Fall 10-31-1995

Intermodal commuter network planning

Maria P. Boile
New Jersey Institute of Technology

ABSTRACT

INTERMODAL COMMUTER NETWORK PLANNING

by
Maria P. Boile

An intermodal commuter network is an integration of passenger transportation systems, or modes, to a single comprehensive system that provides connections among the various modes, and improved travel choices to users. In the system examined in this dissertation, commuters access their final destination via auto, rail, and intermodal auto-to-rail modes. There are numerous highway paths by which a commuter can reach the final destination. Once on the highway, the commuter can switch to rail at stations along the rail route. The commuter may also choose to walk to the rail station closest to the trip's origin.

The main focus of this dissertation is the development of models that can estimate traffic volumes and travel costs on intermodal networks. The particular approach used in the models is demand and supply equilibrium where transportation flows are impacted by the performance of the transportation facilities. Several optimization models are formulated based on sound mathematical and economic principles, and their equilibrium conditions are derived and stated clearly. A rigorous analysis of the mathematical properties of the models proves that these conditions are satisfied from the model solutions. The objective of these models is to alleviate some of the deficiencies encountered in the urban transportation planning process.

A methodological framework is proposed which utilizes the models to analyze and evaluate operating and pricing policies in intermodal networks. The framework is

designed to answer questions of interest to transportation planners, and to investigate the trade-offs between reduction in travel time and the increased cost of capacity improvements.

To link theory and practice, the models are applied, within the proposed framework, to the analysis of a real-world intermodal commuter network. Policies aimed at improving the service quality of the intermodal network are evaluated based on their benefits compared to existing conditions. The models are also used to design an optimal rail transit service by computing rail fares and headways to meet future demands.

The results of the analysis can be used by transportation planners, decision makers, transit operators, and transportation system managers to find effective ways to alleviate congestion on transportation systems. To this end, this dissertation points to areas of future research to further improve the proposed models.

INTERMODAL COMMUTER NETWORK PLANNING

by
Maria P. Boile

A Dissertation
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy

Committee for the Interdisciplinary Program in Transportation

October 1995

APPROVAL PAGE

INTERMODAL COMMUTER NETWORK PLANNING

Maria P. Boile

Dr. Lazar N. Spasovic, Dissertation Advisor — Date
Associate Professor of Industrial Management, NJIT

Dr. Athanassios K. Bladikas, Committee Member — Date
Associate Professor of Industrial and Manufacturing Engineering, NJIT

Dr. Kyriacos C. Mouskos, Committee Member — Date
Assistant Professor of Civil and Environmental Engineering, NJIT

Dr. Louis J. Pignataro, Committee Member — Date
Distinguished Professor and Executive Director, Institute for Transportation, NJIT

Dr. Athanasios K. Ziliaskopoulos, Committee Member — Date
Assistant Professor of Civil, Environmental,
and Engineering Graphics, The Ohio State University

BIOGRAPHICAL SKETCH

Author: Maria P. Boile

Degree: Doctor of Philosophy in Transportation

Date: October 1995

Undergraduate and Graduate Education:

- Doctor of Philosophy in Transportation,
  New Jersey Institute of Technology, Newark, NJ, 1995

- Master of Science in Civil and Environmental Engineering,
  Rutgers University, New Brunswick, NJ, 1992

- Diploma in Civil Engineering,
  National Technical University of Athens, Athens, Greece, 1990

Major: Transportation Engineering

Publications:

Boile M. P., Spasovic L. N., and Bladikas A. K (1994) Modeling Intermodal Auto-Rail Commuter Networks. *Accepted for publication in the Transportation Research Record.*

Spasovic L. N., Boile M. P., and Bladikas A. K. (1994) A Framework for Assessing the Benefits of Highway Traveler Information Services. *Accepted for publication in the Transportation Research Record.*

Spasovic L. N., Boile M. P., and Bladikas A. K. (1993) Bus Transit Service Coverage for Maximum Profit and Social Welfare. *Transportation Research Record,* No. 1451, pp. 12-22

BIOGRAPHICAL SKETCH
(Continued)

Spasovic L. N., and Boile M. P. (1995) A Framework for Modeling Traffic Assignment on Intermodal Auto-Rail Commuter Networks. *Applications of Management Science: Engineering Studies in Manufacturing Engineering, Transportation, and Water Engineering*, Vol. 9.

Presentations:

Boile M. P., Spasovic L. N., and Bladikas A. K. (1995) Modeling Intermodal Auto-Rail Commuter Networks. Presented at the 74th Annual Meeting of the Transportation Research Board, Washington, DC, January 22-28.

Spasovic L. N., Boile M. P., and Bladikas A. K. (1995) A Framework for Assessing the Benefits of Highway Traveler Information Services. Presented at the 74th Annual Meeting of the Transportation Research Board, Washington, DC, January 22-28.

Spasovic L. N., Boile M. P., and Bladikas A. K. (1994) A Methodological Framework for Optimizing Bus Transit Service Coverage. Presented at the 73rd Annual Meeting of the Transportation Research Board, Washington, DC, January 9-13.

Boile M. P., Spasovic L. N., and Mouskos K. C. (1993) Trip Assignment Over an Intermodal Network with Decreasing Rail Supply Function. ORSA/TIMS Joint National Meeting, Phoenix, October 31-November 3.

Spasovic L. N., Boile M. P., and Bladikas A. K. (1993) A Methodological Framework for Optimizing Bus Transit Service Coverage. Presented at the meeting of the Metropolitan Section--New York and New Jersey of the Institute of Transportation Engineers (ITE), October 1993.

This dissertation is dedicated to
my family

ACKNOWLEDGMENT

I would like to express my gratitude to my advisor, Dr. Lazar N. Spasovic, for his guidance throughout the course of the dissertation research. I greatly enjoyed our discussions which I perceive to have been a learning experience. I am indebted to him for his friendship, help, and moral support.

Special thanks to Dr. Athanassios K. Bladikas, Dr. Kyriacos C. Mouskos, Dr. Louis J. Pignataro, and Dr. Athanasios K. Ziliaskopoulos for serving as members of the dissertation committee.

I am grateful to Dr. Athanassios K. Bladikas for editing several versions of this dissertation, and to my colleague, Mr. Kenneth J. Hausman, for his help in editing the final manuscript, although I remain responsible for transgressions of style and substance.

This research was supported by a grant from the U.S. Department of Transportation (US DOT) to the National Center for Transportation and Industrial Productivity (NCTIP), a University Transportation Program Center. This support is gratefully acknowledged but implies no endorsement of the conclusion by the US DOT.

Special thanks to my friends and relatives on both sides of the Atlantic for their love and enduring support.

I am grateful to my late uncle Emmanuel Kouroupas, who I miss a lot, and to my aunt Helen Kouroupas for their support throughout my graduate studies.

Finally, I would like to thank my mother Maro, my father Yiannis, and my sister Natassa, for their spiritual and financial support throughout my academic endeavors. I can hardly find words to express my gratitude for their love and encouragement through the years.

TABLE OF CONTENTS

Chapter **Page**

Chapter **Page**

Chapter	Page

Chapter	Page

Chapter **Page**

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure	Page

CHAPTER 1

INTRODUCTION

1.1 Objectives

The primary objective of this dissertation is to develop models of demand and supply equilibrium over intermodal commuter networks, and to use these models to analyze and evaluate various policies for improving the efficiency and service quality of these networks. An intermodal commuter network is a transportation system served by several modes of travel which allow for transfers among them. The specific system examined here is a network served by multiple highways and a commuter rail line. Travelers, departing from their homes, access their final destination, a Central Business District (CBD), via auto, rail, and intermodal auto-to-rail or park-and-ride modes. If a commuter chooses to begin the trip by auto, then there are numerous paths by which s/he can reach the final destination. Once on the highway, the commuter can switch to rail at stations along the rail route. The commuter may also choose to walk to the rail station closest to the trips' origin.

The marked benefit of this approach is that the intermodal network is considered as an integration of passenger transportation systems (modes) to a single comprehensive system that provides connections among various modes and improved choices to the travelers using these modes. Traffic flows and travel costs are modeled and optimized for the entire system, not for individual modal networks. The models have several other properties as well. First, they consider travelers' preferences in choosing modes and routes for their trips on the intermodal network. They account for travelers' response to travel cost and volume variations on these modes and routes. The models formulate these preferences and behavior in well-defined demand functions. The models also recognize the distinctive nature of each transportation system in terms of performance changes with respect to volume variations. In addition, they consider the behavior of the suppliers and managers of transportation facilities and services, and they formulate their response to expected traffic volumes in well-defined supply functions. The demand and supply functions are combined in quantitative models which are shown to yield well-defined equilibrium solutions. These models consider capacity limitations on the intermodal network, thus they do not overestimate the ability of the systems and facilities of the network to provide mobility.

This dissertation focuses on the development of modeling approaches that can estimate the volume of travel for each mode and route of an intermodal network and the travel costs and operating characteristics (headway and fare) that result from these volumes. These estimates must satisfy certain conditions that are clearly stated for each proposed model. In addition to developing models with sound mathematical and economic properties, the objectives of this dissertation include the rigorous analysis of these models to prove that their solutions satisfy the stated conditions.

Another objective of the dissertation is the development of a methodological framework for evaluating operating and pricing policies in an intermodal network. The purpose of the framework is to analyze the effects of various strategies on both traffic flow patterns and travel costs. The framework is designed to answer questions of interest to transportation planners and to investigate the trade-offs between reductions in travel time and increased costs of capacity improvements.

Finally, the models are used to solve a real-world problem: analyze an existing intermodal commuter network. The models are used to predict traffic flow patterns and travel costs, and to evaluate various operating and pricing policies. The models are also used to predict future traffic volumes and transit managerial responses to these volumes.

1.2 Background

For most of this century, the primary concern in the transportation profession was to provide mobility. The construction of the System of Interstate and Defense Highways, started in the middle of the century, provided travelers with more than forty-two thousand miles of highways on which someone can travel for thousands of miles without encountering a single traffic light (Larson 1993). The interstate highway system was primarily an attempt to provide mobility. Today, highway transportation is plagued with increasing congestion, especially in urban areas, primarily due to the substantial increase in the amount of travel.

This increase is well-reflected in the following trends. In the past decade, according to the Nationwide Personal Transportation Survey (1994), there was a 35 percent increase in persons driving alone to work: from 62 million in 1983 to 84 million in 1990. This increase resulted in an average vehicle occupancy for privately operated vehicles (POV) for work trips of only 1.1. The vehicle miles traveled (VMT) increased 37 percent between 1983 and 1990 with commutes of 8 miles or more accounting for more than 80 percent of commuting VMT. The average work trip by all modes increased from 8.28 miles to 10.14 miles, or 22.4 percent, as people continued to locate their homes farther from their job sites. With more vehicles on the road, and many traveling longer distances, it is easy to see how congestion increased significantly.

Building more highways is not a solution to the increasing congestion problem since it is no longer either efficient nor environmentally and politically acceptable. Transportation professionals need to find alternative solutions, some of which will be discussed in this section.

The use of intelligent transportation systems (ITS) is a promising approach. ITS encompass advanced surveillance, communication, control, and computing systems and engineering management methods, and are envisioned, as described in the Strategic

Plan of Intelligent Vehicle-Highway Systems in the United States (1992), to be able to increase safety, reduce congestion, and improve the productivity of a transportation system. The improvement of transportation systems may be significant in the short-term but is likely to induce more travel. Thus, consideration must be given to demand management as well. This goal can be achieved by comprehensive transportation planning: to integrate all transportation systems into a single system, an intermodal transportation network. The analysis of such a system should be capable of predicting the impacts of changes to one mode on the performance of others, and suggesting fiscally responsible strategies for improving network-wide performance. The purpose of analyzing intermodal networks is to find ways to increase the attractiveness of public transit modes, improve transit operations, and provide commutes between origins and destinations via several modes of transport in a synchronized, seamless way.

The increasing role of public transit in the transportation networks and the integration of the transportation systems are also addressed in the latest legislation. The necessity of inclusion of all forms of transportation in a unified interconnected manner, such as to form a National Intermodal System, is addressed in the Intermodal Surface Transportation Efficiency Act of 1991 (ISTEA). According to the Clean Air Act Amendments of 1990 (CAAA), employers located in non-attainment areas and having more than 100 employees are required to increase the average vehicle occupancy of vehicles arriving at the job site during the morning peak period by 25%. It is suggested that incentives, as well as restrictions, should be imposed to induce commuters to use public transit, and thus decrease air pollution.

To better analyze and improve the operation of an intermodal transportation system, sophisticated planning models are needed. These models should have the properties described in this section, if they are to be useful planning tools and aid the managerial decision-making process.

1.3 Research Significance

The research presented in this dissertation is very significant, both for its theoretical and practical contributions.

From a theoretical viewpoint the research is important for several reasons. It deals with intermodal commuter network planning, integrates various modes of transport in a single comprehensive system, addresses the interdependency among modes serving the same area, and enables the analysis of interactions among modes. It integrates comprehensive demand and supply functions in a network equilibrium context, formulates quantitative models which incorporate the decision process of all entities: user, supplier, and operator. To the best of the author's knowledge, this research is the first attempt to use representative supply functions (highway response to congestion, managerial responses to traffic volumes, operational adjustments in levels of service and prices) in a network equilibrium model.

The models presented in this dissertation can be used in the managerial, planning, and policy decision making processes. To this end, the models make several practical contributions in the areas of transportation systems planning, and transit management and operations. Governmental and regional planning organizations can use these models to analyze and evaluate policies regarding operating and pricing schemes. Planning agencies can predict traffic volumes and travel costs, and analyze price and service quality levels for a transportation system. Transit operators can analyze policies to improve transit service and increase ridership by attracting more highway users to transit. Finally, the users of the transportation system will benefit since the objective of the models is to optimize user travel costs and travel patterns on intermodal networks, and to improve the service provided by the transportation system.

1.4 Plan of the Dissertation

Chapter 2 presents some fundamental issues in the demand and supply intermodal network equilibrium modeling. Chapter 3 reviews the relevant transportation literature. Chapter 4 presents three approaches for modeling intermodal decisions, their similarities and their differences. Chapter 5 contains the mathematical formulation of three models proposed in this dissertation; the models combine comprehensive demand and supply functions in a network equilibrium context. Chapter 6 presents the case study. The assumptions and input data used in the analysis of an intermodal network are presented, and are followed by the development of a methodological framework, designed to analyze various operating and pricing policies on an intermodal network. Chapter 7 gives an application example of the three models within the methodological framework. The analysis is two-fold: the results of the network analysis are used to verify the equilibrium conditions of the models; and the results from various policy analyses are used to suggest directions of improvement on the intermodal network. Chapter 8 presents the formulation of a commuter rail service design model within an intermodal network equilibrium context. The model is used to analyze the case study network presented in Chapter 6. The equilibrium condition of the model is verified from the numerical results of the model, and the model is used to analyze various scenarios representing future traffic conditions. The dissertation concludes with Chapter 9, which contains a summary, conclusions, and directions of future research.

CHAPTER 2

FUNDAMENTALS OF DEMAND AND SUPPLY NETWORK EQUILIBRIUM

2.1 Introduction

The purpose of this chapter is to present and discuss some urban transportation planning fundamentals, specifically the modeling of intermodal transportation networks. The models are used to forecast traffic flows on network links, and compute travel times, costs, and level of service characteristics. The particular approach to be discussed is the demand and supply equilibration over intermodal networks. The basic concept of network equilibrium modeling and its relation to the classical economic equilibrium paradigm is presented first. The problem to be solved using this approach is then stated, and the rationale for choosing such an approach is discussed.

2.2 Demand and Supply Equilibrium

Urban transportation planning consists of estimating the demand for travel between origins and destinations, and the usage of travel modes and routes in the transportation network, given the socioeconomic characteristics of an urban area and the existing or proposed transportation systems and services.

The major problem to be solved in the area of urban transportation planning is estimating traffic flows and the resulting levels of service which will occur on a transportation network. The demand for transportation (V) is derived, since it is the result of an underlying economic or social activity (E). In passenger transportation, people do not travel for the sole purpose of traveling, but to earn a living, visit friends, etc. The demand for transportation on each facility of the network depends on the prevailing conditions on the facility (S), and is basically a reflection of the requirements for transport by users of the system. Conceptually, the demand as a function of the socioeconomic activity and the service provided is written as $V = f[E,S]$.

The supply of transportation (S) represents the characteristics of a facility (T) and is closely related to traffic volumes (V). The supply, as a function of the characteristics of the facility and the traffic volumes, is written as $S = f[T,V]$. Usually, it is given as a price or cost of travel, and is an increasing function of traffic volume. The increasing slope of the function is explained by the fact that at higher traffic volumes there is higher interaction between users, which results in increasing travel times. Travel time usually increases at a faster rate than traffic volume.

Both demand and supply of transport must be described in a similar manner and used jointly in an approach termed demand and supply equilibrium. This approach determines the total amount of travel which will occur under specified conditions, and the associated travel costs and levels of service. The demand and supply equilibrium over a transportation network specifies the volumes (V^*) and the resulting levels of service(S^*) that will actually occur, as:

$$\left.\begin{aligned} V &= f[E,S] \\ S &= f[T,V] \end{aligned}\right\} \rightarrow (V^*, S^*)$$

A graphical representation of the equilibrium between demand and supply is shown in Figure 2.1, which shows demand as a decreasing function with respect to travel cost, and supply as an increasing function with respect to traffic volume. The equilibrium is given as the point of intersection of the two functions. The following section is an example of demand and supply equilibrium on a transportation facility.

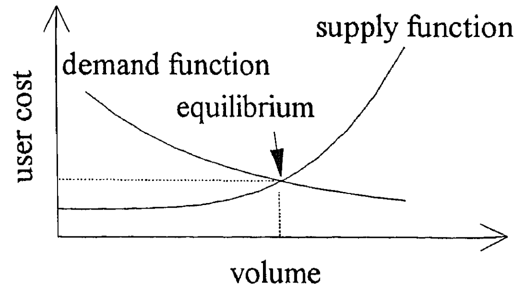


Figure 2.1 Demand and Supply Equilibrium

2.2.1 Example

Figure 2.2 shows a representation of a transportation facility, a one way street, connecting two points termed origin (O) and destination (D).

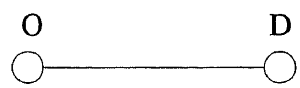


Figure 2.2 Highway Facility

The performance of the highway facility is measured in terms of travel time, and, for the purpose of simplification, is given by the following linear expression:

$$t = a+b*V \tag{2.1}$$

where: t - travel time in minutes,

V - traffic volume in users per hour,

a - free-flow travel time, the time required to traverse a facility under zero volume traffic conditions, and

b - increase in travel time for an additional traveler.

The traffic volume, a function of the travel time, is given by:

$$V = c\text{-}d*t \tag{2.2}$$

where: c - potential number of travelers, and

d - decrease in number of travelers as a result of increase in travel time.

For this example the values of a and b are set at 5 minutes and 0.1 minute per user per hour, respectively, and the values of c and d are set at 100 users per hour and 0.2 users per hour per minute, respectively.

The equilibrium traffic volumes (V^*) and travel times (t^*) that satisfy both equations (2.1) and (2.2) are $V^* = 97$ users per hour and $t^* = 14.7$ minutes. They are shown in Figure 2.3, which contains the demand and supply functions, and identifies the equilibrium point at the intersection of the two functions.

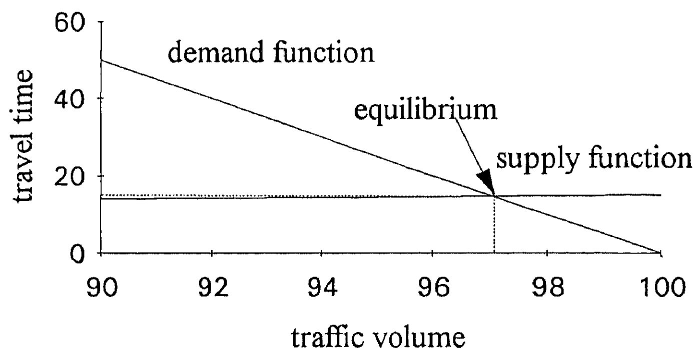


Figure 2.3 Demand and Supply Equilibrium for a Transportation Facility

2.3 Economic Market Equilibrium

The transportation demand and supply equilibrium is very similar to the concept of equilibrium in the analysis of economic markets. This similarity was first observed by Beckmann et al. (1956). In economic theory, equilibrium between demand and supply for a homogeneous commodity occurs at a price such that the total quantity produced equals the total quantity purchased. A graphical representation of economic market equilibrium is shown in Figure 2.4 as the point of intersection of the demand and supply functions.

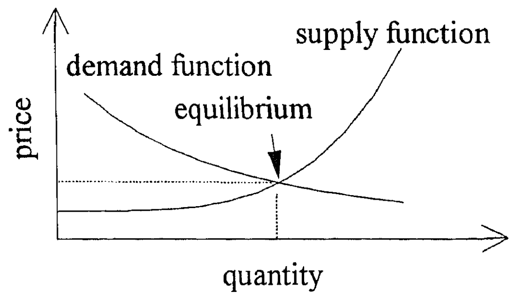


Figure 2.4 Economic Market Demand and Supply Equilibrium

The supply function of the economic market equilibrium specifies the relationship between the price for a commodity and the amount of the commodity that producers are willing to produce and sell. In transportation, the supply function is somewhat different in that it represents the characteristics of a transportation facility.

The demand function specifies the relationship between the price of the commodity and the amount of the commodity that consumers are willing to consume or purchase. In economics, the demand function is a direct function of price, while in transportation the quantity of flow is a function of all costs, such as travel time and out-of-pocket costs, perceived by transportation users.

2.4 Network Equilibrium

The previous sections described the demand and supply equilibrium on a transportation facility and its similarity to economic market equilibrium. In this section, the concept of equilibrium will be extended to apply to transportation networks. Transportation networks consist of links, such as highways, and nodes, such as intersections, each described by its own supply function. The equilibrium problem thus becomes more complicated. Travel demand is given between an origin and destination, while the supply function is composed of the supply functions of all links that are in a path connecting that origin and destination. Travelers are choosing their routes based on the quality of service they encounter on all facilities that are part of the selected route.

Three simple examples are presented in turn to illustrate equilibria on a transportation network, and to identify the difficulties that arise in this process. The first example is a highway network served by two highway links connected to form a series of links or a path. The second example is a highway network served by two parallel highways. The third example is a multi- (bi-) modal network.

2.4.1 Highway Network Equilibrium with Highways in Series

The simple network shown in Figure 2.5 consists of one origin-destination pair connected by two one-way highway links, link 1 and link 2, representing a directed path, path 12, between the origin and destination.

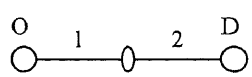


Figure 2.5 Experimental Highway Network with Links in Series

The supply function between origin and destination is composed of the supply functions of the two links. The supply function of each link and the resulting supply function for the path between origin and destination are shown in Figure 2.6. The average cost on path 12 is derived as the sum of the average costs on links 1 and 2. For zero volume, for example, the average cost on path 12 is the sum of OA and OB, with OA and OB being the zero volume costs on link 1 and link 2, respectively.

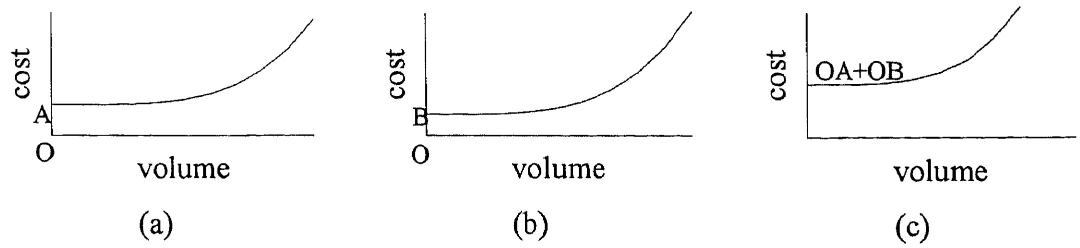


Figure 2.6 Average Cost Functions. (a) Link 1. (b) Link 2. (c) Path 12

The total demand for travel between origin and destination is assumed to be fixed and equal to both the traffic volume on link 1 and the traffic volume on link 2. The equilibrium between the path 12 supply function and the demand function is given as the point of intersection of the two functions. This equilibrium is shown in Figure 2.7.

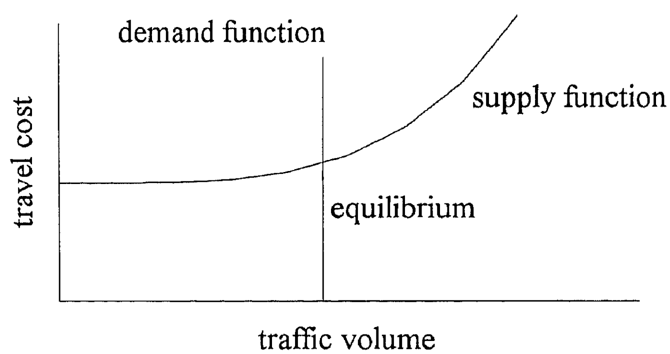


Figure 2.7 Demand and Supply Equilibrium on a Two-Link Sequence Network

2.4.2 Highway Network Equilibrium with Parallel Highways

The simple highway network shown in Figure 2.8 consists of one origin-destination pair connected by two one-way highway links, link 1 and link 2, each one representing a directed highway path. Highway path 1 consists of highway link 1, and highway path 2 consists of highway link 2.

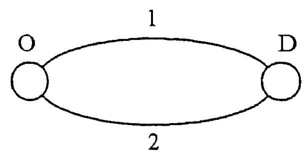


Figure 2.8 Experimental Highway Network with Parallel Links

The performance functions for highways are shown in Figure 2.9. Part (a) of the figure shows the average user cost on link 1, and part (b) shows the average user cost on link 2. The behavioral principle by which travelers select their routes between origin and destination is a generalization of the well known user equilibrium or Wardrops' first principle (Wardrop 1952). According to Wardrops' first principle, each traveler, for a trip between an origin and a destination, selects the route that minimizes his/her travel time. At a certain point (user equilibrium) no traveler is able to further improve his/her travel time by unilaterally changing routes (Sheffi 1985). This principle can be generalized to include, in addition to the travel time cost, other costs (parking fees, transit fares, tolls, vehicle operating costs) as a travel impedance.

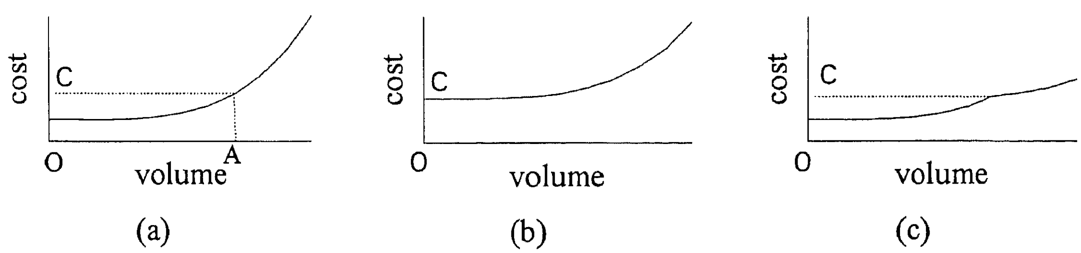


Figure 2.9 Average Cost Functions. (a) Path 1. (b) Path 2. (c) Best Path

Based on the generalized user equilibrium behavioral principle, travelers choose their paths such as to minimize their generalized cost of traveling. Up to volume OA the least cost path is link 1. The cost for volume OA on link 1 is OC, which is equal to the zero volume cost on link 2. As the traffic volume increases, some travelers will continue using link 1 while some will travel on link 2. The resulting user cost-volume relationship for the network is shown in part (c) of Figure 2.9. The network equilibrium assignment when the total demand for travel is fixed is represented graphically in Figure 2.10.

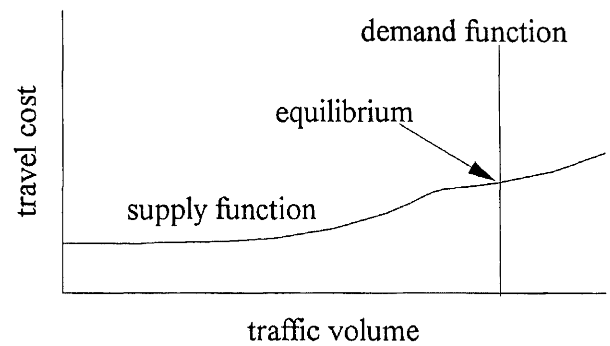


Figure 2.10 Demand and Supply Equilibrium on a Two Parallel Link Highway Network

2.4.2.1 Alternative Method. An alternative method of graphically representing the demand and supply equilibrium for the transportation network of Figure 2.8 is shown in Figure 2.11 where the fixed demand to be accommodated on the two links is indicated by the length of the horizontal axis.

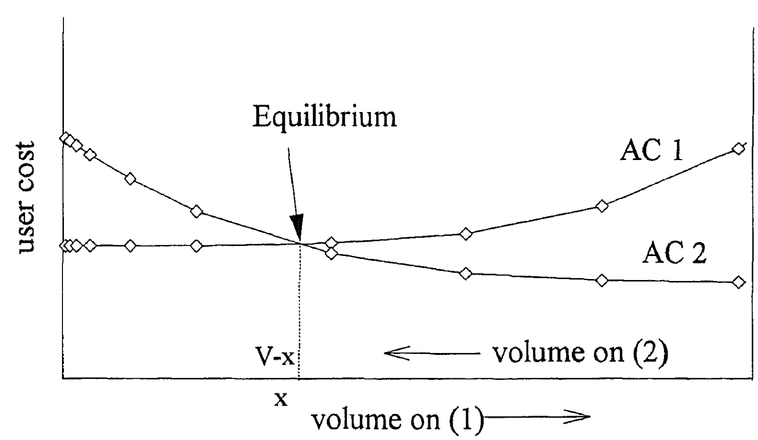


Figure 2.11 Alternative Graphical Representation of Equilibrium on a Two Parallel Link Highway Network

The average cost (or performance) functions for the two links are shown, each beginning at the extreme ends of the horizontal axis; the performance function for link 1 begins on the left end of the axis while the one for link 2 begins on the right end. In this example the equilibrium solution is given at the intersection of the performance functions, splitting the total demand (V) among the two highways. The equilibrium volume on highway 1 is x and on highway 2, V-x.

2.4.3 Multimodal Network Equilibrium

The two-mode network developed for the purpose of this example is shown in Figure 2.12. The network consists of a one-way highway link (link 1), and a one-way rail link (link 2). The first link represents a directed highway path (path 1) and the second link represents a directed rail path (path 2) connecting an origin with a destination.

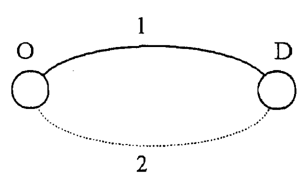
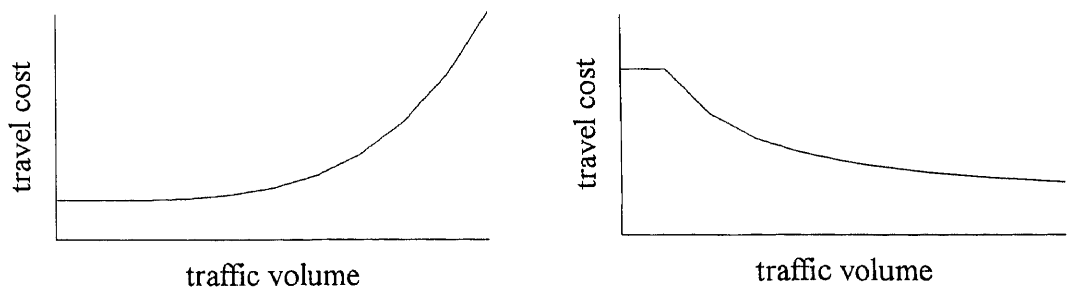


Figure 2.12 Experimental Bimodal Network

The highway performance function is assumed to be an increasing function of traffic volume, while the rail performance function is assumed to be a decreasing function of traffic volume. These functions are shown in part (a) and part (b) of Figure 2.13, respectively.

Travelers select a mode of travel and an actual route between their origin and destination in a way that minimizes their generalized cost of traveling. Assuming a fixed total demand for travel between origin and destination, the problem becomes to assign the total demand among the modes and paths of the network to minimize the individual travelers' cost.



(a) (b)

Figure 2.13 Average Cost Functions. (a) Highway Path (b) Rail Path

The equilibrium assignment for the bi-modal network for fixed demand is represented graphically in Figure 2.14.

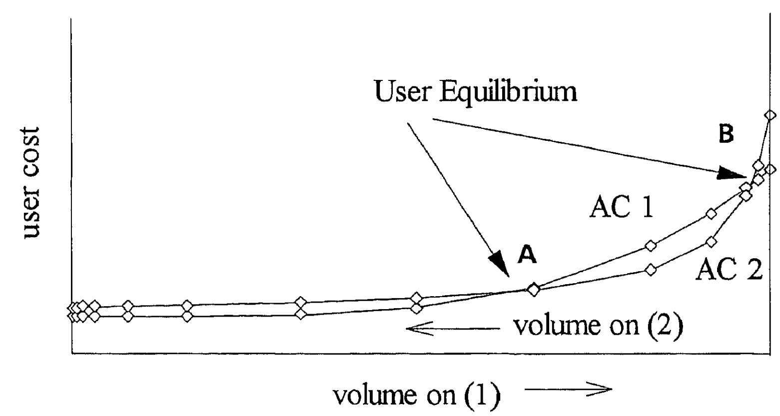


Figure 2.14 Graphical Representation of Equilibrium on a Bi-modal Network

In Figure 2.14 the total demand to be accommodated on the two paths is indicated by the length of the horizontal axis. The average cost functions for the two paths are shown, each beginning at the extreme ends of the horizontal axis; the highway performance function begins on the left end of the axis, while the one for rail begins on the right end. This graphical representation of equilibrium illustrates a problem that may arise in modeling multimodal (and, as an extension, intermodal) networks; i.e., the existence of more than one equilibrium solution. For the two-path, two-mode network shown in Figure 2.14 there are two equilibrium points indicated as A and B. At equilibrium point A, the rail share of the total demand is higher, and the resulting average cost is lower, compared to equilibrium point B. The preference of travelers towards private auto results in network equilibrium according to point B. Obviously, the movement of equilibrium from point B to point A (by either influencing demand or supply) is advantageous. Both highway and rail users will encounter lower travel cost; highway users due to decreased congestion, and rail users due to decreased headways, and thus waiting time. In addition, the increased rail ridership will contribute to increases in fare-box revenue.

