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ABSTRACT

ON ADAPTIVE CODE DIVISION MULTIPLE ACCESS DETECTORS

by
Wenping Chen

Several variations of adaptive CDMA synchronous receiver schemes which can automatically adjust to the system parameters, are studied in this dissertation. Unlike their non-adaptive counterparts, these adaptive detectors require no knowledge of the received signals' energies and have similar or better performance.

Minimum energy and decorrelating criterion are used to update the weights. Since weights are updated according to the changes of the detector's outputs, hence no training sequence is required. The convergences of some of the aforementioned adaptive detectors are also disscussed.

The detectors proposed in this dissertation are near-far resistant,thus rendering the high-precision power control unnecessary. Among them, the two-stage adaptive detector with a soft tentative decision is worth mentioning. It combines the advantages of two different two-stage detectors and approaches the performance of the optimum detector, but with much less complexity.

ON ADAPTIVE CODE DIVISION MULTIPLE ACCESS DETECTORS

by
Wenping Chen

A Dissertation
Submitted to the Faculty of
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APPROVAL PAGE

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This dissertation is dedicated to:

My parents for their support and encouragement,
My wife for her love and patience,
My son for his presence and inspiration.

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CHAPTER 1

INTRODUCTION

Code Division Multiple Access (CDMA) is a technique by which two or more information source users share the same transmission medium and same frequency band, and yet the transmissions occur simultaneously. Based on the spread-spectrum technique for anti-jam and multipath rejection applications, CDMA has been proposed to support simultaneous digital communications among a large community of relatively uncoordinated users.

In CDMA, each user is assigned a different sequence called a signature code. The signal to be transmitted is modulated by this unique wide-band code sequence prior to transmission, spreading the spectrum of the waveform. Several packets of information are transmitted simultaneously over a common channel using preassigned signature code waveforms, which is unlike conventional narrow band cellular systems where signals from different users either transmit in different frequency bands (Frequency Division Multiple Access, or FDMA) or different time units (Time Division Multiple Access, or TDMA).

If one looks at CDMA in either the frequency or time domain, the CDMA signals appear to be on top of each other. A particular user's signal is "separated" in the receiver by correlating the received signal with the original signature code, which only emphasizes signal energy from the selected user. The response of the correlator to the signals from other users represents interference.

Since CDMA transmits signals in the same time and frequency band, its capacity is essentially limited by interference (unlike FDMA and TDMA, capacities of which are primarily bandwidth limited). Any reduction in interference converts directly into an increase in its capacity. By utilizing the human voice activity and some other features, it has been claimed that the net improvement in capacity of CDMA over digital TDMA is on the order of 4 to 6 times [1, 2, 3], and over FDMA about 20 times [3].

The conventional method to detect the CDMA signal is to treat the desired user as the only user presented and to treat the signal from the other users as noise. A conventional detector consists of a bank of matched filters, each one matched to the signature sequence of the particular user, thereby ignoring the multiple-access interference, or equivalently, ignoring the cross-correlations between the modulation sequences of different users. The sampled output of each matched filter contains the desired signal, the residual interference from all other users, and additive noise. This conventional detector is relatively simple to implement, but it is vulnerable to the near-far effect. That is, when strong interference is present, it is very difficult to demodulate the weak signal. Stringent power control and/or low cross-correlation codes have been used to solve the problem. However, power control comes at the price of increased complexity of the system and low cross-correlation between a given number of signals can be achieved only at the expense of an increased bandwidth. It is not surprising that reliable performance from the conventional detector has been possible only for low bandwidth efficiencies [4].

Acknowledging the fact that multiuser interference can not be modeled as an additive white Gaussian process, in recent years, a lot of work has been done in defining a detector whose performance is superior to the performance of the conventional detector. In [5, 6, 7], an optimum CDMA multiuser detector based on the Maximum-Likelihood (ML) sequence detector has been studied. The complexity of the ML detector grows exponentially with the number of users and thus it is impractical unless the number of the users is quite small. A class of sub-optimum detectors was studied in [8]-[14]. Among them are detectors that use the decorrelating detector, which is based on the linear transformation of the sampled matched filters' outputs [10, 11]. The decorrelating decision-feedback detector presented in [12] utilizes the differences in received users' energies, where the decisions of the stronger users are used to eliminate interference on weaker users. Another approach for sub-optimum multiuser detectors with low complexity was proposed in [15, 16], where in order to perform detection of the desired user, tentative decisions on information bits of all other users are made. The estimate of the multiple access interference is then obtained and is subtracted from the desired signal. The performance of some of these sub-optimum schemes is close to the performance of the optimum detector. Particularly when the energy of the interference increases, they become indistinguishable. However, prior estimation of the received signal energies is required for the detectors' proper operation.

Some of the efforts to improve the performance of multiuser CDMA detectors were directed toward using conventional and novel adaptive algorithms for interference cancellation and signal separation. For example, in [17] and [18], the one-stage multiuser CDMA detector was used in succession with linear transformation, decorrelation, tentative decision (hard limiter), followed by an adaptive interference canceler using the minimum energy criterion. Its error performance was studied and shown to be significantly better than the decorrelating detector. A convergence analysis of this adaptive detector can be found in [19]. Further improvement in performance, particularly at a low interference-to-desired-signal ratio, was obtained when the tentative decision was obtained from a soft-limiter [20]. A synchronous CDMA multiuser detector using an algorithm called the "bootstrapped decorrelating algorithm" was proposed in [21].

In this dissertation we propose several versions of synchronous adaptive sub-optimum detector using two different adaptive algorithms. These adaptive detectors have potential of automatically adjusting to the time-varying channel and do not require the knowledge of the received signal energies and yet achieve similar or better performance than their non-adaptive counterparts with similar complexity. Their steady state weights and error probabilities are evaluated. The convergence of the weight training is also studied. All the analytical results are confirmed by the simulations.

1.1 Outline of the Dissertation

The dissertation is organized as follows. Following this introduction, Chapter 2 will give a brief review of some of previous work. Among them are the conventional detector, the optimum detector, and the decorrelating detector. In Chapter 3, two adaptive algorithms used in this dissertation, the minimum energy algorithm and the decorrelating algorithm are studied. One-stage detectors with and without tentative decision are introduced in Chapters 4 and 5. In Chapters 6 and 7, two-stage adaptive detectors are studied. In Chapter 8, a two-stage adaptive detector using the soft tentative decision is proposed. Finally, the conclusion is given in Chapter 9.

CHAPTER 2

PREVIOUS WORK

In this chapter, several types of synchronous CDMA detectors will be introduced and their error performance presented.

2.1 The Conventional Single User Detector

The conventional single user approach to multiuser detection is to demodulate each user's signal as if it were the only one present. It consists of a bank of filters, each one matched to the signature sequence of the particular user, as shown in Fig. 2.1.

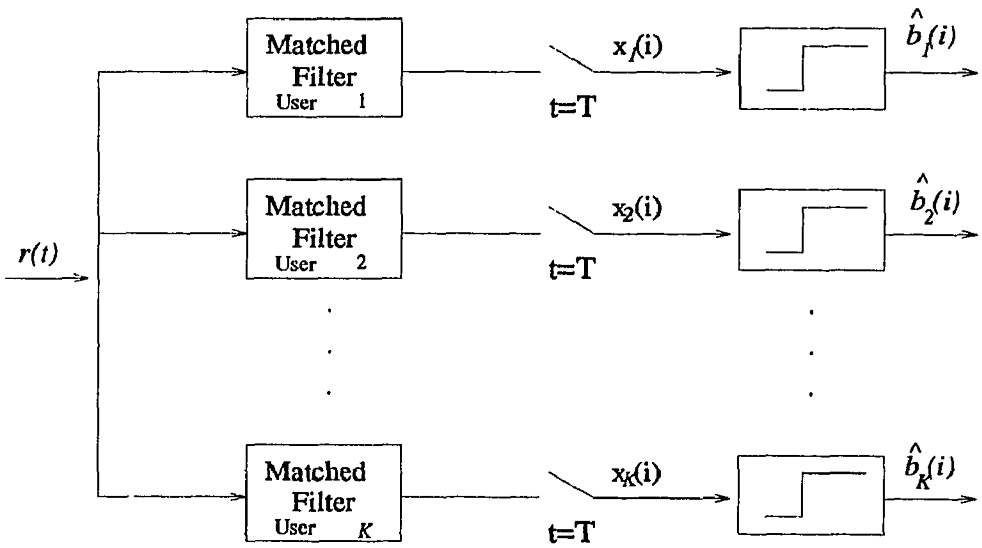

Figure 2.1 Conventional Single User Detector

For the synchronous case, the received signal $r(t)$ is expressed as:

$$r(t) = \sum_{k=1}^{K} \sum_{i} b_k(i) \sqrt{a_k} s_k(t - iT) + n(t), \tag{2.1}$$

where $b_k(i) \in \{-1, +1\}$ is the k-th user's data bit in the i-th time interval and $n(t)$ is the additive, zero mean, white Gaussian noise with a two-sided energy spectral density of $N_0/2$. The received energy of the k-th user signal, unknown to the receiver, is denoted as a_k. The signature sequence $s_k(t)$, of the same duration T as a data bit, is known to the receiver. The sampled outputs of the bank of matched filters in the i-th bit interval can be expressed as:

$$\boldsymbol{x}(i) = \boldsymbol{\mathcal{P}}\boldsymbol{A}\boldsymbol{b}(i) + \boldsymbol{n}(i) \tag{2.2}$$

For the sake of convenience the index i will be omitted elsewhere in the text if the synchronous case is discussed. Then Eq. (2.2) becomes:

$$\boldsymbol{x} = \boldsymbol{\mathcal{P}}\boldsymbol{A}\boldsymbol{b} + \boldsymbol{n}$$

where:

$$\boldsymbol{x} = [x_1, x_2, \dots, x_K]^T$$

$$\boldsymbol{b} = [b_1, b_2, \dots, b_K]$$

$$\boldsymbol{A} = diag[\sqrt{a_1}, \sqrt{a_2}, \dots, \sqrt{a_K}]$$

$$\boldsymbol{n} = [n_1, n_2, \dots, n_K]^T.$$

$$\boldsymbol{\mathcal{P}} = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1K} \\ \rho_{21} & 1 & \dots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \dots & 1 \end{bmatrix}$$

The (k, j)-th element of the symmetric cross-correlation matrix $\boldsymbol{\mathcal{P}}$ is defined as:

$$\rho_{kj} = \int_0^T s_k(t)s_j(t)dt \quad k, j \in (1, 2, \dots, K) \quad \text{with } \rho_{kk} = 1.$$

The covariance matrix of a zero mean Gaussian noise vector $\boldsymbol{n}$ is:

$$E\{\boldsymbol{n}\boldsymbol{n}^T\} = \frac{N_0}{2}\mathcal{P}.$$

From expression (2.2), we can see that the output of each matched filter contains not only the desired signal, but also the interferences from the other users. Due to the existence of interference, the error performance of the conventional CDMA detector is determined by both the signal-to-noise ratio ($SNR_k = a_k/N_o$) and the amount of interference. The expression of error probability of the conventional CDMA detector for the k-th user is given in Eq. (2.3):

$$P_{e_k} = \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k} Q\left(\frac{\sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k}{N_o/2}\right), \tag{2.3}$$

where $\boldsymbol{A}_k$ is a diagonal $(K-1) \times (K-1)$ submatrix of $\boldsymbol{A}$ with its k-th diagonal entry removed, $\boldsymbol{\rho}_k$ is a $(K-1) \times 1$ column vector obtained from $\mathcal{P}$ by deleting the element ρ_{kk}, $\boldsymbol{b}_k$ is a $(K-1) \times 1$ vector obtained from $\boldsymbol{b}$ by deleting the element b_k.

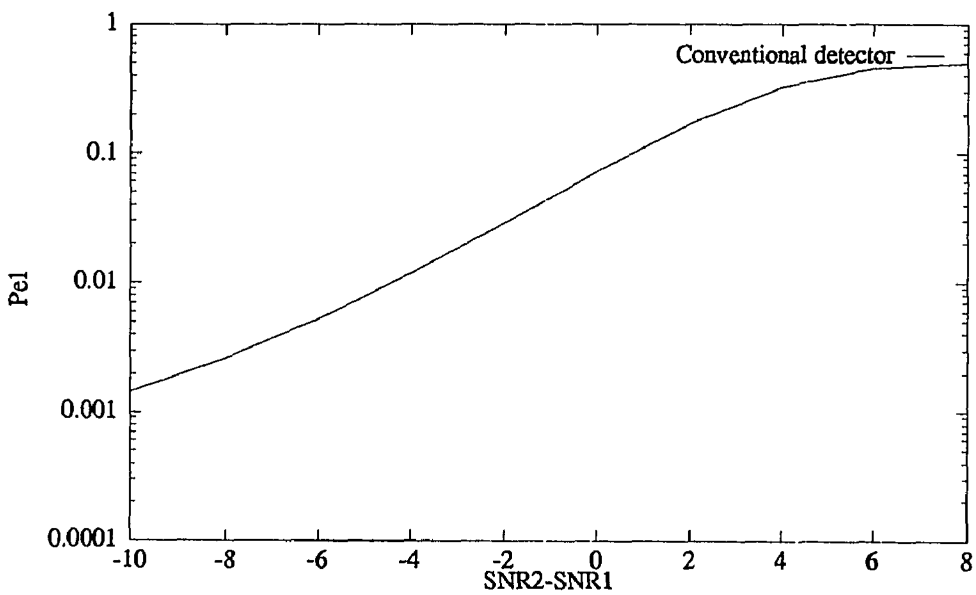

Figure 2.2 Probability of Error for User 1 ($K = 2$, $SNR_1 = 8$ dB, $\rho_{12} = 0.7$)

Fig. 2.2 is a simple, two-user error performance example of the conventional detector. The cross-correlation coefficient is taken to be 0.7. The SNR of user one is set to 8 dB, while the SNR of user two, relative to user one, varies from -10 to 8 dB. From this figure we notice that the stronger the interferer (user two) is, the worse the performance of the desired user (user one). This is referred to as the near-far problem.

2.2 The Optimum Multiuser Detector

Fig. 2.3 shows an optimum detector, proposed in [5], for the multiuser interference environment and it is shown to eliminate the near-far problem and provide a much improved performance.

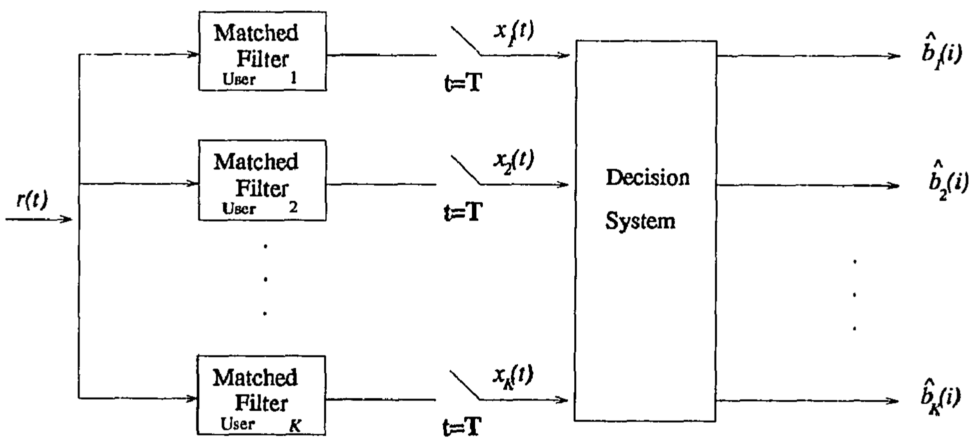

Figure 2.3 Optimum Multiuser Detector

The optimum detector consists of a bank of matched filters followed by a decision system employing the maximum likelihood algorithm. Denote the optimum decision on $\boldsymbol{b}^o$ as:

$$\boldsymbol{b}^o = [b_1^o, b_2^o, \ldots, b_k^o]$$

which maximizes the log-likelihood function [27, 28]. That is, choose $\boldsymbol{b}^o$ such that

$$\left\{2(\boldsymbol{A}\boldsymbol{b}^o)^T\boldsymbol{x} - (\boldsymbol{A}\boldsymbol{b}^o)^T\boldsymbol{\mathcal{P}}(\boldsymbol{A}\boldsymbol{b}^o)\right\}, \tag{2.4}$$

is maximized. The maximum of (2.4) can be achieved by substituting 2^K possible values of $\boldsymbol{b}^o$ into it and comparing their results. The computation complexity grows exponentially with the number of users.

2.3 The Sub-optimum Multiuser Detectors

Several sub-optimum detectors which have much less computational complexity and slightly worse performance than the optimum detector, but better performance than the conventional detector, are introduced in [7, 8, 9, 10, 11, 12, 15, 16]. Among them are the decorrelating detector and the two-stage fixed weights detector.

2.3.1 The Synchronous Decorrelating Detector

The decorrelating detector is first mentioned in [7]. It consists of a a bank of matched filters and a decorrelator, as shown in Fig. 2.4.

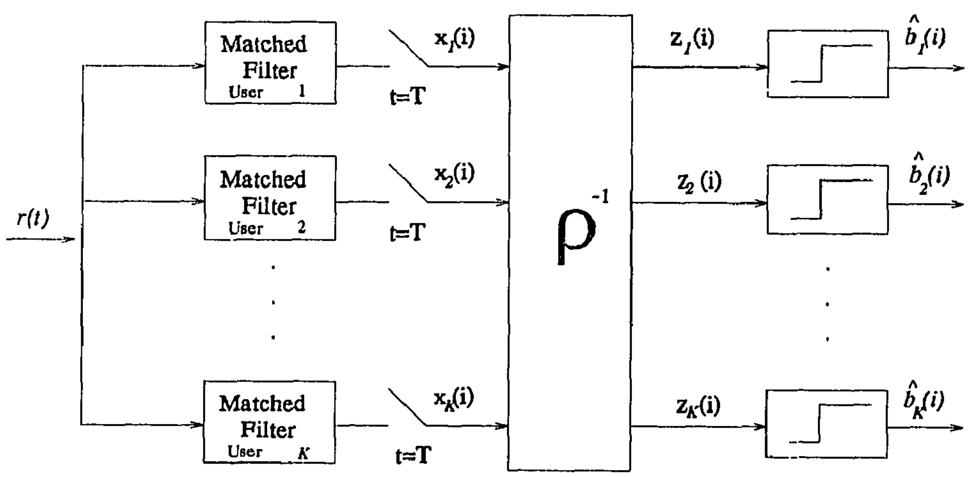

Figure 2.4 Synchronous Decorrelating Detector

The decorrelator is actually a linear transformation using the inverse of the code cross-correlation matrix.

The output vector of the decorrelator is given by:

$$z = \mathcal{P}^{-1}x = Ab + \mathcal{P}^{-1}n = Ab + \Gamma n = Ab + \xi, \tag{2.5}$$

where Γ is the inverse of $\mathcal{P}$, and $\xi = \Gamma n$.

From Eq. (2.5) we can see that the signals at the output of the decorrelator are fully decorrelated and resulted in signal separation.

Hence, the error probability of the decorrelator is given by:

$$P_{e_k} = Q\left(\frac{\sqrt{a_k}}{\sigma_{\xi_k}}\right),$$

where σ_{ξ_k} is the standard deviation of noise ξ_k.

Fig. 2.5 shows the error performance of the two-user decorrelator for $SNR_1 = 8dB$ and $\rho_{12} = 0.7$. The error performance of the conventional and single-user detector are also included for comparison.

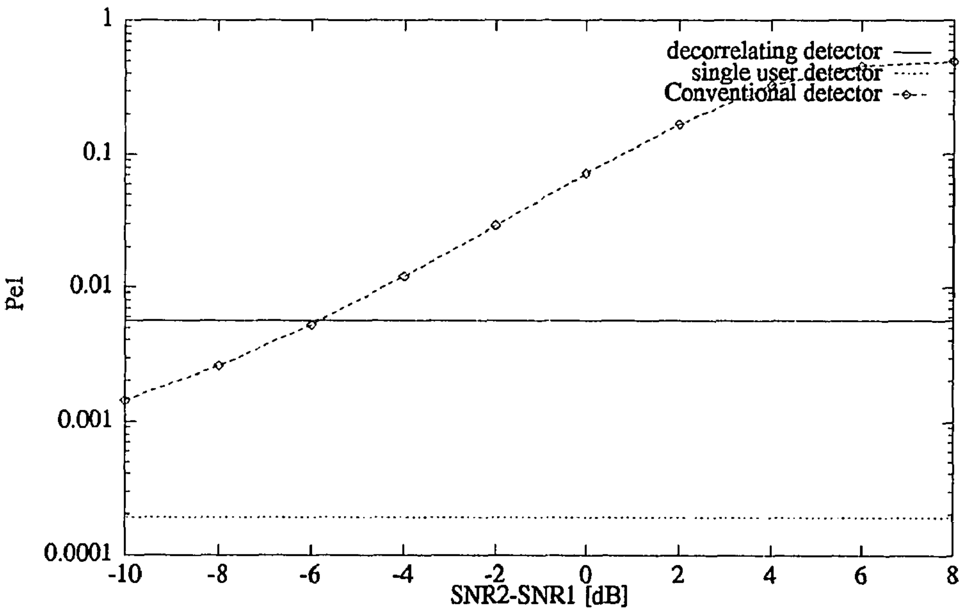

Figure 2.5 Probability of Error for User 1 ($K = 2$, $SNR_1 = 8$ dB, $\rho_{12} = 0.7$)

It is interesting to compare the performance of the decorrelating detector and the conventional detector. The former is generally better than the latter, but this is not necessarily true when the interference is weak. This leaves some space for the soft decision which we will discuss in later chapters.

2.3.2 Two-stage Fixed Weight Detector

Another approach for suboptimum multiuser detectors with low complexity was proposed in [15]. It consists of a bank of matched filters followed by a decorrelator, tentative hard decision and a multiple-access interference (MAI) canceler. In order to perform detection of the desired user, tentative decisions on information bits of all other users are made based on the outputs of the decorrelator. The estimate of the multiple access interference is then obtained and is subtracted from the desired signal.

The output of the detector is given by:

$$\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{W}^T\hat{\boldsymbol{b}}, \tag{2.6}$$

where

$$\boldsymbol{W} = \begin{bmatrix} 0 & w_{12} & \dots & w_{1K} \\ w_{21} & 0 & \dots & w_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ w_{K1} & w_{K2} & \dots & 0 \end{bmatrix}.$$

and $w_{jk} = \rho_{jk}\sqrt{a_j}$, for each $j,\ k,\ j \neq k$. The error probability of the k-th user for a K-user case can be expressed as:

$$P_{e_k} = \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k} Q\left(\frac{\sqrt{a_k} - \left(\boldsymbol{b}_k - \hat{\boldsymbol{b}}_k\right)^T \boldsymbol{A}_k \boldsymbol{\rho}_k}{\sqrt{N_0/2}} \right) Pr\{\hat{\boldsymbol{b}}_k | \boldsymbol{b}_k\}. \tag{2.7}$$

Fig. 2.6 plots the error performance of the detector for a two-user case $\rho_{12} = 0.7$ and $SNR_1 = 8$ dB with SNR_2 changed from -10 to 8 dB. The error probability of the decorrelator is also plotted for comparison.

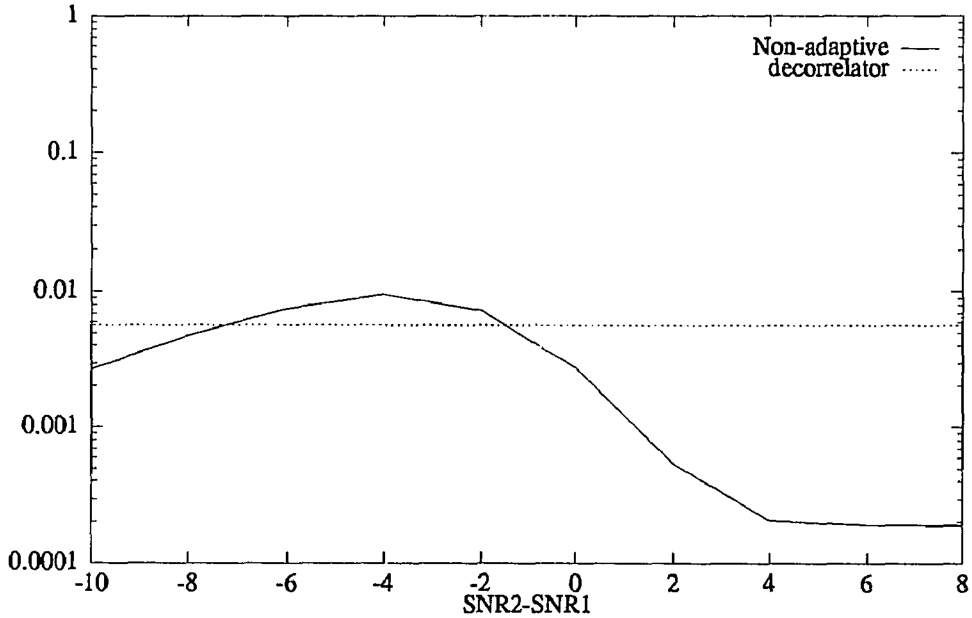

Figure 2.6 Error Probability of User 1 for $K = 2$, $\rho = 0.7$ and $SNR_1 = 8$ dB

The performance of this sub-optimum scheme is close to the performance of the optimum detector, particularly when the energy of the interference increases, they become indistinguishable.

