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Simulation study of particle tracking system

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ABSTRACT

SIMULATION STUDY OF THE PARTICLE TRACKING SYSTEM

by
Prashant B. Patel

A particle tracking system is being developed in the Particle Technology Laboratory of the Mechanical Engineering Department, at NJIT. In this thesis, an attempt is made to understand the effect of various parameters on the errors in the computed position and orientation of the sphere being tracked. Simulation results for a single transmitter assembly as well as a three transmitter assembly with 6, 9 and 17 antennae are presented. The effects of the level of noise in the voltages on the solution for the single transmitter assembly and three transmitter assembly are discussed. The concept of virtual antenna is introduced and experimental results as well as simulations results using the virtual antennae are presented. Simulation results showing the advantages of techniques such as perturbations and extrapolation in the solution code are discussed. The effects of antenna shape, size and aspect ratio on the voltages are studied. A new calibration technique for reducing the systematic errors in the measured voltages is proposed. The accuracy of the particle tracking system is confirmed by a test in which the sphere containing the transmitter is dropped in a free fall and the gravitational constant is calculated.

SIMULATION STUDY OF PARTICLE TRACKING SYSTEM

by
Prashant B. Patel

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SIMULATION STUDY OF PARTICLE TRACKING SYSTEM

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*This thesis is dedicated to
my parents*

ACKNOWLEDGMENT

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CHAPTER 1

INTRODUCTION

1.1 Developments in the Particle Tracking System

This is a study of the some analysis techniques used in the non-intrusive particle tracking system for dry particulate flows. The particle tracking system is based on the principle of magnetic coupling. Three mutually perpendicular transmitters are packed in a one inch diameter sphere. The transmitters emit signals at three different frequencies; 2 MHz., 3.65 MHz., and 4.4 MHz. This sphere is then allowed to flow within a mass of similar spheres in a 120" x 15" x 12.5" chute around which an array of antennae are positioned. Voltages in the antennae are functions of the position and orientation of the transmitters. A theoretical model Ref. [5], predicts the voltage for any position and orientation of the transmitter with respect to the antennae. This model, in conjunction with the Levenberg & Marquardt algorithm Ref. [8] to solve non-linear least square problems, is used to track the position and orientation of the sphere.

The feasibility of the above technique has already been proven by simulations and experiments during its development (See Ref. [1], [2], [4], [10] and [15]). At first, experiments were conducted with one transmitter. In the experiments with one transmitter it was found that results were not consistent and reliable. Accuracy in position and orientation was not sufficient for practical applications. Moreover, information about the rotation of the transmitter about its own axis was impossible to obtain using only one transmitter. It was concluded that two or preferably three transmitters were required for acceptable results. There were many practical problems in the manufacturing of multiple transmitters. Multiple transmitters must fit in the same one inch sphere used for single transmitter. The other constraint was that two transmitters are not permitted to communicate with each other. Eventually a three transmitter system was designed and

fabricated. The code for signal processing with single as well as multiple transmitters was developed. Various techniques such as a 27-point correction scheme (See Ref. [15]) to reduce systematic errors, and a dynamic switching technique [1], perturbations (See Section 2.4) etc., were implemented. In the experiments conducted so far, accuracy of approximately 0.40" in position and about 5° in orientation have been obtained.

1.2 Prior Work

Anshumat Parasar [10], a graduate student working for the Particle Technology Laboratory, contributed towards the development of 'forward-model' code i.e. given the position and orientation of a transmitter, this code finds the voltage in the antenna loop. In continuation to that, Ashok Adhavani [2] set up the *lmdif* non-linear least square fit numerical solution package. Elliotte Harold, a mathematics student, helped to make the code more robust and flexible. Jerry Volcy [15], also a gradate student in Mechanical Engineering, made a major contribution towards the implementation aspects of the particle tracking system. He performed extensive experiments with one transmitter. He also implemented the 27-point correction scheme for reducing the systematic errors in the voltages. Jitesh Agrawal [1], a graduate student of Manufacturing Department, made modifications in the code for the use with three transmitters. He conducted experiments with three transmitters and implemented dynamic switching and initial guess modifications in the codes.

1.3 Getting Ready for Actual Experiments

In this thesis, the analysis techniques are studied in detail using simulations and experiments to make the computations more accurate, reliable and consistent. Errors, both random and systematic, affect the accuracy of the position and orientation of the transmitter. Therefore, the effect of noise in voltages on the performance of particle tracking system is studied. A comparison of errors in solutions using one transmitter and

using three transmitters is performed. A new approach for calibrating the antennae is proposed for the three transmitter configuration. The effects of the number of antennae and the antennae configuration on the solution is studied using simulations. An attempt is made here to understand the types of multiple solutions occurring in the solution due to orientations and local minimums.

In chapter two we study the influence of the number of transmitters and the number of antennae on the solution. We also study the influence of noise, in addition to studying variations in a step size parameter of the Levenberg & Marquardt algorithm code. Topics such as convergence, local minimum, and multiple solutions are also studied. Finally, the simulation study of perturbations and extrapolation techniques is presented.

Chapter three discusses the aspect ratio, size and shape study of antennae. The effect of antennae coupling is also studied.

Systematic errors in signal processing are discussed in Chapter four. An alternative schemes to the 27-point correction method are discussed.

In Chapter five, the experimental results for the free fall trajectory are presented and the accuracy of the particle tracking system is confirmed by calculating the value of the gravitational constant.

In simulations, increasing the number of antennae for one transmitter improves the results. The amount of improvement in results depends upon the type of trajectory and orientation of the transmitter. Both for the one transmitter as well as three transmitters, there is a significant improvement in results by using each piece of data twice. It is found that calibrating the model with experimental voltage values by using calibration points in the antenna plane leads to better results than using calibration points away from antenna plane. A calibration scheme where calibration is done only in the region where the model and measured voltage do not match well is proposed. The absolute difference in measured and model voltages seems to affect the results more than the percentage errors. Average errors in X, Y and Z positions of about 0.1 inch or less are obtained in actual experiments.

The results in general are not always predictable and consistent due to the non-linear nature of the problem.

Further work needs to be done on the virtual antennae concept on actual data. The problem with the multiple solutions in orientations results needs to be solved. The suggested modifications in the 27-point correction scheme and the calibration points can be implemented.

CHAPTER 2

SIMULATION STUDY

In this chapter simulation study of the particle tracking system is performed to understand the effect of such parameters as number of transmitters, number of receiving antenna, noise, perturbations, extrapolations, and multiple solutions on the solution.

2.1 Comparison of One Transmitter vs. Three Transmitters Results

In this section a comparison is made between the performance of a single transmitter assembly and a three transmitter assembly. Results using several antennae configurations employing 6, 9 and 17 antennae are presented for the single transmitter assembly as well as the three transmitter assembly. The study of one transmitter is important as it has practical applications. The processing time of data with one transmitter is less, but this is of no significance to the study described herein. The hardware requirements are reduced with one transmitter. Due to very limited space, small batteries have to be used to power the transmitters. After 20 minutes of use, the three transmitter assembly lowers the battery power by 10 percent every minute. The one transmitter assembly requires one third the power consumption of the three transmitter assembly. Most importantly the data acquisition rate with one transmitter is much higher. As a result it can be used in very high velocity flows. The disadvantage of one transmitter is that information about one orientation variable (rotation about global X) is lost as rotating the transmitter about this axis does not change the voltage induced in the antenna.

For the purpose of simulation model chute of dimensions 20" x 20" x 20" is used. Six antennae are positioned on the six faces of the chute. (See Figure A.1). A right hand Cartesian co-ordinate system is used to describe the positions and orientations of the transmitter assembly, as shown in Figure A.1. The antennae perpendicular to the X axis of the co-ordinate system are termed as X-antennae. Similarly, the antennae perpendicular to

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the Y and Z axis of the co-ordinate system are termed as Y-antennae and Z-antennae. The rotation about the X-axis of the co-ordinate system will be referred to as X-rotation or α hereafter in further discussion. Similarly, the rotations about Y-axis and Z-axis will be called Y-rotation or β and Z-rotation or γ, respectively. A three- dimensional sinusoidal trajectory with an amplitude of seven inches in the Y direction and five inches in the Z direction with a cycle of twenty inches in X is used for simulations. The trajectory has 21 points in one inch increments starting at X=0.0, Y=10.0, Z=10.0 and ending at X=20.0, Y=10.0, Z=10.0. The orientations α, β and γ are incremented by 5.0, 7.5 and 10.0 degrees successively for each point. (See figure A.2). Uniformly distributed 5% of maximum range random noise is added to voltages. Plots for the one transmitter assembly are shown in Figures B.1-6. The maximum errors in positions for single transmitter assembly are 13.41 inches, 18.55 inches and 2.38 inches in X, Y and Z respectively. Using the same trajectory, noise level and antennae configuration, three transmitter results are also obtained. Plots of actual versus predicted positions for three transmitters case are shown in Figures B.7-12 of Appendix B. The maximum errors in positions are X= 0.53 inch, Y= 0.42 inch and Z= 0.80 inch. Table 2.1 gives the summary of results for both the one and three transmitter cases. Clearly, results using the three transmitters are better than using only a single transmitter.

Table 2.1: Uniformly distributed ± 5% of maximum range random noise with 6 antenna.

	3-T Configuration				1-T Configuration			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.53	0.42	0.80	0.97	13.41	18.55	2.38	19.28
Average error	0.20	0.14	0.29	0.41	2.68	4.54	0.88	5.94
Std. deviation	0.18	0.09	0.23	0.26	2.96	5.92	0.70	6.13

For the case of one transmitter there are five variables i.e. X, Y, Z, β and γ (as rotating the transmitter about its axis does not affect the voltage induced in the antennae)

and six equations, while for the three transmitter assembly there are six variables (X, Y, Z, α, β, and γ) and eighteen equations (number of equations = number of variables x number of transmitters). The number of equations for the one transmitter (six) are about the same as the number of variables (five). This may be one of the major factors contributing to the large errors in the one transmitter results. To investigate if increasing the number of antennae, and thus the number of information elements (voltages), improves the solution for the single transmitter case, additional antennae are incorporated. First three are added and then another eight antennae are added to the original six antennae chute.

Three antennae are added in the X-direction at X=5.0, 10.0 and 15.0 inches (See Figure A.1). The results of this study are presented in Table 2.2, tabulating the maximum error, average error and standard deviations in positions and orientations for the previously described sinusoidal trajectory. For the three transmitter assembly, the number of equations is now 9 x 3 = 27. The improvement in the maximum and average errors in X, Y and Z for the three transmitter assembly is about 33% (total distance errors). For the single transmitter assembly there is an improvement of 75% in maximum errors and 66% in average errors in X, Y and Z (total distance errors).

Table 2.2: Uniformly distributed ± 5% of maximum range random noise with 9 antenna.

	3-T Configuration				1-T Configuration			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.43	0.63	0.37	0.66	3.14	4.09	1.97	5.16
Average error	0.11	0.13	0.14	0.26	0.94	0.98	0.58	1.81
Std. deviation	0.12	0.13	0.09	0.16	0.92	1.21	0.56	1.24

Since improvements in the results were obtained by increasing the number of antennae, eight more antennae are added to the current nine antennae configuration. Three antennae each in the Y and Z directions are added at 5, 10 and 15 inches respectively as shown in

Figure A.1. Table 2.3 shows the maximum error, average error and standard deviations in the predicted positions for this new configuration of 17 antennae. Plots of actual versus predicted positions for one transmitter case are shown in Figures B.13-B.18. For the three transmitters case similar plots are shown in Figures B.19-22 of appendix B. Plots for the X rotation and Y rotation are not known as they do not show any significant change from the 9 antenna case.

Table 2.3: Uniformly distributed ± 5% of maximum range random noise with 17 antenna.

	3-T Configuration				1-T Configuration			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.26	0.14	0.22	0.35	2.27	0.77	0.30	2.29
Average error	0.06	0.05	0.06	0.11	0.31	0.19	0.09	0.43
Std. deviation	0.08	0.04	0.06	0.09	0.47	0.17	0.09	0.47

For the three-transmitter assembly, the reduction in maximum as well as mean errors in positions is about 50% which is approximately the same as the increase in the number of antennae (The relation between number of antennae and errors in position needs to be studied in detail by taking different types of trajectories and antennae configurations. This study has not been covered in this thesis). The same trend of improvement is observed for the one transmitter case. The ratio of maximum errors in the total distance (column four of table) for the one transmitter simulation to that in the three transmitter simulation is 19.28/0.97 = 19.87 for the six antennae configuration and 2.29/0.35 = 6.54 for 17 antennae configuration. The results of three transmitters and six antennae and one transmitter and 17 antennae are comparable although the three transmitter results are better. The following observations can be made from the above results.

1. There is improvement in results with the increase in the number of antennae for the single transmitter and three transmitter cases.

2. Around 15 to 20 antennae are required to obtain a solution with errors less than 1" in X, Y and Z positions for the single transmitter case. For the three transmitter assembly, only six antennae are required.

3. Three transmitters always give better results than a single transmitter (with same number of antennae).

2.2 Virtual Antennae

For one transmitter as well as for three transmitters, it is observed that increasing the number of antennae improves the solution for most cases. Therefore a new approach of "doubling the data" is implemented. Essentially, doubling means using the same voltage data twice. We call this "using virtual antennae" as the data from each antenna is used twice. For a linear least squares problem, using the same data twice will give the same solution. The effect on our non-linear problem investigated. In the present study, both simulated as well as experimental data is used. The experimental run uses a straight line, constant orientation trajectory. The summary of experimental results with and without virtual antennae are presented in Table 2.4 and the plots can be seen in Figures B.23-34. There is an improvement of 0.3 inches in the maximum distances by adding virtual antennae. It is observed that rotation about X is consistently off by -4 degrees and rotation about Z by 9 degrees. (The mounted three transmitter assembly does not have perfectly orthogonal transmitters. See Ref. [1] for further details about transmitter corrections. The error in our actual assembly is -4 degrees in X rotation and 9 degrees in Z rotation. This is

Table 2.4: Experimental Results using Virtual Antennae Concept. (3-T & 7 antennae)

	With Virtual Antennae				Without Virtual Antennae			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.40	0.18	0.10	0.42	0.67	0.31	0.14	0.72
Average error	0.20	0.06	0.05	0.23	0.22	0.20	0.09	0.32
Std. deviation	0.13	0.05	0.03	0.13	0.19	0.06	0.04	0.17

the source of these errors).

Tables 2.5, 2.6 and 2.7 show simulation results with 6, 17 and 34 antennae, respectively for the one transmitter system with a 5% of maximum range random noise. The same antennae configuration described in Section 2.1 is used for 6 and 17 antennae (See figure A.1). For the 34 antennae case, the same 17 antennae positions are used twice. The trajectory selected is a straight line from X=0.0 to X=20.0 with Y=10.0, Z=10.0 with an increment of one inch and zero rotations in X, Y and Z.

Table 2.5: Simulated Data with ±5% maximum range random noise. (One Transmitter)

	6 Antennae			
	X inches	Y inches	Z inches	Distance inches
Maximum error	1.08	5.64	8.66	10.16
Average error	0.39	3.15	3.87	5.22
Std. deviation	0.32	1.69	2.75	2.89

Since only one transmitter is used, there are no signals in the Y and Z antennae. Only the X and the diagonal antennae are useful for finding the solution. The improvements in maximum and mean errors in positions with 17 antennae is more than 50% compared to 6 antennae. By using the virtual antennae there is a further reduction in errors by 60%. Although this approach of increasing the number of antennae improves the solution, the factors contributing to the improvement are not understood.

Table 2.6: Simulated Data with ±5% maximum range random noise.(One Transmitter)

	17 Antennae			
	X inches	Y inches	Z inches	Distance inches
Maximum error	0.38	1.53	3.47	3.55
Average error	0.09	0.52	0.39	0.8
Std. deviation	0.08	0.81	0.79	1.03

Table 2.7: Simulated Data with ±5% maximum range random noise.(One Transmitter)

	34 Antennae			
	X inches	Y inches	Z inches	Distance inches
Maximum error	0.12	1.00	0.41	1.02
Average error	0.05	0.25	0.16	0.34
Std. deviation	0.04	0.23	0.12	0.22

2.3 Effect of Level of Noise on Solution

In actual experiments, there are small differences between measured and predicted voltages. This could be due to model inaccuracies and instrumentation errors, external noise from other devices, etc. Such effects can be simulated by corrupting the exact predicted voltages at random by a small amount .

A three dimensional sinusoidal trajectory is used. (See figure A.2). As the transmitter is rotated about the Y and Z axes all the antennae get almost equal signals. A uniformly distributed noise equivalent to ±5%, ±10%, and ±15% of the maximum range of voltage is added. The added noise is a certain percentage of the maximum range, and not of the actual voltage reading. Thus the added voltage may be a larger percentage of the actual reading when the voltage reading is small. Both the one transmitter assembly as well as the three transmitter configurations are considered. It is observed that the effect of noise is more pronounced for the one transmitter configuration, and as the noise increases, the errors in position and orientations also increase. The predicted errors for 5% noise are summarized in Table 2.8

Table 2.8: Uniformly distributed ±5% of maximum range random noise. Single transmitter results are with a 15 antennae configuration.

	Three transmitters				One transmitter			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.75	0.64	0.98	1.18	2.03	0.99	1.40	2.23
Average error	0.21	0.27	0.32	0.56	0.57	0.28	0.35	0.87
Std. deviation	0.19	0.18	0.25	0.21	0.55	0.27	0.41	0.56

For the three transmitter configuration, ±5% error in the voltage reading results in an average position error of 0.32 inch or less in any single coordinate direction. This is equivalent to about 1.6% of the position range of 20 inches. The maximum position error is 0.98 inch or less, which is equivalent to about 4.9%. The diameter of the sphere containing the transmitter assembly used for the particle tracking system is one inch. Therefore maximum errors in positions of less than or equal to one inch are considered to be good.

As shown in Table 2.8, the results for a single transmitter are not as good as those for three transmitters, as a single transmitter provides much less information than three orthogonal transmitters. The accuracy of predicting translation in X, Y and Z are similar. For the one transmitter case the maximum error in X, a translation of 2.03 inches, is equivalent to about 10.15% of the range of 20 inches in X direction. The maximum errors in Y and Z translations of 0.99 inch and 1.40 inches are equivalent to 5.0% and 7.0% of the range of 20 inches in the Y and Z directions respectively.

Table 2.9: Uniformly distributed ±10% of maximum range random noise.
Single transmitter results using 15 antennae.

	Three transmitters				One transmitter			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	1.65	1.16	2.04	2.04	3.05	1.70	8.80	9.18
Average error	0.45	0.55	0.57	1.08	1.04	0.56	1.40	2.22
Std. deviation	0.40	0.36	0.50	0.45	0.84	0.44	2.46	2.30

The effect of added noise for the one transmitter assembly as well as the three transmitter assembly can be understood at least in part through the above examples of ± 5% added noise. In principle, one can theoretically analyze the effect of noise or errors in the voltage readings on the computed position and orientation through analyzing the derivatives of the position and orientation with respect to voltages, but that would require

inverting the functional relationship given by the voltage model. This is not a trivial task. Further understanding of the influence of noise on the solution can be gained by examining the effect of a higher percentage of simulated noise errors. Therefore, ±10% and ±15% uniformly distributed random noise was added to the voltage data, and both the single transmitter and three transmitter configurations were analyzed. The results obtained for the prediction of position are presented in Table 2.9 and Table 2.10. For the three transmitter configuration, both noise levels result in an increase in the errors as compared to the ±5% noise case. The largest average translation error for the ±10% noise is 2.85%, and for ±15% noise is 4.45%. For the one transmitter configuration, the largest average translation error for the ±10% noise is 7.0% and for ±15% noise is 8.05%.

Table 2.10: Uniformly distributed ±15% of maximum range random noise. Single transmitter results are using 15 antennae.

	Three transmitters				One transmitter			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	2.57	2.72	2.93	3.31	4.61	3.79	9.32	9.70
Average error	0.73	0.89	0.89	1.68	1.46	0.88	1.61	2.80
Std. deviation	0.58	0.57	0.75	0.73	1.15	0.84	2.45	2.36

The maximum errors in X, Y and Z positions for the 5% noise case using three transmitter assembly are less than one inch. These errors are comparable to the errors obtained in actual experiments. Because they are within one particle diameter of exact we consider them to be acceptable. For the 10% and 15% noise cases using the three transmitter assembly, the maximum as well as mean errors in positions are approximately double and three times that of 5% noise case. Maximum errors in X, Y and Z positions for the 5% noise case using the single transmitter assembly are between one and two inches (see Table 2.8) which are not acceptable as "good" solutions. The three transmitter assembly

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always gives better results compared to the single transmitter assembly for all levels of noise.

2.4 Improving Global Convergence with Perturbations

Due to the highly non-linear nature of the problem, many local minima may exist near the global minimum. As six variables are involved, it is very difficult to visualize the effect of all the variables at the same time. The solution also depends on the number of information elements (voltages) available. As observed previously, approximately 18 elements are required for good convergence. For three transmitters, this is achieved by providing six antennae. For the one transmitter configuration, approximately 18 antennae must be used to achieve similar results. Another method to improve the solutions is to take more one initial guess. The Levenberg-Marquart method is applied using each one of "k" initial guesses. The method then converges to as many as "k" solutions. The convergent solution with the lowest residual is then selected. This is called the perturbation technique. There are two ways in which perturbations have been used. In the first technique, the initial guess is randomly incremented within a fixed radius which is called the radius of perturbation (See Figure 2.1). This method is called the multiple-initial-guess method. The term initial guess in the figure means the solution of the previous solved data point. In the figure, three perturbations are shown. Perturbation number one gives "Guess 1" which is the initial guess for that perturbation. The solution obtained with this initial guess is called "solution 1". Similarly, the other two perturbations can be explained. It should be noted that although initial guesses for each perturbation are always within the radius of perturbation, the solution is not restricted by the radius of perturbation. In our computation we take one initial guess with a zero radius of perturbation.

In the second method, the successive perturbations are performed at the previously obtained minimum (See Figure 2.2). This is called the floating initial guess method. Here the initial guess for the first perturbation is obtained from the previously solved data point

similar to the multiple initial guess method. The initial guesses for the successive perturbations are the solutions of the previous perturbations. As shown in the figure "Initial guess 2" is obtained from the "Solution 1" of perturbation one. Similarly, "Initial guess 3" is obtained from "Solution 2" of perturbation number two and so on.
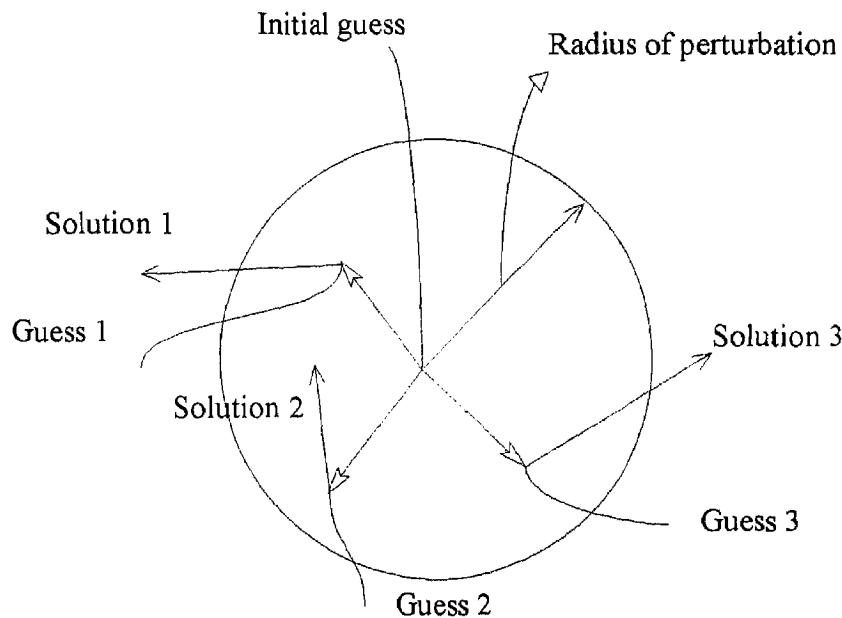
Figure 2.1: Multiple initial guess method for perturbations

In the multiple initial guess method, if the radius of perturbation is small then a global minimum may not be obtained. Also, if the solution is outside the radius of perturbation then it can be difficult to reach a global minimum. A large radius of perturbation may lead to convergence to a local minimum. Also, greater number of perturbations are required as the radius of perturbation is increased. In the floating initial guess method, if the radius of perturbation is small, the solution can get stuck at a prominent local minimum. The radius of perturbation should be made dependent on the velocity of the flow and the speed of the data acquisition system. In these study, six perturbations are used.
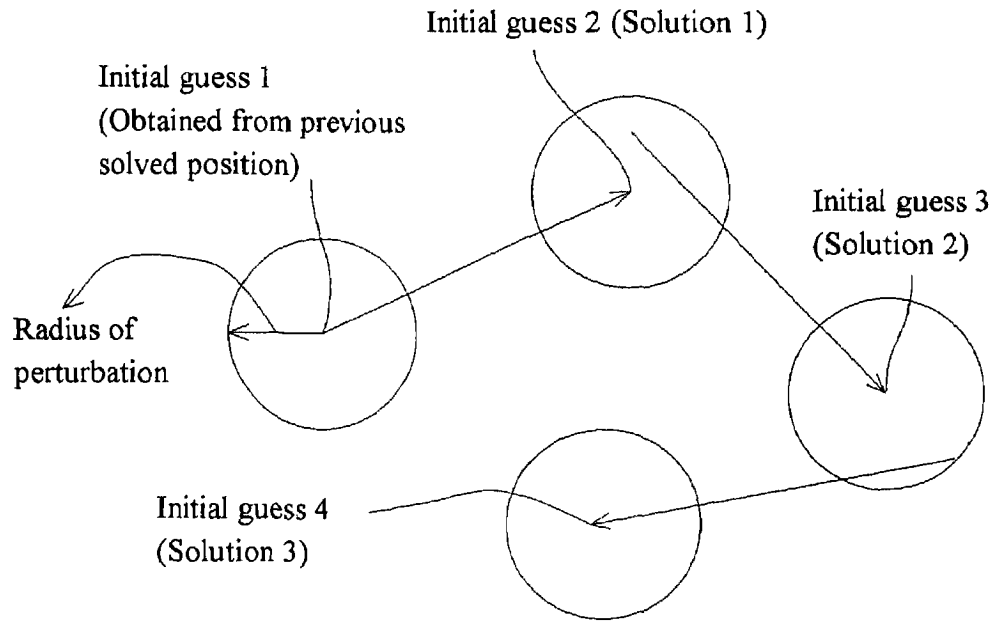
Figure 2.2: Floating initial guess method of perturbations

Table 2.11: Trajectory of Runs 2.1.

	X [inches]	Y [inches]	Z [inches]	Alpha	Beta	Gamma
1	2	5	4	0	0	0
2	3	5.75	4.75	10	15	8
3	4	6.5	5.5	20	30	16
4	5	7.25	6.25	30	45	24
5	6	8	7	40	60	32
6	7	8.75	7.75	50	75	40
7	8	9.5	8.5	60	90	48
8	9	10.25	9.25	70	105	56
9	10	11	10	80	120	64
10	11	11.75	10.75	90	135	72
11	12	12.5	11.5	100	150	80
12	13	13.25	12.25	110	165	88
13	14	14	13	120	180	96
14	15	14.75	13.75	130	195	104
15	16	15.5	14.5	140	210	112
16	17	16.25	15.25	150	225	120
17	18	17	16	160	240	128

Two simulation runs (Run 2.1 & Run 2.2) comparing the results of no perturbation, multiple initial guess method of perturbation, and floating point initial guess method are

presented. The 20" x 20" x 20" model chute with six antennae (explained in Section 2.1) is used. All results are using the three transmitter assembly. The radius of perturbation used for all the results presented below is one inch. Table 2.11 shows the trajectory used in the Run 2.1. This is a straight line trajectory with all the three rotations (α, β, and γ) varying. Table 2.12 summarizes the results obtained without using perturbations. Table C.1 gives the solution for the no perturbation case. Table 2.13 compares the results of multiple initial guess method of perturbation and floating point initial guess method of perturbation. Tables C.2 and C.3 show the solutions of the multiple initial guess method of perturbation and floating initial guess method of perturbation. The average errors in positions for the results with no perturbations are X=0 79, Y=0 57 and Z=0.73. The average errors in positions for the results with the multiple initial guess method of perturbation are X=0.30, Y=0.24 and Z=0.25 and with the floating point initial guess method of perturbation these are X=0.27, Y=0.25 and Z=0.25. There is a significant improvement in results by using perturbations. However, both the multiple initial guess method and floating point initial guess method of perturbation give similar results.

Table 2.12: Results with no perturbations. 5% maximum range random noise, three transmitter configuration. (Run 2.1)

	X inches	Y inches	Z inches	Distance inches
Maximum error	2.48	2.15	3.43	4.44
Average error	0.79	0.57	0.73	1.33
Std. deviation	0.78	0.59	1.01	1.31

Table 2.13: Results with perturbations. 5% maximum range random
noise, three transmitter configuration. 1" radius of perturbation (Run 2.1)

	Multiple Initial Guess Method				Floating Point Initial Guess Method			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.62	0.47	0.66	0.85	0.62	0.47	0.68	0.81
Average error	0.30	0.24	0.25	0.52	0.27	0.25	0.25	0.50
Std. deviation	0.21	0.12	0.20	0.19	0.20	0.11	0.20	0.18

Table 2.14 shows the trajectory for Run 2.2. This is a sinusoidal trajectory with simultaneous rotations in α, β and γ. Table 2.15 and Table 2.16 tabulate the results obtained with no perturbations, multiple initial guess method of perturbation and floating point initial guess method of perturbation. There is a reduction of 0.54 inch in the maximum error in X position by using perturbation. The Y and Z position errors show no improvements in results by using perturbation.

Table 2.14: Trajectory of Runs 2.2.

	X [inches]	Y [inches]	Z [inches]	alpha	beta	gamma
1	0	8.50	11.00	0	0	0
2	1	10.66	12.55	5	7.5	10
3	2	12.61	13.94	10	15	20
4	3	14.16	15.05	15	22.5	30
5	4	15.16	15.76	20	30	40
6	5	15.50	16.00	25	37.5	50
7	6	15.16	15.76	30	45	60
8	7	14.16	15.05	35	52.5	70
9	8	12.61	13.94	40	60	80
10	9	10.66	12.55	45	67.5	90
11	10	8.50	11.00	50	75	100
12	11	6.34	9.45	55	82.5	110
13	12	4.39	8.06	60	90	120
14	13	2.84	6.95	65	97.5	130
15	14	1.84	6.24	70	105	140
16	15	1.50	6.00	75	112.5	150
17	16	1.84	6.24	80	120	160
18	17	2.84	6.95	85	127.5	170
19	18	4.39	8.06	90	135	180
20	19	6.34	9.45	95	142.5	190
21	20	8.50	11.00	100	150	200

Table 2.15: Results with no perturbations. 5% maximum range random noise, three transmitter configuration. (Run 2.2)

	X inches	Y inches	Z inches	Distance inches
Maximum error	1.25	0.50	1.00	1.68
Average error	0.28	0.19	0.36	0.54
Std. deviation	0.31	0.15	0.25	0.37

Table 2.16: Results with perturbations. 5% maximum range random noise, three transmitter configuration. 1″ radius of perturbation. (Run 2.2)

	Multiple Initial Guess Method				Floating Point Initial Guess Method			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.71	0.49	1.09	1.31	0.71	0.50	1.09	1.32
Average error	0.25	0.18	0.36	0.53	0.25	0.19	0.36	0.52
Std. deviation	0.21	0.12	0.29	0.33	0.24	0.12	0.27	0.32

It can be concluded that applying a perturbation method is useful for the solution code to reach acceptable minima. Two reasonable methods were described for perturbation i.e. floating point initial guess method and multiple initial guess method. It is inconclusive whether one method is better as both the methods give similar results.

2.5 Use of Extrapolation in Solution

To reduce the dependency of the initial guesses on the speed of the data acquisition system, a technique of extrapolation is used to make initial guesses. Here, the last two points in the tracking sphere's history are examined and extrapolated to arrive at a new location where the sphere would be if its path were not disturbed (See Ref. [15]). If the speed of the data acquisition system is such that the particle travels only a small distance (0.05 to 0.5 inch) between two samplings, then extrapolation may not help as *lmdif* can easily converge up to a radius of half an inch (This has been noticed in solutions for various trajectories). But in actual flows, the velocity of the flow is not constant. If in

some part of flow, the velocity is so high that the increment between two samplings is around one inch, then *lmdif* may require extrapolation to get to the best solution. The difficulty with the use of the extrapolation technique is that actual flows are not systematic in behavior. Using extrapolation in such cases may result in a poor initial guess for some points and hence poor convergence. In Run 2.3 a trajectory is studied in which the distances between sampling of the particle voltages changes in the X, Y and Z directions. The summary of results for the run with and without extrapolation are shown in Table 2.17. Corresponding plots appear in Figures B.35-46. An analysis of the results in Table 2.16 shows that there is 75% improvement in the maximum error in X position, 50% in Y position, and 33% in Z position with extrapolation. This shows that results are improved using the extrapolation technique.

Table 2.17: Run 2.3 showing the effect of extrapolation on solution.

	Without extrapolation				With extrapolation			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	2.8	2.12	1.09	3.56	0.70	1.0	0.73	1.00
Average error	0.53	0.48	0.33	0.87	0.14	0.21	0.25	0.42
Std. deviation	0.80	0.55	0.33	0.96	0.16	0.22	0.21	0.27

2.6 Effect of *epsfcn* on Convergence

The parameter *epsfcn* is an input variable used in determining a suitable step length for the forward difference approximation used in the Levenberg-Marquart method code. This approximation assumes that the relative errors in the functions are of the order of *epsfcn*. If *epsfcn* is less than machine precision, it is assumed that the relative errors in the functions are of the order of the machine precision, (see Ref. [9]). Table 2.18 shows the results of a simulation with different *epsfcn* values. In the present simulation, perturbations

are not used. Although not presented here, it is found that by using perturbations, all the above cases give the exact solution.

For small values of *epsfcn* it may be difficult for the *lmdif* algorithm to avoid a local minimum. Therefore the solution has larger errors as *epsfcn* is reduced from 1.0 * e - 06 to 1.0 * e-09. When a large value of *epsfcn* such as 0.1 is used, maximum errors in total distance of about 1 inch are observed. With large step lengths, it is possible that *lmdif* goes past the exact solution. A value of *epsfcn* = 0.001 number produces the exact results in this case.

Table 2.18: Effect of *epsfcn*, Results shown are with extrapolation, without perturbations and with zero noise.

	epsfcn = 0.1				epsfcn = 0.001			
	X inches	Y inches	Z inches	Distance inches	X inches	Y inches	Z inches	Distance inches
Maximum error	0.77	0.19	0.58	0.97	0.00	0.00	0.00	0.00
Average error	0.08	0.04	0.07	0.12	0.00	0.00	0.00	0.00
Std. deviation	0.19	0.07	0.14	0.24	0.00	0.00	0.00	0.00
	epsfcn = 1.0 * e - 06				epsfcn = 1.0 * e - 09			
Maximum error	1.43	1.24	3.60	3.78	4.50	2.13	0.00	4.82
Average error	0.21	0.29	0.69	0.86	0.97	0.48	1.20	1.68
Std. deviation	0.35	0.42	1.12	1.19	1.28	0.53	1.10	1.47

2.7 Multiple Solutions

Multiple solutions can arise due to two situations:

1. Multiple solutions due to the antennae configuration.

2. Multiple solutions due to rotations.

Figure A.3 of Appendix A shows the experimental test chute. Consider two points at X=19.0 inches and X=21.0 inches with other positions and orientations the same. The voltages in antennae 2, 4, 5, 6, 7 will be the same for both the positions. The voltages in antennae 1 and 3 will be different but they are weaker than in 2, 4, 5, 6, and 7. Also the

number of elements with same voltage are 5 x 3 = 15 compared to 2 x 3 = 6 with different voltages. Due to this and considering that there is usually some noise in the data, going from X=20.0 inches to X=21.0 inches, there is a chance that the solution will converge at X=19.0 inches.
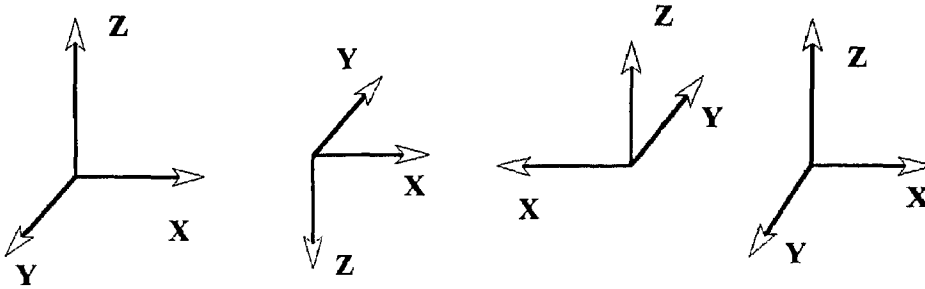

Figure 2.3: Successive rotation about X, Y and Z result in the same position.

As shown in Figure 2.3 rotating the X, Y and Z axes by 180° gets one to the original orientation. The model voltages can be obtained in the form as shown in Figure 2.4a with a symmetry of 180°. Due to the limitation of the data acquisition system, sign changes in the measured voltages cannot be obtained. The measured voltages are of the form shown in the figure 2.4b, and have a symmetry of 90°. The word symmetry here means that voltages between 0-90° and 90-180° have the same magnitude and sign. Therefore, rotating the transmitter 90° results in the exact same value of voltage. This could result in multiple solutions in orientation for the one transmitter assembly.

(a)   (b)
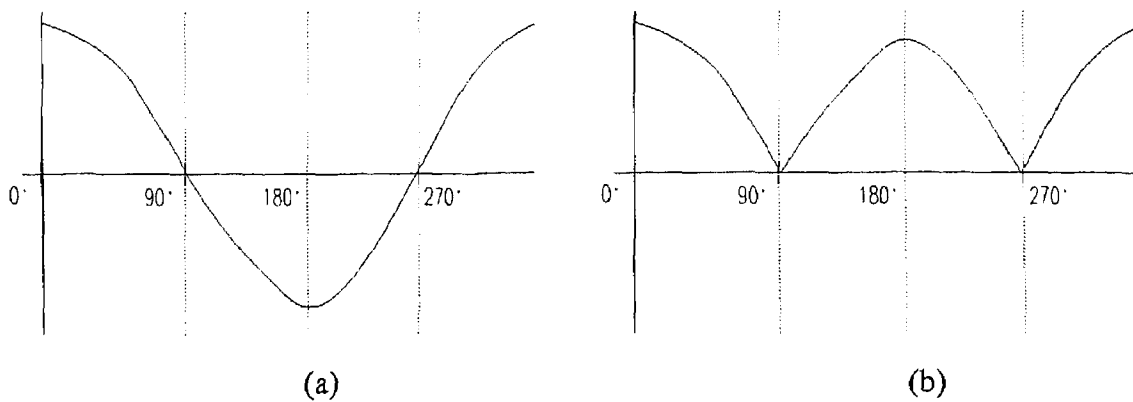
Figure 2.4: (a) Theoretical voltages. (b) Actual measured voltages obtained.

Surprisingly such kinds of multiple solutions rarely occur. More common multiple solutions are those in which the X, Y and Z axes are rotated by 180°. There are many combinations in which these multiple solutions can occur. Unfortunately although these solutions are physically the same, the computer code responds differently when two such solutions are given as initial guesses. However these multiple solutions are easy to detect and correct manually during post processing.

CHAPTER 3

ANTENNAE CONFIGURATION

In this chapter, the effect of aspect ratio, size and shape of the antenna on the model voltages is studied. The errors due to antennae coupling in the measured voltages are studied. It is found that antenna coupling introduces negligible errors if the closest distance between two antennae at least one inch.

3.1 Aspect Ratio, Size & Shape Study of Antennae

3.1.1 Effect of Antennae Shape on Voltage

Determining the position and orientation of the tracking sphere involves solving for six unknowns i.e. X, Y, Z, α, β and γ. If three transmitters are used for tracking, theoretically only two antennae are required for solution. Generally it is observed that 6 to 7 antennae are required for a good solution. The configuration of these antennae, i.e. their location, size, shape and number are very much related to the type of results obtained. Two type of antennae shapes are of interest.

