |  |
| 4 | 120.167 | 120.168 | 120.167 | 120.116 | 120.090 | 120.126 | 120.139 | 120.171 |  |  |  |  |  |  |  |
| 5 | 118.582 | 118.589 | 118.595 | 118.556 | 118.536 | 118.586 | 118.601 | 118.654 |  |  |  |  |  |  |  |
| 6 | 117.229 | 117.243 | 117.261 | 117.249 | 117.246 | 117.311 | 117.343 | 117.407 |  |  |  |  |  |  |  |
| 1 | 116.163 | 116.186 | 116.218 | 116.251 | 116.274 | 116.355 | 116.399 | 116.482 |  |  |  |  |  |  |  |
| 8 | 115.395 | 115.428 | 115.476 | 115.573 | 115.632 | 115.727 | 115.783 | 115.886 |  |  |  |  |  |  |  |



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TME SECI- 300.00:
    TOTAL AREA(C2)= 0,00000
    NEDGE AREA- 0,0000
    CRIGIMAL PENITRATION DEPTH* . }203
    ANGLE(DEG)- 
    ARC LENGTH/TIME INCREMENT
```

    TEAPERATURE HISTGRY OF PQUDER
    AREA
1
2
3

132.091132 .091 132.091 134.220 134.220 134.22t 135.224-134422
132.091 132.0.1 132.092 132.092 232.091132 .079132 .079132 .079

120.205 120.206 120. 206 120. 200 120. 207120.172126 .171128 .170
126.566126 .587126 .588126 .590126 .588126 .542126 .541126 .539
125.179125 .180125 .181125 .183125 .181125 .125125 .125125 .122
124.021 124.022.124.023 124.025 124.023.123.956 123.958 123.955
123.099123 .100123 .202123 .103123 .101123 .029123 .028123 .025
$122.438 .122 .439122 .350 \quad 122.442$ 122.440.122.363 122.362 122.359
122.033122 .034122 .035122 .038122 .035121 .955121 .954121 .951
121.898121 .899121 .900121 .902 .121 .900121 .819121 .818121 .815

TEMPERATURE DF STUCK MATERIAL

| Area | 10 | 11. | 12. | 13 -.. | 14 - | 15. | 16 ..... | 17. | 18. | 19 | 20. | 21 | 22 | 23 - 24 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 134.228 | 134.220 | 134. 228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 134.228 |
| 2 | 131.95 | 131.955 | 131.959 | 131.959 | 131.964 | 131.969 | 132.008 13 | 132.008 | 132.024 | 132.028 | 112.037 | 132.047 | 132.065 | 132.066 132.086 |
| 3 | 129.786 | 129.780 | 129.788 | 129.788 | 129.790 | 129.810 | 129.888 1 | 129.888 | 129.921 | 129.928 | 129.946 | 129.966 | 130.002 | 130.009130 .044 |
| 4 | 127.790 | 127.782 | 127.793 | 127.794 | 127.809 | 127.826 | 127.947 | 127.948. | 127.996 | 128.009 | 128.036 | 128.066 | 128.122 | 128.132.128.186 |
| 5 | 126.018 | 126.007 | 126.021 | 126.022 | 126.042 | 126.0661 | 126.228 | 126.229 | 126.295 | 126.311 | 126.339 | 126.380 | 126.440 | 126.454 126.520 |
| 6 | 124.445 | 124.430 | 124.449 | 124.450 | 124.474 | 124.5061 | 124.7191 | 124.720 | 124.806 | 124.826 | 124.864 | 124.917 | 124.998 | 125.016 125.111 |