CHAPTER 3

ADAPTIVE CANCELERS

In CDMA, several users' signals overlap both in time and in frequency. Unlike the conventional detector, where the desired signal is detected as if it is the only user present and signals from other users are treated as noise, in the sub-optimum detectors proposed in this dissertation, we introduce an adaptive cancellation scheme. The cancellation scheme uses the estimations of the interferences and deducts them from the input. Fig. 3.1 is a model of the multiuser canceler for the k-th user,

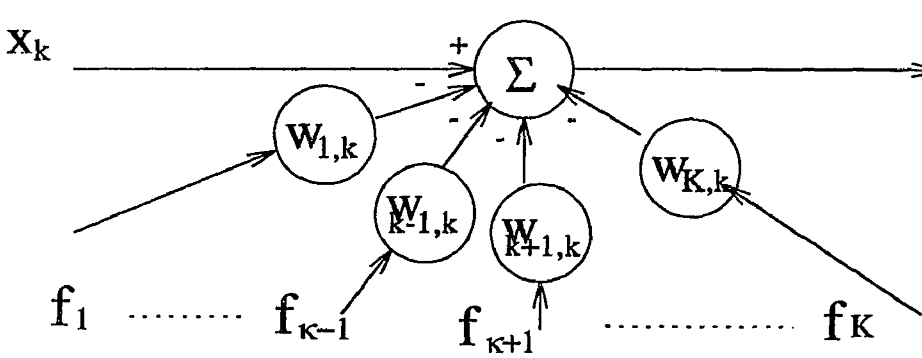

Figure 3.1 Model of Multiuser Canceler

where in the figure, x_k and y_k are the input and output of the canceler of the k-th user, respectively. x_k is usually obtained from the output of the matched filter of the k-th user. $w_{j,k}$, $(j = 1, \ldots, K, j \neq k)$ is the weight from the j-th user to the k-th user, for a adaptive scheme the weights have to be updated by the adaptive algorithm, and f_j is the estimate of the j-th interference. The estimates of the interferences have to be generated by the detector, unlike cases where training sequences are available.

Without loss of generality, we can consider the detector's output for user one, y_1, which is given by:

$$y_1 = x_1 - \boldsymbol{w}_1^T \boldsymbol{f}_1$$

$$\begin{aligned} &= \sqrt{a_1}b_1 + (\rho_{12}\sqrt{a_2}b_2 - w_{21}f_2) + (\rho_{13}\sqrt{a_3}b_3 - w_{31}f_3) \\ &+ \ldots + (\rho_{1K}\sqrt{a_K}b_K - w_{K1}f_K) + n_1, \end{aligned} \tag{3.1}$$

where

$$\boldsymbol{f}_1 = [f_2, f_3, \ldots, f_K]^T$$

$$\boldsymbol{w}_1 = [w_{21}, w_{31}, \ldots, w_{K1}]^T.$$

Now the design of an adaptive detector becomes solving the following two problems:

1. Where and how to get the estimation of the interference signals?

2. Which performance index to use in finding the weight matrix?

We will discuss the first problem in later chapters.

The most commonly used method for adaptation of weights is the steepest descent algorithm. The updating rule is expressed as:

$$\boldsymbol{w}_k(i+1) = \boldsymbol{w}_k(i) - \frac{1}{2}\mu \nabla_{\boldsymbol{w}_k} E\{f^2(\boldsymbol{w}_k)\}, \tag{3.2}$$

where $\boldsymbol{w}_k$ is the weight vector for the k-th user, and $f()$ is a cost function which is a function of $\boldsymbol{w}_k$.

Thus the steady state of the weight vector for user k can be computed from:

$$\nabla_{\boldsymbol{w}_k} E\{f^2(\boldsymbol{w}_k)\} = 0.$$

The troublesome of the Eq. (3.2) is the expectation operator. Although in practice we can substitute the expectation with the time average, it will need some memory and thus it will increase the complexity of the system. An alternative of this is the stochastic gradient (SG) algorithm. The principle behind the SG algorithm is to ignore the expectation. The quantity which is left, while random, has an expected value equal to the desired gradient. Thus, it is an unbiased estimate of the gradient [22]. A disadvantage of the SG algorithm compared to the time average is that in order to reduce the variation of steady state, a smaller step size have to be used and hence it slows down the speed of convergence. Various methods can be used to speed the convergence, such as the varied step size method which uses the large step size in the beginning to expedite the speed and uses the small step size for the steady state to minimize the variation. In all the simulation examples given in this dissertation, the SG algorithm is used.

In this dissertation several versions of adaptive detectors using one of the steepest descent algorithms (Minimum Energy) and decorrelating algorithm are proposed. The steady state performance of these two adaptive algorithms is studied. Their weight convergence issues are also discussed.

3.1 Minimum Energy Algorithm

Minimum Energy is a steepest descent-based algorithm. The cost function of the minimum energy algorithm is the energy of the output signal of the canceler. So it minimizes the output signal energy of the canceler.

Let $\nabla(y_1^2(n))$ denote the value of the gradient vector of output energy of user one with respect to its weight vector $\boldsymbol{w}_1(n)$ at time n. According to the method of steepest descent, the updated value of the weight vector at time $n+1$ is computed by using the simple recursive relation:

$$\boldsymbol{w}_1(n+1) = \boldsymbol{w}_1(n) - \frac{1}{2}\mu[\nabla(y_1^2(n))], \tag{3.3}$$

where μ is a positive real-valued constant referred to as step size. By definition and using Eq. (3.1):

$$\nabla(y_1^2(n)) = \begin{bmatrix} \frac{\partial y_1^2(n)}{\partial w_{21}(n)} \\ \frac{\partial y_1^2(n)}{\partial w_{31}(n)} \\ \vdots \\ \frac{\partial y_1^2(n)}{\partial w_{K1}(n)} \end{bmatrix} = \begin{bmatrix} -2y_1 f_2(n) \\ -2y_1 f_3(n) \\ \vdots \\ -2y_1 f_K(n) \end{bmatrix} = -2y_1 \boldsymbol{f}_1(n). \tag{3.4}$$

By substituting Eq. (3.4) into (3.3), we get:

$$\boldsymbol{w}_1(n+1) = \boldsymbol{w}_1(n) + \mu E\{y_1(n)\boldsymbol{f}_1(n)\}. \tag{3.5}$$

The steady state weight vector can also be analytically computed from a system of (K-1) equations:

$$E\{\nabla(y_1^2)\} = -2E\{y_1 \boldsymbol{f}_1\} = 0. \tag{3.6}$$

To find the optimum weights (minimum energy) by the way of the steepest descent algorithm, we proceed as follows:

1. We begin with an initial value $\boldsymbol{w}_1(0)$ for the weight vector corresponding to user one, which is chosen arbitrarily. The value $\boldsymbol{w}_1(0)$ provides an initial guess as to where the solution point may be located in the output energy surface. Typically, $\boldsymbol{w}_1(0)$ is set to equal to the null vector.

2. Using this initial or present guess, we compute the gradient vector, which is defined as the derivative of the output energy of user one with respect to its weight vector $\boldsymbol{w}_1(n)$ at time n (i.e., the nth iteration). The gradient vector can be computed using 3.6 in the implementation.

3. We compute the next guess at the weight vector by making a change in the initial or present guess in the direction opposite to that of the gradient vector.

4. Repeat the process from step 2.

3.1.1 Convergence Issue

In [19] the weight convergence of one steepest descent-type canceler has been studied. In this section, the results of [19] are extended to other minimum energy schemes considered in this dissertation. The adaptive rule for the weights is given by:

$$\begin{aligned} \boldsymbol{w}_k(i+1) &= \boldsymbol{w}_k(i) - \frac{1}{2}\mu \frac{\partial}{\partial \boldsymbol{w}_k(i)} E\{y_k^2\} \\ &= \boldsymbol{w}_k(i) + \mu E\{x_k \boldsymbol{f}_k\} - E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\}\boldsymbol{w}_k \\ &= (\boldsymbol{I} - \mu E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\})\boldsymbol{w}_k(i) + \mu E\{x_k \boldsymbol{f}_k\}. \end{aligned} \tag{3.7}$$

Since $E\{x_k \boldsymbol{f}_k^T\} = E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\})\boldsymbol{w}_k^o$, where $\boldsymbol{w}_k^o$ is the optimum weight, Eq. (3.7) can be written as:

$$\boldsymbol{w}_k(i+1) = (\boldsymbol{I} - \mu E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\})\boldsymbol{w}_k(i) + \mu E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\}\boldsymbol{w}_k^o.$$

Define

$$\boldsymbol{c}_k(i) = \boldsymbol{w}_k^o - \boldsymbol{w}_k(i). \tag{3.8}$$

and we have:

$$\begin{aligned} \boldsymbol{c}_k(i+1) &= (\boldsymbol{I} - \mu E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\})\boldsymbol{c}_k(i) \\ &= (\boldsymbol{I} - \mu E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\})^{i+1}\boldsymbol{c}_k(0) \\ &= (\boldsymbol{I} - \mu \boldsymbol{R}_k)^{i+1}\boldsymbol{c}_k(0) \\ &= \boldsymbol{H}_k^{i+1}\boldsymbol{c}_k(0), \end{aligned} \tag{3.9}$$

where both $\boldsymbol{R}_k = E\{\boldsymbol{f}_k \boldsymbol{f}_k^T\}$ and $\boldsymbol{H}_k = \boldsymbol{I} - \mu \boldsymbol{R}_k$ are $(K-1) \times (K-1)$ symmetric matrices. Define:

$$d_{max} = \max_i \{e_{ii}\}$$

$$d_{min} = \min_i \{e_{ii}\}$$

and

$$o_{max} = \max_{\substack{i,j \\ i \neq j}} |e_{ij}|,$$

where e_{ij} is the (i,j)-th element of $\boldsymbol{R}_k$. Diagonal elements of $\boldsymbol{H}_k$ range from $(1 - \mu d_{max})$ to $(1 - \mu d_{min})$, and off-diagonal elements range between $(-\mu o_{max}, \mu o_{max})$.

By using similar derivations as [19], it is easy to get that the the necessary and sufficient conditions for the weights to achieve convergence and stability are:

1. $(1 - \mu d_{min}) + \mu o_{max}(K-2) < 1$, which leads to

$$(K-2)o_{max} < d_{min}.$$

   This balances three elements of the system: the number of users that can access the system simultaneously, the SNRs, and the cross-correlation of their signature sequences.

2. $(1 - \mu d_{max}) - \mu(K-2)o_{max} > -1$, which leads to

$$\mu < \frac{2}{d_{max} + (K-2)o_{max}}.$$

   This is the condition on learning rate μ for the system to achieve convergence and stability. When condition 1 is satisfied, it implies that μ can be any value between $(0, \frac{2}{d_{max}+d_{min}})$.

These two conditions are sufficient for the k-th signal to converge, i.e., the weights will converge to the same steady state no matter what their initial values are.

3.2 Decorrelating Algorithm

The decorrelating algorithm is also referred to as the bootstrap algorithm. The decorrelating algorithm is capable of separating mixed signal sources [23]. The received signal of a CDMA receiver is a mixture of the desired user and interferences. Since the signals from the desired user and from the interferences are statistically independent, we expect to blindly separate them using the decorrelating algorithm.

Assume $f(x)$ and $g(x)$ are two odd functions of x, and $f(x) \neq g(x)$. Then the procedure to find the weight matrix by the decorrelating algorithm is as follows [23, 29]:

1. Set initial weights, the step is similar to step 1 of the minimum energy algorithm.
2. Compute the correlations between $f(y_i)$ and $g(\boldsymbol{y}_i)$, for $i = 1, 2, \ldots, K$.
3. Compute the next guess at the weight vector $\boldsymbol{w}_i$ by making a change in the initial or present guess in a direction of the vector $E\{f(y_i)g(\boldsymbol{y}_i)\}$.
4. Repeat the procedure from step 2.

$f(x)$ and $g(x)$ can be any odd functions. In this dissertation, we chose $f(x) = x$, and $g(x) = sgn(x)$ for the implementation reason since every digital detector has to perform $sgn(\cdot)$ to make the final decision.

The steady state weight matrix $\boldsymbol{w}$ can be, in principle, obtained by analytically solving a system of $K(K-1)$ non-linear equations:

$$E\{y_i sgn(\boldsymbol{y}_i)\} = 0, \quad i = 1, 2, \ldots, K. \tag{3.10}$$

From the definition of y_j, it is easy to see that y_j is a function of $\boldsymbol{w}_j$. To find the solution of Eq. (3.10), numerical methods have to be used.

For a reliable communication link, the error probability of the resulting output has to be very low. When the main contribution to the error is caused by the multiuser interference, we can make the following approximation[1]:

$$\begin{aligned} E\{y_i sgn(y_j)\} &\approx E\{y_i b_j\}(1 - 2Pr(b_j \text{ in error})) \\ &= E\{y_i b_j\}(1 - 2Pe_j) \end{aligned}$$

or

$$E\{y_i sgn(\boldsymbol{y}_i)\} \approx E\{y_i \boldsymbol{b}_i\}(\boldsymbol{I} - 2\boldsymbol{P}e_i). \tag{3.11}$$

where $\boldsymbol{P}e_i$ is a $(K-1)(K-1)$ diagonal matrix with its (i,i)-th element is the error probability of the i-th user, Pe_i. So, $\boldsymbol{w}_i$ can be computed from:

$$\begin{aligned} E\{y_i sgn(\boldsymbol{y}_i)\} &\approx E\{y_i \boldsymbol{b}_i\}(I - 2\boldsymbol{P}e_i) \\ &= E\{(x_i - \boldsymbol{f}_i^T \boldsymbol{w}_i)\boldsymbol{b}_i(I - 2\boldsymbol{P}e_i)\} = 0 \end{aligned}$$

$$E\{x_i \boldsymbol{b}_i\} = E\{\boldsymbol{b}_i \boldsymbol{f}_i^T\}\boldsymbol{w}_i. \tag{3.12}$$

This approximation is equivalent to using training sequence, i.e., $sgn(\boldsymbol{y}_i) = \boldsymbol{b}_i$.

3.3 Convergence Issue

The adaptive rule for the weights is given by:

$$\begin{aligned} \boldsymbol{w}_k(i+1) &= \boldsymbol{w}_k(i) + \mu E\{y_k(i) sgn(\boldsymbol{y}_k(i))\} \\ &= \boldsymbol{w}_k(i) + \mu E\{x_k(i) sgn(\boldsymbol{y}_k(i))\} - E\{sgn(\boldsymbol{y}_k(i))\boldsymbol{f}_k^T(i)\}\boldsymbol{w}_k(i) \\ &= (\boldsymbol{I} - \mu E\{sgn(\boldsymbol{y}_k(i))\boldsymbol{f}_k^T(i)\})\boldsymbol{w}_k(i) + \mu E\{x_k(i) sgn(\boldsymbol{y}_k(i))\}. \end{aligned} \tag{3.13}$$

[1]This approximation is due to Y. Bar-Ness

By substituting (3.8) and (3.12) into the above equation, we have:

$$\begin{aligned} \boldsymbol{c}_k(i+1) &= (\boldsymbol{I} - \mu E\{\boldsymbol{b}_k(i)(I - 2\boldsymbol{P}e_i)\boldsymbol{f}_k^T(i)\})\boldsymbol{c}_k(i) \\ &= (\boldsymbol{I} - \mu E\{\boldsymbol{b}_k(i)(I - 2\boldsymbol{P}e_i)\boldsymbol{f}_k^T(i)\})^{i+1}\boldsymbol{c}_k(0) \\ &= (\boldsymbol{I} - \mu \boldsymbol{R}_k)^{i+1}\boldsymbol{c}_k(0) \\ &= \boldsymbol{H}_k^{i+1}\boldsymbol{c}_k(0), \end{aligned} \tag{3.14}$$

where $\boldsymbol{R}_k = E\{\boldsymbol{b}_k(I - 2\boldsymbol{P}e_i)\boldsymbol{f}_k^T(i)\}$ and $\boldsymbol{H}_k = \boldsymbol{I} - \mu\boldsymbol{R}_k$. The (i,j)th element of $\boldsymbol{R}_k$ is equal to $E\{b_i(1 - 2Pe_i)f_j\}$.

Using the same technique as we used for the analysis of the weight convergence of the minimum energy scheme, we get the necessary and sufficient condition of convergence:

$$(K-2)o_{max} < d_{min},$$

and

$$\mu < \frac{2}{d_{max} + (K-2)o_{max}}.$$

Note that this is only true when the approximation we made is sufficiently accurate. In cases where the approximation is not good, the analysis of the convergence could be very complicated.

CHAPTER 4

ONE-STAGE ADAPTIVE DETECTORS

In previous chapter, we raised the question of where and how to get the estimations of the interference.

The output of the matched filter has been used by the conventional detector to detect its information, since it contains the information of the user. For the one-stage detector discussed in this chapter, we take the estimation signals from the output of the matched filters. To detect the k-th user, the estimation signals for the other users are weighted and deducted from the output of the matched filter of the k-th user. A detector consisting of a bank of matched filters and an adaptive canceler is given in Fig. 4.1.

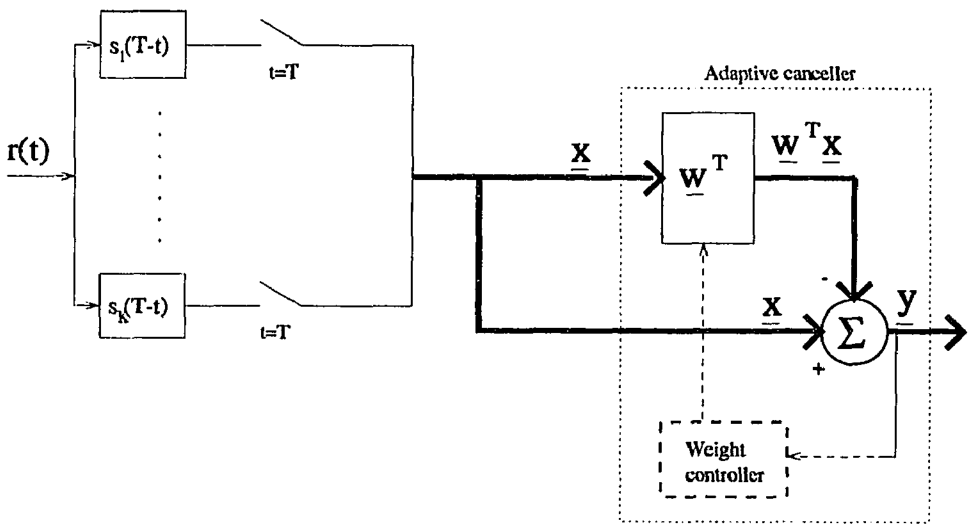

Figure 4.1 One-stage Adaptive Detector

The canceler's vector output is given by:

$$\boldsymbol{y} = (\boldsymbol{I} - \boldsymbol{W}^T)\boldsymbol{x} = \boldsymbol{V}\mathcal{P}\boldsymbol{A}\boldsymbol{b} + \boldsymbol{V}\boldsymbol{n}, \tag{4.1}$$

where $\boldsymbol{W}$ is the weight matrix defined in previous chapter, except that w_{ij}'s are adaptively updated.

$$\boldsymbol{V} = [\boldsymbol{I} - \boldsymbol{W}^T] = \begin{bmatrix} 1 & -w_{21} & \dots & -w_{K1} \\ -w_{12} & 1 & \dots & -w_{K2} \\ \vdots & \vdots & \ddots & \vdots \\ -w_{1K} & -w_{2K} & \dots & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}^{(1)} \\ \boldsymbol{v}^{(2)} \\ \vdots \\ \boldsymbol{v}^{(K)} \end{bmatrix} \tag{4.2}$$

where $\boldsymbol{v}^{(i)}$ is a row vector. Thus:

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{v}^{(1)} \\ \boldsymbol{v}^{(2)} \\ \vdots \\ \boldsymbol{v}^{(K)} \end{bmatrix} [\boldsymbol{\rho}^{(1)}\boldsymbol{\rho}^{(2)}\cdots\boldsymbol{\rho}^{(K)}] \begin{bmatrix} \sqrt{a_1}b_1 \\ \sqrt{a_2}b_2 \\ \vdots \\ \sqrt{a_K}b_K \end{bmatrix} + \boldsymbol{\xi}, \tag{4.3}$$

where $\boldsymbol{\rho}^{(i)}$ is the i-th column of $\boldsymbol{\mathcal{P}}$, and

$$\boldsymbol{\xi} = [\xi_1, \xi_2, \dots, \xi_K]^T = \boldsymbol{V}\boldsymbol{n},$$

with

$$E(\xi_k) = 0 \tag{4.4}$$

$$\begin{aligned} E\{\xi_k^2\} &= \sigma_{\xi_k}^2 \\ &= E\{\boldsymbol{v}^{(k)}\boldsymbol{n}\boldsymbol{n}^T(\boldsymbol{v}^{(k)})^T\} \\ &= \boldsymbol{v}^{(k)}\boldsymbol{\mathcal{P}}(\boldsymbol{v}^{(k)})^T\frac{N_o}{2} \end{aligned} \tag{4.5}$$

$$E\{\xi_k\xi_p\} = \boldsymbol{v}^{(k)}\boldsymbol{\mathcal{P}}(\boldsymbol{v}^{(p)})^T\frac{N_o}{2}.$$

The output for the k-th user is expressed as:

$$y_k = x_k - \boldsymbol{w}_k{}^T\boldsymbol{x}_k, \tag{4.6}$$

where $\boldsymbol{w}_k$ is the k-th column vector of $\boldsymbol{W}$ with the element w_{kk} deleted, and $\boldsymbol{x}_k$ is the vector obtained from $\boldsymbol{x}$ by deleting the element x_k.

y_k can also be expressed as:

$$y_k = \boldsymbol{v}^{(k)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i + \xi_k. \tag{4.7}$$

The error probability for the k-th user is evaluated as follows:

$$\begin{aligned}
P_{e_k} &= E_{b_k, \boldsymbol{b}_k} Pr\{\hat{b}_k \text{ in error} | b_k, \boldsymbol{b}_k\} \\
&= \frac{1}{2} \sum_{\boldsymbol{b}_k} \Bigg[Pr\{\xi_k > \boldsymbol{v}^{(k)} \boldsymbol{\rho}^{(k)} \sqrt{a_k} - \boldsymbol{v}^{(k)} \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\} \\
&+ \; Pr\{\xi_k < -\boldsymbol{v}^{(k)} \boldsymbol{\rho}^{(k)} \sqrt{a_k} - \boldsymbol{v}^{(k)} \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\} \Bigg] Pr\{\boldsymbol{b}_k\} \\
&= 2^{-(K-1)} \sum_{\boldsymbol{b}_k} Pr\{\xi_k > \boldsymbol{v}^{(k)} \boldsymbol{\rho}^{(k)} \sqrt{a_k} - \boldsymbol{v}^{(k)} \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\} \\
&= 2^{-(K-1)} \sum_{\boldsymbol{b}_k} Q \left(\frac{\boldsymbol{v}^{(k)} \boldsymbol{\rho}^{(k)} \sqrt{a_k} - \boldsymbol{v}^{(k)} \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\}}{\sigma_{\xi_k}} \right).
\end{aligned}$$

4.1 Using Minimum Energy Algorithm

From Eq. (4.6), the output for the k-th user can be expressed as:

$$y_k = x_k - {\boldsymbol{w}_k}^T \boldsymbol{x}_k.$$

Using the minimum energy criterion, the steady state values of the weights affecting the k-th output are found from $\frac{\partial}{\partial \boldsymbol{w}_k} E\{y_k^2\} = 0$ and are evaluated as:

$$\frac{\partial}{\partial \boldsymbol{w}_k} E\{y_k^2\} = 0 \;=\; \frac{\partial}{\partial \boldsymbol{w}_k} E\{(x_k - \boldsymbol{w}_k^T \boldsymbol{x}_k)(x_k - \boldsymbol{w}_k^T \boldsymbol{x}_k)^T\}$$

$$
\begin{aligned}
&= \frac{\partial}{\partial \boldsymbol{w}_k} E\{x_k^2 - 2x_k \boldsymbol{x}_k^T \boldsymbol{w}_k + \boldsymbol{w}_k^T \boldsymbol{x}_k \boldsymbol{x}_k^T \boldsymbol{w}_k\} \\
&= -2E\{x_k \boldsymbol{x}_k\} + 2E\{\boldsymbol{x}_k \boldsymbol{x}_k^T\}\boldsymbol{w}_k, \qquad (4.8)
\end{aligned}
$$

the steady state values of the weights affecting the k-th output as:

$$
\boldsymbol{w}_k = \left[E\{\boldsymbol{x}_k \boldsymbol{x}_k^T\}\right]^{-1} E\{x_k \boldsymbol{x}_k\}, \qquad (4.9)
$$

where the diagonal and off-diagonal elements of $E\{\boldsymbol{x}_k \boldsymbol{x}_k^T\}$ are found as:

$$
E\{x_i^2\} = a_i + \sum_{\substack{j=1 \\ j \neq i}}^{K} \rho_{ji}^2 a_j + \sigma^2 \qquad (4.10)
$$

and

$$
E\{x_i x_j\} = \rho_{ij}(a_i + a_j + \sigma^2) + \sum_{\substack{m,n \\ m \neq n \neq i,j}} \rho_{im}\rho_{jn}\sqrt{a_m}\sqrt{a_n} \qquad (4.11)
$$

respectively.