1. Rectangular, and

2. Circular

Circular antennae are more symmetric than square antennae, and could be used in places where the flow area is circular e.g. pipes. In the present study, rectangular antennae are used. As the code has been set up for using rectangular antennae, simulated data for circular antennae is not presented.

3.1.2 Antennae Size & Aspect Ratio Study

Empirically, it is found that smaller antennae with aspect ratios close to 1 work reasonably well experimentally. Two things are to be considered while deciding the antennae size and aspect ratio.

1. The signal levels in the antennae: Figures 3.1 shows the effect of antennae size on the model induced voltage with aspect ratio of one. The voltage induced by a transmitter in the plane of the antenna of dimension 10x10 inches antenna (~0.0046) is approximately double that for 20x20 inches antenna (~0.0023) and about three times that of 30x30 inches antenna (~0.0015). But a 10x10 inch antenna receives lower voltages away from the plane of antenna. A similar trend can be observed in Figure 3.3 in which the trajectory is going away from the antenna plane. Figure 3.2 and Figure 3.4 show plots for trajectories in the antenna plane. It is seen that smaller size antenna have higher voltages in the plane

2. The sensitivity of voltages with respect to changes in position or orientation of the transmitter: Figure 3.1 shows that a 10x10 inch antenna has a larger change in voltage than bigger antennae sizes. The large variations in voltages for the smaller antennae are advantageous to the backward solver. In Figure 3.3, although the 10x20 inches antenna and the 20x40 inches antenna have the same aspect ratio, the voltages in the 10x20 antenna are much higher because of its smaller size.

Figure 3.1 Effect of antenna size on the model voltage. The trajectory is a straight line along the axis of the antenna with one inch increment.

Figure 3.2 Effect of antenna size on the model voltage. The trajectory is a straight line in the plane of the antenna with an increment of one inch.
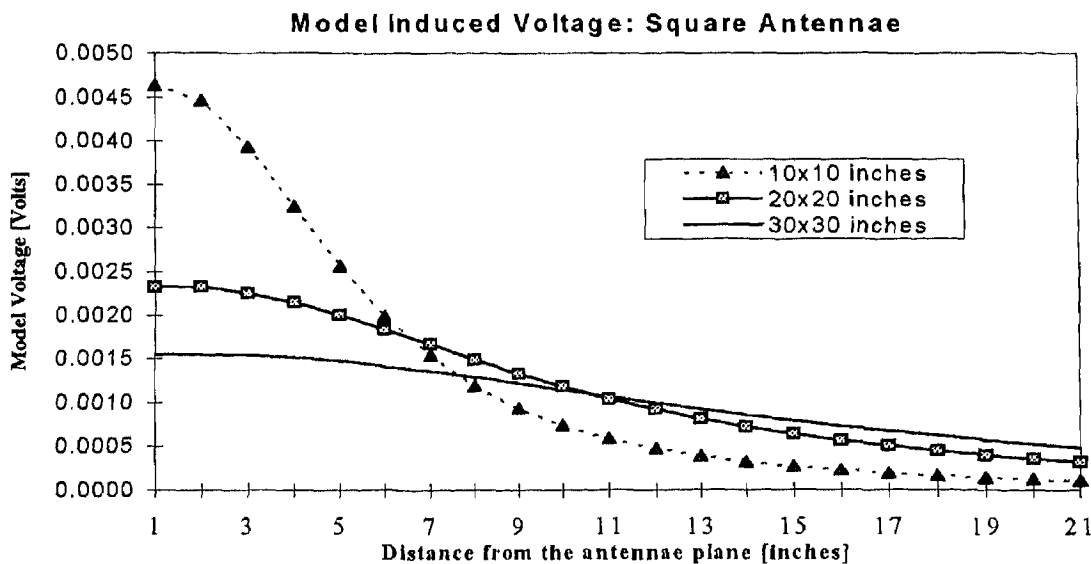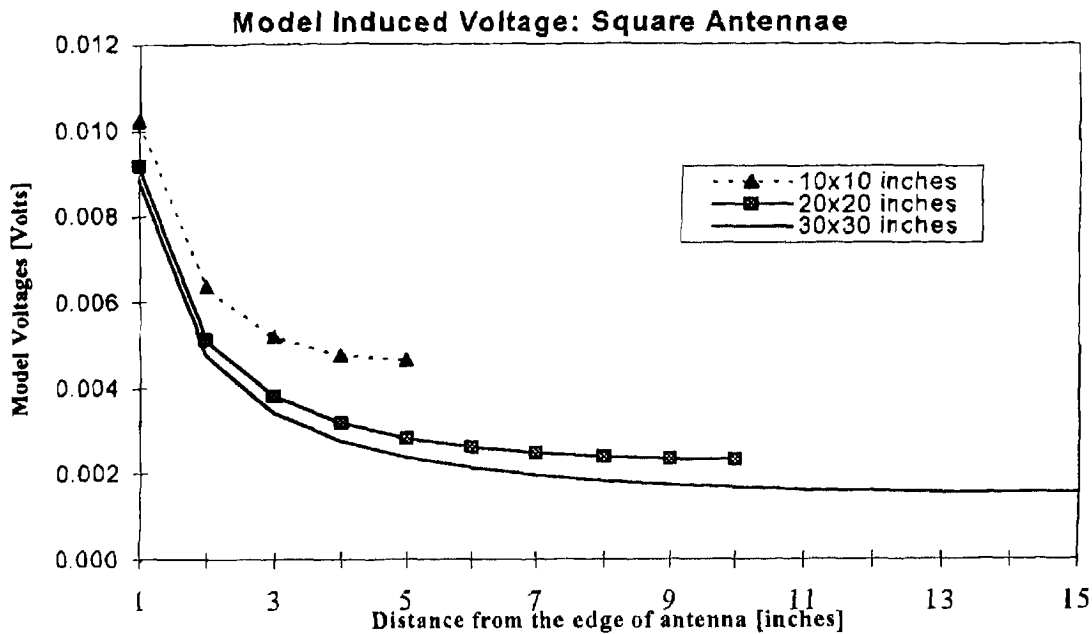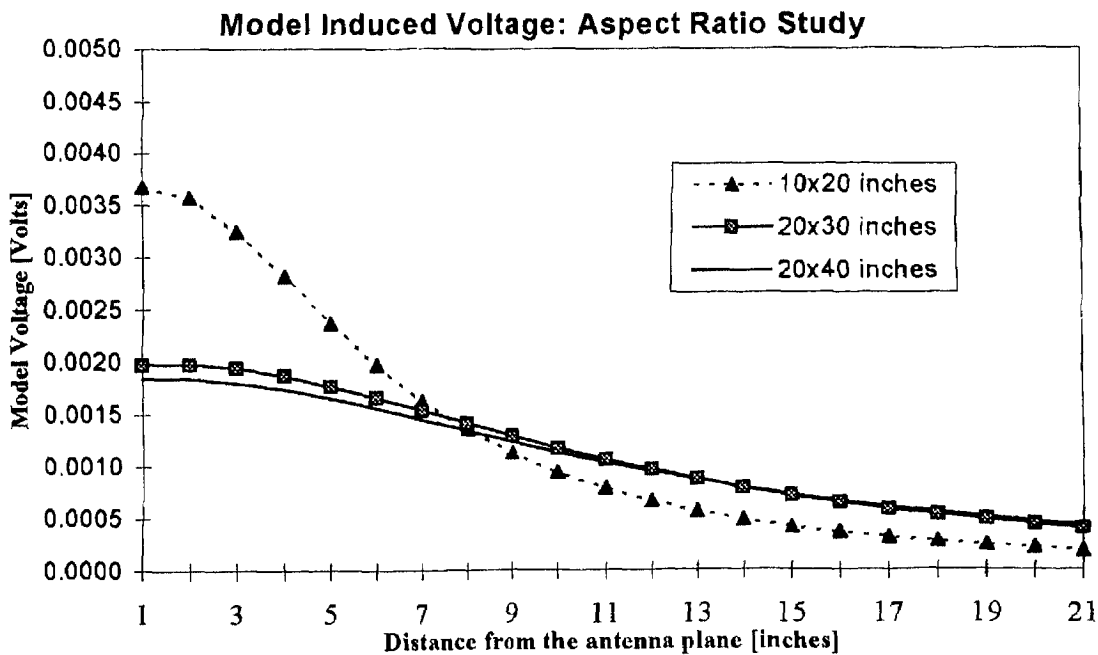

Figure 3.3 Effect of antenna aspect ratio on the model voltage. The trajectory is a straight line along the axis of the antenna with an increment of one inch.

Larger antennae tend to give low signals and thus a small signal to noise ratios. Experimental work has shown that antennae with larger aspect ratios tend to deviate more from the model predicted voltages, resulting in large systematic errors in the final results. Thus the smaller or more square the antennae, the better the results. However, small antennae have high reading in only a small region. Therefore more small antennae are needed to cover the length of the chute.
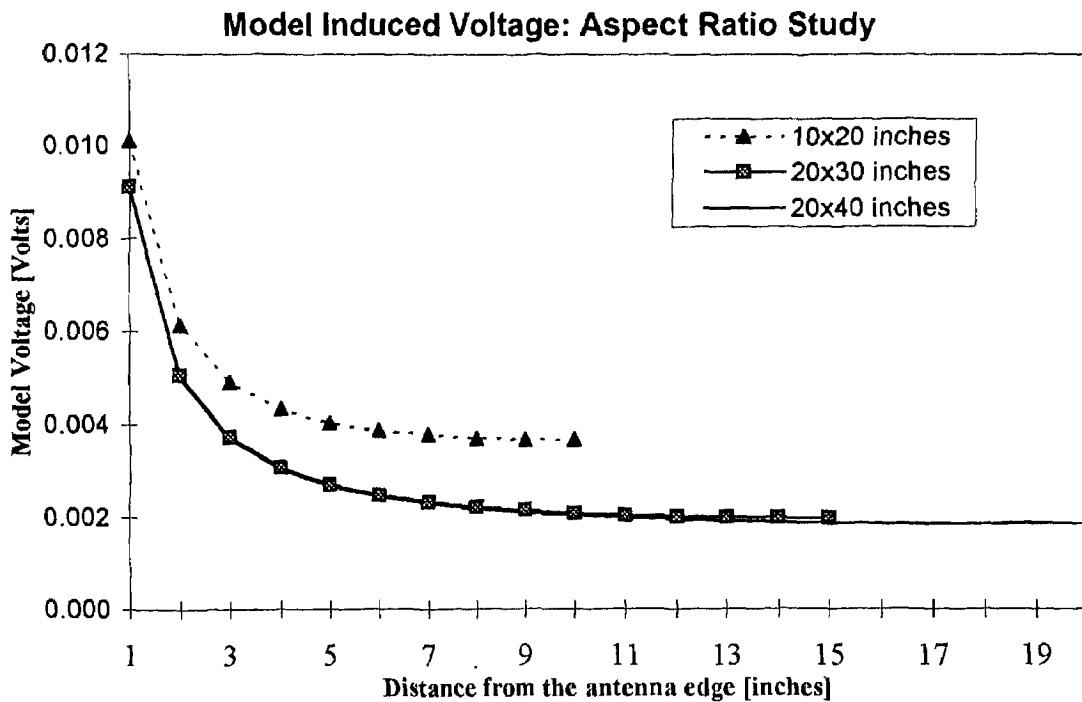
Figure 3.4: Effect of antenna aspect ratio on the model voltage. The trajectory is a straight line in the plane of the antenna with an increment of one inch.

3.2 Antennae Coupling

When a signal is induced in any one antenna, that antenna behaves as a transmitter and induces signals in adjacent antennae. These antennae in turn induce signals in other antennae. In practice, the propagating signal strength drops quickly from antenna to antenna so that only the first transmission is of significance. A detailed description of antennae coupling can be found in Ref. [15]

Antennae coupling depends on two things:

1. Distance between the antennae: Experimental work suggests [Troiano] that coupling signals between adjacent antennae can be neglected if the distance between the antennae is one inch or more.

2. Signal levels in the antennae: As discussed in Ref. [15], the coupling is dependent positively on the signal levels of adjacent antennae and negatively on the signal level in the antenna itself. In other words, when the signals in an antenna are low, it is highly susceptible to coupling from adjacent antennae.
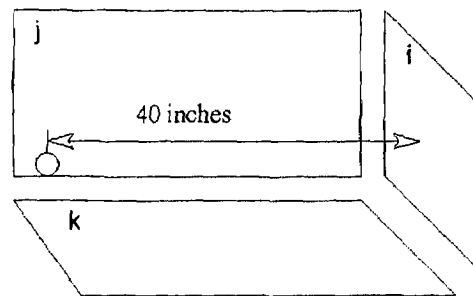
Figure 3.5: Antennae coupling

Consider the worst case scenario for antennae coupling (See Figure 3.5). Antennae j and k have dimensions 40 x 20 inches and antenna i has dimensions 20 x 20 inches. The edges of antenna i are one inch away from the edges of antennae j and k at their closest point. The transmitter is located near the wires and the planes of antennae j and k. It is also approximately 40 inches away from antenna i. The voltage induced in antenna i is found to be about 15 counts which is less than 1.0% of the maximum voltage induced in the i antenna. In most cases this voltage is much less. Also as antenna i has a very low signal, it has a minor contribution towards the solution. This example demonstrates that errors due to antenna coupling are negligible. To get the exact amount of coupling voltage, antennae j and k have to be turned off first while voltage reading in antenna i is taken. This work

has already been done in Ref. [15]. The principal interest in the present study was to look for a significant error in the voltages which may affect the results. The errors due to antenna coupling are not significant. We conclude, therefore, that if the antennae are at least one inch apart then antenna coupling is not significant.

CHAPTER 4

STUDY OF SYSTEMATIC ERRORS

In this chapter the systematic errors in the particle tracking system are studied. That is, we study the difference between model voltages and measured voltages. The errors can result from assumptions made in the model or in the measurements of voltages. Here the errors in the measurement of voltages are discussed.

4.1 Model-Reality Plots

Two types of errors appear in the measurement of any physical entity: random errors and systematic errors. Random errors refer to the unpredictable fluctuations that creep into any measurement. Systematic errors, unlike random errors, are introduced into a system by human biases or imperfections in the equipment used for measuring. Such errors must be corrected (Ref. [15]).

The systematic errors can be attributed to any of the factors given below (Ref. [15]).

- The field of the transmitter is not perfectly symmetric.

- The amplifier boards are not perfectly linear.

- The data acquisition boards are not perfectly linear.

- Magnetic coupling may exist between antennae.

- Incorrect transmitter correction matrix.

- Higher order terms neglected in the derivation of the model may produce systematic errors.

The study of systematic errors in the measured voltage can be found in Ref. [15]. Errors between measured and model voltages as large as 70% (at low signal levels) were found using the one transmitter assembly. The present study is done along the same line with three transmitters. Typical model-reality plots are shown in Figure 4.1a-d. In Figure

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4.1a, the transmitter is initially four inches away from the plane of antenna number four i.e.Y antenna at X=0.0 inches (See Figure A.3) and one inch (perpendicular distance) from the edge. It is then moved towards the center of the antenna. As shown in the figure the errors are largest around X = -18 inches. This is the region where the voltages are highest. For the other regions the errors are almost constants.

In Figure 4.1b, the axis of the transmitter is collinear with the axis of the antenna number 2 i.e. X antenna at X=20 inches. The transmitter travels from within the plane and center of the antenna to a point 20 inches from this plane in a straight line with an increment of one inch. This plot also show maximum deviations near the high voltage regions. As the calibration is done in the plane of antenna, initially the model and reality match well.

In Figure 4.1c, the axis of the transmitter is collinear with the axis of the antenna number 2. The transmitter travels from within the plane and five inches away from the center of the antenna to a point 20 inches from this plane in a straight line with an increment of one inch. In this plot, the errors are the least at Z =10.0 inches. The errors increase as we move towards and away from that point in both directions along the Z axis.

In Figure 4.1d, the transmitter is initially one inch from the edge of and in the plane of antenna number two. The transmitter is then moved towards the center of the antenna, and within its plane. This plot shows large errors when the transmitter is near the antenna wire. As we move away from the antenna wire the errors decrease exponentially.

The calibration point for all four plots is taken at the center of antenna number two. In Figures D.1-D.4 the same four plots with calibration points at a distance fifteen inches away from the antenna plane are shown.
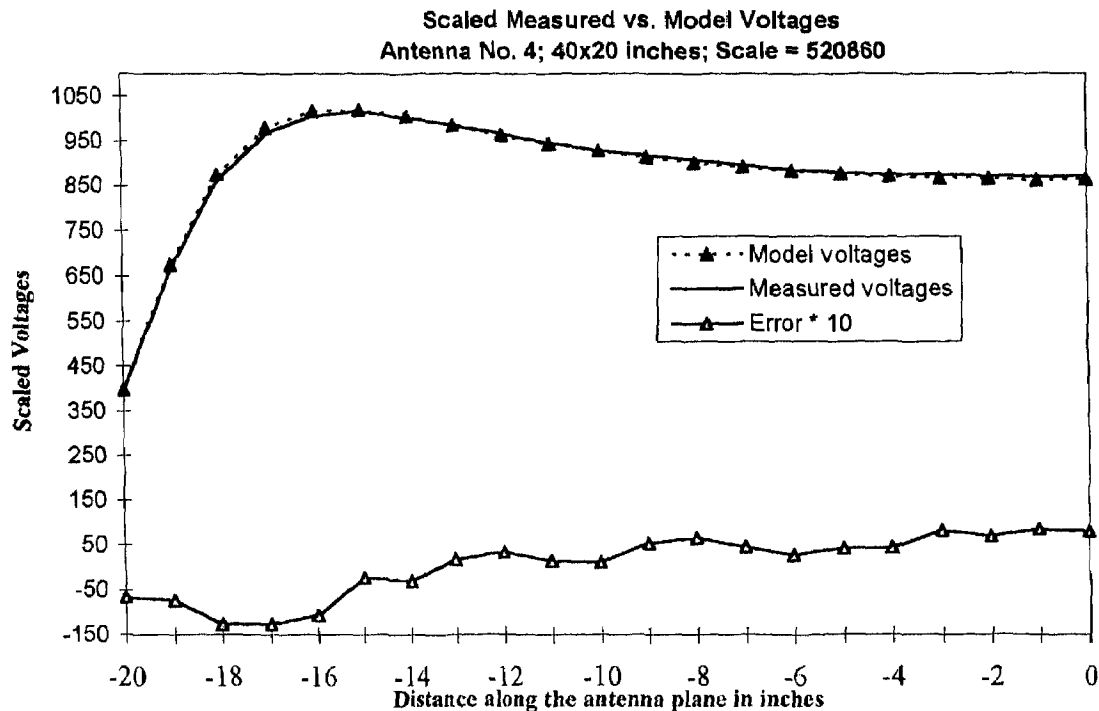
Scaled Measured vs. Model Voltages
Antenna No. 4; 40x20 inches; Scale = 520860

Figure 4.1a: Model-Reality Plot for 2.2 MHz transmitter it is moved along and 4 inch away from the plane of antenna.

The maximum error in Figure 4.1a is 1.5% with the scale as 520860 and 1.9% with the scale of 525353. Figure 4.1b has a maximum error of 1.6% at higher voltages with the scaling factor of 520860 and 2.4% with factor of 525353. In Figure 4.1c the maximum errors are at low voltage readings. The errors are 3.8% for the scaling factor of 520860 and 3.0% with the scaling factor of 525353 respectively. Figure 4.1d shows large deviations in measured voltages near the antenna wire. Errors as high as 13% are found when the transmitter is three inches from the wire and in the plane of antenna with the scale of 520860. The error is only about 2% when the transmitter is located 5 inches from the antenna wire. The maximum error with the scaling factor of 525353 for Figure 4.1d is 13.6%.

Four conclusions can be derived from the above observations:

1. There is a reduction in errors by taking the calibration point at the center of the antenna (since results of Figures 4.1a-d are better than Figures D.1-4).

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2. There is no effect of aspect ratio on the calibration (Figure 4.1a vs. Figure 4.1b).

3. In the region less then 5" from the antenna wire the discrepancy between model and reality is high (Figure 4.1d).

4. In other regions, model and measured voltages match well with maximum error of about ±2% at high signals and ±4% at low signal levels.

Scaled Measured vs. Model Voltages
Antenna No. 2; 20x20 inches; Scale = 520860

Figure 4.1b: Model-Reality Plot for 2.2 MHz transmitter as it is moved away from the plane of antenna in a path coincident with the axis of the antenna.

The accuracy of the measured voltages is about 2%. Therefore the solution may not improve by adjusting the model voltages (i.e. making corrections to the model) in the region where the model-reality match is good. Corrections are required in region less then 5" inches from the antenna wire. These measurements were done for the 20x20 in

Scaled Measured vs. Model Voltages
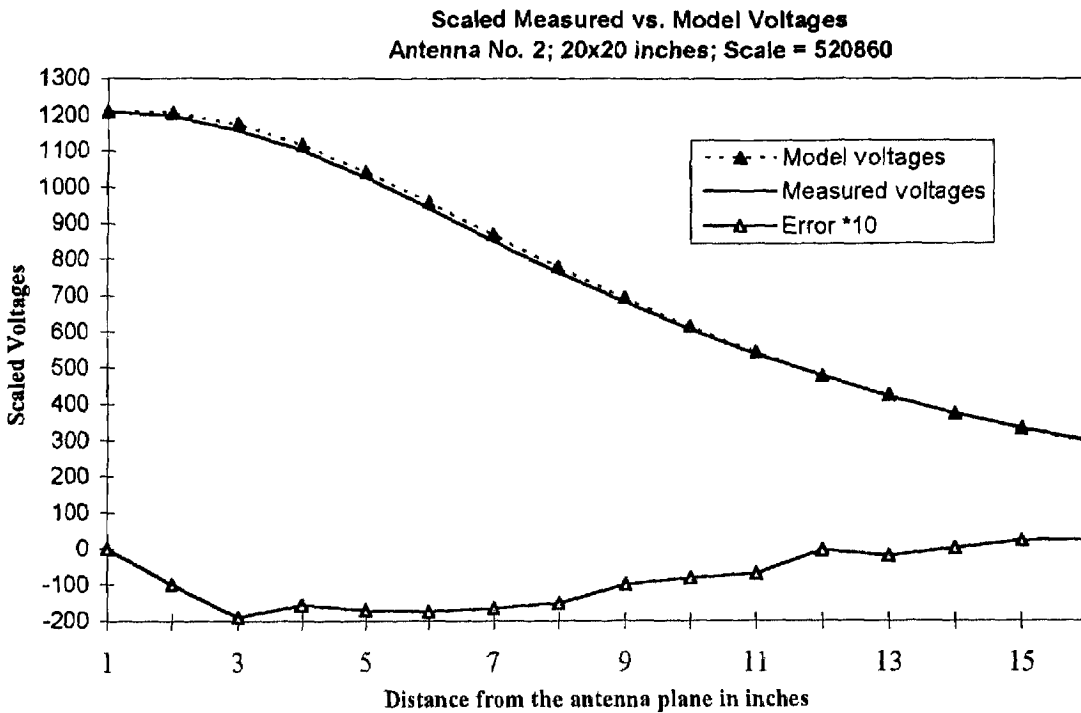**Antenna No. 2; 20x20 inches; Scale = 520860**
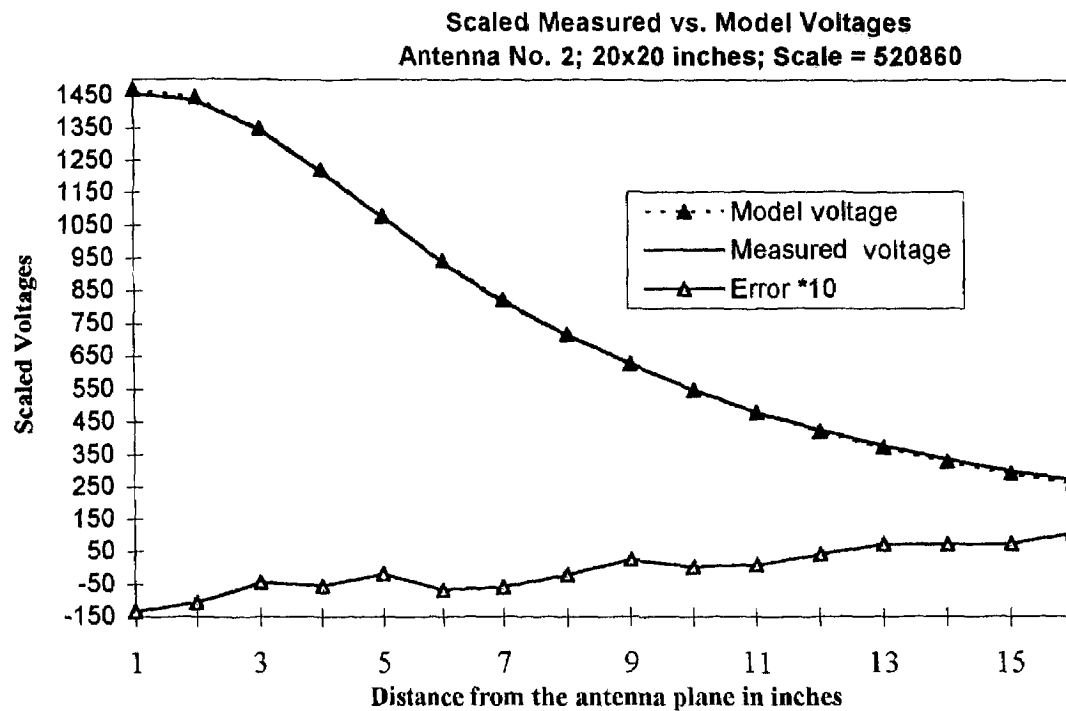
Figure 4.1c: Model-Reality Plot for 2.2 MHz transmitter as the transmitter is moved from the plane of antenna in a path coincident with the axis of the antenna.

ches antenna. Other antenna sizes or aspect ratios may behave differently. The antenna size that will be used in the chute being developed at NJIT for actual experiments is about 18x18 inches. The approximate placement of the 18x18 inch antenna in the chute is shown in the Figure 4.2. Only 2 inches in the Z and 3.5 inches in the Y direction need to be calibrated in the plane of the antenna. Another option is that the measured voltage for a particular antenna lying in the above described region should not be taken into account for the solution, i.e., that antenna is shut-off while the transmitter is in the restricted region (along the lines of the quality information selection scheme suggested by Ref. [1]).

Scaled Measured vs. Model Voltages
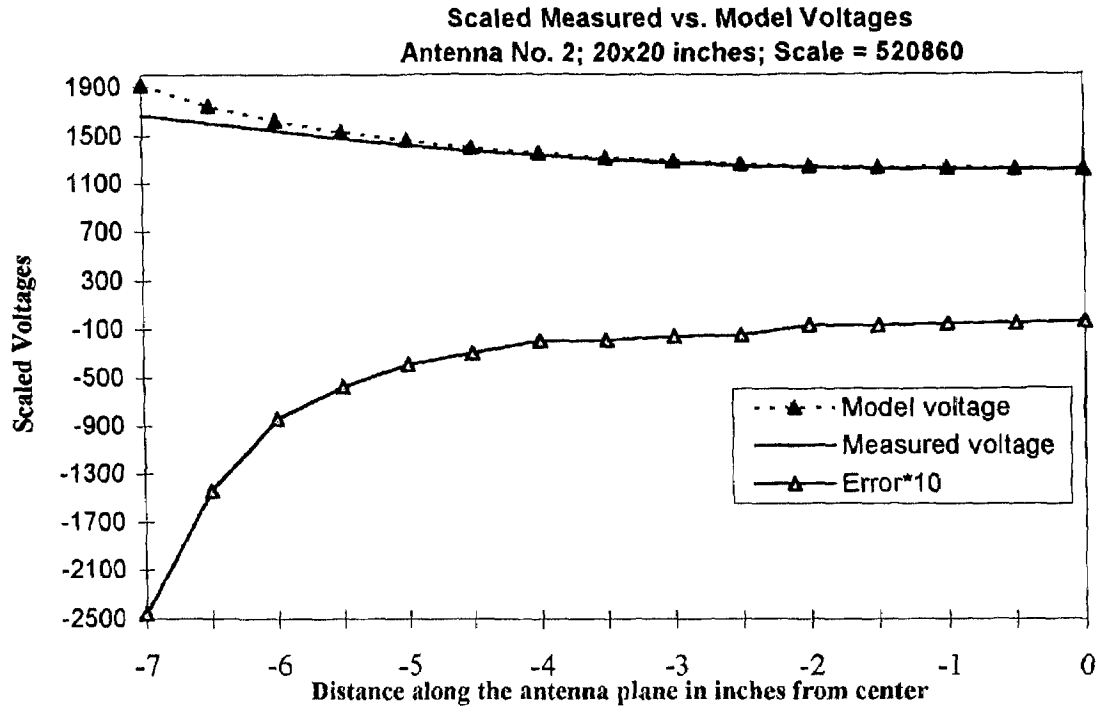**Antenna No. 2; 20x20 inches; Scale = 520860**

Figure 4.1d: Model-Reality Plot for 2.2 MHz transmitter as the transmitter is moved in the plane of antenna.
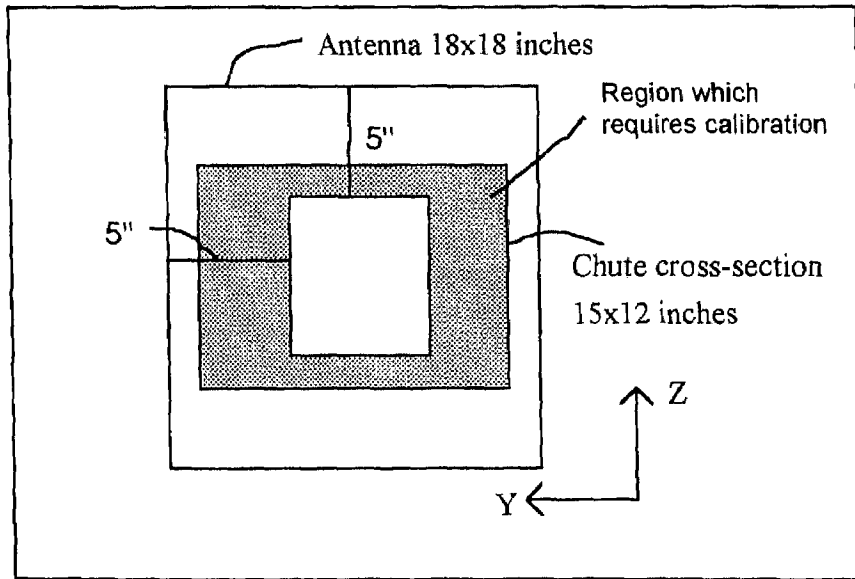

Figure 4.2: X-Antenna configuration in actual chute.

CHAPTER 5

EXPERIMENTAL RESULTS

In this Chapter, the gravitational constant is calculated by dropping the three transmitter assembly in a free fall and solving for the X, Y and Z positions. This type of trajectory is selected to check the speed and accuracy of the fast data acquisition system. The chapter ends with conclusions and suggestions for future work.

5.1 Progress Towards Actual Experiments: Free Fall Test Results

In the experimental results presented so far, the transmitter was kept stationary while taking the measurements. This was done so that the position and the orientation are known and can be compared with the measurements obtained.

However, in actual flow experiments, the sphere containing the transmitter will be moving at high velocity. The purpose of the particle tracking system is to trace the position and orientation of the particle in a continuous flow. It was necessary to test the performance of the tracking system with a fast data acquisition rate. A fast controlled trajectory had to be developed. One option was to pack the transmitter in a cylinder which is then rolled down an inclined surface. Here it was difficult to measure the velocity of the rolling cylinder. Also, obtaining a working test setup was expensive and time consuming. This idea therefore was abandoned. The other option was to drop the sphere containing the transmitter vertically keeping the chute vertical. The rate at which the sphere falls in free fall is known and thereby it was possible to calculate the gravitational constant. The problem with this method was that once the sphere was in free fall there was no restriction on its rotation.

The results of the drop test are presented in Table 5.1. Figure 5.1 shows the plot of measured versus actual position in X. Plots of X, Y, Z positions and the rotation are

shown in Figures D.5-D.10. The exact actual rotations are not known and therefore actual rotations are not presented in the plots. The gravitational constant was found to be 383.97 inch/s^2. The error is 0.63%. The maximum error in X is 0.441 inch, in Y it is 0.58 inch and in Z is 0.48 inch. The errors are similar to those obtained with the slow data acquisition rate. The mean errors in X, Y and Z are less than 0.1 inch. These are slightly better than those obtained with the slow data acquisition rate. The reason for improvement in the results could be because of very low measurement error, more data, and very small increments between each pair of data points.

Table 5.1: Experimental results: Drop Test

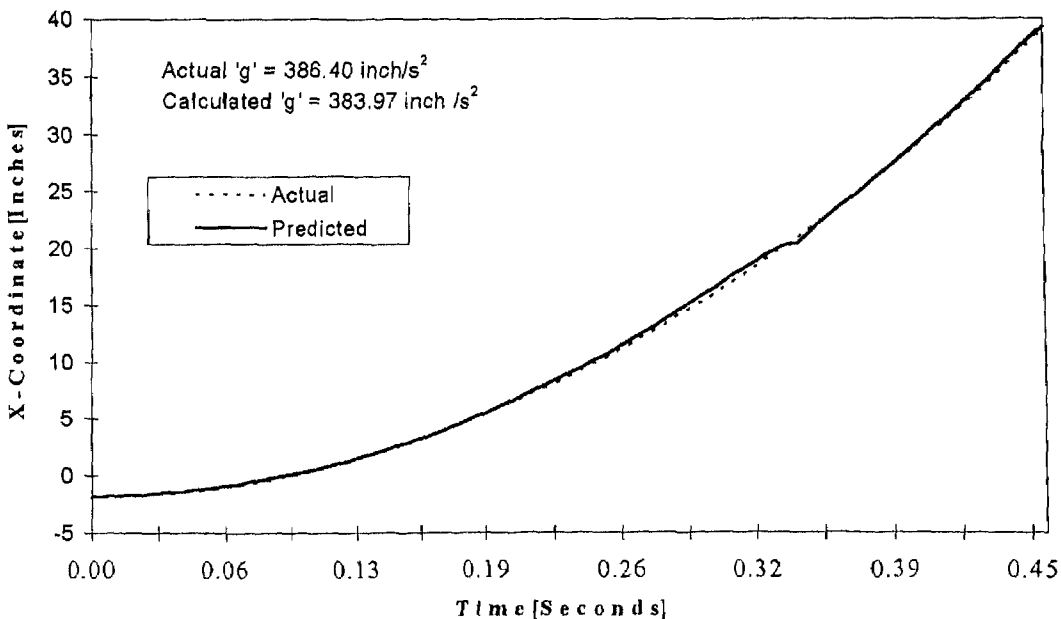|	Drop Test Results		
	X inches	Y inches	Z inches
Maximum error	0.44	0.58	0.48
Average error	0.11	0.15	0.10
Std. deviation	0.10	0.13	0.10

Figure 5.1: Checking the accuracy of the tracking system by calculating the gravitational constant

5.2 Conclusions

We have studied some of the analysis techniques used in the particle tracking system for dry particulate flows. In the one transmitter assembly vs. three transmitter assembly comparison we found that there is an improvement in results with the increase in the number of antennae. Between 15 to 20 antennae are requires to obtain a solution with average errors less than one inch for the single transmitter assembly while the three transmitter assembly require only six antennae for similar results. It was also observed that a three transmitter assembly always gives better results than a single transmitter assembly with the same number of antennae.

It is observed in both simulations as well as actual experiments that doubling the data in the $fvec$ (Refer to Section 2.2) results in improvement in the results for a single transmitter assembly as well as for a three transmitter assembly.

Increasing in the level of noise has similar effects on a one transmitter assembly and a three transmitter assembly. Increasing the noise from 5% to 10% (Refer Section 2.3) doubles the maximum and average errors in positions for both the one transmitter assembly and the three transmitter assembly. Comparing the errors obtained with different noise levels in simulations and errors obtained in actual experiments we can conclude that there are less than 5% random and systematic errors in the measured voltages.

Perturbations help the solution code to reach a global minimum. The floating point initial guess method and multiple initial guess method of perturbations (Refer to Section 2.4) give similar results. A large radius of perturbation can lead to an incorrect solution. A radius of perturbation of one inch is recommended.

In the flows where the velocities are not constant, using an extrapolation technique improves the results.

The value 0.001 of the step length parameter epsfcn of *lmdif* is observed to be optimal for the three transmitter assembly in simulation.

Small antennae give larger voltage signals compared to larger antennae. Signal levels in a 10"x10" antenna are found to be double those in a 20"x20" antenna. Smaller antennae also have the benefit of having larger variations in the voltages. This can improve computational results. The limitation of smaller antennae is that the signal gets weaker as we move away from the plane of antenna. Therefore more such antennae are required.

We can conclude that errors due to antenna coupling are negligible if the closest distance between two antennae is at least one inch.

It is found that there is a reduction in errors by taking the calibration point at the center of the antenna. There is no effect of aspect ratio on the calibration. It is observed that in the regions less then 5" (in the plane of antenn) from the antenna wire the discrepancy between the model and reality is high.

In the experimental results of the drop test, maximum error in total distance of less than 0.5" and average error in total distance of less than 0.3" are obtained.

