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Decentralized reliable control for large-scale LTI systems

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ABSTRACT

DECENTRALIZED RELIABLE CONTROL FOR LARGE-SCALE LTI SYSTEMS

by
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Reliable control concerns the ability of closed loop system to maintain stability and regulation properties during arbitrary sensor, controller, and actuator failure. Reliable control research has been an active research topic for more than 10 years.

Recent approach for reliable control includes the H_∞ method, the algebraic factorization design, and the robust servomechanism control. These methods have been surveyed and discussed in this thesis with the robust servomechanism control methodology serving as the basis of the research development of this work.

In this thesis, the reliable control for large-scale, multi-input/output linear system is considered. Two concepts of reliable control are introduced in this work: (1) Decentralized Robust Servomechanism Problem with Complete Reliability ($DRSPwCR$) and (2) Block Decentralized Robust Servo Problem with Complete Reliability ($BDRSPwCR$). The $DRSPwCR$ solves the reliable control problem by applying strict diagonal decentralized controller configurations. The $BDRSPwCR$ solves the reliable control problem by applying block diagonal decentralized controller configurations.

Research results of solving $DRSPwCR$ for the class of minimum phase systems is first developed in this work. The problem is solved by applying strict decentralized PID^r control to an otherwise unreliable plant and thus significantly extending the class of processes that can be controlled reliably. Research results of solving $BDRSPwCR$ is developed for plants which have a pre-imposed block diagonal structure or plants with non-minimum phase minors. The reliable control conditions for an arbitrary linear system is then analyzed, and a general controller

synthesis for solving the reliable control problem for arbitrary linear system is given in this work.

The *DRSPwCR* can be applied in many industry areas as well as in the transportation area. In this work, the reliable control results are applied in the urban vehicle traffic network. A traffic queue length model is developed, a control algorithm is synthesized, and simulations are made under different traffic subsystem failure modes such as non-functioning traffic lights, traffic accidents, and intersection blockage, etc.

Finally, future research topics such as to relax the constraints of plants to achieve reliable control and to optimize the closed loop system dynamic performances, etc. are proposed.

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This work is dedicated to
my parents and my brothers

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LIST OF SYMBOLS

x	:	state vector of the nominal system, $x \in \Re^n$
u	:	control input vector of the nominal system, $u \in \Re^\nu$
u_i	:	the ith scalor input, $u_i \in \Re^{n_i}$, $i \in [1, 2, \cdots, \nu]$
y	:	output vector of the nominal system, $y \in \Re^\nu$
y_i	:	the ith scalor output, $y_i \in \Re^{n_i}$, $i \in [1, 2, \cdots, \nu]$
ω	:	constant disturbance input, $\omega \in \Re^\Omega$
$T(s)$	:	open loop transfer matrix of the nominal system.
$t_{ij}(s)$	:	the polynomial element corresponding to the ith row and jth column of the transfer matrix $T(s)$. $i, j \in [1, 2, \cdots, \nu]$.
$\overline{T}(s)$	:	normalized transfer matrix, $\overline{T}(s) \triangleq T(s) \times \text{diag}[(t_{11}, \cdots, t_{\nu\nu})]^{-1}$.
$T[i_1, \cdots, i_\phi](s)$	:	a submatrix obtained from $T(s)$ by retaining the $i_1, \cdots, i_\phi$th non-redundant rows and columns.
$\overline{T}[i_1, \cdots, i_\phi](s)$	:	a submatrix obtained from $\overline{T}(s)$ by retaining the $i_1, \cdots, i_\phi$th non-redundant rows and columns.
$N(s)$	:	numerator matrix of $T(s)$, $T(s) = \frac{N(s)}{d(s)}$.
$N[i_1, \cdots, i_\phi](s)$	:	a submatrix obtained from $N(s)$ by retaining the $i_1, \cdots, i_\phi$th non-redundant rows and columns.
$n_{ij}(s)$	:	the polynomial element corresponding to the ith row and jth column of $N(s)$, $i, j \in [1, 2, \cdots, \nu]$.
$\det N(s)$	:	determinant of $N(s)$.
$\det N[\phi](s)$	:	the ϕth order leading principal minor of $N(s)$ defined as $\det N[i_1, \cdots, i_\phi](s)$.

LIST OF SYMBOLS

(Continued)

$K_P(s)$: proportional controller matrix, $K_P(s) \in \mathcal{C}^{\nu \times \nu}$

k_{pi} : the ith proportional controller gain, $i \in [1, 2, \cdots, \nu]$

$K_I(s)$: integral controller matrix, $K_I(s) \in \mathcal{C}^{\nu \times \nu}$

k_i : the ith integral control gain, $i \in [1, 2, \cdots, \nu]$

r : maximum pole-zero excess of the tranfer matrix $T(s)$

$K_D(s)$: the rth order derivative controller matrix, $K_D(s) \in \mathcal{C}^{\nu \times \nu}$

$K_D^i(s)$: the derivative control applied on the ith group of input/output channels.

$k_i^d(s)$: the ith D^r derivative controller.

$K_{PD}(s)$: PD controller, defined as $K_P(s) + K_D(s)$, $K_{PD}(s) \in \mathcal{C}^{\nu \times \nu}$

$K_i(s)$: the controller applied to the ith input/output channel of a multivariable system.

$K_I^i(s)$: the integral controller applied to the ith group of input/output channels.

$T_P(s)$: the transfer matrix of closed-loop system when the proportional control is applied.

$T_{PD}(s)$: the transfer matrix of closed-loop system when the proportional and derivative control is applied.

$D_1(s)$: closed-loop characteristic polynomial of a two-input/output system with channel 1 only operational.

$D_2(s)$: closed-loop characteristic polynomial of a two-input/output system with channel 2 only operational.

$D_{12}(s)$: closed-loop characteristic polynomial of a two-input/output system with channels 1 and 2 both operational.

LIST OF SYMBOLS

(Continued)

v_i	:	a sub-vector of the input vector u, with $u = (v_1, v_2, \cdots, v_\mu)'$.
z_i	:	a sub-vector of the output vector y, with $y = (z_1, z_2, \cdots, z_\mu)'$.
$Re[\sigma(M(\cdot))]$	:	real parts of eigenvalues of the matrix $M(\cdot)$.
$T^B[i_1, \cdots, i_\phi](s)$	:	the transfer matrix under partial block failure, with the fault free input/ouput pairing $(v_{i_1}, z_{i_1}), \cdots, (v_{i_\phi}, z_{i_\phi})$ being remained.
$N^B_{ij}(s)$	:	a sub-block of $N(s)$ corresponding to input vector v_j and output vector z_i.
$T^B_{ij}(s)$	:	a sub-block of $T(s)$ corresponding to input vector v_j and output vector z_i.
$N^B[i_1, \cdots, i_\phi](s)$	:	the numerator matrix of the transfer matrix $T(s)$ under partial block failure, with the fault free input/output pairing $(v_{i_1}, z_{i_1}), \cdots, (v_{i_\phi}, z_{i_\phi})$ being remained.
$K^B[i_1, \cdots, i_\phi](s)$	:	the controller matrix under partial block failure, with the fault free input/ouput pairing $(v_{i_1}, z_{i_1}), \cdots,$ (v_{i_ϕ}, z_{i_ϕ}) being remained.
$\Delta N^B[\phi]$	:	block transmission zeros of $N[i_1, i_2, \cdots, i_\phi](s)$.
$M_\phi[1_1, \cdots, i_\phi]$	:	the ϕth steady state interaction matrix, $\phi = 2, 3, \cdots, \mu$.
η_i	:	integral controller vector of the ith block.
$\mathcal{A}_{i_\phi}$	:	closed-loop state feedback matrix when the $i_1, \cdots i_\phi$ controller blocks are installed.
q^i_x	:	queue length in front of ith intersection in the x direction.
q^i_y	:	queue length in front of ith intersection in the y direction.
f^i_{in}	:	maximum output flow rate of the ith intersection.

LIST OF SYMBOLS

(Continued)

g_i : normalized "go" signal duration at x direction of the ith intersection.

$\overline{g}_i$: nomial value of g_i.

Δg_i : incremental "go" duration fractional right turn traffic flow rate a ith intersection.

θ_i : fractional right turn traffic flow rate at the ith intersection.

β_i : fractional left turn traffic flow rate at the ith intersection.

f^i_{xin} : input flow rate at the x direction of ith of intersection.

f^i_{yin} : input flow rate at the y direction of ith of intersection.

f^i_{xout} : output flow rate at the x direction of ith of intersection.

f^i_{yout} : output flow rate at the y direction of ith of intersection.

N_x : number of streets (x-direction).

N_y : number of avenues (y-direction).

$\mathcal{T}$: a constant matrix consists of θ_is and β_is, characterizes the flow configuration of the network.

J : a constant matrix defined as $\underbrace{[\text{block diagonal}\,(1 \;\; -1)]}_{n \text{ blocks}}$.

q : a $2n \times 1$ vector consists of q^i_xs and q^i_ys.

Δu_i : the ith control input, the incremental traffic flow passing through the ith intersection during the "go" signal.

F_m : a constant vector consists of 0s and f^i_ms.

LIST OF SYMBOLS

(Continued)

$\overline{u}_i$	:	nominal control input values at the ith intersection.
F_c	:	a $2n \times 1$ vector comprising of external input flow rate.
$f(q_i)$	:	a non-linear function to maintain the non-negativity of the ith queue length.
L	:	a $2n \times n$ constant matrix to convert non-standard traffic network into standard configuration.
Φ	:	a constant matrix defined as $(I - T)J'$.
M	:	a constant matrix defined as $F_c - (I - T)(F_m + J\overline{u})$.
$\mathcal{N}$	:	a $n \times 1$ fictitious(or dummy) controller vector.
Δu_f	:	control input vector in the non-standard traffic network queue length model, consists of both real and fictitious control values.
Δq_i	:	queue length difference defined as $q_x^i - q_y^i$.
Δq_{ref}^i	:	the Δq_i reference value.
F_{in}	:	a $2n \times 1$ vector consists of f_{xin}^i and f_{yin}^i.
F_{out}	:	a $2n \times 1$ vector consists of f_{xout}^i and f_{yout}^i.

CHAPTER 1
INTRODUCTION

Reliable control concerns the ability of closed loop system to maintain key properties such as stability and regulation during arbitrary sensor, actuator or partial controller failure. In many industrial control problems, reliability requirements are critical to the long term feasibility of system operations.

Existing approaches in dealing with partial system failure can be classified as 1) fault-sensing and 2) fault-tolerant. The latter is generally considered as "reliable" control where the system with partial failure can remain maximally functional without retuning of the controllers.

In about the recent 10 years, a number of methods in the literature have been developed in the research of reliable control. There are three major research methods in this area: the extended H_∞[13] method, the algebraic factorizations[12] design, and the robust servo control[4] design, etc.

The H_∞-norm has been found as a particularly useful performance measure in solving diverse control problems including disturbance rejection, model reference design, tracking and robust design. The Extended H_∞ design method for the reliable centralized and decentralized control system design was developed by using observer-based output feedback. The leaders in this area are William R. Perkins[13][15] of University of Illinois, Urbana, and M.H.Shor[14][15] of Oregon State University. They present a methodology for design of the reliable centralized and decentralized control systems in which the resulting designs are guaranteed closed-loop stability and an H_∞ disturbance-attenuation bound for the base case as well as any admissible control component failures occur. The design is obtained by including in the nominal plant description additional disturbance inputs or regulated inputs to account for possible control inputs or measurements outages, respectively, and computing basic H_∞ control designs for the augmented plant. The existence of solutions of the design equations are sufficient to guarantee that the reliable design tolerates system component outages within a prespecified set of susceptible sensors or actuators in centralized case or within a prespecified set of susceptible control channels in the decentralized case. The following subsections show the controller structure and application conditions for the decentralized reliable control case.

The algebraic factorization methodology is an algebraic design method which was developed based on diagonalization certain transfer functions of the nominal system which is linear, time-invariant, multi-input multi-output with unity-feedback. The leaders in this area are A.Nazli Gündeş and C.A.Desoer[9][12][11][10] of University of California, Davis. In this research, the reliable control problem was treated as simultaneous stabilization of the nominal plant and the plant multiplied by different failure matrices. The system is stable for all possible failures of at most k of the sensor-connections or at most m of the actuator-connections. In this case the plant and the controller must have certain properties. These properties are explained in terms of denominator-matrices of their coprime-factorizations. A controller design methodology, which does not require the failure to be known in advance, is obtained by diagonalization of certain transfer functions of the nominal system having the above properties.

The robust servo control methodology assumes that no plant model of the system is available. The motivation of this design is mainly directed at process control systems and large-scale systems, where it is difficult to obtain a mathematical model of the system, such system is carried out by using decentralized design, i.e. the multivariable system is treated as though is consists of a number of separate single-input/output systems. This research was first developed by T.N.Chang[4][5] and E.J.Davison[4][41] of University of Toronto in 1986. Interaction analysis of multivariable systems has long been an outstanding issue[4][8]. The need for analyzing interaction is obvious: high loop interaction frequently leads to deterioration of transient response, system stability and regulation characteristics. On the other hand, this issue is complex, since the interactions depend on numerous factors such as stability, control structure, and plant structure, etc. In addition, standard multivariable control design could be carried out only if the mathematical model of the system was known. In the robust servo control design method, the only assumptions made for the plant are that (i) it can be described by a linear time-invariant model and (ii) it is open loop asymptotically stable. Under these assumptions, a set of steady-state interaction indices has been introduced, these indices give a measure of interaction when a decentralized proportional and integral controller configuration is used. The indices may be determined experimentally and thus, the design are quite practical to apply to industrial large-scale systems, which often lack an accurate mathematical description.

The robust servo control has wide applications on the following areas:

- Fault Tolerant Traffic Control Systems: In an urban traffic network where flow efficiency is of prime importance, fault-tolerant capability is critical to the long term operation and integrity of the system.

- Power Systems: In a power system, it is very critical to tolerant arbitrary sub-system failure without affecting other fault free sub-systems operation normally.

- Chemical Process Control: In such multivariable systems, the mathematical model of the plant is usually unknown, the robustness of the reliable control system is very important.

- Data Traffic Network Systems(ATM or other data communication network): data traffic network systems is similar to the vehicle traffic network systems.

- Manufacturing Systems: the example such as multi-dimensional robot motion control systems, the reliability is important to the system operations.

The research work in this dissertation follows the robust servo control approach where the decentralized robust servomechanism problem with complete reliability is considered. The failure pattern considered in this work includes arbitrary sensor, actuator, and controller failure, thereby imposing the restriction that the system is open-loop stable. The primary objectives of the reliable controls are: 1) disturbance rejection and 2) stability of the failed system and regulation of the remaining system.

In the following chapters, a detailed literature survey, including the application conditions, and controller synthesis are provided as well as a comparison of advantages and disadvantages between these existing methodologies. Following the background theory of robust servo reliable control design, some new results of decentralized robust servo problems with complete reliability($DRSPwCR$) are given. Following the background theory of robust servo reliable control design, some new results of decentralized robust servo problems with complete reliability($DRSPwCR$) are given. A special application example of traffic network control using the robust servo approach is given with simulation results.

CHAPTER 2
LITERATURE SURVEY

In this chapter, three major methodologies in reliable control research have been surveyed. There methods are: Extended H_∞, Algebraic Factorization, and Decentralized Robust Servochmechanism Problem. A comparison among these methodologies on their advantages, disadvantages and application limitations is given.

2.1 Extended H_∞ Method

In this section the Extended H_∞ Method is analyzed in detail. In the system description, the state space model of the open loop nominal plant is given, a full state observer based feedback control law is synthesis; The main theoretical results include a set of design equations to be solved for the synthesis of decentralized controllers, such controller design results that the closed loop system is internally stable under the prespecified measurement outages and controller outages, and provides a good performance for the H_∞ norm of the closed loop system transfer matrix.

2.1.1 System Description and Control Law

In this subsection, the state space models of the nominal system and control system, as well as the augmented system are described.

Consider the linear time-invariant plant:

$$\begin{aligned} \dot{x} &= Ax + Bu + Gw_0 \\ Y &= Cx + w \end{aligned} \tag{2.1}$$

where $Y \equiv (y_1, \cdots, y_\nu)' = (C_1, \cdots, C_\nu)'x + (w_1, \cdots, w_\nu)'$ is the locally measured outputs vector. $u \in \Re^\nu$ is the local control input vector, $B = [B_1, \cdots, B_\nu]$.

Also define:

$$S_i = B_i B_i' \quad i \in (1, 2, \cdots, \nu) \tag{2.2}$$

$$S = S_1 + S_2 + \cdots + S_\nu = BB' \tag{2.3}$$

The decentralized control law an be expressed as:

$$\begin{aligned} \dot{\xi}_i &= (A + BK + GK_d - L_iC_i)\xi_i + L_iy_i \\ u_i &= K_i\xi_i \end{aligned} \tag{2.4}$$

where K_i is the state feedback control gain, $i \in (1, 2, \cdots, \nu)$. Apply the ν controllers to plant(2.1) gives a closed-loop system of order $(\nu + 1)n$ described by:

$$\dot{x}_e = F_e x_e + G_e w_e, \quad z = H_e x_e \tag{2.5}$$

where

$$x_e' = (x'\xi') \tag{2.6}$$

$$\xi' = (\xi_1'\xi_2' \cdots \xi_\nu') \tag{2.7}$$

$$w_e' = (w_0'w') \tag{2.8}$$

$$F_e = \begin{pmatrix} A & BK_c \\ L_cC & A_{\alpha c} - L_cC_c \end{pmatrix} \tag{2.9}$$

$$G_e = \begin{pmatrix} G & 0 \\ 0 & L_c \end{pmatrix} \tag{2.10}$$

$$H_e = \begin{pmatrix} H & 0 \\ 0 & K_c \end{pmatrix} \tag{2.11}$$

$$A_{\alpha c} = \mathrm{diag}(A_\alpha, A_\alpha, \cdots, A_\alpha) \tag{2.12}$$

$$A_\alpha = A + BK + GK_d \tag{2.13}$$

$$C_c = \mathrm{diag}(C_1, C_2, \cdots, C_\nu) \tag{2.14}$$

$$K_c = \mathrm{diag}(K_1, K_2, \cdots, K_\nu) \tag{2.15}$$

$$L_c = \mathrm{diag}(L_1, L_2, \cdots, L_\nu) \tag{2.16}$$

Transforming coordinates of (2.5) such that the last νn state variables are the errors $e_i = \xi_i - x, i \in (1, 2, \cdots, \nu)$, gives:

$$\dot{\tilde{x}}_e = \tilde{F}_e\tilde{x}_e + \tilde{G}_e w_e, \quad z = \tilde{H}_e\tilde{x}_e \tag{2.17}$$

where

$$\tilde{F}_e = M_e^{-1} F_e M_e = \begin{pmatrix} A+BK & BK_c \\ G_c K_d & A_c - L_c C_c \end{pmatrix} \tag{2.18}$$

$$\tilde{G}_e = M_e^{-1} G_e = \begin{pmatrix} G & 0 \\ -G_c & L_c \end{pmatrix} \tag{2.19}$$

$$\tilde{H}_e = H_e M_e = \begin{pmatrix} H & 0 \\ K & K_c \end{pmatrix} \tag{2.20}$$

$$M_e = \begin{pmatrix} I & 0 \\ I_c & I \end{pmatrix} \tag{2.21}$$

$$G_c = I_c G \tag{2.22}$$

$$A_c = A_{\alpha c} - I_c B K_c \tag{2.23}$$

$$I_c' = [I, I, \cdots, I] \in \Re^{n \times \nu n} \tag{2.24}$$

The closed loop system transfer matrix is defined as

$$T(s) = \tilde{H}_e (sI - \tilde{F}_e)^{-1} \tilde{G}_e$$

The goal of the design is to select the state feedback gains, the observer gains and the disturbance estimate gain so that the closed loop transfer matrix $T(s)$ satisfies $\|T\|_\infty \leq \alpha$ for some prescribed $\alpha > 0$.

2.1.2 Main Results

The main results of the Extended H_∞ design are listed as the following lemma and theorem:

For a system described by:

$$\dot{x} = Fx + Gw, \quad z = Hx \tag{2.25}$$

Lemma 2.1.1 *Let $T(s) = H(sI - F)^{-1} G$ with (F, H) a detectable pair. If there exist a real matrix $X \geq 0$ and a positive scalar α such that*

$$F'X + XF + \frac{1}{\alpha^2} XGG'X + H'H \leq 0 \tag{2.26}$$

then F is Hurwitz, and $T(s)$ satisfies $\|T\|_\infty \leq \alpha$.

Theorem 2.1.1 *[13] Let (A, H) be a detectable pair and α be a positive scalar. Suppose*

$$K = -B'X, \quad K_d = \frac{1}{\alpha^2}G'X \tag{2.27}$$

where $X \geq 0$ satisfies

$$A'X + XA + \frac{1}{\alpha^2}XGG'X - XSX + H'H = 0 \tag{2.28}$$

and where $A_\alpha = A + BK + GK_d$ is Hurwitz, and $A + GK_d$ had no $j\omega$-axis eigenvalues. Suppose also

$$L_i = W_{ii}C_i' \quad i \in (1, 2, \cdots, \nu) \tag{2.29}$$

where $W > 0$ satisfies the Riccati-like algebraic equation

$$\begin{aligned} WA_c' + A_cW + \frac{1}{\alpha^2}WK_c'K_cW & \\ -WC_c'C_cW + G_cG_c' + (W - W_D)C_c'C_c(W - W_D) &= 0 \end{aligned} \tag{2.30}$$

with W_{ii} and W_D defined by

$$W = \begin{pmatrix} W_{11} & W_{12} & \cdots & W_{1\nu} \\ W_{21} & W_{22} & \cdots & W_{2\nu} \\ \vdots & \vdots & & \vdots \\ W_{\nu 1} & W_{\nu 2} & \cdots & W_{\nu\nu} \end{pmatrix} \tag{2.31}$$

$$W_D = diag(W_{11}, W_{22}, \cdots, W_{\nu\nu}) \tag{2.32}$$

Then the decentralized feedback control law(2.4) stabilizes the plant(2.1) and the closed-loop transfer-function matrix $T(s) = H_e(sI - F_e)^{-1}G_e$, from w_e to z satisfies:

$$\|T\|_\infty \leq \alpha$$

Theorem 2.1.1 indicates that, if the state feedback controller gain K must satisfy the design Equation(2.28), the resulting closed loop system is stable, and the H_∞ noise attenuation bound satisfies the given performance value α.

2.1.3 Design of Reliable Decentralized Control Systems

The following Theorem 2.1.2 and Theorem 2.1.3 are the main results of reliable control design equations to be solved for fault tolerance of measurement outages and control input outages, respectively.

A. Outages Modeled as Measurement Failures:

Let $\Omega \subseteq \{1, 2, \cdots, \dim(y)\}$ corresponding to a selected subset of sensors susceptible to outages. Introduce the decomposition

$$C = C_\Omega + C_{\overline{\Omega}}$$

where $C_{\overline{\Omega}}$ is formed from C by zeroing out rows corresponding to susceptible sensors. Let $\omega \subseteq \Omega$ correspond to a particular subset of the susceptible sensors that actually experience an outage, and let $T_{\overline{\omega}}(s)$ denote the transfer-function matrix of the resulting closed-loop system. It is convenient to adopt the notation

$$C = C_\omega + C_{\overline{\omega}}$$

where C_ω and $C_{\overline{\omega}}$ have meanings analogous to those of C_Ω and $C_{\overline{\Omega}}$. Also, decompose the observer gain as

$$L = L_\omega + L_{\overline{\omega}}$$

so that

$$LC = L_\omega C_\omega + L_{\overline{\omega}} C_{\overline{\Omega}}$$

Suppose the measurement failures takes the form $y_i = 0$. The closed loop system takes the form

$$\dot{x}_e = F_{e\overline{\omega}} x_e + G_{e\overline{\omega}} w_e, \quad z = H_e x_e \tag{2.33}$$

where

$$F_{e\overline{\omega}} = \begin{pmatrix} A & BK \\ L_{\overline{\omega}} C_{\overline{\Omega}} & A_\alpha - LC \end{pmatrix}, G_{e\overline{\omega}} = \begin{pmatrix} G & 0 \\ 0 & L_{\overline{\omega}} \end{pmatrix} \tag{2.34}$$

and

$$A_\alpha = A + BK + GK_d$$

The following theorem describes the reliable design:

Theorem 2.1.2 *[13] With all assumptions and the decentralized design as in Theorem (2.1.1), let $X \geq 0$ satisfy*

$$A'X + XA + \frac{1}{\alpha^2}XGG'X - XSX + H'H + \alpha^2 C_\Omega' C_\Omega = 0 \qquad (2.35)$$

where $\Omega \subseteq (1, 2, \cdots, \nu)$, $C_\Omega' = (C_{t+1}' \cdots C_\nu')$. Then, for the measurements outages corresponding to any $\omega \subseteq \Omega$, the closed-loop system is internally stable, and the closed loop transfer matrix $T_{\overline{\omega}}(s)$ satisfies:

$$\|T_{\overline{\omega}}\|_\infty \leq \alpha$$

In addition, all controllers corresponding to the "susceptible" set Ω are open-loop stable.

Theorem 2.1.2 indicates that, if the control gain design satisfies the design Equation(2.35), then the closed loop system under certain prespecified set of measurement outages is internally stable and H_∞ bound satisfies the given performance.

B. Outages Modeled as Control Input Failures

Assume the controller failures are modeled as $u_i = 0, i \in \omega$.

Theorem 2.1.3 *With all assumptions and the decentralized design as in Theorem (2.1.1), let $X \geq 0$ satisfy*

$$A'X + XA + \frac{1}{\alpha^2}XGG'X - XS_{\overline{\Omega}}X + H'H = 0 \qquad (2.36)$$

and let $W > 0$ satisfy

$$\begin{aligned} WA_{c+}' + A_{c+}W + \frac{1}{\alpha^2}WK_c'K_cW - WC_c'C_cW + G_cG_c' \\ +\alpha^2 I_c S_\Omega I_c' + (W - W_D)C_c'C_c(W - W_D) &= 0 \end{aligned} \qquad (2.37)$$

where

$$A_{c+} = A_c + diag(S_\Omega X, S_\Omega X, \cdots, S_\Omega X,)$$

$$S_\Omega = B_\Omega B_\Omega^{'}, \quad S = S_\Omega + S_{\overline{\Omega}}$$

$$\Omega \subseteq (1, 2, \cdots, \nu), \; B_\Omega = (B_{t+1} \cdots B_\nu)$$

Let the controller be given by:

$$\begin{aligned} \dot{\xi}_i &= (A + BK + G_+ K_{d+} - L_i C_i)\xi_i + L_i y_i \\ u_i &= K_i \xi_i \end{aligned} \tag{2.38}$$

with $i \in (1, 2, \cdots, \nu)$ *and*

$$G_+ = (G\alpha B_\Omega), \quad K_{d+} = \frac{1}{\alpha^2} G_+ X$$

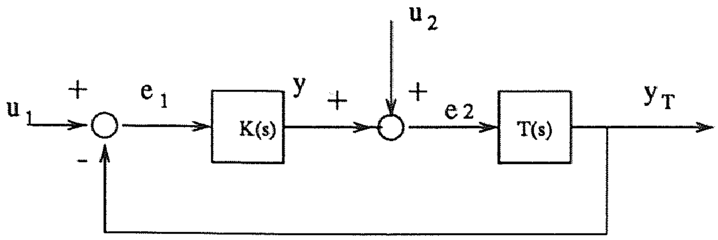
Assuming all controllers are open-loop(internally) stable. Then for controller outages corresponding to any $\omega \subseteq \Omega$*, the closed-loop system is internally stable, and the closed loop transfer matrix* $T_{\overline{\omega}}(s)$ *satisfies:*

$$\|T_{\overline{\omega}}\|_\infty \leq \alpha$$

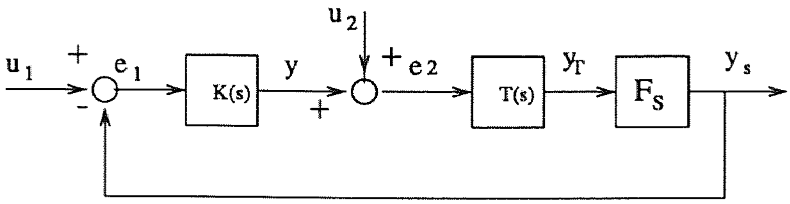
Theorem 2.1.3 indicates that, if the control gain design satisfies the design Equation(2.36), then the closed loop system under certain prespecified set of controller outages is internally stable and H_∞ bound satisfies the given performance.

2.2 Algebraic Factorization

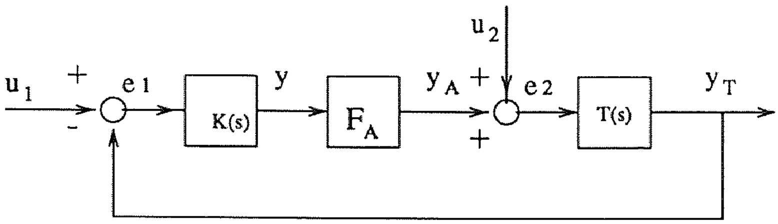
In this section the Algebraic Factorization Method is discussed along with system description, necessary and sufficent conditions of application, and control law synthesis.



The system (T(s), K(s))



The system (FS, T(s), K(s))



The system (T(s), FA, K(s))

Figure 2.1 The System And The Sensor/Actuator Connections

2.2.1 System Descriptions

Consider the LTI, MIMO feedback systems in Figure 2.1 where $T(s)$ represents the plant, $K(s)$ represents the controller, $F_S \in \Re^{\nu \times \nu}$ represents the sensor connections, it is a diagonal matrix whose entries are nominally equal to 1; if jth sensor fails, the jth diagonal entry becomes a stable rational function including 0. $F_A \in \Re^{\nu \times \nu}$ represents the actuator connections, it is a diagonal matrix whose entries are nominally equal to 1; if jth actuator fails, the jth diagonal entry becomes a stable rational function including 0.

If $T(s)$ is stabilizable and detectable, then $T(s)$ is said has no hidden-modes. Let $N_T D_T^{-1}$ denote any right-coprime-factorization(RCF) and $\tilde{D}_T^{-1}\tilde{N}_T$ denote any left-coprime-factorization(LCF) of $T(s) \in \Re^{\nu \times \nu}$, $T(s) = N_T D_T^{-1} = \tilde{D}_T^{-1}\tilde{N}_T$.

Similarly, $N_K D_K^{-1}$ denote any right-coprime-factorization(RCF) and $\tilde{D}_K^{-1}\tilde{N}_K$ denote any left-coprime-factorization(LCF) of $K(s) \in \Re^{\nu \times \nu}$, $K(s) = N_K D_K^{-1} = \tilde{D}_K^{-1}\tilde{N}_K$.

Assumption 2.2.1 *Assuming that:*

- *The plant* $T(s) \in \Re^{\nu \times \nu}$
- *The controller* $K(s) \in \Re^{\nu \times \nu}$
- $T(s)$ *and* $K(s)$ *have no hidden-modes.*

2.2.2 Main Results

Notations: Let U be a subset of field C of complex number, where U is closed and symmetric about the real axis, $\pm\infty \in U$.

$$U \supset C_+ := \{s \in C \| \Re e(s) \geq 0\}$$

Let R_u be the ring of proper rational functions which have no poles in U. The group of units of R_u is ψ. The set of matrices whose entries are in R_u is denoted $M(R_u)$. A matrix M is called R_u stable iff $M \in M(R_u)$; $M \in M(R_u)$ is R_u-unimodular iff $\det M \in \psi$.

Definition 2.2.1 *R_u-stability and integrity:*

(a) The system $(F_S, T(s), K(s))$ is said to be R_u stable iff the transfer matrix $T_S(S) \in M(R_u)$. For a fixed k, where $k \in (1, \cdots, \nu)$, the system $(F_S, T(s), K(s))$ is said to have $k - sensor - integrity$ iff it is R_u-stable for all F_S

(b) The system $(F_A, T(s), K(s))$ is said to be R_u stable iff the transfer matrix $T_A(s) \in M(R_u)$. For a fixed m, where $m \in (1, \cdots, \nu)$, the system $(F_S, T(s), K(s))$ is said to have $m - actuator - integrity$ iff it is R_u-stable for all F_A

Definition 2.2.1 explaines the methematical meaning of reliability to tolerant sensor, and actuator/control failures for the closed loop system described by Figure 2.1.

