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Performance evaluation of spread spectrum system with cochannel interference through a nonlinear channel

Kim, Yong Hun, Ph.D.

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PERFORMANCE EVALUATION OF SPREAD SPECTRUM SYSTEM WITH COCHANNEL INTERFERENCE THROUGH A NONLINEAR CHANNEL

by
Yong H. Kim

A Dissertation
Submitted to the Faculty of New Jersey Institute of Technology
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APPROVAL PAGE

Performance Evaluation of PN Spread
Nonlinear Satellite Systems with Cochannel Interference

by
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ABSTRACT
Performance Evaluation of Spread Spectrum System with Cochannel Interference through a Nonlinear Channel

by

Yong H. Kim

This thesis deals with the problem of more than one subscriber transmitting data signals through a common satellite repeater using code division multiplexing to separate the signals. We are concerned with the problem of amplifying two DS spread spectrum signals, both QPSK or BPSK modulated, in a common device in which limiting occurs. One signal is considered the signal we desire to receive, and the other, having the same nominal carrier frequency with a small random offset, is considered to be a cochannel interferer. The case of a cochannel interferer on the uplink and downlink in QPSK signalling and BPSK signalling systems is analyzed in detail. This is an important practical problem in code division multiple access satellite communication systems, which usually contain limiting in the satellite amplifier, often in the form of a saturated traveling wave tube amplifier.

The satellite repeater is modeled using a bandpass hard limiter. The inverse Fourier transform method, which is applicable to the analysis of PN spread spectrum systems is applied to calculate the output of the bandpass hard limiter. The limiter output plus AWGN is taken to be the input of a correlation receiver for which we calculate the probability of error as function of the signal to noise and, signal to interference ratios.

From these results we can determine the effect on error performance due to the inclusion of a bandpass limiter in the transmission path.

The assumptions made in deriving the theoretical performance of the system have been checked by simulating the entire system using the BOSS software package.

The results of the simulation show good agreement with the theoretical calculations within 1 to 2 *dB* in SNR. In addition by means of simulation we were able to explore some features of the system that could not be addressed analytically, such as the effect of unbalanced codes on system performance.

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CHAPTER 1

INTRODUCTION

Communications serve to transfer information between a source and a user. Terrestrial communications confront long distance communications constraints because they either use guided media — wire lines, coaxial cables, and optical fiber cables — which have in common the fact that they require a physical path between terminals or wireless transmissions such as microwave radio relays which due to propagation problems must be in line of sight.

Satellite communications are the result of research in the field of radio communications with the intention of achieving the greatest coverage and capacity at the lowest cost. Currently, communication satellites in orbit demonstrate that satellite repeaters can provide a large communication capacity without the need of laying copper or fiber cables, and that a single communication satellite can afford many communication channels between widely separated points. For example, for military applications, means are needed for allowing simultaneous access to a single satellite repeater by a number of widely separated stations which may move frequently. With satellite orbits at geostationary altitudes(35,784 km), the repeater is visible over large sections of the earth's surface and hence is potentially able to interconnect a large number of far-flung points.
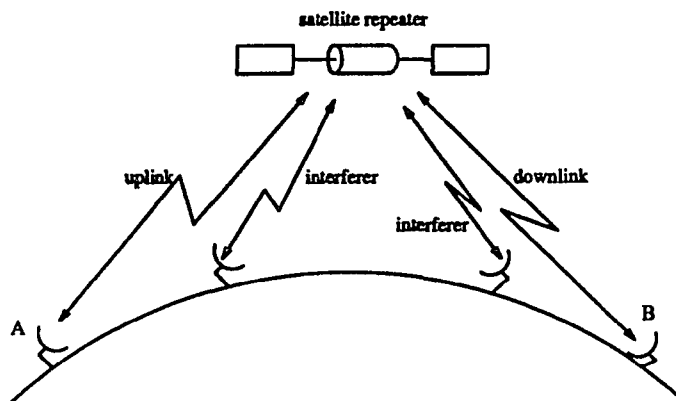
Figure 1.1: Satellite communications.

Fig. 1.1 illustrates communication between stations A and B through a satellite repeater that is subject to interference on both the uplink and the downlink. Ideally it would be desirable to permit any station in view of the satellite to communicate with one or more of the other stations at will and with as little interaction as possible with other users of the repeater.

It is well known that the most common multiple access techniques are frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA) [1]. In FDMA, all users transmit simultaneously, but use disjoint frequency bands. In TDMA, all users occupy the same radio frequency bandwidth, but transmit sequentially in time. In direct sequence (DS) CDMA systems, all users are permitted to transmit simultaneously using the same band of frequencies. Users are each assigned a different spreading code, which are close to orthogonal (i.e., their cross correlation value is much less than their auto correlation value over one symbol period) to each other, so that they can be separated in the receiver despreading process. If a signal is multiplied by the spreading code, the spread signal has a power spectral density approximating white noise, and is therefore called pseudonoise (PN). Spreading sequences having a uniformly low cross correlations such as the Gold codes are used for pseudonoise codes [2]–[11]. When

the receiver is correctly synchronized to the transmitter and has the same despread code, the original signal at receiver is regenerated, but if the two signals have different codes, the received signal is close to white noise.

In this thesis we concerned with the performance [12]-[37] of CDMA system, and in particular how their performance is affected by nonlinearity in the transmission path. Transfer characteristics and the various effects of nonlinear components used in a satellite repeater have been investigated by a number of researchers [38]-[46]. It has been shown that the use of a nonlinear device, for example, a hard limiter, leads to degradation in system performance. This effect was known from previous work in FDMA systems, where the power ratios of the different input signals at the input of a hard limiter may substantially change at the output, favoring the strong input signal, and where intermodulation products may mix into the signal bands. However, nonlinear elements are often employed at the front end of a receiving system to prevent the occurrence of large signals such as impulsive noise from overloading the following stages. Also communication satellite systems usually employ a traveling wave tube amplifier which is operated in a nonlinear region of its input output characteristic to achieve good power efficiency. The nonlinear device investigated in this thesis is a bandpass limiter which consists of a hard limiter and band pass filters. The use of a hard limiter in a communication system leads to the generation of intermodulation products because of its nonlinear characteristics. The transmitter signal, in the form of an electromagnetic wave, propagates through a random or fading medium to the communication receiver. The received signal in such an environment is corrupted by random disturbances such as additive noise, multiplicative multipath fading, and cochannel interference. The additive noise can be caused by atmospherics, automobile ignitions, power lines, industrial equipment, the receiver itself and numerous other sources. In this thesis, we are considering only additive cochannel interference, and additive noise which is assumed a white Gaussian noise.

Several methods have been used for the calculation of the effect of memoryless nonlinearities on the sum of various input signals. Davenport [47] was the first to use the transform method which yields the autocorrelation function of the signal at the output of the nonlinearity from knowledge of the joint characteristic function of the input signal. Since then this method has been the most widely applicable method. But it cannot give a complete analysis because of the absence of phase information in the output signals. Hence, the transform method was modified to enable a useful analysis of PN spread-spectrum systems with the Fourier-expansion and the inverse Fourier transform approach by Jain [48] and Baer [38] respectively. A time domain approach is applied in both modified methods. The theories presented by Baer [38] provide an explicit analysis of the intermodulation terms which are generated when the input signal of the nonlinear device consists of a superposition of several stochastic and deterministic signals.

Baer [38] examined interference effects of hard limiting in PN spread-spectrum systems under the condition that the transmitted and interference signals were binary phase shift keying (BPSK) signals, but did not calculate the performance of the whole satellite system. The cochannel interference in nonlinear quadriphase shift keying (QPSK) satellite systems was analyzed by Kennedy and Shimbo [49], but they did not consider it in spread spectrum systems. Kullstam [50] studied the effect of arbitrary interference on spread-spectrum QPSK signals without considering any nonlinear channel. BPSK and QPSK have been widely used in the mostly power-constrained transponders of existing domestic satellite communications systems, though minimum shift keying (MSK) and combined modulation coding techniques are also getting attention [51].

Whereas the authors mentioned above studied various parts of the overall problem, the contribution of this thesis is that the entire system was considered. That is both the desired signal and the interferer were taken to be spread spectrum

signals, having either BPSK or QPSK modulation. The channel included a bandpass limiter, and additive noise on both the uplink and downlink were included. The overall performance of the system, that is the probability of error as a function of the signal to noise and signal to interference ratios was obtained. We derived the mathematical expressions for the bandpass limiter output by using the inverse Fourier transform method [38, 52] for both cases, QPSK signalling and BPSK signalling. The probability of symbol error was analyzed for three different conditions, which are uplink interference, downlink interference, and up and downlink interference. Fig. 1.1 shows the locations in the system where the interference is introduced.

To check the assumptions made in deriving the theoretical performance of the system and to gain additional insight into its behavior, the system was simulated using the block oriented systems simulator (BOSS) software package. By means of this simulation, we are able to explore some features of the system that could not be addressed analytically, such as the effect of processing gain and unbalanced codes on system performance The outline of this report is organized as follows:

- In Chapter 2, we discuss the proposed DS spread spectrum QPSK satellite channel system which contains a cochannel uplink interferer. Additive white Gaussian noise (AWGN) is added in the up and the downlink of satellite channel. The output of the bandpass limiter is derived and analyzed by using the inverse Fourier transform method. Symbol error rate of the optimum correlation receiver is obtained. Some numerical examples are given and compared with simulation results.

- In Chapter 3, we analyze the DS spread QPSK satellite channel with downlink cochannel interference. The results are presented in the form of error probability curves showing the performance of this channel. We compare the analytical results with the simulation results.

- In Chapter 4, we consider a QPSK spread satellite system with up and downlink interference. We make use of the analytical results derived in the previous two chapters. The output performance is compared with the previous results. Numerical example plots of the two different cases are given for the case in which the total interference power is the same as the uplink interference power and for the case in which it is two times the uplink interference.

- Chapter 5 presents the results for a BPSK signal operating on a DS spread spectrum satellite link channel which includes a bandpass limiter. We find the performance with interference on the uplink, the downlink, and on the both up and downlink.

- Finally, Chapter 6 gives the comparison of the performance between BPSK systems and QPSK systems and the conclusion of our report, containing suggestion for future work.

CHAPTER 2

UPLINK INTERFERENCE IN QPSK SPREAD SPECTRUM SYSTEMS

2.1 Introduction

The effect of a bandpass limiter in CDMA QPSK satellite spread spectrum communication systems is investigated. The case of a cochannel interferer on the uplink is analyzed in detail.

The nonlinear characteristics of the bandpass limiter results in the generation of intermodulation products which become narrow band signals after the despreading process in the spread spectrum receiver. These intermodulation products and the cochannel interference affect the system performance. The symbol error rate of the system is calculated as a function of the signal to noise and signal to interference ratios.

BOSS is used for the analysis of the system and the checking of the system performance. The results of the simulation are plotted together with the theoretical results. In addition, by means of simulation, we were able to explore some features of the system that could not be addressed analytically, such as the effect of processing gain and unbalanced codes on system performance.

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2.2 Mathematical Analysis

2.2.1 Communication Model

A simplified functional block diagram of the system that is analyzed in this work is shown in Fig. 2.1. The bandpass filter BP_0 is assumed to be an ideal filter with bandwidth large enough to pass $x(t)$ with negligible distortion. The bandwidth of the bandpass filter BP_1 is used to pass only the fundamental band around the carrier frequency ω_0 and reject those around all the harmonics. The input to the bandpass
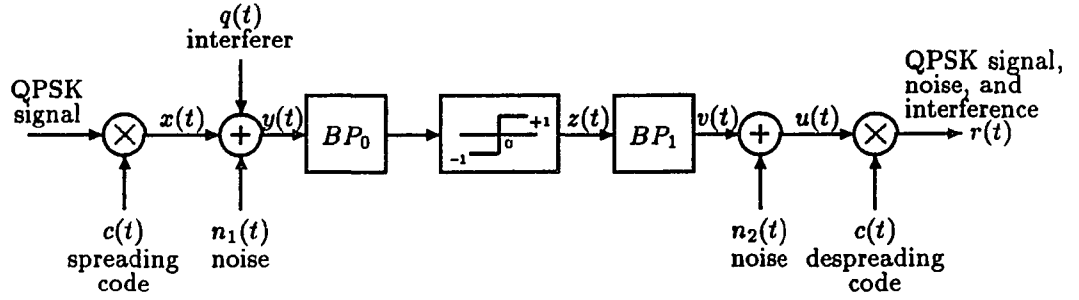
Figure 2.1: Functional Block Diagram(Uplink Interference).

filter BP_0 consists of the DS spread spectrum QPSK signal $x(t)$, the interfering signal $q(t)$, and white Gaussian noise $n_1(t)$. We consider the interference signal $q(t)$ as an another DS spread QPSK signal.

The DS spread spectrum QPSK signal $x(t)$ can be represented over one symbol period, T, as

$$
\begin{aligned}
x(t) &= Ac(t)\cos\left\{\omega_0 t + \phi_x(t) + \theta_x\right\} \\
&= Ac(t)\cos\left\{\omega_0 t + a_i\frac{\pi}{4} + \theta_x\right\} \\
& \qquad 0 \leq t \leq T
\end{aligned}
\tag{2.1}
$$

where a_i are random variables which take on the values 1, 2, 3, 4 with equal probability in each symbol interval, A is the amplitude of the carrier frequency, $c(t)$ denotes the

PN code signal, ω_0 is the carrier frequency, and $a_i\frac{\pi}{4}$ represents the QPSK modulation. It is assumed that PN spreading code signal has only the amplitude values of $+1$ or -1. The interfering signal $q(t)$ can be also expressed over one symbol period, T, as

$$\begin{aligned}
q(t) &= \alpha p(t)\cos\left\{\omega_1 t + \phi_q(t) + \theta_q\right\} \\
&= \alpha p(t)\cos\left\{\omega_1 t + b_i\frac{\pi}{4} + \theta_q\right\} \\
& \quad 0 \leq t \leq T
\end{aligned} \qquad (2.2)$$

where b_i are random variables which take on the values 1, 2, 3, 4 with equal probability in each symbol interval, α is the amplitude of the interferer. The function $p(t)$ is the interfering PN code signal of the same frequency as the PN code signal $c(t)$.

The filters BP_0 and BP_1 in the bandpass limiter are assumed to be wide enough to pass the signal with negligible distortion. The function of BP_0 is to remove as much noise, $n_1(t)$, as possible before limiting. The hard limiter is considered to be ideal, that is, its output is either ± 1. In Fig. 2.1, $n_1(t)$ and $n_2(t)$ are assumed to be statistically independent white Gaussian noise.

The first operation that is performed by the spread spectrum receiver is to eliminate the effects of the spreading function $c(t)$. This can be accomplished by multiplying the received signal by an identical binary sequence that is in time synchronization with it. The resulting output signal is then supplied to a pair of correlators where it is multiplied by a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$ and integrated over a symbol period. The outputs of the correlators are used to make a decision about the value of a_i.

2.2.2   Hard Limiter Output

We are going to examine the effects of the band pass limiter in the spread spectrum system which is depicted in Fig. 2.1 for the given interference. The input of the bandpass limiter, $y(t)$, is the sum of the three signals. These are the information signal $x(t)$, the interference signal $q(t)$, and white Gaussian noise $n_1(t)$.

$$y(t) = x(t) + q(t) + n_1(t)$$

$$= Ac(t)\cos\left(a_i\frac{\pi}{4}+\theta_x\right)\cos\omega_0 t - Ac(t)\sin\left(a_i\frac{\pi}{4}+\theta_x\right)\sin\omega_0 t$$

$$+ \alpha p(t)\cos\omega_1 t\,\cos\left(b_i\frac{\pi}{4}+\theta_q\right) - \alpha p(t)\sin\omega_1 t\,\sin\left(b_i\frac{\pi}{4}+\theta_q\right) + n_1(t) \quad (2.3)$$

The output $z(t)$ of a memoryless hard limiter is related to the input by the transfer characteristic $g(y)$ of the hard limiter. The characteristic of $g(y)$ is taken to be

$$g(y) = \begin{cases} 1 & y(t) > 0 \\ 0 & y(t) = 0 \\ -1 & y(t) < 0 \end{cases} \quad (2.4)$$

$g(y)$, being a signum function, has a Fourier transform $G(j\omega) = \dfrac{2}{j\omega}$, Then by using the inverse Fourier transform of the transfer function, we have

$$z(t) = g(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)e^{j\omega y}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{j\omega}e^{j\omega y}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{j\omega}\exp\Big[j\omega\Big\{ Ac(t)\cos\left(a_i\frac{\pi}{4}+\theta_x\right)\cos\omega_0 t$$

$$- Ac(t)\sin\left(a_i\frac{\pi}{4}+\theta_x\right)\sin\omega_0 t + \alpha p(t)\cos\omega_1 t\,\cos\left(b_i\frac{\pi}{4}+\theta_q\right)$$

$$- \alpha p(t)\sin\omega_1 t\,\sin\left(b_i\frac{\pi}{4}+\theta_q\right) + n_1(t)\Big\}\Big]\,d\omega \quad (2.5)$$

To simplify the calculation, we take $a_i = b_i = 1$ and $\theta_x = \theta_q = 0$; due to symmetry and averaging this does not effect the symbol error rate calculation.

$$z(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{j\omega}e^{\frac{j\omega Ac(t)}{\sqrt{2}}\cos\omega_0 t}\,e^{-\frac{j\omega Ac(t)}{\sqrt{2}}\sin\omega_0 t}$$

$$\cdot e^{\frac{j\omega\alpha p(t)}{\sqrt{2}}\cos\omega_1 t}\,e^{-\frac{j\omega\alpha p(t)}{\sqrt{2}}\sin\omega_1 t}\,e^{j\omega n_1(t)}\,d\omega \quad (2.6)$$

If we expand one of the exponential terms in the above equation, we have

$$e^{\frac{j\omega Ac(t)}{\sqrt{2}}\cos\omega_0 t} = \cos\left\{\frac{\omega Ac(t)}{\sqrt{2}}\cos\omega_0 t\right\} + j\sin\left\{\frac{\omega Ac(t)}{\sqrt{2}}\cos\omega_0 t\right\}$$

$$= \cos\left\{\frac{\omega Ac(t)}{\sqrt{2}}\cos\omega_0 t\right\} + jc(t)\sin\left\{\frac{\omega A}{\sqrt{2}}\cos\omega_0 t\right\} \quad (2.7)$$

Applying standard identities [53], we find that the cosine and sine terms in Eq. 2.7 may be written as

$$\cos\left\{\frac{\omega A}{\sqrt{2}}\cos\omega_0 t\right\} = J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{A\omega}{\sqrt{2}}\right)\cos 2k\omega_0 t \qquad (2.8)$$

$$\sin\left\{\frac{\omega A}{\sqrt{2}}\cos\omega_0 t\right\} = 2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos(2k+1)\omega_0 t \qquad (2.9)$$

where $J_i\left(\frac{\omega A}{\sqrt{2}}\right)$ denotes the Bessel function of the first kind with order i and argument $\frac{\omega A}{\sqrt{2}}$. Hence, if we substitute Eq.s 2.8 and 2.9 into Eq. 2.7, we have

$$e^{\frac{j\omega Ac(t)}{\sqrt{2}}}\cos\omega_0 t = J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty}j^k c^k(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)\cos k\omega_0 t \qquad (2.10)$$

For the expression of $e^{-\frac{j\omega Ac(t)}{\sqrt{2}}}\sin\omega_0 t$, we can obtain a similar expansion as shown below.

$$
\begin{aligned}
e^{-\frac{j\omega Ac(t)}{\sqrt{2}}}\sin\omega_0 t &= J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty}J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right)\cos 2k\omega_0 t \\
&\quad - j2c(t)\sum_{k=1}^{\infty}J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)\sin(2k-1)\omega_0 t \\
&= J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty}(-j)^k c^k(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(k\omega_0 t - \frac{\pi}{2}k\right)
\end{aligned}
$$

$$\qquad (2.11)$$

By replacing ω_0 with ω_1 and $c(t)$ by $p(t)$, we obtain the expression for the interference term. Here, Eq. 2.13 is derived by applying different identities [53].

$$e^{\frac{j\omega\alpha p(t)}{\sqrt{2}}}\cos\omega_1 t = J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty}j^k p^k(t)J_k\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos k\omega_1 t \qquad (2.12)$$

$$
\begin{aligned}
e^{-\frac{j\omega\alpha p(t)}{\sqrt{2}}}\sin\omega_1 t &= J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty}(-j)^k p^k(t)J_k\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
&\quad \cdot \cos\left(k\omega_1 t - \frac{\pi}{2}k\right)
\end{aligned}
$$

$$\qquad (2.13)$$

Substitute Eq.s 2.10, 2.11, 2.12, and 2.13 into Eq. 2.6.

$$z(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty} j^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos k\omega_0 t \right\}$$

$$\cdot \left\{ J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty} (-j)^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \right\}$$

$$\cdot \left\{ J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty} j^k p^k(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos k\omega_1 t \right\}$$

$$\cdot \left\{ J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty} (-j)^k p^k(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\left(k\omega_1 t - \frac{\pi}{2}k\right) \right\} \, d\omega \quad (2.14)$$

After expanding we have

$$z(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \right.$$

$$+ 2J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} j^k p^k(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos k\omega_1 t$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} (-j)^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2}k\right)$$

$$+ 4J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k j^{i+k} c^k(t) p^i(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2}k\right)$$

$$\cdot \cos i\omega_1 t$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} j^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos k\omega_0 t$$

$$+ 4J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} j^{i+k} c^k(t) p^i(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos k\omega_0 t \cos i\omega_1 t$$

$$+ 4J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k j^{i+k} c^{i+k}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos i\omega_0 t \cos\left(k\omega_0 t - \frac{\pi}{2}k\right)$$

$$+ 8J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (-1)^k j^{i+k+l} c^{i+k}(t) p^l(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\cdot \cos i\omega_0 t \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \cos l\omega_1 t$$

$$+ 2J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_o\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} (-j)^k p^k(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\left(k\omega_1 t - \frac{\pi}{2}k\right)$$

$$+ 4J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^{\infty}\sum_{i=1}^{\infty} (-1)^i (j)^{k+i} p^{k+i}(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_i\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos k\omega_1 t \cos\left(i\omega_1 t - \frac{\pi}{2}i\right)$$

$$+ 4J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty}\sum_{i=1}^{\infty} (-j)^{k+i} c^k(t) p^i(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2}k\right)$$

$$\cdot \cos\left(i\omega_1 t - \frac{\pi}{2}i\right)$$

$$+ 8J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k+l}j^{i+k+l}c^k(t)p^{i+l}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\left(k\omega_0 t - \frac{\pi}{2}k\right)\cos i\omega_1 t\cos\left(l\omega_1 t - \frac{\pi}{2}l\right)$$

$$+ 4J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}j^{k+l}(-1)^l c^k(t)p^l(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos k\omega_0 t\cos\left(l\omega_1 t - \frac{\pi}{2}l\right)$$

$$+ 8J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^l j^{i+k+l}c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos k\omega_0 t$$

$$\cdot\cos i\omega_0 t\cos\left(l\omega_1 t - \frac{\pi}{2}l\right)$$

$$+ 8J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k+l}j^{i+k+l}c^{i+k}(t)p^l(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos i\omega_0 t$$

$$\cdot\cos\left(k\omega_0 t - \frac{\pi}{2}k\right)\cos\left(l\omega_1 t - \frac{\pi}{2}l\right)$$

$$+ 16\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}(-1)^{k+m}j^{i+k+l+m}c^{i+k}(t)p^{l+m}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_m\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos i\omega_0 t\cos\left(k\omega_0 t - \frac{\pi}{2}k\right)\cos l\omega_1 t\cos\left(m\omega_1 t - \frac{\pi}{2}m\right)\bigg\}\,d\omega \qquad (2.15)$$

2.2.3 Investigation of the Hard Limiter Output

Looking at the complicated expression for the hard limiter output given in Eq. 2.15, we attempt to simplify matters by identifying various types of terms. We can separate it into information signal terms, cochannel interfering terms, noise, and intermodulation product terms.

Information Signal

The desired information signal from Eq. 2.15 is classified as those terms having a carrier frequency of ω_0 or a harmonics of ω_0. Therefore the information signal $z_m(t)$ can be shown to be

$$z_m(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} (-j)^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \right.$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} j^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos k\omega_0 t$$

$$+ 4J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k j^{i+k} c^{i+k}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\left. \cdot \cos i\omega_0 t \, \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \right\} d\omega \qquad (2.16)$$

Interfering Signal

The interfering signal consists of those terms having carrier frequency ω_1 or harmonics of ω_1. So,

$$z_q(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ 2J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} j^k p^k(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos k\omega_1 t \right.$$

$$+ 2J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} (-j)^k p^k(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\left(k\omega_1 t - \frac{\pi}{2}k\right)$$

$$+ 4J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^{\infty}\sum_{i=1}^{\infty} (-1)^i (j)^{k+i} p^{k+i}(t) J_k\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_i\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\left. \cdot \cos k\omega_1 t \, \cos\left(i\omega_1 t - \frac{\pi}{2}i\right) \right\} d\omega \qquad (2.17)$$

Noise

Noise is simply given by a single term. Here, the noise signal $n_1(t)$ at any time t is a zero mean Gaussian random variable with variance $\sigma^2 = \frac{N_0}{2}B$ where B is the bandwidth of the filter BP_0.

$$z_n(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega \qquad (2.18)$$

Intermodulation Product Terms

Due to the nonlinearity of hard limiter, we have many intermodulation product terms. These are terms whose carrier frequency is of the form $m\omega_0 \pm n\omega_1$, where m and n

are integers.

$$
\begin{aligned}
z_{mq}(t) =\ & \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k j^{i+k} c^k(t) p^i(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \right. \\
& \cdot \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \cos i\omega_1 t \\
& + 4J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} j^{i+k} c^k(t) p^i(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos k\omega_0 t \cos i\omega_1 t \\
& + 8J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (-1)^k j^{i+k+l} c^{i+k}(t) p^l(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos i\omega_0 t \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \cos l\omega_1 t \\
& + 4J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty}\sum_{i=1}^{\infty} (-j)^{k+i} c^k(t) p^i(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \\
& \cdot \cos\left(i\omega_1 t - \frac{\pi}{2}i\right) \\
& + 8J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (-1)^{k+l} j^{i+k+l} c^k(t) p^{i+l}(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \cos i\omega_1 t \cos\left(l\omega_1 t - \frac{\pi}{2}l\right) \\
& + 4J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} j^{k+l}(-1)^l c^k(t) p^l(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos k\omega_0 t \cos\left(l\omega_1 t - \frac{\pi}{2}l\right) \\
& + 8J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (-1)^l j^{i+k+l} c^k(t) p^{i+k}(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos k\omega_0 t \cos i\omega_0 t \cos\left(l\omega_1 t - \frac{\pi}{2}l\right) \\
& + 8J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (-1)^{k+l} j^{i+k+l} c^{i+k}(t) p^l(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos i\omega_0 t \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \cos\left(l\omega_1 t - \frac{\pi}{2}l\right) \\
& + 16 \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} (-1)^{k+m} j^{i+k+l+m} c^{i+k}(t) p^{l+m}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \left. \cdot J_m\left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos i\omega_0 t \cos\left(k\omega_0 t - \frac{\pi}{2}k\right) \cos l\omega_1 t \cos\left(m\omega_1 t - \frac{\pi}{2}m\right) \right\} d\omega
\end{aligned}
$$

$$(2.19)$$

2.2.4 Input of the Spread Spectrum Receiver

The bandpass filter BP_1 removes all the harmonics generated by the nonlinearity of the hard limiter. The received signal $u(t)$, thus, contains only those limiter output components in the fundamental band centered at ω_0 and white Gaussian noise added in the downlink. Using the following conditions, $|i - k| = 1$, $\omega_0 \simeq \omega_1$, and $\Delta\omega = |\omega_1 - \omega_0|$, the output of bandpass limiter $v(t)$ is calculated from the output of the hard limiter $z(t)$. The signal $u(t)$, which consists of $v(t)$ plus AWGN, can be written as the sum of $u_m(t)$, $u_q(t)$, $u_n(t)$, and $u_{mq}(t)$ given by the following equations.

Information Signal

Only those signal components for which the condition $|i - k| = 1$ for the summation variables i and k in the last summation of Eq. 2.16 will appear at the bandpass filter output BP_1. Here, the product of two cosine terms which has a dummy variable i and k can be calculated by using the following trigonometric identity:

$$
\begin{aligned}
\cos i\omega_0 t \cos k\omega_1 t &= \frac{1}{2}\Big\{\cos(i\omega_0 t - k\omega_1 t) + \cos(i\omega_0 t + k\omega_1 t)\Big\} \\
&= \frac{1}{2}\cos\Big\{(i-k)\omega_0 t - k\Delta\omega_1 t\Big\} + \frac{1}{2}\cos\Big\{(i+k)\omega_0 t + k\Delta\omega_1 t\Big\} \quad (2.20)
\end{aligned}
$$

Of the two terms in Eq. 2.20, only the term which is close to ω_0 can pass filter BP_1. Using this condition yields the following expression for $u_m(t)$.

$$
\begin{aligned}
u_m(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\Bigg[&-2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)c(t)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t - \frac{\pi}{2}\right) \\
&+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)c(t)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t \\
&- 2J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}c^{2i+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t - \frac{\pi}{2}(i+1)\right\} \\
&+ 2J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}c^{2k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t + \frac{\pi}{2}k\right)\Bigg]\,d\omega
\end{aligned}
$$

$$(2.21)$$

Interfering Signal

By using the identity of Eq. 2.20 and the condition $|i - k| = 1$, the interfering signal can be calculated as follows:

$$
\begin{aligned}
u_q(t) &= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left[2J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}} \right) p(t) J_1 \left(\frac{\omega \alpha}{\sqrt{2}} \right) \cos \omega_1 t \right. \\
&\quad - 2J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) J_o \left(\frac{\omega \alpha}{\sqrt{2}} \right) p(t) J_1 \left(\frac{\omega \alpha}{\sqrt{2}} \right) \cos \left(\omega_1 t - \frac{\pi}{2} \right) \\
&\quad + 2J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) \sum_{i=1}^{\infty} p(t) J_{i+1} \left(\frac{\omega \alpha}{\sqrt{2}} \right) J_i \left(\frac{\omega \alpha}{\sqrt{2}} \right) \cos \left(\omega_1 t + \frac{\pi}{2} i \right) \\
&\quad - 2J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) \sum_{i=1}^{\infty} p(t) J_k \left(\frac{\omega \alpha}{\sqrt{2}} \right) J_{k+1} \left(\frac{\omega \alpha}{\sqrt{2}} \right) \\
&\quad \left. \cdot \cos \left\{ \omega_1 t - \frac{\pi}{2} (k+1) \right\} \right] d\omega
\end{aligned}
\tag{2.22}
$$

Noise

Noise consists of downlink noise $n_2(t)$ and uplink noise $n_1(t)$ passing the bandpass limiter.

$$
u_n(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{j\omega} e^{j\omega n_1(t)} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}} \right) d\omega + n_2(t)
\tag{2.23}
$$

where $n_2(t)$ is a white Gaussian noise.

Intermodulation Product Terms

The intermodulation product portion of the bandpass limiter output, which has been calculated, consists of a large number of terms. The detailed results are presented in the Eq. A.1 of Appendix A.

2.2.5 Despread Output

The signal $u(t)$ is the wideband signal which enters the spread spectrum receiver. Now, we can obtain the despread output, $r(t)$, from $u(t)$ by multiplying $u(t)$ by the despreading signal code $c(t)$. The despread information signal, interfering signal, noise, and intermodulation product can be obtained from previous Eqs. 2.21, 2.22, 2.23, and A.1. All these terms include the uplink additive Gaussian noise $n_1(t)$ as a factor. Hence, all these signals have a stochastic nature.

Information Signal

The despread information portion of the signal can be expressed as

$$
\begin{aligned}
r_m(t) &= u_m(t)c(t) \\
&= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\left\{-2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t-\frac{\pi}{2}\right)\right. \\
&\quad + 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t \\
&\quad - 2J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t-\frac{\pi}{2}(i+1)\right\} \\
&\quad \left. + 2J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t+\frac{\pi}{2}k\right)\right\}\,d\omega
\end{aligned}
\tag{2.24}
$$

After arranging the above equation, $r_m(t)$ becomes

$$
\begin{aligned}
r_m(t) &= -\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t-\frac{\pi}{2}\right)d\omega \\
&\quad + \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t\,d\omega \\
&\quad - \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t-\frac{\pi}{2}(i+1)\right\}\,d\omega \\
&\quad + \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t+\frac{\pi}{2}k\right)\,d\omega
\end{aligned}
\tag{2.25}
$$

Interfering Signal

Despread interfering terms are given by

$$
\begin{aligned}
r_q(t) \;=\;& u_q(t)c(t) \\
=\;& \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) p(t)c(t)\cos\omega_1 t \, d\omega \\
&- \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_o\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) c(t)p(t)\cos\left(\omega_1 t - \frac{\pi}{2}\right) d\omega \\
&+ \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty} c(t)p(t) J_{i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos\left(\omega_1 t + \frac{\pi}{2}i\right) d\omega \\
&- \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^{\infty} c(t)p(t) J_k\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
&\cdot \cos\left\{\omega_1 t - \frac{\pi}{2}(k{+}1)\right\} d\omega
\end{aligned}
\tag{2.26}
$$

Noise

Noise consists of downlink noise $n_2(t)$ and uplink noise $n_1(t)$ passing the bandpass limiter. The despread noise consists of two terms as follows

$$
\begin{aligned}
r_n(t) \;=\;& u_n(t)c(t) \\
=\;& \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \frac{1}{j} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) c(t)\, d\omega + n_2(t)c(t) \\
=\;& \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) c(t)\, d\omega + n_2'(t) \\
\triangleq\;& r_{n1}(t) + n_2'(t)
\end{aligned}
\tag{2.27}
$$

where $n_2'(t) = n_2(t)c(t)$ and $n_2(t)$ is a white Gaussian noise.

The bandpass limiter passing portion of noise, $r_{n1}(t)$, can be written again

$$
\begin{aligned}
r_{n1}(t) \;=\;& \frac{2}{\pi} \int_0^{\infty} \frac{1}{\omega} \sin\omega n_1(t) J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) c(t)\, d\omega \\
=\;& \int_0^{\infty} \beta(\omega) \sin\omega n_1(t)\, c(t)\, d\omega
\end{aligned}
\tag{2.28}
$$

where $\quad \beta(\omega) = \frac{2}{\pi \omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right)$

The autocorrelation function of $r_{n1}(t)$ is

$$R_{r_{n1}}(\tau) = \int_0^\infty \int_0^\infty \beta(\omega_1)\beta(\omega_2)\overline{\sin\{\omega_1 n_1(t)\} \sin\{\omega_2 n_1(t+\tau)\} c(t) c(t+\tau)} \, d\omega_1 \, d\omega_2$$

(2.29)

The values $\sin\{\omega_1 n_1(t)\}$ and $\sin\{\omega_2 n_1(t+\tau)\}$ are uncorrelated for time t and $t+\tau$. Hence, $r_{n1}(t)$ is white noise. Eq. 2.29 can be rewritten as follows:

$$R_{r_{n1}}(\tau) = \int_0^\infty \int_0^\infty \beta(\omega_1)\beta(\omega_2)\overline{\sin\{\omega_1 n_1(t)\} \sin\{\omega_2 n_1(t+\tau)\}} \cdot \overline{c(t) c(t+\tau)} \, d\omega_1 \, d\omega_2$$

(2.30)

Since it is not easy to calculate this autocorrelation, we derive rather conservative estimates of the true value in which we neglect the spreading code. Therefore, Eq. 2.30 will be

$$R_{r_{n1}}(\tau) \simeq \int_0^\infty \int_0^\infty \beta(\omega_1)\beta(\omega_2)\overline{\sin\{\omega_1 n_1(t)\} \sin\{\omega_2 n_1(t+\tau)\}} \, d\omega_1 \, d\omega_2 \qquad (2.31)$$

Here, the expected value may be rewritten as

$$\overline{\sin\{\omega_1 n_1(t)\} \sin\{\omega_2 n_1(t+\tau)\}} = \frac{1}{2}\overline{\cos\{\omega_1 n_1(t) - \omega_2 n_1(t+\tau)\}}$$
$$- \frac{1}{2}\overline{\cos\{\omega_1 n_1(t) + \omega_2 n_1(t+\tau)\}} \quad (2.32)$$

Since $n_1(t)$ is a stationary Gaussian process, we can notate as follows for any fixed value of τ.

$$n'(t,\tau) \triangleq \omega_1 n_1(t) - \omega_2 n_1(t+\tau)$$
$$n''(t,\tau) \triangleq \omega_1 n_1(t) + \omega_2 n_1(t+\tau) \qquad (2.33)$$

The variance of $n'(t,\tau)$ and $n''(t,\tau)$ can be calculated as follows [54].

$$\sigma'^2(\tau) = \sigma^2 \omega_1^2 + \sigma^2 \omega_2^2 - 2\omega_1 \omega_2 R_n(\tau)$$
$$\sigma''^2(\tau) = \sigma^2 \omega_1^2 + \sigma^2 \omega_2^2 + 2\omega_1 \omega_2 R_n(\tau) \qquad (2.34)$$

$R_n(\tau)$ is the autocorrelation of the bandpass noise $n_1(t)$ which is given by [55]

$$R_n(\tau) = N_0 B \mathrm{sinc}(B\tau)\cos\omega_0\tau \qquad (2.35)$$

Thus the expression about Eq. 2.32 can be written again as

$$\overline{\sin\{\omega_1 n_1(t)\}\sin\{\omega_2 n_1(t+\tau)\}} = \frac{1}{2}e^{-\frac{\sigma'^2(\tau)}{2}} - \frac{1}{2}e^{-\frac{\sigma''^2(\tau)}{2}} \qquad (2.36)$$

If we insert Eq. 2.36 into Eq. 2.31, we have

$$R_{r_{n1}}(\tau) \simeq \int_0^\infty \int_0^\infty \beta(\omega_1)\beta(\omega_2)\left\{\frac{1}{2}e^{-\frac{\sigma'^2(\tau)}{2}} - \frac{1}{2}e^{-\frac{\sigma''^2(\tau)}{2}}\right\}\,d\omega_1\,d\omega_2 \qquad (2.37)$$

Since the power spectral density of a random process is equal to the Fourier transform of its autocorrelation function, we obtain, by using Eq. 2.37,

$$S_n(\omega_0) = \int_{-\infty}^\infty R_{r_{n1}}(\tau)e^{-j2\pi\omega_0\tau}\,d\tau \qquad (2.38)$$

Since $R_{r_{n1}}(\tau)$ is real and even, Eq. 2.38 is written again

$$
\begin{aligned}
S_n(\omega_0) &= 2\int_0^\infty R_{r_{n1}}(\tau)\cos\omega_0\tau\,d\tau \\
&\simeq \int_0^\infty \int_0^\infty \beta(\omega_1)\beta(\omega_2)\int_0^\infty e^{-\frac{\sigma'^2(\tau)}{2}}\cos\omega_0\tau\,d\tau\,d\omega_1\,d\omega_2 \\
&\quad - \int_0^\infty \int_0^\infty \beta(\omega_1)\beta(\omega_2)\int_0^\infty e^{-\frac{\sigma''^2(\tau)}{2}}\cos\omega_0\tau\,d\tau\,d\omega_1\,d\omega_2
\end{aligned}
\qquad (2.39)
$$

After we substitute, we have the following results

$$
\begin{aligned}
S_n(\omega_0) &\simeq \frac{4}{\pi^2}\Bigg[\int_0^\infty \int_0^\infty \frac{1}{\omega_1^2\omega_2^2}J_0^2\left(\frac{\omega_1 A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega_1\alpha}{\sqrt{2}}\right)\frac{2}{\pi\omega_2}J_0^2\left(\frac{\omega_2 A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega_2\alpha}{\sqrt{2}}\right) \\
&\quad \cdot \int_0^\infty e^{-\frac{\omega_1^2\sigma^2(\tau)}{2}}e^{-\frac{\omega_2^2\sigma^2(\tau)}{2}}\exp\{\omega_1\omega_2 N_0 B\mathrm{sinc}(B\tau)\cos\omega_0\tau\} \\
&\quad \cdot \cos\omega_0\tau\,d\tau\,d\omega_1\,d\omega_2 \\
&\quad - \int_0^\infty \int_0^\infty \frac{1}{\omega_1^2\omega_2^2}J_0^2\left(\frac{\omega_1 A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega_1\alpha}{\sqrt{2}}\right)\frac{2}{\pi\omega_2}J_0^2\left(\frac{\omega_2 A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega_2\alpha}{\sqrt{2}}\right) \\
&\quad \cdot \int_0^\infty e^{-\frac{\omega_1^2\sigma^2(\tau)}{2}}e^{-\frac{\omega_2^2\sigma^2(\tau)}{2}}\exp\{-\omega_1\omega_2 N_0 B\mathrm{sinc}(B\tau)\cos\omega_0\tau\} \\
&\quad \cdot \cos\omega_0\tau\,d\tau\,d\omega_1\,d\omega_2\Bigg]
\end{aligned}
\qquad (2.40)
$$

The autocorrelation of $n_2'(t)$ will be

$$
\begin{aligned}
R_{n_2'}(\tau) &= \overline{n_2(t_1)c(t_1)n_2(t_1+\tau)c(t_1+\tau)} \\
&= \overline{n_2(t_1)n_2(t_1+\tau)} \cdot \overline{c(t_1)c(t_1+\tau)} \\
&= R(\tau) \cdot \overline{c(t_1)c(t_1+\tau)}
\end{aligned}
\tag{2.41}
$$

Since $R(\tau)$ is delta function, $R_{n_2'}(\tau)$ is also delta function. Thus, $n_2'(t)$ is also white Gaussian noise.

Intermodulation Product Terms

The intermodulation products at the output of the bandpass limiter contain a great many terms. We omit the details of them here, as we show later they can be neglected in the symbol error rate calculations. Despread outputs of intermodulation product term are given in the Eq. B.1 of the Appendix B.

2.3 Performance Evaluation

2.3.1 Theoretical Derivation

We are going to calculate the performance of the spread spectrum system which is depicted in Fig. 2.1 for the situation where interference is present only on the uplink. The despread signal is supplied to a pair of correlators and multiplied by a locally generated pair of coherent reference signals and integrated from 0 to T. Two basis functions $\phi_1(t)$ and $\phi_2(t)$ are used to demodulate the received signals, namely

$$\phi_1(t) = \frac{1}{T} \cos\omega_0 t \quad 0 \le t \le T \tag{2.42}$$

$$\phi_2(t) = \frac{1}{T} \sin\omega_0 t \quad 0 \le t \le T \tag{2.43}$$

where T is the symbol duration.

Demodulated Information Signal

The outputs of the correlators are m_1 and m_2 respectively, they are given by the following expressions.

$$
\begin{aligned}
m_1 &= \int_0^T r_m(t)\,\phi_1(t)\,dt \\
&= \frac{1}{T}\int_0^T \Bigg[-\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\sin\omega_0 t\,\cos\omega_0 t\;d\omega \\
&\quad + \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t\,\cos\omega_0 t\,d\omega \\
&\quad - \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t-\frac{\pi}{2}(i+1)\right\}\cos\omega_0 t\,d\omega \\
&\quad + \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t+\frac{\pi}{2}k\right)\cos\omega_0 t\,d\omega \Bigg]\,dt
\end{aligned}
\tag{2.44}
$$

From the trigonometric identities, $e^{j\omega n_1(t)} = \cos\omega n_1(t)+j\sin\omega n_1(t)$. Since $\sin\omega n_0(t)$ is a odd function and $\cos\omega n_0(t)$ is a even function in ω, m_1 may now be written as

$$m_1 = -\frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t) \, \sin \omega_0 t \, \cos \omega_0 t \, dt \, d\omega$$

$$+ \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t) \, \cos^2 \omega_0 t \, dt \, d\omega$$

$$- \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^\infty J_i \left(\frac{\omega A}{\sqrt{2}}\right) J_{i+1} \left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t)$$

$$\cdot \cos \left\{\omega_0 t - \frac{\pi}{2}(i+1)\right\} \cos \omega_0 t \, dt \, d\omega$$

$$+ \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^\infty J_{k+1} \left(\frac{\omega A}{\sqrt{2}}\right) J_k \left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t)$$

$$\cdot \cos \left(\omega_0 t + \frac{\pi}{2}\right) \cos \omega_0 t \, dt \, d\omega \qquad (2.45)$$

Gaussian noise $n_1(t)$ is included in uplink but it will be assumed, in the theoretical analysis of the probability of error, that the downlink noise is at least 3 dB greater than that on the uplink [49], which is true for practical satellite systems. The effect of uplink Gaussian noise $n_1(t)$ is thus neglected. Setting $n_1(t)$ equal to zero, the results for m_1 will be

$$m_1 \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$\triangleq \sqrt{\frac{E_1}{2}} \qquad (2.46)$$

where E_1 is a constant and given by

$$E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2 \qquad (2.47)$$

In a similar way, the m_2 demodulated may be represented as

$$m_2 = \int_0^T r_m(t)\,\phi_2(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left[-\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\sin\omega_0 t\,\sin\omega_0 t\,d\omega \right.$$

$$+\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t\,\sin\omega_0 t\,d\omega$$

$$-\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t-\frac{\pi}{2}(i+1)\right\}\sin\omega_0 t\,d\omega$$

$$\left. +\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t+\frac{\pi}{2}k\right)\sin\omega_0 t\,d\omega \right] dt$$

After some calculation, m_2 will be

$$m_2 \simeq -\frac{2}{\pi}\int_0^{\infty}\frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)d\omega$$

$$-\frac{2}{\pi}\sum_{i=1}^{\infty}(-1)^i\int_0^{\infty}\frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega$$

$$-\frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^{k+1}\int_0^{\infty}\frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega$$

$$\triangleq -\sqrt{\frac{E_2}{2}} \tag{2.48}$$

where E_2 is also a constant and defined by

$$E_2 = \left[\frac{2\sqrt{2}}{\pi}\int_0^{\infty}\frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)d\omega \right.$$

$$+\frac{2\sqrt{2}}{\pi}\sum_{i=1}^{\infty}(-1)^i\int_0^{\infty}\frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega$$

$$\left. +\frac{2\sqrt{2}}{\pi}\sum_{k=1}^{\infty}(-1)^{k+1}\int_0^{\infty}\frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega \right]^2 \tag{2.49}$$

Demodulated Interferer

The effect of the demodulated interferer will be treated by considering two cases, the average case and the worst case. These cases depend on the relation between $c(t)$ and

$p(t)$ over a symbol interval as will be explained later. For the average case,

$$
\begin{aligned}
q_1 &= \int_0^T r_q(t)\,\phi_1(t)\,dt = \frac{1}{T}\int_0^T r_q(t)\,\cos\omega_0 t\,dt \\
&= \frac{1}{T}\int_0^T \left[\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) c(t)p(t)\cos\omega_1 t\,d\omega \right. \\
&\quad - \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_o\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) c(t)p(t)\cos\left(\omega_1 t - \frac{\pi}{2}\right) d\omega \\
&\quad + \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty} c(t)p(t) J_{i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos\left(\omega_1 t + \frac{\pi}{2}i\right) d\omega \\
&\quad - \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^{\infty} c(t)p(t) J_k\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
&\quad \left. \cdot \cos\left\{\omega_1 t - \frac{\pi}{2}(k{+}1)\right\} d\omega \right] \cos\omega_0 t\,dt
\end{aligned}
\tag{2.50}
$$

For practical CDMA systems, the chip time of the spreading code is much smaller than symbol time of the message, which is equivalent to saying the processing gain is much larger than one. Since spreading codes, $c(t)$ and $p(t)$, are approximately orthogonal over the one symbol interval of the information signal, the value of $\int_0^{T_s} c(t)p(t)\,dt$ is much less than the one of $\int_0^{T_s} c^2(t)\,dt$. Under this condition the value of q_1 is smaller than the values of m_1 by the processing gain and can be neglected in the average case. Similarly, equation q_2 is also

$$
q_2 = \int_0^T r_q(t)\,\phi_2(t)\,dt = \frac{1}{T}\int_0^T r_q(t)\,\sin\omega_0 t\,dt \simeq 0
\tag{2.51}
$$

For the worst case, the magnitude of $c(t)p(t)$ is one for the entire symbol duration, in other words, processing gain is $0\ dB$. Eq. 2.50 may be written again by using trigonometric identities.

$$q_1 = \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega\alpha}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos\omega n_1(t) \, \cos\omega_1 t \, \cos\omega_0 t \, dt \, d\omega$$

$$- \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega\alpha}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos\omega n_1(t) \, \cos\left(\omega_1 t - \frac{\pi}{2}\right) \cos\omega_0 t \, dt \, d\omega$$

$$+ \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty \frac{1}{T} \int_0^T \cos\omega n_1(t) \, J_{i+1} \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_i \left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos\left(\omega_1 t + \frac{\pi}{2} i\right)$$

$$\cdot \cos\omega_0 t \, dt \, d\omega$$

$$- \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty \frac{1}{T} \int_0^T \cos\omega n_1(t) \, J_k \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+1} \left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_1 t - \frac{\pi}{2}(k+1)\right\} \cos\omega_0 t \, dt \, d\omega \tag{2.52}$$

With the assumption $n_1(t) \approx 0$, and using $|\omega_1 - \omega_0| = \Delta\omega$ and trigonometric identities, Eq. 2.52 is given by

$$q_1 = \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega\alpha}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \left\{\cos(2\omega_0 + \Delta\omega) t + \cos\Delta\omega t\right\} dt \, d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega\alpha}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \left\{\sin(2\omega_0 + \Delta\omega)t + \sin\Delta\omega t\right\} dt \, d\omega$$

$$+ \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty \frac{1}{T} \int_0^T J_{i+1} \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_i \left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \left\{\cos\left(2\omega_0 t + \Delta\omega t + \frac{\pi}{2} i\right) + \cos\left(\Delta\omega t + \frac{\pi}{2} i\right)\right\} dt \, d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty \frac{1}{T} \int_0^T J_k \left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+1} \left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \left[\cos\left\{2\omega_0 t + \Delta\omega t - \frac{\pi}{2}(k+1)\right\} + \cos\left\{\Delta\omega t - \frac{\pi}{2}(k+1)\right\}\right] dt \, d\omega \tag{2.53}$$

The cosine and the sine of $2\omega_0 t$ are included in above equation. This frequency is twice that of the carrier frequency. Since the integration time, one symbol period, is very large compared with one period of this frequency, It is reasonable to assume that this term integrates essentially to 0. With assumption of $\Delta\omega T \ll 1$, the output component thus becomes

$$q_1 \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega$$

$$+ \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty J_{i+1} \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_i \left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos \frac{\pi}{2} i \, d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty J_k \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{k+1} \left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos \left\{\frac{\pi}{2}(k+1)\right\} \, d\omega$$

$$= \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega \tag{2.54}$$

Similarly, final result of q_2 can be written as

$$q_2 \simeq -\frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty (-1)^{i+1} J_{2i-1} \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{2i} \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty (-1)^k J_{2k} \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{2k+1} \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega \tag{2.55}$$

Demodulated Noise Signal

From Eq.s 2.27 and 2.43, demodulation noise, r_{n_i}, will be given by

$$r_{n_i} = \int_0^T r_n(t) \, \phi_i(t) \, dt \quad \text{where} \quad i = 1, 2$$

$$= \frac{1}{j\pi} \int_{-\infty}^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) \int_0^T e^{j\omega n_1(t)} c(t) \phi_i(t) \, dt \, d\omega + \int_0^T n_2' \, \phi_i(t) \, dt$$

$$= \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0^2 \left(\frac{\omega \alpha}{\sqrt{2}}\right) \int_0^T \sin \omega n_1(t) \, c(t) \phi_i(t) \, dt \, d\omega + \int_0^T n_2' \, \phi_i(t) \, dt$$

$$\tag{2.56}$$

With the same assumption as before, that $n_1(t)$ can be neglected compared to $n_2(t)$.

$$r_{n_i} \approx \int_0^T n_2'(t) \phi_i(t) dt \tag{2.57}$$

Since $n_2(t)$ has a zero mean value, the random variable n_2' extracted from n_2 also has a zero mean.

$$E[r_{n_i}] = E\left[\int_0^T n_2'(t) \, \phi_i(t) \, dt\right] = 0 \tag{2.58}$$

To find the variance of r_{n_i}

$$
\begin{aligned}
\sigma_{r_{n_i}}^2 &= E\left[r_{n_i}^2\right] = E\left[\int_0^T n_2'(t)\,\phi_i(t)\,dt \int_0^T n_2'(u)\,\phi_i(u)\,du\right] \\
&= E\left[\int_0^T\int_0^T \phi_i(t)\phi_i(u)n_2'(t)n_2'(u)\,dt\,du\right] \\
&= E\left[\int_0^T\int_0^T \phi_i(t)\phi_i(u)R_w(t-u)\,dt\,du\right]
\end{aligned}
\tag{2.59}
$$

since $R_w(t,u) = \dfrac{N_0}{2}\delta(t-u)$

$$
\sigma_{r_{n_i}}^2 = \frac{N_0}{4T}
\tag{2.60}
$$

Demodulated Intermodulation Products

For demodulated intermodulation products we have that

$$
\begin{aligned}
r_{1mq} &= \int_0^T r_{mq}(t)\,\phi_1(t)\,dt \\
&= -\frac{2}{\pi T}\int_{-\infty}^{\infty}\int_0^T \frac{1}{\omega}\,e^{j\omega n_1(t)}\cos\omega_0 t\left[-J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}c^i(t)p^i(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\right. \\
&\quad \cdot J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\left\{\omega_0 t - \frac{\pi}{2}(i+1) - i\Delta\omega t\right\} \\
&\quad + J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
&\quad \cdot\cos\left\{\omega_0 t - \frac{\pi}{2}k + (k+1)\Delta\omega t\right\} \\
&\quad + J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}(-1)^i c^i(t)p^i(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos(\omega_0 t - i\Delta\omega t) \\
&\quad + J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
&\quad \cdot\cos\left\{\omega_0 t + (k+1)\Delta\omega t\right\} \\
&\quad - J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^i c^{i+k+1}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right) \\
&\quad \cdot\cos\left\{\omega_0 t - \frac{\pi}{2}k - (i+k-1)\,\Delta\omega t\right\} \\
&\quad + J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k+1}(t)p^{1-i+k}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t + \frac{\pi}{2}k + (1-i+k)\,\Delta\omega t\right\} \\
&\quad - J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k+1}(t)p^{-1-i+k}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)
\end{aligned}
$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}k - (-1-i+k)\,\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{i+k}c^{i+k+1}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}k - (i-k-1)\,\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{i+k}c^{i+k+1}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}k + (i-k+1)\,\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}(-1)^i c^i(t)p^i(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\left(\omega_0 t - \frac{\pi}{2} - i\Delta\omega t\right)$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2} - (k+1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{-i+k}c^{k+1}(t)p^{k-1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i+1) - (k-1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{-i+k}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i-1) + (k+1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k-1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(1-i) + (1-k)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(1+i) + (1+k)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{k+1}(t)p^{-k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i+1) - (k-1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i-1) + (k+1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(k+1) - (k+1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{l=1}^{\infty} c^l(t)p^l(t)J_{l+1}\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right) \cos\left\{\omega_0 t + \frac{\pi}{2}l - l\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}k - (k+i-1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}k + (k+i+1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{i-k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}k + (i-k+1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{i-k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}k - (i-k-1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^i c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}k - (k-i-1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^i c^{k+1}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}k + (k-i+1)\Delta\omega t\right\}$$

$$+ J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k c^{i+k+1}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i-1) - (i+k-1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k c^{i+k+1}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i+1) + (i+k+1)\Delta\omega t\right\}$$

$$- J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^{k-i} c^{i+k+1}(t)p^{k-i+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos \left\{ \omega_0 t + \frac{\pi}{2}(i-1) + (k-i+1)\Delta\omega t \right\}$$

$$+ J_0 \left(\frac{\omega\alpha}{\sqrt{2}} \right) \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} (-1)^{k-i} c^{i+k+1}(t) p^{k-i-1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_{k-i-1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(i+1) - (k-i-1)\Delta\omega t \right\}$$

$$+ J_0 \left(\frac{\omega\alpha}{\sqrt{2}} \right) \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} c^{i+k+1}(t) p^{i-k-1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_{i-k-1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t + \frac{\pi}{2}(i-1) - (i-k-1)\Delta\omega t \right\}$$

$$- J_0 \left(\frac{\omega\alpha}{\sqrt{2}} \right) \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} c^{i+k+1}(t) p^{i-k+1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_{i-k+1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(i+1) + (i-k+1)\Delta\omega t \right\}$$

$$+ \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^k c^{i+k+1}(t) p^{i+k-1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{i+k+l-1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(-i-l+1) + (-i-k+1)\Delta\omega t \right\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^k c^{i+k+1}(t) p^{i+k+1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{i+k+l+1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(i+l+1) + (i+k+1)\Delta\omega t \right\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^i c^{i+k+1}(t) p^{-i-k+1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{-i-k+l+1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(-i+l+1) - (i+k-1)\Delta\omega t \right\}$$

$$+ \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^i c^{i+k+1}(t) p^{-i-k-1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{-i-k+l-1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t + \frac{\pi}{2}(-i+l-1) + (i+k+1)\Delta\omega t \right\}$$

$$+ \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^{k-l} c^{i+k+1}(t) p^{i+k-1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{i+k-l-1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(-i+l+1) - (i+k-1)\Delta\omega t \right\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^{k-l} c^{i+k+1}(t) p^{i+k+1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{i+k-l+1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_0 t - \frac{\pi}{2}(i+1) + (i+k+1)\Delta\omega t \right\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^{k-i-l} c^{i+k+1}(t) p^{-i+k+1}(t) J_i \left(\frac{\omega A}{\sqrt{2}} \right) J_k \left(\frac{\omega A}{\sqrt{2}} \right) J_l \left(\frac{\omega\alpha}{\sqrt{2}} \right) J_{-i+k-l+1} \left(\frac{\omega\alpha}{\sqrt{2}} \right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i+l-1) + (-i+k+1)\Delta\omega t\right\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i-l}c^{i+k+1}(t)p^{-i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-i+k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i+l+1) + (i-k+1)\Delta\omega t\right\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}c^{i+k+1}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i+l-1) + (-i+k+1)\Delta\omega t\right\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}c^{i+k+1}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i+l+1) + (i-k+1)\Delta\omega t\right\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i}c^{i+k+1}(t)p^{-i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-i+k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right).$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i-l-1) + (-i+k+1)\Delta\omega t\right\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i}c^{i+k+1}(t)p^{-i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-i+k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i-l+1) + (-i+k-1)\Delta\omega t\right\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{l}c^{i+k+1}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\left\{\omega_0 t + \frac{\pi}{2}(i-l-1) - (i-k-1)\Delta\omega t\right\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{l}c^{i+k+1}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k-l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\left. \cdot \cos\left\{\omega_0 t - \frac{\pi}{2}(i-l+1) + (i-k+1)\Delta\omega t\right\}\right] \, dt \, d\omega \tag{2.61}$$

Similarly r_{2mq} is also given by replacing $\phi_1(t)$ with $\phi_2(t)$. Here, the spreading codes $c(t)$ and $p(t)$ are contained in all terms of Eq. 2.61. In the terms where $c(t)$ and $p(t)$ appear raised to even powers, they do not effect the result. Thus, all these intermodulation terms are narrow-band signals and independent of the signal processing gain. However, according to the investigation of Baer [38] , the power ratio of the information signal and the strongest narrow-band intermodulation product is also bigger than 10 dB if the power ratio of information signal $x(t)$ and interferer $q(t)$ is bigger

than 0 *dB* and hence these intermodulation terms will be neglected. Since odd power terms of $c(t)$ and $p(t)$ are wideband interference, we can neglect them if we assume the processing gain is large.