Further work need to be done on the virtual antennae concept using experimental data. The suggested modifications in the 27-point correction scheme and the calibration points can be implemented. Modifications in the code are required to take care of the multiple solutions in orientations.
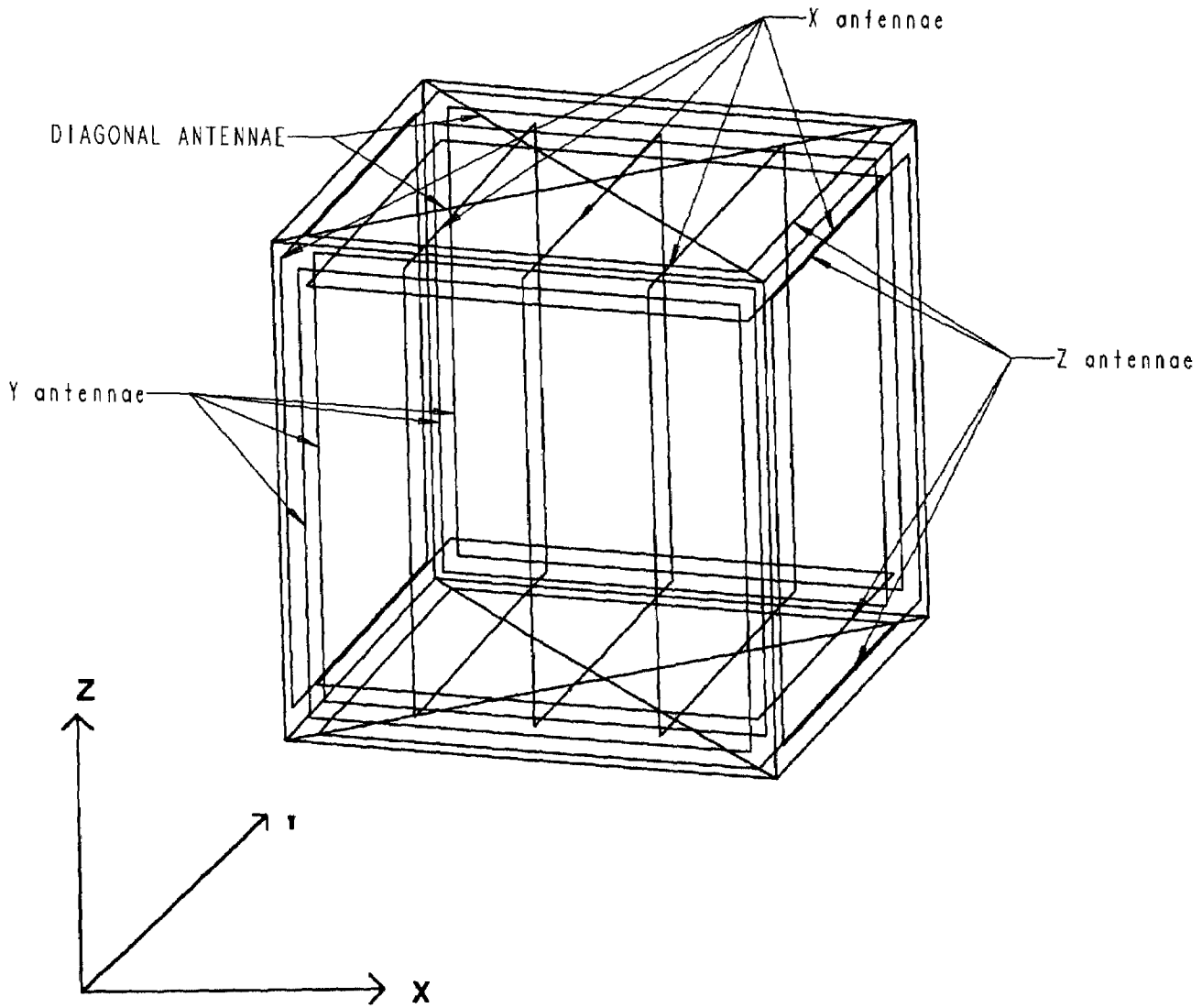
Figure A.1: 20" x 20" x 20" simulation chute with 15 antennae.
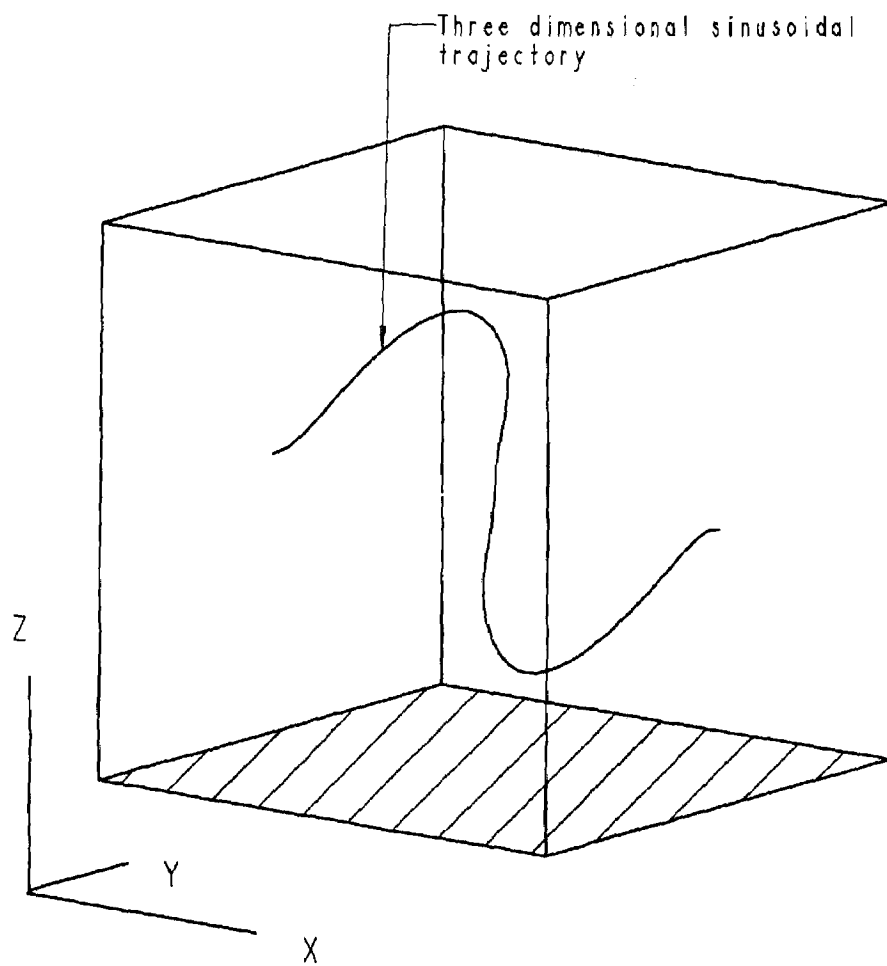
Figure A.2: Three Dimensional Sine Curve Trajectory.
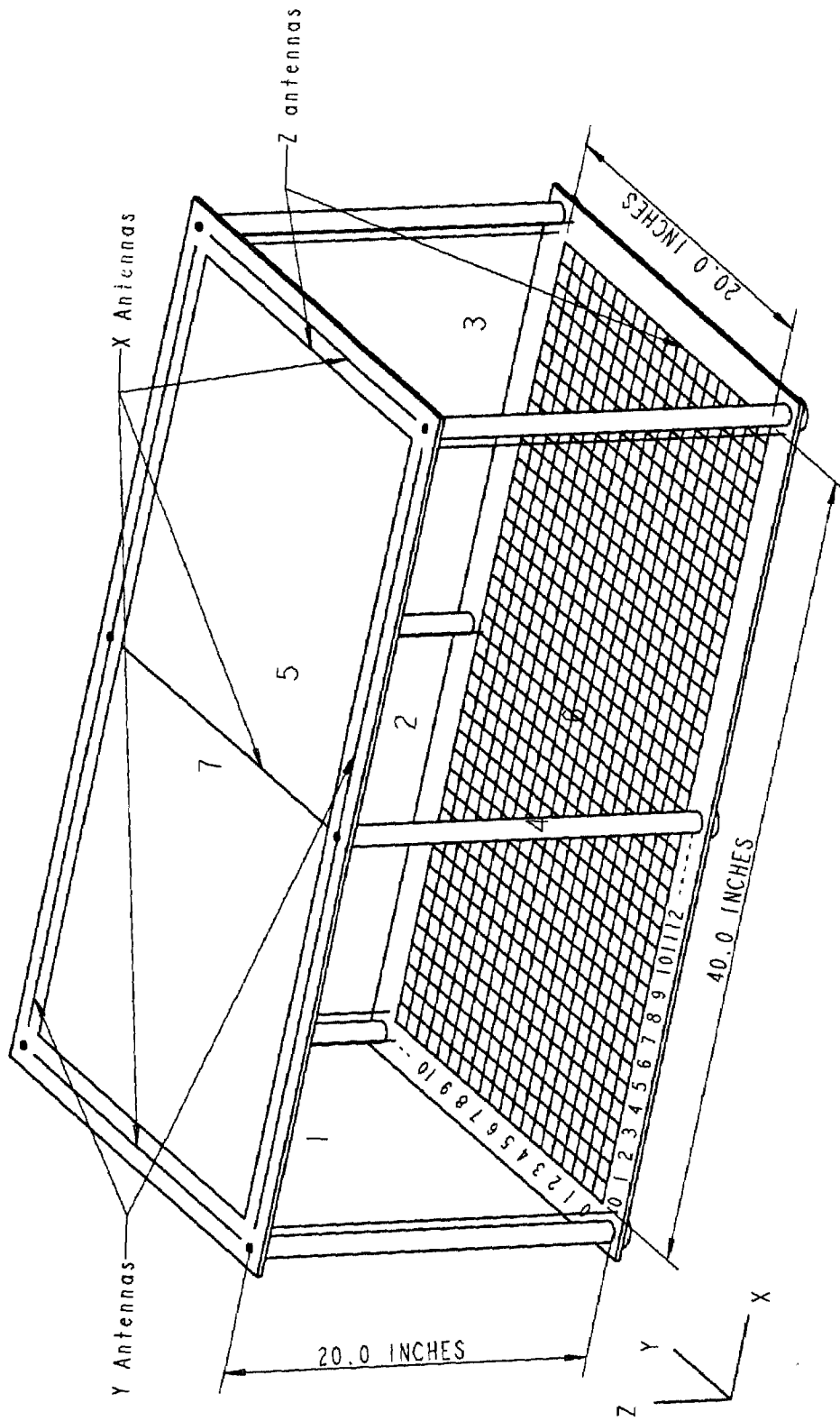
Figure A.3: Experimental Chute
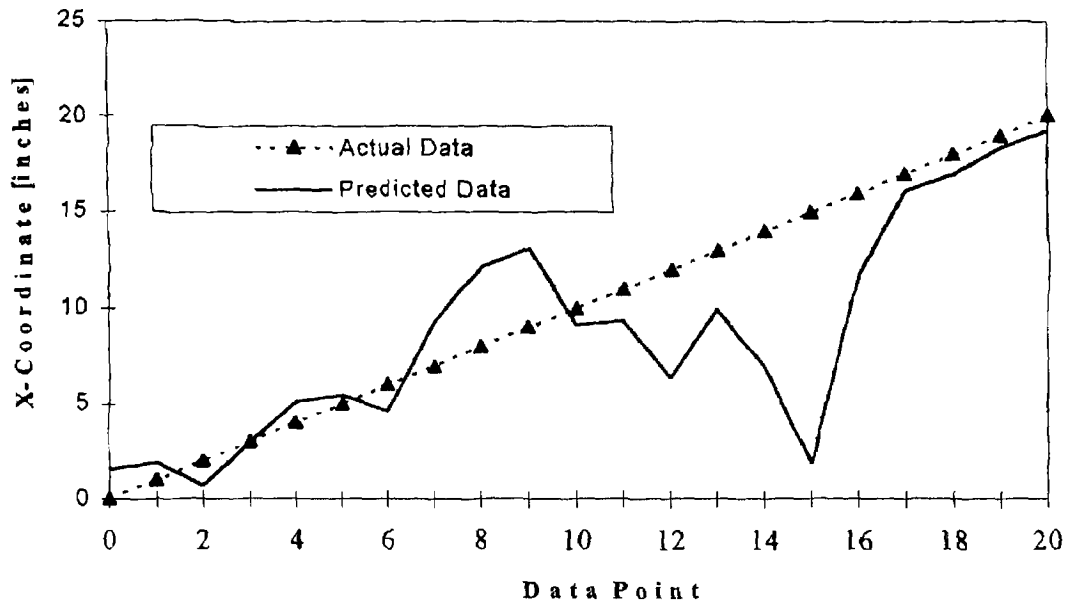
Figure B.1: 1-T & Six antennae. 5% Noise.
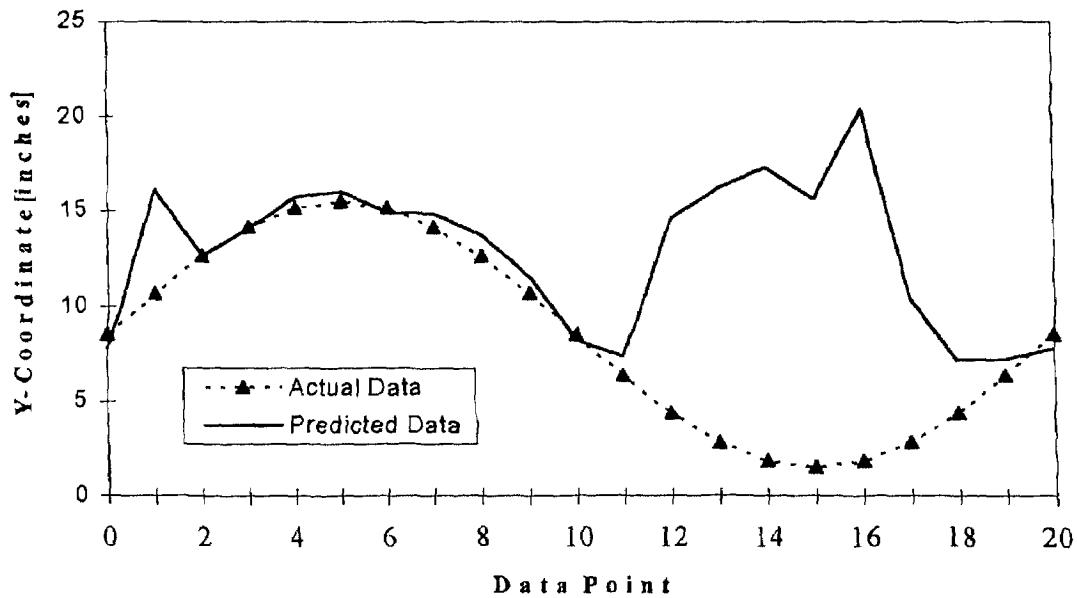
Figure B.2: 1-T & Six antennae. 5% Noise.
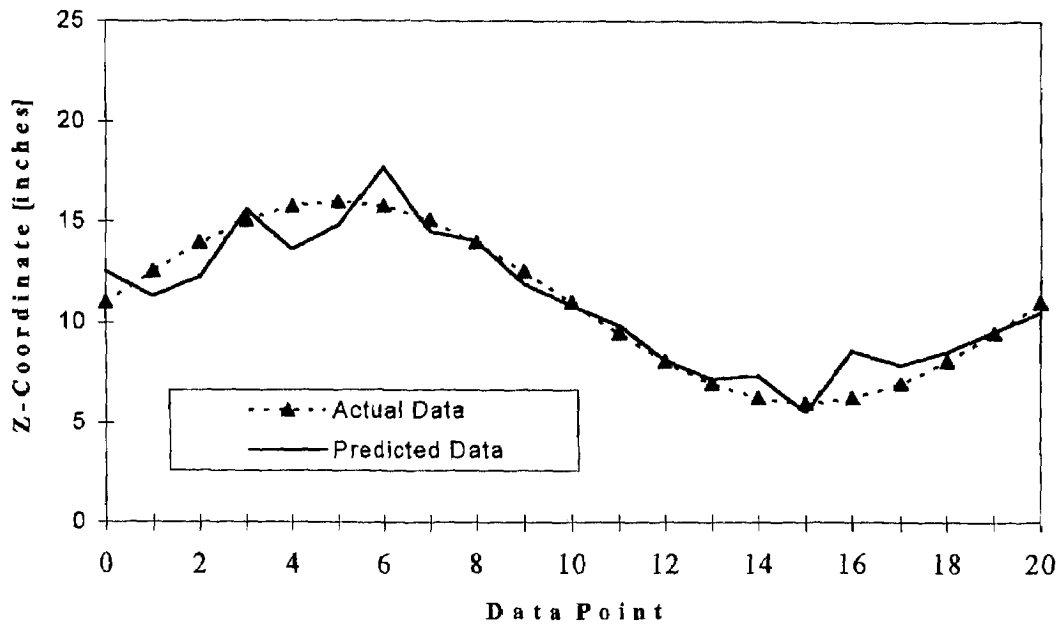
Figure B.3: 1-T & Six antennae. 5% Noise.
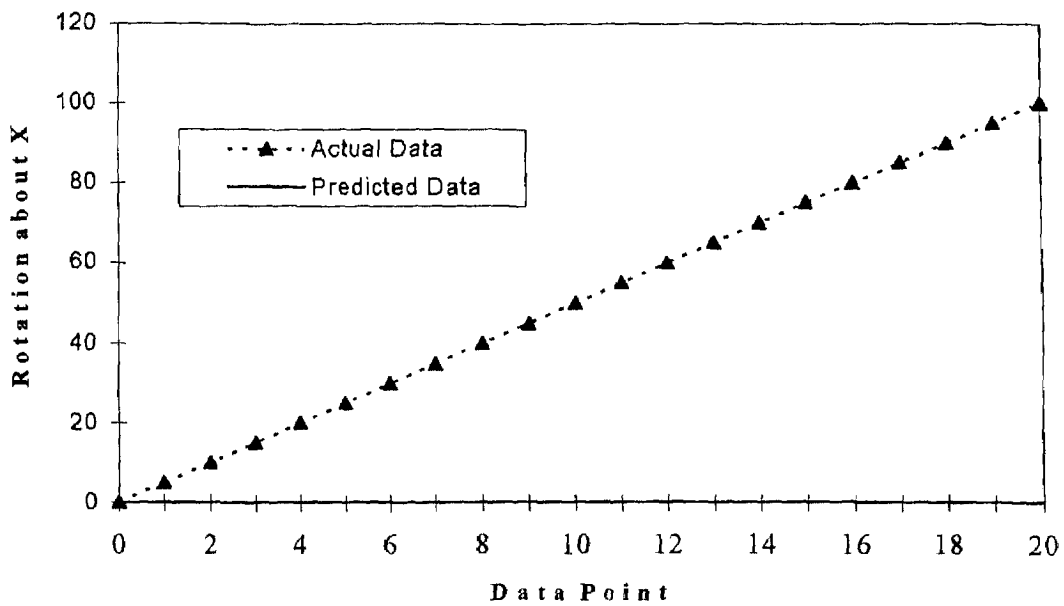
Figure B.4: 1-T & Six antennae. 5% Noise.
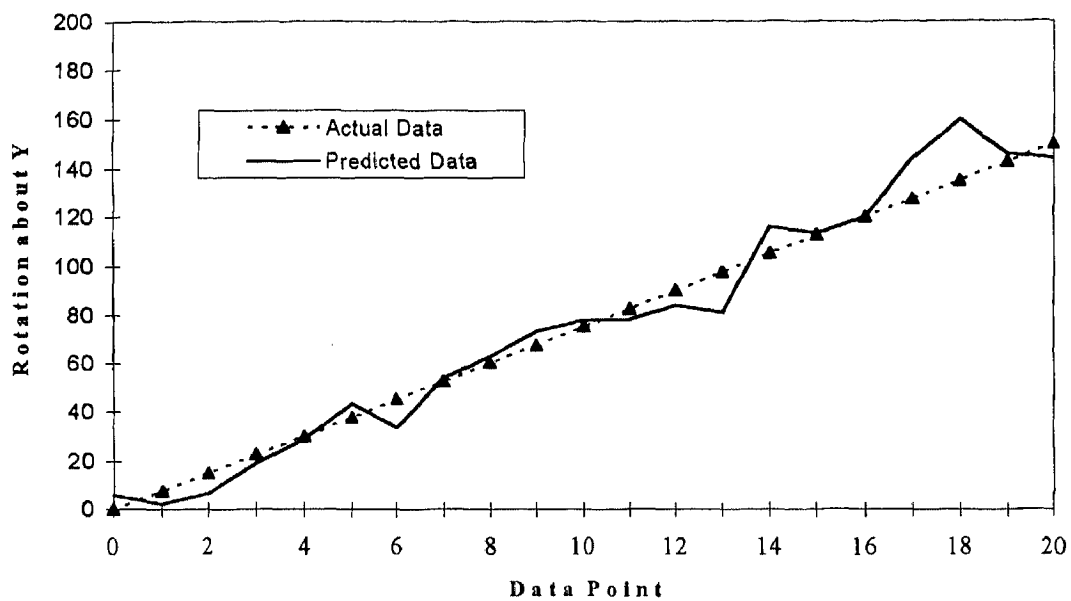
Figure B.5: 1-T & Six antennae. 5% Noise.
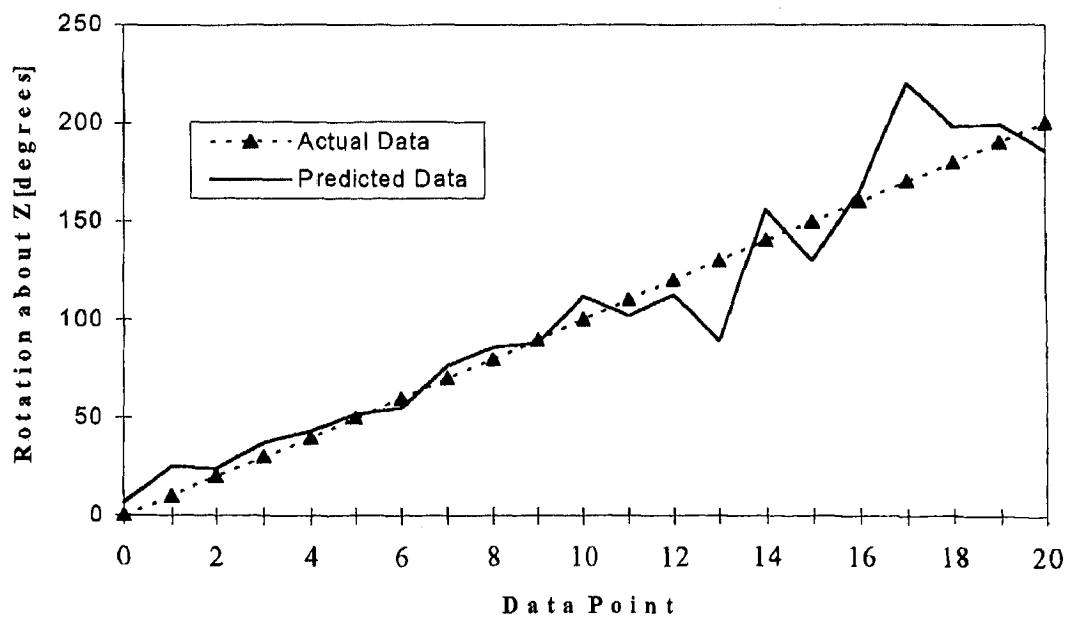
Figure B.6: 1-T & Six antennae. 5% Noise.
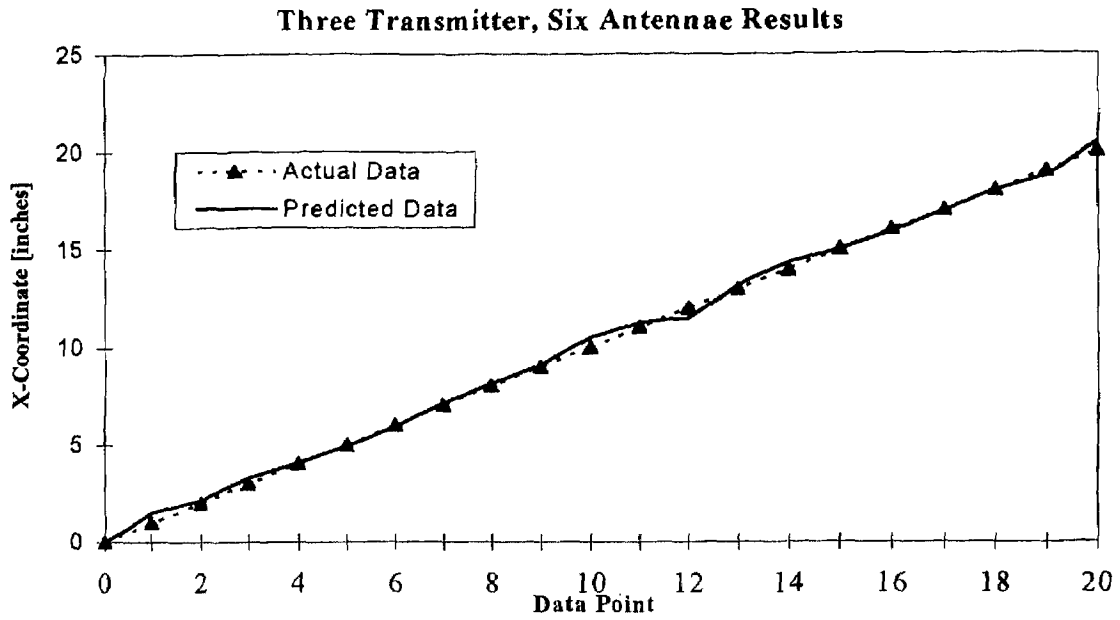
Figure B.7: 3-T & Six antennae. 5% Noise.
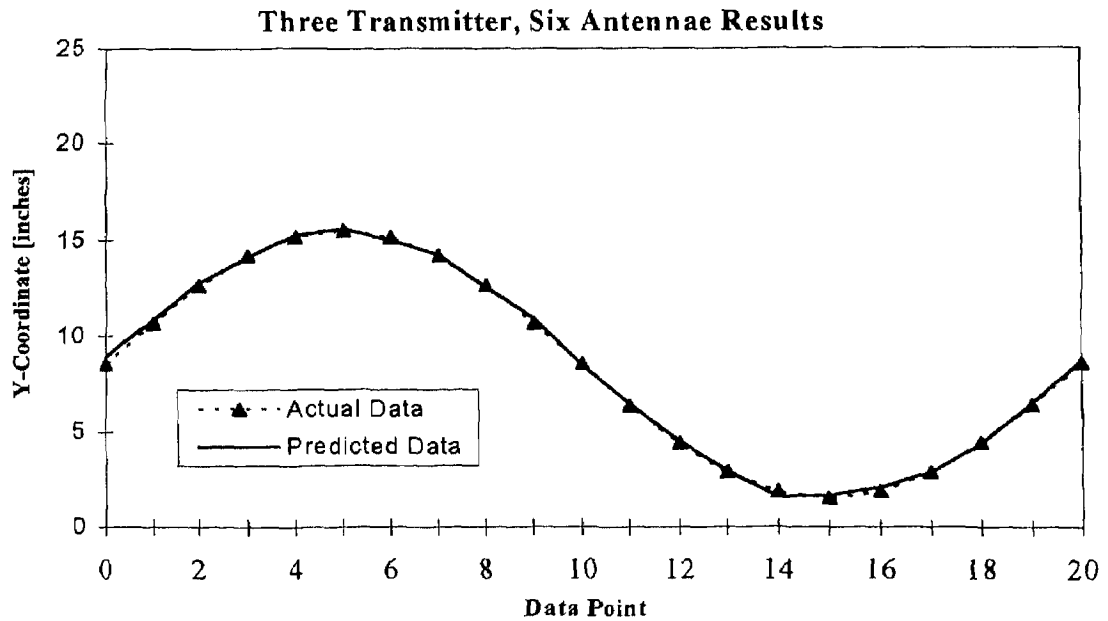
Figure B.8: 3-T & Six antennae. 5% Noise.

Three Transmitter, Six Antennae Results
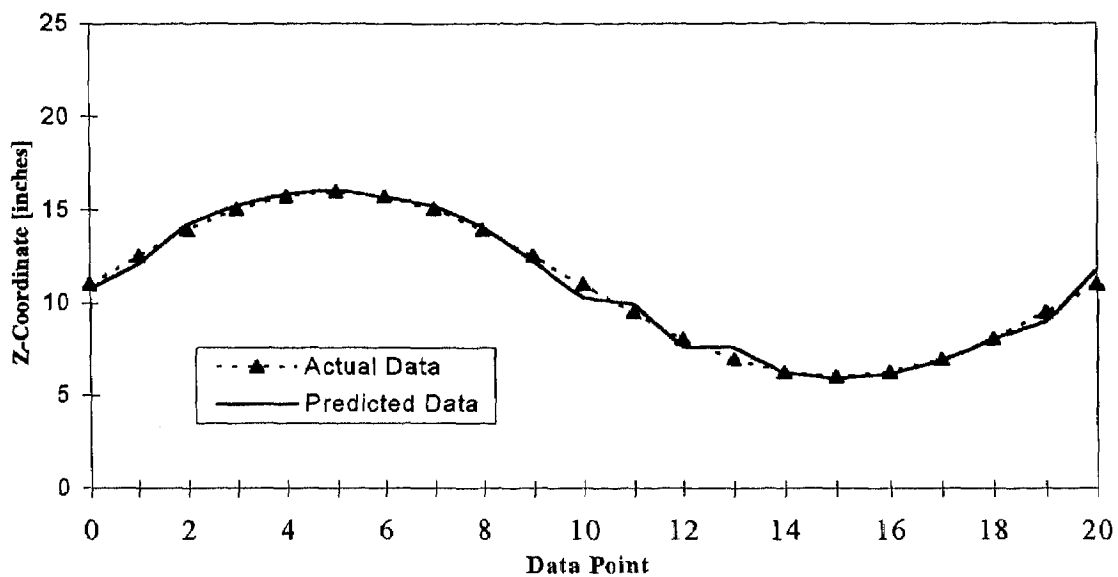
Figure B.9: 3-T & Six antennae. 5% Noise.

Three Transmitter, Six Antennae Results
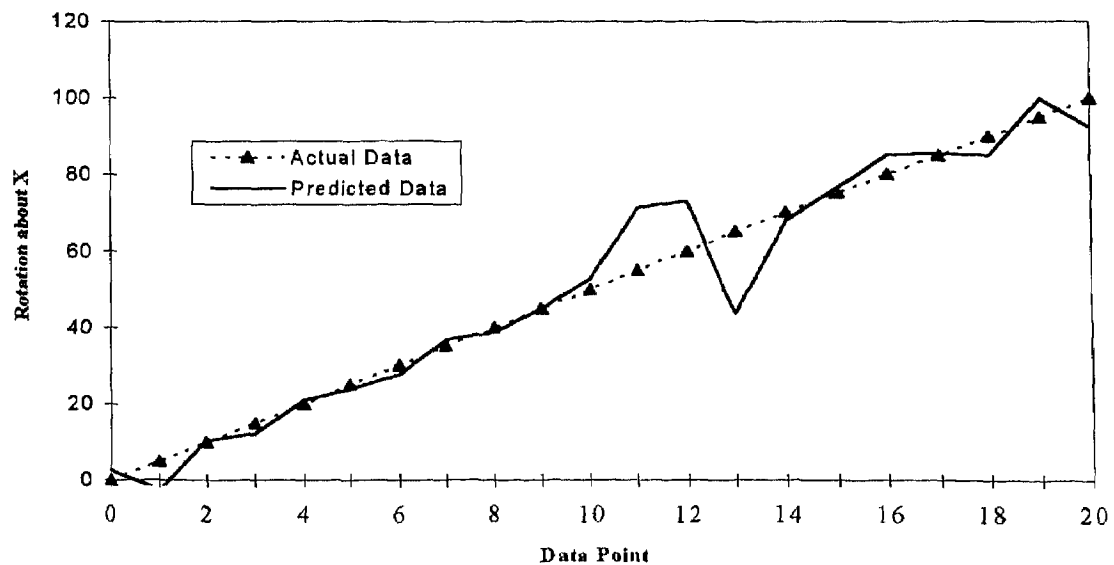
Figure B.10: 3-T & Six antennae. 5% Noise.

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Three Transmitter, Six Antennae Results
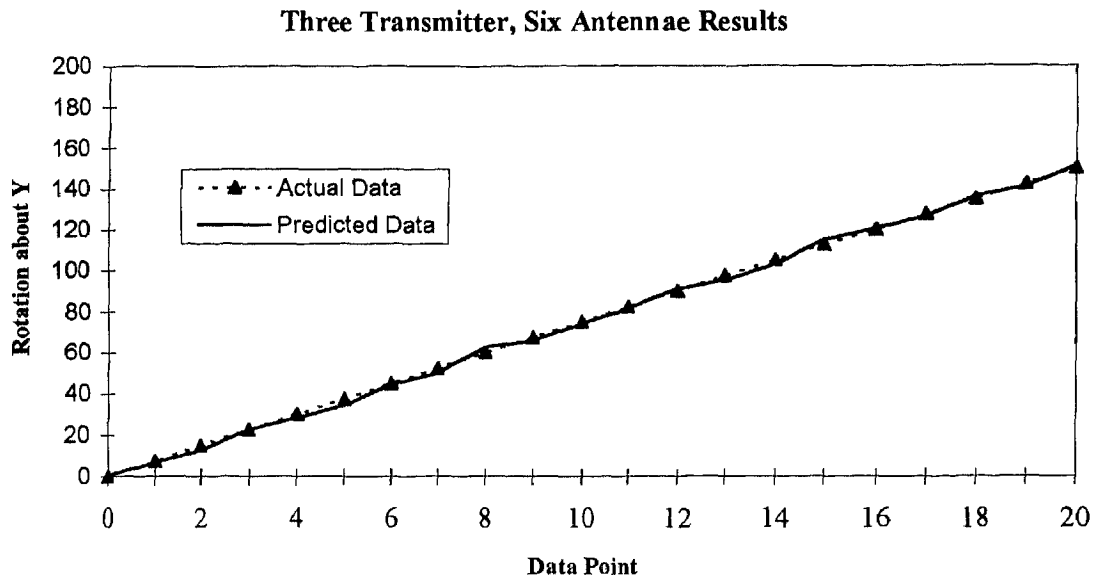
Figure B.11: 3-T & Six antennae. 5% Noise.

Three Transmitter, Six Antennae Results
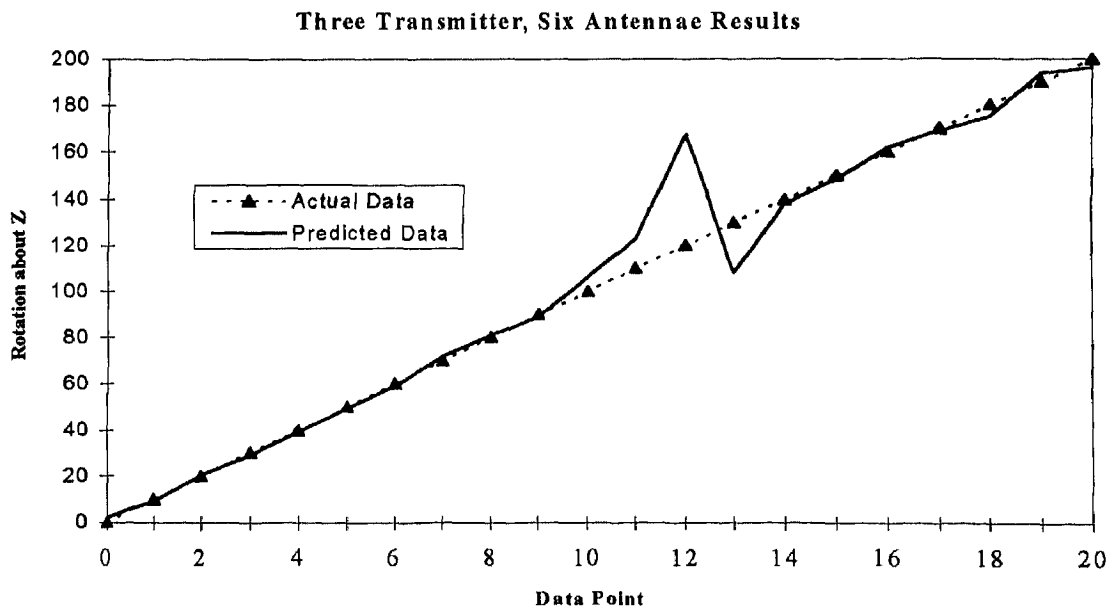
Figure B.12: 3-T & Six antennae. 5% Noise.

One Transmitter, 17 Antennae Results
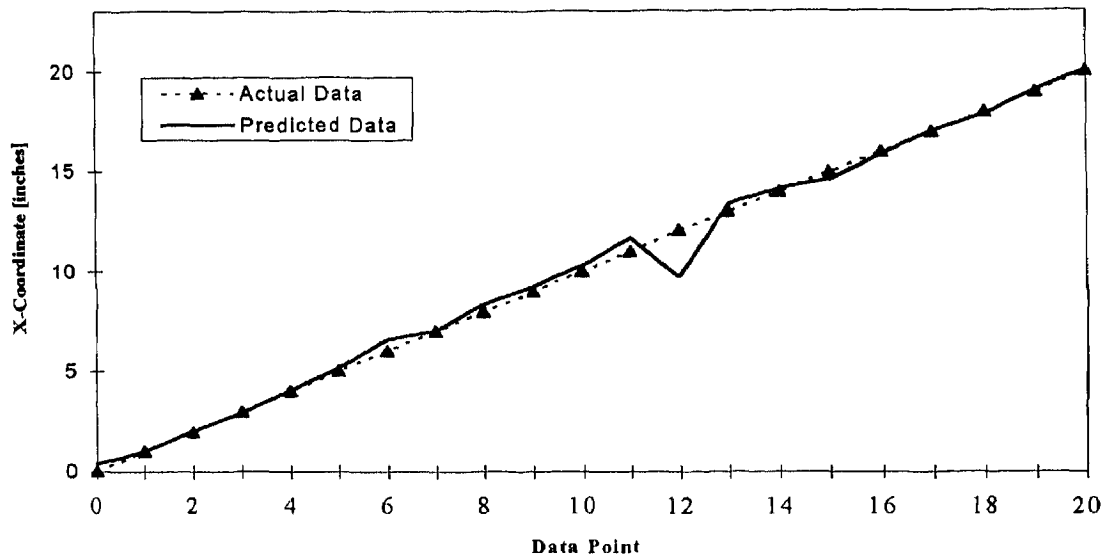
Figure B.13: 1-T & 17 antennae. 5% Noise.

One Transmitter, 17 Antennae Results
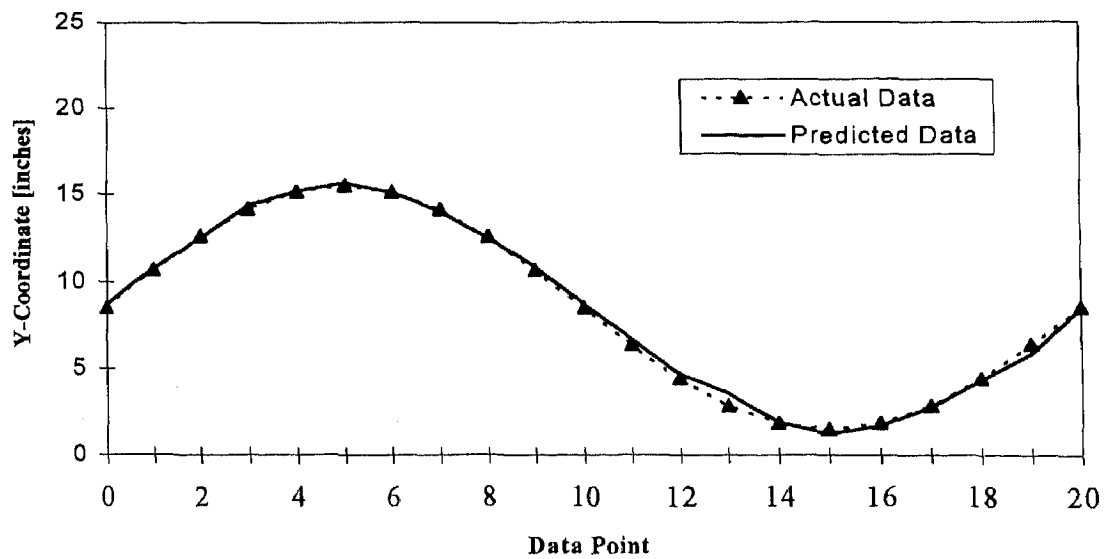
Figure B.14: 1-T & 17 antennae. 5% Noise.

One Transmitter, 17 Antennae Results
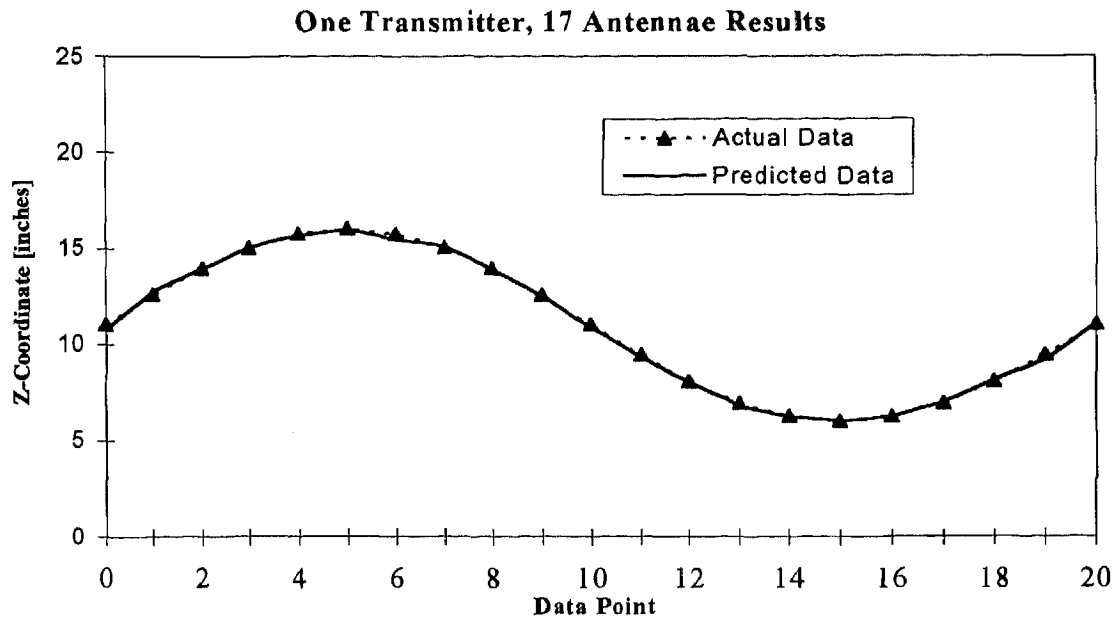
Figure B.15: 1-T & 17 antennae. 5% Noise.

