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Data allocation in disk arrays with multiple raid levels

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ABSTRACT

DATA ALLOCATION IN DISK ARRAYS WITH MULTIPLE RAID LEVELS

by

Jun Xu

There has been an explosion in the amount of generated data, which has to be stored reliably because it is not easily reproducible. Some datasets require frequent read and write access, like online transaction processing applications. Others just need to be stored safely and read once in a while, as in data mining. This different access requirements can be solved by using the RAID (redundant array of inexpensive disks) paradigm, i.e., RAID1 for the first situation and RAID5 for the second situation. Furthermore rather than providing two disk arrays with RAID1 and RAID5 capabilities, a controller can be postulated to emulate both. It is referred as a heterogeneous disk array (HDA).

Dedicating a subset of disks to RAID1 results in poor disk utilization, since RAID1 vs RAID5 capacity and bandwidth requirements are not known a priori. Balancing disk loads when disk space is shared among allocation requests, referred to as virtual arrays - VAs poses a difficult problem. RAID1 disk arrays have a higher access rate per gigabyte than RAID5 disk arrays. Allocating more VAs while keeping disk utilizations balanced and within acceptable bounds is the goal of this study.

Given its size and access rate a VA's width or the number of its Virtual Disks - VDs is determined. VDs allocations on physical disks using vector-packing heuristics, with disk capacity and bandwidth as the two dimensions are shown to be the best. An allocation is acceptable if it does not exceed the disk capacity and overload disks even in the presence of disk failures. When disk bandwidth rather than capacity is the

bottleneck, the clustered RAID paradigm is applied, which offers a tradeoff between disk space and bandwidth.

Another scenario is also considered where the RAID level is determined by a classification algorithm utilizing the access characteristics of the VA, i.e., fractions of small versus large access and the fraction of write versus read accesses.

The effect of RAID1 organization on its reliability and performance is studied too. The effect of disk failures on the X-code two disk failure tolerant array is analyzed and it is shown that the load across disks is highly unbalanced unless in an NxN array groups of N stripes are randomly rotated.

DATA ALLOCATION IN DISK ARRAYS WITH MULTIPLE RAID LEVELS

by
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DATA ALLOCATION IN DISK ARRAYS WITH MULTIPLE RAID LEVELS

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This work is dedicated to my beloved wife, son and family

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CHAPTER 1

INTRODUCTION

There has been a recent explosion in the volume of data being generated by various services, such as video on demand, internet data center, data warehousing, digital imaging, nonlinear video editing. Five exabytes (5×2^{60} bytes) of new information were generated in 2002 and new data is growing annually at the rate of 30% [47]. The economic viability of these services depends on storing data at low cost, while acceptance by their customers depends on their keeping data unaltered and accessible with low latency. Fortunately, this has been accompanied with rapidly increasing magnetic disk capacities and a drop per gigabyte in disk costs.

High data availability is important, because of the high cost of downtime for many applications. Furthermore, the loss of certain data is unacceptable, since it is irreproducible or very costly to reproduce. For example in 1975 the former USSR sent probes Venera 9 and 10 to the surface of Venus to collect data and imagery [73]. Had these data been lost, it would cost tens of millions of dollars to recollect them.

1.1 What is RAID?

The *Redundant Array of Inexpensive Disks - RAID* paradigm [48] is a solution to the disk failure problem. A typical disk array consists of a bunch of *identical hard disk drives* attached to an *array controller*, which is connected to a host computer using high-bandwidth links. The responsibility of the array controller is maintaining address mapping, maintaining redundant information, controlling individual disks, translating host requests, and recovering from disk or link failures. The array controller provides a linear address space to the host. The redundant information is maintained by the

disk array controller and is transparent to the user. The mapping of this host side linear address space to individual disk address space is referred to as the *data layout*.

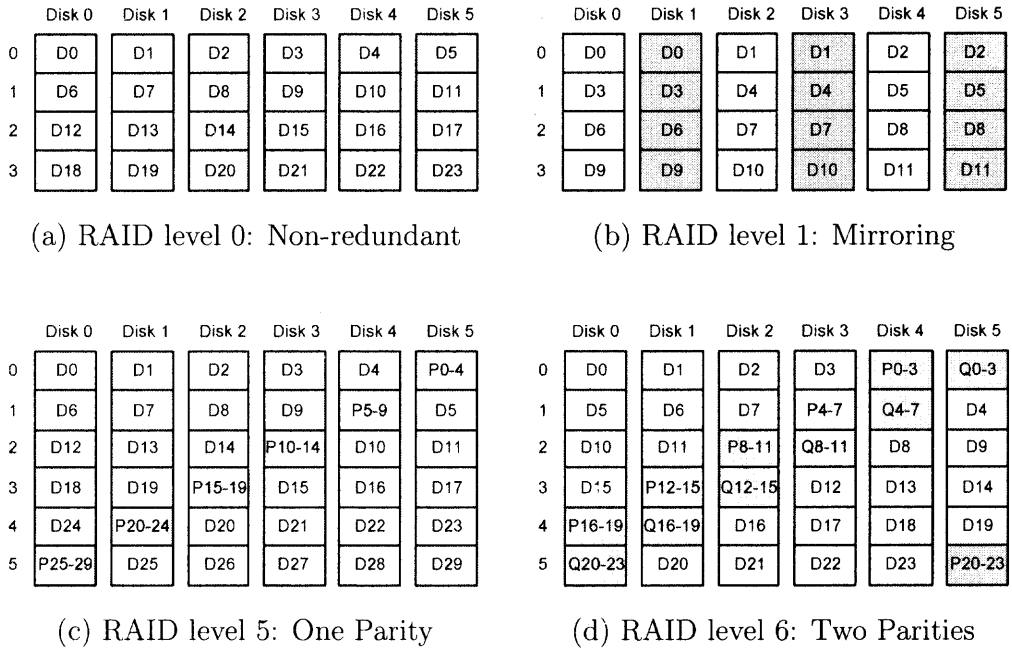
One of the fundamental concepts of RAID is *striping* [51, 28, 43], which partitions the linear address space exported by the array controller into smaller fixed size blocks called a *stripe unit* or *striping unit* - SUs. The benefits of striping include automatic load balancing and high bandwidth for large sequential transfers through parallel accesses.

RAID level 1 through 5 were first described in [51]. Even though not in the original RAID classification, RAID level 0 is often used to indicate a non-redundant disk array with striping, i.e., the failure of any disk in RAID0 results in data loss. RAID1, known as mirroring, provides redundancy by duplicating all data from one drive to another. In spite of the high degree of redundancy, RAID1 only guarantees recovery from a single disk failure [63], [70]. However, the doubling of the disk access bandwidth is beneficial from the viewpoint of the rapid increase in disk capacities, especially for applications requiring reading and writing of small blocks.

RAID5 stripes data at the block level and distributes parity SUs among the drives. In other words, no single disk is dedicated to parity. RAID6 uses two parities to tolerate two disk failures. In Figure 1.1, the data and redundancy information organizations for RAID levels 0, 1, 5, and 6 are illustrated. The RAID5 design shown uses *left-symmetric* organization [40], which repeats placing SUs in left to right diagonals. The group of disks that a parity is computed over is called a *parity group*. For the samples shown in Figure 1.1, there is only one parity group for each RAID level for RAID5.

Two and more disk failures can be tolerated by using Reed-Solomon codes in RAID6 [48] and sophisticated parity codings, such as EVENODD [41], RDP [46], and X-codes [75]. Each data block is protected by two parity groups in these arrays. A

performance analysis methodology is specified in [8], which given a certain workload and a RAID level, develops cost functions for primitive disk operations, which can be converted to service times according to disk characteristics.



(a) RAID level 0: Non-redundant

(b) RAID level 1: Mirroring

(c) RAID level 5: One Parity

(d) RAID level 6: Two Parities

Figure 1.1 Data layout in RAID levels 0, 1, 5 and 6. The shaded blocks are parities. d_i means data are bit or byte interleaved over disks. D_i means data are block interleaved. p_{i-j} means the parity is computed over d_i through d_j, P_{i-j} is defined similarly [30].

When one disk fails in RAID5 or RAID6 the load due to read requests on surviving disks is doubled. The RAID6 read load triples if two disks fail. To minimize the impact of load increase, clustered RAID disassociates the parity group size **G** from the number of disk N, so that the load increase is quantified by the declustering ratio: $\alpha = (G-1)/(N-1)$ [53]. The load increase in clustered RAID5 and RAID6 disk arrays with read and write request is quantified in [3]. Small writes in RAID5 and RAID6 may be processed as the read-modify-writes (RMWs) on reconstruct writes [60].

Rebuild is a systematic reconstruction of the contents of a failed disk on a spare disk in dedicated sparing [68] or empty space at surviving disks in distributed sparing [67]. Distributed sparing is preferable to dedicated sparing, since disk bandwidth is wasted otherwise [42], [67].

1.2 Demand for Heterogeneous Disk Arrays

Compared to the drop per gigabyte in disk costs, data management costs have increased sharply. Recent studies have indicated that storage management costs dominate the cost of large storage system over the course of their lifetime [11, 49]. Therefore it is important to automate this process, while optimizing storage utilization and providing satisfactory disk throughput and response time for user requests.

Different RAID levels have different performance characteristics and perform well only for a relatively narrow range of workloads. Hence, a typical RAID system provides many configuration parameters: data- and parity- layout choice, stripe unit sizes, parity group sizes, cache sizes and cache management policies, and so on. Setting these parameters correctly requires skilled, highly paid personnel and a painful process of trial and error. It should be able to choose the right configuration for different datasets based on their characteristics. It should be gracefully expandable, so that there is no need to provide too much spare storage for future use.

Furthermore different applications have different requirements, including, but not limited to capacity, bandwidth, and reliability. As far as reliability and performance are concerned, there is no single RAID level that can meet all of the requirements. Each RAID level has different characteristics and performs well only for a relatively narrow range of workloads. Hence, a storage system which can combine different RAID levels. i.e., RAID0/1/5/6 etc., is required to meet the requirements of individual

stores. In other words, the storage system should be heterogeneous in terms of the RAID level.

Rapid growth in disk capacity, high management cost and complex application requirements create the demand for the heterogenous disk array, a system that automates decision-making process, while optimizing storage utilization and providing satisfactory disk bandwidth and response time for user requests has been an important area of investigation see e.g., [26],[25].

1.3 Related Studies

HDA is built on many studies on RAID reliability, performance and design variations for parity placement and recovery schemes.

1.3.1 File Placement

Each file (one allocation request) has different characteristics associated with it: size, access frequency. Each system also has it own configuration: disk capacities, maximum disk access rates, disk bandwidths (transfer rates), data path, etc. The objective of the file placement problem is to match file characteristics with system configurations so as to balance disk workloads (by eliminating disk access skew) or to meet response time requirements for certain applications.

The file placement problem - FPP is modeled as 2-D vectors: size and access rate in [52]. *Forum* [18], *Minerva* [12] and *Ergastulum* [13] are three generations of design tools at HP for the "attribute mapping problem" [29] [55] to create a self-configuring and self-managing storage system. Bin-packing is applicable to do the data allocation problem at hand, e.g., the online best-fit bin packing with random order described in [36] is used in HP's Ergastulum frame work.

1.3.2 HP AutoRAID

HP AutoRAID implements two RAID levels (RAID1 and RAID5) inside a single disk array controller [34] to satisfy a wide variety of workloads. RAID1 is used at the upper level to provide full redundancy and excellent performance, while RAID5 is applied at the lower level to provide cost effective storage for less active data. The data blocks can be promoted or demoted between these two levels as access patterns change.

HP AutoRAID uses all disks for a stripe for RAID5. HDA computes the width of a VA dynamically according to the size and estimated access rate of an allocation. The average width of RAID5 VAs in HDA are usually less than the number of disks in the system.

HP AutoRAID only deals with small datasets. The segment (VD in this dissertation) size is fixed, i.e., 128KB. HDA considers large file allocations.

Load balancing in HP AutoRAID may not be achieved because it focuses on the balance of the amount of data on the disks instead of bandwidth usage. HDA explicitly makes load balance as a target because nowadays bandwidth is the bottleneck resource in disk storage systems.

The HP AutoRAID array uses static partition: part of the space is dedicated to RAID1 while the rest is formatted as RAID5. The potential problem is that it may run into a thrashing mode in which each update causes the target Relocation Block (RB) , which is a unit of data migration with the size of 64KB, to be promoted up to RAID1 and a second one demoted to RAID5 if the active write working set exceeds the size of RAID1 storage for long periods of time.

1.3.3 Previous Studies of HDA

In [7] the effect of different allocation strategies for files that need to be allocated inside RAID1 or RAID5 "containers" [30],[5],[19],[7] was investigated. Since space requirements for RAID1 and RAID5 allocations are not known a priori, these RAID arrays are allocated on demand based on disk space availability.

The study of HDA in this dissertation differs from previous ones in that it allocates the *Virtual Disks - VDs* of *Virtual Arrays - VAs* based not only on their storage requirements, but also estimated access rates. The j^{th} allocation request for a VA is specified as: (i) the data volume (D_j), (ii) estimated arrival rate (Λ_j), (iii) data access characteristics (request sizes, random versus sequential access, the read/write - R:W ratio, etc.), (iv) data availability requirements. Given VA attributes a rule-based system can be used to determine the RAID level, but it is assumed in one study (Chapter 2) that the RAID level is pre-specified. An analytic method to determine the more desirable RAID level is give in another study (Chapter 3).

1.3.4 Other Approaches

OceanStore [54] aims for a high availability and high security world-wide peer-to-peer system, buy does not provide high bandwidth. FARSITE [10] stores data in free disk space on workstations in a corporate network. FARM [74] breaks files up into fixed-size blocks: the default size of a block is 1 MB. The size of a block in this thesis is determined on the fly according to bandwidth and capacity usage of disks.

1.4 Organization of the Dissertation

In Chapter 2 data allocation in HDA is discussed. Given its size and access rate the VA's width or the number of its Virtual Disks - VDs is first determined. Then several online single-pass data allocation methods are evaluated . An allocation is acceptable if it does not exceed the disk capacity and overload disks especially in the presence of disk failures. When disk bandwidth rather than capacity is the bottleneck, the clustered RAID paradigm is applied, which offers a tradeoff between disk space and bandwidth.

RAID level classification in HDA is covered in Chapter 3. An analysis has been developed to estimate the load per VA postulating that it is configured as RAID1 or RAID5 and the RAID level which minimizes the load per VA is selected, since in this way more VAs can be allocated in a disk bandwidth bound system.

Chapter 4 discusses four RAID1 organizations: basic mirroring - BM, group rotate declustering - GRD, interleaved declustering - ID, and chained declustering - CD. The last three organizations provide a more balanced disk load than BM when a single disk fails, but are more susceptible to data loss than BM when additional disks fail. The four organizations are compared from the viewpoint of: (a) reliability (results are quoted from [63]), (b) performability, (c) performance. The ranking from the viewpoint of reliability and performability is: BM, CD, GRD, ID (with two clusters). BM and CD provide the worst performance, ID has a better performance than BM and CD, but is outperformed by GRD.

The load increase and imbalance of the X-code method is analyzed in Chapter 5. A general expression is derived for disk loads and graphs to quantify the load imbalance are presented.

Conclusions are given in Chapter 6.

CHAPTER 2

DATA ALLOCATION IN HETEROGENEOUS DISK ARRAYS

2.1 Introduction

There has been a recent explosion in the volume of data being generated, but this has been fortunately accompanied with rapidly increasing magnetic disk capacities and a drop per gigabyte in disk costs. With decreasing storage costs the cost of storage management is gaining more and more importance. Automating this process, while optimizing storage utilization and providing satisfactory disk throughput and response time for user requests has been an important area of investigation see e.g., [26],[25]. This is an attempt to take a fresh look at the problem with a simplified setting.

High data availability is important, because of the high cost of downtime for many applications. Furthermore, the loss of certain data is unacceptable, since it is irreproducible or very costly to reproduce. The RAID paradigm [48] is a solution to the disk failure problem. Single disk failures can be tolerated by mirroring data, as in RAID1, or by erasure coding using parity, as in RAID5. In fact, more than two disks can be allocated to RAID1, in which case the data is striped across multiple arrays in a RAID1/0 configuration. Two and more disk failures can be tolerated by using Reed-Solomon codes in RAID6 [48] and more sophisticated parity codings, such as EVENODD and RDP.

RAID1 or disk mirroring replicates the same data on two disks, so that it can be read from either disk, i.e., the access bandwidth to data is doubled. A further improvement in access bandwidth can be attained by judicious routing of read requests, e.g., accessing the data from the disk which provides the lower service

time. Modified data should be updated at both disks. The read load on the surviving disk is doubled when a single disk fails, but this load can be distributed over multiple disks with a more sophisticated data allocation scheme than basic mirroring. One such method is interleaved declustering, which has implications on the safeness of data allocations, i.e., no overload in degraded mode.

Load balancing in RAID5 is achieved via striping, i.e., partitioning large files into stripe units - SUs, which are allocated in a round-robin manner across the N disks. One of the SUs in a row is the parity computed across the remaining SUs in that row. If one of N disks fails, the contents of its blocks can be reconstructed on demand by issuing a fork-join request to read and exclusive-OR - XOR the corresponding blocks from the $N-1$ surviving disks. This is obviously one of the constraints of the allocation policy, that no two SUs in a stripe can be allocated on a single disk. Since in addition to fork-join read requests, each disk processes its own read requests the load at surviving disks is doubled in degraded mode. The load increase is smaller for write requests [3].

Clustered RAID - CRAID or *parity declustering* solves the load increase problem by setting the parity group size G to be smaller than N, so that only a subset of N disks will be involved in reconstructing a requested data block [53]. The load increase of surviving disks for RAID5 as a result of read requests is given by declustering ratio: $\alpha = (G-1)/(N-1)$ [53]. The size G affects the fraction of redundant data being held; i.e. in RAID5, 1/G of the blocks hold redundant data. Clustering provides a continuum of redundancy levels to minimize the impact of disk failures on performance in degraded mode. In other words the same load increase for a parity group in degraded mode is achieved by using less disks. The load in clustered RAID5 and RAID6 disk arrays in normal and degraded operating modes is given in [3] and is used in this study in estimating disk loads in degraded mode.

The rebuild process in RAID5 is a systematic reconstruction of the contents of a failed disk on a spare disk, which involves the reading of successive rebuild units (tracks), XORing them to recreate the lost track, and writing them to the spare disk. RAID5 is susceptible to data loss if there is a second disk failure, before the rebuild process is completed, or there is a *latent sector failure - LSF*, which is the more likely event.

Datasets with different application access requirements is considered. OLTP applications generate high access rates to read/write small, randomly placed blocks, while some database applications read/write large chunks of data. The former induce the small write penalty in RAID5, while the latter can be processed as full-stripe writes efficiently. It follows that RAID1 is the appropriate configuration in the former case, while RAID5 is more appropriate in the latter case, especially for high-volume datasets. Rather than providing two disk arrays with RAID1 and RAID5 capabilities, a controller emulating both is postulated so that disk space can be shared among RAID1 and RAID5 *virtual arrays - VAs*. This is a more flexible scheme than dedicating $n < N$ disks to RAID1 and $N - n$ disks to RAID5, since resource demands in the two categories are not known a priori. Sharing of disk space among RAID1 and RAID5 arrays has a load balancing effect, since RAID1 arrays have higher access rates per GB.

A synthetic workload is used to compare the effectiveness of several allocation methods. Given its size and access rate, a VA's width or the number of its Virtual Disks - VDs is first determined. Several "single-pass" data allocation methods are proposed, which take into account both the capacity and bandwidth available at each disk. An allocation is acceptable if it does not overload a disk or exceed its capacity and can tolerate a single disk failure. When disk bandwidth, rather than capacity, is the bottleneck, the clustered RAID paradigm is applied to RAID5 disk arrays, which offers a tradeoff between disk space and bandwidth.

The assumption is that the RAID level is determined by its attributes. In addition, there are many parameters associated with each RAID level that need to be specified: stripe unit size, striping width or the width of a VA, parity group size in clustered RAID, etc. Allocating a VA across all disks has the disadvantage that if a disk fails then the load at all remaining disks is doubled. For $N' < N$ the load increase per utilized disk by the VA is higher, but fewer disks are affected, which is in fact a form of CRAID. The HP AutoRAID selects the RAID level based on the observed access pattern [34], i.e., a subset of data with a high access rate is stored in RAID1 format. Objects to be stored on disk have different reliability and performance requirements, so that the selection of the appropriate RAID level poses a dilemma [26].

Since no single RAID level is satisfactory in all cases, rather than acquiring multiple arrays with different RAID levels, a *Heterogeneous Disk Array - HDA* is proposed, which supports heterogeneity at RAID level. Heterogeneity at disk level was considered in an earlier study [7], but heterogeneity at the level of brick level or storage nodes is more relevant nowadays. The earlier study is mainly concerned with the allocation of smaller files inside containers with fixed RAID1 and RAID5 formats. Directory structures developed in this study are applicable to the new HDA. In this study RAIDs with the same level may have different widths, stripe unit sizes, and parity group sizes.

The allocation problem at the VA level can be formulated by a two-dimensional "bin-packing" or vector-packing problem [52], where one dimension is the number of disks and the other dimension is the disk bandwidth utilization. Given rectangles with varying heights and widths the goal is to allocate as many of them as possible into a rectangle consisting of all disks. Branch-and-bound methods are applicable in this case. The problem at hand is made more difficult by the fact that allocation requests are malleable, i.e., the "wider" the allocation the lower the load at each disk.

Limits to capacity and bandwidth of allocations on each disk is used to determines the width of the VA. The setting of these parameters remains an area of further investigation.

In Section 2.2 how to calculate the load increase for a VA in both normal and degraded mode is displayed. Allocation methods next given in Section 2.3. Next the experimental approach used in comparing them is specified in Section 3.5. Sensitivity tests are reported in Section 2.4.1. Following it the methods to compute the parity group size G in clustered RAID is compared in Section 2.5. In Section 3.7 related results are discussed.

2.2 Allocation Requests

There is a disk array with N disks, which may be allocated across multiple bricks. Data allocation in a single brick is considered here for the sake of brevity, but maintaining brick boundaries limits the propagation of overload due to disk failures. Bricks may hold heterogeneous disk and be specialized, fast, small capacity versus slow, large capacity disks. Allocation requests in the form of VAs (virtual arrays), become available one at a time and are processed immediately. Each VA is specified as follows:

- *RAID level*, which is specified by ℓ with $\ell = 1$ for RAID1 and $\ell = 5$ for RAID5.
- *Size of dataset.* The size of the dataset associated with i^{th} VA is denoted by V_i. V_i is used to determine the access rate to the dataset, but its actual size V'_i is larger due to replication (RAID1) or erasure coding (RAID5), i.e., $V'_i = 2V_i$ and $V'_i = V_i(1 + 1/W_i)$, respectively. W_i is determined below.
- *The workload.* The arrival rate from an infinite number of sources is $\Lambda_i = V_i\kappa_\ell$, where κ_ℓ, the I/O intensity per GB is determined by the RAID level ℓ. The sizes of disk requests and the distribution of accesses determines the transfer time and positioning time respectively. Mean disk service time $\overline{x}_{disk}$ is computed

by assuming requests are uniformly distributed over all disk cylinders and that requests are served in FCFS order, which are worst case scenarios. The fraction of writes determines the processing overhead, which is high for RAID5 due to the small write penalty [48], but even higher for RAID6 [3].