2.5 Supply Functions

Three types of models of supply-side characteristics of transport systems have been identified by Morlok (1980). Two types are characterized by explicit user cost-volume relationships; the user cost is a direct function of volume. Type I considers all characteristics of a transport facility, or service, under the control of management to be fixed and only the volume of traffic to vary. Type II includes managerial responses to volume variations. Type III represents managerial behavior that is based on other considerations in addition to traffic volume.

2.5.1 Type I Relationships

A characteristic Type I user cost-volume relationship is shown in Figure 2.15. The user cost is a strictly increasing function of traffic volume.

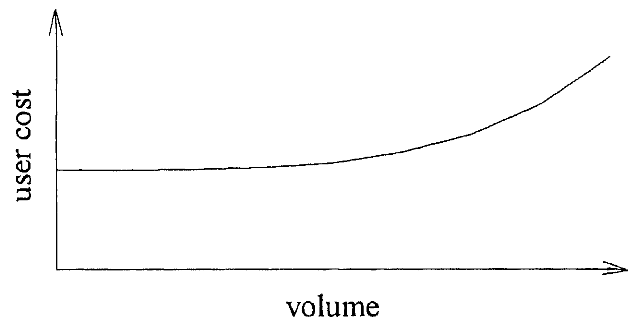


Figure 2.15 Type I Supply Function (Increasing)

This type of function is commonly used to represent the effects of congestion on transportation facilities. User cost-volume relationships of this type have been developed for various kinds of highway facilities by the U.S. Bureau of Public Roads (BPR) (1964). Although the travel time in these relationships is an increasing function of traffic volume, the out-of-pocket costs (highway tolls, parking fees) are assumed to remain constant.

Type I relationships can be used in transit services, such as bus transit, where the travel time is expected to increase with respect to traffic volume, under the assumption of fixed fares and operating characteristics. Transit services, however, especially exclusive right-of-way (commuter rail, light rail, or bus operating on exclusive bus lanes) have travel costs that can be assumed constant over a range of volume from zero to system capacity. A graphical representation of such a function is shown in Figure 2.16. An example of this type of relationship as a supply function for commuter rail in a demand and supply equilibrium context is given in Manheim (1979).

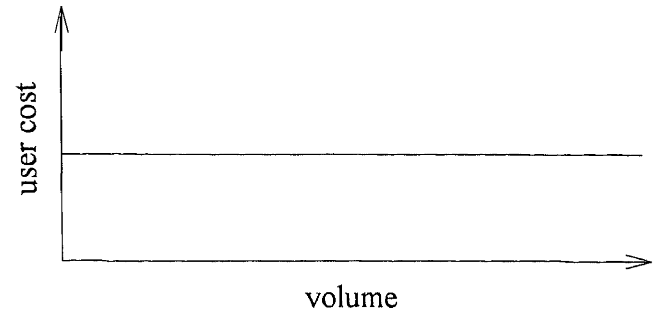


Figure 2.16 Type I Supply Function (Constant)

2.5.2 Type II Relationships

Type II relationships, in addition to capturing the effects of congestion on network performance, consider changes in fees and operating characteristics in response to travel demand. Type II relationships are characteristic of transit systems that operate on exclusive right-of-way and are thus able to adjust operating characteristics to traffic volumes. A graphical representation of a Type II relationship is shown in Figure 2.17.

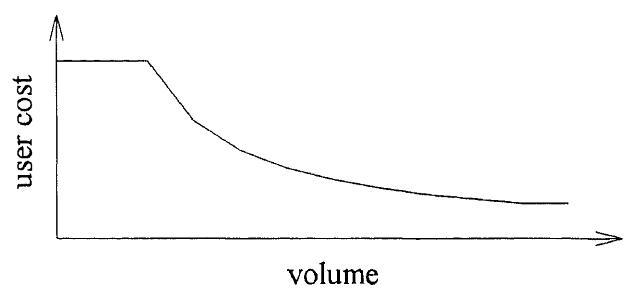


Figure 2.17 Type II Supply Function

For a transit system, this function reflects the constant vehicle running time plus the waiting time estimated at one-half of the headway. Headway is constant until transit ridership reaches the capacity of the current operating regime, and then decreases due to reductions in headway which increase the capacity of the line. In addition to the headway adjustments, a decrease in travel time can be achieved by better tailoring rail service to travel demand using accelerated operating regimes (skip-stop, express-local and zonal). The impact of these regimes on travel times in the network is beyond the scope of this dissertation.

2.5.3 Discussion

The selection of appropriate supply functions in modeling supplier behavior is very important in managerial, planning, and policy formulation decisions. Three planning horizons have been identified by Morlok (1978). Short-run is defined as the planning period in which transit management can not adjust its schedules, fares, frequencies, and resulting headways. Intermediate-run is the period of time required for transit management to adjust operating characteristics to the expected traffic volumes. Long-run is the planning period in which not only operating characteristics, but also technologies used need to be adjusted. Long-run planning will not be examined here, since other considerations in addition to traffic volumes and the resulting costs are needed.

In short-run planning, Type I relationships are adequate to describe the performance of a transportation system as a function of traffic volumes. In intermediate-run, Type II relationships are more appropriate since they provide the means for estimating possible changes in prices and levels of service of a transit system, in addition to future traffic volumes.

Most of the existing network equilibrium models have been designed to solve the highway network equilibrium problem incorporating Type I functions in their supply side. Models of multimodal networks that assume that the operating characteristics for transit are fixed or exogenously determined are also using Type I relationships.

Little attention has been paid to the development of Type II relationships, and even less in their inclusion in network equilibrium models. Type II relationships have been developed by Morlok (1976) with the purpose of including them in network equilibrium models; something that is done for the first time in this dissertation. The problem that arises from using these functions in network equilibrium was briefly illustrated in Section 2.4.3 and will be discussed in more detail later.

2.6 Transportation Demand

The total demand for travel on a network can be either fixed or variable. A fixed demand has a constant value which is not affected by variations in the quality of the provided service. A variable demand is sensitive to changes in the service provided, in addition to other considerations, and is determined as a function of service quality. Although transport demand is not usually fixed, it is a plausible assumption to consider the demand for work trips during the morning peak period as fixed. An example of variable demand is shopping trips; Shoppers can select whether to travel or not at a certain time period based primarily on the quality of the service provided during that time period.

An example of fixed and inelastic demand is given in Figure 2.18. The figure shows that the highway and rail shares of the total demand are fixed and not sensitive to travel cost. The sum of the highway and rail volumes equals total demand.

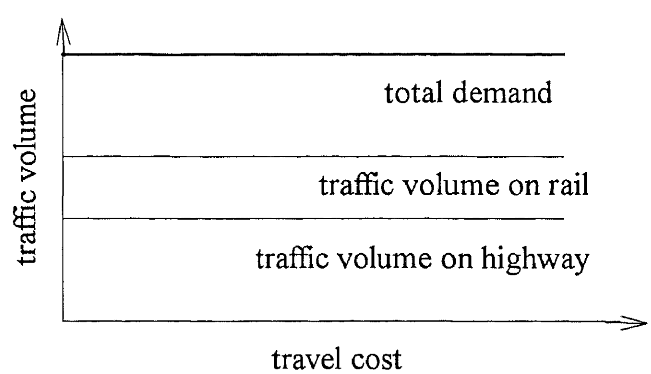


Figure 2.18 Fixed and Inelastic Demand

An example of fixed and elastic demand is given in Figure 2.19. The Figure shows that the highway and rail shares of total demand are sensitive to travel cost. The sum of the highway and rail volumes equals the total demand which is fixed.

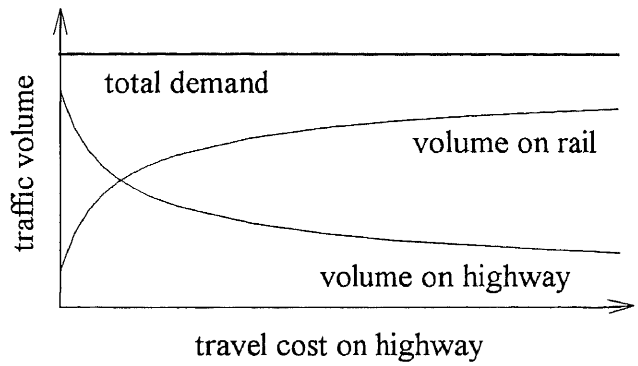


Figure 2.19 Fixed and Elastic Demand

In this dissertation, the total demand between each origin and destination is considered to be fixed and elastic.

2.7 Urban Transportation Modeling System

The existing practice of urban transportation planning, known as the Urban Transportation Modeling System (UTMS), represents the first large-scale use of modern systems analysis methods in transportation. The structure of the UTMS is presented in Figure 2.20.

The first stage of UTMS is trip generation component where the number of trips produced by and attracted to each zone are calculated. When the first step is completed, the trip production from all origin zones of the network (Vi) and the trip attraction from all destination zones (Vj) are known. The second stage is trip distribution. The estimated productions and attractions are used to predict origins and destinations for trip interchanges (Vij). The third step is modal split which projects the portion of trips that will be choosing each of the available modes of transport (Vijm). This split is usually based on service levels that are offered by the choice modes, usually highway and transit. Traffic assignment, which is the fourth and final step, assigns the shares of trips to actual routes in the particular mode-specific network (Vijmr). For example, transit flows are assigned over transit routes and highway flows are assigned over highway routes.

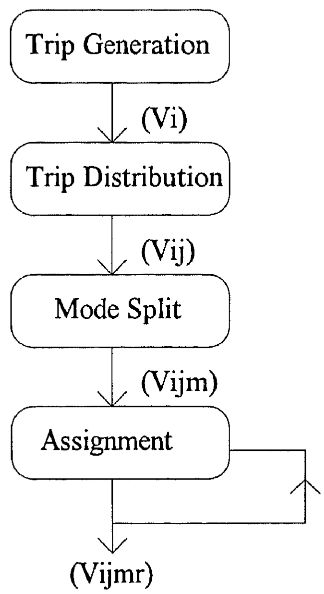


Figure 2.20 The Urban Transportation Modeling System.

The major deficiency of this process is that by having four discrete stages, consistency among the stages is usually not achieved, thus making it very difficult (if not impossible) for the procedure to reach an equilibrium solution. To overcome this problem and achieve consistency, oversimplified models are used within each step, thus lowering the expected accuracy of the results.

Introducing feed-back loops among the four stages (feeding back results from the last stage to previous ones) can be a meaningful way of adjusting the results. In practice, however, this process is usually performed only once since it is time consuming.

A promising approach in urban transportation planning was shown earlier: combining demand and supply in network equilibrium models. These models should be based on well defined principles with a sound theoretical background and a guarantee of convergence to a well-defined equilibrium solution. The development of models is one of the goals of this dissertation.

2.8 Intermodal Networks

This section presents the concept of an intermodal network as it is used in this dissertation, gives an example of such a network, and addresses some issues that must be considered in intermodal network planning.

2.8.1 Concept

Before the concept of the intermodal network is presented, it is essential to give a definition of intermodalism. According to the summary of the discussions during the first Transport Public Policy Forum session on the topic of Intermodalism (1994):

> " The Department of Transportation (DOT) has chosen the following broad and comprehensive definition of intermodalism, including the following aspects:
>
> *Connections*: convenient, rapid, efficient, and safe transfer of people or goods from one mode to another (including end point pick-up and delivery) during a single journey to provide the highest quality and most comprehensive transport service for its cost.
>
> *Choices*: the provision of transportation options through the fair and healthy competition for transportation business between different modes, independently or in combination.
>
> *Coordination and Cooperation*: collaboration among transportation organizations for the purpose of improving transportation service, quality,

safety, and economy for all modes or combinations of modes in an environmentally sound manner. "

According to the same source, this concept of intermodalism reflects a comprehensive and visionary representation of how transportation should work in a perfect world. The goal of any policy choice is to improve the performance or efficiency of the transport system. An intermodal commuter network is defined as the integration of passenger transportation systems (modes) to a single comprehensive system where travelers departing from their homes have several options available. They can use any mode available to them all the way from their origin to their destination, or they can begin their trip using one mode and switch to another mode at any intermediate point between their origin and destination. An intermodal network is considered as one system and traffic flows and travel costs are optimized for the whole system and not for separate auto and transit networks. An intermodal network is distinguished from a multimodal network in that the multimodal network is served by more than one mode, but once a mode is chosen, travelers cannot shift to another mode during their trip.

2.8.2 Example of an Intermodal Commuter Network Equilibrium

A graphical representation of a simple intermodal commuter network is shown in Figure 2.21. The network consists of four links, two highway and two rail. The four links form three paths: auto path (P1) comprising highway link 1; intermodal path (P2) comprising highway link 2 and rail link 4; and rail path (P3) comprising rail link 3 and rail link 4. The three paths connect the origin and destination. Travelers on this network have several options available to them. Departing from their origins, they can access their final destination via auto, rail, and intermodal auto-to-rail modes (e.g., park and ride). A train station (T), connecting links 2, 3 and 4, serves as a transfer point for highway users to shift to rail.

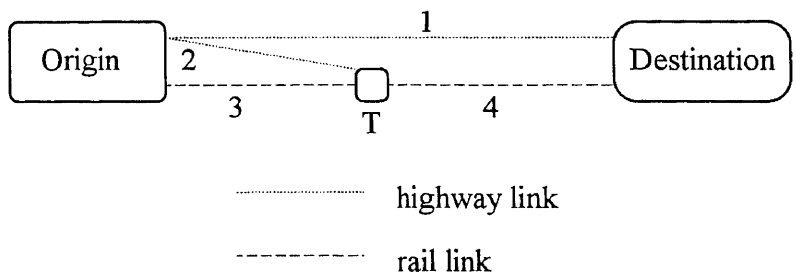


Figure 2.21 Intermodal Commuter Network

A free-flow travel time and a capacity are associated with the links of the network. The free-flow travel time is the time needed to traverse a link under normal (uncongested) conditions. The highway links 1 and 2 have a free flow travel time of 10 and 3 minutes, respectively, and the rail links 3 and 4 have travel times of 5 and 5.5 minutes, respectively. For highway links, the link capacity is assumed to be 2,800 pcph.

The performance function of the highway links is a Type I exponentially increasing function of traffic volume. The travel time on rail links is assumed to be constant while the waiting time is half the headway, which is a decreasing function of traffic volume. The supply function for rail is a Type II function. These functions are link specific and are written for each link of the network as:

$$\text{Link 1: } t_1 = 10*(1+1.15\frac{x_1^4}{2800^4})$$

$$\text{Link 2: } t_2 = 3*(1+1.15\frac{x_2^4}{2800^4})$$

$$\text{Link 3: } t_3 = 5$$

$$\text{Link 4: } t_4 = 5.5+\frac{3000}{100+x_4}$$

where: t_i is the actual travel time on link i,

x_i is the traffic volume on link i

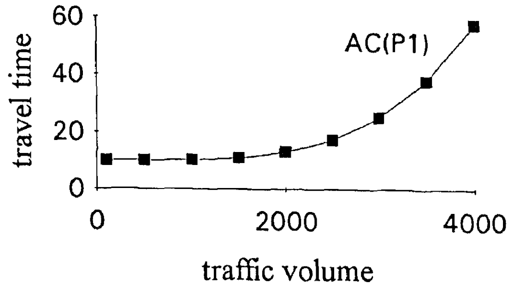
The total demand for travel on the network is assumed to be 3,000 travelers per hour. The problem to be solved is to assign these travelers on the network in a way that minimizes their individual cost of travel (in this case time). According to the user equilibrium principle, an equilibrium solution is reached when the average cost of traveling on all utilized paths is equal and less than the cost on the unutilized paths. To solve the user equilibrium problem, the average cost functions are formulated for each path. The average cost on a path is equal to the sum of the average costs on the path's links. The average cost functions are as follows:

Average Cost on path 1: $10*(1+1.15\frac{x_1^4}{2800^4})$

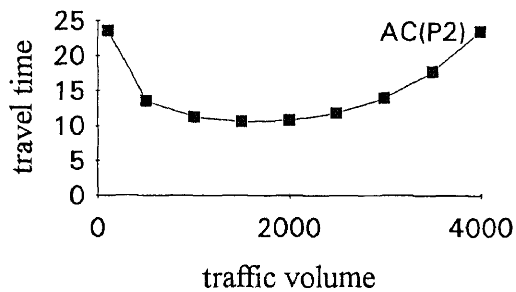
Average Cost on path 2: $3*(1+1.15\frac{x_2^4}{2800^4})+\frac{3000}{100+x_4}+5.5$

Average Cost on path 3: $\frac{3000}{100+x_4}+5+5.5$

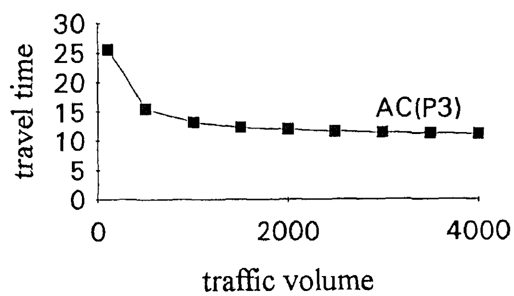
A graphical representation of these functions is shown in Figure 2.22. Part (a) of this figure shows the average cost for the auto path (P1). Part (b) shows the average cost for the intermodal path (P2), and part (c) shows the average cost for the rail path (P3). Travel time on the auto path increases exponentially with traffic volumes. Travel time on the intermodal path decreases as the traffic flow increases up to a certain volume due to the decrease in rail headway. At demand levels greater than this volume, the effects of congestion on the highway portion of the trip are greater than the savings



(a)



(b)



(c)

Figure 2.22 Average Cost Functions. (a) Auto Path. (b) Intermodal Path. (c) Rail Path.

due to headway reduction, and, as a result, the total travel time increases. The average cost on rail decreases with increasing traffic flows up to a certain demand due to the decrease in headway. For greater demands, the headway remains constant at its minimum value and the travel time is constant.

2.8.2.1 Results of the Analysis. The problem is solved analytically by equalizing the average costs of the paths and setting the sum of path flows equal to the total demand. Various systems of equations were developed and solved. The first system equates the average costs on all three paths thus seeking a solution where all three paths will be utilized. This system did not have a solution. Next, systems equalizing the average costs of every two paths were solved. These systems seek solutions where only two paths are utilized and have equal average costs while the third path remains unutilized with a higher average cost. The system that equalized the average costs of paths P1 and P2 had two solutions shown in Table 2.1 as solutions 2 and 3. Finally, the total demand was assigned to each one path and the travel costs were estimated. When path P1 was assigned the total demand, its average cost was lower than the average cost on paths P2 and P3 that were unutilized, thus this assignment was considered as an equilibrium solution designated as solution 1 in Table 2.1. When path P2 was assigned the total demand, its average cost was higher than the average cost on unutilized paths P1 and P3. This was also the case when the total demand was assigned to path P3. Thus, the analytical procedure determined the three equilibrium solutions shown in Table 2.1.

Table 2.1 Feasible Solutions for the Intermodal Network Equilibrium

Equilibrium Solutions	Equilibrium Flows (in pas/hr)			Equilibrium Path Time (min.)			Total Network Time (min.)
	P1	P2	P3	P1	P2	P3	
1	3000	0	0	25.15	38.5	40.5	75,450
2	2,895	105	0	23.14	23.14	40.5	69,420
3	1,355	1,645	0	10.63	10.63	40.5	31,890

According to the first equilibrium solution all 3,000 passengers are assigned to auto path P1 experiencing an average travel time of 25.15 minutes. The average travel times on the unutilized intermodal and rail paths are 38.5, and 40.5 minutes, respectively. The total travel time for all users of the network is 75,450 minutes. The second equilibrium solution is a result of 105 auto users shifting to intermodal, yielding an average travel time of 23.14 minutes. As more travelers shift to rail the third equilibrium solution is reached, according to which there are 1,355 auto users and 1,645 intermodal users. The average travel time decreased further, to 10.63 minutes. This analysis derives the same conclusion as the one presented in Section 2.4.3, and shows that the shift of travelers from auto to transit can be advantageous for all network users.

2.8.3 Issues to be Considered on Intermodal Network Analysis

The scope of analyzing intermodal networks is to find ways to increase the attractiveness of public transit modes by improving transit operations, and provide commutes between origins and destinations via several modes of transport in a synchronized, seamless way. A successful analysis will suggest ways to alleviate highway congestion and its negative social, economic, and environmental effects. However, there are several methodological and policy issues that need to be considered as well.

2.8.3.1 Methodological Issues. In planning for intermodal networks, it is necessary to recognize that each transportation system is part of a broader transportation environment. Each mode should be analyzed within this environment in a way that considers the interactions and interdependency among the different systems. A transit or highway planner or decision-maker should examine the transportation system as a whole and not as separate modal networks.

To better analyze and improve the operation of an intermodal system, effective planning models are needed. An intermodal network equilibrium model should be based on a principle that considers the interactions and allows transfers among various modes, while combining demand and supply to yield well-defined equilibrium solutions.

The distinctive nature of each transportation system must be recognized in the formulation of an intermodal network equilibrium model by utilizing appropriate functions to represent the performance of each system. In addition, various limitations that characterize the capacity and the operation of the systems (i.e., limited number of available parking spaces, limited train seating capacity, and limited rail line capacity) must be considered as well.

Finally, in an intermodal network setting, there are various principles that govern travelers' preferences in choosing their modes, their types of access to various modes, and their actual routes on a network. These principles must be carefully examined and considered in a model formulation.

2.8.3.2 Policy Issues. An intermodal network equilibrium model should be able to evaluate the impacts of changes to one mode on the network-wide performance, as well as the impacts of changes to one mode on the performance of the others. These changes can be either incentives, encouraging the use of public transit, or disincentives, discouraging auto use. Intermodal network equilibrium models should be able to answer questions like:

- How does an increase in highway out of pocket expenses influence the use of transit?
- How does a major transit improvement influence transit ridership?
- What are the effects of a major transit improvement on highway performance?
- Does highway congestion affect transit times?

Not having answers to these questions makes it difficult to evaluate transit improvements and to analyze operating and pricing policies within the broader environment of an intermodal network.

2.9 Problem Statement

The transportation system to be modeled in this dissertation is an intermodal network served by multiple highways and commuter rail. At each origin, travelers can select from any of the available modes and routes. Furthermore, they can choose to begin a trip using one mode and then shift to another mode at any intermediate transfer point between their origin and destination. Physical capacity of links and nodes may limit their choice (i.e., there may not be available space on a commuter parking lot, or seat on train).

Given performance functions and transit service characteristics for the network, the problem is to assign the total demand for travel between each origin and destination of the network to actual modes and routes, based on well-defined behavioral principles. The objective is to minimize the cost of traveling for an individual traveler, while satisfying his or her preferences towards a particular mode or a particular type of access to various modes and considering capacity limitations of the facilities of the network.

Three models are developed, which combine Type I supply functions with various demand functions to determine equilibrium traffic assignments and travel costs over an intermodal network. An additional model of commuter rail service design which utilizes Type II supply functions for transit in an intermodal network equilibrium context is developed. This model estimates values of transit service characteristics (headway, fare) in addition to the equilibrium flows and travel costs over an intermodal network.

After analyzing their mathematical properties and stating the conditions for an equilibrium solution, the models are used to evaluate various operating and pricing policies on the intermodal network, and to assist managerial decision-making regarding operating schemes to meet future demands.

CHAPTER 3

LITERATURE REVIEW

3.1 Introduction

The literature reviewed in this chapter focuses on the formulations of demand and supply network equilibrium models. In addition to the theoretical developments in the area, the state-of-practice is reviewed and deficiencies of currently used transportation planning software are reported.

3.2 Origins of Network Equilibrium Modeling

The formulation of network equilibrium models has its origins in the 1950's. The user equilibrium principle, that was introduced in chapter 2, according to which travelers are choosing their routes while traveling on networks was stated by Wardrop (1952). Several formulations of the network equilibrium problem, based on this principle, have been developed and solution algorithms have been proposed. It has been shown that the conditions defining user equilibrium can be mathematical programming formulations, nonlinear complementarity formulations, or variational inequality formulations.

Beckmann et al. (1956) formulated an equivalent convex programming program for the route choice problem. Based on a theorem developed by Kuhn and Tucker in 1951, and Karush independently, they proved the existence and uniqueness of the solution to this problem and the stability of its equilibrium solution. A computational algorithm for the solution of the problem formulated by Beckmann et al. was developed the same year by Frank and Wolf (1956). This algorithm is easily implemented, but it converges slowly and behaves poorly as it approaches an equilibrium solution. An excellent review of the mathematical formulations and the algorithms used to solve the network equilibrium problem is presented in Sheffi (1985).

Modifications and improvements of the Frank-Wolf algorithm, as well as new algorithmic developments, have been proposed and tested. Some examples are LeBlanc et al. (1975), Evans (1976), LeBlanc et al. (1981), LeBlanc et al. (1985), Dafermos et al. (1969), and Florian et al. (1974). The algorithmic developments are not reviewed and analyzed further since this dissertation deals with the formulation of the network equilibrium problem, not with the development of algorithmic approaches. More specifically, this dissertation concentrates on the development of models which combine mode choice, access choice, and route choice, in an intermodal network equilibrium context.

3.3 Network Equilibrium Models

This section presents the review of several papers that deal with the formulation of network equilibrium models. Only two papers were found that deal with the problem as it was described in Chapter 2. Several papers however, address various aspects of the problem and are discussed in this section.

Dafermos (1972) introduced a multimodal traffic equilibrium model where the interactions between modes were considered. She stated the conditions under which the multimodal traffic equilibrium problem can be reduced to a minimization problem.

Florian (1977) developed an equilibrium model of travel by car and one or more public transit modes. Wardrop's equilibrium holds for drivers, while transit users are assigned to the minimum cost transit route according to the all-or-nothing technique. A single mode (auto) elastic equilibrium assignment problem determines the auto impedances, while transit impedances are parametrically kept fixed during the optimization procedure.

Fisk et al. (1981) determined sufficient conditions for existence and uniqueness of the equilibrium solution of the problem formulated by Florian (1977).

Florian et al. (1978) presented a combined trip distribution, mode choice, and traffic assignment in a multimodal network equilibrium model. The problem is formulated as an equivalent minimization model where transit impedances are exogenously determined.

Abdulaal et al. (1979) presented two models that combine modal split and traffic assignment. In the first model, travelers choose their routes according to Wardrop's equilibrium principle and their modes according to mode choice functions. In the second model, travelers choose modes and routes according to Wardrop's equilibrium principle. The authors present the conditions under which these models can be formulated as equivalent optimization problems.

Dafermos (1982) used the variational inequality approach to formulate the multimodal network equilibrium problem.

Florian et al. (1983) presented a two mode equilibrium road and transit assignment model which incorporates a zonal aggregate mode choice model. The model is formulated as a variational inequality problem, and the conditions for the uniqueness of an equilibrium solution are stated.

Aashtiani (1979) formulated multimodal network equilibrium as a nonlinear complementarity problem, and he derived the sufficient conditions for uniqueness of its solution.

Tatineni et al. (1993) presented a combined trip distribution, modal split, and traffic assignment model. The paper addressed the need for improvements in the forecasting methodology and the capability of combined models to overcome shortcomings of the sequential urban transportation planning procedure, such as inconsistency among various steps. The model considered auto and transit as the two alternative modes and kept transit travel times and costs fixed.

The above studies are relevant to this dissertation because they all deal with the equilibration between demand and supply over transportation networks that are served by more than one mode. The basic difference, however, between these studies and this dissertation is that the former consider only pure modes. Once travelers have chosen modes, they are assigned over modal networks without the possibility of switching modes during their journey.

To the author's knowledge, there are only two recent papers that explicitly consider and analyze intermodal trips in a network equilibrium context. The first paper, by Fernandez et al. (1994), presented three model formulations with auto, metro, and combined modes (auto-to-metro), and analyzed the resulting equilibrium conditions. The underlying assumption is that the combined mode is considered only at those origins where metro is not available. When metro is available, the traveler's choice is limited between auto and metro.

The second paper, by Boile et al. (1994), presented a methodological framework for analyzing and evaluating operating and pricing policies over an intermodal network. Central to the methodological framework is a network equilibrium model which combines mode choice and traffic assignment to assign traffic flows over an intermodal network. The objective is to minimize individual travelers' costs, while considering their preferences towards various modes.

In contrast to Fernandez et al. (1994), the formulations presented in this dissertation consider intermodal trips to be an option at every origin of the network regardless of the availability of a train station. These formulations also consider the choice of access to various modes. They capture the fact that even when a traveler has an option to walk to a near-by train station, she or he may prefer to drive to, or to be dropped off at, a station along the metro route.

3.4 State-of-Practice and Currently Used Urban Transportation Planning Software

The majority of popular mainstream planning software packages such as QRSII (National Cooperative Highway Research Program 1978), MINUTP (Murtagh et. al. 1992) and TRANPLAN (The Urban Analysis Group 1990), follow the Urban Transportation Modeling System (UTMS) four-step procedure. These software packages have several shortcomings.

First, an inexact methodology is used to assign flows over the networks. When assignment is done by the all-or-nothing method, the impact of congestion on travel times is not recognized since travel times are assumed to be constant. When the minimization of network-wide cost is used (QRSII), the assignment is inconsistent with driver behavior. According to Wardrop's First Principle (Wardrop 1952) drivers will attempt to switch paths between their origin and destination as long as this switch can decrease their individual travel times. This inconsistency could lead to unrealistic flows, especially in networks with moderate congestion. In cases when an "equilibrium solution" is computed, it is accomplished using an inexact heuristic (MINUTP).

Second, the software do not recognize the interaction between network performance and modal split. The final equilibrium travel times are not considered in adjusting the initial modal splits. However, even when the computed travel times are fed-back to adjust the modal split, flaws with the traffic assignment usually produce unrealistic flows. The software fail to capture the interrelationship among the various steps and to calculate a valid equilibrium of supply and demand.

Third, the UTMS packages ignore intermodal flows by not permitting trips to shift between the auto and transit networks. This flaw is critical because the highway portion of an intermodal trip will impact highway travel times and thus modal splits. This shortcoming makes it difficult, if not impossible, to use the packages for evaluating impacts of transit on highway network performance.

An exception among the current software in their ability to model intermodal trips is the new version of EMME/2--Release 7 (INRO Consultants 1994). A new module, "Matrix Convolutions", allows the enumeration of intermediate zones between an origin and a destination. By having an intermediate zone serve as a destination zone for the highway network and as an origin zone for the transit network, it is possible to consider intermodal trips.

In marked contrast to the current software, the models presented in this dissertation perform a combined mode choice-traffic assignment over an integrated highway-transit network, wherein different performance functions are used to model travel over each portion of an intermodal trip. The models can be used to evaluate impacts of transit and highway improvement policies on network-wide performance, and to analyze operating and pricing policies within the broader environment of an intermodal network.

CHAPTER 4

MODELING INTERMODAL NETWORKS

4.1 Selection Process for a Trip over an Intermodal Network

Traveling over an intermodal network involves a selection process during which travelers have to choose modes, access types, and routes between their origins and destinations. This selection process is shown in Figure 4.1.

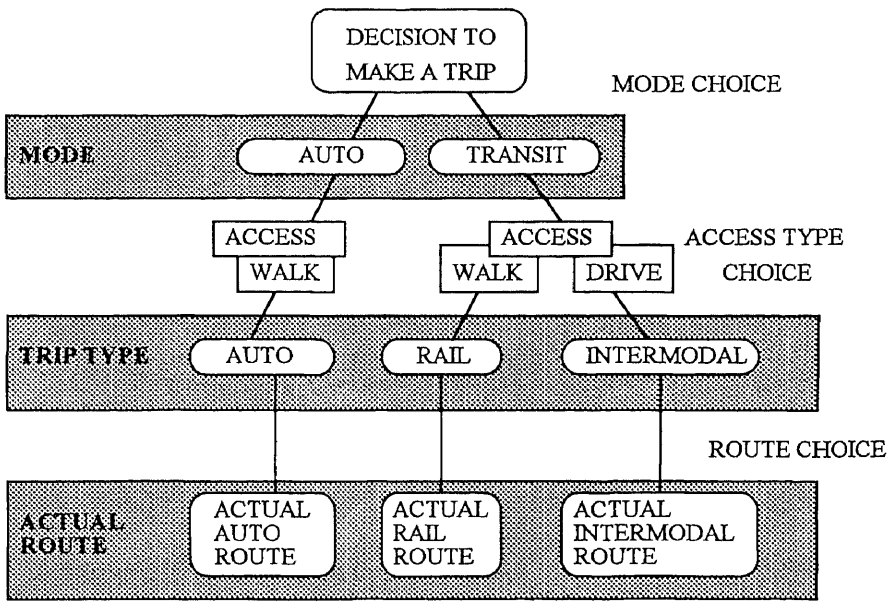


Figure 4.1 Selection Process for a Trip over an Intermodal Network

For the network examined in this dissertation, the first choice a traveler has to make is whether to use private auto or transit for a trip between an origin and a destination. Once this choice is made, travelers have to choose type of access to the chosen mode. Two access types are considered for transit: walk and drive. Based on the selected access type, a transit trip can be characterized as pure rail (walk access) or intermodal (drive access). The only access type considered for auto trips is walk thus only pure auto trips are available within the auto mode. After access type has been selected, travelers have to choose their actual routes in the network.