Reliable Control Conditions: Let $\mathcal{F}_{Sk}$ denotes the class of sensor-connection failures defined by:

$$\mathcal{F}_{Sk} := \{\text{diag}[f_1 \cdots f_\nu] | \text{for } j = 1, \cdots, \nu, \text{at least } (\nu - k) \text{of the } f_j = 1\}$$

k corresponds to the maximum number of failures allowed in the sensor-connections, $k \in (1, \cdots, \nu)$. Similarly, $\mathcal{F}_{Am}$ denotes the class of actuator-connection matrices defined by:

$$\mathcal{F}_{Am} := \{\text{diag}[f_1 \cdots f_\nu] | \text{for } j = 1, \cdots, \nu, \text{at least } (\nu - m) \text{of the } f_j = 1\}$$

Theorem 2.2.1 *Necessary and sufficient conditions for integrity[12]:*
(a) The system $(F_S, T(s), K(s))$ is stable for all F_S iff

$$D_S = \begin{bmatrix} \tilde{D}_T & -\tilde{N}_T N_K \\ F_S & D_K \end{bmatrix} \text{ is } R_u - unimodular, \forall F_S \in \mathcal{F}_{Sk} \tag{2.39}$$

(b) The system $(T(s), F_A, K(s))$ has is stable for all F_A iff

$$D_A = \begin{bmatrix} D_T & -F_A \\ \tilde{N}_K N_T & \tilde{D}_K \end{bmatrix} \text{ is } R_u - unimodular, \forall F_A \in \mathcal{F}_{Am} \tag{2.40}$$

Theorem 2.2.1 indicates the necessary and sufficient conditions for the matrix D_S has to satisfy in order to tolerant sensor failures, and the necessary and sufficient conditions for the matrix D_A has to satisfy in order to tolerant actuator/controller failures.

2.2.3 Controller Synthesis

Figure 2.2 shows the controller design for the case $k = 1$ or $\nu - 1$,

$$\begin{aligned} K(s) &= \tilde{D}_K^{-1} \tilde{N}_K \\ &= (\tilde{D}_{SD} + \tilde{N}_{SD}(Y_{Sk} + Q_{Sk})\tilde{N}_T)^{-1} \\ &\quad (\tilde{N}_{SD} - \tilde{N}_{SD}(Y_{Sk} + Q_{Sk})\tilde{D}_T) \end{aligned}$$

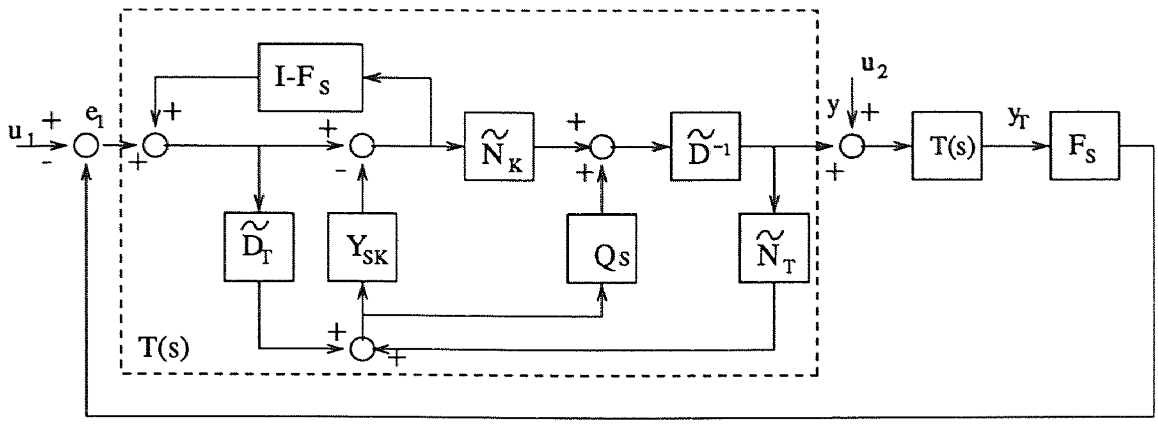


Figure 2.2 The $(F_S, T(s), K(s))$ System Where $K(s)$ Has k-sensor Stable

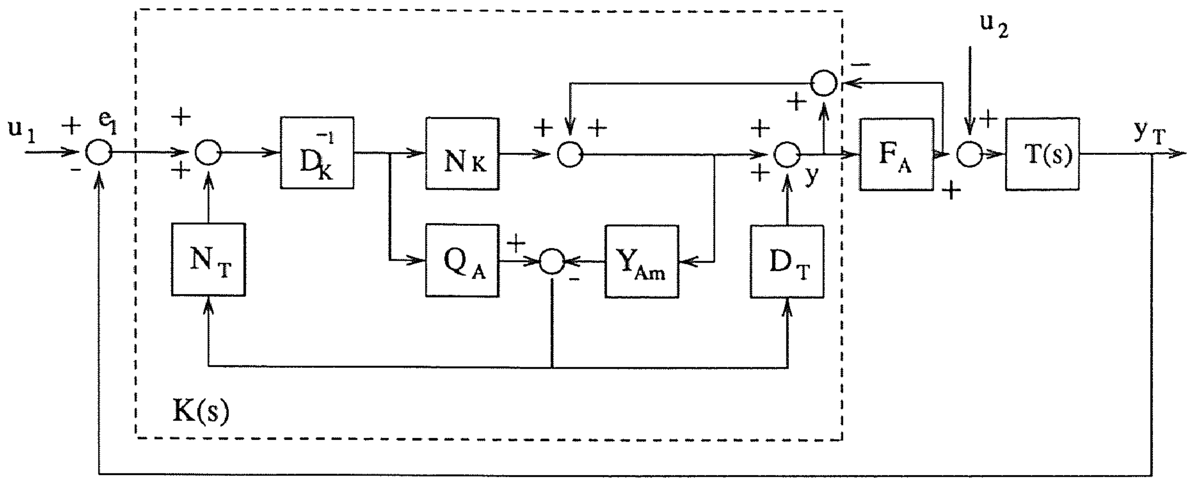


Figure 2.3 The $(T(s), F_A, K(s))$ System Where $K(s)$ Has m-actuator Stable

$$
\begin{aligned}
&= N_K D_K^{-1} = (N_{SD} - D_T \tilde{N}_{SD}(Y_{Sk} + Q_{Sk})) \\
&\cdot \ (D_{SD} + N_T \tilde{N}_{SD}(Y_{Sk} + Q_{Sk}))^{-1} \\
&= K_{SD}(I_\nu + (Y_{Sk} + Q_{Sk})\tilde{N}_T K_{SD})^{-1} \\
&\cdot \ (I_\nu - (Y_{Sk} + Q_{Sk})\tilde{D}_T)
\end{aligned} \tag{2.41}
$$

Figure 2.3 shows the controller design for $m = 1$ or $m = \nu - 1$.

$$
\begin{aligned}
K(s) &= \tilde{D}_K^{-1} \tilde{N}_K \\
&= (\tilde{D}_{AD} + (Y_{Am} + Q_{Am} N_{AD})\tilde{N}_T)^{-1} \\
&\cdot \ (\tilde{N}_{AD} - (Y_{Am} + Q_{Am}) N_{AD} \tilde{D}_T)
\end{aligned}
$$

$$
\begin{aligned}
&= N_K D_K^{-1} = (N_{AD} - D_T(Y_{Am} + Q_{Am})N_{AD}) \\
&\cdot \; (D_{AD} + N_T(Y_{Am} + Q_{Am})N_{AD})^{-1} \\
&= (I_\nu - D_T(Y_{Am} + Q_{Am})) \\
&\cdot \; (I_\nu + K_{AD}N_T(Y_{Am} + Q_{Am})^{-1}K_{AD} \qquad (2.42)
\end{aligned}
$$

The Q_{Sk}, Y_{Sk} matrices are constructed from the matrix and elements which diagonalize the LCF of $T(s)$, and the Q_{Ak}, Y_{Ak} matrices are constructed from the matrix and elements which diagonalize the RCF of $T(s)$. See[11] for their definitions.

2.3 Robust Servo Control

In this section, the Robust Servo Control method is analyzed in detail. In the system description, the state space model of the nominal plant is given, a set of definitions are given including the normalized transfer matrix, the Decentralized Robust Servomechanism Problem, and different system fault modes such as sensor failure, controller/actuator failure and multiple failures. The existing main results on the sufficent conditions of reliable control by using the decentralized robust PI control configuration are given.

2.3.1 System Description

The plant is assumed open loop stable and represented be the following model;

$$
\begin{aligned}
\dot{x} &= Ax + Bu + E\omega \\
y &= Cx \qquad (2.43)
\end{aligned}
$$

where $A \in \Re^{n\times n}$, $B = [b_1, \cdots, b_\nu] \in \Re^{n\times\nu}$,$E \in \Re^{n\times\Omega}$, and $C = [c_1, \cdots, c_\nu] \in \Re^{\nu\times n}$, The variables $x \in \Re^n$, $u = [u_1,\cdots,u_\nu]' \in \Re^\nu$, $y = [y_1,\cdots,y_\nu]' \in \Re^\nu$, and $\omega \in \Re^\Omega$, are the state, input, output and the constant disturbance vectors, respectively.

It is assumed that the controllers to be used to control (2.43) are constrained to be decentralized and has the following type:

$$u_i(t) = k_i \int_0^t y_i(\tau) d\tau \tag{2.44}$$

The open loop transfer matrix can be expressed as:

$$T(s) = C(sI - A)^{-1}B = \frac{N(s)}{d(s)} = \{t_{ij}(s)\}; i, j = 1, 2 \cdots, \nu \tag{2.45}$$

Define the following normalized matrix:

$$\overline{T} = T(s)(\mathrm{diag}(t_{11}, t_{22}, \cdots, t_{\nu\nu}(s))^{-1} \tag{2.46}$$

The matrix $\overline{T}[i_1, i_2, \cdots, i_\phi](s)$ is obtained from $\overline{T}(s)$ by retaining only the i_1,i_2,$\cdots$,i_ϕ non-redundant rows and columns.

Similarly, let $N[i_1, i_2, \cdots, i_\phi](s)$ be obtained from $N(s)$ by retaining i_1,i_2,$\cdots$,i_ϕ non-redundant rows and columns where i_1,i_2,$\cdots$,$i_\phi \in [1, \nu]$,$\phi \in [1, \nu]$.

The set of ϕth order leading principal minors of $N(s)$ is defined as:

$$\det N[\phi](s) = \{p(s) \in \mathcal{C} | p(s) = \det(N[i_1, i_2, \cdots, i_\phi](s)) \tag{2.47}$$

whose roots are just the transmission zeros of the $\phi \times \phi$ principal subsystem of (2.43).

With no loss of generality, it is assumed that the control structure is given by the following input/output pairing:

$$\{(u_1, y_1),\ (u_2, y_2), \cdots,\ (u_\nu, y_\nu)\} \tag{2.48}$$

Definition 2.3.1 *[4] Decentralized Robust Servomechanism Problem (DRSP): Given the plant (2.43) and input/output pairing(2.48), obtain a decentralized controller so that the following conditions all hold:*

1. *The closed loop system is asymptotically stable.*

2. *Asymptotically tracking occur, i.e.* $lim_{t \to \infty} y(t) = 0$, *for all constant* ω.

3. *Property 2) holds for parametric perturbations:* $A \to A + \delta A, B \to B + \delta B$, *and* $C \to C + \delta C$ *provided that the closed loop system remains stable.*

2.3.2 Main Results

The main results of Robust Servo Control method are described by Lemma 2.3.1 and Lemma 2.3.2, where Lemma 2.3.1 concerns with the existence of solution to the $DRSP$ and Lemma 2.3.2 provides a sufficient condition for a solution to $DRSP$ with complete reliability.

Lemma 2.3.1 *[4] There exists a solution to the DRSP iff there exists a controller installation sequence* $\{i_1, i_2, \cdots, i_\nu\} = \{1, 2, \cdots, \nu\}$, *the following conditions all hold:*

- $detN[i_1](0) \neq 0,\ \forall i_1 \in \{1, 2, \cdots, \nu\}$
- $detN[i_1, i_2](0) \neq 0,\ \forall i_1, i_2 \in \{1, 2, \cdots, \nu\}$
- $detN[i_1, i_2, i_3](0) \neq 0,\ \forall i_1, i_2, i_3 \in \{1, 2, \cdots, \nu\}$
- $\vdots$
- $detN(0) \neq 0$

The following channel failure cases are now defined:

Definition 2.3.2 *Sensor Failure: The ith sensor is said to have failed at time* $t_1 > 0$ *if*

$$y_i(t) = 0 \quad \forall t > t_1$$

The ith sensor failure reflects the situation where a sensor in the ith channel ceases to function and generates only a null output for all time thereafter.

Definition 2.3.3 *Controller/Actuator Failure: The ith controller/actuator is said to have failed at time* $t_2 > 0$ *if*

$$u_i(t) = 0 \quad \forall t > t_2$$

The ith controller or actuator failure reflects the situation where a controller or actuator in ith channel ceases to function and generates only a null output for all time thereafter.

Definition 2.3.4 *[7] Plant with Multiple Failures: Given the plant (2.43) where $\nu - \phi$ sensors/actuators/controllers have failed so that the resulting plant is described by:*

$$\begin{aligned} \dot{x} &= Ax + b_{i_1}u_{i_1} + b_{i_2}u_{i_2} + \cdots + b_{i_\phi}u_{i_\phi} \\ y_i &= c_i x \quad i \in \{i_1, i_2, \cdots, i_\phi\} \end{aligned} \tag{2.49}$$

The ϕ-input/output plant given by (2.49) is said to be a plant with partial sensor, actuator or controller failure.

The $\nu - \phi$ sensors/actuators/controllers failure reflect the situation where sensors/actuators/controllers in those $\nu - \phi$ channels cease to function and generate only null outputs.

Definition 2.3.5 *[7] DRSP with Complete Reliability(DRSPwCR): Given the plant(2.43) and the input/output pair(2.48), obtain a decentralized controller so that the following conditions all hold:*

1. *There exists a solution to the DRSP for the normal plant(2.43).*
2. *Under partial channel failure, the controller solves the DRSP for (2.49) without retuning.*

Lemma 2.3.2 *[4] By applying the decentralized control(2.44), there exists a solution to the $DRSPwCR$ if the following conditions all hold:*

1. $det\overline{T}[i_1, i_2](0) > 0, \ \forall i_1, i_2 \in \{1, 2, \cdots, \nu\}$
2. $det\overline{T}[i_1, i_2, i_3](0) > 0, \ \forall i_1, i_2, i_3 \in \{1, 2, \cdots, \nu\}$

3. $\vdots$

4. $det\overline{T}(0) > 0$.

Remark: The above lemma requires that all principal minors of $\overline{T}(0)$ be strictly positive, a condition not always satisfied by an arbitrary plant.

Plant satisfying all the conditions in Lemma 2.3.2 are called "reliable systems". Otherwise, it is referred to as an "unreliable system" in the later chapters.

To solve $DRSPwCR$ for an unreliable system, it is not adequate to use integral control(2.44) only. Existing reliable control results based on the Robust Servo Control method are limited to the reliable systems.

2.4 Summary of Existing Methodologies

From the above introductions of different reliable control design methodologies, their application conditions, advantages and disadvantages can be concluded as following:

The H_∞ methodology focuses on the disturbance-attenuation performances of the reliable control system. Besides the condition of an acurate mathematical model of the plant is known, it is also required that the H_∞ disturbance-attenuation bound is high enough in order to get a positive definite solution to the design equations.It is a sufficient condition. At present, it is not known whether a solution exists for lower H_∞-norm. One of the important advantages of this design is that it addresses the issue of providing guarantees on system performance, it is a robust design. One disadvantage of this design is that the failure mode is prespecified, while in many cases the failure occurs without being noticed. Also, since it is an observer based methodology, the order of the augmented closed-loop system increases rapidly with the increasing of input/output channels, therefore it is not practical in the large-scale industrial control systems.

The algebraic factorization design requires an extremely acurate mathematical model for the nominal plant and controllers and also at least one specific sensor/actuator never fails. The advantage of this design is that it provides the necessary and sufficient conditions for reliable control, and it solves almost complete reliable problems of the plant without the failure being known. A disadvantage is that a strictly acurate mathematical model is required. Since the parameterization of the controllers are completely dependent on the plant transfer matrices, the robustness is very weak. Once the plant model is slightly different due to parametric perturbations, the controller design based on previous nominal plant model may not be reliable to the perturbed plant model.

The robust servo control emphasizes the reliable control without the availability of plant mathematical model. The most important condition is that the steady-state interaction indices must be strictly positive. It is a necessary and sufficient condition. This design not only provides almost all the advantages that other designs provide such as robustness, system performances, complete reliability, etc, but also has a significant advantage that other designs do not have, ie, it is very practical in the real world, large-scale control systems. The disadvantage is that the systems satisfying the conditions of steady-state interaction indices are still limited. Research based on this design methodology for the purpose of relaxing the sufficient condition is a current research topic.

CHAPTER 3

RELIABLE CONTROL FOR MINIMUM PHASE SYSTEMS

In this chapter, new research results of solving Decentralized Robust Servo Problem with Complete Reliability($DRSPwCR$) for a class of open loop stable, minimum phase system is developed. The decentralized PID^r controller configuration is synthesized and simulation results of a numerator example is given.

3.1 New Results

From the discussion of Robust Servo Control Methodology in the previous chapter, it is evident that decentralized integral control(2.44) requires the plant to satisfy all conditions in Lemma 2.3.2, i.e., the $\overline{T}(0)$ matrix of the nominal plant must possess positive principal minors. When a given plant does not satisfy all conditions in Lemma 2.3.2, the integral control(2.44) strategy only cannot solve the $DRSPwCR$. However, for a certain group of unreliable, minimum phase systems, the following new results are now obtained to relax the conditions of Lemma 2.3.2:

Theorem 3.1.1 *For a system whose normalized transfer matrix does not posses positive principal minors and cannot achieve $DRSPwCR$ with control(2.44) only, there exists a solution to the DRSPwCR(Definition 2.3.5) if the following conditions all hold:*

1. *There is a solution to $DRSP$ for (2.43).*
2. *$detN[\phi](s), \phi = 1, 2, \cdots, \nu$ possess no unstable zeros.*

To solve the $DRSPwCR$, a possible controller is the PID^r type given below:

$$K_P(s) = \text{diag}(k_{p1}, k_{p2}, \cdots, k_{p\nu}) \tag{3.1}$$

$$K_I(s) = \text{diag}(\frac{k_1}{s}, \frac{k_2}{s}, \cdots, \frac{k_\nu}{s}) \tag{3.2}$$

$$k_i^d(s) = k_i^1 s + k_i^2 s^2 + \cdots + k_i^r s^r, i = 1, 2, \cdots, \nu \tag{3.3}$$

$$K_D(s) = \text{diag}(k_1^d(s), k_2^d(s), \cdots, k_\nu^d(s)) \tag{3.4}$$

$$K_{PD}(s) = K_P(s) + K_D(s) \tag{3.5}$$

where r is the maximum pole-zero excess defined as:

$$r \triangleq n - \text{minimum order}(\det N(\phi)), \forall \phi \in [1, \nu] \tag{3.6}$$

Remark: Compared with the traditional PID control, a higher derivative control D^r control is applied in this design. The practical implication of using higher derivative term is to let the closed loop system to become diagonal dominant as the frequency increases to a high value.

Given that "r", the maximum pole-zero excess, is generically 1, the PID^r controller reduces to a regular PID type where the derivative action can be indirectly synthesized as follows:

From (2.43), the derivative of the output vector is calculated as:

$$\begin{aligned} \dot{y} &= C\dot{x} = C(Ax + Bu) \\ &= CAx + CBu \end{aligned} \tag{3.7}$$

If the plant model is known, then $\dot{y}$ can be computed without carrying out the differentiation explicitly.

In the event that high order D^r action is required, a description type controller with the following structure may be used:

$$T_i^c \dot{\eta}_i = A_i^c \eta_i + B_i^c y_i, \quad i = 1, \cdots, \nu \tag{3.8}$$

$$u_i = C_i^c \eta_i + D_i^c y_i \quad i = 1, \cdots, \nu \tag{3.9}$$

Where it is assumed that:

$$\text{rank}(T_i^c) - \deg(\det(sT_i^c - A_i^c)) \geq r, \quad i = 1, \cdots, \nu$$

Proof of Theorem(3.1.1):

The proof is carried out by construction. Define $T_P(s)$ to be the transfer matrix of plant (2.43) with proportional feedback K_P; then,

$$T_P(s)^{-1} = T(s)^{-1} - K_P \tag{3.10}$$

as $||K_P||$ becomes sufficiently large, $T_P(0)$ approaches a diagonal matrix and $det\overline{T}[i_1, \cdots, i_\phi](0) = 1$, which satisfies the conditions in Lemma 2.3.2.

Apply now the derivative controller in (3.4) so that the feedback control is described by $K(s)$ in (3.5) and the closed loop transfer function is given by:

$$T_{PD}(s) = (I - T(s)K(s))^{-1}T(s)K(s) \tag{3.11}$$

Denote $K[i_1, \cdots, i_\phi](s)$ be obtained from $K(s)$ with only the $i_1,\cdots,i_\phi$th non-redundant rows and columns. The characteristic polynomial of nominal closed-loop system is given by:

$$d^\nu - d^{\nu-1} \sum_{i\in[1,\nu]} N[i]K[i] + \cdots + (-1)^\nu \det(NK)(s) \tag{3.12}$$

whereas the characteristic polynomial for the failed system is given by:

$$\begin{aligned} d^\phi - d^{\phi-1} \sum_{i\in[i_1,\cdots,i_\phi]} N[i]K[i] + d^{\phi-2} \sum_{i,j\in[i_1,\cdots,i_\phi]} \det(N[i,j]K[i,j]) \\ + \cdots + (-1)^\phi \det(N[i_1, \cdots, i_\phi](s)K[i_1, \cdots, i_\phi](s)) \end{aligned} \tag{3.13}$$

Now since $K(s)$ is of order of r, the maximum value of pole-zero difference, $\det N(s)K(s)$ has the same degree as d^ν and $\det(N[i_1, \cdots, i_\phi](s)K[i_1, \cdots, i_\phi](s))$ has the same degree as $d^\phi(s)$. Therefore, in both cases, as $||K_P|| \to \infty$, the polynomials are dominated by the last term of (3.12) and(3.13) respectively which are always stable from the assumption that $\det N[\phi](s), \phi = 1, 2, \cdots, \nu$ do not possess unstable zeros.

The following algorithm provides a procedure to synthesize a PID^r controller for plant satisfying the conditions of Theorem 3.1.1:

Algorithm 3.1.1 *Synthesis of PID^r Controller:*

1. *Verify that the conditions in Theorem 3.1.1 are satisfied.*

2. *Apply the proportional control(3.1) to the plant so that all principle minors of $\overline{T_p}(0)$ are strictly positive. This can be achieved if $||K_P||$ is large enough.*

3. *Determine r, the maximum pole-zero excess.*

4. *Synthesize a stable PD^r control(3.5) so that the closed loop system are asymptotically stable for all $\phi \in [1, \nu]$. This can always be achieved if the conditions in Theorem 3.1.1 are satisfied and the gain of controllers are high enough.*

5. *Apply the integral control(3.2) sequentially.*

3.2 Example

A 3-input/output system is described by the following transfer matrix:

$$T(s) = \frac{\begin{bmatrix} 0.66842s^2 + 4.1749s + 6.8677 & 9.3044s + 8.4617 & 2 \\ 5.2693s^2 + 0.91965s + 6.5392 & 0.26245s + 0.47465 & 200 \\ 5.2693s^2 + 0.91965s + 6.5392 & 0.26245s + 0.47465 & 100 \end{bmatrix}}{s^3 + 3s^2 + 3s + 1}$$

the DC gain matrix:

$$T(0) = \begin{bmatrix} 6.8677 & 8.4617 & 2 \\ 6.5392 & 0.47465 & 200 \\ 6.5392 & 0.47465 & 100 \end{bmatrix}$$

Now since $\det(\overline{T}[1,2](0)) = -15.975 < 0$,

$\det(\overline{T}[2,3](0)) = -1 < 0$,

$\det(\overline{T}[3,1](0)) = -189 < 0$,

conditions of Lemma 2.3.2 are violated and therefore, $DRSPwCR$ for this system

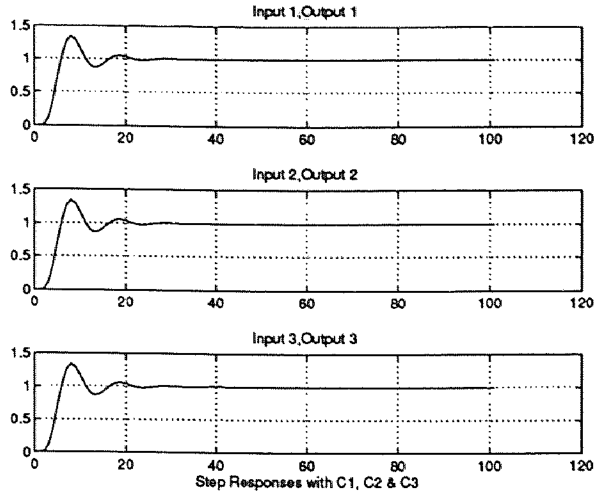


Figure 3.1 System Outputs at Normal Operation

cannot be solved by integral controller only. However, it is noted that the transmission zeros of all the principal minors are minimum phase:

$$\begin{aligned}
\det N[1](s) &= 0.66842s^2 + 4.1749s + 6.8677 \\
\det N[2](s) &= 0.26245s + 0.47465 \\
\det N[3](s) &= 100 \\
\det N[1,2](s) &= -48.852s^3 - 51.731s^2 - 64.841s - 52.073 \\
\det N[1,3](s) &= 119.54s^2 + 426.68s + 752.16 \\
\det N[2,3](s) &= -26.245s - 47.465 \\
\det N(s) &= 4885.2s^3 + 5173.1s^2 + 6484.1s + 5207.3
\end{aligned}$$

Therefore, $DRSPwCR$ can be achieved for this system by PID^r control with $r = 3$ here. In simulation, the following PID^3 is applied:

$$K_1(s) = K_2(s) = K_3(s) = -50 - 100s - 100s^2 - 50s^3 - 20/s$$

The closed loop system disturbance rejection characteristic under the following failure modes are shown in Figures 3.1 to Figure 3.5:

- All 3 channels are operational. Figure 3.1 shows that asymptotic tracking takes place.
- One channel has failed. Figure 3.2 shows the system outputs with integral control only, it is noted that the failed system remains asymptotically stable

with tracking occurring for the remaining operational input/output channels. Figure 3.3 the system outputs with PID^3 control, the failed system also remains asymptotically stable with tracking occurring for the remaining operational input/output channels.

- Two of the three channels have failed. Figure 3.4 shows the system outputs with integral control only, this time, the fault free channel output will not be asymptotically stable. However, with PID^3 control, similar to the single controller failure scenario, asymptotic stability and regulation still hold for the remaining system as shown in Figure 3.5.

The current result shows that complete reliability against sensors and actuators failure can be achieved for the class of open loop stable, minimum phase plant which may not satisfy certain previous known conditions for reliable control. The key towards establishing reliability is the introduction of high derivative control which always stabilizes a minimum phase system without altering its DC gain. The synthesis of reliable control(for an unreliable plant) becomes a fairly straight forward process.

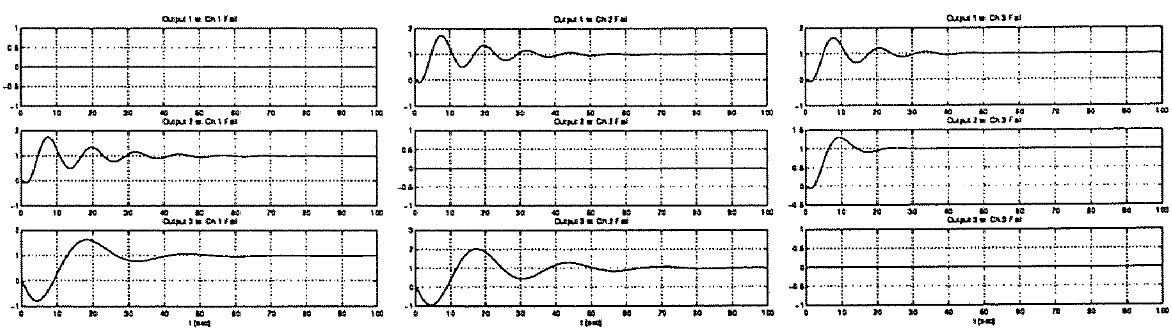


Figure 3.2 Outputs with I-Control, 1 Channel Failed

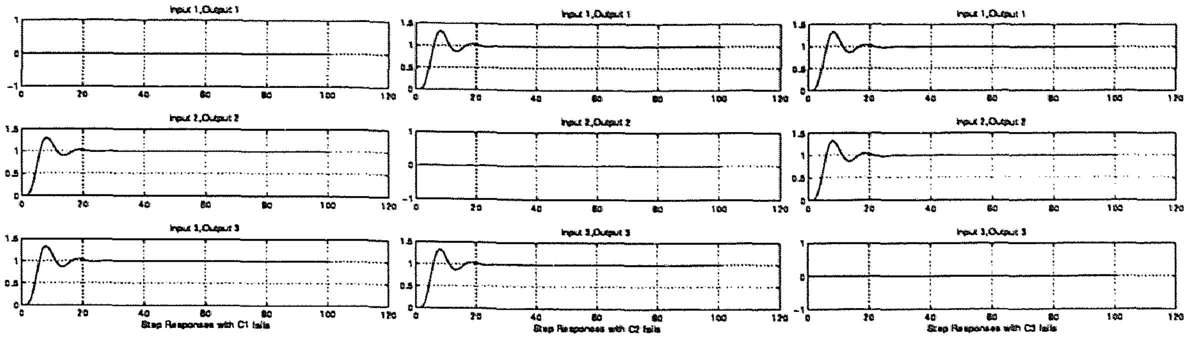


Figure 3.3 Outputs with PID^3 Control, 1 Channel Failed

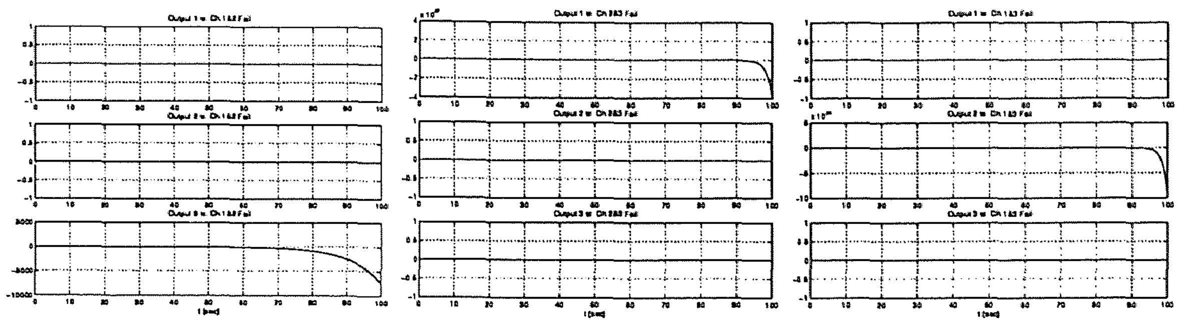


Figure 3.4 Outputs with I-Control, 2 Channels Failed

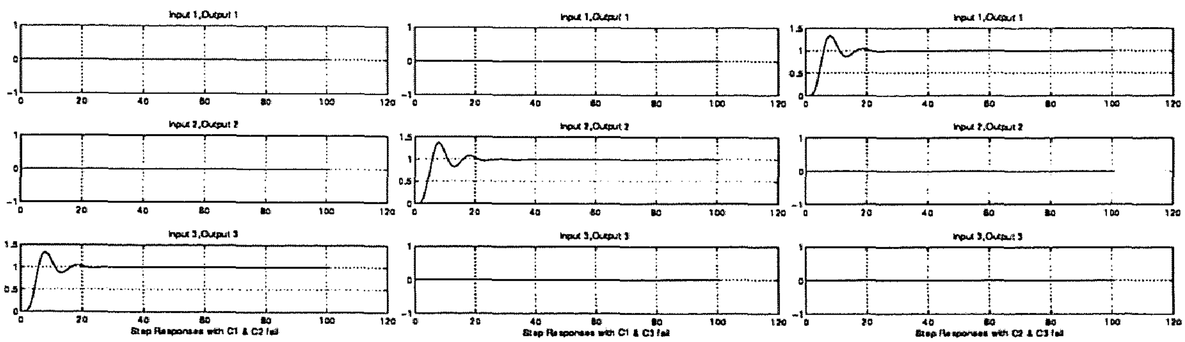


Figure 3.5 Outputs with PID^3 Control, 2 Channels Failed

CHAPTER 4

RELIABLE CONTROL FOR ARBITRARY LINEAR SYSTEM

In this chapter, the necessary conditions to achieve reliable control for a given unreliable plant are analyzed inductively. The general controller synthesis procedures are derived, followed by 2 numerical examples of reliable control using different control strategies.

