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Performance analysis of collaborative hybrid-arq protocols over fading channels

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ABSTRACT

PERFORMANCE ANALYSIS OF COLLABORATIVE HYBRID-ARQ PROTOCOLS OVER FADING CHANNELS

by
Igor Stanojev

Impairments due to multipath signal propagation on the performance of wireless communications systems can be efficiently mitigated with time, frequency or spatial diversity. To exploit spatial diversity, multiple-antenna technology has been thoroughly investigated and emerged as one of the most mature communications areas. However, the need for smaller user terminals, which results in insufficient spacing for antenna collocation, tends to limit the practical implementation of this technology. Without compromising terminal dimensions, future wireless networks will therefore have to exploit their broadcast nature and rely on collaboration between single-antenna terminals for exploiting spatial diversity.

Among cooperative schemes, Collaborative ARQ transmission protocols, prescribing cooperation only when needed, i.e., upon erroneous decoding by the destination, emerge as an interesting solution in terms of achievable spectral efficiency. In this thesis, an information theoretical approach is presented for assessing the performance of Collaborative Hybrid-ARQ protocols based on Space-Time Block Coding. The expected number of retransmissions and the average throughput for Collaborative Hybrid-ARQ Type I and Chase Combining are derived in explicit form, while lower and upper bound are investigated for Collaborative Hybrid-ARQ Incremental Redundancy protocol, for any number of relays. Numerical results are presented to supplement the analysis and give insight into the performance of the considered scheme. Moreover, the issue of practical implementation of Space-Time Block Coding is investigated.

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by
Igor Stanojev

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APPROVAL PAGE

PERFORMANCE ANALYSIS OF COLLABORATIVE HYBRID-ARQ PROTOCOLS OVER FADING CHANNELS

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To my family

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Tanja's children. I hope that someday, instead of living thousands of miles apart, we can be together, as should be. I love you the most.

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CHAPTER 1

INTRODUCTION

Telecommunications, and in particular the wireless market, have recuperated in recent years and are today in constant growth. Internet access and video streaming are well supported by the highly developed wired communication technology. However, the customer demands are growing for a "anywhere, anytime" access to information, thus emphasizing the need for efficient wireless communications. And, while the research in wireless area has achieved remarkable results, it still seems to be seriously challenged by these demands.

Among currently emerging technologies that tend to boost wireless capacity, terminal collaboration is especially promising. Though still facing a preliminary phase of theoretical performance assessment, collaborative technology promises to retain the main performance gain of multiple-antenna technologies, without requiring physical deployment of multiple antennas at the wireless terminal. In particular, a practical and effective way to exploit its benefits is through Collaborative Hybrid-ARQ (Automatic Repeat Request) protocols, carefully designed to use effectively the spectral resource. In the following sections, we will discuss the need and the basics of collaboration, explain Hybrid-ARQ protocols, and finally merge these two concepts to elaborate on Collaborative Hybrid-ARQ protocols.

1.1 Collaborative Transmission

The phenomenon of channel fading presents the greatest challenge for implementation of high capacity wireless networks. Spatial diversity, carried out through multiple collocated antennas at the source and/or the destination terminals is the most powerful alternative for mitigating fading. The technology that consequently emerged, Multiple-Input-Multiple-Output (MIMO), though providing efficient theoretical solutions, is often practically

1

infeasible since, due to the size limitations, mobile stations often cannot support sufficient spacing between antennas.

Collaborative transmission with practical single-antenna stations provide an interesting solution for employing spatial diversity, while avoiding the terminal size issue (see e.g., [2], [3]). In particular, wireless networks benefit from their broadcast nature, since the information transmitted from the source towards the destination can be overheard by any other available surrounding station. If the particular protocol is designed to support the collaboration within the network, these surrounding stations can act as relays and therefore aid the current transmission, forming the distributed antenna array with the source or the destination. This aid can be carried out through the transmission in the dedicated channel (usually reserved time slot), or, more interesting, through the transmission in the same channel with the source, employing some sort of Space-Time (ST) coding. It is the ST, particularly STBC (Space-Time Block Coding) scheme that we focus on throughout this work.

In order to distinguish from collaborative transmission, we will refer to classical non-collaborative MIMO as the direct transmission methods, indicating that only the source and the destination participate in the communication. It is worth noting that two subclasses of MIMO will be used, namely MISO (Multiple-Input-Single- Output) and SISO (Single-Input-Single-Output) direct transmissions, as they provide useful upper (ideal collaboration) and lower (no collaboration) bound, respectively, to the performance of collaborative transmission in our model.

According to the principles that prescribe the relay behavior, we can classify collaborative transmission into two categories. The first one is Amplify and Forward (AF), where the relays simply amplify whatever information they received from the source, and rebroadcast it. Though simple, this scheme suffers from the noise enhancement, since together with the information, the noise at the relay receiving antenna is also amplified.

Moreover, the signal at the destination that comes from the relay can be severely damaged due to its propagation through two serial channels.

The other scheme requires that the relay successfully decodes the source information, before reencoding and retransmitting it. If the error is detected at the relay, the latter sustains from any retransmission. This scheme, named Decode and Forward (DF), though more complex, generally gives better performance than AF scheme, and it is the scheme that we will consider in our work.

1.2 ARQ Protocols

While the different coding and error correction methods in Physical (PHY) layer reduce the probability of erroneous transmission, they cannot provide completely error-free communication. Automatic Repeat Request (ARQ) protocol, embedded in the Medium Access Control (MAC) layer, demands the retransmission of each erroneously received packet. While, with reference to the packet flow control, there are several versions of ARQ protocols, such as Go-Back-N or Selective-Repeat [1], we will focus on the simplest one, Stop-and-Wait ARQ protocol, where the system sustains from the transmission of any following packets until the transmission of the current one is successfully completed. This protocol is typically used in the modern distributed wireless networks.

After receiving and processing the packet transmitted from the source, the destination node typically checks the CRC (Cyclic Redundancy Check) header to determine whether the packet is error-free. If the packet is damaged, the destination sends a NACK (Not Acknowledge) message toward the source, signaling that an error occurred in the previous transmission, and that retransmission of the packet is required. If the source, within some predefined time, does not receive any message from the destination, it will assume that the transmission was unsuccessful and will retransmit the packet. This cycle will proceed until final successful reception, when the destination signals successful event sending an ACK (Acknowledge) message. Moreover, in order to prevent the system outage caused

by numerous consecutive unsuccessful trials typical for very hostile channel environment, a maximum number of retransmissions is usually predefined. If this number is reached, the retransmission is delayed for the time interval during which the channel is expected to change significantly

Using the ARQ protocol without support of some extra coding can be, however, hazardous for the system behavior. Transmitting on permanently hostile communication channel, as mentioned, can lead to numerous unsuccessful retransmission, significantly reducing the system efficiency. To cope with this challenge, ARQ protocols should be backed with some complementary method that enhances the packet resilience toward channel conditions. Hybrid-ARQ protocols present the ARQ protocol upgraded with Forward Error Control (FEC) protection, a coding technique used on packets for increasing their robustness. Usually a low rate protection code is used in combination with interleaving, reducing the effect of fading and increased noise.

When no further enhancement is used, merging of FEC and ARQ concepts is labeled as Type I Hybrid-ARQ Protocol (HARQ-TI). Note that the performance of plain ARQ and HARQ-TI protocols is closely related, since the advantage of FEC technique is parameterized simply by the fixed coding gain. Since the information theoretic approach that we employ throughout this work assumes the use of coding, we will disregard plain ARQ protocol in our work, bearing in mind that if necessary, we can analyze it by applying trivial shift along Signal-to-Noise Ratio (SNR) dimension to HARQ-TI protocol results.

The motivation for proposing Type II HARQ protocols lies in the inability of plain ARQ and Type I HARQ to benefit with every retransmission. Type II HARQ protocols are protocols with memory, since they use buffering to preserve the erroneous packets and combine them in a certain manner with other copies during the detection process, thus gaining significantly with each retransmission. Type II Hybrid-ARQ Chase Combining Protocol (HARQ-CC), named after the pioneer in this area [4] and sometimes referred as Packet Combining , does not introduce any novelty (i.e., more complexity) on the source

side, since the same copies of the original packet are retransmitted upon receiving the NACK message. However, at the destination side packets are buffered and combined with the most recent packet. We will assume that the soft combining (Maximum Ratio Combining - MRC) is used, and will not consider relatively inferior hard combining, usually performed through the bit-wise majority voting.

Type II Hybrid-ARQ Incremental Redundancy Protocol (HARQ-IR), often called Code Combining, represents the most sophisticated HARQ protocol. Upon receiving the retransmission request, the source generates new parity bits (different with each trial) and sends them instead of the original packet. At the destination, received versions of packets are concatenated and processed according to the decoding rule. The effect is equivalent to resending the packet protected with the more powerful code with each trial. We can expect the performance of HARQ-IR to be quite superior to that of HARQ-CC protocol, due to the coding gain advantage of applied coding techniques. However, the complexity of both the transmitting and the receiving side introduced by this scheme, gives the HARQ-CC protocol some more practical value.

1.3   Collaborative Hybrid-ARQ Protocols

Collaborative Hybrid-ARQ protocol [6] is the sublimation of HARQ and collaboration principles performed through STBC coding. Unlike the conventional collaborative schemes that commonly assume that the relays have their own dedicated channel (e.g., through time or frequency division) to forward the information from the source [5], in Collaborative HARQ protocols the first transmission is performed by the source alone, and the collaboration, performed through means of STBC, takes part in the retransmission, if the latter is requested by the destination. Therefore, the network resources, i.e., time and frequency, are not divided among stations but fully exploited, since no dedicated channels are used. Moreover, the cost of including the relays in current communication is reduced, for the relays transmit only if necessary, i.e., if the retransmission is requested.

Original transmission   Retransmission
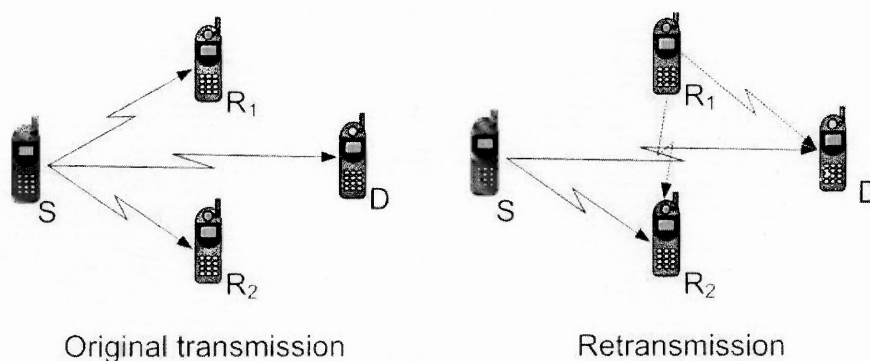
Figure 1.1 Illustration of the collaborative HARQ with two active relays, R_1 and R_2. In this example, relay R_1 decodes successfully the original transmission and cooperates with the source S for the retransmission.

According to the Collaborative HARQ protocol, as mentioned, in the first time-slot the source S broadcasts a packet to destination D and any available relay R_i, $i = 1, ..., M$ (see fig. 1.1 for an example with $M = 2$). If CRC at the destination determines erroneous decoding, packet retransmission is requested by the destination via a NACK message. Then, relays that have successfully decoded in the first time-slot (i.e., relay R_1 in example of fig. 1.1), signal their availability to the source and switch from receiving to transmitting mode. The retransmission is performed by a distributed antenna array consisting in the source and activated relays, through joint transmission of a space-time codeword. The actual codeword can be a copy of the original packet, if HARQ-TI or HARQ-CC is used, or entirely new packet consisted of parity bits generated according to HARQ-IR method used. The destination, as well as any remaining receiving relays (i.e., relay R_2 in fig. 1.1), decode the STBC data and, if HARQ-CC or HARQ-IR is implemented, perform appropriate packet or code-combining with previously received codewords [7], respectively. The procedure repeats until the CRC at the destination reveals successful detection and an ACK message is sent, or a predefined maximum number of retransmissions is reached.

CHAPTER 2

SYSTEM ANALYSIS

In this chapter, we investigate the Collaborative HARQ employing either HARQ-TI, HARQ-CC or HARQ-IR protocol, based on the DF scheme. We consider a block Rayleigh fading model, where the channel stays constant during each transmission slot, but changes independently with each retransmission. The channel gains between any two nodes (the source, the destination and the relays) are modeled as mutually independent, time-uncorrelated identically distributed (iid) symmetric complex Gaussian variables with the power equal to unity

$$E[h^{(k)}h^{(l)^*}] = \delta(k - l), \quad k, l = 1, 2, ... \quad (2.1)$$

where the superscript $^{(k)}$ denotes the time slot.

The model of interest of this work places the source and the relays relatively close and identically spaced from each other. The destination is relatively far from this group, on approximately same distance from the source and any relay. To capture the effect of grouping the nodes in such a manner, we increase the SNR at the relays for the gain parameter α ($\alpha > 1$), as shown on fig. 2.1 for two-relay model. Feedback channels from the destination toward the source and the relays are assumed perfectly reliable for the transmission of short, strongly coded ACK/NACK messages and are therefore not shown on the fig. 2.1. Moreover, since the block-fading provides time diversity with each transmission attempt, our model does not employ predefined maximum number of retransmissions. Two parameters commonly used to provide insight in HARQ protocols are the delay and the throughput of the system. In our analysis, we consider ACK/NACK transmission time, the signal processing delay and the propagation delay negligible comparing to the time needed for the actual transmission of packet. This way, the system delay can be parameterized with the expected number of transmissions, $E[T]$, where T is the actual number of transmissions

7
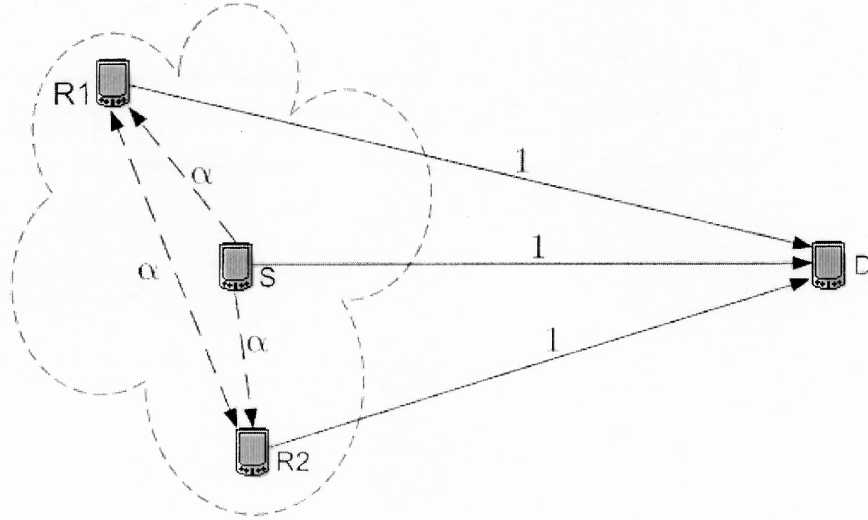
Figure 2.1 Model illustration for system with two relays.

necessary for the successful decoding at the destination node. The ratio $C_0/E[T]$, where $C_0 \, [nat/s/Hz]$ is the transmission rate, determines the throughput of the system.