Calculation of Average Probability of Symbol Error

Owing to the symmetry of the decision regions, the probability of interpreting the received signal point correctly is the same regardless of which particular signal was actually transmitted. Since we assume the phase of the QPSK signal as $\frac{\pi}{4}$, the element x_1, in-phase component, of the observation vector X must be positive and the element x_2, quadrature component, of the observation vector X must be negative for a correct decision when this QPSK signal is transmitted. This means that the probability of a correct decision, P_c, equals the conditional probability of the joint event $x_1 > 0$ and $x_2 < 0$, given that this QPSK signal was transmitted. Both X_1 and X_2 are independent Gaussian random variables with a conditional means equal to $\sqrt{\frac{E_1}{2}}$ and $-\sqrt{\frac{E_2}{2}}$ respectively and a variance equal to $\frac{N_0}{4T}$.

For the average case, since we neglect the interferer terms, probability of symbol error for DS spread spectrum QPSK signal can be expressed as follows.

$$
\begin{aligned}
P(c) &= \int_0^\infty \sqrt{\frac{2T}{\pi N_0}} e^{-\frac{2T\left(x_1-\sqrt{\frac{E_1}{2}}\right)^2}{N_0}} dx_1 \int_{-\infty}^0 \sqrt{\frac{2T}{\pi N_0}} e^{-\frac{2T\left(x_2+\sqrt{\frac{E_2}{2}}\right)^2}{N_0}} dx_2 \\
&= 1 - Q\left(\sqrt{\frac{2E_1T}{N_0}}\right) - Q\left(\sqrt{\frac{2E_2T}{N_0}}\right) + Q\left(\sqrt{\frac{2E_1T}{N_0}}\right) Q\left(\sqrt{\frac{2E_2T}{N_0}}\right) \quad (2.62)
\end{aligned}
$$

where $Q(x)$ denotes Q function[1]. Hence,

$$
\begin{aligned}
P(e) &= 1 - P_c \\
&= Q\left(\sqrt{\frac{2E_1T}{N_0}}\right) + Q\left(\sqrt{\frac{2E_2T}{N_0}}\right) - Q\left(\sqrt{\frac{2E_1T}{N_0}}\right) Q\left(\sqrt{\frac{2E_2T}{N_0}}\right) \quad (2.63)
\end{aligned}
$$

---

[1]The Q function is defined as

$$
Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right), \text{ where } \operatorname{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy
$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right.$$

$$+ \frac{2\sqrt{2}}{\pi} \sum_{i=1}^\infty (-1)^i \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$\left. + \frac{2\sqrt{2}}{\pi} \sum_{k=1}^\infty (-1)^{k+1} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

For the worst case, processing gain 0 dB, we are considering interferer terms only. Therefore,

$$P_1(c) = \int_0^\infty \frac{\sqrt{2T}}{\sqrt{\pi N_0}} e^{-\frac{2T\left(x_1 - \sqrt{\frac{E_1}{2}} - q_1\right)^2}{N_0}} dx_1 \int_{-\infty}^0 \frac{\sqrt{2T}}{\sqrt{\pi N_0}} e^{-\frac{2T\left(x_2 + \sqrt{\frac{E_2}{2}} + q_2\right)^2}{N_0}} dx_2$$

$$= 1 - Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) - Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$+ Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right) \qquad (2.64)$$

where q_1 and q_2 are given in Eq.s 2.54 and 2.55 respectively.

$$P_1(e) = 1 - P_1(c)$$

$$= Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$- Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right) \qquad (2.65)$$

For the worst case where the processing gain is 0 dB, we are considering interferer terms. When processing gain is 0 dB, $c(t)p(t)$ will be

c(t)	p(t)	c(t)p(t)	Probability
1	1	1	1/4
1	-1	-1	1/4
-1	1	-1	1/4
-1	-1	1	1/4

Using the fact that for the case where $|c(t)p(t)| = 1$, the probability that $c(t)p(t) = 1$ and the probability that $c(t)p(t) = -1$ will each be 1/2. The probability that the

value of $c(t)p(t)$ equals to -1 can be given similarly with Eq. 2.65 as follows.

$$P_2(e) = Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$
$$- Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right) \tag{2.66}$$

Finally, the probability of error for the worst case will be

$$P(e) = P\left\{c(t)p(t)=+1\right\}P\left\{e/c(t)p(t)=+1\right\} + P\left\{c(t)p(t)=-1\right\}P\left\{e/c(t)p(t)=-1\right\}$$
$$= \frac{1}{2}P_1(e) + \frac{1}{2}P_2(e)$$
$$= \frac{1}{2}\left[Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right)\right.$$
$$+ Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right) - Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$
$$\left. - Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right) \right] \tag{2.67}$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right.$$
$$+ \frac{2\sqrt{2}}{\pi}(-1)^i \sum_{i=1}^\infty \int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$
$$\left. + \frac{2\sqrt{2}}{\pi}(-1)^{k+1} \sum_{k=1}^\infty \int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

$$q_1 \simeq \frac{2}{\pi}\int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

$$q_2 \simeq -\frac{2}{\pi}\int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$
$$- \frac{2}{\pi}\int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty (-1)^{i+1}J_{2i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{2i}\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$
$$- \frac{2}{\pi}\int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty (-1)^k J_{2k}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{2k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

2.3.2 Numerical Results

The expression for the probability for the uplink QPSK interference derived in the previous section are presented graphically in this section. A Fortran program was written to compute the Eq.s 2.63 and 2.67. Subroutines of the IMSL in mainframe computer was used to calculate all the Bessel functions, the summations, and the integration. Most of the curves show the probability of symbol error, $P(e)$ as a function of average received energy per bit as normalized to the channel noise power density expressed in dB, denoted by E_b/N_0. The average energy per bit is related to the average energy per symbol by the formula, $E_b = \dfrac{E_s}{\log_2 M}$. It should be kept in mind that in calculating these results we have assumed that uplink AWGN is much less than downlink AWGN and the influence of intermodulation products is much less than that of interferer terms.

Some numerical results for a DS spread spectrum QPSK hard limited channel are given in the following figures to illustrate the effect of the interference on system performance. Symbol error rate as a function of E_b/N_0 for the average case and the worst case is given in Figs. 2.2 through C.4.

Performance generally increases with an increasing value of S/I ratio. However, the average case shows that performance no longer improves much once the signal to interference power ratio becomes greater than 10 dB. And, in the worse case, performance is saturated when signal to interference ratio is around 30 dB.

From Figs. 2.2 and 2.3, we observe that a strong interfering signal, that passes through the band pass filter can seriously disturb the performance of the receiver. We can also see from Fig. C.4 that the effect of an interferer in a satellite channel can be neglected when the S/I $\geq$ 40 dB.
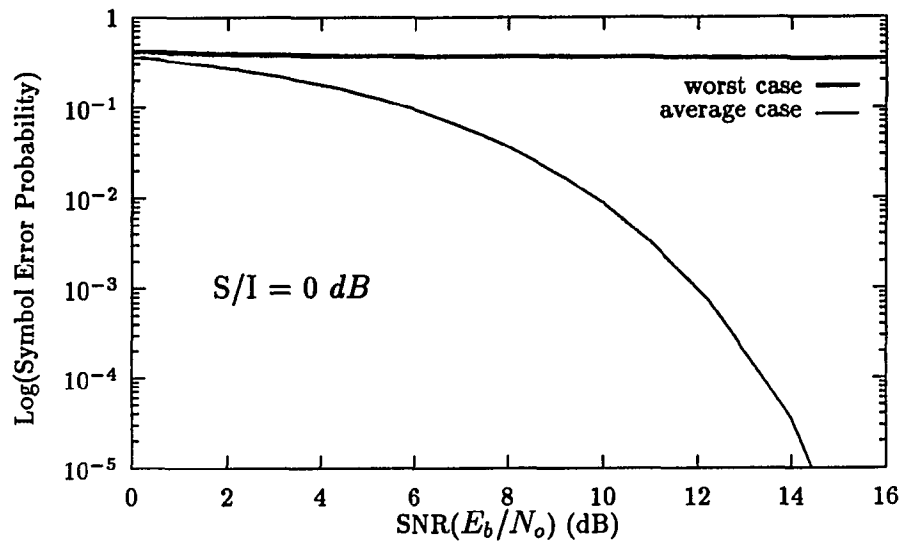
Figure 2.2: Symbol error rate for the average case and the worst case as a function of E_b/N_0 (S/I = 0 dB)
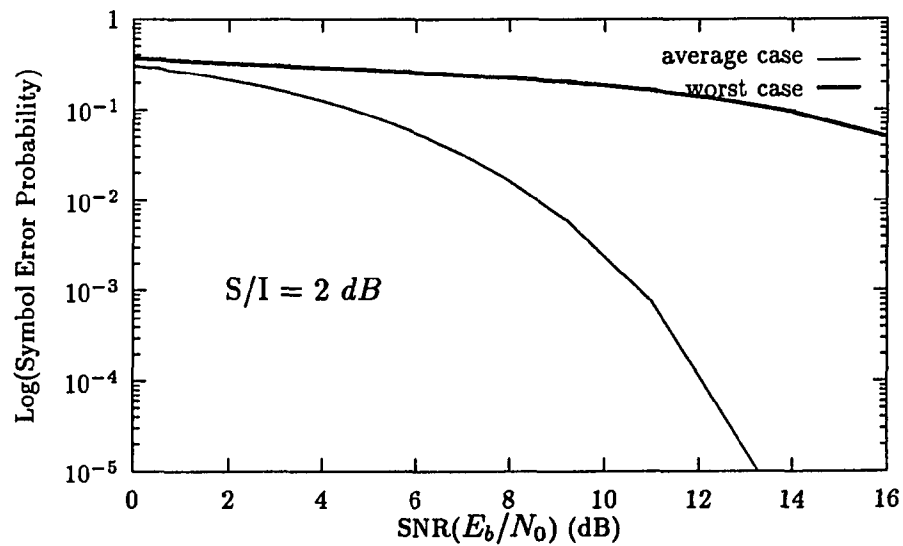
Figure 2.3: Symbol error rate for the average case and the worst case as a function of E_b/N_0 (S/I = 2 dB)

2.4 Simulation for Evaluating of Performance

2.4.1 Simulation Model

This simulation is implemented by using the BOSS software package of Comdisco Systems Inc. The block diagram used in simulation is shown in Fig. 2.4. A BOSS simulation consists of a system block diagram, a particular set of values for the simulation, and system parameters and is done by digital signal processing techniques. Hence, simulation time also increases when the sampling frequency increases in the simulation. All simulation is thus performed in baseband to reduce simulation time. Actually module *SEMI-ANALYTIC MPSK_ERROR ESTIMATOR* is also designed to work in baseband. The various modules in the simulation are described below.

- Module *DS SPREAD QPSK TRANSMITTER* has two outputs. One is used for generating the baseband DS spread QPSK signal, the other is baseband QPSK for measuring the performance of of the system. The output of the *DS SPREAD QPSK TRANSMITTER* plus the output of the interfering transmitter go to the bandpass limiter.

- Intersymbol interference is considered in this simulation. But this output of *DS SPREAD QPSK TRANSMITTER* is transmitted through the baseband limiter whose bandwidth is limited to 1.5 times the first null point in the power spectral density to minimize intersymbol interference. This value was empirically determined to be close to optimum. The baseband limiter consists of a Chebyshev lowpass filter, hard limiter, and Butterworth lowpass filter. In the hard limiter, the output signal equals 1 times the algebraic sign of the input signal.

- *DE_ROTATOR/DE_SPREADER* module is used to despread and for phase and delay synchronization.

- *SEMI-ANALYTIC MPSK_ESTIMATOR* is used for estimating the symbol error rate in conjunction with the module *NOISE BW IMPULSE INJECT*. These
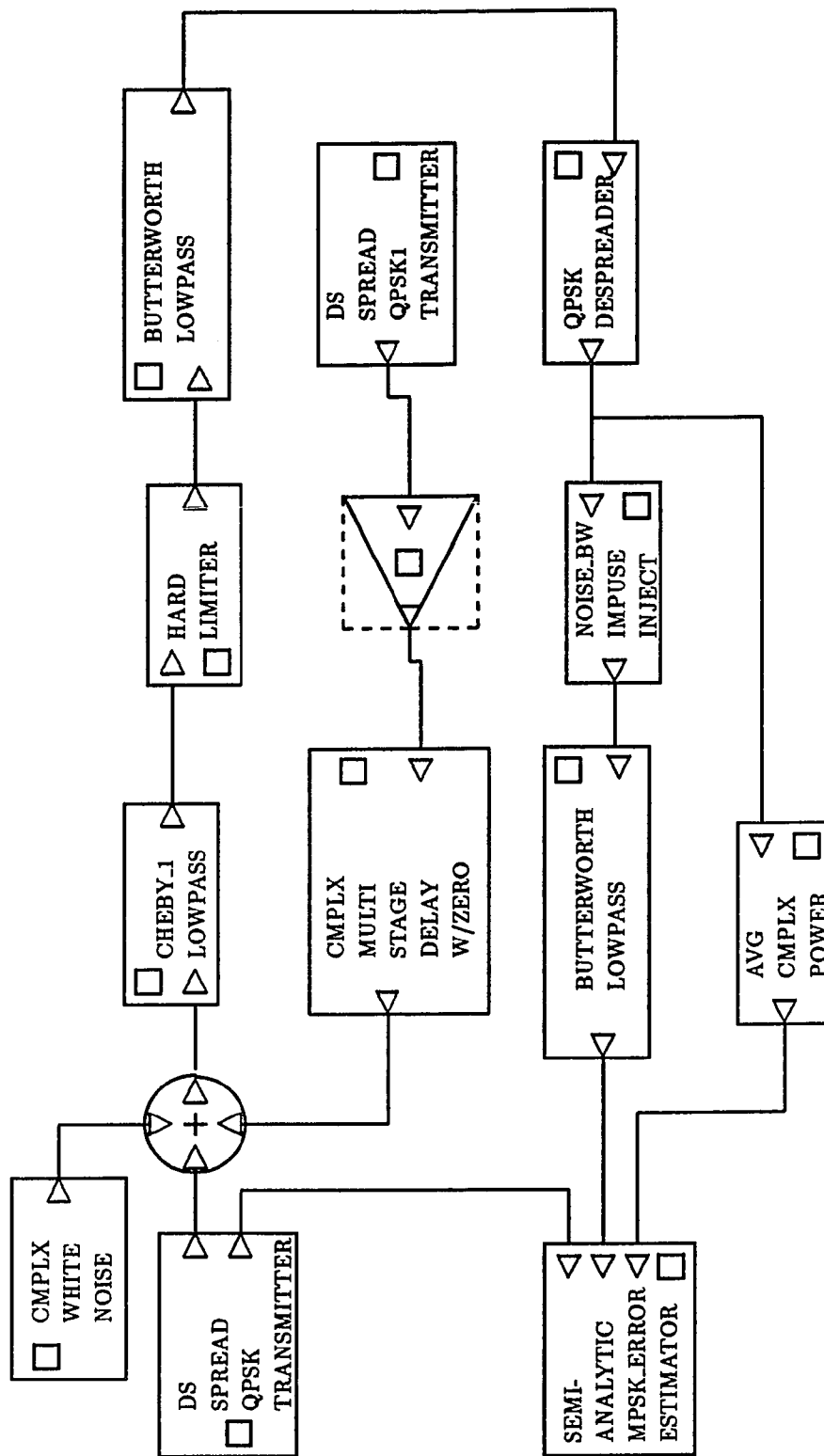
Figure 2.4: Simulation block diagram (uplink interference)

modules automatically measure the equivalent noise bandwidth of the modules downstream from the point where Gaussian noise enters the system.

- *NOISE BW IMPULSE INJECT* module must be placed at the point where noise is supposed to enter the system. However, due to limitations of the semi-analytical methods, we must not include any non-linear devices in the noise's path since the noise is assumed to be Gaussian by the estimator. In the other words, noise bandwidth is defined for the linear portion of the system. In this simulation, the downlink noise bandwidth is determined by the filter following the despreader. The *NOISE BW IMPULSE INJECT* module is placed behind of the despreader module.

2.4.2   Simulation Parameters

The choice of the simulation parameters is the most important and difficult problem encountered in carrying out the simulation because the performance is very sensitive to each parameter value. Some of the key parameters of this example are shown in Table 2.1 and discussed below.

- *Stop time* is set to greater than or equal to the following value

$$
\begin{aligned}
\text{stop time} = {} & \text{calibration start time} + 3 \times (\text{noise BW cal. duration}) + \# \text{ of} \\
& \text{samples in correlation subseq.} + 2 \times (\text{max delay to calculate}) \\
& \times \text{DT} \times \text{symbol time} + (\# \text{ of symbols used for error estimation} \\
& + 1) \times \text{symbol time}
\end{aligned}
$$

where DT is the interval, in seconds, between signal samples for the simulation run.

- DT must be small enough to avoid undue aliasing for the signals being sampled. Ideally, $1/DT$, which is the sampling rate, should be greater than twice the highest frequency present in the signals to be sampled. In practice, many waveforms

have infinite bandwidth, so DT is made small enough that the aliased power falling into the simulation bandwidth is small compared to the noise power in the same bandwidth. In the simulation of communication systems we usually choose the sampling rate so that we have 8 to 16 samples per symbol. However, in many simulations a higher sampling rate is required. Also, it is a good idea to have an integer number of samples per symbol. In the PN DS simulation, the highest frequency is given by the PN code rate. So if the sampling rate is chosen to be 8 samples per symbol, the choice of sampling frequency is

$$\text{sampling frequency} = 8 \times \text{PN code rate} = 8 \times \text{PN code rate} = 8 \times 63$$
$$= 504$$

In this example, the PN code rate was chosen to give 63 chips per symbol. When using the estimator, BOSS computes a symbol error rate plot for every sample in the symbol, corresponding to where in the symbol you sample the incoming signal. If we increase the number of samples per despread symbol to 16, we can get better resolution for that plot.

- *# of samples in correlation subsequence* specifies the number of samples used to perform the correlation in order to estimate the transmitter to receiver delay. The correlation window must be around 10 symbol intervals. A more distorted signal obviously requires more samples for an accurate delay and phase estimation. The value of this parameter can be obtained from the following expression.

  # of samples in correlation subsequence = several symbols/DT

- *Max. delay to calculate* has to be set to a number somewhat larger than the expected delay.

  Max. delay to calculate = block_delay samples + (max_delay symbols)/DT

- *# of samples in correlation subsequence* specifies the number of samples used to perform the correlation in order to estimate the transmitter to receiver delay. The correlation window must be around 10 symbol intervals. A more distorted received signal obviously requires more samples for an accurate delay and phase estimation. If this number is too small, the phase and delay meter module (inside the estimator) computed a delay of zero samples. This computed delay goes as input to the Rotate to First Sector module (inside the estimator), and inside it, to a Variable Delay module (inside the Rotate to First Sector module). The delay input to the Variable Delay module must be greater than zero. If this input is zero, an error occurs and the simulation crashes.

- *Samples of channel delay and derotation phase* can be easily given by evaluating the crosscorrelation magnitude. The correlation requires two input signals. The first input signal specified should be the delayed signal. Hence, in this simulation, the output of the DS spread QPSK transmitter and the input of the QPSK despread can be considered as the first input signal and the delayed signal respectively. The correlation is done in the frequency domain, so even for a long pn sequence, the correlation can be calculated in a small amount of time. The maximum value of the magnitude spectrum can be written as

  Max. value of the magnitude spectrum = DT × length of reference signal

- *Calibration start time* is the absolute time in seconds to begin calibration. Before this time, the estimator module is completely disabled, except for the task of keeping track of the sample number within the current symbol. Calibration start time is in the module noise_bw impulse inject and the module semi-analytic estimator. At the calibration start time, the module noise_bw impulse inject opens the connection between its input and output, and connects the output to a constant zero value. However, if block processing modules such as the fre-

quency domain filter are used, this parameter would be set to start calibration after the blocking delay time of the filter has elapsed.

- _Noise bw calculation duration_ is set to a value larger than the expected time for the impulse response of the modules downstream of the thermal noise to die out. For our simulation, this parameter is set to 10 × symbol time.

- _Semi-analytic error estimator_ provides a multidimensional plot in the BOSS post processor allowing up to four independent axes for the symbol error probability which is a dependent axis. The first three independent axes are allocated internally to the normalized symbol time, $SNR^2 (E_b^3 / N_0)$, and static phase offset(degree), respectively. The resolution of the normalized symbol time axis depends on the number of samples per symbol used in the simulation. The range and resolution of the other independent axes corresponding to SNR and phase offset can be specified at the simulation run time by setting the minimum, increment, and iteration number of these independent variables to the desired value. In this simulation, normalized symbol time and carrier phase offset are chosen 0.5 and 0 respectively, and fourth independent axis is not used.

- _# of symbols for error est._ is the number of symbols which are used in the actual calculation of the probability of error. It is suggested to use at least 60 symbols.

- _# of samples/symbol_ is the number of samples in a symbol interval. Since sampling frequency is 504 and the bit rate is 2.0, we have 252 samples per bit before despreading. Since the QPSK signal is generated using two BPSK signals, the QPSK symbol rate is the same as BPSK symbol rate in this case.

---

[2]SNR stands for signal-to-noise ratio
[3]E_b stands for energy per bit

- *# of possible TX waveform* is the number of waveforms in the MPSK signal space. For instance, since QPSK is used in this simulation, this value equals 4.

- *In-phase and Quad register initial* is an integer which is used to initialize the simple shift register generator(SSRG). This integer is converted to binary, and put in the registers with the least significance bit of the integer going into the right most register in the SSRG (the side of the output).

STOP-TIME	72
DT	0.001984
SAMPLES OF DELAY	1
FILTER ORDER	5
TX2 BW	94.5
TX1 BW	94.5
I INPHASE REGISTER INITIAL	45
I QUAD REGISTER INITIAL	45
CENTER FREQUENCY	1.0
GAIN CONSTANT	(0.125 , 0.125)
UPLINK NOISE POWER	0.001
MPSK ERROR PROB AXIS LABEL	Symbol Error Probability
# OF PHASE OFFSETS	3
PHASE OFFSET INCREMENT	2
PHASE OFFSET MINIMUM	-2
# OF SNRS	9
SNR INCREMENT	2
SNR MINIMUM	0
# OF POSSIBLE TX WAVEFORMS	4
# OF SAMPLES/SYMBOL	252
# OF SYMBOLS FOR ERROR EST	60
MAX DELAY TO CALCULATE	630
# OF SAMPLES IN CORR SUBSEQ	3780
NOISE BW CALC DURATION	10.0
DE-ROTATION PHASE (DEG)	0.010242100805044175
SAMPLES OF CHANNEL DELAY	5
PN SEQUENCE RATE	63
BIT RATE	2
SHIFT REGISTER ORDER	6
IN-PHASE REGISTER INITIALIZATION	63
QUADRATURE REGISTER INITIALIZATION	63
POST RX BW	1.1
POST RX FILTER ORDER	3

Table 2.1: Simulation parameter

2.4.3 Simulation Results

Fig. 2.5 displays the performance result for the processing gain [4] of 18 dB, when the signal to interference ratio varies from 0 dB to 30 dB. Here, we can notice that the performance does not increase when the signal to interference power ratio is greater than 20 dB.

Figs. C.7 to C.16 represent the performance results for the given signal to interference ratio, when the processing gain varies from 0 dB to 23 dB. From these figures, we observe the effect on performance of the processing gain. Generally, performance increases when processing gain increases. However, when the processing gain takes on values greater than 23 dB, the symbol error rate is degraded sharply. The reason for this is the unbalance of the spreading codes used for these large values of processing gain. Figs. C.12 and C.13 give some explanations for this. Here, we can realize that an unbalanced code has larger crosscorrelation value peaks than a balanced code. The normalized maximum value of the magnitude spectrum is 2 because QPSK signal is expressed by the complex form, sum of the in-phase and the quadrature components. These crosscorrelation values amplified through the ideal hard limiter, severely degrade output performance. Consequently, when the total period of the spreading PN sequence is much greater than the duration of a message symbol, then in some message symbols, the number of positive chips and the number of negative chips may not be close. This results in a bias during that particular message, bit which may damage the performance of the receiver.

Figs. C.14 and C.15 show the curves of the optimum processing gain for the given probability of symbol error and signal to interference ratio.

These figures are the results of several simulations for a single interferer placed on the uplink, passing through the common baseband limiter along with the infor-

---

[4]In comparing results from different investigation, it is important to observe how processing gain has been defined. In this simulation processing gain has been defined as $10 \log_{10} \left(2 \frac{T_{symbol}}{T_{chip}} \right)$

mation signal. In each run, the interferer frequency is set to a slightly different value from the desired frequency.

Fig. C.17 gives the comparison between the performance with and without a transmitter filter. Finally, a comparison between theoretical and simulation results is given in Figs C.18 to C.23. Here, the simulation plot corresponds to the results for the processing gain equal to 0 dB, which is called the worst case. From these figures we can see indirectly the degradation effects of the intermodulation product terms on the receiver performance.

Since this simulation is performed in baseband , the effect of higher order harmonics is not generated. However, since the harmonics would be almost completely removed by filters in the repeater and the receiver, their absence should cause a negligible error.
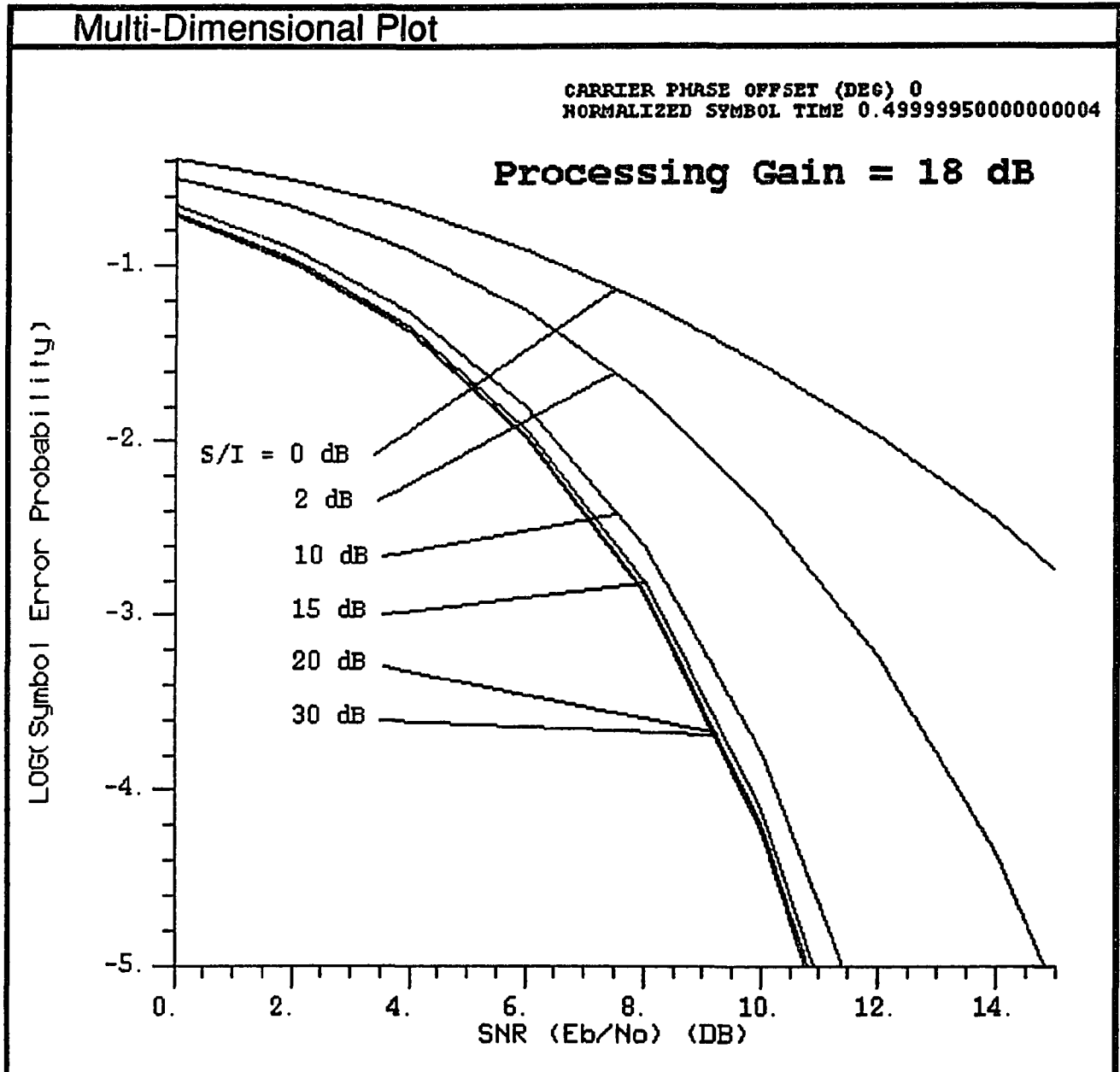
Figure 2.5: Symbol error rate as a parameter of S/I(processing gain = 18 dB)

2.5 Conclusion

The mathematical formulation for evaluating the performance of a DS spread spectrum QPSK system, subject to cochannel interference, through a channel that includes an ideal band pass limiter has been derived using an inverse Fourier transform approach in the time domain. Expressions for the desired signal, interferer, and intermodulation product terms have been obtained for the case where the input to the band pass limiter consists of DS spread spectrum QPSK signal, DS spread spectrum QPSK signal interferer, and noise. Because of the complexity of intermodulation product terms, numerical results were calculated with only interferer terms.

These theoretical results were compared with results of a computer simulation using BOSS. The simulation results vary a little depending on the simulation parameters, for instance, spreading code, interfering code, etc.. The theoretical and simulation results were in close agreement over most of the range of the parameters.

CHAPTER 3

DOWNLINK INTERFERENCE IN QPSK SPREAD SPECTRUM SYSTEMS

In this chapter, attention is focused on the downlink channel where the interference is located. Here the nonlinear characteristic of the bandpass limiter does not effect the interfering signal although it does produce distortion products on the desired signal. It is possible to evaluate the performance for this case and compare it with uplink interference.

Since the interference and the information signal are not mixed in the nonlinear channel produced by the bandpass limiter, the final results are relatively simple compared to the uplink case and it is seen that the performance for the worst case is worse than with uplink interference.

3.1 Mathematical Analysis

3.1.1 Communication Model

A simplified functional block diagram of the system analyzed is shown in Fig. 3.1. This model is the same as Fig. 2.1 except that the location of interferer moves from uplink to downlink[1].

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[1]refer to Ch. 2 for the function of each block in the diagram
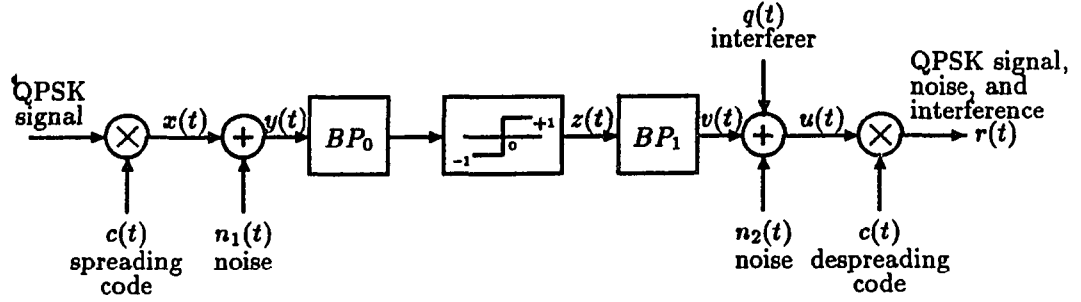
Figure 3.1: Functional Block Diagram(Downlink Interference).

The DS spread spectrum QPSK signal, $x(t)$, and interfering signal, $q(t)$, can be represented over one symbol period as

$$x(t) \;=\; Ac(t)\cos\left\{\omega_0 t + a_i\frac{\pi}{4} + \theta_x\right\} \tag{3.1}$$

$$d(t) \;=\; \beta s(t)\cos\left\{\omega_2 t + d_i\frac{\pi}{4} + \theta_d\right\} \tag{3.2}$$

where (A, β), $(c(t), s(t))$, (ω_0, ω_2), and (θ_x, θ_d) are the amplitudes, PN spreading codes, carrier frequencies, and initial random phases of the desired signal and interferer respectively. a_i and d_i are random variables which take on the values 1, 2, 3, 4 with equal probability in each symbol interval.

3.1.2 Hard Limiter Output

Since the interferer affects the signal only in the downlink, the input of the bandpass limiter, $y(t)$, is just the information signal mixed with Gaussian noise.

$$
\begin{aligned}
y(t) \;&=\; x(t) + n_1(t) \\
&=\; Ac(t)\cos\left(a_i\frac{\pi}{4} + \theta_x\right)\cos\omega_0 t - Ac(t)\sin\left(a_i\frac{\pi}{4} + \theta_x\right)\sin\omega_0 t \\
&\quad + n_1(t) \tag{3.3}
\end{aligned}
$$

The output $z(t)$ of a memoryless hard limiter is related to the input by the transfer characteristic $g(y)$. $g(y)$, being a signum function, has a Fourier transform $G(j\omega) =$

$\frac{2}{j\omega}$, Then by using the inverse Fourier transform of the transfer function, we have

$$
\begin{aligned}
z(t) &= g(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(j\omega) e^{j\omega y} \, d\omega \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{j\omega} e^{j\omega y} \, d\omega \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{j\omega} \exp\left[j\omega \left\{ Ac(t) \cos\left(a_i \frac{\pi}{4} + \theta_x \right) \cos\omega_0 t \right. \right. \\
&\quad \left. \left. - Ac(t) \sin\left(a_i \frac{\pi}{4} + \theta_x \right) \sin\omega_0 t + n_1(t) \right\} \right] \, d\omega
\end{aligned}
\tag{3.4}
$$

To simplify the calculation, we take $a_i = 1$ and $\theta_x = 0$; due to symmetry and averaging this does not effect the symbol error rate calculation.

$$
z(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{j\omega} e^{\frac{j\omega Ac(t)}{\sqrt{2}} \cos\omega_0 t} e^{-\frac{j\omega Ac(t)}{\sqrt{2}} \sin\omega_0 t} e^{j\omega n_1(t)} \, d\omega
\tag{3.5}
$$

If we substitute Eq.s 2.10, and 2.11 into Eq. 3.5, we have

$$
\begin{aligned}
z(t) &= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty} j^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos k\omega_0 t \right\} \\
&\quad \cdot \left\{ J_0\left(\frac{\omega A}{\sqrt{2}}\right) + 2\sum_{k=1}^{\infty} (-j)^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(k\omega_0 t - \frac{\pi}{2} k \right) \right\} \, d\omega
\end{aligned}
$$

After expanding we have

$$
\begin{aligned}
z(t) &= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left\{ J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) + 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^{\infty} (-j)^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \right. \\
&\quad \cdot \cos\left(k\omega_0 t - \frac{\pi}{2} k \right) + 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^{\infty} j^k c^k(t) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos k\omega_0 t \\
&\quad \left. + 4\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^k j^{i+k} c^{i+k}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \cos i\omega_0 t \cos\left(k\omega_0 t - \frac{\pi}{2} k \right) \right\} \, d\omega
\end{aligned}
\tag{3.6}
$$

3.1.3   Input of the Spread Spectrum Receiver

All harmonics caused by the nonlinearity are removed in the bandpass filter BP_1. The input of receiver, $u(t)$, is the superposition of the output of the bandpass limiter, downlink noise, and downlink interference. Hence, the receiver input can be shown that

$$
\begin{aligned}
u(t) &= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left[J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) + 2J_0 \left(\frac{\omega A}{\sqrt{2}}\right) (-j)c(t)J_1 \left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(\omega_0 t - \frac{\pi}{2}\right) \right. \\
&\quad + 2J_0 \left(\frac{\omega A}{\sqrt{2}}\right) (j)c(t)J_1 \left(\frac{\omega A}{\sqrt{2}}\right) \cos \omega_0 t \\
&\quad + 2\sum_{i=1}^{\infty} (-1)^{1+i}(j)^{1+2i} c^{1+2i}(t) J_i \left(\frac{\omega A}{\sqrt{2}}\right) J_{1+i} \left(\frac{\omega A}{\sqrt{2}}\right) \cos\left\{\omega_0 t - \frac{\pi}{2}(1+i)\right\} \\
&\quad \left. + 2\sum_{i=1}^{\infty} (-1)^{k}(j)^{1+2k} c^{1+2k}(t) J_{1+k} \left(\frac{\omega A}{\sqrt{2}}\right) J_k \left(\frac{\omega A}{\sqrt{2}}\right) \cos\left\{\omega_0 t + \frac{\pi}{2}k\right\} \right] d\omega + n_2(t) \\
&\quad + \beta s(t) \cos \omega_2 t \, \cos\left(d_i \frac{\pi}{4} + \theta_d\right) - \beta s(t) \sin \omega_2 t \, \sin\left(d_i \frac{\pi}{4} + \theta_d\right)
\end{aligned}
\tag{3.7}
$$

As an assumption for the simplicity of calculation, if we take $d_i = 1$ and $\theta_d = 0$

$$
\begin{aligned}
u(t) &= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left[J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) - 2jJ_0 \left(\frac{\omega A}{\sqrt{2}}\right) c(t)J_1 \left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(\omega_0 t - \frac{\pi}{2}\right) \right. \\
&\quad + 2jJ_0 \left(\frac{\omega A}{\sqrt{2}}\right) c(t)J_1 \left(\frac{\omega A}{\sqrt{2}}\right) \cos \omega_0 t \\
&\quad - 2j\sum_{i=1}^{\infty} c(t) J_i \left(\frac{\omega A}{\sqrt{2}}\right) J_{1+i} \left(\frac{\omega A}{\sqrt{2}}\right) \cos\left\{\omega_0 t - \frac{\pi}{2}(1+i)\right\} \\
&\quad \left. + 2\sum_{i=1}^{\infty} c(t) J_{1+k} \left(\frac{\omega A}{\sqrt{2}}\right) J_k \left(\frac{\omega A}{\sqrt{2}}\right) \cos\left\{\omega_0 t + \frac{\pi}{2}k\right\} \right] d\omega + n_2(t) \\
&\quad + \frac{\beta s(t)}{\sqrt{2}} \cos \omega_2 t - \frac{\beta s(t)}{\sqrt{2}} \sin \omega_2 t
\end{aligned}
\tag{3.8}
$$

3.1.4 Despread Output

In spread spectrum systems, we need to multiply $u(t)$ by the despreading code $c(t)$ which is synchronized to the desired information signal.

$$r(t) = u(t)c(t)$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\left[\frac{1}{j}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)c(t) - 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t - \frac{\pi}{2}\right)\right.$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t$$

$$- 2\sum_{i=1}^{\infty}J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t - \frac{\pi}{2}(i{+}1)\right\}$$

$$+ 2\sum_{i=1}^{\infty}J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t + \frac{\pi}{2}k\right)\right]d\omega + n_2(t)c(t)$$

$$+ \frac{\beta s(t)c(t)}{\sqrt{2}}\cos\omega_2 t - \frac{\beta s(t)c(t)}{\sqrt{2}}\sin\omega_2 t \tag{3.9}$$

3.1.5 Investigation of the Despread Signal

Differently from uplink interference, we do not have any intermodulaton product terms because the interferer is added after the nonlinearity device. Thus, we have an information signal term, an interfering signal term, and a noise term.

Information Signal

Unlike uplink interference, no interfering component is contained in the information signal.

$$r_m(t) = -\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t - \frac{\pi}{2}\right)d\omega$$

$$+ \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)\cos\omega_0 t\, d\omega$$

$$- \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\sum_{i=1}^{\infty}J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left\{\omega_0 t - \frac{\pi}{2}(i{+}1)\right\}d\omega$$

$$+ \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\sum_{k=1}^{\infty}J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right)\cos\left(\omega_0 t + \frac{\pi}{2}k\right)d\omega \tag{3.10}$$

Interfering Signal

After the interfering signal is despread, it becomes wide band interferer. Hence, if we take a high processing gain, we can neglect this term.

$$r_q(t) = \frac{\beta s(t)c(t)}{\sqrt{2}} \cos \omega_2 t - \frac{\beta s(t)c(t)}{\sqrt{2}} \sin \omega_2 t \qquad (3.11)$$

Noise

Noise consists of downlink noise $n_2(t)$ and uplink noise $n_1(t)$ passing the bandpass limiter.

$$\begin{aligned}
r_n(t) &= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \frac{1}{j} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) c(t) d\omega + n_2(t)c(t) \\
&= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) c(t) d\omega + n_2'(t) \qquad (3.12)
\end{aligned}$$

where $n_2'(t) = n_2(t)c(t)$

3.2 Performance Evaluation

3.2.1 Theoretical Derivation

The despread signal is supplied to a pair of correlators where it is multiplied by a locally generated pair of coherent reference signals and integrated from 0 to T. Two basis functions $\phi_1(t)$ and $\phi_2(t)$ are used for the demodulation of the received signals

$$\phi_1(t) \;=\; \frac{1}{T}\cos\omega_0 t \quad 0 \le t \le T \tag{3.13}$$

$$\phi_2(t) \;=\; \frac{1}{T}\sin\omega_0 t \quad 0 \le t \le T \tag{3.14}$$

where T is the symbol duration.

Demodulated Information Signal

The receiver output is given by

$$
\begin{aligned}
m_1 \;=\;& \int_0^T r_{mx}(t)\,\phi_1(t)\,dt \\
=\;& \frac{1}{T}\int_0^T \Bigg[-\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0\!\left(\frac{\omega A}{\sqrt{2}}\right) J_1\!\left(\frac{\omega A}{\sqrt{2}}\right) \sin\omega_0 t \,\cos\omega_0 t \; d\omega \\
& + \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0\!\left(\frac{\omega A}{\sqrt{2}}\right) J_1\!\left(\frac{\omega A}{\sqrt{2}}\right) \cos\omega_0 t \,\cos\omega_0 t \, d\omega \\
& - \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \sum_{i=1}^{\infty} J_i\!\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+1}\!\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left\{\omega_0 t - \frac{\pi}{2}(i+1)\right\} \cos\omega_0 t \, d\omega \\
& + \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \sum_{k=1}^{\infty} J_k\!\left(\frac{\omega A}{\sqrt{2}}\right) J_{k+1}\!\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(\omega_0 t + \frac{\pi}{2}k\right) \cos\omega_0 t \, d\omega \Bigg]\, dt
\end{aligned}
$$

After some manipulations by using the trigonometric identities, we obtain the following.

$$m_1 = -\frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t) \sin \omega_0 t \cos \omega_0 t \, dt \, d\omega$$

$$+ \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t) \cos^2 \omega_0 t \, dt \, d\omega$$

$$- \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} \sum_{i=1}^\infty J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t) \cos\left\{\omega_0 t - \frac{\pi}{2}(i+1)\right\}$$

$$\cdot \cos \omega_0 t \, dt \, d\omega$$

$$+ \frac{4}{\pi} \int_0^\infty \frac{1}{\omega} \sum_{k=1}^\infty J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \frac{1}{T} \int_0^T \cos \omega n_1(t) \cos\left(\omega_0 t + \frac{\pi}{2}\right)$$

$$\cdot \cos \omega_0 t \, dt \, d\omega \tag{3.15}$$

If we neglect the effect of uplink Gaussian noise $n_1(t)$ for the same reason as in the previous chapter, the final result will be

$$m_1 \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$\triangleq \sqrt{\frac{E_1}{2}} \tag{3.16}$$

$$\text{where} \quad E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2 \tag{3.17}$$

Similarly, the m_2 can be represented as

$$m_2 = \int_0^T r_m(t) \phi_2(t) \, dt$$

$$= \frac{1}{T} \int_0^T \left[-\frac{2}{\pi} \int_{-\infty}^\infty \frac{1}{\omega} e^{j\omega n_1(t)} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) \sin \omega_0 t \sin \omega_0 t \, d\omega \right.$$

$$+ \frac{2}{\pi} \int_{-\infty}^\infty \frac{1}{\omega} e^{j\omega n_1(t)} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) \cos \omega_0 t \sin \omega_0 t \, d\omega$$

$$- \frac{2}{\pi} \int_{-\infty}^\infty \frac{1}{\omega} e^{j\omega n_1(t)} \sum_{i=1}^\infty J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left\{\omega_0 t - \frac{\pi}{2}(i+1)\right\} \sin \omega_0 t \, d\omega$$

$$+ \frac{2}{\pi} \int_{-\infty}^\infty \frac{1}{\omega} e^{j\omega n_1(t)} \sum_{k=1}^\infty J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k+1}\left(\frac{\omega A}{\sqrt{2}}\right) \cos\left(\omega_0 t + \frac{\pi}{2} k\right) \sin \omega_0 t \, d\omega \right] dt$$

After some calculation, we have

$$m_2 \simeq -\frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$-\frac{2}{\pi} \sum_{i=1}^\infty (-1)^i \int_0^\infty \frac{1}{\omega} J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$-\frac{2}{\pi} \sum_{k=1}^\infty (-1)^{k+1} \int_0^\infty \frac{1}{\omega} J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$\stackrel{\triangle}{=} -\sqrt{\frac{E_2}{2}} \tag{3.18}$$

where

$$E_2 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right.$$

$$+\frac{2\sqrt{2}}{\pi} \sum_{i=1}^\infty (-1)^i \int_0^\infty \frac{1}{\omega} \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$+\frac{2\sqrt{2}}{\pi} \sum_{k=1}^\infty (-1)^{k+1} \int_0^\infty \frac{1}{\omega} \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2 \tag{3.19}$$

Demodulated Interferer

Demodulated interferer can be analyzed for two different cases, average case and worst case. For the average case,

$$q_1 = \int_0^T r_q(t)\, \phi_1(t) = \frac{1}{T} \int_0^T r_q(t) \cos\omega_0 t\, dt$$

$$= \frac{1}{T} \int_0^T [\beta s(t) \cos\omega_2 t - \beta s(t) \sin\omega_2 t]$$

$$\cdot \cos\omega_0 t\, dt \tag{3.20}$$

q_1 can be written again by using trigonometric identities.

$$q_1 = \frac{\beta}{2\sqrt{2}T} \left[\int_0^T s(t)c(t)\left\{\cos(2\omega_0 + \Delta\omega)t + \cos\Delta\omega t\right\} dt\right.$$

$$\left. - \int_0^T p(t)c(t)\left\{\sin(2\omega_0 + \Delta\omega)t + \sin\Delta\omega t\right\} dt\right] \tag{3.21}$$

Here we can neglect the component of twice the carrier frequency.

$$q_1 \simeq \frac{\beta}{2\sqrt{2}T} \left[\int_0^T p(t)c(t) \cos\Delta\omega t\, dt - \int_0^T p(t)c(t) \sin\Delta\omega t\, dt\right] \tag{3.22}$$

If the processing gain is large, the value of q_1 will be close to 0. Similarly, the equation for q_2 is

$$q_2 = \int_0^T r_q(t)\,\phi_2(t)\,dt = \frac{1}{T}\int_0^T r_q(t)\,\cos\omega_0 t\,dt \simeq 0 \tag{3.23}$$

For the worst case, 0 dB processing gain, Eq. 3.22 may be written again as following

$$
\begin{aligned}
q_1 &\simeq \frac{\beta}{2\sqrt{2}T}\int_0^T \cos\Delta\omega t\,dt - \frac{\beta}{2\sqrt{2}T}\int_0^T \sin\Delta\omega t\,dt \\
&= \frac{\beta}{2\sqrt{2}T}\left[\frac{\sin\Delta\omega T}{\Delta\omega} + \frac{\cos\Delta\omega T - 1}{\Delta\omega}\right]
\end{aligned}
\tag{3.24}
$$

For the case of $\Delta\omega T \ll 1$, we have

$$q_1 \simeq \frac{\beta}{2\sqrt{2}} \tag{3.25}$$

similarly, equation q_2 is also

$$q_2 \simeq -\frac{\beta}{2\sqrt{2}} \tag{3.26}$$

Demodulated Noise Signal

We can calculate the demodulated noise signal in a manner similar to that of Chapter 2.

$$
\begin{aligned}
r_{n_i} &= \int_0^T r_n(t)\,\phi_i(t)\,dt \quad \text{where } i = 1,2 \\
&= \frac{1}{j\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)\int_0^T e^{j\omega n_1(t)}c(t)\phi_i(t)\,dt\,d\omega + \int_0^T n_2'\,\phi_i(t)\,dt
\end{aligned}
\tag{3.27}
$$

If $n_1(t) \approx 0$, we have

$$r_{n_i} \approx \int_0^T n_2'(t)\phi_i(t)dt \tag{3.28}$$

Its expected value and variance can be given by

$$E\left[r_{n_i}\right] = E\left[\int_0^T n_2'\,\phi_i(t)\,dt\right] = 0 \tag{3.29}$$

$$\sigma_{r_{n_i}}^2 = \frac{N_0}{4T} \tag{3.30}$$

Calculation of Average Probability of Symbol Error

The probability of symbol error will be calculated for two cases, the average case and the worst case. For the average case, the probability of symbol error is simply given by

$$
\begin{aligned}
P(e) &= 1 - P_c \\
&= 1 - \left[\int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x_1 - \sqrt{\frac{E_1}{2}}\right)^2}{N_0}} dx_1 \int_0^{-\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x_2 + \sqrt{\frac{E_2}{2}}\right)^2}{N_0}} dx_2 \right] \\
&= Q\left(\sqrt{\frac{E_1}{N_0}}\right) + Q\left(\sqrt{\frac{E_2}{N_0}}\right) - Q\left(\sqrt{\frac{E_1}{N_0}}\right) Q\left(\sqrt{\frac{E_2}{N_0}}\right)
\end{aligned}
\tag{3.31}
$$

where $Q(x)$ denotes Q function[2].

$$
E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right]^2
\tag{3.32}
$$

$$
\begin{aligned}
E_2 = &\left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right. \\
&+ \frac{2\sqrt{2}}{\pi} \sum_{i=1}^\infty (-1)^i \int_0^\infty \frac{1}{\omega} \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \\
&\left. + \frac{2\sqrt{2}}{\pi} \sum_{k=1}^\infty (-1)^{k+1} \int_0^\infty \frac{1}{\omega} \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right]^2
\end{aligned}
\tag{3.33}
$$

For the worst case, we can calculate the probability of error, $P_1(e)$, when the value of $c(t)p(t)$ is $+1$.

$$
\begin{aligned}
P_1(c) &= \int_{q_1}^\infty \frac{\sqrt{2T}}{\sqrt{\pi N_0}} e^{-\frac{2T\left(x_1 - \sqrt{\frac{E_1}{2}}\right)^2}{N_0}} dx_1 \int_{-\infty}^{q_2} \frac{\sqrt{2T}}{\sqrt{\pi N_0}} e^{-\frac{\left(x_2 + \sqrt{\frac{E_2}{2}}\right)^2}{N_0}} dx_2 \\
&= 1 - Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) - Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right) \\
&\quad + Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right)
\end{aligned}
\tag{3.34}
$$

where q_1 and q_2 are given in Eq.s 3.25 and 3.26, respectively.