| 7 | 123.110 | 123.091 | 123.113 | 123.115 | 123.144 | 123.1831 | 123.451 | 123.453 | 123.560 | 123.585 | 123.633 | 123.699 | 123.802 | 123.624 123.940 |
| 8 | 121.994 | 121.971 | 121.997 | 121.992 | 122.03: | 122.082 | 122.4.10 | 122. $\$ 12$ | 122.542 | 122.573 | 122.632 | 122.712 | 122.839 | 122.866 123.005 |
| 9 | 121.126 | 121.097 | 121.127 | 121.129 | 121.169 | 121.2281 | 121.8221 | 121.624 | 121.780 | 121.817 | 121.886 | 121.982 | 122.134 | 122.165 122.320 |
| 10 | 120.496 | 120.461 | 120.495 | 120.497 | 120.543 | 120.614 .1 | 121.080. | 121.082 | 121.265 | 121.309. | 121.390 | 121.502 | 121.681 | 121.718-121.908 |
| 11 | 119.753 | 119.696 | 119.745 | 119.748 | 119.814 | 119.930 | 120.6401 | 120.643 | 120.900 | 120.961 | 121.070 | 121.223 | 121.459 | 121.506121 .744 |
| ArEA | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37. | $38 \quad 39$ |
| 1 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.2281 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228 | 134.228134 .228 |
| 2 | 132.090 | 132.095 | 132.103 | 132. 106 | 132.111 | 132.115 | 132.120 .1 | 132.121 | 132.120 | 132.135 | 132.137 | 132.141 | 132.148 | 132.148 132.160 |
| 3 | 130.054 | 130.064 | 130.080 | 130.085 | 130.096 | 130.1041 | 130.1141 | 130.116 | 130.129 | 130.144 | 130.148 | 130.154 | 130.169 | $130.169 \quad 130.194$ |
| 4 | 128.201 | 128.217 | 128.240 | 128.248 | 128.265 | 128.277.1 | 128.292 | 128. 296 | 128.314 | 128.338 | 120.344 | 128.355 | 128.379 | 128.379 128.420 |
| 5 | 126.548 | 126.570 | 126.602 | 126.613 | 126.634 | 126.6511 | 126.672 | 126.876 | 126.701 | 126.732 | 126.740 | 126.756 | 126.787 | 126.787 126.842 |
| 6 | 125.136 | 125.165 | 125.206 | 125.219 | 125.246 | 125.267 | 125.293 1 | 125.209 | 125.331 | 125.369 | 125.379 | 125.308 | 125.438 | $125.438-125.506$ |
| 7 | 123.972 | 124.007 | 124.053 | 124.070 | 124.085 | 124.111 | 124.143 1 | 124.149 | 124.188 | 124.234 | 124.247 | 124.269 | 124.318 | 124.317124.400 |
| 8 | 123.043 | 123.086 | 123.141 | 123.162 | 123. | 123.111 | 123.250 | 123.2! | 123.303 | 123.357 | 123.372 | 123.308 | 123.45 | $123.654-123.552$ |
| 9 | 122.374 | 122.425 | 122.491 | 122.515 | 122.539 | 122.5771 | 122.6191 | 122.627 | 122.681 | 122.742 | 122.759 | 122.788 | 122.854 | 122.853 122.955 |
| 10 | 121.960 | 121.620 | 121.723 | 121.761 | 121.798 | 121.857 1 | 121.9161 | 121.928 | 122.007 | 122.093 | 122.119 | 122.156 | 122.249 | 122.245122 .396 |
| 11 | 121.810 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | $0.000 \quad 0.000$ |
| area | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 - 134 | -49 | 50 | -31 | 52 | $53-54$ |