Regardless of the system model (direct or collaborative transmission) and ARQ type, the expected number of transmissions necessary for the successful decoding at destination can be written as

$$E[T] = \sum_{n=1}^{\infty} n P \{T = n\} \,, \tag{2.2}$$

where the probability that exactly n trials are necessary, $P\{T = n\}$, is given by

$$P\{T = n\} = [1 - p_e\,(n)] \prod_{k=1}^{n-1} p_e(k), \tag{2.3}$$

with $p_e(k)$ denoting the probability that kth transmission was erroneous, given that the previous transmissions were also unsuccessful. Furthermore, erroneous transmission is defined as the event when the achievable rate C is smaller than the transmission rate $C_0, C < C_0$. The achievable rate for the simple case of the AWGN (Additive White Gaussian Noise) channel is defined as

$$C_{AWGN} = \log\left(1 + \frac{P}{N_0}\right), \tag{2.4}$$

where P and N_0 are the signal power and the power spectral density of the Additive White Gaussian Noise (AWGN) at the destination, respectively, and the ratio P/N_0 depicts SNR ratio. We will refer to (2.4) to determine the necessary rates and therefore $p_e(k)$ for the more complex scenarios that include HARQ protocols and different network models with fading channels.

This chapter is divided into two main sections. We will first discuss the single-relay model, and in the following section expand it to the multi-relay model. For each network model, we will consider HARQ-TI , HARQ-CC and HARQ-IR protocols. For each protocol, collaborative networks will be compared with direct transmission networks, SISO and MISO, with former providing the lower (no transmission from relays) and latter the upper (ideal collaboration) bound for the quality of collaborative network performance. The final task in each scenario will be to determine $p_e(k)$, since this result can be simply plugged into (2.2)-(2.3) to solve for the delay and the throughput.

2.1 Single-Relay Model

As mentioned, we will discuss three HARQ protocols, starting from the simplest HARQ-TI, following with HARQ-CC, and eventually ending this section with the most complex HARQ-IR protocol.

2.1.1 HARQ-TI

In the HARQ-TI section, as well as in following sections dedicated to the other two HARQ protocols, we will discuss both direct transmission and collaborative models. Direct transmission models provide upper and lower bounds on the performance of collaborative scenarios. In particular, a 1×1 (one transmitting, one receiving antenna) SISO model presents the worst case for collaboration, where the relay is not able to assist the source. On the other hand, a 2×1 (two transmitting, one receiving) MISO model represents the most optimistic collaborative scenario, where the help from the relay is immediate, i.e.,

activating the relay does not require the use of channel resources. It should be remarked that this optimistic scenario cannot be achieved with the collaborative model, due to the first transmission when the relay listens to the source message and cannot assist in transmission.

Direct Transmission For the SISO model and HARQ-TI protocol, the rate achievable at the destination in nth transmission is

$$C_{D,1\times1}(n) = \log\left(1 + \left|h_{SD}^{(n)}\right|^2 \frac{P}{N_0}\right),\tag{2.5}$$

with $h_{SD}^{(n)}$ denoting the channel gain between the source and the destination in the nth transmission $((n-1)$th retransmission). Note that, due to the lack of memory of HARQ-TI protocol, only the current (re)transmission is of importance.

Similarly, MISO rate can be written as

$$C_{D,2\times1}(n) = \log\left[1 + \left(\left|h_{SD,1}^{(n)}\right|^2 + \left|h_{SD,2}^{(n)}\right|^2\right)\frac{P}{N_0}\right],\tag{2.6}$$

where $h_{SD,1}^{(n)}$ and $h_{SD,2}^{(n)}$ present the gains of the independent channels in the MISO model between each of the transmitting antennas and the destination, at the nth transmission trial. Note that summation in (2.6) describes the diversity effect of space-time transmission from two antennas.

The fading channel gains $\left|h_{SD}^{(n)}\right|^2$, $\left|h_{SD,1}^{(n)}\right|^2$, and $\left|h_{SD,2}^{(n)}\right|^2$ in (2.5)-(2.6) are independent identically distributed (iid) exponential variables,i.e, iid chi-square variables with two degrees of freedom. Moreover, the overall fading gain in (2.6), $\left|h_{SD,1}^{(n)}\right|^2 + \left|h_{SD,2}^{(n)}\right|^2$, is a chi-square variable with four degrees of freedom, so the probabilities of error in the nth transmission $((n-1)$th retransmission), for the SISO and MISO models respectively, read

$$
\begin{aligned}
p_{e,1\times1}(n) &= P\left\{C_{D,1\times1}(n) < C_0\right\} &\tag{2.7a}\\
&= F_{\chi^2}(\mu, 2) \\
p_{e,2\times1}(n) &= P\left\{C_{D,2\times1}(n) < C_0\right\} &\tag{2.7b}\\
&= F_{\chi^2}(\mu, 4),
\end{aligned}
$$

where

$$\mu = 2\frac{e^{C_0} - 1}{P/N_0},\tag{2.8}$$

$F_{\chi^2}(x, 0) = 1$, and $F_{\chi^2}(x, \nu)$, $\nu = 1, 2, \ldots$ is the cumulative distribution function of the chi-square variable with ν degrees of freedom, taken at value x,

$$F_{\chi^2}(x, \nu) = \int\limits_0^x \frac{t^{(\nu-2)/2}e^{-t/2}}{2^{\nu/2}\Gamma(\nu/2)}dt\tag{2.9}$$

and $\Gamma(x)$ is the gamma function, defined as

$$\Gamma(x) = \int\limits_0^\infty e^{-t}t^{x-1}dt,\tag{2.10}$$

or, if x is a positive integer, which proves to be always the truth for this analysis,

$$\Gamma(x) = (x - 1)!.\tag{2.11}$$

Collaborative Transmission Since the rate achievable by the destination in a collaborative scenario depends on the state of the relay, i.e., whether it is activated and assisting the source or idle and listening to the source, it is crucial to start the analysis with the performance of the relay node. The rate achievable at the relay after n transmissions is

$$C_R(n) = \log\left(1 + \left|h_{SR}^{(n)}\right|^2 \frac{\alpha P}{N_0}\right),\tag{2.12}$$

with $h_{SR}^{(n)}$ denoting the channel gain between the source and the relay in the nth transmission. Introducing the notation $\{C(1:n) < C_0\}$ for the event $\{C(1) < C_o, \ldots, C(n) < C_o\}$, we can express the probability that the relay has not yet correctly decoded at the nth transmission as

$$\begin{aligned}\bar{p}_R(n) &= P\{C_R(1:n) < C_0\}\\ &= \prod_{i=1}^n P\{C_R(i) < C_0\},\end{aligned}\tag{2.13}$$

or, according to (2.12),

$$\bar{p}_R(n) = F_{\chi^2}^n(\mu_\alpha, 2),\tag{2.14}$$

where $\mu_\alpha = \frac{\mu}{\alpha}$. On the other hand, the probability that the relay received successfully at the trial n is

$$\begin{aligned}p_R(n) &= P\{C_R(1:n-1) < C_0\}[1 - P\{C_R(k) < C_0\}]\\ &= F_{\chi^2}^{n-1}(\mu_\alpha, 2)[1 - F_{\chi^2}(\mu_\alpha, 2)].\end{aligned}\tag{2.15}$$

As mentioned, in the collaborative transmission model the achievable rate at the destination node depends on the relay state:

$$C_D(n; R) = \log\left[1 + \left(\left|h_{SD}^{(n)}\right|^2 + \left|h_{RD}^{(n)}\right|^2\right)\frac{P}{N_0}\right],\tag{2.16a}$$

$$C_D(n; \bar{R}) = \log\left[1 + \left|h_{SD}^{(n)}\right|^2\frac{P}{N_0}\right],\tag{2.16b}$$

with $h_{RD}^{(n)}$ denoting the channel gain between the relay and the destination in the nth transmission. We adopted the notation $C_D(n, R)$ and $C_D(n, \bar{R})$ for the throughputs achievable at the destination in scenarios when the relay has been able and unable to correctly decode, respectively. The probability of error at the nth trial can finally be written as

$$\begin{aligned}p_e(n) &= \bar{p}_R(n-1)P\{C_D(n; \bar{R}) < C_0\} + \sum_{k=1}^{n-1} p_R(k)P\{C_D(n; R) < C_0\}\\ &= F_{\chi^2}^{n-1}(\mu_\alpha, 2)F_{\chi^2}(\mu, 2) + \left(1 - F_{\chi^2}^{n-1}(\mu_\alpha, 2)\right)F_{\chi^2}(\mu, 4).\end{aligned}\tag{2.17}$$

2.1.2 HARQ-CC

Direct Transmission HARQ-CC is a protocol that performs soft combining of all received packets, including the erroneous ones, so the 1×1 SISO and 2×1 MISO rates at the

destination, achievable after n transmissions, can be written respectively as

$$C_{D,1\times1}(n) = \log\left(1 + \sum_{i=1}^{n}\left|h_{SD}^{(i)}\right|^2\frac{P}{N_0}\right),\qquad(2.18\text{a})$$

$$C_{D,2\times1}(n) = \log\left[1 + \sum_{i=1}^{n}\left(\left|h_{SD,1}^{(i)}\right|^2 + \left|h_{SD,2}^{(i)}\right|^2\right)\frac{P}{N_0}\right].\qquad(2.18\text{b})$$

Unlike the memoryless HARQ-TI, for the HARQ-CC protocol the probability of the erroneous reception in the nth transmission has to be conditioned on the previous $(n-1)$ unsuccessful trials:

$$\begin{aligned}p_e(n) &= P\left\{C(n) < C_0 | C(1:n-1) < C_0\right\}\\ &= \frac{p\{C(n) < C_0\}}{p\{C(n-1) < C_0\}}.\end{aligned}\qquad(2.19)$$

Note that in (2.19) we used the fact that the event $\{C(1:n-1) < C_0\}$ is equivalent to the event $\{C(n-1) < C_0\}$. Since the equivalent channel power gains (the terms under summation) in (2.18a) and (2.18b) are summations of n and $2n$ identical exponentially distributed variables, respectively, resulting in chi-square variables with $2n$ and $4n$ degrees of freedom, the required probabilities of error can be written as

$$p_{e,1\times1}(n) = \frac{F_{\chi^2}(\mu, 2n)}{F_{\chi^2}(\mu, 2(n-1))},\qquad(2.20\text{a})$$

$$p_{e,2\times1}(n) = \frac{F_{\chi^2}(\mu, 4n)}{F_{\chi^2}(\mu, 4(n-1))}.\qquad(2.20\text{b})$$

Collaborative Transmission   Since the rate achievable at the destination depends on the relay behavior, we start the analysis with the relay node. At the nth transmission, the rate achievable by relay is

$$C_R(n) = \log\left(1 + \sum_{i=1}^{n}\left|h_{SR}^{(i)}\right|^2\frac{\alpha P}{N_0}\right).\qquad(2.21)$$

The probability that after n transmissions the relay still did not receive successfully can be written as

$$\bar{p}_R(n) = P\{C_R(1:n) < C_0\}$$
$$= P\{C_R(n) < C_0\}, \qquad (2.22)$$

and, according to (2.21),

$$\bar{p}_R(n) = F_{\chi^2}(\mu_\alpha, 2n). \qquad (2.23)$$

The probability that the relay received successfully at the trial k, but not before, is

$$p_R(n) = P\{C_R(1:n-1) < C_0\}[1 - P\{C_R(n) < C_0|C_R(1:n-1) < C_0\}]$$
$$= P\{C_R(n-1) < C_0\}[1 - P\{C_R(n) < C_0|C_R(n-1) < C_0\}]$$
$$= F_{\chi^2}(\mu_\alpha, 2(n-1))\left[1 - \frac{F_{\chi^2}(\mu_\alpha, 2n)}{F_{\chi^2}(\mu_\alpha, 2(n-1))}\right]$$
$$= F_{\chi^2}(\mu_\alpha, 2(n-1)) - F_{\chi^2}(\mu_\alpha, 2n). \qquad (2.24)$$

Unlike HARQ-TI, for the HARQ-CC protocol the destination rate depends not only on whether the relay is transmitting, but also on the time instant when it started transmitting. Therefore, instead of using the notation $C_D(n; \bar{R})$ and $C_D(n; R)$, we switch to the notation $C_D(n; j)$, where j presents the transmission slot when the relay had the final, successful reception,

$$C_D(n; j) = \log\left(1 + \sum_{i=1}^{n}\left|h_{SD}^{(i)}\right|^2\frac{P}{N_0} + \sum_{i=j+1}^{n}\left|h_{RD}^{(i)}\right|^2\frac{P}{N_0}\right). \qquad (2.25)$$

Note that the the equivalent channel power gain $\sum_{i=1}^{n}\left|h_{SD}^{(i)}\right|^2 + \sum_{i=j+1}^{n}\left|h_{RD}^{(i)}\right|^2$ defined in (2.25), valid only for $j \leq n-1$, is a chi-square variable with $(2n-j)$ degrees of freedom. For the case when the relay was not able to receive before the current retransmission, we use the notation $C_D(n; n)$:

$$C_D(n; n) = \log\left(1 + \sum_{i=1}^{n}|h_{SD,i}|^2\frac{P}{N_0}\right). \qquad (2.26)$$

Finally, the probability of error at the nth transmission is

$$
\begin{aligned}
p_e(n) &= \bar{p}_R(n-1)P\left\{C_D(n;n) < C_0 | C_D(1:n-1;n-1) < C_0\right\} + \\
&\quad + \sum_{k=1}^{n-1} p_R(k)P\left\{C_D(n;k) < C_0 | C_D(1:n-1;k) < C_0\right\} \\
&= \bar{p}_R(n-1)P\left\{C_D(n;n) < C_0 | C_D(n-1;n-1) < C_0\right\} + \\
&\quad + \sum_{k=1}^{n-1} p_R(k)P\left\{C_D(n;k) < C_0 | C_D(n-1;k) < C_0\right\}.
\end{aligned}
\tag{2.27}
$$

Combining the last equation with (2.23) and (2.24) gives us the final result for the probability of error conditioned on previous unsuccessful attempts:

$$
\begin{aligned}
p_e(n) &= F_{\chi^2}(\mu_\alpha, 2(n-1)) \frac{F_{\chi^2}(\mu, 2n)}{F_{\chi^2}(\mu, 2(n-1))} + \\
&\quad + \sum_{k=1}^{n-1} \left[F_{\chi^2}(\mu_\alpha, 2(k-1)) - F_{\chi^2}(\mu_\alpha, 2k)\right] \frac{F_{\chi^2}(\mu, 2(2n-k))}{F_{\chi^2}(\mu, 2(2n-k-2))}.
\end{aligned}
\tag{2.28}
$$

2.1.3 HARQ-IR

As the analysis of HARQ-IR can become very tedious, convenient performance bounds are derived in Appendix A. Since, as noted in Appendix A, the performance of HARQ-IR is lower bounded by the performance of HARQ-CC protocol, in the following we will focus on the upper bound on the performance of HARQ-IR protocol.