4.1 Range of Reliability of K_P

In order to develop the theoretical conditions for reliable control by using strict diagonal decentralized control, it is first necessary to analyze the range of reliability of K_P for the general 2×2 open loop stable system.

Given an open loop stable, unreliable plant of the structure:

$$T(s) = \frac{1}{d(s)} \begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix} \tag{4.1}$$

where $d(s)$ is a stable polynomial. Let

$$d(s) = s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0 \tag{4.2}$$

$$n_{ij}(s) = n_{r_{ij}}^{ij} s^{r_{ij}} + n_{r_{ij}-1}^{ij} s^{r_{ij}-1} + \cdots + n_1^{ij} s + n_0^{ij} \tag{4.3}$$

where $i, j \in [1, 2]$, and r_{ij} denote the order of the polynomial $n_{ij}(s)$.

With no loss of generality, assume that,

$$n_{ij}(0) > 0, \quad i, j \in [1, 2]$$

Define now:

$$\Delta n(s) = n_{11}(s)n_{22}(s) - n_{12}(s)n_{21}(s)$$

and assume that a decentralized proportional controller

$$K_P = \begin{pmatrix} k_{p1} & 0 \\ 0 & k_{p2} \end{pmatrix}$$

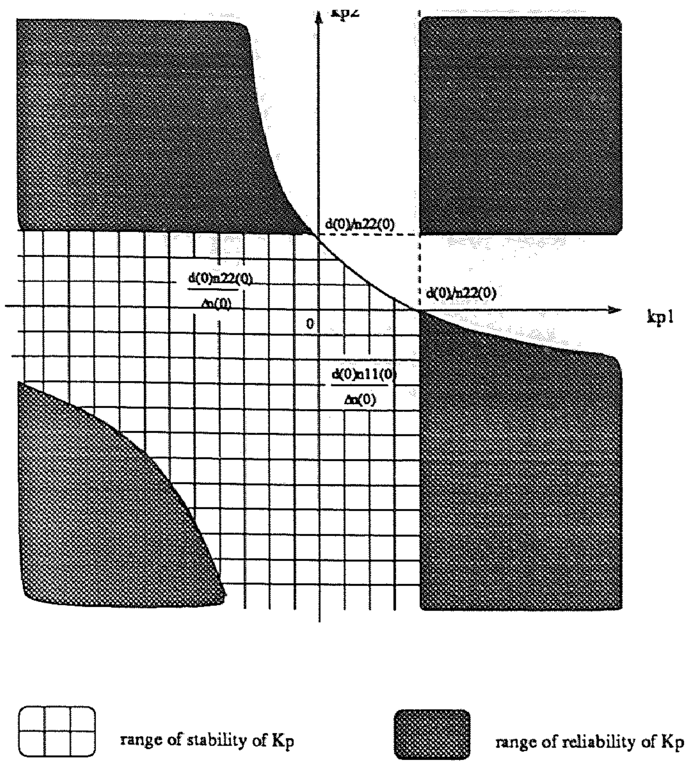


Figure 4.1 K_P Range of Stability and Reliability

has been applied. The regions of reliability for the proportional gain parameters are now analyzed as follows:

As shown in Figure 4.1, in order for the closed loop system to be reliable, the values of k_{p1} and k_{p2} have to be located in the shaded areas described by the following four equations:

Region 1 (Quadrant I):

$$k_{p1} > \frac{d_0}{n_{11}(0)}, \; k_{p2} > \frac{d_0}{n_{22}(0)} \tag{4.4}$$

Region 2 (Quadrant II):

$$\begin{aligned} (\frac{d_0 n_{22}(0)}{\Delta n(0)} - k_{p1})(\frac{d_0 n_{11}(0)}{\Delta n(0)} - k_{p2}) &< \frac{d_0^2 n_{12}(0) n_{21}(0)}{\Delta n(0)^2} \\ k_{p1} > \frac{d_0}{n_{11}(0)}, \quad k_{p2} < 0 & \end{aligned} \tag{4.5}$$

Region 3 (Quadrant III):

$$(\frac{d_0 n_{22}(0)}{\Delta n(0)} - k_{p1})(\frac{d_0 n_{11}(0)}{\Delta n(0)} - k_{p2}) < \frac{d_0^2 n_{12}(0) n_{21}(0)}{\Delta n(0)^2}$$

$$k_{p1} < 0, \quad k_{p2} > \frac{d_0}{n_{22}(0)} \tag{4.6}$$

Region 4 (Quadrant IV):

$$(\frac{d_0 n_{22}(0)}{\Delta n(0)} - k_{p1})(\frac{d_0 n_{11}(0)}{\Delta n(0)} - k_{p2}) > \frac{d_0^2 n_{12}(0) n_{21}(0)}{\Delta n(0)^2}$$
$$k_{p1} < 0, \quad k_{p2} < 0 \tag{4.7}$$

The derivation of the reliable range of K_P can be found in Appendix C. When decentralized PD^r controllers are applied, the closed loop characteristic polynomials are described by the following equations:

$$D_1(s) = d(s) - n_{11}(s)k_{p1} \tag{4.8}$$

$$D_2(s) = d(s) - n_{22}(s)k_{p2} \tag{4.9}$$

$$D_{12}(s) = D_1(s)D_2(s) - n_{12}(s)n_{21}(s)k_{p1}k_{p2} \tag{4.10}$$

where, $D_1(s)$ is the closed loop system characteristic polynomial when k_{p1} only is applied. Similarly $D_2(s)$ is the closed loop system characteristic polynomial when k_{p2} only is applied and $D_{12}(s)$ is the closed loop characteristic polynomial when both proportional controllers are applied.

Define $\overline{T}_P$ the normalized closed loop transfer matrix as described in Equation(3.10). T_1 is the closed loop DC gain matrix when controller 1 only is installed, while T_2 is the closed loop DC gain matrix when controller 2 only is installed. For the reliable control consideration, values of k_{p1}, k_{p2} have to be such that the closed loop system $\overline{T}_P$ possesses positive principal minors under both cases of controllers normal operations and any channel failures, therefore the conditions in one of the following scenarios must be satisfied:

Scenario 1:

$$det(\overline{T}_P) > 0 \tag{4.11}$$

$$T_P(1,1)T_1(1,1) > 0 \tag{4.12}$$

$$T_P(2,2)T_2(2,2) > 0 \tag{4.13}$$

Scenario 2:

$$det(\overline{T}_P) < 0 \quad (4.14)$$

$$T_P(1,1)T_1(1,1) < 0 \quad (4.15)$$

$$T_P(2,2)T_2(2,2) > 0 \quad (4.16)$$

Scenario 3:

$$det(\overline{T}_P) < 0 \quad (4.17)$$

$$T_P(1,1)T_1(1,1) > 0 \quad (4.18)$$

$$T_P(2,2)T_2(2,2) < 0 \quad (4.19)$$

T_P, T_1 and T_2 are related to the values of open loop DC gain matrix and K_P values as described by the following equations:

$$T_1(1,1) = \frac{n_{11}(0)}{D_1(0)} \quad (4.20)$$

$$T_2(2,2) = \frac{n_{22}(0)}{D_2(0)} \quad (4.21)$$

$$T_P(1,1) = \frac{d_0 n_{11}(0) - \Delta n(0)k_{p2}}{D_{12}(0)} \quad (4.22)$$

$$T_P(2,2) = \frac{d_0 n_{22}(0) - \Delta n(0)k_{p1}}{D_{12}(0)} \quad (4.23)$$

$$det(\overline{T}_P) = \frac{D_{12}(0)}{(d_0 n_{11}(0) - \Delta n(0)k_{p2})(d_0 n_{22}(0) - \Delta n(0)k_{p1})} D_{12}(0) \quad (4.24)$$

Furthermore,

$$D_1(0) = d_0 - n_{11}(0)k_{p1} \quad (4.25)$$

$$D_2(0) = d_0 - n_{22}(0)k_{p2} \quad (4.26)$$

$$D_{12}(0) = D_1(0)D_2(0) - n_{12}(0)n_{21}(0)k_{p1}k_{p2} \quad (4.27)$$

substituting Equations(4.20), (4.21), (4.22), (4.23), and (4.24) into the above 3 scenarios, and multiplying all three equations of any one of the scenarios together, the following equation is obtained as a necessary requirement of K_P for reliability consideration:

$$D_1(0)D_2(0)D_{12}(0) < 0 \tag{4.28}$$

Therefore, in order to achieve reliable control, K_P values have to be such that:

1. All closed loop characteristic polynomials in equations(4.8), (4.9) and (4.10) must be stable.

2. K_P values must be located in the ranges described in Equations (4.4), (4.5), (4.6) and (4.7) of Figure 4.1.

4.2 Necessary Conditions of Reliable Control for 2-input/output Systems

For the 2-input/output system, necessary conditions of reliable control are stated as follows:

Theorem 4.2.1 *Necessary Condition of $DRSPwCR$ For 2×2 System Using PID^r: There exists a solution to the $DRSPwCR$(Definition 2.3.5) for an unreliable 2×2 system by using strict decentralized PID^r only if the following conditions all hold:*

1. *There is a solution to DRSP for (4.1).*

2. *order of $(n_{12}(s)n_{21}(s)) \geq$ order of $(n_{11}(s)n_{22}(s))$.*

3. *$n_{12}(s)$, $n_{21}(s)$ hold no real roots that locate in RHP.*

Before the proof of Theorem 4.2.1, the following lemma is required:

Lemma 4.2.1 *Let $P_1(s), P_2(s)$ be stable polynomials and define $P_3(s)$ as*

$$P_3(s) = P_1(s) + P_2(s) \tag{4.29}$$

if

$$P_1(0)P_2(0) > 0$$

then $P_3(s)$ *holds no unstable real roots.*

The proof of Lemma 4.2.1 is given in Appendix D.

Proof of Theorem(4.2.1):

When a decentralized controller

$$K(s) = \begin{pmatrix} K_1(s) & 0 \\ 0 & K_2(s) \end{pmatrix} \tag{4.30}$$

is applied to the open loop stable 2-input/output plant, the closed loop characteristic polynomials are described by the following equations:

$$D_1(s) = d(s) - n_{11}(s)K_1(s) \tag{4.31}$$

$$D_2(s) = d(s) - n_{22}(s)K_2(s) \tag{4.32}$$

$$D_{12}(s) = D_1(s)D_2(s) - n_{12}(s)n_{21}(s)K_1(s)K_2(s) \tag{4.33}$$

Re-write Equation(4.33) as:

$$n_{12}(s)n_{21}(s)K_1(s)K_2(s) = D_1(s)D_2(s) + (-D_{12}(s)) \tag{4.34}$$

Let $P_1(s) = D_1(s)D_2(s)$, $P_2(s) = -D_{12}(s)$,

$P_3(s) = n_{12}(s)n_{21}(s)K_1(s)K_2(s)$,

From Lemma 4.2.1, stability of $D_1(s)D_2(s)$ and $D_{12}(s)$ together with the condition $P_1(0)P_2(0) > 0$, implies that $P_3(s)$ holds no unstable real roots, or, $n_{12}(s)n_{21}(s)K_1(s)K_2(s)$ holds no unstable real roots. Equivalently,

- $n_{ij}(s)$ has no real roots in RHP, for $i, j \in [1, 2]$, $i \neq j$.

- $K_i(s)$ has no real roots in RHP, for $i \in [1, 2]$.

thus establishing the necessary condition 3 of Theorem 4.2.1.

Condition 2 of Theorem 4.2.1 is proved by contradiction. Assume that:

$$\text{order of } (n_{11}n_{22})(s) > \text{order of } (n_{12}n_{21})(s) \tag{4.35}$$

From Equations (4.8) and (4.9),

- order of $D_1(s) \geq$ order of $n_{11}(s)K_1(s)$.
- order of $D_2(s) \geq$ order of $n_{22}(s)K_2(s)$.

Therefore order of $(D_1D_2)(s) \geq$ order of $(n_{11}n_{22}K_1K_2)(s)$,

together with the assumption in Equation(4.35), it is obtained that

$$\text{order of } (D_1D_2)(s) > \text{order of } (n_{12}n_{21}K_1K_2)(s)$$

From Equation(4.10), order of $D_{12}(s)$ will following the order of $(D_1D_2)(s)$, and the highest order coefficients of $D_{12}(s)$ and $(D_1D_2)(s)$ are the same.

Since $D_{12}(s)$ and $(D_1D_2)(s)$ are both stable, therefore, their coefficients must necessarily be of same signs. For the lowest order terms,

$$D_{12}(0)D_1(0)D_2(0) > 0$$

which violates Equation(4.28). Therefore, (4.28) implies that:

$$\text{order of } n_{12}(s)n_{21}(s) \geq \text{order of } n_{11}(s)n_{22}(s)$$

which proves Condition 2 of Theorem 4.2.1.

As shown in Figure 4.1, with proportional control only, the gridded area is the region for all closed loop characteristic polynomials $D_1(s), D_2(s)$ and $D_{12}(s)$ to be stable. This gridded area does not overlaps any shaded area, which is the range of reliability. The following conclusion is also obtained:

Lemma 4.2.2 *With proportional control only, the closed loop system is not able to achieve reliable control for a given unreliable plant(4.1).*

The above lemma indicates that proportional control with gain K_P located in the range of reliability will destabilize the system. It is therefore necessary to induce D^r derivative feedback to restabilize the system.

4.3 Necessary Conditions of Reliable Control by Using PID^r

From the above discussion of 2×2 system, the similar conclusions for an arbitrary LTI system are readily obtained. When a strict diagonal decentralized controller configuration is to be applied to solve the *DRSPwCR*, the system must satisfy certain necessary conditions, these conditions are explained in Theorem 4.3.1 as follows:

Given an open loop stable plant, with the open loop transfer matrix

$$T(s) = \frac{1}{d(s)} \begin{pmatrix} n_{11}(s) & \cdots & n_{1\nu}(s) \\ \cdots & \cdots & \cdots \\ n_{\nu 1}(s) & \cdots & n_{\nu\nu}(s) \end{pmatrix} \tag{4.36}$$

where $d(s)$ is a stable polynomial. Let

$$d(s) = s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0 \tag{4.37}$$

$$n_{ij}(s) = n^{ij}_{r_{ij}} s^{r_{ij}} + n^{ij}_{r_{ij}-1} s^{r_{ij}-1} + \cdots + n^{ij}_1 s + n^{ij}_0 \tag{4.38}$$

where $i, j \in [1, \nu]$, and r_{ij} denote the order of the polynomial $n_{ij}(s)$. Now since $d(s)$ is assumed to be stable,

$$d_i > 0, \ i = 0, 1, \cdots, n-1$$

It is also assumed that

$$n_{ij}(0) > 0, \quad \forall i, j \in [1, \nu]$$

The following results are obtained:

Theorem 4.3.1 *Necessary Condition of DRSPwCR Using PID^r:*

There exists a solution to the DRSPwCR(Definition 2.3.5) using decentralized PID^r only if the following conditions all hold:

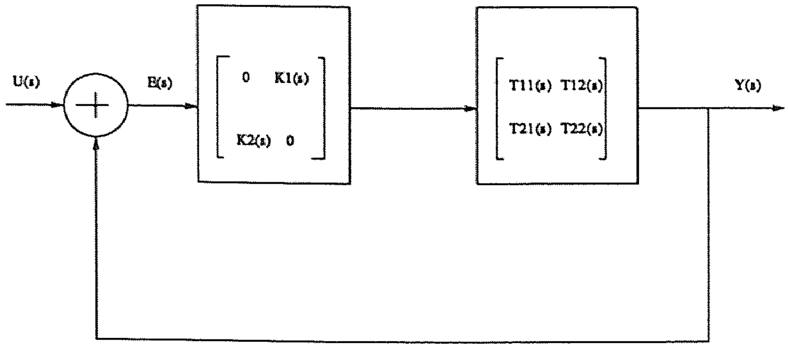


Figure 4.2 A 2-Input/output Plant with Permutation Controllers

1. *There is a solution to DRSP for (2.43).*

2. *For any unreliable* 2×2 *subsystem*

$$T_{ij}(s) = \frac{1}{d(s)} \begin{pmatrix} n_{ii}(s) & n_{ij}(s) \\ n_{ji}(s) & n_{jj}(s) \end{pmatrix} \tag{4.39}$$

order of $(n_{ij}(s)n_{ji}(s)) \geq$ *order of* $(n_{ii}(s)n_{jj}(s))$ *where* $i, j \in [1, \nu]$.

3. *For any unreliable subsystem as (4.39),* n_{ij} *holds no unstable real roots,* $\forall i, j \in [1, \nu]$, *and* $i \neq j$.

4.4 Permutation Strategy in *DRSPwCR*

When a given plant does not satisfy the necessary conditions in Theorem 4.3.1, the *DRSPwCR* cannot be solved by diagonal decentralized controllers. In this case, one possible solution can be the use of input-output permutation strategy. In this way, the decentralized control gains can be off-diagonal. For example, a 2×2 controller structure can be:

$$K(s) = \begin{pmatrix} 0 & K_1(s) \\ K_2(s) & 0 \end{pmatrix} \tag{4.40}$$

When a controller of the above structure is added in the feedforward path of the system, with a unit feedback, as shown in Figure 4.2, the closed loop system is equivalent to one constructed from regular diagonal control and an open loop transfer matrix with the input-output pairing re-assigned. The previously open-loop unreliable plant may be converted to a reliable plant. For the 2-input/output case, the resulting transfer matrix becomes:

$$T(s) = \frac{1}{d(s)} \begin{bmatrix} n_{12}(s) & n_{11}(s) \\ n_{22}(s) & n_{21}(s) \end{bmatrix}$$

The steady-state indice matrix will be the exact complement of the diagonal system. In this way, any unreliable 2×2 input/output system, reliable control can always be achieved by permutation strategy. For higher order systems, this strategy may not work.

4.5 Controller Synthesis

For an arbitrary given open loop stable linear system of ν-input/output system, the following steps can be applied to synthesize a decentralized controller to solve $DRSPwCR$:

1. Obtain the DC gain matrix $T(0)$ experimentally, determine whether the system is reliable by using conditions in Lemma 2.3.2 of Chapter 2.

2. If all conditions in Lemma 2.3.2 are satisfied, then $DRSPwCR$ can be solved by using decentralized integral controllers only with $k_i = -\epsilon t_{ii}$, where t_{ii} is the ith diagonal element of $T(0)$, $i \in [1, \nu]$, and $\epsilon > 0$ is the tuning gain.

3. If one or more conditions in Lemma 2.3.2 are not satisfied, then complete reliable control cannot be achieved by integral controllers only. Check all the principal minors of the transfer matrix to verify if they are minimum phase.

4. If all the principal minors of the unreliable system are minimum phase, then the $DRSPwCR$ can be solved by using PID^r controllers which can be constructed by Algorithm 3.1.1.

5. If one or more principal minors of the system are non-minimum phase, use the permutation strategy to adjust the system structure. The permutation is realized by pairing up certain inputs and outputs from the ν channels.

6. After permutation, if the system can satisfy the conditions of Lemma 2.3.2, then complete reliability can be solved by integral controllers. If the permutated system is unreliable but minimum phase for all principal minors, then PID^r controllers can be selected for reliability.

7. If any of the conditions of Theorem 4.3.1 is violated, the system is not able to solve $DRSPwCR$ by using the strict diagonal decentralized control. Refer to the following chapter for a discussion of block decentralized reliable control synthesis.

4.6 Examples

The following 2 examples illustrate the two different structure of plants with unreliable characteristics. Example 1 is the Headbox Model from [4] , it is an unreliable plant with all principal minors being minimum phase, the $DRSPwCR$ are solved by PID^r controllers. The second example is an unreliable plant, one of its sub-system's is the Rosenbrock's Model[4] with two inputs being flipped thus resulting in an unreliable plant, this sub-system's principal minors have real roots located in the RHP, therefore it does not satisfy the necessary conditions to solve the diagonal $DRSPwCR$ with PD^r feedback compensators. However, the system can be achieved reliability through the permutation strategy.

4.6.1 Example 1: Headbox Model[4]

The linear model of a headbox is given by the following state-space equation:

$$\dot{x} = \begin{pmatrix} -0.395 & 0.01145 \\ -0.011 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 1.038 \\ 0.000966 & 0.03362 \end{pmatrix} u$$

$$y = x \tag{4.41}$$

The transfer matrix is:

$$T(s) = \frac{\begin{bmatrix} 1.1061 \times 10^{-5} & 1.038s + 3.8495 \times 10^{-4} \\ 9.66 \times 10^{-4}s + 3.8157 \times 10^{-4} & 3.362 \times 10^{-2}s + 1.8619 \times 10^{-3} \end{bmatrix}}{s^2 + 0.395s + 1.2595 \times 10^{-4}} \tag{4.42}$$

so that

$$T(0) = \frac{\begin{bmatrix} 1.1061 \times 10^{-5} & 3.8495 \times 10^{-4} \\ 3.8157 \times 10^{-4} & 1.8619 \times 10^{-3} \end{bmatrix}}{1.2595 \times 10^{-4}} = \begin{bmatrix} 8.7821 \times 10^{-2} & 3.0564 \\ 3.0295 & 14.783 \end{bmatrix}$$

and

$$det(\overline{T}(0)) = -6.1323$$

this negative value shows the plant is unreliable, or it cannot achieve reliable control by using decentralized integral control only. Figures 4.3, 4.4 and 4.5 show that, when a controller installation sequence 1, 2 is made, the following integral controller

$$K_1(s) = -10/s, \; K_2(s) = 10/s$$

provides closed loop stability and asymptotic regulation. However, when controller 1 failed, the resultant system becomes unstable. This is because the plant is unreliable and cannot achieve reliable with integral control only.

However, all the principal minors of the plant are minimum phase,

$$\begin{aligned} n_{11}(s) &= 1.1061 \times 10^{-5} \\ n_{22}(s) &= 3.362 \times 10^{-2}s + 1.8619 \times 10^{-3} \\ \Delta n(s) &= -1.0027 \times 10^{-3}s^2 - 3.9607 \times 10^{-4}s - 1.2629 \times 10^{-7} \end{aligned}$$

Therefore, the $DRSPwCR$ can be reached by PID control. For example, the following choices of PID control yield the required reliable control:

$$K_1(s) = K_2(s) = -100s - 50s - 10/s$$

Figures 4.6, 4.7 and 4.8 show the step responses of the closed loop system under different failure modes, it is obvious that the system is now reliable.

4.6.2 Example 2: Plant with Modified Rosenbrock's Model[4]

A linear model of a plant is given by the following transfer matrix:

$$T(s) = \frac{\begin{bmatrix} -s^2+s+2 & -s^2+1 & -s^2+0.1 \\ -s^2+1 & -s^2-0.6667s+0.3333 & -s^2-0.6667s+0.05 \\ -s^2+0.1 & -s^2-0.6667s+0.3333 & -s^2-0.6667s+0.1 \end{bmatrix}}{s^3+3s^2+3s+1}$$

so that

$$T(0) = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0.3333 & 0.05 \\ 1 & 0.3333 & 0.1 \end{bmatrix}$$

and

$$\begin{aligned} det(\overline{T}(1,2)(0)) &= -0.5 \\ det(\overline{T}(1,3)(0)) &= 0.95 \\ det(\overline{T}(2,3)(0)) &= 0.5 \\ det(\overline{T}(0)) &= -0.475 \end{aligned}$$

The negative values indicate that the plant is unreliable and it cannot achieve reliable control by using integral control only. Figures 4.9, 4.10, and 4.11 show the system step responses by using diagonal decentralized integral control only. The integral controller parameters are tuned sequentially for stability and asymptotic tracking with

$$K_1(s) = -0.3/s;\ K_2(s) = 0.3/s;\ K_3(s) = 0.8/s$$

It is noted that when failure of some channel(s) occur such as channel 1 failed, channel 2 failed, and any two channels failed, the resultant system becomes unstable.

Furthermore, in the off-diagonal minors of the transfer matrix, there are real roots that locate in RHP, which indicates that the plant does not satisfy the necessary conditions of $DRSPwCR$ in Theorem 4.3.1. Therefore, for this plant, it is not possible to use the diagonal decentralized PID^r control to achieve reliable control. To solve the $DRSPwCR$ problem for this system, one possibility is then to try the strategy of permutation, resulting in the following structure:

$$K(s) = \begin{bmatrix} 0 & K_1(s) & 0 \\ K_2(s) & 0 & 0 \\ 0 & 0 & K_3(s) \end{bmatrix}$$

Simulation results with

$$K_1(s) = -0.3/s; \; K_2(s) = -0.3/s; \; K_3(s) = -0.8/s$$

are shown in Figures 4.12, 4.13 and 4.14. It is observed that the closed loop system is stable and asymptotic regulation occurs for all output channels. Furthermore, under arbitrary controller failure, the system with partial failure remains stable and asymptotic tracking continues to take place for the fault free channels.