The width W_i of VA_i is determined below. The fraction of read and write requests to VA_i is denoted by r_i and $w_i = 1 - r_i$. The mean service time for single read - SR, single write - SW, and read-modify-write - RMW requests is $\bar{x}_{SR}$, $\bar{x}_{SW}$, $\bar{x}_{RMW}$, respectivly. RMW requests can be processed as an SR followed by a disk rotation to write the data and parity block, or independent SR and SW requests, where the parity is computed at the disk array controller. The parity calculation is carried out at the disks in the first case and the disk array controller in the second case, providing a higher level of integrity.

$$\rho_i = \Lambda_i(r_i\bar{x}_{SR} + 2w_i\bar{x}_{RMW}),$$

$$\rho_i = \Lambda_i[r_i\bar{x}_{SR} + 2w_i(\bar{x}_r + \bar{x}_{SW})].$$

The width of the array can be determined based on a maximum disk utilization (ρ_{max}) or capacity constraint (V_{Max}) per VA:

$$W_i^{bandwidth} \geq \rho_i/\rho_{max}$$

$$W_i^{capacity} \geq (N-1)V_i/V_{max} + 1.$$

$$W_i = min\left[max\left(W_i^{utilization}, W_i^{capacity}\right), N\right]$$

Limiting the utilization of the disk by a VA reduces the possibility of disk overload, when disk loads are underestimated. Maximizing the width of the allocation in RAID5 minimizes the space overhead. Note that this minimizes the volume of data to be written for full stripe writes and allows the maximum level of parallelism for read accesses. The analysis can be extended to incorporate full stripe writes.

A similar analysis is applicable to RAID1 with basic mirroring. It is assumed that read requests are uniformly distributed over the two replicas.

$$\rho_i = \Lambda_i((r_i/2)\bar{x}_{SR} + w_i\bar{x}_{SW}).$$

The RAID level yielding the smaller load is selected in [26], but this method does not take into account the sizes of the allocations, so that a very large dataset may be allocated as RAID1.

Given W_i, $\rho_{i,n} = \rho_i/W_i$. As one of the virtual disks of the i^{th} VA is allocated, the utilization of disks on which VDs are allocated is incremented: $\rho_n = \rho_n + \rho_{i,n}, \forall n$.

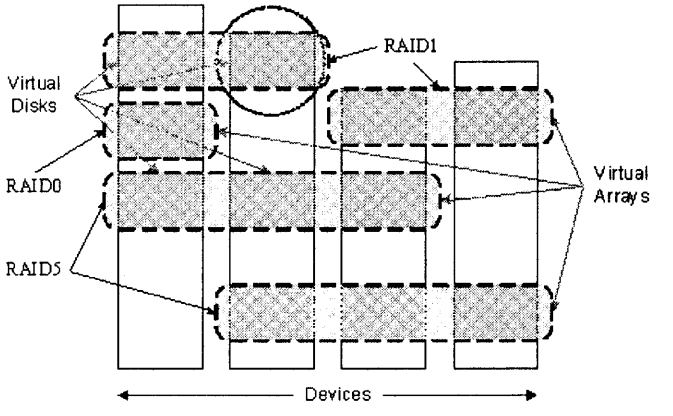


Figure 2.1 HDA with several VAs [5].

2.2.1 Load Increase in Normal Mode

In [3] formulas are given to compute the disk utilization of reads and writes for RAID5 and cluster RAID5. It is given for the worst case with all read requests ($r = 1$). It is the load increase of $\rho_{i,n}$. The subscript is omitted in the following formulas. For RAID1 only Basic Mirroring with width of two is considered.

2.2.1.1 RAID5. In normal mode the cost for the normal RAID and clustered RAID are the same.

$$\rho_{RAID5/F0} = \lambda_i r_i \bar{x}_{SR} + 2\lambda_i w_i \bar{x}_{RMW} \tag{2.1}$$

.

2.2.1.2 RAID1 for BM with W=2. In normal mode the cost for reads is only half of the read requests because the requests can be redirected evenly to the mirrored disks.

$$\rho_{RAID1/F0} = \frac{1}{2}\lambda_i r_i \bar{x}_{SR} + \lambda_i w_i \bar{x}_{SW} \tag{2.2}$$

.

2.2.2 Load Increase in Degraded Mode

In order to make an allocation safe, it is important to make sure disks will not exceed its maximum utilization in degraded mode, i.e. with disk failure(s). However, there is no disk failures in degraded mode. The load increase for a VA is calculated as if one disk fails. All the study discussed here are carried out in the degraded mode except specified explicitly.

2.2.2.1 RAID5. In degraded mode the disk utilization for normal RAID5 is computed as follows.

$$\rho_{read}^{RAID5/F1} = 2\lambda_i r_i \bar{x}_{SR} \tag{2.3}$$

$$\rho_{write}^{RAID5/F1} = \lambda_i w_i \left[\frac{2(W-2)}{W-1} \bar{x}_{RMW} + \frac{2}{W-1} \bar{x}_{SW} + \frac{W-2}{W-1} \bar{x}_{SR} \right] \tag{2.4}$$

For Clustered RAID with group size $G(G \leq W)$, the formulas are

$$\rho_{read}^{CRAID5/F1} = \lambda_i r_i \times \left[\frac{W+G-2}{W-1} \bar{x}_{SR} \right] \tag{2.5}$$

$$\rho_{write}^{CRAID5/F1} = \lambda_i w_i \times \left[\frac{2W-4}{W-1} \bar{x}_{RMW} + \frac{2}{W-1} \bar{x}_{SW} + \frac{G-2}{W-1} \bar{x}_{SR} \right] \tag{2.6}$$

2.2.2.2 RAID1 for BM with W=2. In degraded mode for a read request each disk has to process its own load plus the load of the failed disk, i.e. $W/2(W-1)$. For a write request it only needs to write data to $W-1$ disks instead of W.

$$\rho_{read}^{RAID1/F1} = \lambda_i r_i \bar{x}_{SR} \tag{2.7}$$

$$\rho_{write}^{RAID1/F1} = \lambda_i w_i \bar{x}_{SW} \tag{2.8}$$

2.3 Balancing Allocations

Both disks and allocation requests are modeled as two-dimensional vectors [52], where the first dimension is the access rate and the second dimension is size. The disk bandwidth is determined by the maximum throughput (accesses per second).

Data allocation is modeled as vector addition, so that the sum of the allocations should be less than the disk vector in both coordinates. The problem of balancing the utilization in terms of both throughput and capacity is defined as follows. The focus is on maximizing the number of allocations $I = I_{R1} + I_{R5}$, where I_{R1} and I_{R5} denote the number of allocations in the two categories. The allocation requests appear as a pseudo-random sequence.

The allocation requests considered here are at the level of VDs, so that the allocation of VA_i requires the allocation of W_i VDs, whose sequence is denoted by J, whose elements are denoted by $p_j = (x_j, c_j)$, where x_j is its expected access rate and c_j is the size of data. In fact, J is the concatenation of sequences J_i due to VA_i.

The n^{th} disk is represented by a vector $d_n = (X_n, C_n)$, where X_n denotes the maximum throughput and C_n the capacity of the n^{th} drive. A valid solution allocates from the sequence J of VDs into N sets $J_1, \ldots, J_N$. VDs belonging to the same VA should be allocated on a different disks. A VA is considered allocated if all of its VDs are allocated. Let U_n^x and U_n^c denote the utilization of the bandwidth and capacity of the n^{th} disk. which are defined as follows:

$$U_n^x = \sum_{j \in J_n} x_j / X_n, \quad U_n^c = \sum_{j \in J_n} c_j / X_c.$$

The allocation of VAs is continued until an allocation is unsuccessful, at which point no alternative drive is considered. Allocation policies are judged by the number of allocations, which maximizes the number of allocated VDs.

1. **Round Robin:** Allocate on disk drives sequentially.
2. **Random:** Choose disk drives randomly.
3. **Best Fit:** Scan the disks and choose disks with minimum remaining bandwidth or maximum disk utilization (after the allocation).
4. **First Fit:** Disks are considered in increasing order of their indices and a VD is allocated on the first disk that can hold it.
5. **Worst fit:** Allocate requests on disk with minimum bandwidth utilization provided that disk capacity constraint is satisfied.

   Let U_n^x and U_n^c denote the bandwidth and capacity utilization of the n^{th} disk. Two more sophisticated allocation methods minimizes the following objective functions:
6. **Minimize F1:**

$$F_1 = \max_{1 \le n \le N} \{U_n^x, \beta U_n^c\},$$

   $0 \le \beta \le 1$ is an emphasis factor of capacity utilization.
7. **Minimize F2:**

$$F_2 = \operatorname*{Var}_{1 \le n \le N}(U_n^x) + \beta \operatorname*{Var}_{1 \le n \le N}(U_n^c),$$

   $\mathrm{Var}(x_n)$ is the variance of x_n over all possible N.

The focus is mainly on balanced disk utilizations, rather than disk capacities, since unbalanced throughputs will result in highly variable response times, while disk capacity is cheap.

Given the number of disks, say $N = 12$, the disk drive and disk access characteristics, e.g., access to small randomly placed blocks of data, fraction of RAID1 versus RAID5 requests (RAID level ℓ), distribution of sizes of VAs, several runs are made to determine the average number of allocations for each method. The allocation experiment proceeds as in Algorithm 1:

Algorithm 1 VA allocation algorithm

Initialize VA allocation count: $i = 0$. Generate allocation requests until a request is unsuccessful.

1. Increment i and generate VA_i request with appropriate RAID level ℓ.
2. Determine VA_i size: V_i based on size distribution for RAID level ℓ.
3. Generate estimated access rate: $\Lambda_i = \kappa_\ell V_i$.
4. Calculate load in degraded mode for this VA as discussed in Section 2.2.2.
5. Determine allocation width W_i based on disk capacity and utilization constraints.
6. Determine if a successful allocation of all VDs is possible. If not stop allocation.
7. Increment the utilization of disks to which VA_i is assigned.
8. $i++$ and return to Step 1.

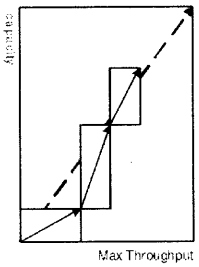


Figure 2.2 Virtual array allocation vectors [5].

2.4 Experimental Results

Table 2.1 Specifications of IBM 18ES Model DNES-309170W Disk Drives

Model	IBM DNES-309170W
Capacity	9.17GB
No. of cylinders	11474
Tracks per cylinder	5
Sectors per track	247-390
Number of zones	11
Rotation speed	7200 RPM
T_{rot}	8.3 ms
T_h	0.14 ms
$\bar{x}_{seek}$	6.9 ms

The following general configurations are used to run experiments. There are twelve IBM 18ES (model DNES-309170W), [1] whose specifications are given in Table 4.7. This type of disk is used in all experiments in this chapter. The VA sizes are exponentially distributed with mean of 256 MB for RAID1 and three times that for RAID5. V_{max} is set to 1/50 of the capacity of all disks and ρ_{max} 1/20 of the bandwidth of each disk. The value of β used in Min F1 and Min F2 is set to *1*. The

[1] `http://www.storage.ibm.com/hdd/prod/ultrastar.htm`.

access rates for RAID1 disk arrays are ten times the rates of RAID5. Three cases are considered for disk requests.

Table 2.2 Comparison of the Allocation Methods with R5:R1=3:1 and $r = 1$ in Normal Mode

Method	Bandwidth Bound				Balanced				Capacity Bound			
		Allocations				Allocations				Allocations		
		Allocations				Allocations				Allocations		
	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5
Min F1	80	47.1	142.4	189.6	90	57.2	173.2	230.4	90	69.2	206.7	275.9
Min F2	78	46.4	142.3	188.6	76	56.2	172.4	228.6	86	68.2	205.6	273.8
Worst Fit	66	45.2	137.6	182.8	15	48.1	150.4	198.5	9	61.3	183.6	244.9
Best Fit	63	41.1	123.1	164.2	12	44.3	135.3	179.6	8	57.4	178.3	235.7
Round Robin	17	33.3	102.3	135.6	10	44.4	133.4	177.8	8	63.2	190.1	253.3
First Fit	13	33.1	100.4	133.5	0	35.1	103.0	138.1	0	38.1	115.3	153.5
Random	13	29.1	90.3	119.4	6	33.0	107.3	140.3	2	57.2	172.2	229.3

Table 2.3 Comparison of the Allocation Methods with R5:R1=3:1 and $r = 0.75$ in Normal Mode

	Bandwidth Bound				Balanced				Capacity Bound			
		Allocations				Allocations				Allocations		
Method	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5
Min F1	78	41.4	125.1	166.5	87	50.2	152.1	202.3	89	68.5	204.7	273.2
Min F2	75	40.7	124.9	165.7	73	49.3	151.4	200.7	85	67.5	203.6	271.1
Worst Fit	63	39.7	120.8	160.5	14	42.3	132.1	174.4	9	60.7	181.7	242.4
Best Fit	61	36.1	108.1	144.2	12	38.9	118.8	157.8	8	56.8	176.6	233.4
Round Robin	16	29.2	89.8	119.1	10	39.0	117.2	156.2	8	62.6	188.2	250.8
First Fit	13	29.1	88.2	117.2	0	30.8	90.5	121.3	0	37.7	114.2	151.9
Random	12	25.5	79.3	104.8	6	29.0	94.3	123.2	2	56.6	170.5	227.1

Table 2.4 Comparison of the Allocation Methods with R5:R1=3:1 and $r = 0.5$ in Normal Mode

Method	Bandwidth Bound				Balanced				Capacity Bound			
		Allocations				Allocations				Allocations		
	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5
Min F1	75	32.4	97.9	130.3	84	39.3	119.1	158.4	89	68.2	203.7	271.9
Min F2	73	31.9	97.8	129.7	70	38.6	118.5	157.2	85	67.2	202.6	269.8
Worst Fit	61	31.1	94.6	125.7	14	33.1	103.4	136.5	9	60.4	180.8	241.2
Best Fit	59	28.3	84.6	112.9	12	30.5	93.0	123.5	8	56.6	175.7	232.3
Round Robin	16	22.9	70.3	93.2	10	30.5	91.7	122.3	8	62.3	187.3	249.5
First Fit	12	22.8	69.0	91.8	0	24.1	70.8	95.0	0	37.6	113.6	151.2
Random	12	20.0	62.1	82.1	6	22.7	73.8	96.5	2	56.3	169.6	225.9

Table 2.5 Comparison of the Allocation Methods with R5:R1=3:1 and $r = 1$ in Degraded Mode

	Bandwidth Bound				Balanced				Capacity Bound			
		Allocations				Allocations				Allocations		
Method	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5
Min F1	71	23.2	72.0	95.2	87	28.2	84.6	112.8	87	34.2	105.3	139.5
Min F2	71	23.2	72.0	95.2	73	27.3	84.6	111.9	83	33.4	102.3	135.7
Worst Fit	56	21.8	71.6	93.4	14	23.7	75.2	98.9	9	29.7	93.4	123.1
Best Fit	54	20.3	68.7	89.0	12	22.0	70.1	92.1	8	27.6	90.2	117.8
Round Robin	19	16.4	50.6	67.0	10	21.1	66.4	87.5	8	30.8	95.1	125.9
First Fit	10	16.0	48.5	64.5	0	18.7	56.8	75.5	0	19.3	59.7	79.0
Random	10	13.7	47.2	60.9	6	20.2	61.7	81.9	2	28.0	86.9	114.9

Table 2.6 Comparison of the Allocation Methods with R5:R1=3:1 and $r = 0.75$ in Degraded Mode

Method	Bandwidth Bound				Balanced				Capacity Bound			
		Allocations				Allocations				Allocations		
	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5
Min F1	82	17.3	64.0	81.3	98	23.2	72.6	95.8	80	33.0	104.0	137.0
Min F2	75	17.3	64.0	81.3	69	23.1	73.5	96.6	83	32.1	101.0	133.1
Worst Fit	66	16.5	59.6	76.1	10	21.3	66.9	88.2	7	29.0	92.0	121.0
Best Fit	63	15.0	58.8	73.8	9	19.6	62.4	82.0	6	27.1	89.0	116.1
Round Robin	12	12.8	45.6	58.4	11	16.8	56.6	73.4	6	30.0	92.1	122.1
First Fit	16	13.7	44.7	58.5	0	17.9	53.8	71.7	0	18.1	58.0	76.1
Random	12	14.0	38.7	52.7	7	17.8	53.6	71.4	2	27.0	85.0	112.1

Table 2.7 Comparison of the Allocation Methods with R5:R1=3:1 and $r = 0.5$ in Degraded Mode

Method	Bandwidth Bound				Balanced				Capacity Bound			
		Allocations				Allocations				Allocations		
	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5	Best	R1	R5	R1+R5
Min F1	78	15.3	57.6	72.9	98	21.7	66.6	88.3	84	33.0	104.0	137.0
Min F2	75	15.3	55.7	71.1	71	20.9	67.4	88.3	90	32.1	101.0	133.1
Worst Fit	63	14.7	51.9	66.6	6	19.9	63.8	83.7	3	29.0	92.0	121.0
Best Fit	61	13.3	51.2	64.5	5	18.2	59.5	77.7	3	27.1	89.0	116.1
Round Robin	13	10.7	39.4	50.1	10	14.0	47.5	68.7	6	30.0	92.1	122.1
First Fit	16	12.0	39.6	51.6	0	16.9	51.8	61.5	0	18.1	58.0	76.1
Random	12	11.2	32.9	44.1	3	15.2	47.2	62.4	2	26.1	83.2	109.3

1. *Bandwidth Bound:* Allocation requests consume disk bandwidth faster than disk capacity, i.e. the bandwidth and capacity utilization ratio of allocation requests is higher than the disk ratio (8.5 accesses/sec. per GB for RAID5).

2. *Balanced:* The allocation requests consume disk capacity at almost the same rate of disk bandwidth. (3.3 accesses/sec. per GB for RAID5).

3. *Capacity Bound:* Allocations consume disk capacity faster than disk bandwidth, i.e. the capacity and bandwidth utilization ratio of allocation requests is greater than that of disks. (2.1 accesses/sec. per GB for RAID5).

100 runs are made for each case and display the number of bests in 100 runs for each method. The number of allocations for RAID1 and RAID5 is the average of 100 runs.

The following conclusions are drawn from experimental results in Tables 2.2,2.3,2.4, 2.5, 2.6 and 2.7.

1. The number of VAs allocated in normal mode is almost double that in degraded mode when $r = 1$. That is because the load in normal mode is roughly half of that in degrade mode.

2. It is necessary to consider both system utilization and capacity to get a robust performance.

3. Minimize F1 and F2 are consistently the best in terms of the number of allocations in all configurations.

4. First Fit, Random and Round Robin are the worst among all methods.

5. Worst Fit is comparable with F1 and F2 when bandwidth bound, but not when capacity bound. The reason is that it balances the bandwidth utilization on each disk, but not the capacity. So it works well in bandwidth bound workload and poorly in capacity bound workload.

Tables 2.8, 2.9 and 2.10 display the comparison of average bandwidth and capacity utilization of all disks and number of RAID1 and RAID5 VAs allocated with

$R5 : R1 = 3 : 1$, $r = 1$ and bandwidth bound workload in degraded mode among methods Min F1, F2 and round robin.

Table 2.8 Disk Utilizations After Allocations with $R5 : R1 = 3 : 1$, $r = 1$ and Bandwidth Bound Workload in Degraded Mode

Method	Bandwidth Util (in %)		Capacity Util (in %)		No. of VAs		
	Avg	Stdv	Avg	Stdv	R1	R5	Total
Min F1	90.7	1.6	51.3	2.6	23	72	95
Min F2	90.4	2.3	51.3	2.2	23	72	95
Round Robin	63.8	20.1	36.8	8.4	16	51	67

Table 2.9 Disk Utilizations After Allocations with R5:R1=3:1, $r = 1$ and Balanced Workload in Degraded Mode

Method	Bandwidth Util (in %)		Capacity Util (in %)		No. of VAs		
	Avg	Stdv	Avg	Stdv	R1	R5	Total
Min F1	89.9	1.6	95.3	0.6	28	85	113
Min F2	89.6	1.9	94.4	0.7	27	84	111
Round Robin	70.4	17.7	74.6	13.1	21	66	87

Table 2.10 Disk Utilizations After Allocations with R5:R1=3:1, $r = 1$ and Capacity Bound Workload in Degraded Mode

Method	Bandwidth Util (in %)		Capacity Util (in %)		No. of VAs		
	Avg	Stdv	Avg	Stdv	R1	R5	Total
Min F1	47.8	0.8	99.2	0.6	34	105	139
Min F2	47.7	0.5	98.4	0.5	33	102	135
Round Robin	45.0	11.7	94.8	3.7	31	95	126

Each disk's utilization after allocations with bandwidth bound workload for Round Robin, Min F1 and Min F2 are shown in Figures 2.3 and 2.4.

Each disk's utilization after allocations with balanced workload for Round Robin, Min F1 and Min F2 are shown in Figures 2.5 and 2.6.

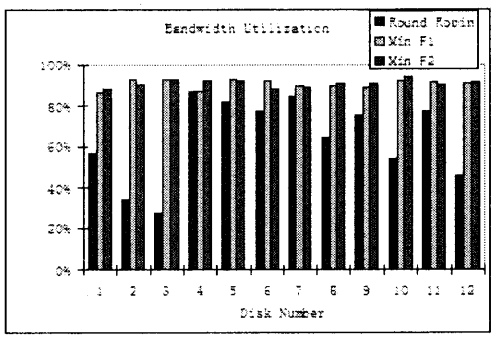


Figure 2.3 Disk bandwidth utilizations with R5:R1=3:1, $r = 1$ and bandwidth bound workload in degraded mode.

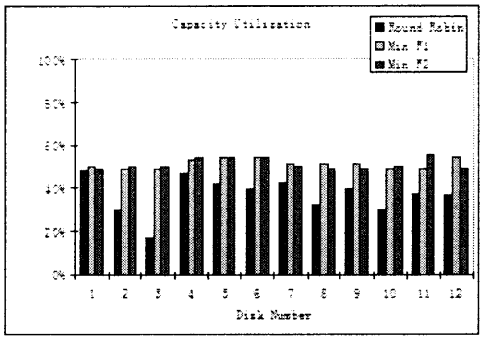


Figure 2.4 Disk capacity utilizations with R5:R1=3:1, $r = 1$ and bandwidth bound workload in degraded mode.

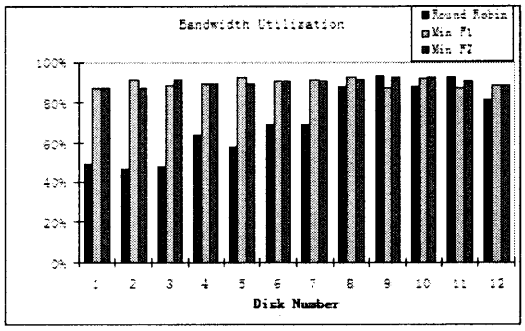


Figure 2.5 Disk bandwidth utilizations with R5:R1=3:1, $r = 1$ and balanced workload in degraded mode.

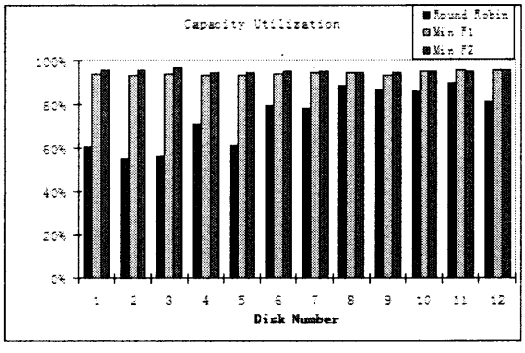


Figure 2.6 Disk capacity utilizations with R5:R1=3:1, $r = 1$ and balanced workload in degraded mode.