Direct Transmission The system that uses the HARQ-IR protocol benefits with each retransmission with the entirely new information [7], and the information (rate) gathered by trial n can be written as

$$
C_{D,1\times1}(n) = \sum_{i=1}^{n} \log\left(1 + \left|h_{SD}^{(i)}\right|^2 \frac{P}{N_0}\right),
\tag{2.29a}
$$

$$
C_{D,2\times1}(n) = \sum_{i=1}^{n} \log\left[1 + \left(\left|h_{SD,1}^{(i)}\right|^2 + \left|h_{SD,2}^{(i)}\right|^2\right) \frac{P}{N_0}\right].
\tag{2.29b}
$$

According to (A.8), (2.29a) and (2.29b) can be bounded as

$$C_{D,1\times1}(n) \leq n\log\left(1 + \sum_{i=1}^{n}\left|h_{SD}^{(i)}\right|^2\frac{P}{nN_0}\right), \tag{2.30a}$$

$$C_{D,2\times1}(n) \leq n\log\left[1 + \sum_{i=1}^{n}\left(\left|h_{SD,1}^{(i)}\right|^2 + \left|h_{SD,2}^{(i)}\right|^2\right)\frac{P}{nN_0}\right]. \tag{2.30b}$$

Note that equivalent channel gains in (2.30a) and (2.30b) are chi-square variables with $2n$ and $4n$ degrees of freedom, respectively. Furthermore, HARQ-IR is a protocol with memory, so the equation (2.19), derived in the section 2.1.2 and repeated here, still holds,

$$p_e(n) = \frac{p\{C(n) < C_0\}}{p\{C(n-1) < C_0\}}. \tag{2.31}$$

Finally, the upper bounds for the quality of the performance of direct-transmission networks are given by the following equations

$$p_{e,1\times1}(n) \geq \frac{F_{\chi^2}\left(\mu(n), 2n\right)}{F_{\chi^2}\left(\mu(n-1), 2\left(n-1\right)\right)}, \tag{2.32a}$$

$$p_{e,2\times1}(n) \geq \frac{F_{\chi^2}(\mu(n), 4n)}{F_{\chi^2}\left(\mu(n-1), 4\left(n-1\right)\right)}, \tag{2.32b}$$

where $\mu(n) = 2n\frac{e^{C_0/n}-1}{P/N_0}$.

Collaborative Transmission   Analysis of Collaborative HARQ-IR can be significantly simplifies if we note that the equations (2.32a)-(2.32b) for the upper bound of achievable rate using the direct transmission HARQ-IR protocol differ from the equations (2.20a)-(2.20b) for HARQ-CC scheme only in the parameter $\mu(n)$. This conclusion will hold in case of any network type, so we can apply (2.28) and directly write the upper bound on quality of the system performance,

$$p_e(n) \geq F_{\chi^2}(\mu_\alpha(n-1), 2(n-1))\frac{F_{\chi^2}(\mu(n), 2n)}{F_{\chi^2}(\mu(n-1), 2(n-1))} + \tag{2.33}$$

$$+ \sum_{k=1}^{n-1} [F_{\chi^2}(\mu_\alpha(k-1), 2(k-1)) - F_{\chi^2}(\mu_\alpha(k), 2k)]\frac{F_{\chi^2}(\mu(n), 2(2n-k))}{F_{\chi^2}(\mu(n-1), 2(2n-k-2))},$$

where $\mu_\alpha(k) = \frac{\mu(k)}{\alpha}$.

2.2   Multi-Relay Model

In the section 2.1 analysis is performed for the network scenario with a single relay. Moreover, basic tools were presented that can now be exploited for more sophisticated case that includes any number M of relays.

2.2.1   HARQ-TI

Direct Transmission   The direct transmission network models that bound the M-relay collaborative model are 1×1 SISO and $(M + 1) \times 1$ MISO models. The respective rates that can be achieved with the memoryless HARQ-TI protocols are

$$C_{D,1\times1}(n) = \log\left(1 + \left|h_{SD}^{(n)}\right|^2 \frac{P}{N_0}\right), \tag{2.34a}$$

$$C_{D,(M+1)\times1}(n) = \log\left(1 + \sum_{i=1}^{M+1} \left|h_{SD,i}^{(n)}\right|^2 \frac{P}{N_0}\right), \tag{2.34b}$$

with $h_{SD}^{(n)}$ denoting the channel gain between the source and the destination and $h_{SD,i}^{(n)}$ the channel gains between ith source antenna, $i = 1,..M+1$, and the destination during the nth transmission ($(n-1)$th retransmission). Obviously, equivalent channel gains in (2.34a) and (2.34b) are chi-square variables with 2 and $2(M+1)$ degrees of freedom, and the respective probabilities of error in nth transmission can be therefore written as:

$$p_{e,1\times1}(n) = F_{\chi^2}(\mu, 2), \tag{2.35a}$$

$$p_{e,(M+1)\times1}(n) = F_{\chi^2}(\mu, 2(M+1)). \tag{2.35b}$$

Collaborative Transmission   As mentioned in section 2.1, the rate achieved by the destination depends on the state of relays, i.e., whether or not they are able to assist the source during retransmissions. Therefore, we need to first analyze the relay nodes and determine the probability of their activation in particular transmission instance, i.e., the probability

$p_R(k_1, ..., k_{n-1})$ that k_1 relays have decoded successfully in the first transmission, k_2 in second (but not before) and so on. For this purpose, we enumerate M available relays, $R_1, ..., R_M$, assuming without loss of generality that the indices of the active relays, i.e. the relays that decoded successfully, precede those of inactive. The rate achievable at relay R_i at the transmission slot n, given that $\tilde{k}_n = \sum_{i=1}^{n-1} k_i$ relays $(\tilde{k}_n < i)$ turned active by the $(n-1)$th slot, is

$$C_{R_i}(n, \tilde{k}_n) = \log\left[1 + \left(\left|h_{SR_i}^{(n)}\right|^2 + \sum_{j=1}^{\tilde{k}_n} \left|h_{R_j R_i}^{(n)}\right|^2\right) \frac{\alpha P}{N_0}\right], \quad (2.36)$$

where $h_{SR_i}^{(n)}$ and $h_{R_j R_i}^{(n)}$, $i, j = 1, ..., M$, $i \neq j$, $i > \tilde{k}_n$ denote the channel gain between the source and the relay R_i, and the channel gain between the relays R_j and R_i, respectively, during nth transmission trial. Notice that with HARQ-TI the achievable rate only depends on the current state of the fading channels since no combining of previously received packets is carried out. Further, since the fading term in (2.36) is the sum of $1 + \tilde{k}_n$ independent exponentially distributed variables, the overall channel fading gain is a chi-square random variable with $2(1 + \tilde{k}_n)$ degrees of freedom and the probability that with $\tilde{k}_n$ the relay R_i does not decode at the step n reads

$$\bar{p}_{R_i}(n|\tilde{k}_n) = P\{C_{R_i}(n, \tilde{k}_n) < C_0\} = F[\mu_\alpha, 2(\tilde{k}_n + 1)], \quad (2.37)$$

and the probability that k_n relays successfully decode in the current slot is

$$p_R(k_n|\tilde{k}_n) = P_{bin}(\bar{p}_{R_i}(n, \tilde{k}_n), M - \tilde{k}_n, k_n) \quad (2.38)$$

where $P_{bin}(p, N, n) = \binom{N}{n} p^{N-n}(1-p)^n$ represents the binomial distribution. Finally, the probability $p_R(k_1, ..., k_{n-1})$ reads

$$p_R(k_1, k_2, ..., k_{n-1}) = \prod_{i=1}^{n} p_R(k_i|\tilde{k}_i), \quad (2.39)$$

where we have $\tilde{k}_1 = 0$.

At the destination, the achievable rate for the nth transmission depends on the number $\tilde{k}_n$ of relays that have decoded correctly by transmission $n-1$ and therefore collaborate with the source via space-time coding in the nth retransmission:

$$C_D(n, \tilde{k}_n) = \log_2 \left[1 + \left(\left| h_{SD}^{(n)} \right|^2 + \sum_{j=1}^{\tilde{k}_n} \left| h_{R_jD}^{(n)} \right|^2 \right) \frac{P}{N_0} \right], \qquad (2.40)$$

with $h_{R_jD}^{(n)}$, $j = 1, ..., M$, denoting the channel gain between relay R_j and the destination during nth transmission trial. The fading term in (2.40) is again the sum of $1 + \tilde{k}_n$ independent exponentially distributed variables, and the probability that the destination does not decode in the nth attempt, with $\tilde{k}_n$ activated relays, reads

$$P\{C_D(n, \tilde{k}_n) < C_0\} = F[\mu, 2(1 + \tilde{k}_n)] \qquad (2.41)$$

The probability of unsuccessful decoding at the n-th transmission (given that the previous $(n-1)$ transmissions were unsuccessful) can be written as

$$p_e(n) = \sum_{\mathcal{K}} p_R(k_1, ..., k_{n-1}) \, P\{C_D(n, \tilde{k}_n) < C_0\}, \qquad (2.42)$$

where the sum is to be carried out over the set $\mathcal{K}$ of tuples $(k_1, ..., k_{n-1})$: $\mathcal{K} = \{(k_1, ..., k_{n-1}) | \tilde{k}_n = \sum_{i=1}^{n-1} k_i \leq M \}$. In the Appendix Bit is shown that this set contains $\sum_{i=0}^{M} \binom{n-2+i}{i}$ terms. Finally, after plugging (2.39) and (2.41) into equation (2.42), the probability $p_e(n)$ reads

$$p_e(n) = \sum_{\mathcal{K}} \prod_{i=1}^{n} P_{bin} \left\{ F\left[\mu_\alpha, 2\left(1 + \tilde{k}_i\right) \right] ; M - \tilde{k}_i, k_i \right\} F\left[\mu, 2\left(1 + \tilde{k}_n\right) \right]. \qquad (2.43)$$

2.2.2 HARQ-CC

Direct Transmission According to HARQ-CC protocol, previously received packets are soft-combined prior to detection. Therefore, achievable SISO 1×1 and MISO $(M+1) \times 1$

rates at the destination can be written respectively as

$$C_{D,1\times1}(n) = \log\left(1 + \sum_{i=1}^{n} \left|h_{SD}^{(i)}\right|^2 \frac{P}{N_0}\right), \qquad (2.44\text{a})$$

$$C_{D,(M+1)\times1}(n) = \log\left(1 + \sum_{i=1}^{n} \sum_{j=1}^{M+1} \left|h_{SD,j}^{(i)}\right|^2 \frac{P}{N_0}\right), \qquad (2.44\text{b})$$

where $h_{SD}^{(i)}$ and $h_{SD,j}^{(i)}$ denote the channel gain between the single-antenna source and the destination, and the channel gain between the jth source antenna and the destination, respectively, during ith transmission trial. Notice in (2.44b) that the effect of space-time combining (inner sum) is of the same nature as packet combining (outer sum). Furthermore, the overall channel gains, $\sum_{i=1}^{n} \left|h_{SD}^{(i)}\right|^2$ and $\sum_{i=1}^{n} \sum_{j=1}^{M+1} \left|h_{SD,j}^{(i)}\right|^2$ are chi-square variable with $2n$ and $2n(M+1)$ degrees of freedom, respectively, and, following the equation 2.19, the conditioned probabilities of erroneous reception can be finally written as

$$p_{e,1\times1}(n) = \frac{F_{\chi^2}(\mu, 2n)}{F_{\chi^2}(\mu, 2(n-1))}, \qquad (2.45\text{a})$$

$$p_{e,(M+1)\times1}(n) = \frac{F_{\chi^2}(\mu, 2n(M+1))}{F_{\chi^2}(\mu, 2(n-1)(M+1))}. \qquad (2.45\text{b})$$

Collaborative Transmission Due to the soft-combining method of HARQ-CC protocol, the achievable rates depend not only on the number of transmission trials n and total number of currently active relays $\tilde{k}_n$, but also on the exact time instants when these relays turned active, i.e., on $k_1, ..., k_{n-1}$ (recall that $\tilde{k}_n = \sum_{i=1}^{n-1} k_i$). In particular, the rate achievable by the relay R_i at transmission n reads:

$$C_{R_i}(n; k_1, .., k_{n-1}) = \log\left[1 + \sum_{j=1}^{n}\left(\left|h_{SR_i}^{(j)}\right|^2 + \sum_{l=1}^{\tilde{k}_j} \left|h_{R_l R_i}^{(j)}\right|^2\right)\frac{\alpha P}{N_0}\right]. \qquad (2.46)$$

Notice that at time-slot n, the fading term in (2.46) is a chi-square random variable with

$$2\sum_{j=1}^{n}(1 + \tilde{k}_j) = 2(n + \sum_{j=1}^{n}\sum_{l=1}^{j-1} k_l)$$

$$= 2(n + \sum_{j=1}^{n-1} k_j(n-j)) \qquad (2.47)$$

degrees of freedom. Therefore, the probability that in the n-th trial the relay R_i still does not successfully decode, given that in attempts i, $i = 1, .., n - 1$, k_i relays turned to active mode (but not before), is

$$
\begin{aligned}
\bar{p}_{R_i}(n, k_1, .., k_{n-1}) &= P\left\{ C_{R_i}(n, k_1, .., k_{n-1}) < C_0 | C_{R_i}(n - 1, k_1, .., k_{n-2}) < C_0 \right\} \\
&= \frac{F\left[\mu_\alpha, 2\left(n + \sum_{j=1}^{n-1} k_j(n - j) \right) \right]}{F\left[\mu_\alpha, 2\left(n - 1 + \sum_{j=1}^{n-2} k_j(n - j) \right) \right]}.
\end{aligned}
\tag{2.48}
$$

It follows that the probability that k_n relays successfully decode in the current slot is

$$
p_R(k_n | k_1, ... k_{n-1}) = P_{bin}(\bar{p}_{R_i}(n, k_1, .., k_{n-1}), M - \tilde{k}_n, k_n),
\tag{2.49}
$$

and, finally, the probability $p_R(k_1, ..., k_n)$ reads

$$
p_R(k_1, k_2, ..., k_n) = \prod_{i=1}^{n} p_R(k_i | k_1, ..., k_{i-1}).
\tag{2.50}
$$

The rate achievable in the n-th transmission at the destination, given that $k_1, ... k_{n-1}$ relays were activated in the previous transmissions, is

$$
C_D(n, k_1, .., k_{n-1}) = \left[1 + \sum_{l=1}^{n} \left(\left| h_{SD}^{(l)} \right|^2 + \sum_{j=1}^{\tilde{k}_n} \left| h_{R_j D}^{(l)} \right|^2 \right) \frac{P}{N_0} \right],
\tag{2.51}
$$

with the fading term as a chi-square random variable with $n + \sum_{j=1}^{n-1} k_j(n - j)$ degrees of freedom. Thus, the probability of unsuccessful decoding at the nth transmission trial conditioned on the event that the previous $(n - 1)$ transmissions were unsuccessful is

$$
\begin{aligned}
P\left\{ C_D(n, k_1, .., k_{n-1}) < C_0 | C_D(n - 1, k_1, .., k_{n-2}) < C_0 \right\} &= \\
= \frac{F\left[\mu, 2\left(n + \sum_{j=1}^{n-1} k_j(n - j) \right) \right]}{F\left[\mu, 2\left(n - 1 + \sum_{j=1}^{n-2} k_j(n - j) \right) \right]}.
\end{aligned}
\tag{2.52}
$$