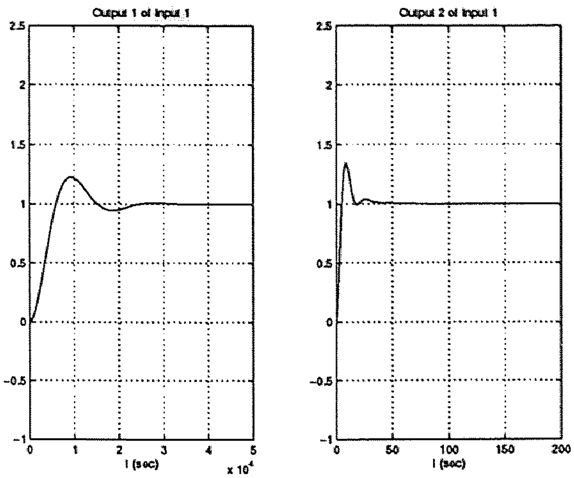


Figure 4.3 The Headbox Model Outputs with I-Control, Normal Operation

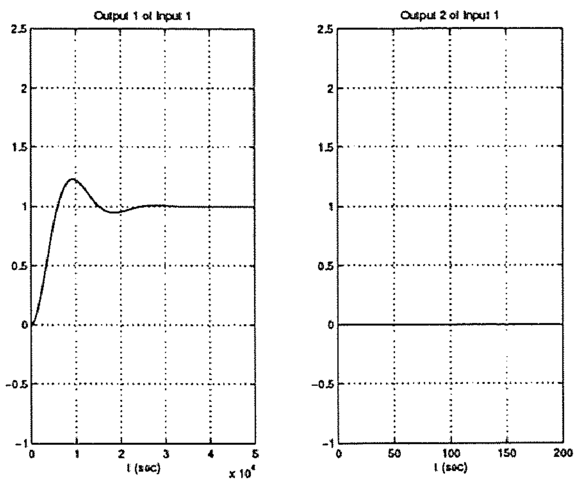


Figure 4.4 The Headbox Model Outputs with I-Control, Channel 2 Failed

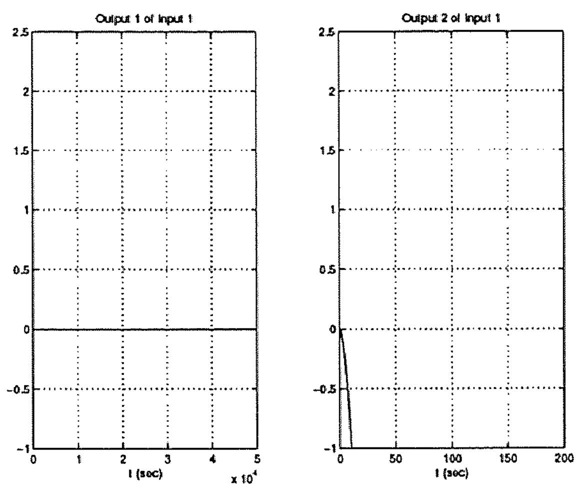


Figure 4.5 The Headbox Model Outputs with I-Control, Channel 1 Failed

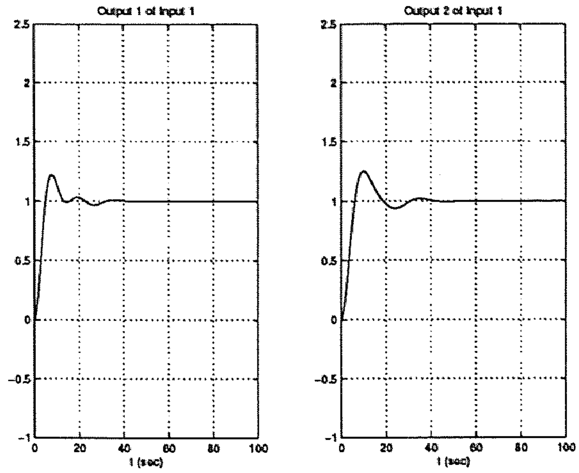


Figure 4.6 The Headbox Model Outputs with PID-Control, Normal Operation

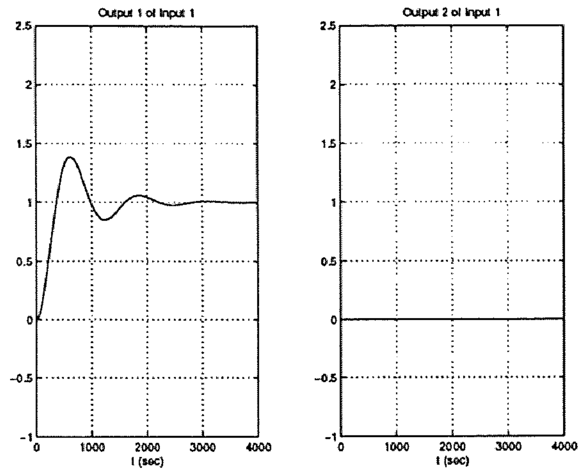


Figure 4.7 The Headbox Model Outputs with PID-Control, Channel 2 Failed

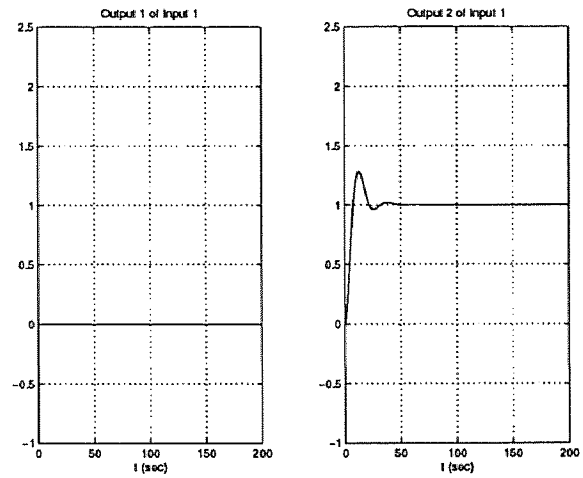


Figure 4.8 The Headbox Model Outputs with PID Control, Channel 1 Failed

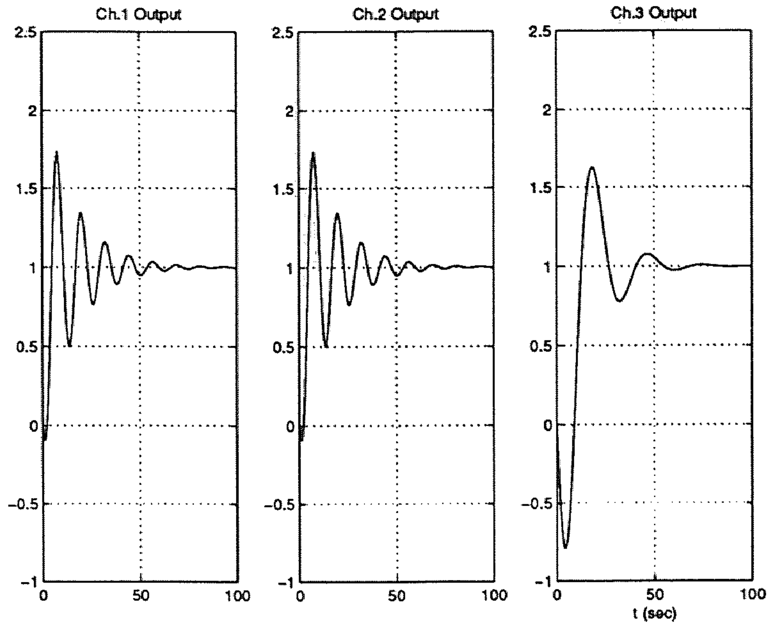


Figure 4.9 The Rosenbrock's Model Outputs with I-Control, Normal Operation

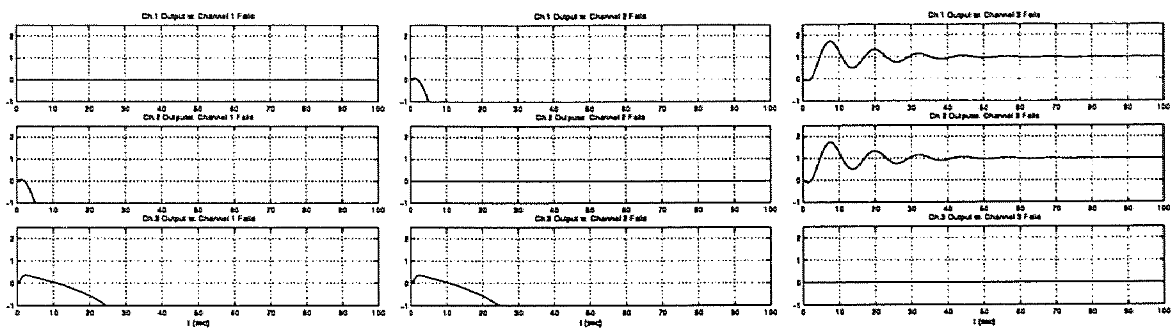


Figure 4.10 The Rosenbrock's Model Outputs with I-Control, 1 Channel Failed

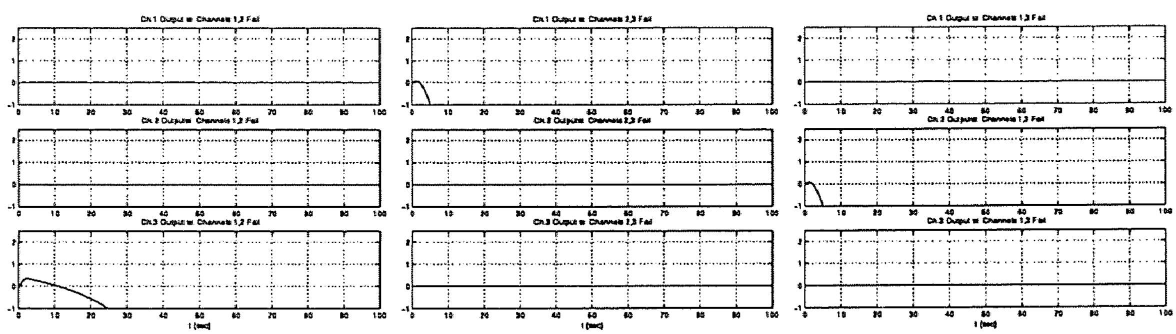


Figure 4.11 The Rosenbrock's Model Outputs with I-Control,2 Channels Failed

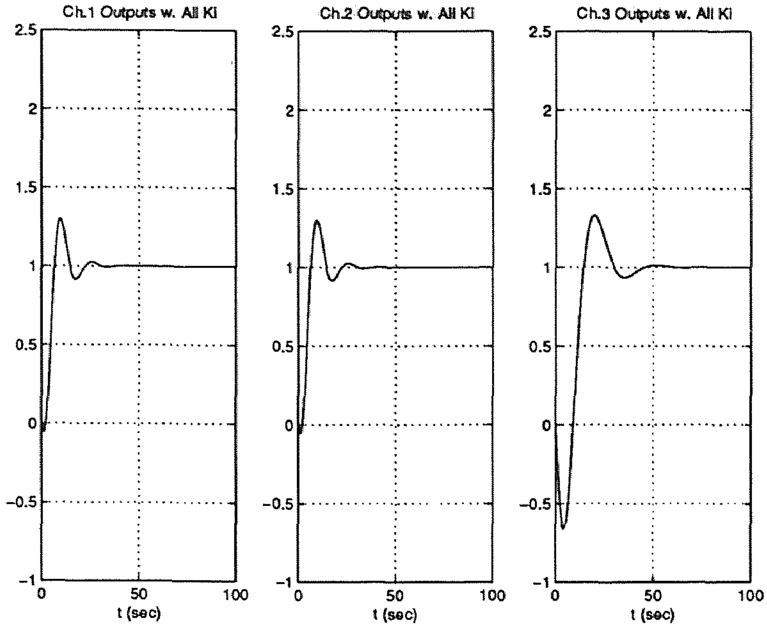


Figure 4.12 The Rosenbrock's Model Outputs with Permutation, Normal Operation

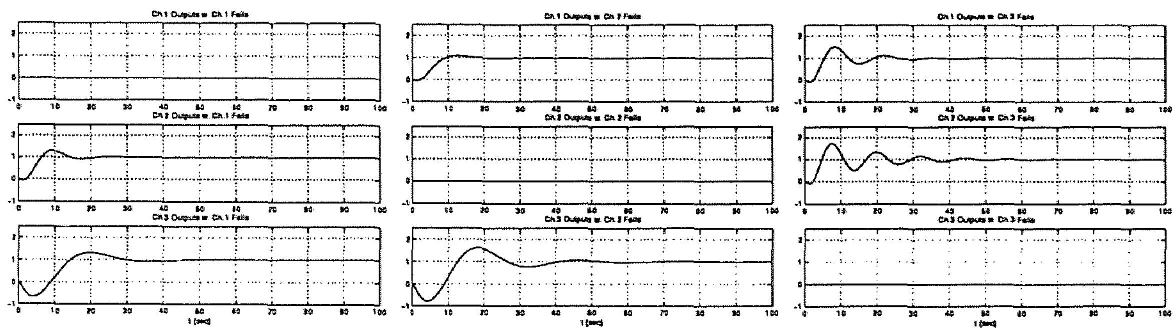


Figure 4.13 The Rosenbrock's Model Outputs with Permutation, 1 Channel Failed

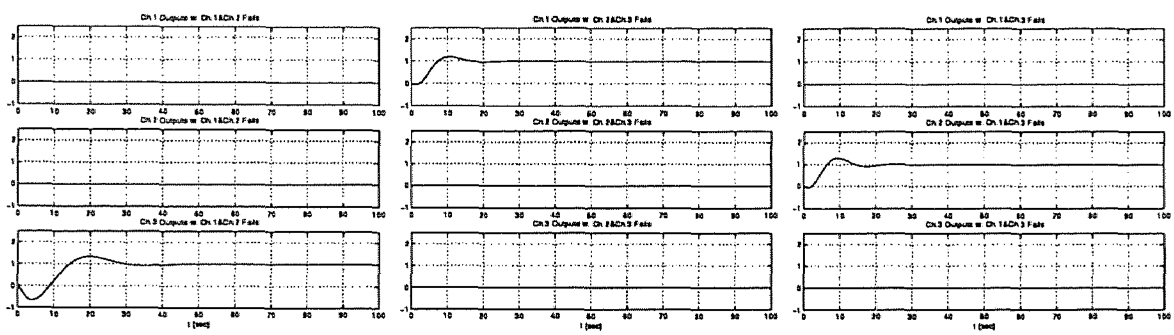


Figure 4.14 The Rosenbrock's Model Outputs with Permutation, 2 Channels Failed

CHAPTER 5

BLOCK DECENTRALIZED RELIABLE CONTROL SYNTHESIS

In this chapter, block decentralized control configuration is considered. That is, the feedback gain matrices possess a block diagonal structure instead of the strict diagonal structure assumed in the previous chapter.

Motivations for adopting a block decentralization include:

1. Physical constraints such as when a number of multivariable, control subsystems are interconnected.

2. For non-minimum phase, unreliable systems, block decentralization offers an alternative way of achieving fault tolerance by grouping together strongly coupled input-output channels.

5.1 System Description

In this section the state space model of the nominal plant is described, the block decentralized controller is configured, and a set of different failure modes are defined. The plant is assumed open loop stable and represented be the same model as in the previous chapters:

$$\begin{aligned} \dot{x} &= Ax + Bu + E\omega \\ y &= Cx \end{aligned} \tag{5.1}$$

To reflect the block decentralized structure, the B, C matrices are partitioned as:

$B = (B_1,\ B_2,\ \cdots,\ B_\mu),\ \ B_i \in \Re^{n \times n_i}\ for\ i = 1, 2, \cdots, \mu$

$C = (C_1,\ C_2,\ \cdots,\ C_\mu)',\ \ C_i \in \Re^{n_i \times n}\ for\ i = 1, 2, \cdots, \mu$

Correspondingly, the inputs vector v and output vector y are partitioned as:

$u = (v_1,\ v_2,\ \cdots,\ v_\mu)',\ v_i \in \Re^{n_i},\ for\ i = 1, 2, \cdots, \mu$

$y = (z_1,\ z_2,\ \cdots,\ z_\mu)',\ z_i \in \Re^{n_i},\ for\ i = 1, 2, \cdots, \mu$

where $\sum_{i=1}^{\mu} = \nu$. Finally, ω is assumed to be an unknown constant disturbance.

The open loop transfer matrix defined in (2.45)

$$T(s) = C(sI - A)^{-1}B = \frac{N(s)}{d(s)}$$

is now partitioned according to the block decentralized control structure as:

$$T(s) = \begin{pmatrix} T_{11}^B(s) & T_{12}^B(s) & \cdots & T_{1\mu}^B(s) \\ \vdots & \cdots & \cdots & \vdots \\ T_{\mu 1}^B(s) & T_{\mu 2}^B(s) & \cdots & T_{\mu\mu}^B(s) \end{pmatrix} \tag{5.2}$$

where $T_{ij}^B \in \mathcal{C}^{n_i \times n_j}$ is the transfer submatrix corresponding to the jth input vector u_j and ith output vector z_i, furthermore, let

$$T_{ij}^B(s) = \frac{N_{ij}^B(s)}{d(s)}$$

where $d(s)$ is the minimal polynomial of $T(s)$ and $N_{ij}^B(s)$ is the corresponding numerator matrix. The above partition (5.2) corresponds to the following input-output blocks pairing:

$$\{(v_1, z_1)\ (v_2, z_2)\ \cdots\ (v_\mu, z_\mu)\} \tag{5.3}$$

and the feedback control is therefore block diagonal, given by:

$$U(s) = K(s)Y(s) \tag{5.4}$$

where

$$K(s) = \text{block diagonal}(K_1^B(s), K_2^B(s), \cdots, K_\mu^B(s)) \tag{5.5}$$

and $K_i^B(s) \in \mathcal{C}^{n_i \times n_j}$.

The following block failure cases are now defined:

Definition 5.1.1 *Sensor Block Failure:*

The ith sensor block is said to have failed at time $t_1 > 0$ if

$$z_i(t) = 0 \quad \forall t > t_1$$

Remark: Similar to the definition in the strict decentralized control structure, the ith block sensors failure reflects the situation where all sensors in ith block ceased to function and generate only a null output for all time thereafter.

Definition 5.1.2 *Controller/Actuator Block Failure:*
The ith controller and/or actuator block is said to have failed at time $t_2 > 0$ if

$$v_i(t) = 0 \quad \forall t > t_2$$

Definition 5.1.3 *System with Partial Block Failure:*
Assume that sensor, actuator and/or controller blocks $\{i_{\phi+1}, i_{\phi+2}, \cdots, i_\mu\} \subset \{1, 2, \cdots, \mu\}$ have failed, then the resultant system is referred to as a system with partial block failure and it is described by the following equations:

$$\begin{aligned} \dot{x} &= Ax + B_{i_1}v_{i_1} + B_{i_2}v_{i_2} + \cdots + B_{i_\phi}v_{i_\phi} + E\omega \\ z_i &= C_i x, \quad i \in \{i_1, i_2, \cdots, i_\phi\} \end{aligned} \tag{5.6}$$

Definition 5.1.4 *ϕ-th Order Leading Principal Minor:*
Let $T^B[i_1, i_2, \cdots, i_\phi](s)$ be the transfer matrix of the system with partial block failure of (5.6), and let

$$T^B[i_1, i_2, \cdots, i_\phi](s) = \frac{N^B[i_1, i_2, \cdots, i_\phi](s)}{d(s)}$$

when $d(s)$ is the minimal polynomial of $T(s)$ and $N^B[i_1, i_2, \cdots, i_\phi](s)$ is the corresponding numerator matrix. Then, $det(N^B[i_1, \cdots, i_\phi](s))$ is called the ϕ-th order leading principal minor.

Definition 5.1.5 *ϕ-th Order Fault-free Block Decentralized PD^r Controller:*
Let $K^B_{PD}[i_1, i_2, \cdots, i_\phi](s)$ be the ϕ-th order fault-free block decentralized PD^r controller applying to the system with partial block failure of (5.6).

Definition 5.1.6 *Block Decentralized Robust Servomechanism Problem (BDRSP): Given the plant (5.1) and blocks of inputs/outputs pairing(5.3), obtain a block decentralized structured controller so that the following conditions all hold:*

1. *The closed loop system is asymptotically stable.*

2. *Asymptotically tracking occur, i.e.* $lim_{t\to\infty}y(t) = 0$*, for all constant disturbance* ω.

3. *Property 2) holds for parametric perturbations:* $A \to A + \delta A, B \to B + \delta B$*, and* $C \to C + \delta C$ *provided that the closed loop system remains stable.*

Definition 5.1.7 *Block Transmission Zeros: Let*

$$\Delta N^B[\phi] = \{\lambda \in C | det N^B[i_1, i_2, \cdots, i_\phi](\lambda) = 0, \forall \{i_1, i_2, \cdots, i_\phi\} \subset \{1, 2, \cdots, \mu\}\}$$

be defined as the block transmission zeros of $N^B[i_1, i_2, \cdots, i_\phi](s)$.

The existing necessary and sufficient condition for $BDRSP$ is stated as the following Lemma 5.1.1:

Lemma 5.1.1 *There exists a solution to the BDRSP iff there exists an installation sequence* $\{i_1, i_2, \cdots, i_\mu\} = \{1, 2, \cdots, \mu\}$ *so that*

$$0 \notin \Delta N^B[\phi], \;\; \phi = 1, 2, \cdots, \mu$$

5.2 Main Results

In this section, the Block Decentralized Robust Servomechanism Problem with Complete Reliability is considered. The failure condition are represented by block sensor failure(Definition 5.1.1), and controller/actuator block failure(Definition 5.1.2). The goal is to maintain the reliability of the system under such block failures.

Definition 5.2.1 *BDRSP with Complete Reliability(BDRSPwCR):*
Given the open loop stable plant(5.1) and the block of inputs/outputs pair(5.3), obtain a block decentralized controller so that the following conditions all hold:

1. *There exists a solution to the BDRSP for the nominal plant(5.1).*

2. *Under partial block failure, the controller solves the BDRSP for the plant (5.6) without retuning.*

Definition 5.2.2 *Steady-state Interaction Matrices:*
Given plant (5.1) with block decentralized control configuration, assume that the controller block installation sequence $\{i_1, i_2, \cdots, i_\mu\}$ is to be applied; then the following $\mu - 1$ steady-state interaction matrices M_i, $i = 2, \cdots, \mu$ with respect to this controller installation sequence are defined:

$$M_2[i_1, i_2] = I - T^B_{i_2 i_1}(0)(T^B_{i_1 i_1}(0))^{-1} T^B_{i_1 i_2}(0)(T^B_{i_2 i_2}(0))^{-1} \tag{5.7}$$

$$\begin{aligned} M_3[i_1, i_2, i_3] = \; & I - [T^B_{i_3 i_1}(0)(T^B_{i_1 i_1}(0))^{-1} \;\; T^B_{i_3 i_2}(0)(T^B_{i_2 i_2}(0))^{-1}] \\ & \begin{bmatrix} I & T^B_{i_1 i_2}(0)(T^B_{i_2 i_2}(0))^{-1} \\ T^B_{i_2 i_1}(0)(T^B_{i_1 i_1}(0))^{-1} & I \end{bmatrix}^{-1} \\ & \begin{bmatrix} T^B_{i_1 i_3}(0) \\ T^B_{i_2 i_3}(0) \end{bmatrix} (T^B_{i_3 i_3}(0))^{-1} \end{aligned} \tag{5.8}$$

$\cdots \quad \cdots \quad \cdots$

$$\begin{aligned} M_\mu[i_1, \cdots, i_\mu] = \; & I - [T^B_{i_\mu i_1}(0)(T^B_{i_1 i_1}(0))^{-1} \;\; \cdots T^B_{i_\mu, i_{\mu-1}}(0)(T^B_{i_{\mu-1}, i_{\mu-1}}(0))^{-1}] \\ & \begin{bmatrix} I & \vdots & T^B_{i_1, i_{\mu-1}}(0)(T^B_{i_{\mu-1}, i_{\mu-1}}(0))^{-1} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ T^B_{i_{\mu-1}, i_1}(0)(T^B_{i_1 i_1}(0))^{-1} & \vdots & I \end{bmatrix}^{-1} \\ & \begin{bmatrix} T^B_{i_1 i_\mu}(0) \\ T^B_{i_2 i_\mu}(0) \\ \vdots \\ T^B_{i_{\mu-1}, i_\mu}(0) \end{bmatrix} (T^B_{i_\mu i_\mu}(0))^{-1} \end{aligned} \tag{5.9}$$

where, $\forall\{i_1, i_2, \cdots, i_\mu\} \subset \{1, 2, \cdots, \mu\}$.

It is noted that M_R is the Schur complement of $N^B[i_1, i_2, \cdots, i_R](0)$.

The following new results on the $BDRSPwCR$ is obtained:

Assume the integral controllers are applied to the system:

$$K_I(s) = \text{block diag}(\frac{K_I^1}{s}, \frac{K_I^2}{s}, \cdots, \frac{K_I^\mu}{s}) \tag{5.10}$$

where $K_I^i \in \Re^{n_i \times n_i}$, $i = [1, 2, \cdots, \mu]$

Theorem 5.2.1 *There exists a solution to the BDRSPwCR by using block diagonal decentralized integral control(5.10) if the following conditions all hold:*

- $0 \notin \Delta N^B[\phi], \phi = 1, 2, \cdots, \mu$
- $Re(\sigma(M_2[i_1, i_2])) > 0,\ \forall\{i_1, i_2\} \subset \{1, 2, \cdots, \mu\}, i_1 \neq i_2$
- $Re(\sigma(M_3[i_1, i_2, i_3])) > 0,\ \forall\{i_1, i_2, i_3\} \subset \{1, 2, \cdots, \mu\}, i_1 \neq i_2 \neq i_3$
- $\vdots$
- $Re(\sigma(M_\mu[i_1, \cdots, i_\mu])) > 0,\ \forall\{i_1, \cdots, i_\mu\} \subset \{1, \cdots, \mu\}, i_1 \neq i_2 \neq \cdots \neq i_\mu$

where $Re(\sigma(M(\cdot)))$ are the real parts of eigenvalues of the matrices $M(\cdot)$.

Remark: In general, if the plant model(5.1) of the plant is not available, the steady-state interaction matrices defined above can be obtained experimentally.

Proof of Theorem 5.2.1:

Given plant(5.1) and the input/output pairings of (5.3), the open loop transfer matrix is described as (5.2). The block decentralized controller structure is given by (5.5) with the installation sequence of $i_1, i_2, \cdots, i_\mu$.

Assume that the integral controller(5.10) is applied to solve the $BDRSP$, the state space realization of the integral controller is:

$$\begin{aligned} \dot{\eta}_i &= z_i \\ v_i &= K_I^i \eta_i \end{aligned} \tag{5.11}$$

where $K_I^i = -\epsilon(T_{ii}^B(0))^{-1}$, and $\epsilon > 0$ is a tuning parameter.

Assuming the $i_1 \sim i_{\phi-1}$ controller blocks have been properly installed and the closed loop system is stable, the nominal closed loop system $\mathcal{A}_{i_{\phi-1}}$ is given by:

$$\mathcal{A}_{i_{\phi-1}} = \begin{bmatrix} A & (\epsilon B_{i_1} K_I^1 \; \epsilon B_{i_2} K_I^2 \cdots \epsilon B_{i_{\phi-1}} K_I^{\phi-1}) \\ \begin{pmatrix} C_{i_1} \\ C_{i_2} \\ \vdots \\ C_{i_{\phi-1}} \end{pmatrix} & 0 \end{bmatrix} \tag{5.12}$$

with the i_ϕth controller block installed, and $\epsilon \to 0$, eigenvalues of the closed loop matrix $\mathcal{A}_{i_\phi}$ are given by:

$$\sigma(\mathcal{A}_{i_\phi}) = \sigma\begin{pmatrix} \mathcal{A}_{i_{\phi-1}} & B_{i_\phi} K_I^\phi \\ C_{i_\phi} & 0 \end{pmatrix} \tag{5.13}$$

$$\sigma(A_{i_{\phi-1}}) \dot{\cup} - \sigma\left[(C_{i_\phi} \;\; 0) A_{i_{\phi-1}}^{-1} \begin{pmatrix} B_{i_\phi} K_I^\phi \\ 0 \end{pmatrix}\right] \tag{5.14}$$

since it is assumed that $Re(\sigma(\mathcal{A}_{i_\phi-1})) < 0$, in order for the closed loop system to be stable, it is required that,

$$\sigma\left[(C_{i_\phi} \;\; 0)\mathcal{A}_{i_{\phi-1}}^{-1} \begin{pmatrix} B_{i_\phi} K_I^\phi \\ 0 \end{pmatrix}\right] > 0 \tag{5.15}$$

From the following matrix inversion lemma:

$$\begin{bmatrix} A & D \\ C & B \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + E\Delta^{-1}F & -E\Delta^{-1} \\ -\Delta^{-1}F & \Delta^{-1} \end{bmatrix}$$

where $\Delta = B - CA^{-1}D$, $E = A^{-1}D$ and $F = CA^{-1}$, and the structure of $\mathcal{A}_{i_{\phi-1}}$ in (5.12), the left hand side matrix of Equation (5.15) becomes:

$$(C_{i_\phi} \;\; 0) \begin{bmatrix} A & (B_{i_1} K_I^1 \; B_{i_2} K_I^2 \cdots B_{i_{\phi-1}} K_I^{\phi-1}) \\ \begin{pmatrix} C_{i_1} \\ C_{i_2} \\ \vdots \\ C_{i_{\phi-1}} \end{pmatrix} & 0 \end{bmatrix} \begin{pmatrix} B_{i_\phi} K_I^\phi \\ 0 \end{pmatrix}$$

$$= C_{i_\phi}[A^{-1} - A^{-1}(B_{i_1} K_I^1 \; B_{i_2} K_I^2 \cdots B_{i_{\phi-1}} K_I^{\phi-1})$$

$$\left[\begin{pmatrix} C_{i_1} \\ C_{i_2} \\ \vdots \\ C_{i_{\phi-1}} \end{pmatrix} A^{-1}(B_{i_1} K_I^1 \; B_{i_2} K_I^2 \cdots B_{i_{\phi-1}} K_I^{\phi-1})\right]^{-1}$$

$$\begin{pmatrix} C_{i_1} \\ C_{i_2} \\ \vdots \\ C_{i_{\phi-1}} \end{pmatrix} A^{-1}](B_{i_\phi}K_I^{\phi}) \tag{5.16}$$

with $-C_iA^{-1}B_j = T_{ij}(0)$, and K_I^i set to $-\epsilon(-C_iA^{-1}B_i)^{-1}$, $\forall i,j \in [i_1, i_2, \cdots i_\phi]$. Upon simplification, Equation (5.16) becomes:

$$\begin{aligned} I - [&T^B_{i_\phi,i_1}(0)(T^B_{i_1i_1}(0))^{-1}\, T^B_{i_\phi,i_2}(0)(T^B_{i_2i_2}(0))^{-1} \cdots T^B_{i_\phi,i_{\phi-1}}(0)(T^B_{i_{\phi-1},i_{\phi-1}}(0))^{-1}] \\ \times &\begin{bmatrix} I & T^B_{i_1i_2}(0)(T^B_{i_2i_2}(0))^{-1} & \cdots & T^B_{i_1,i_{\phi-1}}(0)(T^B_{i_{\phi-1},i_{\phi-1}}(0))^{-1} \\ T^B_{i_2i_1}(0)(T^B_{i_1i_1}(0))^{-1} & I & \cdots & T^B_{i_2,i_\phi-1}(0)(T^B_{i_{\phi-1},i_{\phi-1}}(0))^{-1} \\ \vdots & \vdots & \vdots & \vdots \\ T^B_{i_{\phi-1},i_1}(0)(T^B_{i_1i_1}(0))^{-1} & \cdots & \cdots & I \end{bmatrix}^{-1} \\ \times &\begin{bmatrix} T^B_{i_1,i_\phi}(0) \\ T^B_{i_2,i_\phi}(0) \\ \vdots \\ T^B_{i_{\phi-1},i_\phi}(0) \end{bmatrix} (T^B_{i_\phi,i_\phi}(0))^{-1} = M_\phi[i_1, i_2, \cdots, i_\phi] \end{aligned} \tag{5.17}$$

For stability requirement,

$$Re(\sigma(M_{i_\phi}[i_1, i_2, \cdots, i_\phi])) > 0$$

Similarly, for the $i_{\phi-1}$ I/O blocks system, stability requirement is

$$Re(\sigma(M_{\phi-1}[i_1, i_2, \cdots, i_{\phi-1}]) > 0$$

where the $M_{i_{\phi-1}}$ matrix is obtained by substituting i_ϕ with $i_{\phi-1}$. Similar requirements apply to the $i_{\phi-2}, i_{\phi-3}, \cdots, 2$ block I/O systems and therefore establish the sufficiency of Theorem(5.2.1).

(end of proof)

To solve the $BDRSPwCR$, it is not always adequate to use only integral control. That is, the conditions of Theorem 5.2.1 are not always satisfied by any general process. Therefore, a block PID^r type controller, which is the multivariable extension of Theorem 3.1.1 of Chapter 3, will be applied to weaken the conditions of

Theorem 5.2.1. The structure of the block PID^r controller is given below:

$$
\begin{aligned}
K_P(s) &= \text{block diag}(K_P^1, K_P^2, \cdots, K_P^\mu) && (5.18)\\
K_I(s) &= \text{block diag}(\frac{K_I^1}{s}, \frac{K_I^2}{s}, \cdots, \frac{K_I^\mu}{s}) && (5.19)\\
K_D^i(s) &= K_{di}^1 s + K_{di}^2 s^2 + \cdots + K_{di}^r s^r, i = 1, 2, \cdots, \mu && (5.20)\\
K_D(s) &= \text{block diag}(K_D^1(s), K_D^2(s), \cdots, K_D^\mu(s)) && (5.21)\\
K_{PD}(s) &= K_P(s) + K_D(s) && (5.22)\\
K(s) &= K_P(s) + K_I(s) + K_D(s) && (5.23)
\end{aligned}
$$

where

$$r = \text{order}(d(s)) - \text{min.}\{\Delta N^B[1], \Delta N^B[2], \cdots, \Delta N^B[\mu]\} \quad (5.24)$$

Theorem 5.2.2 *There exists a solution to the BDRSPwCR(Definition 5.2.1) if the following conditions all hold:*

1. *There is a solution to BDRSP for (5.1).*

2. $\Delta N^B[\phi] < 0$, $\phi = 1, 2, \cdots, \mu$

Proof of Theorem 5.2.2:

Define $T_P(s)$ to be the transfer matrix of plant (5.1) with block decentralized proportional feedback $K_P(s) = \text{block diag}(K_P^1, K_P^2, \cdots, K_P^\mu)$; then,

$$T_P(s)^{-1} = T(s)^{-1} - K_P(s) \quad (5.25)$$

as $||K_P^i|| \to \infty, \forall i \in [1, \nu]$, $T_P(0)$ becomes diagonal

and $M_2[i_1, i_2], \cdots, M_\mu[i_1, i_2, \cdots, i_\mu] \to I$, so that $Re(\sigma(M_i)) > 0$, thus satisfying the conditions in Theorem 5.2.1.

For the consideration of stability under partial groups of channels fail, assuming that $T^B[i_1, \cdots, i_\phi](s)$ is the subsystem consists of the fault free blocks.

$$T^B[i_1, \cdots, i_\phi](s) = \frac{1}{d(s)} N^B[i_1, \cdots, i_\phi](s) \quad (5.26)$$

and $K^B_{PD}[i_1, \cdots, i_\phi](s)$ is the fault-free block decentralized PD^r controller. Then the fault-free subsystem closed-loop transfer matrix:

$$T^B_{PD}[i_1, \cdots, i_\phi](s) = (I - T^B[i_1, \cdots, i_\phi](s)K^B_{PD}[i_1, \cdots, i_\phi](s))^{-1}T^B[i_1, \cdots, i_\phi]$$

$$= (d(s)I - N^B[i_1, \cdots, i_\phi]K^B_{PD}[i_1, \cdots, i_\phi])^{-1}N^B[i_1, \cdots, i_\phi]$$

Now set $||K_P|| \to \infty, ||K_D|| \to \infty$, when r be the maximum pole-zero excess of $T(s)$, then

$$T^B_{PD}[i_1, \cdots, i_\phi](s) \to -(K^B_{PD}[i_1, \cdots, i_\phi](s)^{-1}N^B[i_1, \cdots, i_\phi](s)^{-1})N^B[i_1, \cdots, i_\phi](s) \tag{5.27}$$

when the determinant of $N^B[i_1, \cdots, i_\phi](s) \neq 0 \ \ \forall s \in [0, +\infty]$, i.e., $\Delta N^B[\phi] < 0 \ \ \phi = 1, \cdots, \mu$, then

$$N^B[i_1, \cdots, i_\phi](s)^{-1}N^B[i_1, \cdots, i_\phi](s) = I$$

therefore equation (5.27) becomes:

$$T^B_{PD}[i_1, \cdots, i_\phi](s) \to -K^B_{PD}[i_1, \cdots, i_\phi](s)^{-1}$$

Controller $K^B_{PD}[i_1, \cdots, i_\phi](s)$ matrix can always be chosen such that each controller is a polynomial with all roots being located in LHP, therefore, the roots of characteristic polynomial of the closed loop system are located in LHP.

(end of proof)

Remark: Specially, for a two-block system, condition 2 of Theorem 5.2.2 becomes:

$$Re(\Delta N^B[1]) < 0 \tag{5.28}$$

$$Re(\Delta N^B[2]) < 0 \tag{5.29}$$

i.e. $\det N^B[1](s), \det N^B[2](s)$ are Hurwitz.

The following example in the next section is a 4-input/output system with its transfer matrix being partitioned into two blocks and the simulation results confirm the above theorem, i.e., the $BDRSPwCR$ can be solved by using the block decentralized PID^r control.

5.3 Example

In this example, the $BDRSPwCR$ will be solved for a 4-input/output system consisting of 2 blocks with 2 input/output channels. The principal diagonal blocks $T_{11}^B(s)$ and $T_{22}^B(s)$ are modified Rosenbrock's Models (Rosenbrock's Model with the 2 inputs interchanged). The system does not satisfy the conditions in Theorem 5.2.1, however, it satisfies the conditions in Theorem 5.2.2, therefore the $BDRSPwCR$ can be solved by block PID^r controllers.