One Transmitter, 17 Antennae Results
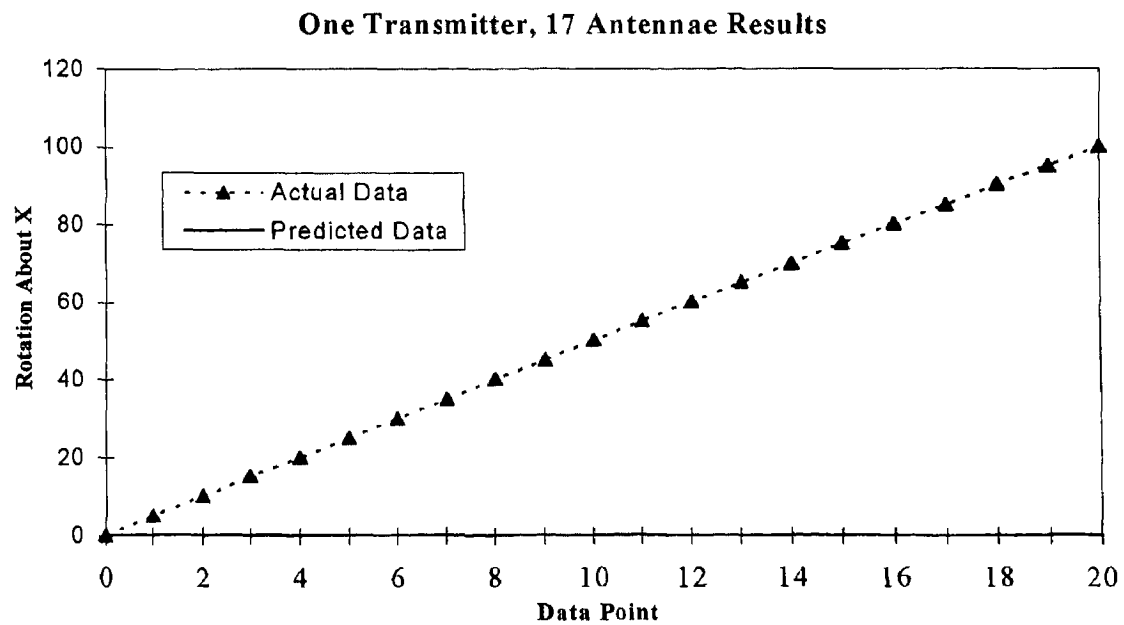
Figure B.16: 1-T & 17 antennae. 5% Noise.
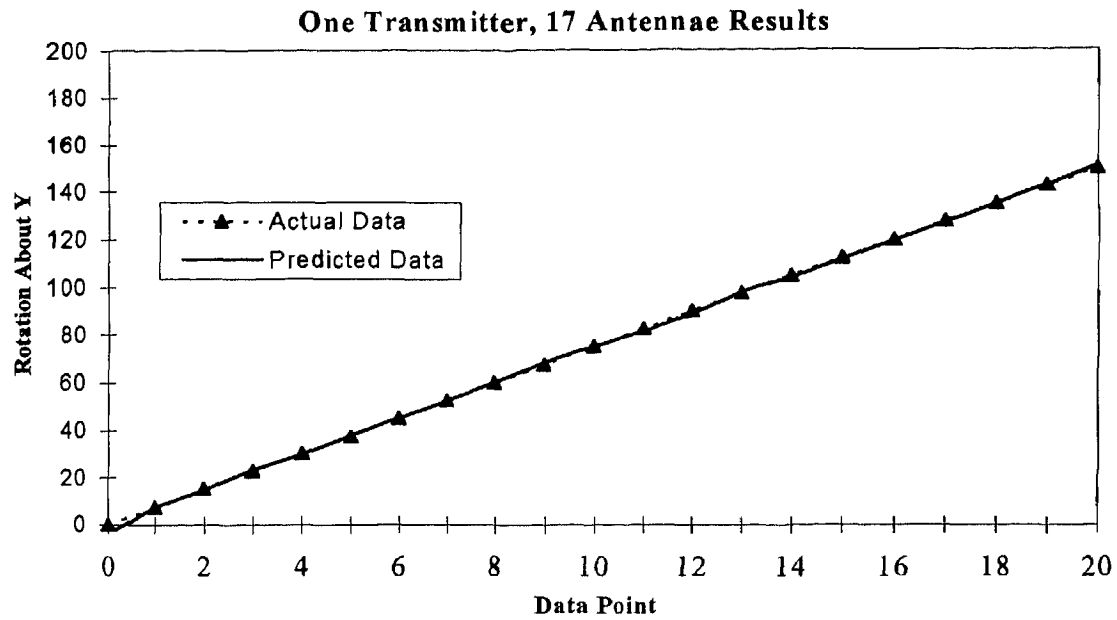
Figure B.17: 1-T & 17 antennae. 5% Noise.
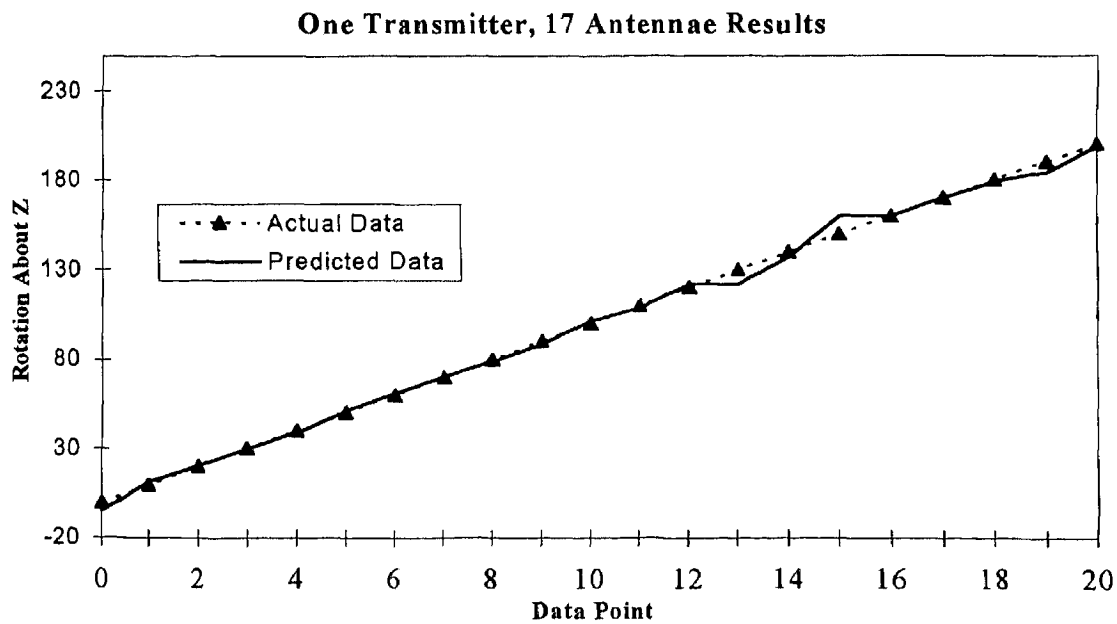
Figure B.18: 1-T & 17 antennae. 5% Noise.

Three Transmitter, 17 Antennae Results
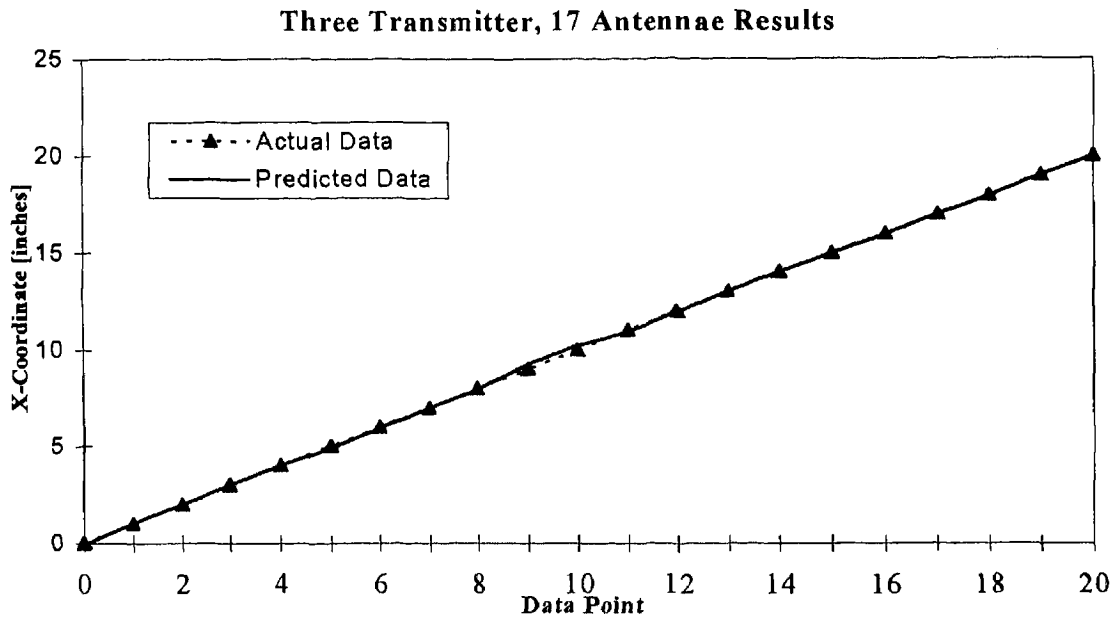
Figure B.19: 3-T & 17 antennae. 5% Noise.

Three Transmitter, 17 Antennae Results
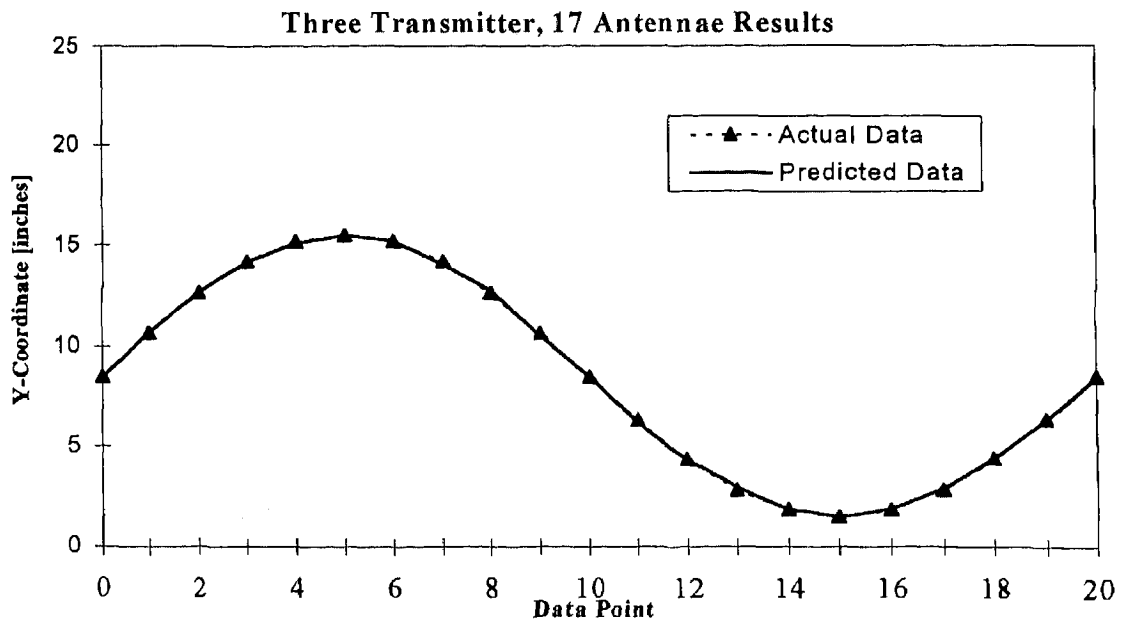
Figure B.20: 3-T & 17 antennae. 5% Noise.

Three Transmitter, 17 Antennae Results
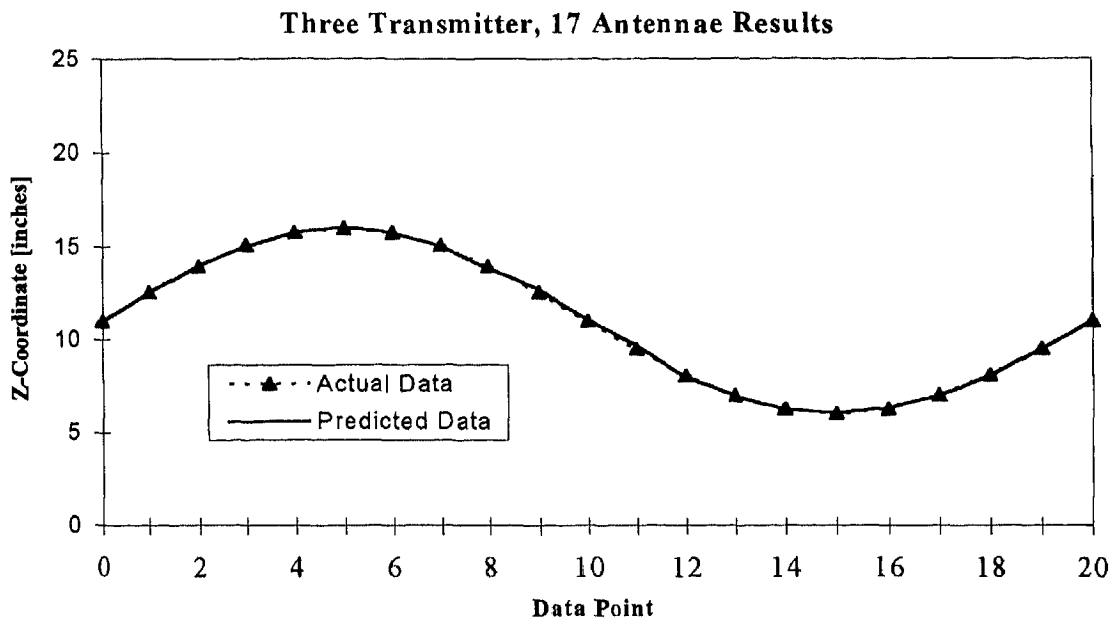
Figure B.21: 3-T & 17 antennae. 5% Noise.

Three Transmitter, 17 Antennae Results
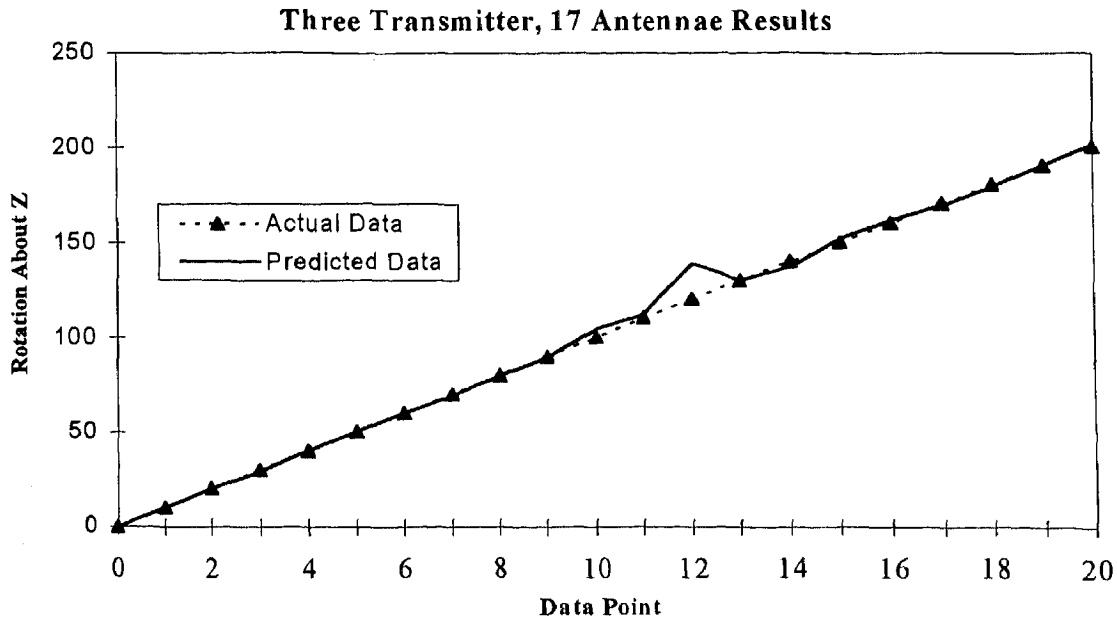
Figure B.22: 3-T & 17 antennae. 5% Noise.

Experimental Results with Virtual Antennae
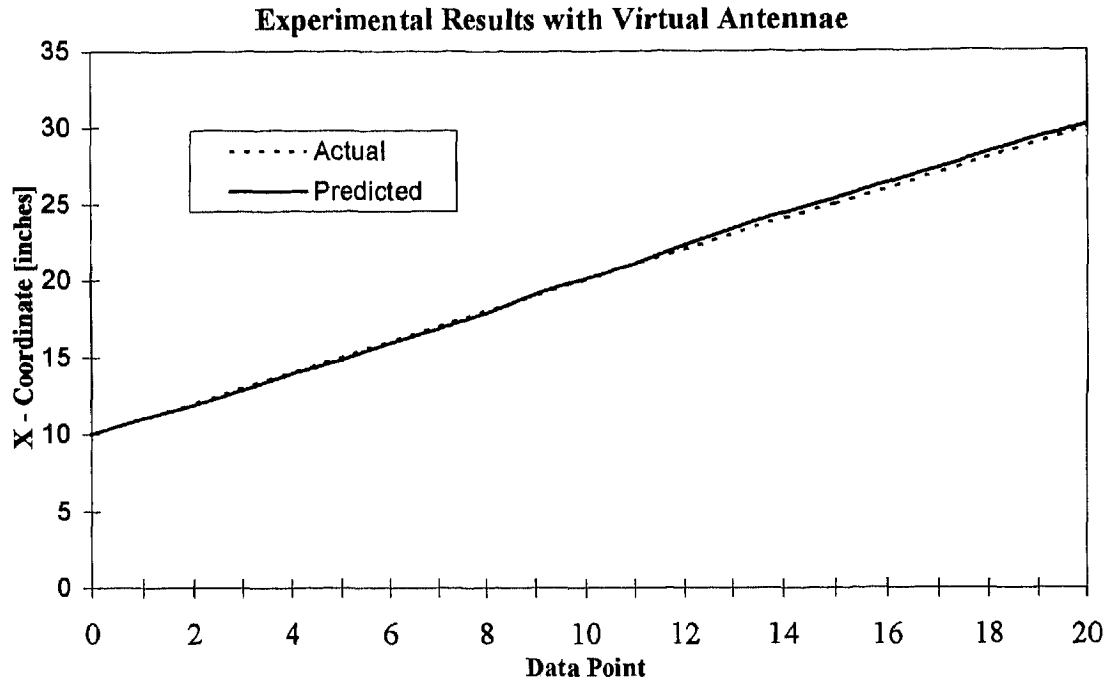
Figure B.23: Experimental Results testing virtual antennae concept.

Experimental Results with Virtual Antennae
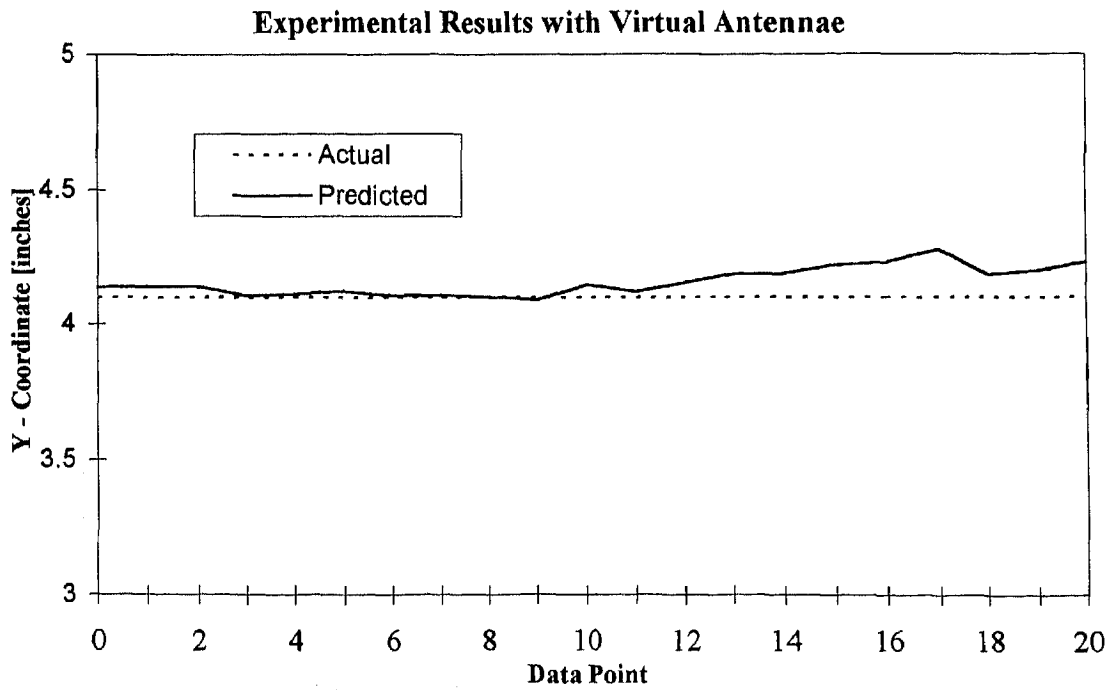
Figure B.24: Experimental Results testing virtual antennae concept.
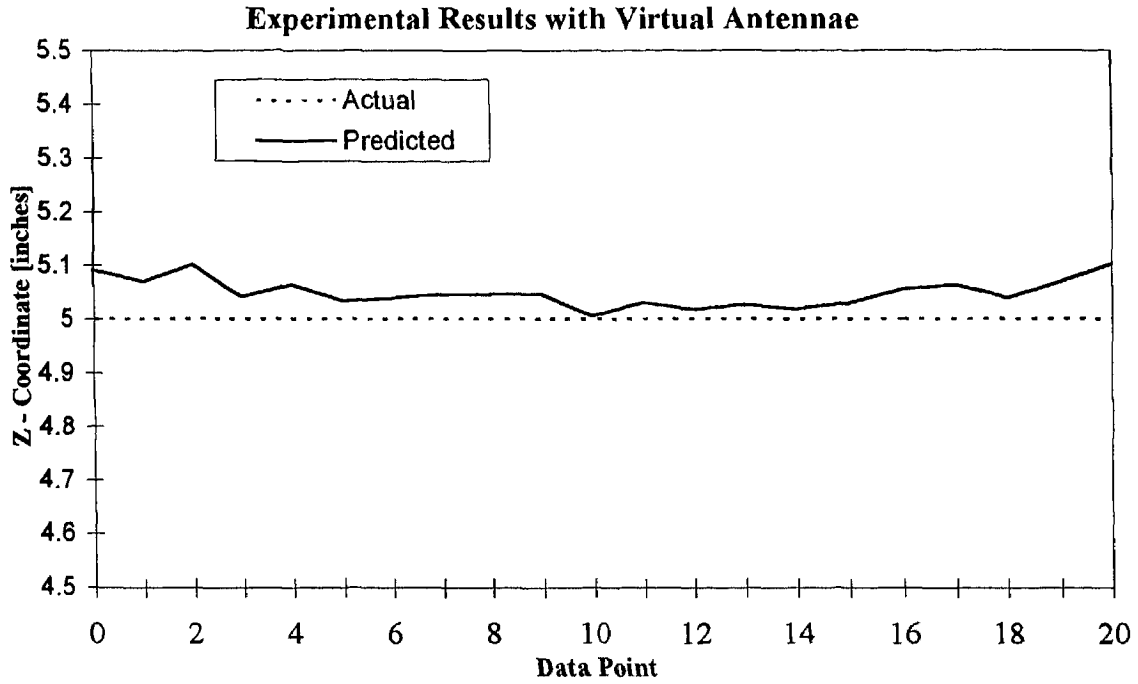
Figure B.25: Experimental Results testing virtual antennae concept.
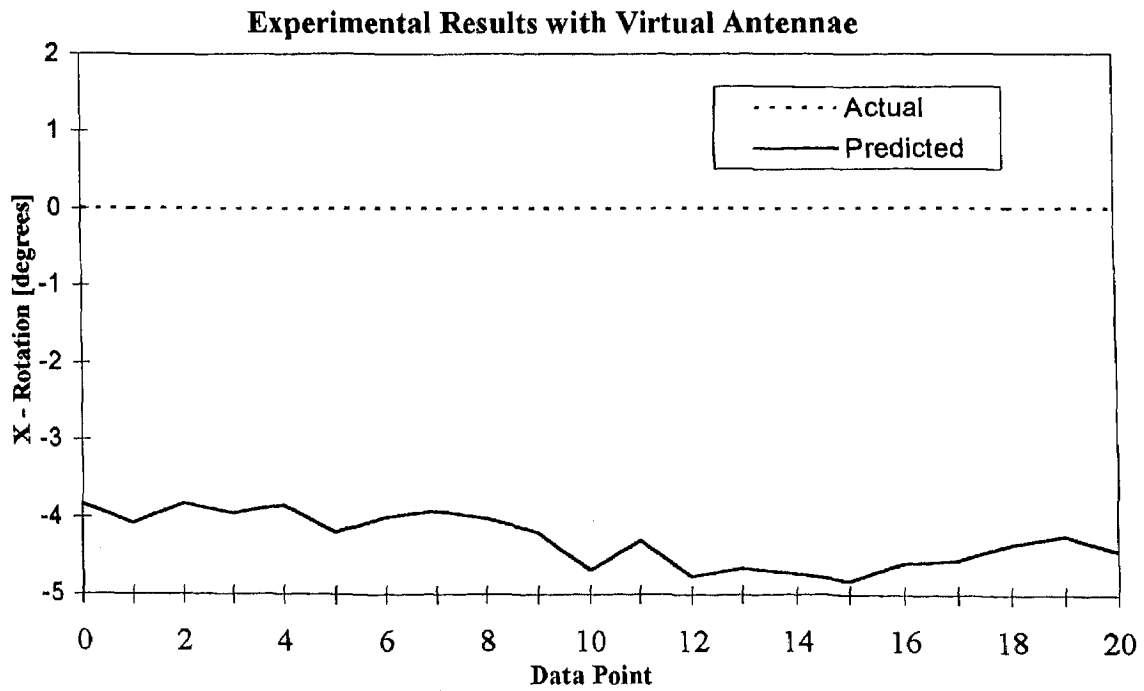
Figure B.26: Experimental Results testing virtual antennae concept.

Experimental Results with Virtual Antennae
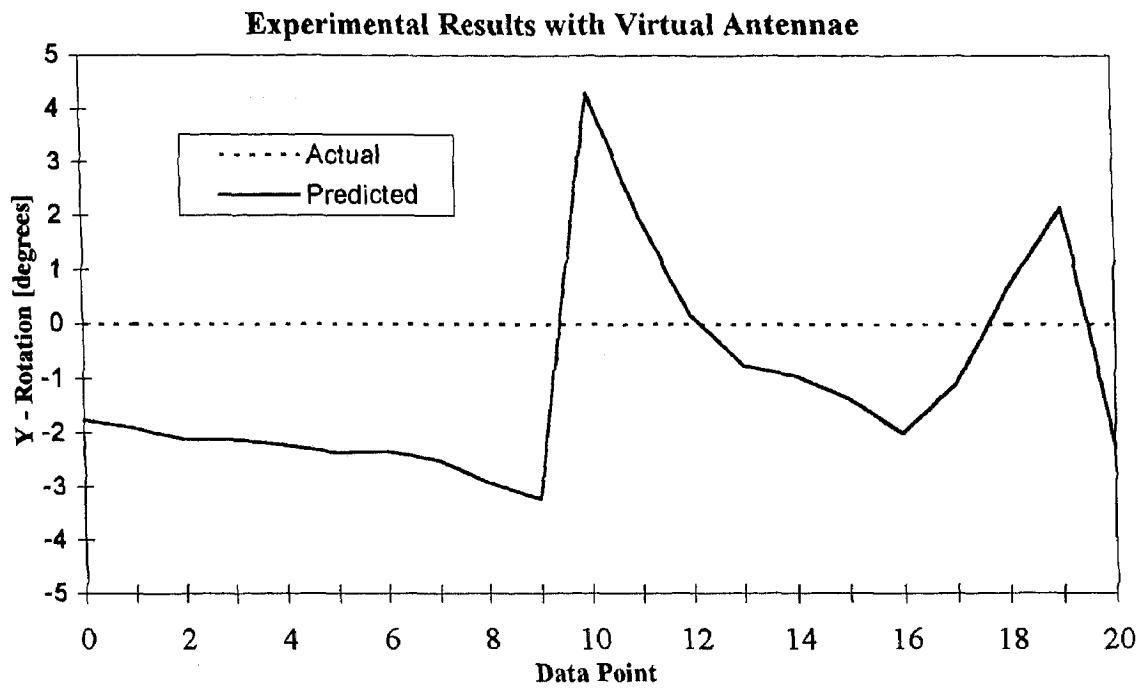
Figure B.27: Experimental Results testing virtual antennae concept.

Experimental Results with Virtual Antennae
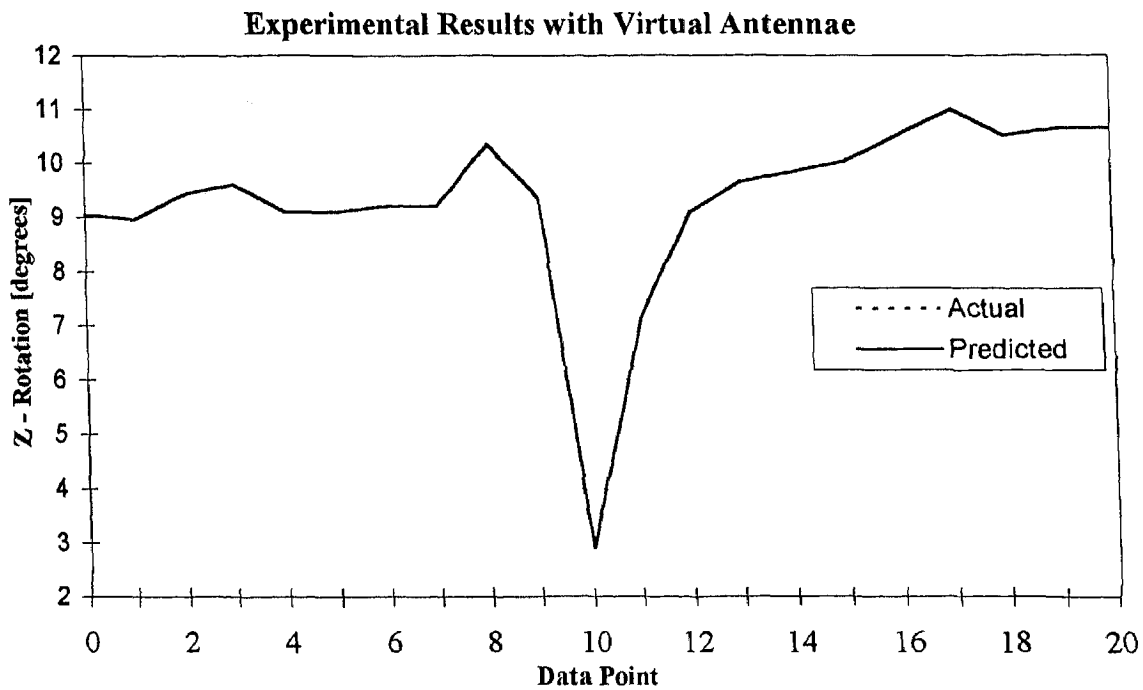
Figure B.28: Experimental Results testing virtual antennae concept.

Without Virtual Antennae

Experimental Results Without Virtual Antennae
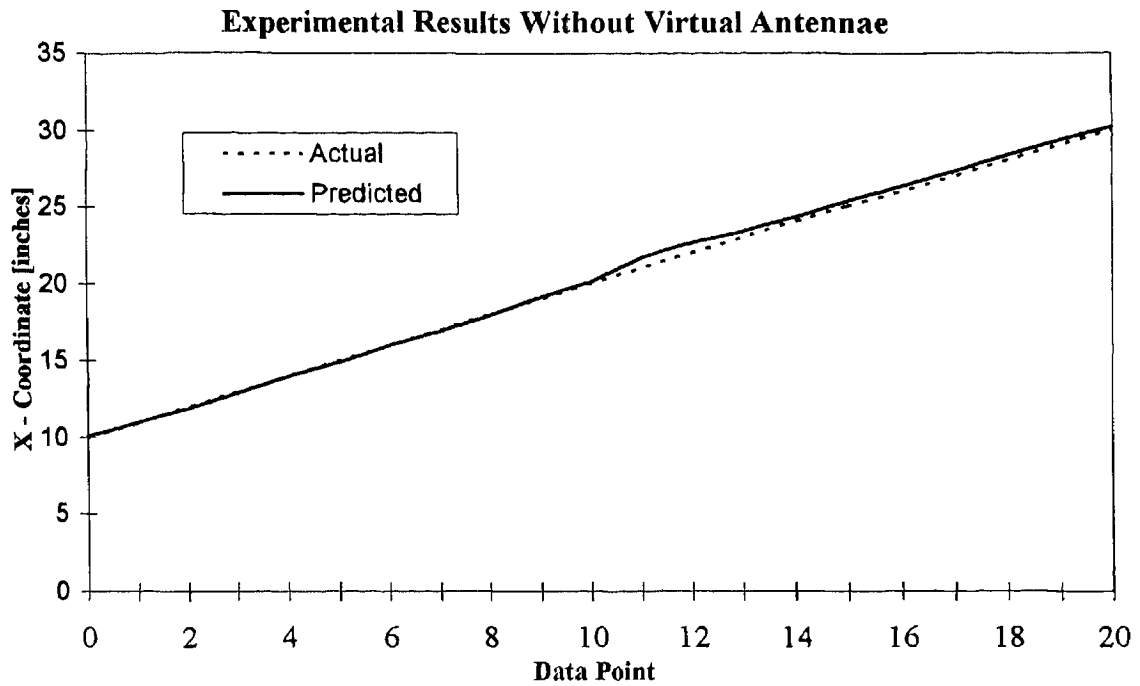
Figure B.29: Experimental Results testing virtual antenna concept.