Accordingly, the probability of unsuccessful decoding at the n-th transmission (given that the previous $(n-1)$ transmissions were unsuccessful) can be written as

$$p_e(n) = \sum_{\mathcal{K}} p_R(k_1, ..., k_{n-1}) \times P\left\{C_D(n, k_1, .., k_{n-1}) < C_0 | C_D(n-1, k_1, .., k_{n-2}) < C_0\right\},$$
(2.53)

or, combining (2.48)-(2.50) and (2.52) with the previous equation

$$p_e(n) = \sum_{\mathcal{K}} \left\{ \prod_{i=1}^{n} P_{bin}\left(\frac{F\left[\mu_\alpha, 2\left(i + \sum_{j=1}^{i-1} jk_{i-j}\right)\right]}{F\left[\mu_\alpha, 2\left(i - 1 + \sum_{j=1}^{i-2} jk_{i-j}\right)\right]}; M - \tilde{k}_i, k_i \right) \times \right.$$
$$\left. \times \frac{F\left[\mu, 2\left(n + \sum_{j=1}^{n-1} jk_{n-j}\right)\right]}{F\left[\mu, 2\left(n - 1 + \sum_{j=1}^{n-2} jk_{n-j}\right)\right]} \right\}.$$
(2.54)

2.2.3 HARQ-IR

As mentioned, the performance of HARQ-IR protocol is lower bounded by the performance of HARQ-CC protocol, discussed in previous section. Therefore, throughout this section we will focus on the upper bound, described in Appendix A and already used in section 2.1.3

Direct Transmission According to Appendix A the upper bounds for the achievable SISO 1×1 and MISO $(M+1) \times 1$ rates at the destination for the HARQ-IR protocol can be written respectively as

$$C_{D,1\times1}(k) \leq k \log\left(1 + \sum_{i=1}^{k} \left|h_{SD}^{(i)}\right|^2 \frac{P}{kN_0}\right),$$
(2.55a)

$$C_{D,(M+1)\times1}(k) \leq k \log\left(1 + \sum_{i=1}^{k} \sum_{j=1}^{M+1} \left|h_{SD,j}^{(i)}\right|^2 \frac{P}{kN_0}\right),$$
(2.55b)

and the respective upper bounds are

$$p_{e,1\times1}(n) \geq \frac{F_{\chi^2}(\mu(n), 2n)}{F_{\chi^2}(\mu(n-1), 2(n-1))},$$
(2.56a)

$$p_{e,(M+1)\times1}(n) \geq \frac{F_{\chi^2}(\mu(n), 2n(M+1))}{F_{\chi^2}(\mu(n-1), 2(n-1)(M+1))}.$$
(2.56b)

Collaborative Transmission   Using the relation between the HARQ-CC protocol and the upper bound for HARQ-IR protocol performance, we can directly apply (2.54), substituting μ with corresponding $\mu(n)$

$$
p_e(n) \geq \sum_{\mathcal{K}} \left\{ \prod_{i=1}^{n} P_{bin} \left(\frac{F\left[\mu_\alpha(i), 2\left(i + \sum_{j=1}^{i-1} jk_{i-j}\right)\right]}{F\left[\mu_\alpha(i-1), 2\left(i-1 + \sum_{j=1}^{i-2} jk_{i-j}\right)\right]} ; M - \tilde{k}_i, k_i \right) \times \right.
$$
$$
\left. \times \frac{F\left[\mu(n), 2\left(n + \sum_{j=1}^{n-1} jk_{n-j}\right)\right]}{F\left[\mu(n-1), 2\left(n-1 + \sum_{j=1}^{n-2} jk_{n-j}\right)\right]} \right\}.
\tag{2.57}
$$

CHAPTER 3

NUMERICAL RESULTS

In order to get insight into the performance of the considered schemes, in this chapter we provide some numerical results based on the analytical solutions derived in the previous chapter. Following the analysis model, the remaining of this chapter is divided into single and multi-relay sections. In each of two sections we will consider the performance of HARQ-IR protocol separately, due to its interesting behavior and the different analysis method, based on the derived bounds. Furthermore, numerical results will be typically presented in terms of the average delay and/or the average achievable throughput versus Signal-to-Noise Ratio.

3.1 Single-Relay Model

Based on the analytical results provided in Section 2.1, numerical results for single-relay protocols are presented in the remaining of this section. We will first focus on the simple HARQ-TI and HARQ-CC protocols, while, as mentioned, the HARQ-IR protocol will be examined in separate section.

3.1.1 HARQ-TI and HARQ-CC Protocols

The influence of the SNR gain at the relay, depicted through the parameter α, on the system performance is investigated first. Fig. 3.1 and fig. 3.2 show the average delay $E[T]$ and the throughput $C_0/E[T]$ versus SNR ratio, respectively, for the Collaborative HARQ-CC scheme with transmission rate $C_o = 2 \, nat/s/Hz$ and different values of the source-relay gain α. Moreover, the direct transmission 1×1 SISO and 2×1 MISO models are presented as a reference. Let us focus first on the relatively low SNR region. As expected, as the quality of the source-relay link increases, the performance of the collaborative model starts
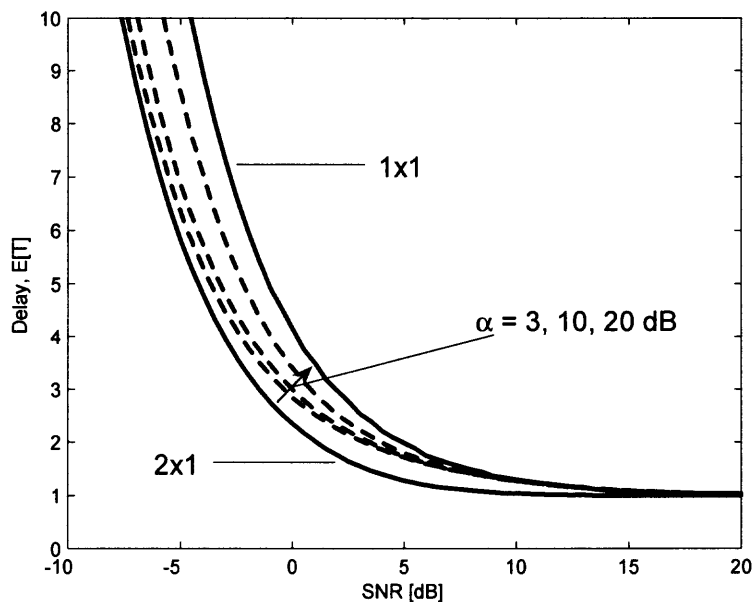
Figure 3.1 Average number of transmissions versus SNR, for different SNR gains at the relay (HARQ-CC scheme, $C_0 = 2\ nat/s/Hz$).

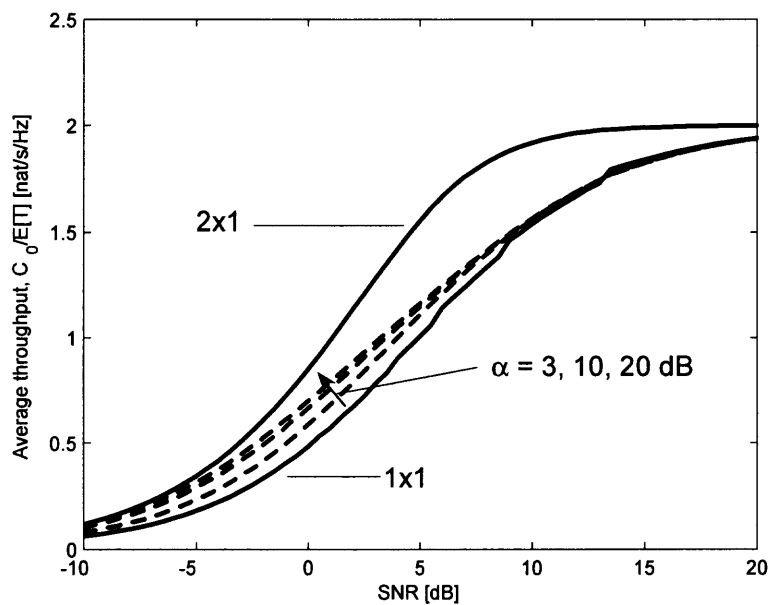**Figure 3.2** Average achievable throughput versus SNR, for different SNR gains at the relay (HARQ-CC scheme, $C_0 = 2\ nat/s/Hz$).

to resemble the performance of 2×1 MISO model, and on the opposite, as α decreases, the collaborative model behaves similar to 1×1 SISO direct transmission model. This SNR region is characterized by numerous retransmissions, and the diversity advantage of MISO in very first transmission is not relevant. On the other hand, in the relatively high SNR region, characterized by an average of two or less transmissions (equivalently, one or less retransmissions), MISO direct transmission clearly outperforms collaborative scheme, due to the increased impact of diversity in the first transmission achieved by the MISO model. Obviously, in order for the relay to significantly aid the source-destination communication, no matter what the quality of the source-relay link is, the system should work in a relatively low SNR region, where the expected number of transmissions is larger than two. In the following, where not stated otherwise, we will use $\alpha = 20dB$.

Fig. 3.3 and fig. 3.4 show the average delay and the throughput versus SNR, respectively, comparing the performance of HARQ-TI and HARQ-CC scheme, for the transmission rates $C_0 = 2\ nat/s/Hz$ and $C_0 = 4\ nat/s/Hz$. The benefit of using the HARQ-CC scheme is quite obvious in low SNR region, where the numerous trials provide the HARQ-CC scheme with enough memory to capitalize on. This advantage starts to fade as the SNR increases (as the available memory decreases), but is still visible until the average number of transmission falls to less than two transmissions (one retransmission). Moreover, the impact of the transmission rate is quite significant. As the rate increases, the number of retransmissions needed to achieve this rate increases significantly, which can lead to the system overload (throughput reduced to zero) in the low SNR region. This is particularly expressed for memoryless HARQ-TI. Also, the SNR required to achieve the maximum throughput C_0, when no retransmissions are needed, is increased. The results above are further confirmed in fig. 3.5, that captures the performance of both HARQ protocols and different transmission models.

From the discussion above, there exists a trade-off between transmission rate C_0 and the average number of transmissions $E[T]$, that determines the average throughput.
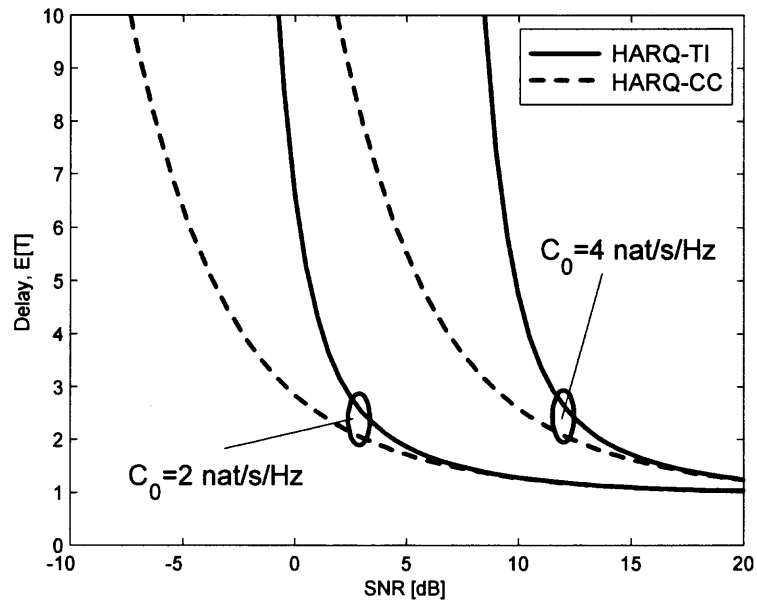
Figure 3.3 Average number of transmissions versus SNR for collaborative HARQ-TI and HARQ-CC protocols and two transmission rates.
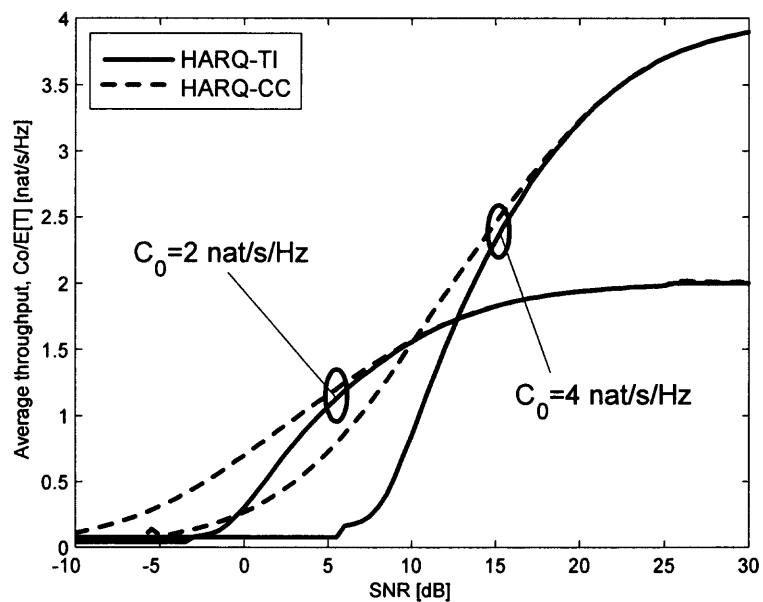

Figure 3.4 Average achievable throughput versus SNR for collaborative HARQ-TI and HARQ-CC protocols and two transmission rates.
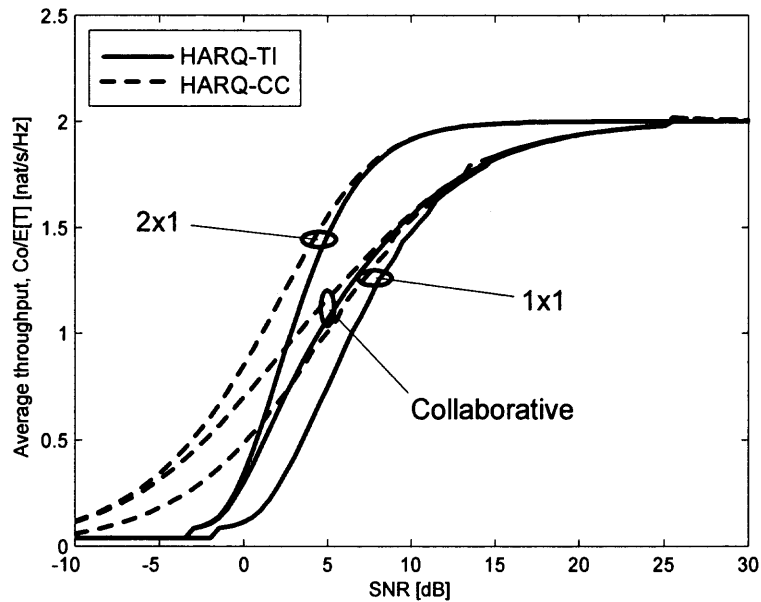
Figure 3.5 Average throughput versus SNR for HARQ-TI and HARQ-CC protocols and different transmission models $(C_0 = 2 \ nat/s/Hz)$.