Each trip taken on an actual route of the intermodal network is characterized as auto, rail, or intermodal. Rail and intermodal trips are transit-mode trips while auto trips are auto-mode trips. The decision process shown in Figure 4.1 is used in the formulation of the mathematical models presented in this dissertation.

4.2 Modeling Approaches

4.2.1 Introduction

Three approaches for modeling intermodal decisions are presented. These approaches formulate the choice of mode, access type, and actual route on an intermodal network, within a network equilibrium context. The distinctive difference among the three modeling approaches depends on the choices that are modeled within the demand side versus the choices that are modeled within the supply side of the formulation.

Within the demand side, a choice is formulated using disagregate choice models, such as binary logit or nested logit. These formulations assume that each alternative is chosen with some finite probability and consider the relative attractiveness of one alternative over the others. Within the supply side a choice is formulated as a route choice (routing) problem, in which a traveler chooses the alternative that minimize his/her generalized travel cost.

4.2.2 First Modeling Approach-Intermodal Network Route Choice Problem

The first modeling approach shown in Figure 4.2 formulates mode, access type, and route choice in the supply side of the model. The generalized user equilibrium approach that was presented in Chapter 2 is used in this formulation, and is extended to perform the choice of access type in addition to the choices of mode and route. The rationale behind this formulation is that a traveler will choose mode, access type, and actual route on a network, such as to minimize his/her generalized cost of traveling.

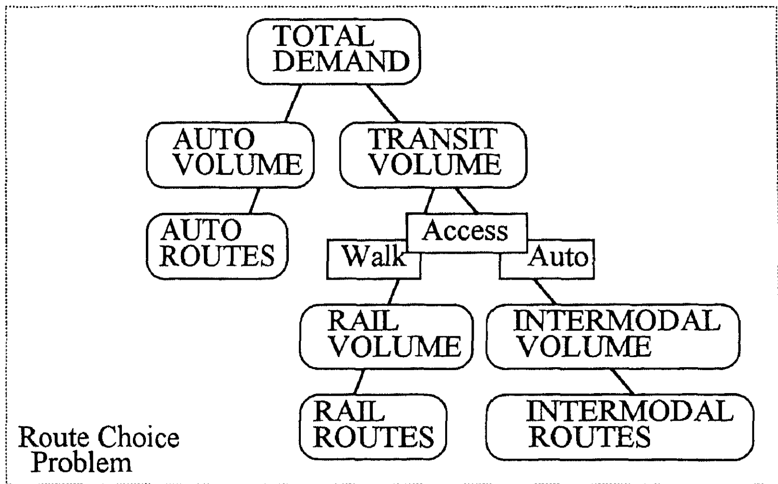


Figure 4.2 Intermodal Network Route Choice Process

4.2.3 Second Modeling Approach-Intermodal Network Mode and Route Choice

The second modeling approach, shown in Figure 4.3, formulates the mode choice in the demand side of the formulation.

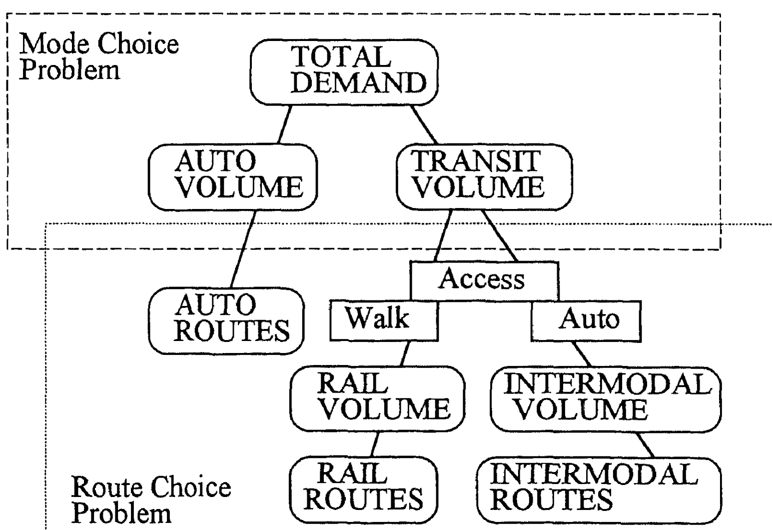


Figure 4.3 Intermodal Network Mode and Route Choice Process

The demand side utilizes a random utility mode choice model (binomial logit model) to choose between auto and transit. The choice of the access type is formulated within the supply side along with the choice of actual routes. This approach assumes that modes are chosen with some finite probability, and access type and actual routes are chosen based strictly on the minimization of the generalized travel cost.

4.2.4 Third Modeling Approach-Intermodal Network Mode, Access Type, and Route Choice

The third modeling approach, shown in Figure 4.4, formulates mode and access type choice within the demand side of the formulation.

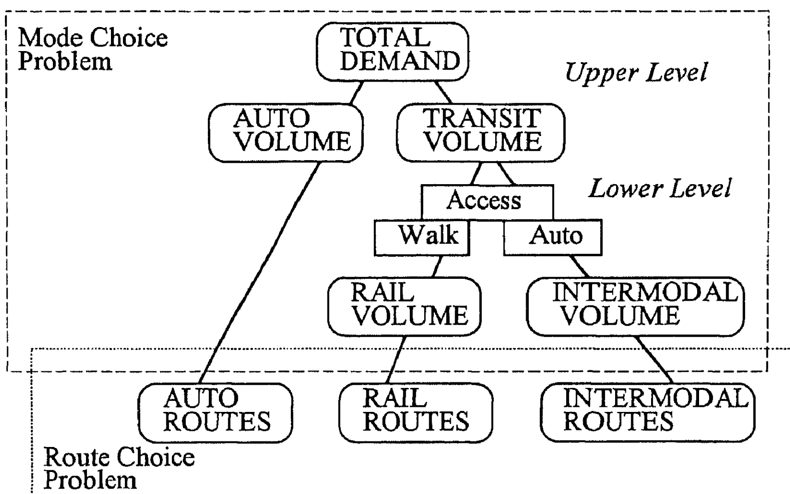


Figure 4.4 Intermodal Network Mode, Access Type, and Route Choice Process

A nested logit model is utilized to perform these choices. The so-called upper level decision of the nested logit model splits the total demand for each origin-destination pair between demand for auto and demand for transit. The lower level decision splits the demand for transit between pure rail and intermodal trips. The choice of actual routes within each mode is formulated in the supply side as a routing problem. In this approach, modes and access types are chosen with some finite probability, and the actual routes of the network are chosen based on the generalized cost minimization.

4.3 Discussion

The reason why several models are formulated is that each of the proposed models requires different levels of information to be available for the estimation of exogenously determined parameters. Transportation planners do not always have detailed information available for the network under study. Thus, based on data availability, one can select the appropriate model to analyze a network considering its assumptions and limitations.

The third formulation is based on the most sound economic and behavioral principles and is, in general, expected to provide more accurate results. The disadvantage of this formulation, however, is that it involves a large number of exogenously determined parameters and requires excessive and detailed information about travel patterns and choices made by commuters on a particular network. When the level of detail of the available data does not allow the estimation of the parameters of the demand model (mode choice and access type choice parameters), one of the other two models can be used. More specifically, the second model can be used if adequate information is available to determine the mode choice parameters, and the first model can be used if the available data is not sufficient for the estimation of any of the demand model parameters.

The results of the three models are not expected to be identical because of their different underlying assumptions. To increase the expected accuracy of the model predictions, it is essential to include the largest possible number of important factors affecting travel decisions including travel time, waiting time, transfer time, and out of pocket costs, in both the demand and supply side models.

CHAPTER 5

NETWORK EQUILIBRIUM MODELS

5.1 Introduction

5.1.1 General Approach

The three demand and supply network equilibrium models are formulated in this chapter as mathematical programs with non-linear objective functions and linear constraints. The general mathematical expression of these models is:

$$\min z, \text{ s.t.: } a_i x = b_i, \quad x \geq 0$$

The equilibrium conditions for each of the problems are stated and the solutions of the problems are proved to satisfy these conditions.

To find the solutions to these problems their Lagrangian is formulated and the first derivatives of the Lagrangian with respect to the decision variables are computed. The Lagrangian of the problems is formulated by multiplying the constraints of the formulations with Lagrangian multipliers u_i, and introducing them in the objective functions. The mathematical programs then become equivalent to:

$$\min L = z + \sum_i u_i (b_i - a_i x), \quad x \geq 0$$

The Lagrangian multiplier u_i represents the shadow price for constraint i. Then, it is shown that, under certain conditions, the Karush-Kuhn-Tucker (K-K-T) conditions for the problem are necessary and sufficient for the optimal solution of the problem. The Karush-Kuhn-Tucker conditions can be expressed as:

$$\frac{\partial L}{\partial x} = \frac{\partial z}{\partial x} - \sum_i u_i a_i \begin{cases} = 0, & \text{if } x > 0 \\ \geq 0, & \text{if } x = 0 \end{cases}$$

By applying these conditions to the Lagrangian of the problems an optimal solution which is unique and which is shown to satisfy the equilibrium conditions is obtained.

5.1.2 Notation

The following notation is used in the model formulation:

Nodes, Links and Modes:

O = origin

D = destination

ij = origin-destination pair

l = link

z, a, r, w, t = highway, access, rail, walking and transfer links respectively

m = mode (auto or transit)

k = trip type (auto, rail, intermodal)

A, T = auto and transit modes respectively

A, R, M = auto, rail and intermodal trip types respectively

p = path

Sets

L = set of all links

$LZ, LA, LR, LCR\ LW, LT$ = sets of all highway, access, rail, critical rail, walking, and transfer links respectively

P = set of all paths

P_A, P_T, P_R, P_M = set of all auto, transit, rail, and intermodal paths respectively

Parameters:

T^{ij} = total demand between origin i and destination j

D_{ij} = demand function

D_{ij}^{-1} = inverted demand function

U_m^{ij} = utility of traveling from origin i to destination j via mode m

U_k^{ij} = utility of type k trip from origin i to destination j

GC_m^{ij} = minimum generalized cost of traveling from origin i to destination j via mode m

GC_k^{ij} = minimum generalized cost of type k trip from origin i to destination j

$GC_{p_m}^{ij}$ = generalized cost of traveling on mode m path p from origin i to destination j

$GC_{p_k}^{ij}$ = generalized cost of type k trip on path p from origin i to destination j

$\delta_{lp_m}^{ij}$ = binary parameter, an element of link-path matrix (1 if link l is in mode m path p between origin i and destination j, and 0 otherwise)

occ = occupancy rate for auto

$space_l$ = existing number of parking spaces at a rail station

$seats$ = number of train seats per peak period

λ = transit vehicle load factor

α, β = exogenously determined parameters of the mode choice model

Choice Variables

x_l = flow on link l

f_{pk}^{ij} = flow on type k path p, from origin i to destination j

$c(x_l)$ = cost of traveling on link l

T_m^{ij} = mode m trip rate between origin i and destination j

T_k^{ij} = type k trip rate between origin i and destination j

5.2 First Model - Intermodal Network Route Choice

5.2.1 Model Assumptions

The following assumptions are made in the first modeling approach:

- Total travel demand between each O-D pair is fixed and known.
- Travel cost is the only service characteristic perceived by travelers when making a trip.
- Travelers' mode and access type preferences are modeled implicitly.
- Travelers have multiple route choices available to them.
- Travelers have perfect information on travel times and costs on all routes.
- Travelers are identical in their behavior.

5.2.2 Model Statement

The problem to be formulated is: given the characteristics of an intermodal network, the total demand for travel between each origin and destination, and the link performance functions; find the link flow patterns for the network. The underlying assumption of the model is that commuters choose the mode, access type, and route that minimize their individual cost of traveling. The model is formulated as a mathematical program with a nonlinear objective function and linear constraints. The general model statement is:

Minimize Total Individual User Cost

subject to:

Demand Conservation Constraints

Link Flow Conservation Constraints

Rail and Transfer Link Capacity Constraints

Non-negativity Constraints

5.2.3 Equilibrium Condition

The model must satisfy the condition that no traveler has an incentive to unilaterally change routes for s/he can not further minimize his/her travel cost. This equilibrium condition is expressed as:

$$GC_p^{ij} - GC^{ij} \begin{Bmatrix} = 0, \text{ if } f_p^{ij} > 0 \\ \geq 0, \text{ if } f_p^{ij} = 0 \end{Bmatrix} \qquad \forall ij \qquad (5.1)$$

This condition indicates that a path p from origin i to destination j is utilized only if the generalized cost on this path (GC_p^{ij}) is equal to the minimum generalized cost of traveling on that O-D pair (GC^{ij}). According to this condition, at equilibrium, all utilized paths between an O-D pair ij have the same generalized cost which is less than or equal to the cost of traveling on the unutilized paths for the same O-D pair.

5.2.4 Model Formulation

The formulation of the first model adopts an objective function of the user equilibrium traffic assignment with fixed demand similar to the one formulated by Beckman (1956). The mathematical expression of this function is:

$$\text{Minimize } z(x_l) = \sum_{l \in L} \int_0^{x_l} c(\omega) d\omega \tag{5.2}$$

The constraints of the formulation can be described as follows:

The demand conservation constraints ensure that all trips between O-D pairs are accounted for by equating the demand for each O-D pair with the sum of the flows on all the paths available to travelers between this O-D pair. This constraint is of the form:

$$T^{ij} = \sum_p f_p^{ij} \qquad \forall\ i,j,p \tag{5.3}$$

The link flow conservation constraints equate the flow on a link with the sum of the flows on all the paths that are going through that link $[x_l = x_l(f_p)]$. Paths are identified by the binary parameter δ_{lp}^{ij} taking on the value of one when link l is included in path p and zero otherwise. The auto occupancy rate (occ) is used to convert person trips into vehicle trips. Thus, for highway, access, and transfer links this constraint is:

$$x_l = \frac{1}{occ} \sum_{ij} \sum_p \delta_{lp}^{ij} * f_p^{ij} \qquad \forall\ l \subseteq LZ, LA, LT \tag{5.4}$$

The link flow conservation constraint for rail and walking links is:

$$x_l = \sum_{ij} \sum_p \delta_{lp}^{ij} * f_p^{ij} \qquad \forall\ l \subseteq LR, LW \tag{5.5}$$

The parking capacity constraint ensures that the number of cars parked at a parking lot does not exceed the available number of parking spaces. The mathematical expression of this constraint is:

$$x_l \leq space_l \qquad \forall\ l \subseteq LT \tag{5.6}$$

The rail capacity constraint insures that the number of rail users does not exceed the train capacity which is defined as the train seating capacity multiplied by an allowable load factor (λ). The mathematical expression of this constraint is:

$$x_l \leq seats^{*}\lambda \qquad \forall\ l \subseteq LCR \tag{5.7}$$

The last constraint of the formulation is the nonegativity constraint. The formulation requires the nonegativity of link flows and path flows. Since one constraint is redundant (because of the relationship of link and path flows given from the link flow conservation constraints) only the nonegativity of the path flows is kept in the mathematical formulation of the model. This constraint is of the form:

$$f_p^{ij} \geq 0 \qquad \forall\ i,j,p \tag{5.8}$$

The complete model statement is shown in Table 5.1.

5.2.5 Derivation of Equilibrium Conditions

To prove that a solution of the proposed mathematical formulation satisfies the equilibrium condition it is sufficient to show that the Karush-Kuhn-Tucker conditions for the minimization program are identical to the equilibrium conditions (Sheffi 1985).

Table 5.1 First Intermodal Network Equilibrium Model

$$\text{Minimize } z(x_l) = \sum_{l \in L} \int_0^{x_l} c(\omega)d\omega$$

Subject to:

$T^{ij} = \sum_p f_p^{ij}$	$\forall\ i,j,p$
$x_l = \frac{1}{occ} \sum_{ij} \sum_p \delta_{lp}^{ij} * f_p^{ij}$	$\forall\ l \subseteq LZ,LA,LT$
$x_l = \sum_{ij} \sum_p \delta_{lp}^{ij} * f_p^{ij}$	$\forall\ l \subseteq LR,LW$
$x_l \le space_l$	$\forall\ l \subseteq LT$
$x_l \le seats * \lambda$	$\forall\ l \subseteq LCR$
$f_p^{ij} \ge 0$	$\forall\ i,j,p$

For this purpose the Lagrangian of the problem is formulated and its first derivatives with respect to the decision variables are set equal to zero. To formulate the Lagrangian, the demand conservation constraint is multiplied by a Lagrangian multiplier (u^{ij}) and introduced in the objective function. In addition, the link flow conservation constraints, equations (5.4)-(5.5), are directly introduced in the objective function by expressing the flow on each link as the sum of the flows on the paths that are using that link. The rail seating capacity and the parking capacity constraints have been introduced in the model to consider the capacity limitations of the network. If, during the analysis of a particular network, these constraints become binding the model will not reach an equilibrium solution. Since these constraints, however, do not affect the mathematical properties of the problem (if unlimited capacity is assumed) they are not considered in the analysis of the equilibrium conditions.

The mathematical expression of the Lagrangian of the problem is:

$$L(f,u) = z[x(f)] + \sum_{ij} u^{ij}(T^{ij} - \sum_{p} f_p^{ij})$$

with $f_p^{ij} \geq 0 \quad \forall p,i,j$

The first derivatives of the Lagrangian with respect to path flows (f_p^{ij}) and Lagrangian multipliers (u^{ij}) are derived and used to state the Karush-Kuhn-Tucker optimality conditions for a stationary point. These conditions can be expressed as:

$$f_p^{ij} \frac{\partial L(f,u)}{\partial f_p^{ij}} = 0 \quad \text{and} \quad \frac{\partial L(f,u)}{\partial f_p^{ij}} \geq 0 \qquad \forall p,i,j \qquad (5.9)$$

$$\frac{\partial L(f,u)}{\partial u^{ij}} = 0 \qquad \forall i,j \qquad (5.10)$$

$$f_p^{ij} \geq 0 \qquad \forall p,i,j \qquad (5.11)$$

The first derivative of the Lagrangian with respect to path flows is:

$$\frac{\partial L(f,u)}{\partial f_p^{ij}} = \frac{\partial z[x(f)]}{\partial f_p^{ij}} + \frac{\partial \sum_{ij} u^{ij}(T^{ij} - \sum_p f_p^{ij})}{\partial f_p^{ij}} \tag{5.12}$$

According to the derivation chain rule:

$$\frac{\partial z[x(f)]}{\partial f_p^{ij}} = \sum_l \frac{\partial z(x)}{\partial x_l} \frac{\partial x_l}{\partial f_p^{ij}}$$

and the link flow conservation constraints:

$$x_l = \frac{1}{occ} \sum_p \delta_{lp}^{ij} f_p^{ij}, \text{ or } \frac{\partial x_l}{\partial f_p^{ij}} = \frac{1}{occ} \delta_{lp}^{ij} \qquad \forall l \in LZ, LA, LT$$

$$x_l = \sum_p \delta_{lp}^{ij} f_p^{ij}, \text{ or } \frac{\partial x_l}{\partial f_p^{ij}} = \delta_{lp}^{ij} \qquad \forall l \in LR, LW$$

equation (5.12) becomes:

$$\frac{\partial z(x)}{\partial x_l} \frac{\partial x_l}{\partial f_p^{ij}} + \frac{\partial \sum_{ij} u^{ij}(T^{ij} - \sum_p f_p^{ij})}{\partial f_p^{ij}}, \text{ or}$$

$$\sum_{l \in LZ, LA, LT} \frac{1}{occ} \delta_{lp}^{ij} \frac{\partial z(x_l)}{\partial x_l} + \sum_{l \in LR, LW} \delta_{lp}^{ij} \frac{\partial z(x_l)}{\partial x_l} - u^{ij} \tag{5.13}$$

The derivative of the objective function with respect to the link flows is:

$$\frac{\partial z(x)}{\partial x_l} = \frac{\partial}{\partial x_l} \sum_l \int_0^{x_l} c(x_l) dx_l = c(x_l)$$

and equation (5.13) becomes:

$$\frac{\partial L(f,u)}{\partial f_p^{ij}} = \sum_{l \in LZ, LA, LT} \frac{1}{occ} \delta_{lp}^{ij} c(x_l) + \sum_{l \in LR, LW} \delta_{lp}^{ij} c(x_l) - u^{ij} \tag{5.14}$$

The two summations in equation (5.14) represent the average generalized cost on path p between origin i and destination j. This average cost can be symbolized as GC_p^{ij}. The Lagrangian multiplier u^{ij} represents the minimum average generalized cost for ij which is the cost of the utilized paths and can be symbolized as GC^{ij}.

The partial derivative of the Lagrangian with respect to the Lagrangian multiplier is:

$$\frac{\partial L(f,u)}{\partial u^{ij}} = \frac{\partial \sum_{ij} u^{ij}(T^{ij} - \sum_p f_p^{ij})}{\partial u^{ij}} = T^{ij} - \sum_p f_p^{ij}$$

Thus the K-K-T conditions become:

$$f_p^{ij}(GC_p^{ij} - GC^{ij}) = 0 \qquad \forall p,i,j \tag{5.15}$$

$$GC_p^{ij} - GC^{ij} \geq 0 \qquad \forall p,i,j \tag{5.16}$$

$$T^{ij} - \sum_{p} f_p^{ij} = 0 \qquad \forall i,j \qquad (5.17)$$

$$f_p^{ij} \geq 0 \qquad \forall\, p,i,j \qquad (5.18)$$

Equations (5.15)-(5.16) state that, if the generalized cost on path p between O-D pair ij (GC_p^{ij}) is greater than the generalized cost on the lowest cost path (GC^{ij}) for the same O-D pair, the corresponding flow on path p is zero. If the average cost on path p is equal to the minimum cost of traveling between i and j then the flow on the path can be greater than or equal to zero. Equation (5.17) is the demand conservation constraint written for each O-D pair and equation (5.18) is the nonegativity constraint. Equations (5.15)-(5.16) are equivalent to the equilibrium condition thus the solution of the mathematical problem presented in Table 5.1 satisfies the equilibrium condition.

5.2.6 Convexity Analysis

To prove that the solution of the problem is unique it must be proved that: the objective function is strictly convex and the feasible region defined by the constraints of the formulation is convex (Sheffi 1985). The latter is satisfied since the constraints are linear equalities. The nonegativity and the rail seating and parking capacity constraints do not alter this characteristic. The objective function is strictly convex under the assumption that the link performance functions are separable (i.e., $\partial c(x_a) / \partial x_b = 0 \;\; \forall a \neq b$, and $dc(x_a) / dx_a > 0 \;\; \forall a$, where a, b are links of the network) and monotonicaly increasing functions of their argument. Under the separability assumption the cost of traveling on a link depends only on the flow on that link and not on the flow on any other link of the network. The objective function is the sum of the integrals of the link cost functions. The link cost functions are monotonicaly increasing and thus strictly convex. The integral of a strictly convex function is strictly convex, and the sum of strictly convex functions is strictly convex. As a result, the objective function is strictly convex meaning that it has a unique minimum. Considering also the convexity of the feasible region the mathematical program presented in Table 5.1 has a unique minimum which is obtained from the Karush-Kuhn-Tucker conditions and shown to satisfy the equilibrium condition.

5.3 Second Model - Intermodal Network Mode and Route Choice

5.3.1 Model Assumptions

The assumptions made in the second modeling approach are:

- Total travel demand for each O-D pair is fixed and known.
- Travel demand is elastic, i.e. sensitive to travel cost on alternative modes.
- Travelers have a range of mode and route choices available to them.
- Travelers' preferences towards various modes are considered in the mode choice selection.
- Travelers have perfect information on travel times and costs on all routes.
- Travelers are identical in their behavior.

5.3.2 Model Statement

The problem to be formulated for the second modeling approach is: given the characteristics of an intermodal network, the total demand between each origin and destination, the link performance functions, and the demand function which represents the mode selection process; find the trip rates for each mode between each origin-destination pair and the link flow patterns for the network.

The difference between the first model and the one presented in this section is that the mode choice process in this model is not based strictly on the minimization of the individual travelers' generalized cost of traveling. Travelers' preferences between auto and transit are considered and the mode selection process is formulated in the demand side of the model.

A binary logit type model is used to perform the mode choice between auto and transit and compute the number of trips for each mode. The binary logit model is of the form:

$$T_A^{ij} = T^{ij} \frac{\exp(U_A^{ij})}{\exp(U_A^{ij}) + \exp(U_T^{ij})} \tag{5.19}$$

The total demand for each origin-destination pair T^{ij} is fixed and is used by the mode choice model to estimate modal shares. The utilities of traveling between i and j via auto and transit are given respectively from the following expressions:

$$U_A^{ij} = -\beta * GC_A^{ij} \text{, and}$$

$$U_T^{ij} = -\alpha_{TA} - \beta * GC_T^{ij}$$

Substituting the expressions for the utilities in equation (5.19) the demand function (D_{ij}) becomes:

$$T_A^{ij} = T^{ij} \frac{\exp(-\beta * GC_A^{ij})}{\exp(-\beta * GC_A^{ij}) + \exp(-\alpha_{TA} - \beta * GC_T^{ij})} \tag{5.20}$$

The demand function generates the number of trips by mode as a function of the disutility (generalized cost) of that mode. The disutility depends on the values of service variables (in-vehicle travel time, out-of-vehicle time, and out-of-pocket costs) which are included in the supply side of the model. Based on the values of these variables and the minimization of the individual travelers' generalized cost principle the supply side of the model estimates the split of transit travelers between pure rail and intermodal trips and the number of travelers using each route of the network.

The model is formulated as a mathematical program with a non linear objective function and linear constraints. The general model statements is:

Minimize total individual user cost minus the integral of the inverse demand function

subject to:

Demand Conservation Constraints

Link Flow Conservation Constraints

Rail and Transfer Link Capacity Constraints

Non-negativity Constraints

5.3.3 Equilibrium Conditions

An equilibrium solution to the model must satisfy two conditions. First, for each mode, no traveler has an incentive to unilaterally change routes for s/he can not reduce his/her travel cost. This equilibrium condition is expressed as:

$$GC_{p_m}^{ij} - GC_m^{ij} \begin{Bmatrix} = 0, \text{ if } f_{p_m}^{ij} > 0 \\ \geq 0, \text{ if } f_{p_m}^{ij} = 0 \end{Bmatrix} \qquad \forall m, i, j \qquad (5.21)$$

This condition indicates that a mode m path p from origin i to destination j is utilized only if the generalized cost on this path is equal to the minimum generalized cost for that O-D pair and for mode m. The condition is written for each mode and for each origin-destination pair.

The second equilibrium condition states that no traveler has an incentive to unilaterally change modes. This condition is expressed as:

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) \tag{5.22}$$

Equation (5.22) gives the relationship between the generalized costs of the two utilized modes.

5.3.4 Model Formulation

The objective function of this model minimizes the total individual user cost minus the integral of the inverse demand function. The mathematical expression of the objective function is:

$$\text{Minimize } z(f,T) = \sum_{l \in L} \int_0^{x_l} c(\phi)d\phi - \sum_{ij} \int_0^{T_m^{ij}} D_{ij}^{-1}(\omega)d\omega \tag{5.23}$$

where D_{ij}^{-1} derived from equation (5.19) (see Exhibit 1 in Appendix A for derivation) is:

$$D_{ij}^{-1} = GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA})$$

Using this expression in the objective function, it becomes:

$$\text{Minimize } z(f,T) = \sum_{l \in L} \int_0^{x_l} c[x_l(f)]dx_l - \sum_{ij} \int_0^{T_T^{ij}} -\frac{1}{\beta}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA})dT_T^{ij}$$

A travel demand function where trips are proportional to a negative exponential function of the travel impedance has been shown to be equivalent to entropy type distribution model (Florian et. al. 1978). Thus, utilizing entropy models the objective function can be expressed as (see Exhibit 2 in Appendix A for derivation):

$$\text{Minimize } z(f,T) = \sum_{l \in L} \int_0^{x_l} c[x_l(f)]dx_l + \frac{1}{\beta} T_T^{ij} (\ln T_T^{ij} + \alpha_{TA}) + \frac{1}{\beta} T_A^{ij} \ln T_A^{ij}$$

The demand conservation constraints ensure that all trips between O-D pairs are accounted for by equating the demand for each O-D pair and the sum of flows on all paths available to travelers between this O-D pair. This constraint is of the form:

$$T^{ij} = T_A^{ij} + T_T^{ij} \qquad \forall\ i,j \qquad (5.24)$$

The auto demand conservation constraint states that the demand for auto for each O-D pair is equal to the sum of flows on all auto paths available to travelers for this O-D pair:

$$T_A^{ij} = \sum_{p_A} f_{p_A}^{ij} \qquad \forall\ i,j \qquad (5.25)$$

The transit demand conservation constraint states that the demand for transit for each O-D pair is equal to the sum of flows on all transit paths available to travelers for this O-D pair:

$$T_T^{ij} = \sum_{PT} f_{PT}^{ij} \qquad \forall\, i,j \qquad (5.26)$$

The link flow conservation constraint equates the flow on a link and the sum of flows on all paths using the link $[x_l = x_l(f_p)]$. Paths are identified by the binary parameter $\delta_{lp_m}^{ij}$; that parameter is equal to one when link l is included in mode m path p and zero otherwise. The auto occupancy rate (occ) is used to convert person trips into vehicle trips. Thus, for highway, access, and transfer links this constraint is:

$$x_l = \frac{1}{occ}\sum_{ij}\sum_{PA} \delta_{lp_A}^{ij} * f_{PA}^{ij} + \frac{1}{occ}\sum_{ij}\sum_{PT} \delta_{lp_T}^{ij} * f_{PT}^{ij} \qquad \forall\, l \subseteq \mathrm{LZ}, LA, LT \qquad (5.27)$$

The link flow conservation constraint for rail and walking links is:

$$x_l = \sum_{ij}\sum_{PA} \delta_{lp_A}^{ij} * f_{PA}^{ij} + \sum_{ij}\sum_{PT} \delta_{lp_T}^{ij} * f_{PT}^{ij} \qquad \forall\, l \subseteq LR, LW \qquad (5.28)$$

The parking capacity constraint ensures that the number of cars parked at a parking lot does not exceed the available number of parking spaces and is expressed as:

$$x_l \leq space_l \qquad \forall\, l \subseteq LT \qquad (5.29)$$

The rail capacity constraint ensures that the number of rail riders does not exceed rail capacity, defined as the train seating capacity multiplied by the load factor (λ). This constraint is expressed as:

$$x_l \leq seats * \lambda \qquad \forall\, l \subseteq LCR \qquad (5.30)$$

The last constraint of the formulation is the non negativity of the path flows:

$$f^{ij}_{p_m} \geq 0 \qquad \forall p,m,i,j \qquad (5.31)$$

The complete model statement is shown in Table 5.2.

5.3.5 Derivation of Equilibrium Conditions

To prove that a solution of the second model satisfies its equilibrium conditions, the Lagrangian of the equivalent minimization problem is formulated. Similar to the first model, the Karush-Kuhn-Tucker conditions of the problem will be shown to be identical to the equilibrium conditions. To formulate the Lagrangian of the problem the demand conservation constraints are multiplied by Lagrangian multipliers (u^{ij}_m) and introduced in the objective function. In addition, the link flow conservation constraints (equations 5.27-5.28) are directly introduced in the objective function by expressing the flow on a link as the sum of the flows of the paths using that link.

The Lagrangian is then as follows:

$$L(f,T,u) = \sum_l \int_0^{x_l} c[x_l(f)]dx_l - \sum_{ij} \int_0^{T^{ij}_T} -\frac{1}{\beta}(\ln \frac{T^{ij}_T}{T^{ij} - T^{ij}_T} + \alpha_{TA})dT^{ij}_T +$$

Table 5.2 Second Intermodal Network Equilibrium Model

$$\text{Minimize } z(f,T)=\sum_{l\in L}\int_0^{x_l} c(\phi)d\phi-\sum_{ij}\int_0^{T_m^{ij}} D_{ij}^{-1}(\omega)d\omega$$

Subject to:

$$T^{ij}=T_A^{ij}+T_T^{ij} \qquad \forall\ i,j$$

$$T_A^{ij}=\sum_{p_A} f_{p_A}^{ij} \qquad \forall\ i,j$$

$$T_T^{ij}=\sum_{p_T} f_{p_T}^{ij} \qquad \forall\ i,j$$

$$x_l=\frac{1}{occ}\sum_{ij}\sum_{p_A}\delta_{lp_A}^{ij}*f_{p_A}^{ij}+\frac{1}{occ}\sum_{ij}\sum_{p_T}\delta_{lp_T}^{ij}*f_{p_T}^{ij} \qquad \forall\ l\subseteq \mathrm{LZ},LA,LT$$

$$x_l=\sum_{ij}\sum_{p_A}\delta_{lp_A}^{ij}*f_{p_A}^{ij}+\sum_{ij}\sum_{p_T}\delta_{lp_T}^{ij}*f_{p_T}^{ij} \qquad \forall\ l\subseteq LR,LW$$

$$x_l\le space_l \qquad \forall\ l\subseteq LT$$

$$x_l\le seats*\lambda \qquad \forall\ l\subseteq LCR$$

$$f_{p_m}^{ij}\ge 0 \qquad \forall p,m,i,j$$

$$\sum_{ij} u^{ij}(T^{ij} - T_A^{ij} - T_T^{ij}) + \sum_{ij} u_A^{ij}(T_A^{ij} - \sum_{pA} f_{p_A}^{ij}) + \sum_{ij} u_T^{ij}(T_T^{ij} - \sum_{pT} f_{p_T}^{ij}) \tag{5.32}$$

with $f_{p_m}^{ij} \geq 0 \qquad \forall\ p,m,i,j$

The first derivatives of the Lagrangian with respect to path flows $f_{p_m}^{ij}$, mode m O-D trip rates T_m^{ij}, and Lagrangian multipliers u_m^{ij} are derived and used to state the optimality conditions for a stationary point.