Experimental Results Without Virtual Antennae
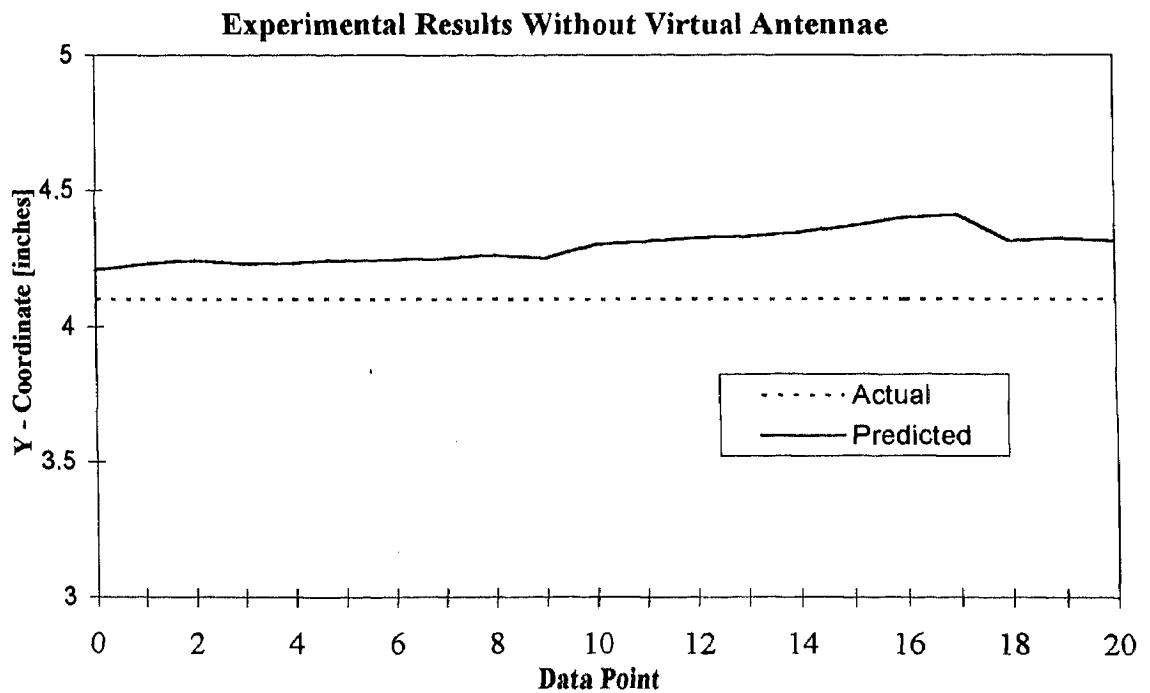
Figure B.30: Experimental Results testing virtual antenna concept.
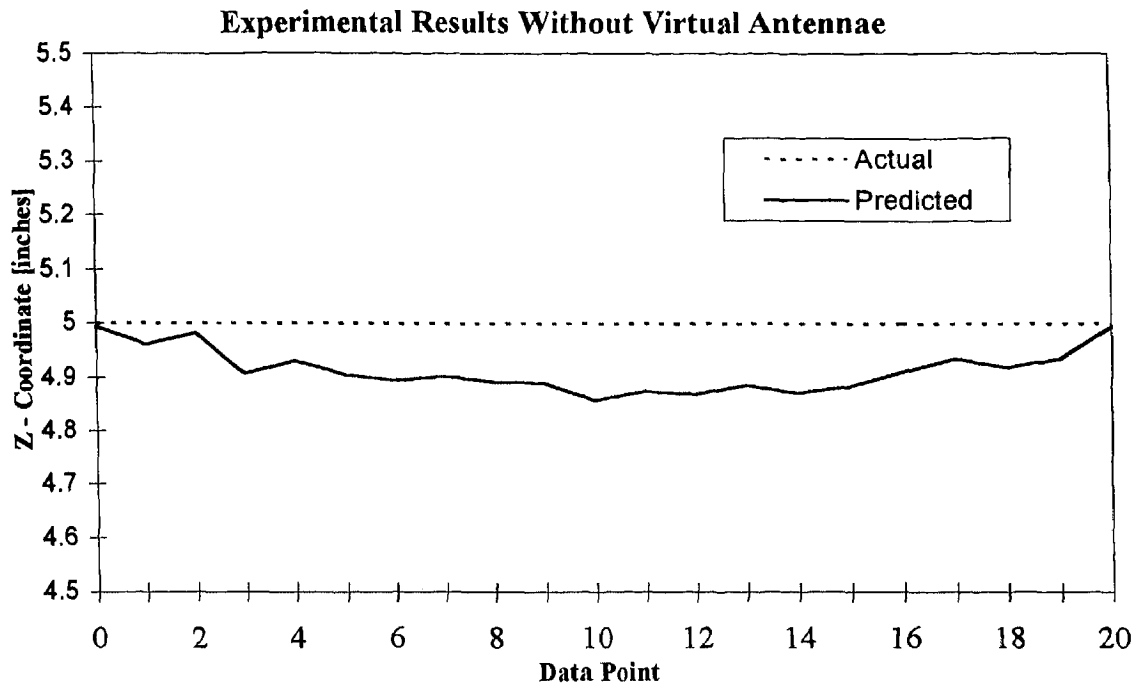
Figure B.31: Experimental Results testing virtual antenna concept.
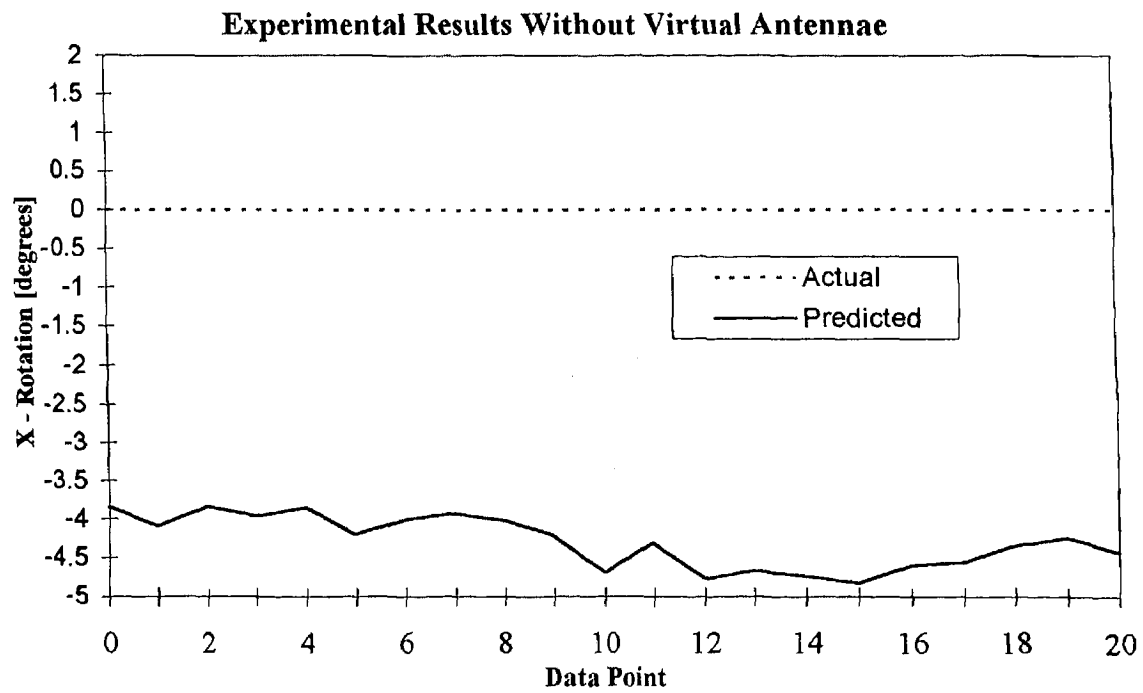
Figure B.32: Experimental Results testing virtual antenna concept.
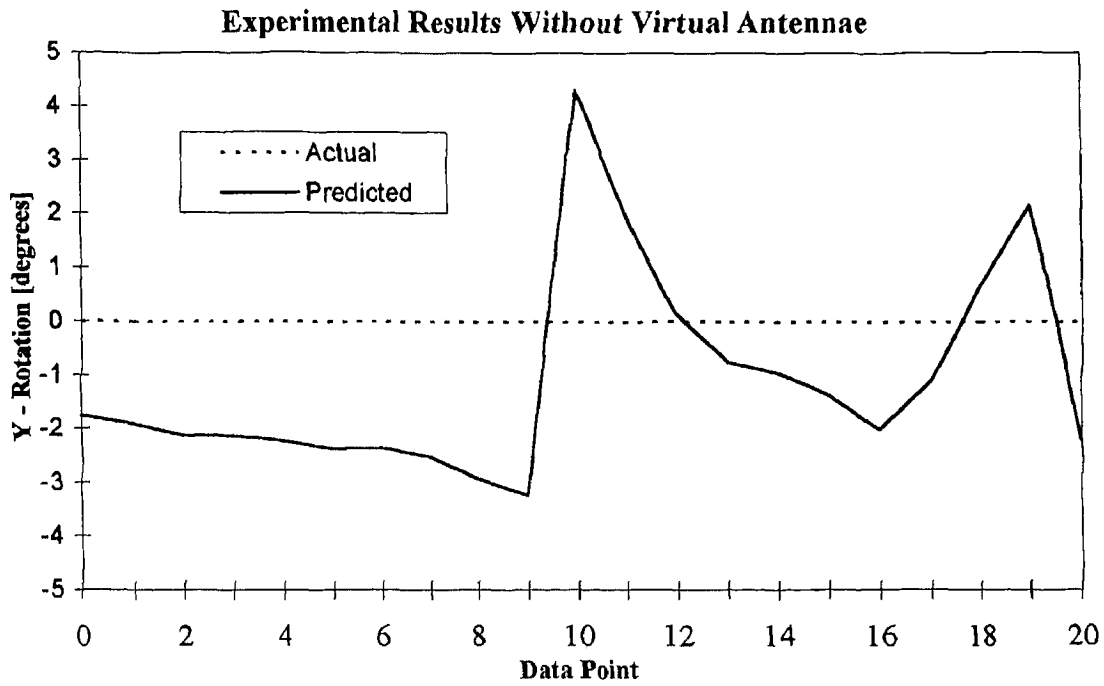
Figure B.33: Experimental Results testing virtual antenna concept.
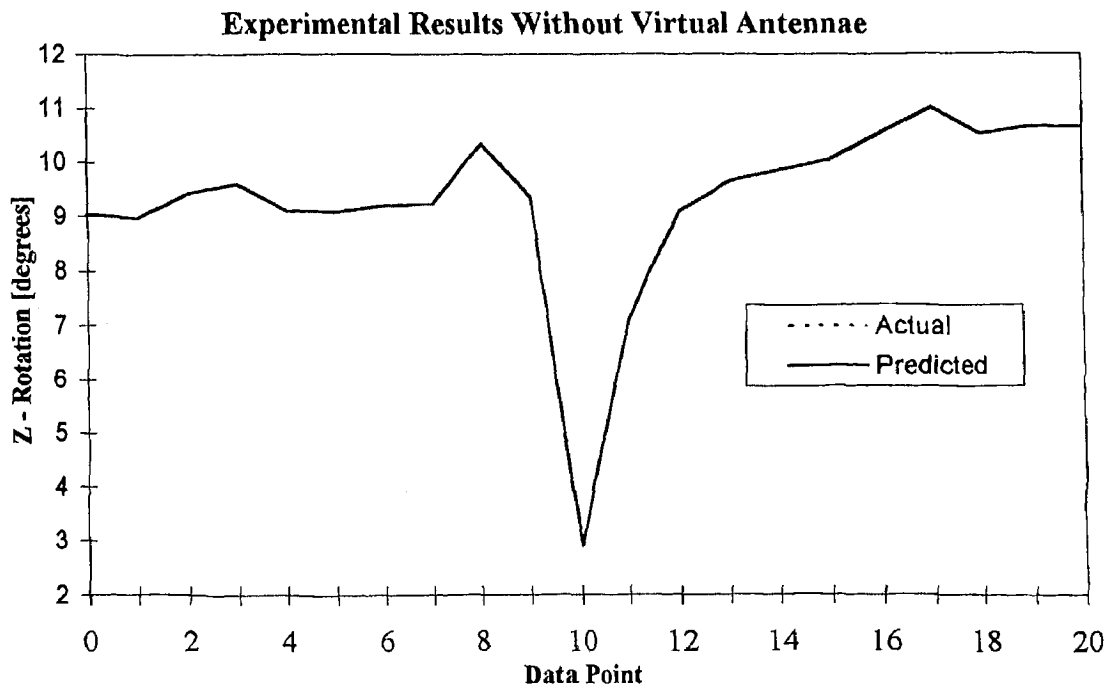
Figure B.34: Experimental Results testing virtual antenna concept.
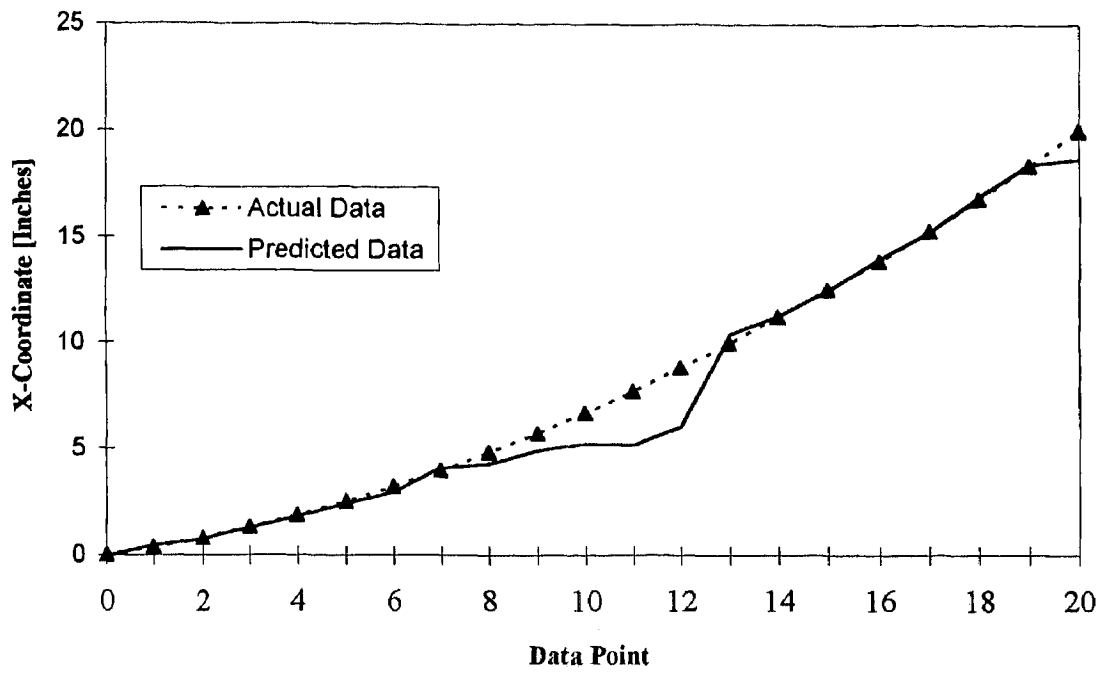
Figure B.35: Simulation plots without extrapolation.
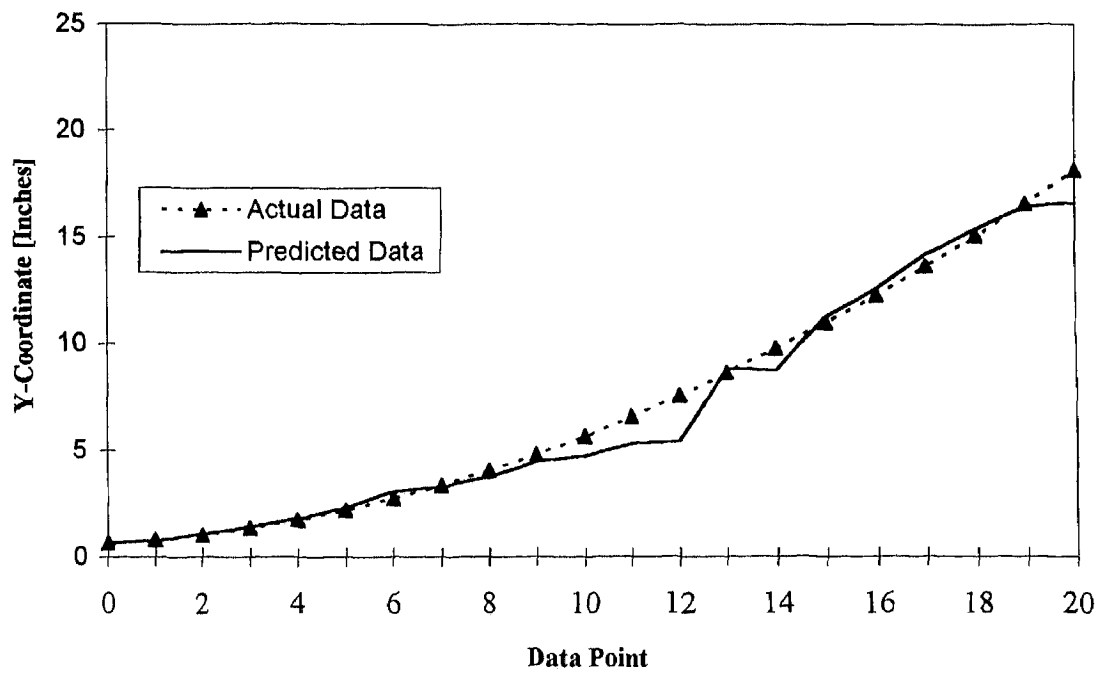
Figure B.36: Simulation plots without extrapolation.
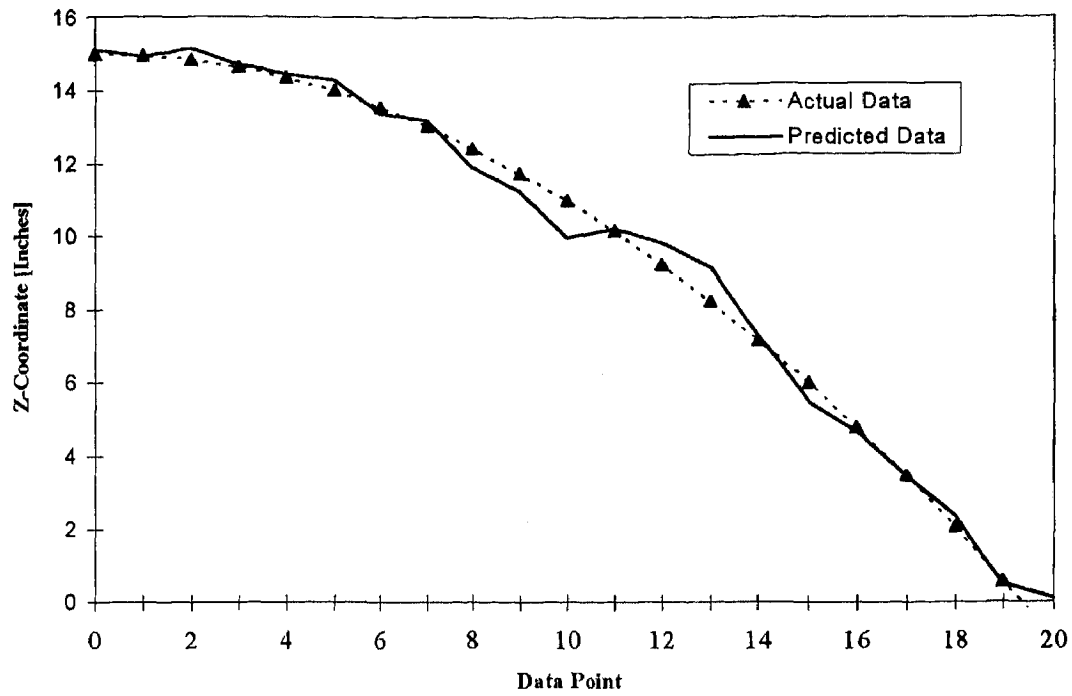
Figure B.37: Simulation plots without extrapolation.
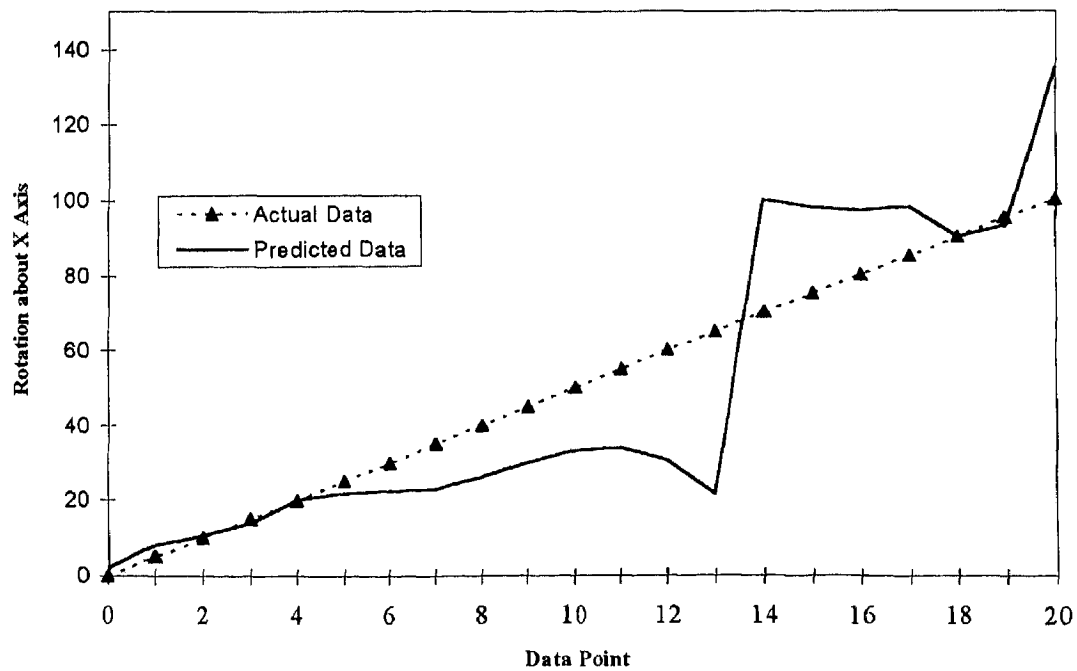
Figure B.38: Simulation plots without extrapolation.
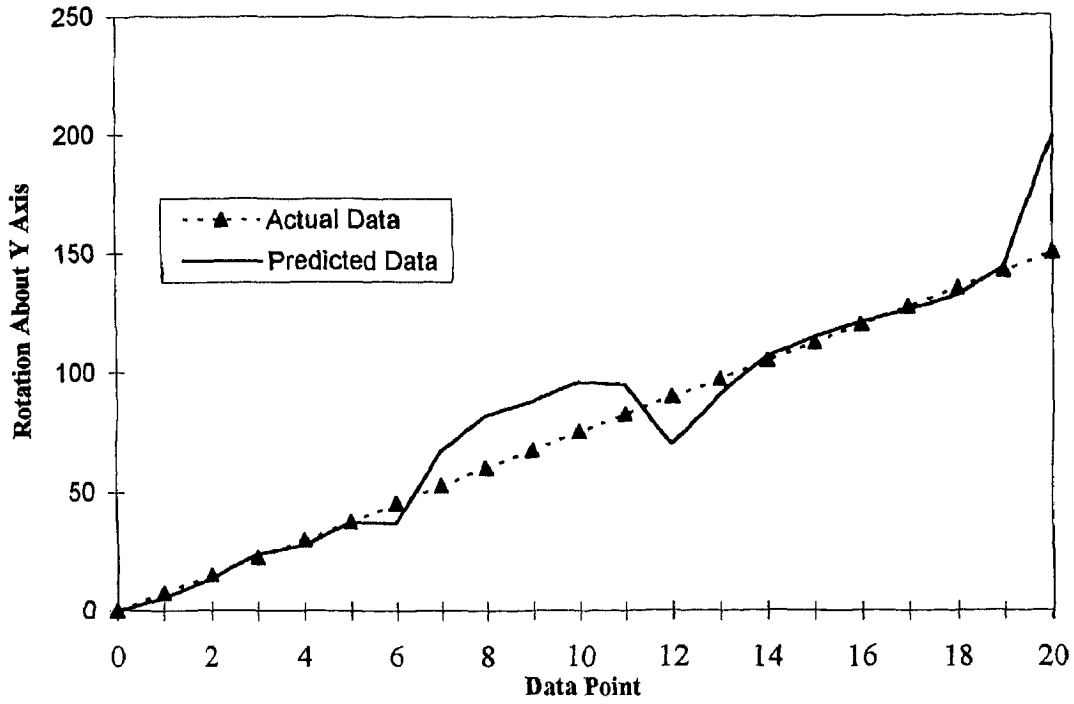
Figure B.39: Simulation plots without extrapolation.
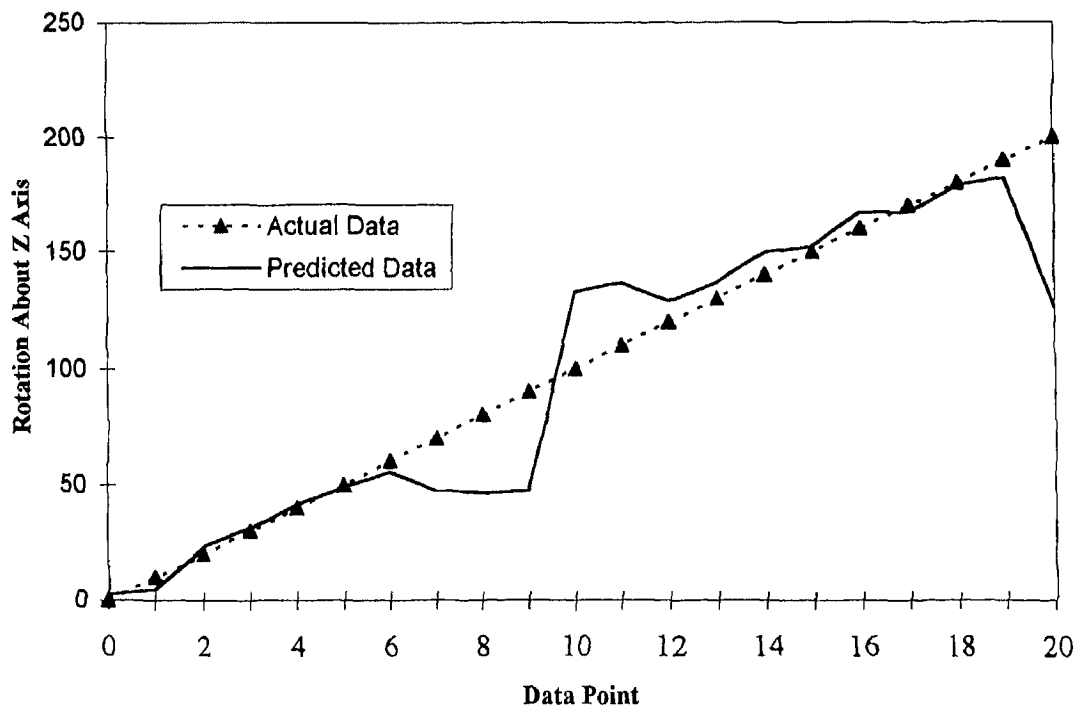
Figure B.40: Simulation plots without extrapolation.
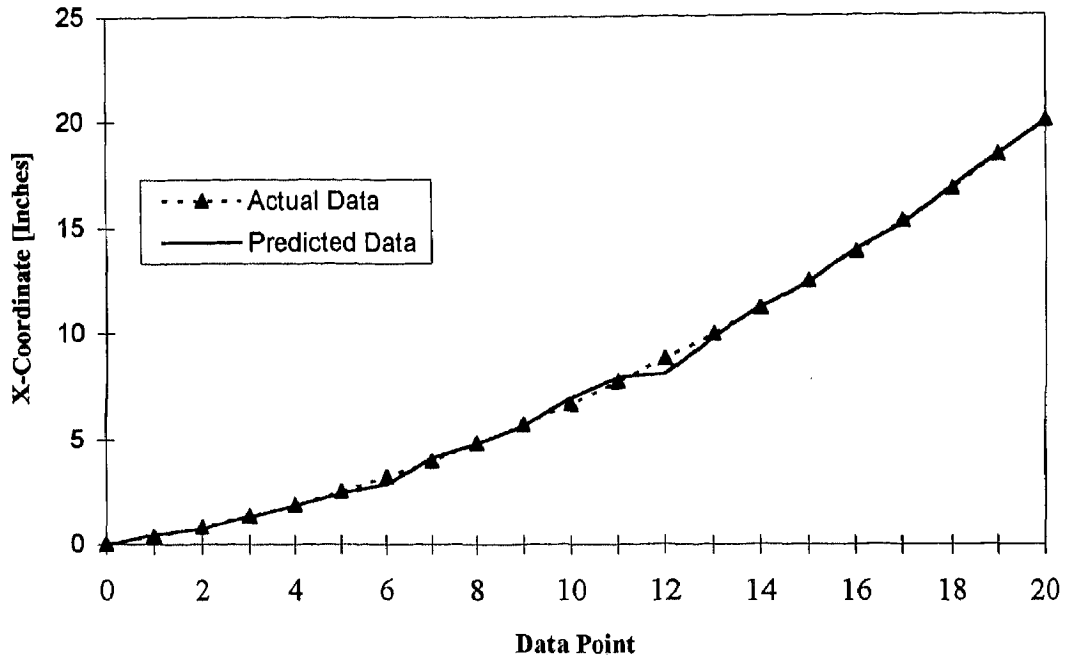
Figure B.41: Simulation plots with extrapolation.
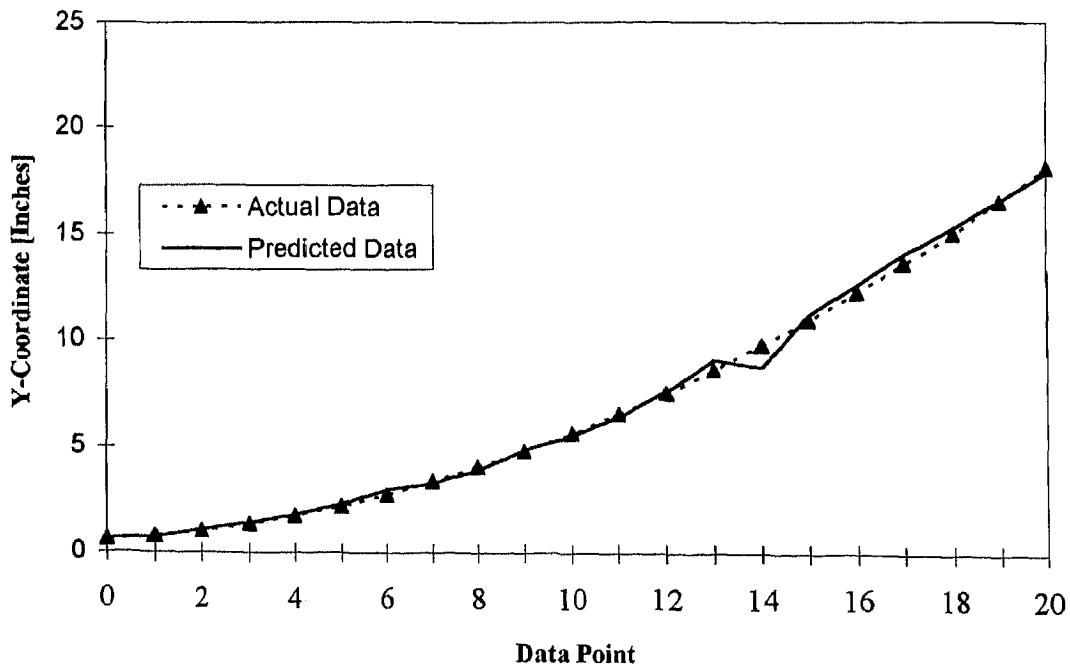
Figure B.42: Simulation plots with extrapolation.
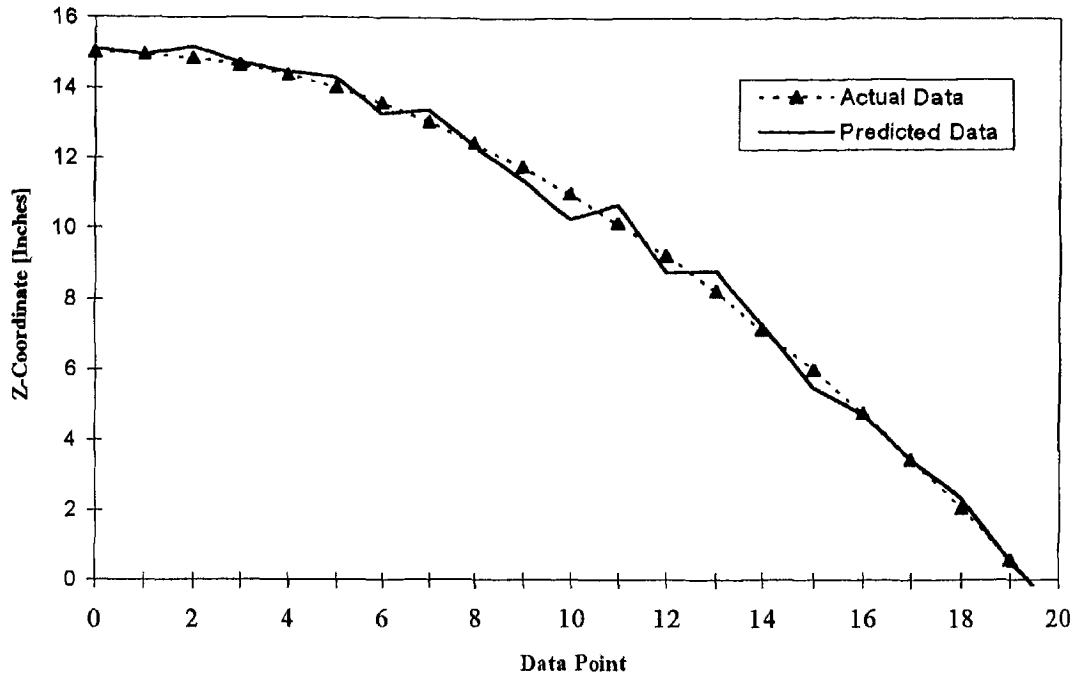
Figure B.43: Simulation plots with extrapolation.
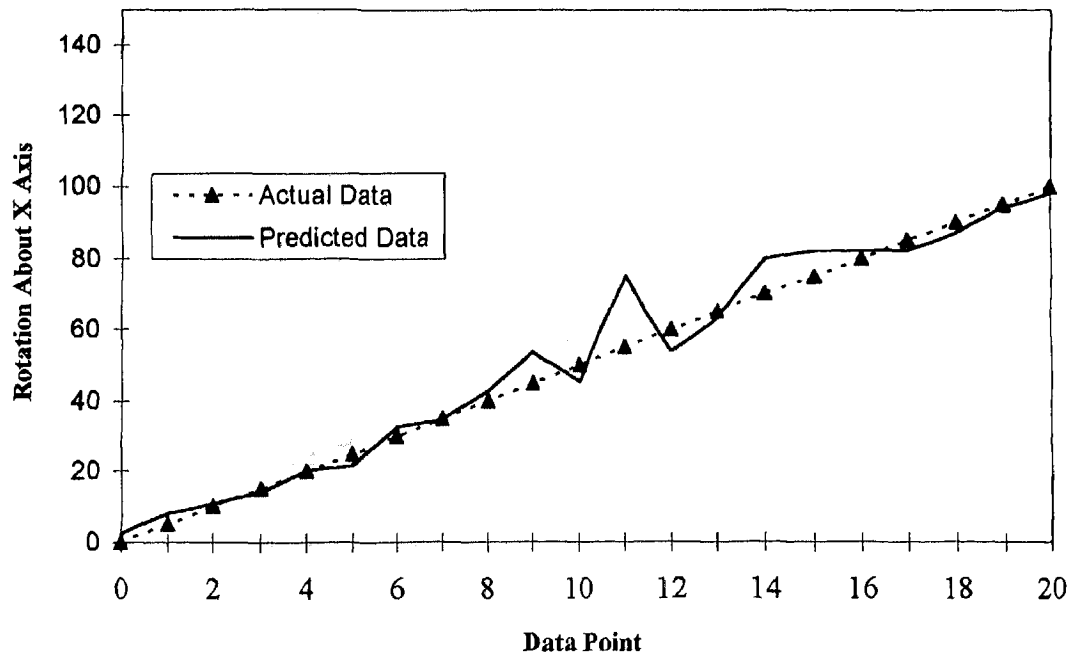
Figure B.44: Simulation plots with extrapolation.
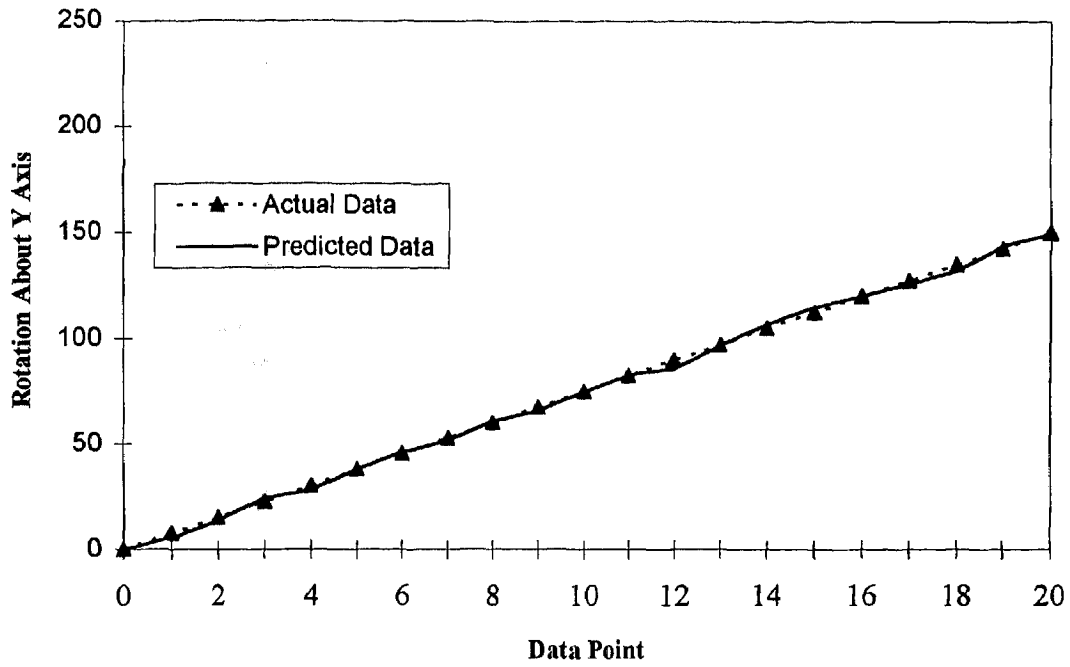
Figure B.45: Simulation plots with extrapolation.
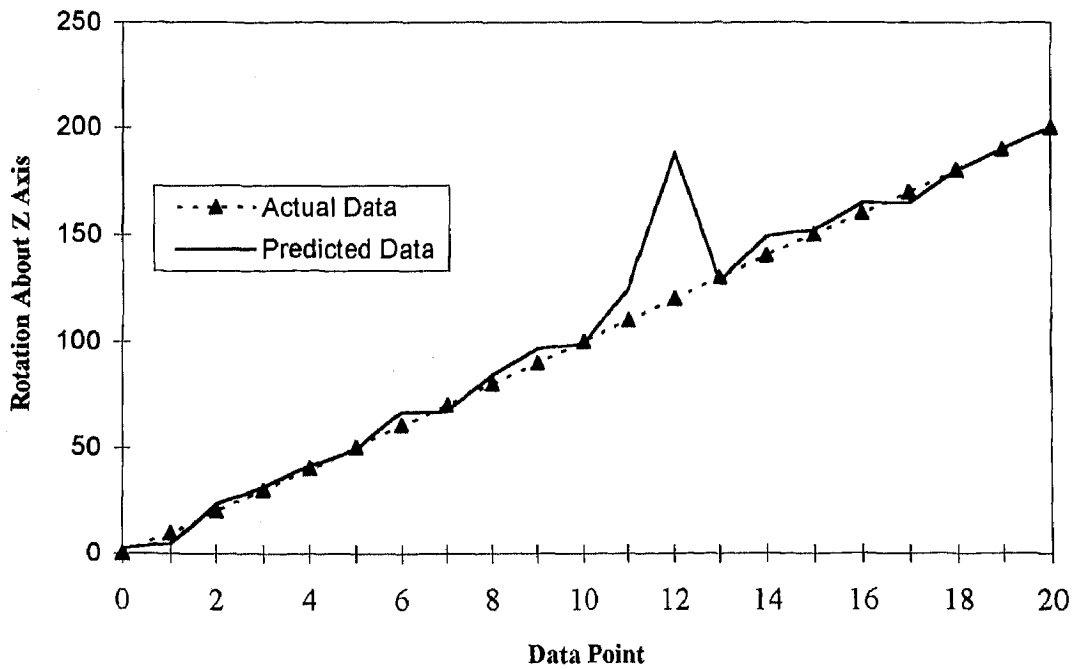
Figure B.46: Simulation plots with extrapolation.