For the two-user case, the weights of user one and user two can be computed as:

$$
w_{21} = \frac{\rho_{12}(a_1 + a_2 + \sigma^2)}{a_2 + \rho_{12}^2 a_1 + \sigma^2} \qquad (4.12)
$$

$$
w_{12} = \frac{\rho_{12}(a_1 + a_2 + \sigma^2)}{a_1 + \rho_{12}^2 a_2 + \sigma^2}. \qquad (4.13)
$$

4.1.1 Numerical Examples

Two numerical examples are given in Figs. 4.2 and 4.3. These two figures show the error probabilities for a two-user system with the signal-to-noise ratio of user one equal to 8 dB, and the signal-to-noise ratio of user two changing from -2 to 16 dB.

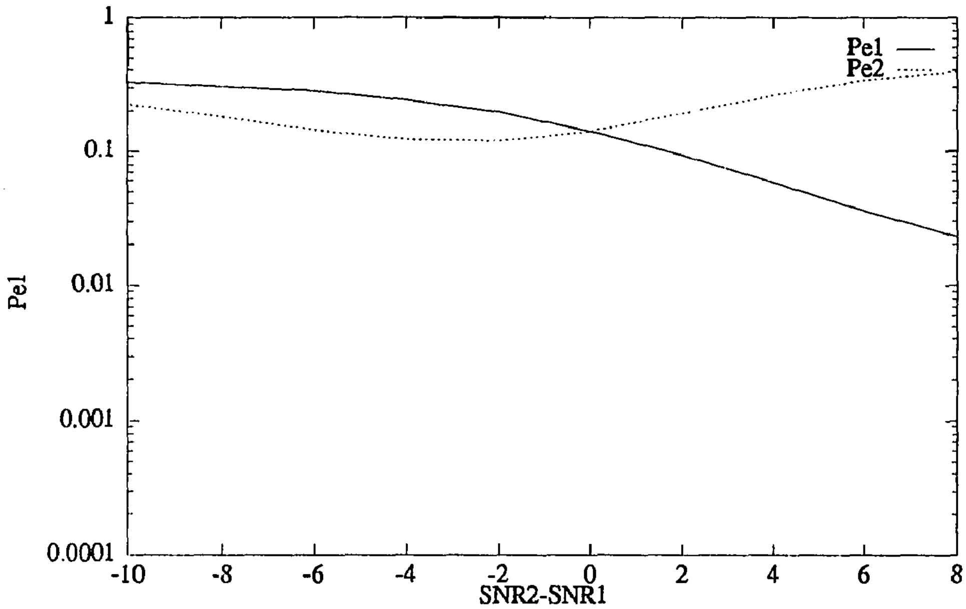

Figure 4.2 Error Probability of User 1, for $K = 2$, $\rho = 0.7$, $SNR_1 = 8$ dB

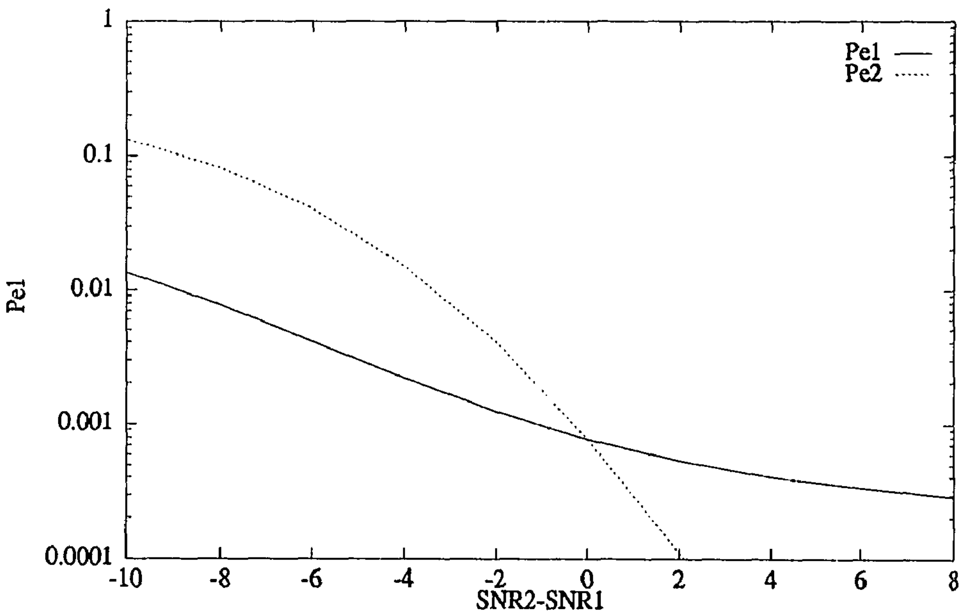

Figure 4.3 Error Probability of User 1, for $K = 2$, $\rho = 1/7$, $SNR_1 = 8$ dB

In Fig. 4.2, the correlation between two users' signature codes is equal to 0.7. In Fig. 4.3, the correlation is equal to 1/7. The results can be explained by the behavior of the weights. From Eqs. (4.12) and (4.13), if ρ_{12} is large, when $a_1 >> a_2$, $w_{12} \approx \rho_{12}$ and $w_{21} \approx 1/\rho_{12}$. When $a_2 >> a_1$, $w_{12} \approx 1/\rho_{12}$ and $w_{21} \approx \rho_{12}$. Because of the result of the weights, the performance of the user with large input SNR is worse than the one with smaller one. If ρ_{12} is small, $w_{12} \approx \rho_{12}(1 + a_2/a_1)$ and $w_{21} \approx \rho_{12}(1 + a_1/a_2)$.

4.2 Using Decorrelating Algorithm

Using the decorrelating criterion,

$$E\{y_k sgn(y_p)\} = 0 \qquad k, p = 1, 2, \ldots, K, k \neq p \tag{4.14}$$

we get:

$$\begin{aligned} & E\left\{\left[\boldsymbol{v}^{(k)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i + \xi_k\right] sgn(y_p)\right\} \\ = \quad & E\left\{\left(\boldsymbol{v}^{(k)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\right) sgn(y_p) + \xi_k sgn(y_p)\right\} = 0. \end{aligned}$$

The first term of the above equation yields:

$$\begin{aligned} & E\left\{\left[\boldsymbol{v}^{(k)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\right] sgn(y_p)\right\} \\ = \quad & E\left\{\left[\boldsymbol{v}^{(k)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i\right] sgn\left(\boldsymbol{v}^{(p)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i + \xi_p\right)\right\} \\ = \quad & E_{\boldsymbol{b}}\left\{\boldsymbol{v}^{(k)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i \left[Pr\left\{\xi_p > -\boldsymbol{v}^{(p)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i | \boldsymbol{b}\right\}\right.\right. \\ - \quad & \left.\left. Pr\left\{\xi_p < -\boldsymbol{v}^{(p)} \sum_{i=1}^{K} \boldsymbol{\rho}^{(i)} \sqrt{a_i} b_i | \boldsymbol{b}\right\}\right]\right\} \end{aligned}$$

and the second term is,

$$
\begin{aligned}
E\{\xi_k sgn(y_p)\} &= E\left\{\xi_k sgn\left(\boldsymbol{v}^{(p)}\sum_{i=1}^{K}\boldsymbol{\rho}^{(i)}\sqrt{a_i}b_i + \xi_p\right)\right\} \\
&= \sum_{\boldsymbol{b}}\left(\int_{-\infty}^{\infty}\int_{-\xi_p^o}^{\infty}\xi_k f_{\xi_k\xi_p}(\xi_k,\xi_p)d\xi_k d\xi_p\right. \\
&- \left.\int_{-\infty}^{\infty}\int_{-\infty}^{-\xi_p^o}\xi_k f_{\xi_k\xi_p}(\xi_k,\xi_p)d\xi_k d\xi_p\right),
\end{aligned}
$$

where $f_{\xi_k\xi_p}(\xi_k,\xi_p)$ denotes the Gaussian density function, and $\xi_p^o = \boldsymbol{v}^{(p)}\sum_{i=1}^{K}\boldsymbol{\rho}^{(i)}\sqrt{a_i}b_i$.

Therefore in order to get w_{kp} $(k,p = 1,2,\ldots,K, k \neq p)$, a system of $K(K-1)$ non-linear equations is to be solved.

A two-user example is given in Appendix A.

An easier way to solve the $K(K-1)$ non-linear equations is to use the approximation in Eq. (3.11) to linearize them:

$$
\begin{aligned}
E\{y_i sgn(y_j)\} &\approx E\{y_i b_j\} \\
&= (\rho_{ij} - w_{ji}) = 0
\end{aligned}
$$

So, finally we will get:

$$
w_{ji} = \rho_{ij} \quad \text{for every } i,\ j, \quad i \neq j
$$

4.2.1 Numerical Example

Two examples are shown in Figs. 4.4 and 4.5. Fig. 4.4 is a two-user example. The correlation of the two users' signature code is equal to 0.7, and the energy of user one is set to 8 dB. Fig. 4.5 is a three-user example. A Gold code of length 7 is used in this example. Again user one's energy is 8 dB. The error performance of its non-adaptive counterpart — decorrelator is also plotted for comparison. One can see that when the interference-to-signal ratio is small, the proposed scheme works better than the decorrelator. When the interference to signal ratio is large, the two schemes have the same performance.

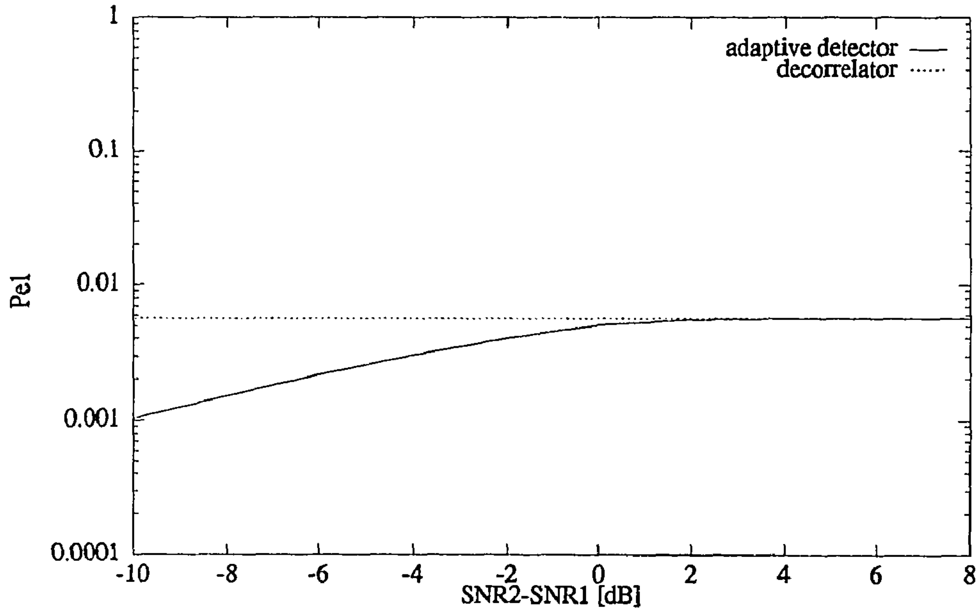

Figure 4.4 Error Probability of User 1, for $K = 2$, $\rho = 0.7$, $SNR_1 = 8$ dB

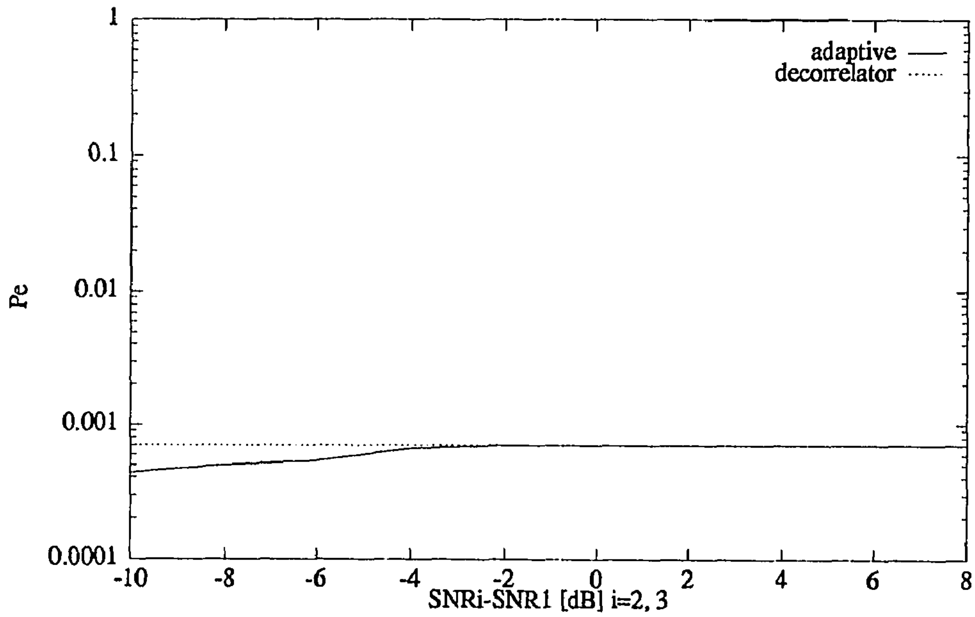

Figure 4.5 Error Probability of User 1, for $K = 3$, $SNR_1 = 8$ dB

CHAPTER 5

ONE-STAGE ADAPTIVE DETECTOR WITH HARD DECISION

The outputs of the matched filter have been used as the estimation signals of the interferences in previous chapter. For the one-stage detector discussed in this chapter, we will still take the estimation of the interference signals from the output of the matched filters. But tentative decisions are made to these signals prior to sending to the canceler. To detect the k-th user, the estimations of the interference are weighted and deducted from the output of the matched filter of the k-th user. A detector consisting of a bank of matched filters, a hard-limiter, and an adaptive canceler is given in Fig. 5.1.

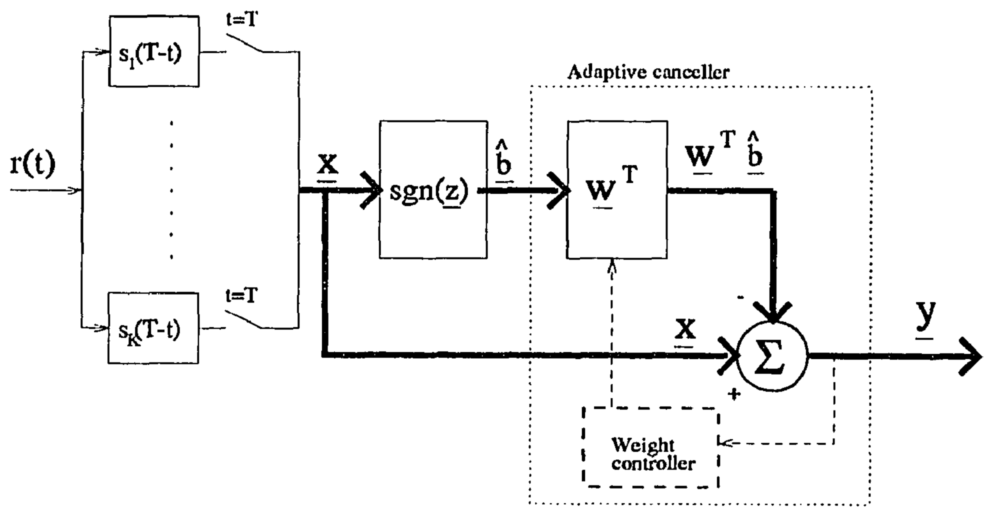

Figure 5.1 One-stage Detector with Hard Decision

The canceler's vector output is given by:

$$\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{W}^T \hat{\boldsymbol{b}}, \tag{5.1}$$

where $\hat{\boldsymbol{b}}$ is a column vector whose i-th element is the decision output of the i-th matched filter. The output for the k-th user can be expressed as:

$$y_k = x_k - \boldsymbol{w}_k{}^T \hat{\boldsymbol{b}}_k = \sqrt{a_k} b_k + \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} \sqrt{a_i} b_i - w_{ik} \hat{b}_i) + n_k. \tag{5.2}$$

The error probability for the k-th user is evaluated as follows:

$$\begin{aligned} Pe_k &= \frac{1}{2}(Pr\{n_k > \sqrt{a_k} - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} \sqrt{a_i} b_i - w_{ik} \hat{b}_i) | b_k = -1\} \\ &+ Pr\{n_k < -\sqrt{a_k} - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} \sqrt{a_i} b_i - w_{ik} \hat{b}_i) | b_k = 1\}). \end{aligned}$$

Since $\hat{\boldsymbol{b}}_k$ is a function of Gaussian noise vector $\boldsymbol{n}_k$ and $E\{n_i n_j\} = \rho_{ij}$. The Pe_k can be computed by the n-dimensional Gaussian integrations. A two-user example is given in Appendix B.

5.1 Using Minimum Energy Algorithm

Using the minimum energy criterion, the steady state values of the weights affecting the k-th output are found from $\frac{\partial}{\partial \boldsymbol{w}_k} E\{y_k^2\} = 0$, and are evaluated as:

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{w}_k} E\{y_k^2\} = 0 &= \frac{\partial}{\partial \boldsymbol{w}_k} E\{(x_k - \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k)(x_k - \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k)^T\} \\ &= \frac{\partial}{\partial \boldsymbol{w}_k} E\{x_k^2 - 2x_k \hat{\boldsymbol{b}}_k^T \boldsymbol{w}_k + \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k \hat{\boldsymbol{b}}_k^T \boldsymbol{w}_k\} \\ &= -2E\{x_k \hat{\boldsymbol{b}}_k\} + 2E\{\hat{\boldsymbol{b}}_k \hat{\boldsymbol{b}}_k^T\} \boldsymbol{w}_k, \end{aligned} \tag{5.3}$$

the steady state values of the weights affecting the k-th output as:

$$\boldsymbol{w}_k = \left[E\{\hat{\boldsymbol{b}}_k\hat{\boldsymbol{b}}_k^T\}\right]^{-1} E\{x_k\hat{\boldsymbol{b}}_k\}, \tag{5.4}$$

where $E\{\hat{\boldsymbol{b}}_k\hat{\boldsymbol{b}}_k^T\}$ and $E\{x_k\hat{\boldsymbol{b}}_k\}$ are computed in Appendix B, Eqs. (B.3), (B.4), and (B.5).

For the two-user case, the weights of the user one and two are given by:

$$w_{21} = \sqrt{a_1}E\{b_1\hat{b}_2\} + \rho_{12}\sqrt{a_1}E\{b_2\hat{b}_2\} + E\{n_1\hat{b}_2\} \tag{5.5}$$

$$w_{12} = \sqrt{a_2}E\{b_2\hat{b}_1\} + \rho_{12}\sqrt{a_2}E\{b_1\hat{b}_1\} + E\{n_2\hat{b}_1\}. \tag{5.6}$$

5.1.1 Numerical Examples

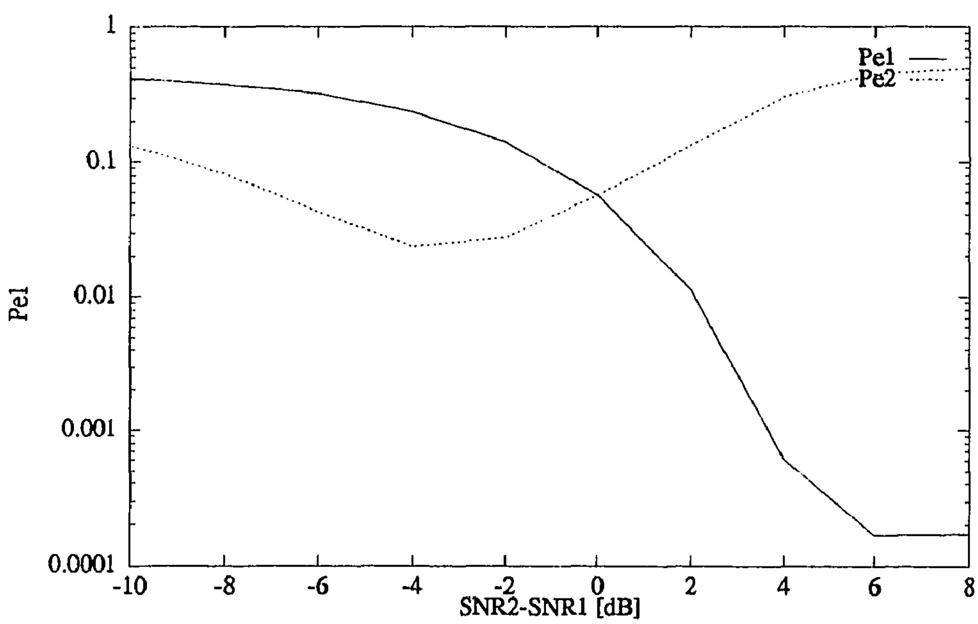

Figure 5.2 Error Probability of User 1, $SNR_1 = 8$ dB, $\rho = 0.7$, $K = 2$

A simulation example of the detector is given in Fig. 5.2. The correlation of the two users' signature code is equal to 0.7, and the SNR of user one is set to 8 dB.

The SNR_2 changes from -2 to 16 dB. As SNR_2 increases, the estimation of user two becomes more accurate and the weight w_{21} becomes close to ρ (Eq. 5.5). The performance of user one reaches the single user bound. The performance of user two becomes worse as its energy increases, since the estimation of user one becomes worse, and the weight w_{12} approaches $\sqrt{a_2}$ (Eq. (5.6)).

5.2 Using Decorrelating Algorithm

Using the decorrelating criterion,

$$E\{y_k sgn(\boldsymbol{y}_k)\} = 0, \tag{5.7}$$

where each element of $E\{y_k sgn(\boldsymbol{y}_k)\}$, $E\{y_k sgn(y_j)\}$, for $j = 1, 2 \ldots, K, j \neq k$, can be computed from:

$$\begin{aligned} E\{y_k sgn(y_j)\} &= E\{(\sqrt{a_k}b_k + \sum_{\substack{i=1 \\ i\neq k}}^{K}(\rho_{ik}\sqrt{a_i}b_i - w_{ik}\hat{b}_i) + n_k)sgn(y_j)\} \\ &= E\{\sqrt{a_k}b_k sgn(y_j)\} + E\{sgn(y_j)\sum_{\substack{i=1 \\ i\neq k}}^{K}\rho_{ik}\sqrt{a_i}b_i\} \\ &- E\{sgn(y_j)\sum_{\substack{i=1 \\ i\neq k}}^{K}w_{ik}\hat{b}_i\} + E\{n_k sgn(y_j)\}. \end{aligned} \tag{5.8}$$

Now calculating the weight matrix becomes solving Eq. (5.8). Note that y_j is a function of b_j, $\boldsymbol{b}_j$ $\hat{\boldsymbol{b}}_j$, and n_j. Since b_j is also a function of $\boldsymbol{n}_j$, y_j is actually a function of $\boldsymbol{n}$ and $\boldsymbol{b}$. Eq. (5.8) can only be solved using a numerical method. A two-user system example is given in Appendix B.