Each disk's utilization after allocations with capacity bound workload for Round Robin, Min F1 and Min F2 are shown in Figures 2.7 and 2.8.

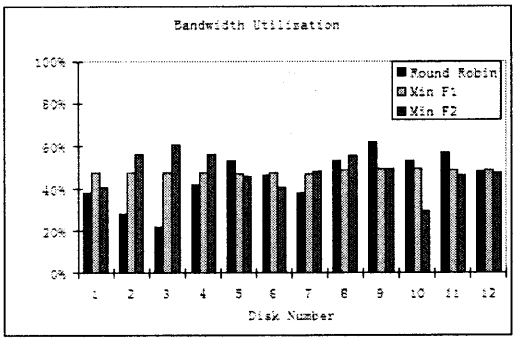


Figure 2.7 Disk bandwidth utilizations with R5:R1=3:1, $r = 1$ and capacity bound workload in degraded mode.

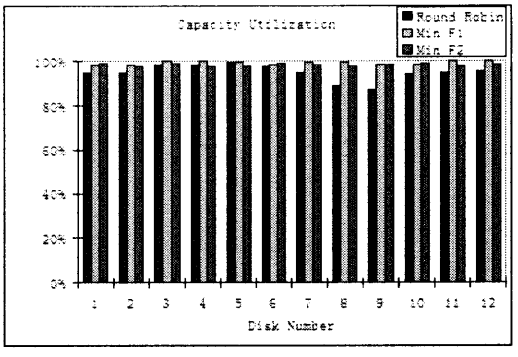


Figure 2.8 Disk capacity utilizations with R5:R1=3:1, $r = 1$ and capacity bound workload in degraded mode.

Conclusions:

1. Minimizing the variation of each disk's utilization can increase the number of VA allocations, like Min F1 and Min F2.

2. Bad allocation methods have large standard deviation of utilization on each disk.

β is an emphasis factor on capacity utilization in Min F1 and F2. With the value changing from 0 to 1, more weights are put on the disk capacity utilization. As a result the methods also balance the capacity utilization on each disk.

The following general configurations are used to run experiments. The total number of disks is 12. The ratio of RAID5 to RAID1 requests is 3:1 and the Read/Write ratio is 1:0, $r = 1$. The VA sizes are exponentially distributed with mean of 256 MB for RAID1 and three times that for RAID5. V_{max} is set to 1/50 of the capacity of all disks and ρ_{max} 1/20 of the bandwidth of each disk. The access rates for RAID1 disk arrays are ten times the rates of RAID5. All three cases are considered, i.e., bandwidth bound, balance and capacity bound.

The results are displayed in Tables 2.11 and 2.12.

Table 2.11 Sensitivity of β in Min F1 with R5:R1=3:1 and $r = 1$ and in Degraded Mode

β	Bandwidth Bound	Balanced	Capacity Bound
≥ 1	116	127	139
0.8	116	127	139
0.5	108	123	133
0.3	108	108	125
0	108	108	123

Table 2.12 Sensitivity of β in Min F2 with R5:R1=3:1 and $r = 1$ and in Degraded Mode

β	Bandwidth Bound	Balanced	Capacity Bound
≥ 1	116	129	139
0.8	116	129	139
0.5	116	129	139
0.3	116	129	139
0	108	108	123

Conclusions

1. β has some effects on all three workloads.
2. In balance and capacity bound workloads the number of VAs allocated increases more than *10* percent when β varies from *0 to 1*. The effects on bandwidth bound workload is less significant.
3. Rule of thumb is making $\beta = 1.0$.

2.4.1 Effects of ρ_{max} and V_{max}

ρ_{max} and V_{max} control the maximum size and estimated access rate of an allocation request. If the values of these two parameters are set too small allocation requests will be small in terms of both size and access rate too, which increases directory space overhead. On the contrary if the values are too large, only a few requests can be allocated. It is inefficiency due to fragmentation.

The following general configurations are used to run experiments. The total number of disks is 12. The ratio of RAID5 to RAID1 requests is 1:0 and the Read/Write ratio is 1:0, $r = 1$. The VA sizes are exponentially distributed with mean of 768 MB for RAID5. Min F1 is used in this experiment because it is one of the best reported in Section 2.3. β is set to *1*. All three cases are considered, i.e., bandwidth bound, balance and capacity bound.

Table 2.13 Effects of ρ_{max} and V_{max} in VAs Allocations

		Bandwidth bound			Balanced			Capacity bound		
	v_{max}	1/25	1/50	1/100	1/25	1/50	1/100	1/25	1/50	1/100
ρ_{max}	1/10	46	46	46	74	93	123	88	104	135
	1/20	49	49	49	77	96	125	88	104	135
	1/40	50	50	50	82	99	128	88	104	135

The following conclusions are drawn from the experiments in Table 2.13:

1. ρ_{max} can influence the number of allocations with both balanced and bandwidth bound workload. The reason is ρ_{max} controls the maximum bandwidth of each allocation, which will decide the size of the VA. As the size of VA varies the affected disks in degraded mode also vary. Hence the number of allocations fluctuates with ρ_{max}.

2. ρ_{max} has no effects on capacity bound workload. That is because with such workload, capacity is the bottleneck. It will reach its limit first before ρ_{max} takes effect.

3. V_{max} can influence the number of allocations with both balanced and capacity bound workload. The reason is ρ_{max} controls the maximum capacity of each allocation, which will decide the size of the VA. As the size of VA varies the affected disks in degraded mode also vary. Hence the number of allocations fluctuates with V_{max}.

4. V_{max} has no effects on bandwidth bound workload. That is because with such workload, bandwidth is the bottleneck. It will reach its limit first before V_{max} takes effect.

2.5 Clustered RAID

For a given disk model the capacity/bandwidth ratio (γ_d) is fixed. If the capacity bandwidth ratio for a VA without clustering (γ_{nc}) is same as that of disk then the capacity/bandwidth utilization of the disk is balanced and more VAs can be allocated. Unfortunately, most of the time γ_{nc} is not equal γ_d. By using clustering, i.e., changing the parity group size G, the capacity and bandwidth of the VA can be changed. It is possible to make the capacity/bandwidth ratio for the clustered VA γ_c close to γ_d.

α shows the load increase in CRAID5 and the parity group size G is related with α ($G = \alpha(N-1)+1$). If the γ_{nc} for a VA is lower than γ_d (bandwidth utilization is higher than capacity), then picking a small α can make the load increase small.

With a small α G is small too, which introduce more over head (1/G). As a result the capacity utilization for this VA increases and bandwidth utilization decreases (γ_c increases). By varying α it is possible to find the best α to make γ_c close to γ_d.

For the IBM 18ES disk the capacity is $9.17GB$ and the bandwidth is 87.5 accesses per second. γ_d is 0.105. The average access rate of a VA for bandwidth bound workload is 8.5 second for the size of 1GB. Using these numbers and all read requests ($r = 1$), results in Table 2.14 show that when α is about 0.25, γ_c is close to γ_d.

Table 2.14 Change of Capacity/Bandiwdth Ratio γ_c

α	G	$Capacity_c$	$Bandwidth_c$	γ_c
0.125	2	1.14	8.56	0.133
0.25	4	1.01	9.51	0.106
0.375	5	0.96	10.46	0.091
0.5	7	0.92	11.42	0.081
0.625	8	0.90	12.37	0.073
0.75	9	0.89	13.32	0.067
0.875	11	0.88	14.27	0.061
1	12	0.87	15.22	0.057

Now simulations described in Section 3.5 with the same general configurations are carried out. Only bandwidth bound workload is considered and Method "Min F1" is used in this experiment. β is set to *1*. But before a VA is allocated, the best α is computed to bring the capacity/bandwidth ratio (γ_c) of this VA close to that (γ_d) of the disk. Once the α is found the new width will be calculated accordingly.

First the α for all VAs is fixed. Three runs are made with α equal to 0.25, 0.5 and 0.75 respectively. The following conclusions can be made from Table 2.15, 2.16, 2.17 and 2.18:

Table 2.15 Number of RAID5 Allocations with Bandwidth Bound Workload in Degraded Mode $N = 12$

α	r:w=1:0	r:w=3:1	r:w=1:1
0.25	108.6 (c)	69.4 (b)	50.6 (b)
0.5	92.0 (b)	61.7 (b)	46.4 (b)
0.75	78.8 (b)	55.5 (b)	42.8 (b)

Table 2.16 Number of RAID5 Allocations with Bandwidth Bound Workload in Normal Mode $N = 12$

α	r:w=1:0	r:w=3:1	r:w=1:1
0.25	108.6 (c)	108.6 (c)	108.6 (c)
0.5	119.2 (c)	119.2 (c)	119.2 (c)
0.75	124.1 (c)	124.1 (c)	105.4 (b)

Table 2.17 Number of RAID5 Allocations with Bandwidth Bound Workload in Degraded Mode with Dynamic Parity Group Size W

α	r:w=1:0 (W=5.8)	r:w=3:1 (W=5.7)	r:w=1:1 (W=5.7)
0.25	94.6 (c)	74.8 (b)	53.4 (b)
0.5	106.3 (c)	70.7 (b)	51.3 (b)
0.75	103.9 (b)	67.0 (b)	49.4 (b)

Table 2.18 Number of RAID5 Allocations with Bandwidth Bound Workload in Normal Mode with Dynamic Parity Group Size W

α	r:w=1:0 (W=10.9)	r:w=3:1 (W=10.2)	r:w=1:1 (W=9.1)
0.25	106.8 (c)	105.6 (c)	103.4 (c)
0.5	117.8 (c)	116.7 (c)	114.8 (c)
0.75	123.0 (c)	122.1 (c)	120.5 (c)

1. If the bandwidth is the bottleneck more allocations can be made with the decreasing of α. That is because with small α the bandwidth consumption for a VA is low. Hence, more VAs can be allocated.

2. If the capacity is the bottleneck more allocations can be made with the increasing of α. That is because with large α the capacity consumption for a VA is low (small overhead $1/G$). Hence, more VAs can be allocated.

3. With α increases from 0 to 1, it is possible the bottleneck changes from capacity to bandwidth. The reason is that α increases goes with the opposition direction of capacity increase ($1/G$), but the same direction of bandwidth increase. As a result the number of allocations goes up while capacity is the bottleneck, reaches the maximum when bottleneck switches from capacity to bandwidth and goes down with while bandwidth is the bottleneck.

Table 2.19 Comparison of Relative Number of Allocations with and without Clustered RAID and R5:R1=1:0 in Degraded Mode

	RAID5 (width=N)	RAID5 (width=W)		Clustering on W disks		
			Avg Width		Avg G	Avg γ
$r:w=1:0$	1	1.45	5.8	1.57	3.9	0.106
$r:w=3:1$	1	1.30	5.7	1.49	2.03	0.086
$r:w=1:1$	1	1.22	5.6	1.34	2.8	0.06

Next α is computed dynamically for each VA during the run. The following conclusions can be made from Table 2.19:

1. Clustered RAID can increase the number of allocations dramatically.

2. With the increase of write requests clustering has less effect, i.e., the increase of number of allocations decreases.

2.6 Conclusions and Related Work

HDA has been described in this study. Also several data allocation policies have been specified and experimental results comparing the efficiency of several data allocation methods have been reported. It is shown that two of these methods, which take into account both disk access bandwidth and capacity outperform others across a wide variety of allocation requests. The sensitivity of allocations to two of the parameter settings, ρ_{max} and V_{max} has also been presented. V_{max} has no effects on the number of VA allocations while ρ_{max} has some impacts. Finally the effect of utilizing Clustered RAID with bandwidth bound workload is analyzed. Results show that clustered RAID can further improve the number of allocations.

HP AutoRAID implements two heterogeneous RAID levels (RAID1 and RAID5) inside a single disk array controller [34] to satisfy a wide variety of workloads. RAID1 is used at the upper level to provide full redundancy and excellent performance while RAID5 is applied at the lower level to improve storage cost for less active data, at somewhat lower performance. The data blocks can be promoted or demoted between these two levels as access patterns change.

HP AutoRAID uses all disks for a stripe (VA in this thesis) for RAID5. It is well know that the performance of a full stripe in RAID5 is worse than a clustered RAID5. HDA computes the width of a VA dynamically according to the size and estimated access rate of an allocation. The average width of RAID5 VAs are usually less than the number of disks in the system.

HP AutoRAID only deals with small datasets. The segment (VD in this study) size is fixed, i.e. 128KB. HDA considers large file allocations.

Load balance in HP AutoRAID is just a hope because it focuses on the balance of the amount of data on the disks instead of bandwidth usage. HDA explicitly makes

load balance as a target because nowadays bandwidth is the bottleneck in storage systems.

The HP AutoRAID array uses static partition: part of the space are RAID1 while the rest are formatted to be RAID5. The potential problem is that it may run into a thrashing mode in which each update causes the target Relocation Block (RB) to be promoted up to RAID1 and a second one demoted to RAID5 if the active write working set exceeds the size of RAID1 storage for long periods of time. HDA decides RAID level of an allocation on the fly so that there is no static partition.

CHAPTER 3

RAID LEVEL SELECTION FOR HETEROGENEOUS DISK ARRAYS

3.1 Introduction

There has been an explosion in the volume of data being generated, but this has been accompanied with an exponential increase in magnetic recording density resulting in larger disk capacities in small form factor disks and dropping cost per gigabyte. Storage represents a growing fraction of total system cost and more importantly storage management ocosts are increasing rapidly. This is a step in simplifying this process.

A method is proposed to maximize the number of allocated datasets, referred to as *Virtual Arrays - VAs* in *Heterogeneous Disk Arrays - HDAs.* [1] HDAs support two levels: RAID1 and RAID5, which both tolerate single disk failures, but have different characteristics in processing database workloads [48]. The number of allocations is constrained by the disk bandwidth, so a VA is allocated at a RAID level, which minimizes the disk access bandwidth. A review of related work follows.

Disk space for RAID1 and RAID5 containers are allocated on demand to make space for file allocation requests, which are tagged as RAID1 or RAID5 in advance, e.g., small files are allocated as RAID1 and large files as RAID5 [7]. RAID1 and RAID5 containers share space on possibly heterogeneous disks.

The single level allocation of VAs with predetermined RAID1 and RAID5 levels is investigated in [69], but only homogeneous disks are considered. Each VA has an associated size and access rate proportional to size, which also depends to its RAID level. VAs are partitioned into multiple *Virtual Disks - VDs* based on

[1] *VAs may be considered to be equivalent to Logical Units -LUs in RAID literature.*

a maximum bandwidth and capacity per disk per VA. VDs are allocated to disks taking into account current disk bandwidth and capacity utilizations in a single pass algorithm, i.e. no optimization is attempted by batching requests. It is shown in [7][69] that allocation methods, which minimize the variation of bandwidth and capacity utilization across disks perform better than others which do not.

In this study the realistic case is considered, where the disk bandwidth is the only limiting resource, i.e., that disk capacity is not a constraining resource. Unlike [69] the RAID level is not known a priori, but rather determined using a simple queueing analysis based on VA's workload characteristics. VAs are subject to two types of requests: (i) accesses to small blocks of data, as in the case of online transaction processing applications, (ii) accesses to large blocks, as in the case of batch applications. Such accesses are processed as full stripe reads and writes for efficiency purposes. The frequencies of the two types of requests and the fraction of reads and writes in each category are assumed to be known.

VAs are classified into two RAID levels based on the level, which provides the lower load. The classification shows that RAID5 is the preferred level when the frequency of full stripe requests spanning all disks is high. RAID1 is the preferred level when the fraction of small writes is high. An allocation study with a synthetic workload is reported, which shows that a combination of RAID levels results in more allocations than a single level, either RAID1 or RAID5.

This chapter is organized as follows. In Section 3.2 the RAID1 and RAID5 levels are described, which are utilized in this study. In Section 3.3 the modeling assumptions, the notation used in this chapter, and expressions for the parameters used in the analysis are provided. The analytical formulas used in classification are developed in Section 3.4. In Section 3.5 the results of a parametric study is presented to gain insight into the results of classification. In Section 3.6 empirical study is

reported that HDA outperform a purely RAID1 or RAID5 allocation. Finally, in Section 3.7 conclusions are presented.

3.2 RAID Levels and Their Operation

The RAID paradigm is necessitated by high data availability requirements due to the high cost of downtime in many applications. Furthermore, the loss of certain data is unacceptable, because the data is irreproducible or very costly to reproduce. The original 1988 RAID proposal had five RAID levels tolerating single disk failures via mirroring in RAID level 1 or RAID1, which introduces 100% redundancy, and parity coding used in RAID levels 3-5 with one disk out of N dedicated to parity [48]. RAID2 based on the Hamming code is excluded from the discussion, because of its high overhead when small blocks are updated. Striping is intended to eliminate disk access skew. It partitions large datasets into *stripe units - SUs*, which are allocated in a round-robin manner across the disks.

RAID3 and RAID4 both allocate parity blocks on one disk, but RAID3 is not suited for general-purpose applications, since it is geared for parallel access to massive datasets via synchronized reading and writing of all disks. RAID0 is a RAID with striping, but no redundancy, as a consequence disks are subjected to the least possible load. The parity disk can become a bottleneck in RAID4 for a write-intensive workload. RAID5 alleviates this problem by using the left-symmetric layout which places parity SUs in left-to-right repeating diagonals [48].

The *Basic Mirroring - BM* configuration of RAID1 replicates the same data on two disks, so that it can be read from either disk [63][70]. The fact that mirroring doubles the access bandwidth to data is important from the viewpoint of rapidly increasing disk capacities, i.e., a single disk may be unable to provide the bandwidth

required for accessing the data that it holds. Access time can be improved by judicious routing of read requests, i.e., data is accessed from the disk providing the lower access time. Updates should be carried out at both disks, but they can first be held in *Non-Volatile Storage - NVS*. This allows read requests which affect application response time to be processed at a higher priority than writes. Repeated updating of dirty blocks in NVS reduces the number of disk accesses for destaging data. Furthermore, destaging dirty blocks in batches reduces disk utilization for writing. Thees effects can be quantified by analyzing I/O trace data, but this is beyond the scope of this study. NVS caches are applicable to RAID5 disk arrays [67].

RAID1/0 stripes data across multiple RAID1 arrays, but when one of the disk fails, the read load on the surviving disk is doubled as in BM. Striping is applicable to RAID1 with multiple pairs of disks. Three RAID1 configurations with a smaller overload than BM are described in [63][70]. For example, the *interleaved declustering* layout distributes the load of a failed disk in a cluster of n disks over $n-1$ disks, so that the increase of read load per disk is $n/(n-1)$ [63][70]. Only BM is considered in this study for the sake of brevity, but the method developed here can be easily applied to other RAID1 configurations.

The updating of small data blocks in RAID5 is costly and is hence referred to as the small write penalty in RAID5. The *reconstruct write* reads the remaining corresponding blocks in a stripe and exclusive-ORs (XORs) them with the modified data block to compute the parity [60]. Reconstruct writes are preferable when the parity group size is small or more than one parity block is to be computed. The alternative read-modify-write method is costly in that it incurs two accesses to read the data and the parity block and two accesses to write them [48]. Updates can be carried out more efficiently by using *Read-Modify-Writes - RMWs*, i.e., the reading of a data and parity block, followed by their overwriting after one disk rotation [8]. Disks have the capability to compute parities and the difference (XOR) of the new

and old data blocks computed at the data disk is sent to the parity disk, where the new parity is computed. The parity is computed at disks given the old data and parity blocks. Processing reads and writes of data and parity blocks as separate requests, with the parity computed at the RAID controller, provides higher data integrity than RMWs [8].

The updating of large data blocks in RAID5, can be carried out efficiently as full stripe writes, i.e., the writing of all SUs in a stripe over all disks. The efficiency results from the fact that the new parity SU is calculated by *exclusive-ORing (XORing)* the data SUs as they are being transmitted to consecutive disks.

If a disk block is requested on a failed disk it can be reconstructed on demand by issuing a fork-join request, which reads the corresponding blocks from the $N-1$ surviving disks and XORs them to reconstruct the missing block. Since in addition to fork-join read requests each disk processes its own read requests the read load of the surviving disks is doubled in degraded mode. Clustered RAID which utilizes a smaller parity group size (G) than the number of disks in the array is a solution to the load increase problem [53]. Parity group layouts to balance the update load are discussed in the Appendix of [3]. The load increase with read and write requests in clustered RAID is given in [3]. Clustered RAID provides a tradeoff between disk bandwidth and capacity utilization. It has been considered in [69], but will not be considered in this study.

The rebuild process in RAID5 is a systematic reconstruction of the contents of a failed disk on a spare disk. This involves the reading of successive rebuild units (e.g., tracks), XORing corresponding tracks to recreate a lost track, and writing the reconstructed track onto a spare disk. RAID5 is susceptible to data loss if there is a second disk failure, before the rebuild process is completed, or the rebuild process

cannot complete because of a *latent sector failure - LSF*, which is the more likely event.

Two disk failure tolerant - 2DFT can be used for storing datasets with very high availability requirements at the cost of extra redundancy, two check disks versus one in RAID5. Two disk failures are rather uncommon, but RAID6 allows the rebuild process after one disk failure to be completed in spite of LSFs. Reed-Solomon codes are utilized in RAID6 disk arrays, while EVENODD, RDP, X-code, and RM2 utilize specialized parity codes. 2DFTs is not considered to shorten the discussion, but the methods described in this chapter can be extended to include 2DFTs in HDA in a straightforward manner. The performance of several 2DFT arrays from the viewpoint of disk accesses is studied in [3][8]. RAID6, EVENODD, and RDP exhibit the same disk access pattern and for example with two disk failures the load on the surviving disks is tripled for read requests.

HP's AutoRAID switches the RAID level between RAID1/0 and RAID5 dynamically, i.e., data with a high access rate is stored as RAID1 [34]. *Log-Structured Arrays -LSAs* accumulate modified files in a cache associated with the RAID controller, which are then written out as full stripe writes. An example, is the Iceberg disk array, which is also a RAID6 [48]. AutoRAID's RAID5 arrays also utilize the LSA paradigm.

The automatic selection of RAID levels into RAID1/0 and RAID5 is addressed in a comprehensive study reported in [26], which is part of a study of data allocation in disk arrays [25]. The two approaches considered in [26] are classified as the *tagging approach* and a *solver-based or integrated approach.* The tagging approach is classified as *rule-based* and *model based.* The rule-based tagging approach is based on a set of rules of thumb. The model based approach selects the RAID level which minimizes the umber of IOPS (I/Os per second). The solver-based approach has two variants: *partially adaptive* and *fully adaptive.* In the former case the RAID level cannot be

reassigned, while this is possible in the latter case. A dozen initial goal functions are proposed, such as minimize the average of capacities and utilizations of all the disks.

An analytic method is developed to select RAID levels for disk arrays and provide empirical evidence that HDA outperforms a pure RAID1 or RAID5 configuration.

3.3 Modeling Assumptions

Disk space constraints are not considered in this study, since this is not an issue with increasing disk capacities, but still relative space requirements are interesting. The SU size is K tracks with $K = 1$ initially, but the allocation size can be varied with K. All VA allocations have the same size for data, i.e., $M - 1$ SUs. In RAID5 arrays with width M, a VA allocation will constitute a stripe. For RAID1 with the BM configuration all $M - 1$ data tracks per allocation are written consecutively on one disk and then replicated on another, i.e., there is no striping.

Variable size allocations will not have an effect on the total number of allocations, since bandwidth bound disks are assumed to have an infinite capacity. To furthermore simplify the discussion it is assumed that all allocation requests have the same arrival rate. Although variable arrival rates of requests to disks can be accommodated, it follows from the discussion that these variability will not effect the classification. The number of tracks per SU (designated as K) is varied since this affects disk access times.