APPENDIX C

Table C.1: Solution of Run 2.1, with no perturbations, 5% of maximum range random noise

```
Data Point 1
=============

Initial guess: 2.000000  5.000000  4.000000  0.000000  0.000000  0.000000
Perturbation 1
Initial start : X = 2.000000   Y = 5.000000    Z = 4.000000    A = 0.000000    B = 0.000000    G = 0.000000
Initial residual: 0.000348
Converged Position : X = 1.908083   Y = 5.262133    Z = 4.051388    A = 357.834251 B = 1.057518    G = 3.910182
Residual for this perturbation : 0.000234
Using perturbation #: 1
Solution to improve: X=1.908083  Y=5.262133  Z=4.051388  A=357.834251  B=1.057518  G=3.910182
Initial residual: 0.000234
Final solution : X=  1.908 Y=  5.262 Z=  4.051 A=357.834 B=  1.058 G=  3.910
Final residual: 0.000234


Data point #: 2
=============

Initial guess: 1.908083  5.262133  4.051388  357.834251  1.057518  3.910182
Perturbation 1
Initial start : X = 1.908083   Y = 5.262133    Z = 4.051388    A = 357.834251    B = 1.057518    G = 3.910182
Initial residual: 0.001212
Converged Position : X = 3.306877   Y = 5.389136  Z = 4.669258    A = 13.561798    B = 14.460684    G = 6.444733
Residual for this perturbation : 0.000290
 Using perturbation #: 1
Solution to improve: X=3.306877  Y=5.389136  Z=4.669258  A=13.561798  B=14.460684  G=6.444733
Initial residual: 0.000290
Final solution : X=  3.307 Y=  5.389 Z=  4.669 A= 13.562 B= 14.461 G=  6.445
Final residual: 0.000290


Data point #: 3
=============

Initial guess: 3.306872  5.389138  4.669265  13.561757  14.460779  6.444720
Perturbation 1
Initial start : X = 3.306872   Y = 5.389138    Z = 4.669265    A = 13.561757    B = 14.460779    G = 6.444720
Initial residual: 0.001155
Converged Position : X = 3.882117   Y = 6.708703  Z = 5.610813  A = 20.696186    B = 27.730686    G = 17.681493
Residual for this perturbation : 0.000285
 Using perturbation #: 1
Solution to improve: X=3.882117  Y=6.708703  Z=5.610813  A=20.696186  B=27.730686  G=17.681493
Initial residual: 0.000285
Final solution : X=  3.882 Y=  6.709 Z=  5.611 A= 20.696 B= 27.731 G= 17.681
Final residual: 0.000285


Data point #: 4
=============

Initial guess: 3.882117  6.708703  5.610813  20.696186  27.730686  17.681493
Perturbation 1
Initial start : X = 3.882117   Y = 6.708703    Z = 5.610813    A = 20.696186    B = 27.730686    G = 17.681493
Initial residual: 0.001070
```

Converged Position : X = 5.623642   Y = 7.638746   Z = 5.905741 A = 28.950388 B = 48.094163   G = 24.236594
Residual for this perturbation : 0.000310
 Using perturbation #: 1
Solution to improve: X=5.623642 Y=7.638746 Z=5.905741 A=28.950388 B=48.094163 G=24.236594
Initial residual: 0.000310
Final solution : X= 5.624 Y= 7.639 Z= 5.906 A= 28.950 B= 48.094 G= 24.237
Final residual: 0.000310

Data point #: 5
================
Initial guess: 5.623642 7.638746 5.905741 28.950388 48.094163 24.236594
Perturbation 1
Initial start : X = 5.623642   Y = 7.638746   Z = 5.905741   A = 28.950388   B = 48.094163   G = 24.236594
Initial residual: 0.000705
Converged Position : X = 5.942331   Y = 8.469713   Z = 7.016146 A = 39.142684 B = 57.571305   G = 32.047738
Residual for this perturbation : 0.000312
 Using perturbation #: 1
Solution to improve: X=5.942331 Y=8.469713 Z=7.016146 A=39.142684 B=57.571305 G=32.047738
Initial residual: 0.000312
Final solution : X= 5.943 Y= 8.470 Z= 7.016 A= 39.142 B= 57.571 G= 32.048
Final residual: 0.000312

Data point #: 6
================
Initial guess: 5.942559 8.469933 7.016101 39.142208 57.571088 32.048022
Perturbation 1
Initial start : X = 5.942559   Y = 8.469933   Z = 7.016101   A = 39.142208   B = 57.571088   G = 32.048022
Initial residual: 0.000674
Converged Position : X = 7.000094   Y = 8.418107   Z = 7.603394 A = 29.284243 B = 71.360718   G = 22.355054
Residual for this perturbation : 0.000350
 Using perturbation #: 1
Solution to improve: X=7.000094 Y=8.418107 Z=7.603394 A=29.284243 B=71.360718 G=22.355054
Initial residual: 0.000350
Final solution : X= 7.000 Y= 8.418 Z= 7.603 A= 29.284 B= 71.361 G= 22.355
Final residual: 0.000350

Data point #: 7
================
Initial guess: 7.000094 8.418107 7.603394 29.284243 71.360718 22.355054
Perturbation 1
Initial start : X = 7.000094   Y = 8.418107   Z = 7.603394   A = 29.284243   B = 71.360718   G = 22.355054
Initial residual: 0.000725
Converged Position : X = 7.885757   Y = 9.157865   Z = 8.573621   A = 40.977452 B = 84.271073   G = 40.581319
Residual for this perturbation : 0.000468
Using perturbation #: 1
Solution to improve: X=7.885757 Y=9.157865 Z=8.573621 A=40.977452 B=84.271073 G=40.581319
Initial residual: 0.000468
Final solution : X= 7.886 Y= 9.158 Z= 8.574 A= 40.977 B= 84.271 G= 40.581
Final residual: 0.000468

Data point #: 8
================
Initial guess: 7.885757 9.157865 8.573621 40.977452 84.271073 40.581319

Perturbation 1

Initial start : X = 7.885757   Y = 9.157865   Z = 8.573621   A = 40.977452   B = 84.271073   G = 40.581319
Initial residual: 0.000787
Converged Position : X = 9.762012   Y = 9.97395 Z = 9.52436   A = 121.554793 B = 111.497284   G = 132.386196
Residual for this perturbation : 0.000390
Using perturbation #: 1
Solution to improve: X=9.762012 Y=9.973952 Z=9.524365 A=121.554793 B=111.497284 G=132.386196
Initial residual: 0.000390
Final solution : X= 9.762 Y= 9.974 Z= 9.524 A=121.555 B=111.497 G=132.386
Final residual: 0.000390

Data point #: 9
================

Initial guess: 9.762012 9.973952 9.524365 121.554793 111.497284 132.386196
Perturbation 1
Initial start : X = 9.762012   Y = 9.973952   Z = 9.524365   A = 121.554793   B = 111.497284   G = 132.386196
Initial residual: 0.000556
Converged Position : X = 10.188115   Y = 10.506131   Z = 9.871031 A = 102.23009 B = 118.842   G = 121.871866
Residual for this perturbation : 0.000323
Using perturbation #: 1
Solution to improve: X=10.188115 Y=10.506131 Z=9.871031 A=102.230093 B=118.842532 G=121.871866
Initial residual: 0.000323
Final solution : X= 10.188 Y= 10.506 Z= 9.871 A=102.230 B=118.843 G=121.872
Final residual: 0.000323

Data point #: 10
================

Initial guess: 10.188115 10.506131 9.871031 102.230093 118.842532 121.871866
Perturbation 1
Initial start : X = 10.188115   Y = 10.506131   Z = 9.871031   A = 102.230093   B = 118.842532   G = 121.871866
Initial residual: 0.000787
Converged Position : X = 10.051780 Y = 11.201101 Z = 10.289061 A = 94.288236 B = 134.9405 G = 107.709345
Residual for this perturbation : 0.000482
Using perturbation #: 1
Solution to improve: X=10.051780 Y=11.201101 Z=10.289061 A=94.288236 B=134.940587 G=107.709345
Initial residual: 0.000482
Final solution : X= 10.052 Y= 11.201 Z= 10.289 A= 94.288 B=134.941 G=107.709
Final residual: 0.000482

Data point #: 11
================

Initial guess: 10.051780 11.201101 10.289061 94.288236 134.940587 107.709345
Perturbation 1
Initial start : X = 10.051780   Y = 11.201101   Z = 10.289061   A = 94.288236   B = 134.940587   G = 107.709345
Initial residual: 0.001063
Converged Position : X = 11.402946   Y = 12.09168 Z = 11.36847 A = 82.90717 B = 150.30188 G = 105.887226
Residual for this perturbation : 0.000754
Using perturbation #: 1

Solution to improve: X=11.402946 Y=12.091683 Z=11.368475 A=82.907172 B=150.301882 G=105.887226
Initial residual: 0.000754
Final solution : X= 11.403 Y= 12.092 Z= 11.368 A= 82.907 B=150.302 G=105.887
Final residual: 0.000754

Data point #: 12
================
Initial guess: 11.402946 12.091683 11.368475 82.907172 150.301882 105.887226
Perturbation 1
Initial start : X = 11.402946   Y = 12.091683   Z = 11.368475   A = 82.907172   B = 150.301882   G = 105.887226
Initial residual: 0.001122
Converged Position : X = 11.225386 Y = 13.223554 Z = 11.531737 A = 70.104376 B = 163.714884 G = 92.342676
Residual for this perturbation : 0.000897
Using perturbation #: 1
Solution to improve: X=11.225386 Y=13.223554 Z=11.531737 A=70.104376 B=163.714884 G=92.342676
Initial residual: 0.000897
Final solution : X= 11.225 Y= 13.224 Z= 11.532 A= 70.104 B=163.715 G= 92.343
Final residual: 0.000897

Data point #: 13
================
Initial guess: 11.225386 13.223554 11.531737 70.104376 163.714884 92.342676
Perturbation 1
Initial start : X = 11.225386   Y = 13.223554   Z = 11.531737   A = 70.104376   B = 163.714884   G = 92.342676
Initial residual: 0.001095
Converged Position : X = 11.520318 Y = 14.16775 Z = 11.031964 A = 61.077838 B = 176.983353   G = 97.166786
Residual for this perturbation : 0.000889
Using perturbation #: 1
Solution to improve: X=11.520318 Y=14.167756 Z=11.031964 A=61.077838 B=176.983353 G=97.166786
Initial residual: 0.000889
Final solution : X= 11.520 Y= 14.168 Z= 11.032 A= 61.078 B=176.983 G= 97.167
Final residual: 0.000889

Data point #: 14
================
Initial guess: 11.520318 14.167756 11.031964 61.077838 176.983353 97.166786
Perturbation 1
Initial start : X = 11.520318   Y = 14.167756   Z = 11.031964   A = 61.077838   B = 176.983353   G = 97.166786
Initial residual: 0.001594
Converged Position : X = 13.007415 Y = 15.791440 Z = 12.097249 A = 51.802306 B = 180.579676 G = 88.549584
Residual for this perturbation : 0.001470
Using perturbation #: 1
Solution to improve: X=13.007415 Y=15.791440 Z=12.097249 A=51.802306 B=180.579676 G=88.549584
Initial residual: 0.001470
Final solution : X= 13.007 Y= 15.791 Z= 12.097 A= 51.802 B=180.580 G= 88.550
Final residual: 0.001470

Data point #: 15
================
Initial guess: 13.007415 15.791440 12.097249 51.802306 180.579676 88.549584
Perturbation 1
Initial start : X = 13.007415   Y = 15.791440   Z = 12.097249   A = 51.802306   B = 180.579676   G = 88.549584
Initial residual: 0.002033
Converged Position : X = 14.175482 Y = 17.653630 Z = 11.073954 A = 24.156717 B = 185.198436 G = 76.559197
Residual for this perturbation : 0.001805
Using perturbation #: 1
Solution to improve: X=14.175482 Y=17.653630 Z=11.073954 A=24.156717 B=185.198436 G=76.559197

Initial residual: 0.001805
Final solution : X= 14.175 Y= 17.654 Z= 11.074 A= 24.157 B=185.198 G= 76.559
Final residual: 0.001805

Data point #: 16
================

Initial guess: 14.175482 17.653630 11.073954 24.156717 185.198436 76.559197
Perturbation 1
Initial start : X = 14.175482   Y = 17.653630   Z = 11.073954   A = 24.156717   B = 185.198436   G = 76.559197
Initial residual: 0.001955
Converged Position : X = 15.567777 Y = 18.237650 Z = 12.558322 A = 23.002636 B = 199.671365 G = 68.636001
Residual for this perturbation : 0.001646
Using perturbation #: 1
Solution to improve: X=15.567777 Y=18.237650 Z=12.558322 A=23.002636 B=199.671365 G=68.636001
Initial residual: 0.001646
Final solution : X= 15.568 Y= 18.238 Z= 12.558 A= 23.003 B=199.671 G= 68.636
Final residual: 0.001646

Data point #: 17
================

Initial guess: 15.567777 18.237650 12.558322 23.002636 199.671365 68.636001
Perturbation 1
Initial start : X = 15.567777   Y = 18.237650   Z = 12.558322   A = 23.002636   B = 199.671365   G = 68.636001
Initial residual: 0.002522
Converged Position : X = 17.884871 Y = 17.140826 Z = 15.82861 A = 339.27933 B = 240.139385 G = 127.315208
Residual for this perturbation : 0.000312
Using perturbation #: 1
Solution to improve: X=17.884871 Y=17.140826 Z=15.828614 A=339.279333 B=240.139385 G=127.315208
Initial residual: 0.000312
Final solution : X= 17.885 Y= 17.141 Z= 15.829 A=339.279 B=240.139 G=127.315
Final residual: 0.000312

Table C.2: Solution for Run 2.1, using the multiple initial guess method of perturbation and 5% of maximum range random noise.

Data point #: 1
================
Initial guess: 2.000000 5.000000 4.000000 0.000000 0.000000 0.000000
Perturbation 1
Initial start : X = 2.000000   Y = 5.000000   Z = 4.000000   A = 0.000000   B = 0.000000   G = 0.000000
Initial residual: 0.000348
Converged Position : X = 1.9080   Y = 5.262133   Z = 4.051388   A = 357.834251   B = 1.057518   G = 3.910182
Residual for this perturbation : 0.000234
Perturbation 2
Initial start : X = 2.076735   Y = 4.091409   Z = 3.239555   A = -13.191068   B = -10.763649   G = 13.565972
Initial residual: 0.001712
Converged Position : X = 1.9080   Y = 5.262129   Z = 4.051385   A = 357.834268   B = 1.057532   G = 3.910174
Residual for this perturbation : 0.000234
Perturbation 3
Initial start : X = 1.677185   Y = 5.219440   Z = 4.353550   A = -0.923207   B = 29.127355   G = 1.845163
Initial residual: 0.001308
Converged Position : X = 2.131078   Y = 4.992070   Z = 4.523408   A = 2.113386   B = 15.023031   G = 1.140154
Residual for this perturbation : 0.000713
Perturbation 4

Initial start : X = 2.860660   Y = 4.057370   Z = 4.646261   A = 0.970073   B = -9.755218   G = 2.364431
Initial residual: 0.001299
Converged Position : X = 1.908131   Y = 5.2620   Z = 4.051328   A = 357.834964   B = 1.057378   G = 3.909931
Residual for this perturbation : 0.000234
Perturbation 5
Initial start : X = 1.326699   Y = 4.613243   Z = 3.643394   A = 15.070934   B = -4.283269   G = -25.207512
Initial residual: 0.001529
Converged Position : X = 1.9081   Y = 5.261850   Z = 4.051260   A = 357.836077   B = 1.057103   G = 3.909590
Residual for this perturbation : 0.000234
Perturbation 6
Initial start : X = 1.340915   Y = 5.865431   Z = 4.044569   A = -23.366262   B = -11.124931   G = 14.628084
Initial residual: 0.000986
Converged Position : X = 1.9081   Y = 5.261800   Z = 4.051252   A = 357.837233   B = 1.056340   G = 3.909157
Residual for this perturbation : 0.000234
Using perturbation #: 6
Solution to improve: X=1.908136 Y=5.261800 Z=4.051252 A=357.837233 B=1.056340 G=3.909157
Initial residual: 0.000234
Final solution : X= 1.908 Y= 5.262 Z= 4.051 A=357.837 B= 1.056 G= 3.909
Final residual: 0.000234

Data point #: 2
================
Initial guess: 1.908136 5.261800 4.051252 357.837233 1.056340 3.909157
Perturbation 1
Initial start : X = 1.908136   Y = 5.261800   Z = 4.051252   A = 357.837233   B = 1.056340   G = 3.909157
Initial residual: 0.001212
Converged Position : X = 3.306876   Y = 5.389135   Z = 4.669257   A = 13.5618   B = 14.460682   G = 6.444742
Residual for this perturbation : 0.000290
Perturbation 2
Initial start : X = 1.213861   Y = 5.296268   Z = 4.033044   A = 332.736646   B = -6.933900   G = 28.171278
Initial residual: 0.001840
Converged Position : X = 2.355221   Y = 6.68962   Z = 5.63338   A = 333.835780   B = 10.083009   G = 17.227520
Residual for this perturbation : 0.001186
Perturbation 3
Initial start : X = 2.545173   Y = 5.486262   Z = 4.536041   A = 363.501667   B = 14.054051   G = -23.463114
Initial residual: 0.001298
Converged Position : X = 3.306867   Y = 5.389187   Z = 4.669291   A = 13.560974   B = 14.4617   G = 6.444619
Residual for this perturbation : 0.000290
Perturbation 4
Initial start : X = 2.492935   Y = 4.339318   Z = 4.976930   A = 353.970287   B = -11.657783   G = 8.307276
Initial residual: 0.001805
Converged Position : X = 3.306869   Y = 5.389186   Z = 4.669288   A = 13.560977   B = 14.4617   G = 6.444603
Residual for this perturbation : 0.000290
Perturbation 5
Initial start : X = 2.626086   Y = 4.917189   Z = 3.451609   A = 367.014062   B = 22.749365   G = -11.850739
Initial residual: 0.001294
Converged Position : X = 3.3068   Y = 5.389174   Z = 4.669275   A = 13.560819   B = 14.461921   G = 6.444681
Residual for this perturbation : 0.000290
Perturbation 6
Initial start : X = 2.316222   Y = 6.136906   Z = 3.743316   A = 330.493453   B = 10.250081   G = 10.801632
Initial residual: 0.001586
Converged Position : X = 3.306868   Y = 5.389186   Z = 4.669289   A = 13.5609   B = 14.461768   G = 6.444613
Residual for this perturbation : 0.000290
Using perturbation #: 1
Solution to improve: X=3.306876 Y=5.389135 Z=4.669257 A=13.561801 B=14.460682 G=6.444742
Initial residual: 0.000290
Final solution : X= 3.307 Y= 5.389 Z= 4.669 A= 13.562 B= 14.461 G= 6.445
Final residual: 0.000290

Data point #: 3
================
Initial guess: 3.306872 5.389138 4.669264 13.561760 14.460777 6.444728
Perturbation 1
Initial start : X = 3.306872   Y = 5.389138   Z = 4.669264   A = 13.561760   B = 14.460777   G = 6.444728
Initial residual: 0.001155
Converged Position : X = 3.8821   Y = 6.708703   Z = 5.610813   A = 20.696186   B = 27.730683   G = 17.681493
Residual for this perturbation : 0.000285
Perturbation 2
Initial start : X = 3.966036   Y = 5.891211   Z = 4.317019   A = 30.327138   B = 9.929665   G = -21.648083
Initial residual: 0.001807
Converged Position : X = 3.787934   Y = 7.3700   Z = 4.936479   A = 10.112886   B = 10.650022   G = 15.557759
Residual for this perturbation : 0.001001
Perturbation 3
Initial start : X = 3.152541   Y = 5.093602   Z = 4.148435   A = 9.121504   B = 44.208233   G = -6.468717
Initial residual: 0.001315
Converged Position : X = 4.124313   Y = 6.241335   Z = 5.5058   A = 10.990509   B = 32.524125   G = 10.453263
Residual for this perturbation : 0.000480
Perturbation 4
Initial start : X = 2.478195   Y = 5.564818   Z = 4.790913   A = 15.568672   B = -1.096089   G = 8.718151
Initial residual: 0.001362
Converged Position : X = 3.9137   Y = 6.720625   Z = 5.586648   A = 20.439302   B = 28.164270   G = 17.384468
Residual for this perturbation : 0.000284
Perturbation 5
Initial start : X = 2.694218   Y = 5.707652   Z = 4.074918   A = -15.280082   B = -0.425397   G = -2.693205
Initial residual: 0.001901
Converged Position : X = 4.0157   Y = 6.8086   Z = 4.803008   A = 359.518764   B = 14.195867   G = 13.092759
Residual for this perturbation : 0.001046
Perturbation 6
Initial start : X = 2.911640   Y = 5.367521   Z = 3.865442   A = -14.785606   B = 39.005354   G = 24.037694
Initial residual: 0.001685
Converged Position : X = 3.905008   Y = 6.717930   Z = 5.566202   A = 20.4011   B = 28.277214   G = 17.401783
Residual for this perturbation : 0.000284
Using perturbation #: 4
Solution to improve: X=3.913771 Y=6.720625 Z=5.586648 A=20.439302 B=28.164270 G=17.384468
Initial residual: 0.000284
Final solution : X= 3.914 Y= 6.721 Z= 5.587 A= 20.439 B= 28.164 G= 17.384
Final residual: 0.000284

Data point #: 4
================
Initial guess: 3.913771 6.720625 5.586648 20.439302 28.164270 17.384468
Perturbation 1
Initial start : X = 3.913771   Y = 6.720625   Z = 5.586648   A = 20.439302   B = 28.164270   G = 17.384468
Initial residual: 0.001062
Converged Position : X = 5.6236   Y = 7.638746   Z = 5.905741   A = 28.950388   B = 48.094163   G = 24.236594
Residual for this perturbation : 0.000310
Perturbation 2
Initial start : X = 3.380363   Y = 6.240027   Z = 6.372612   A = 24.491982   B = 52.337023   G = -11.859724
Initial residual: 0.001007
Converged Position : X = 5.268275   Y = 8.152554   Z = 6.4787   A = 0.458213   B = 51.191748   G = 340.219311
Residual for this perturbation : 0.000349
Perturbation 3
Initial start : X = 3.905763   Y = 5.789760   Z ≈ 5.247628   A = -0.921723   B = 37.538454   G = -3.053789
Initial residual: 0.001086
Converged Position : X = 5.002979   Y = 7.312470   Z = 6.679734   A = 6.542554   B = 48.096547   G = 1.128846
Residual for this perturbation : 0.000620
Perturbation 4
Initial start : X = 3.726542   Y = 6.107992   Z = 5.338826   A = 12.566733   B = 16.205973   G = 32.651469

Initial residual: 0.001685
Converged Position : X = 5.623642   Y = 7.638746   Z = 5.905741   A = 28.9503   B = 48.094163   G = 24.236594
Residual for this perturbation : 0.000310
Perturbation 5
Initial start : X = 3.810472   Y = 7.448794   Z = 4.782061   A = 24.381064   B = 31.323459   G = 40.288663
Initial residual: 0.001242
Converged Position : X = 5.623642   Y = 7.638746   Z = 5.9057   A = 28.950389   B = 48.094163   G = 24.236596
Residual for this perturbation : 0.000310
Perturbation 6
Initial start : X = 3.654541   Y = 7.170784   Z = 5.734016   A = 36.897829   B = 7.884653   G = 22.577102
Initial residual: 0.001556
Converged Position : X = 5.6236   Y = 7.6387   Z = 5.905741   A = 28.950388   B = 48.094163   G = 24.236594
Residual for this perturbation : 0.000310
Using perturbation #: 1
Solution to improve: X=5.623642 Y=7.638746 Z=5.905741 A=28.950388 B=48.094163 G=24.236594
Initial residual: 0.000310
Final solution : X= 5.624 Y= 7.639 Z= 5.906 A= 28.950 B= 48.094 G= 24.237
Final residual: 0.000310

Data point #: 5
================

Initial guess: 5.623642 7.638746 5.905741 28.950388 48.094163 24.236594
Perturbation 1
Initial start : X = 5.623642   Y = 7.638746   Z = 5.905741   A = 28.950388   B = 48.094163   G = 24.236594
Initial residual: 0.000705
Converged Position : X = 5.942331   Y = 8.469713   Z = 7.016146   A = 39.1426   B = 57.57130   G = 32.047738
Residual for this perturbation : 0.000312
Perturbation 2
Initial start : X = 4.937593   Y = 6.859446   Z = 5.999124   A = 47.096756   B = 69.575764   G = -1.596036
Initial residual: 0.001430
Converged Position : X = 6.002048   Y = 7.84825   Z = 6.68145   A = 9.024906   B = 67.668495   G = 357.893619
Residual for this perturbation : 0.000512
Perturbation 3
Initial start : X = 4.626252   Y = 6.682269   Z = 6.290396   A = 2.087540   B = 37.157969   G = 13.906191
Initial residual: 0.001434
Converged Position : X = 6.003698   Y = 7.58037   Z = 6.689358   A = 10.732582   B = 66.437186   G = 3.522338
Residual for this perturbation : 0.000514
Perturbation 4
Initial start : X = 6.441978   Y = 7.439164   Z = 6.689703   A = 13.523015   B = 66.659791   G = 6.694940
Initial residual: 0.000532
Converged Position : X = 6.0043   Y = 7.579700   Z = 6.689894   A = 10.765574   B = 66.434461   G = 3.552401
Residual for this perturbation : 0.000514
Perturbation 5
Initial start : X = 5.290714   Y = 8.519918   Z = 5.625869   A = 36.659351   B = 77.311963   G = 23.097343
Initial residual: 0.000755
Converged Position : X = 6.003170   Y = 7.821065   Z = 6.66516   A = 9.29016   B = 67.190280   G = 359.714202
Residual for this perturbation : 0.000517
Perturbation 6
Initial start : X = 6.591715   Y = 6.928599   Z = 5.679725   A = 5.955805   B = 28.848514   G = 17.382729
Initial residual: 0.001704
Converged Position : X = 6.0038   Y = 7.580236   Z = 6.689603   A = 10.733378   B = 66.436936   G = 3.522753
Residual for this perturbation : 0.000514
Using perturbation #: 1
Solution to improve: X=5.942331 Y=8.469713 Z=7.016146 A=39.142684 B=57.571305 G=32.047738
Initial residual: 0.000312
Final solution : X= 5.943 Y= 8.470 Z= 7.016 A= 39.142 B= 57.571 G= 32.048
Final residual: 0.000312

Data point #: 6

```
===============
Initial guess: 5.942559 8.469933 7.016101 39.142208 57.571088 32.048022
Perturbation 1
Initial start : X = 5.942559   Y = 8.469933   Z = 7.016101   A = 39.142208   B = 57.571088   G = 32.048022
Initial residual: 0.000674
Converged Position : X = 7.000094   Y = 8.4181   Z = 7.603394   A = 29.284243   B = 71.360718   G = 22.355054
Residual for this perturbation : 0.000350
Perturbation 2
Initial start : X = 6.298052   Y = 8.122888   Z = 6.030008   A = 35.647229   B = 67.516248   G = 10.381190
Initial residual: 0.000755
Converged Position : X = 7.1315   Y = 8.913380   Z = 7.303326   A = 21.298973   B = 74.398589   G = 8.783805
Residual for this perturbation : 0.000255
Perturbation 3
Initial start : X = 5.380883   Y = 9.008115   Z = 7.917767   A = 25.136210   B = 84.084289   G = 2.780375
Initial residual: 0.000764
Converged Position : X = 6.950847   Y = 9.0609   Z = 7.010336   A = 28.082047   B = 79.531802   G = 16.076979
Residual for this perturbation : 0.000305
Perturbation 4
Initial start : X = 6.816992   Y = 8.485373   Z = 7.910529   A = 17.823709   B = 29.871828   G = 18.120626
Initial residual: 0.001519
Converged Position : X = 7.000173   Y = 8.4177   Z = 7.603585   A = 29.287757   B = 71.360670   G = 22.356776
Residual for this perturbation : 0.000350
Perturbation 5
Initial start : X = 5.067729   Y = 8.179993   Z = 7.448733   A = 31.541763   B = 56.254893   G = 47.768848
Initial residual: 0.001073
Converged Position : X = 6.9994   Y = 8.419323   Z = 7.603238   A = 29.253484   B = 71.357288   G = 22.335000
Residual for this perturbation : 0.000350
Perturbation 6
Initial start : X = 5.130055   Y = 8.114810   Z = 7.789589   A = 13.356587   B = 28.194909   G = 61.602197
Initial residual: 0.002253
Converged Position : X = 7.0188   Y = 8.463049   Z = 7.607702   A = 29.192172   B = 71.792164   G = 21.801111
Residual for this perturbation : 0.000351
Using perturbation #: 2
Solution to improve: X=7.131569 Y=8.913380 Z=7.303326 A=21.298973 B=74.398589 G=8.783805
Initial residual: 0.000255
Final solution : X= 7.132 Y= 8.913 Z= 7.303 A= 21.299 B= 74.399 G= 8.784
Final residual: 0.000255


Data point #: 7
===============
Initial guess: 7.131569 8.913380 7.303326 21.298973 74.398589 8.783805
Perturbation 1
Initial start : X = 7.131569   Y = 8.913380   Z = 7.303326   A = 21.298973   B = 74.398589   G = 8.783805
Initial residual: 0.000648
Converged Position : X = 7.8582   Y = 9.5258   Z = 8.677915   A = 350.616993   B = 85.502105   G = 338.056655
Residual for this perturbation : 0.000326
Perturbation 2
Initial start : X = 7.280106   Y = 8.629349   Z = 6.392544   A = 37.277793   B = 74.445977   G = -11.985411
Initial residual: 0.001418
Converged Position : X = 7.9917   Y = 9.818094   Z = 8.631749   A = 32.023683   B = 85.522404   G = 20.309814
Residual for this perturbation : 0.000351
Perturbation 3
Initial start : X = 6.775210   Y = 8.513150   Z = 7.079672   A = 26.190085   B = 50.569586   G = 10.346900
Initial residual: 0.001332
Converged Position : X = 7.8986   Y = 9.86446   Z = 8.6752   A = 40.1932   B = 85.245420   G = 28.714550
Residual for this perturbation : 0.000348
Perturbation 4
Initial start : X = 7.660983   Y = 9.002788   Z = 6.398990   A = 33.410121   B = 75.003877   G = 0.619472
Initial residual: 0.000999
```

Converged Position : X = 7.9101   Y = 9.878117   Z = 8.698239   A = 46.735462   B = 84.445077   G = 35.269872
Residual for this perturbation : 0.000348
Perturbation 5
Initial start : X = 7.988395   Y = 9.769779   Z = 6.360356   A = 38.280627   B = 82.885488   G = -11.382421
Initial residual: 0.001358
Converged Position : X = 7.862798   Y = 9.544222   Z = 8.6839   A = 345.6814   B = 85.8021   G = 333.485237
Residual for this perturbation : 0.000326
Perturbation 6
Initial start : X = 7.692616   Y = 8.868041   Z = 7.953686   A = 19.436412   B = 54.988849   G = -21.050685
Initial residual: 0.001414
Converged Position : X = 7.875342   Y = 9.528707   Z = 8.6831   A = 359.7603   B = 85.6691   G = 347.307090
Residual for this perturbation : 0.000328
Using perturbation #: 5
Solution to improve: X=7.862798 Y=9.544222 Z=8.683978 A=345.681446 B=85.802172 G=333.485237
Initial residual: 0.000326
Final solution : X= 7.863 Y= 9.544 Z= 8.684 A=345.681 B= 85.802 G=333.485
Final residual: 0.000326

Data point #: 8
================
Initial guess: 7.862798 9.544222 8.683978 345.681446 85.802172 333.485237
Perturbation 1
Initial start : X = 7.862798   Y = 9.544222   Z = 8.683978   A = 345.681446   B = 85.802172   G = 333.485237
Initial residual: 0.000741
Converged Position : X = 8.731791   Y = 9.7035   Z = 9.0201   A = 54.380879   B = 111.890493   G = 59.089621
Residual for this perturbation : 0.000335
Perturbation 2
Initial start : X = 8.154820   Y = 9.523823   Z = 9.658995   A = 373.741423   B = 80.445411   G = 315.498602
Initial residual: 0.001590
Converged Position : X = 9.6576   Y = 9.87081   Z = 10.4072   A = 61.441834   B = 68.614443   G = 50.632676
Residual for this perturbation : 0.000389
Perturbation 3
Initial start : X = 8.844473   Y = 8.623506   Z = 7.935148   A = 344.394125   B = 109.302088   G = 322.061302
Initial residual: 0.000826
Converged Position : X = 9.1625   Y = 9.8788   Z = 8.8621   A = 60.248699   B = 111.761760   G = 49.607484
Residual for this perturbation : 0.000266
Perturbation 4
Initial start : X = 7.696074   Y = 9.793607   Z = 8.484663   A = 343.781225   B = 67.548694   G = 343.996029
Initial residual: 0.000817
Converged Position : X = 9.7742   Y = 9.9475   Z = 9.521867   A = 305.707206   B = 68.720438   G = 316.553113
Residual for this perturbation : 0.000396
Perturbation 5
Initial start : X = 7.248802   Y = 9.337569   Z = 8.289234   A = 339.361064   B = 58.547783   G = 303.894230
Initial residual: 0.001206
Converged Position : X = 9.8130   Y = 9.88988   Z = 9.49972   A = 314.692461   B = 69.184009   G = 305.137790
Residual for this perturbation : 0.000425
Perturbation 6
Initial start : X = 8.590961   Y = 8.955460   Z = 9.253393   A = 332.697453   B = 109.352845   G = 320.984796
Initial residual: 0.000592
Converged Position : X = 9.39541   Y = 10.095327   Z = 9.70512   A = 323.58958   B = 110.0735   G = 314.646362
Residual for this perturbation : 0.000448
Using perturbation #: 3
Solution to improve: X=9.162514 Y=9.878893 Z=8.862137 A=60.248699 B=111.761760 G=49.607484
Initial residual: 0.000266
Final solution : X= 9.163 Y= 9.879 Z= 8.862 A= 60.249 B=111.762 G= 49.607
Final residual: 0.000266

Data point #: 9
================

Initial guess: 9.162514 9.878893 8.862137 60.248699 111.761760 49.607484
Perturbation 1
Initial start : X = 9.162514   Y = 9.878893   Z = 8.862137   A = 60.248699   B = 111.761760   G = 49.607484
Initial residual: 0.000664
Converged Position : X = 10.530633   Y = 10.818143 Z = 9.966880 A = 77.807547 B = 118.9673   G = 57.927295
Residual for this perturbation : 0.000273
Perturbation 2
Initial start : X = 9.372677   Y = 9.349186   Z = 8.289058   A = 44.340149   B = 116.851873   G = 74.334265
Initial residual: 0.000857
Converged Position : X = 10.1437   Y = 10.8624   Z = 10.7308   A = 76.440906   B = 116.190274   G = 94.908518
Residual for this perturbation : 0.000509
Perturbation 3
Initial start : X = 9.402053   Y = 10.461027   Z = 8.714594   A = 55.850034   B = 134.570202   G = 35.754086
Initial residual: 0.000842
Converged Position : X = 10.5304   Y = 10.8179   Z = 9.9660   A = 77.801074   B = 118.967064   G = 57.912989
Residual for this perturbation : 0.000273
Perturbation 4
Initial start : X = 9.652421   Y = 9.923209   Z = 8.082506   A = 38.283634   B = 133.274619   G = 73.416632
Initial residual: 0.001144
Converged Position : X = 10.063899   Y = 10.5320   Z = 10.0764   A = 57.6924   B = 115.854666   G = 74.514024
Residual for this perturbation : 0.000515
Perturbation 5
Initial start : X = 8.549810   Y = 9.632830   Z = 8.915007   A = 36.301175   B = 129.597830   G = 59.580329
Initial residual: 0.000947
Converged Position : X = 10.0642   Y = 10.53209   Z = 10.0767   A = 57.6971   B = 115.852504   G = 74.514173
Residual for this perturbation : 0.000515
Perturbation 6
Initial start : X = 9.320601   Y = 10.404081   Z = 9.174083   A = 49.203409   B = 129.612957   G = 64.874455
Initial residual: 0.000776
Converged Position : X = 10.063   Y = 10.5318   Z = 10.07613   A = 57.693334   B = 115.855671   G = 74.513066
Residual for this perturbation : 0.000515
Using perturbation #: 1
Solution to improve: X=10.530633 Y=10.818143 Z=9.966880 A=77.807547 B=118.967347 G=57.927295
Initial residual: 0.000273
Final solution : X= 10.531 Y= 10.818 Z= 9.967 A= 77.808 B=118.967 G= 57.927
 Final residual: 0.000273

Data point #: 10
===============
Initial guess: 10.530633 10.818143 9.966880 77.807547 118.967347 57.927295
Perturbation 1
Initial start : X = 10.530633   Y = 10.818143   Z = 9.966880   A = 77.807547   B = 118.967347   G = 57.927295
Initial residual: 0.000764
Converged Position : X = 10.7498   Y = 11.44817   Z = 11.2126   A = 94.7490   B = 136.104748   G = 78.801223
Residual for this perturbation : 0.000310
Perturbation 2
Initial start : X = 9.990402   Y = 11.500293   Z = 9.636205   A = 62.225015   B = 110.083009   G = 38.643753
Initial residual: 0.001089
Converged Position : X = 10.7633   Y = 11.4466   Z = 11.20545   A = 94.9606   B = 136.050837   G = 79.099070
Residual for this perturbation : 0.000310
Perturbation 3
Initial start : X = 9.715993   Y = 11.273780   Z = 10.242521   A = 58.931450   B = 145.807931   G = 64.009482
Initial residual: 0.000682
Converged Position : X = 10.4529   Y = 11.9724   Z = 10.7051   A = 78.102082   B = 137.011694   G = 72.028188
Residual for this perturbation : 0.000394
Perturbation 4
Initial start : X = 10.313760   Y = 9.904602   Z = 10.578689   A = 75.480691   B = 94.360060   G = 84.258081
Initial residual: 0.001432
Converged Position : X = 10.0479   Y = 11.2007   Z = 10.2889   A = 94.31381   B = 134.948903   G = 107.734021

Residual for this perturbation : 0.000482
Perturbation 5
Initial start : X = 10.972997   Y = 10.548490   Z = 10.923733   A = 89.064749   B = 129.750523   G = 40.513270
Initial residual: 0.001223
Converged Position : X = 10.756074   Y = 11.451103   Z = 11.2208   A = 94.8475   B = 136.2100   G = 78.949818
Residual for this perturbation : 0.000310
Perturbation 6
Initial start : X = 11.295251   Y = 10.800111   Z = 9.901207   A = 78.589058   B = 148.846973   G = 64.700812
Initial residual: 0.000714
Converged Position : X = 10.4595   Y = 11.5213   Z = 11.2216   A = 86.844355   B = 136.321374   G = 72.943522
Residual for this perturbation : 0.000328
Using perturbation #: 2
Solution to improve: X=10.763308 Y=11.446670 Z=11.205457 A=94.960654 B=136.050837 G=79.099070
Initial residual: 0.000310
Final solution : X= 10.763 Y= 11.447 Z= 11.205 A= 94.961 B=136.051 G= 79.099
Final residual: 0.000310

Data point #: 11
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Initial guess: 10.763308 11.446670 11.205457 94.960654 136.050837 79.099070
Perturbation 1
Initial start : X = 10.763308   Y = 11.446670   Z = 11.205457   A = 94.960654   B = 136.050837   G = 79.099070
Initial residual: 0.000859
Converged Position : X = 12.6091   Y = 12.8233   Z = 11.7881   A = 103.0188   B = 150.065519   G = 78.730218
Residual for this perturbation : 0.000289
Perturbation 2
Initial start : X = 10.623735   Y = 12.004937   Z = 10.814247   A = 105.944633   B = 159.203031   G = 61.343822
Initial residual: 0.000858
Converged Position : X = 12.58243 Y = 12.5830   Z = 11.6578   A = 101.4980   B = 150.689429   G = 75.730984
Residual for this perturbation : 0.000295
Perturbation 3
Initial start : X = 10.955573   Y = 11.066923   Z = 10.730950   A = 88.327619   B = 121.424191   G = 69.624475
Initial residual: 0.001220
Converged Position : X = 12.6080   Y = 12.8236   Z = 11.7865   A = 103.0249   B = 150.059171   G = 78.737300
Residual for this perturbation : 0.000289
Perturbation 4
Initial start : X = 10.483527   Y = 12.293411   Z = 10.300275   A = 104.514230   B = 142.787355   G = 86.981601
Initial residual: 0.000750
Converged Position : X = 12.6103   Y = 12.8221   Z = 11.7898   A = 103.0112   B = 150.061705   G = 78.736059
Residual for this perturbation : 0.000289
Perturbation 5
Initial start : X = 10.643346   Y = 11.660604   Z = 10.927038   A = 91.695931   B = 153.326061   G = 84.941641
Initial residual: 0.000845
Converged Position : X = 12.6108   Y = 12.8220   Z = 11.7913   A = 103.0001   B = 150.066664   G = 78.729613
Residual for this perturbation : 0.000289
Perturbation 6
Initial start : X = 10.141894   Y = 12.321534   Z = 11.428712   A = 73.376744   B = 133.743752   G = 78.816593
Initial residual: 0.001126
Converged Position : X = 10.8821   Y = 12.31411   Z = 11.73209   A = 73.3338   B = 150.652794   G = 96.249964
Residual for this perturbation : 0.000719
Using perturbation #: 3
Solution to improve: X=12.608017 Y=12.823659 Z=11.786575 A=103.024976 B=150.059171 G=78.737300
Initial residual: 0.000289
Final solution : X= 12.608 Y= 12.824 Z= 11.787 A=103.025 B=150.059 G= 78.737

Final residual: 0.000289

Data point #: 12
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Initial guess: 12.608017 12.823659 11.786575 103.024976 150.059171 78.737300
Perturbation 1
Initial start : X = 12.608017   Y = 12.823659   Z = 11.786575   A = 103.024976   B = 150.059171   G = 78.737300
Initial residual: 0.000768
Converged Position : X = 13.4606   Y = 13.445   Z = 12.4244   A = 110.262036   B = 168.279122   G = 85.912766
Residual for this perturbation : 0.000339
Perturbation 2
Initial start : X = 13.592379   Y = 13.213896   Z = 11.648257   A = 126.257323   B = 121.062592   G = 97.968359
Initial residual: 0.001588
Converged Position : X = 13.460614   Y = 13.445490   Z = 12.4245   A = 110.2620   B = 168.2788   G = 85.915193
Residual for this perturbation : 0.000339
Perturbation 3
Initial start : X = 11.667795   Y = 12.764968   Z = 10.894567   A = 96.647180   B = 156.004829   G = 56.736661
Initial residual: 0.001056
Converged Position : X = 13.4606   Y = 13.44548   Z = 12.4245   A = 110.2620   B = 168.278859   G = 85.915220
Residual for this perturbation : 0.000339
Perturbation 4
Initial start : X = 12.395272   Y = 13.602720   Z = 11.581524   A = 96.135803   B = 146.329288   G = 60.161485
Initial residual: 0.001083
Converged Position : X = 13.4606   Y = 13.4454   Z = 12.4245   A = 110.2620   B = 168.278902   G = 85.915259
Residual for this perturbation : 0.000339
Perturbation 5
Initial start : X = 12.624604   Y = 11.862517   Z = 12.578127   A = 112.289758   B = 123.472480   G = 66.017961
Initial residual: 0.001714
Converged Position : X = 13.460620   Y = 13.4454   Z = 12.4245   A = 110.2620   B = 168.2789   G = 85.915287
Residual for this perturbation : 0.000339
Perturbation 6
Initial start : X = 13.298244   Y = 13.732392   Z = 11.680367   A = 104.908423   B = 130.318619   G = 51.386554
Initial residual: 0.001616
Converged Position : X = 13.460621   Y = 13.445473   Z = 12.4245   A = 110.2620   B = 168.27891   G = 85.915280
Residual for this perturbation : 0.000339
Using perturbation #: 1
Solution to improve: X=13.460653 Y=13.445697 Z=12.424479 A=110.262036 B=168.279122 G=85.912766
Initial residual: 0.000339
Final solution : X= 13.461 Y= 13.446 Z= 12.424 A=110.262 B=168.279 G= 85.913
Final residual: 0.000339

Data point #: 13
================
Initial guess: 13.460653 13.445697 12.424479 110.262036 168.279122 85.912766
Perturbation 1
Initial start : X = 13.460653   Y = 13.445697   Z = 12.424479   A = 110.262036   B = 168.279122   G = 85.912766
Initial residual: 0.000846
Converged Position : X = 13.481663   Y = 14.2290   Z = 12.4072   A = 120.0568   B = 181.649270   G = 95.061055
Residual for this perturbation : 0.000314
Perturbation 2
Initial start : X = 12.986191   Y = 12.727929   Z = 12.789175   A = 130.002107   B = 151.222997   G = 64.576504
Initial residual: 0.001450
Converged Position : X = 13.463126   Y = 14.227632   Z = 12.437610   A = 119.805681   B = 181.625301   G = 95.081117
Residual for this perturbation : 0.000314
Perturbation 3
Initial start : X = 13.492917   Y = 14.250377   Z = 11.571036   A = 130.759181   B = 165.614448   G = 71.766347
Initial residual: 0.001173
Converged Position : X = 13.4833   Y = 14.21723   Z = 12.3642   A = 120.2503   B = 181.754461   G = 95.224307
Residual for this perturbation : 0.000314
Perturbation 4
Initial start : X = 12.763290   Y = 14.143648   Z = 11.600769   A = 95.212321   B = 154.370189   G = 61.953985
Initial residual: 0.001521