The transfer function matrix is partitioned into two blocks, with each block consists a 2-input/output subsystem.

$$\begin{aligned} T_{11}^B(s) &= \frac{1}{d(s)}N_{11}^B(s) \\ &= \frac{1}{s^3+3s^2+3s+1}\begin{bmatrix} -s^2+s+2 & -s^2+1 \\ -s^2+1 & -s^2-0.6667s+0.3333 \end{bmatrix} \end{aligned} \tag{5.30}$$

$$\begin{aligned} T_{12}^B(s) &= \frac{1}{d(s)}N_{12}^B(s) \\ &= \frac{1}{s^3+3s^2+3s+1}\begin{bmatrix} 100s^2+100s+100 & 10s^2+10s+20 \\ 10s^2+10s+30 & 10s^2+10s+10 \end{bmatrix} \end{aligned} \tag{5.31}$$

$$\begin{aligned} T_{21}^B(s) &= \frac{1}{d(s)}N_{21}^B(s) \\ &= \frac{1}{s^3+3s^2+3s+1}\begin{bmatrix} 100s^2+100s+100 & 10s^2+10s+20 \\ 10s^2+10s+30 & 10s^2+10s+10 \end{bmatrix} \end{aligned} \tag{5.32}$$

$$\begin{aligned} T_{22}^B(s) &= \frac{1}{d(s)}N_{22}^B(s) \\ &= \frac{1}{s^3+3s^2+3s+1}\begin{bmatrix} -s^2+s+2 & -s^2+1 \\ -s^2+1 & -s^2-0.6667s+0.3333 \end{bmatrix} \end{aligned} \tag{5.33}$$

The DC gain sub-matrices are:

$$T_{11}^B = \begin{bmatrix} 2 & 1 \\ 1 & 0.3333 \end{bmatrix} \tag{5.34}$$

$$T_{12}^B = \begin{bmatrix} 100 & 20 \\ 30 & 10 \end{bmatrix} \quad (5.35)$$

$$T_{21}^B = \begin{bmatrix} 100 & 20 \\ 30 & 10 \end{bmatrix} \quad (5.36)$$

$$T_{22}^B = \begin{bmatrix} 2 & 1 \\ 1 & 0.3333 \end{bmatrix} \quad (5.37)$$

It is noted that for the $T_{11}^B(s), T_{22}^B(s)$ blocks, reliable control for strict diagonal decentralized configuration cannot be obtained as discussed in the previous chapter.

The steady-state interaction matrix

$$M_2 = I - T_{21}^B(0)(T_{11}^B(0))^{-1}T_{12}^B(0)(T_{22}^B(0))^{-1} = \begin{bmatrix} 1.599 \times 10^3 & 1.8 \times 10^3 \\ 0 & -8.99 \times 10^2 \end{bmatrix} \quad (5.38)$$

The eigenvalues of the M_2 matrix are calculated as -1.599×10^3, -8.99×10^2, which are all negative, and the condition in Theorem 5.2.1 is not satisfied.

However, the transmission zeros of all principal minor blocks are given by:

$$\begin{aligned} \text{roots}(\det N_{11}^B(s)) &= -1,\ -1,\ -1 \\ \text{roots}(\det N_{22}^B(s)) &= -1,\ -1,\ -1 \\ \text{roots}(\det N(s)) &= -0.52491 \pm j1.0989 \\ &\quad -0.47891 \pm j0.99090 \\ &\quad -0.49546 \pm j0.29998 \\ &\quad -0.50408 \pm j0.27644 \end{aligned}$$

The real parts of all the block transmission zeros are all negative, therefore the conditions in Theorem 5.2.2 are satisfied and the $BDRSPwCR$ can be solved by block decentralized PID^r structured controllers. In the example, the maximum pole-zero excess is 1, therefore, the PID^1 controllers are selected as:

$$K_1(s) = \begin{bmatrix} -50s - 50 - 1/s & 0 \\ 0 & -50s - 50 - 1/s \end{bmatrix} \quad (5.39)$$

$$K_2(s) = \begin{bmatrix} -50s - 50 - 1/s & 0 \\ 0 & -50s - 50 - 1/s \end{bmatrix} \quad (5.40)$$

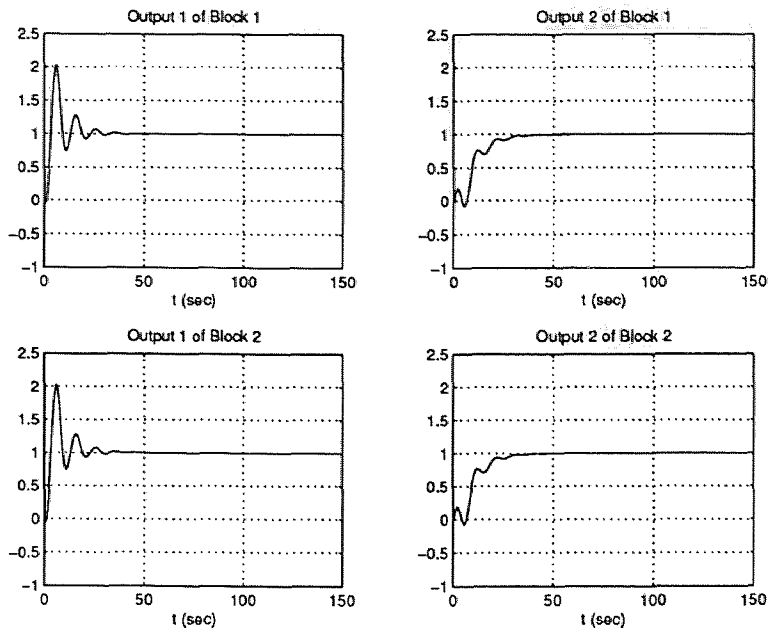


Figure 5.1 Step Responses of The Plant at Normal Operation

5.3.1 Simulation Results

The following simulations are made for two cases: (1)Without PD^1 feedback controllers and (2) With PD^1 controller enhancement. These situations are considered:

- Both blocks are normally operational.
- Block 2 failed.
- Block 1 failed.

Case 1: Without PD^1 feedback control:

Block decentralized integral controller with

$$K_I^1(s) = K_I^2(s) = \begin{bmatrix} 1/s & 0 \\ 0 & 1/s \end{bmatrix}$$

are applied to the plant. As shown in Figure 5.1, asymptotic regulation takes place for the nominal plant. However, the closed loop system becomes explosively unstable when controller 1 fails (Figure 5.2) or controller 2 fails (Figure 5.3).

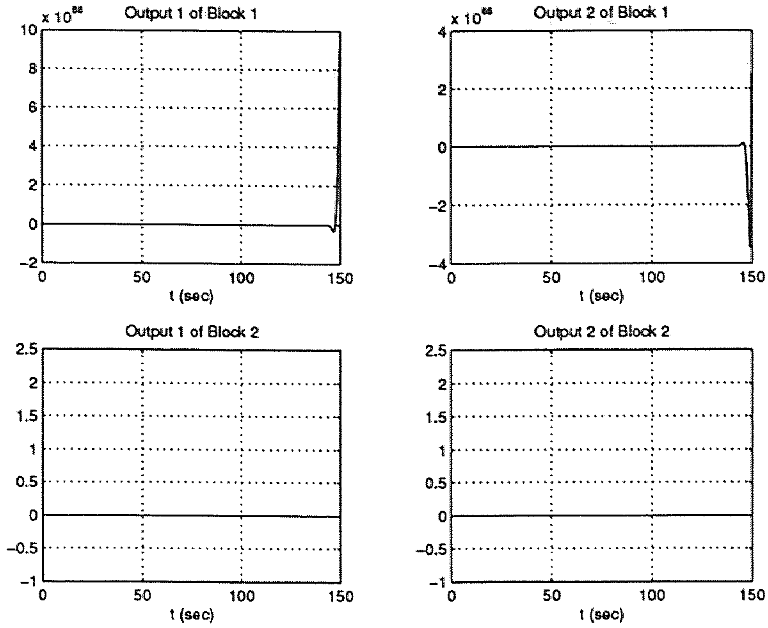


Figure 5.2 Step Responses of The Plant with Block 2 Failed

Case 2: With PD^1 feedback control:

As shown in Figures 5.4, 5.5 and 5.6, with the block decentralized PID^1 controllers (5.39) and (5.40) added, the closed loop system is reliable. The failure of any block does not affect the stability of the system and the fault-free block continues to produce asymptotic regulation.

The conclusions that can be obtained from this chapter are: To deal with the closed loop reliability for non-minimum phase, unreliable, open-loop stable linear systems, it is always significant to group those channels that have severe internal interactions, and treat this group as a single sub-system. Failures of one such sub-system does not affect the stability and regulation of other sub-systems.

It should be noted that the responses of the closed loop system have not been optimized. In the event that faster speed of response is desired, the parameter optimization method[6] can be applied.

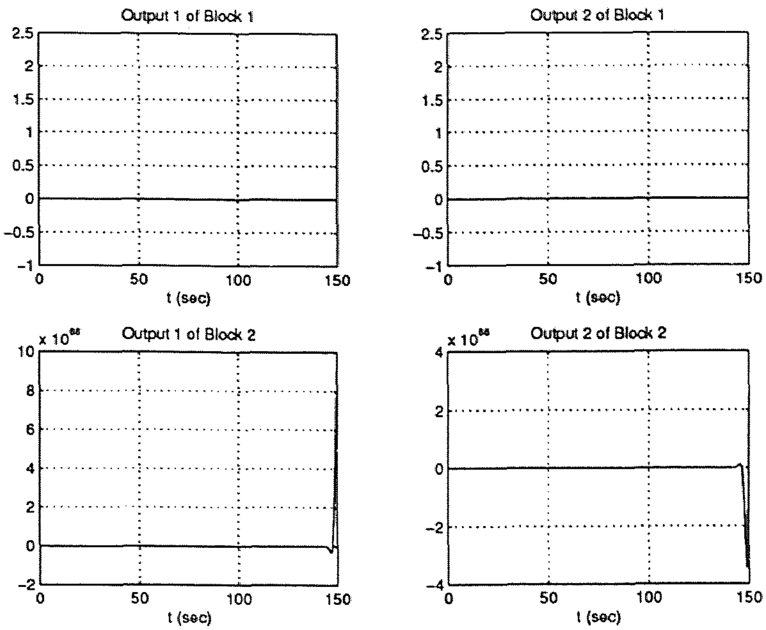


Figure 5.3 Step Responses of The Plant with Block 1 Failed

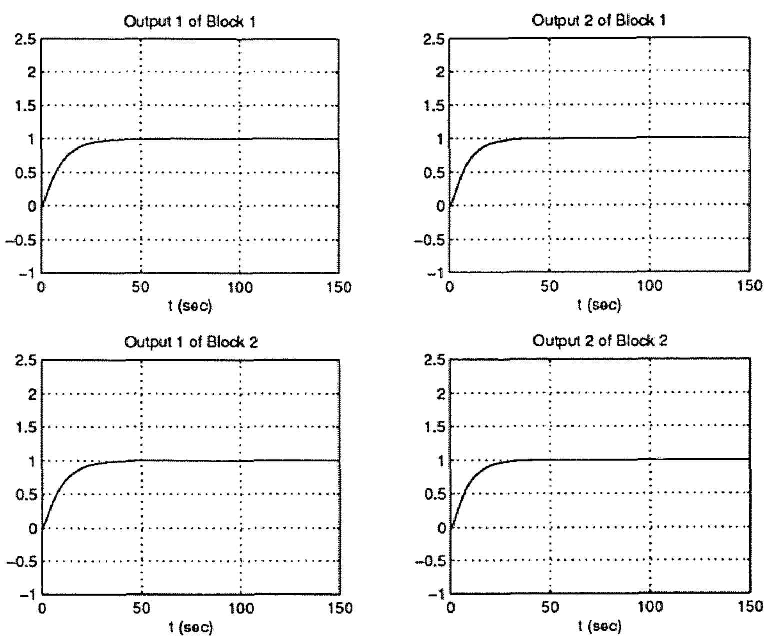


Figure 5.4 Step Responses of The Plant at Normal Operation

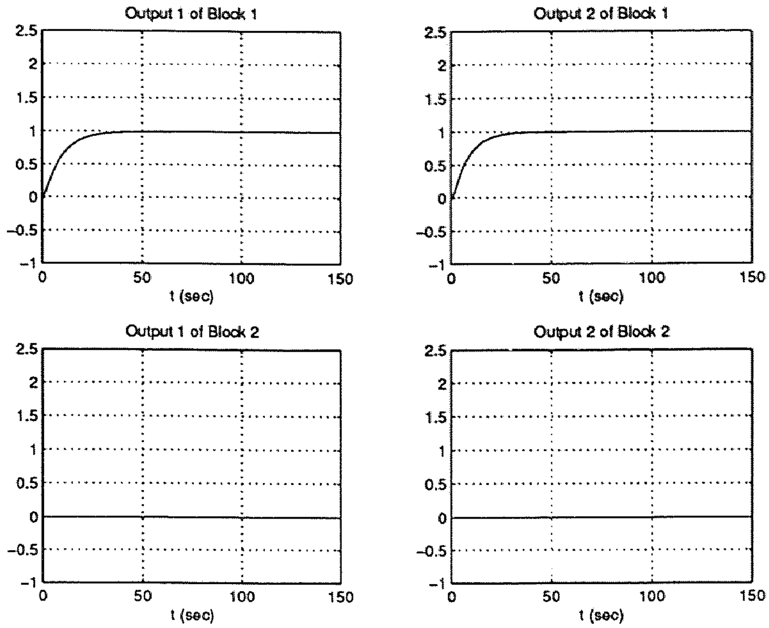


Figure 5.5 Step Responses of The Plant with Block 2 Failed

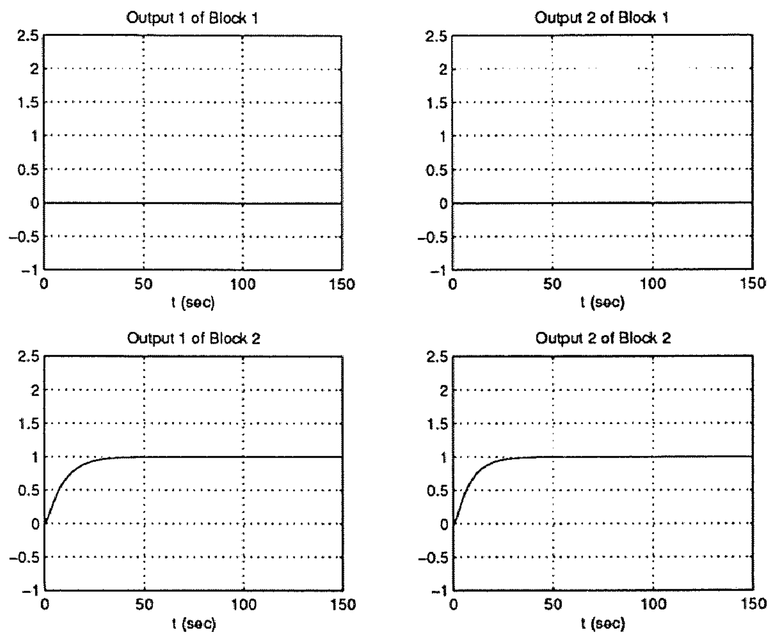


Figure 5.6 Step Responses of The Plant with Block 1 Failed

CHAPTER 6

RELIABLE CONTROL OF MARGINALLY STABLE SYSTEMS AND APPLICATION TO FAULT TOLERANT URBAN TRAFFIC CONTROL PROBLEM

The previous chapters deals with relaible control of open loop stable systems. In this chapter, the reliable control analysis are extended to systems with special structure, e.g., systems with marginal stability. This is followed by an application to fault tolerant urban traffic network, which is a marginally stable system. The example includes the traffic network queue length model development, decentralized controller synthesis, and simulations results on the system under normal operations as well as under different failure modes.

6.1 Reliable Control of Marginally Stable Systems

The application of decentralized reliable control can be extended to systems with marginally stability. Consider the system described as:

$$\begin{aligned} \dot{x} &= Bu + E\omega \\ y_i &= x_i \;\; i = 1, 2, \cdots, n \qquad (6.1) \\ e_i &= y_i - y_i^{ref}, \;\; i = 1, 2, \cdots, n \qquad (6.2) \end{aligned}$$

where $B = [b_1, \cdots, b_n] \in \Re^{n \times n}$, $E \in \Re^{n \times \Omega}$, and $C = [c_1, \cdots, c_n] \in \Re^{n \times n}$, The variables $x \in \Re^n$, $u = [u_1, \cdots, u_n]' \in \Re^n$, $y = [y_1, \cdots, y_n]' \in \Re^n$, and $\omega \in \Re^{\Omega}$, are the state, input, output and the constant disturbance vectors, respectively. Furthermore, the error vector $e = [e_1, \cdots, e_n]' \in \Re^n$ and the constant reference $y_i^{ref} \in \Re^n$ vector are assumed known.

This is a marginally stable system since the minimal polynomial of this system possesses only a single root at the origin.

The following decentralized robust PI control is applied to achieve closed loop stability and asymptotic regulation, i.e. $lim_{t\to\infty}e(t) = 0$:

$$\begin{aligned} u &= K_P e + K_I \eta \\ \dot{\eta} &= e \end{aligned} \tag{6.3}$$

Definition 6.1.1 *Strictly Diagonally Dominant: An n by n matrix A is said to be strictly diagonally row dominant if*

$$a_{ii} \neq 0, \quad |a_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}|$$

It is said to be strictly diagonally column dominant if A' is strictly diagonally row dominant.

The following result is obtained:

Theorem 6.1.1 *There exists a solution to the reliable control of the marginally stable system(6.1) by using the decentralized control structure(6.3) iff B is strictly diagonally row or column dominant.*

Proof of Theorem 6.1.1:

The necessity follows from Theorem 1 of [41]. The sufficiency is established on noting that as $||K_P|| \to 0$, the eigenvalues of the closed loop system are given by the eigenvalues BK_P together with those of $-K_P^{-1}K_I$. The reliable properties can now be obtained by choosing a diagonal K_P and a diagonal K_I so that:

1. The nominal closed loop eigenvalues are stable.
2. The eigenvaluses of failed closed loop system (excluding those corresponding to the failed subsystem dynamics) are stable.

This is always possible if the B matrix is diagonally dominant.

(end of proof)

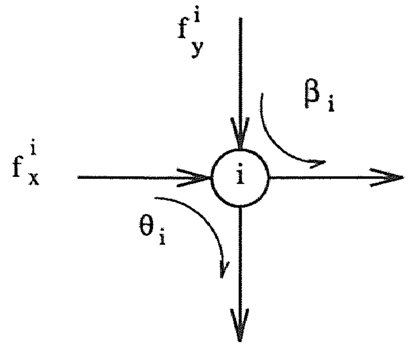


Figure 6.1 A Standard 2-Input/Output Intersection

6.2 Fault Tolerant Traffic Control Applicaiton

Traffic-responsive area traffic control are steadily gaining popularity in many cities where road congestion is becoming a serious problem.

A number of strategies such as SCAT(Sydney Co-ordinated Adaptive Traffic) and SCOOT(Split, Cycle and Offset Optimization Technique) are currently in use in a number of British Commonwealth Countries such as Australia and The United Kingdom.

In an urban traffic network where flow efficiency is of prime importance, fault-tolerant capability is critical to the long term operation and integrity of the system. Most importantly, a fault-tolerant traffic control system does not become unstable when unpredictable faults occur, thereby ensuring the integrity of the fault-free subsystem until full fault recovery takes place.

The dynamics of the urban traffic networks can be analyzed from a number of existing approaches: e.g., mass-balance[41], continuum flow[44] and microscopic , discrete-time model[45]. The approach of [41] is used in this work in that it directly addresses the issues of traffic queue dynamics.

In this section, a model based on the mass-balance approach [41] is developed. Standard traffic networks, consisting solely of 2-input, 2-output nodes shown in Figure 6.1, is first considered. This is followed by an analysis of "non-standard" traffic networks which may include merges, splits, and alternate routes.

To facilitate the discussion of the model, the following symbols are introduced below:

q_x^i	:	queue length in front of the ith intersection in the x direction
q_y^i	:	queue length in front of the ith intersection in the y direction
f_m^i	:	maximum output flow rate of the ith intersection
g_i	:	normalized "go" signal duration of the ith intersection in the x direction
$1 - g_i$	:	normalized "go" signal duration of the ith intersection in the y direction
$\overline{g}_i$	:	nominal value of g_i
Δg_i	:	incremental go duration ($= g_i - \overline{g}_i$)
θ_i	:	fractional right turn traffic flow rate at the ith intersection
β_i	:	fractional left turn traffic flow rate at the ith intersection
f_{xin}^i	:	input flow rate at the x direction of the ith intersection
f_{yin}^i	:	input flow rate at the y direction of the ith intersection

The x or y direction of the ith intersection is said to have an external feed if the source is external to the network (rather than being other routes inside the network). Otherwise, the input is called internal. Furthermore, the ith intersection is defined as undersaturated if:

$$f_{xin}^i + f_{yin}^i < f_m^i$$

it is said to be saturated if:

$$f_{xin}^i + f_{yin}^i = f_m^i$$

and it is said to be oversaturated if:

$$f_{xin}^i + f_{yin}^i > f_m^i$$

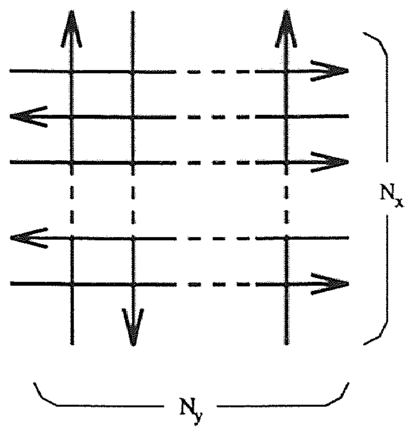


Figure 6.2 Standard Traffic Network Structure

6.2.1 General Model of Queue Lengths in Traffic Network

As shown in Figure 6.2, assume a standard structure of traffic network consists of N_x horizontal streets and N_y vertical avenues, each street or avenue is one-way, any two neighboring streets or avenues have opposite flow directions, the traffic light cycle length is fixed, and a no-turn-on-red traffic rule is applied. The total number of intersections in the network is therefore $n = N_x \times N_y$.

The queue dynamics of the network, taking into account of vehicles making right or left turns, is derived in the Appendix A and given by the following equation:

$$\dot{q} = (I_{2n} - \mathcal{T})J'\Delta u + M - (I_{2n} - \mathcal{T})f(q) \tag{6.4}$$

where

- $q = [q_x^1 \ q_y^1 \ q_x^2 \ q_y^2 \cdots q_x^n \ q_y^n]' \in \Re^{2n}$
- $\mathcal{T} \in \Re^{2n\times 2n}$ characterizes the flow configuration between any two intersections in the network. The structure of this matrix is given in Table A.1 of Appendix A.

- $J \in \Re^{n\times 2n}$ is defined as

$$J = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}$$

- $\Delta u = [\Delta u_1, \Delta u_2, \Delta u_3, \cdots, \Delta u_n]' \in \Re^n$ is the control input vector whose elements are given by $\Delta u_i = \Delta g_i f_m^i,\ i = 1, 2, \cdots, n$. The physical meaning of Δu_i is the adjusted amount of traffic flow passing through the ith intersection at the green light duration from x direction of that intersection.

- $M \epsilon \Re^{2n\times 1}$ is defined as:

$$M = F_c - (I - T)(F_m + J'\overline{u})$$

where

$$\overline{u} = [\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_i, \cdots, \overline{u}_n]'$$

$$\overline{u}_i = \overline{g}_i f_m^i,\ i = 1, 2, \cdots, n$$

$$F_m = [0, f_m^1, 0, f_m^2, \cdots, 0, f_m^i, \cdots, 0, f_m^n]'$$

and $F_c \in \Re^{2n}$ is determined by the following conditions:

$$F_c(2i-1) = \begin{cases} 0 & \text{if } x\text{-input is internal} \\ f_{xin}^i & \text{if } x\text{-input is external} \end{cases}$$

$$F_c(2i) = \begin{cases} 0 & \text{if } y\text{-input is internal} \\ f_{yin}^i & \text{if } y\text{-input is external} \end{cases}$$

$i = 1, 2, \cdots, n$

- Finally,

$$f(q) = [f(q_x^1)\ f(q_y^1)\ \cdots\ f(q_y^n)]' \in \Re^{2n}$$

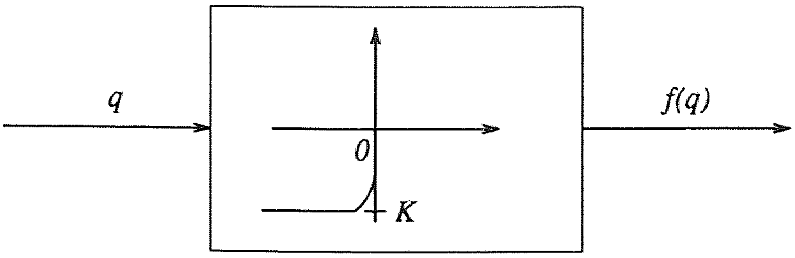


Figure 6.3 $f(q)$ Function

where $f(q_i)$ is a nonlinear function defined as:

$$f(q_i) = \begin{cases} 0 & q_i > \varepsilon \\ -K & q_i < \varepsilon \end{cases}$$

The role of $f(q)$ is to maintain the non-negativity of the queues by keeping $\dot{q}$ non-negative when $q = 0$. In the limiting case, this can be achieved by letting $K \to \infty$, $\varepsilon \to 0$. It is noted that $f(\cdot)$ can be readily approximated by a relay or a sigmoidal function such as

$$f(q) = K(\frac{1}{1+e^{-Kq}} - 1) = \frac{-Ke^{-Kq}}{1+e^{-Kq}}$$

as shown in Figure 6.3.

6.2.2 16-Intersection Example of Queue Model

Consider now the 16-intersection network shown in Figure 6.4. For this network,

$$q = \begin{pmatrix} q_x^1 \\ q_y^1 \\ \vdots \\ q_x^{16} \\ q_y^{16} \end{pmatrix}, \dot{q} = \begin{pmatrix} \dot{q}_x^1 \\ \dot{q}_y^1 \\ \vdots \\ \dot{q}_x^{16} \\ \dot{q}_y^{16} \end{pmatrix}, \Delta u = \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_{15} \\ \Delta u_{16} \end{pmatrix}$$

and

$$J = \underbrace{[\text{block diagonal}\,(1 \ \ -1)]}_{16 \text{ blocks}}$$

F_c is given by:

$$\begin{array}{llll} F_c(1) = f_{xin}^1 & F_c(2) = f_{yin}^1 & F_c(6) = f_{yin}^3 & F_c(15) = f_{xin}^8 \\ F_c(17) = f_{xin}^9 & F_c(28) = f_{yin}^{14} & F_c(31) = f_{xin}^{16} & F_c(32) = f_{yin}^{16} \end{array}$$

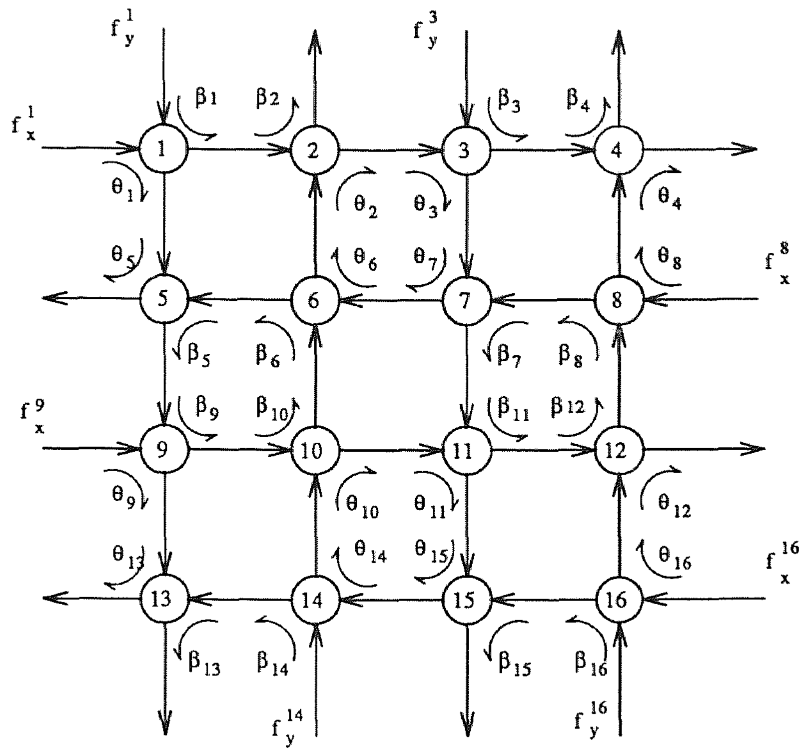


Figure 6.4 Standard 16-Node traffic network structure

while other elements of F_c are set to zero. Furthermore,

$$F_m = [0, f_m^1, 0, f_m^2, \cdots, 0, f_m^{16}]'$$

$$\overline{u} = [\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_{16}]'$$

$$M = F_c - (I - \mathcal{T})(F_m + J\overline{u})$$

$\mathcal{T}$ matrix is given in Figure 6.5:

6.2.3 Special Case: Saturated Intersections

For a saturated or over-saturated network with non-zero initial conditions, $q_i > 0, \quad i = 1, 2, \cdots, 2n$, therefore $f(q) = 0$ and (6.4) becomes:

$$\dot{q} = (I - \mathcal{T})J'\Delta u + M \tag{6.5}$$

or equivalently,

$$
T=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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1-\theta_1 & \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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\theta_1 & 1-\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\beta_7 & \theta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{10} & 1-\theta_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & \theta_3 & 1-\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_5 & 1-\theta_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\theta_9 & \beta_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\theta_{11} & \beta_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{16} & 1-\beta_{16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\theta_{14} & \beta_{14} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_9 & 1-\beta_{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\beta_{15} & \theta_{15} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\theta_{16} & \beta_{16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{11} & 1-\beta_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Figure 6.5 $\mathcal{T}$ Matrix for the 16-Intersection Example

$$\dot{q} = \Phi \Delta u + M \tag{6.6}$$

where

$$\Phi = (I - \mathcal{T})J' \tag{6.7}$$

$$M = F_c - (I - \mathcal{T})(F_m + J\bar{u}) \tag{6.8}$$

It is noted that (6.6) is equivalent to the model described in [41].

6.2.4 Treatment of Non-standard Structure Networks

Although the traffic model (6.4) is derived for standard rectangular networks, this model can also be extended to include non-standard networks by first converting them into equivalent standard structures. The conversion procedure consists of adding fictitious nodes and fictitious streets. The following examples illustrate this construction:

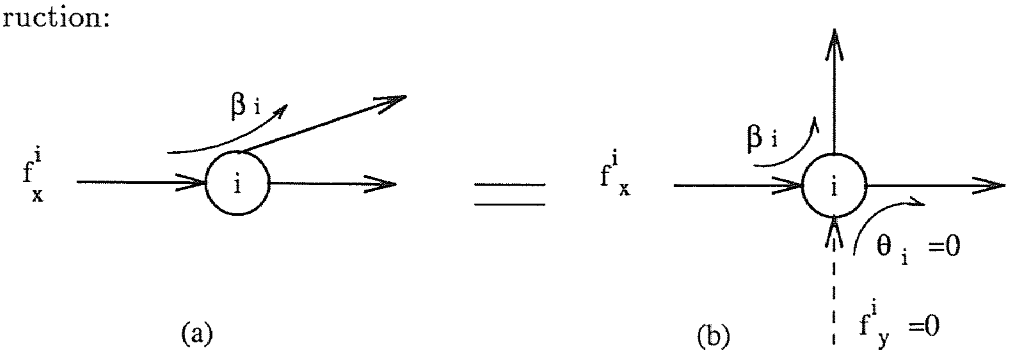

Figure 6.6 Split Node and Its Standard Equivalent Node

Figure 6.6a is a split node, i.e., one street is split into two without traffic light control . Figure 6.6b is its equivalent standard node (fictitious) with $f_y^i = 0$, $\theta_i = 0$.

Figure 6.7a is a merge node, i.e., two streets merge into one street , also without traffic light control. Figure 6.7b is its equivalent standard node(fictitious) with $\theta_i = 0$.

Further examples are shown in Figures 6.8 and 6.9, where the latter is taken from reference [41].