| $1$ | 134.220 | 134.228 | 134.228 | 134.228 | 134.220 | 134.228 1 | 134.2281 | 134.228 | 134.228 | 134.22\% | 134.228 | 134.228 | 134.228 | 134.220 134.228 |
| 2 | 132.162 | $132.166$ | $132.17$ | $132.174$ | 132.189 | 132.103 1 | 132.196 | 132.201 | 132.221-1 | $132+224$ | 132.263 | 132.257 | 132.269 | $132.272 \quad 132.260$ |
| 3 | 130.197 | 130.205 | 130.218 | 130.221 | 130.250 | 130.2571 | 130.2591 | 130.272 | 130.3041 | 130.311 | 130.340 | 130.354 | 130.378 | 130.385130 .400 |
| 4 | 128.424 | 128.441 | 128.471 | 128.476 | 128.531 | 126.542 | 120.8441 | 128.563 | 128.610 | 128.621 | 128.662 | 128.676 | 128.712 | 128.722 128.746 |
| 5 | 126.847 | 126.869 | 126.906 | 126.913 | 126.985 | 127.001 | 127.0041 | 127.030 | 127.099 | 127.114 | 127.177 | 127.207 | 127.254 | 127.268 127.299 |
| 6 | 125.513 | 125.540 | 125.584 | 125.594 | 125.685 | 125.705 | 125.708 | 125.743 | 125.835 | 125.856 | 125.939 | 125.985 | 126.04 | 126.062-126.100 |
| 7 | 124.408 | 124.441 | 124.492 | 124.504 | 124.617 | 124.641 | 124.645 1 | 124.688 | 124.805 | 124.828 | 124.938 | 125.001 | 125.072 | 125.093 125.138 |
| . | 123.561 | 123.600 | 123.659 | 123.673 | 123.810 | 123.838-1 | 123.843-1 | 123.895 | 124.038 | 124.065 | 124.202 | 124-281 | 124.364 | 124.389 124.442 |
| 0 | 122.966 | 123.011 | 123.079 | 123.095 | 123.259 | 123.2911 | 123.2961 | 123.357 | 123.5291 | 123.560 | 123.726 | 123.823 | 123.918 | 123.947124 .006 |
|  |  |  |  |  | 133.190 | 122.878 | 122.884 1 | 122.970 | 123.211 | 123.251 | 123.484 |  | 123.729 | 123.764 123.834 |




```
TIME(SEC): 330.030
    TOTAL AREA(CZ)= 0.000O.000
    HEDGE AREAR O:0000
    ANGIE(DEG)m 0,000
    ANGLE(DEG)m
    NO LEMGTHITLME INCREHENT:
    TEMPERATURE HISTORY OF PDNDER
```



```
    143.243.143.243 143*243.143.243 143.243.143.284.143.246 153.280
    142.371 142.371 142.371 142.372 1442.372 142.315 142.315 142.315
    141.374 141.374 141.375 141.375 141.375 141.514 141.514 141.514
    140.641 140.641 140.641 140.642 140.642 140.491 140.491 140.491
    139.714 139.714 139.714 139.716 139.715 139.824.139.824 139.824
    139.093 139.093 139.093 139.095 139.095 138.994 130.993 138.993
    136.965 136.965-136.965 136.968_136.096 136.900. 136.000-136.889
    135.594 135.594 135.594 135.598 135.597 135.536 135.536 135.535
    134.833 134.834 134.834 134.838 134.838 134.793.134.793 134.702
    134.591 134.591 134.591 134.596 134.595 134.553 134.553 134.552
```

TEMPERATURE DF STUCK MATERIAL





TEMPERATURE OF STUCK MATERIAL





WITH OVEN CONVECTION HEATING