We elaborate on this issue in fig. 3.6, where the average throughputs of the collaborative HARQ-CC systems with different transmission rates are presented. It can be noticed that the higher transmission rates fully exploit the system resources at high SNR values, but can lead to the system overload in the low SNR region. On the other hand, low transmission rate systems show excellent performance in lower SNR regions, but are unable to exploit the benefits of high SNR. The envelope of the fixed-rate curves (solid line) in fig. 3.6, which shows the maximum throughputs at each SNR value, depicts the possible performance of the system that adaptively selects the transmission rate C_0, based on the knowledge of the average SNR at the transmitter. Such a system would fully exploit the system resources, but would require the knowledge of the average SNR at the source.

Fig. 3.7 extends the example depicted in fig. 3.6 by showing the average throughputs for the collaborative, the 1×1 and the 2×1 systems for adaptive rate allocation scheme mentioned above. Since the ability to adapt the transmission rate yields a relatively low expected number of retransmissions, following the previous discussion, both collaborative
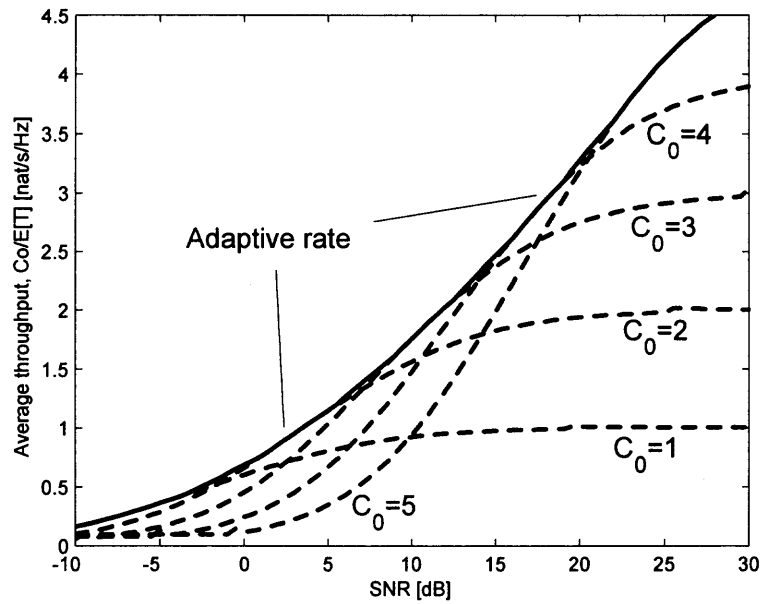
Figure 3.6 Average throughput of systems with fixed transmission rates and of the system with the adaptive rate allocation ability (Collaborative HARQ-CC scheme).
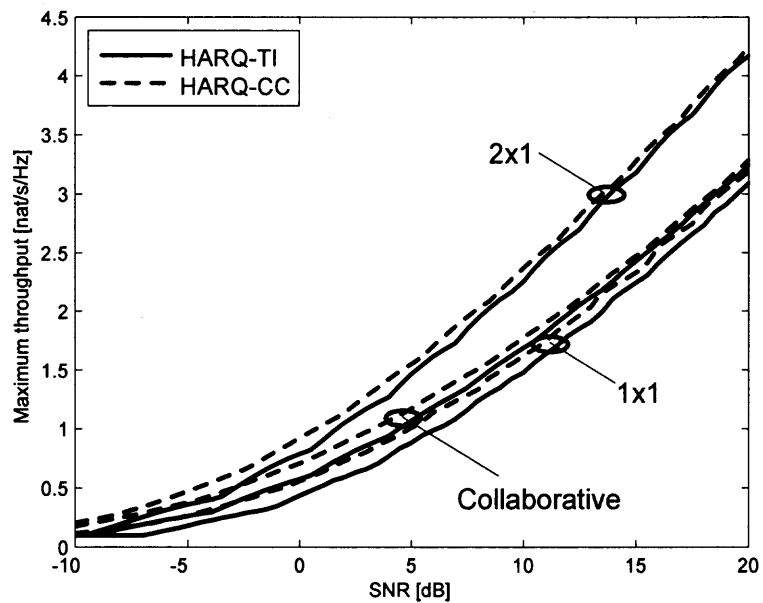

Figure 3.7 Average throughput with adaptive rate allocation based on the average SNR at the transmitter (HARQ-CC protocol for different transmission models).

HARQ protocols perform close to the 1×1 lower bound, while HARQ-CC provides only a slightly increased throughput in comparison to HARQ-TI protocol.

3.1.2 HARQ-IR Protocol

As we mentioned at the beginning of the chapter, HARQ-IR should be examined individually, due to its interesting behavior and the fact that the bounds are used for the performance assessment. While the former reason will be discovered as we proceed with system investigation, we start with the latter, checking the validity of the bounds. Fig. 3.8 and fig. 3.9 show the simulated average delay and the average throughput versus SNR, respectively, for different transmission models using HARQ-IR protocol, along with corresponding upper and lower bounds derived in the Section 2.1.2. It is seen that, while the lower bound that coincides with the performance of HARQ-CC is very loose, the upper bound matches well with the actual simulated system delay or the throughput. Henceforth, we will describe the performance of this protocol through its upper bound.

Fig. 3.10 shows the average delay versus SNR for collaborative HARQ-IR systems using the different transmission rates. Besides the expected increase of the delay with the more demanding rate, we notice a slight curve flooring around the two-transmissions area, more visible as the transmission rate increases. In fact, in order to further reduce the number of transmissions from $T = 2$ to $T = 1$, a large power increase is required due to the effectiveness of the second transmission as compared to the first. We could expect that the flooring effect appears not only in the area of two, but for three and more transmission. As we will realize later, this is true, but as the number of transmissions increases, the relative gain decreases, and this behavior is harder to observe. Furthermore, this behavior is not noticed for HARQ-CC and HARQ-TI protocols, since, though the retransmission brings possible diversity and power gains, these protocols are not as powerful in exploiting the memory, or not exploiting the memory at all. On the other hand, increased transmission rate has an exponential impact on system performance, as visible in (2.8), and has a 'stretching'
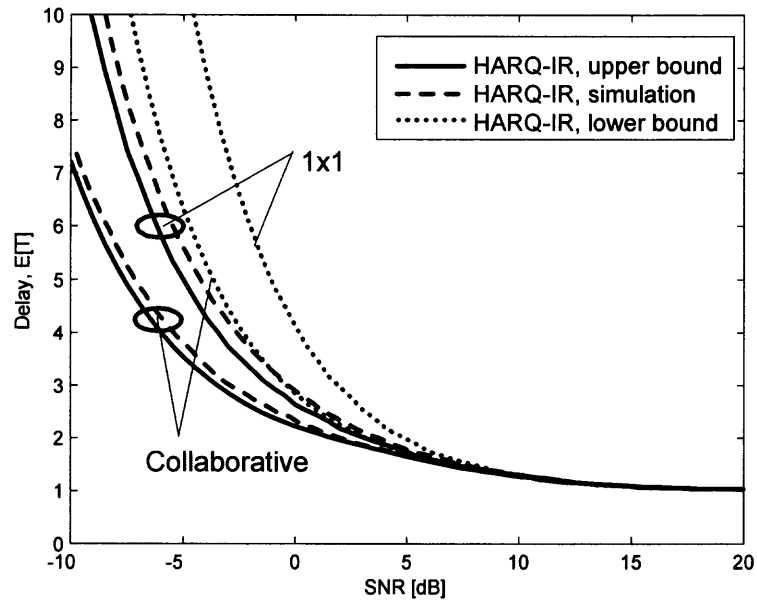
Figure 3.8 Average number of transmissions versus SNR, upper and lower bound and simulated delay (HARQ-IR scheme, $C_0 = 2\ nat/s/Hz$).

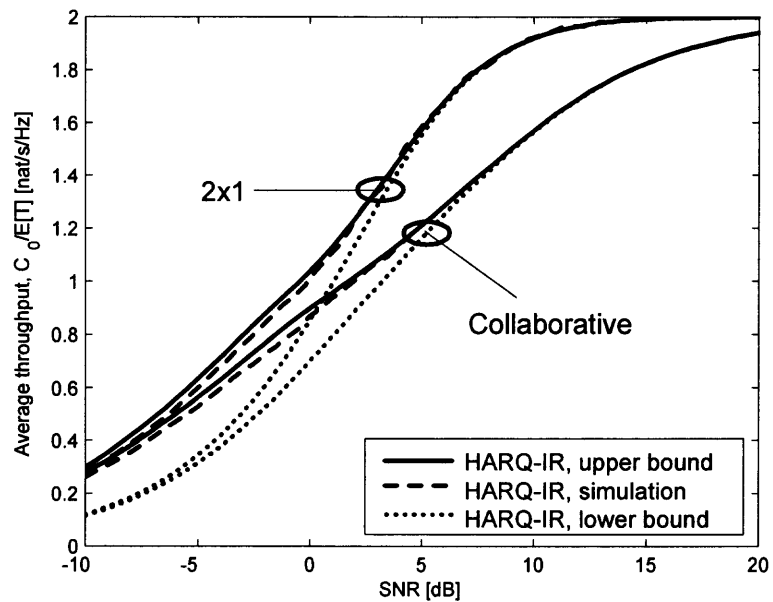**Figure 3.9** Average throughput versus SNR, upper and lower bound and simulated delay (HARQ-IR scheme, $C_0 = 2\ nat/s/Hz$).
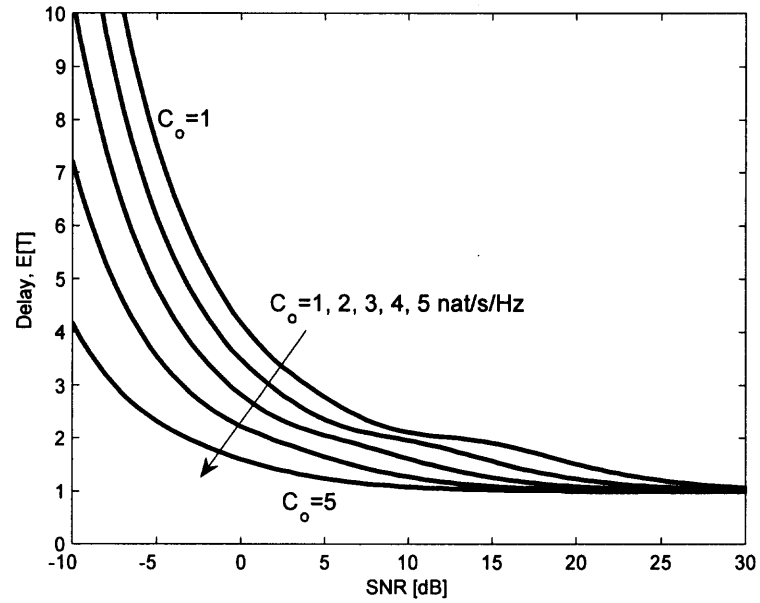
Figure 3.10 Average number of transmissions versus SNR for collaborative HARQ-IR with different transmission rates C_0.
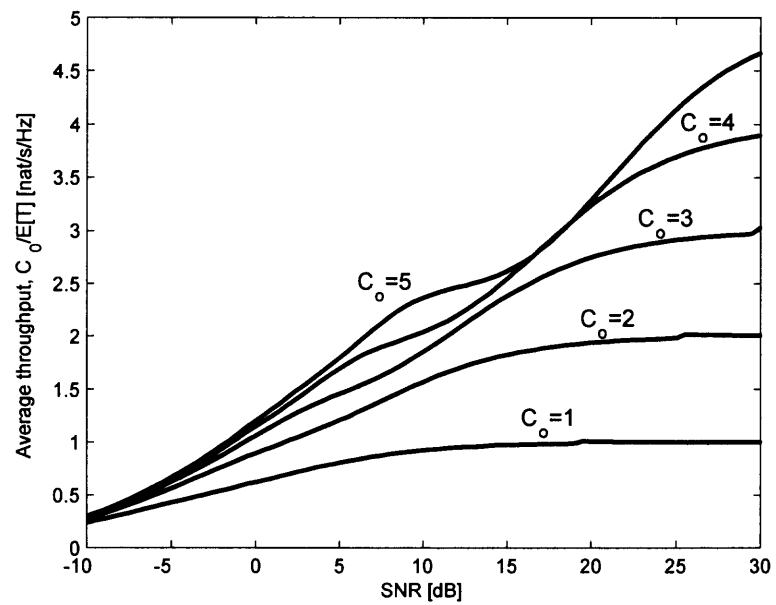

Figure 3.11 Average throughput versus SNR for collaborative HARQ-IR with different transmission rates C_0 $[nat/s/Hz]$.

effect on the SNR ratio ($\frac{SNR}{C_0}$), making the floored periods longer. The described effect is even more visible if instead of the delay, we present the throughput curves of the same systems (fig. 3.11).

Fig. 3.11 reveals that increasing the transmission rate does not imply a reduction of the average throughput, which is another unique feature of the HARQ-IR protocol. This behavior is notably different from that of less powerful HARQ schemes, such as HARQ-TI or HARQ-CC (fig. 3.6). Even though the increased rate leads to more retransmissions (fig. 3.10), due to the code-combining effectiveness of the HARQ-IR retransmission, the average throughput is in general not decreased. As a conclusion, the HARQ-IR protocol has the remarkable ability of resilience towards low SNR, or equivalently, towards increased starting rate. Notice that, however, this result (due to our assumption) does not take into account the impact of signaling overhead due to retransmissions.

The following figures, fig. 3.12 and fig. 3.13 show the throughputs of a 1×1 SISO and a 2×1 MISO models respectively, and stress the effectiveness of the HARQ-IR protocol. Notice that, though not as emphasized as in MISO case due to the lack of power and diversity gain, the flooring effect is visible for SISO model also. In general, we should expect this phenomenon to be more visible as we encounter the model with more relays or transmitting antennas, as the following section shows.