The general non-standard queue model can now be expressed as:

$$\dot{q} \;=\; (I - \mathcal{T})J'L\Delta u + (I - \mathcal{T})J'\mathcal{N} + M - (I - \mathcal{T})f(q) \tag{6.9}$$

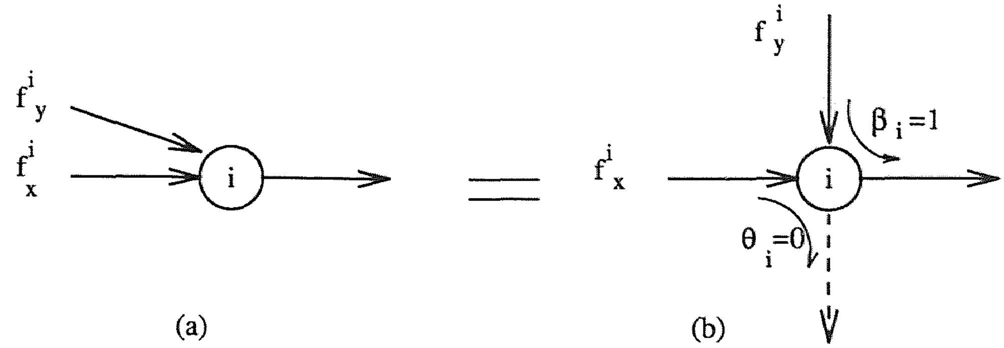

Figure 6.7 Merge Node and Its Standard Equivalent Node

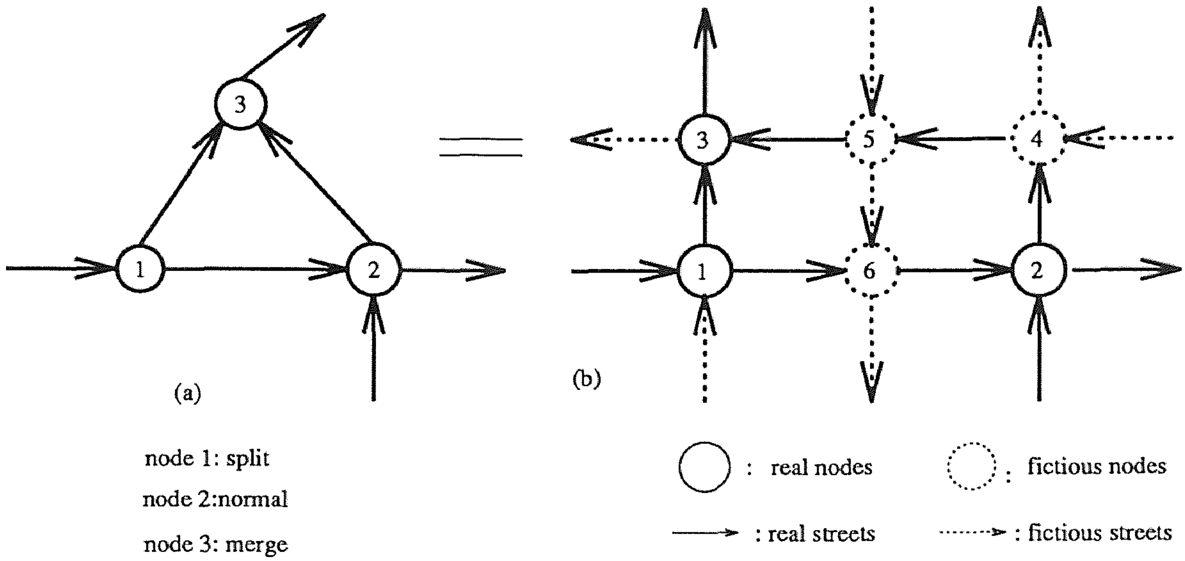

Figure 6.8 A Non-standard Network and Its Equivalent Structure

$$Y = Cq$$

where L is determined as following:

$$L(i,j) = 0 \quad (i \neq j)$$

$$L(i,i) = \begin{cases} 1 & \text{the ith node is real} \\ 0 & \text{the ith node is fictitious} \end{cases}$$

C is determined by: $C(i,j) = 0$ for $i \neq j$, and $C(i,i) = 1$ if the ith queue is real, and 0 if it is fictitious. $\mathcal{N} \in \Re^n$ is the fictitious controller vector, $\mathcal{N}(i) = 0$ if the i th node is real, and $\mathcal{N}(i) = f_m^i - \overline{u}_i$ if the node is fictitious.

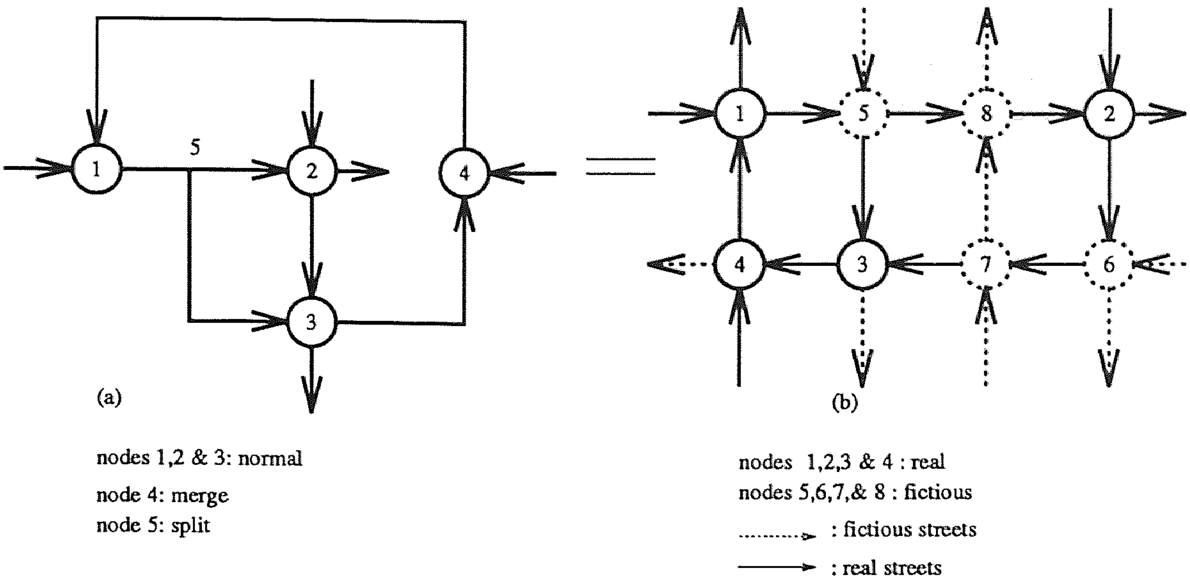

Figure 6.9 A Non-standard Network and Its Equivalent Structure

6.3 Traffic Control System Design

For modern area traffic control systems such as SCAT or SCOOT, the primary goal is the regulation of queue length. However, given that there are two queues (x and y) and only one traffic control, it is not possible to regulate the queue length individually. Instead, the difference of the queue length is regulated to a set of prespecified values according to optimal area traffic conditions. In the development to follow, it is assumed that the network is either saturated or oversaturated so that the queue dynamics are given by (6.6).

6.3.1 Nominal Control Objectives

Let the queue difference and queue difference error at the ith intersection to be

$$\Delta q^i = q_x^i - q_y^i \tag{6.10}$$

and define

$$\Delta q = \left[\Delta q^1 \; \Delta q^2 \; \cdots \; \Delta q^n\right]' \tag{6.11}$$

$$\Delta q_{ref} = \left[\Delta q_1^{ref} \; \Delta q_2^{ref} \; \cdots \; \Delta q_n^{ref}\right]' \tag{6.12}$$

The process error is given as:

$$e = \Delta q - \Delta q^{ref} \tag{6.13}$$

The model representing the queue difference is readily obtained from (6.4) as:

$$\Delta \dot{q} = B\Delta u + E \tag{6.14}$$

where $B = J\Phi$ and $E = JM$.

From (6.14) and (6.13), the nominal control objectives can be stated as:

1. For all constant Δq^{ref}, $e(t) \to 0$ as $t \to \infty$.

2. The closed loop system is asymptotically stable.

6.3.2 Controller Structure

For this work, a robust PI controller is selected for fault-tolerant traffic control:

$$\Delta u_i = k_{pi} e_i + k_i \eta \quad i = 1, \cdots, n \tag{6.15}$$

$$\dot{\eta}_i = e_i \quad i = 1, \cdots, n \tag{6.16}$$

where k_{pi} and k_i correspond to the proportional gain and integral gain of the ith controller, respectively.

This controller has the advantages that 1) it readily admits a decentralized information distribution structure, 2) it has low dynamic order and hence is efficient to implement, and 3) its frequency response characteristics can be directly shaped to match a given set of performance and robustness specifications.

6.3.3 Traffic Faults

The types of system fault that may arise in such traffic networks include:

1. ith sensor fault: $\Delta q^i = 0$, $i = 1, 2, \ldots, n$.

2. ith actuator fault: $\Delta u^i = 0$, $i = 1, 2, \ldots, n$.

3. Flow blockage due to accidents and other road conditions: Such faults result in parametric and structural changes in the B, E and Φ, M matrices.

4. Grid-lock at the ith intersection is defined as $f_m^i \to 0$, i.e., the maximum flow rate of the intersection drops to zero.

5. Communication fault where signal transmission between the intersections is partially blocked. For example, a communication blockage from the jth sensor (Δq^j) to the ith intersection control (Δu^i) results in the ijth entries of the gain matrices K_P, K_I becoming zero.

6.3.4 Fault-tolerant Traffic Control

The performance objectives of a fault-tolerant traffic control system are:

1. Stabilization of the nominal traffic network so that Δq, the incremental queue difference, is bounded.

2. Regulation of queue length so that $e \to 0$ and $q < \infty$ for saturated intersections and $q \to 0$ for a non-saturated intersection.

3. Under partial system fault, the remaining subsystem is stable and the queues corresponding to the fault-free subsystem continue to be regulated. Furthermore, no readjustment of the controllers are required.

Condition 3 above implies that the traffic control system can tolerate local faults without relying on fault detection techniques that may further increase system complexity and sensitivity. Moreover, a fault-tolerant system does not become exponentially unstable so that during the period between the fault occurrence and fault recovery, the traffic network remains maximally functional.

6.3.5 Closed Loop System and Fault Tolerant Control Synthesis

Two closed loop dynamic models are generated by applying the controller (6.15) to the queue difference (6.11) and the queue length (6.4).

The equations describing the closed loop dynamics of the queue difference is given as:

$$\begin{bmatrix} \dot{\Delta q} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} BK_P & BK_I \\ I_n & 0 \end{bmatrix} \begin{bmatrix} \Delta q \\ \eta \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{6.17}$$

while the closed loop dynamics of the queue length are described by:

$$\begin{bmatrix} \dot{q} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} \Phi K_P J & \Phi K_I \\ J & 0 \end{bmatrix} \begin{bmatrix} q \\ \eta \end{bmatrix} + \begin{bmatrix} M \\ 0 \end{bmatrix} \tag{6.18}$$

It should be noted that the model (6.18) always contains n uncontrollable eigenvalues at the origin.

Let V diagonalize the closed loop matrix

$$\begin{bmatrix} \Phi K_P J & \Phi K_I \\ J & 0 \end{bmatrix}$$

so that

$$V^{-1} \begin{bmatrix} \Phi K_P J & \Phi K_I \\ J & 0 \end{bmatrix} V = D$$

where

$$D = \mathrm{diag}(\underbrace{0, 0, 0, \ldots, 0,}_{n} \lambda_1, \ldots, \lambda_{2n}) \tag{6.19}$$

The traffic control problem may be considered as an extension of the basic reliable control prolbem discussed in Section 6.1 since the dynamics of both Δq and q are considered. Therefore, in addition to the conditions in Theorem 6.1.1, it is necessary to further characterize the properties of (6.17) and (6.18). Moreover, for traffic networks, $B = J\Phi = J(I - \mathcal{T})J'$ is always strictly diagonally dominant and thereby satisfying the condition of Theorem 6.1.1. It remains to establish the boundedness of Δq and q. This is given by Theorem 6.3.1 below:

Theorem 6.3.1 *There exists a solution to the fault tolerant traffic control problem if the following conditions all hold:*

1. *$Re(\lambda_i) < 0$, $i = 1, 2, \ldots, 2n$ where λ_i's are the controllable eigenvalues given in (6.19).*

2. *The first n elements of the vector $V^{-1}M$ do not possess positive constants.*

Proof of Theorem 6.3.1:

Condition 1 is obvious. Boundedness of q is established on noting that $q \geq 0$ so that the non positivity of the first n constant elements of the vector $V^{-1}M$ implies the elements of q either stay constant or reach zero.

6.4 Simulation Results for Fault Tolerant Control

The 16-intersection traffic network developed in the previous chapter is now simulated for the following operating conditions:

- Normal operation
- Failure mode 1: sensor failure
- Failure mode 2: actuator failure
- Failure mode 3: flow blockage
- Failure mode 4: grid lock

The controller is given in (6.15), with $k_{pi} = 5$, $k_i = 5$, $i = 1, \cdots, 16$ for each controller. The simulation block diagram is given in Figure 6.10.

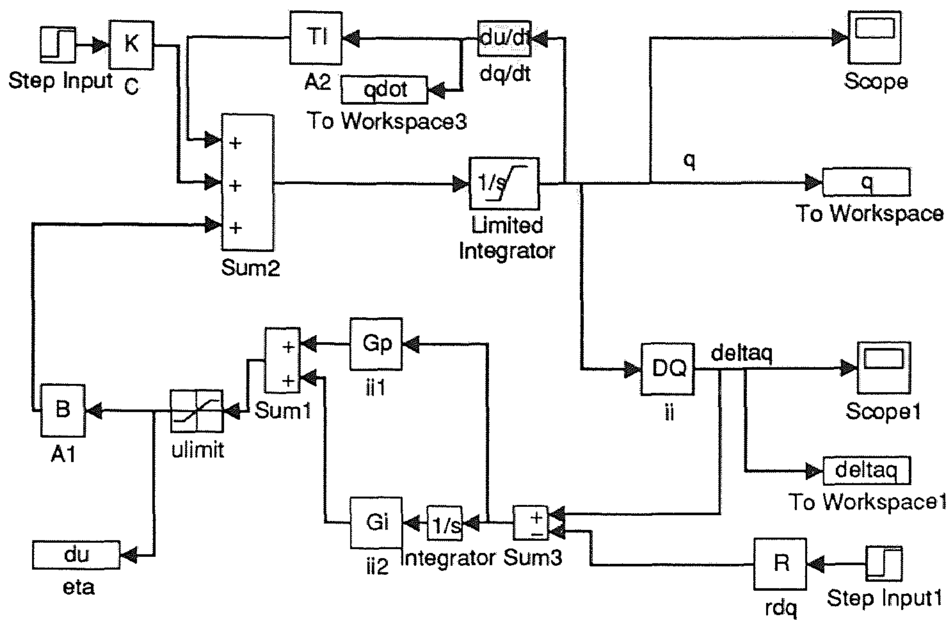

Figure 6.10 Simulation Diagram of 16-Node Traffic Network

6.4.1 Normal Operation

For normal operations, input flow rate at boundary nodes are assumed to be saturated, i.e.

$$f^i_{xin} + f^i_{yin} = f^i_m$$

the sensors and actuators function normally. where:

$$\begin{aligned} f^i_m &= 400 \ \text{cars/2min}, \ i = 1, \cdots, 16 \\ \theta_i, \beta_i &= 0.25, \ i = 1, \cdots, 16 \\ \overline{u} &= 200 \ \text{cars/2min}, \ i = 1, \cdots, 16 \end{aligned}$$

Two cases are simulated: $\Delta q^{ref} = 0$ and $\Delta q^{ref} = 10$. The results are plotted in Figures 6.11 and 6.12. It is observed that in both cases, the queue lengths are bounded and Δq_2, Δq_9 approach the reference levels asymptotically.

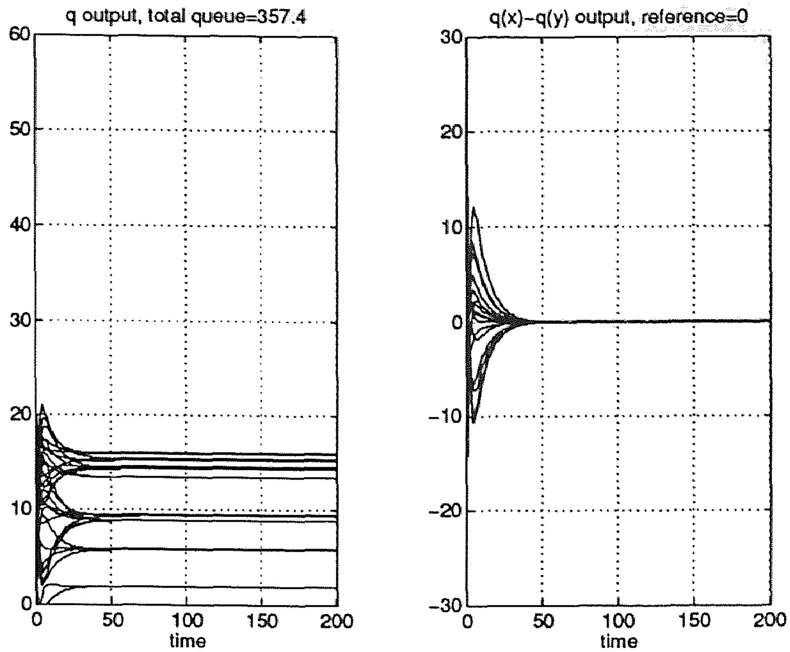

Figure 6.11 q and Δq Outputs When $\Delta q^{ref} = 0$

6.4.2 Sensor Failure

Assume now that $\Delta q^{ref} = 0$ and the queue length sensors at intersections 2 and 9 have failed so that $\Delta q^2 \equiv 0$ and $\Delta q^9 \equiv 0$. As shown in Figure 6.13, the fault tolerant control system continue to regulate all Δqs except Δq_2 and Δq_9 which settled to 3.06 and -11.8, respectively. It is further observed that, despite the sensor failure, all queue lengths are bounded.

6.4.3 Actuator Failure

Similar to the sensor failure mode, it is now assumed that the actuators at intersections 1, 3, and 7 has failed, resulting in $\Delta u_1 = 0$, $\Delta u_3 = 0$, and $\Delta u_7 = 0$. The simulation results with $\Delta q^{ref} = 0$ are shown in Figure 6.14. Again, all the queue in the traffic network are bounded and asymptotic regulation occurs for all Δqs except at $\Delta q^1, \Delta q^3$ and Δq^7 which approach -8.5, 8.1, and 1.4 respectively.

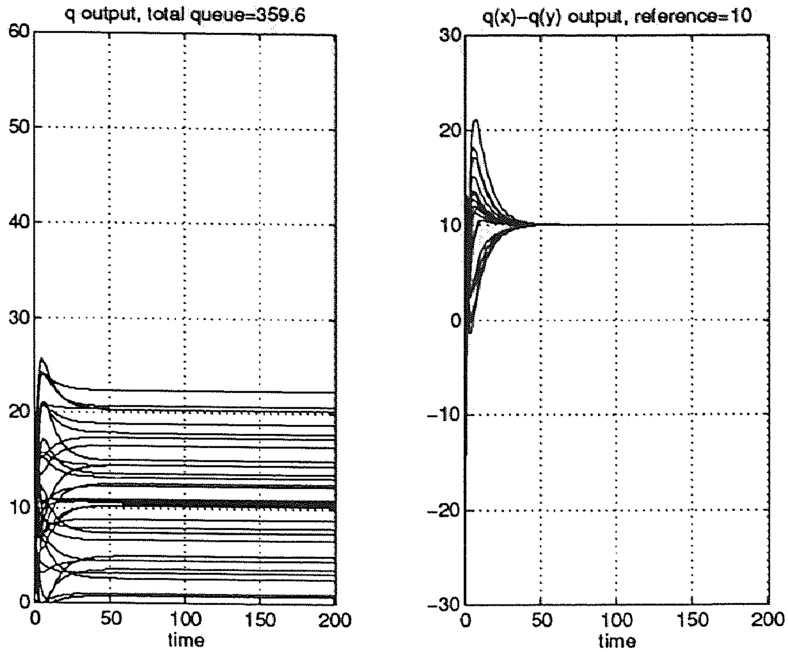

Figure 6.12 q and Δq Outputs When $\Delta q^{ref} = 10$

6.4.4 Operation with Flow Blockage

Assume now a blockage occurs between intersections 1 and 2, so that the vehicle exiting x direction of intersection 1 must turn right, and all the vehicle exiting y direction of intersection 1 must go straight, i.e., $\theta_1 = 1$, $\beta_1 = 0$. The simulation results are plotted in Figure 6.15.

It is observed that the queue in front of intersection 5 keeps increasing as the intersection is oversaturated. All other intersections, on the other hand, continue to function properly under fault tolerance control.

6.4.5 Operation with Gridlock

This is the case when an intersection is totally blocked so that its flow rate effectively drops to zero. For this simulation study, it is assumed that a gridlock occurs at intersection 6 and $f_m^6 = 0$. Due to the gridlock condition, the queues at intersection 6 grow linearly as shown in Figure 6.16. All other intersections continue to operate properly.

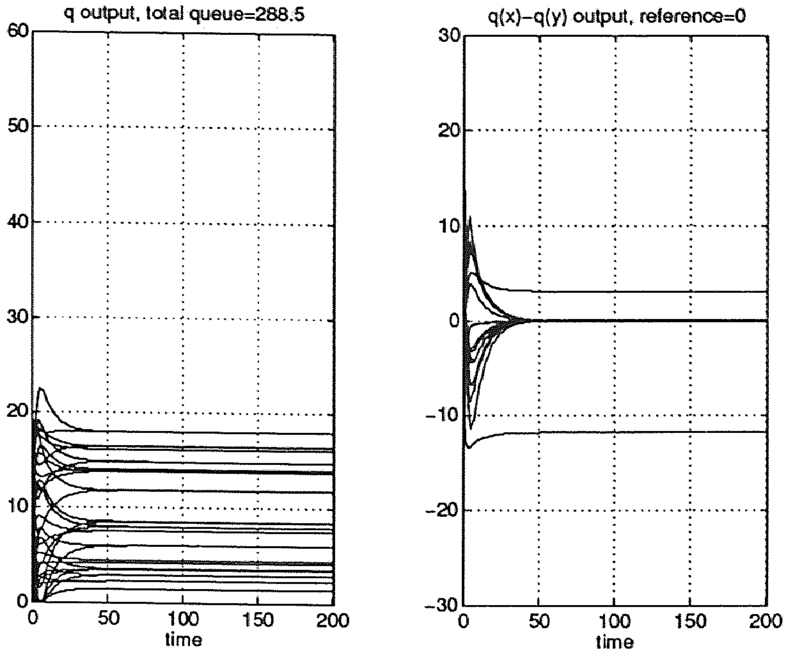

Figure 6.13 q and Δq Outputs with Sensors at Intersections 2 and 9 Failed

6.4.6 Effects of Δq^{ref} on Total Queue Length

Frequently, it is the primary objective of an area traffic network to minimize a weighted sum of the queue length over a period of time. For example, the sum of queue length $\sum q$ may be used to measure the efficiency of the network. In this simulation study, it is shown that the reference queue difference (Δq^{ref}) plays an important role in influencing the value of $\sum q$.

A family of $\sum q$ under different Δq_{ref} are computed and the results are plotted in Figure(6.17)

It is observed that, to minimize $\sum q$, the reference queue difference should be around zero which is perhaps not surprising, given the high degree of symmetry of this example network.

However, for a more general network topology, the choice of Δq^{ref} will have a significant impact on the overall vehicle density or equivalently, $\sum q$. The adjustment of Δq^{ref} as a daily schedule has been utilized in SCOOT but such adjustment is based strictly on past traffic data only.

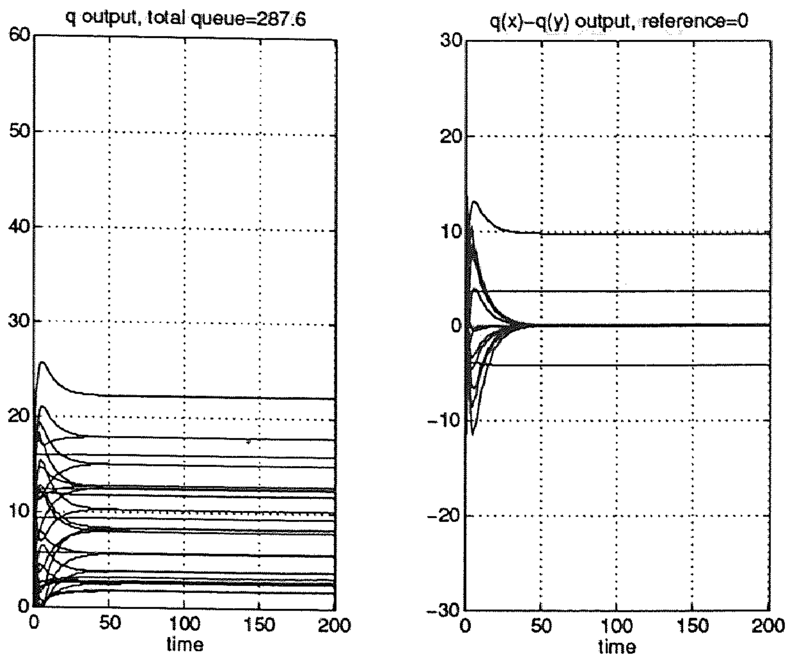

Figure 6.14 q and Δq Outputs with Actuators in Intersections 1,3,7 Failed

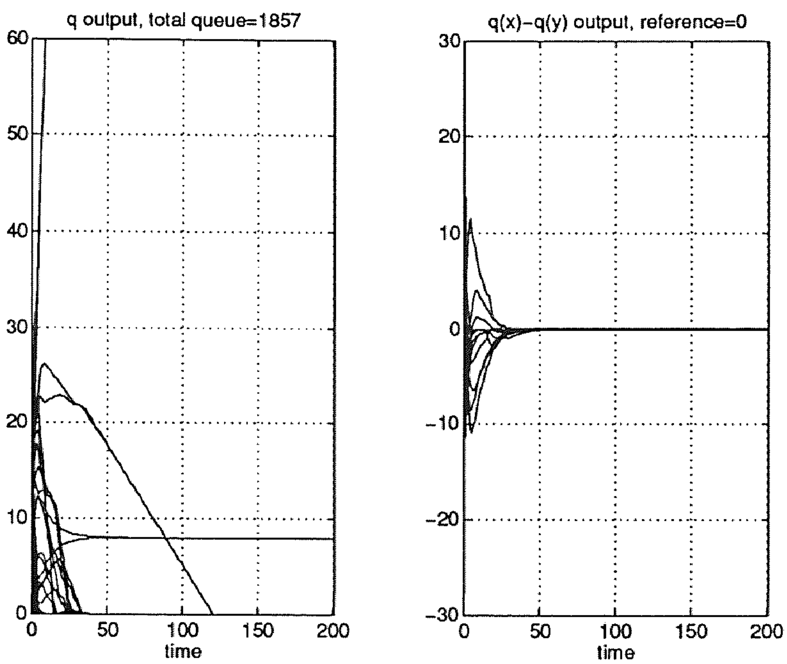

Figure 6.15 q and Δq with Flow Blockage Between intersection 1 and 2

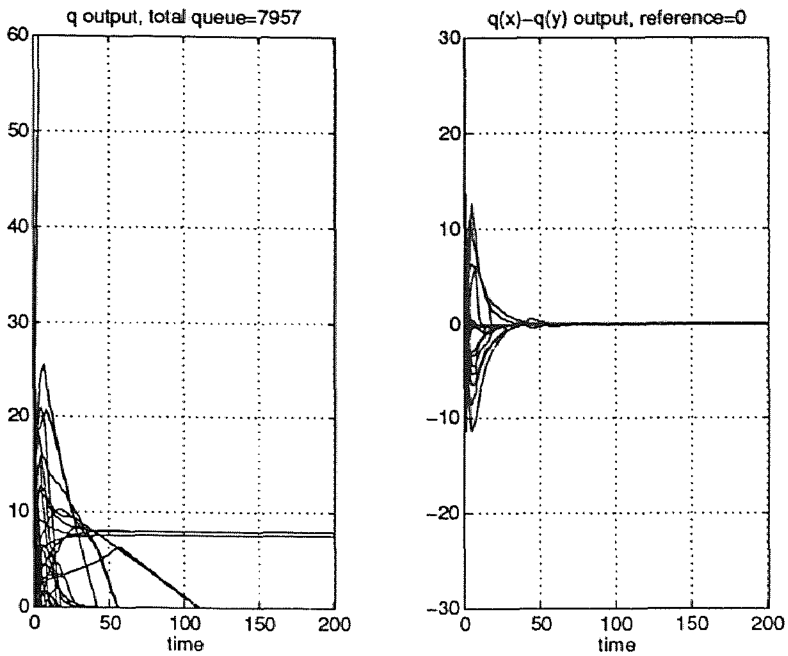

Figure 6.16 q and Δq Values with Gridlock Occurs at Intersection 6

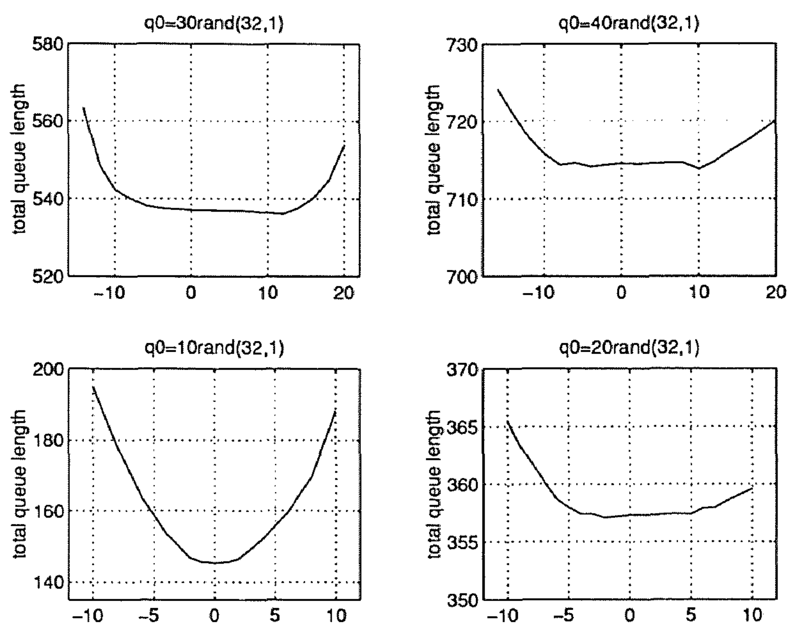

Figure 6.17 $\sum q$ Values Under Different Δq Reference Values

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH ISSUES

In the previous chapters, the research of decentralized reliable control on large-scale, multi-input/output linear plant without mathematical model available is developed. A $DRSPwCR$ is solved for the class of plants that are open loop stable, unreliable, and minimum phase by applying the strict decentralized PID^r algorithm. A $BDRSPwCR$ is solved for the plants with non-minimum phase minors by applying block diagonal decentralized controllers. A general controller synthesis is provided for an arbitrary linear plant. The application of urban vehicle traffic network fault tolerant control is also developed in this work. The following conclusions are obtained:

- Reliable control can be achieved for a class of open loop linear system with the steady-state interaction indices satisfying certain conditions by using decentralized integral controller configurations.

- By applying the decentralized proportional feedback controllers, the steady-state interaction indices can be adjusted to satisfy the reliable control conditions but will cause the closed loop system unstable under certain input-output channels failure.

- The introduction of the decentralized D^r controller re-stabilize the closed loop system without affecting the steady-state interaction indices and therefore the synthesis of decentralized PID^r controllers solve the $DRSPwCR$ for a certain class of unreliable systems with minimum phase minor characteristics.

- Certain class of unreliable systems can achieve reliable control by the input-output permutation strategy.

- The block diagonal decentralized control configuration is an alternative way to solve the reliable control problem for the systems which have non-minimum phase minors and cannot achieve reliable control by strict diagonal decentralized control.
- Most of the arbitrary open loop stable system can achieve reliable control by using $DRSPwCR$, $BDRSPwCR$, or permutation strategy.
- The $DRSPwCR$ solves fault tolerant control problem for urban vehicle traffic networking system under multiple failure modes.

Future research will focus on the following issues:

- To solve reliable control problem for open-loop unstable systems.
- To find the necessary and sufficient conditions of achieving reliable control for an arbitrary linear multi-input/output system by using the decentralized controller configurations.
- To synthesize the controller configuration without D^r term, which tends to introduce noise with the high order derivative algorithm.
- To optimize the controller parameters to improve the system outputs dynamic performances.