---

[2]For the definition of Q function, refer to Ch. 2

$$P_1(e) = 1 - P_1(c)$$

$$= Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$- Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right) \qquad (3.35)$$

Similarly, the probability of error, $P_2(e)$, when the value of $c(t)p(t)$ is -1 can be calculated.

$$P_2(e) = Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$- Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right) \qquad (3.36)$$

Finally the performance for the worst case will be

$$P(e) = \frac{1}{2}P_1(e) + \frac{1}{2}P_2(e)$$

$$= \frac{1}{2}\left[Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right)\right.$$

$$+ Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right) - Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$\left. - Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)\right] \qquad (3.37)$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right.$$

$$+ \frac{2\sqrt{2}}{\pi}(-1)^i \sum_{i=1}^\infty \int_0^\infty \frac{1}{\omega} \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$\left. + \frac{2\sqrt{2}}{\pi}(-1)^{k+1} \sum_{k=1}^\infty \int_0^\infty \frac{1}{\omega} \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

$$q_1 \simeq \frac{\beta}{2\sqrt{2}}$$

$$q_2 \simeq -\frac{\beta}{2\sqrt{2}}$$

3.2.2 Numerical Results

The expressions for the probability of error derived in the previous section have been used to evaluate the performance of the satellite channel. The situation examined in this chapter is where cochannel interference is on the down link. In this case, we have simpler results compared to the uplink interference case since we do not have the mixed terms between the signal and the interferer.

For the average case, the expression for the performance is not given by the function of the interferer term. Hence, if we neglect the downlink interferer term by taking a high processing gain, performance for the average case is always same regardless of the power of interference.

Several numerical results for the performance of DS spread spectrum QPSK in a hard limited channel with downlink cochannel interference are given in following Figs. 3.2 to C.28. These results can be compared to the case where the interference is on the uplink cochannel interference.

In the worst case, the performance with downlink interference is much worse than with the uplink interference when the interference power is close to the signal power. However, when the signal power is much greater than the interference power, the performance with interference on the uplink and with interference on the downlink are very similar. Symbol error rate as a parameter of S/I for the worst case is given in Fig. C.29.
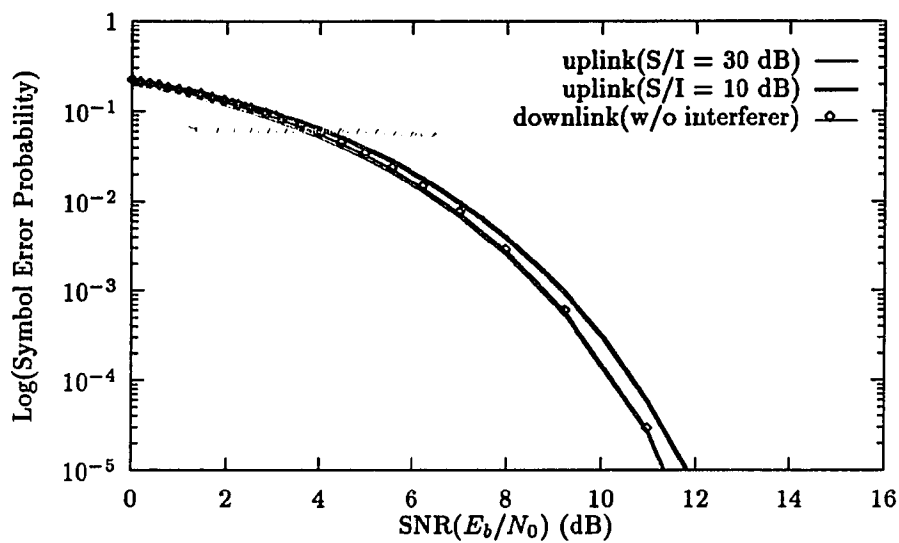
Figure 3.2: Symbol error rate comparison between the uplink interference and the downlink interference for the average case (downlink)
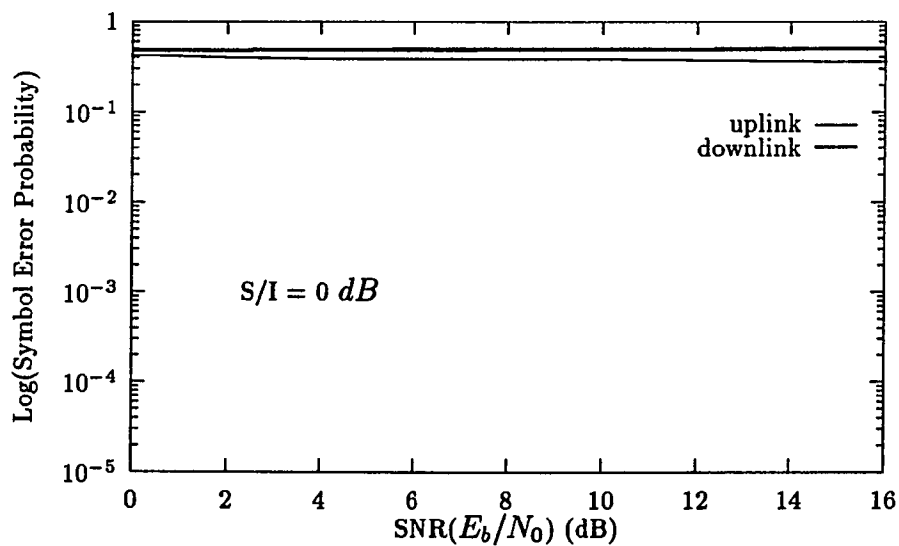

Figure 3.3: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (S/I = 0 dB)

3.3   Simulation for Evaluation of Performance

3.3.1   Simulation Model

The simulation model is very close to the case of uplink interference except for the location of the interferer. The block diagram used in the simulation is shown in Fig. 3.4.

3.3.2   Simulation Results

All simulation results given in this section are the results of several simulations for a single interferer placed on the downlink from the baseband limiter. In Fig. C.30, we depict the average probability of symbol error as a function of SNR for different values of signal to interference ratio in the range from 0 dB to 30 dB. Unlike uplink interference, performance increases until signal to interference power ratio approaches 30 dB. This means that the effect of downlink interference is greater than that of uplink interference.

In Figs. C.31 to C.36 we present the average probability of symbol error as a parameter of signal to interference ratio with different processing gains, 0 dB, 3 dB, 8 dB, 13 dB, 18 dB, and 23 dB, respectively. From these figures, we observe the effect on the performance of the processing gain.

Generally, performance increases when processing gain increases. However, when the processing gain becomes more than 23 dB, symbol error rate is degraded sharply because of the unbalance of code[3]. Figs. C.37 and C.38 show the curves of the optimum processing gain for the given probability of symbol error and signal to interference ratio.

Finally, we compare the analytical results obtained in Eq.s 3.31 and 3.37 with simulation results. We present the average probability of symbol error as a function of SNR with different SIR, 0 dB, 10 dB, 15 dB, 20 dB, and 30 dB for a fixed processing
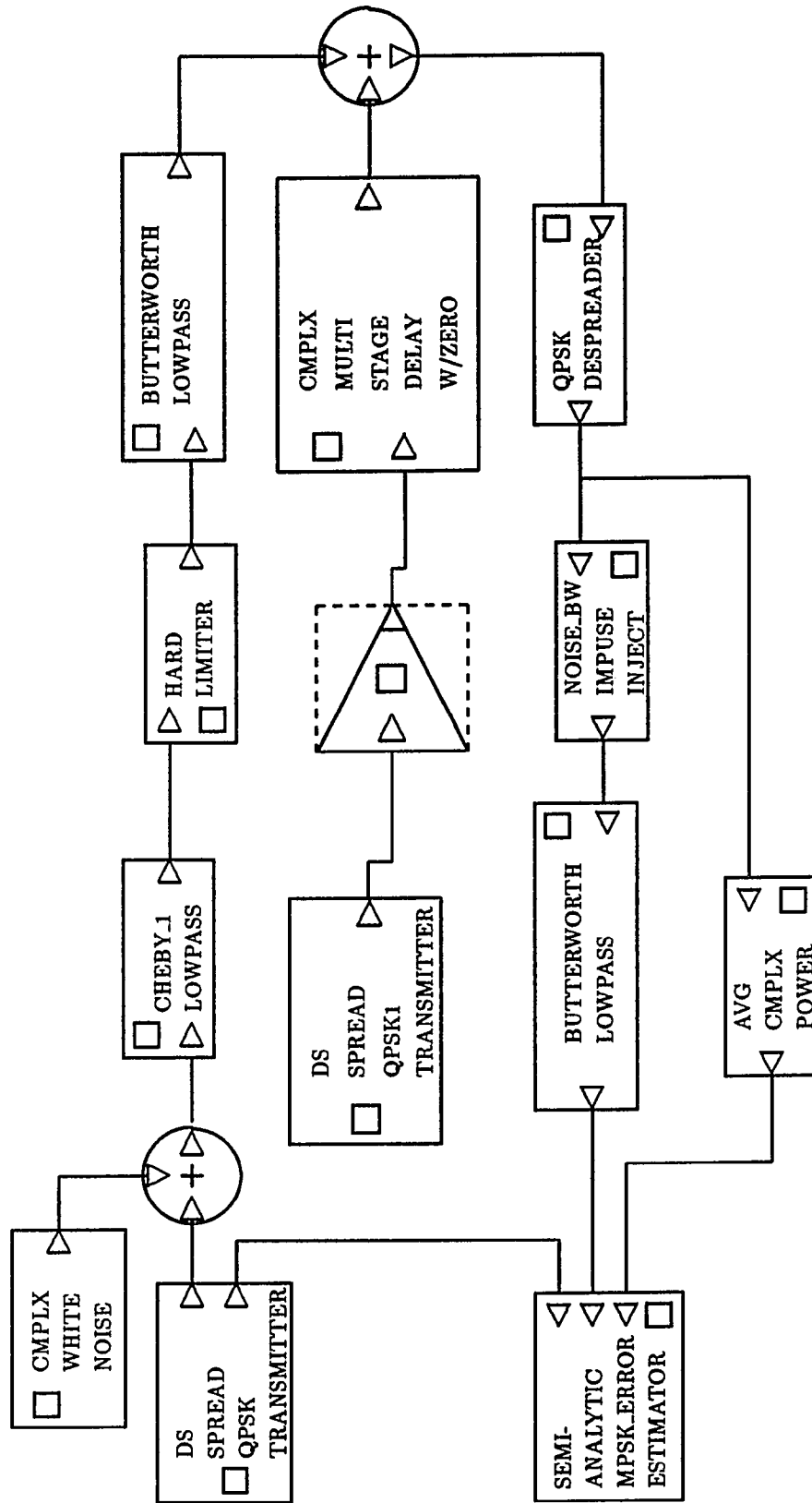
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[3]For the detail, see page 47

Figure 3.4: Simulation block diagram (downlink interference)

gain of 0 *dB* in Figs. C.39, C.40, C.41, C.42, and C.43, respectively.

3.4 Conclusion

The mathematical formulation for evaluating the performance of a DS spread spectrum QPSK signal with single downlink interferer transmitting through the satellite channel has been derived using the inverse Fourier transform method in the time domain. Expressions for the desired signal, interferer, and noise terms have been obtained for the case when the input consists of DS spread spectrum QPSK signal and noise.

These theoretical results were compared with results of a computer simulation and with the case of an uplink interferer. The results of the simulation show good agreement with the theoretical calculations within 1 or 2 dB in SNR. It is also shown that the performance with downlink interference is worse than that with uplink interference.

CHAPTER 4

UP AND DOWNLINK INTERFERENCE IN QPSK SPREAD SPECTRUM SYSTEMS

In this chapter, we propose a system with an interferer in both the up and down links of the satellite channel. The mathematical analysis is based on the results of the uplink interference calculations. Expressions for the probability of error are obtained for the correlation receiver and compared to the results of the previous two chapters.

4.1   Mathematical Analysis

4.1.1   Communication Model

A simplified functional block diagram of the system analyzed is shown in Fig. 4.1. This model is the same as Fig. 2.1 except that an interferer is added to the downlink. If interference is in the up and down link, we need to add the result of the uplink interference to the result of the downlink interference to get the performance of this satellite channel. Hence, the information signal term, noise, and intermodulation product term are the same as derived in Ch. 2. The only difference is in the interference term. The interferer on the downlink can be expressed as

$$d(t) = \beta s(t) \cos\left\{\omega_2 t + d_i \frac{\pi}{4} + \theta_d\right\}$$

$$0 \leq t \leq T$$

where β is the amplitude, $s(t)$ is the spreading code, ω_2 is carrier frequency, and θ_d is the initial random phases. We are only considering the case of $d_i = 1$ and $\theta_d = 0$ for the simplicity of calculation, it will not effect the final result.

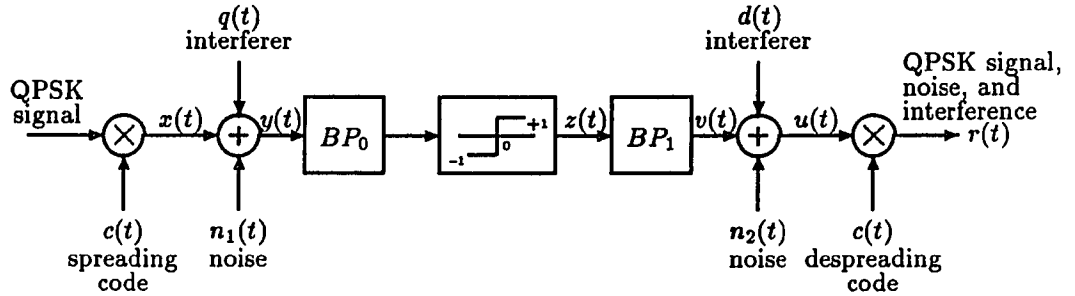$$d(t) = \frac{\beta}{\sqrt{2}} s(t) \cos\omega_2 t - \frac{\beta}{\sqrt{2}} s(t) \sin\omega_2 t \tag{4.1}$$

Figure 4.1: Functional Block Diagram(Up and Down link Interference).

4.1.2 Hard Limiter Output

The output of hard limiter is the same as Eq. 2.15 of the Ch. 2

4.1.3 Input of the Spread Spectrum Receiver

In the downlink channel, the interfering signal and white Gaussian noise are added to the output the of bandpass limiter. Hence the signal $u(t)$, which is a corrupted version of $v(t)$, is the same as Eq.s 2.21, 2.22, and 2.23 in Ch. 2. except for the added

interfering signal. Hence, the input of the receiver, $u(t)$, is given by

$$u(t) = Eq.\ 2.21 + Eq.\ 2.22 + Eq.\ 2.23 + Appendix\ A.1$$

$$+ \frac{\beta}{\sqrt{2}} s(t) \cos \omega_2 t - \frac{\beta}{\sqrt{2}} s(t) \sin \omega_2 t \qquad (4.2)$$

4.1.4 Despread Output

The superposition of the all input signals are despread by multiplying by the despreading code $c(t)$ in the spread spectrum receiver.

$$r(t) = Eq.\ 2.25 + Eq.\ 2.26 + Eq.\ 2.12 + Appendix\ B.1$$

$$+ \frac{\beta}{\sqrt{2}} s(t)c(t) \cos \omega_2 t - \frac{\beta}{\sqrt{2}} s(t)c(t) \sin \omega_2 t \qquad (4.3)$$

4.1.5 Investigation of the Despread Signal

Since information signal terms, noise terms, and intermodulation product terms are investigated in Ch. 2, we omit them in this section. The Interfering signal alone can be expressed as

$$r_q(t) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}} \right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}} \right) p(t)c(t) \cos \omega_1 t \, d\omega$$

$$- \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) J_o \left(\frac{\omega \alpha}{\sqrt{2}} \right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}} \right) c(t)p(t) \cos \left(\omega_1 t - \frac{\pi}{2} \right) \, d\omega$$

$$+ \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) \sum_{i=1}^{\infty} c(t)p(t) J_{i+1} \left(\frac{\omega \alpha}{\sqrt{2}} \right) J_i \left(\frac{\omega \alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left(\omega_1 t + \frac{\pi}{2} i \right) \, d\omega$$

$$- \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0^2 \left(\frac{\omega A}{\sqrt{2}} \right) \sum_{k=1}^{\infty} c(t)p(t) J_k \left(\frac{\omega \alpha}{\sqrt{2}} \right) J_{k+1} \left(\frac{\omega \alpha}{\sqrt{2}} \right)$$

$$\cdot \cos \left\{ \omega_1 t - \frac{\pi}{2} (k+1) \right\} \, d\omega + \frac{\beta}{\sqrt{2}} s(t)c(t) \cos \omega_2 t - \frac{\beta}{\sqrt{2}} s(t)c(t) \sin \omega_2 t \qquad (4.4)$$

4.2   Performance Evaluation

4.2.1   Theoretical Derivation

The despread signal is multiplied by two basis functions $\phi_1(t)$ and $\phi_2(t)$. $\phi_1(t)$ and $\phi_2(t)$ are given by the same expressions as in the previous chapter.

$$\phi_1(t) \;=\; \frac{1}{T}\cos\omega_0 t \quad 0 \le t \le T \tag{4.5}$$

$$\phi_2(t) \;=\; \frac{1}{T}\sin\omega_0 t \quad 0 \le t \le T \tag{4.6}$$

where T is the symbol duration.

Demodulated Information Signal

These terms are the same as the Eq.s 2.46 and 2.48 of the Ch. 2.

Demodulated Interferer

The demodulated interferer terms are given by the sum of uplink interference and downlink interference. Therefore, q_1 can be written as

$$
\begin{aligned}
q_1 \;=\;& \int_0^T r_q(t)\,\phi_1(t) = \frac{1}{T}\int_0^T r_q(t)\,\cos\omega_0 t\,dt \\
\;=\;& \frac{1}{T}\int_0^T \Bigg[\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right)c(t)p(t)\cos\omega_1 t\,d\omega \\
& -\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)J_o\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right)c(t)p(t)\cos\left(\omega_1 t - \frac{\pi}{2}\right)d\omega \\
& +\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}c(t)p(t)J_{i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\left(\omega_1 t + \frac{\pi}{2}i\right)d\omega \\
& -\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{k=1}^{\infty}c(t)p(t)J_k\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos\left\{\omega_1 t - \frac{\pi}{2}(k+1)\right\}d\omega\Bigg]\cos\omega_0 t\,dt \\
& +\frac{\beta}{\sqrt{T}}\left[\int_0^T c(t)s(t)\left(\cos\omega_2 t - \sin\omega_2 t\right)\cos\omega_0 t\,dt\right]
\end{aligned}
\tag{4.7}
$$

By using the results of Ch.s 2 and 3, q_1 approximately equals to 0 for the average case when the processing gain is large. It can be seen that q_1 and q_2 are close to 0

for the average case when the processing gain is large, hence they will set equal to 0. For the worst case, q_1 is given by the sum of Eq. 2.54 of Ch. 2 and Eq. 3.25 of Ch. 3. Hence q_1 can be written as

$$q_1 \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega + \frac{\beta}{2\sqrt{2}} \tag{4.8}$$

q_2 is calculated in a similar manner, as shown below

$$\begin{aligned} q_2 \simeq & -\frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) J_0 \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1 \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega \\ & - \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty (-1)^{i+1} J_{2i-1} \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{2i} \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega \\ & - \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2 \left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty (-1)^k J_{2k} \left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{2k+1} \left(\frac{\omega \alpha}{\sqrt{2}}\right) d\omega \\ & - \frac{\beta}{2\sqrt{2}} \end{aligned} \tag{4.9}$$

Demodulated Intermodulation Products

These terms are the same as the case of uplink interference. Results are given in Eq. 2.61 of Ch. 2.

Demodulated Noise Signal

This part is also the same as the uplink interference case. The details are given in Eq.s 2.56 through 2.60 of Ch. 2.

Calculation of Average Probability of Symbol Error

For the average case which corresponds to a high processing gain, the expression for the probability of symbol error has the same form as in the case of uplink interference. Hence, $P(e)$ yields

$$\begin{aligned} P(e) &= 1 - P_c \\ &= Q\left(\sqrt{\frac{2E_1 T}{N_0}}\right) + Q\left(\sqrt{\frac{2E_2 T}{N_0}}\right) - Q\left(\sqrt{\frac{2E_1 T}{N_0}}\right) Q\left(\sqrt{\frac{2E_2 T}{N_0}}\right) \end{aligned} \tag{4.10}$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\right]^2$$

$$\begin{aligned}E_2 = &\left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\right.\\ &+\frac{2\sqrt{2}}{\pi}\sum_{i=1}^\infty(-1)^i\int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right)\sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\\ &\left.+\frac{2\sqrt{2}}{\pi}\sum_{k=1}^\infty(-1)^{k+1}\int_0^\infty \frac{1}{\omega}J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right)\sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\right]^2\end{aligned}$$

However, for the worst case, performance is much worse than the case of downlink interference with the superposition of up and down link interference. The probability of symbol error when $c(t)p(t) = 1$ can be calculated by

$$\begin{aligned}P_1(e) &= 1 - P_1(c)\\ &= Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right)+Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)\\ &\quad -Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right)Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)\end{aligned}$$

Similarly, we can get the probability, $P_2(e)$, when $c(t)p(t) = -1$.

$$\begin{aligned}P_2(e) &= Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right)+Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)\\ &\quad -Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right)Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)\end{aligned} \tag{4.11}$$

Final result will be

$$\begin{aligned}P(e) &= \frac{1}{2}P_1(e)+\frac{1}{2}P_2(e)\\ &= \frac{1}{2}\left[Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right)+Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)+Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right)\right.\\ &\quad +Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)-Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right)Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)\\ &\quad \left.-Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right)Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)\right]\end{aligned} \tag{4.12}$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\right]^2$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega}J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\right.$$

$$+\frac{2\sqrt{2}}{\pi}(-1)^i\sum_{i=1}^\infty\int_0^\infty\frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega$$

$$\left.+\frac{2\sqrt{2}}{\pi}(-1)^{k+1}\sum_{k=1}^\infty\int_0^\infty\frac{1}{\omega}J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right)d\omega\right]^2$$

$$q_1 \simeq \frac{2}{\pi}\int_0^\infty\frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega\beta}{\sqrt{2}}\right)d\omega+\frac{\beta}{2\sqrt{2}}$$

$$q_2 \simeq -\frac{2}{\pi}\int_0^\infty\frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right)d\omega$$

$$-\frac{2}{\pi}\int_0^\infty\frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^\infty(-1)^{i+1}J_{2i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{2i}\left(\frac{\omega\alpha}{\sqrt{2}}\right)d\omega$$

$$-\frac{2}{\pi}\int_0^\infty\frac{1}{\omega}J_0^2\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{k=1}^\infty(-1)^k J_{2k}\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{2k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)d\omega$$

$$-\frac{\beta}{2\sqrt{2}}$$

4.2.2 Numerical Results

In this section, we present several numerical examples to illustrate the comparison of the three cases: uplink interference, downlink interference, and up and downlink interference. The first set of examples depict the performances as the interference power is varied. These examples are given in Figs. 4.2 to C.47. Then we evaluate the performance of PN QPSK with uplink and downlink interference for the worst case in Fig. C.48.

Fig. C.45 illustrates the fact that the performance with up and downlink interference is about 2 dB inferior to that of uplink interference. However, it is shown, in Figs. C.49 through C.53, that the performance where the total interference power in the up and downlink is equal to the uplink interference and is equally divided between the uplink and downlink is better than downlink interference case. From Fig. C.47, it can be observed that the performance with up and downlink interference equals to that of uplink interference at around 40 dB of signal to interference power ratio.
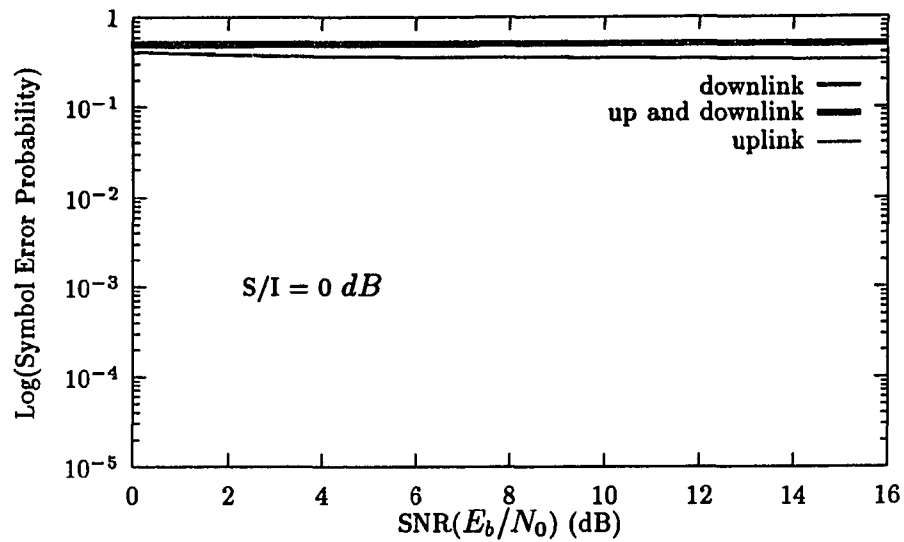
Figure 4.2: Symbol error rate comparison in worst case (: uplink interference, downlink interference, and up and downlink interference) as a function of E_b/N_0 (S/I = 0 dB)
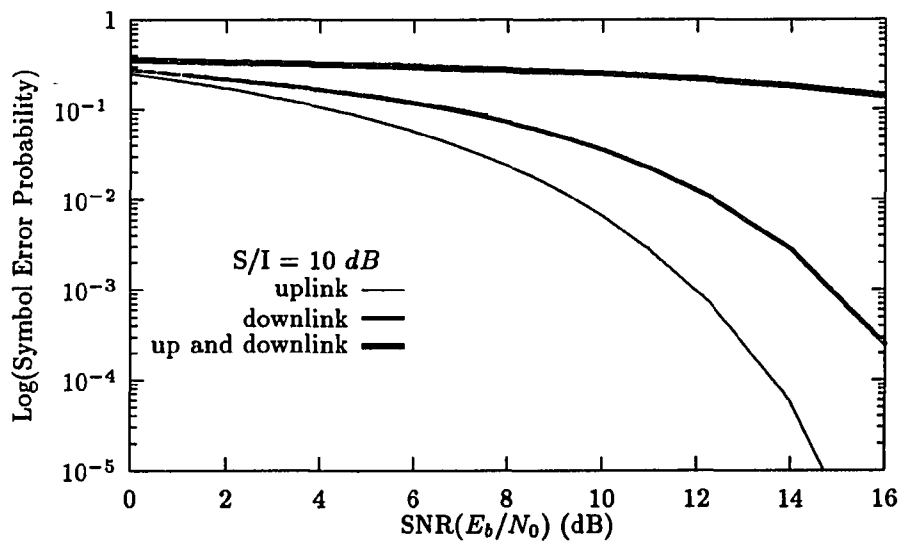

Figure 4.3: Symbol error rate comparison in worst case (: uplink interference, downlink interference, and up and downlink interference) as a function of E_b/N_0 (S/I = 10 dB)

4.3   Simulation for Evaluation of Performance

4.3.1   Simulation Model

The simulation model is the same as the case of uplink interference except the location of interferer. Block diagram used in simulation is shown in Fig. 4.4.

4.3.2   Simulation Results

The performance results for the processing gain of 18 dB are shown in Fig. C.54 when the signal to interference ratio varies from 0 dB to 30 dB. Figs. C.7 to C.16 represent the performance results for the given signal to interference ratio, when the processing gain varies from 0 dB to 23 dB. From these figures, we observe the effect on the performance of the processing gain. Generally, performance increases when processing gain increases. However, when processing gain takes on values greater than 23 dB, the symbol error rate is degraded sharply like the uplink interference case.

We plotted the SNR as a function of processing gain for the $S/I = 20$ dB and 30 dB in Figs. C.61 and C.62 separately. It is shown that optimal processing gain is near 18 dB.

These figures are the simulation results for interferers placed on the uplink and downlink. Here, the uplink interferer passes through the common baseband limiter along with the information signal. In each run, the interferer frequency is set to a slightly different value from the desired frequency.

Finally, a comparison between theoretical and simulation results is given in Figs. C.18 to C.23. From these figures we can see indirectly the degradation effects of the intermodulation product terms on the receiver performance.
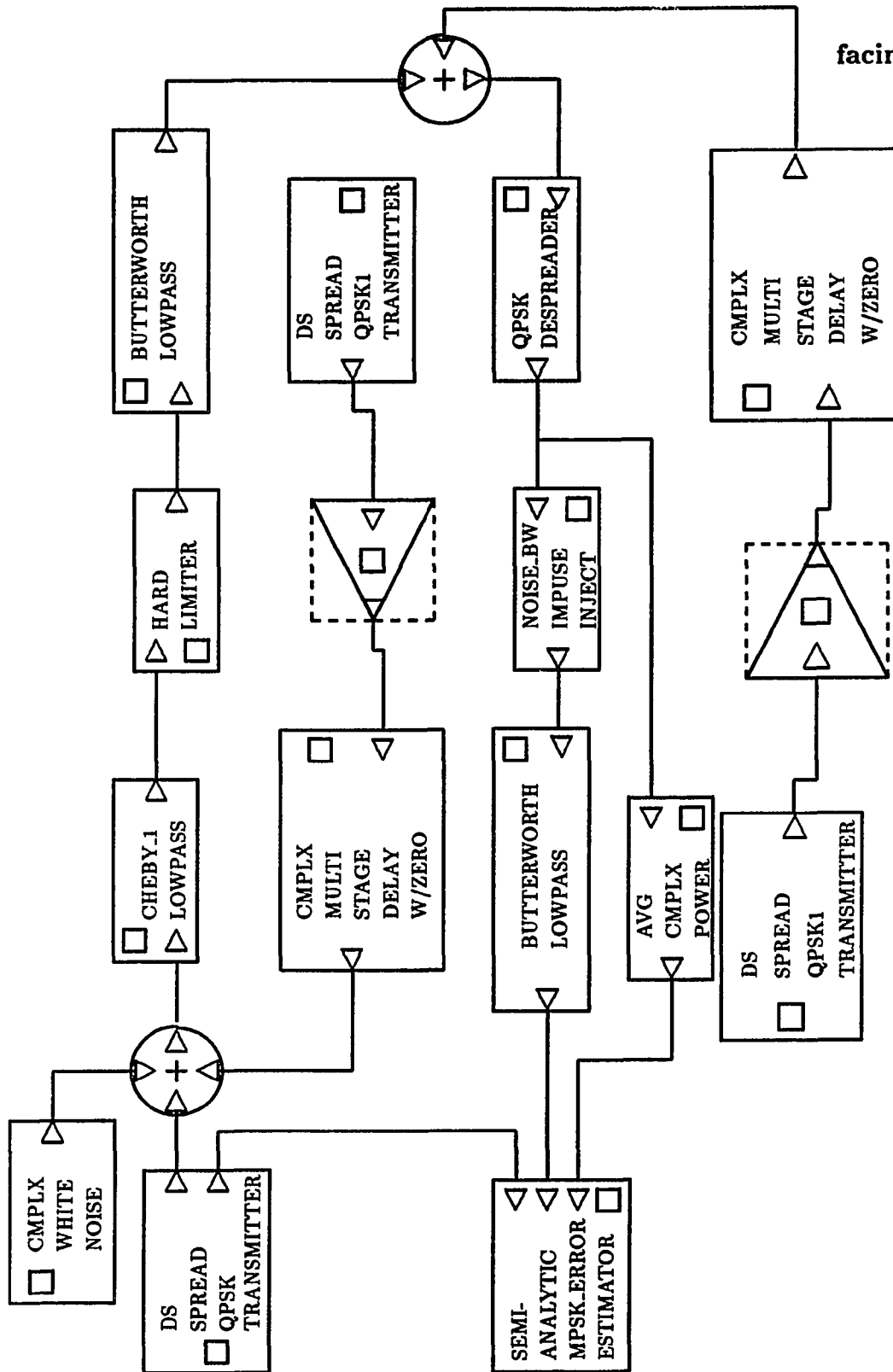
Figure 4.4: Simulation block diagram (up and downlink)

4.4 Conclusion

The mathematical formulation for evaluating the performance of a DS spread spectrum QPSK system, subject to cochannel interference, through a channel that includes an ideal band pass limiter has been derived using an inverse Fourier transform approach in the time domain. Many of the results were based on the analyses done in Ch. 2. Expressions for the desired signal, interferer, and intermodulation product terms have been obtained for the case where the input to the band pass limiter consists of DS spread spectrum QPSK signal, DS spread spectrum QPSK signal interferer, and noise. Because of the complexity of intermodulation product terms, numerical results were calculated with only interferer terms.

These theoretical results were compared with results of a computer simulation using BOSS. and also compared with the cases of only alongside with uplink interference and only downlink interference.

CHAPTER 5

COCHANNEL INTERFERENCE IN BPSK SIGNAL SYSTEM

5.1 Introduction

In this chapter, the performance of PN spread BPSK in a satellite channel is investigated by using the same mathematical procedures and assumptions as used in the previous three chapters. It is expected that the BPSK system will outperform the QPSK system because both systems are evaluated on the basis of symbol error rate.

This chapter is organized as follows: in Sec. 5, the performance with uplink interference is discussed and the case of downlink interference and up and downlink interference are analyzed in Secs. 5.2.5 and 5.3.4 respectively. Finally, In Sec 5.4.4, the results of the chapter are summarized.

5.2 Uplink Interference

5.2.1 Introduction

A communication model analyzed in this section is the same as Fig. 2.1 except for the changing of the signal used, from QPSK to BPSK. A functional block diagram is given in Fig. 5.1. The DS spread spectrum BPSK signal, $x(t)$, and cochannel interference signal, $q(t)$, can be represented as follows
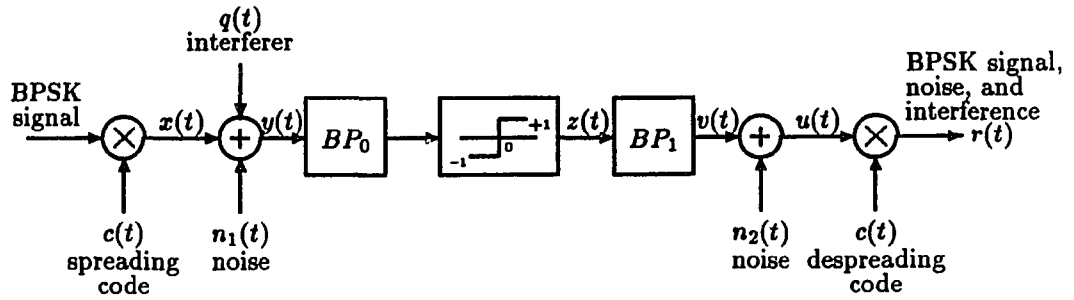
Figure 5.1: Functional Block Diagram(Uplink Interference).

$$x(t) = Ac(t)\cos\{\omega_0 t + \phi_x(t) + \theta_x\} \tag{5.1}$$

$$q(t) = \alpha p(t)\cos\{\omega_1 t + \phi_q(t) + \theta_q\} \tag{5.2}$$

5.2.2 Analysis of the Hard Limiter Output

If we take $\phi_x(t)$, $\phi_q(t) = 0$, input of the hard limiter is given by

$$y(t) = x(t) + q(t) + n_1(t)$$

$$= Ac(t)\cos\omega_0 t + \alpha p(t)\cos\omega_1 t + n_1(t) \tag{5.3}$$

where $n_1(t)$ is the white Gaussian noise. Hence, output of the hard limiter is

$$
\begin{aligned}
z(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)e^{j\omega y}\,d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{j\omega}\exp\Big[j\omega\,\{Ac(t)\cos\omega_0 t + \alpha p(t)\cos\omega_1 t + n_1(t)\}\Big]\,d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{j\omega}e^{j\omega Ac(t)\cos\omega_0 t}\,e^{j\omega\alpha p(t)\cos\omega_1 t}\,e^{j\omega n_1(t)}\,d\omega
\end{aligned}
\tag{5.4}
$$

Now, by using mathematical identities for the Bessel functions, we can express $z(t)$ as

$$z(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \Big\{ J_0(\omega A) J_0(\omega \alpha)$$

$$+ 2J_0(\omega \alpha) \sum_{i=1}^{\infty} j^i c^i(t) J_i(\omega A) \cos i\omega_0 t$$

$$+ 2J_0(\omega A) \sum_{k=1}^{\infty} j^k p^k(t) J_k(\omega \alpha) \cos k\omega_1 t$$

$$+ 4 \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} j^{i+k} c^i(t) p^k(t) J_i(\omega A) J_k(\omega \alpha) \cos i\omega_0 t \cos k\omega_1 t \Big\} \, d\omega \qquad (5.5)$$

Information Signal

The desired information signal from Eq. 5.5 is classified as those terms in the equation having a frequency ω_0. Therefore the information signal $z_m(t)$ can be shown to be

$$z_m(t) = \frac{2}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega \alpha) \sum_{i=1}^{\infty} j^i c^i(t) J_i(\omega A) \cos i\omega_0 t \, d\omega \qquad (5.6)$$

Interfering Signal

The interfering signal consists of the terms containing a carrier frequency ω_1. So,

$$z_q(t) = \frac{2}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A) \sum_{k=1}^{\infty} j^k p^k(t) J_k(\omega \alpha) \cos k\omega_1 t \, d\omega \qquad (5.7)$$

Noise

Noise is simply given by a single term.

$$z_n(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A) J_0(\omega \alpha) \, d\omega \qquad (5.8)$$

Intermodulation Product Terms

Due to the nonlinearity of the hard limiter in the bandpass limiter, we have intermodulation product terms.

$$z_{mq}(t) = \frac{4}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} j^{i+k} c^i(t) p^k(t) J_i(\omega A) J_k(\alpha \omega) \cos i\omega_0 t \cos k\omega_1 t \, d\omega$$

$$(5.9)$$

5.2.3   Analysis of the Despread Output

The output of hard limiter passes through the bandpass filter BP_1. The high order harmonics are removed in this filter. The received signal $u(t)$ thus contains only signal components in the fundamental band centered at ω_0 and white Gaussian noise added in downlink. We can express the output of the bandpass limiter by imposing the conditions of $|i - k| = 1$, $\omega_0 \simeq \omega_1$, and $\Delta\omega = |\omega_1 - \omega_0|$. This output passing through the downlink of the satellite channel is despread in the spread spectrum receiver by multiplying it by the despreading signal code, $c(t)$.

Information Signal

Finally the despread signals can be expressed as

$$r_m(t) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega\alpha) \, J_1(\omega A) \cos \omega_0 t \, d\omega \qquad (5.10)$$

Interfering Signal

Interfering signal are those terms containing a carrier frequency ω_1. So,

$$r_q(t) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A) \, J_1(\omega\alpha) \, c(t) p(t) \cos \omega_1 t \, d\omega \qquad (5.11)$$

Noise

Noise consists of downlink noise $n_2(t)$ and uplink noise $n_1(t)$ passing the bandpass limiter. The despread noise terms can be written as follows

$$r_n(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A) \, J_0(\omega\alpha) \, c(t) d\omega + n_2'(t) \qquad (5.12)$$

where $n_2'(t) = n_2(t) c(t)$ and $n_2(t)$ is a white Gaussian noise. $n_2'(t)$ is also white Gaussian noise.

Intermodulation Product Terms

The intermodulation product parts of the bandpass limiter output which contain many terms are spread in the receiver.

$$
\begin{aligned}
r_{mq}(t) &= \frac{2}{\pi}\sum_{i=1}^{\infty}(-1)^i c^i(t)p^i(t)\cos(\omega_0 - i\Delta\omega)t \int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_{i+1}(A\omega)J_i(\alpha\omega)\,d\omega \\
&\quad + \frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k+1}(t)\cos\{\omega_0 - (k+1)\Delta\omega\}t \\
&\quad \cdot \int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_k(A\omega)J_{k+1}(\alpha\omega)\,d\omega
\end{aligned}
\tag{5.13}
$$

5.2.4 Performance Evaluation

Despread signal is supplied to the correlator receiver where it is multiplied by a locally generated coherent reference signal and integrated over a symbol period. One basis functions $\phi(t)$ is used to demodulate a pair of received signals, that is

$$
\phi(t) = \frac{1}{T}\cos\omega_0 t \quad 0 \le t \le T
\tag{5.14}
$$

where T is the bit symbol duration.

Demodulated Information Signal

If the output of the correlator is denoted by m, m is given by

$$
m = \frac{2}{\pi T}\int_0^T \int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0(\omega\alpha)\,J_1(\omega A)\cos^2\omega_0 t\,d\omega\,dt
\tag{5.15}
$$

If $n_1(t) \approx 0$, above equation can be simplified as follows

$$
\begin{aligned}
m &\simeq \frac{2}{\pi}\int_0^{\infty}\frac{1}{\omega}J_0(\omega\alpha)\,J_1(\omega A)\,d\omega \\
&\triangleq \sqrt{\frac{E}{2}}
\end{aligned}
\tag{5.16}
$$

where

$$
E = \left[\frac{2\sqrt{2}}{\pi}\int_0^{\infty}\frac{1}{\omega}J_0(\omega\alpha)J_1(\omega A)\,d\omega\right]^2
\tag{5.17}
$$

Demodulated Interfering Signal

Demodulated interferer in spread spectrum receiver is represented by

$$q = \frac{2}{\pi T} \int_0^T \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(A\omega) J_1(\omega\alpha) c(t) p(t) \cos \omega_1 t \cos \omega_0 t \, d\omega \, dt \qquad (5.18)$$

If $n_1(t) \approx 0$, Eq. 5.18 can be written again as

$$q \simeq \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \frac{1}{2T} \int_0^T c(t) p(t) \cos \Delta\omega t \, dt \, d\omega \qquad (5.19)$$

For the high processing gain $q \approx 0$. However, for processing gain $= 0$ and $\Delta\omega T \ll 1$, we obtain

$$q \simeq \frac{2}{\pi} \int_0^{\infty} \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \, d\omega \qquad (5.20)$$

Demodulated Noise

The received noise will be demodulated in the correlator receiver. The demodulated output is

$$
\begin{aligned}
r_{n_i} &= \frac{2}{j\pi} \int_0^T \int_{-\infty}^{\infty} c(t) \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A) J_0(\omega\alpha) \, d\omega \, dt + \frac{1}{T} \int_0^T n_2'(t) \, dt \\
&\simeq \frac{1}{T} \int_0^T n_2'(t) \, dt \qquad (5.21)
\end{aligned}
$$

The expected value of r_{n_i} is

$$E[r_{n_i}] = E\left[\int_0^T n_2'(t) \, \phi_i(t) \, dt \right] = 0 \qquad (5.22)$$

The variance of r_{n_i} can be calculated as

$$
\begin{aligned}
\sigma_{r_{n_i}}^2 &= E[r_{n_i}^2] = E\left[\frac{1}{T^2} \int_0^T n_2'(t) \, \cos \omega_0 t \, dt \int_0^T n_2'(u) \, \cos \omega_0 u \, du \right] \\
&= E\left[\frac{1}{T^2} \int_0^T \int_0^T \cos \omega_0 t \, \cos \omega_0 u \, R_w(t - u) \, dt \, du \right] \\
&= \frac{N_0}{4T} \qquad (5.23)
\end{aligned}
$$

Demodulated Intermodulation Products

Intermodulation product terms passing the band pass limiter are given by

$$
\begin{aligned}
r_{mq}(t) &= \frac{2}{\pi}\sum_{i=1}^{\infty}(-1)^i c^i(t)p^i(t)\,\cos(\omega_0 - i\Delta\omega)t \int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_{i+1}(A\omega)J_i(\alpha\omega)\,d\omega \\
&\quad + \frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k+1}(t)\,\cos\{\omega_0 - (k+1)\Delta\omega\}t \\
&\quad \cdot \int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_k(A\omega)J_{k+1}(\alpha\omega)\,d\omega
\end{aligned}
\tag{5.24}
$$

Hence, demodulated intermodulation product terms are

$$
\begin{aligned}
r_{mq}' &= \frac{1}{T}\int_0^T r_{mq}(t)\,\cos\omega_0 t\,dt \\
&\simeq \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}\sum_{i=1}^{\infty}(-1)^i J_{i+1}(A\omega)J_i(\alpha\omega)\frac{1}{T}\int_0^T e^{j\omega n_1(t)}c^i(t)p^i(t)\,\cos i\Delta\omega t\,dt\,d\omega \\
&\quad + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}\sum_{k=1}^{\infty}(-1)^k J_k(A\omega)J_{k+1}(\alpha\omega) \\
&\quad \cdot \frac{1}{T}\int_0^T e^{j\omega n_1(t)}c^{k+1}(t)p^{k+1}(t)\,\cos(k+1)\Delta\omega t\,dt\,d\omega
\end{aligned}
\tag{5.25}
$$

If $n_1(t) \approx 0$, Eq. 5.25 can be written as

$$
\begin{aligned}
r_{mq}' &\simeq \frac{1}{\pi T}\int_{-\infty}^{\infty}\frac{1}{\omega}\sum_{i=1}^{\infty}(-1)^i J_{i+1}(A\omega)J_i(\alpha\omega)\int_0^T c^i(t)p^i(t)\,\cos i\Delta\omega t\,dt\,d\omega \\
&\quad + \frac{1}{\pi T}\int_{-\infty}^{\infty}\frac{1}{\omega}\sum_{k=1}^{\infty}(-1)^k J_k(A\omega)J_{k+1}(\alpha\omega) \\
&\quad \cdot \int_0^T c^{k+1}(t)p^{k+1}(t)\,\cos(k+1)\Delta\omega t\,dt\,d\omega
\end{aligned}
\tag{5.26}
$$

When $i = even$ and $k = odd$, both terms become a narrowband interferer. Since narrowband interferer term is relatively much greater than a wideband interferer when the processing gain is high, the above equation will be

$$
\begin{aligned}
r_{mq}' &\simeq \frac{2}{\pi}\int_0^{\infty}\frac{1}{\omega}\sum_{i=1}^{\infty}J_{2i+1}(A\omega)J_{2i}(\alpha\omega)\,d\omega \\
&\quad - \frac{2}{\pi}\int_0^{\infty}\frac{1}{\omega}\sum_{k=1}^{\infty}J_{2k-1}(A\omega)J_{2k}(\alpha\omega)\,d\omega
\end{aligned}
\tag{5.27}
$$

According to the previous investigation [38], if power ratio of information signal $r(t)$ and interferer $q(t)$ is greater than 5 dB, the power of intermodulation product can be neglected even for the large signal to noise ratios.

Calculation of Average Probability for Symbol Error

For the average case, average probability for symbol error can be derived and expressed by using Q^1 function.

$$
\begin{aligned}
P(e) &= 1 - P_c \\
&= 1 - \left\{ \int_0^\infty \sqrt{\frac{2T}{\pi N_0}} e^{-\frac{2T\left(x-\sqrt{\frac{E}{2}}\right)^2}{N_0}} dx \right\} \\
&= Q\left(\sqrt{\frac{2TE}{N_0}}\right)
\end{aligned}
\tag{5.28}
$$

where E is given in Eq. 5.17.

In the worst case, we need to consider interferer terms. For the case where $c(t)p(t) = 1$,

$$
\begin{aligned}
P_1(e) &= 1 - \left\{ \int_0^\infty \sqrt{\frac{2T}{\pi N_0}} e^{-\frac{2T\left(x-\sqrt{\frac{E}{2}-q}\right)^2}{N_0}} dx \right\} \\
&= Q\left(\frac{\sqrt{2TE}-\sqrt{4T}q}{\sqrt{N_0}}\right)
\end{aligned}
\tag{5.29}
$$

For the case where $c(t)p(t) = -1$ in whole bit symbol period,

$$
P_2(e) = Q\left(\frac{\sqrt{2TE}+\sqrt{4T}q}{\sqrt{N_0}}\right)
\tag{5.30}
$$

Therefore, performance for the worst case is given by

$$
\begin{aligned}
P(e) &= P\{c(t)p(t) = +1\}P\{e/c(t)p(t) = +1\} + P\{c(t)p(t) = -1\}P\{e/c(t)p(t) = -1\} \\
&= \frac{1}{2}Q\left(\frac{\sqrt{2TE}-\sqrt{4T}q}{\sqrt{N_0}}\right) + \frac{1}{2}Q\left(\frac{\sqrt{2TE}+\sqrt{4T}q}{\sqrt{N_0}}\right)
\end{aligned}
\tag{5.31}
$$

where E and q are

$$
E = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A)\, d\omega\right]^2
\tag{5.32}
$$

$$
q \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha)\, d\omega
\tag{5.33}
$$

---

[1] For the definition of Q function, refer to the Section 2.3.1 in Ch. 2

5.2.5 Numerical Results

This section consists of numerical results obtained from the symbol error probability calculating for the correlation receiver when we use a BPSK signal in the satellite link. Cochannel interference on the uplink of the satellite channel is considered.

The primary objective is to observe the influence of the interferer as the power of the interferer varies. Numerical calculations are made for the average case wherein the power of interference is negligible and the worst case wherein the power of interference is maximum.

The comparison between the average case and the worst case for the symbol error rate when signal to interference power ratio ranges from 0 *dB* to 30 *dB* is shown in Figs. 5.2 through C.70. Figs. C.71 and C.72 each shows the variation of performance for the average case and the worst case respectively when signal to interference power ratio changes.

It is shown that overall symbol error rate performance of BPSK is a little better than that of QPSK. We observe that once the the signal to interference power ratio becomes greater than 20 *dB*, the interference has a negligible effect on the performance.
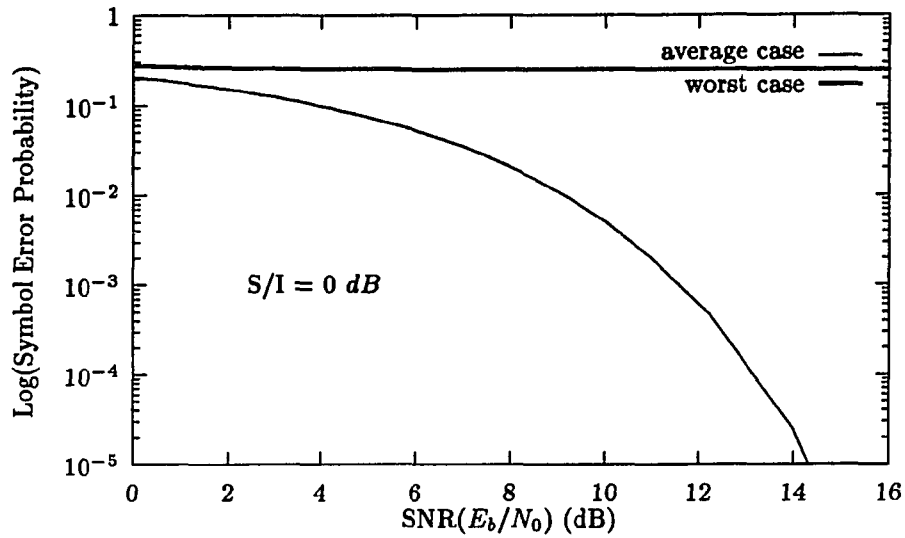
Figure 5.2: Symbol error rate comparison between the average case and the worst case as a function of E_b/N_0 (S/I = 0 dB)
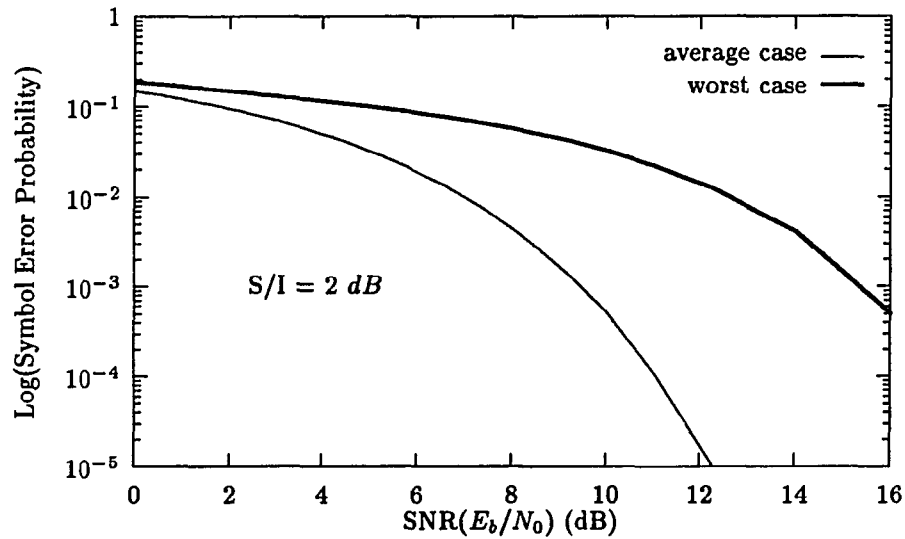
Figure 5.3: Symbol error rate comparison between the average case and the worst case as a function of E_b/N_0 (S/I = 2 dB)

5.3 Downlink Interference Analysis

In case of downlink interference, the information carrier signal is given in Eq. 5.1. This information signal and white Gaussian noise go into the bandpass limiter together. Hence the input of the bandpass limiter, $y(t)$, is the sum of two signals, These are the information signal $x(t)$ and white Gaussian noise $n_1(t)$.

$$
\begin{aligned}
y(t) &= x(t) + n_1(t) \\
&= Ac(t)\cos\{\omega_0 t + \phi_x(t) + \theta_x\} + n_1(t)
\end{aligned}
\tag{5.34}
$$

5.3.1 Analysis of the Hard Limiter Output

The output $z(t)$ of a memoryless hard limiter is related to the input by the transfer characteristic $g(y)$. $g(y)$, being a signum function, has a Fourier transform $G(j\omega) = \frac{2}{j\omega}$, Then by using the inverse Fourier transform of the transfer function, we have

$$
\begin{aligned}
z(t) &= g(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)e^{j\omega y}\,d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2}{j\omega}e^{j\omega y}\,d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2}{j\omega}\exp\Big[j\omega\left\{Ac(t)\cos\{\omega_0 t + \phi_x(t) + \theta_x\} + n_1(t)\right\}\Big]\,d\omega
\end{aligned}
$$

If we take $\phi_x(t) = 0$ and θ_x, $z(t)$ can be rewritten as

$$
z(t) = \frac{1}{j\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\left\{J_0(\omega A) + 2\sum_{k=1}^{\infty}(j)^k c^k(t)J_k(\omega A)\cos k\omega_0 t\right\}d\omega
\tag{5.35}
$$

After $z(t)$ passes bandpass filter BP_1, $v(t)$ will be

$$
\begin{aligned}
v(t) &= \frac{1}{j\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\left\{J_0(\omega A) + 2jc(t)J_1(\omega A)\cos\omega_0 t\right\}d\omega \\
&= \frac{1}{j\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0(\omega A)\,d\omega + \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}c(t)e^{j\omega n_1(t)}J_1(\omega A)\cos\omega_0 t\,d\omega
\end{aligned}
$$

$$
\tag{5.36}
$$

Hence, $u(t)$ will be

$$u(t) = v(t) + q(t) + n_2(t)$$

$$= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A)\, d\omega + \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} c(t) e^{j\omega n_1(t)} J_1(\omega A) \cos \omega_0 t\, d\omega$$

$$+ \beta s(t) \cos\{\omega_2 t + \phi_q(t) + \theta_q\} + n_2(t) \tag{5.37}$$

Here, we take again $\phi_q(t) = \theta_q = 0$

$$u(t) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_0(\omega A)\, d\omega + \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} c(t) e^{j\omega n_1(t)} J_1(\omega A) \cos \omega_0 t\, d\omega$$

$$+ \beta s(t) \cos \omega_2 t + n_2(t) \tag{5.38}$$

Now, this signal can be despread and the despread signal, $r(t)$, is given by

$$r(t) = u(t) c(t)$$

$$= \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{c(t)}{\omega} e^{j\omega n_1(t)} J_0(\omega A)\, d\omega + \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_1(\omega A) \cos \omega_0 t\, d\omega$$

$$+ \beta s(t) c(t) \cos \omega_2 t + n_2(t) c(t) \tag{5.39}$$

5.3.2 Demodulation

This despread signal is sent to the correlator receiver where it is multiplied by a locally generated coherent reference signal. One basis functions $\phi(t)$ which is given in Eq. 5.14 is used to demodulate a pair of received signals.