Reads and writes are allowed to small and large blocks. The latter are processed as full stripe writes or reads in RAID5. According to the LSA paradigm, full stripe reads are used to reclaim the storage space for the old versions of datasets, but the latest versions of datasets are copied first into a new stripe, so that stripes which are

read become available for writing. For example, a 3:1 read/write ratio implies that it takes the contents of three stripes to fill one full stripe to be written. Note that the RAID5 system under consideration is a hybrid system, which behaves both as a traditional RAID5, allowing small writes, and LSA, since LSA is not expected to perform well when small blocks are being updated.

The notation used in Chapter 3 and expressions for variables used in the analysis is listed below.

1. N: Number of disks.
2. M: Width of RAID5 ($M \leq N$). As M is varied it is assumed that full stripe requests span all M disks. This analysis differs from [69], where the width of a VA is determined based on the maximum load of a VA per disk. In this chapter the number of disks for RAID1 is always two.
3. K: number of tracks per SU.
4. Λ: arrival rate of disk accesses to a VA, which includes accesses to small and large blocks.
5. f_{SB}, f_{FS}: fraction of accesses to small and large blocks, with the latter processed as full stripe accesses in RAID5: $f_{FS} + f_{SB} = 1$.
6. f_R, f_W: fraction of full stripe reads and writes: $f_R + f_W = 1$.
7. f_r, f_w: fraction of small block reads and writes: $f_r + f_w = 1$.
8. The mean time for *single read - SR*, *single write - SW*, and *read-modify-write - RMW* accesses to small blocks are as follows:

   $\bar{x}_{SR} = \bar{x}_{seek} + \bar{x}_{lat} + \bar{x}_{xfer}$

   $\bar{x}_{SW} = \bar{x}_{SR} + T_h$

   $\bar{x}_{RMW} = \bar{x}_{SR} + T_{rot}$.

   $\bar{x}_{seek}$, $\bar{x}_{lat}$ and $\bar{x}_{xfer}$ denote the mean seek, rotational latency, and transfer time for disks, respectively. T_h is the head settling time for writes and T_{rot} is the disk rotation time.

9. For full stripe accesses, it is assumed that full tracks are read or written. In both cases a zero-latency capability is assumed, i.e., once the head seeks to the appropriate track, the access starts at the next sector boundary, so that the rotational latency is almost eliminated. In this study the effect of track and cylinder skews [14] are ignored, since they introduce a negligible delay. The SU size is set to the average track size, so setting the transfer time to T_{rot} is an approximation.

   For RAID5:

   $\bar{x}_{R5}^{FS} = \bar{x}_{seek} + K \times T_{rot}$.

   For RAID1 (Basic Mirroring) with all $M-1$ SUs on one disk.

   $\bar{x}_{R1}^{FS} = \bar{x}_{seek} + K(M-1)T_{rot}$.

10. U: relative disk bandwidth utilization.

11. C: relative disk capacity utilization.

3.4 Analytical Model

Two modes for RAID operation are considered.

Normal mode: VA loads are computed without considering the possibility of disk failures, so that the allocations are carried out in normal mode.

Degraded mode: The load increase with one broken disk is calculated and used when VAs are allocated. So that after the allocations the system will not be overloaded because of a disk failure.

Accesses to small and large blocks are considered. The analysis for small blocks appeared in [3], but necessary equations are repeated here for the sake of completeness. Since VAs process accesses to small and large blocks, disk utilizations are the sum of utilizations over the two types of requests.

In what follows ρ is used to denote disk utilization, but also as the total load per VA or sum of utilizations of its VDs, in which case ρ can be greater than one.

3.4.1 Operation in Normal Mode

Disk utilizations for RAID5.

Utilization for small block requests.

$$\rho_{R5}^{SB} = \frac{\Lambda}{M} f_{SB}(f_r \bar{x}_{SR} + 2 f_w \bar{x}_{RMW}). \tag{3.1}$$

Utilization for full stripe accesses.

$$\rho_{R5}^{FS} = \Lambda f_{FS} \bar{x}_{R5}^{FS} (\frac{M-1}{M} f_R + f_W). \tag{3.2}$$

Total utilization per VA:

$$\rho_{R5}^{Normal} = M(\rho_{R5}^{SB} + \rho_{R5}^{FS}) \tag{3.3}$$

Disk utilizations for RAID1.

Utilization for small block requests:

$$\rho_{R1}^{SB} = \Lambda f_{SB}(\frac{1}{2} f_r \bar{x}_{SR} + f_w \bar{x}_{SW}). \tag{3.4}$$

Utilization for full stripe accesses:

$$\rho_{R1}^{FS} = \Lambda f_{FS} \bar{x}_{R1}^{FS} (\frac{1}{2} f_R + f_W). \tag{3.5}$$

Total utilization per VA:

$$\rho_{R1}^{Normal} = 2(\rho_{R1}^{SB} + \rho_{R1}^{FS}). \tag{3.6}$$

3.4.2 Operation in Degraded Mode

Disk utilizations in RAID5.

As far as read requests are concerned, fork-join requests to reconstruct missing data blocks access $M-1$ disks in RAID5, while one disk access is required for RAID1. The read load on surviving disks is doubled in both cases. Write requests in RAID5 spawn fork-join requests to compute the parity when the data block is missing, but otherwise the data block is simply written.

Utilization due to small block requests: This is the sum of utilizations due to read and write requests.

$$\rho_{R5}^{SB} = \frac{\Lambda}{M} f_{SB}[\frac{2(M-1)}{M} f_r \bar{x}_{SR} + f_w(\frac{M-2}{M}\bar{x}_{SR} + \frac{2}{M}\bar{x}_{SW} + \frac{2(M-2)}{M}\bar{x}_{RMW})] \quad (3.7)$$

Utilization due to full stripe accesses: Both reads and writes access $M-1$ disks.

$$\rho_{R5}^{FS} = \Lambda f_{FS} \bar{x}_{R5}^{FS}. \quad (3.8)$$

Total utilization per VA:

$$\rho_{R5}^{Degraded} = (M-1)(\rho_{R5}^{SB} + \rho_{R5}^{FS}). \quad (3.9)$$

Overall utilization per VA: In degraded mode the chance that a RAID5 VA is affected by a disk failure is M/N. Otherwise with probability (1-M/N) it is not affected by a disk failure, so that its load is the same as in normal mode.

$$\rho_{R5} = \frac{M}{N}\rho_{R5}^{Degraded} + (1 - \frac{M}{N})\rho_{R5}^{Normal}. \quad (3.10)$$

Disk utilizations in RAID1

As far as read requests are concerned the load on the surviving disk is doubled, while writes are processed on one disk.

Utilization due to small block requests:

$$\rho_{R1}^{SB} = \Lambda f_{SB}(f_r \bar{x}_{SR} + f_w \bar{x}_{SW}). \quad (3.11)$$

Utilization due to full stripe accesses:

$$\rho_{R1}^{FS} = \Lambda f_{FS} \bar{x}_{R1}^{FS}. \quad (3.12)$$

Total utilization per VA:

$$\rho_{R1}^{Degraded} = \rho_{R1}^{SB} + \rho_{R1}^{FS}. \quad (3.13)$$

Overall utilization per VA: In degraded mode the chance that a RAID5 is affected is $2/N$. Otherwise with probability $(1-2/N)$ it is not affected by a disk failure (treated as in normal mode).

$$\rho_{R1} = \frac{2}{N}\rho_{R1}^{Degraded} + (1 - \frac{2}{N})\rho_{R1}^{Normal}. \tag{3.14}$$

The RAID level selected is the one that minimizes the sum of disk loads, two disks in RAID1 and M disks in RAID5.

3.5 Experimental Results

First the configuration of the disk subsystem is specified. This is followed by classification results and the effect of the following parameters on classification: (i) RAID5 width (M), (ii) the number of sectors per track (K), (iii) the mean seek time.

Table 3.1 Specifications of IBM 18ES Model DNES-309170W Disk Drives

Model	IBM DNES-309170W
Capacity	9.17GB
No. of cylinders	11474
Tracks per cylinder	5
Sectors per track	247-390
Number of zones	11
Rotation speed	7200 RPM
T_{rot}	8.3 ms
T_h	0.14 ms
$\bar{x}_{seek}$	6.9 ms

3.5.1 Disk Array Configuration

The disk array has $N = 12$ disk drives, where N determines the maximum width of RAID5 arrays. IBM 18ES (model DNES-309170W) disk drives is used, [2] whose specifications are given in Table 4.7. Particularly important is the seek time characteristic, [3] which is required in calculating its moments. The mean access time to small blocks is the sum of mean seek time, rotational latency (approximately half a disk rotation), and transfer time. The results of this study are expected to be applicable to other disk drives as well.

The mean seek time given in Table 4.7 is based on the unrealistic assumption that all disk blocks are accessed uniformly. A much lower seek time is observed in operational systems for the following reasons: (i) disk cylinders are not fully populated and in zoned disks it is the outer disk cylinders, which have a higher capacity that are written first; (ii) spatial locality of reference; (iii) seek time can be reduced by reorganizing disk data. For example, the organ pipe organization places files with higher access frequencies on contiguous disk cylinders. The results given in this chapter are based on $\bar{x}_{seek} = 1.2$ milliseconds (ms), but the sensitivity study of classification results to seek time is presented in Section 3.5.5.

The SU size (K) is initially set to one track, but K is varied to study its effect on classification results (see Section 3.5.4). The Read/Write ratio for large accesses is assumed to be $R : W = 3 : 1$, but in Section 3.5.6 the sensitivity of the classification to this ratio is investigated.

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[2] `http://www.storage.ibm.com/hdd/prod/ultrastar.htm`.
[3] `http://www.pdl.cmu.edu/DiskSim/diskspecs.html`.

3.5.2 Classification Results

Table 3.2 provides the RAID level assignments in normal mode for $N = 12$, $M = 6$ and $K = 1$ as f_{FS} (fraction of full stripe accesses - left column of the table) and f_r (fraction of reads accesses to small blocks - the first row in the table) are varied. It can be observed that when the fraction of full stripe accesses is high, RAID5 is preferable to RAID1 (the lower right hand corner of the tables). RAID1 is preferable to RAID5 when the fraction of writes for small data blocks is high, which is due to the small write penalty (upper left hand corner of the tables).

Table 3.3 is the counterpart of Table 3.2 in degraded mode.

In Table 3.4 and Table 3.5 the classification is repeated with $M = 4$. The results are similar to the results for $M = 6$, but it is observed that a smaller M width favors RAID5. The classification with $M = 2$ is RAID5 only when all accesses are full stripe accesses or they are all reads. For $M = 2$ RAID5 in normal mode requires two RMWs, while RAID1 requires two writes.

Figure 3.1 displays the variation in overall disk utilization for RAID1 and RAID5 in degraded mode, which determines the classification. Disk loads per VA allocation or the total utilization is computed setting $\Lambda = 8.7$ accesses/second. From the figure it is shown the load for RAID1 and RAID5 decreases as f_r increases. The reason is that the load increase in degraded mode is a weighted average of load for VAs in normal mode and for VAs with a single disk failure. This follows from Equation (3.10) and Equation (3.14). As f_r increases the disk load in normal mode for RAID5 decreases, since the effect of the small write penalty is diminished. In RAID1 the load is smaller for read requests than writes, since only one disk is involved in processing a read request. Also with an increase in the fraction of full stripe accesses (f_{FS}) the lines shift upward in parallel, which is simply because full stripe accesses result in a higher load.

Table 3.2 Classification of RAID Levels for an Allocation Request with Different Parameters in Normal Mode $N = 12$, $M = 6$, and $K = 1$

$f_{FS}/\ f_r$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.2	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.3	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.4	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5
0.5	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5	R5
0.6	R1	R1	R1	R1	R1	R1	R1	R5	R5	R5	R5
0.7	R1	R1	R1	R1	R1	R5	R5	R5	R5	R5	R5
0.8	R1	R1	R5	R5	R5	R5	R5	R5	R5	R5	R5
0.9	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5
1.0	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5

Table 3.3 Classification of RAID Levels for an Allocation Request with Different Parameters in Degraded Mode $N = 12$, $M = 6$, and $K = 1$

$f_{FS}/\ f_r$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1
0.1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1
0.2	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1
0.3	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.4	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.5	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5
0.6	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5	R5
0.7	R1	R1	R1	R1	R1	R1	R5	R5	R5	R5	R5
0.8	R1	R1	R5	R5	R5	R5	R5	R5	R5	R5	R5
0.9	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5
1.0	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5

Table 3.4 Classification of RAID Levels for an Allocation Request with Different Parameters in Normal Mode $N = 12$, $M = 4$, and $K = 1$

$f_{FS}/\ f_r$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.2	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.3	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.4	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.5	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5
0.6	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5
0.7	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5	R5
0.8	R1	R1	R1	R1	R1	R1	R5	R5	R5	R5	R5
0.9	R1	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5
1.0	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5

Table 3.5 Classification of RAID Levels for an Allocation Request with Different Parameters in Degraded Mode $N = 12$, $M = 4$, and $K = 1$

$f_{FS}/\ f_r$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1
0.1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1
0.2	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.3	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.4	R1	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5
0.5	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5
0.6	R1	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5
0.7	R1	R1	R1	R1	R1	R1	R1	R1	R5	R5	R5
0.8	R1	R1	R1	R1	R1	R1	R5	R5	R5	R5	R5
0.9	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5
1.0	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5

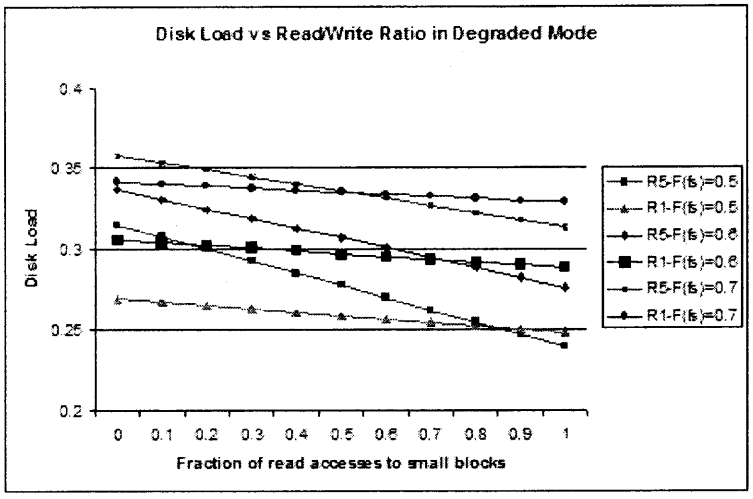

Figure 3.1 Disk loads per VA for RAID1 and RAID5 in degraded mode versus the fraction of small reads f_r. The fraction of full stripe writes (f_{FS}) is set to 0.5, 0.6 and 0.7 and for each value there are two intersecting lines one for RAID1 and the other for RAID5. $N = 12$, $M = 6$, $R : W = 3 : 1$, and $\Lambda = 8.7$ accesses/second.

3.5.3 The Effect of RAID5 Width on Classification

Next the effect of the width of RAID5 disk arrays (M) on the ratio of the number of RAID1 and RAID5 disk arrays is explored, which is denoted by R1/R5. In all cases the fraction of full stripe accesses (f_{FS}) and the fraction of small reads (f_r) change, both of which vary over 11 values, so that there are 121 entries in Table 3.2, of which 55 are RAID5 arrays and 66 are RAID1 arrays, so that R1/R5=6:5.

Figure 3.2 gives the R1/R5 ratio based on Tables 3.2 and 3.3. It is observed that this ratio decreases as M increases. For $M = 2$ RAID5 is inferior to RAID1, because accesses to small blocks in RAID5 incur the small write penalty, while only two writes to tracks are required in RAID1. RAID1 in normal mode requires two seeks and $2M - 2$ disk rotations to write both copies, while one seek and $M - 1$ rotations are required in degraded mode. In RAID5 M (resp. $M - 1$) accesses are required for full stripe processing of reads and writes in normal and degraded modes. The R1/R5 ratio is close to one as M increases.

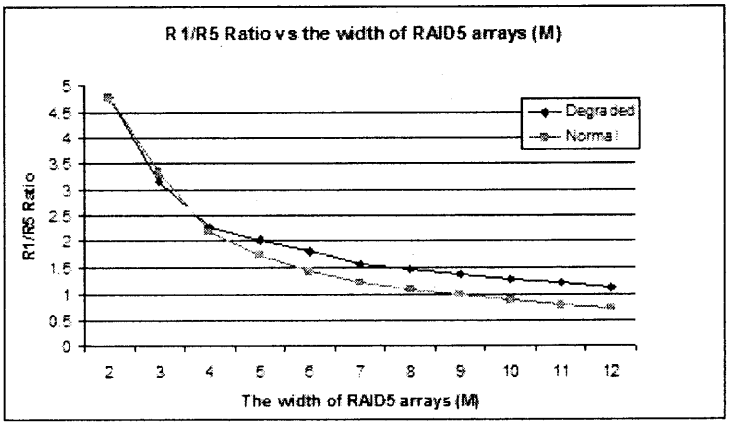

Figure 3.2 The R1/R5 ratio versus M with $N = 12$, $K = 1$. The Read/Write ratio for full stripe accesses $R : W = 3 : 1$.

3.5.4 The Effect of Number of Tracks per Stripe Unit

The size of the SU is specified with K, which is the number of tracks per SU. Figure 3.3 displays the effect of varying K on the classification using the R1/R5 ratio as a metric. As K increases the R1/R5 ratio decreases both in normal and degraded mode. For writes RAID5 requires M seeks and MK disk rotations, while RAID1 requires two seeks and $2(M-1)K$ disk rotations. For reads RAID5 requires $M-1$ seeks and $(M-1)K$ disk rotations, while RAID1 requires one seek and $(M-1)K$ disk rotations.

3.5.5 The Effect of Mean Seek Time on Classification

The sensitivity of the classification with respect to mean seek time is studied. A mean seek time $\bar{x}_{seek} = 6.9$ ms and even half of it yield R1/R5=1:0 in both degraded and normal mode (with one exception). Table 3.6 and Table 3.7 summarize the results of the classification for $M = 6$ and $M = 4$, respectively. The seek times are given as multiples of $\bar{x}_{seek}$: 1/16 (0.4 ms), 1/8 (0.9 ms), 1.2 ms and 1/4 (1.7 ms). From the tables it is observed that larger seek times favor RAID1, since in normal and

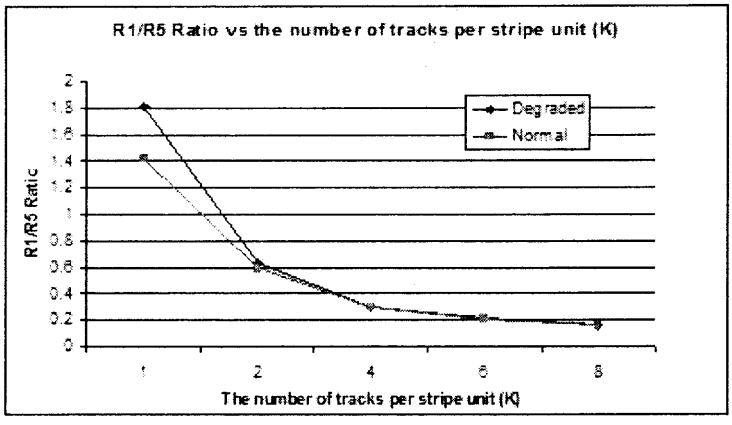

Figure 3.3 Variation of R1/R5 ratio versus K with $N = 12$ and $M = 6$. The Read/Write ratio for full stripe accesses is $R : W = 3 : 1$.

degraded mode full stripe accesses in RAID5 incur about $O(M)$ seeks for RAID5, but $O(1)$ seeks are required for RAID1.

Table 3.6 Classification of RAID Levels for an Allocation Request with Different Seek Time in Normal Mode (N) and Degraded Mode (D)

Seek time (ms)	0.4	0.9	1.2	1.7
Number of R1(N)	57	66	71	87
Number of R5(N)	64	55	50	34
Number of R1(D)	59	68	78	100
Number of R5(D)	62	53	43	21

3.5.6 The Effect of Read/Write Ratio for Full Stripe Accesses

Up to this point it has been assumed that R:W=3:1, but in this section the sensitivity of the classification to this ratio is investigated. The R1/R5 ratio is extracted from tables similar to those given in Section 3.5.2 with no and one disk failure. In fact the R1/R5 ratios in the two cases are quite similar. This is in spite of the fact that if two RAID1 disks fails, only one disk has to be written so that the RAID1 load

Table 3.7 Classification of RAID Levels for Allocation Requests with Different Seek Time in Normal (N) and Degraded (D) Mode

Seek time (ms)	0.4	0.9	1.2	1.7
Number of R1(N)	72	79	83	95
Number of R5(N)	49	42	38	26
Number of R1(D)	71	79	84	101
Number of R5(D)	50	42	37	20

is significantly reduced. On the other hand given that there are $N = 12$ disks, the chances that a particular RAID1 array is affected by a disk failure is small and the RAID1 load in normal mode dominates. It is observed from Table 3.8 and Table 3.9 that RAID1 incurs less load for a large fraction of full stripe reads, since only one seek is required. For a large fraction of writes RAID1 incurs too much overhead since each track has to be written twice, while the overhead for RAID5 is the writing of one additional SU (out of M).

Table 3.8 The Ratio of RAID1/RAID5 Allocations with Varying f_R in Normal Mode $N = 12$, $M = 6$ and $K = 1$

f_R	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
Number of R1	120	120	82	63	52	45	39	36	33	30	28
Number of R5	1	1	39	58	69	76	82	85	88	91	93

Table 3.9 The RAID1/RAID5 Ratio with Varying f_R in Degraded Mode $N = 12$, $M = 6$ and $K = 1$

f_R	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
Number of R1	121	121	92	68	55	47	41	37	33	30	28
Number of R5	0	0	29	53	66	74	80	84	88	91	93

3.6 Effectiveness of Classification on HDA Performance

Experiments are carried out to ascertain the effectiveness of the classification method developed in this chapter and that an HDA populated with VAs (Virtual Arrays) classified according to this method outperforms a disk array consisting solely of RAID1 or RAID5 arrays.

A large number of allocation requests for VAs are generated by using a synthetic workload based on randomly generated input parameters, classify them into RAID1 and RAID5, and allocate them on disks. The steps of the experiment are outlined as Algorithm 2. The experiments which are run with the following input parameters, measures the ratio of disk bandwidth and capacity utilization.

$N = 12$ disks.

One track per SU ($K = 1$).

The read/write ratio for full stripe accesses is $R : W = 3 : 1$ or $f_R = 0.75$.

The fraction of reads for small blocks is $r : w = 3 : 1$ or $f_r = 0.75$.

The fraction of full stripe accesses (f_{FS}) is uniform in the range $(0, 1)$.

The width of RAID5 is $M = 6$ VDs.

The width of RAID1 arrays is two VDs.

The experiment was run for $I_{max} = 1000$ allocations to yields statistically accurate results.

The arrival rate per VA (Λ) is sufficiently small so that the disk bandwidth limit is not reached.

Disk capacities were assumed to be infinite.

In operation in degraded mode the overall laod in degraded mode is considered, rather than assuming that all VAs are affected by a disk failure, which is overly pessimistic.

Algorithm 2 HDA allocation experiment with VAs classified as RAID1 or RAID5.

Generate and allocate VA_i for $1 \leq i \leq I_{max}$.

1. For VA_i generate the fraction of full stripe reads and writes (f_{FS}). The remaining requests are accesses to small blocks. This is not done in all experiments.

2. For VA_i generate fraction of read requests to small blocks (f_r). The remaining requests are writes to small blocks. This is not done in all experiments.