DIMENSION TN（ $1001, T(200,100), O C(200), P S(200), N 2(200), H 3(200)$
C，HZ
COMMON SS，SM，AL，AM，DX，N，TM，AA，AT，AD，TH，DL，NZ，CT，NC，N3，RAD，TS COMMON TAFR．BMF．MT
INTEGER CT，KL

DATA ANECK，3NECK／20000＊0．0，100＊0．01
rlossary
AA TOTAL SROSS SECTICNAL PDOL AREA（CM2）

ACONST OJMMY VARIABLE
AO－PENIFZATON FHIEKNESS（EH）
AH CUNVEITION こDEFFICIENT OF OVEN AIR（J／CMZ－SEC－K）

ALCP SPECIFIC HEAT OF ALUMINEM MOLD（J／KG－C）

aLK THERMAL COVOUCTIVITY OF MOLD（J／CM－SEC－K）

AM THERMAL OIFFUSIVITY AT COMPLETE MELT（CPISEG）
ANEEK NESKRAOIUSAT EACH－NGEE（EHI）
ANEK NECK RADIJS OF EACH NODAL AREA AT THE TIME
ANK JUMMY VARIAGLE $=\operatorname{ANECK}(1,1)$
AR－THMMY VAZIABLE HSE日 IN CALEULAT－ING REHA．JNING こROSS SECTICNAL POOL AREA
AS－TOTAL－AREA ENCOMPANSING NEDAL ANAL＊SIS
（ HEIGHT OF NCDES $x$ TOTAL HIDTH）
AT－ $\overrightarrow{\text { ROSS SENFまONAL－AREA JFーONE HEOGE TEMZ }}$
ATEST CUMYY VARIABLE
AV JUTHY 甘APIABLE－USEG TO OETERMINE HEN ANGLE TMETHA FHAT－QE－ENTERS－THE－POGL IN－THE NE HT TIME INGREMENT H
A4 THE こZOSS SECTIONAL AREA OF THE STUCK GCLUMN OF HAFERI AL THAF REMAINS OUTSIQETHE POOL JURIN：THE NEXT TIME INGREMENT（CMZ）
9A ANGLE－AF RESOONSE OF THE FOOL（RAA）
BR JUMMY VAPIABLE USED IN TEMPERATURE EQUATION
ЭE OUMHY VARI ARL E USEE－SURFAGE NOGAL NECK SIZE EQHAFION－．．．－－－
BMT EXTERTOR TEMP OF MCLD（DEG C）
BMTN EXTERIJRーTEMP OF MOLT DURING THE NEXT TIME INCREMENT（DEG C）

CC half LENGTH OF POOL CORD（CM）
CT－－－OUNFE？HSEG－TO ECUNF－THE NUMBER－OF－SFUCK COLUMNS
C1 SONSTANT USED IN TEMPFRATURE EQUATIONS CETERMINED IN SUBROUFINE－CONS干
G2 JONSTAYT USEO IN NODAL TEMPERATURE EQUATIONS
CZ－जONSTAYF－USEO FN－NOEAT－FEMPERATURE EQUATIJNS
DA AVERAGE DIFEUSIVITY BETWEEN NODES I＋1 AND NODE I（CYZ／SEC）

DC $\triangle R C$ WITTH OF STUCK NODAL COLUMNS（RAD）







$M T=M T N$
 IF（T（NC，N）．GE．SM）GOTO 8 ¢1
IF（FMotFotKMPT）－6070－892

PRINTーROUTINE

CALL PRINT（T，LK，I3）
1－3＝1
CALEPRINF（ANEEK．I2．13）
IF（T（NC，N）．ET．SM）SOTO29

IF（N．LT．NMAX）5OTO992
－－WRITE゙古，30）于M TREM＝0
$\Delta T=0.0$
T
$t K=L K-1$
GOTO 89？
WRITE $(5,31)$

$\rightarrow 3 \theta-33-j=17 \mathrm{~N}$
WRITE（6，32）J，ANEK（J）
3．CONTINUE
STOP
ITM＝I于M＋1
IF（IREM．EQ．O1．JR．（ITM．LT．ITMX））TM＝TM＋TS

ITM＝0

$-G-\cdots * * * *-S E G F I-N \quad \forall$
calculate mix mean temperatjpe when no material has stuck
$110 ? \mathrm{TP}=0.0$

CONFINUE
$T P=(T P+M T / 2) /(N-0.5)$
$F T=(A S * F+(A T-A S T * T T+A T$
301146 III＝？，N
$\mathrm{I}=\mathrm{AC} \mathrm{C}-\mathrm{III}+2$
$T T, J)=T T I-I, J$








FUNCTION RADX $\quad 74 / 825 \quad$ OPT $=0$, ROUN［ $=A / S / M /-D,-D S \quad F T N 5.1+601 \quad 176$
 $N 5$.