The first derivative of the Lagrangian with respect to path flows is:

$$\frac{\partial L(f,T,u)}{\partial f_{p_m}^{ij}} = \frac{\partial z_1[x(f)]}{\partial f_{p_m}^{ij}} + \frac{\partial \sum_{ij} u_m^{ij}(T_m^{ij} - \sum_{pm} f_{p_m}^{ij})}{\partial f_{p_m}^{ij}} =$$

$$\frac{\partial z_1(x)}{\partial x_l}\frac{\partial x_l}{\partial f_{p_m}^{ij}} + \frac{\partial \sum_{ij} u_m^{ij}(T_m^{ij} - \sum_{pm} f_{p_m}^{ij})}{\partial f_{p_m}^{ij}} \tag{5.33}$$

where:

$$z_1[x(f)] = \sum_{l} \int_0^{x_l} c[x(f)]dx_l \tag{5.34}$$

From the link flow conservation constraints the following relationships are obtained:

$$x_l = \frac{1}{occ}\sum_p\sum_m \delta^{ij}_{lp_m} f^{ij}_{p_m} \text{ or } \frac{\partial x_l}{\partial f^{ij}_{p_m}} = \frac{1}{occ}\delta^{ij}_{lp_m} \qquad \forall l \in LZ, LA, LT \qquad (5.35)$$

$$x_l = \sum_p\sum_m \delta^{ij}_{lp_m} f^{ij}_{p_m} \text{ or } \frac{\partial x_l}{\partial f^{ij}_{p_m}} = \delta^{ij}_{lp_m} \qquad \forall l \in LR, LW \qquad (5.36)$$

Using (5.35) and (5.36) equation (5.33) for auto paths becomes:

$$\sum_{l \in LZ,LA,LT} \frac{1}{occ}\delta^{ij}_{lp_A}\left[\frac{\partial z_1(x_l)}{\partial x_l}\right] + \sum_{l \in LR,LW} \delta^{ij}_{lp_A}\left[\frac{\partial z_1(x_l)}{\partial x_l}\right] - u^{ij}_A =$$

$$\sum_{l \in LZ,LA,LT} \frac{1}{occ}\delta^{ij}_{lp_A}\left[c(x_l)\right] + \sum_{l \in LR,LW} \delta^{ij}_{lp_A}\left[c(x_l)\right] - u^{ij}_A \qquad \forall p_A, i, j \qquad (5.37)$$

Equation (5.33) for transit paths is written as:

$$\sum_{l \in LZ,LA,LT} \frac{1}{occ}\delta^{ij}_{lp_T}\left[c(x_l)\right] + \sum_{l \in LR,LW} \delta^{ij}_{lp_T}\left[c(x_l)\right] - u^{ij}_T \qquad \forall p_T, i, j \qquad (5.38)$$

The summations in equations (5.37) and (5.38) represent the average generalized cost on a mode m path p between an origin-destination pair ij which is the sum of the costs on all links that compose the path, and is symbolized as $GC^{ij}_{p_m}$. Thus, expression (5.37) becomes:

$$GC^{ij}_{p_A} - u^{ij}_A \qquad \forall p_A, i, j \qquad (5.39)$$

and expression (5.38) becomes:

$$GC_{p_T}^{ij} - u_T^{ij} \qquad \forall p_T, i, j \tag{5.40}$$

The derivative of the Lagrangian with respect to the auto trip rate T_A^{ij} is:

$$\frac{\partial L(f,T,u)}{\partial T_A^{ij}} = \frac{\partial \sum_{ij} u^{ij}(T^{ij} - T_A^{ij} - T_T^{ij})}{\partial T_A^{ij}} + \frac{\partial \sum_{ij} u_A^{ij}(T_A^{ij} - \sum_{p_A} f_{p_A}^{ij})}{\partial T_A^{ij}} = -u^{ij} + u_A^{ij} \tag{5.41}$$

The derivative of the Lagrangian with respect to the transit trip rate T_T^{ij} is:

$$\frac{\partial L(f,T,u)}{\partial T_T^{ij}} = \frac{\partial z_2(T)}{\partial T_T^{ij}} + \frac{\partial \sum_{ij} u^{ij}(T^{ij} - T_A^{ij} - T_T^{ij})}{\partial T_T^{ij}} + \frac{\partial \sum_{ij} u_T^{ij}(T_T^{ij} - \sum_{p_T} f_{p_T}^{ij})}{\partial T_T^{ij}} =$$

$$\frac{\partial}{\partial T_T^{ij}}\left[-\sum_{ij}\int_0^{T_T^{ij}} D_{ij}^{-1}(T_T^{ij})dT_T^{ij} + \sum_{ij} u^{ij}(T^{ij} - T_A^{ij} - T_T^{ij}) + \sum_{ij} u_T^{ij}(T_T^{ij} - \sum_{p_T} f_{p_T}^{ij})\right] =$$

$$\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) - u^{ij} + u_T^{ij} \tag{5.42}$$

The partial derivatives of the Lagrangian with respect to the Lagrangian multipliers are:

$$\frac{\partial L(f,T,u)}{\partial u^{ij}} = \frac{\partial \sum_{ij} u^{ij}(T^{ij} - T_A^{ij} - T_T^{ij})}{\partial u^{ij}} = T^{ij} - T_A^{ij} - T_T^{ij} \tag{5.43}$$

$$\frac{\partial L(f,T,u)}{\partial u_A^{ij}} = \frac{\partial \sum_{ij} u_A^{ij}(T_A^{ij} - \sum_{p_A} f_{p_A}^{ij})}{\partial u_A^{ij}} = T_A^{ij} - \sum_{p_A} f_{p_A}^{ij} \tag{5.44}$$

$$\frac{\partial L(f,T,u)}{\partial u_T^{ij}} = \frac{\partial \sum_{ij} u_T^{ij}(T_T^{ij} - \sum_{p_T} f_{p_T}^{ij})}{\partial u_T^{ij}} = T_T^{ij} - \sum_{p_T} f_{p_T}^{ij} \tag{5.45}$$

The Lagrangian multipliers u_m^{ij} represent the minimum average cost for mode m and for O-D pair ij (GC_m^{ij}). Substituting the first derivatives of the Lagrangian in the Karush-Kuhn-Tucker conditions the following equations are obtained.

From equations (5.39) and (5.40):

$$GC_{p_A}^{ij} - GC_A^{ij} \begin{cases} = 0, \text{ if } f_{p_A}^{ij} > 0 \\ \geq 0, \text{ if } f_{p_A}^{ij} = 0 \end{cases} \qquad \forall p_A, i, j \tag{5.46}$$

$$GC_{p_T}^{ij} - GC_T^{ij} \begin{cases} = 0, \text{ if } f_{p_T}^{ij} > 0 \\ \geq 0, \text{ if } f_{p_T}^{ij} = 0 \end{cases} \qquad \forall p_T, i, j \tag{5.47}$$

From expression (5.41):

$$-u^{ij} + u_A^{ij} = 0, \text{ or } u^{ij} = u_A^{ij} \qquad \forall i,j \tag{5.48}$$

Considering (5.48), expression (5.42) becomes:

$$u_T^{ij} - u_A^{ij} = GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) \qquad \forall i,j \qquad (5.49)$$

From expressions (5.43)-(5.45):

$$T^{ij} = T_A^{ij} + T_T^{ij} \qquad \forall i,j \qquad (5.50)$$

$$T_A^{ij} = \sum_{pA} f_{pA}^{ij} \qquad \forall i,j \qquad (5.51)$$

$$T_T^{ij} = \sum_{pT} f_{pT}^{ij} \qquad \forall i,j \qquad (5.52)$$

Equations (5.46) and (5.47) state that if the generalized cost on a mode m path p between i and j is greater than the generalized cost on the lowest cost mode m path between the same i and j (GC_m^{ij}), then, the corresponding flow on path p is zero. If the generalized costs are equalized, the flow on path p can be greater than or equal to zero. These conditions are the expressions of the first equilibrium condition for mode m (auto and transit).

Equation (5.49) states that the difference in the generalized cost of the two utilized modes are given from the inverted demand function. This equation is the expression of the second equilibrium condition.

Equations (5.50)-(5.52) are the demand conservation constraints for each O-D pair ij and for each mode m.

5.3.6 Convexity Analysis

To prove that the equivalent optimization program has a unique solution it is sufficient to show that the objective function is strictly convex, and that the feasible region defined by the constraints of the formulation is convex. Similar to the first model, the feasible region defined by the linear equality constraints is convex.

The first part of the objective function is:

$$z_1(x) = \sum_{l} \int_{0}^{x_l} c(x_l) dx_l$$

In this expression the link cost functions are assumed separable (i.e., $\partial c(x_a) / \partial x_b = 0 \ \forall a \neq b$, and $dc(x_a) / dx_a > 0 \ \forall a$, where a, b are links of the network) and monotonicaly increasing functions of their argument. The integral of a monotonicaly increasing function is strictly convex and the sum of strictly convex functions is strictly convex. Thus the first part of the objective function is strictly convex.

The second part of the objective is:

$$z_2(T) = -\sum_{ij} \int_{0}^{T_m^{ij}} D_{ij}^{-1}(\omega) d\omega$$

The demand function for each O-D pair (D_{ij}) is a monotonicaly decreasing function of its argument. The inverse of a monotonicaly decreasing function (D_{ij}^{-1}) is also a decreasing function. The integral of a decreasing function is strictly concave and the sum of strictly concave functions is strictly concave. The negative of a strictly concave function is strictly convex thus the second part of the objective function is strictly convex.

The objective function $z(f,T)$ is strictly convex as the sum of two strictly convex functions $z(f,T) = z_1(f) + [-z_2(T)]$. The strict convexity of $z(f,T)$ implies that the model has unique solution in terms of O-D trip rates and link flows. This solution is given by the Karush-Kuhn-Tucker conditions and was shown to satisfy the equilibrium conditions.

5.4 Third Model - Intermodal Network Mode, Access Type, and Route Choice

5.4.1 Model Assumptions

The assumptions in the third modeling approach are:

- Total travel demand for each O-D pair is fixed and known.
- Travel demand is elastic, i.e. sensitive to the travel cost on alternative modes and access types.
- Travelers have a range of modes, access types, and route choices available to them.
- Travelers' preferences are considered in the mode choice, and in the choice of access type.
- Travelers have perfect information on travel times and costs of all routes.
- Travelers are identical in their behavior.

5.4.2 Review of Demand Model Formulations

To combine demand and supply in a network equilibrium context, choice models are included in the formulation of the demand side to estimate the demand for each mode, access type, or transfer point. The scope of this section is to review various demand model formulations.

Ortuzar (1983) presented a nested logit model which formulated mode choice in cases where mixed-modes (park-and-ride, or kiss-and-ride) are options available to users. The nested logit model formulation was compared to a multinomial logit model formulation and the deficiencies of the latter in formulating intermodal decisions stated.

Forinash et. al. (1993) presented the application and interpretation of nested logit models in formulating intercity mode choice. It was shown that the nested logit, compared to the multinomial logit, better formulated mode choice in cases where some of the alternatives share common components.

Miller (1993) used a nested logit model to estimate central area mode choice and parking demand. The paper presented the work trip mode and parking location choice formulations and statistical estimation results for the Toronto Central Area.

Fan et al. (1993) modeled the access mode and station choice for commuter rail using the morning peak period work trip commute. The choices were modeled within a nested logit model. The paper addressed the inability of multinomial logit models to deal with mixed-modes of travel and the necessity of improved network modeling software for dealing with these modes.

The formulations presented above deal only with demand modeling and they have not been included in demand and supply network equilibrium formulations. In the formulation presented in this section, demand models are utilized within a network equilibrium context. The inadequacy of the multimodal logit models to formulate choices between non independent alternatives is considered and a nested logit model is utilized within the demand side of the formulation.

5.4.3 Model Statement

The problem to be formulated from the third modeling approach is: given the characteristics of an intermodal network, the total demand between each origin and destination, the link performance functions, and the demand functions which represent the mode selection process and the type of access to a mode selection process, find the trip rates for each mode between each origin-destination pair, the trip rates for each access type, and the link flow patterns for the network.

In this model, the mode choice and access type choice are formulated within the demand side of the formulation. Travelers' preferences between auto and transit, as well as between walk and drive access to transit are considered in the decision process. A nested logit model has been utilized within the demand side to formulate these choices. A representation of this model is shown in Figure 5.1.

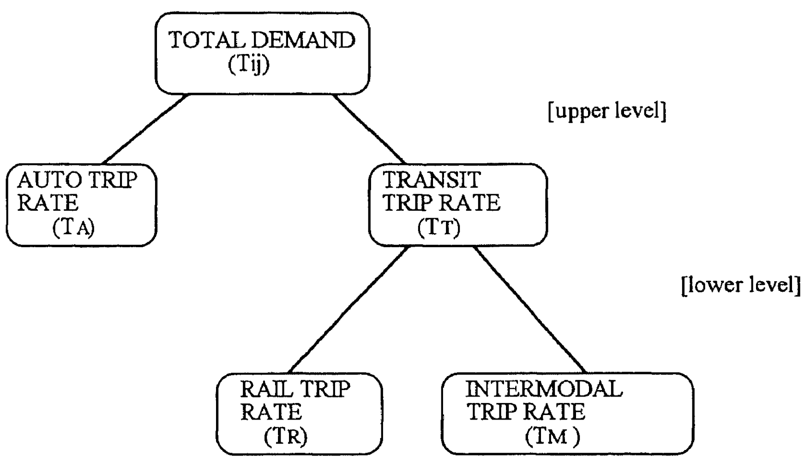


Figure 5.1 Nested Logit Model Structure

The upper level of the nested logit formulation performs the choice between auto and transit modes. The lower level performs the choice, within transit, between rail and intermodal (auto-to-rail) trips. The basic idea in this formulation is that despite the type of access to transit, which can be either walk or drive, pure rail and intermodal trips are both considered as transit trips. This structure acknowledges the greater similarity between walk-to-rail and drive-to-rail trips and the distinct nature of auto trips.

The utility of an alternative is determined from the generalized cost of using the particular alternative. The expression of the utility for each mode *m* or trip type *k* is given as:

$$U_n^{ij} = -\beta_i{*}GC_n^{ij} - \alpha_n \qquad n{=}k,\ m, \text{ and } i{=}1{,}2 \qquad (5.53)$$

Since aggregate zonal data is generally available rather than disagregate information, and under the assumption of a homogeneous group of travelers, the utility of an individual is given from the utility of the alternative that s/he uses.

The demand function used to formulate the upper level choice between auto and transit is:

$$T_A^{ij} = T^{ij}\frac{1}{1+\exp(U_T^{ij}-U_A^{ij})} \qquad (5.54)$$

with: $U_T^{ij} = -\beta_2 GC_T^{ij} - \alpha_{TA}$, and

$U_A^{ij} = -\beta_2 GC_A^{ij}$

The demand function used to formulate the lower level choice between rail and intermodal is:

$$T_R^{ij} = T_T^{ij}\frac{1}{1+\exp(U_M^{ij}-U_R^{ij})} \qquad (5.55)$$

with: $U_M^{ij} = -\beta_1 GC_M^{ij} - \alpha_{MR}$, and

$U_R^{ij} = -\beta_1 GC_R^{ij}$

Based on the properties of the nested logit model (Hartley et al. 1980) the generalized cost for transit (the non elementary alternative) can be expressed as:

$$GC_T^{ij} = -\frac{1}{\beta_1}\ln[e^{U_R^{ij}} + e^{U_M^{ij}}] \tag{5.56}$$

Expressions (5.54) and (5.55) are the demand functions D1 and D2 respectively. For a proper formulation of the nested logit model, the values of β_1 and β_2 must be between zero and one and $\beta_1 > \beta_2$ (McFadden 1979). If $\beta_1 = \beta_2$ the model collapses to a simple multinomial logit model.

The generalized cost is a function of in-vehicle time, out-of-vehicle waiting time, operating cost, and out-of-pocket cost. The same value of generalized cost is used in both mode choice and route choice. This consistency between mode choice and route choice increases the accuracy of the predictions eliminating a major deficiency of the conventional planning models.

In the supply side of the formulation the number of travelers for each mode and access type, determined from the demand side, are assigned over the actual routes of the network. The travelers are assumed to choose their routes based on the minimization of their individual generalized cost of traveling.

The proposed demand and supply network equilibrium model is a mathematical model with a non linear objective function and linear constraints. The general model statement is:

Minimize total individual user cost minus the integral of two inverse demand functions subject to:

Demand Conservation Constraints

Link Flow Conservation Constraints

Rail and Transfer Link Capacity Constraints

Non-negativity Constraints

5.4.4 Equilibrium Conditions

An equilibrium solution to the proposed model must satisfy three conditions. First, for each trip type, no traveler has an incentive to unilaterally change routes for s/he can not reduce her/his travel cost. This condition takes the mathematical form:

$$GC_{pk}^{ij} - GC_{k}^{ij} \begin{cases} = 0 & \text{if } f_{pk}^{ij} > 0 \\ \geq 0 & \text{if } f_{pk}^{ij} = 0 \end{cases} \qquad \forall k,i,j \tag{5.57}$$

This condition indicates that a type k path p from origin i to destination j is utilized only if the generalized cost on this path is equal to the minimum generalized cost for type k trips for that O-D pair.

Second, no transit user has an incentive to change trip type within each mode (i.e., no traveler has an incentive to change access type (walk or drive) to transit) for s/he can not further reduce his/her travel cost. In this case, for each O-D pair, the difference between the generalized cost for rail and intermodal trips is given as:

$$GC_{M}^{ij} - GC_{R}^{ij} = -\frac{1}{\beta_1}(\ln \frac{T_{M}^{ij}}{T_{R}^{ij}} + \alpha_{MR}) \tag{5.58}$$

Finally, no traveler has an incentive to change mode for s/he can not reduce his/her travel cost. The difference between the generalized cost for auto and transit trips is given as:

$$GC_{T}^{ij} - GC_{A}^{ij} = -\frac{1}{\beta_2}(\ln \frac{T_{T}^{ij}}{T_{A}^{ij}} + \alpha_{TA}) \tag{5.59}$$

5.4.5 Model Formulation

The objective function of this model minimizes the total individual user cost minus the integrals of the two inverse demand functions. The mathematical expression of the objective function is:

$$\min z = \sum_{l \in L} \int_0^{x_l} c(\psi) d\psi - \sum_{ij} \int_0^{T_T^{ij}} D1_{ij}^{-1}(\omega) d\omega - \sum_{ij} \int_0^{T_M^{ij}} D2_{ij}^{-1}(\varphi) d\varphi \tag{5.60}$$

where $D1^{-1}$ and $D2^{-1}$ are derived from (5.54) and (5.55) as:

$$D1_{ij}^{-1} = GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) \tag{5.61}$$

$$D2_{ij}^{-1} = GC_M^{ij} - GC_R^{ij} = -\frac{1}{\beta_1}(\ln \frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR}) \tag{5.62}$$

Using (5.61) and (5.62), (5.60) becomes:

$$\min z = \sum_{l \in L} \int_0^{x_l} c(x_l) dx_l - \sum_{ij} \int_0^{T_T^{ij}} -\frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) dT_T^{ij} - \sum_{ij} \int_0^{T_M^{ij}} -\frac{1}{\beta_1}(\ln \frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR}) dT_M^{ij}$$

Utilizing entropy models the objective function can be expressed as (see Exhibit 3 in Appendix A for derivation):

$$\min z = \sum_l \int_0^{x_l} c(x_l)dx_l + \sum_{ij}[\frac{1}{\beta_2}T_A^{ij}(\ln T_A^{ij} - \alpha_{TA}) + \frac{1}{\beta_1}T_R^{ij}\ln T_R^{ij} +$$

$$\frac{1}{\beta_1}T_M^{ij}(\ln T_M^{ij} + \alpha_{MR}) + (\frac{1}{\beta_2} - \frac{1}{\beta_1})T_T^{ij}\ln T_T^{ij}$$

The total demand conservation constraint indicates that the total demand between each origin-destination (O-D) pair is equal to the sum of the auto and transit trip rates for this O-D pair.

$$T^{ij} = T_A^{ij} + T_T^{ij} \qquad \forall\, i,j \qquad (5.63)$$

The auto demand conservation constraint indicates that the auto trip rate for an O-D pair is equal to the sum of flows on all auto paths of this O-D pair.

$$T_A^{ij} = \sum_{PA} f_{PA}^{ij} \qquad \forall\, i,j \qquad (5.64)$$

The same constraint is written for the rail and intermodal trip rates.

$$T_R^{ij} = \sum_{PR} f_{PR}^{ij} \qquad \forall\, i,j \qquad (5.65)$$

$$T_M^{ij} = \sum_{PM} f_{PM}^{ij} \qquad \forall\, i,j \qquad (5.66)$$

The demand for transit conservation constraint indicates that the transit trip rate between each O-D pair is equal to the sum of rail and intermodal trip rates between this O-D pair.

$$T_T^{ij} = T_R^{ij} + T_M^{ij} \qquad \forall\, i,j \qquad (5.67)$$

The link flow conservation constraints are written for every link on the network (highway, access, rail, walking, and transfer). The flow on each link is equated with the sum of flows on all paths using the link. Paths are identified by the binary parameter $\delta_{lp_k}^{ij}$; this parameter is equal to one when link l is included in path p_k, and zero otherwise. The auto occupancy rate (occ) is used to convert person trips to vehicle trips. Thus for each highway, access, and transfer link (i.e., $l \in LZ, LA, LT$) this constraint is:

$$x_l = \frac{1}{occ}\sum_{ij}\sum_{PA}\delta_{lp_A}^{ij} * f_{PA}^{ij} + \frac{1}{occ}\sum_{ij}\sum_{PR}\delta_{lp_R}^{ij} * f_{PR}^{ij} + \frac{1}{occ}\sum_{ij}\sum_{PM}\delta_{lp_M}^{ij} * f_{PM}^{ij} \qquad (5.68)$$

For rail and walking links (i.e., $l \in LR, LW$), the constraint is:

$$x_l = \sum_{ij}\sum_{PA}\delta_{lp_A}^{ij} * f_{PA}^{ij} + \sum_{ij}\sum_{PR}\delta_{lp_R}^{ij} * f_{PR}^{ij} + \sum_{ij}\sum_{PM}\delta_{lp_M}^{ij} * f_{PM}^{ij} \qquad (5.69)$$

The parking capacity constraint ensures that the number of cars parked at a parking lot does not exceed the available number of parking spaces.

$$x_l \le Space_l \qquad \forall\, l \in LT \qquad (5.70)$$

The rail capacity constraint insures that the number of riders in a train does not exceed the train capacity which is the seating capacity of the train multiplied with a load factor (λ).

$$x_l \leq Seats^* \lambda \qquad \forall\, l \in LCR \qquad (5.71)$$

The last constraint of the formulation is the nonegativity constraint:

$$f_{pk}^{ij} \geq 0 \qquad \forall p \in P_A, P_R, P_M \qquad (5.72)$$

The complete model statement is shown in Table 5.3.

5.4.6 Derivation of Equilibrium Conditions

To prove that a solution of the third model satisfies the equilibrium conditions, the Lagrangian of the equivalent minimization problem is formulated. As it was done in the previous models the Karush-Kuhn-Tucker conditions of the problem will be shown to be identical to the equilibrium conditions. To formulate the Lagrangian of the problem, the demand conservation constraints are multiplied by Lagrangian multipliers (u_k^{ij}) and introduced in the objective function. In addition, the link flow conservation constraints (equations 5.68-5.69) are directly introduced in the objective function by expressing the flow on a link as the sum of the flows of the paths that are using that link.

The Lagrangian of the problem takes the form:

$$L(f,T,u) = \sum_l \int_0^{x_l} c(x_l)dx_l + \sum_{ij} \int_0^{T_T^{ij}} \frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA})dT_T^{ij} +$$

Table 5.3 Third Intermodal Network Equilibrium Model

$$\min z = \sum_{l \in L} \int_0^{x_l} c(\psi)d\psi - \sum_{ij} \int_0^{T_T^{ij}} D1_{ij}^{-1}(\omega)d\omega - \sum_{ij} \int_0^{T_M^{ij}} D2_{ij}^{-1}(\varphi)d\varphi$$

subject to:

$$T^{ij} = T_A^{ij} + T_T^{ij} \qquad \forall\ i,j$$

$$T_A^{ij} = \sum_{p_A} f_{p_A}^{ij} \qquad \forall\ i,j$$

$$T_R^{ij} = \sum_{p_R} f_{p_R}^{ij} \qquad \forall\ i,j$$

$$T_M^{ij} = \sum_{p_M} f_{p_M}^{ij} \qquad \forall\ i,j$$

$$T_T^{ij} = T_R^{ij} + T_M^{ij} \qquad \forall\ i,j$$

$$x_l = \frac{1}{occ} \sum_{ij} \left[\sum_{p_A} \delta_{lp_A}^{ij} * f_{p_A}^{ij} + \sum_{p_R} \delta_{lp_R}^{ij} * f_{p_R}^{ij} + \sum_{p_M} \delta_{lp_M}^{ij} * f_{p_M}^{ij} \right] \qquad \forall\ l \in \mathrm{LZ}, LA, LT$$

$$x_l = \sum_{ij} \left[\sum_{p_A} \delta_{lp_A}^{ij} * f_{p_A}^{ij} + \sum_{p_R} \delta_{lp_R}^{ij} * f_{p_R}^{ij} + \sum_{p_M} \delta_{lp_M}^{ij} * f_{p_M}^{ij} \right] \qquad \forall\ l \in LR, LW$$

$$x_l \le Space_l \qquad \forall\ l \in LT$$

$$x_l \le Seats * \lambda \qquad \forall\ l \in LCR$$

$$f_{p_k}^{ij} \ge 0 \qquad \forall p \in P_A, P_R, P_M$$

$$\sum_{ij}\int_0^{T_M^{ij}}\frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_T^{ij}-T_M^{ij}}+\alpha_{MR})dT_M^{ij}+\sum_{ij}u^{ij}(T^{ij}-T_A^{ij}-T_T^{ij})+\sum_{ij}u_T^{ij}(T_T^{ij}-T_R^{ij}-T_M^{ij})+$$

$$\sum_{ij}u_A^{ij}(T_A^{ij}-\sum_{pA}f_{pA}^{ij})+\sum_{ij}u_R^{ij}(T_R^{ij}-\sum_{pR}f_{pR}^{ij})+\sum_{ij}u_M^{ij}(T_M^{ij}-\sum_{pM}f_{pM}^{ij})$$

with: $f_{pk}^{ij}\geq 0, \qquad \forall p\in P_A,P_R,P_M$

The variables of the above formulation are the path flows (f_{pk}^{ij} , for $k=A,R,M$), and the demand variables (T_A^{ij}, T_T^{ij}, T_R^{ij}, T_M^{ij}).

The partial derivatives of the Lagrangian with respect to the variables are (see Exhibit 4 in Appendix A for derivations):

$$\frac{\partial L}{\partial f_{pA}^{ij}}=GC_{pA}^{ij}-u_A^{ij} \qquad \forall ij \qquad (5.73)$$

$$\frac{\partial L}{\partial f_{pR}^{ij}}=GC_{pR}^{ij}-u_R^{ij} \qquad \forall ij \qquad (5.74)$$

$$\frac{\partial L}{\partial f_{pM}^{ij}}=GC_{pM}^{ij}-u_M^{ij} \qquad \forall ij \qquad (5.75)$$

$$\frac{\partial L}{\partial T_A^{ij}}=-u^{ij}+u_A^{ij} \qquad \forall ij \qquad (5.76)$$

$$\frac{\partial L}{\partial T_T^{ij}}=\frac{1}{\beta_2}(\ln\frac{T_T^{ij}}{T^{ij}-T_T^{ij}}+\alpha_{TA})-u^{ij}+u_T^{ij}+\frac{1}{\beta_1}\ln\frac{T_T^{ij}-T_M^{ij}}{T_T^{ij}} \qquad \forall ij \qquad (5.77)$$

$$\frac{\partial L}{\partial T_R^{ij}} = -u_T^{ij} + u_R^{ij} \qquad \forall ij \qquad (5.78)$$

$$\frac{\partial L}{\partial T_M^{ij}} = \frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_T^{ij} - T_M^{ij}} + \alpha_{MR}) - u_T^{ij} + u_M^{ij} \qquad \forall ij \qquad (5.79)$$

$$\frac{\partial L}{\partial u^{ij}} = T^{ij} - T_A^{ij} - T_T^{ij} \qquad \forall ij \qquad (5.80)$$

$$\frac{\partial L}{\partial u_T^{ij}} = T_T^{ij} - T_R^{ij} - T_M^{ij} \qquad \forall ij \qquad (5.81)$$

$$\frac{\partial L}{\partial u_A^{ij}} = T_A^{ij} - \sum_{pA} f_{pA}^{ij} \qquad \forall ij \qquad (5.82)$$

$$\frac{\partial L}{\partial u_R^{ij}} = T_R^{ij} - \sum_{pR} f_{pR}^{ij} \qquad \forall ij \qquad (5.83)$$

$$\frac{\partial L}{\partial u_M^{ij}} = T_A^{ij} - \sum_{pM} f_{pM}^{ij} \qquad \forall ij \qquad (5.84)$$

Setting the above derivatives equal to zero and considering that the Lagrangian multipliers u_k^{ij} represent the minimum average cost for mode k and for O-D pair ij (GC_k^{ij}) the optimality conditions are obtained. From equations (5.73), (5.74) and (5.75) we have:

$$GC_{pA}^{ij} - GC_A^{ij} \begin{cases} = 0 & \text{if } f_{pA}^{ij} > 0 \\ \geq 0 & \text{if } f_{pA}^{ij} = 0 \end{cases} \qquad \forall ij$$

$$GC_{pR}^{ij} - GC_R^{ij} \begin{cases} = 0 & \text{if } f_{pR}^{ij} > 0 \\ \geq 0 & \text{if } f_{pR}^{ij} = 0 \end{cases} \qquad \forall ij$$

$$GC_{pM}^{ij} - GC_M^{ij} \begin{cases} = 0 & \text{if } f_{pM}^{ij} > 0 \\ \geq 0 & \text{if } f_{pM}^{ij} = 0 \end{cases} \qquad \forall ij$$

which are the expressions of the <u>first equilibrium condition</u>.

From equation (5.78) we have: $u_T^{ij} = u_R^{ij}$, and thus equation (5.79) becomes:

$$u_R^{ij} - u_M^{ij} = \frac{1}{\beta_1}(\ln \frac{T_M^{ij}}{T_T^{ij} - T_M^{ij}} + \alpha_{MR})$$

which for positive rail and intermodal path flows is the expression of the <u>second equilibrium condition</u>:

$$GC_{pM}^{ij} - GC_{pR}^{ij} = -\frac{1}{\beta_1}(\ln \frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR})$$

Assuming a positive transit trip rate (T_T^{ij}>0) and considering $u^{ij} = u_A^{ij}$ from equation (5.76) and $u_T^{ij} = u_R^{ij}$ from equation (5.78), equation (5.77) becomes:

$$GC_R^{ij} - GC_A^{ij} + \frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA}) + \frac{1}{\beta_1}\ln \frac{T_T^{ij} - T_M^{ij}}{T_T^{ij}} = 0, \text{ or}$$

$$[GC_R^{ij} + \frac{1}{\beta_1}\ln \frac{T_R^{ij}}{T_T^{ij}}] - GC_A^{ij} + \frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA}) = 0 \qquad (5.85)$$

By definition:

$$T_R^{ij} = T_T^{ij} \frac{e^{U_R^{ij}}}{e^{U_R^{ij}} + e^{U_M^{ij}}}, \text{ or } \ln \frac{T_R^{ij}}{T_T^{ij}} = U_R^{ij} - \ln(e^{U_R^{ij}} + e^{U_M^{ij}}), \text{ or}$$

$$\ln \frac{T_R^{ij}}{T_T^{ij}} = -\beta_1 * GC_R^{ij} - \ln(e^{U_R^{ij}} + e^{U_M^{ij}})$$

Substituting in equation (5.85) we obtain:

$$\{GC_R^{ij} + \frac{1}{\beta_1}[-\beta_1 * GC_R^{ij} - \ln(e^{U_R^{ij}} + e^{U_M^{ij}})]\} - GC_A^{ij} + \frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA}) = 0, \text{ or}$$

$$-\frac{1}{\beta_1}\ln(e^{U_R^{ij}} + e^{U_M^{ij}}) - GC_A^{ij} + \frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA}) = 0$$

considering equation (5.56), the above equation becomes:

$$GC_T^{ij} - GC_A^{ij} + \frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA}) = 0, \text{ or}$$

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta_2}(\ln \frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA})$$

which is the expression of the <u>third equilibrium condition</u>.

Setting the derivatives of the Lagrangian with respect to the Lagrangian multipliers (5.80)-(5.84) equal to zero the demand conservation constraints are obtained as they are given in equations (5.63)-(5.67).