3.1.3 Comparison of HARQ Protocols

Comparison of the three HARQ schemes and different transmission models is presented in fig. 3.14, showing the average throughput versus SNR (the performance of 1×1 SISO model is omitted, for the sake of clarity). The superiority of the HARQ-IR schemes is clearly visible, especially in the low SNR region characterized by the numerous transmission. Of course, the performances of the three protocols converge at high SNR where only a few retransmissions are needed. Note also that the relation between different transmission models does not depend significantly on HARQ protocol, since there is always the

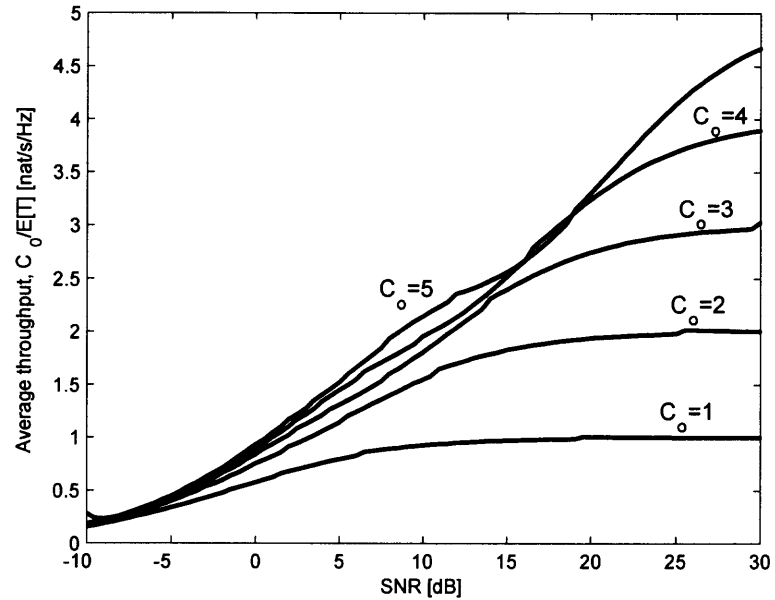**Figure 3.12** Average throughput versus SNR for SISO 1×1 HARQ-IR with different transmission rates C_0 $[nat/s/Hz]$.

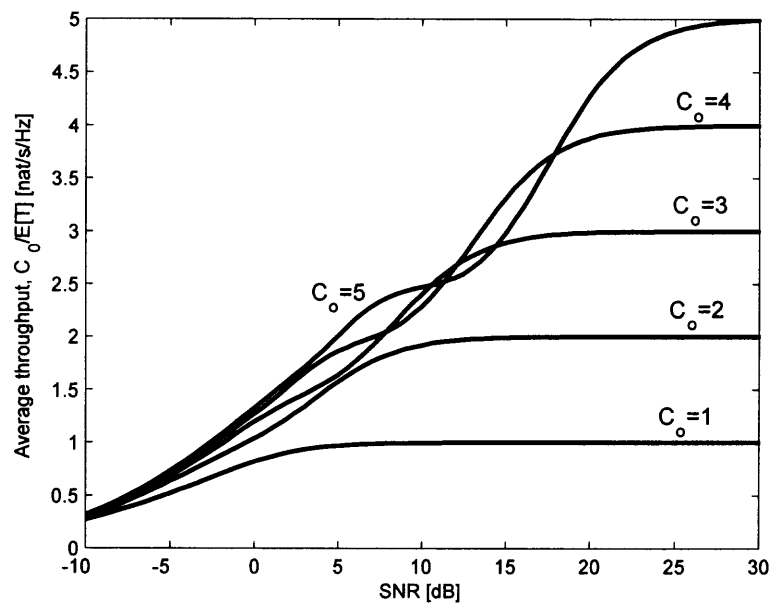**Figure 3.13** Average throughput versus SNR for MISO 2×1 HARQ-IR with different transmission rates C_0 $[nat/s/Hz]$.
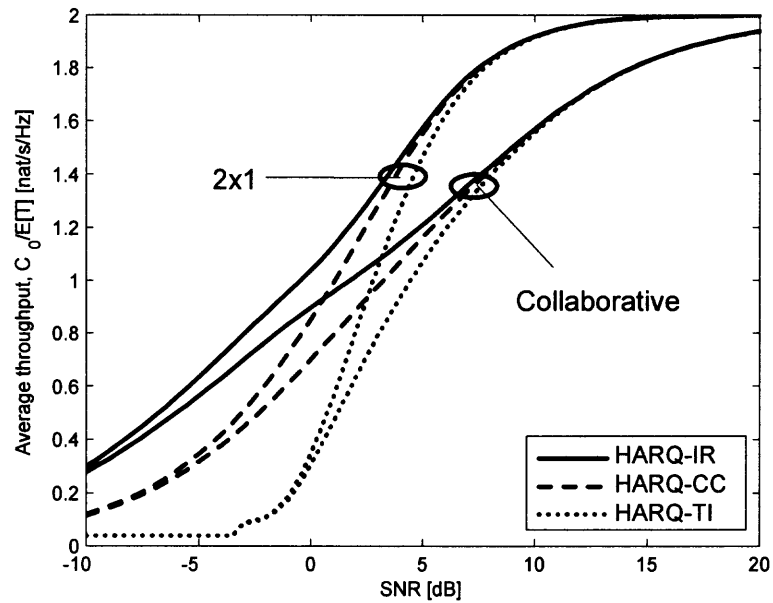
Figure 3.14 Average throughput versus SNR, for different HARQ protocols and transmission models ($C_0 = 2 \, nat/s/Hz$).

convergence of collaborative and 2×1, and collaborative and 1×1 transmission (not shown here), at the relatively low and high SNR, respectively, as long as the same HARQ protocol is used.

Note that the comparison of protocols in terms of transmission rate was already discussed in Section 3.1.2.

3.2 Multi-Relay Model

3.2.1 HARQ-TI and HARQ-CC Protocols

Fig. 3.15 and fig. 3.16 present the average throughputs of the multi-relay Collaborative HARQ-TI and HARQ-CC systems versus SNR, respectively, with different number of relays and transmission rate $C_0 = 3 \, nat/sHz$. For comparison, the throughput of the lower bound, SISO, and the upper bound MISO (the maximum number of relays 10 yields a 11×1 bound) are also shown. Moreover, fig. 3.17 is provided as a reference for the delay of the Collaborative HARQ-CC scheme. Adding relay stations yields relevant performance
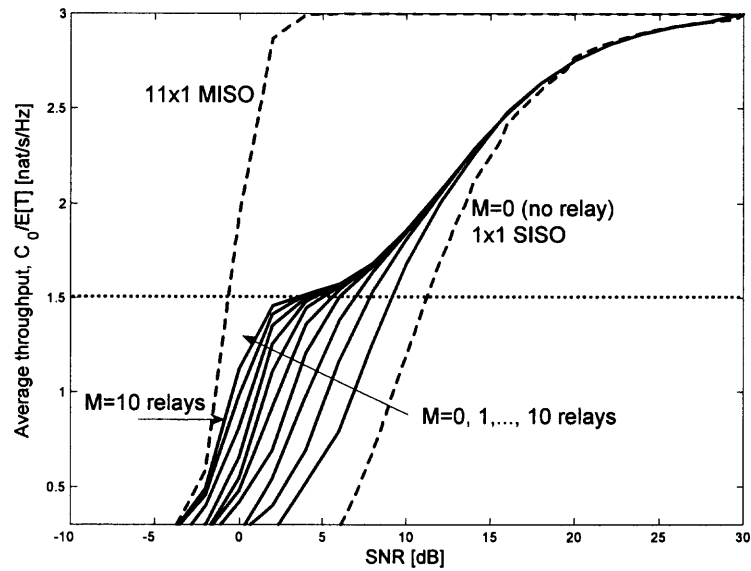
Figure 3.15 Average throughput versus SNR, multi-relay model (collaborative HARQ-TI scheme, $C_0 = 3 \, nat/s/Hz$).
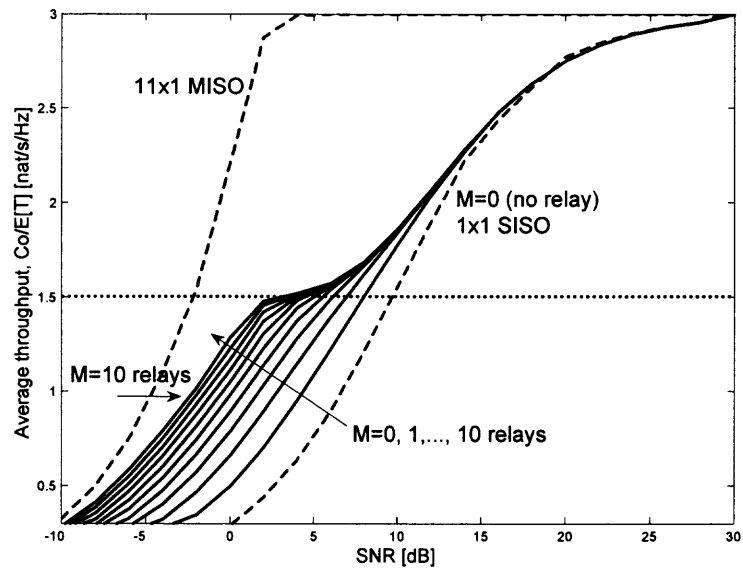

Figure 3.16 Average throughput versus SNR, multi-relay model (collaborative HARQ-CC scheme, $C_0 = 3 \, nat/s/Hz$).
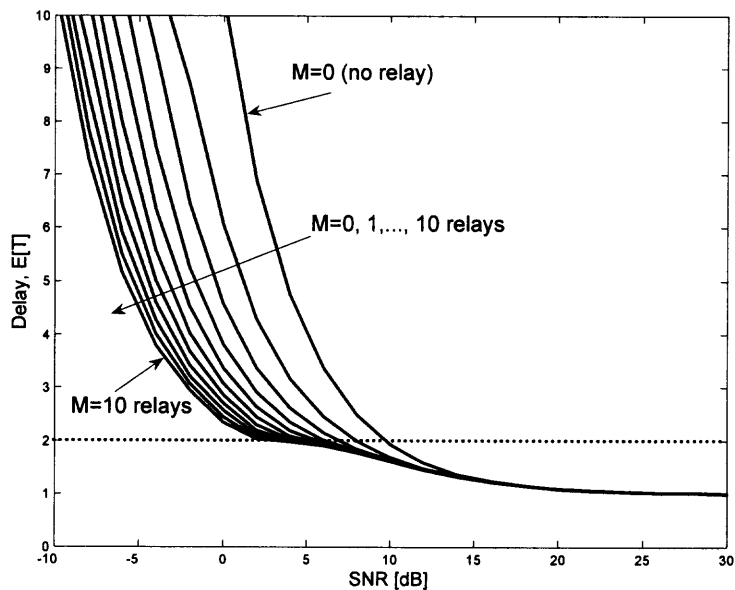
Figure 3.17 Average number of transmissions versus SNR, multi-relay model (collaborative HARQ-CC scheme, $C_0 = 3 \ nat/s/Hz$).

benefits in the relatively low SNR region, where the number of retransmissions is relevant. However, as the average number of retransmissions decreases to less than one (i.e., for the average throughput larger than $C_0/2 = 1.5 \ nat/s/Hz$), cooperation does not actually occur and the performance reduces to the lower bound given by the SISO scenario. Furthermore, increasing the number of relays M results in better performance, but less so as M increases, due to the diminishing diversity gains. Moreover, note that with increasing the number of relays, the flooring phenomenon for HARQ-TI and HARQ-CC becomes more expressed, due to the increased power and diversity gain. However, none of these protocols exploits memory sufficiently, or not at all, in order for this effect to be observable for number of retransmissions greater than one, i.e., for $C_0/3$, $C_0/4$ and so on.

3.2.2 HARQ-IR Protocol

The flooring performance that we encountered in Sections 3.1.2 and 3.2.1 is confirmed and emphasized on fig. 3.18-3.21, that show the average delay and the throughput for
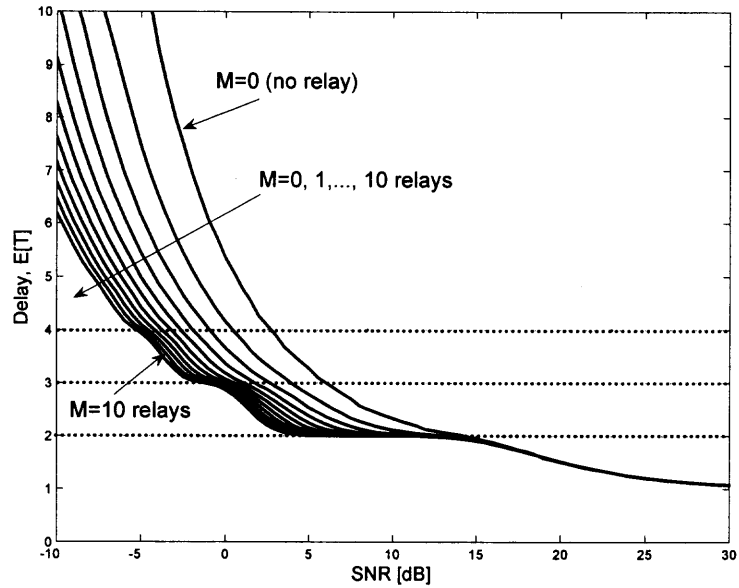
Figure 3.18 Average number of transmissions versus SNR, multi-relay model (collaborative HARQ-IR scheme, $C_0 = 5\ nat/s/Hz$).

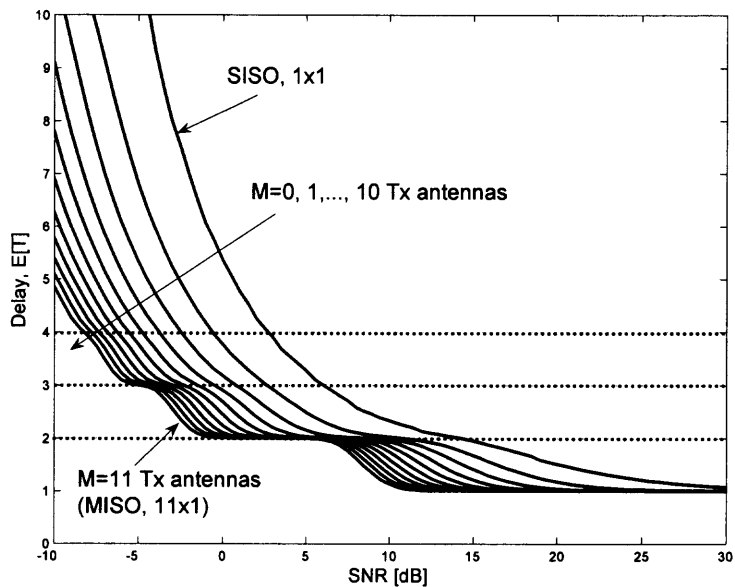**Figure 3.19** Average number of transmissions versus SNR, MISO model (HARQ-IR scheme, $C_0 = 5\ nat/s/Hz$).
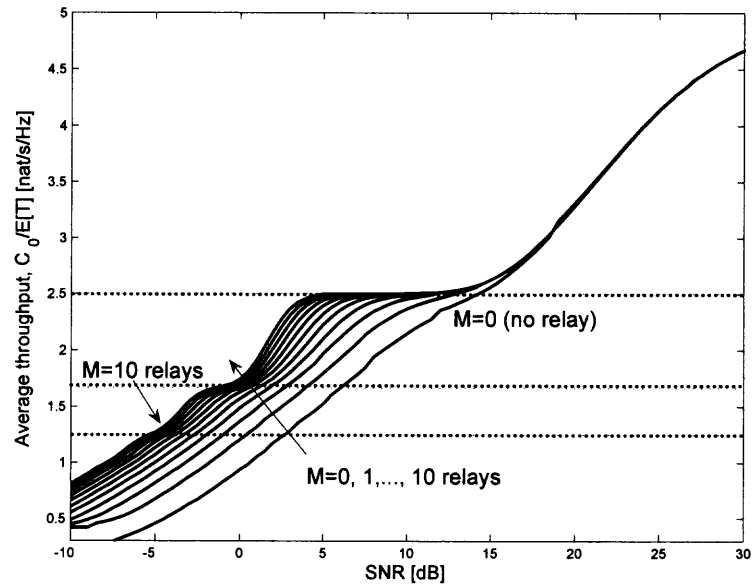
Figure 3.20 Average throughput versus SNR, multi-relay model (collaborative HARQ-IR scheme, $C_0 = 5$ $nat/s/Hz$).
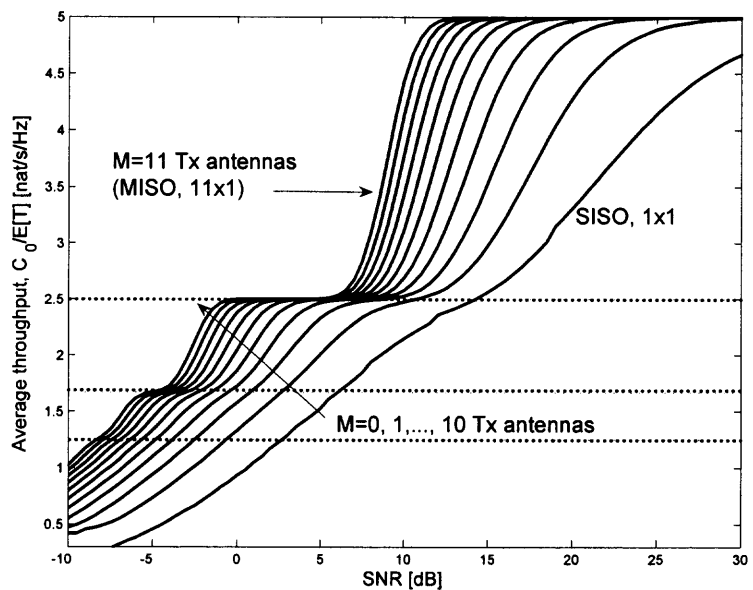
Figure 3.21 Average throughput versus SNR, MISO model (HARQ-IR scheme, $C_0 = 5$ $nat/s/Hz$).