Converged Position : X = 13.4767   Y = 14.2025   Z = 12.466   A = 119.9614   B = 182.192269   G = 95.428574
Residual for this perturbation : 0.000313
Perturbation 5
Initial start : X = 12.957298   Y = 13.695419   Z = 13.104946   A = 108.107357   B = 164.999788   G = 82.839405
Initial residual: 0.001007
Converged Position : X = 13.4616   Y = 14.2102   Z = 12.3698   A = 120.0453   B = 182.174455   G = 94.914460
Residual for this perturbation : 0.000312
Perturbation 6
Initial start : X = 14.416647   Y = 12.573720   Z = 13.096350   A = 86.104251   B = 186.194902   G = 107.067592
Initial residual: 0.001142
Converged Position : X = 12.9701   Y = 14.0318   Z = 12.2911   A = 118.0543   B = 187.635980   G = 102.348058
Residual for this perturbation : 0.000433
Using perturbation #: 5
Solution to improve: X=13.461674 Y=14.210253 Z=12.369820 A=120.045352 B=182.174455 G=94.914460
Initial residual: 0.000312
Final solution : X= 13.462 Y= 14.210 Z= 12.370 A=120.045 B=182.174 G=94.914
Final residual: 0.000312

Data point #: 14
===============
Initial guess: 13.461674 14.210253 12.369820 120.045352 182.174455 94.914460
Perturbation 1
Initial start : X = 13.461674   Y = 14.210253   Z = 12.369820   A = 120.045352   B = 182.174455   G = 94.914460
Initial residual: 0.000973
Converged Position : X = 15.078792   Y = 14.8742   Z = 14.4102   A = 125.8627   B = 195.1766   G = 102.524591
Residual for this perturbation : 0.000230
Perturbation 2
Initial start : X = 12.923042   Y = 13.2284   Z = 11.519655   A = 121.473014   B = 171.962151   G = 116.000184
Initial residual: 0.001476
Converged Position : X = 15.0338   Y = 14.8157   Z = 14.3507   A = 125.1666   B = 197.966110   G = 105.720305
Residual for this perturbation : 0.000273
Perturbation 3
Initial start : X = 14.165060   Y = 13.502278   Z = 11.393910   A = 109.165382   B = 165.618407   G = 93.665824
Initial residual: 0.001656
Converged Position : X = 15.0775   Y = 14.8747   Z = 14.4091   A = 125.8666   B = 195.173673   G = 102.523450
Residual for this perturbation : 0.000230
Perturbation 4
Initial start : X = 14.131863   Y = 13.653452   Z = 13.155074   A = 111.790978   B = 179.843229   G = 75.262999
Initial residual: 0.001509
Converged Position : X = 15.1134   Y = 14.8292   Z = 14.4646   A = 125.1481   B = 197.611952   G = 105.589151
Residual for this perturbation : 0.000276
Perturbation 5
Initial start : X = 14.368052   Y = 14.960177   Z = 11.824229   A = 145.441335   B = 205.106535   G = 96.249219
Initial residual: 0.001098
Converged Position : X = 14.9874   Y = 14.7822   Z = 14.3573   A = 125.1934   B = 198.022853   G = 105.569676
Residual for this perturbation : 0.000272
Perturbation 6
Initial start : X = 13.726263   Y = 14.285146   Z = 13.162253   A = 129.670079   B = 182.896701   G = 108.477788
Initial residual: 0.000813
Converged Position : X = 15.0762   Y = 14.8766   Z = 14.4101   A = 125.8710   B = 195.169976   G = 102.532831
Residual for this perturbation : 0.000230
Using perturbation #: 6
Solution to improve: X=15.076263 Y=14.876672 Z=14.410167 A=125.871083 B=195.169976 G=102.532831
Initial residual: 0.000230
Final solution : X= 15.076 Y= 14.877 Z= 14.410 A=125.871 B=195.170 G=102.533
Final residual: 0.000230

Data point #: 15

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Initial guess: 15.076263  14.876672  14.410167  125.871083  195.169976  102.532831
Perturbation 1
Initial start : X = 15.076263   Y = 14.876672   Z = 14.4101   A = 125.871083   B = 195.169976   G = 102.532831
Initial residual: 0.000963
Converged Position : X = 16.2539   Y = 15.2725   Z = 14.4205   A = 135.9964   B = 208.782620   G = 114.929334
Residual for this perturbation : 0.000229
Perturbation 2
Initial start : X = 14.124172   Y = 14.966204   Z = 15.057796   A = 141.644449   B = 204.005954   G = 90.693114
Initial residual: 0.001242
Converged Position : X = 16.2540   Y = 15.2725   Z = 14.4208   A = 135.9972   B = 208.782797   G = 114.928195
Residual for this perturbation : 0.000229
Perturbation 3
Initial start : X = 16.059083   Y = 15.255112   Z = 14.971642   A = 143.529018   B = 187.407109   G = 73.741298
Initial residual: 0.001579
Converged Position : X = 15.7914   Y = 13.6692   Z = 15.5381   A = 143.0259   B = 208.088650   G = 79.844145
Residual for this perturbation : 0.001296
Perturbation 4
Initial start : X = 14.398482   Y = 15.0639   Z = 14.2067   A = 146.567689   B = 220.926715   G = 114.370430
Initial residual: 0.001079
Converged Position : X = 16.252761   Y = 15.273881  Z = 14.419138  A = 135.997  B = 208.7810  G = 114.928563
Residual for this perturbation : 0.000229
Perturbation 5
Initial start : X = 15.739107   Y = 15.203087   Z = 14.751491   A = 116.527881  B = 173.977883   G = 103.191968
Initial residual: 0.001598
Converged Position : X = 16.2533   Y = 15.2739   Z = 14.4197   A = 136.0007   B = 208.780081   G = 114.927281
Residual for this perturbation : 0.000229
Perturbation 6
Initial start : X = 15.785660   Y = 13.902222   Z = 13.621044  A = 112.876555   B = 184.384112   G = 110.944480
Initial residual: 0.001644
Converged Position : X = 16.2529   Y = 15.2731   Z = 14.4203   A = 136.0050   B = 208.780579   G = 114.924930
Residual for this perturbation : 0.000229
Using perturbation #: 1
Solution to improve: X=16.253940  Y=15.272541  Z=14.420570  A=135.996461  B=208.782620  G=114.929334
Initial residual: 0.000229
Final solution : X= 16.254 Y= 15.273 Z= 14.421 A=135.996 B=208.783 G=114.929
Final residual: 0.000229


Data point #: 16
==============
Initial guess: 16.253940  15.272541  14.420570  135.996461  208.782620  114.929334
Perturbation 1
Initial start : X = 16.253940   Y = 15.272541   Z = 14.4205   A = 135.9964   B = 208.782620   G = 114.929334
Initial residual: 0.001175
Converged Position : X = 16.3933   Y = 16.3054   Z = 15.3531   A = 112.3399   B = 222.784619   G = 126.523874
Residual for this perturbation : 0.000635
Perturbation 2
Initial start : X = 15.622537   Y = 15.673835   Z = 15.351612  A = 106.386082   B = 181.348821   G = 121.277879
Initial residual: 0.002143
Converged Position : X = 16.396214   Y = 16.3059   Z = 15.3539   A = 112.3544   B = 222.7915   G = 126.540898
Residual for this perturbation : 0.000635
Perturbation 3
Initial start : X = 16.016342   Y = 15.335770   Z = 15.3462   A = 153.129898   B = 185.320282   G = 115.762640
Initial residual: 0.001897
Converged Position : X = 16.762809   Y = 16.782263   Z = 15.071611   A = 141.464192   B = 184.517459   G = 86.312959
Residual for this perturbation : 0.000761
Perturbation 4
Initial start : X = 16.582867   Y = 14.397504   Z = 13.841159   A = 146.315799   B = 187.845047   G = 91.897527
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Initial residual: 0.001425
Converged Position : X = 16.7610 Y = 16.7815   Z = 15.0739   A = 141.4637   B = 184.547375   G = 86.331640
Residual for this perturbation : 0.000761
Perturbation 5
Initial start : X = 15.753038 Y = 14.436422   Z = 15.156183   A = 111.136326   B = 183.238196 G = 110.790324
Initial residual: 0.002094
Converged Position : X = 16.761953 Y = 16.7802 Z = 15.07518 A = 141.4435   B = 184.577815 G = 86.327853
Residual for this perturbation : 0.000761
Perturbation 6
Initial start : X = 15.338707 Y = 14.561492   Z = 14.065078 A = 132.771564 B = 183.386129   G = 132.876268
Initial residual: 0.002194
Converged Position : X = 16.392811 Y = 16.3054 Z = 15.3541 A = 112.317030 B = 222.785028 G = 126.538661
Residual for this perturbation : 0.000635
Using perturbation #: 2
Solution to improve: X=16.396214 Y=16.305962 Z=15.353953 A=112.354482 B=222.791555 G=126.540898
Initial residual: 0.000635
Final solution : X= 16.396 Y= 16.306 Z= 15.354 A=112.354 B=222.792 G=126.541
Final residual: 0.000635

Data point #: 17
===============
Initial guess: 16.396214 16.305962 15.353953 112.354482 222.791555 126.540898
Perturbation 1
Initial start : X = 16.396214 Y = 16.305962   Z = 15.3539   A = 112.3544   B = 222.7915   G = 126.540898
Initial residual: 0.002123
Converged Position : X = 16.940815 Y = 16.2727   Z = 16.4478   A = 95.0646   B = 227.8238   G = 141.083482
Residual for this perturbation : 0.001969
Perturbation 2
Initial start : X = 15.578040 Y = 17.229849   Z = 14.557742   A = 125.33964 B = 205.174593   G = 150.437801
Initial residual: 0.002508
Converged Position : X = 17.8846 Y = 17.1406   Z = 15.8291   A = 159.2370   B = 240.143685   G = 127.351530
Residual for this perturbation : 0.000312
Perturbation 3
Initial start : X = 16.654727 Y = 16.534759   Z = 14.836541   A = 99.241145   B = 211.134488   G = 121.710042
Initial residual: 0.002484
Converged Position : X = 16.93444 Y = 16.2836   Z = 16.4540   A = 94.9683   B = 227.756270   G = 141.047949
Residual for this perturbation : 0.001969
Perturbation 4
Initial start : X = 17.322431   Y = 16.553038   Z = 15.814263 A = 136.205100   B = 212.430885   G = 126.075131
Initial residual: 0.002555
Converged Position : X = 17.884664   Y = 17.14069 Z = 15.829   A = 159.23   B = 240.143685   G = 127.351530
Residual for this perturbation : 0.000312
Perturbation 5
Initial start : X = 16.212464   Y = 16.569311   Z = 15.210130   A = 100.350233   B = 225.684907   G =
113.626418
Initial residual: 0.002499
Converged Position : X = 16.935932 Y = 16.283921 Z = 16.454307 A = 94.994864 B = 227.76826 G = 141.050683
Residual for this perturbation : 0.001969
Perturbation 6
Initial start : X = 16.746172   Y = 16.781097   Z = 14.406678   A = 125.253728   B = 242.545420 G = 115.801821
Initial residual: 0.002547
Converged Position : X = 16.93434 Y = 16.28255 Z = 16.454642 A = 94.923218 B = 227.762615 G = 141.094127
Residual for this perturbation : 0.001969
Using perturbation #: 2
Solution to improve: X=17.884664 Y=17.140695 Z=15.829161 A=159.237080 B=240.143685 G=127.351530
Initial residual: 0.000312
Final solution : X= 17.885 Y= 17.141 Z= 15.829 A=159.237 B=240.144 G=127.352
Final residual: 0.000312

Table C.3: Solution of Run2.1, using floating point initial guess method of perturbation and 5% of maximum range random noise.

```
Data point #: 1
==============
Initial guess: 2.000000  5.000000  4.000000  0.000000  0.000000  0.000000
Perturbation 1
Initial start : X = 2.0000   Y = 5.000000   Z = 4.000000   A = 0.000000   B = 0.000000   G = 0.000000
Initial residual: 0.000348
Converged Position : X = 1.9080  Y = 5.2621   Z = 4.0513   A = 357.8342   B = 1.0575   G = 3.910182
Residual for this perturbation : 0.000234
Perturbation 2
Initial start : X = 1.984819  Y = 4.353542   Z = 3.290943   A = 344.643183   B = -9.706131   G = 17.476154
Initial residual: 0.001648
Converged Position : X = 1.90809   Y = 5.262086   Z = 4.051380   A = 357.834561   B = 1.057576   G = 3.910046
Residual for this perturbation : 0.000234
Perturbation 3
Initial start : X = 1.790725   Y = 6.237622   Z = 3.985877   A = 354.983207   B = 29.925837   G = -22.514466
Initial residual: 0.001540
Converged Position : X = 2.131044   Y = 4.992253   Z = 4.523149   A = 2.107794   B = 15.024540   G = 1.140616
Residual for this perturbation : 0.000713
Perturbation 4
Initial start : X = 1.857477   Y = 4.352205   Z = 4.647368   A = 27.331646   B = -9.303924   G = -25.614725
Initial residual: 0.001661
Converged Position : X = 1.5924   Y = 5.628037   Z = 4.138003   A = 9.018907   B = 356.807426   G = 6.045661
Residual for this perturbation : 0.000495
Perturbation 5
Initial start : X = 1.613611   Y = 4.973958   Z = 4.921749   A = 5.178057   B = 361.303701   G = 6.803992
Initial residual: 0.000694
Converged Position : X = 1.90808   Y = 5.26213   Z = 4.051390   A = 357.834298   B = 1.057499   G = 3.910166
Residual for this perturbation : 0.000234
Perturbation 6
Initial start : X = 1.825974   Y = 5.612551   Z = 4.032180   A = 381.070243   B = 6.180684   G = -25.248185
Initial residual: 0.001247
Converged Position : X = 1.9081   Y = 5.26187   Z = 4.051269   A = 357.835866   B = 1.057147   G = 3.909654
Residual for this perturbation : 0.000234
Using perturbation #: 6
Solution to improve: X=1.908170  Y=5.261876  Z=4.051269  A=357.835866  B=1.057147  G=3.909654
Initial residual: 0.000234
Final solution : X= 1.908 Y= 5.262 Z= 4.051 A=357.836 B= 1.057 G= 3.910
Final residual: 0.000234


Data point #: 2
==============
Initial guess: 1.908170  5.261876  4.051269  357.835866  1.057147  3.909654
Perturbation 1
Initial start : X = 1.908170   Y = 5.261876   Z = 4.051269   A = 357.835866   B = 1.057147   G = 3.909654
Initial residual: 0.001212
Converged Position : X = 3.30687  Y = 5.38913   Z = 4.669257   A = 13.561800   B = 14.460683   G = 6.444740
Residual for this perturbation : 0.000290
Perturbation 2
Initial start : X = 2.867500   Y = 5.451090   Z = 5.530814   A = 2.192510   B = 8.474871   G = 29.247386
Initial residual: 0.001342
Converged Position : X = 3.30685   Y = 5.38915   Z = 4.669305   A = 13.561950   B = 14.460896   G = 6.444789
Residual for this perturbation : 0.000290
Perturbation 3
Initial start : X = 3.966660   Y = 5.005500   Z = 4.365122   A = -11.111774   B = 6.107380   G = 5.384673
Initial residual: 0.001372
Converged Position : X = 3.30684   Y = 5.38916   Z = 4.669309   A = 13.560530   B = 14.462172   G = 6.444541
Residual for this perturbation : 0.000290
```

Perturbation 4
Initial start : X = 2.548192   Y = 5.099515   Z = 5.425079   A = 25.333583   B = 27.784883   G = 16.938046
Initial residual: 0.001392
Converged Position : X = 3.30683   Y = 5.38917   Z = 4.669283   A = 13.561051   B = 14.461697   G = 6.444954
Residual for this perturbation : 0.000290
Perturbation 5
Initial start : X = 2.734629   Y = 4.420624   Z = 5.567673   A = 7.964254   B = 19.137580   G = 9.737587
Initial residual: 0.000954
Converged Position : X = 3.30700   Y = 5.38903   Z = 4.669406   A = 13.564847   B = 14.460180   G = 6.444464
Residual for this perturbation : 0.000290
Perturbation 6
Initial start : X = 4.115293   Y = 6.202733   Z = 3.901016   A = 32.557261   B = 42.753823   G = 14.639628
Initial residual: 0.001591
Converged Position : X = 3.3068   Y = 5.389179   Z = 4.669293   A = 13.561320   B = 14.461553   G = 6.444619
Residual for this perturbation : 0.000290
Using perturbation #: 5
Solution to improve: X=3.307003 Y=5.389036 Z=4.669406 A=13.564847 B=14.460180 G=6.444464
Initial residual: 0.000290
Final solution : X= 3.307 Y= 5.389 Z= 4.669 A= 13.565 B= 14.460 G= 6.444
Final residual: 0.000290

Data point #: 3
================
Initial guess: 3.307003 5.389036 4.669406 13.564847 14.460180 6.444464
Perturbation 1
Initial start : X = 3.307003   Y = 5.389036   Z = 4.669406   A = 13.564847   B = 14.460180   G = 6.444464
Initial residual: 0.001155
Converged Position : X = 3.8819   Y = 6.70869   Z = 5.610868   A = 20.696493   B = 27.729385   G = 17.681538
Residual for this perturbation : 0.000285
Perturbation 2
Initial start : X = 3.869137   Y = 5.715853   Z = 6.368484   A = 22.954650   B = 29.013982   G = -1.498488
Initial residual: 0.001001
Converged Position : X = 3.89711   Y = 6.7160   Z = 5.569283   A = 20.344760   B = 28.243535   G = 17.281133
Residual for this perturbation : 0.000285
Perturbation 3
Initial start : X = 3.222521   Y = 7.408816   Z = 5.800416   A = 26.975603   B = 24.129022   G = -10.926989
Initial residual: 0.001120
Converged Position : X = 3.9122   Y = 6.721268   Z = 5.588800   A = 20.449883   B = 28.183604   G = 17.172662
Residual for this perturbation : 0.000285
Perturbation 4
Initial start : X = 3.378341   Y = 5.911019   Z = 4.757320   A = 8.191154   B = 47.872415   G = -4.353492
Initial residual: 0.001183
Converged Position : X = 4.1243   Y = 6.241335   Z = 5.505839   A = 10.990512   B = 32.524127   G = 10.453263
Residual for this perturbation : 0.000480
Perturbation 5
Initial start : X = 4.955422   Y = 7.206458   Z = 4.855944   A = 5.213016   B = 5.938153   G = -12.383134
Initial residual: 0.001526
Converged Position : X = 3.78873   Y = 7.36984   Z = 4.935159   A = 10.082268   B = 10.668306   G = 15.576612
Residual for this perturbation : 0.001001
Perturbation 6
Initial start : X = 2.968290   Y = 8.180579   Z = 4.232298   A = 27.911704   B = 6.243949   G = -11.017664
Initial residual: 0.001561
Converged Position : X = 3.7884   Y = 7.370333   Z = 4.935831   A = 10.106778   B = 10.658932   G = 15.560882
Residual for this perturbation : 0.001001
Using perturbation #: 2
Solution to improve: X=3.897119 Y=6.716007 Z=5.569283 A=20.344760 B=28.243535 G=17.281133
Initial residual: 0.000285
Final solution : X= 3.897 Y= 6.716 Z= 5.569 A= 20.345 B= 28.244 G= 17.281
Final residual: 0.000285

Data point #: 4
================

Initial guess: 3.897119 6.716007 5.569283 20.344760 28.243535 17.281133
Perturbation 1
Initial start : X = 3.897119   Y = 6.716007   Z = 5.569283   A = 20.344760   B = 28.243535   G = 17.281133
Initial residual: 0.001063
Converged Position : X = 5.6236   Y = 7.638746   Z = 5.905741   A = 28.950388   B = 48.094163   G = 24.236594
Residual for this perturbation : 0.000310
Perturbation 2
Initial start : X = 5.840245   Y = 6.815086   Z = 4.989030   A = 44.512184   B = 46.170884   G = 5.731213
Initial residual: 0.000964
Converged Position : X = 6.0141   Y = 8.4835   Z = 6.182355   A = 31.701456   B = 54.074718   G = 8.733205
Residual for this perturbation : 0.000511
Perturbation 3
Initial start : X = 6.212611   Y = 9.242466   Z = 6.712030   A = 5.988261   B = 83.105797   G = -10.857457
Initial residual: 0.001107
Converged Position : X = 4.7621   Y = 8.6967   Z = 5.413756   A = 24.295475   B = 76.132527   G = 5.308829
Residual for this perturbation : 0.000900
Perturbation 4
Initial start : X = 4.349899   Y = 9.154615   Z = 4.507056   A = 15.600802   B = 102.378166   G = -20.210081
Initial residual: 0.001535
Converged Position : X = 4.7621   Y = 8.6967   Z = 5.413756   A = 24.295475   B = 76.132527   G = 5.308829
Residual for this perturbation : 0.000900
Perturbation 5
Initial start : X = 3.905435   Y = 7.987881   Z = 4.982026   A = 35.269812   B = 63.010327   G = -15.588809
Initial residual: 0.001373
Converged Position : X = 5.26843   Y = 8.1529   Z = 6.478881   A = 0.448681   B = 51.188160   G = 340.212571
Residual for this perturbation : 0.000349
Perturbation 6
Initial start : X = 5.288591   Y = 8.565356   Z = 6.817020   A = -23.500802   B = 33.534556   G = 364.062470
Initial residual: 0.001178
Converged Position : X = 5.003353   Y = 7.311928   Z = 6.679893   A = 6.542898   B = 48.096928   G = 1.124168
Residual for this perturbation : 0.000620
Using perturbation #: 1
Solution to improve: X=5.623642 Y=7.638746 Z=5.905741 A=28.950388 B=48.094163 G=24.236594
Initial residual: 0.000310
Final solution : X= 5.624 Y= 7.639 Z= 5.906 A= 28.950 B= 48.094 G= 24.237
Final residual: 0.000310

Data point #: 5
================

Initial guess: 5.623642 7.638746 5.905741 28.950388 48.094163 24.236594
Perturbation 1
Initial start : X = 5.623642   Y = 7.638746   Z = 5.905741   A = 28.950388   B = 48.094163   G = 24.236594
Initial residual: 0.000705
Converged Position : X = 5.9423   Y = 8.469713   Z = 7.016146   A = 39.142684   B = 57.571305   G = 32.047738
Residual for this perturbation : 0.000312
Perturbation 2
Initial start : X = 6.330003   Y = 9.191268   Z = 6.968991   A = 10.499841   B = 72.349733   G = 57.304039
Initial residual: 0.001455
Converged Position : X = 5.9455   Y = 8.473672   Z = 7.015669   A = 39.129915   B = 57.575993   G = 32.043896
Residual for this perturbation : 0.000312
Perturbation 3
Initial start : X = 6.230257   Y = 8.689134   Z = 7.601086   A = 67.908272   B = 35.424892   G = 37.194708
Initial residual: 0.001031
Converged Position : X = 5.9461   Y = 8.470439   Z = 7.015630   A = 39.194410   B = 57.557384   G = 32.087541
Residual for this perturbation : 0.000312
Perturbation 4

```
Initial start : X = 5.805261   Y = 8.364381   Z = 6.438469   A = 48.866612   B = 34.017162   G = 40.977411
Initial residual: 0.000968
Converged Position : X = 5.9458   Y = 8.473144   Z = 7.017364   A = 39.137978   B = 57.572122   G = 32.049375
Residual for this perturbation : 0.000312
Perturbation 5
Initial start : X = 6.535147   Y = 8.097973   Z = 6.512708   A = 22.049773   B = 79.232797   G = 52.259074
Initial residual: 0.001184
Converged Position : X = 5.9455   Y = 8.473672   Z = 7.015669   A = 39.129915   B = 57.575993   G = 32.043896
Residual for this perturbation : 0.000312
Perturbation 6
Initial start : X = 5.628516   Y = 9.446430   Z = 7.172737   A = 48.736157   B = 50.195637   G = 48.212524
Initial residual: 0.000604
Converged Position : X = 5.9456   Y = 8.473535   Z = 7.016336   A = 39.136653   B = 57.573989   G = 32.048169
Residual for this perturbation : 0.000312
Using perturbation #: 3
Solution to improve: X=5.946142 Y=8.470439 Z=7.015630 A=39.194410 B=57.557384 G=32.087541
Initial residual: 0.000312
Final solution : X= 5.946 Y= 8.470 Z= 7.016 A= 39.194 B= 57.557 G= 32.088
Final residual: 0.000312


Data point #: 6
==================
Initial guess: 5.946142 8.470439 7.015630 39.194410 57.557384 32.087541
Perturbation 1
Initial start : X = 5.946142   Y = 8.470439   Z = 7.015630   A = 39.194410   B = 57.557384   G = 32.087541
Initial residual: 0.000674
Converged Position : X = 7.0000   Y = 8.4181   Z = 7.6033   A = 29.284331   B = 71.360746   G = 22.355096
Residual for this perturbation : 0.000350
Perturbation 2
Initial start : X = 6.451140   Y = 7.477227   Z = 7.759531   A = 30.586329   B = 96.872730   G = 45.900931
Initial residual: 0.001098
Converged Position : X = 6.9532   Y = 9.033757   Z = 7.000015   A = 28.539329   B = 79.523542   G = 16.454232
Residual for this perturbation : 0.000306
Perturbation 3
Initial start : X = 7.331387   Y = 8.596911   Z = 6.049776   A = 39.104152   B = 89.422493   G = 1.067268
Initial residual: 0.001073
Converged Position : X = 7.1188   Y = 9.4616   Z = 6.848038   A = 321.066278   B = 83.606383   G = 311.577731
Residual for this perturbation : 0.000460
Perturbation 4
Initial start : X = 6.649665   Y = 9.091640   Z = 6.552971   A = 312.852406   B = 57.141441   G = 311.605507
Initial residual: 0.000954
Converged Position : X = 7.23914 Y = 9.5840   Z = 7.6511   A = 330.505064   B = 73.021586   G = 321.265913
Residual for this perturbation : 0.000464
Perturbation 5
Initial start : X = 6.825264   Y = 9.303881   Z = 7.377274   A = 339.608244   B = 81.089909   G = 292.135954
Initial residual: 0.001418
Converged Position : X = 6.9515   Y = 9.059062   Z = 7.003439   A = 27.945508   B = 79.547283   G = 15.923920
Residual for this perturbation : 0.000305
Perturbation 6
Initial start : X = 7.504240   Y = 8.622339   Z = 7.742949   A = 52.913044   B = 53.764670   G = 31.963367
Initial residual: 0.000877
Converged Position : X = 7.167614   Y = 8.9043   Z = 7.297709   A = 21.813123   B = 74.424148   G = 9.300653
Residual for this perturbation : 0.000255
Using perturbation #: 6
Solution to improve: X=7.167614 Y=8.904380 Z=7.297709 A=21.813123 B=74.424148 G=9.300653
Initial residual: 0.000255
Final solution : X= 7.168 Y= 8.904 Z= 7.298 A= 21.813 B= 74.424 G= 9.301
Final residual: 0.000255
```

Data point #: 7
================
Initial guess: 7.167614 8.904380 7.297709 21.813123 74.424148 9.300653
Perturbation 1
Initial start : X = 7.167614   Y = 8.904380   Z = 7.297709   A = 21.813123   B = 74.424148   G = 9.300653
Initial residual: 0.000647
Converged Position : X = 7.8581   Y = 9.5254   Z = 8.6779   A = 351.675692   B = 85.513413   G = 339.106830
Residual for this perturbation : 0.000326
Perturbation 2
Initial start : X = 8.709958   Y = 10.143006   Z = 9.372653   A = 380.575362   B = 89.323648   G = 348.348943
Initial residual: 0.000799
Converged Position : X = 7.9215   Y = 9.520937   Z = 8.643052   A = 66.468688   B = 88.895488   G = 54.189786
Residual for this perturbation : 0.000359
Perturbation 3
Initial start : X = 7.538433   Y = 9.004928   Z = 8.516784   A = 73.914772   B = 108.676465   G = 57.508509
Initial residual: 0.000720
Converged Position : X = 7.8602   Y = 9.5337   Z = 8.675447   A = 163.562552   B = 94.448620   G = 151.435336
Residual for this perturbation : 0.000325
Perturbation 4
Initial start : X = 8.808853   Y = 9.663490   Z = 7.948788   A = 175.972554   B = 84.643174   G = 174.676452
Initial residual: 0.000630
Converged Position : X = 7.8547   Y = 9.5318   Z = 8.680549   A = 171.068812   B = 94.356499   G = 158.860552
Residual for this perturbation : 0.000326
Perturbation 5
Initial start : X = 7.551012   Y = 9.905971   Z = 9.322319   A = 181.298602   B = 99.665884   G = 150.253310
Initial residual: 0.000755
Converged Position : X = 7.855989   Y = 9.533264   Z = 8.678566   A = 166.8734   B = 94.4204   G = 154.705526
Residual for this perturbation : 0.000325
Perturbation 6
Initial start : X = 8.744899   Y = 9.766133   Z = 9.261065   A = 179.158056   B = 79.531163   G = 139.918094
Initial residual: 0.000999
Converged Position : X = 7.9621   Y = 9.5312   Z = 8.685590   A = 230.781768   B = 93.276692   G = 217.845809
Residual for this perturbation : 0.000356
Using perturbation #: 3
Solution to improve: X=7.860277 Y=9.533711 Z=8.675447 A=163.562552 B=94.448620 G=151.435336
Initial residual: 0.000325
Final solution : X= 7.860 Y= 9.534 Z= 8.675 A=163.563 B= 94.449 G=151.435
Final residual: 0.000325

Data point #: 8
================
Initial guess: 7.860277 9.533711 8.675447 163.562552 94.448620 151.435336
Perturbation 1
Initial start : X = 7.860277   Y = 9.533711   Z = 8.675447   A = 163.562552   B = 94.448620   G = 151.435336
Initial residual: 0.000742
Converged Position : X = 9.1555   Y = 9.8870   Z = 8.8745   A = 243.128352   B = 68.369457   G = 232.567832
Residual for this perturbation : 0.000270
Perturbation 2
Initial start : X = 8.810907   Y = 8.974363   Z = 8.504220   A = 235.132695   B = 69.798168   G = 210.204906
Initial residual: 0.000580
Converged Position : X = 9.1575   Y = 9.88995   Z = 8.8720   A = 242.8891   B = 68.314033   G = 232.232920
Residual for this perturbation : 0.000269
Perturbation 3
Initial start : X = 9.128542   Y = 9.810556   Z = 8.935035   A = 235.363495   B = 96.371589   G = 241.225637
Initial residual: 0.000712
Converged Position : X = 9.534783   Y = 9.9288   Z = 10.3191   A = 237.5129   B = 110.819928   G = 247.344471
Residual for this perturbation : 0.000422
Perturbation 4
Initial start : X = 9.102415   Y = 10.635187   Z = 9.552580   A = 254.955188   B = 99.240319   G = 250.364723

Initial residual: 0.000644
Converged Position : X = 8.958745   Y = 10.0430   Z = 10.3488   A = 271.1320   B = 108.9505   G = 259.780968
Residual for this perturbation : 0.000469
Perturbation 5
Initial start : X = 8.203125   Y = 10.788164   Z = 11.170982   A = 261.269938   B = 128.027758   G = 252.312439
Initial residual: 0.000825
Converged Position : X = 8.97461   Y = 10.0782   Z = 10.3464   A = 269.87578   B = 109.197565   G = 258.126686
Residual for this perturbation : 0.000467
Perturbation 6
Initial start : X = 8.579713   Y = 10.570893   Z = 11.038055   A = 291.345361   B = 104.506587   G = 253.428405
Initial residual: 0.001001
Converged Position : X = 9.2422   Y = 10.1628   Z = 9.80271   A = 308.53198   B = 110.659490   G = 299.094212
Residual for this perturbation : 0.000403
Using perturbation #: 2
Solution to improve: X=9.157532 Y=9.889952 Z=8.872066 A=242.889100 B=68.314033 G=232.232920
Initial residual: 0.000269
Final solution : X= 9.158 Y= 9.890 Z= 8.872 A=242.889 B= 68.314 G=232.233
Final residual: 0.000269

Data point #: 9
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Initial guess: 9.157526 9.889936 8.872072 242.888799 68.314249 232.233390
Perturbation 1
Initial start : X = 9.157526   Y = 9.889936   Z = 8.872072   A = 242.888799   B = 68.314249   G = 232.233390
Initial residual: 0.000655
Converged Position : X = 10.5293   Y = 10.8187   Z = 9.9667   A = 257.242143   B = 61.004545   G = 237.287513
Residual for this perturbation : 0.000274
Perturbation 2
Initial start : X = 11.508370   Y = 11.303646   Z = 9.386379   A = 277.927374   B = 40.843098   G = 222.381161
Initial residual: 0.001334
Converged Position : X = 10.5296   Y = 10.8193   Z = 9.9668   A = 257.244554   B = 61.003191   G = 237.295005
Residual for this perturbation : 0.000274
Perturbation 3
Initial start : X = 10.556750   Y = 11.086301   Z = 9.032008   A = 232.816476   B = 48.402556   G = 211.235849
Initial residual: 0.000763
Converged Position : X = 10.5305   Y = 10.8172   Z = 9.9654   A = 257.539951   B = 61.094927   G = 237.624897
Residual for this perturbation : 0.000273
Perturbation 4
Initial start : X = 9.709448   Y = 11.225006   Z = 10.579471   A = 260.466052   B = 83.486684   G = 211.124950
Initial residual: 0.001277
Converged Position : X = 10.7038   Y = 10.7380   Z = 9.8646   A = 301.592463   B = 64.269584   G = 284.507988
Residual for this perturbation : 0.000485
Perturbation 5
Initial start : X = 11.417419   Y = 10.423178   Z = 10.473387   A = 288.768690   B = 89.118996   G = 278.980323
Initial residual: 0.000983
Converged Position : X = 11.0713   Y = 11.38161   Z = 11.8701   A = 317.9984   B = 89.352945   G = 298.270208
Residual for this perturbation : 0.000843
Perturbation 6
Initial start : X = 11.708743   Y = 11.471079   Z = 11.202948   A = 328.401664   B = 97.120666   G = 309.714719
Initial residual: 0.000884
Converged Position : X = 10.4890   Y = 10.3346   Z = 10.0256   A = 301.5593   B = 115.709011   G = 284.673005
Residual for this perturbation : 0.000515
Using perturbation #: 3
Solution to improve: X=10.530519 Y=10.817212 Z=9.965465 A=257.539951 B=61.094927 G=237.624897
Initial residual: 0.000273
Final solution : X= 10.531 Y= 10.817 Z= 9.965 A=257.540 B= 61.095 G=237.625
Final residual: 0.000273

Data point #: 10

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Initial guess: 10.530519 10.817212 9.965465 257.539951 61.094927 237.624897
Perturbation 1
Initial start : X = 10.530519   Y = 10.817212   Z = 9.965465   A = 257.539951   B = 61.094927   G = 237.624897
Initial residual: 0.000768
Converged Position : X = 10.7423   Y = 11.4584   Z = 11.2156   A = 274.0756   B = 43.958287   G = 257.746023
Residual for this perturbation : 0.000311
Perturbation 2
Initial start : X = 10.195219   Y = 10.879549   Z = 10.640336   A = 284.493910   B = 16.199735   G = 228.872090
Initial residual: 0.001490
Converged Position : X = 10.7378   Y = 11.4507   Z = 11.21996   A = 274.15011   B = 43.840385   G = 257.805173
Residual for this perturbation : 0.000311
Perturbation 3
Initial start : X = 10.618507   Y = 11.917107   Z = 10.481353   A = 259.595960   B = 58.762465   G = 279.834396
Initial residual: 0.000699
Converged Position : X = 10.0560   Y = 11.2027   Z = 10.2891   A = 273.8535   B = 45.260223   G = 287.402696
Residual for this perturbation : 0.000482
Perturbation 4
Initial start : X = 10.728482   Y = 11.807226   Z = 11.111768   A = 280.313063   B = 37.931860   G = 272.634738
Initial residual: 0.000538
Converged Position : X = 10.7472   Y = 11.4303   Z = 11.1927   A = 274.8583   B = 44.059940   G = 258.607281
Residual for this perturbation : 0.000310
Perturbation 5
Initial start : X = 11.616502   Y = 11.117054   Z = 10.782353   A = 261.500304   B = 54.232615   G = 269.029264
Initial residual: 0.000671
Converged Position : X = 10.05624   Y = 11.2038   Z = 10.2889   A = 273.8741   B = 45.256534   G = 287.417771
Residual for this perturbation : 0.000482
Perturbation 6
Initial start : X = 10.251428   Y = 11.425266   Z = 9.558977   A = 269.901748   B = 72.593687   G = 259.654057
Initial residual: 0.001028
Converged Position : X = 10.7423   Y = 11.4439   Z = 11.20781   A = 274.1722   B = 43.986791   G = 257.895422
Residual for this perturbation : 0.000311
Using perturbation #: 4
Solution to improve: X=10.747281 Y=11.430383 Z=11.192759 A=274.858392 B=44.059940 G=258.607281
Initial residual: 0.000310
Final solution : X= 10.747 Y= 11.430 Z= 11.193 A=274.858 B= 44.060 G=258.607
Final residual: 0.000310


Data point #: 11
===========
Initial guess: 10.747281 11.430383 11.192759 274.858392 44.059940 258.607281
Perturbation 1
Initial start : X = 10.747281   Y = 11.430383   Z = 11.192759   A = 274.858392   B = 44.059940   G = 258.607281
Initial residual: 0.000860
Converged Position : X = 12.623587   Y = 12.8105   Z = 11.7841   A = 283.3034   B = 29.8861   G = 259.428232
Residual for this perturbation : 0.000290
Perturbation 2
Initial start : X = 13.402680   Y = 12.598202   Z = 11.850524   A = 306.757164   B = 43.805504   G = 287.415552
Initial residual: 0.000954
Converged Position : X = 12.6272   Y = 12.8078   Z = 11.7864   A = 283.3448   B = 29.882393   G = 259.547697
Residual for this perturbation : 0.000290
Perturbation 3
Initial start : X = 13.038057   Y = 13.206002   Z = 12.172889   A = 268.545096   B = 28.615053   G = 273.845192
Initial residual: 0.000820
Converged Position : X = 12.6273   Y = 12.8078   Z = 11.7864   A = 283.3449   B = 29.882343   G = 259.548133
Residual for this perturbation : 0.000290
Perturbation 4
Initial start : X = 13.616875   Y = 12.762104   Z = 11.450962   A = 308.232734   B = 6.187710   G = 247.479081
Initial residual: 0.001080
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Converged Position : X = 13.3823   Y = 12.7311   Z = 10.5509   A = 291.3866   B = 358.630181   G = 249.881155
Residual for this perturbation : 0.000800
Perturbation 5
Initial start : X = 13.998136   Y = 13.436821   Z = 10.330558   A = 319.99858   B = 334.508079   G = 247.511988
Initial residual: 0.001435
Converged Position : X = 13.32832   Y = 11.12753   Z = 10.9649   A = 288.6269   B = 337.6195   G = 259.300943
Residual for this perturbation : 0.000747
Perturbation 6
Initial start : X = 13.96475   Y = 11.86279   Z = 11.067311   A = 308.339839   B = 337.179438   G = 248.047760
Initial residual: 0.001040
Converged Position : X = 13.3291   Y = 11.1258   Z = 10.9643   A = 288.6118   B = 337.618992   G = 259.329443
Residual for this perturbation : 0.000747
Using perturbation #: 1
Solution to improve: X=12.623587 Y=12.810577 Z=11.784197 A=283.303475 B=29.886140 G=259.428232
Initial residual: 0.000290
Final solution : X= 12.624 Y= 12.811 Z= 11.784 A=283.303 B= 29.886 G=259.428
Final residual: 0.000290