5.2.1 Simulation Example

A simulation example of $K = 2$, $SNR_1 = 8$ dB and $\rho = 0.7$ is given in Fig. 5.3. As the interference energy increases, both users' error probabilities approach 0.5.

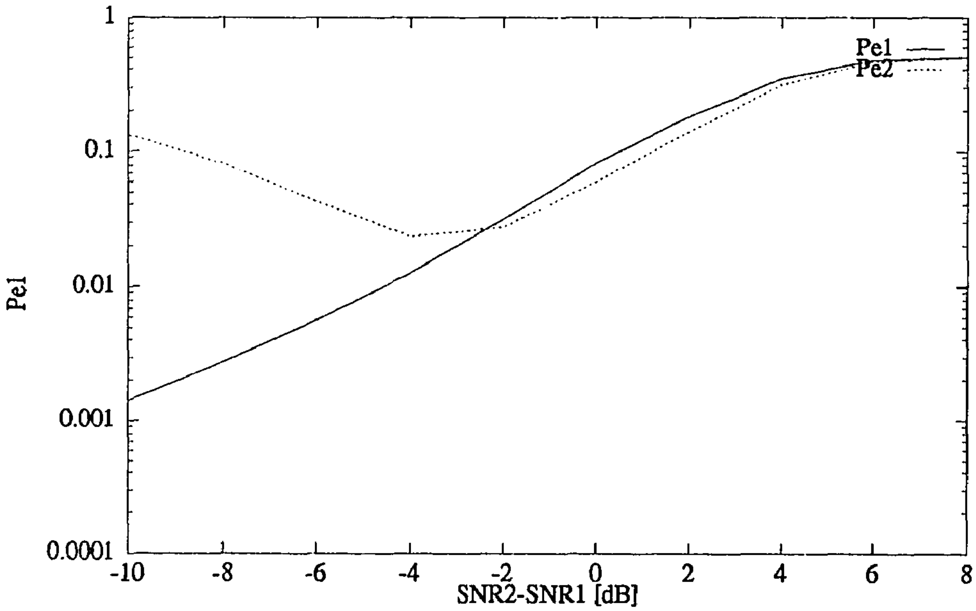

Figure 5.3 Error Probability of User 1 for $K = 2$, $SNR_1 = 8$ dB and $\rho_{12} = 0.7$

CHAPTER 6

TWO-STAGE ADAPTIVE DETECTORS

In this chapter, a two-stage adaptive detector will be introduced. It consists of a bank of matched filters, followed by a decorrelator and a canceler. The estimations to the interference are obtained from the output of the decorrelator. The estimation of the interferences are weighted and subtracted from the output of the matched filter of the desired user. The study of this scheme is important toward better understanding of the scheme which will be introduced in Chapter 8. The structure of this detector is shown in Fig. 6.1.

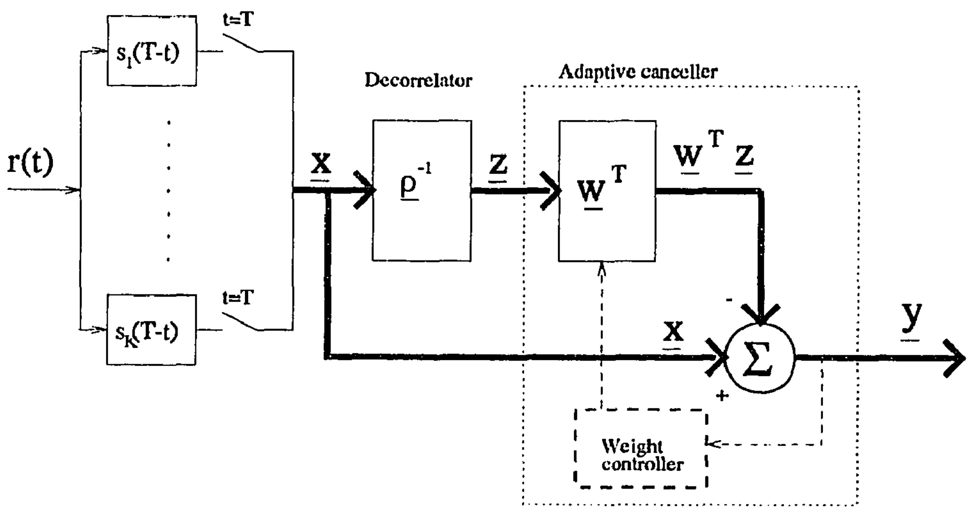

Figure 6.1 Two-stage Adaptive Detector

The output of the detector can be expressed as:

$$y = x - W^T z.$$

The output for the k-th user can be expressed as:

$$y_k = x_k - \boldsymbol{w}_k{}^T \boldsymbol{z}_k = \sqrt{a_k} b_k - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} - w_{ik}) \sqrt{a_i} b_i + \eta_k, \tag{6.1}$$

where $\boldsymbol{z}_k$ is the vector obtained from $\boldsymbol{z}$ by deleting the element z_k, and η_k is the k-th user's noise at the output of the detector:

$$\eta_k = n_k - \sum_{\substack{i=1 \\ i \neq k}}^{K} w_{ik} \xi_i.$$

The error probability of the k-th user can be evaluated as follows:

$$\begin{aligned}
P_{e_k} &= E_{b_k, \boldsymbol{b}_k} Pr\{\hat{y}_k \text{in error} | b_k, \boldsymbol{b}_k\} \\
&= \frac{1}{2} \sum_{\boldsymbol{b}_k} \Bigg[Pr\{\eta_k > \sqrt{a_k} - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} - w_{ik}) \sqrt{a_i} b_i\} \\
&+ Pr\{\eta_k < -\sqrt{a_k} - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} - w_{ik}) \sqrt{a_i} b_i\} \Bigg] Pr\{\boldsymbol{b}_k\} \\
&= \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k} Q \left(\frac{\sqrt{a_k} - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} - w_{ik}) \sqrt{a_i} b_i}{\sigma_{\eta_k}} \right),
\end{aligned}$$

where σ_{η_k} is the standard deviation of the Gaussian noise η_k:

$$\begin{aligned}
\sigma_{\eta_k}^2 &= E\{\eta_k^2\} \\
&= E\{(n_k - \sum_{\substack{i=1 \\ i \neq k}}^{K} w_{ik} \xi_i)^2\} \\
&= \sigma^2 + \sum_{\substack{i=1 \\ i \neq k}}^{K} w_{ik}^2 \sigma_{\xi_i}^2 - \sum_{\substack{i,j=1 \\ i \neq j \neq k}}^{K} w_{ik} w_{jk} E\{\xi_i \xi_j\}.
\end{aligned}$$

$\boldsymbol{\xi} = \boldsymbol{\Gamma n}$ is defined in Eq. (2.5).

$$E\{\boldsymbol{\xi\xi}^T\} = E\{\boldsymbol{\Gamma n n}^T\boldsymbol{\Gamma}^T\} = \boldsymbol{\Gamma}\sigma^2. \tag{6.2}$$

So,

$$\sigma_{\eta_k}^2 = \sigma^2 + \sum_{\substack{i=1 \\ i\neq k}}^{K} w_{ik}^2\gamma_{ii}\sigma^2 - \sum_{\substack{i,j=1 \\ i\neq j\neq k}}^{K} w_{ik}w_{jk}\gamma_{ij}\sigma^2.$$

6.1 Using Minimum Energy Algorithm

Using the minimum energy criterion, the adaptive rule for the weights is given by:

$$\boldsymbol{w}_k \leftarrow \boldsymbol{w}_k - \mu\frac{\partial}{\partial \boldsymbol{w}_k}E\{y_k^2\}.$$

The steady state values of the weights affecting the k-th output are found from $\frac{\partial}{\partial \boldsymbol{w}_k}E\{y_k^2\} = 0$, and are evaluated as:

$$\begin{aligned}
\frac{\partial}{\partial \boldsymbol{w}_k}E\{y_k^2\} = 0 &= \frac{\partial}{\partial \boldsymbol{w}_k}E\{(x_k - \boldsymbol{w}_k^T\boldsymbol{z}_k)(x_k - \boldsymbol{w}_k^T\boldsymbol{z}_k)^T\} \\
&= \frac{\partial}{\partial \boldsymbol{w}_k}E\{x_k^2 - 2x_k\boldsymbol{z}_k^T\boldsymbol{w}_k + \boldsymbol{w}_k^T\boldsymbol{z}_k\boldsymbol{z}_k^T\boldsymbol{w}_k\} \\
&= -2E\{x_k\boldsymbol{z}_k\} + 2E\{\boldsymbol{z}_k\boldsymbol{z}_k^T\}\boldsymbol{w}_k. \qquad (6.3)
\end{aligned}$$

It is easy to show that:

$$E\{x_k\boldsymbol{z}_k\} = E\left\{\left[\sqrt{a_k}b_k + \boldsymbol{\rho}_k^T\boldsymbol{A}_k\boldsymbol{b}_k + n_k\right]\boldsymbol{z}_k\right\}$$

$$= E\{\boldsymbol{z}_k[\boldsymbol{\rho}_k^T \boldsymbol{A}_k \boldsymbol{b}_k]^T\} + E\{n_k \boldsymbol{z}_k\}$$

$$= E\{\boldsymbol{A}_k \boldsymbol{b}_k \boldsymbol{b}_k^T \boldsymbol{A}_k^T\}\boldsymbol{\rho}_k + 0, \tag{6.4}$$

the steady state values of the weights affecting the k-th output as:

$$\boldsymbol{w}_k = \left[E\{\boldsymbol{z}_k \boldsymbol{z}_k^T\}\right]^{-1} \boldsymbol{A}_k \boldsymbol{A}_k^T \boldsymbol{\rho}_k, \tag{6.5}$$

where the diagonal and off-diagonal elements of $E\{\boldsymbol{z}_k \boldsymbol{z}_k^T\}$ are computed as:

$$E\{z_i^2\} = E\{(\sqrt{a_i} b_i + \xi_i)^2\} = a_i + \sigma_{\xi_i}^2, \tag{6.6}$$

and

$$E\{z_i z_j\} = E\{(\sqrt{a_i} b_i + \xi_i) E\{(\sqrt{a_j} b_j + \xi_j) == E\{\xi_i \xi_j\} = \gamma_{ij} \sigma^2, \tag{6.7}$$

respectively.

6.1.1 Numerical Examples

A two-user example is given in Fig. 6.2. The cross-correlation coefficient is taken to be 0.7. The SNR of user one is set to 8 dB, while the SNR of user two, relative to user one, varies from -10 to 8 dB. For comparison, the performance of the decorrelator is also included.

From this figure, one will notice that the error performance of this adaptive scheme is better than the decorrelator when the interference is weaker. As the energy of interference increases, its performance approaches that of decorrelator.

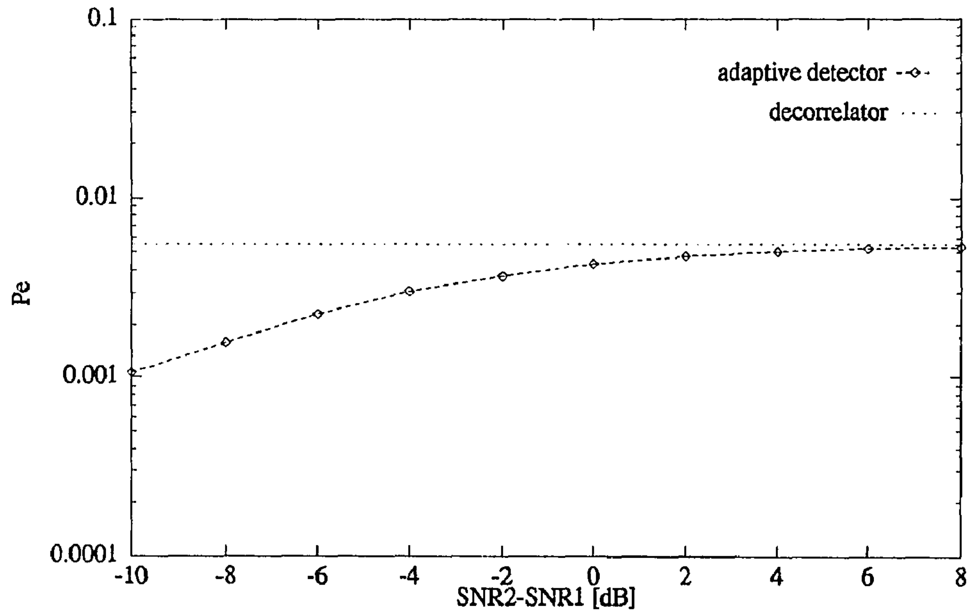

Figure 6.2 Error Probability of User 1 for $K = 2$, $\rho = 0.7$ and $SNR_1 = 8$ dB

6.2 Using Decorrelating Algorithm

From Eq. (6.1),

$$y_k = x_k - \boldsymbol{w}_k^T \boldsymbol{z}_k = \sqrt{a_k} b_k - \sum_{\substack{i=1 \\ i \neq k}}^{K} (\rho_{ik} - w_{ik}) \sqrt{a_i} b_i + \eta_k. \tag{6.8}$$

Using the decorrelating criterion, we have:

$$E\{y_k \, sgn(\boldsymbol{y}_k)\} = 0 \quad k = 1, 2, \ldots, K. \tag{6.9}$$

In order to compute w's, $K(K-1)$ nonlinear equations have to be solved. For the case of $K = 2$, we have to solve the following two equations:

$$\begin{cases} E\{y_1 \, sgn(y_2)\} = 0 \\ E\{y_2 \, sgn(y_1)\} = 0 \end{cases}.$$

These are two similar equations. We take the first one as an example.

$$
\begin{aligned}
& E\{y_1\, sgn(y_2)\} && (6.10)\\
= \;& E\{(\sqrt{a_1}b_1 + \rho_{12}\sqrt{a_2}b_2 - w_{21}z_2 + n_1)sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}z_1 + n_2)\} && \\
= \;& E\{(\sqrt{a_1}b_1 + \rho_{12}\sqrt{a_2}b_2)sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}z_1 + n_2)\} && (6.11)\\
- \;& E\{w_{21}z_2 sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}z_1 + n_2) && (6.12)\\
+ \;& E\{n_1 sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}z_1 + n_2)\}, && (6.13)
\end{aligned}
$$

where:

$$
\begin{aligned}
(6.11) \;=\;& E_{b_1,b_2}(\sqrt{a_1}b_1 + \rho_{12}\sqrt{a_2}b_2))(Pr\{\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}z_1 + n_2 > 0\}\\
-\;& Pr\{\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}z_1 + n_2 < 0\}),
\end{aligned}
$$

after averaging over b_1 and b_2, finally we get:

$$
\begin{aligned}
(6.11) \;=\;& \rho_{12}\sqrt{a_2} + (\sqrt{a_1} - \rho_{12}\sqrt{a_2})Q\left(\frac{\sqrt{a_2} + (\rho_{12} - w_{12}(-1\rho_{12}^2))\sqrt{a_1}}{\sigma\sqrt{1 - w_{12}^2\rho_{12}^2 + w_{12}^2}}\right)\\
+\;& (\sqrt{a_1} - \rho_{12}\sqrt{a_2})Q\left(\frac{\sqrt{a_2} - (\rho_{12} - w_{12}(-1\rho_{12}^2))\sqrt{a_1}}{\sigma\sqrt{1 - w_{12}^2\rho_{12}^2 + w_{12}^2}}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
(6.12) \;=\;& E\{z_2 sgn(\sqrt{a_2} + \rho_{12}\sqrt{a_1}b_1 - w_2 z_1 + n_1)\}\\
=\;& E\{(1 - \rho_{12}^2)\sqrt{a_2}b_2 sgn(\sqrt{a_2} + \rho_{12}\sqrt{a_1}b_1 - w_2 z_1 + n_1)\}\\
+\;& E\{(n_2 - \rho_{12}n_1)sgn(\sqrt{a_2} + \rho_{12}\sqrt{a_1}b_1 - w_2 z_1 + n_1)\} \qquad (6.14)
\end{aligned}
$$

$$(6.13) = E\{n_1 sgn(\sqrt{a_2} + \rho_{12}\sqrt{a_1}b_1 - w_2 z_1 + n_1)\}. \tag{6.15}$$

Eq. (6.14) and (6.15) involve numerical integration. For the number of users equal to K, we have to solve a family of $K(K-1)$ non-linear equations, which is very hard to do even with the aid of the computer. From the numerical examples shown in the next sub-section, we will find that the error probability of this scheme is relatively low with a reasonable value of SNR, so we can use the approximation derived in Eq. (3.11):

$$E\{y_k \hat{\hat{\boldsymbol{b}}}_k\} \approx E\{y_k \boldsymbol{b}_k\}(\boldsymbol{I} - 2\boldsymbol{P}e_k) \tag{6.16}$$

Substituting (6.16) into (6.9) and (6.1), we get:

$$\begin{aligned} E\{y_k \mathrm{sgn}(\boldsymbol{y}_k)\} &\approx E\{(x_k - {\boldsymbol{w}_k}^T \boldsymbol{z}_k)\boldsymbol{b}_k\} \\ &= E\{x_k \boldsymbol{b}_k\} + E\{{\boldsymbol{w}_k}^T \boldsymbol{z}_k \boldsymbol{b}_k\} \\ &= -\boldsymbol{\rho}_k + {\boldsymbol{w}_k}^T. \end{aligned}$$

So finally, we get:

$${\boldsymbol{w}_k}^T = \boldsymbol{\rho}_k. \tag{6.17}$$

A two-user example is given in Fig. 6.3. From this figure, we can see that the performance of the detector is close to the performance of the decorrelator. This is because the decorrelating criterion is used to separate the uncorrelated signals. When the signals at the output of the decorrelator are separated already, the decorrelating criterion does not help much since we can show that two stages of the decorrelator are equivalent to one stage of it:

$$z = \rho^{-1} x$$

$$y = x - wz + n,$$

where the diagonal elements of the w are equal to zero and the (i, j)-th element is equal to ρ_{ij} if the second stage is also a decorrelator (or the weights of the second stage converge to the weights of the decorrelator).

Then we have:

$$W = \mathcal{P} - I,$$

and

$$\begin{aligned} y &= x - Wz = x - (\mathcal{P} - I)\mathcal{P}^{-1}x \\ &= x - x + \mathcal{P}^{-1}x \\ &= z. \end{aligned}$$

6.2.1 Numerical Examples

A numerical example for the two-user system is shown in Fig. 6.3. The signal-to-noise ratio of user one is equal to 8 dB, and user two is changing from -2 to 16 dB. The error probability of the decorrelator is also included. We can see that the performance of the proposed adaptive detector is very close to the performance of the decorrelator. This is explained in the last sub-section.

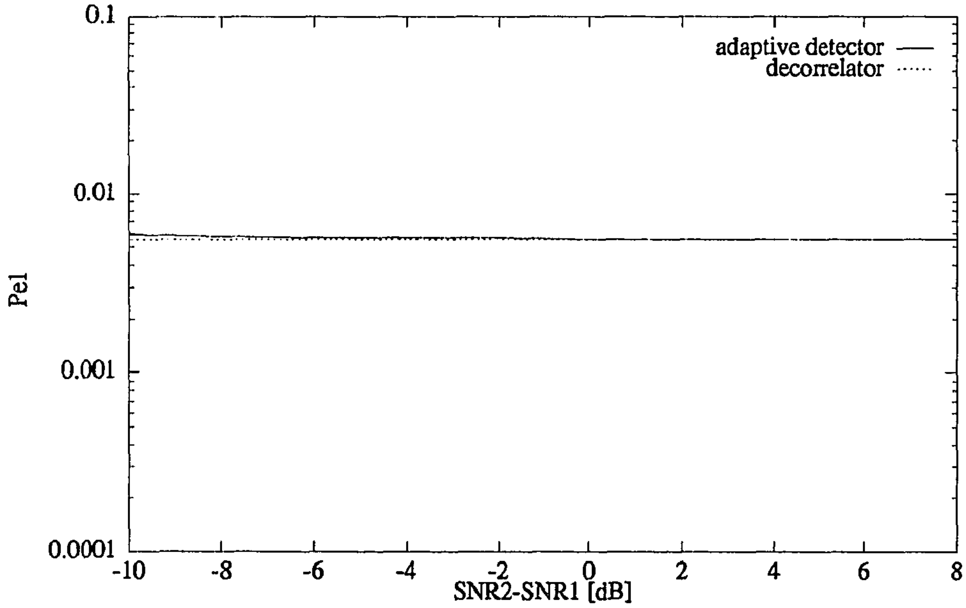

Figure 6.3 Error Probability of User 1, for $K = 2$, $\rho = 0.7$, $SNR_1 = 8$ dB

CHAPTER 7

TWO-STAGE ADAPTIVE DETECTOR WITH TENTATIVE DECISION

The synchronous CDMA receiver considered here consists of a conventional detector, a decorrelator, a hard limiter and a canceler, as depicted in Fig. 7.1.

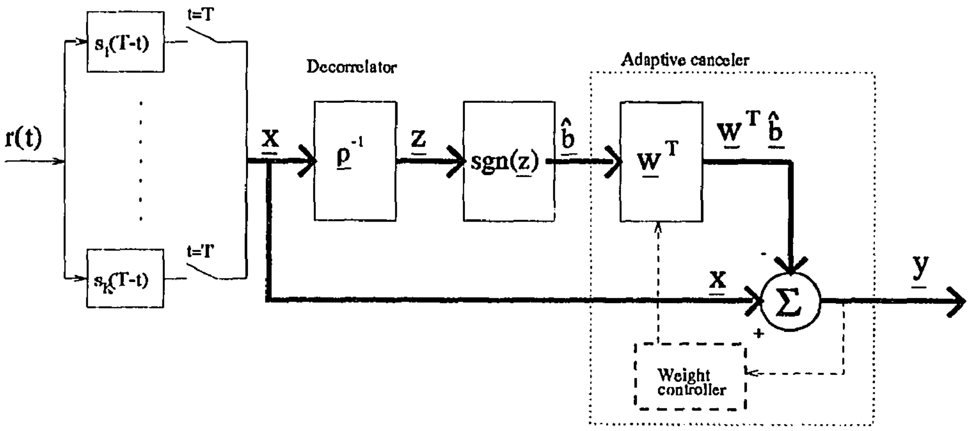

Figure 7.1 Two-stage Adaptive Detector with Tentative Decision

In this section, an adaptive sub-optimum detector is proposed and analyzed. Let $\hat{\boldsymbol{b}} = \left[\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_K\right]^T$ be the decision output of the decorrelator defined as:

$$\hat{\boldsymbol{b}} = \text{sgn}(\boldsymbol{z}).$$

Then the canceler's output is given by:

$$\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{W}^T\hat{\boldsymbol{b}}. \tag{7.1}$$

The output for the k-th user can be expressed as:

$$y_k = x_k - \boldsymbol{w}_k{}^T \hat{\boldsymbol{b}}_k.$$

The output error probability for the k-th user is evaluated as follows.

$$\begin{aligned}
P_{e_k} &= E_{b_k, \boldsymbol{b}_k, \hat{\boldsymbol{b}}_k} Pr\{\hat{\hat{b}}_k \text{ in error} | b_k, \boldsymbol{b}_k, \hat{\boldsymbol{b}}_k\} \\
&= \frac{1}{2} \sum_{\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k} \Big[Pr\{n_k > \sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k + \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\} \\
&+ \; Pr\{n_k < -\sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k + \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\} \Big] \cdot \\
&\quad Pr\{\hat{\boldsymbol{b}}_k | \boldsymbol{b}_k\} Pr\{\boldsymbol{b}_k\}.
\end{aligned}$$

Since $Pr\{\boldsymbol{b}_k\} = 2^{-(K-1)}$ and n_k is a zero mean Gaussian random variable, we can write:

$$\begin{aligned}
P_{e_k} = 2^{-K} \sum_{\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k} & \Big[Pr\{n_k > \sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k + \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\} \\
+ \; Pr\{n_k > \sqrt{a_k} + \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k - \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\} & \Big] Pr\{\hat{\boldsymbol{b}}_k | \boldsymbol{b}_k\}. \qquad (7.2)
\end{aligned}$$

Also

$$\begin{aligned}
Pr\{\hat{\boldsymbol{b}}_k | \boldsymbol{b}_k\} &= Pr\{\text{sgn}(\sqrt{a_1} b_1 + \xi_1), \ldots, \text{sgn}(\sqrt{a_{k-1}} b_{k-1} + \xi_{k-1}), \\
&\quad \text{sgn}(\sqrt{a_{k+1}} b_{k+1} + \xi_{k+1}), \ldots, \text{sgn}(\sqrt{a_K} b_K + \xi_K)\} \\
&= Pr\{\hat{b}_1 \xi_1 > -\sqrt{a_1} b_1 \hat{b}_1, \ldots, \hat{b}_{k-1} \xi_{k-1} > \\
&\quad -\sqrt{a_{k-1}} b_{k-1} \hat{b}_{k-1}, \hat{b}_{k+1} \xi_{k+1} > -\sqrt{a_{k+1}} b_{k+1} \hat{b}_{k+1}, \\
&\quad \ldots, \hat{b}_K \xi_K > -\sqrt{a_K} b_K \hat{b}_K\}
\end{aligned}$$

$$= Pr\{\hat{b}_1\xi_1 < \sqrt{a_1}b_1\hat{b}_1, \ldots, \hat{b}_{k-1}\xi_{k-1} < \sqrt{a_{k-1}}b_{k-1}\hat{b}_{k-1},$$
$$\hat{b}_{k+1}\xi_{k+1} < \sqrt{a_{k+1}}b_{k+1}\hat{b}_{k+1}, \ldots, \hat{b}_K\xi_K < \sqrt{a_K}b_K\hat{b}_K\}$$
$$= Pr\{-\hat{\boldsymbol{b}}_k| - \boldsymbol{b}_k\}. \tag{7.3}$$

Applying the result of (7.3) to the second term in (7.2) enables splitting the latter into two equal terms. Therefore,

$$P_{e_k} = \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k} \left[Pr\{n_k > \sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k + \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\}\right] Pr\{\hat{\boldsymbol{b}}_k | \boldsymbol{b}_k\}. \tag{7.4}$$

Substituting (7.1) into (7.4), we finally obtain:

$$P_{e_k} = \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k} Q\left(\frac{\sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k - \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k}{\sqrt{N_0/2}} \right) Pr\{\hat{\boldsymbol{b}}_k | \boldsymbol{b}_k\}, \tag{7.5}$$

where $Pr\{\hat{\boldsymbol{b}}_k|\boldsymbol{b}_k\}$ is the integral of the $(K-1)$ variate Gaussian density function.