Demodulated Information Signal

Let m be demodulated signals generated by $\phi(t)$. m can be calculated as follows.

$$m = \frac{2}{\pi T} \int_0^T \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} J_1(\omega A) \cos^2 \omega_0 t\, d\omega\, dt$$

$$= \frac{4}{\pi T} \int_0^T \int_0^{\infty} \frac{1}{\omega} J_1(\omega A) \cos^2 \omega_0 t\, \cos \omega n_1(t)\, d\omega\, dt \tag{5.40}$$

If $n_1(t) \approx 0$, above equation will be

$$m = \frac{4}{\pi T} \int_0^T \int_0^\infty \frac{1}{\omega} J_1(\omega A) \cos^2 \omega_0 t \, d\omega \, dt$$

$$= \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_1(\omega A) \, d\omega$$

$$\triangleq \sqrt{\frac{E}{2}} \tag{5.41}$$

where E is a constant and can be expressed by

$$\sqrt{E} = \frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_1(\omega A) \, d\omega \tag{5.42}$$

Demodulated Interfering Signal

The interferer which is added in the downlink can be demodulated as follows.

$$q = \frac{2}{T} \int_0^T \int_0^\infty \beta s(t) c(t) \cos \omega_2 t \cos \omega_0 t \, dt \, d\omega \tag{5.43}$$

If information bit duration is much bigger than the chip time, q is approximately 0. If $p(t) = c(t) = 1$ for the whole bit duration T, q is given by

$$q = \frac{1}{T} \int_0^T \beta \cos \omega_2 t \cos \omega_0 t \, dt \, d\omega$$

$$= \frac{\beta}{2T} \int_0^T \cos \Delta \omega t \, dt \, d\omega = \frac{\beta}{2T} \cdot \frac{\sin \Delta \omega T}{\Delta \omega} \tag{5.44}$$

If $\Delta \omega T \ll 1$, equation q can be simplified as below

$$q \simeq \frac{\beta}{2} \tag{5.45}$$

5.3.3 Performance Evaluation

The calculation of symbol error probability is calculated for the average case and the worst case. For the average case, the average probability of symbol error is

$$P(e) = Q\left(\sqrt{\frac{2TE}{N_0}}\right) \tag{5.46}$$

This result is of the same form as the uplink interference case. The only difference is the expression for the constant E, which is given by,

$$E = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_1(\omega A)\, d\omega\right]^2 \tag{5.47}$$

From above equation, we see that performance is not affected by the interferer terms. In other words, if we have a high processing gain, we can neglect interference. The performance in the worst case when we are considering interferer terms can be calculated by

$$P(e) = \frac{1}{2} Q\left(\frac{\sqrt{2TE} - \sqrt{4T}q}{\sqrt{N_0}}\right) + \frac{1}{2} Q\left(\frac{\sqrt{2TE} + \sqrt{4T}q}{\sqrt{N_0}}\right) \tag{5.48}$$

where E and q are

$$E = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_1(\omega A)\, d\omega\right]^2$$

$$q \simeq \frac{\beta}{2} \tag{5.49}$$

5.3.4   Results and Discussion

The numerical results are analyzed in a similar manner to the uplink interference case. Figs. 5.4 through C.76 show the performance comparison for the worst case between the uplink interference case and the downlink case for each different signal to interference power ratio.

From these results, it is observed that downlink interference degrades performance more severely than does uplink interference. However, as the signal to interference power ratio approaches 30 dB, the effect on the performance by the interferer is negligible in both cases, uplink interference and downlink interference.

Fig. C.77 indicates all the performance of the worst case for different S/I power ratios. For the average case, since interferer is decreased by the high processing gain, the output result is always same even if signal to interference power ratio changes.
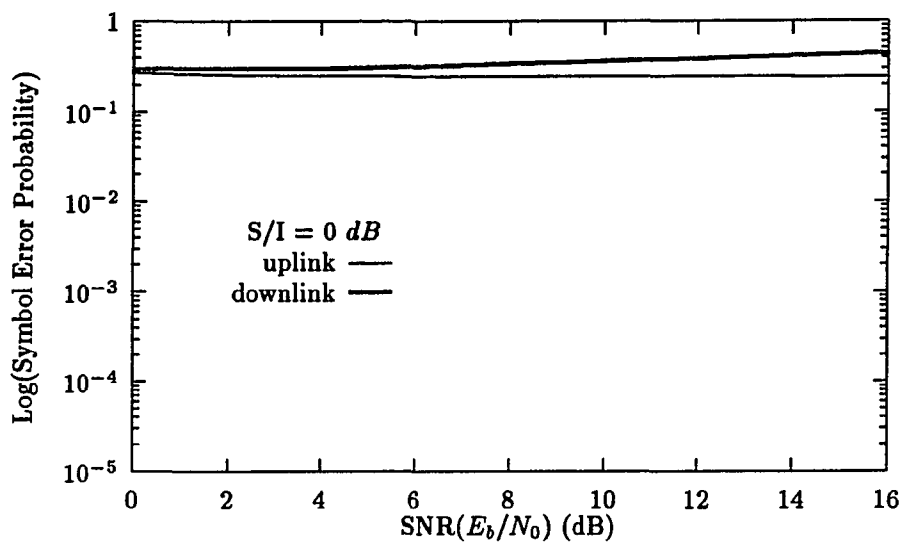
Figure 5.4: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (BPSK, S/I = 0 dB)
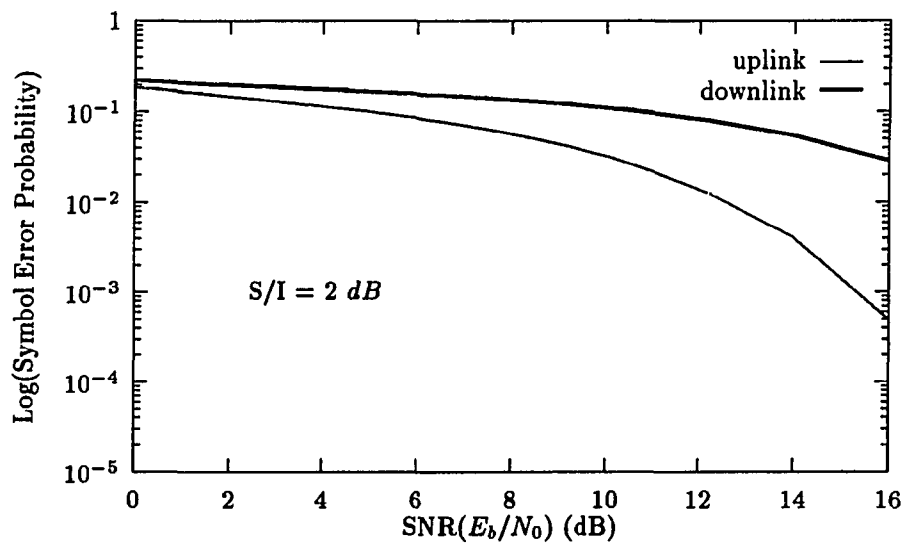
Figure 5.5: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (BPSK, S/I = 2 dB)

5.4 Up and Downlink Interference Analysis

In BPSK signalling system, performance analysis under the condition of up and downlink interference is very similar to the analysis done for the cases of uplink interference and downlink interference. In this section we can express the information signal, the uplink interferer, and the downlink interferer as follows

$$x(t) = Ac(t)\cos\{\omega_0 t + \phi_x(t) + \theta_x\} \tag{5.50}$$

$$q(t) = \alpha p(t)\cos\{\omega_1 t + \phi_q(t) + \theta_q\} \tag{5.51}$$

$$d(t) = \beta s(t)\cos\{\omega_2 t + \phi_d(t) + \theta_d\} \tag{5.52}$$

The input of spread spectrum receiver, $u(t)$, can be manipulated by using the result of the Sec. 5.

$$
\begin{aligned}
u(t) = {} & \frac{2}{\pi}\int_{-\infty}^{\infty} e^{j\omega n_1(t)} J_0(\omega\alpha)J_1(\omega A)\cos\omega_0 t\, d\omega \\
& + \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}c(t)e^{j\omega n_1(t)}J_0(\omega A)J_1(\omega\alpha)\cos\omega_1 t\, d\omega \\
& + \frac{1}{j\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_0(\omega A)J_0(\omega\alpha)\, d\omega + n_2(t) \\
& + \frac{2}{\pi}\sum_{i=1}^{\infty}(-1)^i c^{i+1}(t)p^i(t)\cos(\omega_0 - i\Delta\omega)t\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_{i+1}(A\omega)J_i(\omega\alpha)\, d\omega \\
& + \frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^k c^k(t)p^{k+1}(t)\cos(\omega_0 - (k+1)\Delta\omega)t\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}J_k(A\omega) \\
& \cdot J_{k+1}(\omega\alpha)\, d\omega + \beta s(t)\cos\{\omega_2 t + \phi_d(t) + \theta_d\} \tag{5.53}
\end{aligned}
$$

5.4.1 Despread Output Signal

The first operation which is performed in spread spectrum receiver is despreading. The received signal can be despread by multiplying it with the despreading code, $c(t)$, which is exactly same as the spreading code.

5.4.2 Demodulated Signal

The spread signal is demodulated by multiplying with a basis function. This demodulated signal can be separated into four signal groups which are demodulated

information signal, demodulated interfering signal, demodulated noise, and demodulated intermodulation product terms.

Demodulated Information Signal

The information signal term is the same as in the case of uplink interference. From the previous section,

$$
\begin{aligned}
m &\simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A) \, d\omega \\
&\triangleq \sqrt{\frac{E}{2}}
\end{aligned}
\tag{5.54}
$$

where

$$
E = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A) \, d\omega \right]^2
\tag{5.55}
$$

This equation is equal to Eq. 5.17.

Demodulated Interfering Signal

Interferer terms are the sum of uplink interferer and downlink interferer.

$$
\begin{aligned}
q &\simeq \frac{2}{\pi T} \int_0^T \int_{-\infty}^\infty \frac{1}{\omega} e^{j\omega n_1(t)} J_0(A\omega) J_1(\omega\alpha) c(t) p(t) \cos \Delta\omega t \, dt \, d\omega \\
&\quad + \frac{1}{T} \int_0^T \beta s(t) c(t) \cos\{\omega_2 t + \phi_d(t) + \theta_d\} \cos \omega_0 t \, dt \, d\omega
\end{aligned}
\tag{5.56}
$$

For simplicity, $\phi_d(t) = \theta_d = 0$. If $n_1(t) \approx 0$, Eq. 5.56 will be

$$
\begin{aligned}
q &\simeq \frac{2}{\pi} \int_{-\infty}^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \frac{1}{2T} \int_0^T c(t) p(t) \cos \Delta\omega t \, dt \, d\omega \\
&\quad + \frac{1}{2T} \int_0^T \beta s(t) c(t) \cos\{\omega_2 t + \phi_d(t) + \theta_d\} \cos \omega_0 t \, dt \, d\omega
\end{aligned}
\tag{5.57}
$$

where $|\omega_2 - \omega_0| = \Delta\omega_1$ If the symbol time T is much greater than T_c, $q \approx 0$. However, if chip time T_c has the same time period as the bit symbol period T, q yields

$$
\begin{aligned}
q &\simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \frac{1}{T} \int_0^T \cos \Delta\omega t \, dt \, d\omega \\
&\quad + \frac{1}{2T} \int_0^T \beta \cos \Delta\omega_1 t \, dt \\
&= \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \frac{1}{T} \frac{\sin \Delta\omega_1 T}{\Delta\omega_1}
\end{aligned}
\tag{5.58}
$$

If $\Delta\omega_1 T \ll 1$, q of Eq. 5.58 thus simplifies to

$$q \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \, d\omega + \frac{\beta}{2} \qquad . \qquad (5.59)$$

5.4.3 Performance Evaluation

For practical CDMA systems, the chip time of the spreading code is much smaller than the bit time. Under this condition the correlator output due to the interferer is smaller than the correlator output due to the desired signal by the processing gain and can be neglected in the average case. Since we neglect the interferer terms, the probability of symbol error for DS spread spectrum BPSK signal can be expressed as follows

$$
\begin{aligned}
P(e) &= 1 - P_c \\
&= 1 - \left\{ \int_0^\infty \frac{2T}{\sqrt{\pi N_0}} e^{-\frac{2T\left(x - \sqrt{\frac{E}{2}}\right)^2}{N_0}} dx \right\} \\
&= Q\left(\sqrt{\frac{2TE}{N_0}} \right)
\end{aligned}
\qquad (5.60)
$$

where E is given in Eq. 5.17.

For the worst case, we have to consider the demodulated interferer term for the two cases which are the case of $|c(t)p(t)| = 1$ and the case of $|c(t)p(t)| = -1$ for the whole symbol period. For $|c(t)p(t)| = 1$,

$$
\begin{aligned}
P_1(e) &= 1 - \left\{ \int_0^\infty \frac{2T}{\sqrt{\pi N_0}} e^{-\frac{2T\left(x - \sqrt{\frac{E}{2} - q}\right)^2}{N_0}} dx \right\} \\
&= Q\left(\frac{\sqrt{2TE} - \sqrt{4Tq}}{\sqrt{N_0}} \right)
\end{aligned}
\qquad (5.61)
$$

For $|c(t)p(t)| = -1$, the probability of symbol error can be obtained as

$$P_2(e) = Q\left(\frac{\sqrt{2TE} + \sqrt{4Tq}}{\sqrt{N_0}} \right) \qquad (5.62)$$

Hence, the performance for the worst case is presented by

$$P(e) = \frac{1}{2} Q \left(\frac{\sqrt{2TE} - \sqrt{4T}q}{\sqrt{N_0}} \right) + \frac{1}{2} Q \left(\frac{\sqrt{2TE} + \sqrt{4T}q}{\sqrt{N_0}} \right) \qquad (5.63)$$

where E and q are

$$E = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A) \, d\omega \right]^2 \qquad (5.64)$$

$$q \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) d\omega + \frac{\beta}{2} \qquad (5.65)$$

5.4.4 Numerical Examples

This section consists of numerical results obtained from the computation of the bit error probability for the up and downlink interference. The primary objective is a comparative study to compare the performance with the separate cases of only uplink or only downlink interference as a function of signal to interference power ratio. The performance for the uplink interference and downlink interference is drawn together to facilitate comparison. The performances for the average case in which interferer terms are neglected are exactly same as the Figs. 5.2 to C.71 of Sec. 5.

We are considering two conditions for the worst case. For the first condition, the total interference power is two times the uplink interference since interferer is added in up and down both link. The curves of Figs. 5.6 through C.82 show the performance when signal to interference power ratio changes from 0 dB to 30 dB. Fig. C.83 gives accumulated results for the up and downlink interference. For the second case, the total interference power is divided equally into the up and downlink in order to get the same total interference power with the uplink interference power. The results of this case are given in Figs. C.84 through C.86.
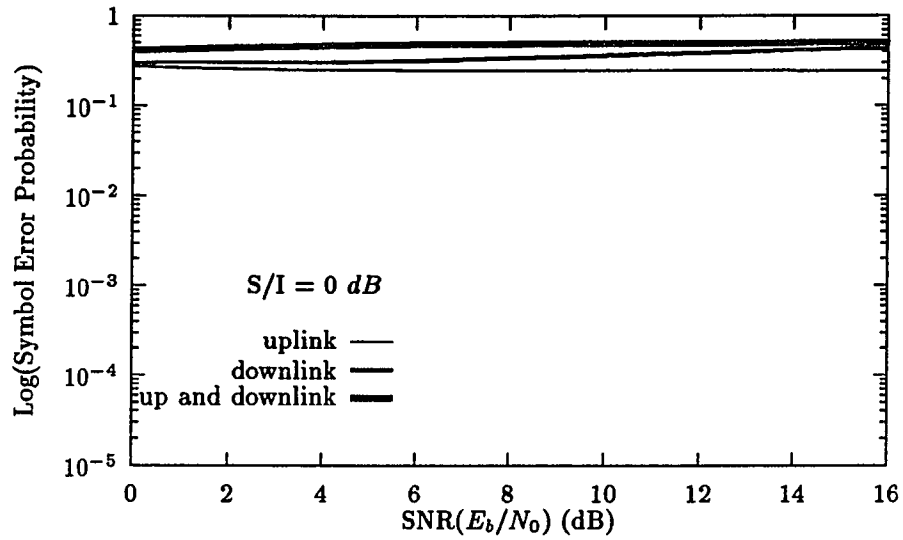
Figure 5.6: Symbol error rate comparison as a function of E_b/N_0 (S/I $= 0$ dB)
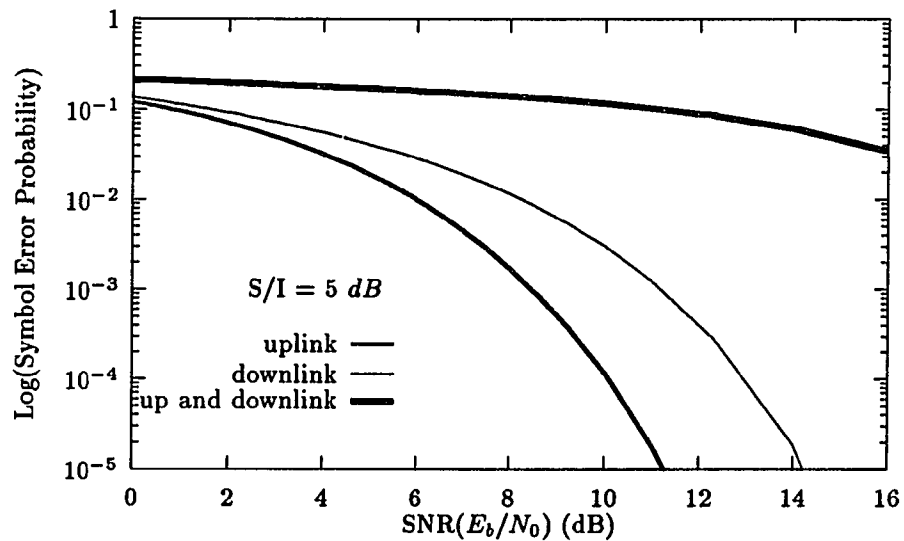
Figure 5.7: Symbol error rate comparison as a function of E_b/N_0 (S/I $= 5$ dB)

5.5   Conclusions

In this Chapter, we have investigated the performance of PN spread BPSK signal which operates in satellite bandpass limiter channel. The performance has been derived for three different conditions which are uplink, downlink, and up and downlink cochannel interference. Numerical results are exhibited for several cases of interest.

CHAPTER 6

CONCLUSION

We review the results and compare the performance of QPSK and BPSK signalling in this chapter. Also, we suggest topics for further research.

6.1 Comparison between QPSK and BPSK Signalling

We considered the performance of QPSK and BPSK signalling for the average and the worst cases. The overall performance of the systems is characterized by means of the following main parameters

- Symbol error rate: compared the performance of QPSK and BPSK on the basis of the systems transmitting equal numbers of symbols per second(two bits per QPSK phase). For BPSK, bit error rate is equal to the symbol error rate. For QPSK, most symbol errors will result in a 1 bit error so the bit error rate is approximately equal to 1/2 of the symbol error rate.

- E_b/N_0: compared on the basis of signal to noise ratio, E_b/N_0, where E_b is the average energy per bit.

Figs. 6.1 through C.102 show the performance for both signalling schemes under different conditions. Here, we can summarize again the expressions for the performance of QPSK and BPSK. The results are given in the following subsections, 6.1.1 and 6.1.2. In section 6.1.3, several numerical examples are shown.

6.1.1　The Average Case

- Uplink Interference

1. QPSK

$$P(e) = Q\left(\sqrt{\frac{2E_1 T}{N_0}}\right) + Q\left(\sqrt{\frac{2E_2 T}{N_0}}\right) - Q\left(\sqrt{\frac{2E_1 T}{N_0}}\right) Q\left(\sqrt{\frac{2E_2 T}{N_0}}\right) \quad (6.1)$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2 \quad (6.2)$$

$$\begin{aligned}
E_2 = &\left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right.\\
&+ \frac{2\sqrt{2}}{\pi}\sum_{i=1}^\infty (-1)^i \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \\
&+ \left.\frac{2\sqrt{2}}{\pi}\sum_{k=1}^\infty (-1)^{k+1} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2
\end{aligned}$$

$$(6.3)$$

2. BPSK

$$P(e) = Q\left(\sqrt{\frac{2TE}{N_0}}\right) \quad (6.4)$$

where

$$E = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A)\, d\omega\right]^2 \quad (6.5)$$

- Downlink Interference

1. QPSK

$$P(e) = Q\left(\sqrt{\frac{2TE_1}{N_0}}\right) + Q\left(\sqrt{\frac{2TE_2}{N_0}}\right) - Q\left(\sqrt{\frac{2TE_1}{N_0}}\right) Q\left(\sqrt{\frac{2TE_2}{N_0}}\right) \quad (6.6)$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2 \quad (6.7)$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi}\int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right.$$

$$+ \frac{2\sqrt{2}}{\pi} \sum_{i=1}^{\infty} (-1)^i \int_0^{\infty} \frac{1}{\omega} \sum_{i=1}^{\infty} J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$+ \frac{2\sqrt{2}}{\pi} \sum_{k=1}^{\infty} (-1)^{k+1} \int_0^{\infty} \frac{1}{\omega} \sum_{k=1}^{\infty} J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \Bigg]^2 \quad (6.8)$$

2. BPSK

$$P(e) = Q\left(\sqrt{\frac{2TE}{N_0}}\right) \quad (6.9)$$

where

$$E = \left[\frac{2\sqrt{2}}{\pi} \int_0^{\infty} \frac{1}{\omega} J_1(\omega A)\, d\omega\right]^2 \quad (6.10)$$

- **Up and Downlink Interference**

1. QPSK

$$P(e) = Q\left(\sqrt{\frac{2E_1 T}{N_0}}\right) + Q\left(\sqrt{\frac{2E_2 T}{N_0}}\right) - Q\left(\sqrt{\frac{2E_1 T}{N_0}}\right) Q\left(\sqrt{\frac{2E_2 T}{N_0}}\right) \quad (6.11)$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^{\infty} \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2 \quad (6.12)$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi} \int_0^{\infty} \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right.$$

$$+ \frac{2\sqrt{2}}{\pi} \sum_{i=1}^{\infty} (-1)^i \int_0^{\infty} \frac{1}{\omega} J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty} J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$+ \left. \frac{2\sqrt{2}}{\pi} \sum_{k=1}^{\infty} (-1)^{k+1} \int_0^{\infty} \frac{1}{\omega} J_0^2\left(\frac{\omega \alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega\right]^2$$

$$(6.13)$$

2. BPSK

$$P(e) = Q\left(\sqrt{\frac{2TE}{N_0}}\right) \quad (6.14)$$

where

$$\sqrt{E} = \frac{2\sqrt{2}}{\pi} \int_0^{\infty} \frac{1}{\omega} J_0(\omega \alpha) J_1(\omega A)\, d\omega \quad (6.15)$$

6.1.2 The Worst Case

- Uplink Interference

1. QPSK

$$P(e) = \frac{1}{2} \left[Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right) \right.$$

$$+ Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$- Q\left(\frac{\sqrt{2TE_1}-\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}+\sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$\left. - Q\left(\frac{\sqrt{2TE_1}+\sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2}-\sqrt{4T}q_2}{\sqrt{N_0}}\right) \right] \qquad (6.16)$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right]^2 \qquad (6.17)$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right.$$

$$+ \frac{2\sqrt{2}}{\pi}(-1)^i \sum_{i=1}^\infty \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$+ \frac{2\sqrt{2}}{\pi}(-1)^{k+1} \sum_{k=1}^\infty \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\left. \cdot J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right]^2 \qquad (6.18)$$

$$q_1 \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega \qquad (6.19)$$

$$q_2 \simeq -\frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty (-1)^{i+1} J_{2i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{2i}\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty (-1)^k J_{2k}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{2k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega \quad (6.20)$$

2. BPSK

$$P(e) = \frac{1}{2} Q\left(\frac{\sqrt{2TE}-\sqrt{4T}q}{\sqrt{N_0}}\right) + \frac{1}{2} Q\left(\frac{\sqrt{2TE}+\sqrt{4T}q}{\sqrt{N_0}}\right) \qquad (6.21)$$

where

$$\sqrt{E} \quad = \quad \frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A) \, d\omega \qquad (6.22)$$

$$q \quad \simeq \quad \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha) \, d\omega \qquad (6.23)$$

- **Downlink Interference**

1. QPSK

$$P(e) = \frac{1}{2} \left[Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right) \right.$$

$$+ Q\left(\frac{\sqrt{2TE_1} + \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} - \sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$- Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$\left. - Q\left(\frac{\sqrt{2TE_1} + \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} - \sqrt{4T}q_2}{\sqrt{N_0}}\right) \right] \qquad (6.24)$$

where

$$E_1 \quad = \quad \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) \, d\omega \right]^2 \qquad (6.25)$$

$$E_2 \quad = \quad \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) \, d\omega \right.$$

$$+ \frac{2\sqrt{2}}{\pi} (-1)^i \sum_{i=1}^\infty \int_0^\infty \frac{1}{\omega} \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) \, d\omega$$

$$\left. + \frac{2\sqrt{2}}{\pi} (-1)^{k+1} \sum_{k=1}^\infty \int_0^\infty \frac{1}{\omega} \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) \, d\omega \right]^2 \; (6.26)$$

$$q_1 \quad \simeq \quad \frac{\beta}{2\sqrt{2}} \qquad (6.27)$$

$$q_2 \quad \simeq \quad -\frac{\beta}{2\sqrt{2}} \qquad (6.28)$$

2. BPSK

$$P(e) = \frac{1}{2} Q\left(\frac{\sqrt{2TE} - \sqrt{4T}q}{\sqrt{N_0}}\right) + \frac{1}{2} Q\left(\frac{\sqrt{2TE} + \sqrt{4T}q}{\sqrt{N_0}}\right) \qquad (6.29)$$

where

$$E \quad = \quad \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_1(\omega A) \, d\omega \right]^2 \qquad (6.30)$$

$$q \quad \simeq \quad \frac{\beta}{2} \qquad (6.31)$$

● **Up and Downlink Interference**

1. **QPSK**

$$P(e) = \frac{1}{2} \left[Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right) \right.$$

$$+ Q\left(\frac{\sqrt{2TE_1} + \sqrt{4T}q_1}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2TE_2} - \sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$- Q\left(\frac{\sqrt{2TE_1} - \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} + \sqrt{4T}q_2}{\sqrt{N_0}}\right)$$

$$\left. - Q\left(\frac{\sqrt{2TE_1} + \sqrt{4T}q_1}{\sqrt{N_0}}\right) Q\left(\frac{\sqrt{2TE_2} - \sqrt{4T}q_2}{\sqrt{N_0}}\right) \right] \qquad (6.32)$$

where

$$E_1 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right]^2 \qquad (6.33)$$

$$E_2 = \left[\frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right.$$

$$+ \frac{2\sqrt{2}}{\pi}(-1)^i \sum_{i=1}^\infty \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^\infty J_{2i}\left(\frac{\omega A}{\sqrt{2}}\right) J_{2i+1}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega$$

$$+ \frac{2\sqrt{2}}{\pi}(-1)^{k+1} \sum_{k=1}^\infty \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^\infty J_{2k-1}\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\left. \cdot J_{2k}\left(\frac{\omega A}{\sqrt{2}}\right) d\omega \right]^2 \qquad (6.34)$$

$$q_1 \simeq \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\beta}{\sqrt{2}}\right) d\omega$$

$$+ \frac{\beta}{2\sqrt{2}} \qquad (6.35)$$

$$q_2 \simeq -\frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_1\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{i=1}^\infty (-1)^{i+1} J_{2i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{2i}\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0^2\left(\frac{\omega A}{\sqrt{2}}\right) \sum_{k=1}^\infty (-1)^k J_{2k}\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{2k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) d\omega$$

$$- \frac{\beta}{2\sqrt{2}} \qquad (6.36)$$

2. **BPSK**

$$P(e) = \frac{1}{2} Q\left(\frac{\sqrt{2TE} - \sqrt{4T}q}{\sqrt{N_0}}\right) + \frac{1}{2} Q\left(\frac{\sqrt{2TE} + \sqrt{4T}q}{\sqrt{N_0}}\right) \qquad (6.37)$$

where

$$\sqrt{E} \;=\; \frac{2\sqrt{2}}{\pi} \int_0^\infty \frac{1}{\omega} J_0(\omega\alpha) J_1(\omega A)\, d\omega \qquad (6.38)$$

$$q \;\simeq\; \frac{2}{\pi} \int_0^\infty \frac{1}{\omega} J_0(A\omega) J_1(\omega\alpha)\, d\omega + \frac{\beta}{2} \qquad (6.39)$$

6.1.3 Numerical Comparison

In order to compare the performance of the two signalling schemes, the probability of symbol error is evaluated for both schemes. Figs. 6.1 through C.102 show the performance for both signalling schemes under different conditions.

- Figs. 6.1 to C.89 are the performance for the average case with interference on the uplink.

- Figs. C.90 to C.94 are the symbol error rate for the worst case with uplink interference.

- Figs. C.95 to C.99 are the probability of symbol error rate for the worst case with downlink interference.

- Figs. C.100 to C.102 are the performance for the worst case with interference on the uplink and the downlink. Here, Figs. C.100 and C.101 are the case of performance where interference power is accumulated in up and down satellite link. However, Fig. C.102 is the case of performance where the interference power is divided equally into uplink and downlink.

From an examination of these curves we can conclude that to achieve the same bit error rate a QPSK system requires 1 to 2 dB more energy per bit than a BPSK system. This conclusion holds over a wide range of signal to noise and signal to interference ratios. It is shown that BPSK signalling systems are better than QPSK signalling systems by on the average more than 1 dB in most conditions on the basis of symbol error rate.
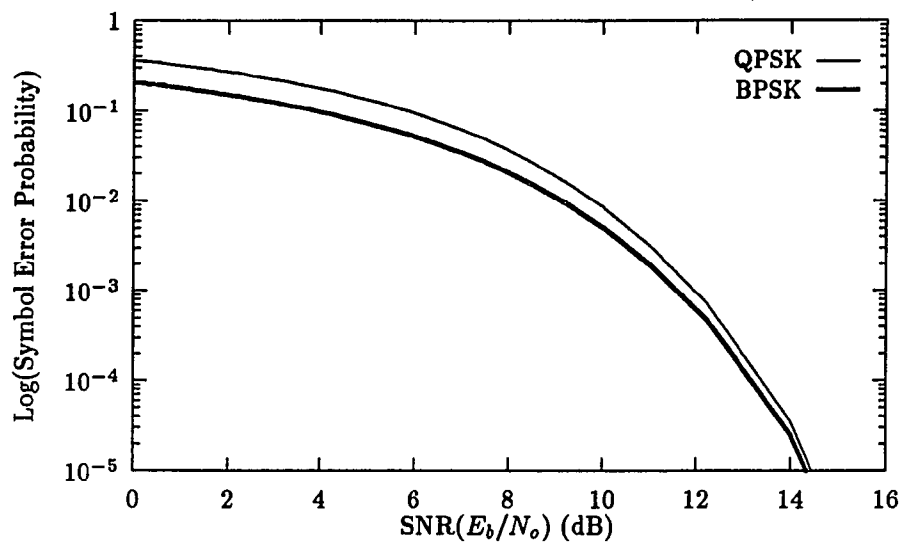
Figure 6.1: Symbol error rate comparison in the average case between QPSK and BPSK for S/I = 0 dB(uplink)
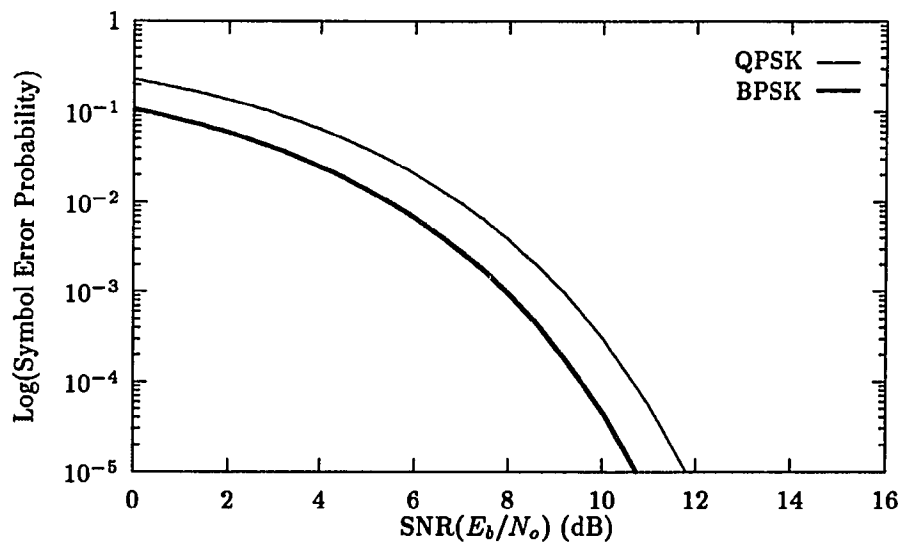
Figure 6.2: Symbol error rate comparison in the average case between QPSK and BPSK for S/I = 10 dB(uplink)

6.2 Summary

This thesis has addressed the problem of communicating over satellite repeater channels in the presence of AWGN and interference. The proposed systems use DS spread QPSK signalling and DS spread BPSK signalling. The performance of these two methods is examined in detail. The systems are analyzed and evaluated mathematically for three conditions; uplink interference, downlink interference, and up and downlink interference. For each condition, the probability of symbol error is evaluated for the average case and the worst case. The major contributions of this report are:

- An analysis of the output of a band pass limiter whose input consists of a P-N DS spread QPSK signal plus cochannel interference by a similar signal plus white Gaussian noise. Expressions are derived for the signal, interference, intermodulation product, and noise terms in the limiter output. These expressions are then used to evaluate the error performance of a correlation receiver whose input consists of the limiter output plus possible additional interference plus noise.

- A similar analysis was performed for BPSK signals.

- The entire system was simulated using the BOSS software package to check the assumptions made in the theoretical analysis.

- With simulation, we were able to explore the effect of processing gain and unbalanced codes on system performance which could not be addressed analytically.

- We could find optimum processing gain. The optimum processing gain observed for three different conditions (uplink, downlink, and up and downlink interference), is typically around 18 dB.

- It is shown that the performance is worse when we use an unbalanced spreading code compared to a balanced spreading code.

- From the numerical results, it is found that the performance saturates at around 15 dB of S/I for the average case and 40 dB for the worst case in both signalling systems.

- The results for the BPSK systems were compared with those for the QPSK systems. They are compared on the basis of symbol error rate and it is shown that the performance of BPSK systems is typically better than QPSK systems by 1 to 2 dB.

- For the same signal to interference power ratio, it was observed that the performance with uplink interference is better than with downlink interference, for S/I > 0 dB. And when the total interference power is fixed, the probability of error for up and downlink is worse than the one for the uplink interference and is better than the one for the downlink interference.

6.3   Suggestions for the Future Work

While we investigated in detail the performance of the satellite channel with cochannel interference, some topics could be pursued further. We list them as possible topics for further research as follows:

- The mathematical analysis assumes many simplifications, as mentioned earlier. The removal of some of these assumptions will improve the accuracy of the analytical approach. If the uplink white Gaussian noise is considered in the final expression for the probability of symbol error, it is expected that the performance expression will be more complicated and the performance worse than the value we obtained.

- In our work, performance is investigated as a function of interference, but it is suggested to analyze the probability of symbol error as a function of the processing gain in future study.

- Performance analysis when we use channel coding in this system model.

- Interference power calculation for the even order powers of $c(t)$ in intermodulation product terms of QPSK.

- In this work, performance with single cochannel interference was analyzed. A more realistic analysis of cochannel interference is the problem of multiple interference.

- Generalized performance expression for the M-ary PSK.

APPENDIX A

• **Intermodulation Product Terms in Bandpass Limiter Output Stage**

By using the condition of $|i-k| = 1$, $\omega_0 \simeq \omega_1$, and $\Delta\omega = |\omega_1 - \omega_0|$, intermodulation product terms of previous section can be calculated as followings

$$
\begin{aligned}
u_{mq}(t) \;=\; & \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{j\omega n_1(t)} \left[-2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty} c^{i+1}(t)p^i(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \right. \\
& \cdot \cos\{\omega_0 t - \frac{\pi}{2}(i+1) - i\Delta\omega t\} \\
& + 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} c^k(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos\{\omega_0 t - \frac{\pi}{2}k + (k+1)\Delta\omega t\} \\
& + 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty} (-1)^i c^{i+1}(t)p^i(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos(\omega_0 t - i\Delta\omega t) \\
& + 2J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{k=1}^{\infty} (-1)^k c^k(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot \cos\{\omega_0 t + (k+1)\Delta\omega t\} \\
& + 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^i c^{i+k}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \\
& \cdot \cos\{\omega_0 t + \frac{\pi}{2}k + (i+k+1)\Delta\omega t\} \\
& - 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} (-1)^i c^{i+k}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \\
& \cdot \cos\{\omega_0 t - \frac{\pi}{2}k - (i+k-1)\Delta\omega t\} \\
& + 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right) \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{i+k}(t)p^{1-i+k}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) \\
& \left. \cdot \cos\{\omega_0 t + \frac{\pi}{2}k + (1-i+k)\Delta\omega t\} \right.
\end{aligned}
$$

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$$- 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k}(t)p^{-1-i+k}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}k-(-1-i+k)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{i+k}c^{i+k}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}k-(i-k-1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{i+k}c^{i+k}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}k+(i-k+1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}(-1)^{i}c^{i+1}(t)p^{i}(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos(\omega_0 t-\frac{\pi}{2}-i\Delta\omega t)$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}(-1)^{k}c^{k}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}-(k+1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{-i+k}c^{k}(t)p^{k-1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(i+1)-(k-1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{-i+k}c^{k}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}(i-1)+(k+1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k}c^{k}(t)p^{k-1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(1-i)+(1-k)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k}c^{k}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(1+i)+(1+k)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}c^{k}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(k+1)-(k+1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{l=1}^{\infty}c^{l+1}(t)p^{l}(t)J_{l+1}\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}l - l\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}k - (k+i-1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}k + (k+i+1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}k + (i-k+1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}k - (i-k-1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^i c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}k - (k-i-1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^i c^k(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}k + (k-i+1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{i+k}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}(i-1) - (i+k-1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{i+k}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{i+k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(i+1) + (i+k+1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k-i} c^{i+k}(t)p^{k-i+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}(i-1) + (k-i+1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k-i} c^{i+k}(t)p^{k-i-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(i+1) - (k-i-1)\Delta\omega t\}$$

$$+ 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{i-k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t + \frac{\pi}{2}(i-1)-(i-k-1)\Delta\omega t\}$$

$$- 2J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{i-k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t - \frac{\pi}{2}(i+1)+(i-k+1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^k c^{i+k}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{i+k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t - \frac{\pi}{2}(-i-l+1)+(-i-k+1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^k c^{i+k}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{i+k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t - \frac{\pi}{2}(i+l+1)+(i+k+1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^i c^{i+k}(t)p^{-i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-i-k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t - \frac{\pi}{2}(-i+l+1)-(i+k-1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^i c^{i+k}(t)p^{-i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-i-k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t + \frac{\pi}{2}(-i+l-1)+(i+k+1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-l} c^{i+k}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i+k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t - \frac{\pi}{2}(-i+l+1)-(i+k-1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-l} c^{i+k}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i+k-l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t - \frac{\pi}{2}(-i+l-1)+(i+k+1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i-l} c^{i+k}(t)p^{-i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k-l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t + \frac{\pi}{2}(i+l-1)+(-i+k+1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i-l} c^{i+k}(t)p^{-i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t - \frac{\pi}{2}(i+l+1)+(i-k+1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} c^{i+k}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}(i+l-1)+(-i+k+1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} c^{i+k}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(i+l+1)+(i-k+1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i} c^{i+k}(t)p^{-i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t+\frac{\pi}{2}(i-l-1)+(-i+k+1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i} c^{i+k}(t)p^{-i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t-\frac{\pi}{2}(i-l+1)+(-i+k-1)\Delta\omega t\}$$

$$+ 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{l} c^{i+k}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}(i-l-1)-(i-k-1)\Delta\omega t\}$$

$$- 2\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{l} c^{i+k}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i-k-l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\left. \cdot\cos\{\omega_0 t-\frac{\pi}{2}(i-l+1)+(i-k+1)\Delta\omega t\}\right] d\omega \tag{A.1}$$

APPENDIX B

● **Despread output of intermodulation product term**

$$
\begin{aligned}
r_{mq}(t) \;=\; & -\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{j\omega n_1(t)}\left[J_0\!\left(\frac{\omega A}{\sqrt{2}}\right)J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}c^i(t)p^i(t)J_{i+1}\!\left(\frac{\omega A}{\sqrt{2}}\right)J_i\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\right. \\
& \cdot\cos\{\omega_0 t-\frac{\pi}{2}(i{+}1)-i\Delta\omega t\} \\
& + J_0\!\left(\frac{\omega A}{\sqrt{2}}\right)J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}c^{k+1}(t)p^{k+1}(t)J_k\!\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\!\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot\cos\{\omega_0 t-\frac{\pi}{2}k+(k{+}1)\Delta\omega t\} \\
& + J_0\!\left(\frac{\omega A}{\sqrt{2}}\right)J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}(-1)^i c^i(t)p^i(t)J_{i+1}\!\left(\frac{\omega A}{\sqrt{2}}\right)J_i\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos(\omega_0 t-i\Delta\omega t) \\
& + J_0\!\left(\frac{\omega A}{\sqrt{2}}\right)J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}(-1)^k c^{k+1}(t)p^{k+1}(t)J_k\!\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\!\left(\frac{\omega\alpha}{\sqrt{2}}\right) \\
& \cdot\cos\{\omega_0 t+(k{+}1)\Delta\omega t\} \\
& - J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^i c^{i+k+1}(t)p^{i+k-1}(t)J_i\!\left(\frac{\omega A}{\sqrt{2}}\right)J_k\!\left(\frac{\omega A}{\sqrt{2}}\right) \\
& \cdot\cos\{\omega_0 t-\frac{\pi}{2}k-(i{+}k{-}1)\Delta\omega t\} \\
& + J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k+1}(t)p^{1-i+k}(t)J_i\!\left(\frac{\omega A}{\sqrt{2}}\right)J_k\!\left(\frac{\omega A}{\sqrt{2}}\right) \\
& \cdot\cos\{\omega_0 t+\frac{\pi}{2}k+(1{-}i{+}k)\Delta\omega t\} \\
& - J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{i+k+1}(t)p^{-1-i+k}(t)J_i\!\left(\frac{\omega A}{\sqrt{2}}\right)J_k\!\left(\frac{\omega A}{\sqrt{2}}\right) \\
& \cdot\cos\{\omega_0 t-\frac{\pi}{2}k-(-1{-}i{+}k)\Delta\omega t\} \\
& \left. - J_0\!\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{i+k}c^{i+k+1}(t)p^{i-k-1}(t)J_i\!\left(\frac{\omega A}{\sqrt{2}}\right)J_k\!\left(\frac{\omega A}{\sqrt{2}}\right)\right.
\end{aligned}
$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}k - (i-k-1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{i+k}c^{i+k+1}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}k + (i-k+1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}(-1)^{i}c^{i}(t)p^{i}(t)J_{i+1}\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos(\omega_0 t - \frac{\pi}{2} - i\Delta\omega t)$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}(-1)^{k}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2} - (k+1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{-i+k}c^{k+1}(t)p^{k-1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(i+1) - (k-1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{-i+k}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}(i-1) + (k+1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k}c^{k+1}(t)p^{k-1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(1-i) + (1-k)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(1+i) + (1+k)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{k+1}(t)p^{-k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(i+1) - (k-1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{-k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}(i-1) + (k+1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{k=1}^{\infty}c^{k+1}(t)p^{k+1}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(k+1) - (k+1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right) J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{l=1}^{\infty} c^l(t)p^l(t)J_{l+1}\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t+\frac{\pi}{2}l-l\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}k-(k+i-1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k+i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}k+(k+i+1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{i-k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}k+(i-k+1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{i-k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}k-(i-k-1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^i c^{1+k}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}k-(k-i-1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega A}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^i c^{k+1}(t)p^{i+k}(t)J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_i\left(\frac{\omega\alpha}{\sqrt{2}}\right) J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}k+(k-i+1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{i+k+1}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}(i-1)-(i+k-1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^k c^{i+k+1}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{i+k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(i+1)+(i+k+1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k-i} c^{i+k+1}(t)p^{k-i+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k-i+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}(i-1)+(k-i+1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}(-1)^{k-i} c^{i+k+1}(t)p^{k-i-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_{k-i-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(i+1)-(k-i-1)\Delta\omega t\}$$

$$+ J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{i+k+1}(t)p^{i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{i-k-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t+\frac{\pi}{2}(i-1)+(i-k-1)\Delta\omega t\}$$

$$- J_0\left(\frac{\omega\alpha}{\sqrt{2}}\right)\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} c^{i+k+1}(t)p^{i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_{i-k+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(i+1)+(i-k+1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^k c^{i+k+1}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i+k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(-i-l+1)+(-i-k+1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^k c^{i+k+1}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)J_{i+k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot\cos\{\omega_0 t-\frac{\pi}{2}(i+l+1)+(i+k+1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^i c^{i+k+1}(t)p^{-i-k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i-k+l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t-\frac{\pi}{2}(-i+l+1)+(i+k-1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^i c^{i+k+1}(t)p^{-i-k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i-k+l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t+\frac{\pi}{2}(-i+l-1)+(i+k+1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-l} c^{i+k+1}(t)p^{i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{i+k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t-\frac{\pi}{2}(-i+l+1)+(i+k-1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-l} c^{i+k+1}(t)p^{i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{i+k-l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t-\frac{\pi}{2}(i+1)+(i+k+1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i-l} c^{i+k+1}(t)p^{-i+k+1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k-l+1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t+\frac{\pi}{2}(i+l-1)+(-i+k+1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{k-i-l} c^{i+k+1}(t)p^{-i+k-1}(t)J_i\left(\frac{\omega A}{\sqrt{2}}\right)J_k\left(\frac{\omega A}{\sqrt{2}}\right)J_l\left(\frac{\omega\alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k-l-1}\left(\frac{\omega\alpha}{\sqrt{2}}\right)\cos\{\omega_0 t-\frac{\pi}{2}(i+l+1)+(i-k+1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} c^{i+k+1}(t) p^{i-k-1}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{i-k+l-1}\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}(i+l-1) + (-i+k+1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} c^{i+k+1}(t) p^{i-k+1}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{i-k+l+1}\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t - \frac{\pi}{2}(i+l+1) + (i-k+1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^{k-i} c^{i+k+1}(t) p^{-i+k+1}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k+l+1}\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\{\omega_0 t + \frac{\pi}{2}(i-l-1) + (-i+k+1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^{k-i} c^{i+k+1}(t) p^{-i+k-1}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\cdot J_{-i+k+l-1}\left(\frac{\omega \alpha}{\sqrt{2}}\right) \cos\{\omega_0 t - \frac{\pi}{2}(i-l+1) + (-i+k-1)\Delta\omega t\}$$

$$+ \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^l c^{i+k+1}(t) p^{i-k-1}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{i-k-l-1}\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\cdot \cos\{\omega_0 t + \frac{\pi}{2}(i-l-1) - (i-k-1)\Delta\omega t\}$$

$$- \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} (-1)^l c^{i+k+1}(t) p^{i-k+1}(t) J_i\left(\frac{\omega A}{\sqrt{2}}\right) J_k\left(\frac{\omega A}{\sqrt{2}}\right) J_l\left(\frac{\omega \alpha}{\sqrt{2}}\right) J_{i-k-l+1}\left(\frac{\omega \alpha}{\sqrt{2}}\right)$$

$$\left. \cdot \cos\{\omega_0 t - \frac{\pi}{2}(i-l+1) + (i-k+1)\Delta\omega t\} \right] \, d\omega \tag{B.1}$$