3. Compute the VA load assuming it is RAID1 as the sum of two VD utilizations in normal mode and with one failed disk. The total load is ρ_{R1} (see Equation (3.14)).

4. Compute the VA load assuming it is RAID5 as the sum of M VD utilizations in normal mode and with one failed disk failure. The total load is ρ_{R5} (see Equation (3.10)).

5. The RAID level for a VA is selected to be the one with the lower overall load, i.e., RAID1 or RAID5 depending on which level yields the same organization, i.e., $min(\rho_{R1}, \rho_{R5})$.

6. Allocate the VDs of VA_i to disks with the minimum utilization of their bandwidth, i.e., the *worst fit* method in [7][69].

7. Increment the bandwidth utilization of disks on which VDs of VA_i are assigned. Also increment allocated disk capacity: $(M-1)K$ tracks per disk for RAID1 and K tracks per disk in RAID5.

Repeat the above with the same pseudo-random sequence, without using the classification method this time, but assuming that all VAs are allocated as RAID1 and RAID5. The outcome of allocations is compared from the viewpoint of relative disk bandwidth U and disk capacity utilizations C. Some observations from experimental results in Table 3.10 are as follows.

1. With a mixture of RAID1 and RAID5, as obtained by the classification method, the relative disk bandwidth utilization U is the lowest among the three runs. This shows that a combination of RAID1 and RAID5 can supplement each other and get the best usage of disk bandwidth utilization.

2. With RAID5 only the relative capacity utilizations C of this run is the lowest and equal to $M/(M-1)$, while RAID1 doubles the space requirements of a dataset.

Table 3.10 Comparison of Relative Disk Bandwidth and Capacity with R1 Only and R5 Only, with respect to R1+R5 in Degraded Mode

	R1 + R5	R1	R5
U	1	1.16	1.07
C	1	1.31	0.79

Table 3.11 Comparison of Relative Disk Bandwidth and Capacity Utilizations for R1+R5, R1 Only, and R5 Only with respect to R0 in Normal Mode

	R0	R1 + R5	R1	R5
U	1	1.06	1.09	1.12
C	1	1.64	2	1.2

The allocation experiment is repeated using utilizations derived in normal mode and use a RAID0 array for comparison purposes. RAID0 requires $M-1$ SUs to hold the data and there is no update overhead. Results in Table 3.11 lead to similar conclusions.

Table 3.12 Comparison of Relative Disk Bandwidth and Capacity Utilization with R1 Only and R5 Only, with respect to R1+R5 in Degraded Mode

	R1 + R5	R1	R5
U	1	1.04	1.06
C	1	1.22	0.73

Table 3.13 Comparison of Relative Disk Bandwidth and Capacity Utilizations for R1+R5, R1 Only, R5 Only with respect to R0 in Normal Mode

	R0	R1 + R5	R1	R5
U	1	1.08	1.10	1.13
C	1	1.71	2	1.20

Next the fraction of full stripe accesses $f_{FS} = 0.75$ is fixed, but the fraction of reads for small blocks is varied uniformly in the range of $(0, 1)$. Tables 3.12 and 3.13 show the results. The results in this case are consistent with previous results.

3.7 Conclusions

A simple model has been used to select RAID levels based on the following parameters: (i) the fraction of accesses to small versus large blocks. The latter can be processed as full stripe writes in a RAID5 environment. (ii) the fraction of reads and writes for small block requests and to large requests. Allocation are based on operation in degraded mode, since a higher load is incurred in this mode. This model can be extended to accommodate different arrival rates. Differences in space requirements for allocation requests require a rule based system and cannot be handled analytically.

The parametric study yields insight as to which RAID level is more appropriate for different parameter settings. It has been shown that a combination of RAID1 and RAID5 levels in one disk array results in a reduction in bandwidth utilization.

CHAPTER 4

RELIABILITY AND PERFORMANCE OF MIRRORED DISK ORGANIZATIONS

4.1 Introduction

Disk mirroring or RAID level 1 (RAID1) predates the *Redundant Arrays of Independent Disks - RAID* classification [51], but is still a very popular paradigm as exemplified by EMC's Symmetrix disk array. It was used in two large-scale systems with a large number of disks: Teradata's DBC/1012 [59] and Tandem's NonStop SQL system [58]. The initial five RAID level classification (RAID1-5) [51] was later extended to seven levels by adding RAID0 and RAID6 [48]. RAID0 is based on striping, which partitions large files into fixed size stripe units allocated in round-robin manner on the disks in the array. Striping balances disk loads, since files with different access rates share disk space. In this study it is assumed that disk loads in multi-disk RAID1 configurations are balanced by via either striping or manual load balancing. RAID3-5 (resp. RAID6) dedicate the capacity of one (resp. two) disks to allow recovery from one (resp. two) disk failures. RAID5 disks can hold more user data, but this issue is ignored since disk capacities are not fully utilized.

In *basic mirroring - BM* the data held on $N/2$ primary disks is replicated on $N/2$ secondary disks. When data is modified the updating of one of two disks can be deferred as long as one disk is updated. When modified data is written onto *nonvolatile storage - NVS*, the updating of both disks can be deferred (see Section 4.2). The fact that data can be read from either disk is advantageous since: (i) disk workloads tend to be dominated by read requests; (ii) advances in magnetic recording density have resulted in several orders of magnitude increase in disk capacity in the last decade, but it is difficult to exploit the increased capacity since the access rate to

each disk is proportional to the volume of data stored on that disk; (iii) disk access time is improving very slowly due to its mechanical nature.

BM can tolerate up to $N/2$ disk failures, as long as one disk in each pair survives. On the other hand the failure of a disk results in the doubling of the read load on the surviving disk. In this chapter the performance, and performability of several RAID1 organizations, which alleviate this shortcoming of BM, i.e., distribute the load of a failed disk on multiple disks is investigated. These organizations are described in Section 4.3. Expressions for the reliability of these organizations, which turn out to be less reliable than basic mirroring are given in [63] and summarized in Section 4.4. An approximate method based on asymptotic expansions of disk unreliabilities is given in [62]. This method allows a quick comparison of RAID reliabilities without resorting to numerical methods. The performability measure combines reliability and performance metrics [15].

The maximum throughput attainable by different RAID1 organizations is obtained and an M/G/1 queueing model is utilized to compare their mean response times in normal or fault-free mode and also degraded mode operation. RAID1 performance is also compared with RAID0, RAID5, and RAID6 organizations when the total number of disks is fixed (equal to N). The case when the number of *data* disks in RAID1 is the same as the number of data disks in RAID5 has been considered in [22], but this case is not considered here for sake of brevity and especially that RAID1 outperforms RAID5 significantly. The performance comparison is carried out with read and write requests to small randomly placed data blocks.

The chapter is organized as follows. Work related to disk arrays is discussed in Section 4.2. Section 4.3 provides the description of four RAID1 organizations, whose reliabilities are derived in [63]. The reliability expressions are summarized in Section 4.4. The reliability and performability in the four cases is compared in

Table 4.1 Notation Used in Chapter 4

N	number of disks in a disk array
M	number of pairs of disks in a disk array, i.e., N/2
c	number of clusters in chained declustering
n	number of disks per cluster $n = N/c$
Λ	arrival rate to the disk array
x_r	service time of a read request
x_w	service time of a write request
$\overline{x}_r, \overline{x_r^2}$	first and second moment of service time of read requests
$\overline{x}_w, \overline{x_w^2}$	first and second moment of service time of write requests
f_r, f_w	fraction of read and write in the requests, $f_r = 1 - f_w$
λ_r	arrival rate for read requests to each disk, $\lambda_r = \frac{\Lambda f_r}{N}$
λ_w	arrival rate for write requests to each disk, $\lambda_w = \frac{\Lambda f_w}{N}$

Section 4.5. The details of the derivation of performability are lengthy and are given in the Appendix. In Section 4.6 the analysis to obtain the mean response time for read and write requests in RAID1 in both normal and degraded modes is provided. Graphs for read response times, which incorporate the effect of write requests are given in Section 4.7. This is followed by the conclusions given in Section 4.8. The notation used in this chapter is summarized in Table 4.1.

4.2 Related Work

Disk arm scheduling is one method to improve disk performance. Shortest seek time first - SSTF and SCAN are two policies to minimize seek time by reducing the seek distance. The Shortest Positioning/Access Time First - SPTF or SATF significantly outperforms these policies, since the contribution of seek time to positioning time

(the sum of seek time and rotational latency) has become less significant (see e.g., [6]).

With the advent of mirrored disks SSTF was extended by routing an incoming request to the *Nearest Server - NS*, i.e., disk cylinder. When seek distances are uniformly distributed, the expected value of the minimum of two seek distances is $5/24 = 0.2083$, which is much smaller than the mean seek distance on a single disk $(1/3)$. The seek minimization issue is discussed in Section 3 in [16], which also presents some simulation results. A notable but not well known analysis [1] obtains the average motion over N randomly selected points. When the points are placed linearly, i.e., not in a circle, the average motion is approximately 0.1626, while the optimal policy yields 0.1598.

Mirrored disk scheduling algorithms can be classified as static (or offline) and dynamic (or online) [9]. Online or dynamic algorithms require knowledge of the current state of the system, e.g., the position of the disk arms to apply the NS algorithm. Other online algorithms are proposed in [9][4]. Static algorithms are applied when the state of the system is difficult to determine. An example of a static routing policy is the probabilistic or uniform policy, while the cyclical or round robin policy can be shown to have superior performance for Poisson arrivals and exponential service times [9]. Routing requests to the outer disk cylinders on one disk and the inner disk cylinders at the other is a form of affinity based routing. The delineation of inner and outer cylinders for this purpose in disks with zoning was considered in [65]. Dynamic routing of requests in RAID1 with the *chained declustering - CD* organization [33] was evaluated in [39].

A greedy policy to minimize the mean seek distance in mirrored disks is to choose the arm nearer to the target cylinder t $(0 \leq t \leq 1)$ and place the other arm at $t/3$, if $t \geq 1/2$, and $1-(1-t)/3$, otherwise. With independent, uniformly distributed

requests the new mean seek distance is 5/36, which is much smaller than 5/24 (the expected minimum of two seek distances) [31]. One of the two arms may be dedicated to serving the inner cylinders of the disk and the other arm to the outer cylinders, but even better performance is attainable without this restriction [27]. It is easy to see that the mean seek distance will equal 0.125 if the arms of are placed at 1/4 and 3/4, but such arm placement might delay the processing of external requests. Optimal disk arm placement in single and mirrored zoned disks is investigated in [64].

Instead of two mirrored disks, higher disk bandwidth can be achieved by a single disk with two arms. Two R/W heads with a fixed separation of d cylinders is considered in [2]. With the movement of both arms restricted to (0,1) the left (resp. right) arm is restricted to the interval (0,1-d) and (d,1), respectively. The paper determines the optimum separation of the arms (d) to minimize the seek distance. In addition an optimal policy is defined which slightly outperforms NS.

The interaction of read and write requests in mirrored disks has been studied extensively, see e.g., [20]. When a cache is not available, performance improvement can be attained by using the write-anywhere policy for writes, so that positioning time is alleviated. When an NVS cache is available, then the processing of writes can be deferred, as noted in Section 4.1, i.e., writes are processed at a lower priority than reads. The processing of batches of writes can be carried out efficiently by taking advantage of disk geometry. A two-phase method for processing reads and writes was described and evaluated in [21], where one disk processes reads, while the other processes writes. Extensions to this method are presented in [66].

The analysis and simulation of various mirrored disk scheduling policies was given in [24]. A performance comparison of RAID1 and RAID5 disk arrays in fault-free mode was considered in [22]. Three RAID1 organizations: BM (basic mirroring), CD (chained declustering), and *group rotate declustering - GRD* were considered.

The workload consists of accesses to small data blocks and full stripe reads and writes. The performance comparison was carried out with the same total number of disks and also the same number of *data* disks. The following conclusions were drawn by considering random routing, join the shortest queue - JSQ, and minimum seek distance policies: (i) JSQ provides the best performance, especially when the load on the I/O system is high. (ii) Given that the total number of disks is the same, RAID1 outperforms RAID5 to a significant degree. (3) RAID1 outperforms RAID5 even for I/O applications where requests are to large data blocks and the same number of disks. The only exception occurs for full stripe writes, which write all the stripe units in a row, so that the parity stripe unit can be computed on the fly. In this case RAID5 writes approximately half of the data written by RAID1. This work is extended to include operation in degraded mode in all cases with the same number of disks, since otherwise the comparison is unfair.

It is argued in [45] that latency is a significant component of disk service time. In the *dual copy* approach both mirrored disks participate in satisfying a single request. Assuming that both arms reach the target cylinder simultaneously, the arm closest to the requested block reads the data.

The *synchronized dual copy approach* rotates the two disks so that they are 180 degrees apart, so that the mean rotational latency is reduced from 1/2 to 1/4 of disk rotation time. The *dual copy on a single disk* approach stores two copies of data 180 degrees apart on neighboring tracks on the same cylinder. In the *dual actuator approach*, there is only one copy of data, but the two actuators are 180 degrees apart.

4.3 RAID1 Organizations

The four RAID1 organizations considered in this study are described below. The number of disks is set to $N = 8$ in all cases.

Basic Mirroring - BM This is the most common type of mirroring, which is shown in Figure 4.2 with striping in effect. Each nonprimed letter denotes a primary SU, while primed letters denote secondary SUs.

Load balancing of read requests across two disks is attained by appropriate request routing, e.g. uniform versus round-robin routing. Other forms of routing in BM are discussed in [24][9]. When a disk fails, the read load at the mirroring disk is doubled.

Table 4.2 Basic Mirroring with Striping with N=8 Disks

Primary Disks				Secondary Disks			
1	2	3	4	5	6	7	8
A	B	C	D	A'	B'	C'	D'
E	F	G	H	E'	F'	G'	H'
I	J	K	L	I'	J'	K'	L'
M	N	O	P	M'	N'	O'	P'

Group Rotate Declustering - GRD. This RAID1 data layout was proposed in [22] to balance disk loads when a single disk failure occurs. Data is striped on $M = N/2$ primary disks and the SUs on the primary disks are replicated in a rotated manner on an equal number of secondary disks as shown in Figure 4.3. With no disk failures reads are routed equally to primary and secondary disks. When a primary disk fails its load will be distributed evenly on the secondary disks, but a smaller fraction of the read load of surviving primary disks needs to be routed to secondary disks to balance disk loads, e.g., with $N = 8$ each surviving primary disk will process a

fraction 4/7 of the read load, rather than one half of it. Each secondary disk processes 1/4 of the read load of a failed primary disk and $3 \times (3/7)/4 = 9/28$ of the load on surviving disks, so that the read load at each secondary disk is also $1/4 + 9/28 = 4/7$. A more general discussion appears in Section 4.6.

Table 4.3 Group Rotate Declustering with N=8 Disks

Primary Disks				Secondary Disks			
1	2	3	4	5	6	7	8
A	B	C	D	A'	B'	C'	D'
E	F	G	H	H'	E'	F'	G'
I	J	K	L	K'	L'	I'	J'
M	N	O	P	N'	O'	P'	M'

Interleaved Declustering - ID. This RAID1 organization shown in Figure 4.4 first appeared in the Teradata DBC/1012 database computer [59]. Each disk is divided into a primary and a secondary area, which have equal capacities. The primary data on each disk in a cluster of disks is distributed evenly on the secondary areas of other disks in the cluster. Given N disks and c clusters, there are $n = N/c$ disks per cluster. When a single disk fails, the read load at the surviving disks of the cluster is balanced and is increased by a factor of $n/(n-1)$.

Chained Declustering - CD. This data organization was proposed in [33] to attain a higher reliability level than ID. Each disk is partitioned into a primary and secondary area, which have equal capacities. Data in primary areas, labeled as A, B, C, ..., is replicated on the next disk in the array, labeled as A', B', C', ..., modulo the number of disks as shown in Figure 4.5.

Consider the failure of disk 1 (D_1). Since a copy of A is only available at D_2, D_2 will processes all of the reads to A and $1/(N-1)$ for accesses to B at D_2. The read load on surviving disks is balanced and is is increased by a factor $N/(N-1)$.

Table 4.4 Interleaved Declustering with N=8 Disks in One Cluster

Disks							
1	2	3	4	5	6	7	8
A	B	C	D	E	F	G	H
b_7	a_1	a_2	a_3	a_4	a_5	a_6	a_7
c_6	c_7	b_1	b_2	b_3	b_4	b_5	b_6
d_5	d_6	d_7	c_1	c_2	c_3	c_4	c_5
e_4	e_5	e_6	e_7	d_1	d_2	d_3	d_4
f_3	f_4	f_5	f_6	f_7	e_1	e_2	e_3
g_2	g_3	g_4	g_5	g_6	g_7	f_1	f_2
h_1	h_2	h_3	h_4	h_5	h_6	h_7	g_1

Table 4.5 Chained Declustering with N=8 Disks

Disks							
1	2	3	4	5	6	7	8
A	B	C	D	E	F	G	H
H'	A'	B'	C'	D'	E'	F'	G'

The failure of two disks is considered next: e.g., disk 1 (D_1) and disk k (D_k,$k < N$). The two disk failures partition the array into two subarrays with $k-2$ and $N-k$ disks with $k-1$ and $N-k+1$ unique data items per subarray, respectively. For $k = 4$ there are items (A,B,C) in subarray 1 with two surviving disks and items (D,E,F,G,H) in subarray 2 with four disks. The load increase is $(k-1)/(k-2)$ and $(N-k+1)/(N-k)$ or 3/2 and 5/4, respectively. Thus the method of balancing read requests only applies locally at the level of subarrays.

4.4 RAID1 Reliability Analysis

The basics for reliability analysis are reviewed. Then the reliability expressions for the four RAID1 organizations under consideration are summarized, which were derived in [63]. An alternative approach to derive the reliabilities is given at the end.

Given the cumulative distribution function of time to component failure $F(t)$, its reliability function is $R(t) = 1-F(t)$ [71]. It is concluded in [28] that the reliability of a single disk can be approximated by an exponential distribution: $R_{disk}(t) = e^{-\lambda t}$, so that its *mean time to failure* - *MTTF* is $MTTF_{disk} = \int_0^\infty e^{-\lambda t} = 1/\lambda$.

The reliability of a system without redundancy is the product of the reliabilities of its components [71], while the reliability of a system with redundancies is determined by the subset of its components that are required for its operation [71]. For example, the reliability of an array of N disks is given as $R_{N-disks} = e^{-N\lambda t}$ and its MTTF as $MTTF_{N-disks} = 1/(N\lambda)$.

The RAID paradigm utilizes redundancy to attain a sufficiently high reliability. Let $A(N,i)$ denote the number of cases that a RAID disk array with N disks can tolerate i disk failures, so that $A(N,0) = 1$ and $A(N,i) = 0, i > i_{max}$, where i_{max} is the maximum number of disk failures that can be tolerated. $i_{max} = 1$ for RAID5,

$i_{max} = 2$ for RAID6, $i_{max} = N/2$ for BM, GRD, and CD, $i_{max} = c$ (the number of clusters) for ID. The following formula applies to any of the above disk arrays.

$$R_{RAID}(t) = \sum_{i=0}^{i_{max}} A(N,i) R_{disk}^{N-i}(t)(1 - R_{disk}(t))^i. \tag{4.1}$$

RAID5 and RAID6 can tolerate one and two disk failures, by dedicating one and two disks out of N disks to check disks, respectively. $A(N,i) = \binom{N}{i}$ for $0 \leq i \leq 1$ for RAID5 and $0i \leq i \leq 2$ for RAID6.

Expressions for $A(N,i)$ for RAID1 with BM, GRD, ID, and CD organizations are given in [63], but are repeated here for the sake of completeness. Asymptotic expansions for reliability equations given in [62] are not repeated here.

$$A_{BM}(N,i) = \binom{N/2}{i} 2^i, \ 0 \leq i \leq N/2. \tag{4.2}$$

$$A_{GRD}(N,i) = 2\binom{N/2}{i}, \quad 1 \leq i \leq N/2. \tag{4.3}$$

$$A_{ID}(N,i) = \binom{c}{i} n^i, 0 \leq i \leq c. \tag{4.4}$$

$$A_{CD}(N,i) = \binom{N-i-1}{i-1} + \binom{N-i}{i}. \tag{4.5}$$

A Markov chain model [71] can be used to derive the reliability of RAID1 organizations. The probability that a RAID survives i disk failures is given as:

$$\mathcal{P}(N,i) = A(N,i)/\binom{N}{i} \tag{4.6}$$

The state S_i of the Markov chain in Figure 4.1. denotes the number of failed disks, so that the failure rate at S_i is $(N-i)\lambda$. The probability that the i^{th} disk failure does not lead to data loss is $\mathcal{P}(N,i) = P[S_{i-1} \rightarrow S_i]$ and the probability of data loss is $Q(N,i) = 1 - P(N,i) = P[S_{i-1} \rightarrow F]$. The mean number of visits to each state is denoted by V_i. $V_0 = 1$ and $V_i = V_{i-1}\mathcal{P}(N,i)$.

In the case of BM with $N = 8$, $P[S_0 \rightarrow S_1] = 1$, $P[S_1 \rightarrow S_2] = 6/7$, $P[S_2 \rightarrow S_3] = 2/3$, $P[S_3 \rightarrow S_4] = 2/5$. It follows that the mean number of visits to the states are given as: $V_0 = V_1 = 1$, $V_2 = 6/7$, $V_3 = 4/7$, $V_4 = 8/35$. Table 4.6 gives the probabilities for all four RAID1 organizations.

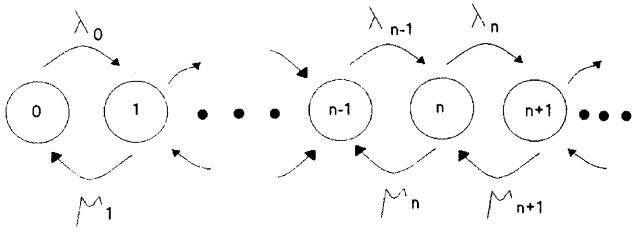

Figure 4.1 Markov chain for a mirrored disk array with N=8 disks (the state specifies the number of failed disks, F is the failed state).

Table 4.6 Transition Probabilities and Number of Visits to Markov Chain States

RAID	$P[S_0 \rightarrow S_1]$	$P[S_1 \rightarrow S_2]$	$P[S_2 \rightarrow S_3]$	$P[S_3 \rightarrow S_4]$	V_0	V_1	V_2	V_3	V_4
BM	1	6/7	2/3	2/5	1	1	6/7	4/7	8/35
GRD	1	3/7	1/3	1/5	1	1	3/7	1/7	1/35
ID	1	4/7			1	1	4/7		
CD	1	5/7	2/5	1/10	1	1	5/7	2/7	1/35

4.5 Comparison of the Four RAID1 Organizations

The four RAID1 organizations are compared from the viewpoint of their *mean time to data loss - MTTDL* and their performability measure.

4.5.1 Reliability Comparison

Given that disk failure rates are exponential with rate $\lambda = 1/MTTF_{disk}$, the MTTDL (without disk repair) can be expressed as $MTTDL = \sum_{i=1}^{i_{max}} V_i/[(N-i)\lambda]$. For the four organizations with $N = 8$ disks and $c = 2$ clusters for ID they are: $MTTDL_{BM} = (163/280)/\lambda = 0.582/\lambda$. $MTTDL_{GRD} = (3/8)/\lambda = 0.375/\lambda$, $MTTDL_{ID} = (61/168)/\lambda = 0.363/\lambda$, $MTTDL_{CD} = (379/840)/\lambda = 0.451/\lambda$. The MTTDL for the RAID5 and RAID6 disk arrays with $N = 8$ disks is $MTTDL_{RAID5} = (15/56)/\lambda = 0.268/\lambda$. and $MTTDL_{RAID6} = (13/28)/\lambda = 0.464/\lambda$.