FUNCFION－RAQ＊\＆
DIMENSIJN N3（200）

C

$X=Y / R A D$

PAOX＝1． 3
GO FO 1
1：IF（X．GT．1．G）GOTO 20

SO TO 1

1 CONTINUE
REFIRN
END
－FUNCTIONTEPK
C

C
$5 P=2302.7$
RETURN
END

FUNCFION－Sぜーチ

```
    C
    C
    \(c\)
            \(S U=31.00-0.059 *(T-105)\)
            RETURN
            END
```

FUNCFION VEうかF－
VISCOSITY =UNETEON - FFOESE)

RETURN
ENQ－－
(1) Frenkel, J., "Viscous Flow of Crystalline Bodies Under The Action of Surface Tension", Journal of Physics, Vol IX, NO 5, 1945, pp 385-391.
(2) Kuczynski, G. C., "Study of the Sintering of Glass", Journal of Applied Physics, Vol 20, Dec 1949, pp 1160-1163.
(3) Kuczynski, Neuville and Toner, "Study of Sintering of Poly(methyl Methacrylate)", Journal of Applied Polymer Science, Vol 14, 1970 , pp 2069-2077.
(4) Lontz, J. F., "Sintering of Polymer Materials", Fundamental Phenomena in the Material Sciences, Vol l, (L.J. Bonis and H.H. Hausner, eds), Plenvn Press, New York, 1964, pp 25-47.
(5) Dillion, R. E., Matheson, L. A. and Bradford, E. B., "Sintering of Synthetic Latex Particles", J. Colloid Sci 6, 1951, pp 108-117.
(6) Tadmor and Gogos, Principles of Polymer Processing, John Wiley \& Sons, New York, NY, 1979, pg 147 .
(7) Bigg, D.M. and Epstein, M. M., "Molding Polymeric Powders by Compaction and Sintering", Science Technology Polymer Process, Processing Int. Conf 1979, pp 897-919.
(8) Narkis, M., "Sintering Behavior of Poly (Methyl Methacrylate) Particles", Polymer Engineering and Science, Oct 1979, Vol 19, No 13, pp 889-892.
(9) Steiner, G. F., Manson, J. A. and Nippert, C. R., "Characterization of Polymer Glasses By Sintering Techniques", Paper presented to American Chemical Society, Chicago, 13-18 Sept 1970, pp 105-111.
(10) Schonhorn, H., Frisch, H. I., Kwei, T. K., "Kinetics of Wetting of Surfaces by Polymer Melts", Journal of Applied Physics, Vol 17, No 13, Nov 66, pp 4967-4973.
(11) Halldin, G. W. and Kamel, I.L., "Powder Processing of UHMV-PE, II. Sintering", Society of Plastic Engineers, 35 th Annual Technical Conference, Apr 1977, pp 298-300.
(12) Ross, Thomas, J., "Review of Polymer Powder Sintering", SpE 37th Annual Technical Conference, Vol 15, May 1979, pp 909-912.
(13) Kuczynski, G. C. and Zaplatynskyj, I., "Sintering of Glass", Journal of the American Ceramic Society, Vol 39, No 10, pp 249-350.
(14) Rao, M. and Throne, J. L., "Theory of Rotational Molding - Part III: Sintering - Melting and Degradation", in Proceedings 30 th Annual SPE Technical Conference: Shaping the Future with Plastics, 15-18 May 1972, Chicago, pp 759-761.
(15) Rosenzweig, N. and Narkis, M., "Coalescence Phenomenology of Spherical Polymer Particles By Sintering", Polymer, Sep 1980, Vol 2l, pp 988-989.
(16) Jayaraman, G. S., Wallace, J. F., Geil, P. H. and Baer, E., "Cold Compaction Molding and Sintering of Polystyrene", Polymer Engineering and Science, Aug 1976, Vol l6, No 8, pp 529-536.