APPENDIX C

• **Figures listed in this appendix are given as follows :**

- QPSK uplink interference

    - Numerical results: Fig.s C.1 to C.6

    - Simulation results: Fig.s C.7 to C.17

    - Comparison: Fig.s C.18 to C.23

- QPSK downlink interference

    - Numerical results: Fig.s C.24 to C.29

    - Simulation results: Fig.s C.31 to C.38

    - Comparison: Fig.s C.39 to C.43

- QPSK up and downlink Interference

    - Numerical results: Fig.s C.44 to C.53

    - Simulation results: Fig.s C.54 to C.62

    - Comparison: Fig.s C.63 to C.66

- BPSK uplink interference: Fig.s C.67 to C.72

- BPSK downlink interference: Fig.s C.73 to C.78

- BPSK up and downlink Interference: Fig.s C.79 to C.86

- Comparison between QPSK and BPSK: Fig.s C.87 to C.102
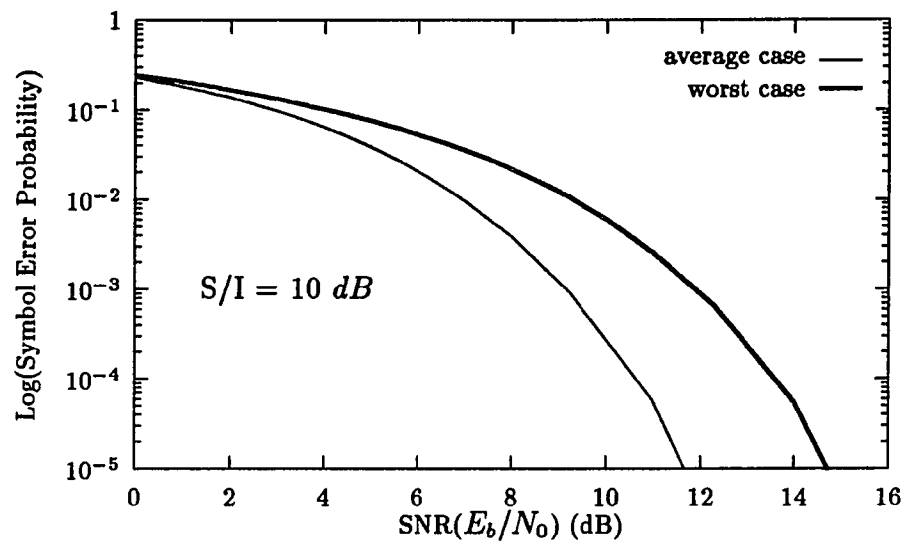
Figure C.1: Symbol error rate for the average case and the worst case as a function of E_b/N_0 (S/I = 10 dB)
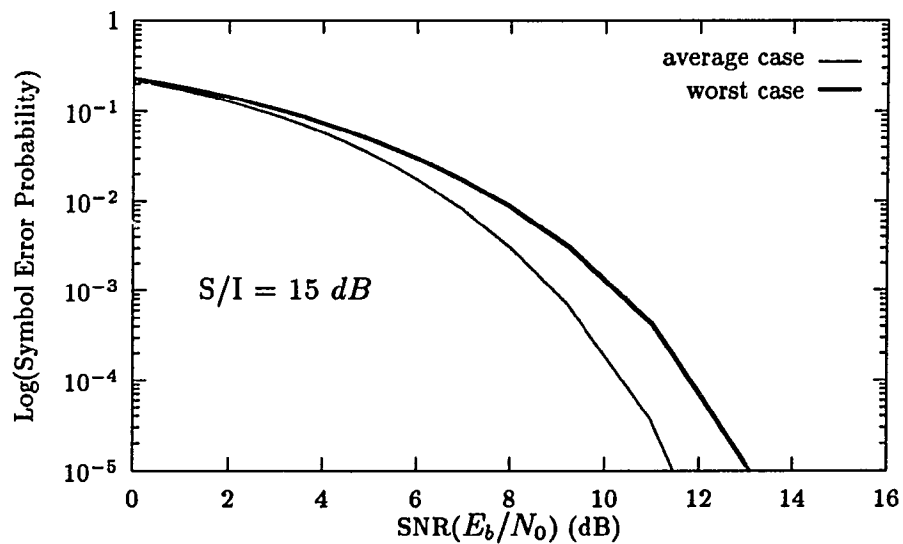
Figure C.2: Symbol error rate for the average case and the worst case as a function of E_b/N_0 (S/I = 15 dB)
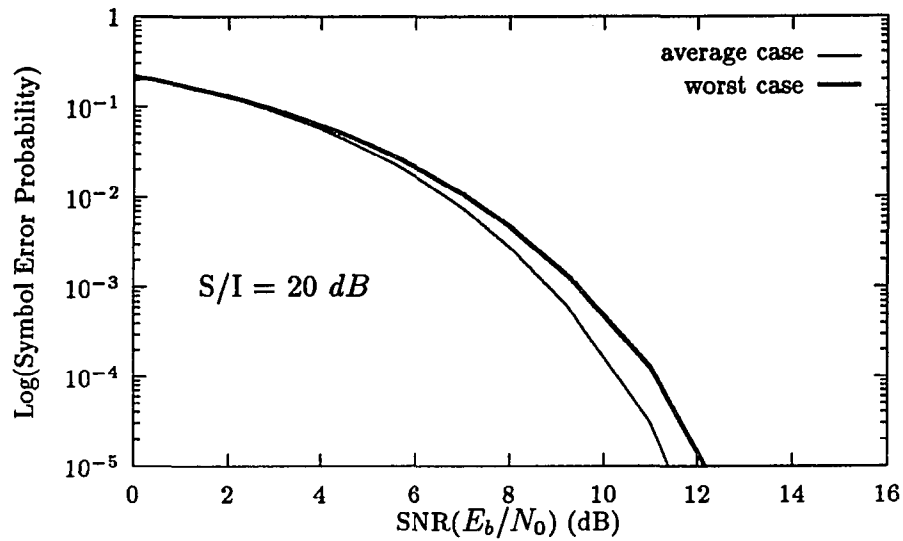
Figure C.3: Symbol error rate for the average case and the worst case as a function of E_b/N_0 (S/I = 20 dB)
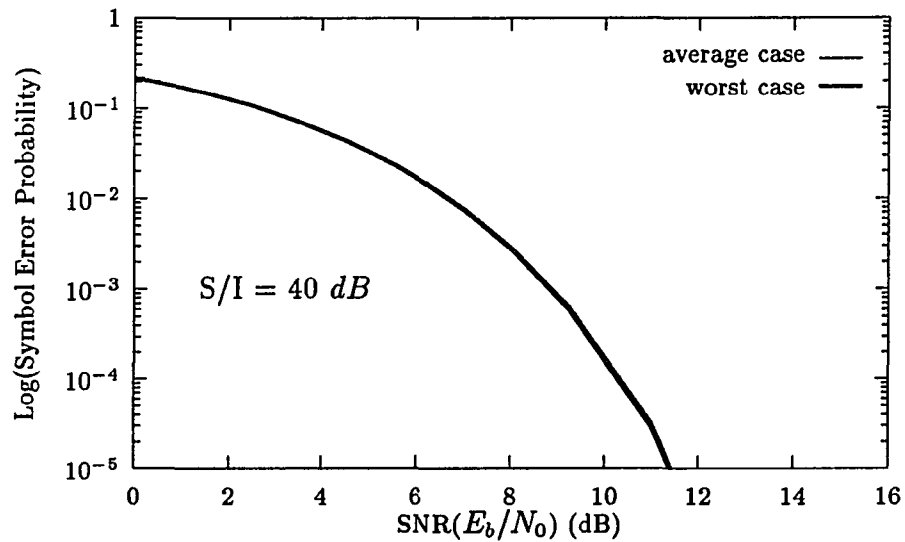
Figure C.4: Symbol error rate for the average case and the worst case as a function of E_b/N_0 (S/I = 40 dB)
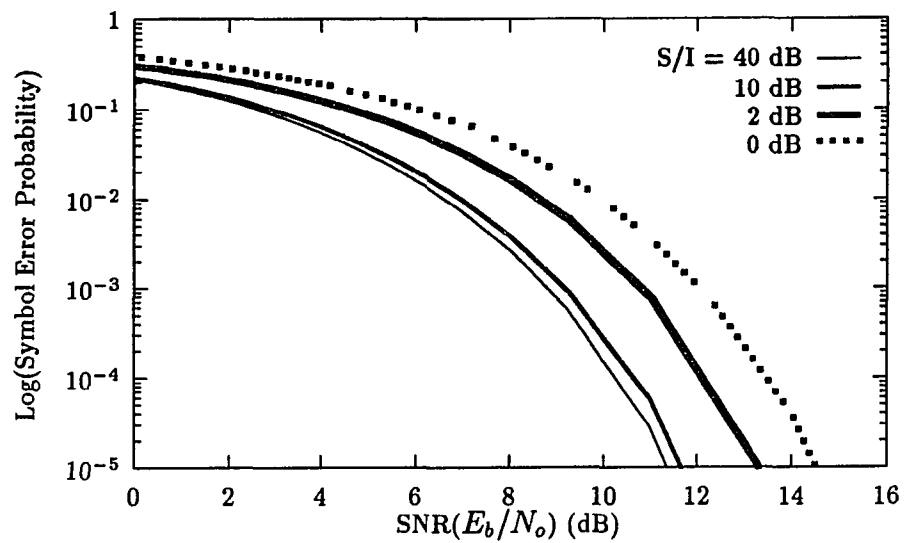
Figure C.5: Symbol error rate as a function of E_b/N_0 for the average case with S/I as a parameter (average case)
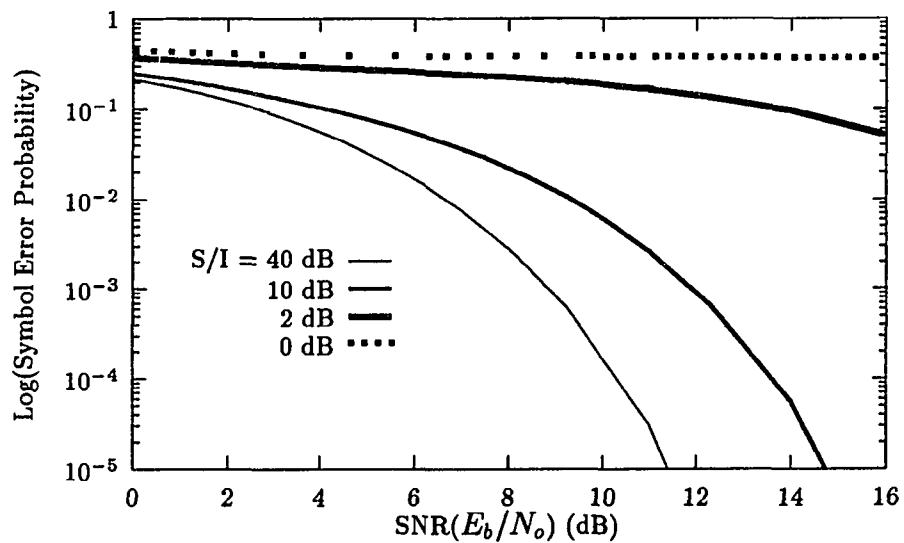
Figure C.6: Symbol error rate as a function of E_b/N_0 for the worst case with S/I as a parameter (worst case)
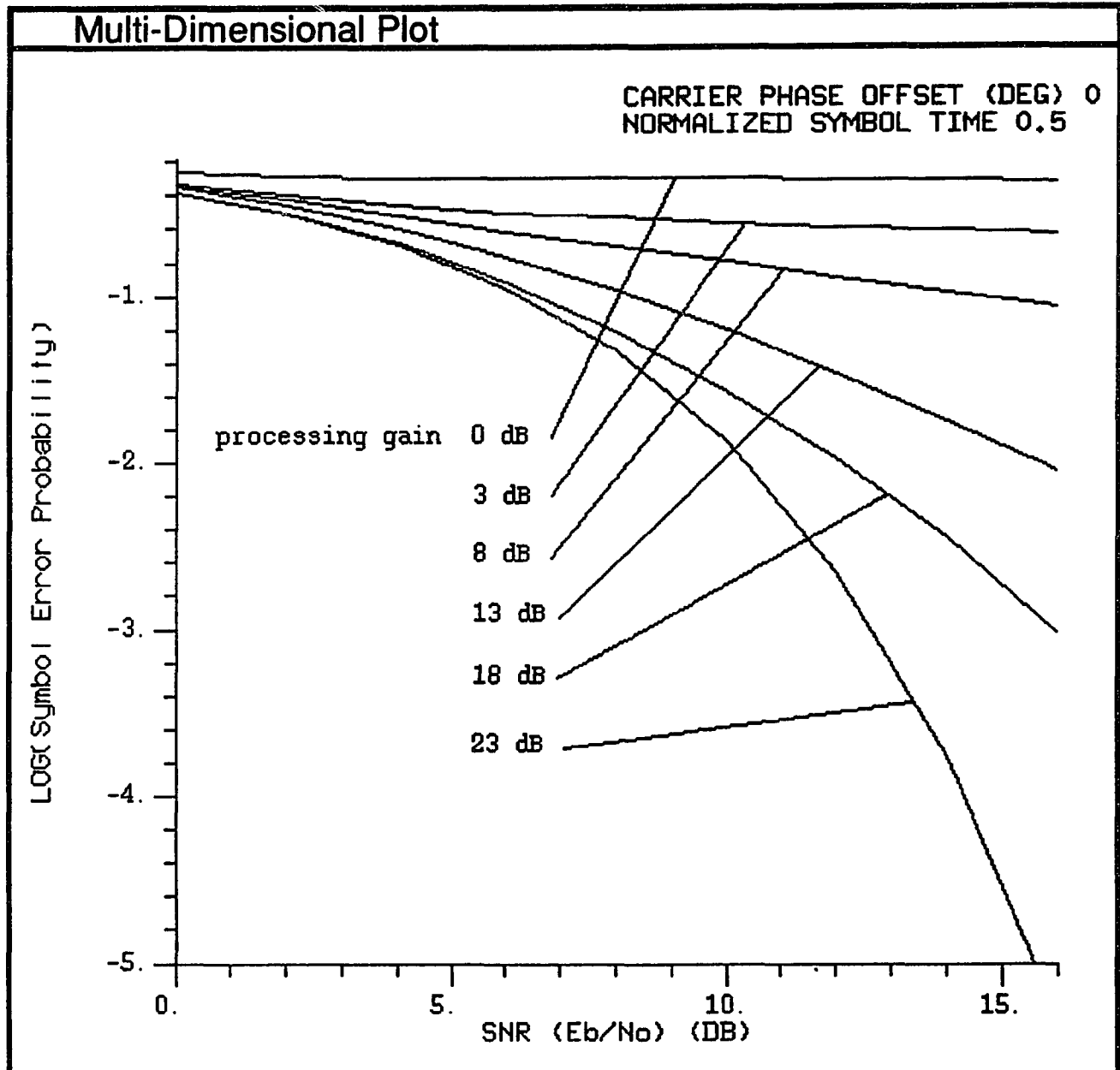
Figure C.7: Symbol error rate as a parameter of processing gain(S/I = 0 dB, uplink)
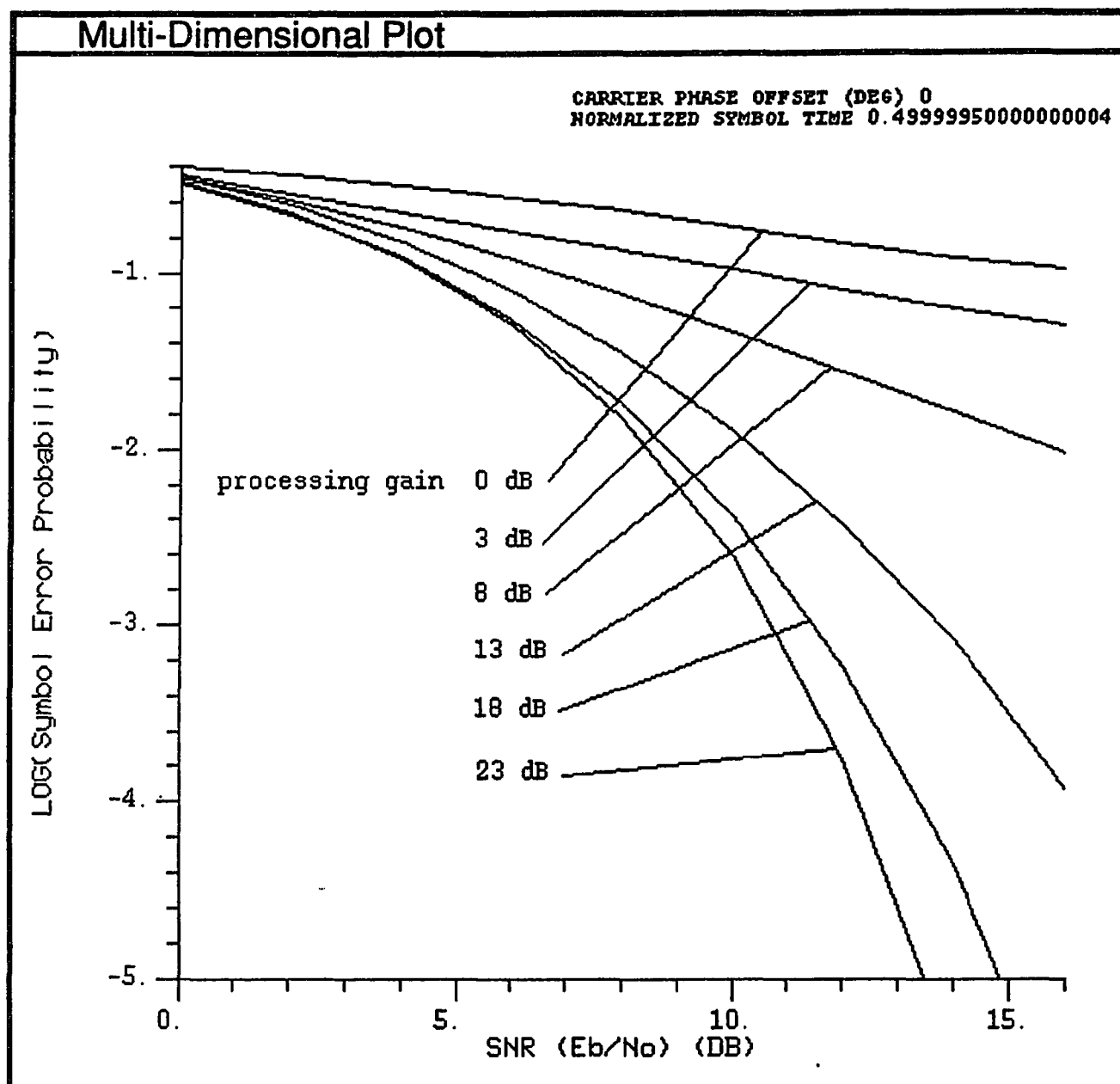
Figure C.8: Symbol error rate as a parameter of processing gain(S/I = 2 dB, uplink)
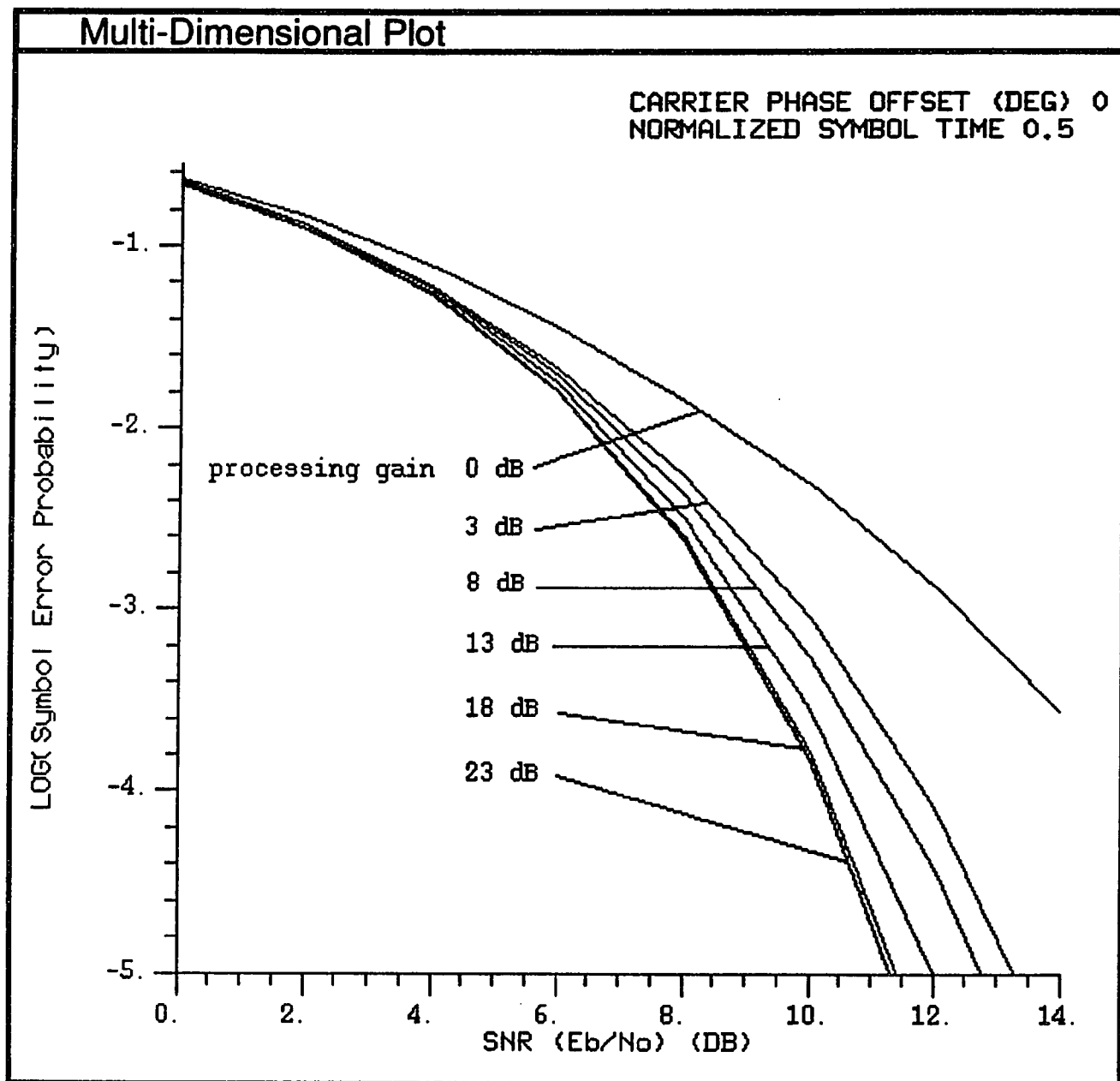
Figure C.9: Symbol error rate as a parameter of processing gain(S/I = 10 dB, uplink)
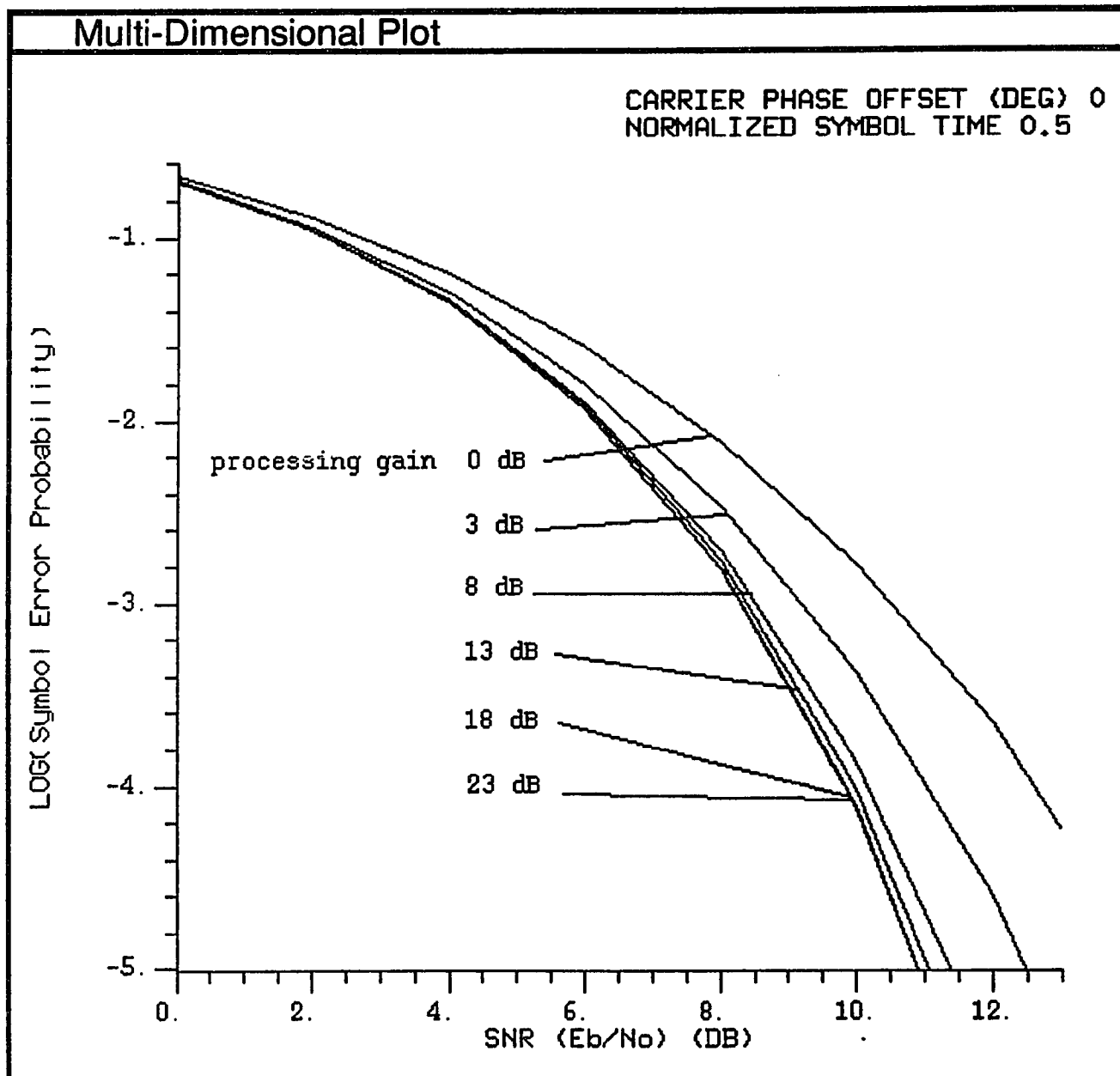
Figure C.10: Symbol error rate as a parameter of processing gain(S/I = 15 dB, uplink)
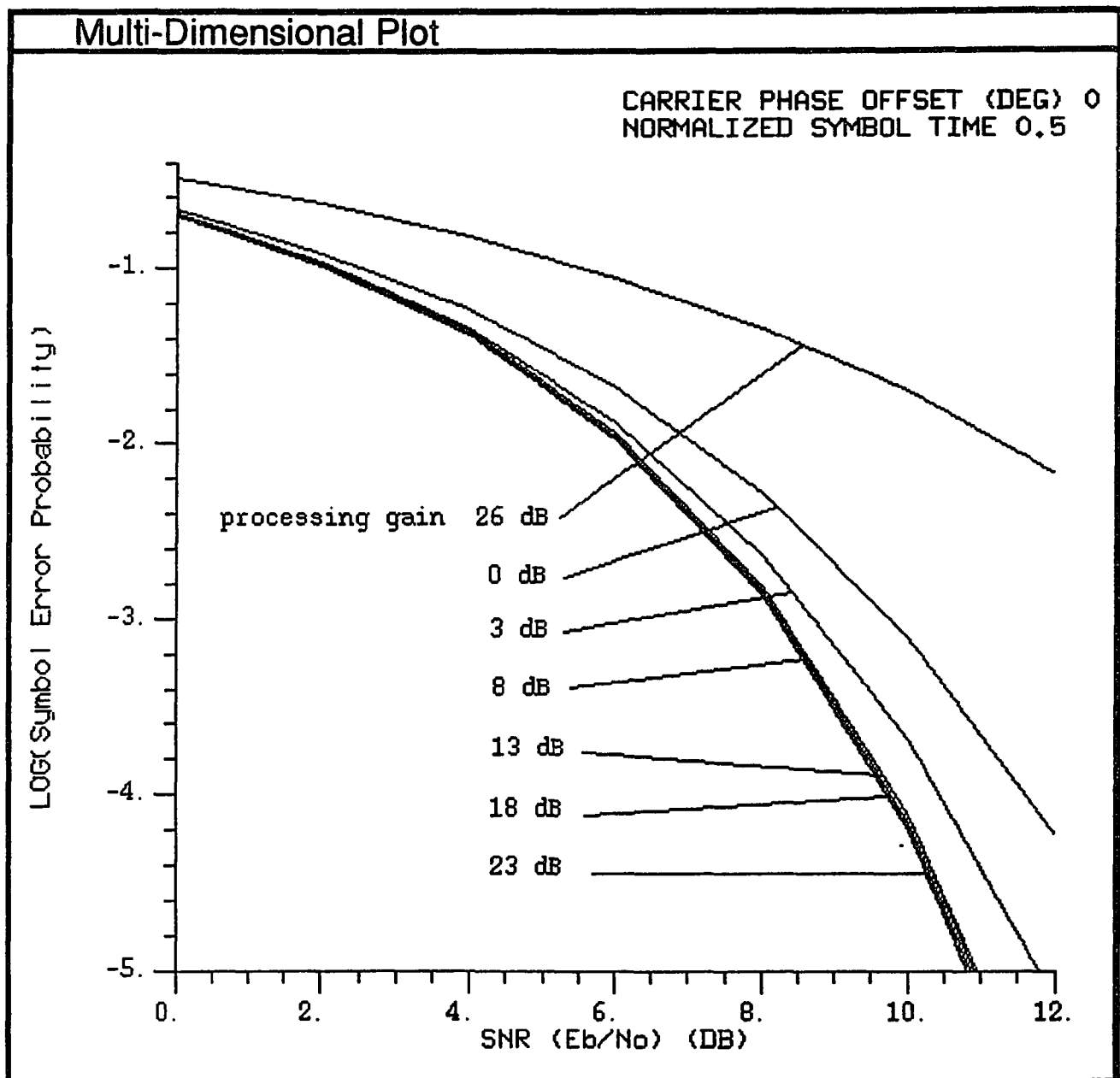
Figure C.11: Symbol error rate as a parameter of processing gain(S/I = 20 dB, uplink)
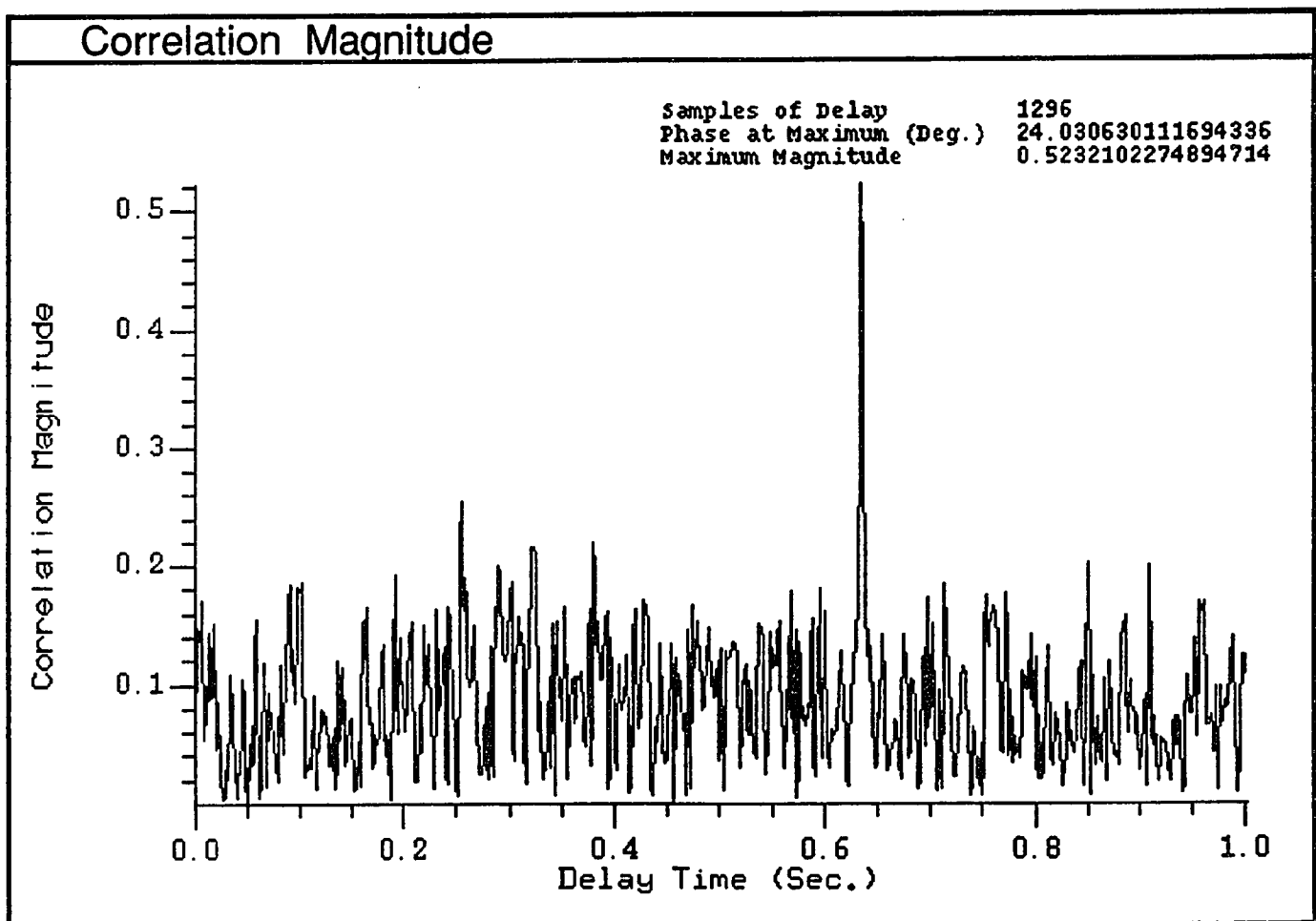
Figure C.12: Crosscorrelation magnitude of balanced code(P.G. = 24 dB, uplink)
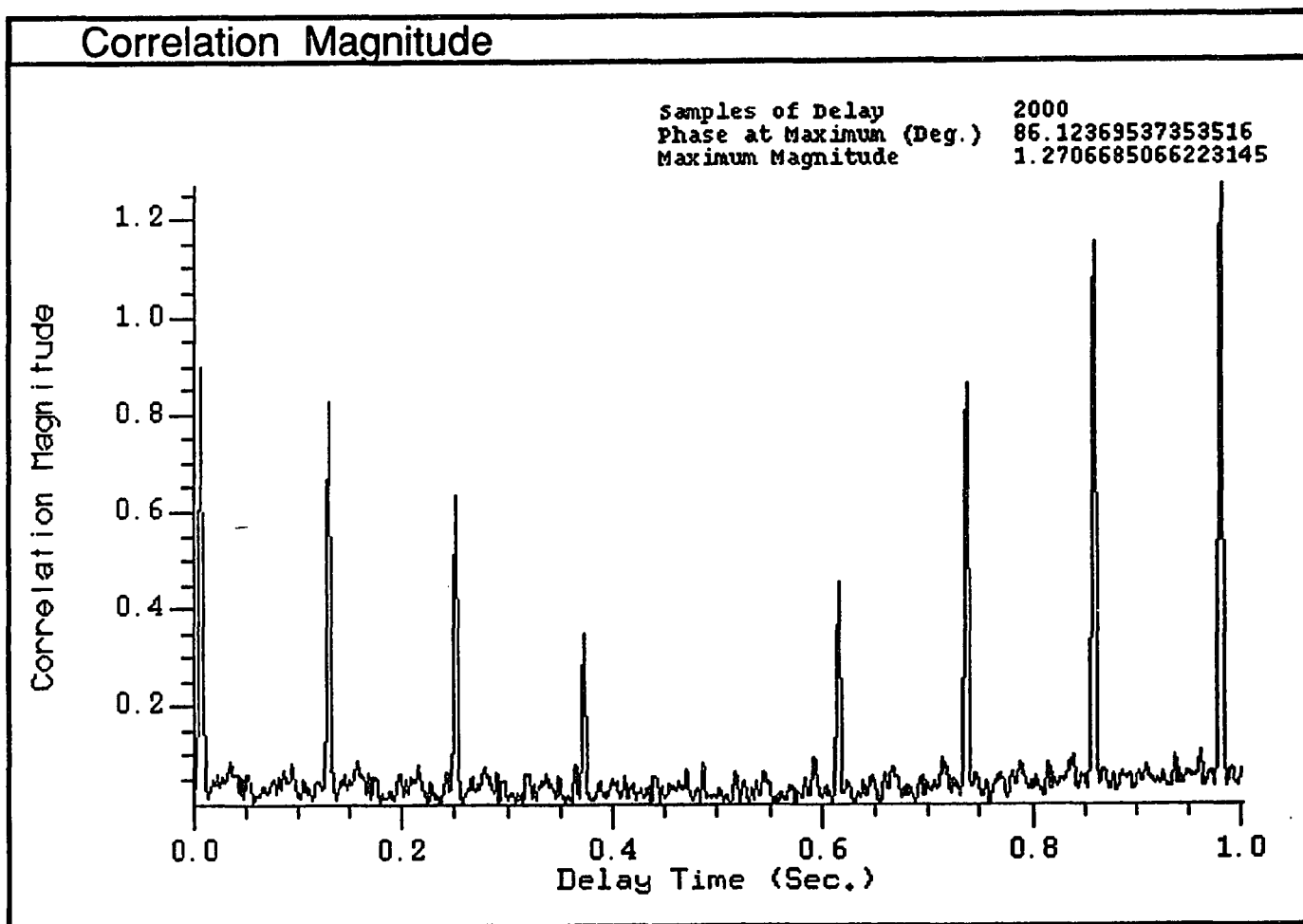
Figure C.13: Crosscorrelation magnitude of unbalanced code(P.G. $= 24$ dB, uplink)
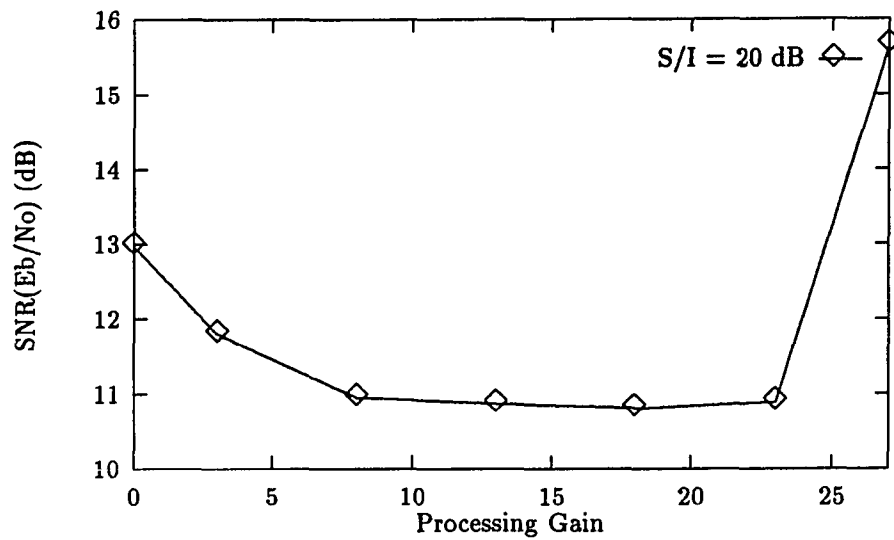
Figure C.14: SNR as a function of processing gain for symbol error rate = 10^{-5} & S/I = 20 dB, uplink
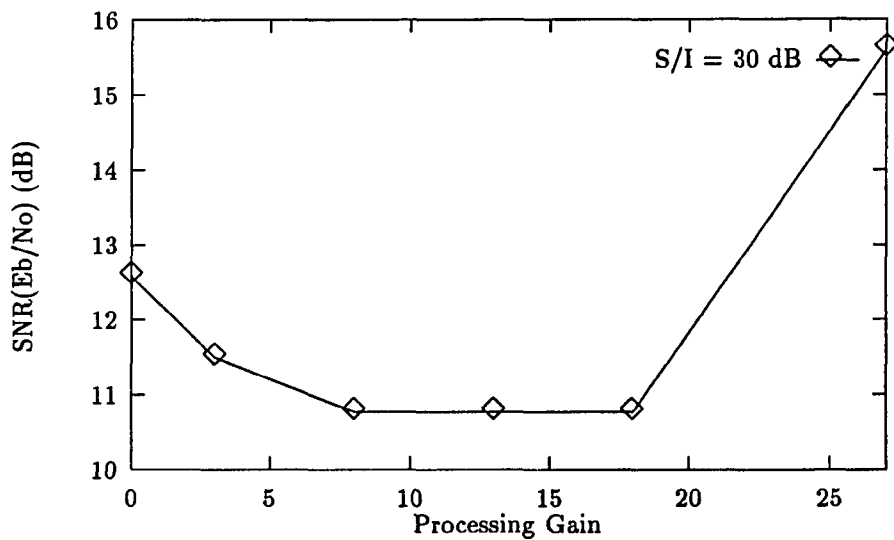
Figure C.15: SNR as a function of processing gain for symbol error rate = 10^{-5} & S/I = 30 dB, uplink
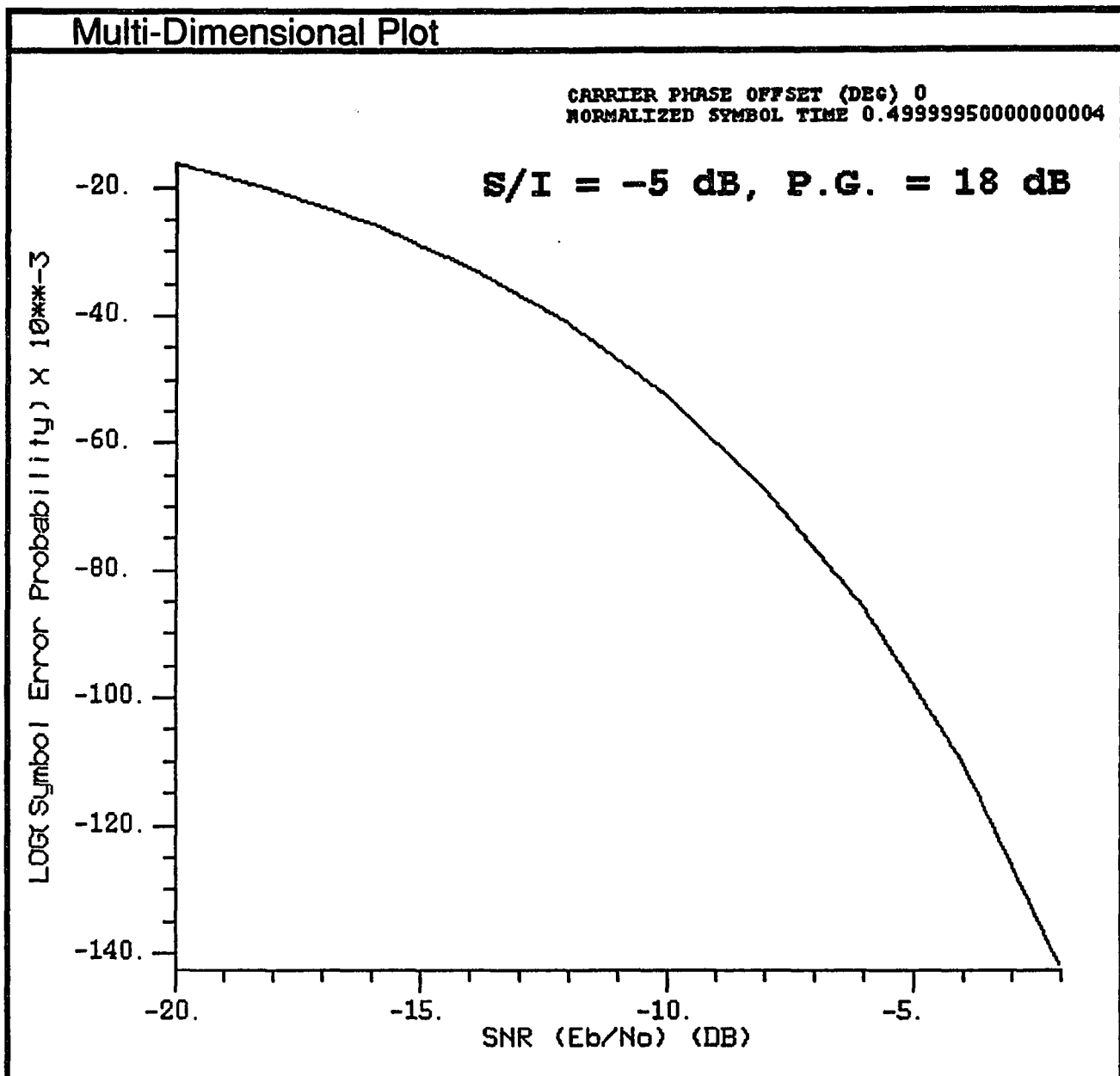
Figure C.16: Symbol error rate as a function of E_b/N_0 (S/I = –5 dB, P.G. = 18 dB)
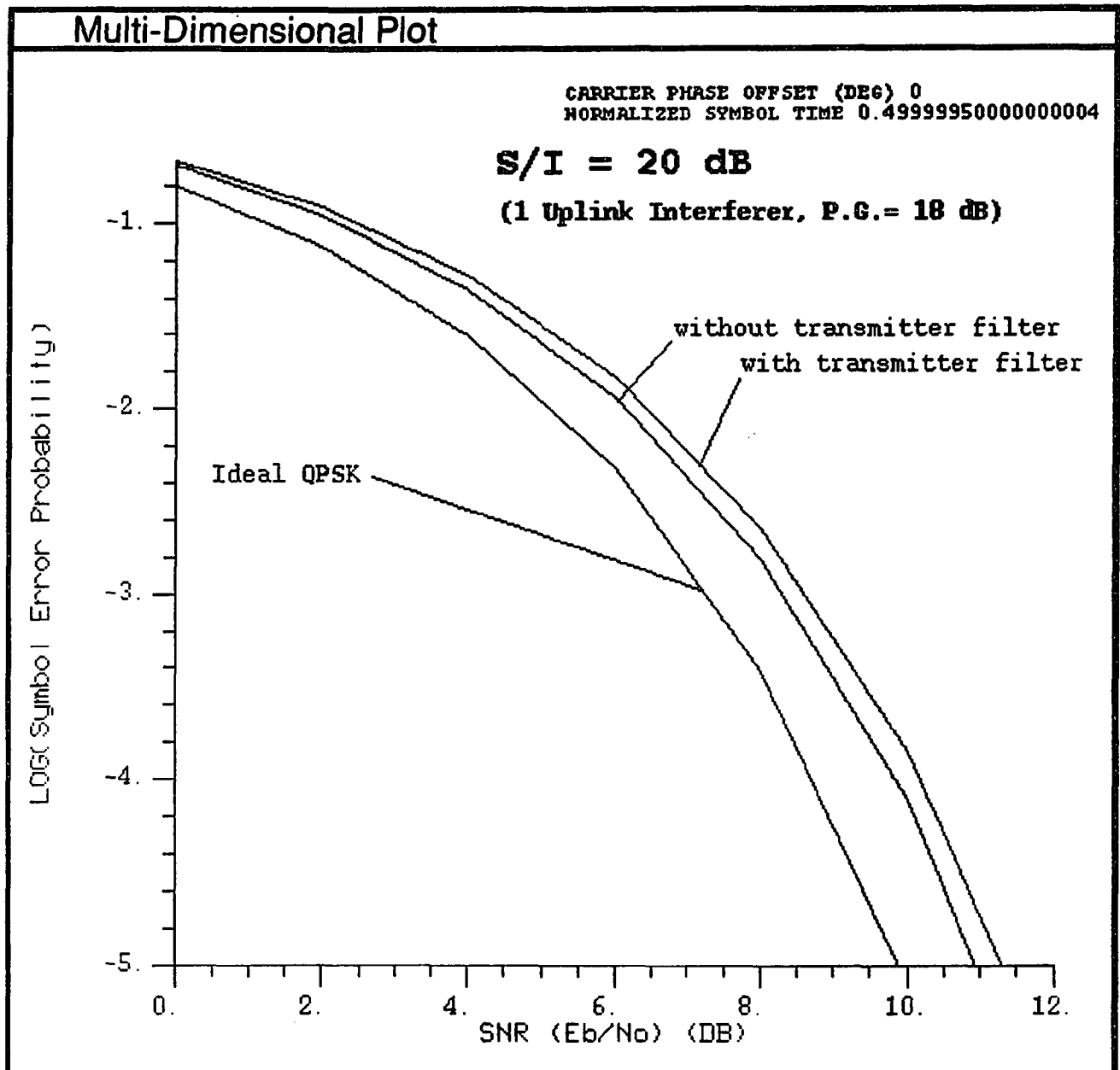
Figure C.17: Symbol error rate comparison between with and without transmitter filter (S/I = 20 dB, P.G. = 18 dB)
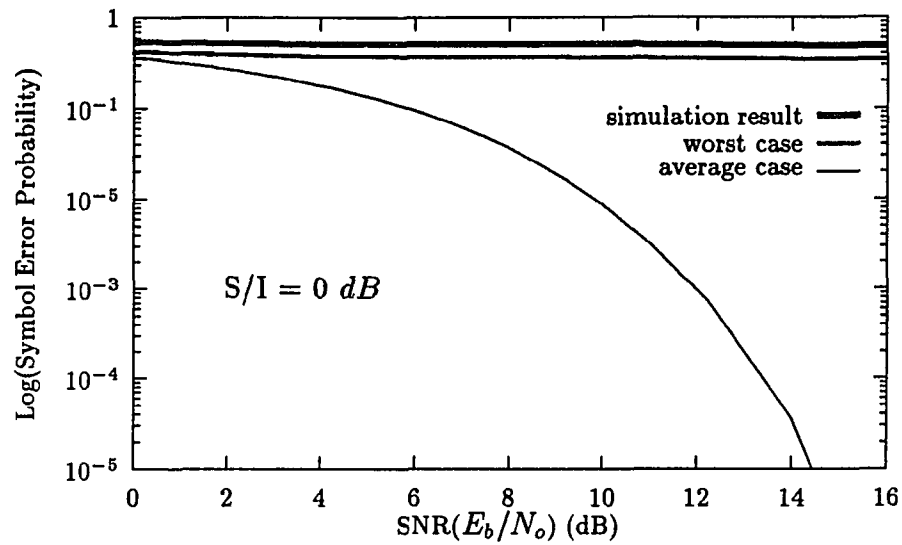
Figure C.18: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
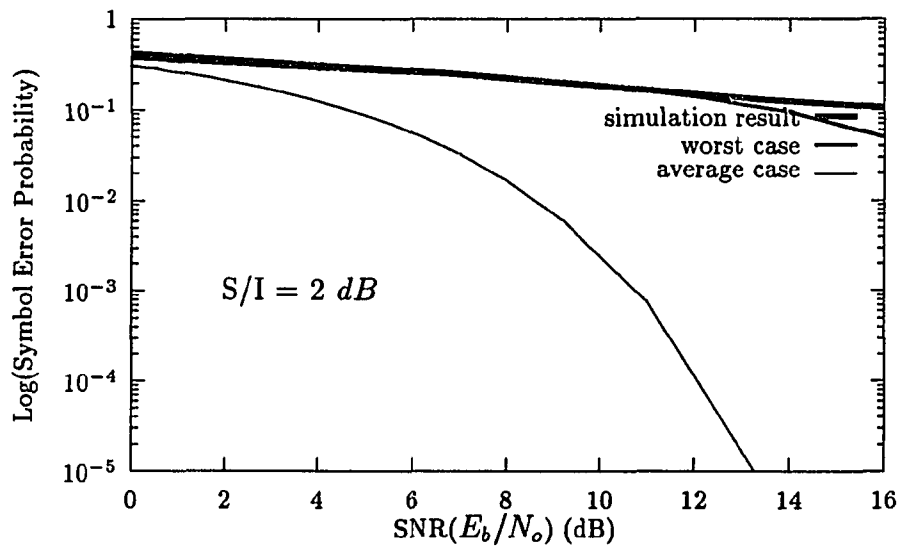
Figure C.19: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
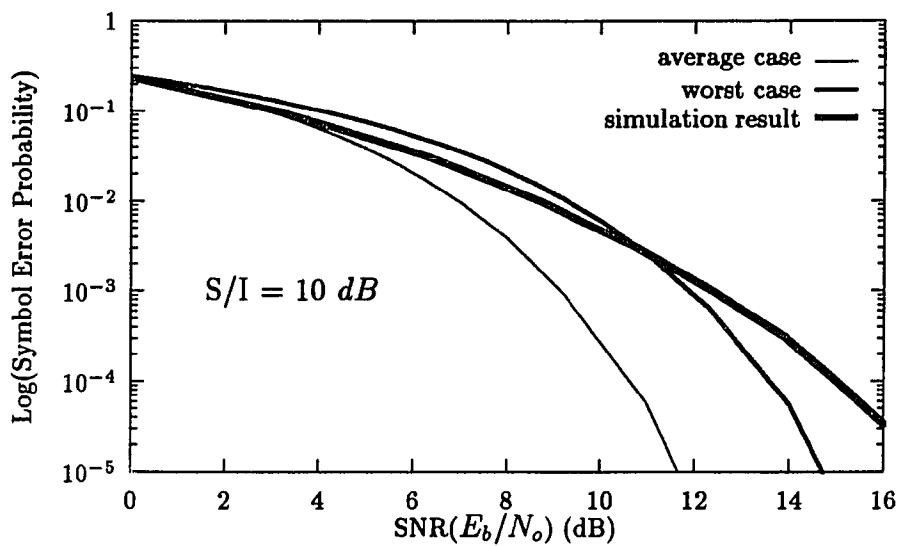
Figure C.20: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
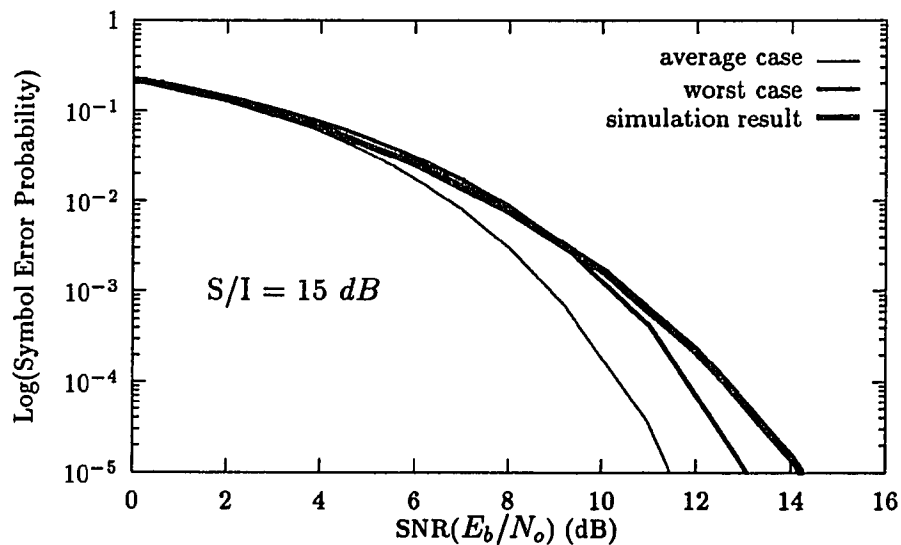
Figure C.21: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
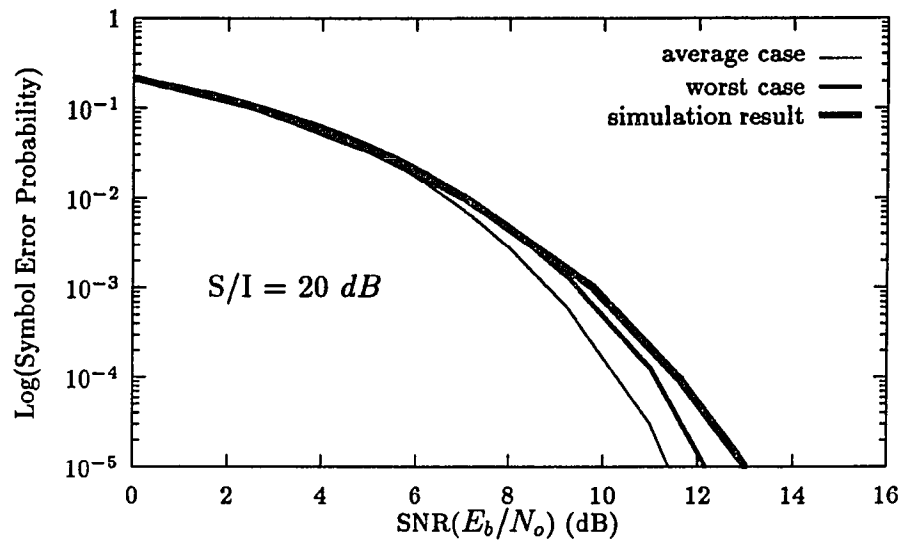
Figure C.22: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
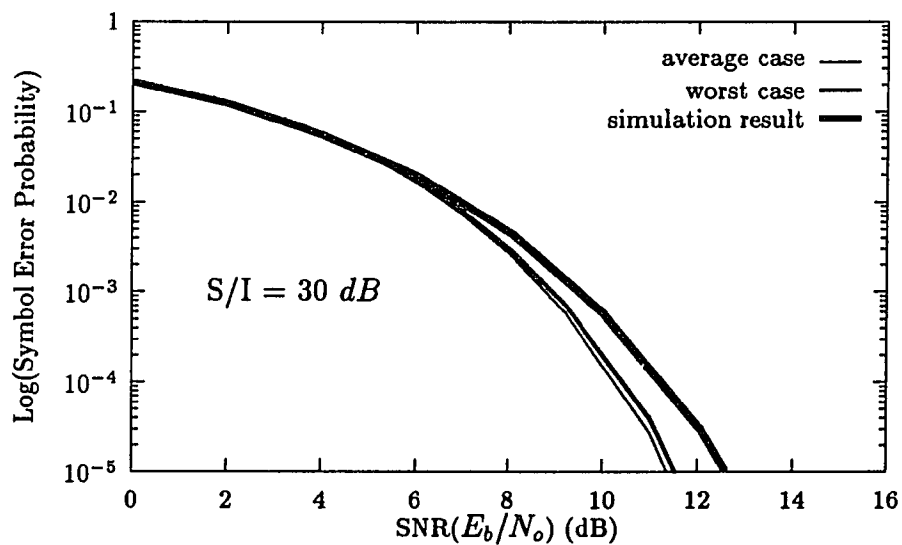
Figure C.23: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
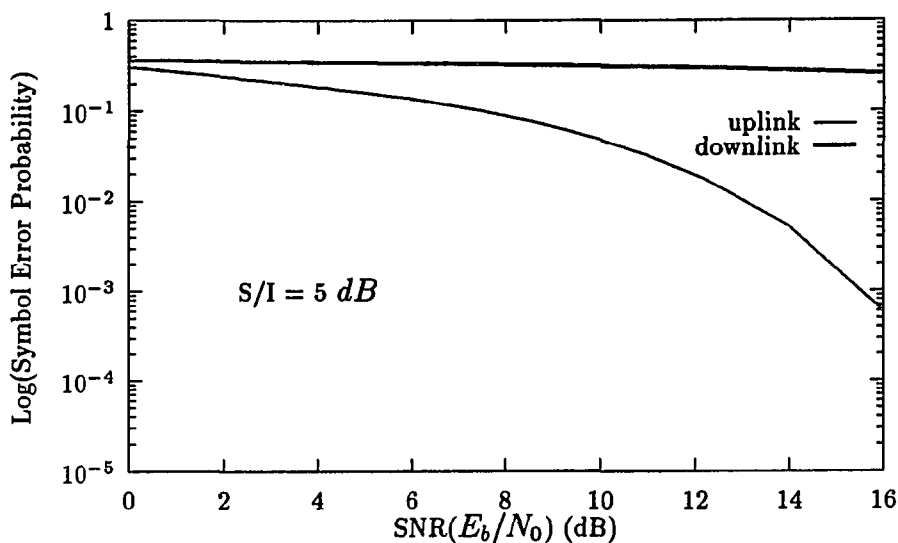
Figure C.24: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (S/I = 5 dB)
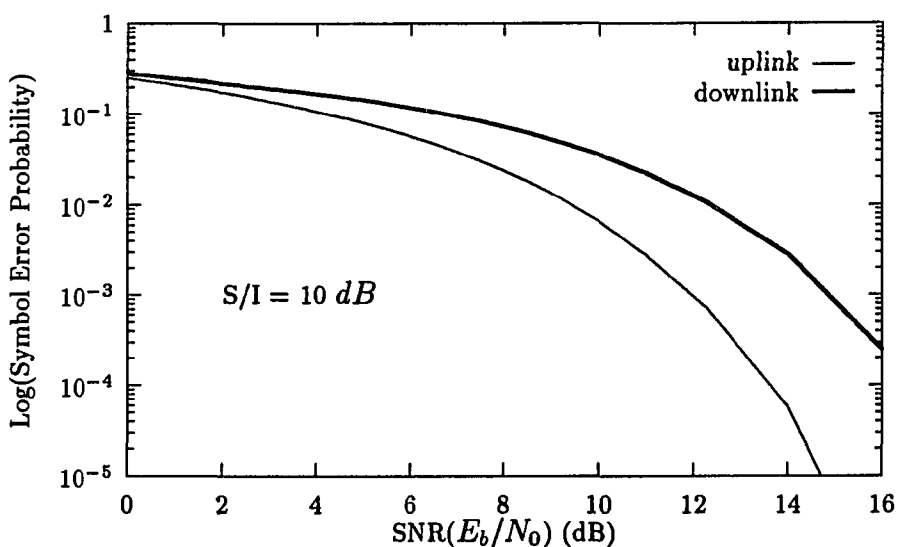
Figure C.25: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (S/I = 10 dB)
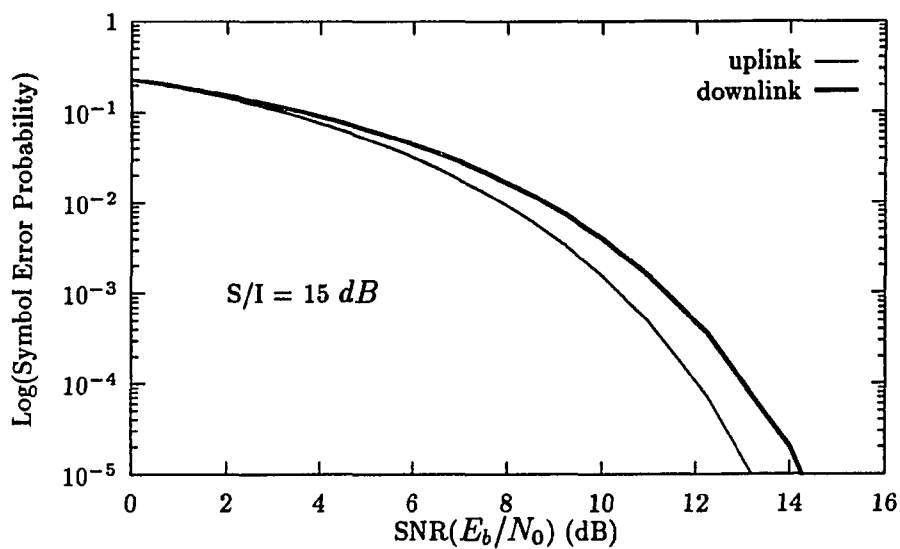
Figure C.26: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (S/I = 15 dB)
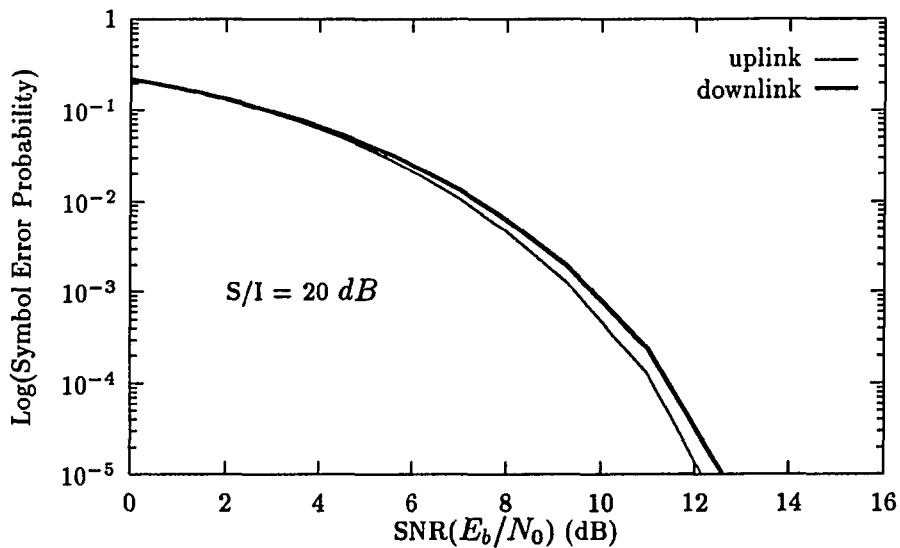
Figure C.27: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (S/I = 20 dB)
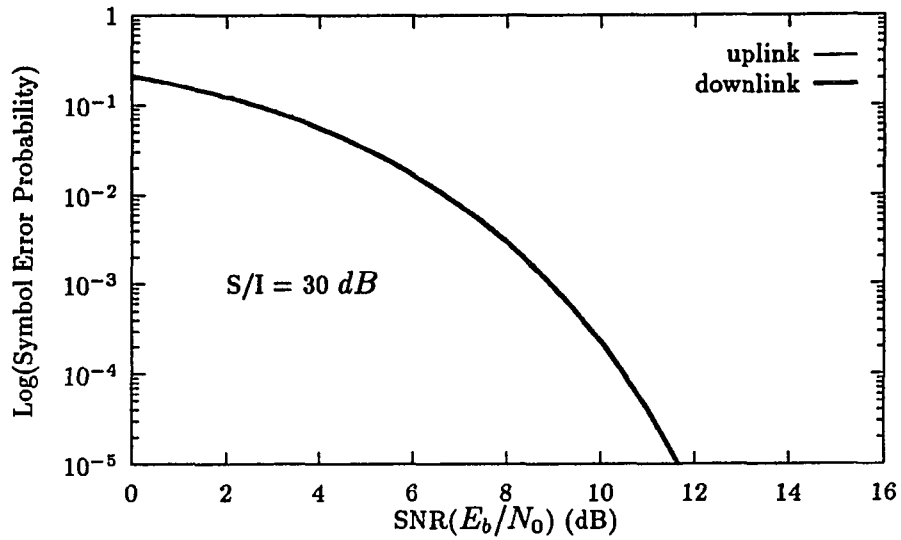
Figure C.28: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (S/I = 30 dB)
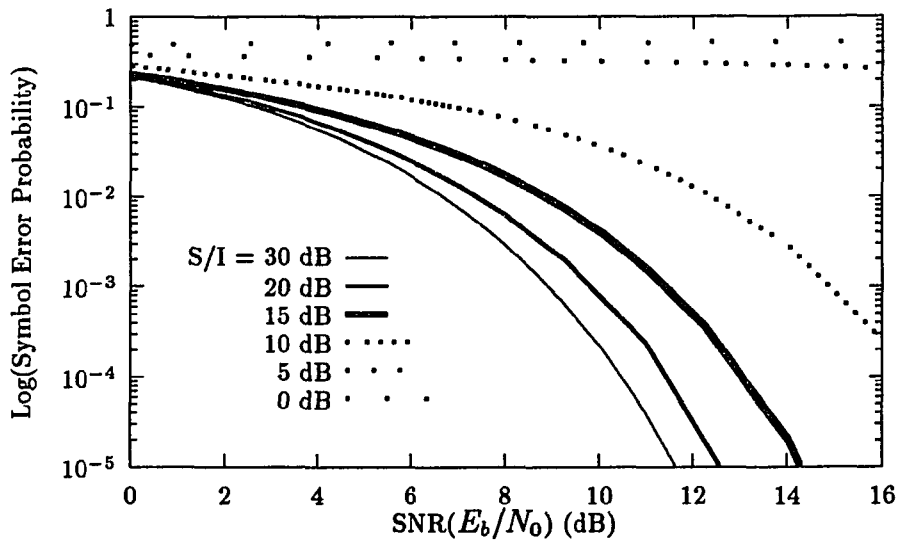
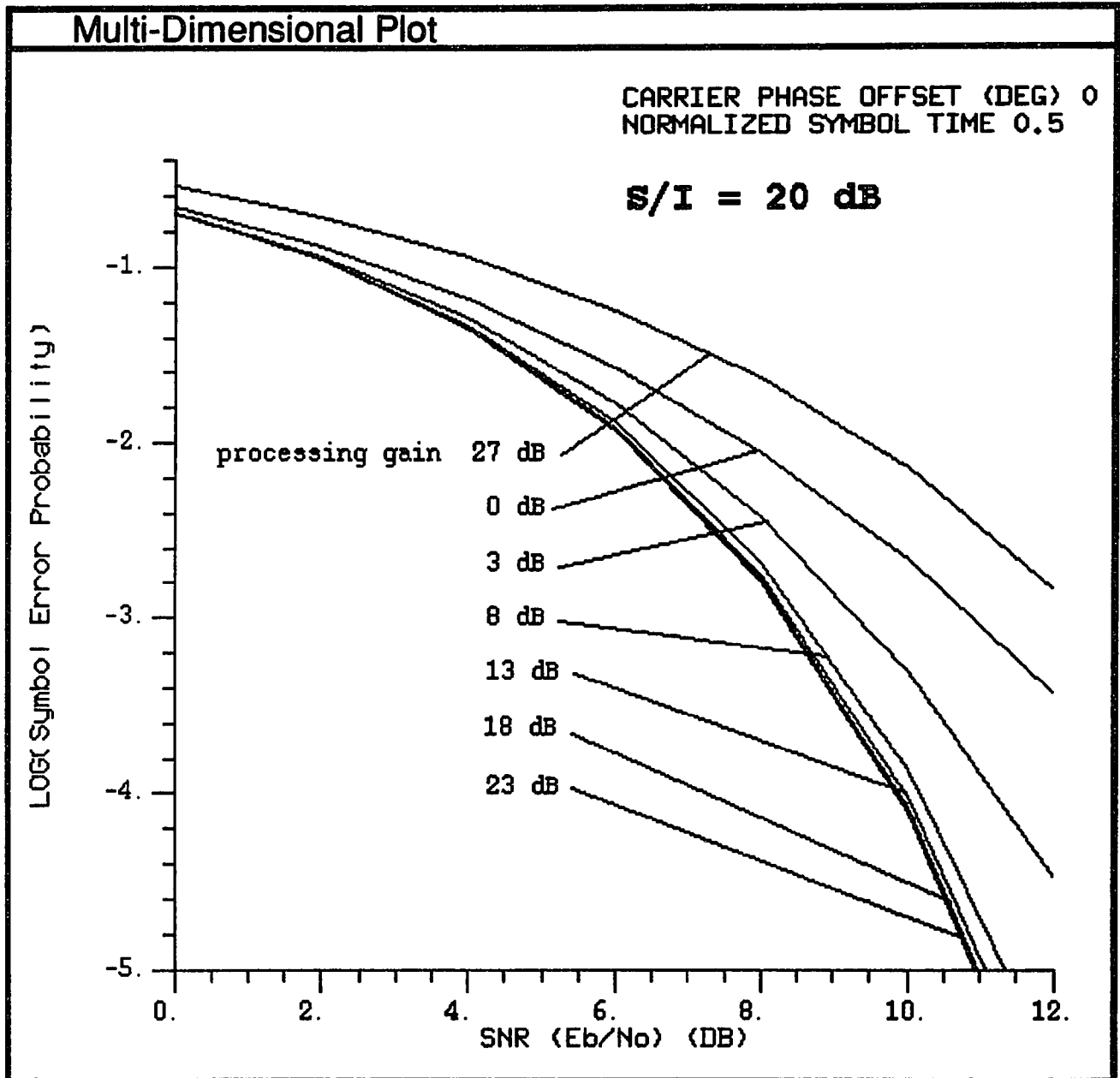Figure C.29: Symbol error rate as a function of E_b/N_0 for the worst case (down link)
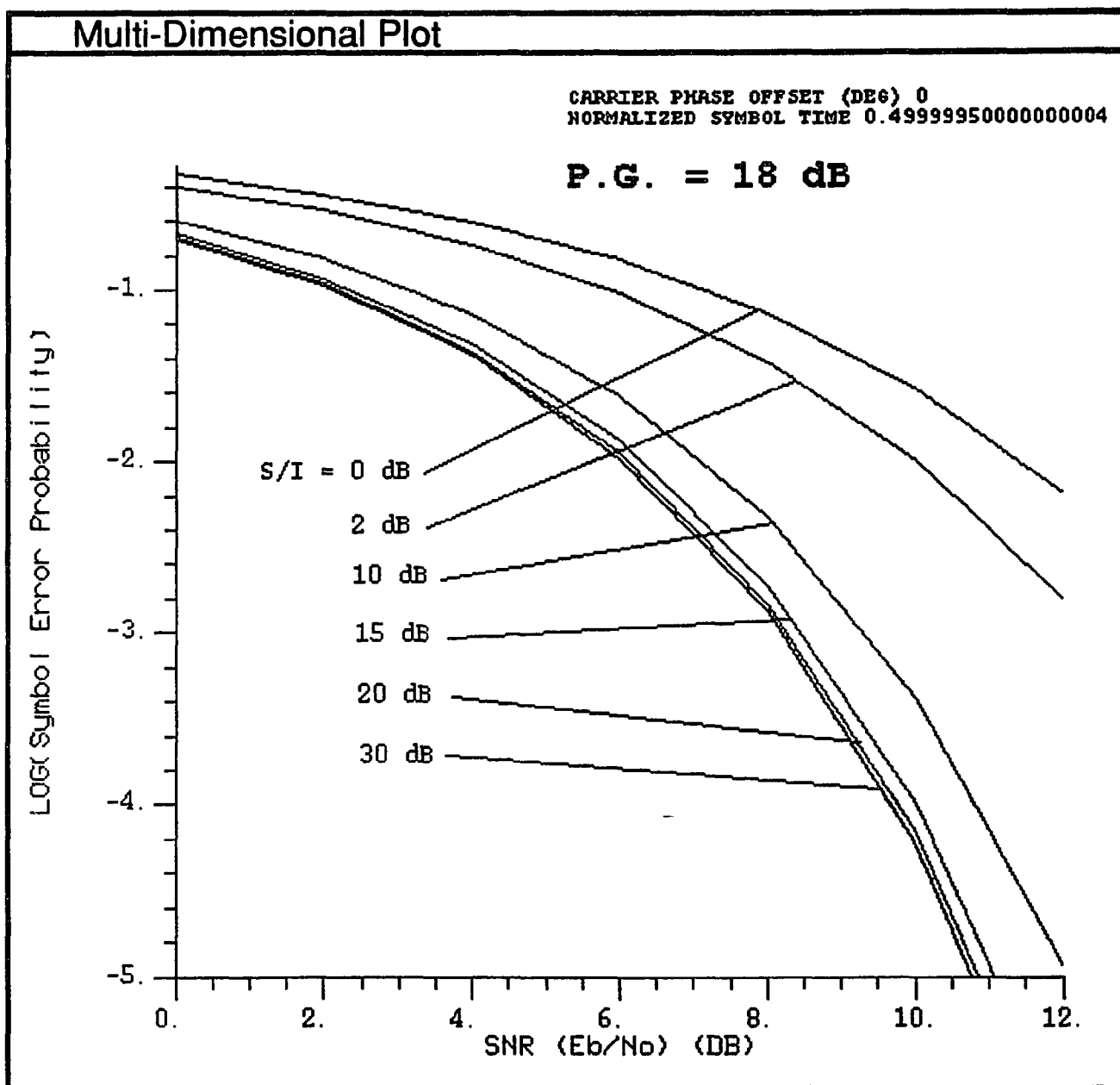
Figure C.30: Symbol error rate as a parameter of S/I(processing gain = 18 dB, downlink)
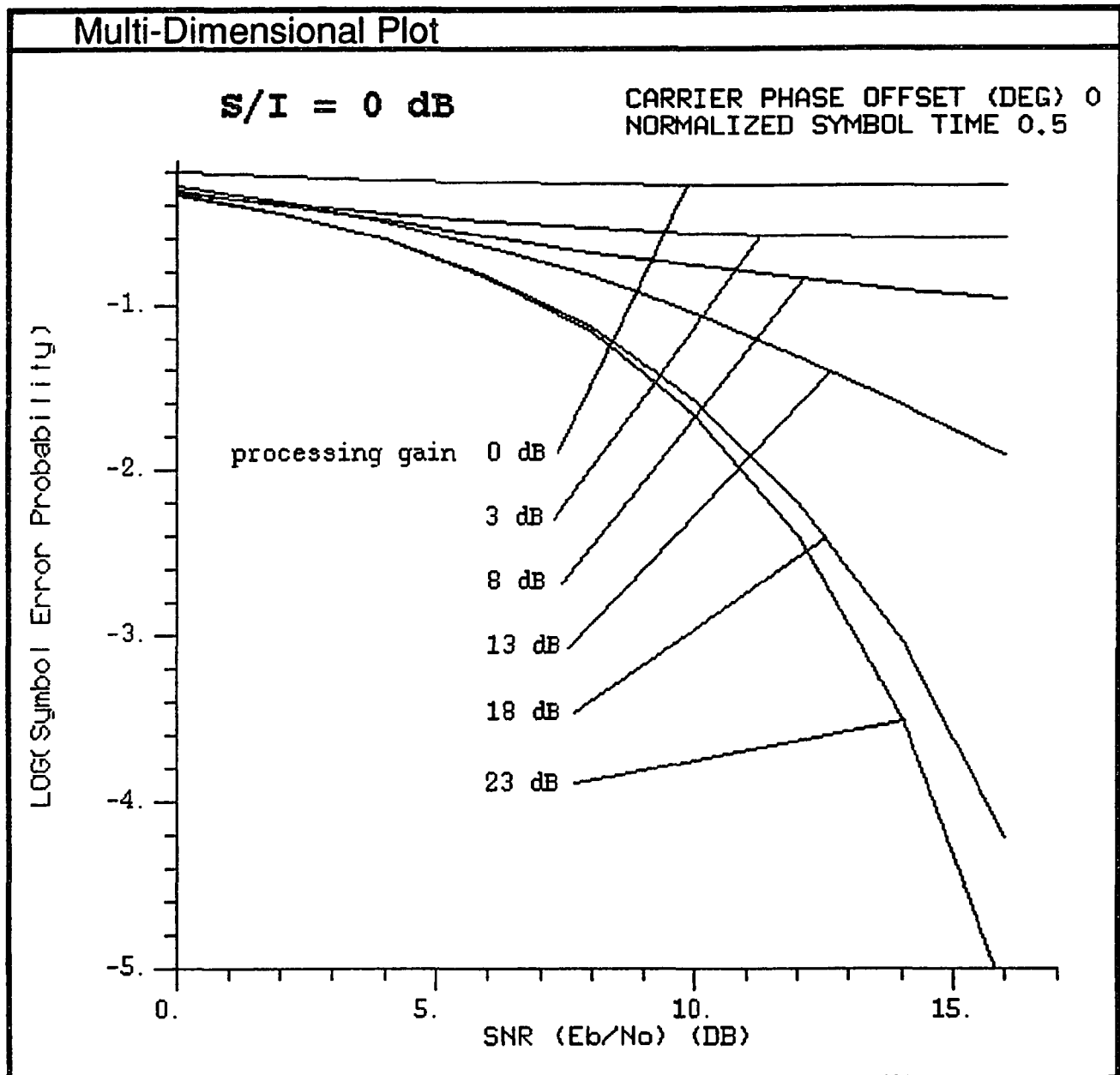
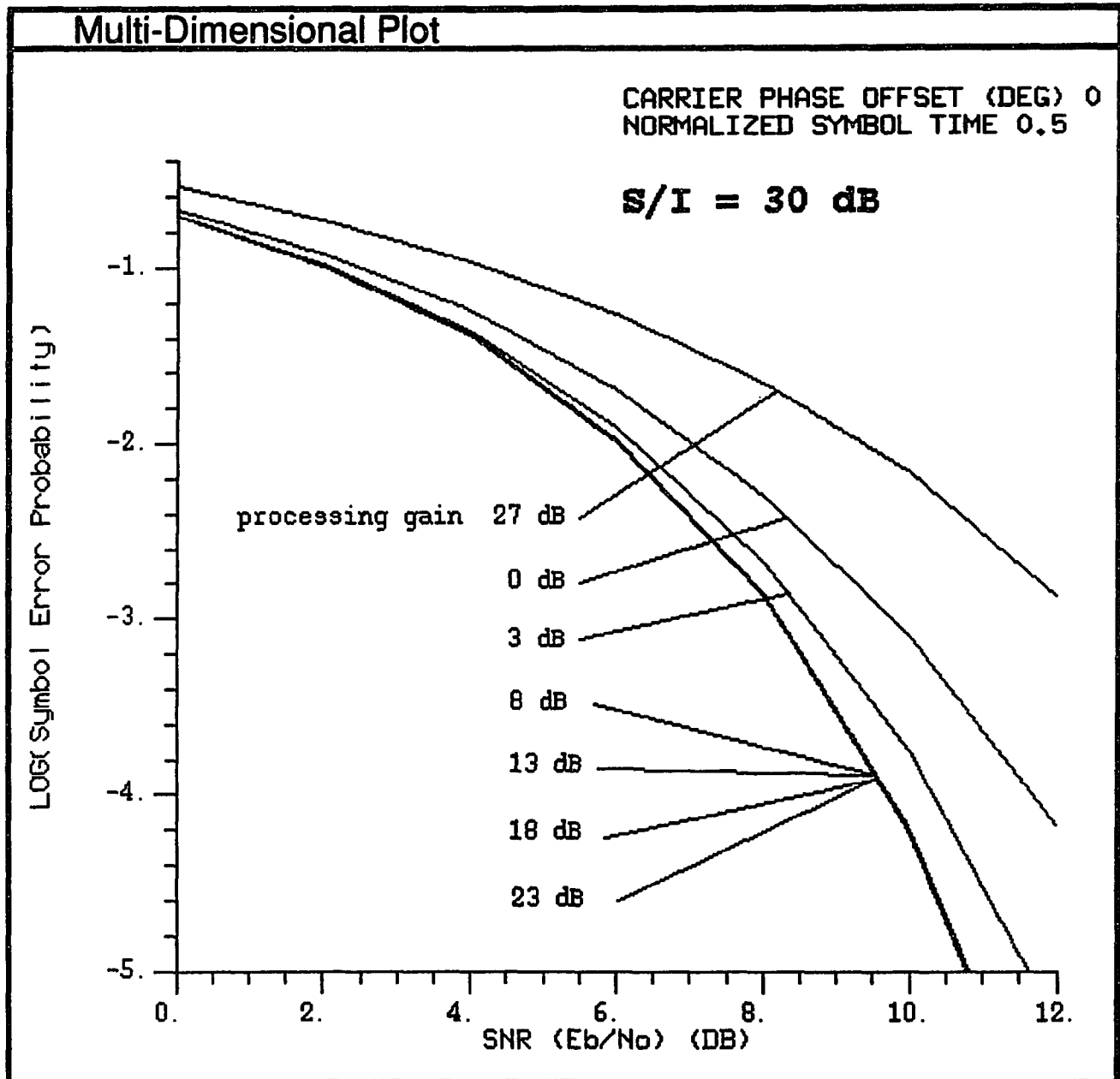Figure C.31: Symbol error rate as a parameter of processing gain(S/I = 0 dB, downlink)