It should be noted that MTTDL is not a good measure of reliability, in the same way that the MTTF might be misleading. For example, the MTTF of *Triple Modular Redundancy - TMR* system: $MTTF = 5/(6\lambda)$ is smaller than the MTTF of a simplex system, which is $1/\lambda$. TMR systems are more reliable for missions with duration $t < (ln2)/\lambda)$ [71].

The MTTDL of disk arrays is usually computed when repair is allowed. For example, the analysis of the Markov chain model in [28] leads to a disk failure if a second disk failure occurs before the repair of the first disk is completed. This analysis is simplified in that it does not take into account *latent sector failures - LSFs* [35], which is the reason for the introduction of RAID6 disk arrays [41].

Figure 5 in [63] plots the reliability of the four RAID1 organizations with $N = 8$ (with $c = 2$ for the ID organization) plus RAID5 and RAID6 disks arrays versus decreasing disk reliabilities. RAID6 is the most reliable and RAID5 the least reliable, with RAID1 reliabilities lying between these two extremes. BM is the most

reliable because it allows the largest number of failed disk configurations without data loss. ID with $c = 2$ clusters with $n = N/2$ disks per cluster is less reliable than BM, since only one disk failure can be tolerated per cluster. GRD is less reliable than ID with two clusters. CD is more reliable than ID since it allows more disk failures without data loss.

4.5.2 Performability Comparison

Performability analysis is a means of characterizing the performance of systems with several operating modes [15], i.e., gracefully degradable systems. In a system with repair given the probability $\pi(S_i)$ that the system is at state S_i and has a performance metric T_i, then the overall performability metric is $T = \sum_{\forall i} T_i \pi(S_i)$. The performability of a system is compared without repair by taking into account the mean time the system spends in each state.

Multiple Sources: Each pair of disks has its own source, which allows all disk pairs to be driven at their maximum bandwidth. When all requests are reads and a single disk fails in BM, then the bandwidth of the pair of disks is halved.

Single Source: Disk requests originate from one source with disk loads balanced in fault-free mode with striping (all disk pairs are accessed uniformly) and without striping.

The expected number of completed disk requests are obtained as the system proceeds through various states of degradation. The number of requests processed in each state is the product of the duration of the state, i.e., $1/((N-i)\lambda)$, the probability that the system tolerates i disk failures without data loss, i.e., $V_i = \mathcal{P}(N, i)$, and the system throughput T_i. $T_i = (N - i)\mu, i > 0$ when requests are from multiple sources. When a single disk failure occurs and requests are from a single source then: $T_i = M\mu$ for BM, $T_i = (n - 1)\mu$ for ID with $n = N/c$ disks in each one of c clusters, In the case

of CD, as exemplified in Section 4.3 and explicated in the Appendix, T_i is determined by the smallest number of contiguous disks. It is

$$N_i = T_i V_i / ((N - i)\lambda). \tag{4.7}$$

The performability metrics for the four RAID1 organizations are derived in detail in the Appendix. Starting with a fault-free system performability is determined by the expected number of read requests processed by the RAID1 system up to the point where a disk failure leads to data loss. All requests are reads is assumed to attain the most discrimination among the four RAID organizations.

The results are summarized in Figure 4.2. BM completes the largest number of requests although its throughput drops from $N\mu$ to $(N/2)\mu$ when arrivals are from a single source. CD is the second best organization. ID with $c = 2$ outperforms ID with $c = 1$, since it can tolerate two disk failures versus one.

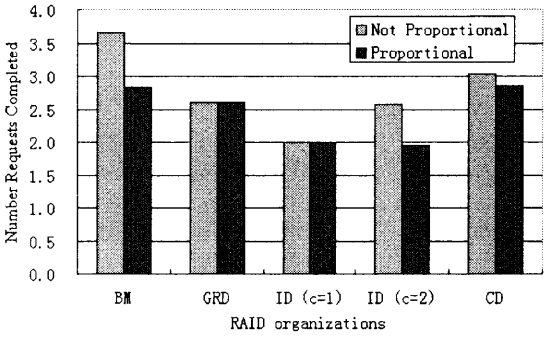

Figure 4.2 Comparison of the expected number of requests processed by the four RAID1 organizations.

4.6 RAID1 Performance Analysis

Formulas are developed to compare the relative performance of the four RAID1 organizations in fault-free mode and with disk failures. Modeling assumptions are presented first.

4.6.1 Modeling Assumptions

- The arrivals of read and write accesses are according to a Poisson process. Probabilistic routing is used to balance the load due to reads, so that the arrival process at the disks remains Poisson.
- Reads and writes are unformly distributed over all primary data, This assumption is not required for the analysis, but simplifies the specification of the model.
- Discrete disk requests are to small randomly placed data blocks on disk, as in *online transaction processing - OLTP* applications.
- Disk requests are processed in FCFS order since this allows a quick comparison of the various RAID1 organizations via a closed form expression for the mean waiting time of the M/G/1 queueing model [37][57].
- In the case of BM and GRD reads are evenly split between the two disks holding the same data to balance their read load. The read load for ID and CD organizations is already balanced. Routing of requests in the presence of disk faults is discussed in Section 4.6.3.

The 9.17 GB, 7200 RPM, IBM 18ES (model DNES-309170W) disk drive is considered in this studies. [1] The detailed characteristics of disk drive is used [2] whose characteristics are summarized in Table 4.7.

The mean response times of discrete disk requests takes into account the effect of zoning and detailed seek time characteristics [65][61]. Read requests which affect

[1] `http://www.storage.ibm.com/hdd/prod/ultrastar.htm`
[2] `http://www.pdl.cmu.edu/DiskSim/diskspecs.`

Table 4.7 Specifications for IBM 18ES Disk Drives

Model	IBM DNES-309170W
Capacity	9.17GB
No. of cylinders (C)	11474
Tracks per cylinder	5
Rotation speed	7200 rpm
Time to rotate (T_{rot})	8.3 ms
Sectors per track	247-390
Number of zones	11
Mean seek time	6.9 ms

application response time may be given a higher priority than writes. This case can be analyzed using the head-of-the-line priority queueing model [38][57].

The two assumptions that disk requests are uniformly distributed over disk blocks is pessimistic, since one or few disk regions are active at any time, which results in shorter seek distances than attained with a uniform distribution of requests. The FCFS disk scheduling is outperformed by SATF, but is considered here since it is amenable to a simple analysis. This is acceptable since interest is given in the relative, rather than the absolute performance of the various RAID organizations.

4.6.2 Fault-Free or Normal Mode of Operation

The four RAID1 organizations have a similar performance when they are operating in fault-free or normal mode. It is assumed that the load is balanced across disk pairs, so the performance of one pair of disks with the BM organization is analyzed. Otherwise, the mean response time overall disks would have to be weighted by the arrival rate to each pair.

Given $M = N/2$ disk pairs, an arrival rate Λ to the disk array, and f_r and f_w as the fractions of reads and writes, the arrival rate of reads and writes to each disk pair is $\lambda_r = f_r\Lambda/M$ and $\lambda_w = f_w\Lambda/M$, respectively. With uniform routing of reads the arrival rate to each disk is:

$$\lambda_{disk} = \lambda_r + \lambda_w = \frac{\Lambda}{M}(\frac{f_r}{2} + f_w) = \frac{\Lambda}{2M}(1 + f_w). \tag{4.8}$$

Given the actual fraction of reads and writes to each disk: $f'_r = (f_r/2)/(f_r/2 + f_w) = f_r/(1 + f_w)$ and $f'_w = 1 - f'_r = 2f_w/(1 + f_w)$, the i^{th} moment of the overall disk service time is:

$$\overline{x^i} = f'_r\overline{x^i_r} + f'_w\overline{x^i_w}. \tag{4.9}$$

The utilization of each disk is:

$$\rho = \lambda_{disk}(f'_r\overline{x}_r + f'_w\overline{x}_w) = \frac{1}{2}\lambda_r\overline{x}_r + \lambda_w\overline{x}_w. \tag{4.10}$$

With a fast-write capability with an NVS cache the time to complete a write is negligibly small. The analysis is simple when the data has to be written onto one disk, but becomes more complicated when a variation of write-anywhere policy is in effect [20].

When a data block is to be written to both disks before a write is considered completed, the write response time equals that of a 2-way fork-join request: $R_2^{F/J}$, which is approximated by the easy to compute R_2^{max}. It is shown in [44] that in the case of the exponential distribution the mean n-way fork-join response time is upper-bounded by R_n^{max}.

R_2^{max} the write response time is the time to write both disk blocks. It can be estimated by first approximating individual writes with the *extreme-value distribution - EVD*: $P[Y < y] = exp(-e^{-\frac{y-a}{b}})$, which has a mean $\overline{Y} = a + \gamma b$ and a variance $\sigma_Y^2 = (\pi b)^2/6$. The maximum of n random variables with EVD is: $Y_{max}^n = (a+\gamma b) + bln(n)$, where $\gamma = 0.57721$ is the Euler-Mascheroni constant. Let R_w and σ_w^2 denote the mean and variance of write requests, then $\overline{Y} = R_w$ and $b = \sqrt{6}\sigma_w/\pi$. It follows that $R_2^{max} = R_w + \sqrt{6}\sigma_w ln(2)/\pi$.

The current system is different in that each disk in addition to writes processes reads (in FCFS order). This results in a higher variability of fork-join response times and this makes R_2^{max} a better approximation. Write response times are not reported in Section 4.7, since they do not affect the application response time.

4.6.3 Degraded Mode Analysis

The arrival rate of reads to surviving disks depends on the organization of the disk array. In fact, the BM organization cannot balance disk loads in degraded mode, while this is possible with the other RAID1 organizations.

4.6.3.1 Basic Mirroring. The arrival rate of reads to the surviving disk is λ_r, so that its utilization factor is given as:

$$\rho = \lambda_r \overline{x}_r + \lambda_w \overline{x}_w. \tag{4.11}$$

With k failed disks the mean overall response time for reads is:

$$R_r = \frac{k}{M} R_r^d + \frac{M-k}{M} R_r, \tag{4.12}$$

where the superscript d denotes the higher response time of reads in a pair with a failed disk.

The response time for writes to failed disk pairs is computed differently, since there are no fork-join requests, except that the individual write response time is higher, since the read load is doubled.

$$R_w = \frac{k}{M} R_w^d + \frac{M-k}{M} R_w^{max}. \tag{4.13}$$

4.6.3.2 Group Rotate Declustering. In GRD the utilizations of primary and secondary disks with k primary (or secondary) disks fail can be balanced as follows. Let α_k denote the fraction of reads processed by the $M-k$ surviving primary disks. Then a fraction $1-\alpha_k$ of reads from these disks are routed to secondary disks (the value of α_k is determined below). The read load of a failed disk is fully routed to secondary disks. The routed reads in both cases are uniformly distributed.

Denoting the utilization of primary and secondary disks as ρ_p and ρ_s, respectively, formula is:

$$\rho_p = \alpha_k \lambda_r \overline{x}_r + \lambda_w \overline{x}_w. \tag{4.14}$$

$$\rho_s = [\frac{M-k}{M}(1-\alpha_k) + \frac{k}{M}]\lambda_r \overline{x}_r + \lambda_w \overline{x}_w. \tag{4.15}$$

Disk utilizations are balanced for:

$$\alpha_k = \frac{M}{2M-k}. \tag{4.16}$$

The mean read response times will be approximately the same, since the utilizations of the primary and secondary disks are equal and reads and writes have about the same response time. Write response times are computed similarly to BM.

4.6.3.3 Interleaved Declustering. With N disks and c clusters the increase in read load on surviving disks in a cluster with $n = N/c$ disks and one failed disk is:

$$\alpha = \frac{n}{n-1}. \tag{4.17}$$

The arrival rate of reads to the surviving disks in the cluster is then:

$$\lambda_r = \frac{f_r \Lambda \alpha}{N}. \tag{4.18}$$

The increase in the read load will affect the maximum throughout sustainable by the cluster. When there are multiple clusters, the response times should be weighed according to the number of clusters in the two categories, i.e., without or with failed disks.

4.6.3.4 Chained Declustering. When a single disk fails in CD, its read load can be distributed evenly over the surviving disks so that the arrival rate of reads increases by a factor $N/(N-1)$. When two non-consecutive disks fail, there will be two groups of disks with sizes g_1 and g_2 and $g_1 + g_2 = N - 2$. The read load in group $i = 1, 2$ is increased by a factor $\alpha_i = (g_i + 1)/g_i$ and the arrival rate of reads to the i^{th} group is then $\lambda_r^i = f_r \Lambda \alpha_i / N$. This calculation generalizes to more than two disk failures.

As shown in Figure B.1 in the Appendix, there are 20 configurations with $k = 2$ disk failures for $N = 8$, but only $I = 3$ different group sizes as follows: (i) In

eight out of twenty cases ($w_1 = 8/20$) $g_1 = 1$ and $g_2 = 5$. (ii) In eight out of twenty cases ($w_2 = 8/20$) $g_1 = 2$ and $g_2 = 4$. (iii) In four out of twenty cases ($w_3 = 4/20$) $g_1 = g_2 = 3$.

Let $R_r^{j,i}$ denote the mean Read response of group i in configuration j. The mean Read response time for CD with N disks, k failed disks, and J configurations, is as follows:

$$R_r^k = \sum_{j=1}^{J} w_j \sum_{i=1}^{k} \frac{g_i + 1}{N} R_r^{j,i}. \tag{4.19}$$

What is not immediately clear from this equation is that R_r^k is dominated by response times from small group sizes, such that numerical results show that CD response time with two disk failures is only slightly better than BM and in fact both disk organizations attain the same maximum throughput.

The response times in each group are determined by its size, rather than its membership in a certain configuration, so that the computation of response times can be simplified by finding the fractions of different group sizes, say f_k. The overall mean response time is given as $R_r = \sum_{\forall k} f_k R_r^k$.

4.7 Performance Results

The performance of RAID1 organizations is compared with RAID0, RAID5, and RAID6 disks arrays. The ID organization has one cluster ($c = 1$). The case when the total number of disks in the arrays is the same and equal to N is considered.

The workload consists of accesses to small (4 or 8 KB) blocks of data, so that the transfer times is negligibly small compared to positioning time. This is because disk requests are uniformly distributed over the blocks and are served in FCFS order,

so that they incur a high positioning time. In addition, the workload incurs the small write penalty, i.e., two (resp. three) read-modify-writes - RMWs to update data and parity blocks in RAID5 (resp. RAID6) [67]. The read to write ratio (R:W) and the number of failed disks are varied from zero to two.

Although some RAID1 organizations can tolerate up to $N/2$ disk failures, there is no guarantee that more than one disk failure can be tolerated. RAID1 results with two disk failures are only valid when the two disk failures do not lead to data loss. In practice, if a spare disk is provided and the rebuild process is started immediately, then the mean time to the completion of the rebuild process is much smaller than the $MTTF_{disk}$, so that the possibility of encountering a system with two failed disks is negligibly small. The results given here are applicable to a system with no repair as long as data loss does not occur. Storage bricks is a new paradigm where an array of disks constitutes the smallest replaceable unit [72], so that spare disks for rebuild processing may not be provided at the brick level.

The analysis of RAID1 is based on the analysis in Section 4.6, while the analysis of RAID5 and RAID6 in [8] is utilized to obtain the response times in this section.

Figures 4.3, 4.4 and 4.5 provide the mean Read response time with no disk failures in RAID0, RAID1, RAID5 and RAID6 when the R:W ratio is 1:0, 3:1 and 1:1, respectively.

RAID1 has the best performance, especially for higher R:W ratios, because it provides twice the access bandwidth of other RAID levels for reading data. RAID0, RAID5 and RAID6 exhibit the same mean response time when the read/write (R:W) ratio is 1:0. In the presence of writes, the read response time for RAID0 is lower than RAID5 and RAID6, since in addition to parallelism in processing reads it incurs less

overhead in updating check disks. RAID5 outperforms RAID6, since only one, rather than two check blocks need to be updated.

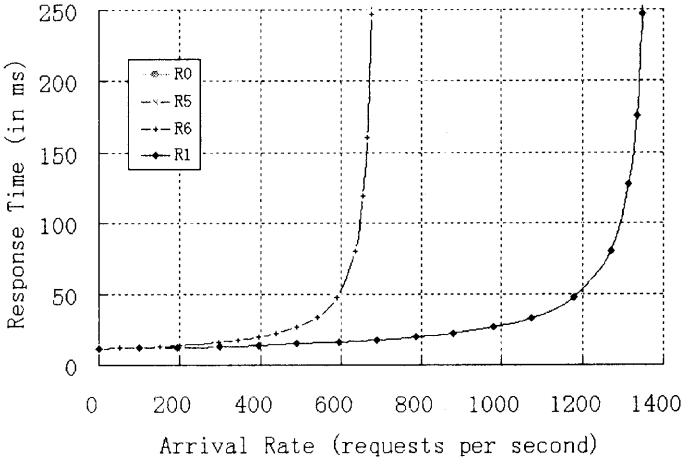

Figure 4.3 Response time of read requests in normal mode R:W=1:0, N=8. The graphs for RAID0 and RAID5 overlap with RAID6, so that only RAID6 is shown.

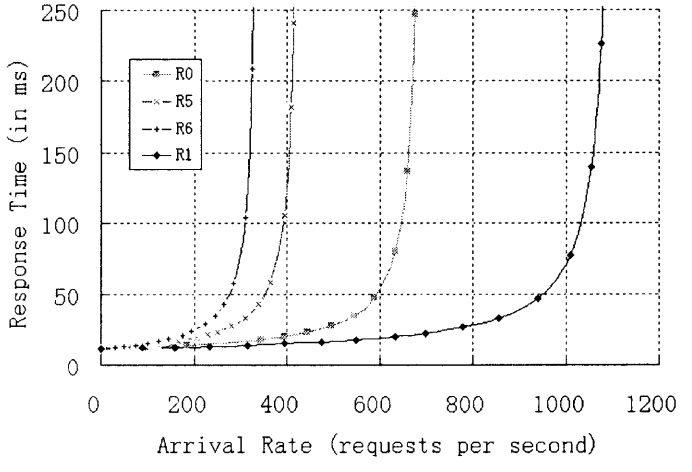

Figure 4.4 Response time of read requests in normal mode, R:W=3:1, N=8. Note that GRD, ID, and CD have the same response time.

Figures 4.6, 4.7 and 4.8 provide the Read response time for RAID1, RAID5 and RAID6 with one disk failure when the R:W ratios are 1:0, 3:1 and 1:1. For R:W=1:0 the maximum throughput attained by RAID5 and RAID6 is half of the throughput attained in normal mode (this is approximately so for RAID6). The maximum throughput for BM is halved, GRD, ID (with $c = 1$), and CD attain

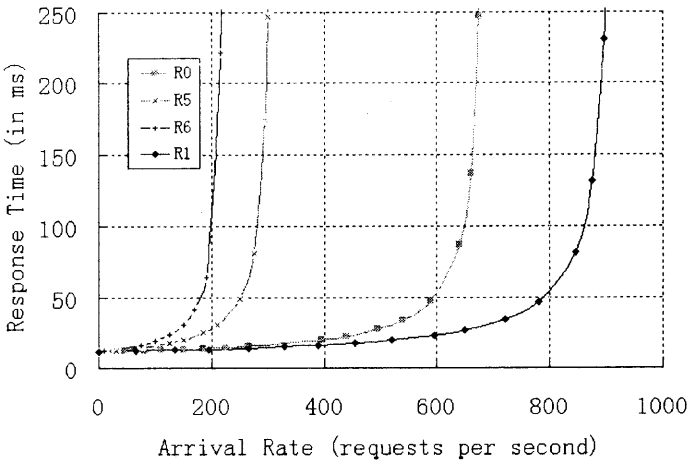

Figure 4.5 Response time for read requests in normal mode, R:W=1:1, N=8. Note that GRD, ID, and CD have the same response time.

$(N-1)/N$ of the throughput in normal mode by utilizing routing to balance the loads.

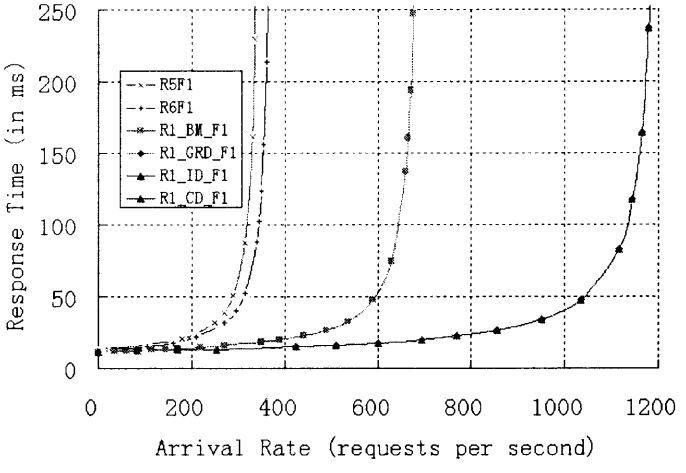

Figure 4.6 Response time for read requests with one disk failure, R:W=1:0, N=8.

Figures 4.9, 4.10 and 4.11 provide the mean Read response times for BM, GRD, CD and RAID6 with two disk failures when the R:W ratios are 1:0, 3:1 and 1:1. Here there are only four systems because RAID5 and ID with one cluster cannot tolerate two disk failures. The maximum throughput for RAID6 with read requests is one third of the throughput with no disk failures, since each surviving disk processes

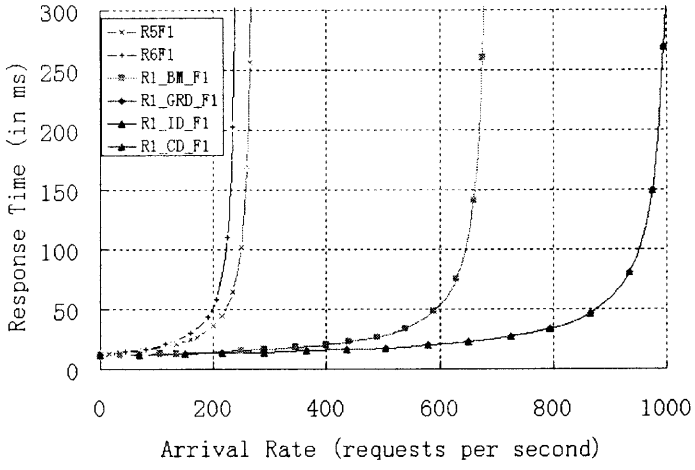

Figure 4.7 Response time for read requests with one disk failure, R:W=3:1, N=8.

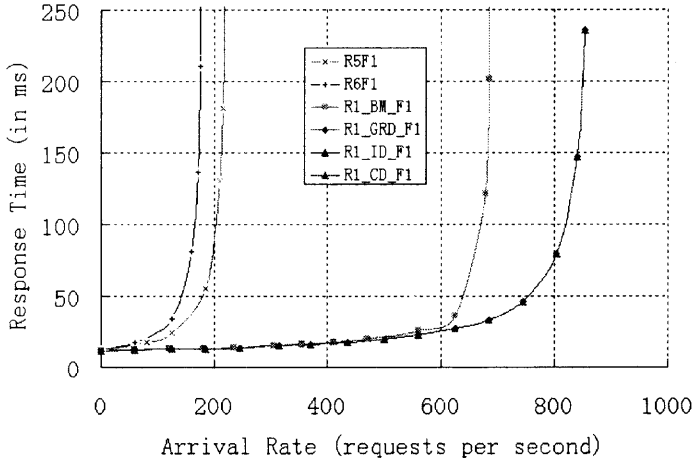

Figure 4.8 Response time for read requests with one disk failure, R:W=1:1, N=8.

it own load and the load of the two failed disks. The response time for RAID1 organizations when they survive two disk failures is lower than RAID6. ID with $c = 1$ cannot tolerate two disk failures.