(17) Mackenzie, J. K. and Shuttleworth, R., "A Phenomenological Theory of Sintering", Proc Phys Society, LXII, 12, pp 833-852.
(18) Throne, James L., Plastic Process Engineering, Marcel Dekker, inc., New York, NY, 1979, pg 60.
(19) Mohan, A., Soni, N. C. and Moorthy, V. K., "Refinements in Determination of Sintering Mechanisms", Transactions of the Indian Ceramic Society, Vol 38, July-Aug 1979, pp ll7-124.
(20) Menges, G., Schulze-Kadelbach, R., Reichstein, H., Thebing, U. and Schmity, J., "Examination of the Principles for Sintering Plastic Materials", Rheology Acta, Mar 1980, pp 633-641.
(21) Rosenzweig, N. and Narkis, M., "Dimensional Variations of Two Spherical Polymeric Particles During Sintering", Polymer Engineering and Science, July 1981, Vol 21, No 10 , pp 582-585.
(22) Ramamoorthy, P. and Treybal, R., "Drop Coalescence in Liquid-Liquid Fluidized Beds", A.I.CH.E. Journal, Vol 24, No 6, Nov 1978, pp 985-992.
(23) Lang, S. B. and Wilke, C. R., "A Hydrodynamic Mechanism for the Coalescence of Liquid Drops. I. Theory of Coalescence at a Planar Interface", Industrial Engineering, Chemical Fundamentals, Vol 10, No 3, 1971, pp 328-340.
(24) Lang, S. B. and Wilke, C. R., "A Hydrodynamic Mechanism for the Coalescence of Liquid Drops. II. Experimental Studies", Industrial Engineering Chemical Fundamental, Vol 10, No 3, pp 341-352.
(25) Shaler, A. J., "Seminar on the Kinetics of Sintering (with Discussions)", Metals Transactions, Vol 185 Nov 1949, pp 796-804.
(26) Vanderhoff, J. W., Tarkowski, H. L., Jenkins, M. C. and Bradford, E. B., "Theoretical Consideration of the Interfacial Forces Involved in the Coalescence of Latex Particles", Journal of Macromolecular Chemistry, Apr 1966, pp 361-397.
(27) Brown, George, "Formation of Films from Polymer Dispersions", Journal of Polymer Science, Vol XXII, 1956, pp 423-434.
(28) Leidi, M., Hartland, S., "The Effect of Vertical Forces on the Coalescence of Two Dimensional Drops", Proc. R. Soc. Lond. 349, 1976, pp 343-354.
(29) Myer, Glen E., Analytical Methods in Heat Transfer, McGraw - Hill Book Co., 1971, pg 198.
(30) Vanderbeck, William E., The Prediction of Sintering Time in the Roto-molding of Polymeric Materials, Thesis, NJIT, Newark, NJ, 1979, pg 71.
(31) Rao, M. and Throne, J. L., "Theory of Rotational Molding - Part I: Heat Transfer", Soc. Plast. Eng, , Tech Pap., 72, 18(Pt. 2), pp 752-756
(32) Rao, M. and Throne, J. L., "Theory of Rotational Molding - Part II: Fluid Flow", Soc. Plast. Eng., Tech Pap., 72, 18(Pt. 2), pp 757-761.
(33) Progelhof, R. C., Throne, J. L., and Ruetsch, R. R., "Methods for Preaicting the Thermal Conductivity of Composite Systems: A Review", Polymer Engineering And Science, Sept 76, Vol 16, No 9, pp 615-625.
(34) Yagi, Sakae, Kunii, and Daizo, "Studies of Effective Thermal Conductivies in Packed Beds", A.I.Ch.E. Journal, Sept 57.
(35) Werley, K. and Gilligan, J., "The Temperature Distribution of a Sphere Placed in a Directed Uniform Heat Flux", Journal of Heat Transfer, May 81, Vol 103, pg. 399.

Sass, Allan, "Simulation of the Heat Transfer Phenomena in a Rotary Kiln", I \& ED Process Design \& Development, Vol 6, No 4, Oct 67, pp 532-535.