Data point #: 12
===============
Initial guess: 12.623587 12.810577 11.784197 283.303475 29.886140 259.428232
Perturbation 1
Initial start : X = 12.623587   Y = 12.810577   Z = 11.784197   A = 283.303475   B = 29.886140   G = 259.428232
Initial residual: 0.000760
Converged Position : X = 13.464683   Y = 13.28194   Z = 12.3465   A = 290.76216   B = 14.4470   G = 266.358584
Residual for this perturbation : 0.000344
Perturbation 2
Initial start : X = 12.610607   Y = 14.204333   Z = 11.821253   A = 283.267636   B = -6.185136   G = 254.088255
Initial residual: 0.000769
Converged Position : X = 13.4549   Y = 13.4459   Z = 12.4276   A = 290.1409   B = 11.703719   G = 265.816039
Residual for this perturbation : 0.000339
Perturbation 3
Initial start : X = 14.345458   Y = 13.754448   Z = 13.329042   A = 303.033225   B = -6.473634   G = 260.069773
Initial residual: 0.000864
Converged Position : X = 13.454875   Y = 13.44754   Z = 12.42847   A = 290.1327   B = 11.7102   G = 265.812577
Residual for this perturbation : 0.000339
Perturbation 4
Initial start : X = 14.199498   Y = 13.057703   Z = 12.517983   A = 261.869581   B = 26.447648   G = 277.135328
Initial residual: 0.001130
Converged Position : X = 13.2682   Y = 13.64082   Z = 10.89187   A = 263.08194   B = 16.37784   G = 265.446564
Residual for this perturbation : 0.000838
Perturbation 5
Initial start : X = 13.174465   Y = 13.379094   Z = 11.430746   A = 252.576020   B = -4.466609   G = 265.571059
Initial residual: 0.000892
Converged Position : X = 12.453732   Y = 13.85176   Z = 11.1738   A = 251.1186   B = 9.6899   G = 258.009256
Residual for this perturbation : 0.000762
Perturbation 6
Initial start : X = 13.000252   Y = 13.752831   Z = 11.352429   A = 239.126765   B = 26.364835   G = 245.548755
Initial residual: 0.001078
Converged Position : X = 12.405100   Y = 13.4070   Z = 11.0519   A = 249.7088   B = 8.594261   G = 254.011225
Residual for this perturbation : 0.000768
Using perturbation #: 2
Solution to improve: X=13.454953 Y=13.445913 Z=12.427653 A=290.140920 B=11.703719 G=265.816039
Initial residual: 0.000339
Final solution : X= 13.455 Y= 13.446 Z= 12.428 A=290.141 B= 11.704 G=265.816
Final residual: 0.000339

Data point #: 13
===============

Initial guess: 13.454953 13.445913 12.427653 290.140920 11.703719 265.816039
Perturbation 1
Initial start : X = 13.454953   Y = 13.445913   Z = 12.427653   A = 290.140920   B = 11.703719   G = 265.816039
Initial residual: 0.000849
Converged Position : X = 13.4674   Y = 14.209968   Z = 12.4526   A = 299.7232   B = 357.4144   G = 275.131741
Residual for this perturbation : 0.000312
Perturbation 2
Initial start : X = 13.799055   Y = 15.039888   Z = 12.167042   A = 325.302246   B = 373.1793   G = 280.128536
Initial residual: 0.001010
Converged Position : X = 13.4674   Y = 14.2099   Z = 12.45268   A = 299.72324   B = 357.4144   G = 275.131741
Residual for this perturbation : 0.000312
Perturbation 3
Initial start : X = 13.053115   Y = 14.122010   Z = 12.09677   A = 277.737566   B = 331.964680   G = 262.559225
Initial residual: 0.001255
Converged Position : X = 13.4715   Y = 14.21242   Z = 12.461738   A = 299.65812   B = 357.3504   G = 275.131456
Residual for this perturbation : 0.000311
Perturbation 4
Initial start : X = 12.636224   Y = 15.022704   Z = 11.923859   A = 316.097625   B = 360.598092   G = 268.923028
Initial residual: 0.000735
Converged Position : X = 13.4674   Y = 14.2099   Z = 12.4526   A = 299.7232   B = 357.414419   G = 275.131741
Residual for this perturbation : 0.000312
Perturbation 5
Initial start : X = 12.839646   Y = 13.659087   Z = 11.8814   A = 324.538807   B = 381.915149   G = 290.366685
Initial residual: 0.001137
Converged Position : X = 13.467415   Y = 14.2099   Z = 12.4526   A = 299.7232   B = 357.4144   G = 275.131741
Residual for this perturbation : 0.000312
Perturbation 6
Initial start : X = 12.869695   Y = 14.607319   Z = 13.13731   A = 302.273355   B = 374.668788   G = 261.897194
Initial residual: 0.000928
Converged Position : X = 13.4674   Y = 14.2099   Z = 12.4526   A = 299.7232   B = 357.414419   G = 275.131741
Residual for this perturbation : 0.000312
Using perturbation #: 3
Solution to improve: X=13.471519 Y=14.212425 Z=12.461738 A=299.658122 B=357.350458 G=275.131456
Initial residual: 0.000311
Final solution : X= 13.472 Y= 14.212 Z= 12.462 A=299.658 B=357.350 G=275.131
Final residual: 0.000311

Data point #: 14
===============
Initial guess: 13.471519 14.212425 12.461738 299.658122 357.350458 275.131456
Perturbation 1
Initial start : X = 13.471519   Y = 14.21242   Z = 12.461738   A = 299.658122   B = 357.350458   G = 275.131456
Initial residual: 0.000955
Converged Position : X = 15.07991   Y = 14.86074   Z = 14.43056   A = 305.68153   B = 344.5631   G = 282.710330
Residual for this perturbation : 0.000230
Perturbation 2
Initial start : X = 15.384355   Y = 15.756189   Z = 14.91359   A = 318.014114   B = 318.910503   G = 298.822252
Initial residual: 0.001381
Converged Position : X = 15.045481   Y = 15.2351   Z = 15.0711   A = 330.8974   B = 331.6979   G = 301.162742
Residual for this perturbation : 0.001033
Perturbation 3
Initial start : X = 14.979642   Y = 14.42831   Z = 15.08089   A = 352.055463   B = 333.856919   G = 273.146403
Initial residual: 0.001558
Converged Position : X = 15.0723   Y = 15.2321   Z = 15.0662   A = 331.1377   B = 331.626046   G = 301.159470
Residual for this perturbation : 0.001033
Perturbation 4
Initial start : X = 15.675047   Y = 14.48548   Z = 15.278927   A = 307.203219   B = 355.300522   G = 278.678130
Initial residual: 0.000757
Converged Position : X = 15.0805   Y = 14.86156   Z = 14.42781   A = 305.69893   B = 344.56194   G = 282.670252

Residual for this perturbation : 0.000230
Perturbation 5
Initial start : X = 14.863890   Y = 15.464800   Z = 13.98860   A = 280.589756   B = 331.820455   G = 295.254765
Initial residual: 0.000824
Converged Position : X = 15.0803   Y = 14.8615   Z = 14.4269   A = 305.6985   B = 344.552505   G = 282.670460
Residual for this perturbation : 0.000230
Perturbation 6
Initial start : X = 14.525690   Y = 14.387049   Z = 14.43983   A = 287.181107   B = 365.473982   G = 302.529166
Initial residual: 0.001463
Converged Position : X = 15.0810   Y = 14.86197   Z = 14.4266   A = 305.7079   B = 344.5655   G = 282.650090
Residual for this perturbation : 0.000230
Using perturbation #: 1
Solution to improve: X=15.079911 Y=14.860746 Z=14.430564 A=305.681531 B=344.563171 G=282.710330
Initial residual: 0.000230
Final solution : X= 15.080 Y= 14.861 Z= 14.431 A=305.682 B=344.563 G=282.710
Final residual: 0.000230

Data point #: 15
================
Initial guess: 15.079911 14.860746 14.430564 305.681531 344.563171 282.710330
Perturbation 1
Initial start : X = 15.079911   Y = 14.860746   Z = 14.430564   A = 305.681531   B = 344.56311   G = 282.710330
Initial residual: 0.000964
Converged Position : X = 16.2534   Y = 15.2761   Z = 14.4156   A = 316.0878   B = 331.305525   G = 294.877896
Residual for this perturbation : 0.000229
Perturbation 2
Initial start : X = 16.346005   Y = 15.947354   Z = 14.915969   A = 318.235328   B = 344.88409   G = 273.235148
Initial residual: 0.000831
Converged Position : X = 16.3796   Y = 15.5108   Z = 14.51319   A = 321.8600   B = 343.2219   G = 285.080070
Residual for this perturbation : 0.000573
Perturbation 3
Initial start : X = 16.9456   Y = 14.5442   Z = 15.4223   A = 318.5476   B = 345.547083   G = 301.791214
Initial residual: 0.001059
Converged Position : X = 16.2542   Y = 15.27591   Z = 14.4156   A = 316.0849   B = 331.3045   G = 294.879935
Residual for this perturbation : 0.000229
Perturbation 4
Initial start : X = 16.829298   Y = 15.284362   Z = 14.692645   A = 326.252914   B = 303.9403   G = 298.941775
Initial residual: 0.001299
Converged Position : X = 16.327027   Y = 15.4791   Z = 14.5221   A = 326.3029   B = 330.7393   G = 295.928610
Residual for this perturbation : 0.000535
Perturbation 5
Initial start : X = 15.909587   Y = 16.303032   Z = 14.998665   A = 326.68893   B = 342.002722   G = 315.220347
Initial residual: 0.001291
Converged Position : X = 15.737448   Y = 15.3401   Z = 15.0578   A = 320.7398   B = 332.3200   G = 304.957035
Residual for this perturbation : 0.000639
Perturbation 6
Initial start : X = 15.0701   Y = 16.124268   Z = 15.974370   A = 298.287176   B = 361.013768   G = 275.997567
Initial residual: 0.001344
Converged Position : X = 16.2541   Y = 15.2755   Z = 14.4162   A = 316.0857   B = 331.303122   G = 294.883914
Residual for this perturbation : 0.000229
Using perturbation #: 6
Solution to improve: X=16.254651 Y=15.275593 Z=14.416261 A=316.085774 B=331.303122 G=294.883914
Initial residual: 0.000229
Final solution : X= 16.255 Y= 15.276 Z= 14.416 A=316.086 B=331.303 G=294.884
Final residual: 0.000229

Data point #: 16
================
Initial guess: 16.254651 15.275593 14.416261 316.085774 331.303122 294.883914

Perturbation 1
Initial start : X = 16.254651   Y = 15.2755   Z = 14.416261   A = 316.085774   B = 331.303122   G = 294.883914
Initial residual: 0.001177
Converged Position : X = 16.387629   Y = 16.31004   Z = 15.35584   A = 292.2420   B = 317.28033   G = 306.476799
Residual for this perturbation : 0.000635
Perturbation 2
Initial start : X = 17.009787   Y = 16.600439   Z = 15.8724   A = 295.410255   B = 300.961780   G = 303.349893
Initial residual: 0.001327
Converged Position : X = 16.387363   Y = 16.30999   Z = 15.35614   A = 292.2393   B = 317.28   G = 306.473347
Residual for this perturbation : 0.000635
Perturbation 3
Initial start : X = 16.4695   Y = 15.4909   Z = 15.9044   A = 311.684367   B = 334.727873   G = 335.267570
Initial residual: 0.001753
Converged Position : X = 16.3571   Y = 16.3473   Z = 15.6012   A = 328.6247   B = 318.23773   G = 309.963147
Residual for this perturbation : 0.000756
Perturbation 4
Initial start : X = 15.501393   Y = 16.778933   Z = 15.577294   A = 339.058185   B = 323.5331   G = 313.512364
Initial residual: 0.001332
Converged Position : X = 16.325   Y = 16.33782   Z = 15.5273   A = 327.5127   B = 318.379271   G = 311.019944
Residual for this perturbation : 0.000752
Perturbation 5
Initial start : X = 15.853892   Y = 16.0987   Z = 16.518176   A = 337.672068   B = 341.655609   G = 304.008539
Initial residual: 0.001936
Converged Position : X = 16.3646   Y = 16.3541   Z = 15.51911   A = 327.9097   B = 318.2414   G = 310.445557
Residual for this perturbation : 0.000753
Perturbation 6
Initial start : X = 17.334862   Y = 16.3004   Z = 15.129625   A = 340.305884   B = 338.275027   G = 292.402463
Initial residual: 0.001429
Converged Position : X = 16.9401   Y = 16.4816   Z = 14.9382   A = 337.0529   B = 315.441138   G = 295.503454
Residual for this perturbation : 0.000214
Using perturbation #: 6
Solution to improve: X=16.940157 Y=16.481628 Z=14.938281 A=337.052960 B=315.441138 G=295.503454
Initial residual: 0.000214
Final solution : X= 16.940 Y= 16.482 Z= 14.938 A=337.053 B=315.441 G=295.503
Final residual: 0.000214

Data point #: 17
===============
Initial guess: 16.940138 16.481596 14.938298 337.053050 315.441169 295.503332
Perturbation 1
Initial start : X = 16.940138   Y = 16.4815   Z = 14.9382   A = 337.053050   B = 315.441169   G = 295.503332
Initial residual: 0.001616
Converged Position : X = 17.8848   Y = 17.1413   Z = 15.82776   A = 339.3461   B = 299.8639   G = 307.248010
Residual for this perturbation : 0.000312
Perturbation 2
Initial start : X = 18.335885   Y = 17.8790   Z = 16.5106   A = 364.861318   B = 283.517561   G = 298.588966
Initial residual: 0.002495
Converged Position : X = 17.8848   Y = 17.1413   Z = 15.8277   A = 339.34585   B = 299.8638   G = 307.248384
Residual for this perturbation : 0.000312
Perturbation 3
Initial start : X = 17.275896   Y = 18.1235   Z = 15.134436   A = 320.272258   B = 329.259684   G = 281.252263
Initial residual: 0.002921
Converged Position : X = 17.8848   Y = 17.1413   Z = 15.8277   A = 339.3460   B = 299.864132   G = 307.247990
Residual for this perturbation : 0.000312
Perturbation 4
Initial start : X = 17.5124   Y = 16.512873   Z = 15.361935   A = 321.054070   B = 299.372274   G = 321.509400
Initial residual: 0.000856
Converged Position : X = 17.8849   Y = 17.1414   Z = 15.8277   A = 339.3430   B = 299.862868   G = 307.253001
Residual for this perturbation : 0.000312

Perturbation 5
Initial start : X = 17.303701   Y = 17.7279   Z = 16.527430   A = 369.226885   B = 276.175849   G = 279.213646
Initial residual: 0.001812
Converged Position : X = 17.8601   Y = 16.6256   Z = 16.48538   A = 10.0962   B = 285.312368   G = 278.767087
Residual for this perturbation : 0.001102
Perturbation 6
Initial start : X = 18.486782   Y = 16.463889   Z = 15.550147   A = 29.137819   B = 309.733679   G = 251.221826
Initial residual: 0.001829
Converged Position : X = 17.859362   Y = 16.624064 Z = 16.48277   A = 10.2142   B = 285.363866   G = 278.632406
Residual for this perturbation : 0.001102
Using perturbation #: 4
Solution to improve: X=17.884922 Y=17.141495 Z=15.827786 A=339.343024 B=299.862868 G=307.253001
Initial residual: 0.000312
Final solution : X= 17.885 Y= 17.141 Z= 15.828 A=339.343 B=299.863 G=307.253
Final residual: 0.000312

Scaled Measured Voltages vs. Model Voltages
Antenna No. 4; 40x20 inches; Scale = 525353
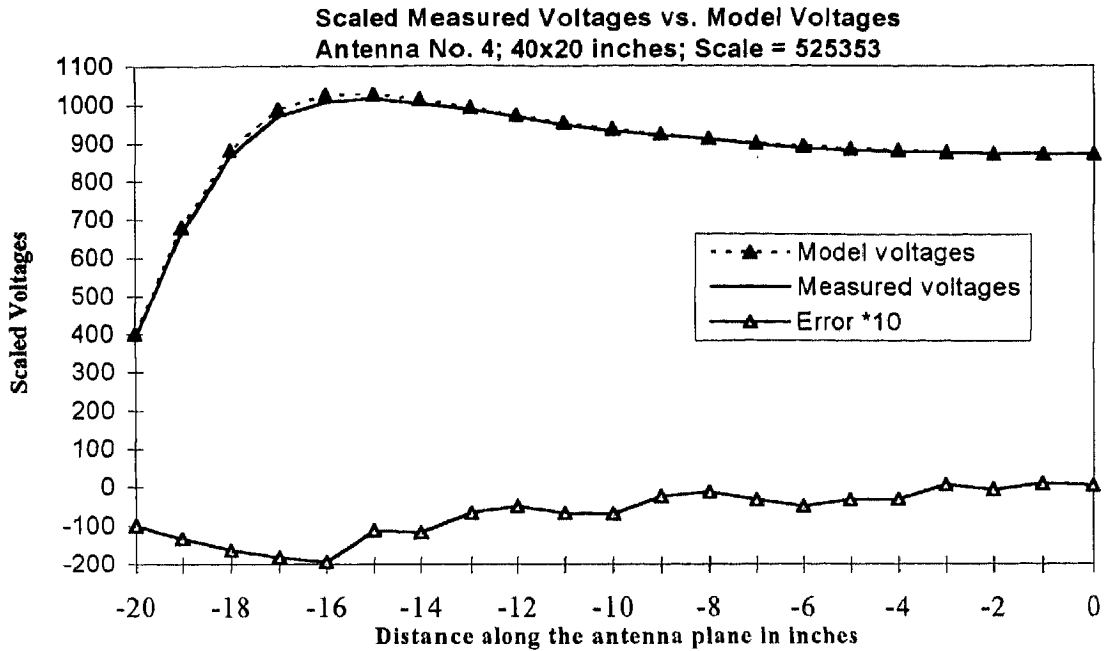
Figure D.1: Model-Reality Plot for 2.2 Mhz transmitter as it is moved along and four inches away from the plane of the antenna.

Scaled Measured vs. Model Voltages
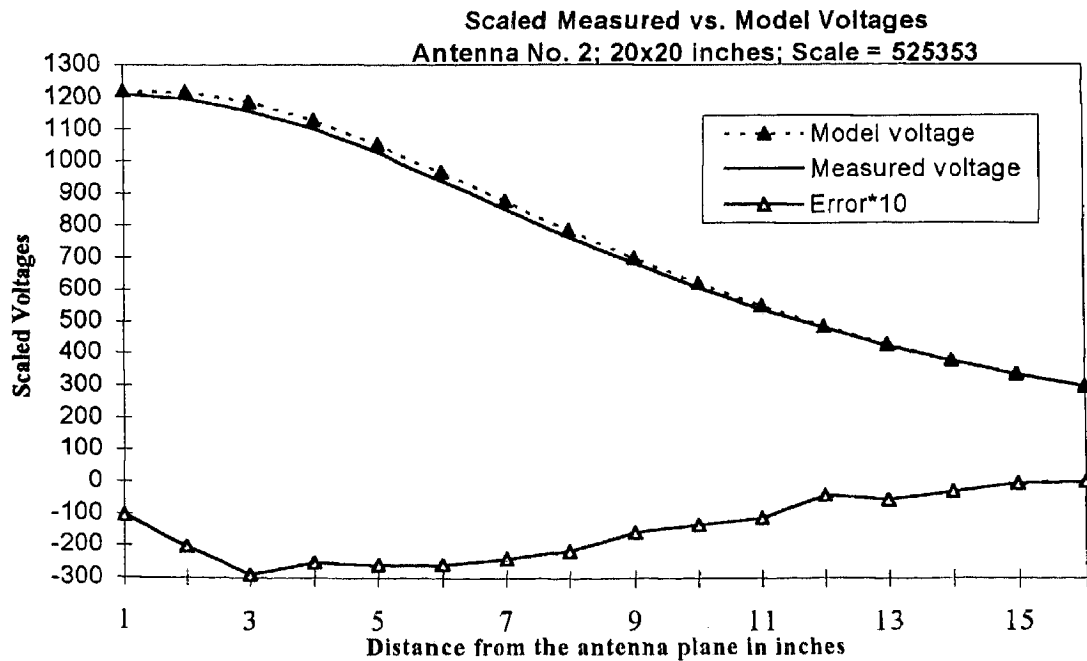**Antenna No. 2; 20x20 inches; Scale = 525353**

Figure D.2: Model-Reality Plot for 2.2 Mhz transmitter as the transmitter is moved from the plane of antenna in a path coincident with the axis of the antenna.
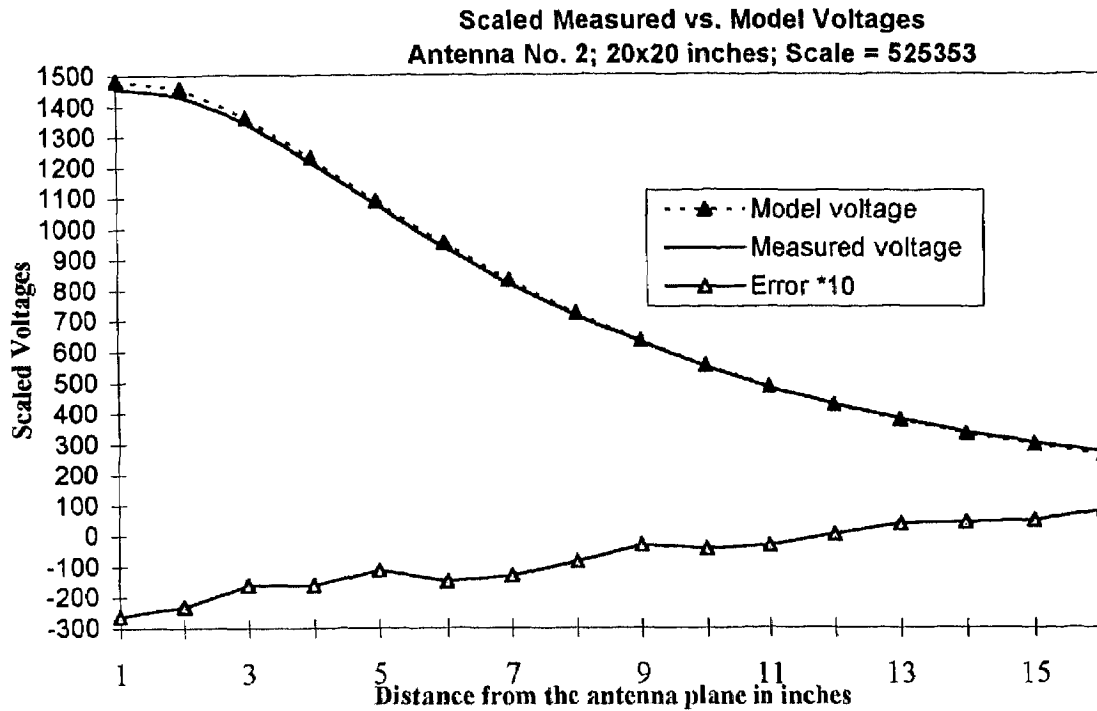
Scaled Measured vs. Model Voltages
Antenna No. 2; 20x20 inches; Scale = 525353

Figure D.3: Model-Reality Plot for 2.2 Mhz transmitter as the transmitter is moved from the plane of antenna.

Scaled Measured vs. Model Voltages
Antenna No. 2; 20x20 inches; Scale = 525353
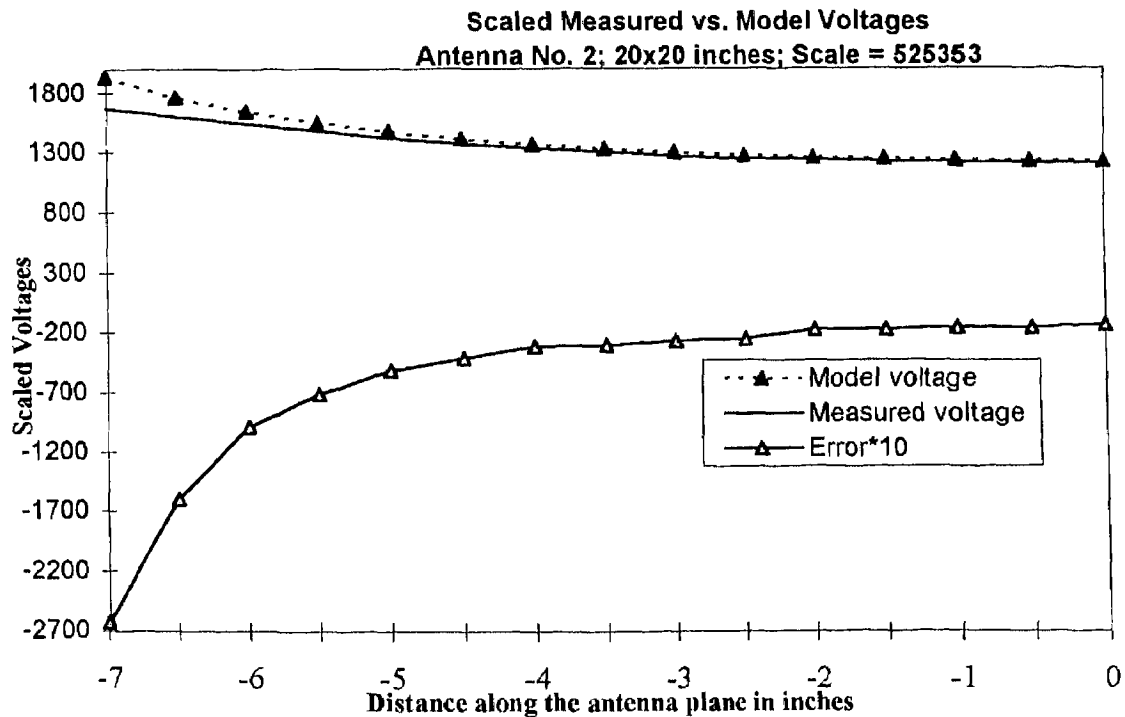
Figure D.4: Model-Reality Plot for 2.2 Mhz transmitter as the transmitter is moved along the plane of antenna.
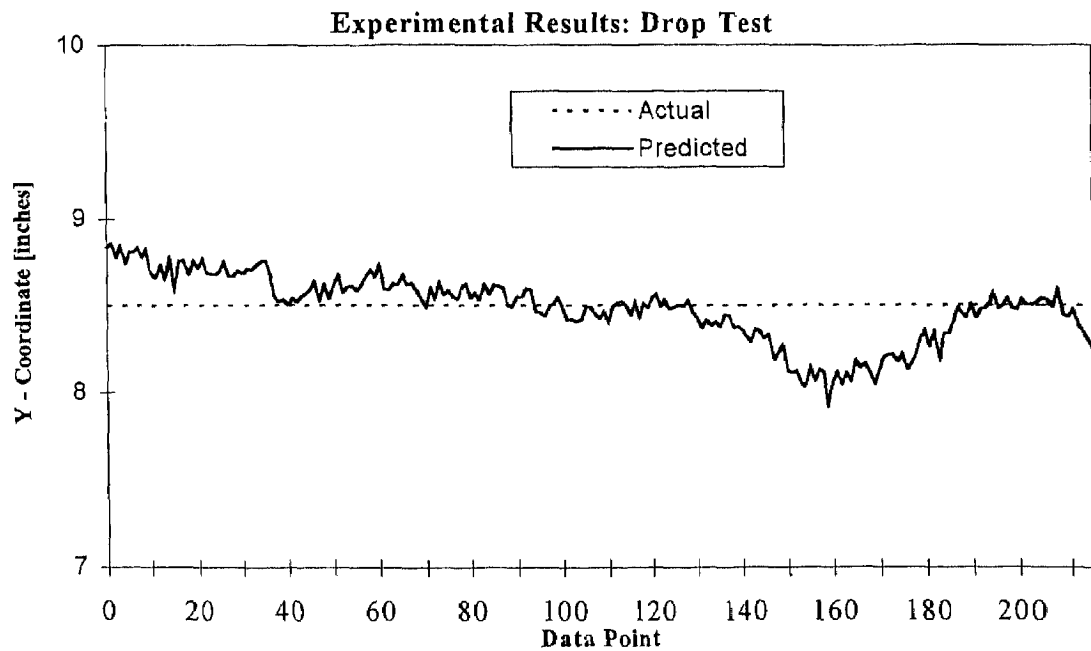
Experimental Results: Drop Test
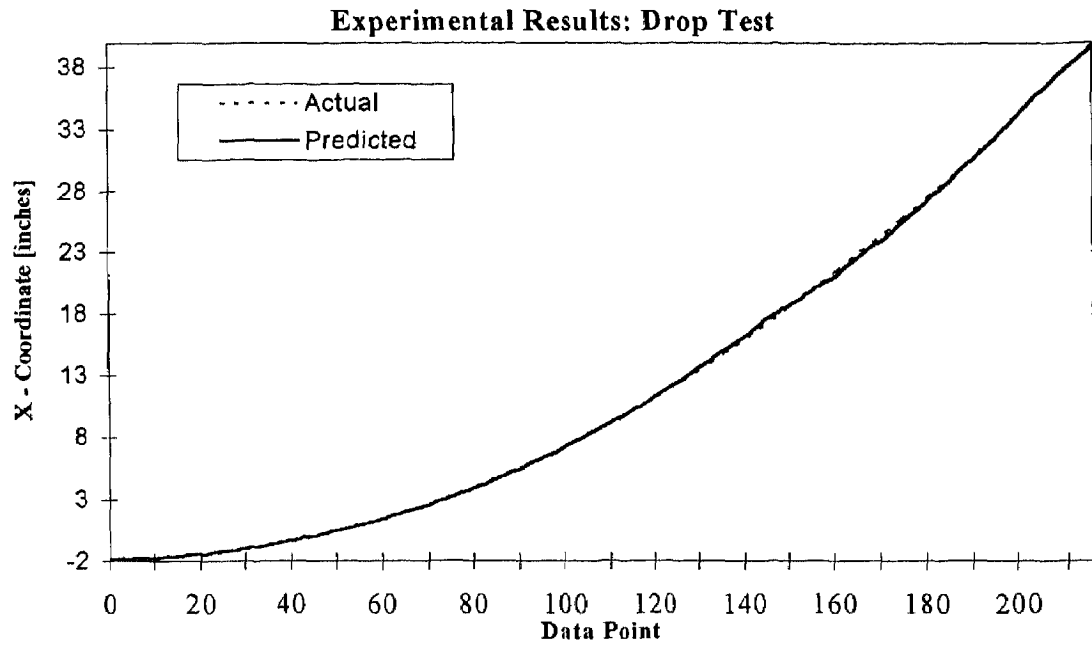
Figure D.5: Experimental Results: Drop Test
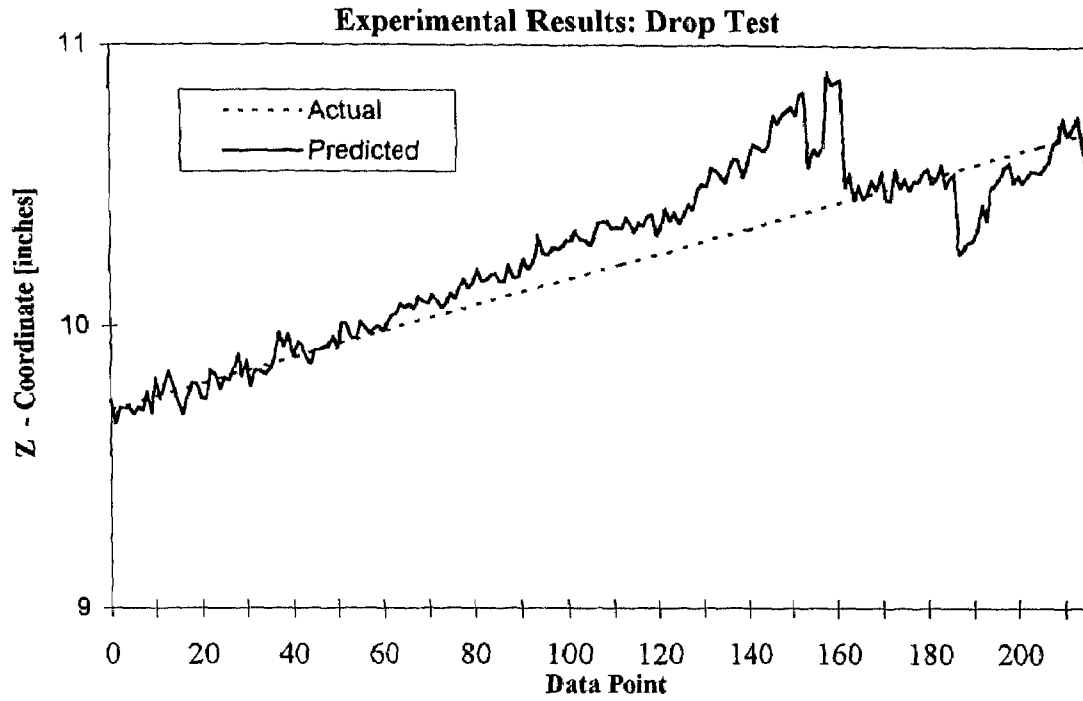
Experimental Results: Drop Test

Figure D.6: Experimental Results: Drop Test

Figure D.7: Experimental Results: Drop Test

Figure D.8: Experimental Results: Drop Test
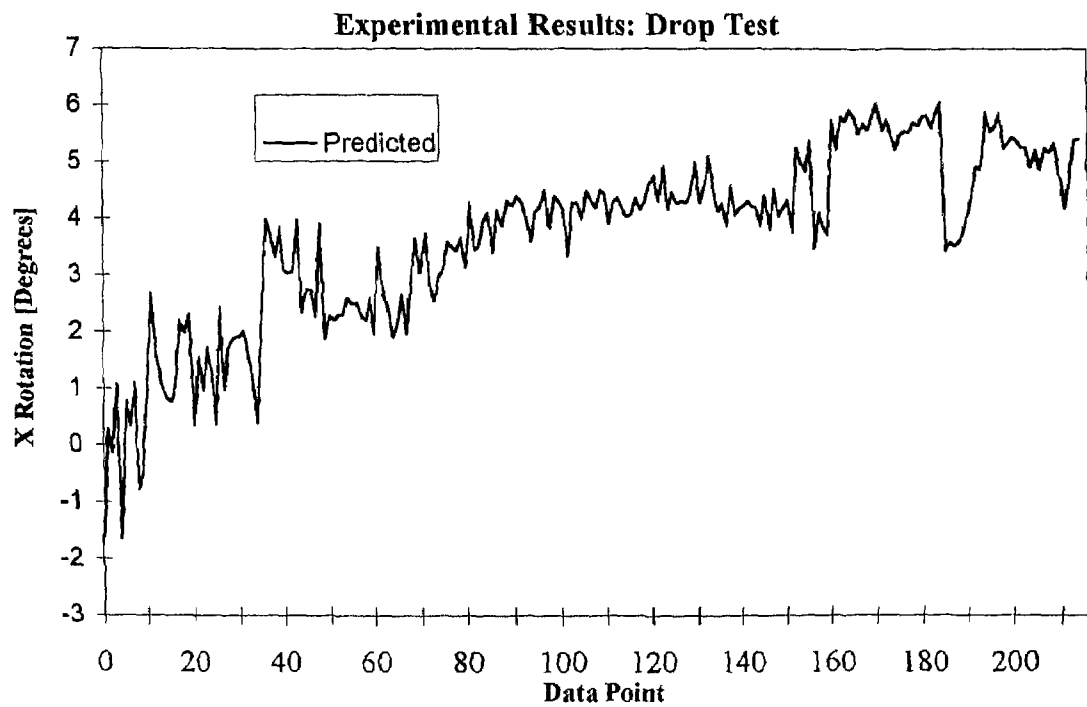
Experimental Results: Drop Test
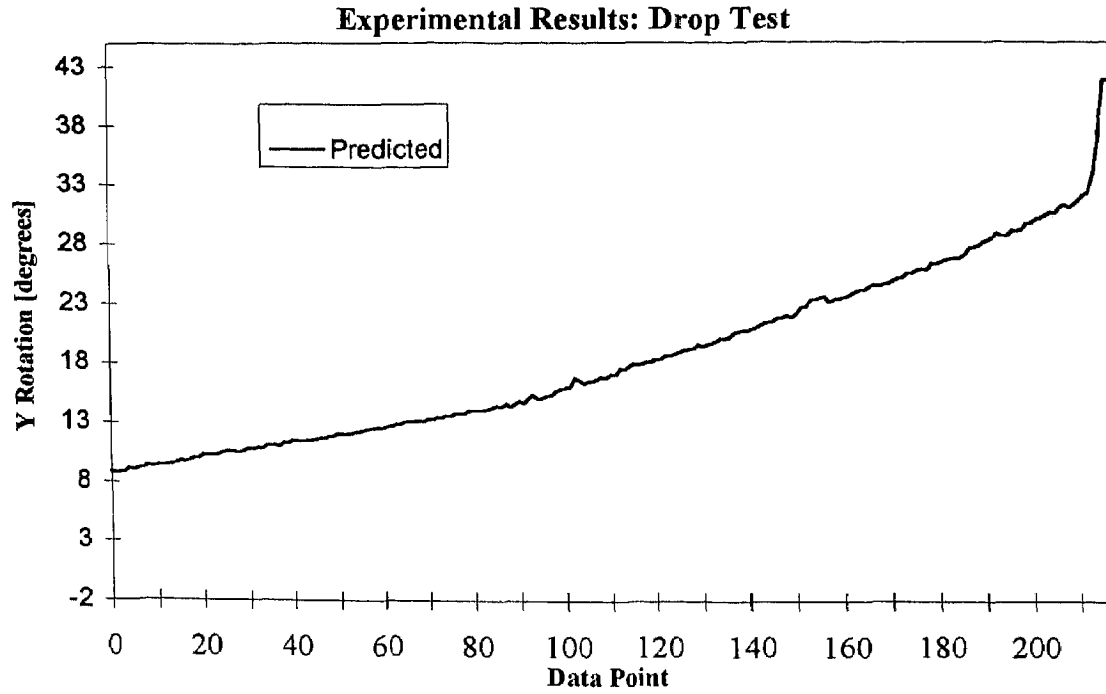

Figure D.9: Experimental Results: Drop Test
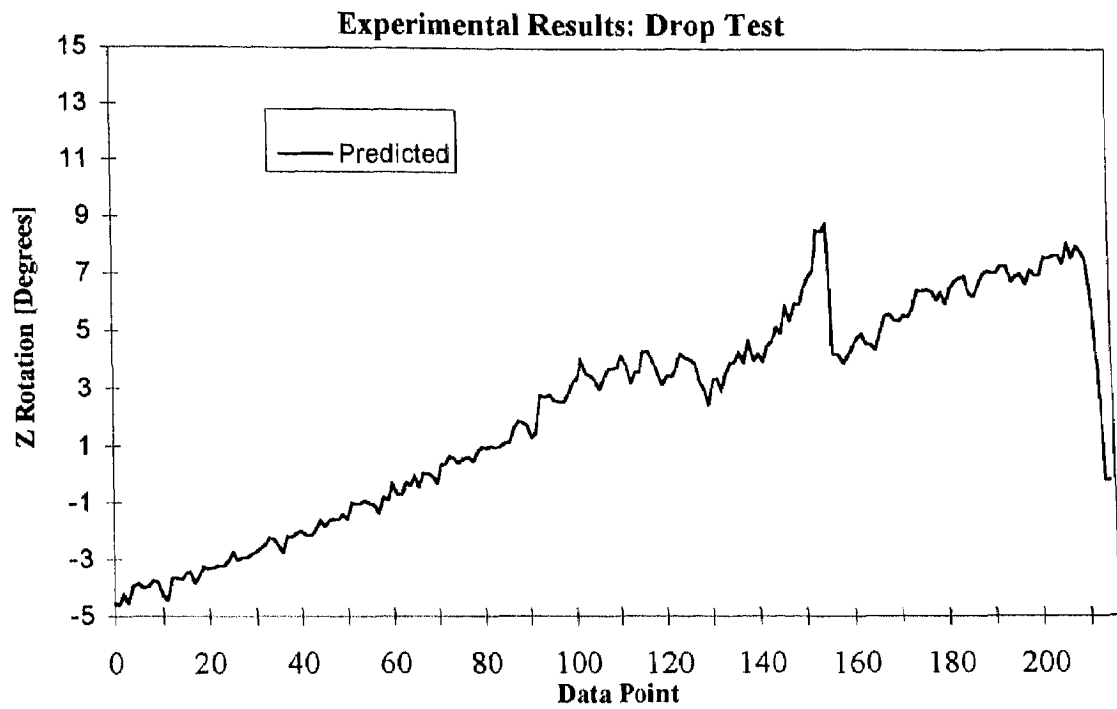
Experimental Results: Drop Test

Figure D.10: Experimental Results: Drop Test

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