BM has the worst among response time among the RAID1 organizations, because load balancing through routing is not possible. CD is outperformed by GRD due to the fact that the failed disks separate the disk array into two groups and the routing of reads is not as efficient as in GRD. However, the probability that CD can tolerate more than one disk failure is higher than GRD. It is emphasized that disk response times are meaningful when the disk array survives disk crashes.

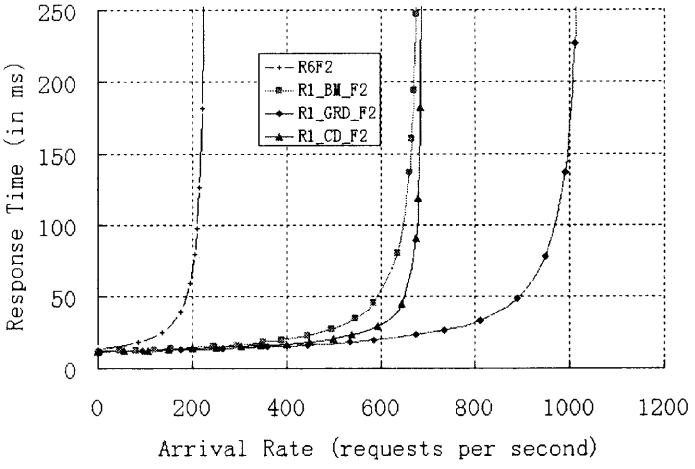

Figure 4.9 Response time for read requests with two disk failures, R:W=1:0, N=8.

4.8 Conclusions

Four mirrored disk organizations have been described and compared their reliability, performability, and performance. Some RAID1 organizations allow up to $N/2$ disk failures, but are restrictive as to which $N/2$ disks can fail. Basic mirroring - BM is the most reliable of the four organizations, but has the disadvantage that it leads to more unbalanced disk loads. Group rotate declustering - GRD requires that all disk failures

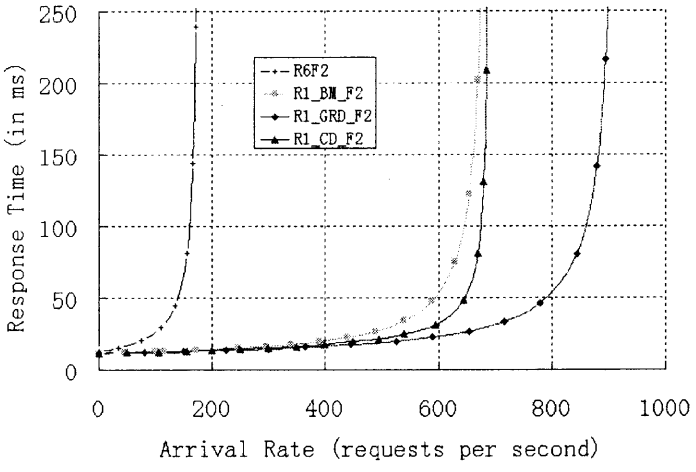

Figure 4.10 Response time for read requests with two disk failures, R:W=3:1, N=8.

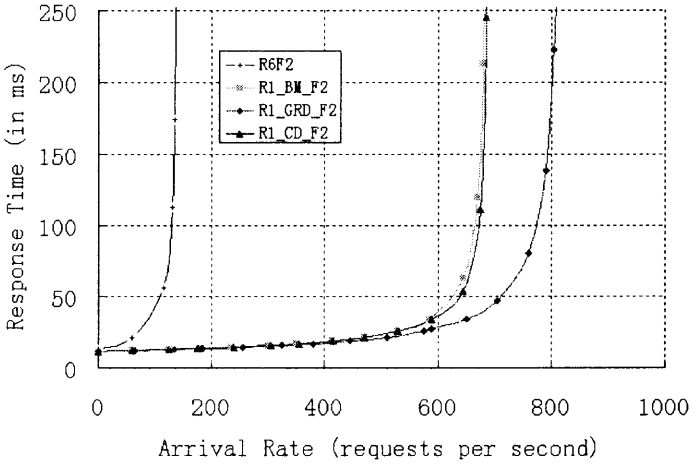

Figure 4.11 Response time for read requests with two disk failures, R:W=1:1, N=8.

are either all primary or all secondary, but the loads can be balanced across surviving disks. The reliability of Interleaved Declustering - ID can be improved by reducing the number of disks in each cluster, but similarly to BM this introduces load imbalance across clusters when disks fail. Chained declustering - CD is an improvement on ID, since up to $N/2$ non-consecutive disks can fail and the load can be balanced among surviving disks.

Table 4.8 Comparison of Different Mirrored Disk Organizations with N=2M Disks

Organization	Basic	Interleaved	Chained	Group Rotate
Load after a disk fails	Not balanced	Balanced	Balanced	Balanced
Degradation in throughput for 100% reads	1/2	$(n-1)/n$	(N-1)/N	(M-1)/M
Balancing	Impossible	Easy	Complicated	Complicated
Max failed	M	c	M	M

When all requests are from a single source, BM attains half of the maximum throughput of the other methods when a single disk fails, which is because load balancing is not possible in this case. With two disk failures GRD outperforms CD (when there is no data loss), because it can route reads among surviving disks more efficiently than CD. On the other hand GRD is more susceptible than CD to data loss. Some conclusions of this study are summarized in Table 4.8.

When the number of disks is the same for all RAID levels, RAID1 outperforms RAID0, RAID5, and RAID6 in all cases. This is especially so with a high fraction of reads, since RAID1 provides twice the access bandwidth of the other RAID levels.

The assumption that the number of data disks is the same, yields an even better performance for RAID1 [22].

The performability of mirrored disks is compared by using the expected number of completed read requests as the performance metric. The comparison is made under the assumption that disk loads are initially balanced and the best attempt is made to maximize the array bandwidth. BM outperforms the other organizations followed by the ID organization when the number of clusters is larger, since in the limit with $n = 2$ disks per cluster they are the same.

The analytical model of RAID1 organizations in this study is quite naive from the viewpoint of routing of requests and disk scheduling. Static probabilistic routing is required to keep the analysis tractable, while simulation is required to study the effect of dynamic routing. The comparison of request routing methods in mirrored disks with BM (basic mirroring) showed that routing has little effect on performance and that performance is dominated by the disk scheduling policy.

The RAID1 organizations discussed in this study are applicable to *storage nodes - SNs* or bricks. A technique such as CD (chained declustering) should be helpful in balancing brick loads when brick failures occur. Distributed SPTF or D-SPTF is a new algorithm which improves read performance by multicasting request to pertinent SNs [23]. Once an SN claims a request, its processing at the other nodes is cancelled. Additional work remains to be done to investigate disk scheduling, including the schemes proposed in [24], for various RAID1 organizations.

CHAPTER 5

ANALYSIS OF X-CODES

5.1 Coding for Multiple Disk Failure Tolerant Arrays

Parity coding to recover from single disk failures was part of the original RAID proposal [51], which introduced five RAID levels. The popular RAID level 5 (RAID5) utilizes striping, which partitions large files into fixed size *stripe units - SUs* which are allocated in a round-robin manner across the disks of the array. The capacity of one disk out of N is dedicated to parity blocks, whose SUs according to the left symmetric organization are placed in left to right repeating diagonals. Distributing the parity blocks has the advantage of balancing the disk load due to updating activity.

The redundancy due to parities in RAID5 allows the on demand reconstruction of the blocks of a single failed disk. A data access to the failed disk entails fork-join requests to the corresponding blocks on the surviving disks. The load on these disks doubles when all requests are reads, so that there is a degradation in response times. More importantly, a RAID5 disk array with a single disk failure is susceptible to data loss if another disk fails. The rebuild process in RAID5 systematically reconstructs the contents of a failed disk on the spare disk, by reading and XORing the contents of surviving disks. In spite of the very small bit error rates for the RAID5 configuration considered in [41] the rebuild process cannot be completed in 4% of cases due to *latent sector failures - LSFs.*

Two-disk failure tolerant - 2DFTs arrays classified as RAID6 [48] are a solution to this problem. There are several schemes to recover from multiple disk failures, which from a coding viewpoint correspond to erasure correction. *Maximum Distance Separable - MDS* codes meeting the Singleton bound require r parity disks in order

to correct r erasures. The most common class of MDS codes are Reed-Solomon (RS) codes, which have been implemented in StorageTek's Iceberg [48] and HP's RAID 5DP (double parity) [32].

RS codes which are based on arithmetic on finite fields are quite expensive to implement [17]. Several parity based methods have been proposed to reduce the computational cost. The EVENODD code has two parity columns that are independent from each other [41]. The Row-Diagonal Parity (RDP) family of codes, optimizes the number of XORs at the encoding [46]. With an appropriate choice of symbol sizes EVENODD and RDP incur the same disk access pattern as RAID6, i.e., three read-modify-writes to update data and parity blocks. The P and Q parity SUs are organized according to the left symmetric organization to balance disk loads due to updates.

RM2 [50] and the X-code [75] are distributed parity schemes, which have an advantage over dedicated parity in that there are no bottlenecks. No disk gets more accesses than others, so that there is no need to rotate the parity columns as in RAID5 and RAID6. While RM2 allows an odd or an even number of disks, the distributed parity codes cannot be shortened in general. For instance, the X-code consists of N columns, where N is a prime. Although EVENODD requires a prime number of disks, empty virtual disks with zero contents can be used to fill the gap.

As noted earlier the load of RAID arrays in processing read requests doubles with a single disk failure and triples in RAID6 with two disk failures. The load increase in RM2 is a function of the redundancy ratio and disk loads in RM2 tend to quite unbalanced [8]. This study concentrates on the X-code organization. It is shown that the load increase with two disk failures is a function of the distance of the failed disks and derive a closed form expression in this case.

5.2 X-code Organization

Xcode [75] requires the number of disks (N) to be a prime number. The stripe unit (SU) on disks are divided into arrays of size $N \times N$, which are repeated to fill the disks. Data SUs are placed in the top $N-2$ rows and the two parity SUs (p and q) are stored in the bottom two rows, one parity per row. Parity groups are constructed from the SUs along several diagonal by the XOR operation (see Figures 5.1 and 5.2).

p(i) denotes the parity group i starting with i^{th} column in the $(N-1)^{th}$ row with slope = 1 [1]. *q(i)* denotes the parity group i starting with i^{th} column in the N^{th} row with slope = -1 ($0 \leq i \leq (N-1)$). Data blocks are protected by p and q parity groups so that two disk failures can be tolerated. Figures 5.1 and 5.2 display the p and q parity groups with $N = 7$. Block (1,2) (1 is the row number and 2 is the column number) is protected by parity group p(3) in Figure 5.1 and parity group q(6) in Figure 5.2. Each parity group contains $(N-1)$ blocks, $(N-2)$ data blocks and one parity block. Parity blocks are only protected by one parity group.

Let $B_{i,j}$ be the parity at the i^{th} row and j^{th} column. For the parity group p with slope = 1, the formula is:

$$B_{N-2,i} = B_{0,<i-2>_N} \oplus B_{1,<i-3>_N} \oplus \ldots \oplus B_{N-3,<i-N+1>_N}.$$

For the parity group q with slope = -1, the formula is:

$$B_{N-1,i} = B_{0,<i+2>_N} \oplus B_{1,<i+3>_N} \oplus \ldots \oplus B_{N-3,<i+N-1>_N},$$

where $0 \leq i \leq N-1$, $< X >_N = X$ module N.

Table 5.1 shows the notation used in Chapter 5.

[1] Slope defined in this chapter is different from the original proposal [75].

	Disk Number						
Row	1	2	3	4	5	6	7
1	2	3	4	5	6	0	1
2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	3
4	5	6	0	1	2	3	4
5	6	0	1	2	3	4	5
6	0	1	2	3	4	5	6
7							

Figure 5.1 p parities with slope=1 and $N = 7$. 7^{th} row is not considered.

	Disk Number						
Row	1	2	3	4	5	6	7
1	5	6	0	1	2	3	4
2	4	5	6	0	1	2	3
3	3	4	5	6	0	1	2
4	2	3	4	5	6	0	1
5	1	2	3	4	5	6	0
6							
7	0	1	2	3	4	5	6

Figure 5.2 q parities with slope=-1 and $N = 7$. 6^{th} row is not considered.

Table 5.1 Notation Used in Chapter 5

N	Number of disks.
$p(i)$	Parity group i with slope = 1, $0 \leq i \leq (N-1)$.
$q(i)$	Parity group i with slope = -1, $0 \leq i \leq (N-1)$.
d	Distance of two failed disks.
$C_{1F}^{Read/A}$	Read cost of an available data block with one disk failure.
$C_{1F}^{Read/U}$	Read cost of an unavailable data block with one disk failure.
C_{1F}^{Read}	Mean read cost of a data block with one disk failure.
C_{1F}^{Write}	Mean write cost of a data block with one disk failure.
$C_{2F}^{Read/A}$	Read cost of an available data block with two disk failures.
$C_{2F}^{Read/U}$	Mean read cost of an unavailable data block with two disk failures.
C_{2F}^{Read}	Mean read cost of a data block with two disk failures.
C_{2F}^{Write}	Mean write cost of a data block with two disk failures.
C_d	Sum of read cost of a failed disk with distance d.
$C_d^{Read/U}$	Mean read cost of an unavailable data block with distance d.
$block$	one cell in the array.
D_{SR}	Cost of read access.
D_{RMW}	Cost of updating data and parity block.

5.3 Cost with One Failed Disk

5.3.1 Read Cost

Read requests only access data blocks. With one failed disk there are $N-2$ unavailable and $(N-1)(N-2)$ available data blocks. Let f_A and f_U denote the probability of accessing an available and unavailable data block respectively. There are two cases for reading a block.

1. **The block is available.**

$$f_A = \frac{N-1}{N} \Rightarrow C_{1F}^{Read/A} = D_{SR}.$$

2. **The block is unavailable.**

$$f_U = \frac{1}{N} \Rightarrow C_{1F}^{Read/U} = (N-2)D_{SR}.$$

The read cost for an unavailable data block is $N-2$ because either parity p(i) or q(i) can be used to reconstruct the unavailable data block.

The mean cost to read a data block is

$$C_{1F}^{Read} = f_A C_{1F}^{Read/A} + f_U C_{1F}^{Read/U} = \frac{2N-3}{N} D_{SR}. \tag{5.1}$$

5.3.2 Write Cost

With one disk failure, there are three cases for writes.

1. **Data and parity blocks are all available.**

$$f_1 = \frac{N-3}{N} \Rightarrow C_1 = 3D_{RMW}.$$

2. **Data and one of the parity blocks are available.**

$$f_2 = \frac{2}{N} \Rightarrow C_2 = 2D_{RMW}.$$

3. **Data block is unavailable.**

$$f_3 = \frac{1}{N} \Rightarrow C_3 = (N-3)D_{SR} + 2D_{RMW}.$$

Data blocks and parity in one of the parity groups (designated as group 1) can be read to reconstruct d_{old}, to compute d_{diff}, and compute $p^1_{new} = p^1_{old} \oplus d_{diff}$. d_{diff} computed in this manner is applied to both parity blocks.

The mean cost for write operation with one disk failure is then the weighted sum over the three cases.

$$C_{1F}^{Write} = \sum_{i=1}^{3} f_i C_i = \frac{N-3}{N} D_{SR} + \frac{3(N-1)}{N} D_{RMW}. \tag{5.2}$$

5.4 Cost with Two Failed Disks

5.4.1 Read Cost

For read cost with two failed disks there are two cases depending on whether the target block is available or not.

1. **The block is available.**

$$f_A = \frac{N-2}{N} \Rightarrow C_{2F}^{Read/A} = D_{SR}.$$

2. **The block is unavailable.**

$$f_U = \frac{2}{N} \Rightarrow C_{2F}^{Read/U}.$$

$C_{2F}^{Read/U}$ will be obtained in the following section.

With two disk failures the read cost for an unavailable data block depends on the distance of two failed disks. The distance between disks i and j is defined as

$$d = min(j - i, i + N - j), \quad 1 \leq i \leq j \leq N.$$

For example, the distance of disk 1 and disk 6 for $N = 7$ is 2, not 5. It is easy to see that the maximum distance is $(N-1)/2$, N is a prime and hence, an odd number.

Cost to read an unavailable data block Some unavailable data blocks can be reconstructed by one parity group access and others need several parity groups accesses, i.e., a recovery path.

For example in Figures 5.3 and 5.4 block (3,1) need to be reconstructed when disk 1 and disk 2 are failed. Block (2,2) needs to be reconstructed for p(4) or block (4,2) for q(3) first. So the read cost for block (3,1) is $(N-2)$ plus the minimum read cost of block (2,2) or block (4,2). Then the same analysis for block (2,2) and block (4,2) will be carried out until the block that can be reconstructed by one parity group access is reached. However, it is impossible to reconstruct block (4,2) because it needs block (5,1) which has two unavailable blocks (one is parity block) in parity group q(1). So only block (2,2) can be used. Block (2,2) needs block (1,1) which can be reconstructed by $(N-2)$ data block accesses. The total cost for block (3,1) is p(2), q(5), and p(4), i.e., $3(N-2)$. Table 5.2 displays the recovery path and read cost of all unavailable data blocks with $N = 7$ and $d = 1$. Block (5,1) can not be recovered by parity group q(1). It separates other data blocks on disk 1 into two groups. However, since block (5,1) is the last data block, it only has one group.

Table 5.3 displays the recovery path and read cost of all unavailable data blocks with $N = 7$ and $d = 3$. This time the recovery path is differently. It shows the

slope = 1	Disk Number						
Row #	1	2	3	4	5	6	7
1	2	3	4	5	6	0	1
2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	3
4	5	6	0	1	2	3	4
5	6	0	1	2	3	4	5
6	0	1	2	3	4	5	6
7							

Figure 5.3 Possible recovery path for block (3,1) with p parity group and $N = 7$.

slope = −1	Disk Number						
Row #	1	2	3	4	5	6	7
1	5	6	0	1	2	3	4
2	4	5	6	0	1	2	3
3	3	4	5	6	0	1	2
4	2	3	4	5	6	0	1
5	1	2	3	4	5	6	0
6							
7	0	1	2	3	4	5	6

Figure 5.4 Possible recovery path for block (3,1) with q parity group and $N = 7$.

Table 5.2 Read Cost for the Failed Disks 1 and 2 with N=7 and d=1

Block	Recovery path	Read cost $(N-2)D_{SR}$
(1,1)	p(2)	1
(2,1)	q(6) + p(3)	2
(3,1)	p(2) + q(5) +p(4)	3
(4,1)	q(6) + p(3) + q(4) + p(5)	4
(5,1)	p(2) + q(5) + p(4) + q(3) + p(6)	5
(1,2)	q(6)	1
(2,2)	p(2) + q(5)	2
(3,2)	q(6) + p(3) +q(4)	3
(4,2)	p(2) + q(5) +p(4) +q(3)	4
(5,2)	q(6) + p(3) + q(4) + p(5) + q(2)	5

other unavailable data blocks that need to be reconstructed first in the same parity group.

One observation is that the recovery path for an unavailable data block is an alternation of p and q parity. It is like p, q, p, q ... until the unavailable data block can be reconstructed by one parity group access. The long the path the higher the cost. So it is very important for the positions of the block that can be reconstructed by one parity group access. For example block (3,1) can not be used to reconstruct other unavailable blocks even through it only needs one party group access (p(4)) to be reconstructed. This is because the parity group before p(4) in the recovery path is q(3). However, the other unavailable block in q(3) is block (7,4) which is a parity block. Since parity blocks are only protected by one parity group (p or q), they can not be used to reconstruct the data block in other parity group (q or p). Block (3,1) is the one separates the data blocks on disk 1 into two groups. One group includes

block (3,1) and the other group includes the rest of data blocks. Blocks in the same group help each other with the reconstruction.

Table 5.3 Read Cost for the Failed Disks 1 and 4 with N=7 and d=3

Block	Recovery path	Read cost $(N-2)D_{SR}$
(1,1)	(4,4) + q(5)	2
(2,1)	(5,4) + q(4)	4
(3,1)	p(4)	1
(4,1)	q(2)	1
(5,1)	(1,4) + q(1)	3
(1,4)	(4,1) + p(5)	2
(2,4)	(5,1) + p(6)	4
(3,4)	q(6)	1
(4,4)	p(1)	1
(5,4)	(1,1) + p(2)	3

Because of existence of an unrecoverable block the read cost of unavailable data blocks on a failed disk shows the pattern in Table 5.4. The $N-2$ columns ($N-2$ data blocks) show the parity group(s) needed to reconstruct an unavailable data block. There are $(N-1)/2$ rows, each row represents a distance. Detailed analysis of read costs for unavailable data blocks for other distances are given in Appendix C.

Mean cost to read an unavailable data block for a given distance For a given distance d of two failed disks, $1 \leq d \leq (N-1)/2$, the sum of read cost for one group (sum of one unshadowed row in Table 5.4) is

$$(N-2)(\sum_{i=1}^{N-1-d} i)D_{SR}$$

Table 5.4 Read Cost for Data Blocks (1:$N-2$) on One Failed Disk with Different Distance and N Disks

The list of parity groups needed to reconstruct an unavailable data block							
1	2	3	4	...	...	$N-3$	$N-2$
1	1	2	3	...	...	$N-4$	$N-3$
1	2	1	2	...	...	$N-5$	$N-4$
1	2	3	1	...	...	$N-6$	$N-5$
				...			
1	2	...	$(N-1)/2-2$	1	2	...	$(N-1)/2+1$
1	2	3	...	$(N-1)/2-1$	1	...	$(N-1)/2$

and for the other group

$$(N-2)(\sum_{j=0}^{d-1} j)D_{SR}.$$

Hence, the read cost for all data blocks on a failed disk for a given distance d is the sum of two groups, i.e., the sum of one row in Table 5.4.

$$C_d = (N-2)(\sum_{i=1}^{N-1-d} i + \sum_{j=0}^{d-1} j)D_{SR}.$$

Because there are $N-2$ data blocks on the failed disk, the mean read cost per data block on a failed disk with distance d is:

$$C_d^{Read/F} = \frac{C_d}{N-2} = (\sum_{i=1}^{N-1-d} i + \sum_{j=0}^{d-1} j)D_{SR}.$$

Mean cost to read an unavailable data block for all distances The mean read cost of an unavailable data block for all distances is ($a = (N-1)/2$):

$$C_{2F}^{Read/U} = \frac{1}{a}\sum_{d=1}^{(N-1)/2} C_d^{Read/F} = \frac{1}{a}\sum_{d=1}^{(N-1)/2}\left(\sum_{i=1}^{N-1-d} i + \sum_{j=0}^{d-1} j\right)D_{SR} = \frac{4a^2-1}{3}D_{SR}.$$

Replacing a with $(N-1)/2$

$$C_{2F}^{Read/U} = \frac{N^2-2N}{3}D_{SR}. \tag{5.3}$$

Mean cost to read a data block Adding the read cost for an available data block the mean read cost for a data block is

$$C_{2F}^{Read} = f_A C_{2F}^{Read/A} + f_U C_{2F}^{Read/U} = \frac{2N^2-N-6}{3N}D_{SR}. \tag{5.4}$$

5.4.2 Write Cost

With two failed disks, there are five cases for writes depending on the availability of the data and two parity blocks. Figure 5.5 shows the specification for all cases. D, P and Q indicate that data and parity blocks are available, while $\overline{D}$, $\overline{P}$ and $\overline{Q}$ indicate that they are not.