5.4.7 Convexity Analysis

To prove that the equivalent optimization program has a unique solution it is sufficient to show that the objective function is strictly convex everywhere and that the feasible region defined by the constraints of the formulation is convex. As it was shown in the first two models, the feasible region defined by the linear equality constraints is convex.

The first part of the objective function which is expressed as

$$\sum_{l \in L} \int_0^{x_l} c(\psi) d\psi$$

is strictly convex everywhere as it was shown in the first two formulations. The link cost functions are separable (i.e., $\partial c(x_a) / \partial x_b = 0 \;\; \forall a \neq b$, and $dc(x_a) / dx_a > 0 \;\; \forall a$, where a, b are links of the network) and monotonicaly increasing functions of their argument. The integral of a monotonicaly increasing function is strictly convex and the sum of strictly convex functions is strictly convex.

The demand functions are monotonicaly decreasing functions of their argument. Their inverse are also decreasing functions. The integrals of decreasing functions are strictly concave and the sum of strictly concave functions is a strictly concave function. The negative of a strictly concave function is strictly convex. Thus the second and third parts of the objective function expressed as:

$$-\sum_{ij} \int_{0}^{T_T^{ij}} D1_{ij}^{-1}(\omega)d\omega, \text{ and}$$

$$-\sum_{ij} \int_{0}^{T_M^{ij}} D2_{ij}^{-1}(\varphi)d\varphi$$

respectively, are also strictly convex. The objective function is strictly convex as the sum of strictly convex functions. The strict convexity of the objective function implies that the problem has a unique solution in terms of link flows and trip rates. Therefore, a solution to the third problem is unique and satisfies the equilibrium conditions.

CHAPTER 6

CASE STUDY--AN INTERMODAL COMMUTER NETWORK

6.1 Introduction

The models presented in Chapter 5 can be used to make predictions regarding the modal shares, flow patterns, and associated travel costs on an intermodal network. The application of the models is accomplished by developing an initial methodological framework that utilizes the network equilibrium models presented in Chapter 5 to analyze a real-world intermodal commuter network. The scope of this chapter is to describe the network to be analyzed and present the methodological framework and the method of solution.

6.2 Assumptions and Input Data

The intermodal commuter network to be analyzed is shown in Figure 6.1. This network is a realistic representation of a portion of the Raritan Valley Corridor located in Union County, NJ. Its five origins are Westfield, Garwood, Cranford, Kenilworth, and Roselle Park (designated O1 through O5), and the destination is Newark (D). The network is composed of three major highways (I-78, Route 22, and Garden State Parkway (GSP)), local county roads which run between the major highways, and a NJ Transit commuter rail line.

The case study required that the following data, classified into two groups (geometric and demand/supply), were collected:

Geometric Data:

- origin-destination pair locations
- centroids of origins and destinations

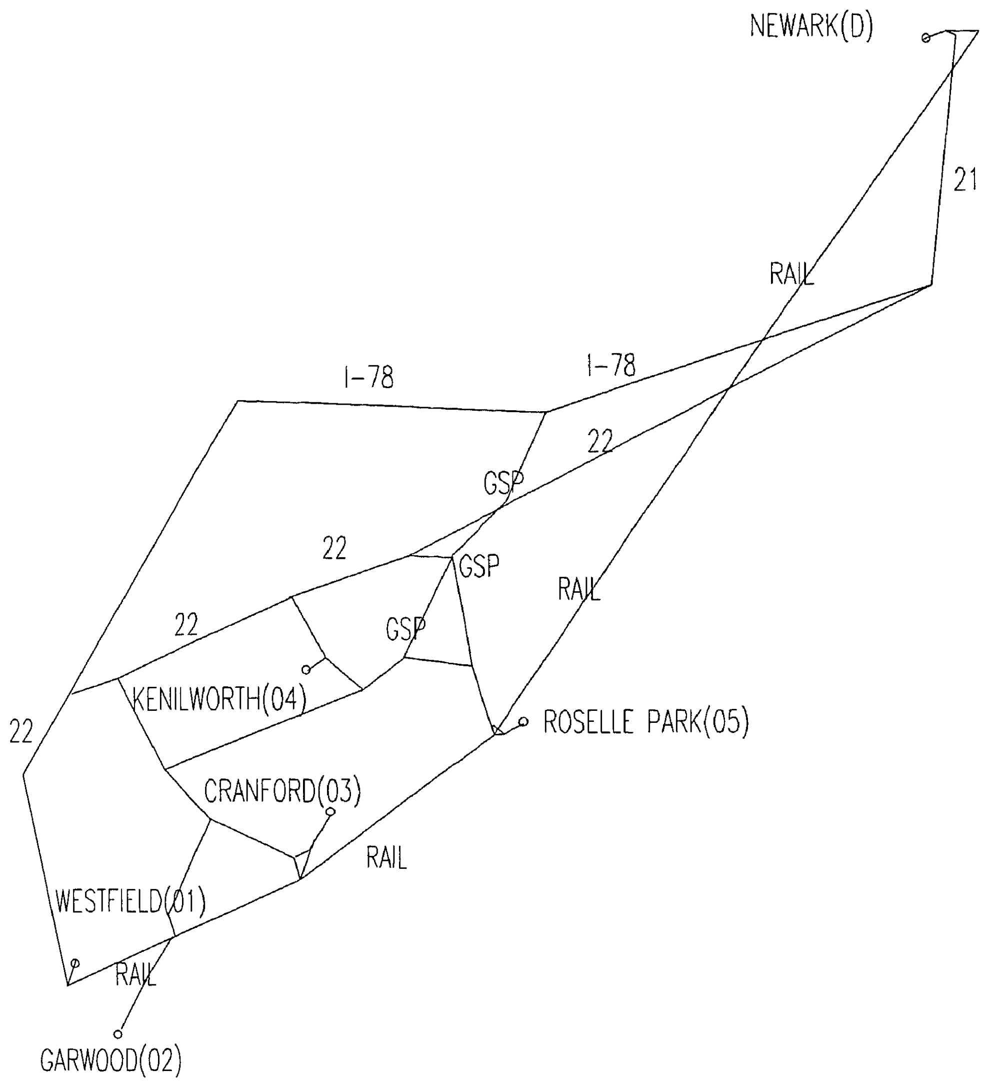


Figure 6.1 Intermodal Commuter Network

- link locations
- link capacities
- link free-flow travel times
- link costs
- paths between O-D pairs.

Demand/Supply:

- origin-destination demands
- background flows originating outside the study network
- frequency and capacity of trains
- rail station parking capacities.

The data that were used as model inputs, their sources, and the assumptions made are presented in detail in the sub-sections that follow.

6.2.1 Origin-Destination Pair Locations

The first step taken in defining the study network is to identify areas where commuters originate their trips and the area were they are destined to. Once the origin (Westfield, Garwood, Cranford, Kenilworth, and Roselle Park) and destination (Newark) areas are identified, it is necessary to define the centroids of these areas. For modeling purposes the population of origin and destination areas is condensed into centroids and it is assumed that all trips originate from, and are destined to, a centroid.

6.2.2 Links and Paths

The network consists of 60 links. These links are grouped into 26 paths. The most traveled paths between each O-D pair are identified and used in the analysis. This process eliminates many paths between O-D pairs, primarily because they either utilize local roads or are circuitous. The paths are divided into auto, rail, and intermodal paths.

The type and number of paths between each origin and destination is given in Table 6.1.

Table 6.1 Links and Paths of the Study Network

	Destination -- Newark		
	Number of Paths		
Origin	Auto	Rail	Intermodal
Westfield	3	1	4
Garwood	2	1	3
Cranford	2	1	2
Kenilworth	2	0	1
Roselle Park	2	1	1

6.2.3 Link Capacities

Highway link capacities depend on the facility type (i.e., arterial, freeway). The highway link capacities are calculated using the methodologies set forth in the Highway Capacity Manual (1985) for each classification. Access links are also considered as arterial links and have the corresponding capacities. All highway links are computed assuming a level of service "C" which represents stable flow on the network.

The highway link capacities are calculated as follows:

Arterial Links:

Assumptions:
- green to cycle ratio g/c = 0.65
- 2-Lane Roadways

Capacity = 1,600*0.65 = 1040 vph

I-78 and Garden State Parkway Links:

Assumptions:
- 70 mph Design Speed
- 10-Lane Roadways

Capacity = 1,550 pcphpl*5 lanes = 7,750 pcph

Route 22 Links:

Assumptions: • 60 mph Design Speed

• 4-Lane Roadway

Capacity = 1,300 pcphpl*2 lanes = 2,600 pcph

Route 21 Links

Assumptions: • green to cycle ratio g/c = 0.65

• 4-Lane Roadway

Capacity = 1,600*0.65*2 = 2080 vph

6.2.4 Link Free-Flow Travel Times

Highway link free-flow travel times are determined by taking the free-flow speed of each highway link and dividing by the distance of that link. Free-flow speeds are assumed to be 55 mph for I-78 and GSP, 50 mph for Routes 21 and 22, and 30 mph for all arterials.

Transfer link free-flow travel times are assumed to be four minutes which represents parking time and access to the train platform. Walking link free-flow travel times are determined by the distances of each walking link. Based on these distances, free-flow travel times are calculated assuming an average walking rate of 4.5 ft/sec. Rail link travel times are derived from the train schedule. The trains on the route stop at each station.

6.2.5 Origin-Destination Demands

Demand between each origin and destination pair is taken from U.S. Bureau of the Census (1990) data. Data is aggregated, and the following O-D demands are used for the 1995 baseline year:

Table 6.2 Origin to Destination Demands

Origin	Destination -- Newark (trips)
Westfield	540
Garwood	130
Cranford	620
Kenilworth	220
Roselle Park	920

6.2.6 Background Volumes

Background volumes represent the vehicles that either have their origin or their destination outside the study area. These vehicles contribute to the congestion of the network, and their interaction with the assigned trips will impact the travel times.

Since there is no comprehensive trip Origin-Destination matrix for background traffic in the study area, the background volumes are estimated. Background volumes are determined by consulting the New Jersey Highway Straight Line Diagrams (1988) a publication that contains volume data for I-78, Route 22, and Route 21 links. In addition, volume data for the Garden State Parkway is obtained through a network study performed by Vollmer Associates (1987). A 1995 baseline year is established, and a two percent per year compounded growth rate is used to expand volume data that were recorded before 1995.

Volume data for all the local highway links is unavailable. Therefore, background volumes for these links is assumed to be sixty percent of their capacities.

Rail link background volumes are determined by consulting ridership data that was provided by NJ Transit. All demands at stations west of Westfield are summed, and the total is used as the background volume for the rail links of the study network.

6.2.7 Frequency and Capacity of Trains

The rail service frequency in the study network is determined from the train schedule to be three trains per hour. The capacity of each train is estimated at 500 seats. This estimate is based on each train consisting of four cars, each car having a seating capacity of approximately 125 seats.

6.2.8 Rail Station Parking Capacities

The number of parking spaces for each rail station, shown in Table 6.3, was obtained from NJ Transit data.

Table 6.3 Current Parking Capacities at Stations

Rail Station	Available Parking (spaces)
Westfield	759 spaces
Garwood	0 spaces
Cranford	373 spaces
Roselle Park	239 spaces

6.2.9 Time-Volume Function

The average travel time on a highway link is assumed to be a function of volume on the link. Therefore, each link is characterized by its performance or volume-travel time function. The time-volume function is assumed to be a modified Bureau of Public Roads (1965) congestion curve of the following form:

$$t_l = t_{fl} * [1 + 0.15(\frac{x_l}{cap_l})^4]$$

where:

t_l = average travel time

t_{fl} = free-flow travel time

6.2.10 Out-of- Pocket Costs

Out-of-pocket costs, such as Garden State Parkway (GSP) tolls, NJ Transit rail fares, parking fees, and vehicle operating costs, are also considered in the model. A standard toll of $0.35 is introduced at the GSP link. The rail fares and parking fees were obtained from NJ Transit and are shown in Table 6.4.

Table 6.4 Rail Fares and Parking Fees

Origin	Rail Fare ($)	Parking Fee ($)
Westfield	2.15	3.00
Garwood	2.15	2.00
Cranford	1.75	1.15
Roselle Park	1.50	1.50
Newark	N/A	5.00

A charge of $0.25 per vehicle-mile is included to calculate vehicle operating cost. The total operating cost for a vehicle trip is then obtained by multiplying this charge by the distance traveled. To translate travel time into time-based costs so that the total network assignment costs can be compared, the travel time is multiplied by the value of time. An average value of time of $20 per hour was chosen.

While actual demand and traffic volume data are used in the analysis, the binary logit type and nested logit type mode choice model coefficients for the models presented in Chapter 5 were estimated to be $\beta = 0.06$ and $\alpha = 0.8$ for the second model and $\beta_1 = 0.06$, $\beta_2 = 0.03$, $a_{TA} = 0.6$, and $\alpha_{MR} = 0.45$ for the third model. Obviously, the mode choice-traffic assignment model can be easily re-run once accurate and origin-specific mode choice coefficients are available.

6.3 Methodological Framework

The framework introduced in this section is designed to answer questions of interest to transportation planners (whether is it possible to assign travelers to the network more efficiently, what are the benefits of the improved assignment, and at what cost are these benefits achieved) and to investigate the trade-offs between the reduction in travel time and the cost of capacity increases. The model answers the above questions by providing:

- the equilibrium assignment of flows over an intermodal network,
- the total network travel cost for each policy considered,
- the incremental changes in cost,
- the estimation of benefits for each alternative,
- the rail service and parking capacity additions that are needed to accommodate an increase in rail ridership.

The framework for analyzing the effects of various policies on the network flow patterns and associated travel costs in an intermodal network is shown in Figure 6.2.

Given an intermodal highway-commuter rail transportation network, the framework starts with the collection of input data. This data consists of the available technology, network characteristics, functions that relate traffic volumes, capacities, and travel times, time impedances, out-of-pocket costs (rail fares, tolls, parking fees, auto operating cost), commuter rail data (frequency and capacity of trains and capacities of station parking facilities), total traffic volumes for each O-D, and travel demand parameters. In addition, background traffic volumes that originate and/or terminate outside the study network are also collected.

The data is used to formulate the demand and supply functions which, together with an assumed behavioral principle, are entered into a combined mode choice-traffic assignment network equilibrium model. The model to be used at this part of

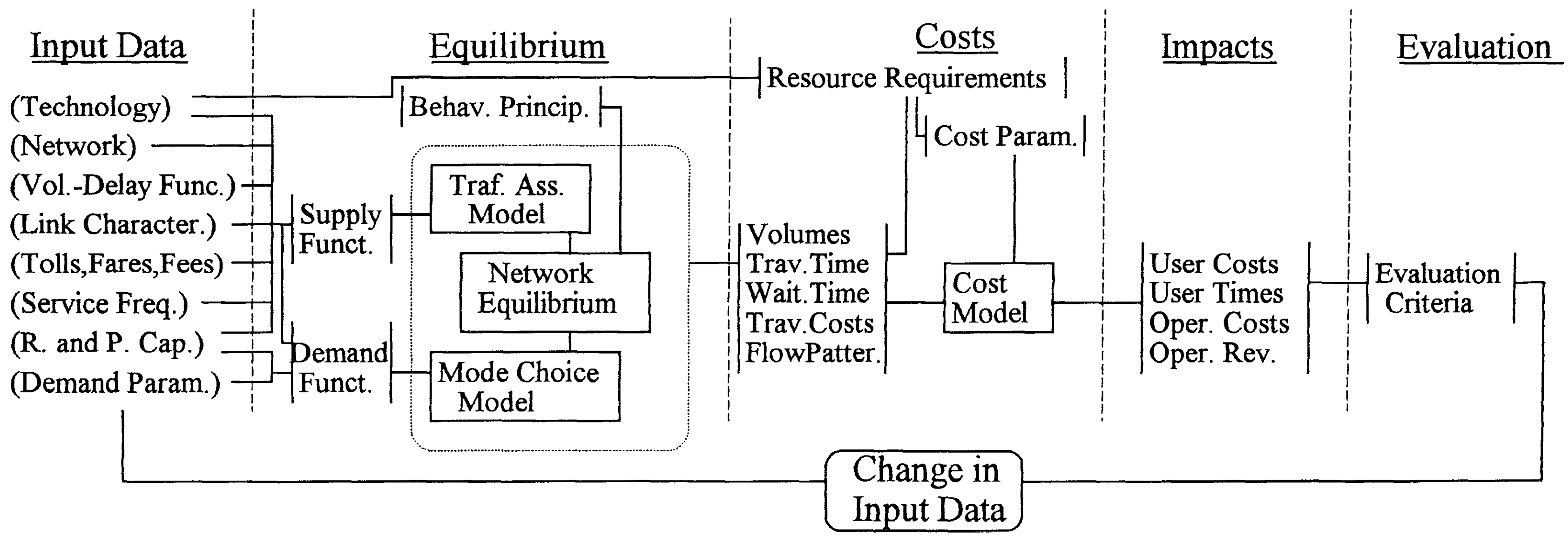


Figure 6.2 Methodological Framework

the network can be any of the combined mode choice-traffic assignment models described in Chapter 5.

The model calculates modal shares, equilibrium flow patterns, and the resulting generalized cost (a sum of out-of-pocket costs, in-vehicle, and out-of-vehicle time costs) of the network assignment. These values, together with various cost parameters estimated on the basis of resource requirements, are entered in a cost model which determines the user monetary and time costs and the costs and revenues of the operator of the network facilities.

The final part of the framework is the evaluation module which evaluates the impact of the analyzed policy by trading off user costs and travel times with operator costs and revenues, and determines the desirability of the policy.

The first policy to be analyzed is the existing situation on the network. Results of the analysis of this policy can serve as a basis for comparison with various alternative policies. The alternative policies can be developed by selecting (and then later changing) the input parameters. Examples of such a change in input parameters can be an increase in the parking spaces at a station parking lot, a decrease in rail fares, an increase in highway tolls, an increase in rail service frequency, or a combination of the above. The new changes are introduced in the input module of the methodological framework, formulating the new policy. The new policy is then analyzed following the procedure that was described in this section and evaluated according to its benefits in comparison with the existing situation.

6.4 Method of Solution

The General Algebraic Modeling System (GAMS) (Brooke et. al. 1988) is used to solve the problems formulated in this dissertation. GAMS is designed to easily represent and solve large scale linear, nonlinear, and integer mathematical programs, utilizing appropriate solvers for each type of problem.

To solve the linearly constrained nonlinear problems a powerful nonlinear solver, called MINOS 5.1 (Murtagh et al. 1987), is implemented within GAMS. MINOS 5.1 solves such problems using well known techniques (reduced-gradient algorithm in conjunction with a quasi-Newton algorithm).

Two problems arise in the nonlinear optimization. The first is to find a solution to the nonlinear program, and the second is to show that this solution is the best (global optimum) among all possible solutions. The second problem is an unresolved research topic. However, this problem does not exist in the formulations presented in Chapter 5 since it was shown that the models have only one solution. MINOS 5.1 deals with the first problem by finding a solution to the linearly constrained nonlinear programs as it will be shown later.

CHAPTER 7

RESULTS OF CASE STUDY ANALYSIS

7.1 Introduction

The three models presented in Chapter 5 are used to analyze the intermodal network described in Chapter 6. The scope of this analysis is twofold. First the numerical results are used to verify the equilibrium conditions, and second, the models are used within the methodological framework to analyze various policies on the intermodal network.

7.2 First Model - Intermodal Network Route Choice

7.2.1 Verification of First Models' Equilibrium Condition

Results of the analysis of the network using the first model are shown in Table 7.1. These results are used to verify the equilibrium condition. The purpose is to show that the numerical solver (MINOS 5.1) used within GAMS was able to reach the equilibrium solution. Table 7.1 shows the generalized cost of traveling on each path of the network and the flow on each utilized path.

The rail and parking capacity constraints are not considered at this point of the analysis for reasons explained in Chapter 5.

The numerical results from Table 7.1 show that for each origin-destination pair the cost of traveling on all utilized paths is the same and it is less than or equal to the cost of traveling on any unutilized path.

It is shown, for example, that the utilized paths P3, P9, and P21 are the minimum cost paths for O1-D, O2-D, and O4-D, respectively. Every other path in the same O-D pair has higher travel cost. O3-D and O5-D have three utilized paths each.

For each of the two O-D pairs, the utilized paths have the same travel cost which is less than the cost on the unutilized paths.

Table 7.1 First Model Results to Verify Equilibrium Condition

O-D pair	Path	Travel Cost	Path Flow
O1-D	P1 (auto)	24.08	
	P2 (auto)	33.22	
	P3 (auto)	23.25	540
	P4 (intermodal)	28.87	
	P5 (intermodal)	34.04	
	P6 (intermodal)	31.51	
	P23 (intermodal)	26.79	
	P7 (rail)	25.34	
O2-D	P8 (auto)	31.10	
	P9 (auto)	22.14	130
	P10 (intermodal)	23.70	
	P11 (intermodal)	24.91	
	P24 (intermodal)	24.66	
	P12 (rail)	23.53	
O3-D	P13 (auto)	31.35	
	P14 (auto)	22.38	65
	P15 (intermodal)	25.16	
	P25 (intermodal)	22.38	373
	P16 (rail)	22.38	182
O4-D	P20 (auto)	24.66	
	P21 (auto)	19.84	220
	P22 (intermodal)	22.62	
	P27 (rail)	59.96	
O5-D	P17 (auto)	22.79	
	P18 (auto)	21.12	253
	P26 (intermodal)	21.12	581
	P19 (rail)	21.12	86

7.2.2 Policies Analyzed from the First Model

Results from the analysis of the network according to the first model are shown in Table 7.2. The table shows the flows for each path type (Auto, Rail, Intermodal) in users/peak hour and the resulting path costs in $/user for various policies.

The analyzed policies are the following:

Policy 1: Baseline Case

This policy models the current situation and represents the "Do Nothing" alternative. It serves as a basis for comparison.

Policy 2: Increase Train Frequency

This policy doubles the frequency from three to six trains per hour, thus, halving the average waiting time.

Policy 3: Increase Parking Capacity

The analysis of the baseline case indicated that some parking lots operate at capacity preventing an increase in intermodal commute. This policy expands parking capacity by adding 60 new spaces to the lot at O5.

Policy 4: Increase Tolls

Highway tolls are doubled (from $0.35 to $ 0.70) to induce travelers to shift from auto to rail.

Policy 5: Increase Parking Fees at the CBD parking lot.

The CBD parking fee is increased by 20% ($1.00). The objective is to make the auto trip less attractive and reduce the number of auto commuters.

Policy 6: Decrease Rail Fares.

The rail fare for each trip is reduced by 20% to decrease the cost of rail and intermodal travel and attract more people to rail.

Policy 7: Combination of policies.

A combination of some of the above described policies is used to formulate policy 7. Results from policy three indicate that, even after the addition of 60 spaces at O5, the parking lot is still fully utilized. According to the dual price of the parking capacity constraint, which is provided by the model, the user cost can further

Table 7.2 Path Flows and Costs for Each Policy According to the First Model

		Policy 1		Policy 2		Policy 3		Policy 4		Policy 5		Policy 6		Policy 7	
O-D Pair	Path Type*	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost
O1-D	A	540	23.26	540	22.67	540	23.25	540	23.55	540	24.08	540	23.21	540	23.67
	R														
	I														
O2-D	A	130	22.14	130	20.64	130	22.138	130	21.15	130	22.20	130	21.81	130	21.64
	R														
	I														
O3-D	A	49	22.38			51	22.38	21	22.38						
	R	198	22.38	247	20.72	196	22.38	226	22.38	247	22.38	247	22.03	247	20.72
	I	373	22.38	373	20.71	373	22.38	373	22.38	373	22.37	373	22.03	373	20.71
O4-D	A	220	19.81	220	18.37	220	22.62	220	19.87	220	19.93	220	19.55	220	19.37
	R														
	I														
O5-D	A	272	21.12			269	21.12	244	21.12	160	21.12	270	20.82		
	R	409	21.12	681	19.46	352	21.12	437	21.12	521	21.12	411	20.82	581	19.46
	I	239	21.09	239	19.42	299	21.10	239	21.09	239	21.09	239	20.80	339	19.42
Total	A	1211		890		1210		1155		1050		1160		890	
	R	607		928		548		663		768		658		828	
	I	612		612		672		612		612		612		712	

* A=Auto, R=Rail, I=Intermodal

decrease by increasing the number of parking spaces at O5. As a result, policy 7 adds 40 spaces to the lot at O5 in addition to those added under policy 3. Also, policy 7 increases the CBD parking fee by 20% (from \$5 to \$6) and doubles the rail frequency from three to six trains per hour.

Table 7.2 shows how travelers shift from auto to transit as a result of transit improvements and/or increased highway impedances and how this shift affects their travel times. The last three rows of the table show the number of auto, rail, and intermodal users on the entire network.

7.2.3 Policy Analysis

7.2.3.1 Policy Effects on Modal Shares. The policy effects in terms of modal shares (in percentages) are shown in Table 7.3 which contains the modal shares for each policy and for each origin-destination pair. It is shown, for example, that as a result of policy 2 (doubling the rail frequency) 8% of auto users from O3 and 29.50% of auto users from O5 are shifted to rail. There is no shift to intermodal paths since the parking lots that are attractive for intermodal commute are fully utilized. Policy 3 added capacity to the parking lot at O5 and resulted in a 6.5% increase in intermodal trips. In general, the policies result in auto travelers shifting to rail and only for those policies where additional parking spaces are provided there is an increase in intermodal trips.

The percent increase or decrease of the total network modal shares for each policy in comparison with the baseline case are shown in Figure 7.1. The auto share decreased for every policy while the intermodal share either increased or remained unchanged. Rail share increased also, except for Policy 3.

7.2.3.2 User and Operator Impacts. The effects of policies in terms of user and operator impacts are presented in Table 7.4. The second and third columns of the table present the user cost for the whole network in $/peak hour and the user time for the whole network in min/peak hour, respectively. The user cost is estimated from the model as the sum of the travel costs of all the travelers on the network and the user time as the sum of the travel times of all the travelers on the network.

The fourth column represents the capital investment required for the addition of parking lot capacity in $/peak hour. The cost of parking space addition per peak hour is estimated by assuming that the cost of acquisition per space is $ 10,000 and its life is 30 years. Using 7% interest (Capital Recovery Factor 0.08059), the annual capital cost of expanding parking capacity by one space is $ 805.90. With a 0.5 peak allocation factor and 265 working days per year the daily cost per peak hour is $1.52 per space.

The fifth column represents the five origin rail station parking revenues in $/peak hour. These revenues are estimated as the number of the parking lot users multiplied by the parking fee for each particular parking lot.

The rail operating cost, presented in the next column, includes maintenance and overhead as well as the more direct cost of operation (operator wages, fuel, spare parts, etc.) and is represented by the all inclusive hourly operating cost per vehicle c. The total hourly operator cost is obtained by multiplying the active fleet with the hourly operating cost per vehicle. The total round trip time is the round trip length divided by the average speed.

The total hourly operator cost is given as:

$$C = \frac{2cL}{HV}$$

where the variables and their values are:

Table 7.3 Modal Shares for each O-D Pair According to the First Model

Policy	O-D Pair	Auto (%)	Rail (%)	Intermodal (%)
1--Baseline	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D	8.00	31.80	60.20
	O4 - D	100.00		
	O5 - D	29.50	44.50	26.00
2--Double Rail Frequency	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D		39.80	60.20
	O4 - D	100.00		
	O5 - D		74.00	26.00
3--Increase Parking Capacity	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D	8.30	31.50	60.20
	O4 - D	100.00		
	O5 - D	29.20	38.30	32.50
4--Increase Highway Tolls	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D	3.40	36.40	60.20
	O4 - D	100.00		
	O5 - D	26.50	47.50	26.00
5--Increase CBD Parking Fee	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D		39.80	60.20
	O4 - D	100.00		
	O5 - D	17.40	56.60	26.00
6--Decrease Rail Fare	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D		39.80	60.20
	O4 - D	100.00		
	O5 - D	29.40	44.60	26.00
7--Increase rail frequency and CBD parking fee and parking capacity	O1 - D	100.00		
	O2 - D	100.00		
	O3 - D		39.80	60.20
	O4 - D	100.00		
	O5 - D		63.20	36.80

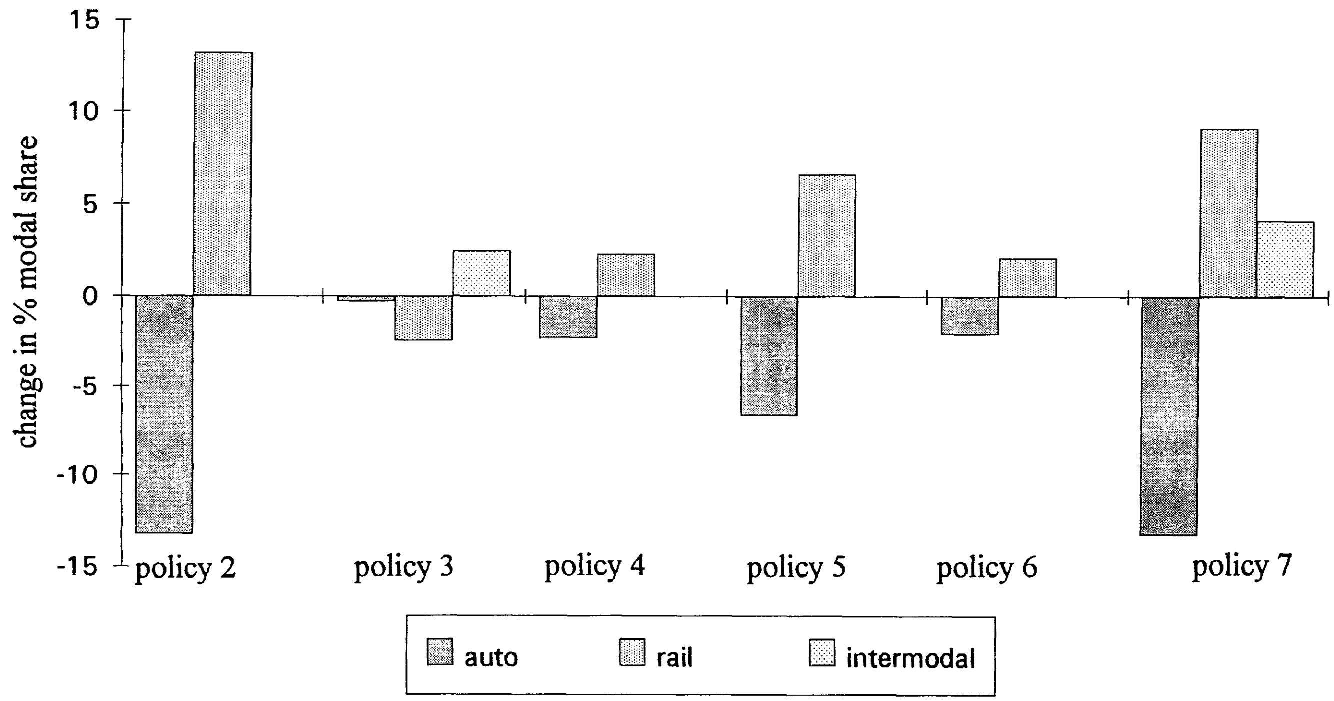


Figure 7.1 First Model-Percent Change in Modal Shares Compared to the Baseline

Table 7.4 User and Operator Impacts Resulted from the First Model

Policy	User Cost ($/peak hour)	User Time (min/ peak hour)	Parking Capital Investm. ($/peak hour)	Station Parking Revenues ($/peak hour)	Rail Operating Cost ($/peak hour)	Rail Fare-Box Revenues ($/peak hour)	Net Rail Operator Revenues ($/peak hour)	Highway Toll Revenues ($/peak hour)	CBD Parking Revenues ($/peak hour)	Net User and Oper. Cost ($/peak hour)
Policy 1	53107	121610	0	787.45	1760	1972.1	999.55	423.6	6051.9	45631.9
Policy 2	49698	117581	0	787.45	3520	2465	-267.55	290.6	4450	45224.9
Policy 3	53105	121331	91.2	877.45	1760	1972.2	998.45	423.5	6049.9	45633.1
Policy 4	53267	121966	0	787.45	1760	2061.8	1089.25	808.7	5776.2	45592.8
Policy 5	53575	123016	0	787.45	1760	2224.9	1252.35	367.5	6300.4	45654.7
Policy 6	52477	121960	0	787.45	1760	1647.9	675.35	406.0	5800.4	45595.2
Policy 7	50585	117106	152	937.45	3520	2465	-269.55	290.6	5340	45223.9

c = vehicle operating cost	400 [$/vehicle-hour]
L = length of transit route	14.66 [miles]
H = route headway	0.333 [hours/vehicle]
V = average transit speed	20 [miles/hour]

As a result, the total hourly operating cost (C) for operating three trains per hour is $1760/hour.

The seventh column gives the rail fare-box revenue in $/peak hour. This revenue is estimated as the sum of the rail fares paid by all rail and intermodal users. Since it is assumed that the rail operator is responsible for the rail line and the station parking lots, the net rail operator revenues in the eighth column are computed by subtracting rail operating costs and parking capital investments from the sum of fare-box and station parking revenues.

The highway toll revenues in $/peak hour in column nine are estimated as the number of toll facility users multiplied by the toll fee.

The CBD parking revenues in $/peak hour are estimated as the number of CBD parking lot users multiplied by the CBD parking fee and are reported in column ten.

The net user and operator costs is the sum of all operator (rail highway, and CBD parking) revenues which are included in the three preceding columns minus the user costs (second column).