APPENDIX A

STANDARD NETWORK QUEUE MODEL DERIVATION

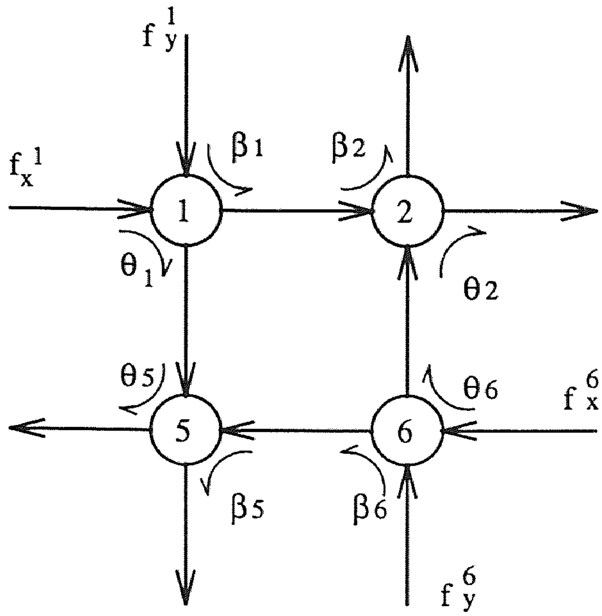

Figure A.1 Standard 4-Node Traffic Network

Figure(A.1) of 4-node network is a part of any standard square structure of traffic network, from the figure we can derive the relationship between f_{in} and f_{out}:

$$f_{xin}^2 = \beta_1 f_{yout}^1 + (1-\theta_1) f_{xout}^1 \qquad (A.1)$$

$$f_{yin}^2 = \theta_6 f_{xout}^6 + (1-\beta_6) f_{yout}^6 \qquad (A.2)$$

It is noted that all f_{xin}^i, f_{xout}^i and f_{yin}^i, f_{yout}^i are functions of time t.

Define:

$$\dot{q} = F_{in} - F_{out}$$

or

$$F_{out} = F_{in} - \dot{q} \qquad (A.3)$$

In fact, each f_{xin}^i, f_{yin}^i of the ith node can be expressed as a function of linear combination of its neighboring nodes' input flow rate and queue increasing rate.

Let

$$F_{in} = \begin{pmatrix} f^1_{xin} \\ f^1_{yin} \\ f^2_{xin} \\ f^2_{yin} \\ \vdots \end{pmatrix}, \quad F_{out} = \begin{pmatrix} f^1_{xout} \\ f^1_{yout} \\ f^2_{xout} \\ f^2_{yout} \\ \vdots \end{pmatrix}$$

See Subsection(6.2.1) for F_c structure. Then, the F_{in}, F_{out} relationship can be expressed as:

$$F_{in} = \mathcal{T} F_{out} + F_c \tag{A.4}$$

Substitute (A.4) into (A.3) , the F_{in} can be solved as :

$$F_{in} = \mathcal{T} F_{in} + F_c - \mathcal{T}\dot{q} \tag{A.5}$$

$\mathcal{T}$ matrix characterizes the flow configuration between two intersections in the network.

From equations (A.1) ,(A.2) and refer to Figure(A.1), the input flow into intersection (2) is described by:

$$f^2_{xin} = \mathcal{T}(3,1) f^1_{xout} + \mathcal{T}(3,2) f^1_{yout} \tag{A.6}$$

$$f^2_{yin} = \mathcal{T}(4,11) f^6_{xout} + \mathcal{T}(4,12) f^6_{yout} \tag{A.7}$$

where

$$\begin{array}{lll} \mathcal{T}(3,1) = 1-\theta_1 & & \mathcal{T}(3,2) = \beta_1 \\ \mathcal{T}(4,11) = \theta_6 & & \mathcal{T}(4,12) = 1-\beta_6 \end{array}$$

Let n_i denotes intersection i, $i = 1, 2, \cdots, n$, then the flow pattern and the corresponding $\mathcal{T}$ matrix elements are:

n_i, x direction $\rightarrow n_j, x$ direction : $\mathcal{T}(2j-1, 2i-1)$

n_i, y direction $\rightarrow n_j, x$ direction : $\mathcal{T}(2j-1, 2i)$

n_i, x direction $\rightarrow n_j, y$ direction : $\mathcal{T}(2j, 2i-1)$

n_i, y direction $\rightarrow n_j, y$ direction : $\mathcal{T}(2j, 2i)$

Table A.1 shows the $\mathcal{T}$ matrix structure:

Table A.1 Structure of $\mathcal{T}$ Matrix

	From	$1x$	$1y$	$\cdots$	nx	ny
To		1	2	$\cdots$	$2n-1$	$2n$
$1x$	1	$\mathcal{T}(1,1)$	$\mathcal{T}(1,2)$	$\cdots$	$\mathcal{T}(1,2n-1)$	$\mathcal{T}(1,2n)$
$1y$	2	$\mathcal{T}(2,1)$	$\mathcal{T}(2,2)$	$\cdots$	$\mathcal{T}(2,2n-1)$	$\mathcal{T}(2,2n)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
nx	$2n-1$	$\mathcal{T}(2n-1,1)$	$\mathcal{T}(2n-1,2)$	$\cdots$	$\mathcal{T}(2n-1,2n-1)$	$\mathcal{T}(2n-1,2n)$
ny	$2n$	$\mathcal{T}(2n,1)$	$\mathcal{T}(2n,2)$	$\cdots$	$\mathcal{T}(2n,2n-1)$	$\mathcal{T}(2n,2n)$

The elements of T may be determined by enumerating intersection 1 over intersections $1, 2, \cdots, n$. The elements of T are otherwise set to zeros.

From Equation (A.5) :

$$(I - \mathcal{T})F_{in} = F_c - \mathcal{T}\dot{q} \tag{A.8}$$

Define η as the maximum rate of change of queue length, i.e.

$$\eta = F_{in} - Gm \tag{A.9}$$

or

$$F_{in} = \eta + Gm \tag{A.10}$$

where

$$\eta = \begin{pmatrix} \eta_x^1 \\ \eta_y^1 \\ \eta_x^2 \\ \eta_y^2 \\ \vdots \end{pmatrix} \quad , \quad Gm = \begin{pmatrix} g_1 f_m^1 \\ (1-g_1) f_m^1 \\ g_2 f_m^2 \\ (1-g_2) f_m^2 \\ \vdots \end{pmatrix}$$

substitute (A.10) into (A.8) to remove F_{in}:

$$(I - \mathcal{T})(\eta + Gm) = F_c - \mathcal{T}\dot{q}$$

or

$$\mathcal{T}\dot{q} = F_c - (I - \mathcal{T})\eta - (I - \mathcal{T})Gm \tag{A.11}$$

Since

$$Gm = \begin{pmatrix} g_1 f_m^1 \\ (1-g_1)f_m^1 \\ g_2 f_m^2 \\ (1-g_2)f_m^2 \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ f_m^1 \\ 0 \\ f_m^2 \\ \vdots \end{pmatrix} + \begin{pmatrix} g_1 f_m^1 \\ -g_1 f_m^1 \\ g_2 f_m^2 \\ -g_2 f_m^2 \\ \vdots \end{pmatrix}$$

Define $F_m = [0, f_m^1, 0, f_m^2, \cdots]'$, i.e. $F_m(2i-1) = 0,\ F_m(2i) = f_m^i$.

and $g_1 = \overline{g}_i + \Delta g_i$,

also $g_i f_m^i = u_i$

then,

$$u_i = (\overline{g}_i + \Delta g_i) f_m^i = \overline{g}_i f_m^i + \Delta g_i f_m^i = \overline{u}_i + \Delta u_i$$

so,

$$Gm = F_m + [\overline{u}_1 + \Delta u1, -\overline{u}_1 - \Delta u_1, \overline{u}_2 + \Delta u_2, -\overline{u}_2 - \Delta u_2, \cdots]'$$

or,

$$Gm = F_m + J\overline{U} - J\Delta U \tag{A.12}$$

where

$$J = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}'_{2n \times n}$$

n is the total number of intersections of the network, and

$$\overline{u} = [\overline{u}_1, \overline{u}_2, \cdots, \overline{u}_i, \cdots]'$$

$$\Delta u = [\Delta u_1, \Delta u_2, \cdots, \Delta u_i, \cdots]'$$

Since η is the net queue increase rate, and q is a non-negative value, so $q =$ limit integrate(η), i.e.

$$q(t + \Delta t) = \begin{cases} q(t) & q(t) = 0 \text{ and } \eta \leq 0 \\ \int_t^{t+\Delta t} \eta & \text{otherwise} \end{cases}$$

let

$$f(q_i) = \begin{cases} 0 & q_i > \varepsilon \\ -K & q_i \leq \varepsilon \end{cases}$$

where ε is a small value(10^{-17} in simulation),

K is a large positive number(10^3 in simulation).

then,

$$q(s) = \frac{1}{s}[\eta - f(q)]$$

$$\dot{q} = \eta - f(q)$$

or

$$\eta = \dot{q} + f(q) \tag{A.13}$$

substitute (A.13), (A.12) into (A.11):

$$\mathcal{T}\dot{q} = F_c - (I - \mathcal{T})(\dot{q} + f(q)) - (I - \mathcal{T})(F_m + J\overline{u} - J\Delta u)$$

or

$$\dot{q} = (I - \mathcal{T})J\Delta u + (F_c - (I - \mathcal{T})(F_m + J\overline{u})) - (I - \mathcal{T})f(q)$$

let

$$M = F_c - (I - \mathcal{T})(F_m + J\overline{u})$$

then

$$\dot{q} = (I - \mathcal{T})J\Delta u + M - (I - \mathcal{T})f(q) \tag{A.14}$$

which is the general queue length model of standard structure traffic network.

The dimension for each matrix in the above equation is the following:

q : $2n \times 1$ vector

$\mathcal{T}$: $2n \times 2n$ square matrix

J : $2n \times n$ matrix

Δu : $n \times 1$ vector

M : $2n \times 1$ vector

The key in the development of the queue length model of the standard traffic network structure is the introduction of the non-linear function $f(q)$, which keeps the queue length to be non-negative. The term $\dot{(q_i)}$ and the term η_i are exactly the same as the queue length in front of the ith intersection are non-zero, the only difference between these two terms is: the former can still be negative when q_i is zero, reflecting the queue length changing direction (postive means the increasing direction, and negative means the decreasing direction), while the later is always zero as long as $q_i = 0$.

APPENDIX B

NON-STANDARD STRUCTURE MODEL DERIVATION

This appendix is the derivation of the general non-standard structure network queue length model by converting the structure into standard equivalent. Figure B.1 is a typical non-standard structure example.

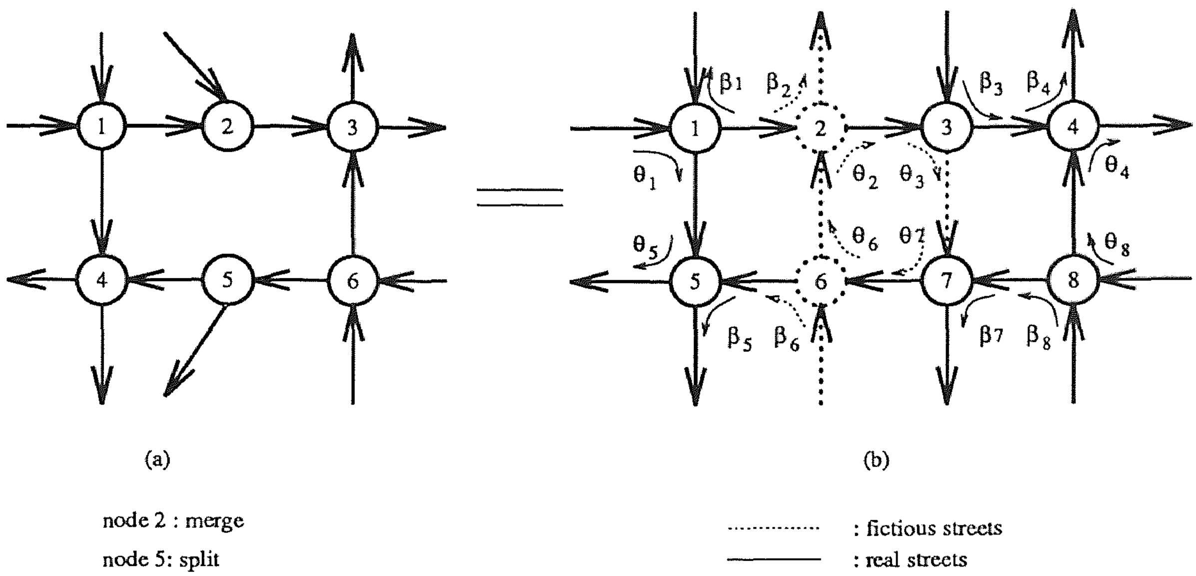

Figure B.1 Example of General Non-standard Structure

nodes 1, 3, 4and 5 : normal nodes with 2 – input/output

node 2 : merge node

node 5 : split node

Figure B.1b is the equivalent standard structure with nodes 2, 3, 6 and 7 be the fictitious nodes and the dotted lines the fictitious streets.

By adding these fictitious nodes on the non-standard traffic network, the network has been converted into a standard structure and can be analyzed by the standard queue length model as discussed in Appenidx A.

The fictitious nodes have the following special characteristics:

node 2 : $\theta_2 = 0, \beta_2 = 0, g_2 = 1$ always green at x direction
$\Delta u_2 = f_m^2 - \overline{u}_2$

node 6 : $\theta_6 = 0, \beta_6 = 0, g_6 = 1$ always green at x direction
$\Delta u_6 = f_m^6 - \overline{u}_6$ $f_y^6 = 0$

node 3 : $\theta_3 = 0, \beta_3 = 1$

node 7 : $\theta_7 = 0$

From the standard model:

$$\dot{q} = (I - \mathcal{T})J\Delta u_f + M - (I - \mathcal{T})f(q) \tag{B.1}$$

Here, $\Delta u_f = [\Delta u_1, \Delta u_2, \cdots, \Delta u_8]'$, a vector consists of both "real" and "fictitious" controllers.

Now,

$$\begin{pmatrix} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \\ \Delta u_4 \\ \Delta u_5 \\ \Delta u_6 \\ \Delta u_7 \\ \Delta u_8 \end{pmatrix} = \begin{pmatrix} \Delta u_1 \\ 0 \\ 0 \\ \Delta u_4 \\ \Delta u_5 \\ 0 \\ 0 \\ \Delta u_8 \end{pmatrix} + \begin{pmatrix} 0 \\ f_m^2 - \overline{u}_2 \\ f_m^3 - \overline{u}_3 \\ 0 \\ 0 \\ f_m^6 - \overline{u}_6 \\ f_m^7 - \overline{u}_7 \\ 0 \end{pmatrix}$$

or,

$$\Delta u_f = L\Delta u + \mathcal{N} \tag{B.2}$$

where L is determined by:$L(i,j) = 0 \ \ (i \neq j)$ and

$$L(i,i) = \begin{cases} 1 & \text{the } i\text{th node is real} \\ 0 & t\text{th } i\text{th node is fictious} \end{cases}$$

and

$$\Delta u = [\Delta u_1, \Delta u_4, \Delta u_5, \Delta u_8]'$$

which is the controller vector of real nodes, and $\mathcal{N}$ is a constant vector consists of 0s, f_m^i s and $\overline{u}_i$s, $i \in [1, \cdots, n]$ denotes a real node number.

Since q vector includes the fictitious queue lengths, only interested in those come from the real ones are to be measured, so the output vector can be expressed as:

$$Y = Cq$$

similar to L, C is determined by: $C(i,j) = 0$ for $i \neq j$, and $C(i,i) = 1$ if the ith queue is real, and 0 if it's fictitious. Substitute (B.2) into (B.1),the general model becomes:

$$\dot{q} = (I - \mathcal{T})JL\Delta u + (I - \mathcal{T})J\mathcal{N} + M - (I - \mathcal{T})f(q) \quad \text{(B.3)}$$

$$Y = Cq \quad \text{(B.4)}$$

Let a non-standard structure traffic network is converted into an a-node standard structure, where there exist m fictitious nodes, the matrices dimensions in Equation (B.3) are:

q	:	$2n \times 1$ vector
$\mathcal{T}$	:	$2n \times 2n$ square matrix
J	:	$2n \times n$ matrix
L	:	$n \times (n-m)$ matrix
Δu	:	$(n-m) \times 1$ vector
$\mathcal{N}$	:	$n \times 1$ vector
M	:	$2n \times 1$ vector
$f(q)$	:	$2n \times 1$ vector
C	:	$(2n - 2m)n$ matrix
Y	:	$(2n - 2m)$ vector

In fact, Equations (B.3) and (B.4) is a general expression for all the standard and non-standard topology. The standard queue model is a special case when $m = 0$.

It is noted that the traffic network model developed in this work matches the road map of the downtown Manhattan of New York City. It will be significant to apply the decentralized robust PI algorithm to the traffic lights control system to reduce the traffic problem in New York City which is becoming more and more critical in the our daily life.

APPENDIX C

K_P RANGE OF RELIABILITY DERIVATION

In this appendix, the range of K_P of reliability and stability is derived for the 2-Input/Output unreliable systems.

Given a system with the transfer matrix:

$$T(s) = \frac{1}{d(s)} \begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix} \tag{C.1}$$

where

$$d(s) = d_n s^n + d_{n-1} s^{n-1} + \cdots + d_1 s + d_0 \tag{C.2}$$

$$n_{ij}(s) = n^{ij}_{r_{ij}} s^{r_{ij}} + n^{ij}_{r_{ij}-1} s^{r_{ij}-1} + \cdots + n^{ij}_1 s + n^{ij}_0 \tag{C.3}$$

where $i, j \in [1, 2]$, and r_{ij} denotes the order of the polynomial $n_{ij}(s)$.

Define

$$\Delta n(s) = n_{11}(s) n_{22}(s) - n_{12}(s) n_{21}(s)$$

The system has the following characteristics:

- Open loop stable, i.e., $d(s)$ possesses no unstable roots. Assuming:

$$d_i > 0 \quad i \in [1, n]$$

- It is unreliable, ie.,

$$det(\overline{T}) < 0 \tag{C.4}$$

where

$$T = \frac{1}{d(0)} \begin{bmatrix} n_{11}(0) & n_{12}(0) \\ n_{21}(0) & n_{22}(0) \end{bmatrix} \tag{C.5}$$

is the DC gain matrix of the open loop system.

$$\overline{T} = T \times \begin{bmatrix} T(1,1) & 0 \\ 0 & T(2,2) \end{bmatrix}^{-1} = \begin{bmatrix} 1 & \frac{n_{12}(0)}{n_{22}(0)} \\ \frac{n_{21}(0)}{n_{11}(0)} & 1 \end{bmatrix}$$

Equation(C.4) implies that

$$1 - \frac{n_{12}(0)n_{21}(0)}{n_{11}(0)n_{22}(0)} < 0$$

or

$$\frac{\Delta n(0)}{n_{11}(0)n_{22}(0)} < 0 \tag{C.6}$$

Without loss of generality, assuming:

$$n_{11}(0) > 0, \quad n_{22}(0) > 0 \tag{C.7}$$

then

$$\Delta n(0) < 0 \tag{C.8}$$

C.1 K_P Range of Reliability

When decentralized proportional controllers are added to the system, the closed loop system DC gain matrix T_P becomes:

$$T_P^{-1} = T^{-1} - K_P \tag{C.9}$$

where

$$K_P = \begin{bmatrix} k_{p1} & 0 \\ 0 & k_{p2} \end{bmatrix} \tag{C.10}$$

substitute Equations(C.5) and (C.10) into Equation(C.9), the closed loop DC gain matrix is obtained as following:

$$T_P = \frac{1}{D_C(0)} \begin{bmatrix} n_{11}(0)d(0) - k_{p2}\Delta n(0) & n_{12}(0)d(0) \\ n_{21}(0)d(0) & n_{22}(0)d(0) - k_{p1}\Delta n(0) \end{bmatrix} \tag{C.11}$$

where $D_C(0)$ is the constant part of closed loop system characteristic polynomial:

$$D_C(0) = d(0)^2 - (n_{11}(0)k_{p1} + n_{22}(0)k_{p2})d(0) + k_{p1}k_{p2}\Delta n(0) \tag{C.12}$$

then $\overline{T}_P$ matrix is obtained as follows:

$$\overline{T}_P = T_P \times (\mathrm{diag}(T_P))^{-1} = \begin{bmatrix} 1 & \frac{n_{12}(0)d(0)}{n_{22}(0)d(0) - k_{p1}\Delta n(0)} \\ \frac{n_{21}(0)d(0)}{n_{11}(0) - k_{p2}\Delta n(0)} & 1 \end{bmatrix} \tag{C.13}$$

the determinant of T_P is obtained as following:

$$\det(\overline{T}_P) = \frac{\Delta n(0)(d(0)^2 - (n_{11}(0)k_{p1} + n_{22}(0)k_{p2})d(0) + k_{p1}k_{p2}\Delta n(0))}{(n_{11}(0)d(0) - k_{p2}\Delta n(0))(n_{22}(0)d(0) - k_{p1}\Delta n(0))} \tag{C.14}$$

rewrite the determinant of T_P as:

$$\det(\overline{T}_P) = \frac{\Delta n(0)D_C(0)}{(n_{11}(0)d(0) - k_{p2}\Delta n(0))(n_{22}(0)d(0) - k_{p1}\Delta n(0))} \tag{C.15}$$

where $D_C(0)$ is known from Equation(C.12). From Equation(C.11), the following equations are obtained:

$$T_P(1,1) = \frac{n_{11}(0)d(0) - k_{p2}\Delta n(0)}{D_C(0)} \tag{C.16}$$

$$T_P(2,2) = \frac{n_{22}(0)d(0) - k_{p1}\Delta n(0)}{D_C(0)} \tag{C.17}$$

Define also

T_1: DC gain matrix when k_{p1} only is added,

T_2: DC gain matrix when k_{p2} only is added, then, from Equation(C.11),

$$T_1(1,1) = \frac{n_{11}(0)}{d(0) - k_{p1}n_{11}(0)} \tag{C.18}$$

$$T_2(2,2) = \frac{n_{22}(0)}{d(0) - k_{p2}n_{22}(0)} \tag{C.19}$$

Define $D_1(0)$ as the constant part of characteristic polynomial when k_{p1} is added in the closed loop system, and $D_2(0)$ as the constant part of characteristic polynomial when k_{p2} is added in the closed loop system, then

$$D_1(0) = d(0) - k_{p1}n_{11}(0) \tag{C.20}$$

$$D_2(0) = d(0) - k_{p2}n_{22}(0) \tag{C.21}$$

then, equations(C.18) and (C.19) can be rewritten as:

$$T_1(1,1) = \frac{n_{11}(0)}{D_1(0)}, \quad T_2(2,2) = \frac{n_{22}(0)}{D_2(0)} \tag{C.22}$$

From the reliable control conditions, K_P values have to be such that the equations all hold in 3 different controller installation sequences:

C.1.1 For Any Installation Sequence:

$$det(\overline{T}_P) > 0 \tag{C.23}$$

$$T_P(1,1)T_1(1,1) > 0 \tag{C.24}$$

$$T_P(2,2)T_2(2,2) > 0 \tag{C.25}$$

substitute Equations(C.15), (C.16), (C.17), (C.18), and (C.19) into the above equations,

$$\frac{n_{11}(0)(n_{11}(0)d(0) - \Delta n(0)k_{p2})}{D_C(0)D_1(0)} > 0 \tag{C.26}$$

$$\frac{n_{22}(0)(n_{22}(0)d(0) - \Delta n(0)k_{p1})}{D_C(0)D_2(0)} > 0 \tag{C.27}$$

$$\frac{\Delta n(0)D_C(0)}{(n_{11}(0)d(0) - \Delta n(0)k_{p2})(n_{22}(0)d(0) - \Delta n(0)k_{p1})} > 0 \tag{C.28}$$

The product of left hand sides of equations(C.26), (C.27) and (C.28) yields:

$$\frac{n_{11}(0)n_{22}(0)\Delta n(0)}{D_1(0)D_2(0)D_C(0)} > 0 \tag{C.29}$$

since the system is unreliable, from the given assumptions (C.7) and (C.8),

$$n_{11}(0)n_{22}(0)\Delta n(0) < 0$$

therefore, a necessary condition of reliable control for 2-input/output system is obtained as following:

$$D_1(0)D_2(0)D_C(0) < 0 \tag{C.30}$$

To satisfy the above inequity, there are 4 different cases which is discussed as following cases (I), (II), (III) and (IV):

<u>Case(I): $D_C(0) < 0, \;\; D_1(0) > 0, \;\; D_2(0) > 0$</u>

From(C.26),

$$n_{11}(0)d(0) - k_{p2}\Delta n(0) < 0$$

and this inequity yields:

$$k_{p2} < \frac{n_{11}(0)d(0)}{\Delta n(0)} \tag{C.31}$$

From(C.27),

$$n_{22}(0)d(0) - k_{p1}\Delta n(0) < 0$$

and this inequity yields:

$$k_{p1} < \frac{n_{22}(0)d(0)}{\Delta n(0)} \tag{C.32}$$

Also from $D_C(0) < 0$,

$$d(0)^2 - (n_{11}(0)k_{p1} + n_{22}(0)k_{p2})d(0) + k_{p1}k_{p2}\Delta n(0) < 0$$

$$\begin{aligned} left\ side &= \Delta n(0)(k_{p1}k_{p2} - k_{p1}\frac{n_{11}(0)d(0)}{\Delta n(0)} - k_{p2}\frac{n_{22}(0)d(0)}{\Delta n(0)} + \frac{d(0)^2}{\Delta n(0)}) \\ &= \Delta n(0)(k_{p1} - \frac{d(0)n_{22}(0)}{\Delta n(0)})(k_{p2} - \frac{d(0)n_{22}(0)}{\Delta n(0)}) \\ &\quad - \frac{d(0)^2 n_{21}(0)n_{12}(0)}{\Delta n(0)} \end{aligned} \tag{C.33}$$

the above expression should be < 0, therefore,

$$\Delta n(0)(k_{p1} - \frac{d(0)n_{22}(0)}{\Delta n(0)})(k_{p2} - \frac{d(0)n_{22}(0)}{\Delta n(0)}) < \frac{d(0)^2 n_{21}(0)n_{12}(0)}{\Delta n(0)}$$

by dividing $\Delta n(0)$ (which is a negative value) at both sides, the following inequity is obtained:

$$(k_{p1} - \frac{d(0)n_{22}(0)}{\Delta n(0)})(k_{p2} - \frac{d(0)n_{22}(0)}{\Delta n(0)}) > \frac{d(0)^2 n_{21}(0)n_{12}(0)}{\Delta n(0)^2} \tag{C.34}$$

Combine the K_P range from Equations(C.31), (C.32) and (C.34), one of the K_P range of reliability is obtained as plotted in Figure C.1.

Case(II): $D_C(0) > 0, \ \ D_1(0) < 0, \ \ D_2(0) > 0$

From(C.26),

$$n_{11}(0)d(0) - k_{p2}\Delta n(0) < 0$$

and this inequity yields:

$$k_{p2} < \frac{n_{11}(0)d(0)}{\Delta n(0)} \tag{C.35}$$

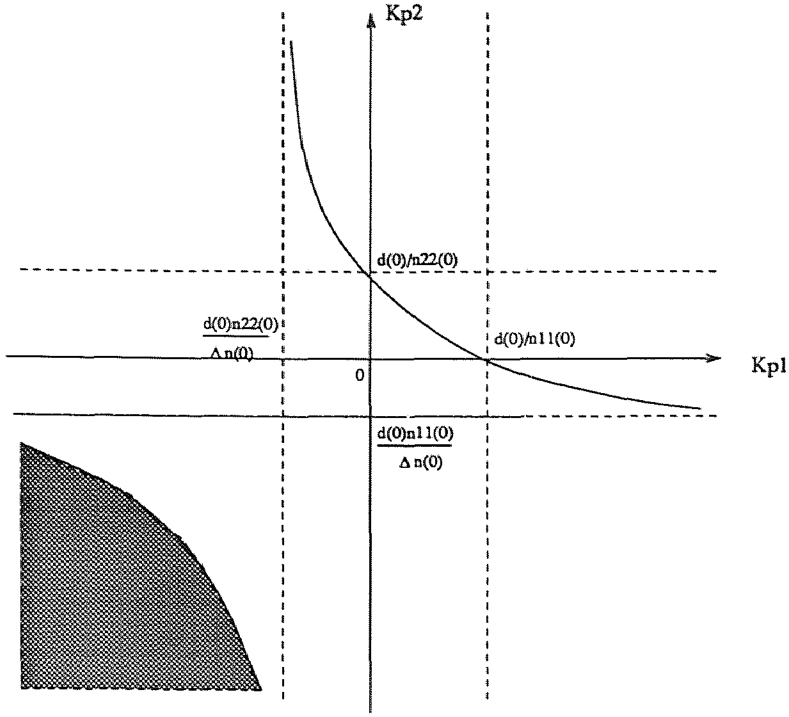

Figure C.1 K_P Range of Reliability in Quadrant III

From $D_1(0) < 0$,

$$d(0) - k_{p1} n_{11}(0) < 0$$

and this inequity yields:

$$k_{p1} > \frac{d(0)}{n_{11}(0)} \tag{C.36}$$

And the above K_P range is included in the range $D_C(0) > 0$, which is shown in Figure C.2.

<u>Case(III): $D_C(0) > 0, \;\; D_1(0) > 0, \;\; D_2(0) < 0$</u>

From(C.27),

$$n_{22}(0)d(0) - k_{p1}\Delta n(0) < 0$$

and this inequity yields:

$$k_{p1} < \frac{n_{22}(0)d(0)}{\Delta n(0)} \tag{C.37}$$

From $D_2(0) < 0$,

$$d(0) - k_{p2} n_{22}(0) < 0$$

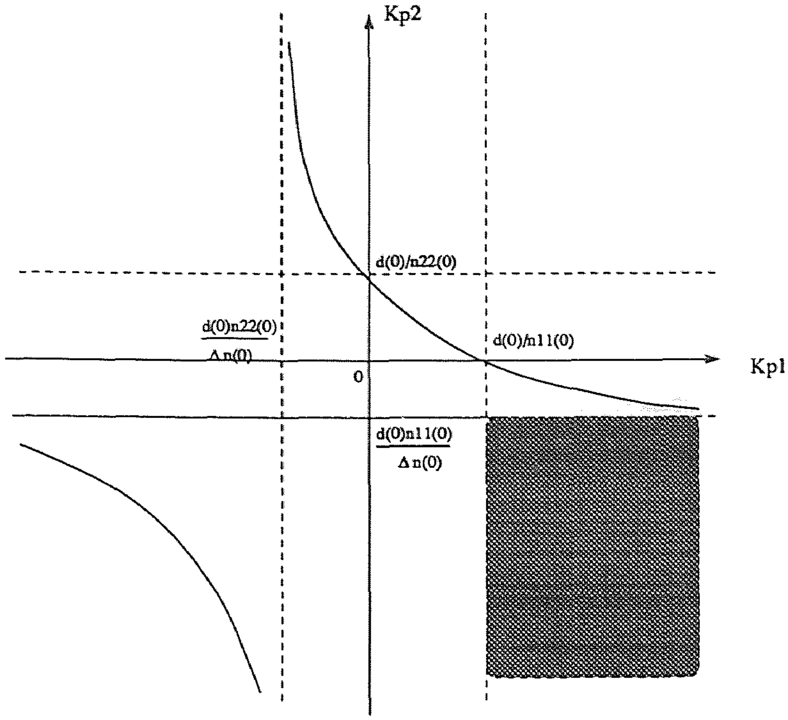

Figure C.2 K_P Range of Reliability in Quadrant IV

and this inequity yields:

$$k_{p2} > \frac{d(0)}{n_{22}(0)} \tag{C.38}$$

And the above K_P range is included in the range $D_C(0) > 0$, which is shown in Figure C.3.