7.1 Using Minimum Energy Algorithm

Using the minimum energy criterion, $\frac{\partial}{\partial \boldsymbol{w}_k} E\{y_k^2\} = 0$, the steady state values of the weights affecting the k-th output are:

$$\boldsymbol{w}_k = \left[E\{\hat{\boldsymbol{b}}_k \hat{\boldsymbol{b}}_k^T\}\right]^{-1} \boldsymbol{A}_k E\{\boldsymbol{b}_k \hat{\boldsymbol{b}}_k^T\} \boldsymbol{\rho}_k, \tag{7.6}$$

where the computation of $E\{\hat{\boldsymbol{b}}_k \hat{\boldsymbol{b}}_k^T\}$ and $E\{\boldsymbol{b}_k \hat{\boldsymbol{b}}_k^T\}$ can be found in Appendix C.

7.1.1 Numerical Examples

Two sets of numerical examples are given in the following figures. For comparison purposes, the error performance of the corresponding fixed-weights detector in [15] (in which the weights are set, based on the knowledge of the received signals' energies, to $w_{ji} = \rho_{ij}\sqrt{a_j}$), and those of the decorrelating detector, are included.

The first example depicted in Fig. 7.2 is a two-user case. The cross-correlation coefficient is taken to be 0.7. The SNR of user one is set to 8 dB, while the SNR of user two, relative to user one, varies from -10 to 8 dB.

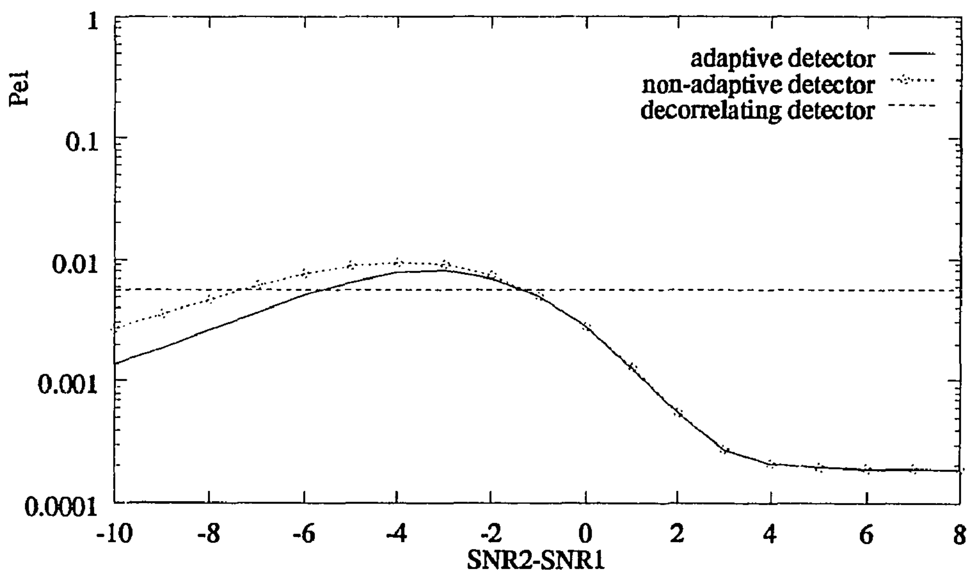

Figure 7.2 Error Probability of User 1 for $K = 2$, $\rho = 0.7$ and $SNR_1 = 8$ dB

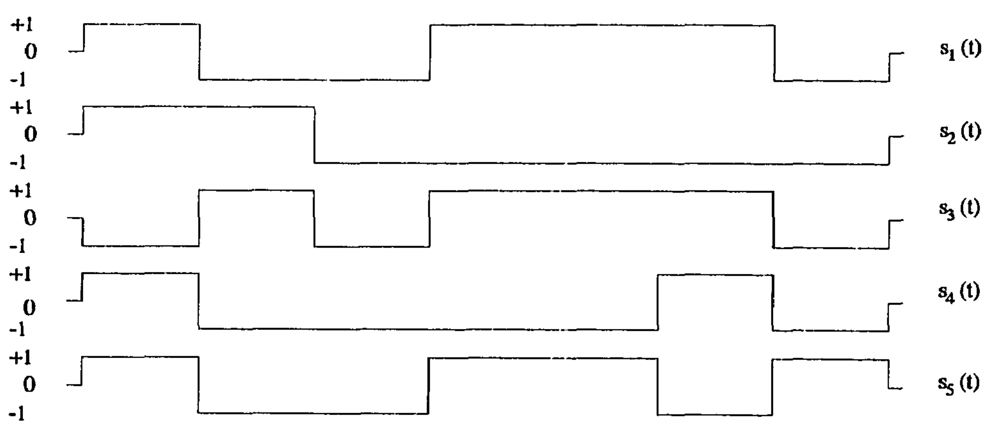

Figure 7.3 Gold Sequences of Length 7

In the second set of examples, Gold sequences (Fig. 7.3) of length seven are chosen for signature waveforms. The cross-correlation matrix $\mathcal{P}$ of the given Gold sequences is:

$$\mathcal{P} = \frac{1}{7}\begin{bmatrix} 7 & -1 & 3 & 3 & 3 \\ -1 & 7 & -1 & 3 & -1 \\ 3 & -1 & 7 & -1 & -1 \\ 3 & 3 & -1 & 7 & -1 \\ 3 & -1 & -1 & -1 & 7 \end{bmatrix}$$

The two-user case with Gold sequences is shown in Fig. 7.4. As expected in this low bandwidth efficiency scenario ($\rho_{12} = -1/7$), the decorrelating detector performs as well as the other two schemes. By adding an additional user, as shown in Fig. 7.5, the decorrelating detector begins to exhibit its inadequacy. The adaptive and fixed-weights schemes show virtually identical performance, with the former being only slightly better for weak interferers.

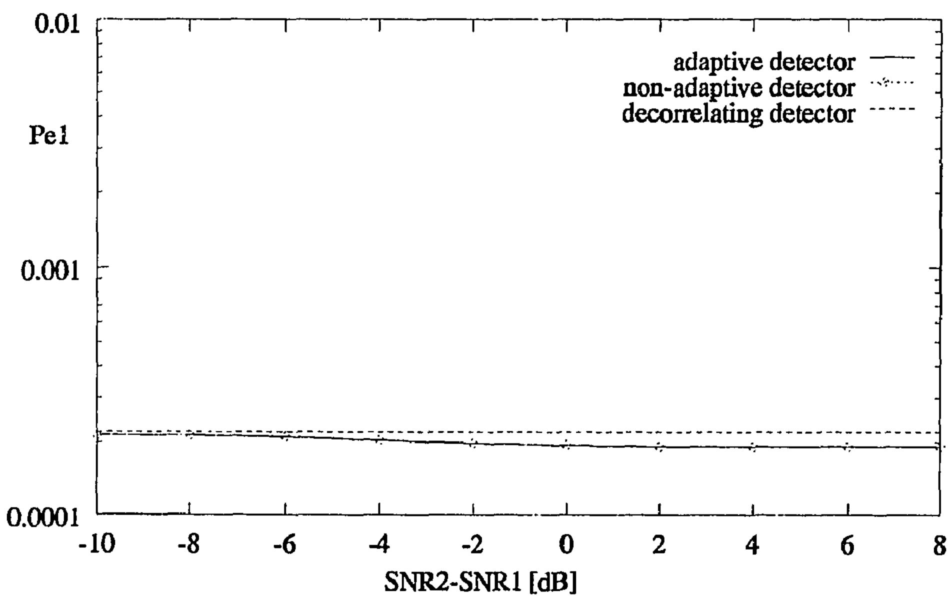

Figure 7.4 Error Probability of User 1 for $K = 2$ and $SNR_1 = 8$ dB

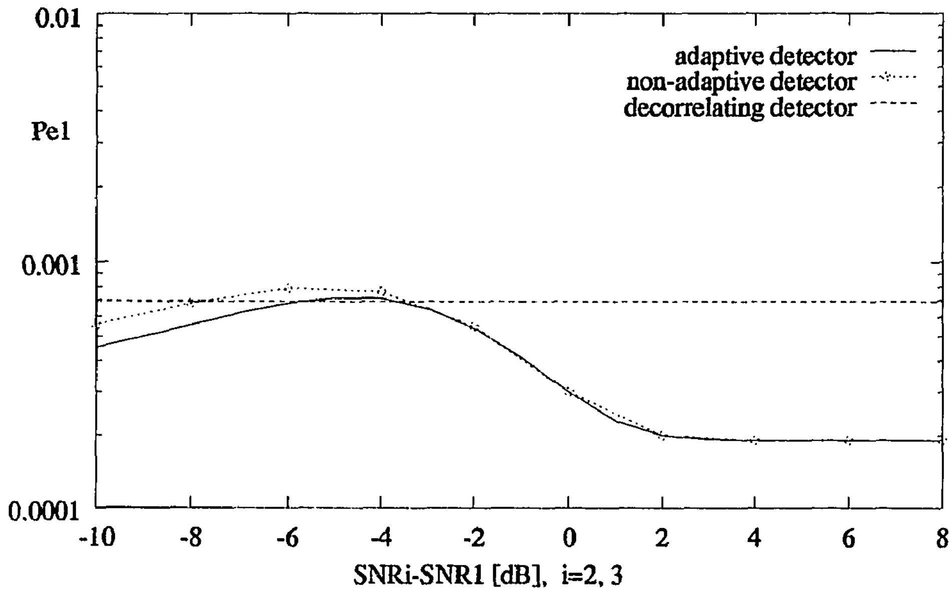

Figure 7.5 Error Probability of User 1 for $K = 3$ and $SNR_1 = 8$ dB

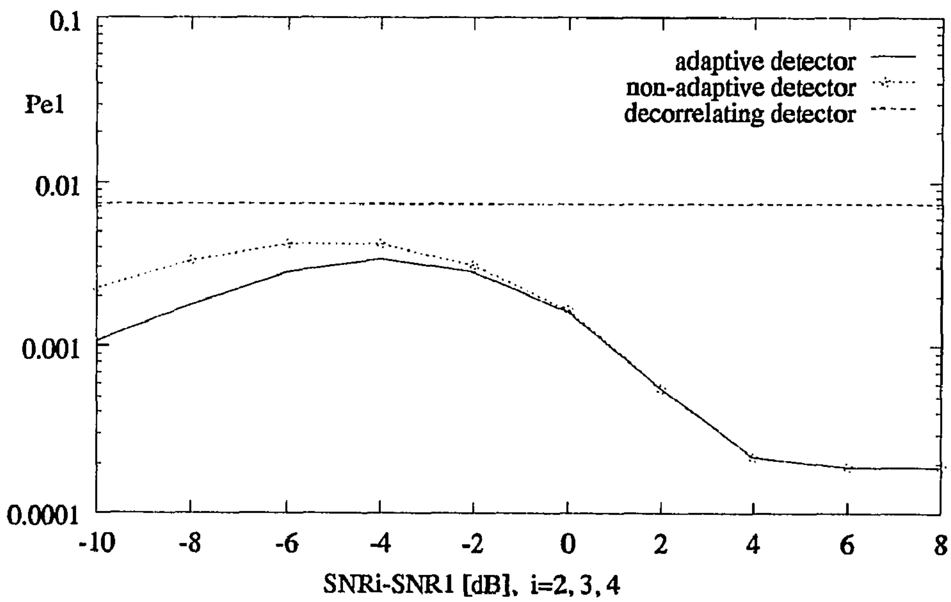

Figure 7.6 Error Probability of User 1 for $K = 4$ and $SNR_1 = 8$ dB

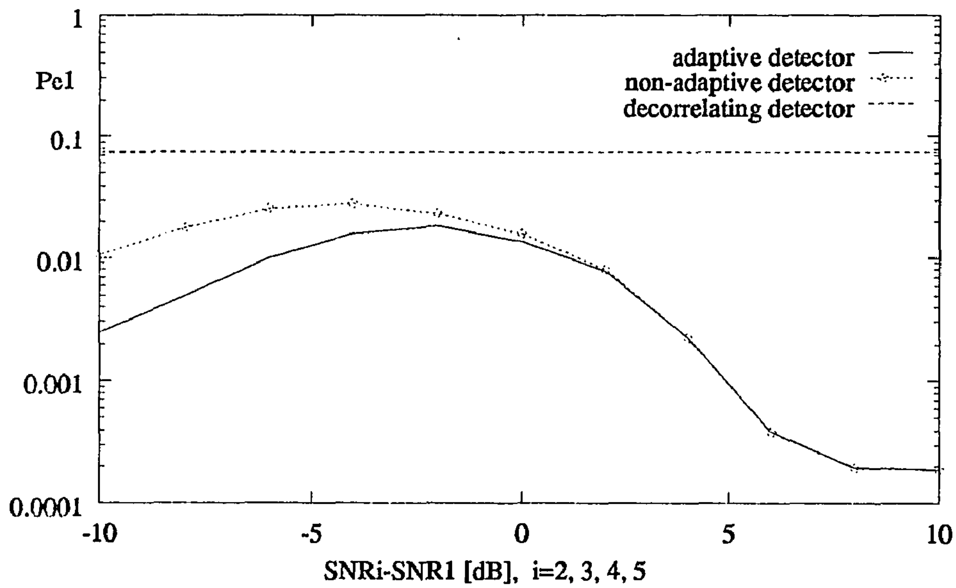

Figure 7.7 Error Probability of User 1 for $K = 5$ and $SNR_1 = 8$ dB

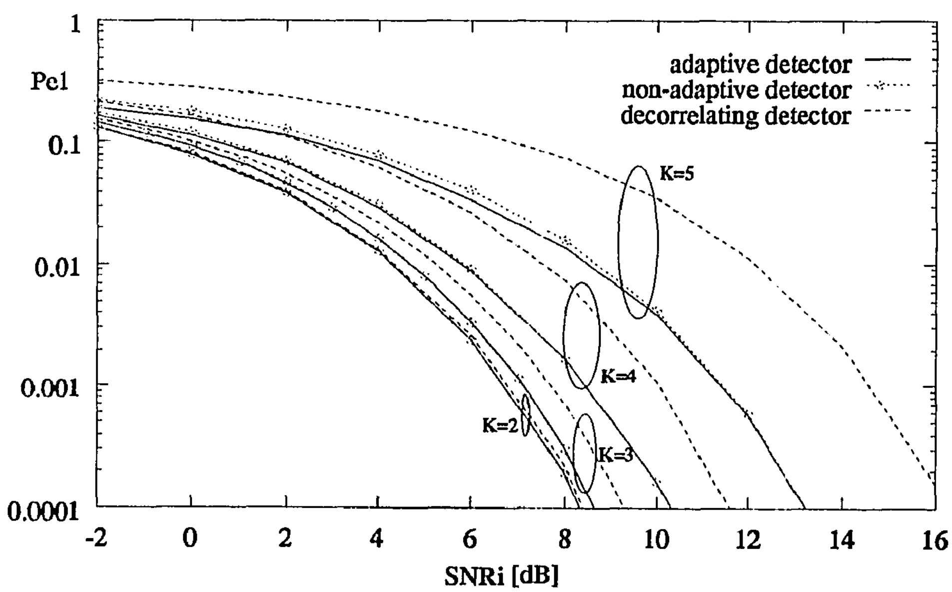

Figure 7.8 Error Probability of User 1 for $K = 5$

With the number of simultaneous users increasing further to $K = 4$ and $K = 5$, as in Figs. 7.6 and 7.7 (note the change of the vertical axis scaling), certain trends become more obvious. Due to its unacceptable high probability of error, the decorrelating detector clearly does not represent an appropriate choice. When the interferers are strong, both the adaptive and the fixed-weights schemes achieve the performance of the single user. The former provides better error performance with weak interferers. Fig. 7.8 shows the error probability of user one for $K = 2$ to 5 when all the users have same SNR.

7.2 Using Decorrelating Algorithm

The canceler's output is given by:

$$\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{W}^T \hat{\boldsymbol{b}}. \tag{7.7}$$

The output for the k-th user can be expressed as:

$$y_k = x_k - \boldsymbol{w}_k{}^T \hat{\boldsymbol{b}}_k. \tag{7.8}$$

The steady state would be reached if

$$E\{y_k \mathrm{sgn}(\boldsymbol{y}_k)\} = 0, \quad \text{for } k = 1, 2, \ldots, K.$$

This means that $K(K-1)$ nonlinear equations have to be evaluated.

Using the approximation derived in Eq. (3.11), we get:

$$E\{y_k \hat{\hat{\boldsymbol{b}}}_k\} \approx E\left\{\left(\sqrt{a_k} b_k + \boldsymbol{\rho}_k^T \boldsymbol{A}_k \boldsymbol{b}_k + n_k - \boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\right) \boldsymbol{b}_k\right\} \cdot,$$

and the algorithm will result in having:

$$\sqrt{a_k} E\left\{b_k \boldsymbol{b}_k\right\} + E\left\{\left(\boldsymbol{\rho}_k^T \boldsymbol{A}_k \boldsymbol{b}_k\right) \boldsymbol{b}_k\right\} + E\left\{n_k \boldsymbol{b}_k\right\} - E\left\{\left(\boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\right) \boldsymbol{b}_k\right\} = 0.$$

It is easy to show that $E\{b_k \boldsymbol{b}_k\} = 0$, $E\left\{\left(\boldsymbol{\rho}_k^T \boldsymbol{A}_k \boldsymbol{b}_k\right) \boldsymbol{b}_k\right\} = \boldsymbol{A}_k^T \boldsymbol{\rho}_k$, $E\{n_k \boldsymbol{b}_k\} = 0$, and $E\left\{\left(\boldsymbol{w}_k^T \hat{\boldsymbol{b}}_k\right) \boldsymbol{b}_k\right\} = \boldsymbol{B}_k \boldsymbol{w}_k$, where $\boldsymbol{B}_k = \mathrm{diag}\left[E\left\{b_j \hat{b}_j\right\}\right]$, $j = 1, 2, \ldots, K \quad j \neq k$. Therefore,

$$\boldsymbol{w}_k = \boldsymbol{B}_k^{-1} \boldsymbol{A}_k \boldsymbol{\rho}_k, \tag{7.9}$$

where we used the fact that $\boldsymbol{A}_k$ is diagonal.

7.2.1 Computational and Simulation Results

Two sets of examples are used to examine the performance of the proposed canceler. In the first set, we consider the two-user case with cross-correlation coefficient $\rho_{12} = 0.7$. It can certainly represent a high bandwidth-efficiency case. Here we find by computation the error probability as a function of the SNR's difference of the two users while the SNR of user one is kept constant at 8 dB and 12 dB. This is shown in Figs. 7.9 and 7.10, respectively.

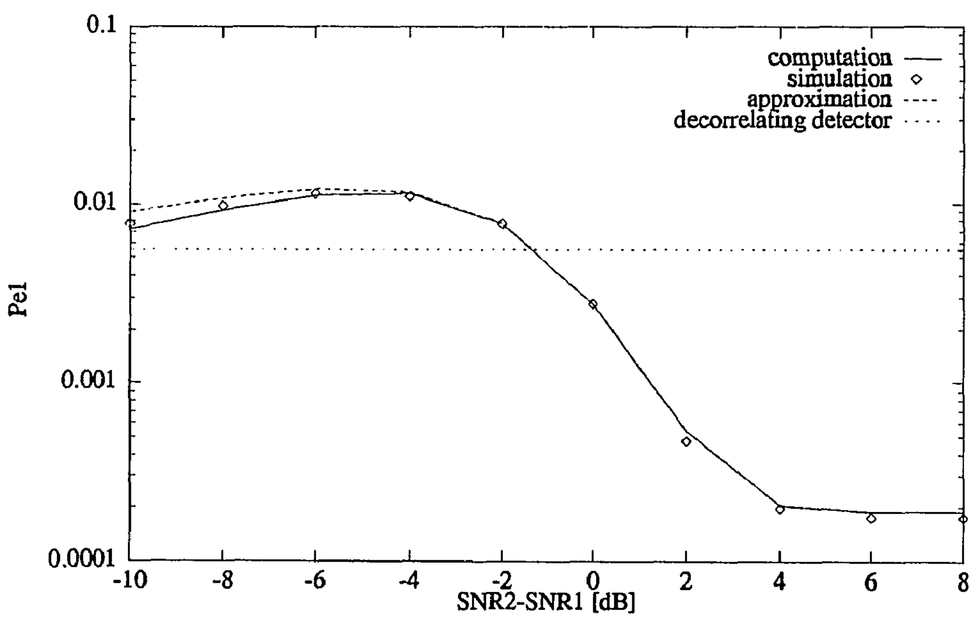

Figure 7.9 Probability of User 1 (K=2, $SNR_1 = 8$ dB, $\rho_{12} = 0.7$)

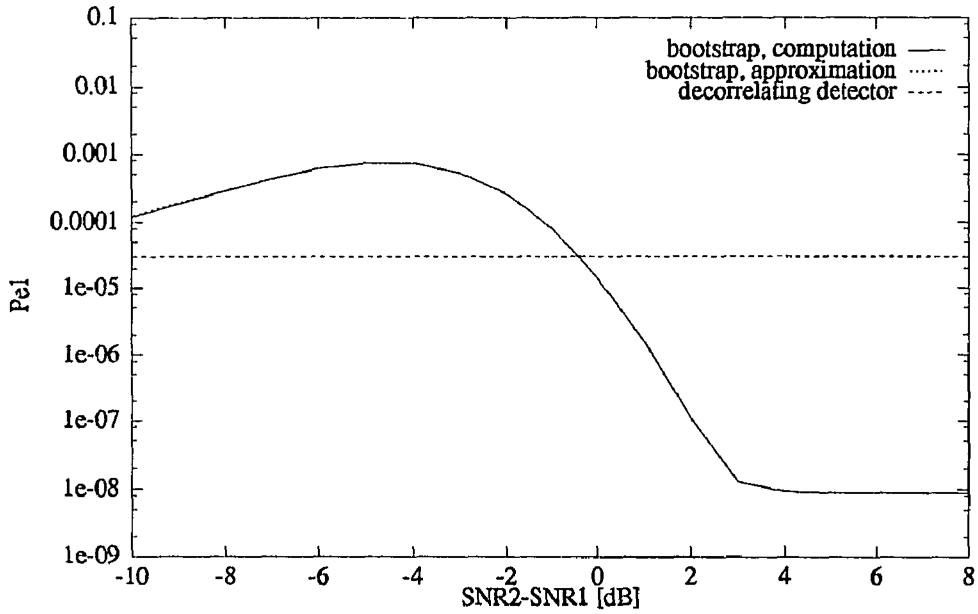

Figure 7.10 Error Probability of User 1 (K=2, $SNR_1 = 12$ dB, $\rho_{12} = 0.7$)

In the figures, we added the results of simulation, those obtained by using the approximation of Eq. (3.11), and those resulting from the decorrelating detector of [11].