(37) Murphy, K., "The Melting Process in Polyethylene Powder Beds", ME 490 Paper, NJIT, Newark, NJ, Dec 81.
(38) Merrill, Ronald, "Thermal Conduction Through an Evacuated Idealized Powder Over the Temperature Range of 100 to $500 \mathrm{K"}$, NASA Technical Note \#D-5063, March 1969.
(39) Mac Adams, John L., "How to Predict Physical Properties of Rotomolded Parts", SPE Tech Paper, Oct 1975, pp 64-71.
(40) Luikov, Shashkov, Vasiliev, and Fraiman, "Thermal Conductivity of Porous Systems", Heat Mass Transfer, Vol 11, Apr 66, pp 117-140.
(41) Lehmberg, J., Hehl, M., and Schugerl, K., "Transverse Mixing And Heat Transfer in Horizontal Rotary Drum Reactors",
(42) Kramers, H., and Croockewit, P., "The Passage of Granular Solids Through Inclined Rotary Kilns", Chem. Eng. Sci., Vol 1, No 6, 1952.
(43) Wes, G., Drinkenburg, A., and Stemerding, S., "Heat Transfer in a Horizontal Rotary Drum Reactor", Powder Tech., 1976
(44) Kunii, Daizo and Smith, "Heat Transfer Characteristics of Porous Rocks", A.I.Ch.E. Journal, March 1960.
(45) Cho, S.H., and Sunderland, J.E., "Heat Conduction Problems With Melting and Freezing", J. of Heat Transfer, Aug 1969, pg 421.
(46) Fraiman, Y., Luikow, A., and Shashkov, A., "Thermal Conductivity of Porous Systems", J. of Heat Mass Transfer, Vol ll, 1968, pp. 117-140.
(47) Beek, W.J. and Stammers, E., "The Melting of a Polymer on a Hot Surface", Polymer Eng \& Science, Jan 69, Vol 9, No 1, pg 49.
(48) Baer, E., Geil, P., Jayaraman, G., and Wallace, J., "Cold Compaction Molding and Sintering of Polystyrene", Polymer Eng \& Sci, Aug 1976, Vol 16, No 8, pg 529.
(49) Hartland, S., and Leidi, M., "The Effect of Vertical Force on the Coalescence of Two Dimensional Drops", proc. R. Soc. Lond. A., Vol 349, 1976, pp 343-354.
(50) Carrow, G.E., "Crosslinkable Rotational Molding High Density Polyethylene", S.P.E. 30th Annual Tech. Conf., May 1972, pp 762-765.
(51) Rao and Toor, "Heat Transfer Between Particles in Packed Beds", Ind. Eng. Chem. Fund., 1984, 23, pp. 294-298.
(52) Ristic, "Kuczynski's Theory of Sintering and Recent Investigations of the Process", International Journal of Science of Sintering, Vol 16, 1984, No 2.
(53) Lewandowski, A., "A Study of Surface Kinetic Angles of Plastic Powders In a Rotating Drum", NCE Thesis, Newark, NJ .
(54) Mercadante, F., "Flow Characteristics of A Granular Matter In a Rotating Cylinder", NCE Thesis, Newark, NJ, 1973.
(55) Krieth, Frank, Principles of Heat Transfer, 2nd Ed., Scranton, Pa., The Ronald Press Company, 1963.
(56) Myers, Glen E., Analytical Methods in Conduction Heat Transfer, New York, McGraw-Hill, 1971.
(57) Technical Information Sheet(TIS) of Marlex LX470 Phillips Chemical Company, Bartlesville, OK, May 1981.
(58) Technical Service Memorandum No. 243 (TSM-243), Marlex Polyolefin Resins, Phillips Chemical Company, Bartlesville, OK, June 1983.
(59) Schenck, Hildebrand, Jr., Fortran Methods in Heat Flow, New York, The Ronald Press Company, 1963 .