Multi-Dimensional Plot

S/I = 2 dB

CARRIER PHASE OFFSET (DEG) 0
NORMALIZED SYMBOL TIME 0.49999950000000004

LOG(Symbol Error Probability)

-1.

-2.   processing gain   0 dB

        3 dB

        8 dB

-3.   13 dB

        18 dB

-4.   23 dB

-5.

0.   5.   10.   15.

SNR (Eb/No) (DB)
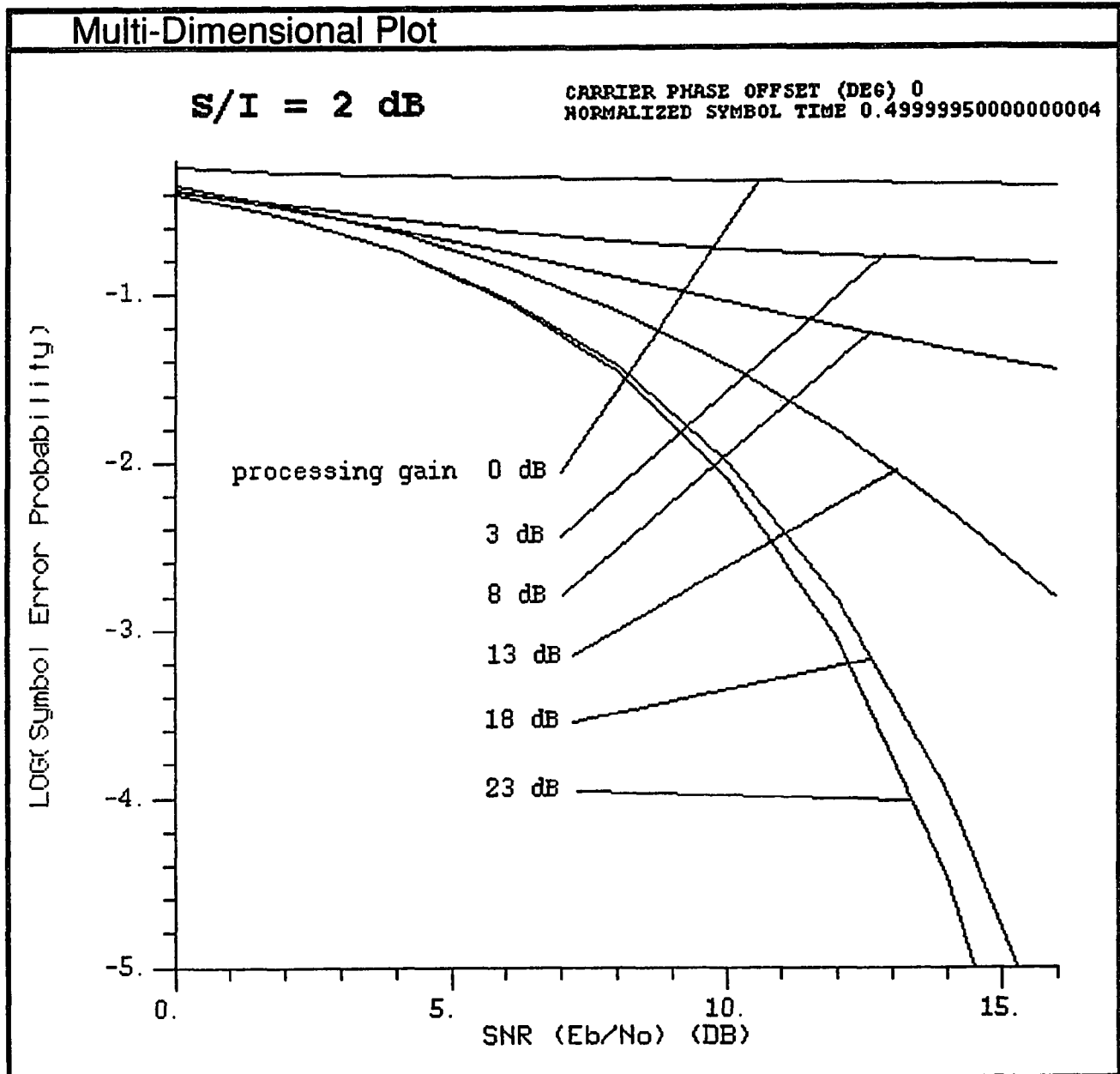
Figure C.32: Symbol error rate as a parameter of processing gain(S/I $= 2$ dB, down-link)
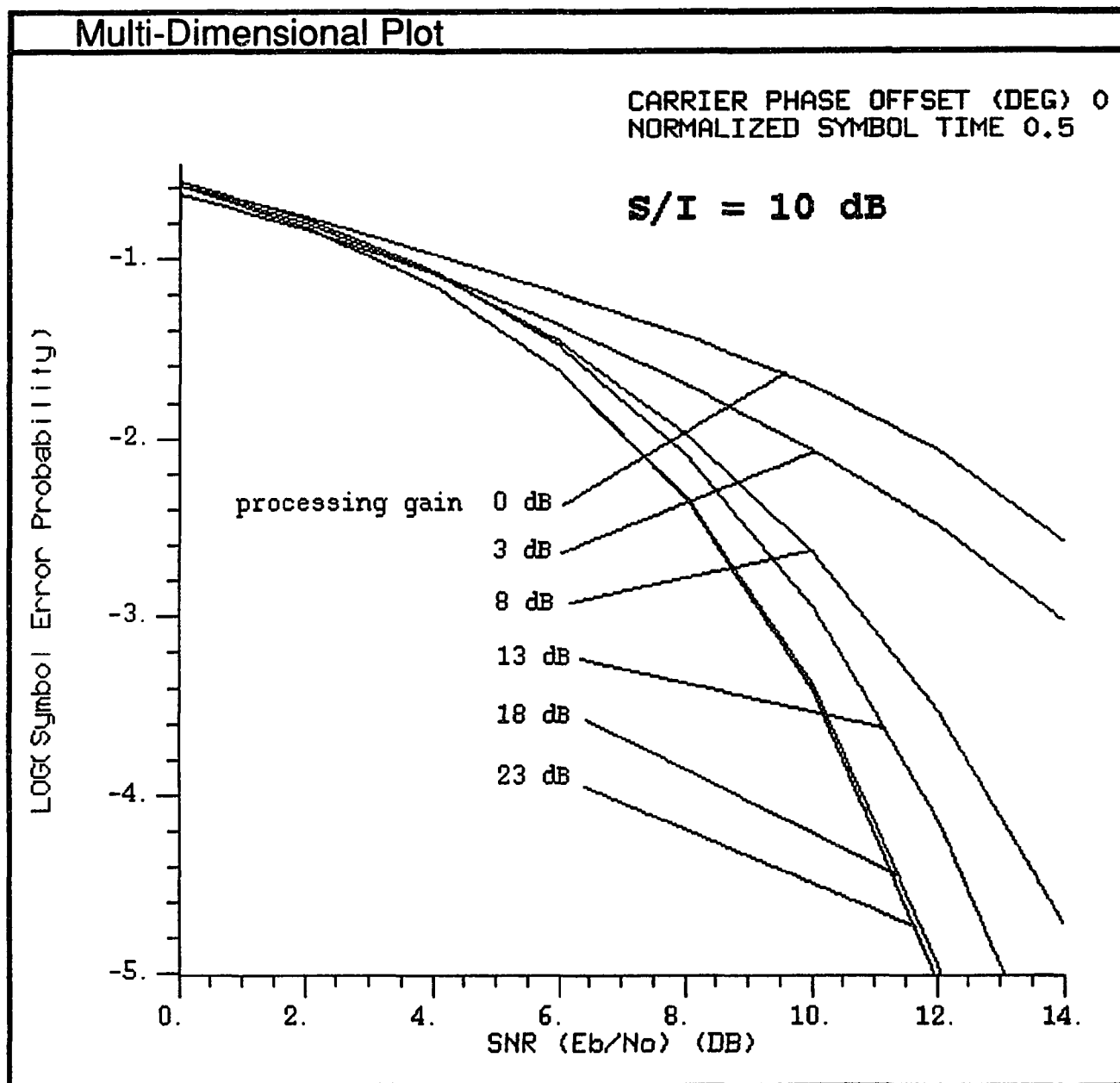
Figure C.33: Symbol error rate as a parameter of processing gain(S/I = 10 dB, downlink)
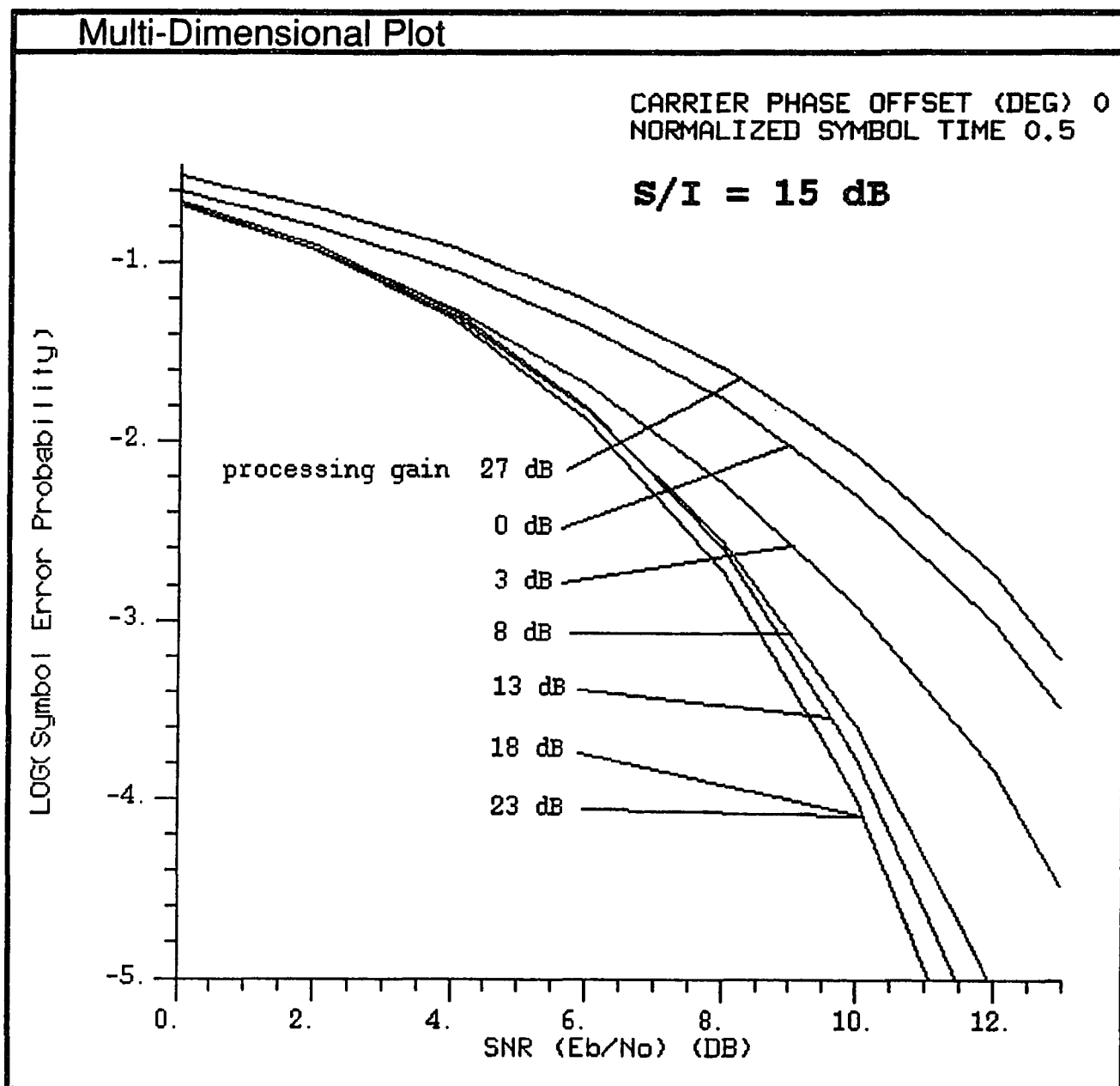
Figure C.34: Symbol error rate as a parameter of processing gain(S/I = 15 dB, downlink)
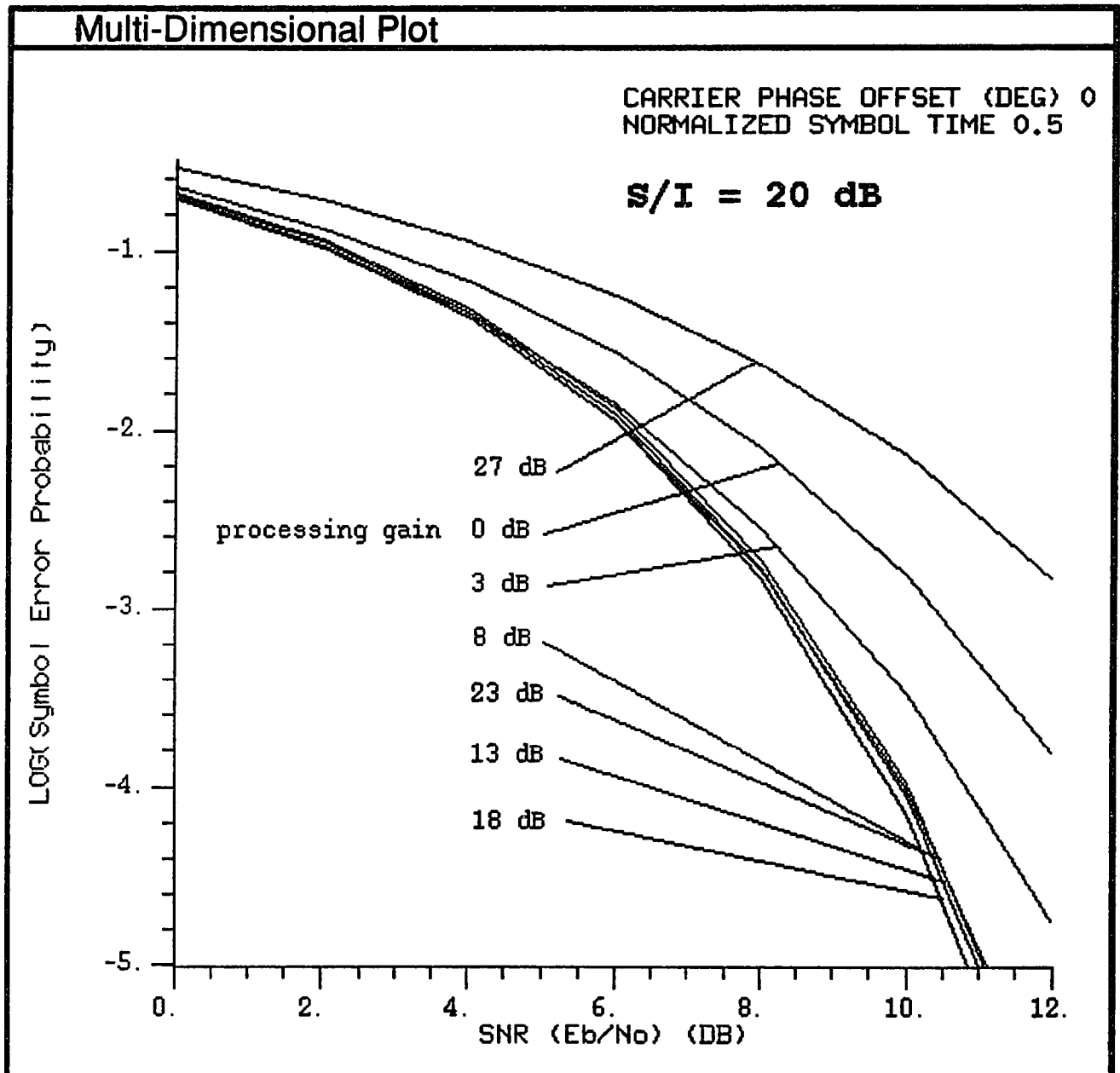
Figure C.35: Symbol error rate as a parameter of processing gain(S/I = 20 dB, downlink)
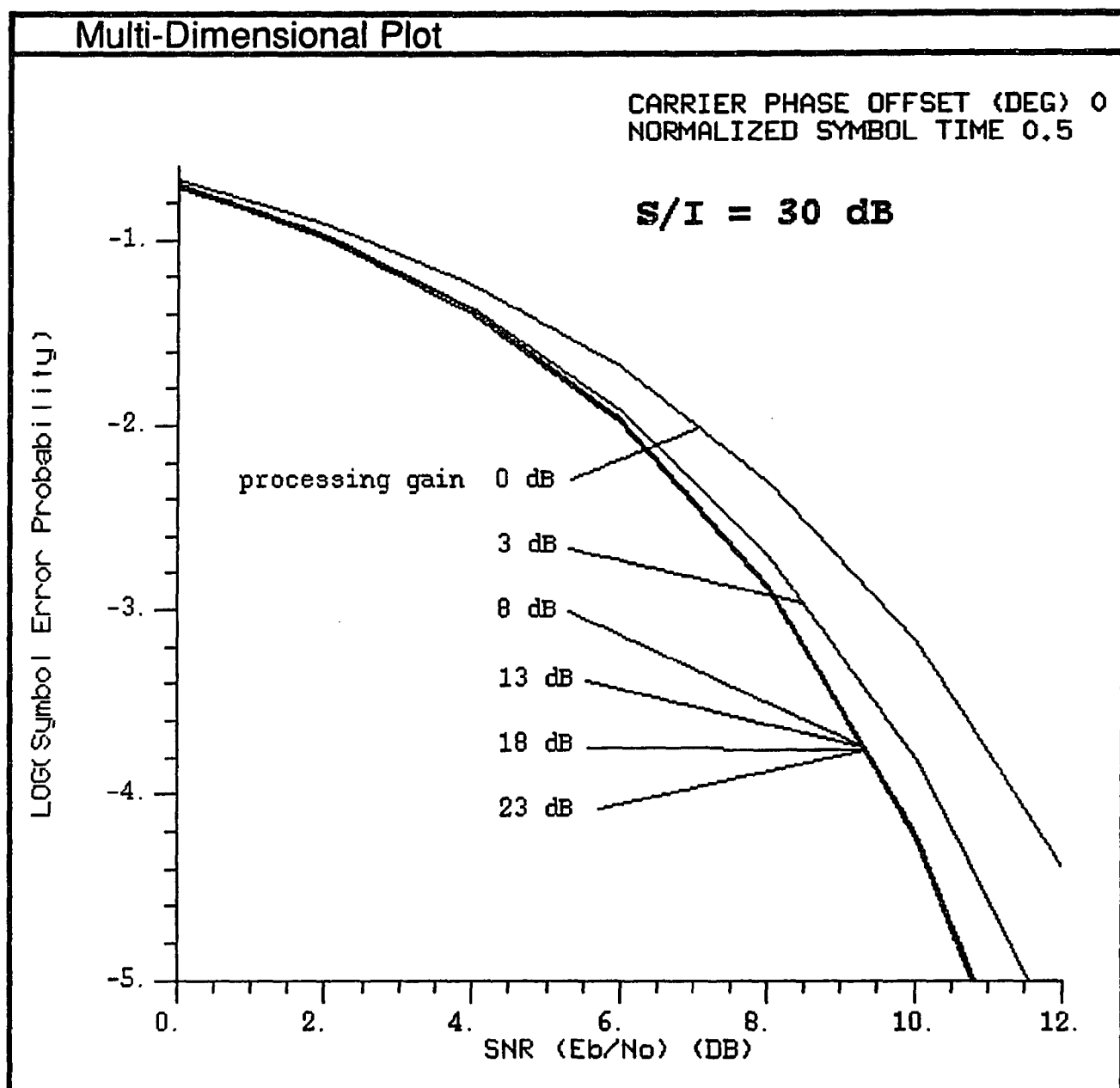
Figure C.36: Symbol error rate as a parameter of processing gain(S/I = 30 *dB*, downlink)
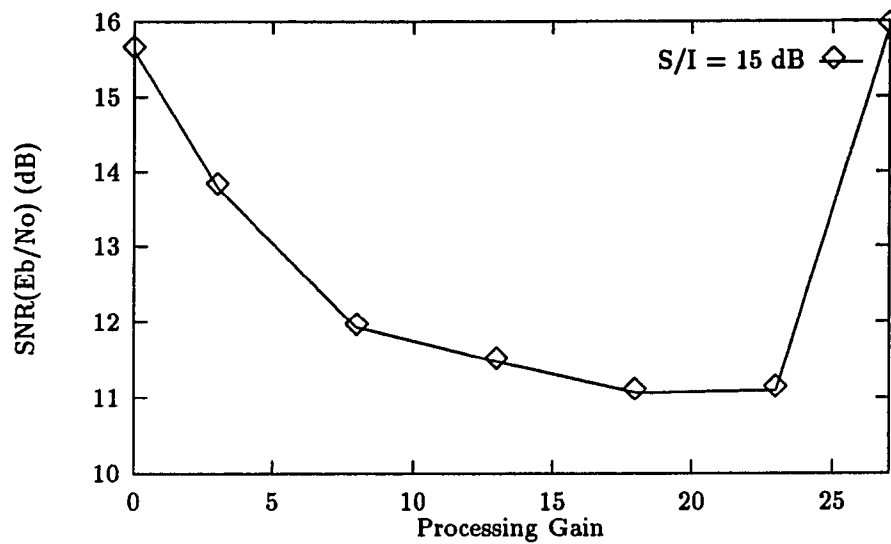
Figure C.37: SNR as a function of processing gain for symbol error rate = 10^{-5} & S/I = 15 dB
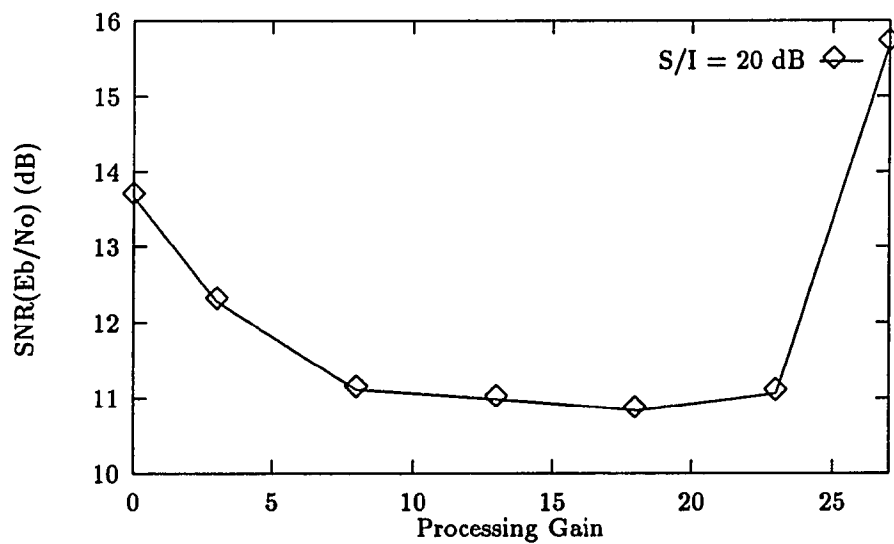
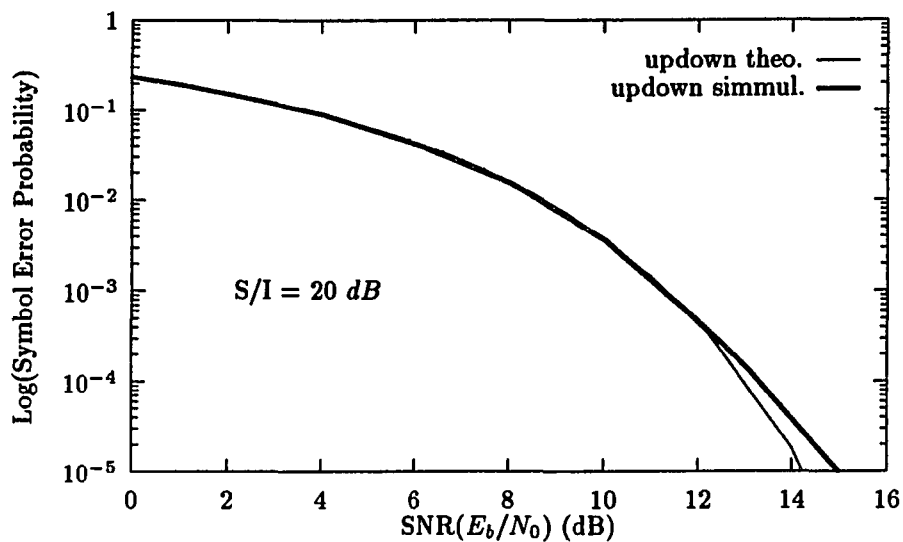Figure C.38: SNR as a function of processing gain for symbol error rate = 10^{-5} & S/I = 20 dB
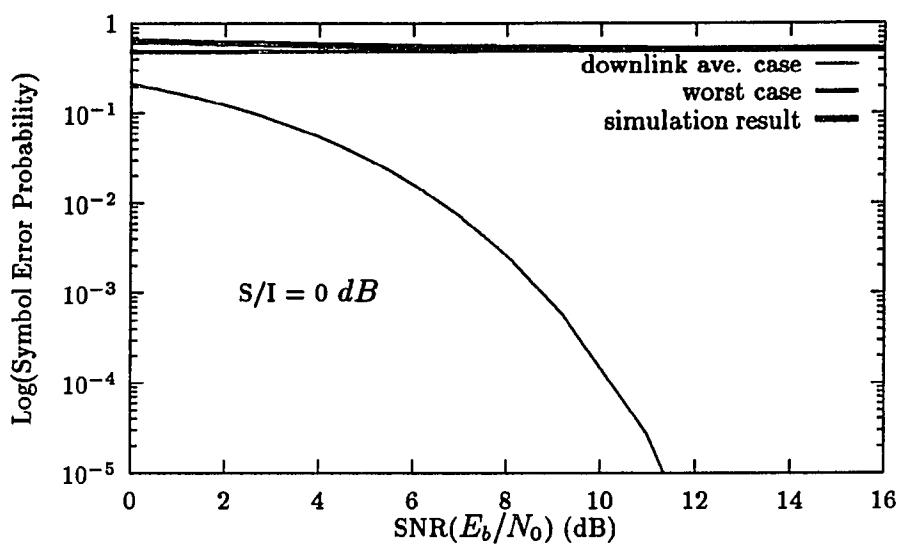
Figure C.39: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
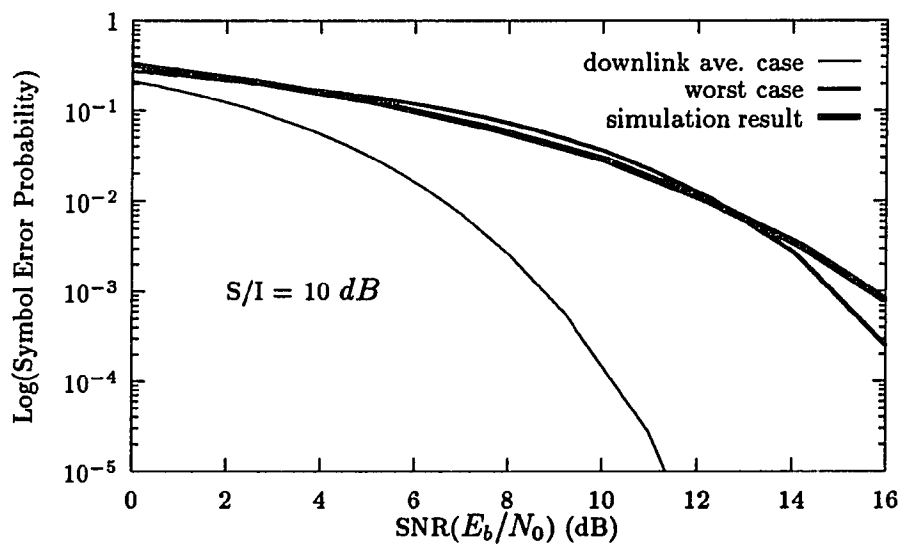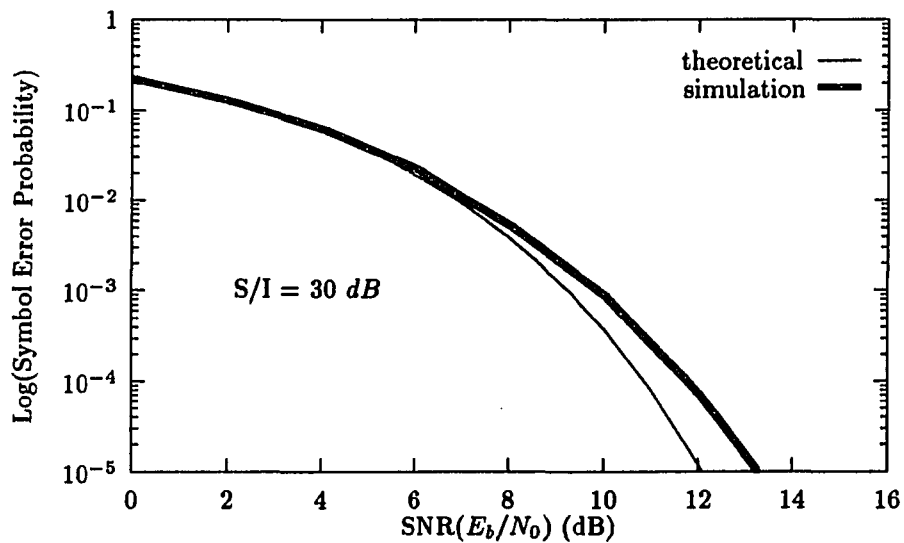