In the second set of examples, Gold sequences of length 7 are again chosen for signature waveforms. The rationale for such a choice is that Gold sequences are regularly used in an asynchronous CDMA environment, and the study of its proposed synchronous counterpart may provide a useful indication of the performance of the former.

In Fig. 7.11, we depict the result of error probability of user one having SNR=8dB as a function of the SNR of the other four users (taken to be the same). These results are obtained by simulations and computation with the approximation. For comparison, the results of the decorrelating detector are also included. Fig. 7.12 is the same except for user one, where SNR=12 dB. Finally, Fig. 7.13 depicts the error probabilities of user one, whose interferences are determined by the first column of $\mathcal{P}$ and having the same energy as interfering users. For comparison purposes we also show the results obtained by using the minimum energy criterion and, as reference, those obtained when using the decorrelating detector.

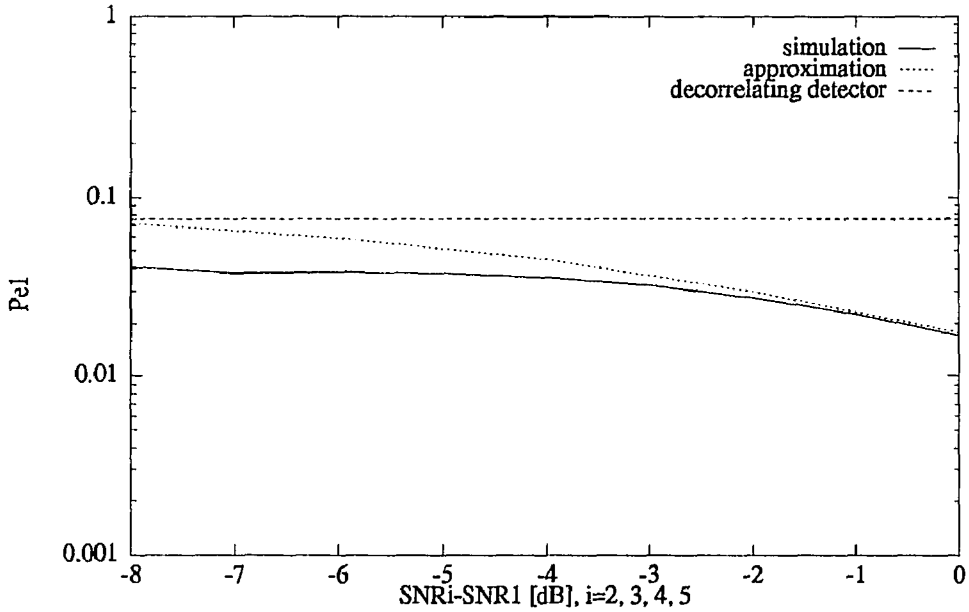

Figure 7.11 Error Probability of User 1 (K=5, SNR_1 = 8 dB, Gold codes)

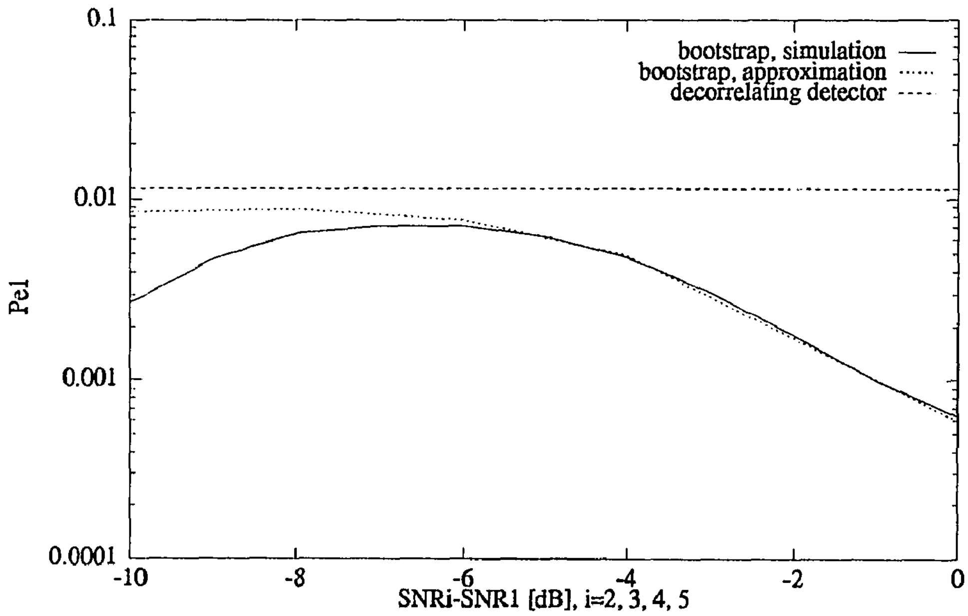

Figure 7.12 Error Probability of User 1 (K=5, SNR_1 = 12 dB, Gold codes)

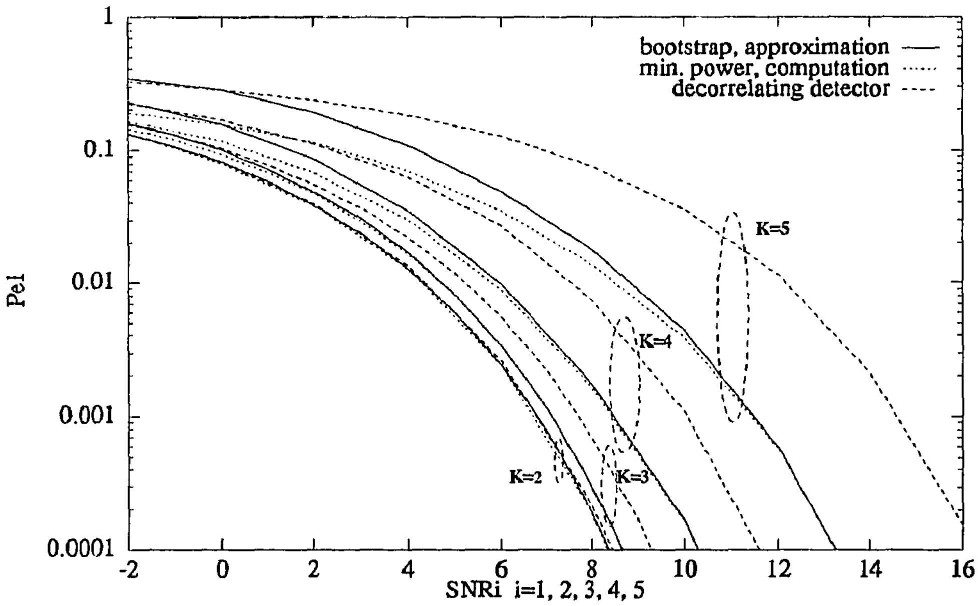

Figure 7.13 Error Probability of User 1 (K=5, Gold codes)

It was established that the case of equal-user energy represents approximately the worst case scenario for user one. Also notice from $\mathcal{P}$ that other users suffer lower levels of interference by remaining users.

From these figures, we notice that the error performance of the approximation curve is worse than the simulation curve, especially in the region where the interference is weak. The approximation is actually equivalent to using the training sequence, since we made the assumption that $sgn(y_i) \approx b_i$. Why is the performance of the scheme with a training sequence worse than the blind one without a training sequence? There are two reasons for this:

1. The weights of the simulation are smaller than those of the approximation ($E\{b_i sgn(y_i)\} < E\{b_i b_i\} = 1$).
2. When the interference is small, the estimation of the interference is bad.

When the estimations of the interference are not good, using them to cancel the interferences makes it worse. This is also the reason why the adaptive scheme introduced in previous section is better than its non-adaptive counterpart.

CHAPTER 8

TWO-STAGE ADAPTIVE DETECTOR WITH SOFT TENTATIVE DECISION

We can plot the error performance of the detector introduced in sections 6.1 and 7.1 on top of each other, as shown in Fig. 8.1.

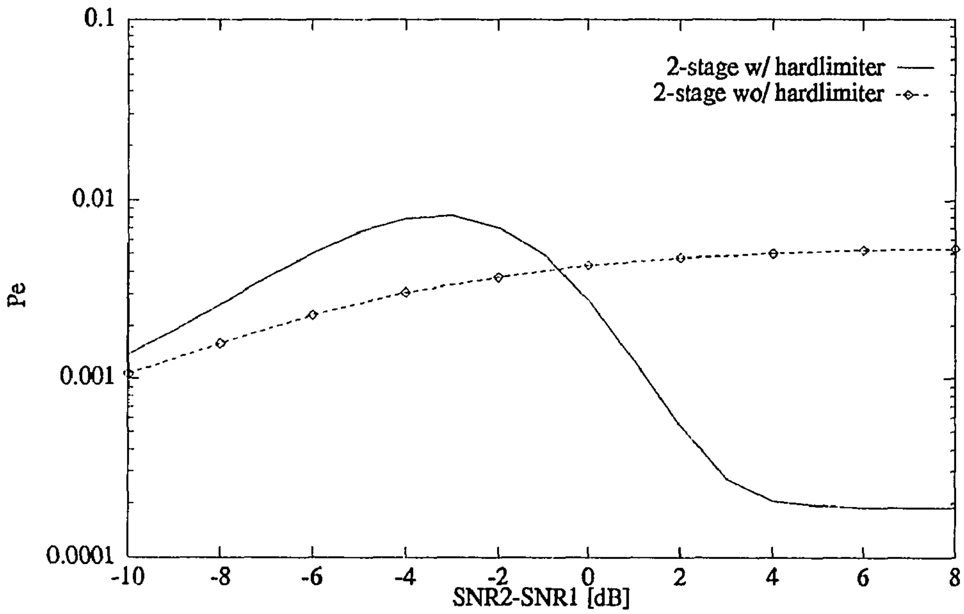

Figure 8.1 Error Probability of User 1 for $K = 2$, $\rho = 0.7$ and $SNR_1 = 8$ dB

If one compares the two curves, he or she will find that when the energy of the interference signal is weaker than the desired signal energy, the performance of the two-stage adaptive detector introduced in Chapter 6 (decorrelator + canceler) is better than the two-stage adaptive detector with tentative decision (decorrelator + decision + canceler), and in the other region, the latter is better. In this chapter, a combination of the two schemes proposed in previous two chapters will be introduced, and the steady state error performance of the detector will be evaluated. In the combined scheme, soft tentative decisions are used.

8.1 Model and Analysis

The detector considered here is shown in Fig. 8.2. It consists of three parts: a bank of matched filters, a decorrelator with soft tentative decision, and the MAI canceler. The difference between this detector and the detector proposed in previous chapter is in the use of a soft decision instead of a hard decision.

Where in the figure H is a nonlinear transformation matrix, there are many selections for a nonlinear transformation. Dead zone, multilevel quantization, and linear clipper are some of them. For reasons concerning performance and computational complexity, in this chapter only the linear clipper is studied.

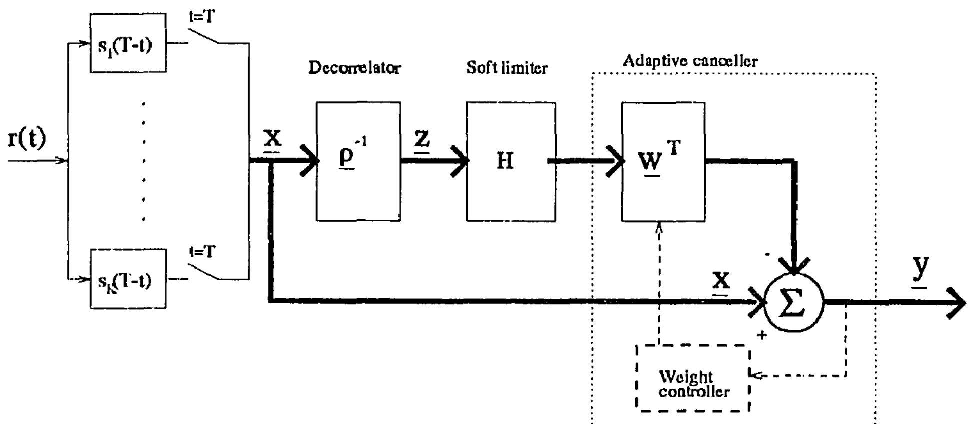

Figure 8.2 Two-stage Adaptive Detector with Soft Tentative Decision

The output for the k-th user can be expressed as:

$$y_k = x_k - \boldsymbol{w}_k{}^T \boldsymbol{h}_k = x_k - \sum_{\substack{l=1 \\ l \neq k}}^{K} w_{lk} h_{lk}. \tag{8.1}$$

$\boldsymbol{h}_k$ is the k-th column vector of a $K \times K$ matrix $\boldsymbol{H}$, with the element h_{kk} deleted. The (l, k)-th element of the matrix $\boldsymbol{H}$ represents the k-th output of the soft limiter to the input z_l, and, for $l, k = 1, \ldots, K \quad l \neq k$, is defined as:

$$h_{lk}(z_l) = \begin{cases} z_l / T_{lk} & |z_l| < T_{lk} \\ sgn(z_l) & \text{otherwise} \end{cases} .$$

Comparing the results of the previous two chapters, we can conclude that when the interference is stronger than the desired signal, the hard decision is preferred. When the interference is weaker than the desired signal, the soft decision is preferred. The selection of the limiter's threshold T_{lk} has to satisfy these constraints.

A few experiments are done to find out the threshold. Fig. 8.3 to Fig. 8.5 are examples of the two-user case.

Considering the above facts and according to the results of the experiment, the threshold is given by:

$$T_{lk} = \frac{\rho_{lk}[E\{|z_k|\}]^2}{E\{|z_l|\}} = \rho_{lk}\boldsymbol{R},$$

where

$$\boldsymbol{R} = \frac{[E\{|z_k|\}]^2}{E\{|z_l|\}}, \tag{8.2}$$

and

$$E\{|z_k|\} = \sqrt{a_k}\left(1 - 2Q\left(\frac{\sqrt{a_k}}{\sigma_{\xi_k}}\right)\right) + \frac{2\sigma_{\xi_k}}{\sqrt{2\pi}}e^{-\frac{a_k}{2\sigma^2_{\xi_k}}},$$

and $\sigma^2_{\xi_k}$ is the variance of ξ_k.

T_{lk} can be determined from the observed values of the decorrelator outputs.

In the figures, the error probability of user one for a two-user system, versus the SNR of user two, are given. The correlations between the two users' signature codes are 0.3, 0.5, and 0.7, respectively. The limiter's threshold T_{lk} is given as the dummy variable, where $\boldsymbol{R}$ is defined in Eq. (8.2). These figures justify the selection of the limiter's threshold T_{lk}.

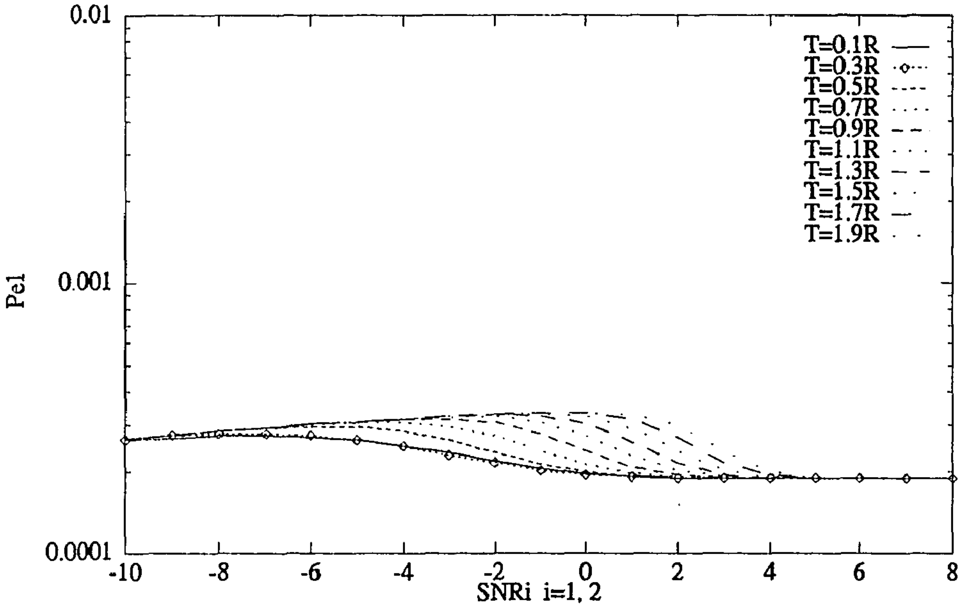

Figure 8.3 Error Probability of User 1 for $K = 2$, $SNR_1 = 8$ dB and $\rho_{12} = 0.3$

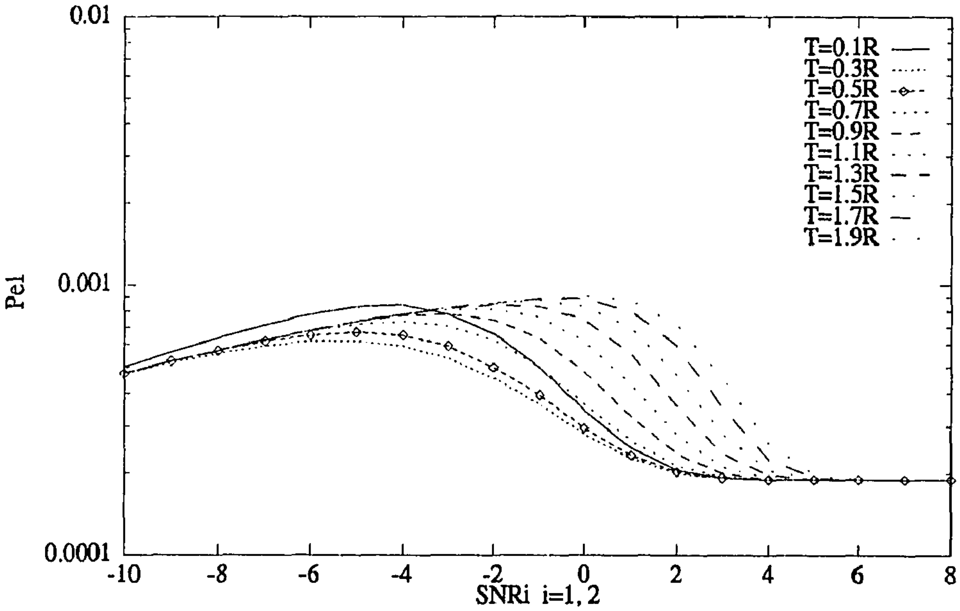

Figure 8.4 Error Probability of User 1 for $K = 2$, $SNR_1 = 8$ dB and $\rho_{12} = 0.5$

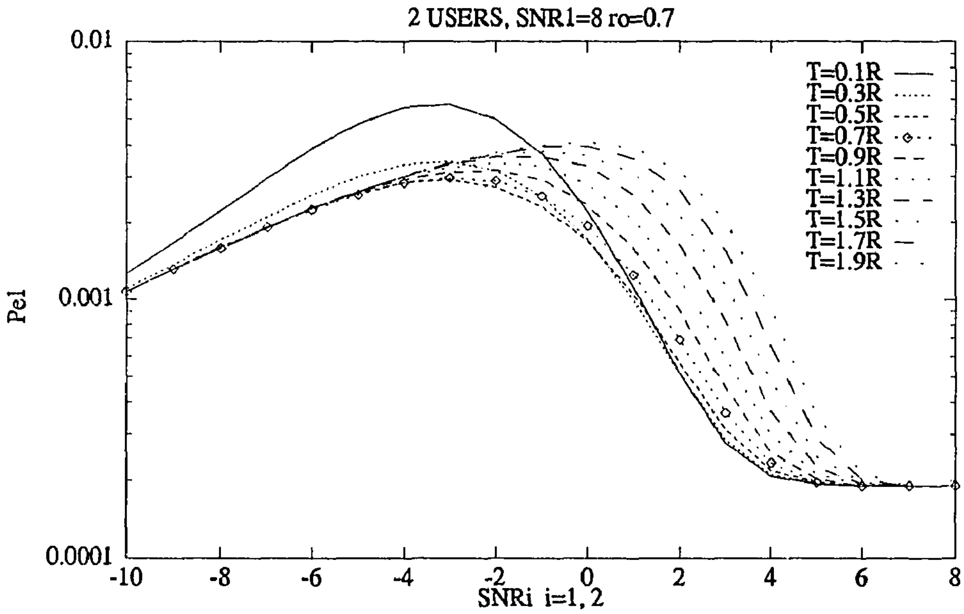

Figure 8.5 Error Probability of User 1 for $K = 2$, $SNR_1 = 8$ dB and $\rho_{12} = 0.7$

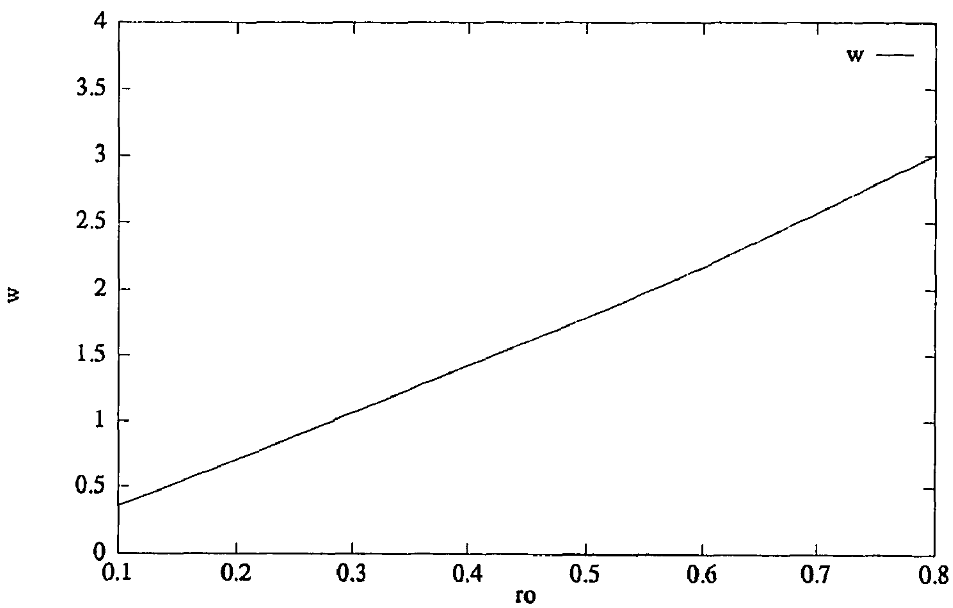

Figure 8.6 Weight of User 1, $SNR_1 = SNR_2 = 8$ dB, with $\rho = 0.1$ to 0.8

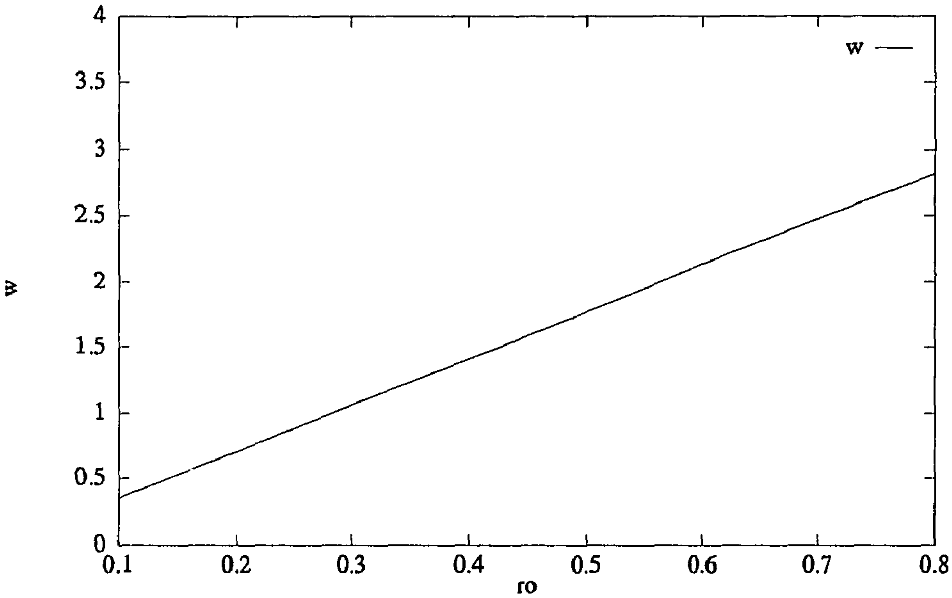

Figure 8.7 Weight of User 1, $SNR_1 = 8$, $SNR_2 = 2$ dB, with $\rho = 0.1$ to 0.8

Fig. 8.6 and Fig. 8.7 plot the weight changes versus the changes of the correlation coefficient. In Fig. 8.6, both SNR_1 and SNR_2 are equal to 8 dB. It gives the weight of user one when the voltage of user two is greater than the threshold. In Fig. 8.7, SNR_1 is equal to 8 dB, and SNR_2 is equal to 2 dB. It gives the weight of user one when the voltage of user two is less than the threshold. In both cases, the weight is nearly a linear function of the correlation coefficient, which is what we need since only so can the soft effect be utilized.