7.2.3.3 Evaluation of Policies. The results presented in Table 7.4 are used to estimate, for each policy, the percent increase or reduction of various measures compared to the baseline case. These percentages are shown in Table 7.5 and are used to evaluate the policies based on their effects on user and operator costs and revenues.

Table 7.5 shows that the highest reduction in user cost results from doubling the rail frequency - Policy 2. The user cost decreased by 6.42% for this policy compared to the baseline case. The policy reduced the rail, highway toll, and CBD parking operator revenues, by 126.76%, 31.40%, and 26.47% respectively. The user time decreased by 3.31%. The high decrease in net rail operator revenues is a result of the increased rail operating cost which was doubled, from \$1760/peak hour to \$3520/peak hour. The net user and operator cost was reduced by 0.892% which is the second highest reduction of this cost for a policy.

The highest reduction in the net user and operator cost resulted from increasing the rail frequency, the parking capacity at O5, and the CBD parking fee - Policy 7. The decrease in net user and operator cost resulted from this policy is 0.894% compared to the baseline case. Policy 7 decreased user cost by 4.75% and user time by 3.70%. This is the highest reduction in user time generated by a policy. The net rail operator revenue decreased substantially, by 126.97%, due to the increased rail operating cost and the cost of the additional parking capacity. Highway toll and CBD parking operator revenues were decreased by 31.4% and 11.76%, respectively, due to the shift of 953 auto users to transit.

The increase in highway tolls - Policy 4 and the reduction in rail fares - Policy 6 had almost the same effects in net user and operator cost. They decreased this cost by 0.085% and 0.080%, respectively. Although Policy 4 diverted 753 travelers from auto to transit, the highway toll revenue increased by 90.91% due to the increase in highway tolls.

Although the parking capacity expansion - Policy 3 diverted 662 travelers from auto to transit, it had almost no effects in user cost and highway toll revenues. The net user and operator costs remained practically unchanged (\$45633.1/peak hour).

Table 7.5 First Model-Percent Change in User and Operator Impacts for each Policy Compared to the Baseline Case

% Change over Baseline	User Cost	User Time	Parking Capital Investm.	Station Parking Revenue	Rail Operating Cost	Rail Fare-Box Revenue	Net Rail Operator Revenue	Highway Toll Revenue	CBD Parking Revenue	Net User & Oper. Cost
Policy 2	-6.42	-3.31	0	0	+100	+24.99	-126.76	-31.40	-26.47	-0.892
Policy 3	-0.0037	-0.23	N/A	+11.43	0	+0.005	-0.11	~ 0	-0.033	+0.0026
Policy 4	+0.30	+0.29	0	0	0	+4.55	+8.97	+90.91	-4.56	-0.085
Policy 5	+0.88	+1.15	0	0	0	+12.82	+25.29	-13.24	+4.11	+0.05
Policy 6	-1.18	+0.29	0	0	0	+16.44	-32.43	-4.15	-4.16	-0.080
Policy 7	-4.75	-3.70	N/A	+19.05	+100	+24.99	-126.97	-31.40	-11.76	-0.894

In terms of their effect on the net user and operator costs, the order of preference of the analyzed policies is as follows:

1. Policy 7 - Double rail frequency, increase station parking capacity and increase CBD parking fee.
2. Policy 2 - Double rail frequency.
3. Policy 4 - Increase highway tolls.
4. Policy 6 - Decrease rail fare.
5. Policy 3 - Increase station parking capacity.
6. Policy 5 - Increase CBD parking fee.

7.3 Second Model - Intermodal Network Mode and Route Choice

7.3.1 Verification of Second Models' Equilibrium Conditions

In this sub-section the numerical results of the analysis of the network using the second model, which are shown in Table 7.6, will be used to verify the models' equilibrium conditions. The table shows the generalized cost of traveling on each path of the network and the flow on each utilized path according to the second models' predictions. For the purpose of the verification of the equilibrium conditions, the path flows have been reported as they were given from the model output and they have not been rounded to the nearest integer.

According to the first equilibrium condition of the second model, for each origin-destination pair the cost of traveling on all utilized auto paths must be the same and less than or equal to the cost of traveling on any unutilized auto path. In addition, the cost of traveling on all utilized transit paths (rail and intermodal) must be the same and less than or equal to the cost of traveling on any unutilized transit path.

Table 7.6 Second Model Results to Verify Equilibrium Conditions

O-D pair	Path	Travel Cost	Path Flow
O1-D	P1 (auto)	27.51	
	P2 (auto)	35.99	
	P3 (auto)	22.90	389
	P4 (intermodal)	28.56	
	P5 (intermodal)	33.21	
	P6 (intermodal)	31.19	
	P23 (intermodal)	26.79	
	P7 (rail)	25.34	151
O2-D	P8 (auto)	34.23	
	P9 (auto)	26.16	85.18
	P10 (intermodal)	23.69	
	P11 (intermodal)	25.19	
	P24 (intermodal)	24.66	
	P12 (rail)	23.53	44.82
O3-D	P13 (auto)	34.55	
	P14 (auto)	26.48	393.69
	P15 (intermodal)	25.51	
	P25 (intermodal)	22.38	37.93
	P16 (rail)	22.38	188.38
O4-D	P20 (auto)	28.07	
	P21 (auto)	23.67	149.04
	P22 (intermodal)	22.69	70.96
	P27 (rail)	59.96	
O5-D	P17 (auto)	26.62	
	P18 (auto)	25.17	584.91
	P26 (intermodal)	21.12	168.04
	P19 (rail)	21.12	167.05

For example, Table 7.6 shows that for O3-D pair the two utilized transit paths (P25 intermodal path and P16 rail path) have the same travel cost (22.38 min/pass) which is less than the travel cost on the unutilized (P15 intermodal) transit path. In addition, the travel cost on the utilized auto path (P14) is less than the travel cost on the unutilized auto path (P13). The same analysis can be performed for every O-D pair on the network and it shows that the numerical results satisfy the first equilibrium condition.

For the second equilibrium condition to be satisfied, it has to be shown that the numerical results satisfy the expression:

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) \qquad \forall ij \qquad (7.1)$$

with $\beta = 0.06$ and $\alpha_{TA} = 0.8$

The second models' numerical results from Table 7.6 are used to verify equation (7.1) as it is shown in Table 7.7. It can be concluded that the numerical results from the second models' output satisfy the second equilibrium condition.

Table 7.7 Verification of Second Models' Second Equilibrium Condition

O-D pair	$GC_T^{ij} - GC_A^{ij}$	$-\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA})$
O1-D	25.34-22.90 = 2.44	-16.67*[ln(151/389)+0.8]=2.44
O2-D	23.53-26.16 = -2.63	-16.67*[ln(44.82/85.18)+0.8]=-2.63
O3-D	22.38-26.48 = -4.1	-16.67*[ln(226.31/393.69)+0.8]=-4.1
O4-D	22.69-23.67 = -0.98	-16.67*[ln(70.96/149.04)+0.8]=-0.98
O5-D	21.12-25.17 = -4.05	-16.67*[ln(335.09/584.91)+0.8]=-4.05

7.3.2 Policies Analyzed from the Second Model

Results from the analysis of the network according to the second model are shown in Table 7.8. The table shows the flows for each path type (Auto, Rail, Intermodal) in users/peak-hour and the resulting path costs in $/user for various policies.

The policies that were analyzed are the same policies analyzed from the first model, as they are described in Section 7.2.2. These policies are:

Table 7.8 Path Flows and Costs for Each Policy According to the Second Model

		Policy 1		Policy 2		Policy 3		Policy 4		Policy 5		Policy 6		Policy 7	
O-D Pair	Path Type*	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost
O1-D	A	389	22.90	379	22.76	389	22.90	387	23.22	383	23.82	386	22.87	373	23.68
	R	151	25.34	161	23.68	151	25.34	153	25.34	157	25.34	154	24.91	167	23.68
	I														
O2-D	A	85	26.16	83	25.83	85	26.16	85	26.44	84	26.96	85	26.10	81	26.63
	R	45	23.53	47	22.99	45	23.53	45	23.53	46	23.53	45	23.10	49	21.87
	I														
O3-D	A	394	26.48	382	26.15	394	26.48	391	26.77	387	27.28	391	26.42	375	26.95
	R	187	22.38	184	20.71	187	22.38	185	22.38	186	22.38	187	22.03	187	20.71
	I	39	22.38	54	20.71	39	22.38	44	22.38	47	22.38	42	22.03	58	20.71
O4-D	A	149	23.67	145	23.35	149	23.67	148	23.95	147	24.48	148	23.60	143	24.16
	R														
	I	71	22.69	75	21.02	71	22.69	72	22.69	73	22.69	72	22.39	77	21.02
O5-D	A	585	25.17	568	24.83	585	25.18	581	25.45	575	25.97	582	25.10	557	25.63
	R	167	21.12	188	19.46	107	21.12	172	21.12	179	21.12	171	20.82	101	19.46
	I	168	21.11	164	19.45	228	21.12	167	21.11	166	21.11	167	20.81	262	19.46
Total	A	1602		1557		1602		1592		1576		1592		1529	
	R	550		580		490		555		568		557		504	
	I	278		293		338		283		286		281		397	

* A=Auto, R=Rail, I=Intermodal

Policy 1: Baseline Case

Policy 2: Double Train Frequency

Policy 3: Increase Parking Capacity at the O5 parking lot by 60 spaces.

Policy 4: Increase Highway Tolls from $0.35 to $ 0.70.

Policy 5: Increase Parking Fees at the CBD parking lot by 20%.

Policy 6: Decrease Rail Fares by 20%.

Policy 7: Combination of policies (add 40 more spaces at O5 in addition to the 60 added under Policy 3, increase CBD parking fee by 20%, and double rail frequency).

7.3.3 Policy Analysis

7.3.3.1 Policy Effects on Modal Shares. The policy effects in terms of modal shares (in percentages) are shown in Table 7.9. According to the table, the increase of rail frequency - policy 2 resulted in a decrease in auto commute from every origin of the network. The increase in parking capacity - Policy 3 did not impact the auto shares but it shifted 6.6% of the O5 rail users to intermodal. The highest percentage reduction in auto commute was a result of the last policy which doubled the rail frequency and increased parking capacity and CBD parking fee.

The percent increase or decrease of the total network modal shares for each policy in comparison with the baseline case are shown in Figure 7.2. The figure shows that every policy resulted in a decrease in auto's share (with the exception of Policy 3 which did not change it) and an increase in intermodal's share. The effects on rail shares vary for each policy. Policies 2, 4, 5, and 6 for example increased rail's share while Policies 3 and 7 reduced it.

7.3.3.2 User and Operator Impacts. The effects of policies in terms of user and operator impacts are presented in Table 7.10. The second and third columns of the table present the user cost and user time in $/peak hour and min/peak hour respectively. The parking capital investment, in $/peak hour, required for the addition of new parking spaces is estimated in column four. The net rail operator revenues in column eight are estimated as the sum of station parking and rail fare-box revenues (from columns five and seven respectively) minus the parking capital investment and the rail operating cost (from columns four and six respectively). The highway toll and CBD parking revenues are reported in columns nine and ten respectively. Finally, column eleven gives the net user and operator costs as the sum of all operator (rail highway, and CBD parking) revenues which are included in the three preceding columns minus the user costs (second column).

The method of estimation of the results for each column of the table is described in Section 7.2.3.2.

7.3.3.3 Evaluation of Policies. The results presented in Table 7.10 are used to estimate, for each policy, the percent increase or reduction of various measures compared to the baseline case. These percentages are shown in Table 7.11 and are used to evaluate the policies based on their effects on user and operator costs and revenues.

Table 7.11 shows that the highest reduction in user cost results from doubling the rail frequency - Policy 2. This policy reduced user cost by 3.43% and the user time by 3.98% compared to the baseline case. The net rail operator revenues decreased from $69.47/peak hour to -$1595.07/peak hour as a result of the high increase in rail operating costs. The highway toll and CBD parking operator revenues were

Table 7.9 Modal Shares for each O-D Pair According to the Second Model

Policy	O-D Pair	Auto (%)	Rail (%)	Intermodal (%)
1--Baseline	O1 - D	72.00	28.00	
	O2 - D	65.50	34.50	
	O3 - D	63.50	30.20	6.30
	O4 - D	67.70		32.30
	O5 - D	63.60	18.20	18.20
2--Double Rail Frequency	O1 - D	70.20	29.80	
	O2 - D	63.70	36.30	
	O3 - D	61.60	29.70	8.70
	O4 - D	65.90		34.10
	O5 - D	61.70	20.50	17.80
3--Increase Parking Capacity	O1 - D	72.00	28.00	
	O2 - D	65.50	34.50	
	O3 - D	63.50	30.20	6.30
	O4 - D	67.70		32.30
	O5 - D	63.60	11.60	24.80
4--Increase Highway Tolls	O1 - D	71.60	28.40	
	O2 - D	65.20	34.80	
	O3 - D	63.10	29.80	7.10
	O4 - D	67.40		32.60
	O5 - D	63.20	18.60	18.20
5--Increase CBD Parking Fee	O1 - D	70.90	29.10	
	O2 - D	64.40	35.60	
	O3 - D	62.40	30.00	7.60
	O4 - D	66.70		33.30
	O5 - D	62.50	19.50	18.00
6--Decrease Rail Fare	O1 - D	71.60	28.40	
	O2 - D	65.00	35.00	
	O3 - D	63.10	30.10	6.80
	O4 - D	67.40		32.60
	O5 - D	63.30	18.50	18.20
7--Increase rail frequency and CBD parking fee and parking capacity	O1 - D	69.00	31.00	
	O2 - D	62.60	37.40	
	O3 - D	60.50	30.20	9.30
	O4 - D	64.80		35.20
	O5 - D	60.60	11.00	28.40

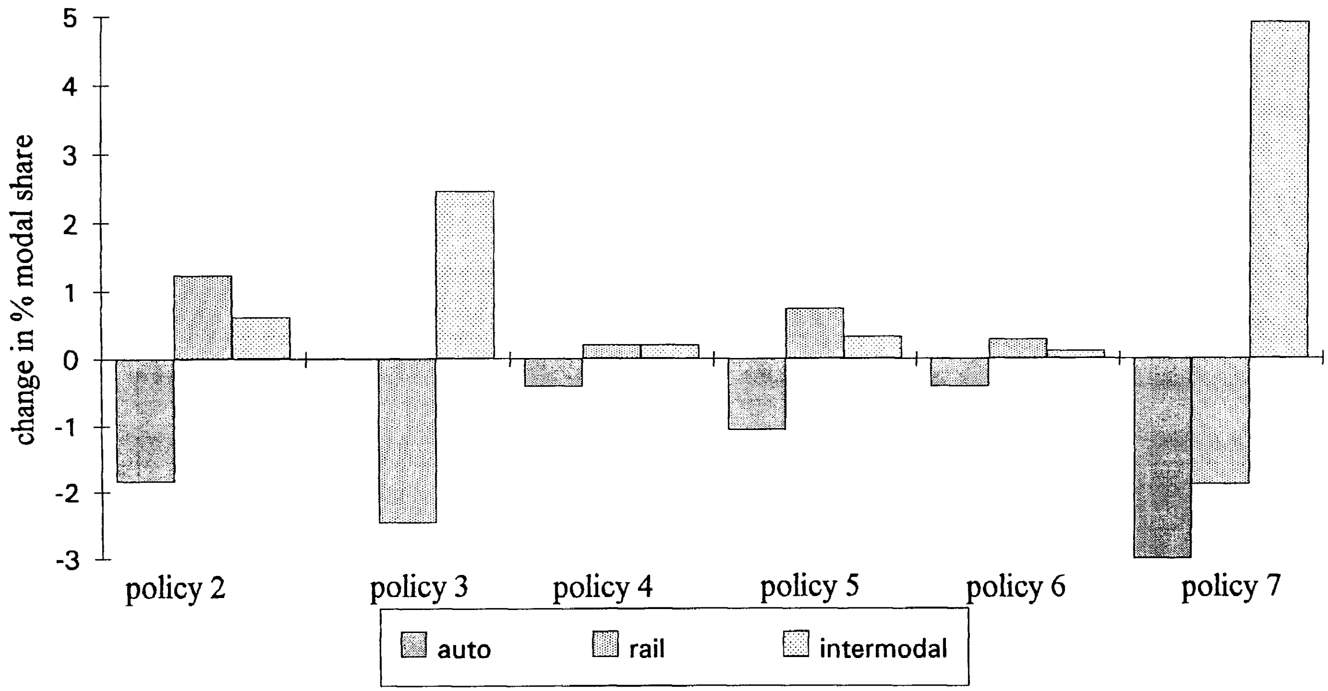


Figure 7.2 Second Model-Percent Change in Modal Shares Compared to the Baseline

Table 7.10 User and Operator Impacts Resulted from the Second Model

Policy	User Cost ($/peak hour)	User Time (min/ peak hour)	Parking Capital Investm. ($/peak hour)	Station Parking Revenues ($/peak hour)	Rail Operating Cost ($/peak hour)	Rail Fare-Box Revenues ($/peak hour)	Net Rail Operator Revenues ($/peak hour)	Highway Toll Revenues ($/peak hour)	CBD Parking Revenues ($/peak hour)	Net User and Oper. Cost ($/peak hour)
Policy 1	58450	130698	0	403.37	1760	1426.1	69.47	560.6	8009.1	49810.8
Policy 2	56446	125488	0	420.10	3520	1504.83	-1595.07	544.8	7783.56	49712.7
Policy 3	58454	130430	91.2	493.4	1760	1426.3	68.5	560.6	8008.7	49816.2
Policy 4	58890	130505	0	409.2	1760	1442.5	91.7	1114.72	7962.3	49721.3
Policy 5	59691	130181	0	412.5	1760	1473.2	125.7	551.19	9449.1	49565.0
Policy 6	58051	130532	0	406.8	1760	1154.3	-198.9	557.4	7962.4	49730.1
Policy 7	57615	124459	152	574.8	3520	1553.47	-1543.7	535.1	9172.93	49450.7

decreased by 2.82% each. Despite the high decrease in operator revenues, the policy reduced the user costs substantially and resulted in the third highest decrease of net user and operator costs (0.314% decrease) compared to the baseline case.

The highest decrease in net user and operator cost resulted from the increase of rail frequency and parking capacity in addition to the increase of the CBD parking fee - Policy 7. The policy decreased the net user and operator revenues by 0.84% compared to the baseline case. The user cost decreased by 1.43% while the highest reduction in user time was achieved (4.77%). As it was the case in Policy 2, the capital investment requirements resulted in a very high decrease of net rail operator revenues (2,322%). In addition, the highway toll revenues were reduced by 4.55% while the CBD parking revenue increased by 14.53%.

The second highest reduction in net user and operator costs (0.49%) resulted from increasing the CBD parking fee - Policy 5. This policy increased user cost by 2.12% while it decreased the user time by 0.39%. The net rail operator revenue increased by 80.94% (from $69.47/peak hour to $125.7/peak hour) and the CBD parking revenue increased by 17.98% (from $8009.1/peak hour, to $9449.1/peak hour). The highway toll revenue decreased by 1.68%.

The increase in highway toll - Policy 4 reduced the net user and operator cost by 0.18% and the decrease in rail fare - Policy 6 reduced the same cost by 0.162%.

The increase in parking capacity - Policy 3 reduced the net rail operator revenue by 1.4% and had almost no effects in highway toll and CBD parking operator revenues. Overall, the net user and operator cost increased slightly by 0.01%.

In terms of their effect on the net user and operator costs the order of preference of the analyzed policies is as follows:

Table 7.11 Second Model-Percent Change in User and Operator Impacts for each Policy Compared to the Baseline Case

% Change over Baseline	User Cost	User Time	Parking Capital Investm.	Station Parking Revenue	Rail Operating Cost	Rail Fare-Box Revenue	Net Rail Operator Revenue	Highway Toll Revenue	CBD Parking Revenue	Net User & Oper. Cost
Policy 2	-3.43	-3.98	0	+4.15	+100	+5.52	-2396	-2.82	-2.82	-0.314
Policy 3	+0.007	-0.205	N/A	+22.32	0	~ 0	-1.40	~ 0	~ 0	+0.01
Policy 4	+0.75	-0.15	0	+1.44	0	+1.15	+31.99	+98.84	-0.58	-0.18
Policy 5	+2.12	-0.39	0	+2.26	0	+3.30	+80.94	-1.68	+17.98	-0.49
Policy 6	-0.68	-0.13	0	+0.85	0	-19.06	-386.31	-0.57	-0.58	-0.162
Policy 7	-1.43	-4.77	N/A	+42.50	+100	+8.93	-2322	-4.55	+14.53	-0.84

1. Policy 7 - Double rail frequency, increase station parking capacity and CBD parking fee.
2. Policy 5 - Increase CBD parking fee.
3. Policy 2 - Double the rail frequency.
4. Policy 4 - Increase highway tolls.
5. Policy 6 - Reduce rail fare.
6. Policy 3 - Increase parking capacity.

7.4 Third Model - Intermodal Network Mode, Access Type, and Route Choice

7.4.1 Verification of Third Models' Equilibrium Conditions

In this sub-section the numerical results of the analysis of the network using the third model, which are shown in Table 7.12, will be used to verify the equilibrium conditions. Table 7.12 shows the generalized cost of traveling on each path of the network and the flow on each utilized path according to the third models' predictions.

According to the first equilibrium condition of the third model, for each origin-destination pair and for each type of trip k (k = auto, rail, intermodal), the cost of traveling on all utilized type k paths between a certain O-D pair must be the same and less than or equal to the cost of traveling on the unutilized type k paths of the same O-D pair. The first condition is automatically verified for rail paths since there is only one rail path in each O-D pair. Table 7.12 shows that for auto and intermodal paths the condition is also verified since for each O-D pair the utilized auto paths have lower travel cost than the unutilized ones and the utilized intermodal paths have lower travel cost than the unutilized ones.

Table 7.12 Third Model Results to Verify Equilibrium Conditions

O-D pair	Path	Travel Cost	Path Flow
O1-D	P1 (auto)	26.41	
	P2 (auto)	34.67	
	P3 (auto)	22.28	331.23
	P4 (intermodal)	28.46	
	P5 (intermodal)	32.91	
	P6 (intermodal)	31.10	
	P23 (intermodal)	26.79	77.03
	P7 (rail)	25.34	131.74
O2-D	P8 (auto)	32.99	
	P9 (auto)	25.06	74.98
	P10 (intermodal)	23.70	21.29
	P11 (intermodal)	25.12	
	P24 (intermodal)	24.66	
	P12 (rail)	23.53	33.73
O3-D	P13 (auto)	33.31	
	P14 (auto)	25.37	350.65
	P15 (intermodal)	25.43	
	P25 (intermodal)	22.384	104.86
	P16 (rail)	22.380	164.49
O4-D	P20 (auto)	26.97	
	P21 (auto)	22.61	149.39
	P22 (intermodal)	22.67	70.61
	P27 (rail)	59.96	
O5-D	P17 (auto)	25.49	
	P18 (auto)	24.01	520.93
	P26 (intermodal)	21.108	155.475
	P19 (rail)	21.124	243.596

For the second equilibrium condition to be satisfied it needs to be shown that the numerical results satisfy the expression:

$$GC_M^{ij} - GC_R^{ij} = -\frac{1}{\beta_1}(\ln \frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR}) \qquad \forall ij \qquad (7.2)$$

with $\beta_1 = 0.06$, and $\alpha_{MR} = 0.45$

The third models' numerical results from Table 7.12 are used to verify equation (7.2) as it is shown in Table 7.13. Results from Table 7.13 indicate that the second equilibrium condition is satisfied.

Table 7.13 Verification of Third Models' Second Equilibrium Condition

O-D pair	$GC_M^{ij} - GC_R^{ij}$	$-\frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR})$
O1-D	26.79-25.34=1.45	-16.67*[ln(77.03/131.74)+0.45]=1.45
O2-D	23.70-23.53=0.17	-16.67*[ln(21.29/33.73)+0.45]=0.17
O3-D	22.384-22.38=0.004	-16.67*[ln(104.86/164.49)+0.45]=0.004
O4-D	N/A	N/A
O5-D	21.108-21.124=-0.016	-16.67*[ln(155.47/243.59)+0.45]=-0.016

For the third equilibrium condition to be satisfied it has to be shown that the numerical results satisfy the expression:

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta_2}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA}) \qquad \forall ij \qquad (7.3)$$

where GC_T^{ij} is given from equation (5.56) as:

$$GC_T^{ij} = -\frac{1}{\beta_1}\ln[\exp(-\beta_1 * GC_R^{ij}) + \exp(-\beta_1 * GC_M^{ij} - \alpha_{MR})]$$

and the values of the parameters are: $\beta_1 = 0.06$, $\beta_2 = 0.03$, $\alpha_{TA} = 0.6$, and $\alpha_{MR} = 0.45$.

Based of the results from Table 7.12 the generalized cost for transit is estimated as shown in Table 7.14.

Table 7.14 Estimation of the Generalized Cost for Transit

O-D pair	$-\frac{1}{\beta_1}\ln[\exp(-\beta_1 * GC_R^{ij}) + \exp(-\beta_1 * GC_M^{ij} - \alpha_{MR})]$
O1-D	-16.67*ln[exp(-0.06*25.34)+exp(-0.06*26.79-0.45)]=17.67
O2-D	-16.67*ln[exp(-0.06*23.53)+exp(-0.06*23.70-0.45)]=15.38
O3-D	-16.67*ln[exp(-0.06*22.38)+exp(-0.06*22.384-0.45)]=14.16
O4-D	-16.67*ln[exp(-0.06*59.96)+exp(-0.06*22.67-0.45)]=27.59
O5-D	-16.67*ln[exp(-0.06*21.124)+exp(-0.06*21.108-0.45)]=12.89

The results of Tables (7.12) and (7.14) provide a verification of expression (7.3) as is shown in Table 7.15.

Table 7.15 Verification of Third Models' Third Equilibrium Condition

O-D pair	$GC_T^{ij} - GC_A^{ij}$	$-\frac{1}{\beta_2}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA})$
O1-D	17.67-22.28=-4.61	-33.33*[ln(208.772/331.228)+0.6]=-4.61
O2-D	15.38-25.06=-9.68	-33.33*[ln(55.018/74.982)+0.6]=-9.68
O3-D	14.16-25.37=-11.21	-33.33*[ln(269.353/350.647)+0.6]=-11.21
O4-D	27.59-22.61=4.98	-33.33*[ln(70.61/149.391)+0.6]=4.98
O5-D	12.89-24.01=-11.12	-33.33*[ln(399.071/520.93)+0.6]=-11.12

7.4.2 Policies Analyzed by the Third Model

Results from the analysis of the network according to the third model are shown in Table 7.16. The table contains the flows for each path type (Auto, Rail, Intermodal) in users/peak hour and the resulting path costs in $/user for each of the policies.

The policies that were analyzed are similar to those analyzed from the first and second model with the exception of the parking capacity expansion. Results of the third model indicated that parking lots are not fully utilized thus parking capacity addition was not necessary. The policies analyzed from the third model are:

Table 7.16 Path Flows and Costs for Each Policy According to the Third Model

		Policy 1		Policy 2		Policy 3		Policy 4		Policy 5		Policy 6	
O-D Pair	Path Type*	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost	flow	cost
O1-D	A	331	22.28	325	22.21	330	22.62	328	23.24	330	22.27	321	23.17
	R	132	25.34	136	23.68	133	25.34	134	25.34	133	24.91	138	23.68
	I	77	26.79	79	25.12	77	26.79	78	26.79	77	26.36	81	25.12
O2-D	A	75	25.06	74	24.88	75	25.37	74	25.95	75	25.02	73	25.78
	R	34	23.53	34	21.87	34	23.53	34	23.53	34	23.10	35	21.87
	I	21	23.70	22	22.03	21	23.70	22	23.70	21	23.35	22	22.04
O3-D	A	351	25.37	344	25.19	349	25.68	347	26.26	349	25.33	340	26.09
	R	164	22.38	169	20.71	165	22.38	167	22.38	165	22.03	171	20.71
	I	105	22.38	107	20.72	106	22.38	106	22.38	106	22.03	109	20.72
O4-D	A	149	22.61	148	22.45	149	22.93	148	23.51	149	22.58	146	23.34
	R												
	I	71	22.67	72	21.00	71	22.67	72	22.67	71	22.37	74	21.00
O5-D	A	521	24.01	511	23.83	519	24.32	515	24.90	519	23.98	505	24.73
	R	244	21.12	250	19.46	245	21.12	247	21.12	245	20.82	253	19.46
	I	155	21.11	159	19.44	156	21.11	158	21.11	156	20.81	162	19.44
Total	A	1427		1402		1422		1412		1422		1385	
	R	574		589		577		582		577		597	
	I	429		439		431		436		431		448	

* A=Auto, R=Rail, I=Intermodal

Policy 1: Baseline Case

Policy 2: Double Train Frequency

Policy 3: Increase Highway Tolls from $0.35 to $0.70

Policy 4: Increase parking fees at the CBD parking lot by 20%

Policy 5: Decrease Rail Fare by 20%

Policy 6: Combination of Policies (double rail frequency and increase CBD parking fee by 20%).

7.4.3 Policy Analysis

7.4.3.1 Policy Effects on Modal Shares. The policy effects in terms of modal shares according to the third model are shown in Table 7.17. The table shows the modal shares for each policy and for each origin-destination pair. It is shown that, for every policy and for every O-D pair, more than half of the travelers use auto. As a result of the policies the transit modal shares increased, in general, compared to the baseline case.

The percent increase or decrease of the total network modal shares for each policy in comparison with the baseline case are shown in Figure 7.3. The figure shows that every policy decreased auto's shares while rail's and intermodal's shares were always increased.

7.4.3.2 User and Operator Impacts. The effects of policies in terms of user and operator impacts are presented in Table 7.18. The table presents user cost and time, station parking revenues, rail operating costs, rail fare-box revenues, net rail operator revenues, highway toll, and CBD parking revenues, and finally net user and operator costs in consecutive columns.

Table 7.17 Modal Shares for each O-D Pair According to the Third Model

Policy	O-D Pair	Auto (%)	Rail (%)	Intermodal (%)
1--Baseline	O1 - D	61.30	24.40	14.30
	O2 - D	57.70	25.90	16.40
	O3 - D	56.60	26.50	16.90
	O4 - D	67.90	4.60	27.50
	O5 - D	56.60	26.50	16.90
2--Double Rail Frequency	O1 - D	60.20	25.20	17.40
	O2 - D	56.60	26.60	16.80
	O3 - D	55.50	27.20	17.30
	O4 - D	67.10	4.30	28.60
	O5 - D	55.50	27.20	17.30
3--Increase Highway Tolls	O1 - D	61.10	24.50	14.40
	O2 - D	57.40	26.10	16.50
	O3 - D	56.30	26.70	17.00
	O4 - D	67.70	4.60	27.70
	O5 - D	56.40	26.60	17.00
4--Increase CBD Parking Fee	O1 - D	60.70	24.80	14.50
	O2 - D	57.10	26.30	16.60
	O3 - D	55.90	26.90	17.20
	O4 - D	67.30	4.70	28.00
	O5 - D	56.00	26.80	17.20
5--Decrease Rail Fare	O1 - D	61.00	24.60	14.40
	O2 - D	57.40	26.20	16.40
	O3 - D	56.30	26.70	17.00
	O4 - D	67.80	4.50	27.70
	O5 - D	56.40	26.60	17.00
6--Increase Rail Frequency and CBD Parking Fee	O1 - D	59.50	25.60	14.90
	O2 - D	55.90	27.00	17.10
	O3 - D	54.80	27.60	17.60
	O4 - D	66.50	4.40	29.10
	O5 - D	54.80	27.60	17.60

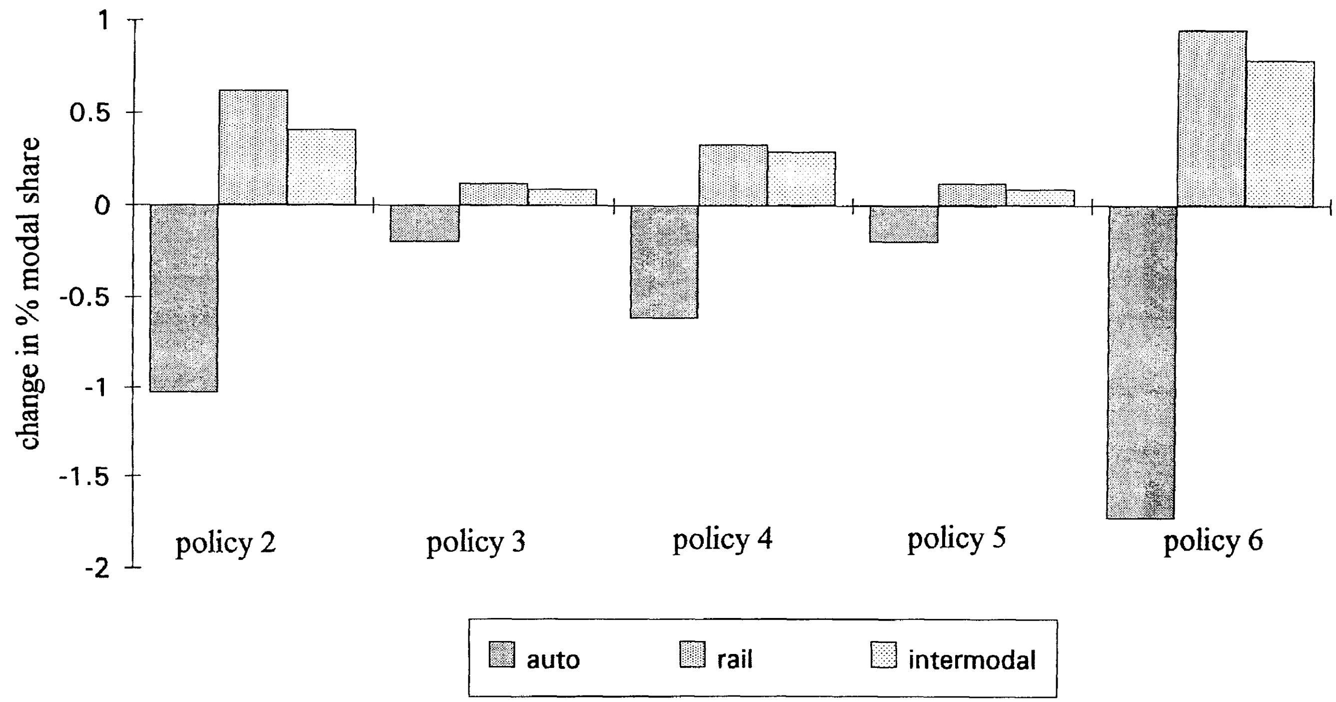


Figure 7.3 Third Model-Percent Change in Modal Shares Compared to the Baseline

Table 7.18 User and Operator Impacts Resulted from the Third Model

Policy	User Cost ($/peak hour)	User Time (min/ peak hour)	Station Parking Revenues ($/peak hour)	Rail Operating Cost ($/peak hour)	Rail Fare-Box Revenues ($/peak hour)	Net Rail Operator Revenues ($/peak hour)	Highway Toll Revenues ($/peak hour)	CBD Parking Revenues ($/peak hour)	Net User and Oper. Cost ($/peak hour)
Policy 1	57178	128765	700.1	1760	1719.3	659.4	499.5	7135.9	48883.2
Policy 2	55215	123345	720.16	3520	1766.1	-1033.74	490.3	7004.2	48754.2
Policy 3	57625	128705	704.1	1760	1728.9	673	995.1	7107.9	48849.4
Policy 4	58447	128599	711.6	1760	1746.8	698.4	493.9	8467.2	48787.5
Policy 5	56780	128706	704.3	1760	1383.5	327.8	497.6	7108.4	48846.2
Policy 6	56437	123119	731.8	3520	1793.9	-994.3	484.7	8308.4	48638.2

7.4.3.3 Evaluation of Policies. Table 7.19 presents for each policy the percent increase or reduction of various measures compared to the baseline case. The percentages presented in this table were estimated based on the results shown in Table 7.18. Based on the percentages reported in Table 7.19 the policies were evaluated according to their effects on user and operator costs and revenues.