Figure C.40: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
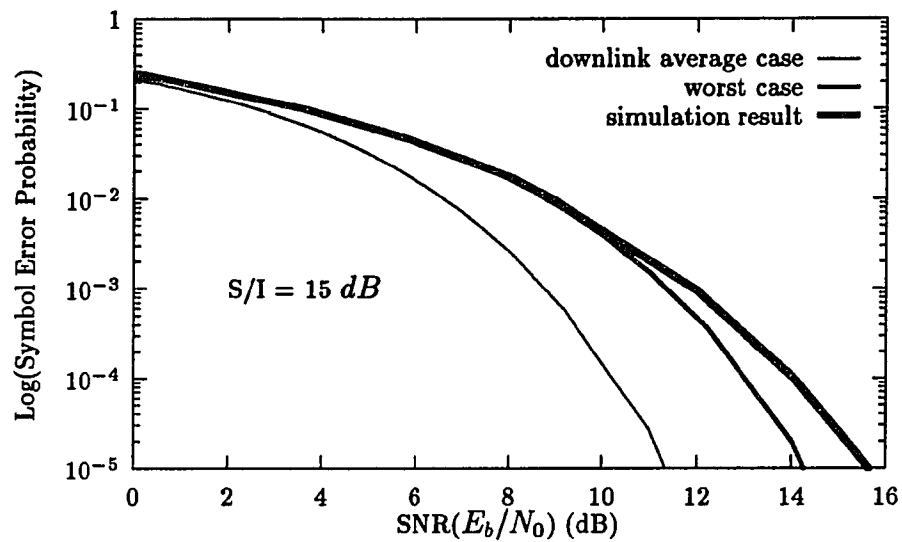
Figure C.41: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
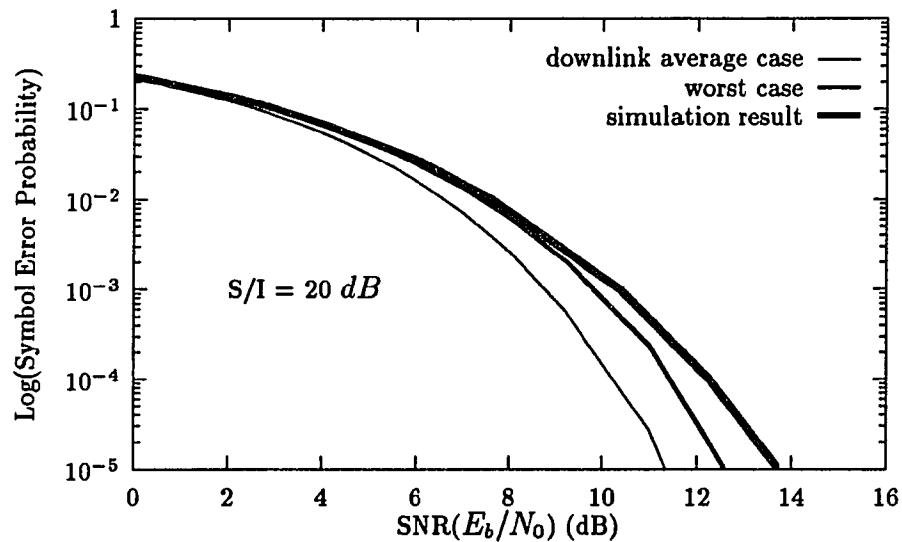
Figure C.42: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
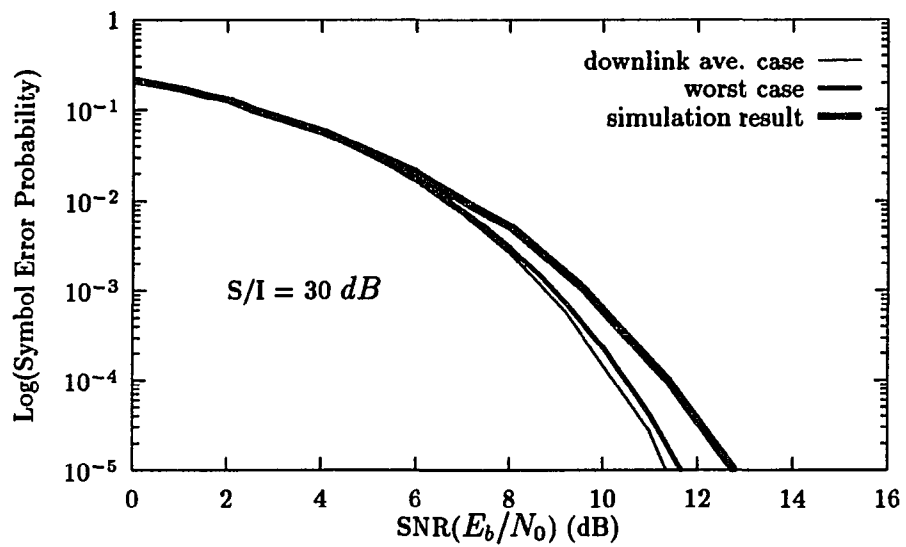
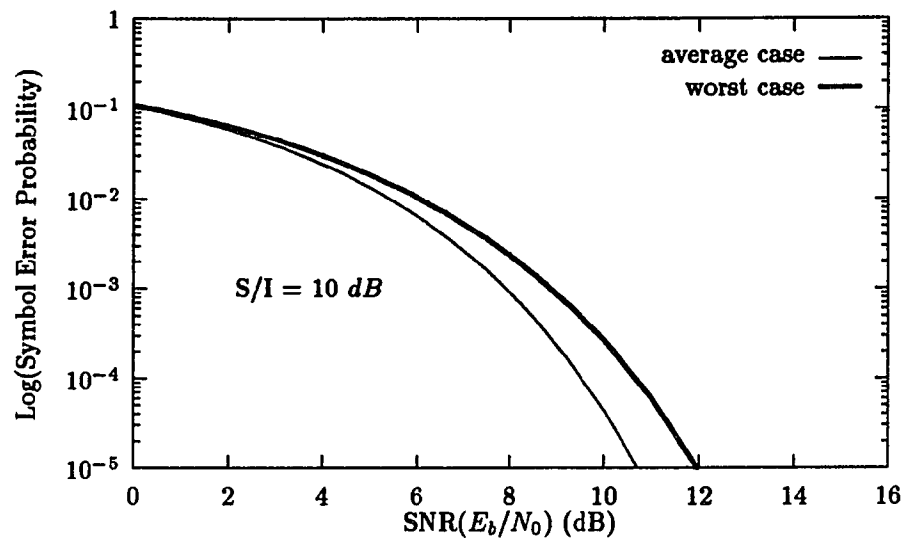Figure C.43: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0
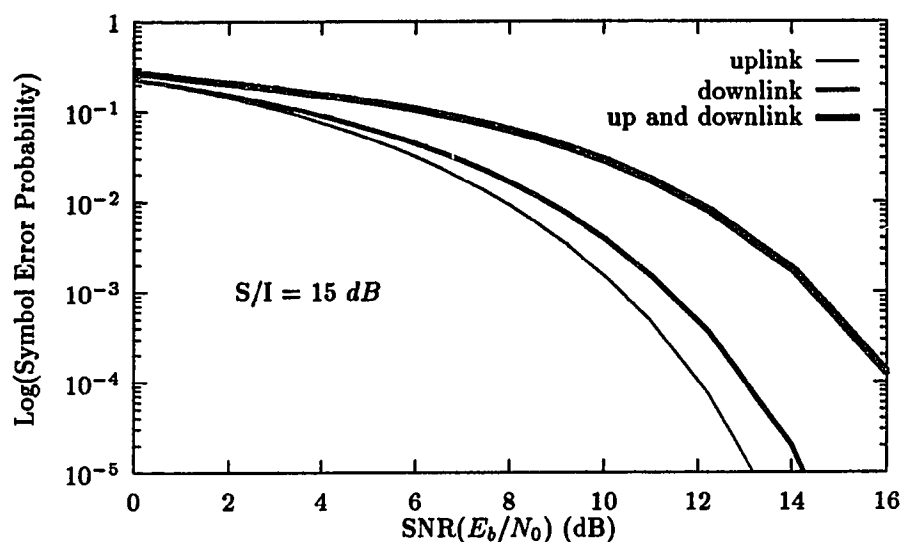
Figure C.44: Symbol error rate comparison in worst case (: uplink interference, downlink interference, and up and downlink interference) as a function of E_b/N_0 (S/I = 15 dB)
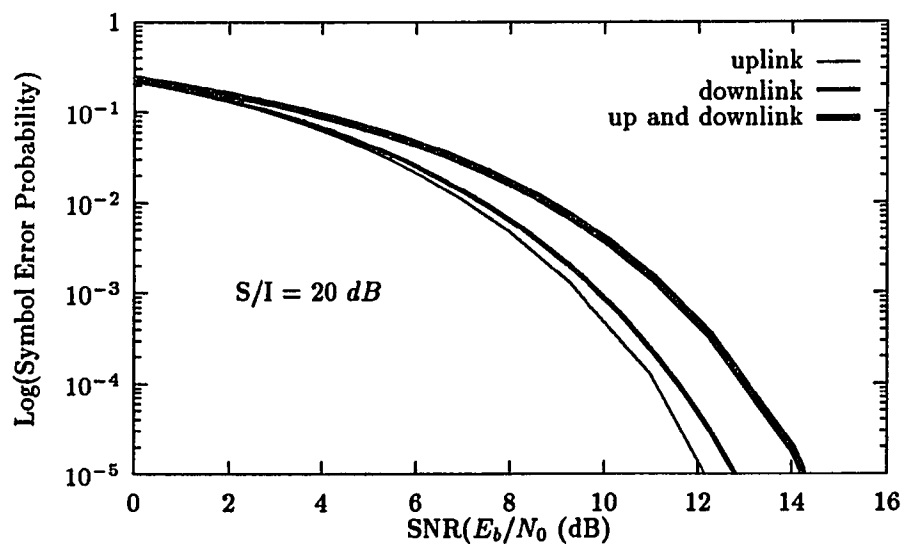
Figure C.45: Symbol error rate comparison in worst case (: uplink interference, downlink interference, and up and downlink interference) as a function of E_b/N_0 (S/I = 20 dB)
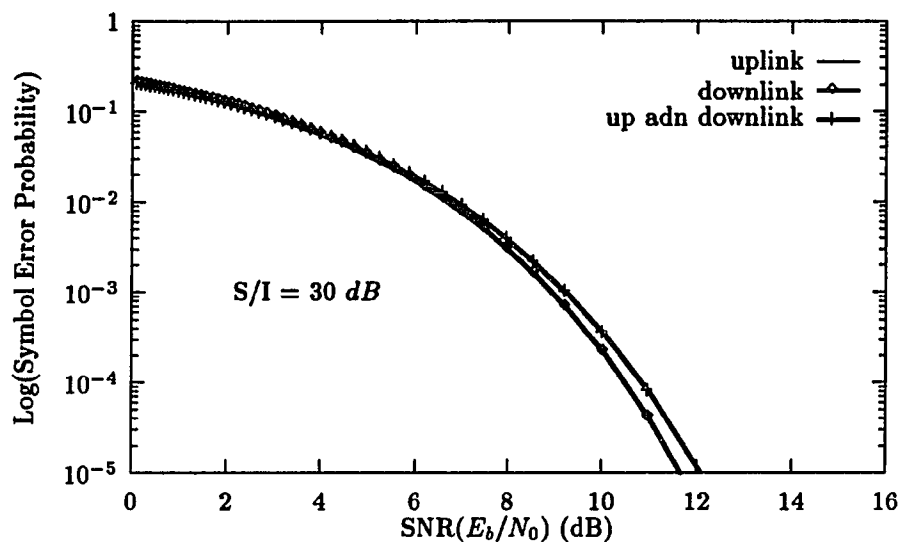
Figure C.46: Symbol error rate comparison in worst case (: uplink interference, downlink interference, and up and downlink interference) as a function of E_b/N_0 (S/I = 30 dB)
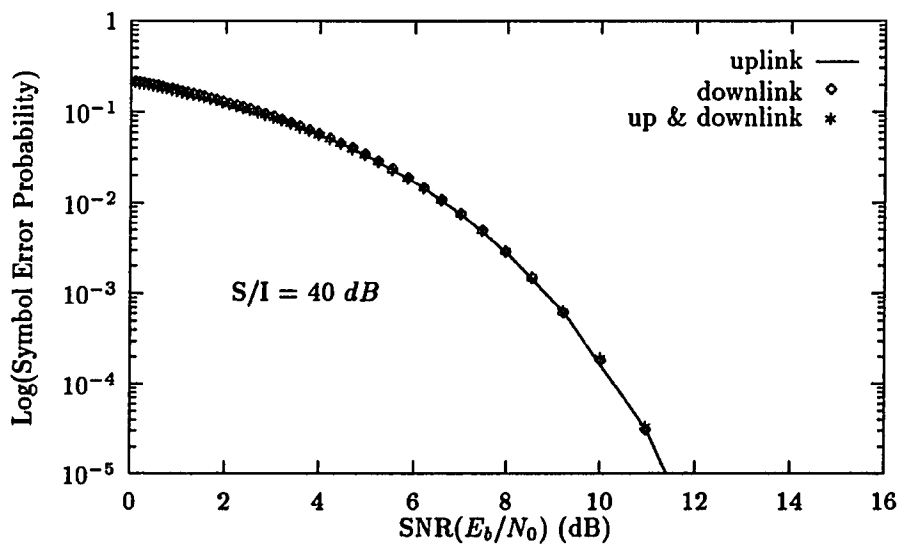
Figure C.47: Symbol error rate comparison in worst case (: uplink interference, downlink interference, and up and downlink interference) as a function of E_b/N_0 (S/I = 40 dB)
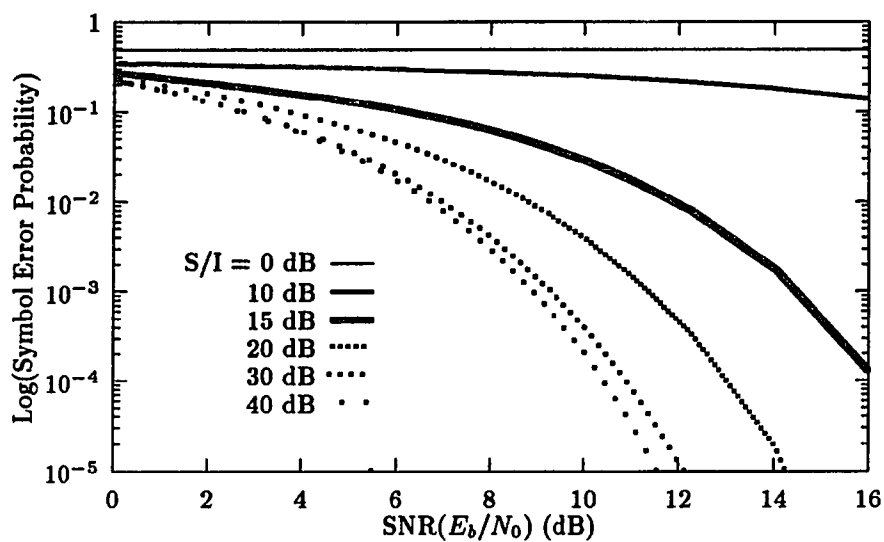
Figure C.48: Symbol error rate as a parameter of S/I for the worst case (up & downlink)
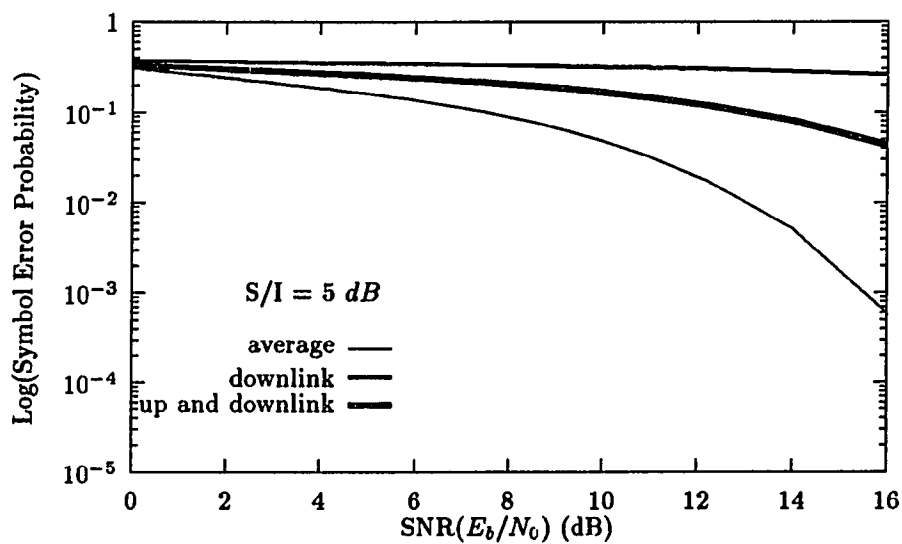
Figure C.49: Symbol error rate for S/I = 5 dB when the total interference power is fixed
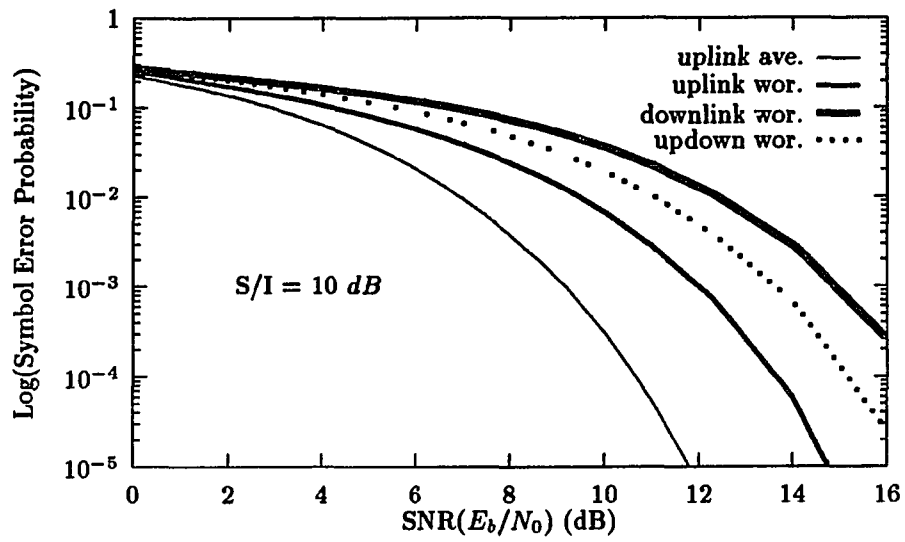
Figure C.50: Symbol error rate for S/I = 10 dB when the total interference power is fixed
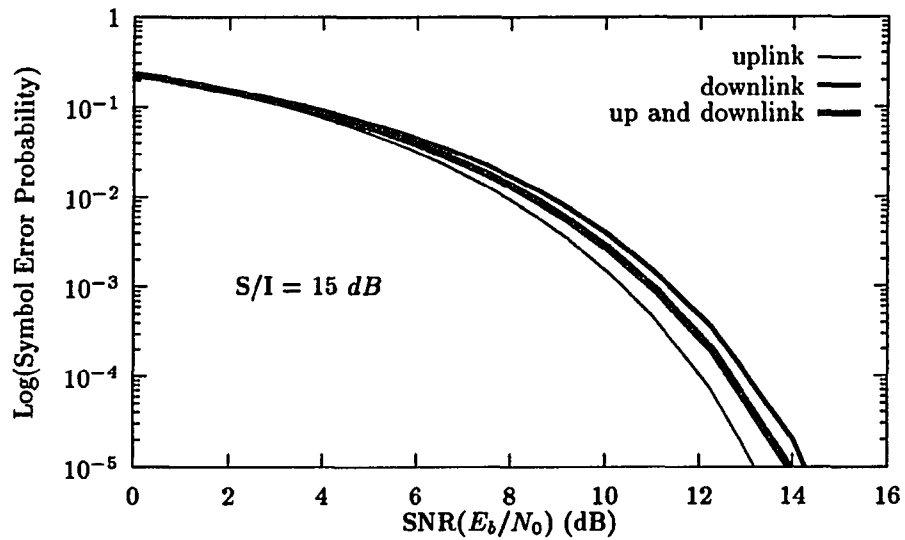
Figure C.51: Symbol error rate for S/I = 15 dB when the total interference power is fixed
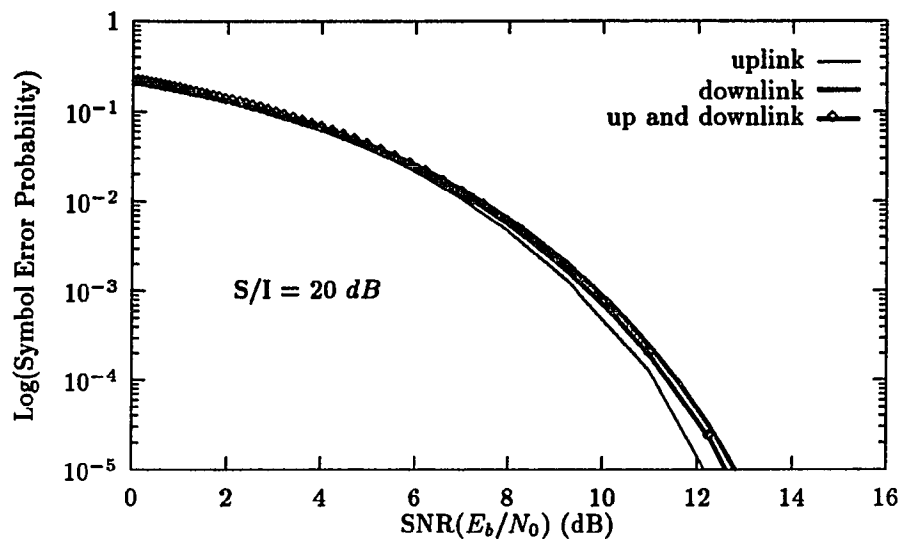
Figure C.52: Symbol error rate for S/I = 20 dB when the total interference power is fixed
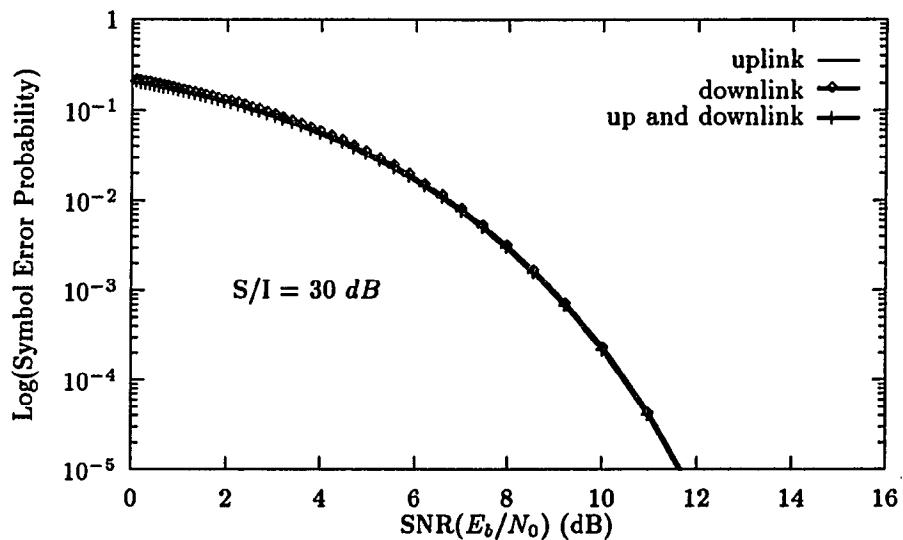
Figure C.53: Symbol error rate for S/I = 30 dB when the total interference power is fixed
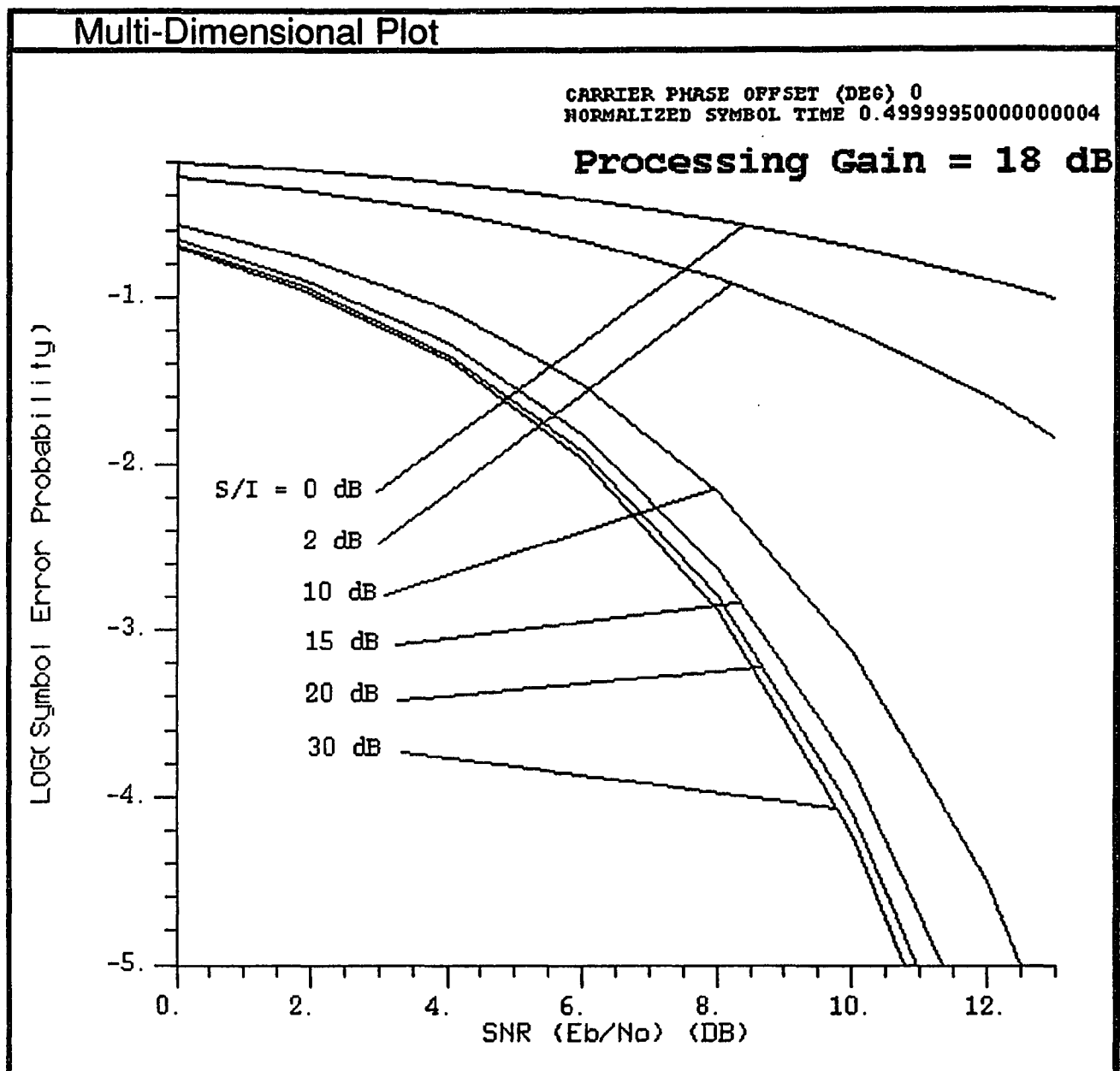
Figure C.54: Symbol error rate as a parameter of S/I(processing gain = 18 *dB*, up & downlink)
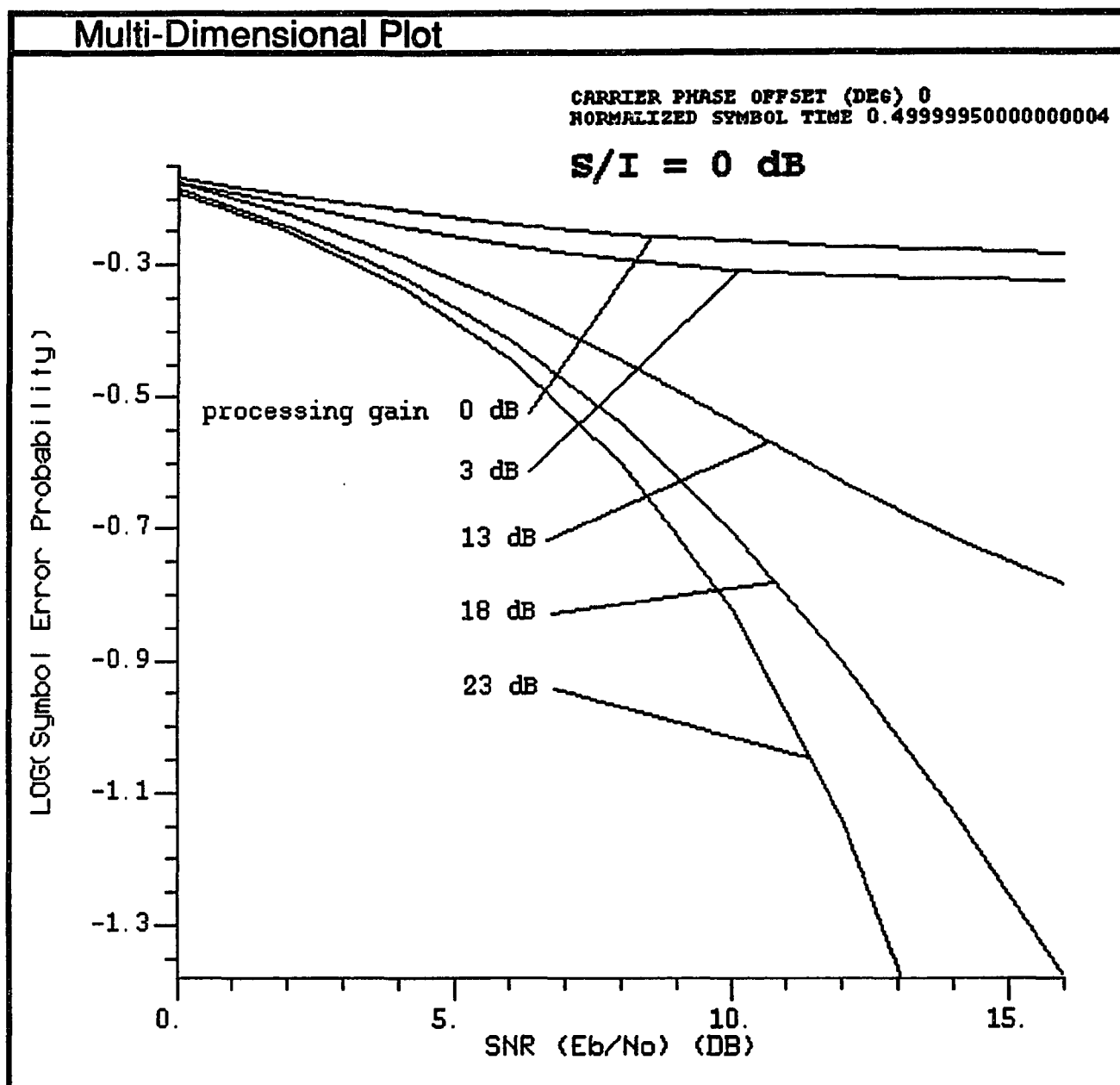
Figure C.55: Symbol error rate as a parameter of processing gain(S/I = 0 dB, up & downlink)
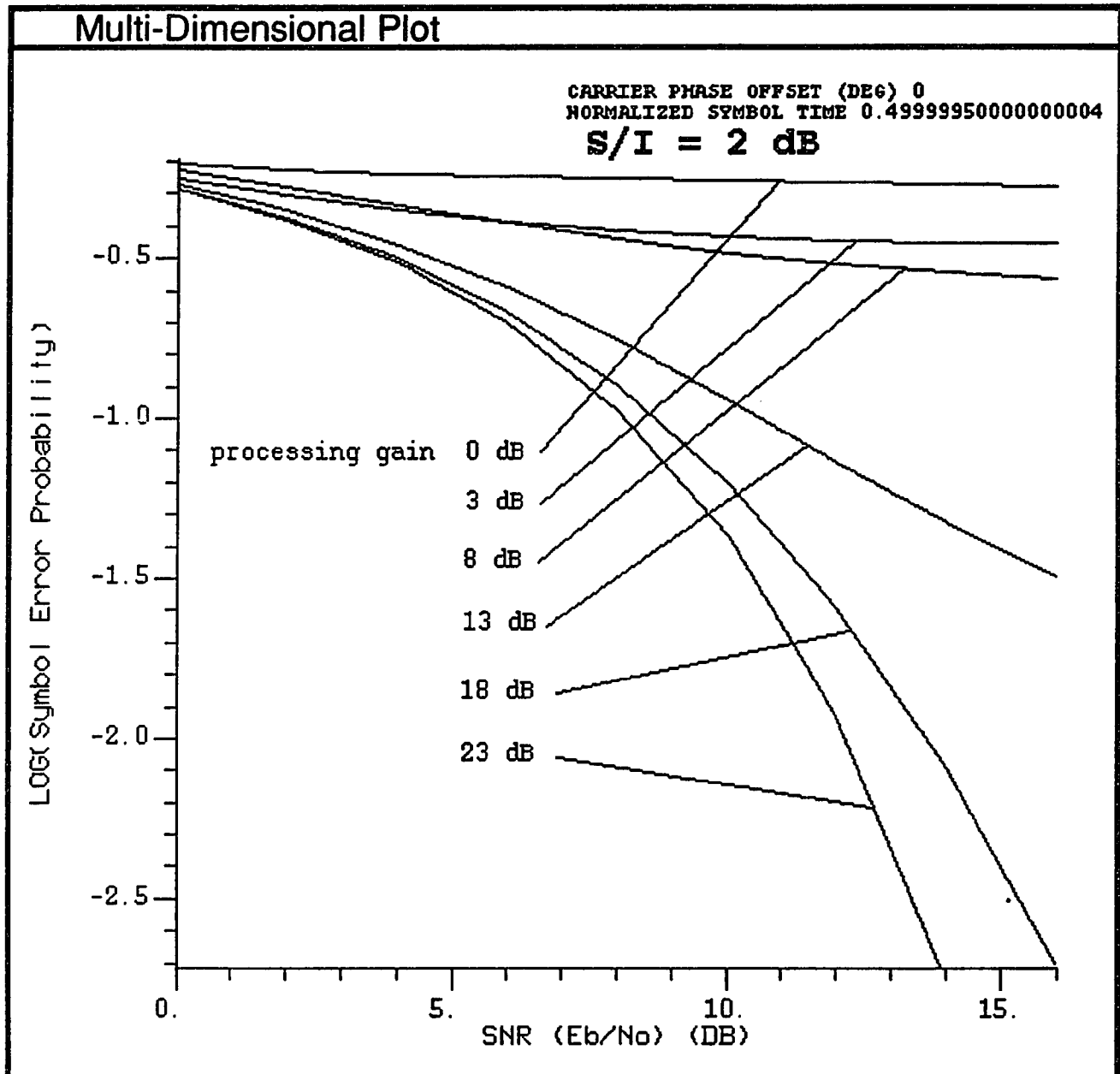
Figure C.56: Symbol error rate as a parameter of processing gain(S/I = 2 *dB*, up & downlink)
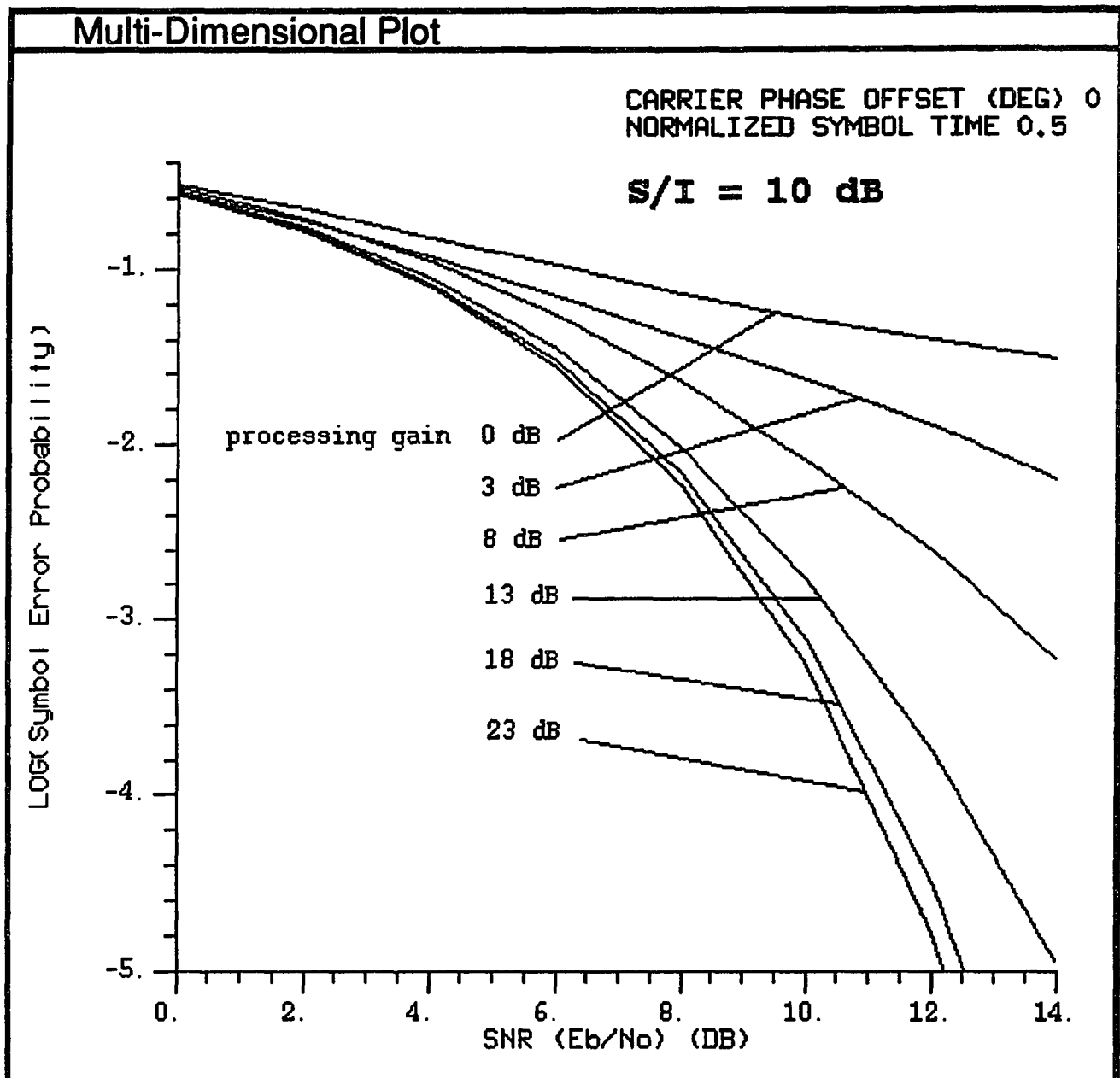
Figure C.57: Symbol error rate as a parameter of processing gain(S/I = 10 dB, up & downlink)
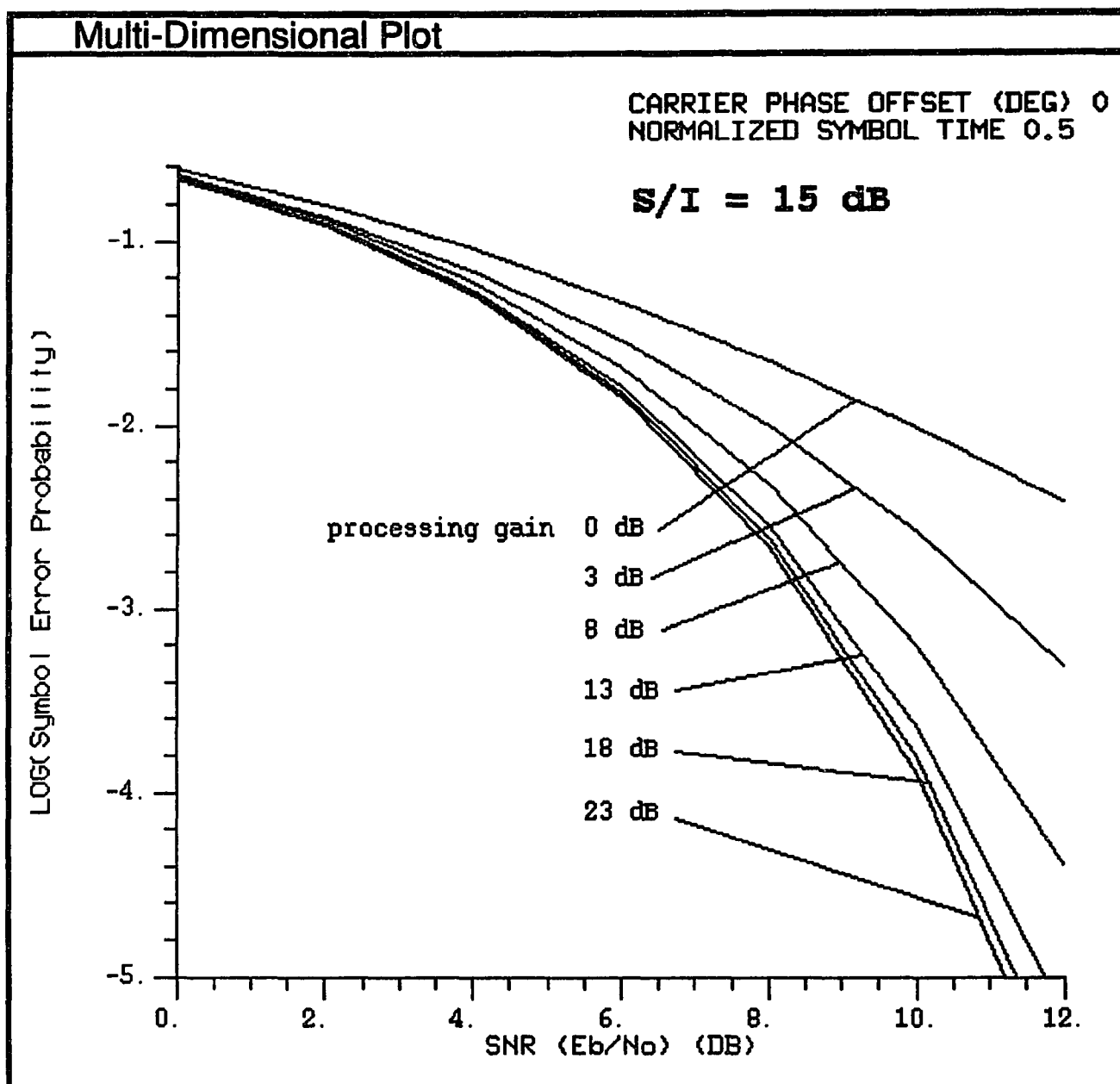
Figure C.58: Symbol error rate as a parameter of processing gain(S/I = 15 dB, up & downlink)

Figure C.59: Symbol error rate as a parameter of processing gain(S/I = 20 dB, up & downlink)

Figure C.60: Symbol error rate as a parameter of processing gain(S/I = 30 dB, up & downlink)
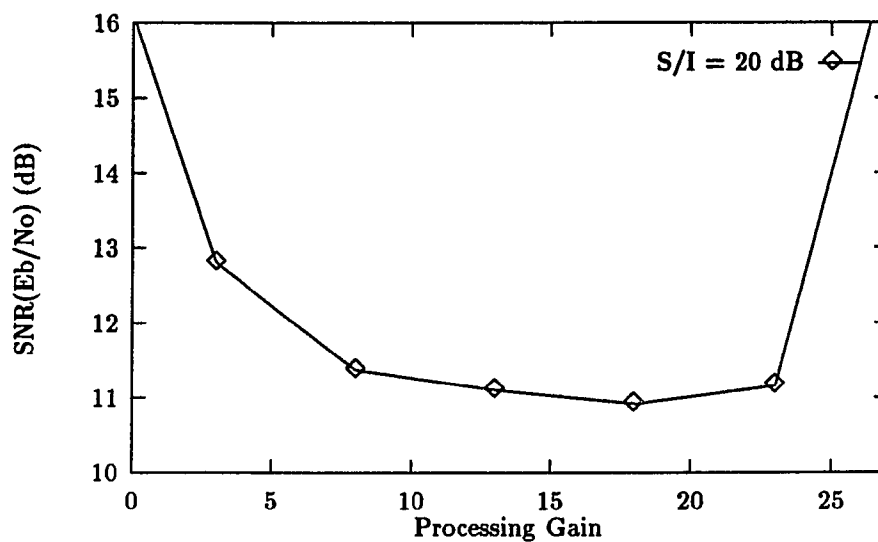
Figure C.61: SNR as a function of processing gain for symbol error rate $= 10^{-5}$ & S/I $= 20$ dB, up & downlink
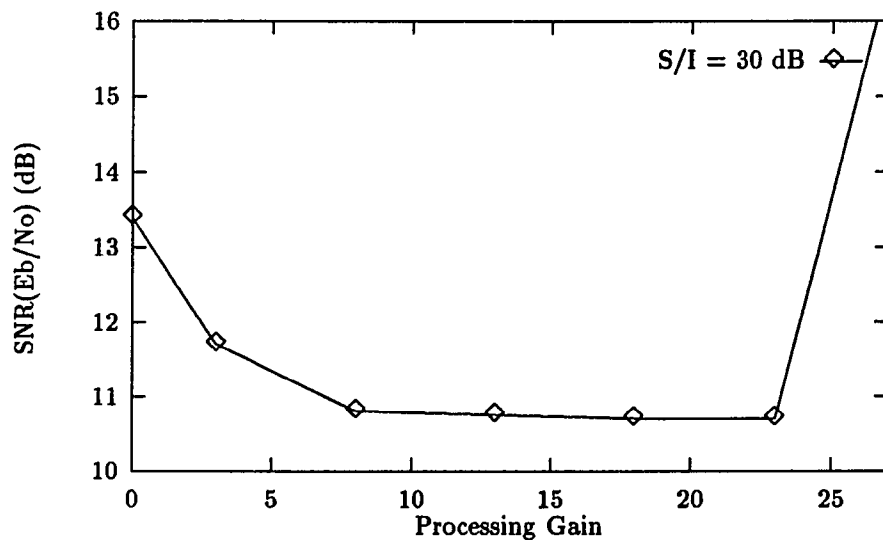
Figure C.62: SNR as a function of processing gain for symbol error rate $= 10^{-5}$ & S/I $= 30$ dB, up & downlink
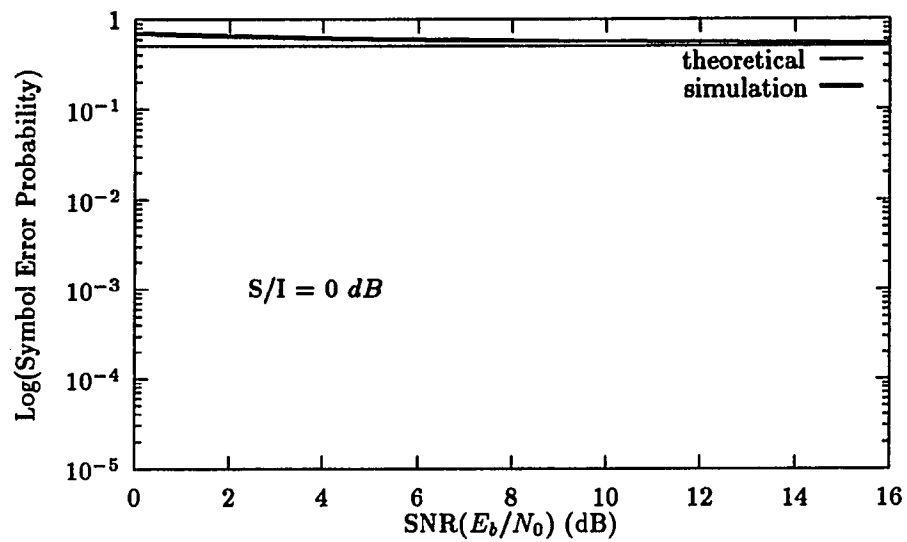
Figure C.63: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0 (S/I = 0 dB)
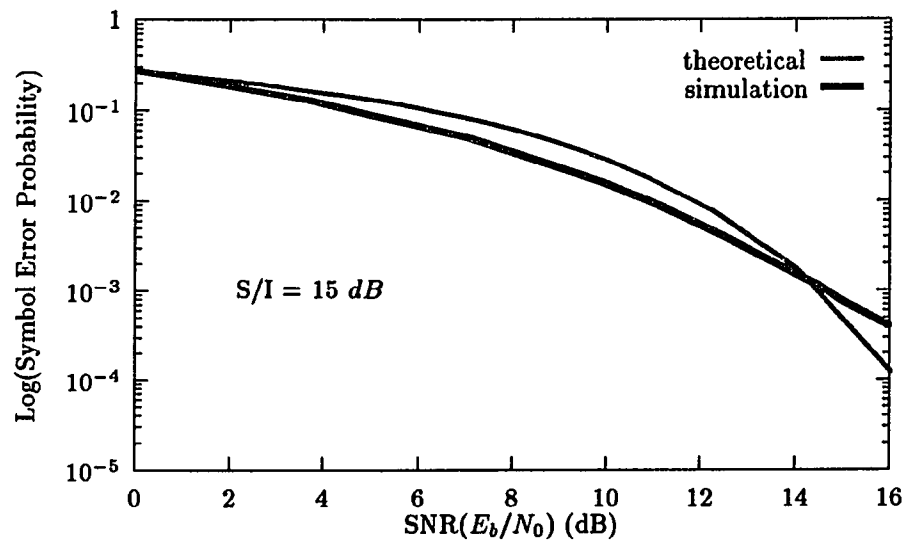
Figure C.64: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0 (S/I = 15 dB)

Figure C.65: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0 (S/I = 20 dB)

Figure C.66: Symbol error rate comparison between theoretical result and simulation result as a function of E_b/N_0 (S/I = 30 dB)