For controlling the weights, the steepest descent algorithm, which simultaneously minimizes the output signal energies $E\{y_k^2\}$, is used. That is, for the k-th output, the optimum weights are obtained by the iterative search:

$$\begin{aligned} \boldsymbol{w}_k(i+1) &= \boldsymbol{w}_k(i) - \frac{\mu}{2}\frac{\partial}{\partial \boldsymbol{w}_k(i)} E\{y_k^2(i)\} \\ &= (\boldsymbol{I} - \mu E\{\boldsymbol{h}_k \boldsymbol{h}_k^T\})\boldsymbol{w}_k(i) + \mu E\{x_k \boldsymbol{h}_k\}. \end{aligned}$$

The steady state values can be evaluated from:

$$\frac{\partial}{\partial \boldsymbol{w}_k} E\{y_k^2\} = 0 = -E\{x_k \boldsymbol{h}_k\} + E\{\boldsymbol{h}_k \boldsymbol{h}_k^T\}\boldsymbol{w}_k. \tag{8.3}$$

It is easily shown that:

$$\begin{aligned} E\{x_k \boldsymbol{h}_k\} &= E\left\{\left[\sqrt{a_k} b_k + \boldsymbol{\rho}_k^T \boldsymbol{A}_k \boldsymbol{b}_k + n_k\right] \boldsymbol{h}_k\right\} \\ &= \left[\boldsymbol{A}_k E\{\boldsymbol{b}_k \boldsymbol{h}_k^T\}\right]^T \boldsymbol{\rho}_k, \end{aligned}$$

where again $\boldsymbol{A}_k$ is a diagonal $(K-1)\times(K-1)$ submatrix of $\boldsymbol{A}$ with its k-th diagonal entry removed, $\boldsymbol{\rho}_k$ is a $(K-1)\times 1$ k-th column vector obtained from $\boldsymbol{\mathcal{P}}$ by deleting the element ρ_{kk}, and $\boldsymbol{b}_k$ is a $(K-1)\times 1$ vector obtained from $\boldsymbol{b}$ by deleting the element b_k. Clearly $E\{\boldsymbol{b}_k \boldsymbol{h}_k^T\}$ is diagonal; therefore, the system of $K-1$ linear equations (8.3) gives the steady state values of the weights affecting the k-th output as:

$$\boldsymbol{w}_k = \left[E\{\boldsymbol{h}_k \boldsymbol{h}_k^T\}\right]^{-1} \boldsymbol{A}_k E\{\boldsymbol{b}_k \boldsymbol{h}_k^T\} \boldsymbol{\rho}_k. \tag{8.4}$$

Let f_{z_i} be the density function of z_i, and $\boldsymbol{f}_{z_i z_j}$ the joint density function of z_i and z_j. Then:

$$\begin{aligned} E\{h_{ik}^2\} &= \frac{1}{2}\sum_{b_i}\left[\int_{\boldsymbol{L}_1} \frac{z_i^2}{T_{ik}^2} f_{z_i} dz_i + \int_{\boldsymbol{L}_2} f_{z_i} dz_i\right] \\ E\{h_{ik}h_{jk}\} &= \frac{1}{4}\sum_{b_i,b_j}\left[\int\int_{\boldsymbol{Z}_1} \frac{z_i z_j}{T_{ik}T_{jk}} \boldsymbol{f}_{z_i z_j} dz_i dz_j + \int\int_{\boldsymbol{Z}_2} \frac{z_i}{T_{ik}} sgn(z_j) \boldsymbol{f}_{z_i z_j} dz_i dz_j \right. \\ &+ \left. \int\int_{\boldsymbol{Z}_3} \frac{z_j}{T_{jk}} sgn(z_i) \boldsymbol{f}_{z_i z_j} dz_i dz_j + \int\int_{\boldsymbol{Z}_4} sgn(z_i) sgn(z_j) \boldsymbol{f}_{z_i z_j} dz_i dz_j\right] \\ E\{b_i h_{ik}\} &= \frac{1}{2}\sum_{b_i}\left[\int_{\boldsymbol{L}_1} b_i \frac{z_i}{T_{ik}} f_{z_i} dz_i + \int_{\boldsymbol{L}_2} b_i sgn(z_i) f_{z_i} dz_i\right], \end{aligned}$$

where $\boldsymbol{L}_i$, $i = 1, 2$ correspond to the appropriate intervals of z_i, and $\boldsymbol{Z}_i$, $i = 1, \ldots, 4$ correspond to the appropriate rectangular regions in the (z_i, z_j) plane.

Defining the k-th user's final decision output as $\hat{b}_k = sgn(y_k)$, its probability of error is evaluated as follows:

$$\begin{aligned} P_{e_k} &= E_{b_k,\boldsymbol{b}_k,\boldsymbol{h}_k} Pr\{\hat{b}_k \neq b_k | b_k, \boldsymbol{b}_k, \boldsymbol{h}_k\} \\ &= \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k} E_{\boldsymbol{h}_k} Pr\{-\sqrt{a_k} + \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k - \boldsymbol{w}_k^T \boldsymbol{h}_k + n_k > 0 | \boldsymbol{h}_k\}. \end{aligned}$$

Introducing the transformed Gaussian random variable ψ_k,

$$\psi_k = n_k - \sum_{\substack{i=1 \\ i \neq k}}^{K} c_i \frac{w_{ik}}{T_{ik}} \xi_i,$$

where

$$c_i = \begin{cases} 1 & |z_i| < T_{ik} \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2, \ldots, K,\ i \neq k,$$

the error probability becomes:

$$P_{e_k} = \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}_k} E_{\boldsymbol{h}_k} Pr\{\psi_k > \sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k + \boldsymbol{w}_k^T \boldsymbol{g}_k | \boldsymbol{h}_k\},$$

where

$$\boldsymbol{g}_k = [g_{1k}, g_{2k}, \ldots, g_{k-1,k}, g_{k+1,k}, \ldots, g_K]^T \text{ with}$$

$$g_{ik} = \begin{cases} \sqrt{a_i} b_i / T_{ik} & |z_i| < T_{ik} \\ sgn(z_i) & \text{otherwise} \end{cases} \quad i = 1, 2, \ldots, K,\ i \neq k.$$

Defining the vector $\boldsymbol{\zeta}$,

$$\boldsymbol{\zeta} = [\xi_1, \xi_2, \ldots, \xi_{k-1}, \psi_k, \xi_{k+1}, \ldots, \xi_K]^T,$$

the final expression for the error probability is obtained as:

$$P_{e_k} = 2^{-(K-1)} \sum_{\boldsymbol{b}_k} \sum_{n=1}^{2^{K-1}} \int_{\boldsymbol{D}_n} \boldsymbol{f}_{\boldsymbol{\zeta}} d\boldsymbol{\zeta}, \tag{8.5}$$

where $\boldsymbol{f}_{\boldsymbol{\zeta}}$ is a K-variate Gaussian density function, $\boldsymbol{D}_n$ is a hyper cube defined by:

$$\sqrt{a_k} - \boldsymbol{b}_k^T \boldsymbol{A}_k \boldsymbol{\rho}_k + \boldsymbol{w}_k^T \boldsymbol{g}_k < \zeta_k,$$

and for $i = 1, 2, \ldots, K$, $i \neq k$,

$$\begin{array}{ll} -T_{ik} - \sqrt{a_i}b_i < \zeta_i < T_{ik} - \sqrt{a_i}b_i & |z_i| < T_{ik} \\ \zeta_i > T_{ik} - \sqrt{a_i}b_i,\ \zeta_i < -T_{ik} - \sqrt{a_i}b_i & \text{otherwise.} \end{array}$$

The two-user examples are given in Appendix D.

8.1.1 Numerical Example and Discussion

Two sets of numerical examples are given in the following figures. In Fig. 8.8, SNR_1 is set to 8 dB, while SNR_2/SNR_1 varies from -10 to 8 dB. The value of the cross-correlation coefficient $\rho_{12} = 0.7$ represents a high bandwidth-efficiency case. Compared with the error performance of the detector in [15], which utilizes hard decisions, and the performance of the decorrelator, a significant improvement has been obtained. In the next two examples, Gold codes of length 7 (e.g., [15] and [17]) were used for signature waveforms. For the two-user case, in a low bandwidth efficiency case with $\rho_{12} = -1/7$ (Fig. 8.9) as expected, performance of the decorrelator is very close to the single-user bound. Therefore, negligible improvement is obtained with either the hard or soft tentative decisions. The three-user case with $\rho_{13} = 3/7$ and $\rho_{23} = -1/7$ is depicted in Fig. 8.10. Here, the one-stage detector with a soft limiter clearly shows the best performance. For a different set of cross-correlation coefficient values ($\rho_{12} = 0.5$, $\rho_{13} = 0.5$, and $\rho_{23} = 0.2$) in Fig. 8.11, the difference in the performance between the decorrelator and the one-stage detector is larger, with the soft limiter again being a better one. Finally, Fig. 8.12 depicts the error probabilities in the case when all users maintain the same $SNR's$ and Gold codes are used.

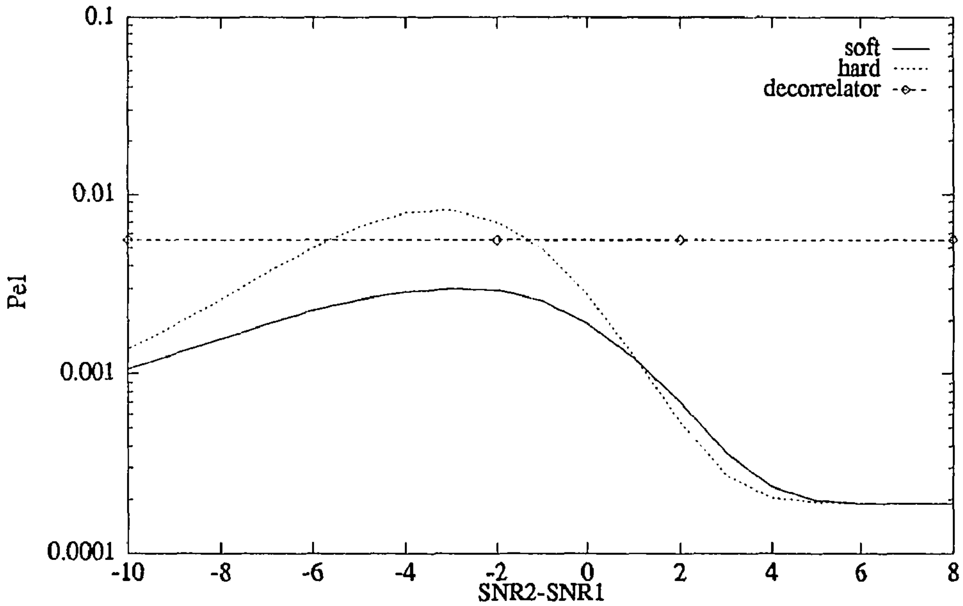

Figure 8.8 Error Probability of User 1 for $K = 2$, $SNR_1 = 8$ dB and $\rho_{12} = 0.7$

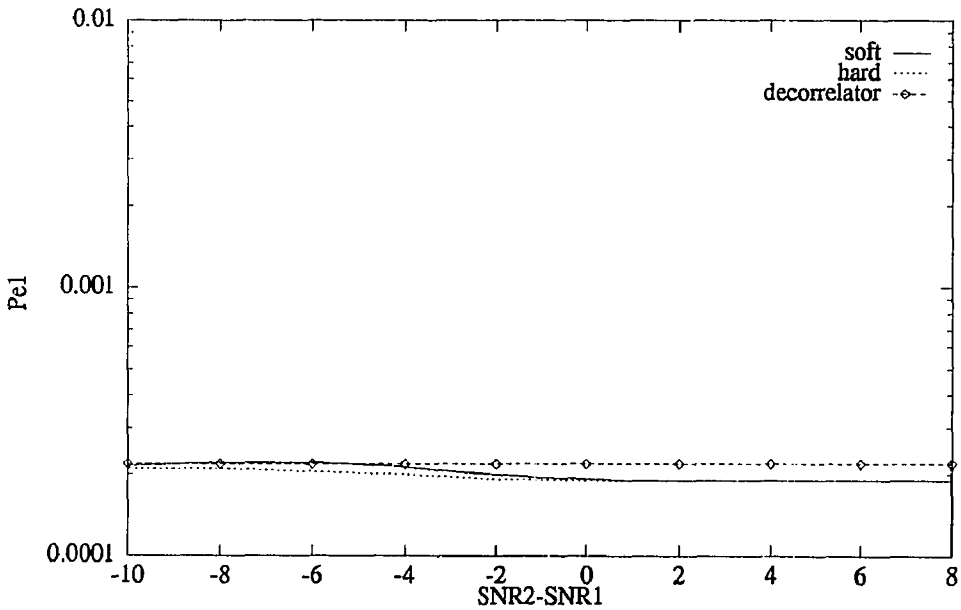

Figure 8.9 Error Probability of User 1 for $K = 2$

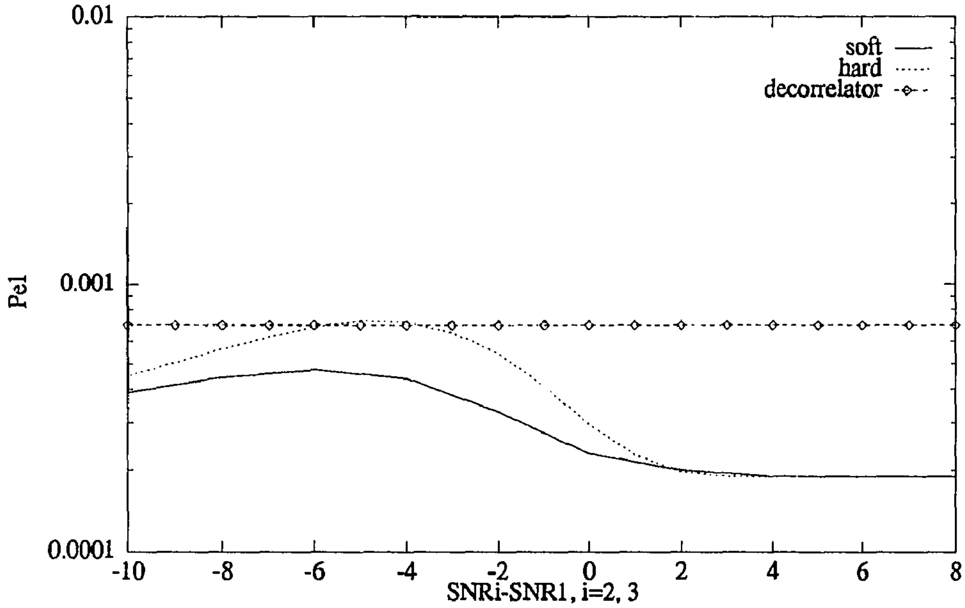

Figure 8.10 Error Probability of User 1 for $K = 3$

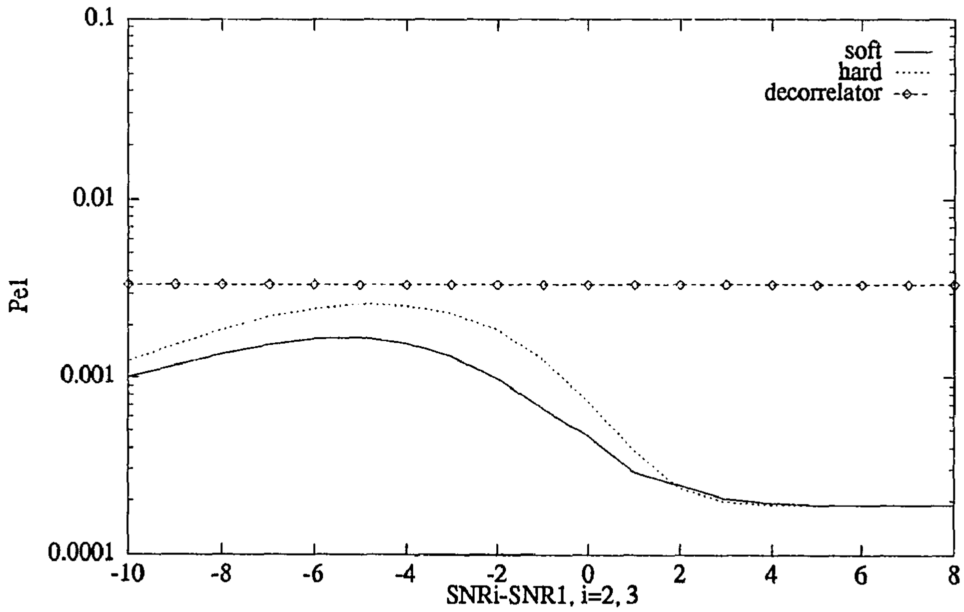

Figure 8.11 Error Probability of User 1 for $K = 3$

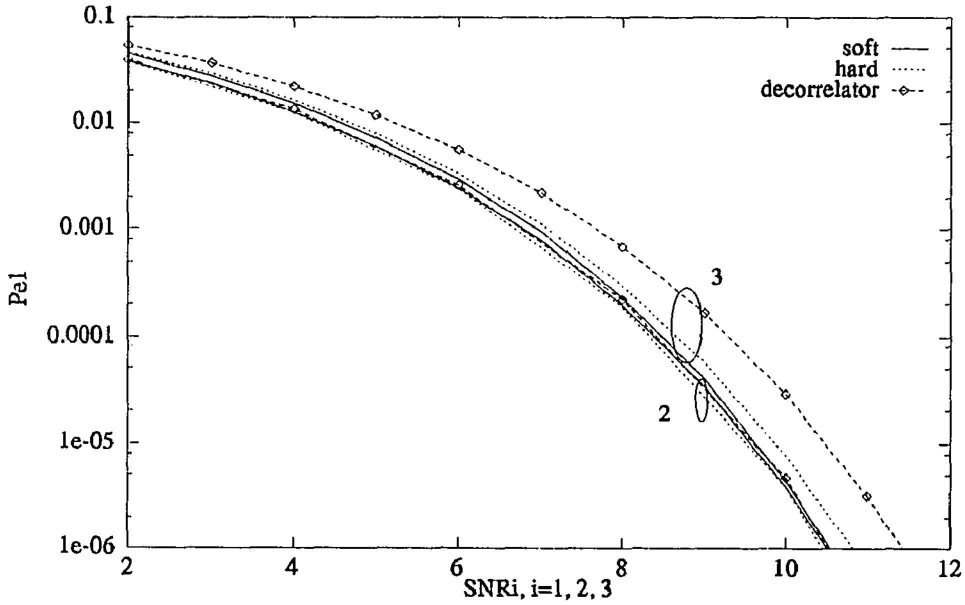

Figure 8.12 Error Probability of User 1 for $K = 2$ and $K = 3$

CHAPTER 9

CONCLUSION

One- and two-stage CDMA adaptive detectors using minimum energy and decorrelating adaptation algorithms are studied in this dissertation. These adaptive detectors do not require the knowledge of the received signals' energies and get better than or similar performance to their non-adaptive counterparts.

Among the detectors using the minimum energy algorithm, the error performance of the user with lower input SNR of a one-stage detector with a hard tentative decision approaches the single-user bound as the interferences' energies increase. The two-stage detectors proposed were shown to provide significantly better performance than the decorrelator. It is especially true for the ones with a tentative decision and under the most critical conditions for the multiuser environment, such as near-far situations and high bandwidth-efficiency utilization. In the presence of strong interference those detectors achieve the performance of the single-user bound. The two-stage detector with a soft tentative decision approaches the performance of the optimum detector but with much less complexity.

Among the detectors using the decorrelating algorithm, the one- and two-stage detector has similar performance to the decorrelator. Two-stage detector with hard tentative decision achieves similar performance as the one using minimum energy algorithm.

APPENDIX A

TWO-USER EXAMPLE OF CHAPTER 4

The sampled outputs of the bank of matched filters in the i-th bit interval can be expressed as:

$$x_1 = \sqrt{a_1}b_1 + \rho\sqrt{a_2}b_2 + n_1 \tag{A.1}$$

$$x_2 = \sqrt{a_2}b_2 + \rho\sqrt{a_1}b_1 + n_2. \tag{A.2}$$

The canceler's output is given by:

$$\begin{aligned} y_1 &= x_1 - w_{21}x_2 \\ &= (1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2 + n_1 - w_{21}n_2 \\ &= (1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2 + \xi_1 \\ y_2 &= x_2 - w_{12}x_1 \\ &= (1 - w_{12}\rho)\sqrt{a_2}b_2 + (\rho - w_{12})\sqrt{a_1}b_1 + n_2 - w_{12}n_1 \\ &= (1 - w_{12}\rho)\sqrt{a_2}b_2 + (\rho - w_{12})\sqrt{a_1}b_1 + \xi_2, \end{aligned}$$

where

$$\begin{aligned} \xi_1 &= n_1 - w_{21}n_2 \\ \xi_2 &= n_2 - w_{12}n_1. \end{aligned}$$

Using the decorrelating criterion,

$$E\{y_k sgn(y_j)\} = 0 \qquad k, j = 1, 2, k \neq j \tag{A.3}$$

$$\begin{aligned} E\{y_1 sgn(y_2)\} &= E\left\{\left[(1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2 + \xi_1\right] sgn(y_2)\right\} \\ &= E\left\{\left((1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2\right) sgn(y_2) + \xi_1 sgn(y_2)\right\}. \end{aligned}$$

The first and second terms of the above equation can be computed from:

$$\begin{aligned} & E\left\{\left[(1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2\right] sgn(y_2)\right\} \\ = \; & E_{b_1,b_2}\left\{\left((1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2\right) \times \right. \\ & \left\{P_r\left[\xi_2 > -(1 - w_{12}\rho)\sqrt{a_2}b_2 - (\rho - w_{12})\sqrt{a_1}b_1\right]\right. \\ - \; & \left.\left. P_r\left[\xi_2 < -(1 - w_{12}\rho)\sqrt{a_2}b_2 - (\rho - w_{12})\sqrt{a_1}b_1\right]\right\}\right\} \\ = \; & E_{b_1,b_2}\left\{\left((1 - w_{21}\rho)\sqrt{a_1}b_1 + (\rho - w_{21})\sqrt{a_2}b_2\right) \times \right. \\ & \left.\left[2Q\left(\frac{-(1 - w_{12}\rho)\sqrt{a_2}b_2 - (\rho - w_{12})\sqrt{a_1}b_1}{\sigma_{\xi_2}}\right) - 1\right]\right\} \\ = \; & (\rho - w_{21})\sqrt{a_2} + \left[(1 - w_{21}\rho)\sqrt{a_1} - (\rho - w_{21})\sqrt{a_2}\right] Q\left(\frac{(1 - w_{12}\rho)\sqrt{a_2} - (\rho - w_{12})\sqrt{a_1}}{\sigma_{\xi_2}}\right) \\ - \; & \left[(1 - w_{21}\rho)\sqrt{a_1} + (\rho - w_{21})\sqrt{a_2}\right] Q\left(\frac{(1 - w_{12}\rho)\sqrt{a_2} + (\rho - w_{12})\sqrt{a_1}}{\sigma_{\xi_2}}\right), \end{aligned}$$

and

$$\begin{aligned} & E\left\{\xi_1 sgn(y_2)\right\} \\ = \; & E_{b_1,b_2}\left\{\xi_1\left[P_r\left[\xi_2 > -(1 - w_{12}\rho)\sqrt{a_2}b_2 - (\rho - w_{12})\sqrt{a_1}b_1\right]\right.\right. \\ & \left.\left. - P_r\left[\xi_2 < -(1 - w_{12}\rho)\sqrt{a_2}b_2 - (\rho - w_{12})\sqrt{a_1}b_1\right]\right]\right\} \\ = \; & E_{b_1,b_2}\left\{\xi_1\left[P_r\left[\xi_2 > C\right] - P_r\left[\xi_2 < C\right]\right]\right\}, \end{aligned}$$

where

$$C = -(1 - w_{12}\rho)\sqrt{a_2}b_2 - (\rho - w_{12})\sqrt{a_1}b_1.$$

Denote

$$\begin{aligned}
E\{\xi_1\xi_2\} &= (\rho + \rho w_{12}w_{21} - w_{12} - w_{21})\frac{N_o}{2} = R\frac{N_o}{2} \\
E\{\xi_1^2\} &= (1 - 2w_{21}\rho + w_{21}^2)\frac{N_o}{2} = h_1^2\frac{N_o}{2} \\
E\{\xi_2^2\} &= (1 - 2w_{12}\rho + w_{12}^2)\frac{N_o}{2} = h_2^2\frac{N_o}{2},
\end{aligned}$$

where

$$R = \rho + \rho w_{12}w_{21} - w_{12} - w_{21}$$

$$h_1^2 = 1 - 2w_{21}\rho + w_{21}^2$$

$$h_2^2 = 1 - 2w_{12}\rho + w_{12}^2.$$

The transformation of (ξ_1, ξ_2) to (u, v) such that $E\{uv\} = 0$ is accomplished by

$$\begin{aligned}
\xi_2 &= v \\
\xi_1 &= \frac{R}{h_2^2}v + \frac{\sqrt{h_1^2h_2^2 - R^2}}{h_2^2}u.
\end{aligned}$$

Then we have

$$
\begin{aligned}
& E\left\{\xi_1 sgn(y_2)\right\} \\
= \; & \frac{1}{2}\sum_{b_1 b_2}\int_{u=-\infty}^{\infty}\int_{v=C}^{\infty}\left(\frac{R}{h_2^2}v+\frac{\sqrt{h_1^2h_2^2-R^2}}{h_2^2}u\right)f_u f_v du dv \\
= \; & \frac{1}{2}\sum_{b_1 b_2}\int_{u=-\infty}^{\infty} f_u du\int_{v=C}^{\infty}\frac{R}{h_2^2}v f_v dv+\int_{u=-\infty}^{\infty}\frac{\sqrt{h_1^2h_2^2-R^2}}{h_2^2}u f_u du\int_{v=C}^{\infty} f_v dv \\
= \; & \frac{1}{2}\sum_{b_1 b_2}\frac{R}{h_2^2}\int_{v=C}^{\infty} v f_v du dv \\
= \; & \frac{1}{2}\sum_{b_1 b_2}\frac{R\sigma_v}{h_2^2\sqrt{2\pi}}e^{-\frac{C^2}{2\sigma_v^2}}.
\end{aligned}
$$

Averaging over b_1 and b_2 one gets

$$
E\{\xi_1 sgn(y_2)\}=\frac{R\sigma_v}{h_2^2\sqrt{2\pi}}\left[e^{-\frac{((1-w_{12}\rho)\sqrt{a_2}+(\rho-w_{12})\sqrt{a_1})}{2\sigma_v^2}}+e^{-\frac{((1-w_{12}\rho)\sqrt{a_2}+(\rho-w_{12})\sqrt{a_1})}{2\sigma_v^2}}\right].
$$

Note here $\sigma_v^2=\sigma_u^2=h_2^2\sigma^2=h_2^2N_o/2$. Now we have two non-linear equations:

$$
y_1 sgn(y_2)=F\left(w_{12},w_{21},a_1,a_2,\rho,N_o\right)=0
$$

and

$$
y_2 sgn(y_1)=G\left(w_{12},w_{21},a_1,a_2,\rho,N_o\right)=0.
$$

We can solve these two equations to get w_{12} and w_{21}.