The highest reduction (0.501%) in net user and operator revenues resulted from the increase of rail frequency in addition to the increase in the CBD parking fee - Policy 6. This policy reduced user costs by 1.296% and user time by 4.38% which is the highest reduction in user time achieved from a policy. The net rail operator revenues were decreased by 250.79% because of the high increase in rail operating cost. The highway toll revenues decreased by 0.029% due to the reduction in auto shares. Although the number of auto users, compared to the baseline case, decreased by 35 users/peak hour the CBD parking revenues increased by 16.43% due to the increase in CBD parking fee.

The highest reduction in user costs (3.43%) resulted from doubling the rail frequency - Policy 2. This policy yields the second highest reduction in net user and operator costs (0.264%) and the second highest reduction in user time (4.21%). This policy reduced the net rail operator revenues by 256.7% and the highway toll and CBD parking operator revenues by 1.84% each.

The increase in CBD parking fee - Policy 4 resulted in the third highest reduction in net user and operator cost (0.196%). The reduction in rail fare - Policy 5 and the increase in highway toll - Policy 3 reduced the same cost by 0.076% and 0.069%, respectively.

In terms of their effect on the net user and operator costs the order of preference of the analyzed policies is as follows:

Table 7.19 Third Model-Percent Change in User and Operator Impacts for each Policy Compared to the Baseline Case

% Change over Baseline	User Cost	User Time	Station Parking Revenue	Rail Operating Cost	Rail Fare-Box Revenue	Net Rail Operator Revenue	Highway Toll Revenue	CBD Parking Revenue	Net User & Oper. Cost
Policy 2	-3.43	-4.21	+2.86	+100	+2.72	-256.7	-1.84	-1.84	-0.264
Policy 3	+0.78	-0.046	+0.57	0	+0.56	+2.06	+99.22	-0.392	-0.069
Policy 4	+0.022	-0.129	+1.64	0	+1.60	+5.91	-1.12	+18.66	-0.196
Policy 5	-0.696	-0.046	+0.60	0	-19.53	+50.29	-0.38	-0.385	-0.076
Policy 6	-1.296	-4.38	+4.52	+100	+0.043	-250.79	-0.029	+16.43	-0.501

1. Policy 6 - Double rail frequency, and increase CBD parking fee.
2. Policy 2 - Double the rail frequency.
3. Policy 4 - Increase CBD parking fee.
4. Policy 5 - Reduce rail fare.
5. Policy 3 - Increase highway tolls.

7.5 Conclusion

The policies analyzed from the three models are a small sample of all possible policies that the outlined methodological framework is capable of evaluating. The results can be used to screen policies aimed at affecting travel demand. Policies aimed at improving rail service and making it more accessible were in general the best in terms of their ability to divert auto drivers to transit. The best deterrent to driving appears to be an increase in the CBD parking fee. It needs to be recognized, however, that the final decision on policy selection is likely to depend on social, political, economic, and environmental considerations as well.

CHAPTER 8

COMMUTER RAIL SERVICE DESIGN UNDER INTERMODAL NETWORK FLOW EQUILIBRATION

8.1 Introduction

The three models presented in Chapter 5 alleviate some of the deficiencies of the existing practice in the area of urban transportation planning by considering intermodal networks, proposing combined demand-supply network equilibrium models well founded in micro-economic theory, and calculating well defined equilibrium flows.

In the supply side of these model formulations, performance functions have been adopted for each facility of the network, such as highway or rail links. In the case of highway links these functions assumed that travel time increases exponentially with traffic volume. For rail, the travel time is assumed to be constant. In addition, the operating characteristics of the facility represented by these functions are assumed to remain constant, an assumption that is plausible for a short-run planning horizon.

The scope of this chapter is to develop a model that can be used for intermediate-term planning considering appropriate supply functions for various systems in the supply side of the formulation. In planning for transit operations, managers should be able to adjust transit service to the expected traffic volumes. In addition, they should be able to analyze the effects of transit pricing and operating policies on competing modes, usually auto, and determine transit fares and headways that meet certain goals. Usually, these goals include the provision of a transit service which is attractive to potential users (low cost, frequent service) and at the same time is profitable or requires very low amounts of subsidy.

To this end, a model is proposed in this chapter that will be able to forecast the impact of changes in transit service design on the performance and flows on highways and rail for an intermodal network. The model formulates the network equilibrium problem considering variable headway and fare for the transit system. The formulation considers headway as a known decreasing function of flow, of a form similar to the one presented in Morlok (1979). The model is formulated according to the first approach for modeling intermodal networks, presented in Section 4.2.2.

The model presents an efficient approach for analyzing public transit within its competitive environment of highway systems and suggesting operating schemes that will improve transit service and increase its attractiveness. The can serve as a powerful tool in the intermediate-run analysis of intermodal networks.

8.2 Literature Review

A review of the transportation literature indicated that there is no equivalent optimization formulation of the network equilibrium problem which utilizes intermediate-term supply functions for transit.

The analysis of the performance of various systems is presented in Morlok (1978) and (1979). Morlok was the first to suggest and derive various intermediate-run supply functions for public transit. These functions were developed with a view toward their inclusion in network equilibrium models (Morlok 1980).

Morlok et. al. (1970) presented a methodology to schedule and select fares for a linear transit facility in response to various kinds of system objectives. The model presented in this methodology recognizes the effects of transit patronage and highway level of service on changes in fares and schedules. Modal split and traffic assignment are iteratively performed for the feasible combinations of choice variables (fare, headway) until the road travel time and volume are consistent with those values used in the modal split model. The optimum policy under various objectives such as system profit, total system ridership, or peak period ridership maximization is obtained by finding the best combination of fare and headway from a set of enumerated combinations.

Holden (1989) stated the fact that since late 1960 researchers believed that congestion could be tackled by appropriate disincentives to use auto and incentives to use public transport, which would as a result make average journey speeds higher. This was demonstrated in an example of traffic assignment over a two-link network that is served by one highway and one rail link. The supply function for highway is assumed to be an exponentially increasing function of traffic volume and the supply function for rail is assumed to be independent of flow (a straight line that represents a constant user cost for any level of transit ridership).

8.3 Commuter Rail Service Design with Flow Equilibration

8.3.1 Problem Statement

The problem to be solved is: given the characteristics of a network, the total demand between each origin and destination, and the link supply functions with headway for commuter rail being a decreasing function of flow (Morlok 1979); find the link flows on an intermodal network, the rail service headway, and fare that minimize the individual users' cost.

The model is formulated as a mathematical program with a non-linear objective function and linear constraints. The objective function minimizes the total individual user cost of traveling on the intermodal network. The constraints are demand conservation, link flow conservation, rail seating capacity, parking capacity, and non-negativity constraints. The general model statement is:

Minimize:

Total Individual User Cost

subject to:

Demand Conservation Constraints

Link Flow Conservation Constraints

Rail Seating Capacity Constraints

Parking Capacity Constraints

Non-negativity Constraints

The network representation and the notation used in this model is similar to the one used in Chapter 5. The main difference is that the rail headway h_R is introduced in the notation as a decision variable, and the parameter *seats* now refers to the number of seats per train and not to the number of seats per analysis period.

8.3.2 User Costs

The costs that are considered in this model are in-vehicle travel time, out-of-vehicle waiting time, operating cost, and out of pocket costs. These costs are encountered on both highway and rail.

8.3.2.1 Highway Costs. The average generalized cost of travel on a highway link z is given by the following expression:

$$g_z(x_z) = OOP_z + VOTT * ff_z * (1 + \alpha \frac{x_z^\beta}{cap_z^\beta}) \tag{8.1}$$

where: OOP_z represents the highway tolls, parking fees and auto operating cost experienced by auto users on highway link z, and

$VOTT$ represents the value of time.

The travel time is given by the Bureau of Public Roads (BPR) (1964) congestion curve the mathematical expression of which is:

$$t_z = ff_z * [1 + a * (\frac{x_z}{cap_z})^b] \tag{8.2}$$

where: t_z - travel time on highway link z,

ff_z - free flow travel time on highway link z, and

a, b - parameters

The travel time is multiplied by the value of time (*VOTT*) to be expressed in monetary terms.

Expression (8.1) is an exponentially increasing function of flow and it is also used to represent the average cost on access, transfer, and walking links when the appropriate values are used for link capacity.

8.3.2.2 Rail Costs. The cost of travel for rail users, is given by the following expression:

$$g_R(x_R, h_R) = VOTT * (ff_R + \frac{h_R}{2}) + fare_R \tag{8.3}$$

In this expression the rail in-vehicle travel time (ff_R) is assumed to be constant and includes the travel time between stations and the in-vehicle waiting time at a station. The out-of-vehicle waiting time is assumed to be equal to half of the headway ($h_R / 2$) (Manheim 1979). The expression for headway used in this model formulation is similar to the one given in Morlok (1979):

$$h_R = \begin{Bmatrix} \min h \rightarrow x_R \geq m \\ \lambda * seats / x_R, \rightarrow n \leq x_R \leq m \\ \max h \rightarrow x_R \leq n \end{Bmatrix} \quad (8.4)$$

In this expression the vehicle load factor λ is prespecified by the transit operator. Minimum and maximum headways are also predefined. The headway is fixed to its minimum or maximum allowable value if the flow on the maximum loading rail link (critical rail link) is greater than a certain value *m* or lower than a certain value *n*, respectively.

The in-vehicle travel and the wait time in expression (8.3) are multiplied by a value of time to be expressed in monetary terms.

The out of pocket cost for a rail user (*fare*) is assumed to be the amount that makes the fare-box revenue equal to operating cost (or a proportion of the operating cost). In this dissertation it is assumed that the fare-box revenue covers entirely the cost of operation (*K*) which is assumed equal to the operating cost per vehicle-hour *(cost)* multiplied by the turnaround travel time *(time)* and divided by the headway (h_R). This assumption can easily be modified to consider a portion of the operating cost to be covered from the fare-box revenue and to estimate the amount of subsidy which is necessary to yield the proposed service. The operating cost is given by the expression:

$$K = \frac{(\cos t)*(time)}{h_R} \quad (8.5)$$

The out of pocket cost per user is then given as:

$$fare_R = \frac{\cos t * time}{h_R * x_R} \quad (8.6)$$

By introducing (8.4) in (8.6), it becomes:

$$fare_R = \frac{\cos t * time}{h_R * x_R} = \frac{\cos t * time * x_R}{\lambda * seats * x_R} = \frac{\cos t * time}{\lambda * seats} \tag{8.7}$$

According to this expression the rail fare for the first model is constant and depends on the allowable load factor which in this case is predetermined.

After substituting expressions (8.4) and (8.7) into Equation (8.3) the generalized cost for rail becomes:

$$g_R(x_R) = VOTT * (ff_R + \frac{\lambda * seats}{2 * x_R}) + \frac{\cos t * time}{\lambda * seats} \tag{8.8}$$

Expression (8.8) is a decreasing function of link flow. Thus, as the flow on rail increases the cost of traveling decreases. When the rail headway *(min h)* can not decrease further due to rail line capacity limitations, the rail cost is constant.

8.3.3 Equilibrium Condition

Since the model is formulated according to the first approach for modeling intermodal networks, presented in Section 4.2.2, an equilibrium solution to the model must satisfy the following condition.

$$GC_p^{ij} - GC^{ij} \begin{Bmatrix} = 0, & \text{if } f_p^{ij} > 0 \\ \geq 0, & \text{if } f_p^{ij} = 0 \end{Bmatrix} \qquad \forall i, j \tag{8.9}$$

This condition states that no traveler has an incentive to unilaterally change routes for s/he can not further minimize her/his travel cost. According to this condition, a path p from origin i to destination j is utilized only if the generalized cost on this path is equal to the minimum generalized cost of traveling on that O-D pair. An equilibrium solution of the problem is obtained if all utilized paths between an O-D pair ij have the same generalized cost which is less than or equal to the cost of traveling on the unutilized paths of the same O-D pair.

8.3.4 Model Formulation

The general expression of the objective function of the model is:

$$z[x(f),h] = \sum_{l \in Z,A,W,T} \int_0^{x_l} [g_l(x_l)]dx_l + \sum_{l \in R} \int_0^{x_l} [g_l(x_l,h_l)]dx_l \tag{8.10}$$

Replacing in (8.10) the headway from expression (8.4) the objective function becomes a function of only the link flow and is expressed as:

$$z[x(f)] = \sum_{l \in Z,A,W,T,R} \int_0^{x_l} [g_l(x_l)]dx_l \tag{8.11}$$

The demand conservation constraint insures that all trips between O-D pairs are accounted for by equating the demand for each O-D pair with the sum of the flows on all the paths available to travelers between this O-D pair. This constraint is expressed as:

$$T^{ij} = \sum_p f_p^{ij} \qquad \forall i,j \tag{8.12}$$

The link flow conservation constraint equates the flow on a link with the sum of the flows on all the paths that are going through that link. Paths that are going through a particular link are identified by the binary parameter δ_{lp}^{ij} taking on the value of one when link l is included in path p and zero otherwise. This constraint for highway, access, and transfer links is:

$$x_l = \frac{1}{occ} \sum_{ij} \sum_{p} \delta_{lp}^{ij} f_p^{ij} \qquad \forall l \in LZ, LA, LT \qquad (8.13)$$

The link flow conservation constraint for rail and walking links is:

$$x_l = \sum_{ij} \sum_{p} \delta_{lp}^{ij} f_p^{ij} \qquad \forall l \in LR, LW \qquad (8.14)$$

The rail seating capacity constraint states that the number of passengers per train does not exceed the allowable train capacity and can be expressed as:

$$x_R * h_R \le \lambda * seats$$

Since this constraint is redundant, i.e., it is automatically satisfied by the headway equation (8.4) which is introduced in the objective function, it will not be considered in the model formulation.

The parking capacity constraint insures that the number of cars parked at a parking lot does not exceed the available number of parking spaces. The mathematical expression of this constraint is:

$$x_l \le space_l \qquad \forall\ l \subseteq LT \qquad (8.15)$$

The last constraint of the formulation is the non negativity constraint according to which the flows on the paths and the flows on the links of the network can not be negative. Since one of these constraints is redundant, only the non negativity of the path flow is kept in the formulation. The constraint is of the form:

$$f_p^{ij} \geq 0 \qquad \forall p,i,j \tag{8.16}$$

The complete model statement is shown in Table 8.1.

8.3.5 Derivation of Equilibrium Conditions

This section shows that the Karush-Kuhn-Tucker conditions derived from the Lagrangian of the model are equivalent to the equilibrium condition stated earlier. For this purpose, the Lagrangian is formulated by introducing the demand conservation constraint, multiplied by a Lagrangian multiplier u^{ij} in the objective function. In addition, the link flow conservation constraints are introduced directly in the objective function by replacing the flow on each link with the sum of the flows on the paths that are using that link. The Lagrangian is then of the form:

$$L(f,u) = z[x(f)] + \sum_{ij} u_{ij}(T^{ij} - \sum_{p} f_p^{ij}) \tag{8.17}$$

with $f_p^{ij} \geq 0 \qquad \forall p,i,j$

Table 8.1 Intermodal Network Equilibrium Model with Decreasing Rail Supply Function

Minimize: $z(x(f)) = \sum_{l \in Z,A,W,T,R} \int_0^{x_l} [g_l(x_l)] dx_l$

subject to:

$$T^{ij} = \sum_p f_p^{ij} \qquad \forall i,j$$

$$x_l = \frac{1}{occ} \sum_{ij} \sum_p \delta_{lp}^{ij} f_p^{ij} \qquad \forall l \in LZ, LA, LT$$

$$x_l = \sum_{ij} \sum_p \delta_{lp}^{ij} f_p^{ij} \qquad \forall l \in LR, LW$$

$$x_l \leq space_l \qquad \forall\, l \subseteq LT$$

$$f_p^{ij} \geq 0 \qquad \forall p,i,j$$

The Karush-Kuhn-Tucker conditions for the problem can be expressed as:

$$f_p^{ij} \frac{\partial L(f,u)}{\partial f_p^{ij}} = 0 \quad \text{and} \quad \frac{\partial L(f,u)}{\partial f_p^{ij}} \geq 0 \qquad \forall p,i,j \tag{8.18}$$

$$\frac{\partial L(f,u)}{\partial u^{ij}} = 0 \qquad \forall i,j \tag{8.19}$$

$$f_p^{ij} \geq 0 \qquad \forall p,i,j \tag{8.20}$$

The derivative of the Lagrangian with respect to the path flows is (see Exhibit 1 in Appendix B for derivation):

$$\frac{\partial L(f,u)}{\partial f_p^{ij}} = \sum_{l \in LZ,LA,LT} \frac{1}{occ} \delta_{lp}^{ij} * g_l(x_l) + \sum_{l \in LW} \delta_{lp}^{ij} * g_l(x_l) + \sum_{l \in LR} \delta_{lp}^{ij} * g_l +$$

$$\sum_{l \in LCR} \delta_{lp}^{ij} [VOTT \frac{h_l(x_l)}{2} + fare] - u_{ij} \qquad (8.21)$$

The first term in (8.21) is the generalized cost of traveling on a highway, access, or transfer link. The second term is the generalized cost of traveling on a walking link. The third term is the in-vehicle travel cost on rail links. The fourth term is the expression of the waiting and out-of-pocket costs for rail. The generalized cost of traveling on a path is the sum of the generalized cost of traveling on the links that this path consists of.

The derivative of the Lagrangian with respect to the multiplier is:

$$\frac{\partial L(f,u)}{\partial u^{ij}} = T^{ij} - \sum_p f_p^{ij} \qquad \forall i,j \qquad (8.22)$$

Setting the first derivatives of the Lagrangian equal to zero and symbolizing the generalized cost on a path as GC_p^{ij} and the minimum generalized cost for O-D pair ij as GC^{ij} the Karush-Kuhn-Tucker conditions become:

$$f_p^{ij}(GC_p^{ij} - GC^{ij}) = 0 \qquad \text{and} \qquad GC_p^{ij} - GC^{ij} \geq 0 \qquad \forall p,i,j \qquad (8.23)$$

$$T^{ij} = \sum_p f_p^{ij} \qquad \forall i,j \quad (8.24) \qquad (8.24)$$

and

$$f_p^{ij} \geq 0 \qquad \forall\, p,i,j \text{ (8.25)} \qquad (8.25)$$

Equation (8.23) states that, at equilibrium, the average cost on all utilized paths p from origin i to destination j (GC_p^{ij}) are equal to the minimum average cost (GC^{ij}) for the same O-D pair and there is no unutilized path with average cost less than GC^{ij}. This equation is equivalent to the equilibrium condition. Equation (8.24) is the demand conservation constraint and Equation (8.25) is the non negativity constraint.

8.3.6 Convexity Analysis

In the previous section it was shown that a solution to the problem obtained from the Karush-Kuhn-Tucker conditions satisfies the equilibrium condition. However, the solution is not unique as the convexity analysis will show.

For the first order conditions to represent a unique (and thus global) minimum the objective function must be strictly convex. This is the case when the Hessian matrix is positive definite (Minoux 1986), meaning that all the elements of the matrix are positive for every value of their argument.

The elements of the Hessian matrix are the second derivatives of the objective function with respect to the decision variables, in this case link flows. A non diagonal element of the matrix is of the form:

$$\frac{\partial^2 z(x_a)}{\partial x_a \partial x_b}$$

and is equal to zero under the separability assumption which states that the cost on a link is a function of the flow on that link only and not of the flow on any other link of the network. The diagonal elements of the matrix are of the form:

$$\frac{\partial^2 z(x_a)}{\partial x_a^2}$$

As a result, the Hessian matrix is diagonal, of the form:

$$\nabla^2 z(x_a) = \begin{vmatrix} \frac{\partial^2 z(x_1)}{\partial x_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 z(x_2)}{\partial x_2^2} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & \frac{\partial^2 z(x_a)}{\partial x_a^2} \end{vmatrix}$$

This diagonal matrix is positive definite if all the diagonal elements are positive. This is not the case since at least one of the diagonal elements of the matrix is not positive (see Exhibit 2 in Appendix B for derivation):

$$\frac{\partial^2 z(x_{CR})}{\partial x_{CR}^2} = -\frac{\lambda * seats * VOTT}{2 * x_{CR}^2} < 0$$

The analysis of the Hessian matrix shows that the problem does not have a strictly convex objective function. The shape of the objective function is not known.

This implies that a solution that satisfies the equilibrium condition is not necessary unique, as it was shown in Chapter 2, and it does not always represent the global minimum of the minimization problem.

8.4 Example - Analysis of Intermodal Commuter Network

The model developed in Section 8.3 was used to analyze the intermodal network described in Chapter 6. The first part of the analysis deals with the verification of the equilibrium condition from the numerical results of the model. The second part uses the model to analyze the network under three scenarios. The first scenario, termed Baseline case, represents the existing situation on the network. Scenarios 2 and 3 represent future situations in years 2000 and 2005. The analysis estimates the equilibrium flows on the network and the resulting travel costs, while suggesting the rail headway and fare that yield these flows. The scope of the analysis is to show how the transit operation can be adjusted to meet future demand.

8.4.1 Verification of Equilibrium Conditions

Results of the analysis of the intermodal network for the baseline year (1995) using the service design model presented in Section 6.3 are shown in Table 8.2. For the reason explained in Section 7.2.1, rail and parking capacity constraints were not considered for the verification of the equilibrium conditions. The assumed load factor was $\lambda=0.8$. Table 8.2 shows the equilibrium path flows and the resulting path costs. The headway was estimated to be 17.76 min. and fare was $1.47 per passenger.

Table 8.2 Results used to Verify the Equilibrium Condition

O-D pair	Path	Travel Cost	Path Flow
01-D	P1 (auto)	23.68	
	P2 (auto)	32.83	
	P3 (auto)	23.19	540
	P4 (intermodal)	28.22	
	P5 (intermodal)	33.63	
	P6 (intermodal)	30.45	
	P23 (intermodal)	25.74	
	P7 (rail)	24.28	
02-D	P8 (auto)	30.70	
	P9 (auto)	21.69	130
	P10 (intermodal)	23.05	
	P11 (intermodal)	24.47	
	P24 (intermodal)	23.60	
	P12 (rail)	22.47	
03-D	P13 (auto)	30.93	
	P14 (auto)	21.92	
	P15 (intermodal)	24.70	
	P25 (intermodal)	21.72	373
	P16 (rail)	21.72	247
04-D	P20 (auto)	24.26	
	P21 (auto)	19.43	220
	P22 (intermodal)	22.21	
05-D	P17 (auto)	22.39	
	P18 (auto)	20.72	250
	P26 (intermodal)	20.72	86
	P19 (rail)	20.72	584

The equilibrium condition is satisfied when for each O-D pair of the network the travel costs of all utilized paths are equal and less than or equal to the travel cost on any unutilized path. Results from Table 8.2 indicate that there is only one utilized path in each of the O-D pairs: 01-D, 02-D, and 04-D. This utilized path has the lowest travel cost among all paths for the same O-D. For the 03-D pair the two utilized paths have the same travel cost which is less than the travel cost on any unutilized path in 03-D. In addition, all three utilized paths of 05-D have the same travel cost which is less than the travel cost on any unutilized path in 05-D. As a result of this analysis it is concluded that the equilibrium condition is satisfied from the numerical solution of the problem.

8.4.2 Results of the Analysis of Future Year Scenarios

Three scenarios were developed for the analysis of the intermodal network. These scenarios involve varying degrees of highway congestion. The first scenario is termed baseline and represents existing levels of congestion on the network. The other two scenarios were generated by taking the baseline scenario of 1995 and applying a two percent per year compounded growth rate to the baseline background volumes. The scenarios consist of years 1995, 2000, and 2005. Results of the analysis of the network for the three scenario years for a load factor λ=0.8 are shown in Table 8.3. The table shows the equilibrium flows and the resulting travel costs as well as the headway and fare for each scenario year.

Table 8.3 shows that the increasing levels of highway congestion have induced commuters to switch to transit. The table shows that, as congestion increases, auto users shift to rail and intermodal paths. This shift resulted in a more efficient (least cost) travel on the transportation network as headway decreased to accommodate the increasing transit demand. The decreased headway resulted in a decrease in average waiting time and average cost of traveling on rail and intermodal paths. For example, the equilibrium travel cost for the O1-D pair in 2005 is \$22.96/user, lower than that in 1995 which is \$23.19/user.

8.4.3 Sensitivity of the Model with Respect to the Rail Load Factor

For the analysis of the intermodal network presented in Section 8.4.2 a load factor equal to 0.8 was selected meaning that the trains will have an average occupancy

Table 8.3 Equilibrium Network Flows and Costs

		1995		2000		2005	
		λ=0.8					
O-D pair	Path Type	flow (pas/hr)	cost ($/hr)	flow (pas/hr)	cost ($/hr)	flow (pas/hr)	cost ($/hr)
O1-D	A	540	23.19	66	23.15		
	R			474	23.15	540	22.96
	I						
O2-D	A	130	21.73				
	R			130	21.33	130	21.13
	I						
O3-D	A						
	R	247	21.74	247	20.59	338	20.40
	I	373	21.73	373	20.59	282	20.40
O4-D	A	220	19.47	220	19.51		
	R						
	I					220	20.98
O5-D	A	256	20.73				
	R	423	20.73	681	19.60	901	19.43
	I	239	20.70	239	19.56	19	19.38
headway(min/train)		17.82		10.86		9.66	
fare($/passenger)		1.47		1.47		1.47	

equal to 80%. Then the frequency of these 80% occupied trains was determined from the optimization procedure. In this section a sensitivity analysis is performed to determine the results of various load factors in equilibrium flows and headways and the resulting travel costs. The baseline year was analyzed for load factors that vary between 0.5 and 1.0. Results of the analysis are shown in Tables 8.4 and 8.5.

Table 8.4 shows the average travel cost of an individual user of the network, for the baseline year and for each origin-destination pair. The table shows that the equilibrium travel cost for an individual user is minimized for a load factor between 0.55 and 0.60.

The values of headway, fare, and total user cost for various load factors are shown in Table 8.5. Using the values of headway, fare, and total user cost from Table

8.5, the total user cost is plotted as a function of headway in Figure 8.1, and as a function of fare in Figure 8.2.

Table 8.4 Sensitivity of Travel Cost with respect to Load Factor

load factor	Travel Cost ($/hr)				
	O1-D	O2-D	O3-D	O4-D	O5-D
0.50	23.14	21.47	21.44	19.21	20.43
0.55	23.13	21.44	21.40	19.17	20.39
0.60	23.13	21.44	21.40	19.17	20.39
0.65	23.14	21.47	21.43	19.21	20.43
0.70	23.15	21.53	21.51	19.27	20.50
0.75	23.17	21.62	21.61	19.36	20.60
0.80	23.19	21.73	21.74	19.47	20.73
0.85	23.22	21.87	21.89	19.60	20.88
0.90	23.26	22.02	22.07	19.76	21.07
0.95	23.30	22.21	22.28	19.94	21.28
1.00	23.35	22.41	22.52	20.15	21.52

Table 8.5 Sensitivity of Headway, Fare, and Total Travel Cost with respect to the Load Factor

load factor λ	Headway (min/train)	Fare ($/passenger)	Total User Cost ($)
0.50	10.80	2.35	51,599
0.55	11.82	2.13	51,515
0.60	12.90	1.95	51,518
0.65	14.04	1.80	51,592
0.70	15.24	1.67	51,732
0.75	16.50	1.56	51,926
0.80	17.82	1.47	52,174
0.85	19.32	1.38	52,476
0.90	20.82	1.30	52,830
0.95	22.50	1.23	53,241
1.00	24.36	1.17	53,714

The figures show that the total user cost is a convex function of headway and fare. The minimum total user cost is equal to $51,507 and the service that yields this minimum cost has a characteristic headway equal to 12.24 minutes per train and fare equal to 2.06 dollars per user.

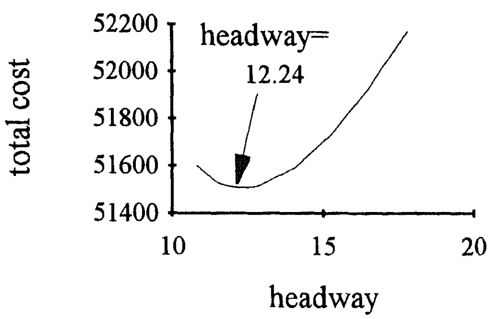


Figure 8.1 Total User Cost vs. Headway

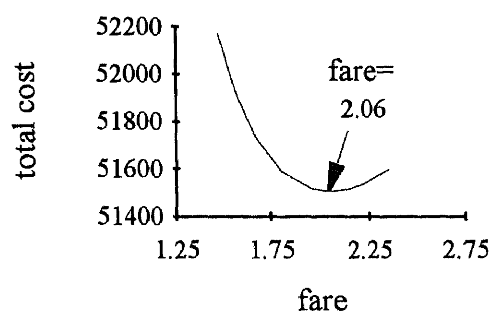


Figure 8.2 Total User Cost vs. Fare

8.5 Conclusion

The model that was formulated in this chapter is very useful in transit operations since it allows the selection of an optimal transit service. From the operator's point of view, the model suggests a service, the expected ridership of which will pay the operating cost through the fare-box revenue. From the users' perspective, the model can be used to determine the headway and fare that minimize the cost of traveling for an individual user on the intermodal network. The example showed that there is a substantial network-wide gain in average user cost brought about by a modal shift from auto to transit. It should be noted that even though congestion increased, the average user cost decreased. This is a sound policy for reducing user cost.

In the mathematical formulation of the short-term network equilibrium problem where the link flows are the only variables to be optimized and the link cost functions are monotonicaly increasing, the objective function to be minimized is strictly convex, a property which ensures the uniqueness of an equilibrium solution (Sheffi 1985). In the mathematical formulation of the intermediate-term network equilibrium problem, where rail headway and fare are optimized along with the link flows, the property of the strict convexity of the objective function is lost and the optimal solution of the problem is not unique. The lack of algorithmic approaches for the solution of this problem made the intermediate-run traffic assignment model unattractive, although from a mathematical perspective this problem is rather challenging and from a practical viewpoint its solution is very important.

The challenge that this formulation puts forward is the development of an algorithm that will be able to guarantee convergence to the global minimum of a function that is not convex, but has a random shape, with local minima and/or maxima.

CHAPTER 9

CONCLUSIONS AND FUTURE RESEARCH

9.1 Summary

What was presented already in this dissertation, accomplished four objectives. First, a theoretically sound approach for modeling intermodal commuter networks was developed. This approach resulted in several (four) models. Second, the mathematical and economic properties of these models were analyzed. Third, a methodological framework that utilizes the network equilibrium models to analyze various policies on an intermodal network was developed. Fourth, the models were used within the methodological framework to analyze a real-world network.

Four models were developed to represent intermodal commuter networks. The first three models combine demand and supply functions to make predictions on the traffic volumes and the resulting travel costs on an intermodal network assuming that the operating characteristics (headway and fare) of the transit system are fixed. The models increase in their degree of complexity as they take into account travelers' preferences towards various modes and types of access to these modes.

The fourth model does not consider the transit service characteristics as fixed, but models implicitly the adjustments of these characteristics to the expected traffic volumes. This model uses more sophisticated supply functions to make predictions regarding pricing and service quality adjustments.