Figure C.67: Symbol error rate comparison between the average case and the worst case as a function of E_b/N_0 (S/I = 10 dB)
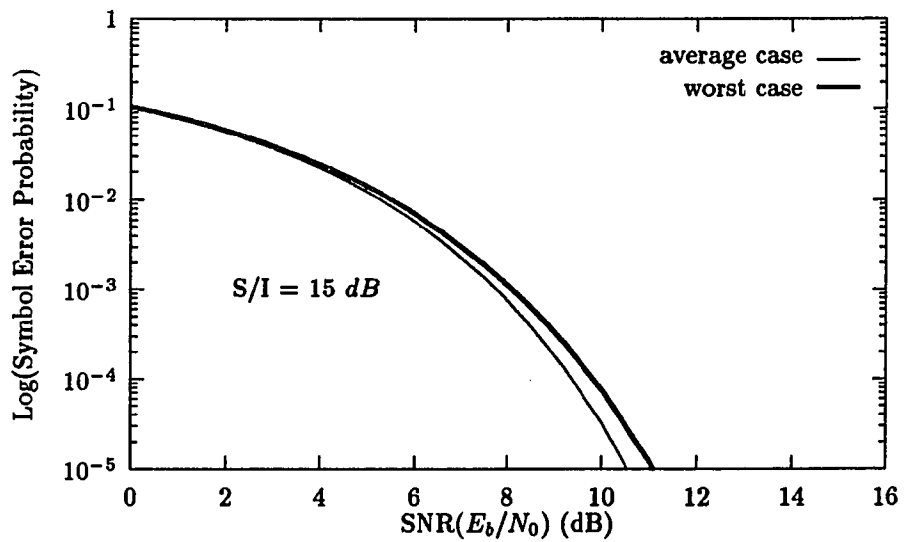
Figure C.68: Symbol error rate comparison between the average case and the worst case as a function of E_b/N_0 (S/I = 15 dB)
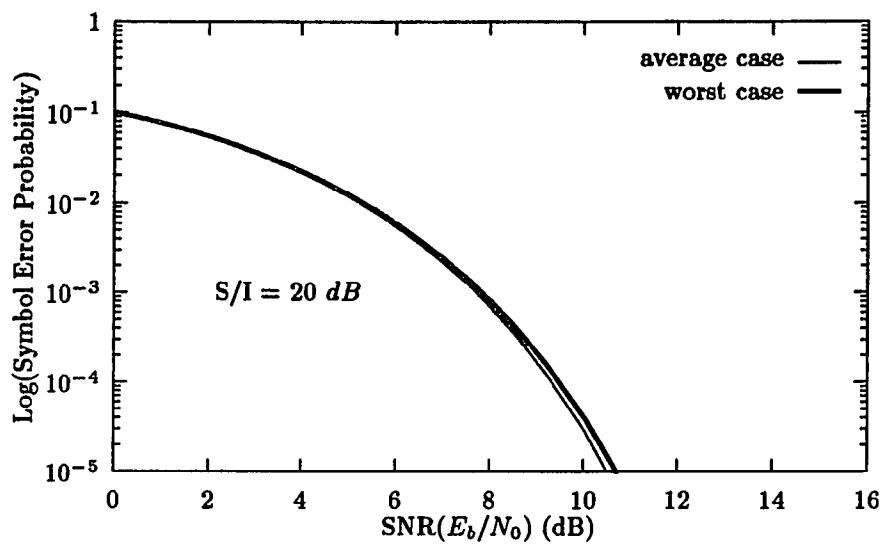
Figure C.69: Symbol error rate comparison between the average case and the worst case as a function of E_b/N_0 (S/I = 20 dB)
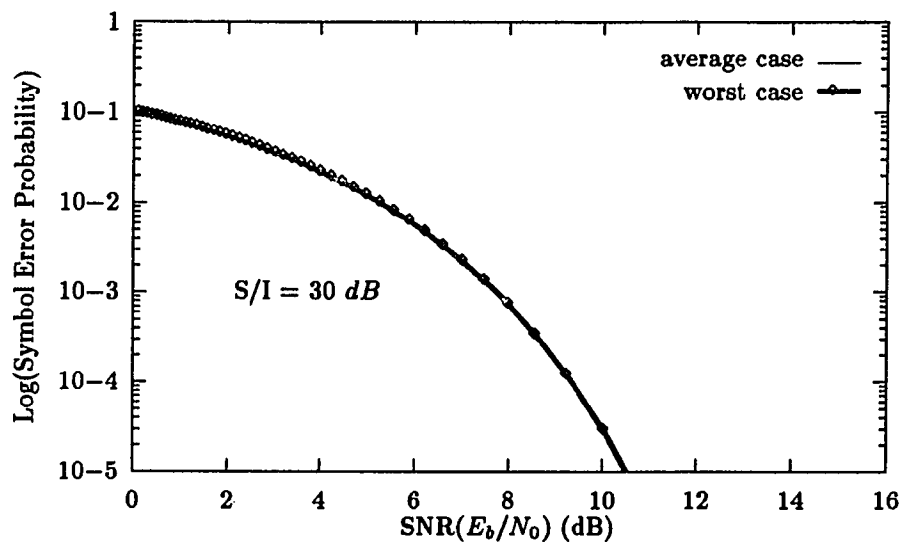
Figure C.70: Symbol error rate comparison between the average case and the worst case as a function of E_b/N_0 (S/I = 30 dB)
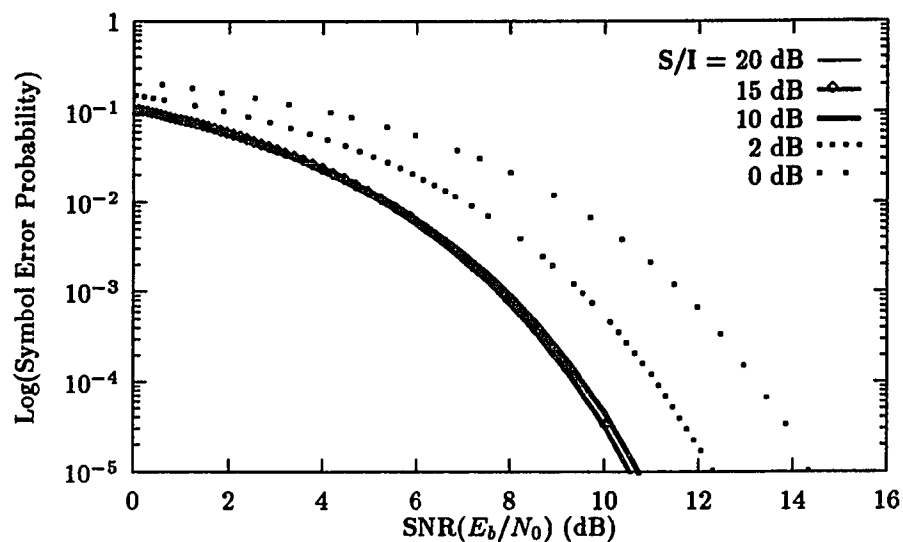
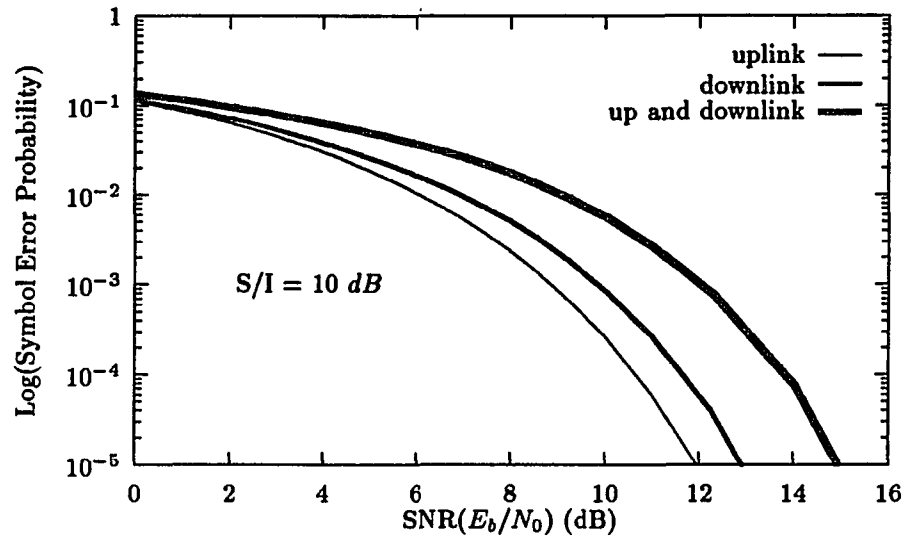Figure C.71: Symbol error rate as a function of E_b/N_0 for the average case (BPSK, uplink)
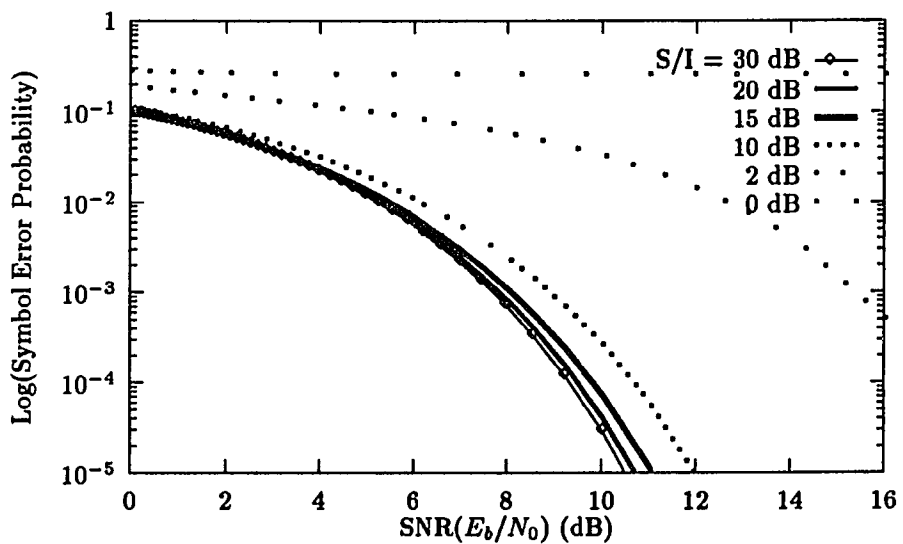
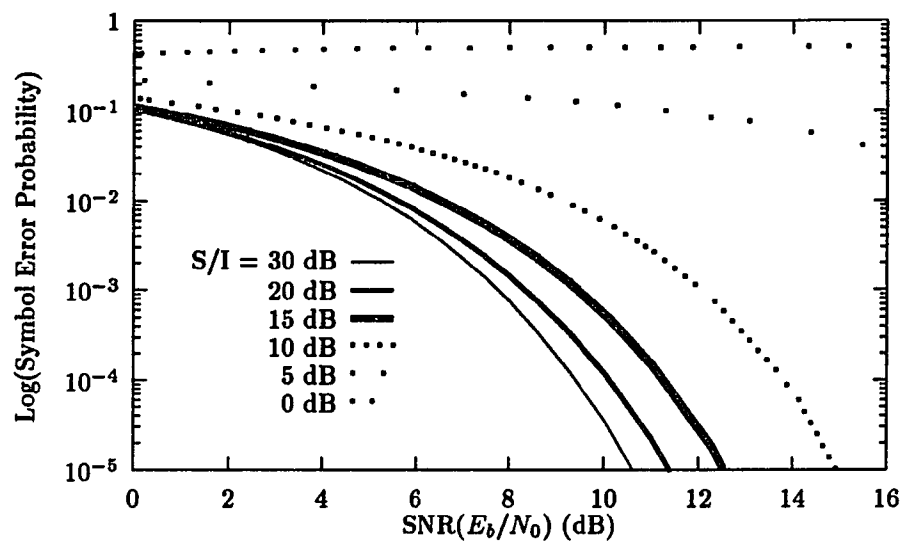Figure C.72: Symbol error rate as a function of E_b/N_0 for the worst case (BPSK, uplink)
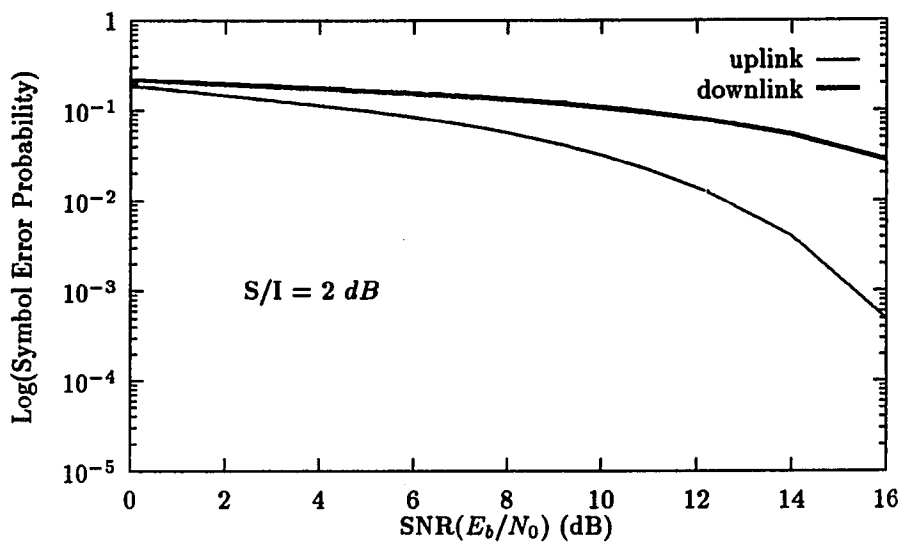
Figure C.73: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (BPSK, S/I = 2 dB)
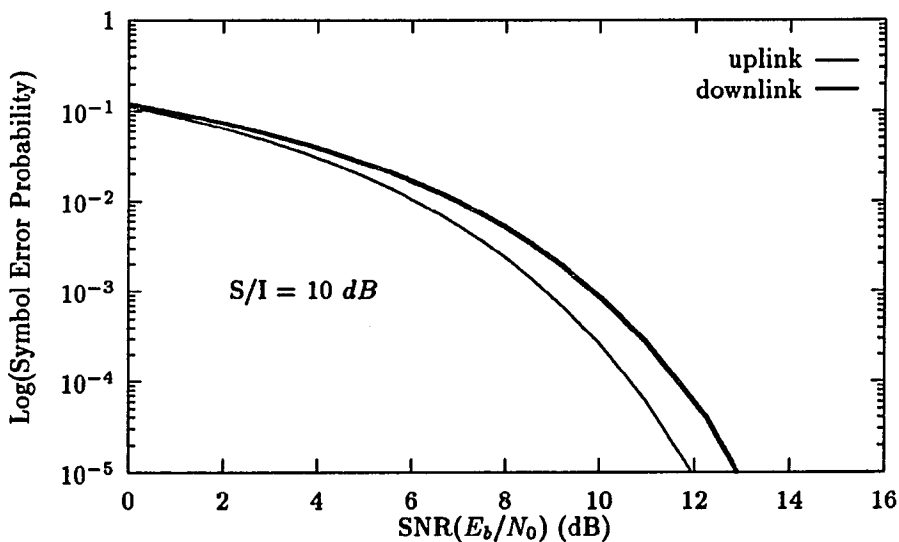
Figure C.74: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (BPSK, S/I = 10 dB)
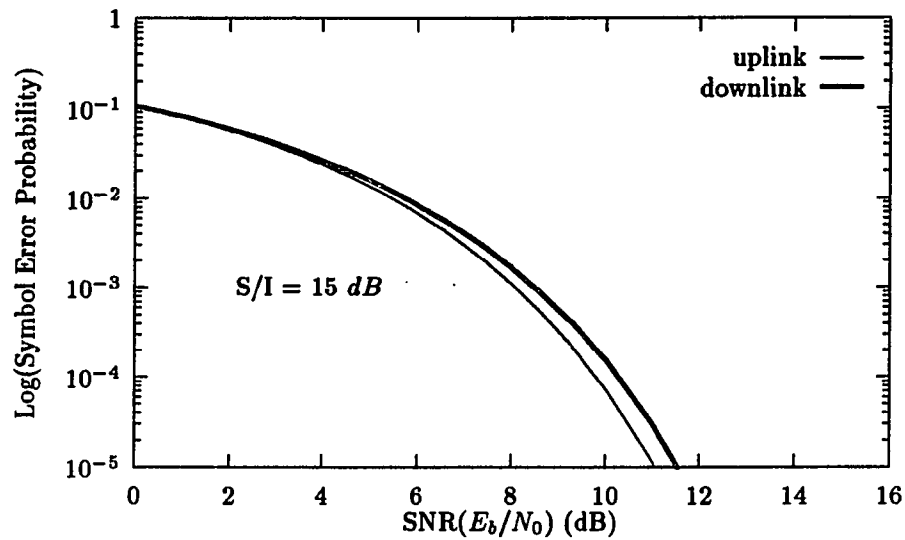
Figure C.75: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function E_b/N_0 (BPSK, S/I = 15 dB)
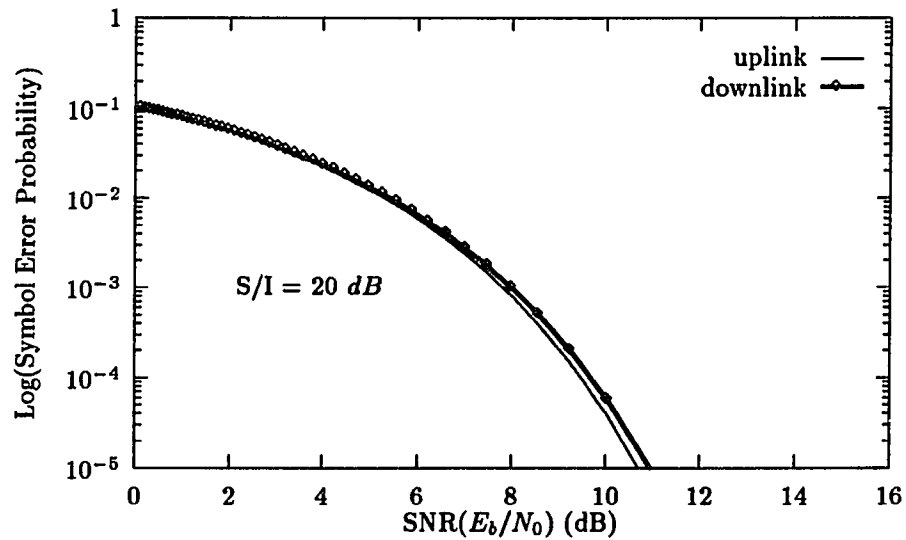
Figure C.76: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (BPSK, S/I = 20 dB)
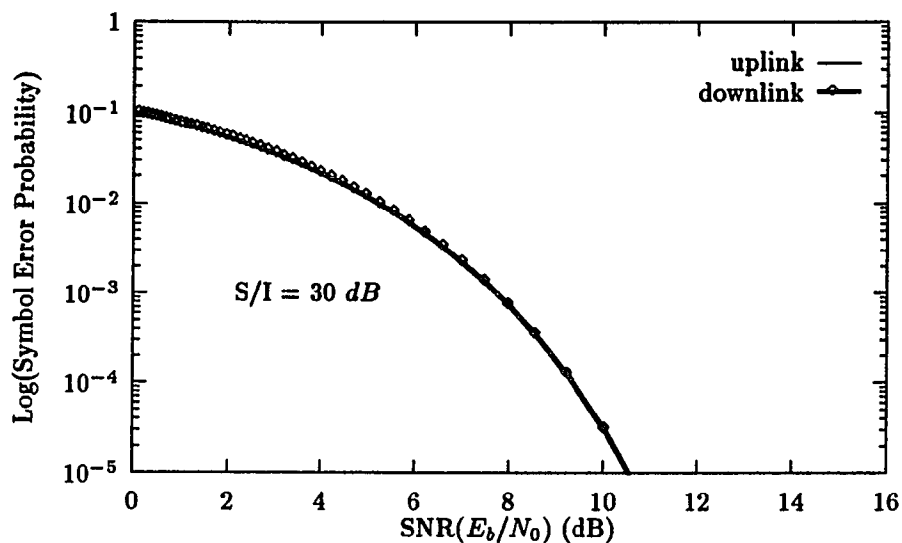
Figure C.77: Symbol error rate comparison between the uplink interference and the downlink interference for the worst case as a function of E_b/N_0 (BPSK, S/I = 30 dB)
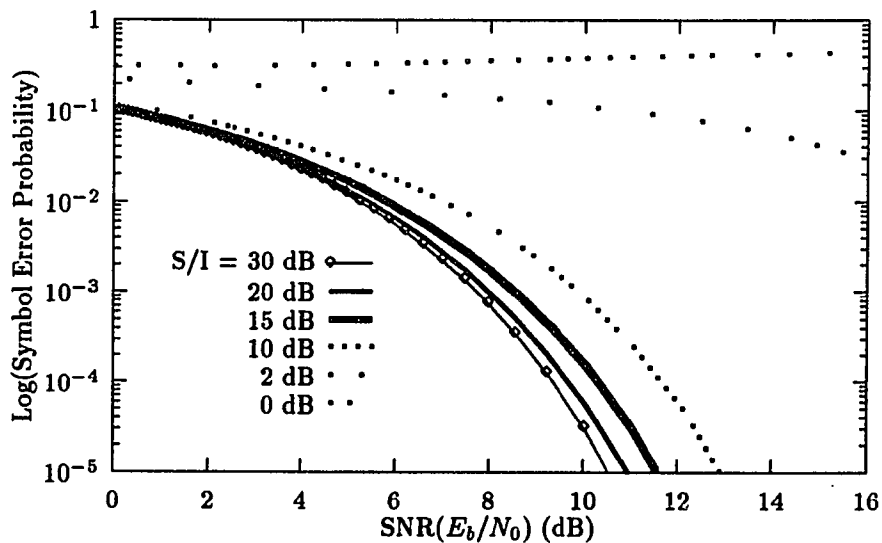
Figure C.78: Symbol error rate as a function of E_b/N_0 for the worst case (BPSK, downlink)

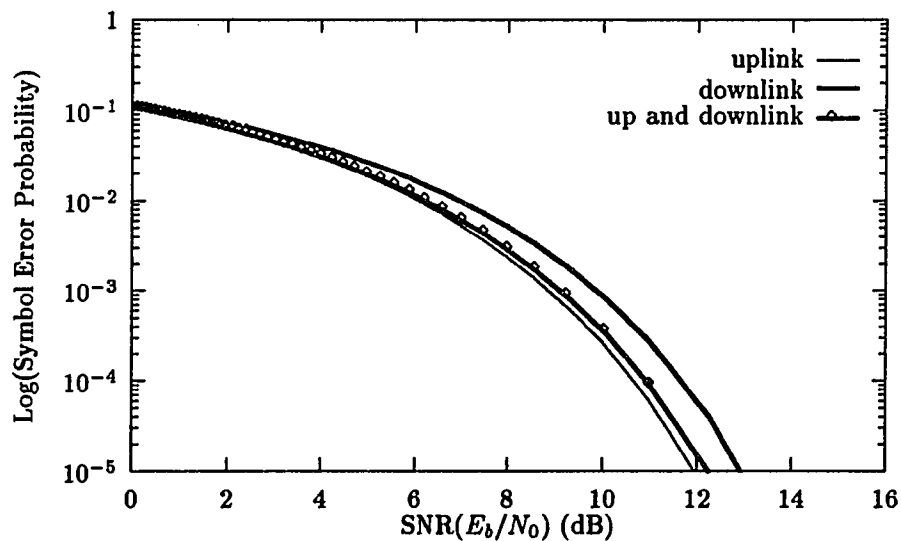Figure C.79: Symbol error rate comparison as a function of E_b/N_0 (S/I $= 10$ dB)
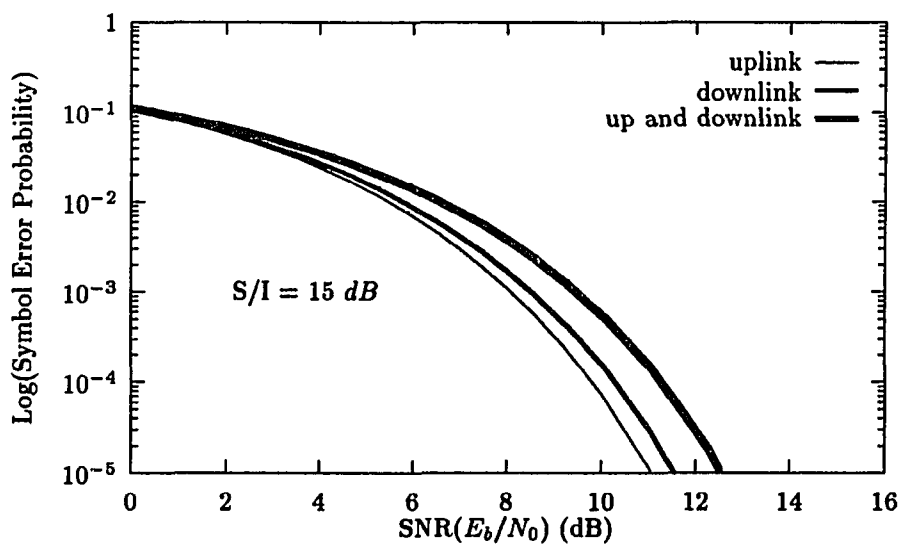
Figure C.80: Symbol error rate comparison as a function of E_b/N_0 (S/I $= 15$ dB)
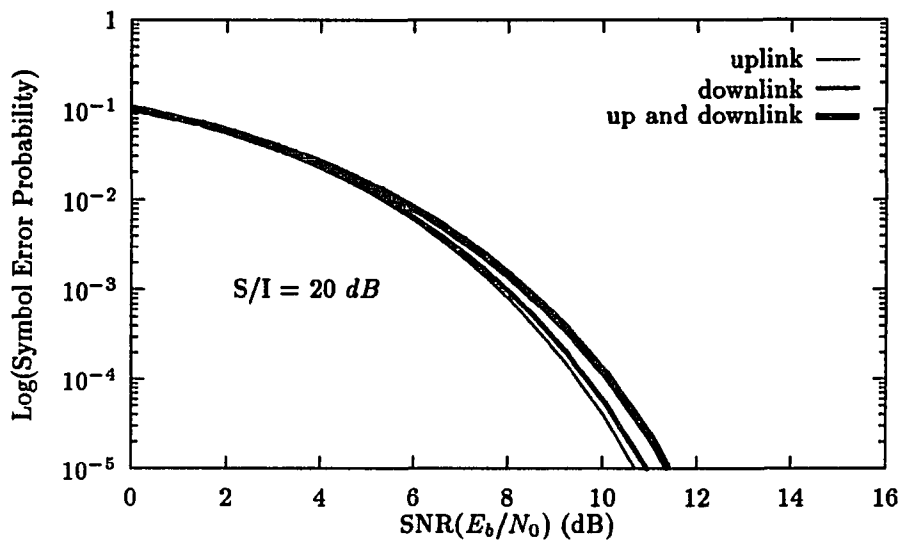
Figure C.81: Symbol error rate comparison as a function of E_b/N_0 (S/I = 20 dB)
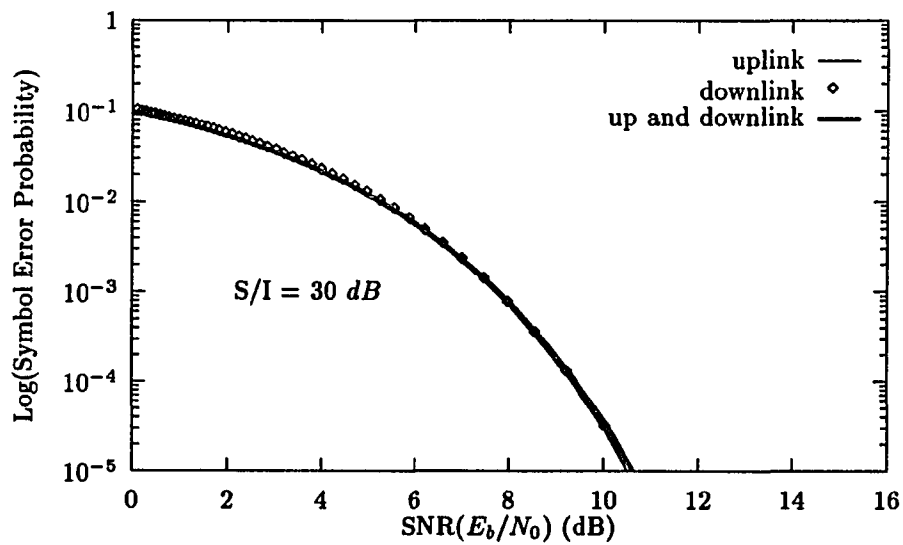
Figure C.82: Symbol error rate comparison as a function of E_b/N_0 (S/I = 30 dB)

Figure C.83: Symbol error rate as a function of E_b/N_0 for the worst case (BPSK, up & downlink)

Figure C.84: Symbol error rate for S/I = 10 dB when the total interference power is fixed (BPSK)
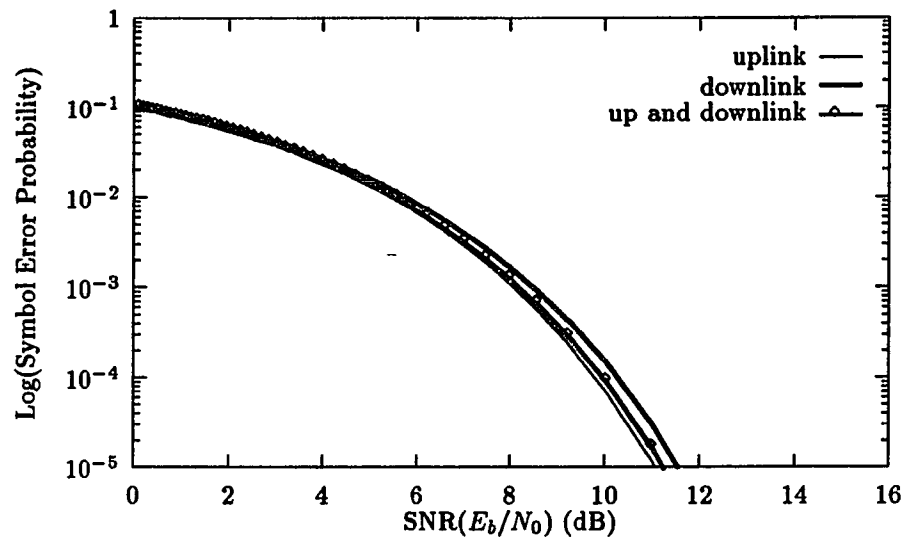
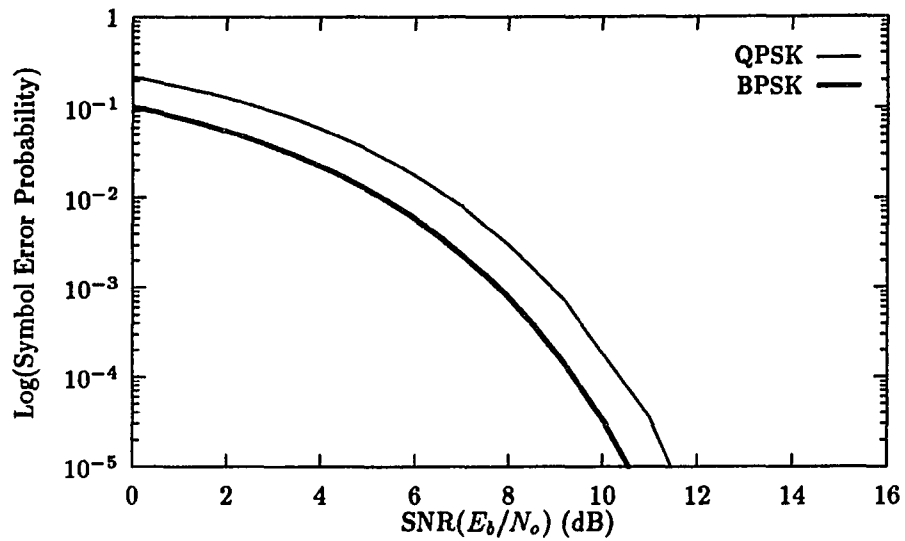Figure C.85: Symbol error rate for S/I = 15 dB when the total interference power is fixed (BPSK)
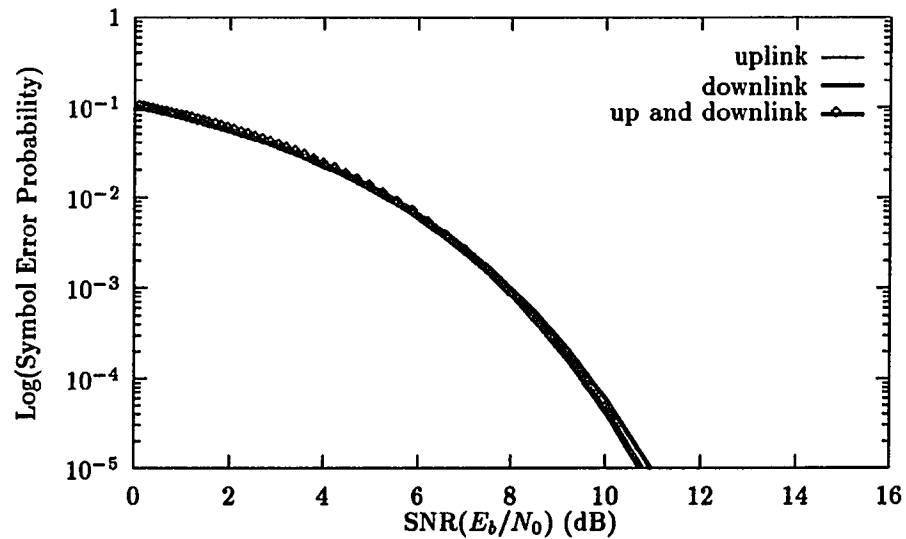
Figure C.86: Symbol error rate for S/I = 20 dB when the total interference power is fixed (BPSK)

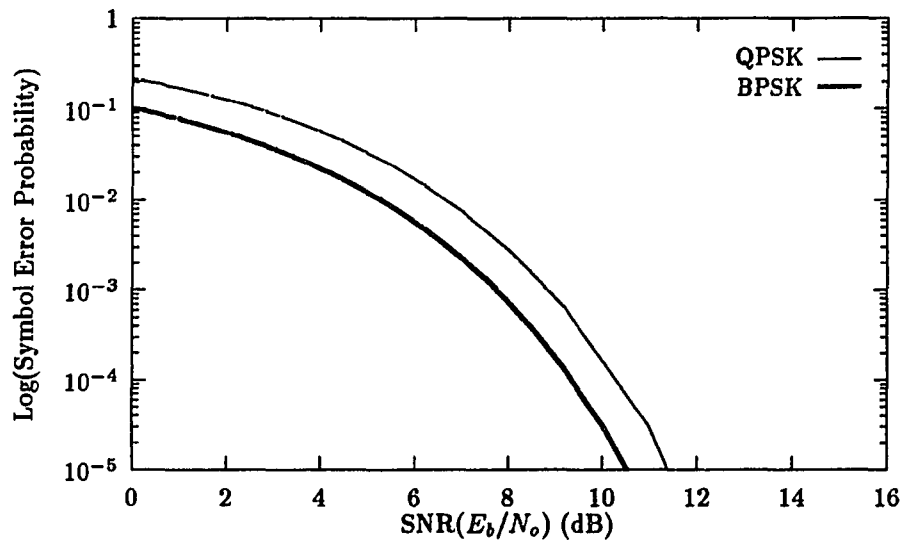Figure C.87: Symbol error rate comparison in the average case between QPSK and BPSK for S/I = 15 dB(uplink)

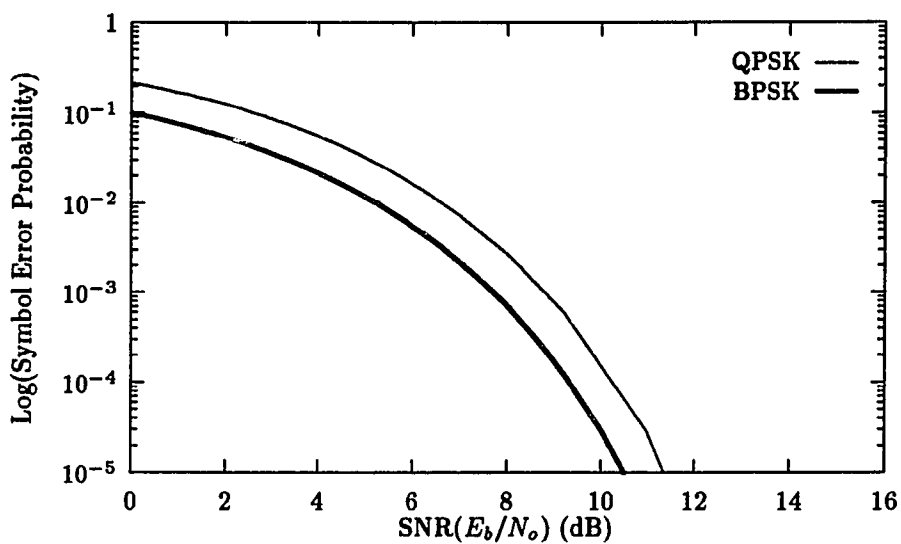Figure C.88: Symbol error rate comparison in the average case between QPSK and BPSK for S/I = 20 dB(uplink)

Figure C.89: Symbol error rate comparison in the average case between QPSK and BPSK for S/I = 30 dB(uplink)

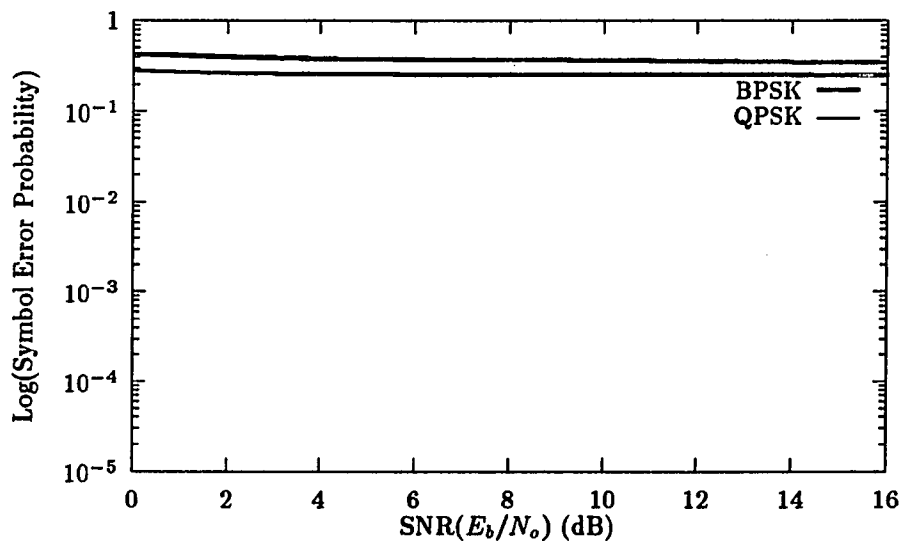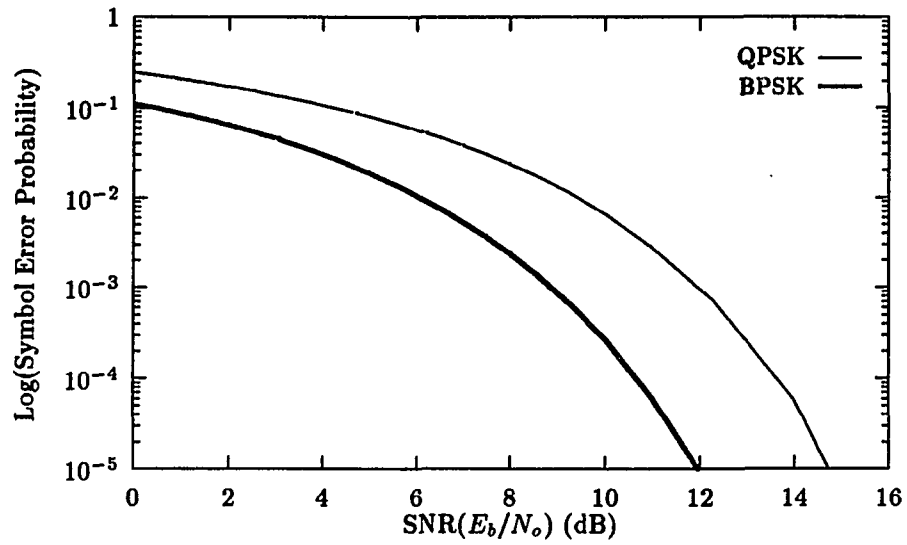Figure C.90: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 0 dB(uplink)

Figure C.91: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 10 dB(uplink)

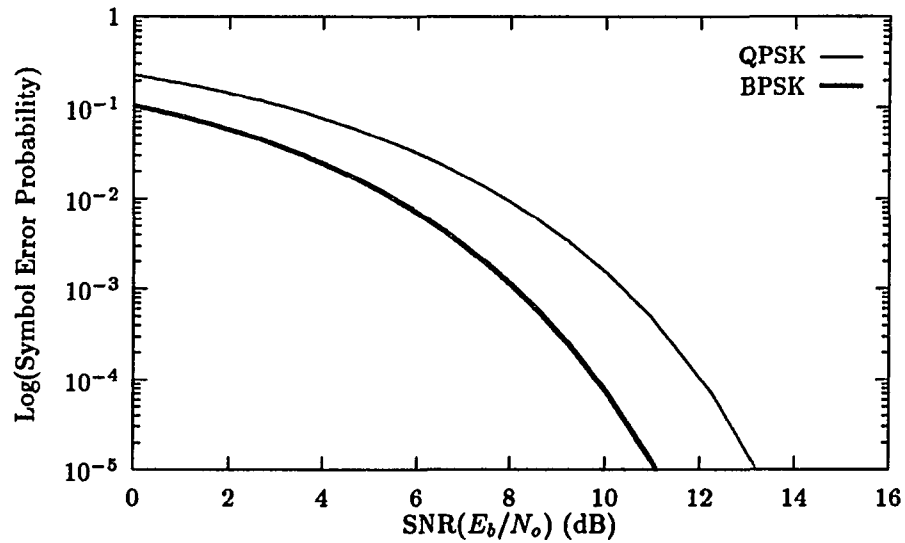Figure C.92: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 15 dB(uplink)

Figure C.93: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 20 dB(uplink)

Figure C.94: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 30 dB(uplink)
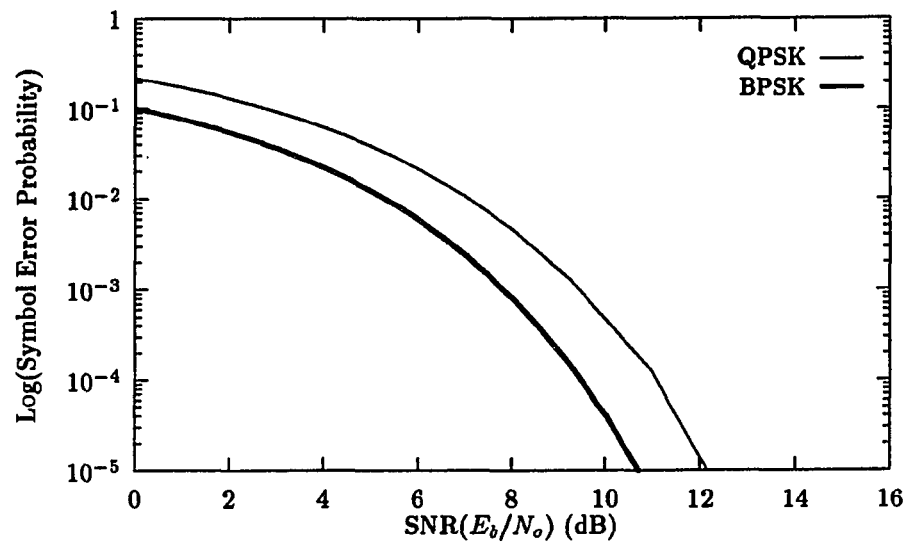
Figure C.95: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 0 dB(downlink)

Figure C.96: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 10 dB(downlink)

Figure C.97: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 15 dB(downlink)
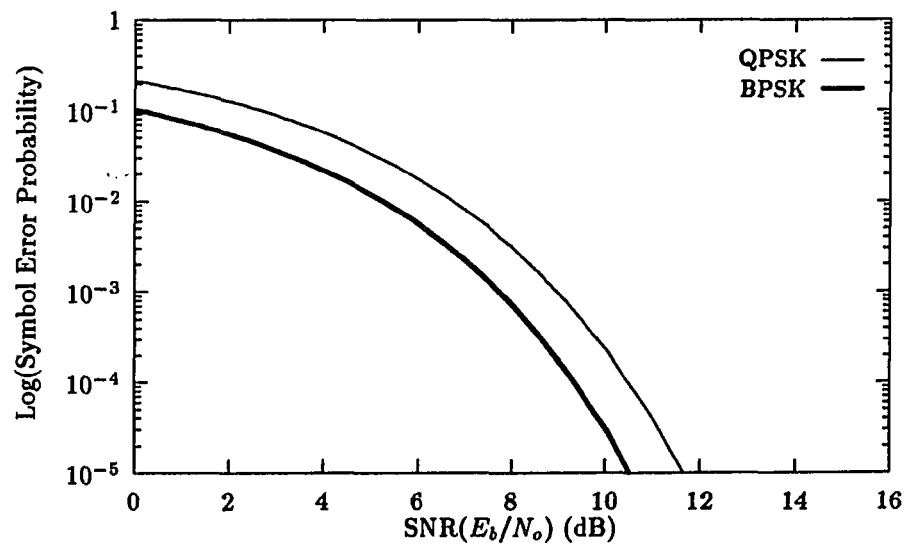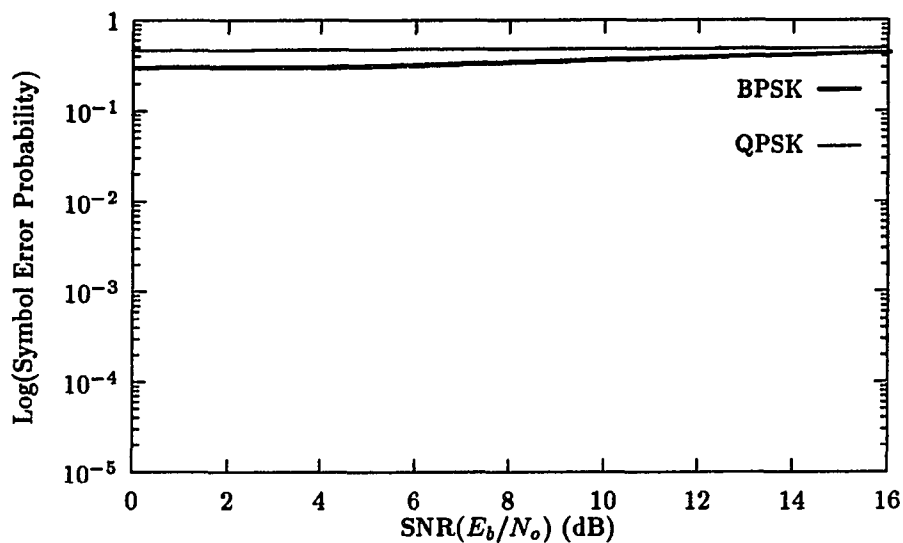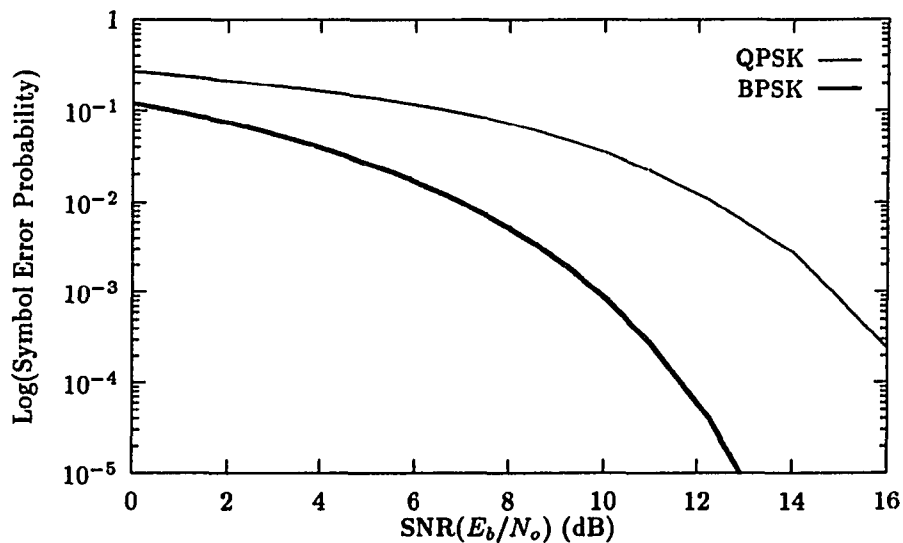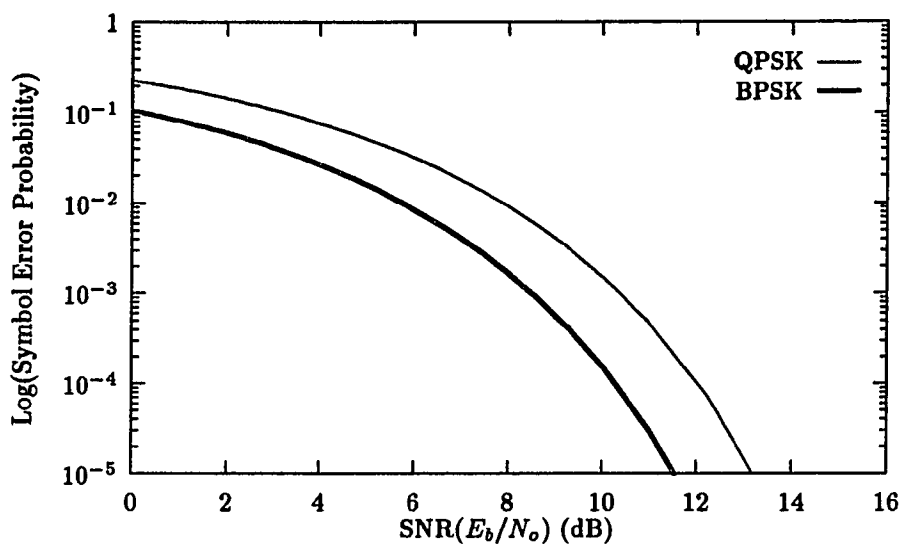

Figure C.98: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 20 dB(downlink)

Figure C.99: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 30 dB(downlink)

Figure C.100: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 0 dB(up and downlink)

Figure C.101: Symbol error rate comparison in the worst case between QPSK and BPSK for S/I = 20 dB(up and downlink, S/I = 20 dB on uplink, S/I = 20 dB on downlink)
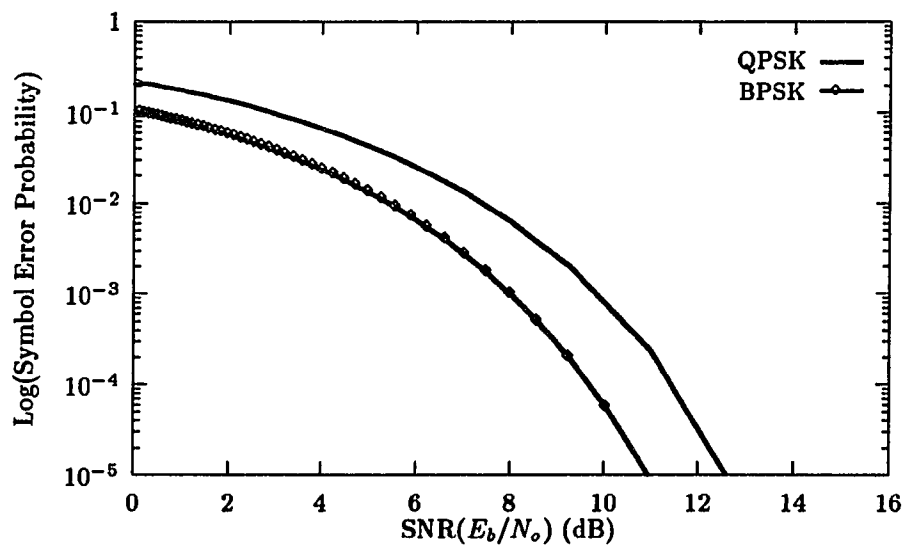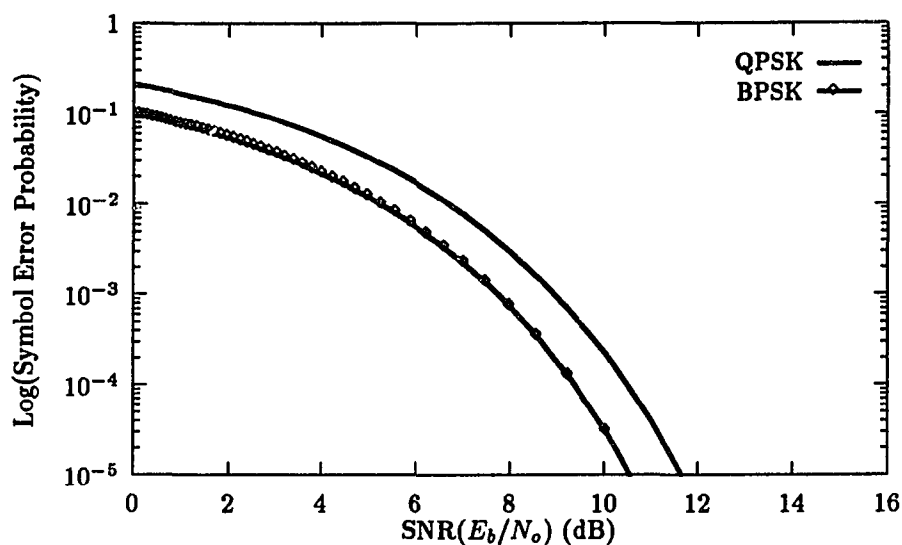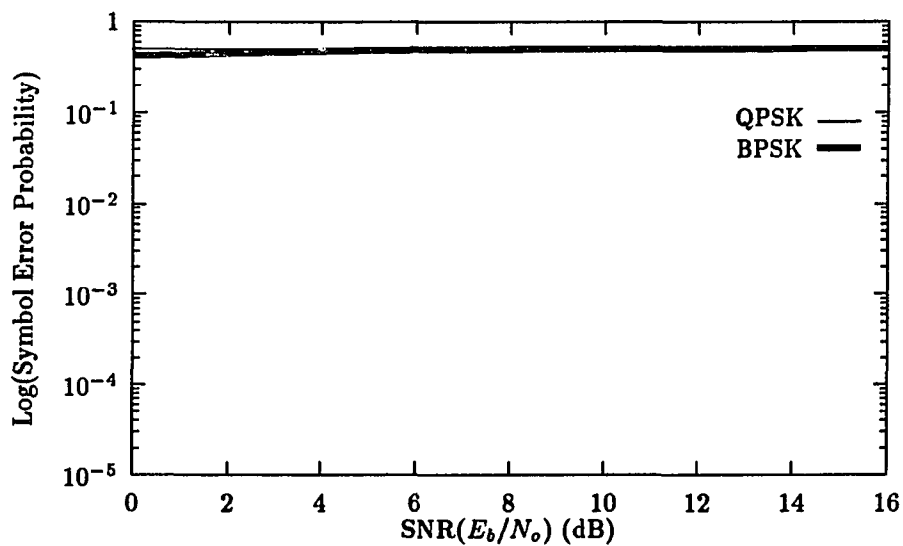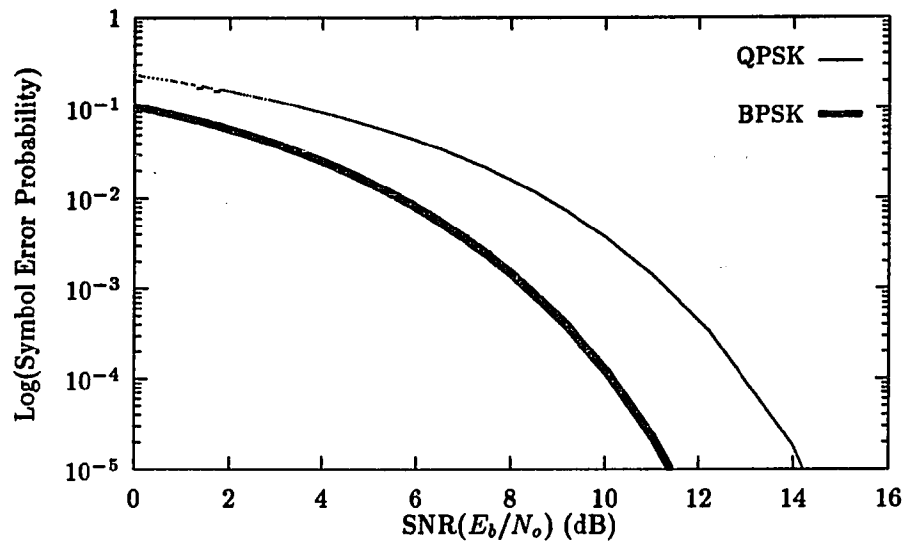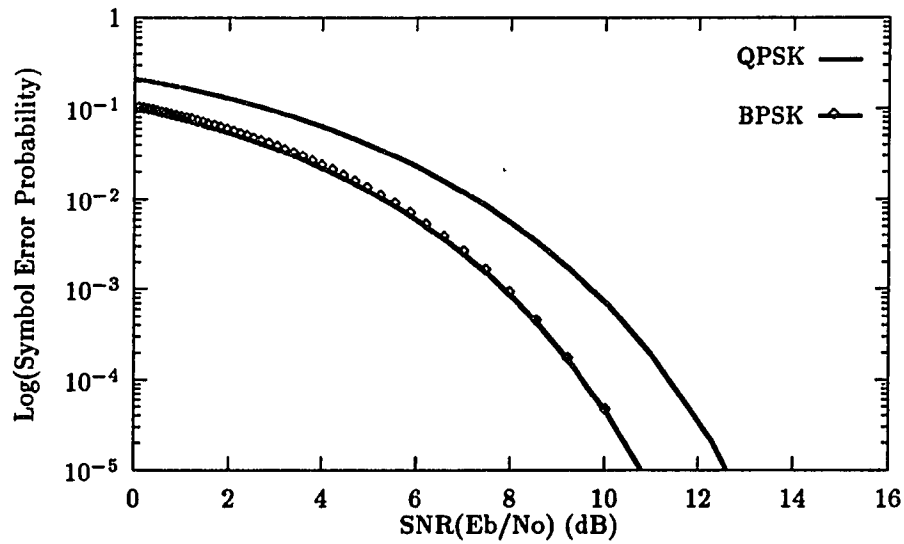

Figure C.102: Symbol error rate comparison for S/I = 20 dB when the total interference power is fixed(up and downlink, S/I = 23 dB on uplink, S/I = 23 dB on downlink)

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