Case(IV): $D_C(0) < 0, \;\; D_1(0) < 0, \;\; D_2(0) < 0$

From $D_1(0) < 0$,

$$d(0) - k_{p1}n_{11}(0) < 0$$

and this inequity yields:

$$k_{p1} > \frac{d(0)}{n_{11}(0)} \tag{C.39}$$

From $D_2(0) < 0$,

$$d(0) - k_{p2}n_{22}(0) < 0$$

and this inequity yields:

$$k_{p2} > \frac{d(0)}{n_{22}(0)} \tag{C.40}$$

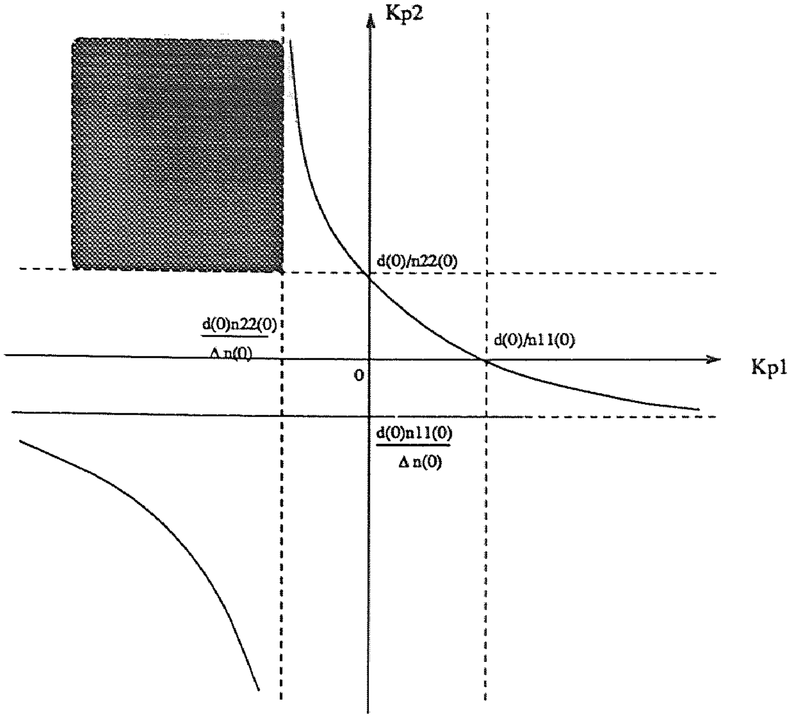

Figure C.3 K_P Range of Reliability in Quadrant II

And the above K_P range is included in the range $D_C(0) < 0$, which is shown in Figure C.4.

By combining the above four ranges of reliability, As shown in Figure C.5, this is the K_P range of reliability is completed at any controller installation sequence.

C.1.2 For Installation Sequence: Controller 2, 1

$$det(\overline{T}_P) < 0 \quad (C.41)$$

$$T_P(1,1)T_1(1,1) < 0 \quad (C.42)$$

$$T_P(2,2)T_2(2,2) > 0 \quad (C.43)$$

substitute Equations(C.15), (C.16), (C.17), (C.18), and (C.19) into the above equations,

$$\frac{n_{11}(0)(n_{11}(0)d(0) - \Delta n(0)k_{p2})}{D_C(0)D_1(0)} < 0 \quad (C.44)$$

$$\frac{n_{22}(0)(n_{22}(0)d(0) - \Delta n(0)k_{p1})}{D_C(0)D_2(0)} > 0 \quad (C.45)$$

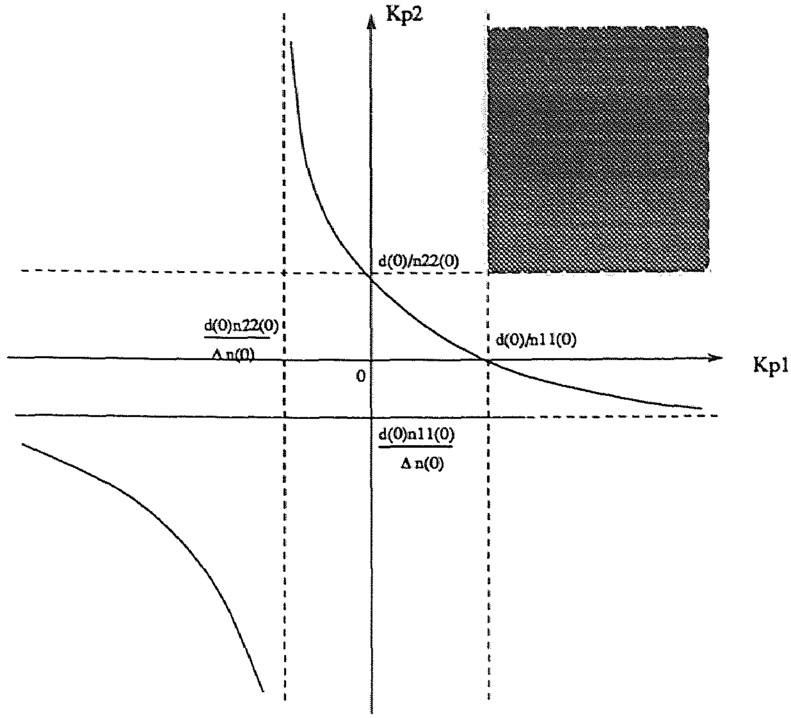

Figure C.4 K_P Range of Reliability in Quadrant I

$$\frac{\Delta n(0)D_C(0)}{(n_{11}(0)d(0)-\Delta n(0)k_{p2})(n_{22}(0)d(0)-\Delta n(0)k_{p1})} < 0 \tag{C.46}$$

The product of left hand sides of equations(C.44), (C.45) and (C.46) still yields the necessary condition as Equation(C.30):

$$D_1(0)D_2(0)D_C(0) < 0$$

To satisfy the above inequities, there is only one solution under the case

$$D_C(0) > 0, \quad D_1(0) < 0, \quad D_2(0) > 0$$

From $D_C(0) > 0$,

$$(k_{p1} - \frac{d(0)n_{22}(0)}{\Delta n(0)})(k_{p2} - \frac{d(0)n_{22}(0)}{\Delta n(0)}) < \frac{d(0)^2 n_{21}(0)n_{12}(0)}{\Delta n(0)^2} \tag{C.47}$$

From $D_1(0) < 0$, the Equation (C.39) shows:

$$k_{p1} > \frac{d(0)}{n_{11}(0)}$$

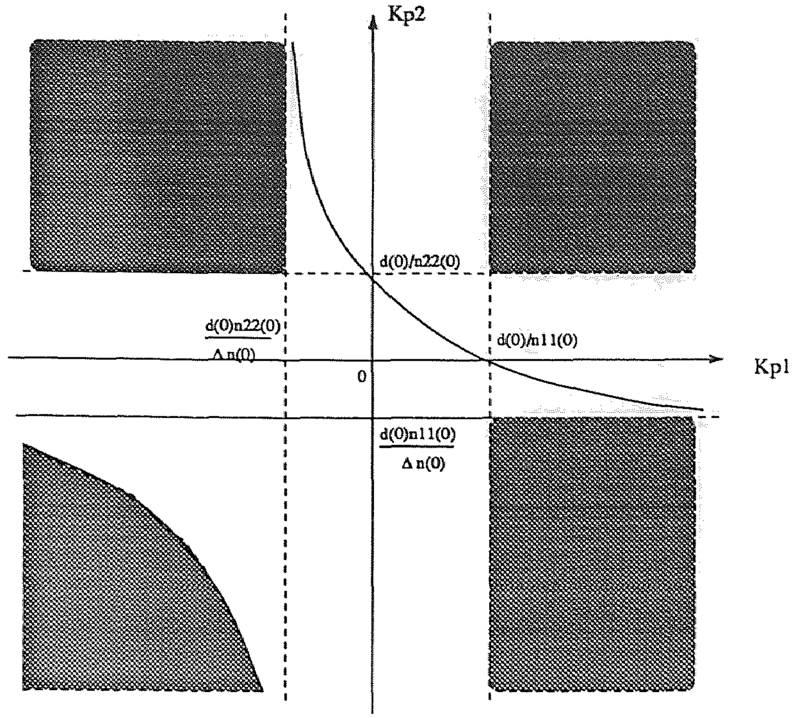

Figure C.5 K_P Range of Reliability at Any installation sequence

From $D_2(0) > 0$, the Equation (C.35) shows:

$$k_{p2} < \frac{n_{11}(0)d(0)}{\Delta n(0)}$$

Figure C.6 shows the range of reliability under this condition:

C.1.3 For Installation Sequence: Controller 1, 2

$$det(\overline{T}_P) < 0 \tag{C.48}$$

$$T_P(1,1)T_1(1,1) > 0 \tag{C.49}$$

$$T_P(2,2)T_2(2,2) < 0 \tag{C.50}$$

substitute Equations(C.15), (C.16), (C.17), (C.18), and (C.19) into the above equations,

$$\frac{n_{11}(0)(n_{11}(0)d(0) - \Delta n(0)k_{p2})}{D_C(0)D_1(0)} < 0 \tag{C.51}$$

$$\frac{n_{22}(0)(n_{22}(0)d(0) - \Delta n(0)k_{p1})}{D_C(0)D_2(0)} < 0 \tag{C.52}$$

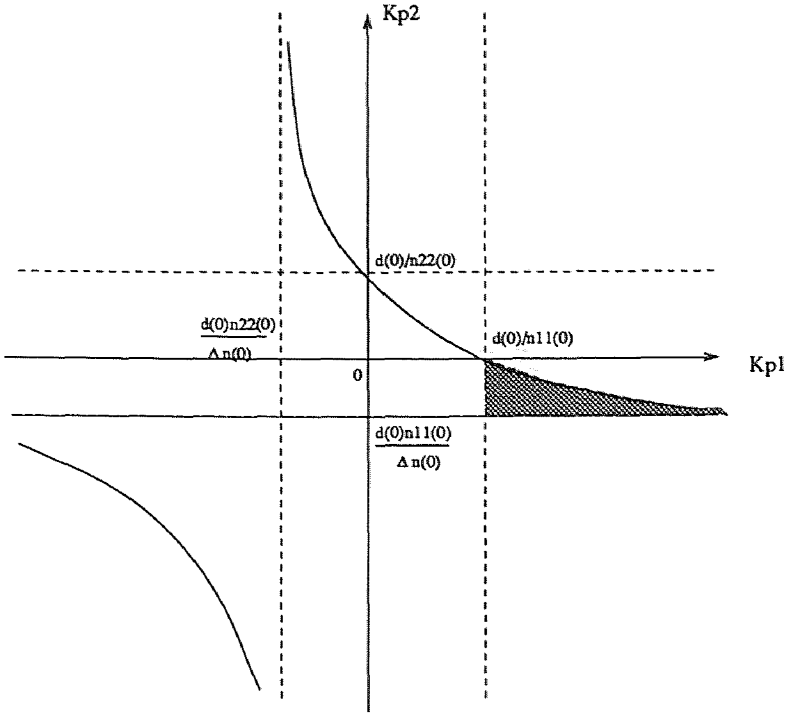

Figure C.6 K_P Range of Reliability at Installation Sequence 2, 1

$$\frac{\Delta n(0) D_C(0)}{(n_{11}(0)d(0) - \Delta n(0)k_{p2})(n_{22}(0)d(0) - \Delta n(0)k_{p1})} \;>\; 0 \tag{C.53}$$

The product of left hand sides of Equations(C.51), (C.52) and (C.53) still yields the necessary condition as Equation(C.30):

$$D_1(0)D_2(0)D_C(0) < 0$$

To satisfy the above inequities, there is only one solution under the case

$$D_C(0) > 0, \;\; D_1(0) < 0, \;\; D_2(0) > 0$$

From $D_C(0) > 0$,

$$(k_{p1} - \frac{d(0)n_{22}(0)}{\Delta n(0)})(k_{p2} - \frac{d(0)n_{22}(0)}{\Delta n(0)}) < \frac{d(0)^2 n_{21}(0)n_{12}(0)}{\Delta n(0)^2} \tag{C.54}$$

From $D_1(0) > 0$, the Equation (C.37) shows:

$$k_{p1} < \frac{d(0)}{n_{11}(0)}$$

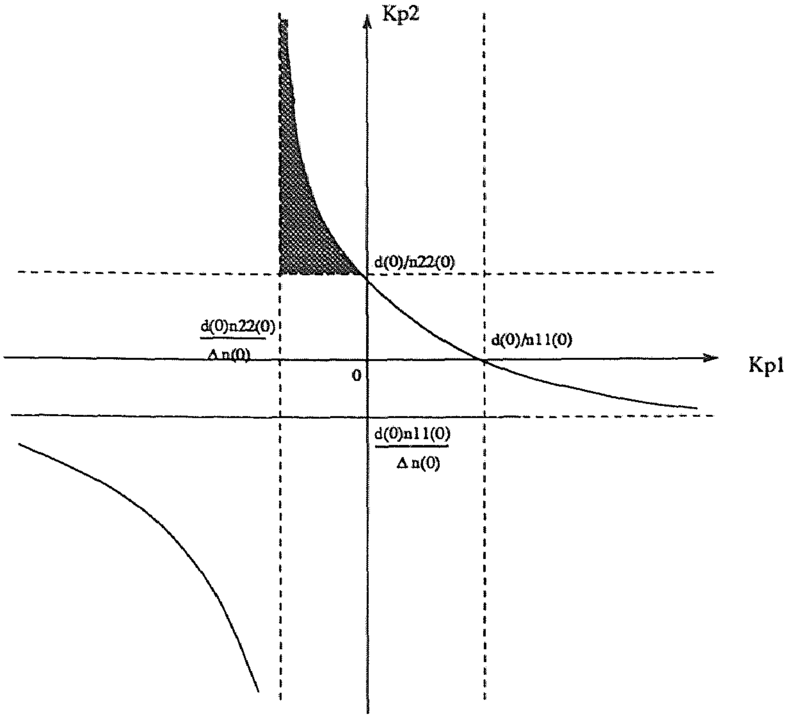

Figure C.7 K_P Range of Reliability at Installation Sequence 1, 2

From $D_2(0) < 0$, the Equation (C.40) shows:

$$k_{p2} > \frac{n_{11}(0)d(0)}{\Delta n(0)}$$

Figure C.7 shows the range of reliability under this condition.

Figure C.8 combines all the above K_P range of reliability.

C.2 K_P Range of Stability Derivation

For the given plant with transfer matrix (C.1), when decentralized feedback proportional controllers(C.10) are add, the K_p range of stability can be obtained by plotting the characteristic loci of the matrix $K_PT(s)$, where,

$$K_PT(s) = -k_{p1} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon \end{bmatrix} \begin{bmatrix} \frac{n_{11}(s)}{d(s)} & \frac{n_{12}(s)}{d(s)} \\ \frac{n_{21}(s)}{d(s)} & \frac{n_{22}(s)}{d(s)} \end{bmatrix} \tag{C.55}$$

where

$$\epsilon = \frac{k_{p1}}{k_{p2}}$$

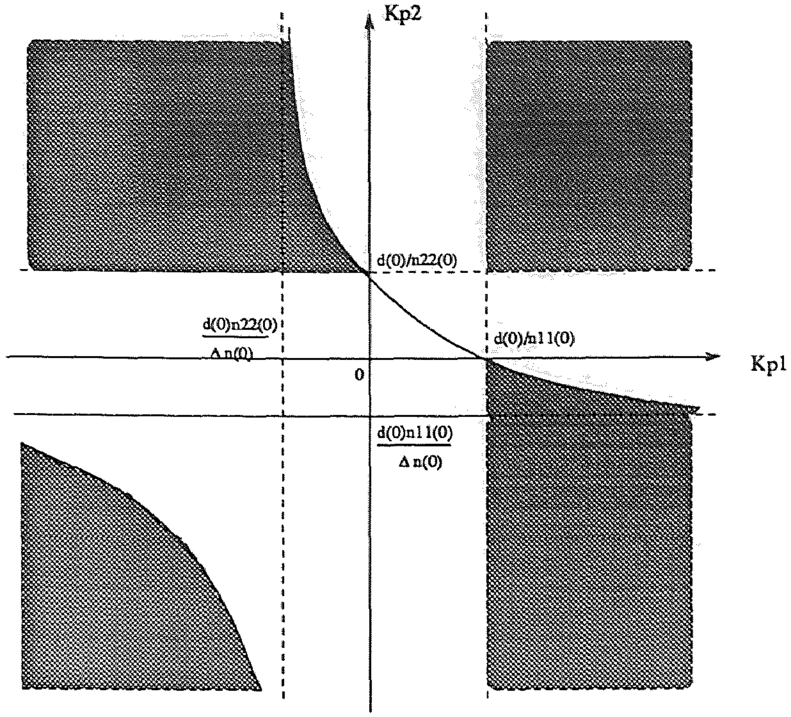

Figure C.8 K_P Range of Reliability

Case 1: both controllers are operational

In this case, the characteristic roots values are obtained as follows:

$$\lambda(j\omega) = \frac{(n_{11}(j\omega) + \epsilon n_{22}(j\omega)) \pm \sqrt{(n_{11}(j\omega) + \epsilon n_{22}(j\omega))^2 - 4\epsilon\Delta n(j\omega)}}{2d(j\omega)} \tag{C.56}$$

When $s = j\omega = j0$, DC values of characteristic roots are:

$$\lambda_1(j0) = \frac{(n_{11}(j0) + \epsilon n_{22}(j0)) \pm \sqrt{(n_{11}(j0) + \epsilon n_{22}(j0))^2 - 4\epsilon\Delta n(j0)}}{2d(j0)} > 0 \tag{C.57}$$

$$\lambda_2(j0) = \frac{(n_{11}(j0) + \epsilon n_{22}(j0)) \pm \sqrt{(n_{11}(j0) + \epsilon n_{22}(j0))^2 - 4\epsilon\Delta n(j0)}}{2d(j0)} < 0 \tag{C.58}$$

Figure C.9 shows the characteristic loci with $\omega \in [-\infty, +\infty]$

Since the open loop plant is stable, the closed loop stability range is:

$$\lambda_1(j0) < -\frac{1}{k_{p1}} < +\infty \tag{C.59}$$

$$-\infty < -\frac{1}{k_{p1}} < \lambda_2(j0) \tag{C.60}$$

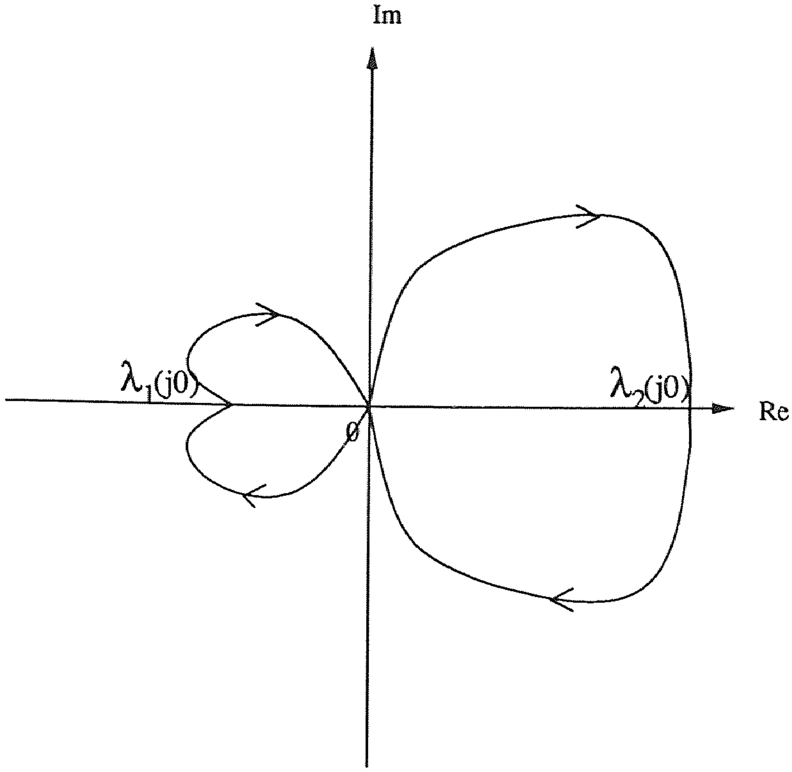

Figure C.9 Characteristic Loci with Both Controllers Installed

From(C.59),

$$k_{p1} > -\frac{1}{\lambda_1(j0)}$$

substitute (C.57) into the above equation:

$$k_{p1} > -\frac{2d(0)}{(n_{11}(0) + \frac{k_{p2}}{k_{p1}} n_{22}(0)) + \sqrt{(n_{11}(0) + \frac{k_{p2}}{k_{p1}} n_{22}(0))^2 - 4\frac{k_{p2}}{k_{p1}} \Delta n(0)}}$$

both sides $\times \frac{1}{k_{p1}}$,

$$1 < -\frac{2d(0)}{(n_{11}(0)k_{p1} + k_{p2}n_{22}(0)) + k_{p1}\sqrt{(n_{11}(0) + \frac{k_{p2}}{k_{p1}} n_{22}(0))^2 - 4\frac{k_{p2}}{k_{p1}} \Delta n(0)}}$$

$$-(n_{11}(0)k_{p1} + k_{p2}n_{22}(0)) - k_{p1}\sqrt{(n_{11}(0) + \frac{k_{p2}}{k_{p1}} n_{22}(0))^2 - 4\frac{k_{p2}}{k_{p1}} \Delta n(0)} < 2d(0)$$

$$2d(0) + (n_{11}(0)k_{p1} + k_{p2}n_{22}(0)) > \sqrt{(n_{11}(0)k_{p1} + k_{p2}n_{22}(0))^2 - 4k_{p1}k_{p2}\Delta n(0)}$$

take square operation at both sides and after simplifying, the following equation is obtained:

$$d(0)^2 + (n_{11}(0)k_{p1} + n_{22}(0)k_{p2})d(0) + k_{p1}k_{p2}\Delta n(0) > 0 \qquad \text{(C.61)}$$

the left hand side of the Equation (C.61) is exactly the same as the $D_C(0)$ in Equation (C.12). Therefore Equation (C.61) can be re-written as

$$D_C(0) > 0$$

From(C.60), substitute (C.58) into,

$$-\infty < -\frac{1}{k_{p1}} < \frac{(n_{11}(j0) + \epsilon n_{22}(j0)) \pm \sqrt{(n_{11}(j0) + \epsilon n_{22}(j0))^2 - 4\epsilon\Delta n(j0)}}{2d(j0)}$$

both sides $\times k_{p1}$,

$$-1 < \frac{(n_{11}(0)k_{p1} + k_{p2}n_{22}(0)) - \sqrt{(n_{11}(0)k_{p1} + k_{p2}n_{22}(0))^2 - 4k_{p2}k_{p1}\Delta n(0)}}{2d(0)}$$

$$-2d(0) - (n_{11}(0)k_{p1} + k_{p2}n_{22}(0)) < -\sqrt{(n_{11}(0)k_{p1} + k_{p2}n_{22}(0))^2 - 4k_{p1}k_{p2}\Delta n(0)}$$

both sides $\times(-1)$,

$$2d(0) + (n_{11}(0)k_{p1} + k_{p2}n_{22}(0)) > \sqrt{(n_{11}(0)k_{p1} + k_{p2}n_{22}(0))^2 - 4k_{p1}k_{p2}\Delta n(0)}$$

take square operation at both sides and after simplifying, the following equation is again obtained:

$$d(0)^2 + (n_{11}(0)k_{p1} + n_{22}(0)k_{p2})d(0) + k_{p1}k_{p2}\Delta n(0) > 0$$

again, the left hand side of the Equation (C.61) is exactly the same as the $D_C(0)$ in equation (C.12), hence

$$D_C(0) > 0$$

is again obtained.

Case 2: controller 1 only is installed

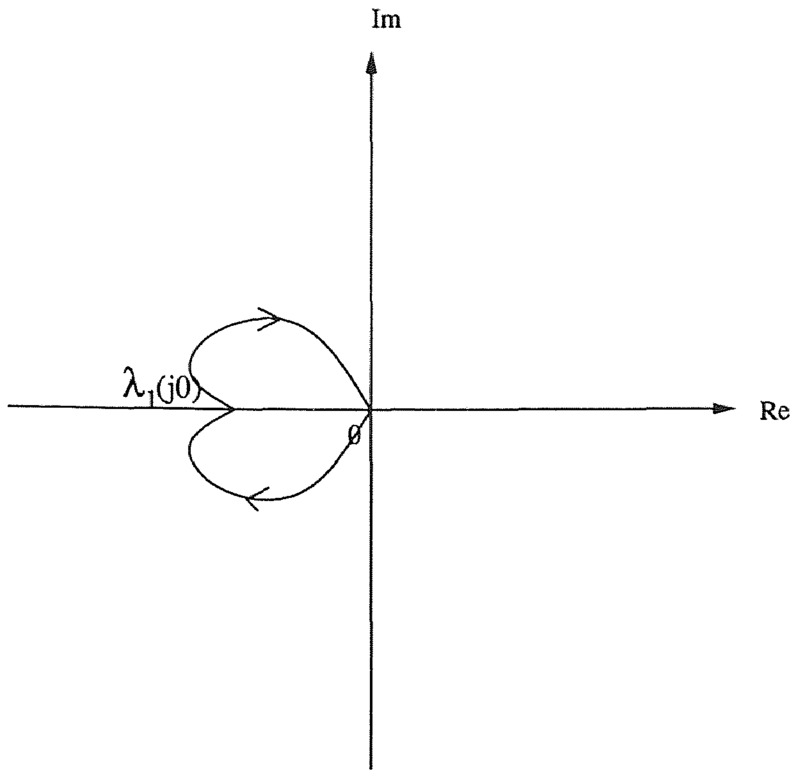

Figure C.10 Characteristic Loci Controller 1 Only Installed

In this case, $K_P T(s)$ becomes:

$$K_P T(s) = -k_{p1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{n_{11}(s)}{d(s)} & \frac{n_{12}(s)}{d(s)} \\ \frac{n_{21}(s)}{d(s)} & \frac{n_{22}(s)}{d(s)} \end{bmatrix} \quad \text{(C.62)}$$

Obviously, one of the characteristic root always stays at the origin. The other root is equal to:

$$\lambda(j\omega) = -\frac{n_{11}(j\omega)}{d(j\omega)} \quad \text{(C.63)}$$

when $\omega = 0$,

$$\lambda(j0) = -\frac{n_{11}(j0)}{d(j0)} < 0 \quad \text{(C.64)}$$

Figure C.10 shows the characteristic loci with $\omega \in [-\infty, +\infty]$

K_{p1} range of stability:

$$0 < -\frac{1}{k_{p1}} < +\infty \; and \; -\infty < -\frac{1}{k_{p1}} < -\frac{n_{11}(0)}{d(0)} \quad \text{(C.65)}$$

or,

$$-\infty < k_{p1} < \frac{d(0)}{n_{11}(0)} \tag{C.66}$$

Case 3: controller 2 only is installed

In this case, $K_P T(s)$ becomes:

$$K_P T(s) = -k_{p2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{n_{11}(s)}{d(s)} & \frac{n_{12}(s)}{d(s)} \\ \frac{n_{21}(s)}{d(s)} & \frac{n_{22}(s)}{d(s)} \end{bmatrix} \tag{C.67}$$

Obviously, one of the characteristic root always stays at the origin. The other root is equal to:

$$\lambda(j\omega) = -\frac{n_{22}(j\omega)}{d(j\omega)} \tag{C.68}$$

when $\omega = 0$,

$$\lambda(j0) = -\frac{n_{22}(j0)}{d(j0)} < 0 \tag{C.69}$$

Figure C.11 shows the characteristic loci with $\omega \in [-\infty,\ +\infty]$

K_{p2} range of stability:

$$0 < -\frac{1}{k_{p2}} < +\infty \ and \ -\infty < -\frac{1}{k_{p2}} < -\frac{n_{22}(0)}{d(0)} \tag{C.70}$$

or,

$$-\infty < k_{p2} < \frac{d(0)}{n_{22}(0)} \tag{C.71}$$

For the fault tolerant consideration, conditions (C.61), (C.66) and (C.71) must all hold. The overlapped range of the 3 conditions is the K_P range of stability as shown in Figure C.12.

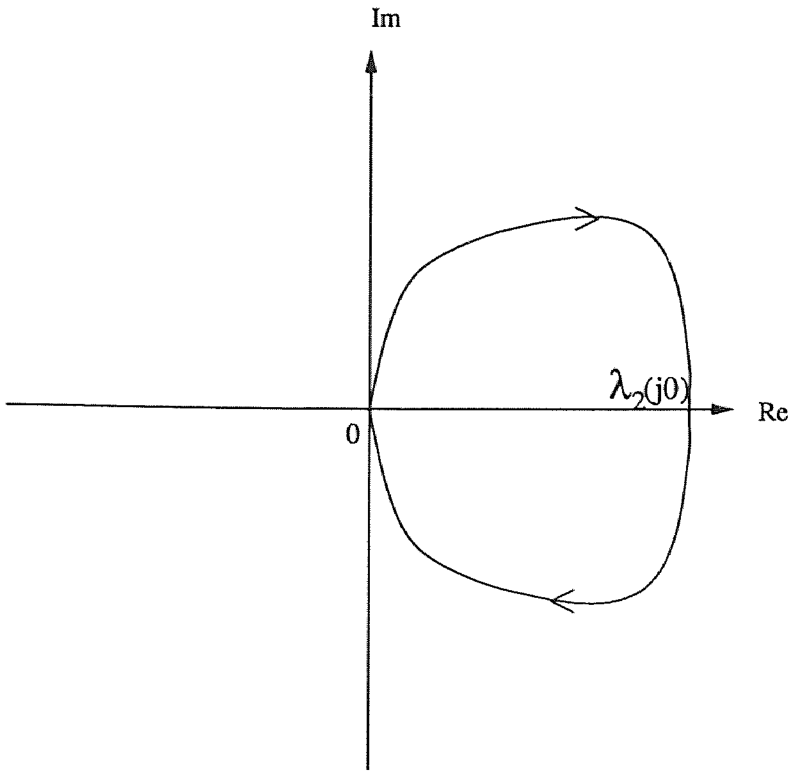

Figure C.11 Characteristic Loci Controller 2 Only Installed

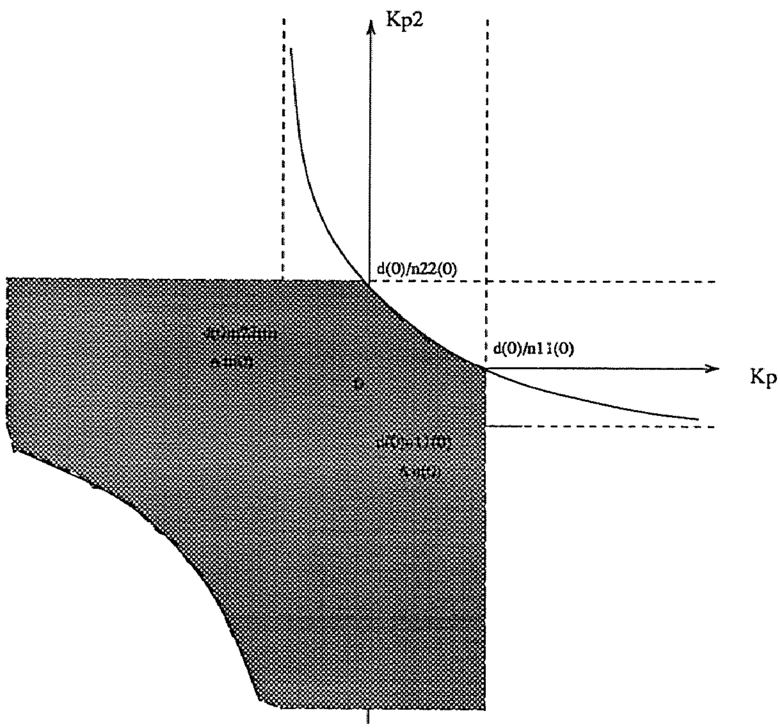

Figure C.12 K_P Range of Stability

APPENDIX D

LEMMA (4.2.1) DERIVATION

In this appendix, the following lemma is proved:

Let $P_1(s), P_2(s)$ be stable polynomials,

$$P_3(s) = P_1(s) + P_2(s) \tag{D.1}$$

if

$$P_1(0)P_2(0) > 0$$

then $P_3(s)$ holds no unstable real roots.

Proof

Let

$$P_1(s) = p_m s^m + p_{m-1}s^{m-1} + \cdots + p_1 s + p_0 \tag{D.2}$$

$$P_2(s) = q_n s^n + q_{n-1}s^{n-1} + \cdots + q_1 s + q_0 \tag{D.3}$$

Assuming $P_3(s)$ holds unstable real roots, then there exists $a > 0$ such that

$$P_3(s)|_{s=a} = 0$$

From equation(4.29),

$$P_1(s)|_{s=a} + P_2(s)|_{s=a} = 0$$

Or, equivalently,

$$(p_m a^m + p_{m-1}a^{m-1} + \cdots + p_1 a + p_0) + (q_n a^n + q_{n-1}a^{n-1} + \cdots + q_1 a + q_0) = 0 \tag{D.4}$$

Since $P_1(0)P_2(0) > 0$, and $P_1(s), P_2(s)$ are all stable, this implies that all coefficients of $P_1(s)$ and $P_2(s)$ are of the same sign. Therefore, Equation(D.4) is not true under this given condition, so the assumption that $P_3(s)$ holds unstable real roots is not true, or, $P_3(s)$ holds no unstable real roots.

(end of Proof)

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