collaborative, SISO and MISO HARQ-IR schemes. As the number of relays (or, in MISO case, antennas) increase, the flooring effect becomes visible for $E[T] = 3, 4$, and possibly more average transmissions. In general, increasing the number of transmitters leads to a more discrete shape of delay and throughput curve, due to the increased importance of each retransmission. Note that at the low SNR region, curves for the MISO direct transmission and the collaborative transmission model behave very similar. The reason behind is, of course, that the starting advantage (the diversity in the first transmission) of the MISO model is diminished by the numerous retransmissions. Consequently, at the higher SNR area, where less than one retransmission is needed, the collaborative model starts to converge towards the SISO model, while the MISO model shows the superiority of the first-trial diversity.

Finally, fig. 3.22 and fig. 3.23 present the throughput versus SNR for the Collaborative and MISO HARQ-IR with different transmission rates, showing the robustness of HARQ-IR protocol to the trade-off issue, more emphasized on the collaborative model. Though there is some observable trade-off between the throughput and the transmission rate, HARQ-IR protocol is completely resilient to transmission overload that results from the high transmission rate demands in a relatively low SNR region.

Figure 3.22 Average throughput versus SNR for different transmission rates ($M = 10$ relays, collaborative HARQ-IR scheme).

Figure 3.23 Average throughput versus SNR for different transmission rates (11×1 MISO HARQ-IR scheme).
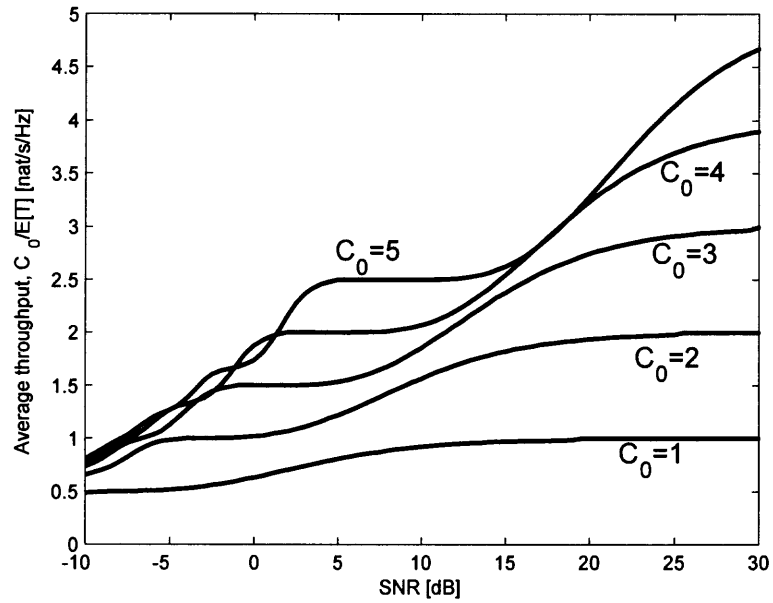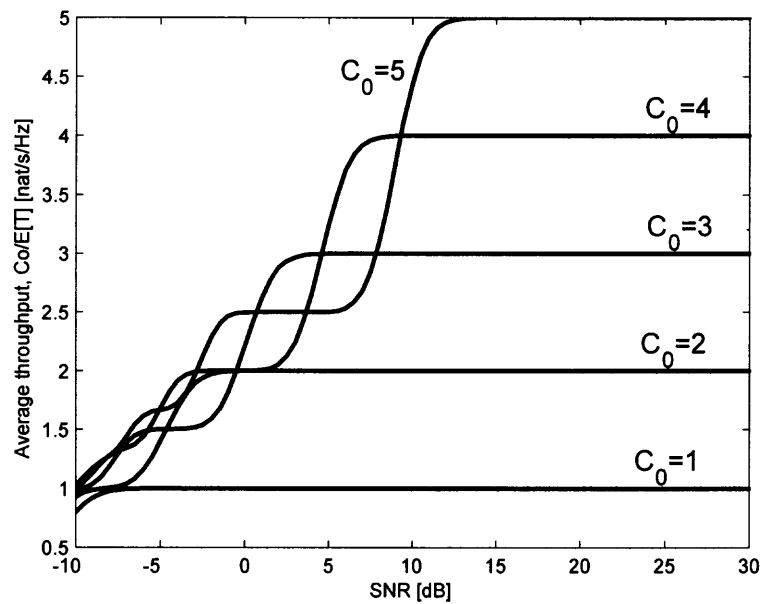
CHAPTER 4

PRACTICAL IMPLEMENTATION OF SPACE-TIME BLOCK CODING

In the previous chapters, it was assumed that during the retransmission, the source and all the active relays transmit simultaneously, achieving full theoretical rate and diversity according to the information theoretic setting. In practice, however, the complex orthogonal Space-Time Block Codes (STBC) that can achieve full diversity and maximum rate do not exist for more than two transmitting antennas [8]. This implies that the results of Section 2.2 for the multi-relay model, numerically presented in Section 3.2, cannot be fully reached by using STBC codes. This chapter we therefore dedicate to the issue of practical implementation of STBC in the Collaborative HARQ protocols.

In the following, we first provide an overview of STBC codes for two transmitting antennas (single-relay case), then proceed with the proposed solutions for reducing the effect of non-orthogonal reception in the multi-relay scenario, and in the final section corroborate these solutions through numerical results.

4.1 STBC Codes for the Single-Relay Model

In the pioneering work [9], Alamouti presented the STBC code that achieves the maximum rate and diversity for two transmitting antennas. Thus, for the single-relay system, where the maximum number of the simultaneous active nodes is two (source and relay), a simple practical solution is available.

Fig. 4.1 illustrates the two transmitting antennas example, that corresponds to the single-relay case. The packet consists of symbols S_1 and S_2, transmitted in two time slots. The first transmitting antenna, e.g., the source in collaborative scenario, sends the symbol S_1 in the first and S_2 in the second time slot, while the second antenna, or the relay, sends the symbols S_2^* and $-S_1^*$ in the first and the second slot, respectively. The signal received

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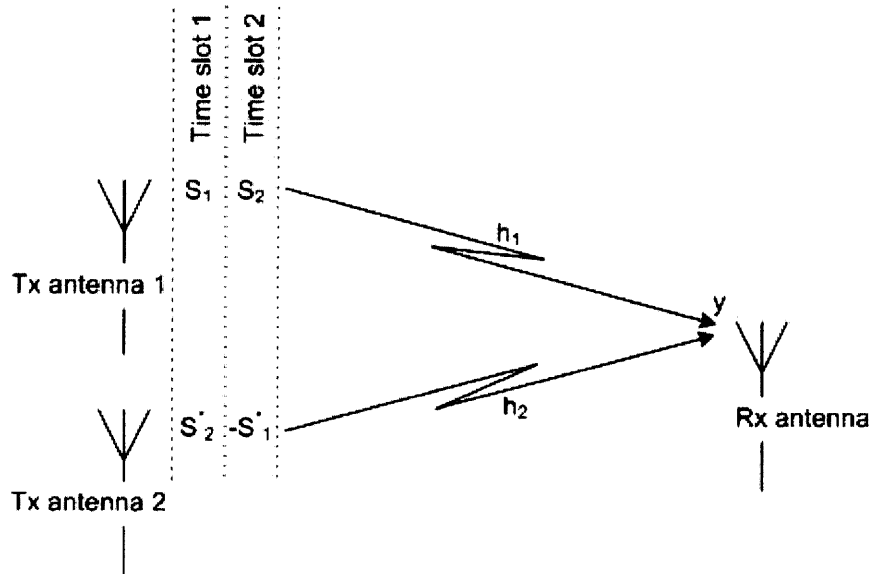
Figure 4.1 Space Time Block Coding scheme for two transmitting antennas.

in ith time slot, r_i, at the receiving antenna (destination) reads

$$r_1 = h_1 s_1 + h_2 s_2^* + n_1 \tag{4.1a}$$

$$r_2 = h_1 s_2 - h_2 s_1^* + n_2 \tag{4.1b}$$

where n_1 and n_2 stand for the uncorrelated white noise at the receiving antenna in the first and the second time slot, respectively. Note that the channels h_1 and h_2 are constant during two time slots, i.e. during one retransmission. After processing in the destination, interference-free estimates of the sent symbols are

$$y_1 = h_1^* r_1 + h_2 r_2^* = (|h_1|^2 + |h_2|^2)s_1 + h_1^* n_1 + h_2 n_2^* \tag{4.2a}$$

$$y_2 = h_2^* r_1 - h_1 r_2^* = (|h_1|^2 + |h_2|^2)s_2 - h_1 n_2^* + h_2^* n_1 \tag{4.2b}$$

The last equation can be written in the matrix form, if we introduce the matrix $\mathbf{H}_\nu$,

$$\mathbf{H}_\nu = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}, \tag{4.3}$$

called the equivalent channel matrix, and the vector form for the sent symbols and the noise

$$\mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}, \mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}. \tag{4.4}$$

The received signal (recall (4.1a)-(4.1b)) can be now written as

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \mathbf{H}_\nu \mathbf{s} + \mathbf{n}, \tag{4.5}$$

and the estimates (4.2a)-(4.2b),

$$
\begin{aligned}
\mathbf{y} &= \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H}_\nu^H \mathbf{r} \\
&= \mathbf{H}_\nu^H \mathbf{H}_\nu \mathbf{s} + \mathbf{H}_\nu^H \mathbf{n} \\
&= \begin{bmatrix} (|h_1|^2 + |h_2|^2)s_1 + h_1^* n_1 + h_2(-n_2) \\ (|h_1|^2 + |h_2|^2)s_2 - h_1(-n_2) + h_2^* n_1 \end{bmatrix}
\end{aligned}
\tag{4.6}
$$

Note that the noise term n_2 does not appear in the same manner in (4.6) and (4.2a)-(4.2b). However, since the inphase and the quadrature components of the additive Gaussian noise have the same statistics, it is justified to use (4.6) as a matrix equivalent of (4.2a)-(4.2b). Furthermore, the equivalent channel matrix $\mathbf{H}_\nu$ is orthogonal, $\mathbf{H}_\nu^H \mathbf{H}_\nu = \left(|h_1|^2 + |h_2|^2 \right) \mathbf{I}_2$, where $\mathbf{I}_2$ is 2×2 identity matrix, signifying that the estimates are interference-free and the full diversity of two is achieved.

4.2 STBC Codes for the Multi-Relay Model

4.2.1 Extended Alamouti Code

The problem of designing the orthogonal STBC codes for more than two transmitting antennas has recently become an intensive area of research. As mentioned, it was shown that the full rate orthogonal block codes do not exist for the practical complex environment [8], and some solutions were presented for nearly orthogonal full diversity rate, or orthogonal but not full-rate codes. One of the most interesting solutions is 'Extended

Alamouti' [10], full diversity non-orthogonal coding scheme. It is based on the Alamouti principle, by recognizing the expansion pattern from the single antenna (1×1 SISO) transmission (recall fig. 4.1 for the sent symbols and the equivalent channel matrix $\mathbf{H}_\nu$ in (4.3))

$$s_1 \rightarrow \begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix}, h_1 \rightarrow \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}. \qquad (4.7)$$

By exploiting this pattern

$$A_s \rightarrow \begin{bmatrix} A_s & B_s \\ B_s^* & -A_s^* \end{bmatrix}, \quad A_h \rightarrow \begin{bmatrix} A_h & B_h \\ -B_h^* & A_h^* \end{bmatrix}, \qquad (4.8)$$

for the sent symbols and the equivalent channel matrix, respectively, Alamouti scheme can be extended on four-antenna case as

$$\begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix} \rightarrow \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}, \qquad (4.9)$$

and

$$\begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \rightarrow \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}. \qquad (4.10)$$

Similarly to (4.4)-(4.6), the final matrix form reads

$$\mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}, \ \mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix},$$

$$\mathbf{H}_\nu = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}, \tag{4.11}$$

$$\mathbf{y} = \mathbf{H}_\nu^H \mathbf{H}_\nu \mathbf{s} + \mathbf{H}_\nu^H \mathbf{n} \tag{4.12}$$

The Extended Alamouti scheme for four transmitting antennas, i.e. for the system with three relays, often called ABBA according to the extension pattern (4.8), is illustrated in fig. 4.2.
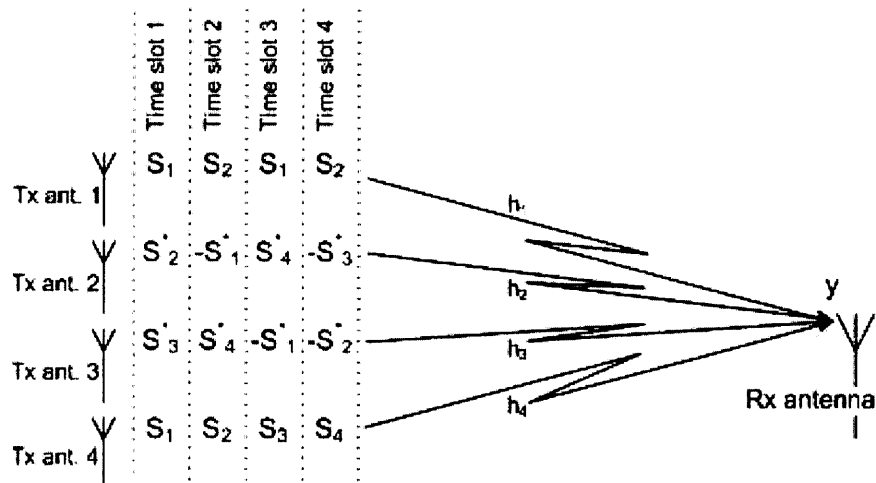

Figure 4.2 Extended Alamouti (ABBA) scheme for four transmitting antennas.