APPENDIX B

COMPUTATIONS FOR CHAPTER 5

B.1 Error Probability of User 1 for a Two-user Case

The canceler's vector output is given by:

$$y_1 = x_1 - w_{21}\hat{b}_2. \tag{B.1}$$

The error probability for user one is evaluated as follows:

$$\begin{aligned}
Pe_1 &= \frac{1}{2}E_{b_2,\hat{b}_2}(Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2}b_2 + w_{21}\hat{b}_2 | b_1 = -1, b_2, \hat{b}_2\} \\
&+ Pr\{n_1 < -\sqrt{a_1} - \rho_{12}\sqrt{a_2}b_2 + w_{21}\hat{b}_2 | b_1 = 1, b_2, \hat{b}_2\}) \\
&= \frac{1}{2}E_{b_2}(Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2}b_2 + w_{21}, n_2 > -\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2}b_2 - w_{21}, n_2 < -\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 < -\sqrt{a_1} - \rho_{12}\sqrt{a_2}b_2 + w_{21}, n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 < -\sqrt{a_1} - \rho_{12}\sqrt{a_2}b_2 - w_{21}, n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}\}) \\
&= \frac{1}{4}(Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2} + w_{21}, n_2 > -\sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2} - w_{21}, n_2 < -\sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 < -\sqrt{a_1} - \rho_{12}\sqrt{a_2} + w_{21}, n_2 > -\sqrt{a_2} - \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 < -\sqrt{a_1} - \rho_{12}\sqrt{a_2} - w_{21}, n_2 < -\sqrt{a_2} - \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} + \rho_{12}\sqrt{a_2} + w_{21}, n_2 > \sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} + \rho_{12}\sqrt{a_2} - w_{21}, n_2 < \sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 < -\sqrt{a_1} + \rho_{12}\sqrt{a_2} + w_{21}, n_2 > \sqrt{a_2} - \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 < -\sqrt{a_1} + \rho_{12}\sqrt{a_2} - w_{21}, n_2 < \sqrt{a_2} - \rho_{12}\sqrt{a_1}\})
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2}(Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2} + w_{21}, n_2 > -\sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} - \rho_{12}\sqrt{a_2} - w_{21}, n_2 < -\sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} + \rho_{12}\sqrt{a_2} + w_{21}, n_2 > \sqrt{a_2} + \rho_{12}\sqrt{a_1}\} \\
&+ Pr\{n_1 > \sqrt{a_1} + \rho_{12}\sqrt{a_2} - w_{21}, n_2 < \sqrt{a_2} + \rho_{12}\sqrt{a_1}\}). \qquad \text{(B.2)}
\end{aligned}$$

B.2 Computations of Expectations

$$E\{\hat{b}_i\hat{b}_i\} = 1 \tag{B.3}$$

$$\begin{aligned}
E\{\hat{b}_i\hat{b}_j\} &= E\{sgn(\sqrt{a_i}b_i + \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k + n_i)sgn(\sqrt{a_j}b_j + \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k + n_j)\} \\
&= \frac{1}{2^K}\sum_{\boldsymbol{b}}(Pr\{n_i > -\sqrt{a_i}b_i - \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k, n_j > -\sqrt{a_j}b_j - \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k\} \\
&+ Pr\{n_i < -\sqrt{a_i}b_i - \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k, n_j < -\sqrt{a_j}b_j - \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k\} \\
&- Pr\{n_i < -\sqrt{a_i}b_i - \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k, n_j > -\sqrt{a_j}b_j - \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k\} \\
&- Pr\{n_i > -\sqrt{a_i}b_i - \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k, n_j < -\sqrt{a_j}b_j - \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k\}) \\
&= \frac{1}{2^{K-1}}\sum_{\boldsymbol{b}}(Pr\{n_i > -\sqrt{a_i}b_i - \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k, n_j > -\sqrt{a_j}b_j - \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k\} \\
&- Pr\{n_i < -\sqrt{a_i}b_i - \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k, n_j > -\sqrt{a_j}b_j - \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k\} \qquad (B.4)
\end{aligned}$$

Every term of the above equation involves a two-dimensional Gaussian integration.

$$E\{x_i\hat{b}_j\} = E\{(\sqrt{a_i}b_i + \sum_{\substack{k=1\\k\neq i}}^{K}\rho_{ik}\sqrt{a_k}b_k + n_i)sgn(\sqrt{a_j}b_j + \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k + n_j)\}, \tag{B.5}$$

where

$$\begin{aligned}
&E\{b_i sgn(\sqrt{a_j}b_j + \sum_{\substack{k=1\\k\neq j}}^{K}\rho_{jk}\sqrt{a_k}b_k + n_j)\} \\
&= \frac{1}{2^K}(Pr\{n_j > -\sqrt{a_j}b_j - \rho_{ij}\sqrt{a_i} - \sum_{\substack{k=1\\k\neq j\neq i}}^{K}\rho_{jk}\sqrt{a_k}b_k)\}
\end{aligned}$$

$$+ \quad Pr\{n_j < -\sqrt{a_j}b_j + \rho_{ij}\sqrt{a_i} - \sum_{\substack{k=1 \\ k\neq j\neq i}}^{K} \rho_{jk}\sqrt{a_k}b_k)\}$$

$$- \quad Pr\{n_j > -\sqrt{a_j}b_j + \rho_{ij}\sqrt{a_i} - \sum_{\substack{k=1 \\ k\neq j\neq i}}^{K} \rho_{jk}\sqrt{a_k}b_k)\}$$

$$- \quad Pr\{n_j < -\sqrt{a_j}b_j - \rho_{ij}\sqrt{a_i} - \sum_{\substack{k=1 \\ k\neq j\neq i}}^{K} \rho_{jk}\sqrt{a_k}b_k)\})$$

$$= \quad \frac{1}{2^{K-}}(Q\left(\frac{-\sqrt{a_j}b_j - \rho_{ij}\sqrt{a_i} - \sum_{\substack{k=1 \\ k\neq j\neq i}}^{K} \rho_{jk}\sqrt{a_k}b_k)}{\sigma}\right)$$

$$- \quad Q\left(\frac{-\sqrt{a_j}b_j + \rho_{ij}\sqrt{a_i} - \sum_{\substack{k=1 \\ k\neq j\neq i}}^{K} \rho_{jk}\sqrt{a_k}b_k)}{\sigma}\right), \qquad \text{(B.6)}$$

and

$$E\{n_i sgn(\sqrt{a_j}b_j + \sum_{\substack{k=1 \\ k\neq j}}^{K} \rho_{jk}\sqrt{a_k}b_k + n_j)\}$$

$$= \quad E\{(\sqrt{1-\rho_{ij}}u + \rho_{ij}n_j)sgn(\sqrt{a_j}b_j + \sum_{\substack{k=1 \\ k\neq j}}^{K} \rho_{jk}\sqrt{a_k}b_k + n_j)\}$$

$$= \quad E\{\rho_{ij}n_j sgn(\sqrt{a_j}b_j + \sum_{\substack{k=1 \\ k\neq j}}^{K} \rho_{jk}\sqrt{a_k}b_k + n_j)\}$$

$$= \quad \frac{\rho_{ij}}{\sqrt{2\pi}\sigma}\left(\int_{-\sqrt{a_j}b_j - \sum_{\substack{k=1 \\ k\neq j}}^{K} \rho_{jk}\sqrt{a_k}b_k}^{\infty} n_j exp(-n_j^2/2\sigma^2)dn_j\right.$$

$$- \quad \left.\int_{-\infty}^{-\sqrt{a_j}b_j - \sum_{\substack{k=1 \\ k\neq j}}^{K} \rho_{jk}\sqrt{a_k}b_k} n_j exp(-n_j^2/2\sigma^2)dn_j\right)$$

$$= \quad \frac{2\rho_{ij}\sigma}{\sqrt{2\pi}} exp\left(\frac{(-\sqrt{a_j}b_j - \sum_{\substack{k=1 \\ k\neq j}}^{K} \rho_{jk}\sqrt{a_k}b_k)^2}{2\sigma^2}\right), \qquad \text{(B.7)}$$

where $E\{un_j\} = 0$.

$$E\{\hat{b}_2 sgn(y_2)\}$$

$$
\begin{aligned}
&= E\{sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 + n_2)sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}\hat{b}_1 + n_2)\} \\
&= \frac{1}{4}\sum_{b_1,b_2}(Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, \\
&\quad n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&+ Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, \\
&\quad n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&+ Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, \\
&\quad n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&+ Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, \\
&\quad n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&- Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, \\
&\quad n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&- Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, \\
&\quad n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&- Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, \\
&\quad n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&- Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, \\
&\quad n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\}, \qquad \text{(B.8)}
\end{aligned}
$$

where w_{12} can be either positive or negative depending on ρ_{12}. If $w_{12} > 0$, we get:

$$
\begin{aligned}
& E\{\hat{b}_2 sgn(y_2)\} \\
&= \frac{1}{4}\sum_{b_1,b_2}(Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&+ Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&+ Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&- \quad 0 \\
&- \quad Pr\{-\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12} < n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, \\
&\quad\quad n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&- \quad Pr\{-\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 < n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, \\
&\quad\quad n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&- \quad 0) \\
&= \quad \frac{1}{4}\sum_{b_1,b_2}(Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&+ \quad Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&+ \quad Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&+ \quad Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&- \quad Pr\{-\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12} < n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1, \\
&\quad\quad n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\} \\
&- \quad Pr\{-\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 < n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, \\
&\quad\quad n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2,\}). \qquad (B.9)
\end{aligned}
$$

Every term of the above equation involves a two-dimensional Gaussian integration.

$$
\begin{aligned}
E\{b_1 sgn(y_2)\} &= E\{b_1 sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}\hat{b}_1 + n_2)\} \\
&= \frac{1}{4}\sum_{b_1,b_2}(b_1 Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1} + w_{12}, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}\} \\
&+ b_1 Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1} - w_{12}, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}\} \\
&- b_1 Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1} + w_{12}, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}\} \\
&- b_1 Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1} - w_{12}, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}\} \qquad (B.10)
\end{aligned}
$$

and

$$\begin{aligned}
E\{n_1 sgn(y_2)\} &= E\{n_1 sgn(\sqrt{a_2}b_2 + \rho_{12}\sqrt{a_1}b_1 - w_{12}\hat{b}_1 + n_2)\} \\
&= E\{n_1 Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}\hat{b}_1\} \\
&- E\{n_1 Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}\hat{b}_1\} \\
&= E\{n_1 Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&+ E\{n_1 Pr\{n_2 > -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&- E\{n_1 Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 + w_{12}, n_1 > -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\} \\
&- E\{n_1 Pr\{n_2 < -\sqrt{a_2}b_2 - \rho_{12}\sqrt{a_1}b_1 - w_{12}, n_1 < -\sqrt{a_1}b_1 - \rho_{12}\sqrt{a_2}b_2\}.
\end{aligned} \tag{B.11}$$

B.3 Two-user Example — Decorrelating Algorithm

The sampled outputs of the bank of matched filters in the i-th bit interval can be expressed as:

$$x_1 = \sqrt{a_1}b_1 + \rho\sqrt{a_2}b_2 + n_1 \quad \text{(B.12)}$$

$$x_2 = \sqrt{a_2}b_2 + \rho\sqrt{a_1}b_1 + n_2. \quad \text{(B.13)}$$

The canceler's output is given by:

$$\begin{aligned} y_1 &= x_1 - w_{21}\hat{b}_2 \\ &= \sqrt{a_1}b_1 + \rho\sqrt{a_2}b_2 - w_{21}\hat{b}_2 + n_1 \\ y_2 &= \sqrt{a_2}b_2 + \rho\sqrt{a_1}b_1 - w_{12}\hat{b}_1 + n_2. \end{aligned}$$

Using the decorrelating criterion,

$$E\{y_1 sgn(y_2)\} = 0$$

and

$$E\{y_2 sgn(y_1)\} = 0.$$

We have two equations and two unknowns, we will be able to solve the equations:

$$\begin{aligned} E\{y_1 sgn(y_2)\} &= E\{(\sqrt{a_1}b_1 + \rho\sqrt{a_2}b_2 - w_{21}\hat{b}_2 + n_1)sgn(y_2)\} \\ &= \sqrt{a_1}E\{b_1 sgn(y_2)\} + \rho\sqrt{a_2}E\{b_2 sgn(y_2)\} \\ &- w_{21}E\{\hat{b}_2 sgn(y_2)\} + E\{n_1 sgn(y_2)\}, \end{aligned}$$

where $E\{b_1 sgn y_2\}$, $E\{n_1 sgn(y_2)\}$ and $E\{\hat{b}_2 sgn(y_2)\}$ have been computed in (B.6), (B.7), (B.8) respectively, and $E\{b_2 sgn(y_2)\}$ can be computed similarly as $E\{b_1 sgn(y_2)\}$.

APPENDIX C

COMPUTATIONS FOR CHAPTER 7

$$E\{\hat{b}_i\hat{b}_i\} = 1 \tag{C.1}$$

and

$$\begin{aligned}
E\{\hat{b}_i\hat{b}_j\} &= E\{sgn(\sqrt{a_i}b_i + \xi_i)sgn(\sqrt{a_j}b_j + \xi_j)\} \\
&= \frac{1}{4}\sum_{b_i,b_j}\{Pr\{\xi_i > -\sqrt{a_i}b_i, \xi_j > -\sqrt{a_j}b_j\} + Pr\{\xi_i < -\sqrt{a_i}b_i, \xi_j < -\sqrt{a_j}b_j\} \\
&- Pr\{\xi_i < -\sqrt{a_i}b_i, \xi_j > -\sqrt{a_j}b_j\} - Pr\{\xi_i > -\sqrt{a_i}b_i, \xi_j < -\sqrt{a_j}b_j\}\} \\
&= \frac{1}{4}(Pr\{\xi_i > \sqrt{a_i}, \xi_j > \sqrt{a_j}\} + Pr\{\xi_i < \sqrt{a_i}, \xi_j < \sqrt{a_j}\} \\
&- Pr\{\xi_i < \sqrt{a_i}, \xi_j > \sqrt{a_j}\} - Pr\{\xi_i > \sqrt{a_i}, \xi_j < \sqrt{a_j}\} \\
&+ Pr\{\xi_i > \sqrt{a_i}, \xi_j > -\sqrt{a_j}\} + Pr\{\xi_i < \sqrt{a_i}, \xi_j < -\sqrt{a_j}\} \\
&- Pr\{\xi_i < \sqrt{a_i}, \xi_j > -\sqrt{a_j}\} - Pr\{\xi_i > \sqrt{a_i}, \xi_j < -\sqrt{a_j}\} \\
&+ Pr\{\xi_i > -\sqrt{a_i}, \xi_j > \sqrt{a_j}\} + Pr\{\xi_i < -\sqrt{a_i}, \xi_j < \sqrt{a_j}\} \\
&- Pr\{\xi_i < -\sqrt{a_i}, \xi_j > \sqrt{a_j}\} - Pr\{\xi_i > -\sqrt{a_i}, \xi_j < \sqrt{a_j}\} \\
&+ Pr\{\xi_i > -\sqrt{a_i}, \xi_j > -\sqrt{a_j}\} + Pr\{\xi_i < -\sqrt{a_i}, \xi_j < -\sqrt{a_j}\} \\
&- Pr\{\xi_i < -\sqrt{a_i}, \xi_j > -\sqrt{a_j}\} - Pr\{\xi_i > -\sqrt{a_i}, \xi_j < -\sqrt{a_j}\}) \\
&= \frac{1}{2}(Pr\{\xi_i > \sqrt{a_i}, \xi_j > \sqrt{a_j}\} + Pr\{\xi_i < \sqrt{a_i}, \xi_j < \sqrt{a_j}\} \\
&- Pr\{\xi_i < \sqrt{a_i}, \xi_j > \sqrt{a_j}\} - Pr\{\xi_i > \sqrt{a_i}, \xi_j < \sqrt{a_j}\} \\
&+ Pr\{\xi_i > \sqrt{a_i}, \xi_j > -\sqrt{a_j}\} + Pr\{\xi_i < \sqrt{a_i}, \xi_j < -\sqrt{a_j}\} \\
&- Pr\{\xi_i < \sqrt{a_i}, \xi_j > -\sqrt{a_j}\} - Pr\{\xi_i > \sqrt{a_i}, \xi_j < -\sqrt{a_j}\}.
\end{aligned} \tag{C.2}$$

Every term of the above equation is a two-dimensional Gaussian integration.

$$\begin{aligned}
E\{\hat{b}_i b_i\} &= E\{sgn(\sqrt{a_i} b_i + \xi_i) b_i\} \\
&= \frac{1}{2}(Pr\{\xi_i > -\sqrt{a_i}\} + Pr\{\xi_i < \sqrt{a_i}\} \\
&- Pr\{\xi_i > \sqrt{a_i}\} - Pr\{\xi_i < -\sqrt{a_i}\}) \\
&= (Pr\{\xi_i > -\sqrt{a_i}\} - Pr\{\xi_i > \sqrt{a_i}\} \\
&= 1 - 2Q\left(\frac{\sqrt{a_i}}{\sigma_{\xi_i}}\right). \qquad \text{(C.3)}
\end{aligned}$$

$$E\{\hat{b}_i b_j\} = 0. \qquad \text{(C.4)}$$

APPENDIX D

TWO-USER EXAMPLES FOR CHAPTER 8

The error probability of user one, Pe_1, for $K = 2$ is given by:

$$
\begin{aligned}
P_{e_1} &= Pr\{\hat{\hat{b}}_1 \neq b_1\} = Pr\{\hat{\hat{b}}_1 \neq b_1, |z_2| < T_{21}\} + Pr\{\hat{\hat{b}}_1 \neq b_1, |z_2| \geq T_{21}\} \\
&= \frac{1}{4}\sum_{b_2}\{Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}z_2/T_{21} + n_1 < 0, |z_2| < T_{21}\} \\
&+ Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}sgn(z_2) + n_1 < 0, |z_2| \geq T_{21}\} \\
&+ \{Pr\{-\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}z_2/T_{21} + n_1 > 0, |z_2| < T_{21}\} \\
&+ Pr\{-\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}sgn(z_2) + n_1 > 0, |z_2| \geq T_{21}\}\} \\
&= \frac{1}{2}\sum_{b_2}\{Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}z_2/T_{21} + n_1 < 0, |z_2| < T_{21}\} \\
&+ Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}sgn(z_2) + n_1 < 0, |z_2| \geq T_{21}\}\} \\
&= \frac{1}{2}\sum_{b_2}\{Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2} - w_{21}(\sqrt{a_2}b_2 + \xi_2)/T_{21} + n_1 < 0, |\sqrt{a_2}b_2 + \xi_2| < T_{21}\} \\
&+ Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}sgn(z_2) + n_1 < 0, z_2 \geq T_{21}\} \\
&+ Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2}b_2 - w_{21}sgn(z_2) + n_1 < 0, z_2 < -T_{21}\}\} \\
&= \frac{1}{2}\{Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2} - w_{21}(\sqrt{a_2} + \xi_2)/T_{21} + n_1 < 0, -T_{21} < \sqrt{a_2} + \xi_2 < T_{21}\} \\
&+ Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2} - w_{21} + n_1 < 0, \sqrt{a_2} + \xi_2 \geq T_{21}\} \\
&+ Pr\{\sqrt{a_1} + \rho_{12}\sqrt{a_2} + w_{21} + n_1 < 0, \sqrt{a_2} + \xi_2 < -T_{21}\} \\
&+ Pr\{\sqrt{a_1} - \rho_{12}\sqrt{a_2} - w_{21}(-\sqrt{a_2} + \xi_2)/T_{21} + n_1 < 0, -T_{21} < -\sqrt{a_2} + \xi_2 < T_{21}\} \\
&+ Pr\{\sqrt{a_1} - \rho_{12}\sqrt{a_2} - w_{21} + n_1 < 0, -\sqrt{a_2} + \xi_2 \geq T_{21}\} \\
&+ Pr\{\sqrt{a_1} - \rho_{12}\sqrt{a_2} + w_{21} + n_1 < 0, -\sqrt{a_2} + \xi_2 < -T_{21}\}\}
\end{aligned}
$$

Denote:

$$\zeta_1 = n_1 - w_{21}\xi_2/T_{21}$$

above equations become:

$$
\begin{aligned}
P_{c_1} &= \frac{1}{2}\{Pr\{\sqrt{a_1}+\rho_{12}\sqrt{a_2}-w_{21}\sqrt{a_2}/T_{21}+\zeta_1<0,-\sqrt{a_2}-T_{21}<\xi_2<-\sqrt{a_2}+T_{21}\} \\
&+ Pr\{\sqrt{a_1}+\rho_{12}\sqrt{a_2}-w_{21}+n_1<0\}Pr\{\xi_2\geq T_{21}-\sqrt{a_2}\} \\
&+ Pr\{\sqrt{a_1}+\rho_{12}\sqrt{a_2}+w_{21}+n_1<0\}Pr\{\xi_2<-\sqrt{a_2}-T_{21}\} \\
&+ Pr\{\sqrt{a_1}-\rho_{12}\sqrt{a_2}+w_{21}\sqrt{a_2}/T_{21}+\zeta_1<0,\sqrt{a_2}-T_{21}<\xi_2<\sqrt{a_2}+T_{21}\} \\
&+ Pr\{\sqrt{a_1}-\rho_{12}\sqrt{a_2}-w_{21}+n_1<0\}Pr\{\xi_2\geq\sqrt{a_2}+T_{21}\} \\
&+ Pr\{\sqrt{a_1}-\rho_{12}\sqrt{a_2}+w_{21}+n_1<0\}Pr\{\xi_2<\sqrt{a_2}-T_{21}\}\}
\end{aligned}
$$

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