The four models have been designed to realistically represent the travelers' choice process in choosing modes and routes in an intermodal network. Travelers are assumed to choose the modes and the routes that minimize their cost of traveling while satisfying their preferences towards particular modes and types of access to these modes. The models give a realistic representation of the transportation system performance under varying levels of congestion. The performance is measured in terms of cost of traveling on the system. Appropriate functions have been selected to represent the performance of each system. In addition, the models formulate managerial responses regarding pricing and level of service adjustments to variations in congestion levels.

The models have been formulated as mathematical programs with non linear objective functions and linear constraints. The mathematical properties of the models were analyzed. The first three models have a convex objective function and their constraints form a convex set. These properties come primarily from the assumption that user costs are increasing with increasing volume, and they guarantee the uniqueness of the equilibrium solution to the problems. These properties have been advantageous in the development of solution algorithms. Existing algorithms have been used herein to solve the problems and they were efficient. The properties of the fourth model are not as good from a mathematical point of view as those of the first three models. The supply relationships used for transit service assume decreasing travel cost with increasing traffic volume, thus loosing the models' advantageous mathematical properties. The algorithms used to solve this problem were shown to reach an equilibrium solution, which is not unique. Even though one is not able to guarantee that the equilibrium solution obtained from the model is the one that yields the lowest travel costs, the model provides useful insights for the prices and levels of service to be provided on the network to achieve certain objectives. These objectives are the provision of a low cost, frequent service for the travelers, which at the same time is economically efficient for the operator.

A methodological framework was developed to analyze and evaluate the effects of various pricing and operating policies on the flow patterns and the associated travel costs on an intermodal network. The framework is designed to answer questions of interest to transportation planners and to investigate the trade-offs between the reduction in travel time and the cost of capacity increases.

A case study commuter network was selected for the application of the models. The commuter network is a representation of the Raritan Valley Corridor network in Union county, New Jersey. The network is served by multiple highways and a rail line. The analysis of the network indicated that the most efficient improvements on the user travel costs come primarily from improvements in the transit level of service. In addition, the analysis shows that substantial reduction in travel costs may occur as congestion on the network increases if transit level of service and fares are properly adjusted to meet an increasing demand.

9.2 Conclusions

The conclusions derived from this dissertation are associated with the developed methodology, the contributions of the dissertation, and directions of future research.

9.2.1 Developed Methodology

A review of the literature and the state of practice in the area of demand and supply network equilibrium and the broader area of transportation planning revealed the necessity for models that will assign travelers over a network based on well defined principles. It is difficult to predict and mathematically represent human behavior in choosing modes and routes over a network. Using inexact methodologies worsens the problem by making predictions even more unrealistic.

With a continuously increasing congestion problem, the need for good planning models is even more pressing. In addition, the capabilities of the transportation system as a whole, not as distinct modal networks need to be realized. The proposed models present new and unique modeling approaches in the network equilibrium area. The models analyze intermodal networks, equilibrate demand and supply to yield well defined equilibrium solutions, enable more realistic predictions of traffic flows and travel costs, evaluate various improvement policies, and suggest efficient operating schemes for transit.

9.2.2 Contributions of the Research

The methodology and findings of the proposed dissertation would contribute in two fields (1) Transportation Systems Planning and (2) Transit Management and Operations.

9.2.2.1 Contributions to Transportation Systems Planning. This dissertation contributes to the transportation systems planning by developing the modeling approaches which fill a gap in the literature in the demand and supply equilibrium on intermodal networks. It also opens new areas for research in variable demand and in real time intermodal network traffic assignment. The proposed research reflects an ambitious effort to model an important problem which arises in intermodal passenger transportation. The primary goal was to conceptualize the model which will capture the complexity of intermodal network equilibrium. A mathematical programming formulation was utilized because of its ability to handle problems with large number of variables. Such a large number of variables is the consequence of our attempt to realistically represent the network in levels that are necessary for a planning model. The relationships between cost and travel volumes are carefully selected to be representative of each system that is considered in the analysis.

The product of this dissertation--the models--are applied mathematical programs which can serve as powerful tools in urban and regional planning for evaluating various planning schemes. The models can be used by public and private planning agencies in predicting traffic volumes and pricing and service quality levels for various transportation systems which are part of an intermodal network. Thus, the models can be part of a decision support system to aid planning of intermodal transportation systems.

9.2.2.2 Contribution to Transit Management and Operations. The models developed in this dissertation can be a powerful tool aiding policy makers in transit operations to evaluate various system designs as well as aiding their decision-making and, therefore, further improving the planning and operation of transit systems. The results of the models should provide the decision makers with a range of insights in two areas: (1) operations planning and (2) systems management.

Model applications in Operations Planning: The models provide the optimal schedules and fares under which transit should operate, to optimize both user and operator costs. By decreasing user costs the service becomes more attractive and the increasing ridership increases system revenues.

Model applications in Systems Management: The models are able to provide an estimate of the magnitude of savings which could result from the introduction of an efficient operation. The models evaluate the change in the cost of traveling for every mode of the network under consideration. Thus, they are able to estimate the effects of decreasing fares, increasing parking availability, and increasing service frequency, to the cost of the system.

9.2.3 Directions for Future Research

Several directions for future research have been identified in the course of this dissertation.

First, the models can be further improved to take into account travelers' preferences in the selection of transit stations (transfer points) in addition to the preferences towards modes and access types to these modes.

The models can be modified to consider variable demand. Appropriate demand functions can be introduced in the formulation of the models to represent the total demand for travel as a function of the service provided on the network.

Accelerated regimes, such as skip stop or express service, can be introduced as an option in the transit service design, and the models can be modified to select the operating regime that is appropriate for each particular network under certain conditions. These regimes would be advantageous for rail because they will further improve the service, thus shifting more people to it.

New efficient algorithms need to be developed to solve the model that optimizes transit operating characteristics. These algorithms should be able to guarantee convergence to the global minimum of a function that is not convex, but has a random shape with local minima and/or maxima.

The coefficients of the demand models can be estimated once detailed data is obtained. The models will thus become more representative of the particular case study network. Once the coefficients of the demand models are estimated, results of the models can be compared to determine the differences in the predictions of the models.

Finally, the models can provide the basis for a real-time, intermodal network traffic assignment model formulation.

APPENDIX A

FORMULA DERIVATIONS FOR CHAPTER 5

Exibit 1 Derivation of the Inverted Demand Function

$$T_A^{ij} = T^{ij} \frac{\exp(U_A^{ij})}{\exp(U_A^{ij}) + \exp(U_T^{ij})} \text{ or}$$

$$\frac{T_A^{ij}}{T^{ij}} = \frac{1}{1 + \exp(U_T^{ij} - U_A^{ij})} \text{ or}$$

$$\exp(U_T^{ij} - U_A^{ij}) = \frac{T^{ij}}{T_A^{ij}} - 1 \text{ or}$$

$$\exp(U_T^{ij} - U_A^{ij}) = \frac{T^{ij} - T_A^{ij}}{T_A^{ij}} \text{ or}$$

$$U_T^{ij} - U_A^{ij} = \ln \frac{T_T^{ij}}{T_A^{ij}} \text{ or}$$

$$-\alpha_{TA} - \beta * GC_T^{ij} + \beta * GC_A^{ij} = \ln \frac{T_T^{ij}}{T_A^{ij}} \text{ or}$$

$$GC_T^{ij} - GC_A^{ij} = -\frac{1}{\beta}(\ln \frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA})$$

Exibit 2 Expression of the Second Models' Objective Function, Utilizing Entropy Models

The objective function of the model is:

$$\text{Minimize } z(f,T) = \sum_{l \in L} \int_0^{x_l} c[x_l(f)]dx_l - \sum_{ij} \int_0^{T_T^{ij}} -\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA})dT_T^{ij}$$

The second part of the function is expressed as:

$$-\sum_{ij} \int_0^{T_T^{ij}} -\frac{1}{\beta}(\ln\frac{T_T^{ij}}{T^{ij} - T_T^{ij}} + \alpha_{TA})dT_T^{ij} =$$

$$\sum_{ij}\frac{1}{\beta} \int_0^{T_T^{ij}} [\ln T_T^{ij} - \ln(T^{ij} - T_T^{ij}) + \alpha_{TA}]dT_T^{ij} =$$

$$\frac{1}{\beta}[T_T^{ij} \ln T_T^{ij} - T_T^{ij} + (T^{ij} - T_T^{ij})\ln(T^{ij} - T_T^{ij}) - (T^{ij} - T_T^{ij}) - T^{ij} \ln T^{ij} + T^{ij} + \alpha_{TA} T_T^{ij}] =$$

$$\frac{1}{\beta} T_T^{ij}(\ln T_T^{ij} + \alpha_{TA}) + \frac{1}{\beta} T_A^{ij} \ln T_A^{ij} - T^{ij} \ln T^{ij}$$

Thus the objective function can be expressed as:

$$z(f,T) = \sum_{l \in L} \int_0^{x_l} c[x_l(f)]dx_l \frac{1}{\beta} T_T^{ij}(\ln T_T^{ij} + \alpha_{TA}) + \frac{1}{\beta} T_A^{ij} \ln T_A^{ij} - T^{ij} \ln T^{ij}$$

Expression: $-T^{ij} \ln T^{ij}$ is constant, and does not affect the minimization procedure, thus it is dropped from the objective function, which finally takes the form:

$$z(f,T) = \sum_{l \in L} \int_0^{x_l} c[x_l(f)]dx_l \frac{1}{\beta} T_T^{ij}(\ln T_T^{ij} + \alpha_{TA}) + \frac{1}{\beta} T_A^{ij} \ln T_A^{ij}$$

Exibit 3 Expression of the Third Models' Objective Function, Utilizing Entropy Models

The objective function of the model is:

$$\min z = \sum_{l\in L}\int_{0}^{x_l} c(x_l)dx_l - \sum_{ij}\int_{0}^{T_T^{ij}} -\frac{1}{\beta_2}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA})dT_T^{ij} - \sum_{ij}\int_{0}^{T_M^{ij}} -\frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR})dT_M^{ij}$$

The second and third part of the function is expressed as:

$$-\sum_{ij}\int_{0}^{T_T^{ij}} -\frac{1}{\beta_2}(\ln\frac{T_T^{ij}}{T_A^{ij}} + \alpha_{TA})dT_T^{ij} - \sum_{ij}\int_{0}^{T_M^{ij}} -\frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_R^{ij}} + \alpha_{MR})dT_M^{ij} =$$

$$\sum_{ij}\frac{1}{\beta_2}\int_{0}^{T_T^{ij}}[\ln T_T^{ij} - \ln(T^{ij} - T_T^{ij}) + \alpha_{TA}]dT_T^{ij} + \sum_{ij}\frac{1}{\beta_1}\int_{0}^{T_M^{ij}}[\ln T_M^{ij} - \ln(T_T^{ij} - T_M^{ij}) + \alpha_{MR}]dT_M^{ij} =$$

$$\frac{1}{\beta_2}[T_T^{ij}\ln T_T^{ij} - T_T^{ij} + (T^{ij} - T_T^{ij})\ln(T^{ij} - T_T^{ij}) - (T^{ij} - T_T^{ij}) - T^{ij}\ln T^{ij} + T^{ij} + \alpha_{TA}T_T^{ij}] +$$

$$\frac{1}{\beta_1}[T_M^{ij}\ln T_M^{ij} - T_M^{ij} + (T_T^{ij} - T_M^{ij})\ln(T_T^{ij} - T_M^{ij}) - (T_T^{ij} - T_M^{ij}) - T_T^{ij}\ln T_T^{ij} + T_T^{ij} + \alpha_{MR}T_M^{ij}] =$$

$$\frac{1}{\beta_2}T_T^{ij}\ln T_T^{ij} - \frac{1}{\beta_2}T_T^{ij} + \frac{1}{\beta_2}T_A^{ij}\ln(T_A^{ij}) - \frac{1}{\beta_2}T_A^{ij} - \frac{1}{\beta_2}T^{ij}\ln T^{ij} + \frac{1}{\beta_2}T^{ij} + \frac{\alpha_{TA}}{\beta_2}T_T^{ij} +$$

$$\frac{1}{\beta_1}T_M^{ij}\ln T_M^{ij} - \frac{1}{\beta_1}T_M^{ij} + \frac{1}{\beta_1}T_A^{ij}\ln T_A^{ij} - \frac{1}{\beta_1}T_A^{ij} - \frac{1}{\beta_1}T_T^{ij}\ln T_T^{ij} + \frac{1}{\beta_1}T_T^{ij} + \frac{\alpha_{MR}}{\beta_1}T_M^{ij} =$$

Exhibit 3 Expression of the Third Models' Objective Function, Utilizing Entropy Models (Continued)

$$\frac{1}{\beta_2} T_T^{ij} \ln T_T^{ij} + \frac{1}{\beta_2} T_A^{ij} \ln(T_A^{ij}) - \frac{1}{\beta_2} T^{ij} \ln T^{ij} + \frac{\alpha_{TA}}{\beta_2} T_T^{ij} +$$

$$\frac{1}{\beta_1} T_M^{ij} \ln T_M^{ij} + \frac{1}{\beta_1} T_A^{ij} \ln T_A^{ij} - \frac{1}{\beta_1} T_T^{ij} \ln T_T^{ij} + \frac{\alpha_{MR}}{\beta_1} T_M^{ij} =$$

Thus the objective function can be expressed as:

$$\min z = \sum_{l \in L} \int_0^{x_l} c(x_l) dx_l + \frac{1}{\beta_2} T_A^{ij} (\ln T_A^{ij} - a_{TA}) + \frac{1}{\beta_1} T_R^{ij} \ln T_R^{ij} - \frac{1}{\beta_2} T^{ij} (\ln T^{ij} - a_{TA})$$

$$\frac{1}{\beta_1} T_M^{ij} (\ln T_M^{ij} + a_M) + (\frac{1}{\beta_2} - \frac{1}{\beta_1}) T_T^{ij} \ln T_T^{ij}$$

Expression: $-\frac{1}{\beta_2} T^{ij} (\ln T^{ij} - a_{TA})$ is constant, and does not affect the minimization procedure, thus it is dropped from the objective function, which finally takes the form:

$$\min z = \sum_{l} \int_0^{x_l} c(x_l) dx_l + \sum_{ij} [\frac{1}{\beta_2} T_A^{ij} (\ln T_A^{ij} - \alpha_{TA}) + \frac{1}{\beta_1} T_R^{ij} \ln T_R^{ij} +$$

$$\frac{1}{\beta_1} T_M^{ij} (\ln T_M^{ij} + \alpha_{MR}) + (\frac{1}{\beta_2} - \frac{1}{\beta_1}) T_T^{ij} \ln T_T^{ij}$$

Exibit 4 Partial Derivatives of the Lagrangian of the Third Problem

Recall: $\frac{\partial x_l}{\partial f^{ij}_{pk}} = \frac{1}{occ}\delta^{ij}_{lp_k} \quad \forall l \in LZ, LA, LT$, and

$$\frac{\partial x_l}{\partial f^{ij}_{pk}} = \delta^{ij}_{lp_k} \quad \forall l \in LR, LW, \text{ and}$$

$$GC^{ij}_{pA} = \sum_{l \in LZ,LA,LT} \frac{1}{occ}\delta^{ij}_{lp_A} c(x_l) + \sum_{l \in LR,LW} \delta^{ij}_{lp_k} c(x_l)$$

$$\frac{\partial L}{\partial f^{ij}_{pA}} = \frac{\partial \sum_l \int_0^{x_l} c(x_l)dx_l}{\partial x_l} \frac{\partial x_l}{\partial f^{ij}_{pA}} + \frac{\partial \sum_{ij} u^{ij}_A (T^{ij}_A - \sum_{pA} f^{ij}_{pA})}{\partial f^{ij}_{pA}} =$$

$$\sum_{l \in LZ,LA,LT} \frac{1}{occ}\delta^{ij}_{lp_A} c(x_l) + \sum_{l \in LR,LW} \delta^{ij}_{lp_A} c(x_l) - u^{ij}_A = GC^{ij}_{pA} - u^{ij}_A$$

$$\frac{\partial L}{\partial f^{ij}_{pR}} = \frac{\partial \sum_l \int_0^{x_l} c(x_l)dx_l}{\partial x_l} \frac{\partial x_l}{\partial f^{ij}_{pR}} + \frac{\partial \sum_{ij} u^{ij}_R (T^{ij}_R - \sum_{pR} f^{ij}_{pR})}{\partial f^{ij}_{pR}} =$$

$$\sum_{l \in LZ,LA,LT} \frac{1}{occ}\delta^{ij}_{lp_R} c(x_l) + \sum_{l \in LR,LW} \delta^{ij}_{lp_R} c(x_l) - u^{ij}_R = GC^{ij}_{pR} - u^{ij}_R$$

Exhibit 4 Partial Derivatives of the Lagrangian of the Third Problem (Continued)

$$\frac{\partial L}{\partial f^{ij}_{pM}} = \frac{\partial \sum_{l} \int_{0}^{x_l} c(x_l)dx_l}{\partial x_l}\frac{\partial x_l}{\partial f^{ij}_{pM}} + \frac{\partial \sum_{ij} u^{ij}_M (T^{ij}_M - \sum_{pM} f^{ij}_{pM})}{\partial f^{ij}_{pM}} =$$

$$\sum_{l \in LZ, LA, LT} \frac{1}{occ}\delta^{ij}_{lpM} c(x_l) + \sum_{l \in LR, LW} \delta^{ij}_{lpM} c(x_l) - u^{ij}_M = GC^{ij}_{pM} - u^{ij}_M$$

$$\frac{\partial L}{\partial T^{ij}_A} = \frac{\partial \sum_{ij} u^{ij}(T^{ij} - T^{ij}_A - T^{ij}_T)}{\partial T^{ij}_A} + \frac{\partial \sum_{ij} u^{ij}_A (T^{ij}_A - \sum_{pA} f^{ij}_{pA})}{\partial T^{ij}_A} = -u^{ij} + u^{ij}_A$$

$$\frac{\partial L}{\partial T^{ij}_T} = \frac{\partial \sum_{ij} \int_{0}^{T^{ij}_T} \frac{1}{\beta_2}(\ln \frac{T^{ij}_T}{T^{ij} - T^{ij}_T} + \alpha_{TA})dT^{ij}_T}{\partial T^{ij}_T} + \frac{\partial \sum_{ij} u^{ij}(T^{ij} - T^{ij}_A - T^{ij}_T)}{\partial T^{ij}_T} +$$

$$\frac{\partial \sum_{ij} \int_{0}^{T^{ij}_M} \frac{1}{\beta_1}(\ln \frac{T^{ij}_M}{T^{ij}_T - T^{ij}_M} + \alpha_{MR})dT^{ij}_M}{\partial T^{ij}_T} + \frac{\partial \sum_{ij} u^{ij}_T (T^{ij}_T - T^{ij}_R - T^{ij}_M)}{\partial T^{ij}_T} \qquad (5.A)$$

Exhibit 4 Partial Derivatives of the Lagrangian of the Third Problem (Continued)

$$\frac{\partial \sum_{ij} \int_0^{T_M^{ij}} \frac{1}{\beta_1}(\ln \frac{T_M^{ij}}{T_T^{ij} - T_M^{ij}} + \alpha_{MR}) dT_M^{ij}}{\partial T_T^{ij}} =$$

$$\frac{\partial \sum_{ij} \frac{1}{\beta_1} \int_0^{T_M^{ij}} [\ln T_M^{ij} - \ln(T_T^{ij} - T_M^{ij}) + \alpha_{MR}] dT_M^{ij}}{\partial T_T^{ij}} =$$

$$\frac{\partial \sum_{ij} \frac{1}{\beta_1} \int_0^{T_M^{ij}} [\ln T_M^{ij} - \ln(T_T^{ij} - T_M^{ij}) + \alpha_{MR}] dT_M^{ij}}{\partial T_T^{ij}} =$$

$$\frac{\partial \sum_{ij} \frac{1}{\beta_1} [\int_0^{T_M^{ij}} \ln T_M^{ij} dT_M^{ij} + \int_0^{T_M^{ij}} \ln(T_T^{ij} - T_M^{ij}) d(T_T^{ij} - T_M^{ij}) + \int_0^{T_M^{ij}} \alpha_{MR} dT_M^{ij}]}{\partial T_T^{ij}} =$$

$$\frac{\partial \sum_{ij} \frac{1}{\beta_1} [T_M^{ij} \ln T_M^{ij} - T_M^{ij} + (T_T^{ij} - T_M^{ij}) \ln(T_T^{ij} - T_M^{ij}) - (T_T^{ij} - T_M^{ij}) - T_T^{ij} \ln T_T^{ij} + T_T^{ij} + a_{MR} T_M^{ij}]}{\partial T_T^{ij}} =$$

$$\frac{1}{\beta_1} [\ln(T_T^{ij} - T_M^{ij}) + (T_T^{ij} - T_M^{ij}) \frac{1}{(T_T^{ij} - T_M^{ij})} - 1 - \ln T_T^{ij} - T_T^{ij} \frac{1}{T_T^{ij}} + 1] =$$

Exibit 4 Partial Derivatives of the Lagrangian of the Third Problem (Continued)

$$\frac{1}{\beta_1}\ln\frac{T_T^{ij}-T_M^{ij}}{T_T^{ij}}$$

Substituting in equation (5.A) and deriving, we obtain:

$$\frac{\partial L}{\partial T_T^{ij}}=\frac{1}{\beta_2}(\ln\frac{T_T^{ij}}{T^{ij}-T_T^{ij}}+\alpha_{TA})-u^{ij}+\frac{1}{\beta_1}\ln\frac{T_T^{ij}-T_M^{ij}}{T_T^{ij}}+u_T^{ij}$$

$$\frac{\partial L}{\partial T_R^{ij}}=\frac{\partial\sum_{ij}u_T^{ij}(T_T^{ij}-T_R^{ij}-T_M^{ij})}{\partial T_R^{ij}}+\frac{\partial\sum_{ij}u_R^{ij}(T_R^{ij}-\sum_{pR}f_{pR}^{ij})}{\partial T_R^{ij}}=-u_T^{ij}+u_R^{ij}$$

$$\frac{\partial L}{\partial T_M^{ij}}=\frac{\partial\sum_{ij}\int_0^{T_M^{ij}}\frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_T^{ij}-T_M^{ij}}+\alpha_{MR})dT_M^{ij}}{\partial T_M^{ij}}+\frac{\partial\sum_{ij}u_T^{ij}(T_T^{ij}-T_R^{ij}-T_M^{ij})}{\partial T_M^{ij}}+$$

$$\frac{\partial\sum_{ij}u_M^{ij}(T_M^{ij}-\sum_{pM}f_{pM}^{ij})}{\partial T_M^{ij}}=$$

$$\frac{1}{\beta_1}(\ln\frac{T_M^{ij}}{T_T^{ij}-T_M^{ij}}+\alpha_{MR})-u_T^{ij}+u_M^{ij}$$

APPENDIX B

FORMULA DERIVATIONS FOR CHAPTER 8

Exibit 1 Derivative of the Lagrangian with Respect to Path-Flows

$$\frac{\partial L(f,u)}{\partial f_p^{ij}} = \frac{\partial z[x(f)]}{\partial f_p^{ij}} + \frac{\partial \sum_{ij} u_{ij}(T^{ij} - \sum_p f_p^{ij})}{\partial f_p^{ij}}$$

According to the derivation chain rule:

$$\frac{\partial z[x(f)]}{\partial f_p^{ij}} = \sum_l \frac{\partial z(x)}{\partial x_l} \frac{\partial x_l}{\partial f_p^{ij}}$$

and the link flow conservation constraints:

$$x_l = \frac{1}{occ}\sum_p \delta_{lp}^{ij} f_p^{ij}, \text{ or } \frac{\partial x_l}{\partial f_p^{ij}} = \frac{1}{occ}\delta_{lp}^{ij} \qquad \forall l \in LZ, LA, LT, \text{ and}$$

$$x_l = \sum_p \delta_{lp}^{ij} f_p^{ij}, \text{ or } \frac{\partial x_l}{\partial f_p^{ij}} = \delta_{lp}^{ij} \qquad \forall l \in LR, LW$$

the derivative of the Lagrangian becomes:

$$\frac{\partial L(f,u)}{\partial f_p^{ij}} = \sum_{l \in LA, LZ, LT} \frac{1}{occ}\delta_{lp}^{ij} \frac{\partial z(x)}{\partial x_l} + \sum_{l \in LW, LR} \delta_{lp}^{ij} \frac{\partial z(x)}{\partial x_l} - u_{ij}$$

According to the generalized cost functions given from Equations (8.1) and (8.3) the derivative of the Lagrangian becomes:

$$\frac{\partial L(f,u)}{\partial f_p^{ij}} = \sum_{l \in LA, LZ, LT} \frac{1}{occ}\delta_{lp}^{ij} g_l(x_l) + \sum_{l \in LW} \delta_{lp}^{ij} g_l(x_l) +$$

Exibit 1 Derivative of the Lagrangian with Respect to Path-Flows (Continued)

$$\sum_{l \in LR} \delta_{lp}^{ij} g_l + \sum_{l \in LCR} \delta_{lp}^{ij} [VOTT \frac{h_l(x_l)}{2} + fare] - u_{ij}$$

Exibit 2 Analysis of a Diagonal Element of the Hessian Matrix

$$z(x_{CR}) = \int_0^{x_{CR}} [VOTT * (ff_{CR} + \frac{\lambda * seats}{2 * x_{CR}}) + \frac{\cos t * time * x_{CR}}{\lambda * seats * x_{CR}}] dx_{CR}$$

$$\frac{\partial z(x_{CR})}{\partial x_{CR}} = VOTT * (ff_{CR} + \frac{\lambda * seats}{2 * x_{CR}}) + \frac{\cos t * time}{\lambda * seats}$$

$$\frac{\partial^2 z(x_{CR})}{\partial x_{CR}^2} = -\frac{\lambda * seats * VOTT}{2 * x_{CR}^2} < 0$$

REFERENCES

Aashtiani, H. Z. (1979) "The Multi-Modal Traffic Assignment Problem." Ph.D. Dissertation, Operations Research Center, M.I.T., Cambridge, Massachusetts.

Abdulaal, M., and L. J. LeBlanc (1979) Methods for Combining Modal Split and Equilibrium Assignment Models. *Transportation Science* 13, pp. 292-314.

Beckmann, H. M., C. B. McGuire, and C. B. Winsten (1956) *Studies in the Economies of Transportation*. Yale University Press, New Haven, Connecticut.

Boile, M. P., L. N. Spasovic, and A. K. Bladikas (1994) Modeling Intermodal Auto-Rail Commuter Networks. Accepted for publication in the *Transportation Research Record.*

Brooke, A., D. Kendrick, and A. Meeraus (1988) *GAMS A User's Guide*. The Scientific Press.

Comsis (1992) *MinUTP Technical User Manual*, 92 B-Version.

Dafermos, S. C., and F. T. Sparrow (1969) The Traffic Assignment Problem for a General Network. *Journal of Res.* National Bureau of Standards-*B* 73B, No. 2, pp. 91-117.

Dafermos, S. C. (1972) The Traffic Assignment Problem for Multiclass User Transportation Networks. *Transportation Science* 6, pp. 73-87.

Dafermos, S. C. (1982) The General Multimodal Network Equilibrium Problem with Elastic Demand. *Networks* 12, pp. 57-72.

Evans, S. P. (1976) Derivation and Analysis of Some Models for Combining Trip Distribution and Assignment. *Transportation Research*, Vol 10 pp.37-57.

Fernandez, E., J. DeCea, M. Florian, and E. Cabrera (1994) Network Equilibrium Models with Combined Modes *Transportation Science* 28, No. 3, pp. 182-192.

Fisk, C., and S. Nguyen (1981) Existence and Uniqueness Properties of an Asymmetric Two-Mode Equilibrium Model, *Transportation Science* 15, pp. 318-328.

Florian, M. (1977) Traffic Equilibrium Model of Travel by Car and Public Transit Modes. *Transportation Science*, Vol. 11, No 2, pp. 166-179.

Florian, M., and S. Nguyen (1974) A Method for Computing Network Equilibrium with Elastic Demands. *Transportation Science* Vol. 8, No 4, pp 321-332.

Florian, M., and S. Nguyen (1978) A Combined Distribution Modal Split and Trip Assignment Model. *Transportation Research*, Vol 12-B, pp. 241-246.

Florian, M., and H. Spiess (1983) On Binary Mode Choice / Assignment Models. *Transportation Science* 17, No. 1, pp. 32-47.

Forinash, C. V. and F. S. Koppelman (1993) Application and Interpretation of Nested Logit Models of Intercity Mode Choice. *Transportation Research Record* 1413, pp. 98-105.

Frank, M. and P. Wolfe (1956) An Algorithm for Quadratic Programming. *Naval Res. Logist. Quart.* 13, pp. 95-110.

Hartley, T. M., and J. de D. Ortuzar (1980) Aggregate Modal Split Models: Is Current U.K. practice Warranted? *Traffic Engineering and Control* 21(1), pp. 7-13.

Holden, D. J. (1989) Wardrop's Third Principle, Urban Traffic Congestion and Traffic Policy, *Journal of Transport Economics and Policy* pp239-262.

INRO Consultants (1994), Montreal, Canada. *Users Manual EMME/2 Release 7.*

Larson, T. D. (1993) IVHS: That Vision Thing. Lessons from the Interstate Program. *IVHS Review* pp.3-10.

LeBlanc, L. J,. and K. Farhangian (1981) Efficient Algorithms for Solving Elastic Demand Traffic Assignment Problems and Mode Split-Assignment Problems. *Transportation Science*, 15, No 4, pp. 306-317.

LeBlanc, L. J., R. V. Helgason, and D. E. Boyce (1985) Improved Efficiency of the Frank-Wolfe Algorithm for Convex Network Programs. *Transportation Science* Vol. 19, No 4, pp. 445-462.

Leblanc, L. J., E. K. Morlok, and W. P. Pierskalla (1975) An Efficient Approach to Solving the Road Network Equilibrium Traffic Assignment Problem. *Transportation Research* 9, 309-318.

Manheim, M. L. (1979) *Fundamentals of Transportation Systems Analysis: Volume 1: Basic Concepts*. MIT Press Series in Transportation Studies, Cambridge, Massachusetts.

McFadden, D. (1979) *Quantitative Methods for Analyzing Travel Behaviour of Individuals: Some Recent Developments in Behavioural Travel Modeling* (Edited by D. A. Hensher and P. R. Stopher). Croom Helm, London.

Miller, E. (1993) Central Area Mode Choice and Parking Demand. *Transportation Research Record*, 1413, pp. 60-69.

Minoux, M. (1986) *Mathematical Programming Theory and Algorithms*. John Wiley and Sons Ltd., New York.

Morlok, E. K. (1976) *Traffic Equilibrium Methods* (edited by M. Florian) (Lecture Notes in Economics and Mathematical Systems 118), pp.322-367. Springer-Verlag, Berlin.

Morlok, E. K. (1978) *Introduction to Transportation Planning and Engineering*. McGraw Hill, New York.

Morlok, E. K. (1979) Short Run Supply Functions with Decreasing User Costs. *Transportation Research*-B 13B, pp. 183-187.

Morlok, E. K. (1980) Types of Transportation Supply Functions and their Applications, *Transportation Research*-B Vol. 14B, pp 9-27.

Morlok, E. K., and J. A. Warner III (1970) Optimal Scheduling and Fare Selection on Urban Corridor Rapid Transit Systems. Department of Civil Engineering, Thetechnological Institute and The Transportation Center. Northwestern University, Evanston, Illinois.

Murtagh, B. A., and M. A. Saunders (1987). *MINOS 5.1 User's Guide*, Technical Report SOL83-20R, Stanford University, Stanford, California.

National Cooperative Highway Research Program, Report 187 (1978). *Quick-Response Urban Travel Estimation Techniques and Transferable Parameters: User's Guide*. Transportation Research Board, Washington, D.C.

NPTS Urban Travel Patterns (1994) 1990 *Nationwide Personal Transportation Survey (NPTS)*. FHWA-PL-94-018.

Ortuzar, J. de D. (1983) Nested Logit Models for Mixed-Mode Travel in Urban Corridors. *Transportation Research A*, Vol. 17A, No. 4, pp. 283-299.

Sheffi, Y. (1985) *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*, Prentice-Hall Inc., Englewood Cliffs, New Jersey.

New Jersey Highway Straight Line Diagrams (1988) New Jersey Department of Transportation.

Strategic Plan for Intelligent-Highway Systems in the United States (1992) Prepared by IVHS America, Report No: IVHS-AMER-92-3.

Tatineni, M.R., M. R. Lupa, D. B. Englund, D. E. Boyce (1993) Transportation Policy Analysis Using a Combined Model of Travel Choice, submitted for publication to the *Transportation Research Record.*

The Urban Analysis Group (1990). *TRANPLAN: User Manual,* Version 7.0.

Transport Public Policy Forum-Intermodalism (1994) *Summary of First Session.* Eno Headquarters, Lansdowne, Virginia.

Transportation Research Board (1985) *Highway Capacity Manual* Special Report 209.

U.S. Bureau of Public Roads, Department of Commerce (1965) *The National System of Interstate and Defense Highways: Status of Improvements as of* Dec.31 1965, Washington, DC.

US Bureau of Census. *1990 Census of Population.* US Department of Commerce.

US Bureau of Public Roads (1964). *Traffic Assignment Manual.* US Department of Commerce.

Vollmer Associates (1987) *Garden State Parkway Comprehensive Traffic Study.*

Wardrop, J. G. (1952) Some Theoretical Aspects of Road Traffic Research. *Proceedings Institute of Civil Engineers Part II.*