As mentioned, ABBA coding results in the full rate non-orthogonal reception,

$$H_\nu^H H_\nu = \sum_{i=1}^{4} |h_i|^2 \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\ 0 & -X & 1 & 0 \\ X & 0 & 0 & 1 \end{bmatrix},$$ (4.13)

with the non-orthogonal factor

$$X = \frac{1}{\sum_{i=1}^{4} |h_i|^2} 2\operatorname{Re}\left(h_1 h_4^* - h_2 h_3^*\right)$$ (4.14)

System with three transmitting antennas can be easily achieved using the presented scheme with one of the channels, say the fourth one, in deep fade, $h_4 = 0$, equivalent to turning off one of antennas. The same ABBA extension scheme and puncturing principle can be used to employ STBC for more than four transmitting antennas, i.e. for the collaborative system with more than three relays.

4.2.2 Extended Alamouti Code with Feedback

In the processing matrix $H_\nu^H H_\nu$ for four antennas (4.13), only the cross-diagonal terms are non-zero, so the amount of the interference resulting from the non-orthogonality is relatively small. However, as proposed in [11], the interference can be further reduced by attempting to minimize the factor X (4.14). Simple way of altering the value X can be achieved by altering the pattern of the sent symbols, for example inverting the sign or conjugating the symbols from one or more antennas. This would effect the equivalent channel matrix H_ν, which would in turn change the value of factor X.

In [11], the following four modifications of ABBA model were proposed:

$$\bullet\ \mathbf{S}_1 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \rightarrow \mathbf{H}_{\nu1} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix},$$

$$\bullet \ \mathbf{S}_2 = \begin{bmatrix} -s_1 & s_2 & s_3 & s_4 \\ -s_2^* & -s_1^* & s_4^* & -s_3^* \\ -s_3^* & s_4^* & -s_1^* & -s_2^* \\ -s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \rightarrow \mathbf{H}_{\nu 2} = \begin{bmatrix} -h_1 & h_2 & h_3 & h_4 \\ -h_2^* & -h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_{4^*} & -h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & -h_1 \end{bmatrix},$$

$$\bullet \ \mathbf{S}_3 = \begin{bmatrix} js_1 & -js_2 & s_3 & s_4 \\ js_2^* & js_1^* & s_4^* & -s_3^* \\ js_3^* & -js_4^* & -s_1^* & -s_2^* \\ ks_4 & js_3 & -s_2 & s_1 \end{bmatrix} \rightarrow \mathbf{H}_{\nu 3} = \begin{bmatrix} jh_1 & -jh_2 & h_3 & h_4 \\ -jh_2^* & -jh_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & -jh_1^* & jh_2^* \\ h_4 & -h_3 & jh_2 & jh_1 \end{bmatrix} \text{ and}$$

$$\bullet \ \mathbf{S}_4 = \begin{bmatrix} js_1 & js_2 & s_3 & s_4 \\ js_2^* & -js_1^* & s_4^* & -s_3^* \\ js_3^* & js_4^* & -s_1^* & -s_2^* \\ js_4 & -js_3 & -s_2 & s_1 \end{bmatrix} \rightarrow \mathbf{H}_{\nu 4} = \begin{bmatrix} jh_1 & jh_2 & h_3 & h_4 \\ jh_2^* & -jh_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & -jh_1^* & -jh_2^* \\ h_4 & -h_3 & -jh_2 & jh_1 \end{bmatrix},$$

resulting respectively in

$\bullet \ X_1 = \dfrac{1}{\sum_{i=1}^{4} |h_i|^2} 2 \operatorname{Re}\left(h_1 h_4^* - h_2 h_3^* \right)$ (regular ABBA),

$\bullet \ X_2 = \dfrac{1}{\sum_{i=1}^{4} |h_i|^2} 2 \operatorname{Re}\left(-h_1 h_4^* - h_2 h_3^* \right),$

$\bullet \ X_3 = -\dfrac{1}{\sum_{i=1}^{4} |h_i|^2} 2 \operatorname{Im}\left(h_1 h_4^* + h_2 h_3^* \right)$ and

$\bullet \ X_4 = -\dfrac{1}{\sum_{i=1}^{4} |h_i|^2} 2 \operatorname{Im}\left(h_1 h_4^* - h_2 h_3^* \right).$

Since the destination is the only node in the Alamouti scheme assumed to know the channel states, it chooses the pattern that yields minimum non-orthogonal factor, $X = min$ $\{X_1, X_2, X_3, X_4\}$, and broadcasts the decision to the source and the relays. Transmitters act accordingly and choose the pattern requested by the destination. Note that this optimization requires negligible extra resources, since the feedback is only 2 bits long. Moreover, the proposed modifications are purely empirical (i.e., there might be better solutions), but, as the following section shows, they yield very good results.
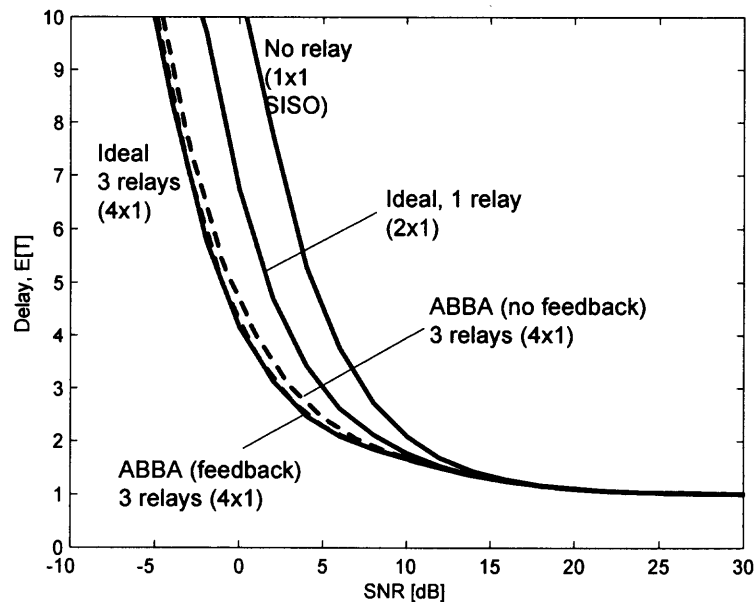
Figure 4.3 Simulated average number of transmissions versus SNR for the Collaborative HARQ-CC scheme using theoretical and Extended Alamouti scheme with and without feedback ($C_0 = 2\ nat/s/Hz$).

4.3 Simulation Results

The average number of transmissions versus SNR, for the no-relay (SISO), one- and three-relays Collaborative HARQ-CC scheme using the theoretical (ideal) and the extended Alamouti scheme, with and without feedback, for the transmission rate $C_0 = 2\ nat/s/Hz$, are presented in fig. 4.3. For the simulation purposes, uncoded packets transmitted through a complex Gaussian channel are assumed to be successfully decoded at the receiving station if less than 1% of its bits are erroneously received. When using the extended Alamouti (ABBA) scheme without feedback, the SNR loss is approximately $1dB$ comparing to the theoretical bound. On the other hand, the optimized ABBA (with feedback) for the three-relay system yields very good performance, with the negligible SNR loss comparing to the theoretical bound. Whether the SNR gain of $1dB$ or simplified protocol with no feedback is preferred, depends on the actual system requirements. As a conclusion, there exist practical solutions, such as Extended Alamouti, that could produce the performance

with the quality comparable to the theoretical multi-relay scenario examined in Sections 2.2 and 3.2.

CHAPTER 5

FINAL REMARKS

5.1 Conclusion

Future wireless systems are expected to rely on collaboration between terminals in order to mimic MIMO links in scenarios where multiple antenna technology is infeasible due to terminal size limitations. Collaborative Hybrid-ARQ emerged as one of the most promising collaboration protocols, involving the relay participation only *if needed,* thus saving system resources.

In this work, we investigated three different protocols, namely Collaborative HARQ-TI, HARQ-CC and HARQ-IR protocols, each of them employing STBC codes for the collaboration during the retransmission. Closed form expressions for the expected number of transmissions and average throughput for each protocol and any number of relays are derived in Chapter 2. For the Collaborative HARQ-IR protocol, upper and lower bounds were used. In the same chapter, direct-transmission models, namely SISO and MISO, were also analyzed for each of HARQ protocols, providing performance bounds on the Collaborative scenario.

Analytical results were corroborated by numerical results in Chapter 3, revealing some interesting characteristics of Collaborative HARQ protocols. It is shown that in the low SNR region collaboration yields performance close to the upper bound set by MISO system, while it tends to the performance of no collaboration (SISO) as the SNR increases. HARQ protocols were compared, confirming the superiority of HARQ-IR and the relatively poor performance of memoryless Hybrid-TI protocol. The influence of transmission rate was investigated, revealing the trade-off between the transmission rate and the average throughput in the low SNR region for HARQ-TI and HARQ-CC protocols, with the same effect negligible for HARQ-IR protocols, if signalling overhead is not considered.

Furthermore, it was confirmed that if the transmitting stations have the knowledge of *average* SNR at the destination, system does not need to employ collaboration, since the number of retransmissions is greatly reduced.

In Chapter 4, the issue of practical implementation of STBC codes for multi-relay scenario was approached. While STBC codes that achieve full diversity and transmission rate do not exist for the realistic complex environment and multi-relay transmission, it was shown that there exist suboptimal solutions with negligible degradation, thus justifying the multi-relay analysis performed in Chapter 2.

5.2   Open Issues

Here we list some possible directions for future research in the field of Collaborative HARQ:

- *Designing feasible MAC protocol.* Among the difficulties that can emerge when facing practical implementation,only the STBC issue was investigated in this thesis. Detailed design of Medium Access Control (MAC) protocol needs to be addressed, facing the challenges of the distributed nature of collaborative transmission.

- *Transmitting and processing power consumption.* Besides its complexity, Collaborative HARQ model has the disadvantage of the increased power consumption. In each transmission (or retransmission), relays either transmit or receive and decode the message. While transmission is usually regarded as the primary cause of the power consumption, the complexity of STBC processing can be quite demanding and its impact shouldn't be neglected. The research on power consumption issue should reveal whether this model saves power at a system level or actually consumes more then the simple SISO does.

APPENDIX A

DERIVATION OF PERFORMANCE BOUNDS FOR HARQ-IR PROTOCOL

According to [7], system using HARQ-IR protocol can be considered as a set of parallel links, where each of the links represents one retransmission attempt. In other words, each retransmission brings entirely new information, and the achievable rate of incremental redundancy protocol is equivalent to the summation of rates achieved during each of those retransmissions:

$$C(k) = \sum_{i=1}^{k} C_i, \qquad (A.1)$$

where C_i is the rate of memoryless system, achievable at receiving node in the ith attempt

$$C_i = \log\left(1 + x_i\right), \quad i = 1, 2, ..., \qquad (A.2a)$$

and the positive factor x_i is of the following form

$$x_i = \sum_{l=1}^{m} |h_l|^2 \frac{P}{N_0}, \qquad (A.3)$$

where m is the number of transmitting antennas. While assessing the performance of the system with achievable rate

$$C(k) = \sum_{i=1}^{k} \log\left(1 + x_i\right), \qquad (A.4)$$

proves to be too complex, closed-form expressions can be derived if following lower and upper bounds of (A.4) are used.

A.1 Lower bound

We can write (A.4) as

$$
\begin{aligned}
C(k) &= \log \prod_{i=1}^{k} (1 + x_i) \\
&= \log \left(1 + \sum_{i=1}^{k} x_i + \sum_{\substack{i,j=1 \\ i \neq j}}^{k} x_i x_j + \ldots + \prod_{i=1}^{k} x_i \right),
\end{aligned} \tag{A.5}
$$

or

$$
C(k) \geq \log\left(1 + \sum_{i=1}^{k} x_i\right). \tag{A.6}
$$

Notice that the lower bound for the general case of HARQ-IR, given by (A.6), is equivalent to the exact performance of HARQ-CC model. This equality between HARQ-IR lower bound and HARQ-CC performance will hold for any network model.

A.2 Upper bound

For the upper bound, we can use Jensen's inequality

$$
\frac{1}{k} \sum_{i=1}^{k} \log x_i \leq \log \sum_{i=1}^{k} \frac{x_i}{k}, \tag{A.7}
$$

equivalent to (recall (A.4))

$$
C(k) \leq k \log \left(1 + \sum_{i=1}^{k} \frac{x_i}{k} \right), \tag{A.8}
$$

which is the general form for the upper bound for rate achievable in the system using HARQ-IR protocol.

APPENDIX B

DERIVATION OF CARDINALITY OF SET $\mathcal{K}$

Recall that the set $\mathcal{K}$ is consisted of tuples $(k_1, ..., k_{n-1})$, $k_i = 0, 1, .., M$, such that $\sum_{i=1}^{n-1} k_i \leq M$, or

$$\mathcal{K} = \left\{ (k_1, ..., k_{n-1}) \,|\, \tilde{k}_n = \sum_{i=1}^{n-1} k_i \leq M \right\}. \tag{B.1}$$

The cardinality of the set $\mathcal{K}$, $|\mathcal{K}|$, is equal to the number of solutions $(k_1, ..., k_{n-1})$ of the following inequality

$$k_1 + ... + k_{n-1} \leq M, \tag{B.2}$$

where $k_1, .., k_{n-1}$ are *nonnegative* integers. However, let us examine (B.2) in the following *equality* form

$$x_1 + ... + x_p = t, \tag{B.3}$$

where x_i are *positive* integers. It is a textbook combinatorics example, equivalent to placing $(p - 1)$ objects of one type (say, lines) between t objects of other type (say, balls). For example, if $p = 4$, $t = 9$, one of solutions, say $3 + 2 + 1 + 3 = 9$ can be depicted as OOO|OO|O|OOO. Number of different ways to place $(p - 1)$ lines between t balls is obviously $\binom{t-1}{p-1}$. Now we can expand the problem depicted by (B.3) to *nonnegative* integers x_i. For this purpose, let us add p to both sides of (B.3):

$$(x_1 + 1) + ... + (x_p + 1) = t + p, \tag{B.4}$$

or, for *positive* integers y_i, $y_i = x_i + 1$,

$$y_1 + ... + y_p = t + p. \tag{B.5}$$

55

Obviously, number of solutions of equation (B.5) is $\binom{t+p-1}{p-1} = \binom{t+p-1}{t}$. Finally, the number of solutions of inequality (B.2) is equal to the summation of number of solutions of (B.5) for $t = 0, .., M$, and $p = n - 1$, $|\mathcal{K}| = \sum_{t=0}^{M} \binom{n+t-2}{t}$.

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