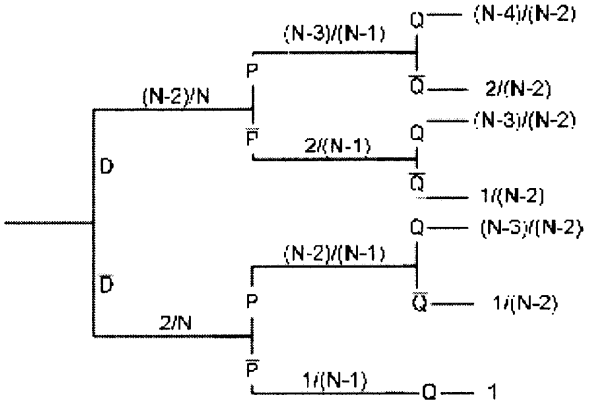

Figure 5.5 The case analysis for Xcode with two disk failures.

So the costs for all cases are:

1. **All three blocks are available.**

$$f_1 = \frac{(N-3)(N-4)}{N(N-1)} \Rightarrow C_1 = 3D_{RMW}.$$

2. **Data and one of the parity blocks are available.**

$$f_2 = \frac{4(N-3)}{N(N-1)} \Rightarrow C_2 = 2D_{RMW}.$$

3. **Data is available, but both parity blocks are unavailable.**

$$f_3 = \frac{2}{N(N-1)} \Rightarrow C_3 = D_{SW}.$$

4. **Data block is unavailable, but both parity blocks are available.**

$$f_4 = \frac{2(N-3)}{N(N-1)} \Rightarrow C_4 = (\frac{N^2-2N}{3} - 1)D_{SR} + 2D_{RMW}.$$

5. **Data and one of the parity blocks are unavailable.**

$$f_5 = \frac{4}{N(N-1)} \Rightarrow C_5 = (\frac{N^2-2N}{3} - 1)D_{SR} + D_{RMW}.$$

In cases 4 and 5 when the data block is not available, it is treated as a read request and reconstruct at a cost $C_{2F}^{Read/U} = (N^2-2N)D_{SR}/3$. If both parities survive they are written as $2D_{RMW}$ minus D_{SR}. If there is one surviving parity it is written as a $2D_{RMW}$ minus D_{SR}.

The overall mean cost for write operation with two disks failure are then the weighted mean over the five cases.

$$C_{2F}^{Write} = \sum_{i=1}^{5} f_i C_i = \frac{2}{N(N-1)} D_{SW} + \frac{2(N-3)(N+1)}{3N} D_{SR} + \frac{(3N^2-9N+4)}{N(N-1)} D_{RMW}.$$

5.5 Summary

The cost of operation for an Xcode disk array is summarize in Table 5.5.

Table 5.5 Cost of Operation for Xcode with N Disks

Mode	Read	Write
0	D_{SR}	$3D_{RMW}$
1	$\frac{2N-3}{N}D_{SR}$	$\frac{N-2}{N}D_{SR} + \frac{3(N-1)}{N}D_{RMW}$
2	$\frac{2N^2-N-6}{3N}D_{SR}$	$\frac{2}{N(N-1)}D_{SW} + \frac{2(N-3)(N+1)}{3N}D_{SR} + \frac{(3N^2-9N+4)}{N(N-1)}D_{RMW}$

5.6 Discussion of Performance Results

The mean read cost with one and two failed disks are plotted.

5.6.1 One Failed Disk

Figure 5.6 shows the mean read cost of a data block with one disk failure versus number of disks by using Equation(5.1). From the figure it is shown the increase of cost decreases as the number of disks increases.

5.6.2 Two Failed Disks

Figure 5.7 shows the mean read cost of an unavailable data block with two failed disks versus number of disks by using Equation(5.3).

Figure 5.8 displays the mean read cost of a data block with two failed disks versus number of disks by using Equation(5.4).

The mean read cost of an unavailable data block increase at a fast pace as the total number of disks increases. However, this effect is outweighed by the ratio of the number of unavailable data blocks and the total number of data blocks. In other words, as the number of disks increases there are higher number of available data blocks compared to failed data blocks.

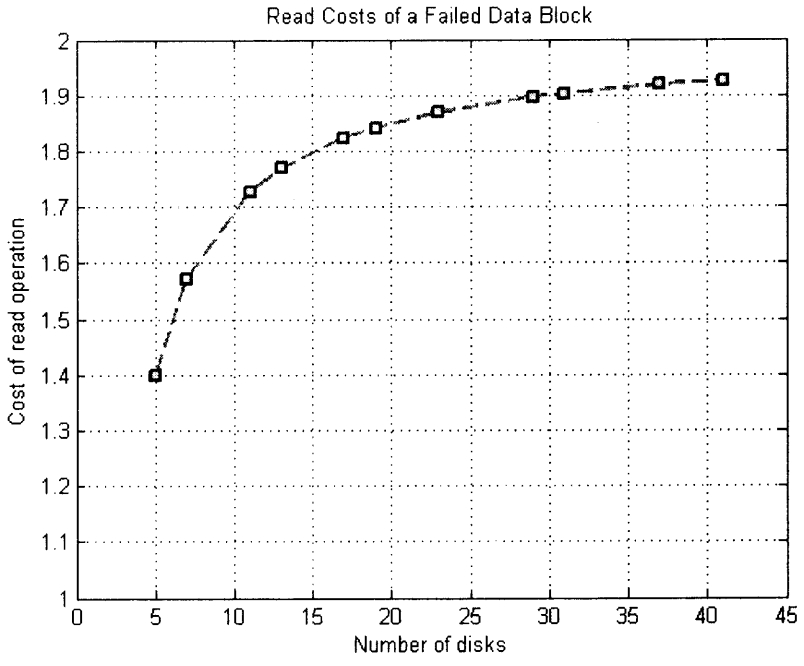

Figure 5.6 Mean cost of read operation with one disk failure versus number of disks.

5.7 Load Imbalance with Disk Failures

In Xcodes N disks are divided into an $N \times N$ arrays with the top $N-2$ rows for data blocks and bottom 2 rows for parity blocks. If the load on one block is treated as a unit, the load for reads on each disk is $N-2$ with no disk failures because there are $N-2$ data blocks and reads only access data blocks. When the disk array operates with disk failures, the surviving disks process some of the read requests for the failed disks due to data block reconstructions. However, the load increase on disks is not the same.

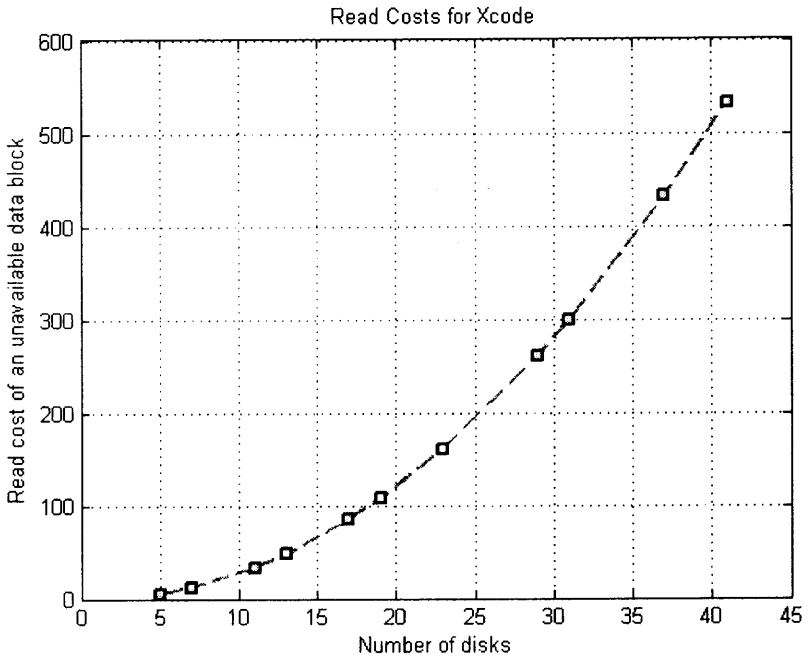

Figure 5.7 Mean read cost of an unavailable data block with two failed disks versus number of disks.

5.7.1 Load Imbalance on Each Disk with One Disk Failure

With one disk failure the blocks on the failed disk can be reconstructed by one parity group access (either p(i) or q(i)). Due to the symmetric allocation of parity groups, one of the neighboring disks of the failed disk has $N-2$ data blocks in the same parity groups as those on the failed disks while the rest surviving disks have $N-3$ data blocks. So the load increase is almost balanced on each of surviving disks with the load increase on one disk is $N-2$ and the rest is $N-3$. Comparing the load of $N-2$ on one disk with no disk failure the load on each disk is almost doubled with one failed disk.

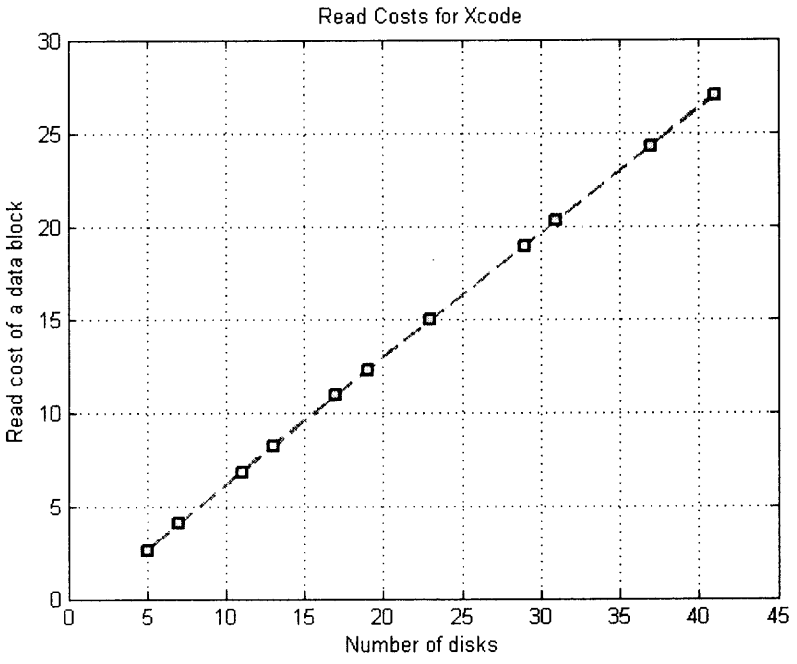

Figure 5.8 Mean read cost of a data block with two failed disks versus number of disks.

5.7.2 Load Imbalance on Each Disk with Two Disk Failures

With two disk failures the reconstruction of unavailable blocks is complicated. Some blocks can be reconstructed by one parity group access. Some blocks have another unavailable block in the same parity group, which needs to be reconstructed first. As discussed in the previous section the reconstruction may need to go down several levels until the unavailable block can be reconstructed. Hence, the total cost is the sum of accesses for each reconstruction.

A recursive algorithm is developed to compute the load increase on each disk with different distance (see Appendix D). The algorithm is based on encoding rules given in [75].

Figure 5.9 shows the load increase on each disk with different distances for $N = 7$ and two disks failed. X axis is i^{th} disk, Y axis is the distance. Z axis is the

times of load increase comparing that in the normal mode. Load increase is computed as follows. First compute the extra disk accesses on each surviving disk due to the failed disks. Then compute the total accesses on a surviving disk by adding its normal disk accesses to the extra accesses. Finally dividing this number by the normal disk accesses it is obtained how many times load increase are comparing to its normal accesses.

For example the maximum load increase on one disk with $d = 1$ is 26 read accesses besides its own 5 read accesses. Hence, the load increase is 6.2 folds of the load with no failed disk.

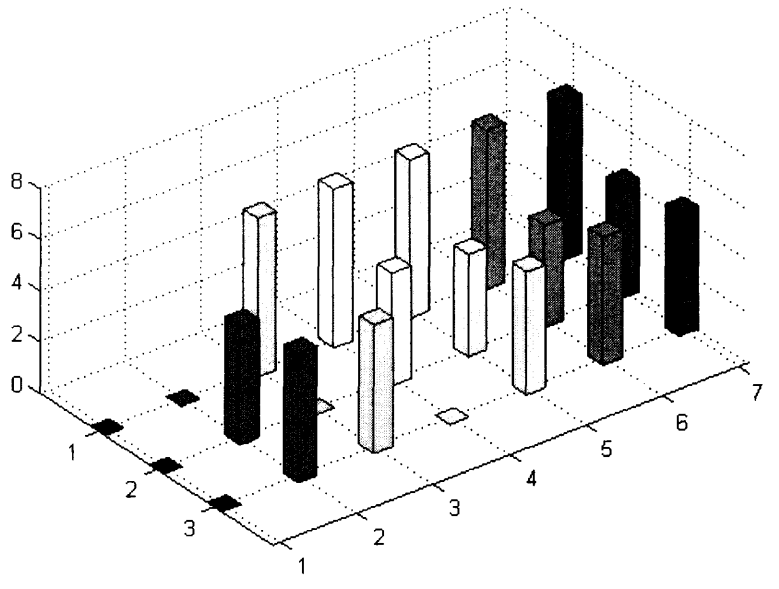

Figure 5.9 Load increase on each disk for reads with two failed disks and $N = 7$. X axis is i^{th} disk, Y axis is the distance. Z axis is the times of load increase comparing that in the normal mode.

Some observations from the result are as follows:

1. With two disk failures the load increase on each surviving disk is balanced for $d = 1$. This is because the overload is symmetrically distributed.

2. The load on the surviving disks is skewed when $d > 1$.
3. However, the load increase on each disk is the largest when $d = 1$.

Mean Load Increase on a Disk versus Distance: Mean load increase on a disk varies with the distance. Figures 5.10, 5.11 and 5.12 show the mean load increase on each disk with different distances when N is 7, 19, 37 and two disk failures. Y axis shows the mean load increase of a disk for a given distance. It is computed as follows. First obtain load increase on each surviving disk as discussed above. Then get the mean of the load increase on each surviving disk to get the mean load increase on a disk with a given distance.

Mean Load Increase vs Number of Disks: Figure 5.13 shows mean load increase, best and worst cases among all distances, the maximum load increase on a disk in both best and worst cases versus number of disks. To obtain the values of these variables the load increase on each surviving disk for a given distance ($LI_{d,j}, 1 \leq j \leq N, 1 \leq d \leq (N-1)/2$) is computed first. Then the mean load increase for each distance is calculated by the formula

$$MLI_d = average(LI_{d,j}).$$

The best and worst cases are

$$MLI_{best} = min(MLI_d), MLI_{worst} = max(MLI_d).$$

Mean load increase is

$$MLI = average(MLI_d).$$

The maximum load increase on a disk in best case (d is the distance in MLI_{best}) is

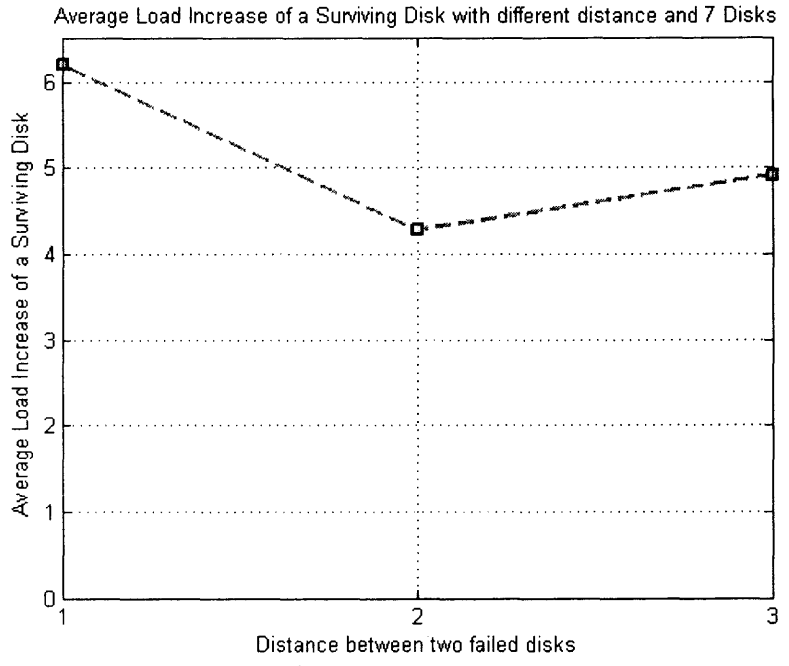

Figure 5.10 Mean load increase on a disk for reads with two disk failures and $N = 7$.

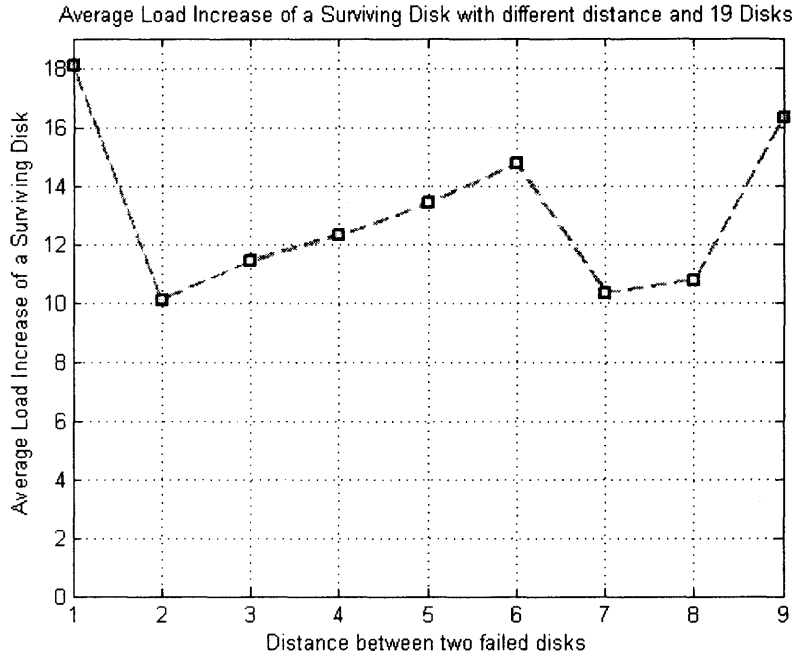

Figure 5.11 Mean load increase on a disk for reads with two disk failures and $N = 19$.

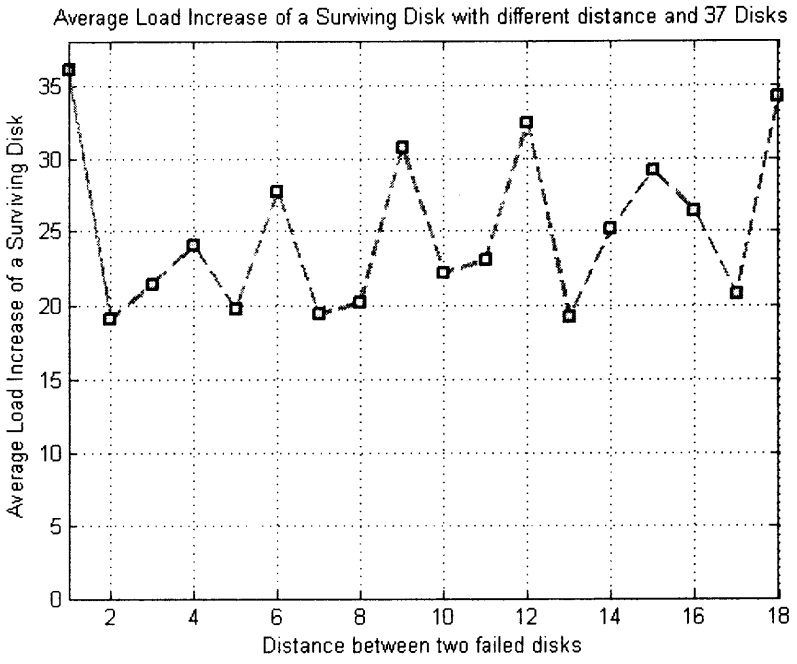

Figure 5.12 Mean load increase on a disk for reads with two disk failures and $N = 37$.

$$LI_{max} = max(LI_{d,j}).$$

Same is true for the maximum load increase on a disk in worst case (d is the distance in MLI_{worst}).

Due to the symmetric allocations of parity groups, the load increase on all disks are the same in the worst case which happens with $d = 1$. Therefore the lines for worst case and the maximum load increase on a disk in the worst case are overlapped as shown the top line in the Figure 5.13. The load in the worst case increases proportion to the increase of number of disks. The line in the middle is the mean load increase versus number of disks. The bottom two lines are the mean load increase in the best case and the maximum load increase on a disk in this case. There is only a little difference between them.

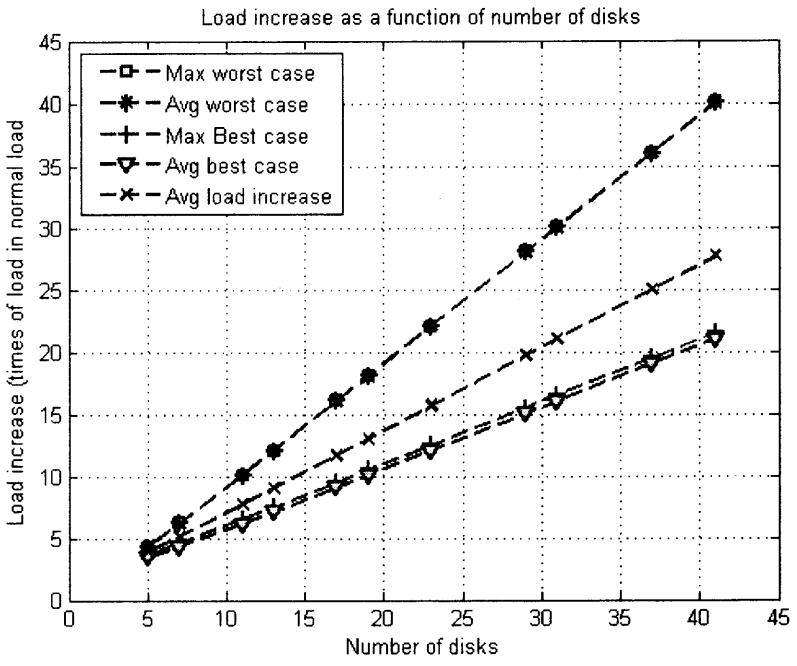

Figure 5.13 Mean load increase vs number of disks.

5.8 RM2 Coding Scheme and its Load Increase for Reads

RM2 is defined as [50]: "Given a redundancy ratio p and the number of disks N, construct N parity groups each of which consists of $2(M-1)$ data blocks and one parity block such that each data block should be included in two groups, where $M = 1/p$." Each disk contains one parity and $M-1$ data blocks, so that the parity blocks are distributed evenly.

In [8] a method is given to construct the RM2. **Begin quote.** An algorithmic solution to this problem is based on an $N \times N$ *redundancy matrix (RM)*, where each column corresponds to a disk and each row corresponds to a parity group. The columns of RM are called *placement vectors.* Values of the elements of RM, $RM_{i,j}$, are defined as follows:

- $RM_{ij} = -1$ A parity block of the disk j belongs to parity group i.
- $RM_{ij} = 0$ Nothing (none of the blocks on disk j belongs to parity group i).
- $RM_{ij} = k$ The kth data block of disk j belongs to group i. $(1 \leq k \leq M-1)$

The redundancy matrix RM must have the following properties: (1) There is only one -1 in each row, i.e., each parity group (PG) has one parity block. (2) There is only one -1 in each column, i.e., one parity block for each disk in a segment. (3) Each column has $2(M-1)$ positive entries with each number appearing exactly twice, i.e., each data block is protected by exactly two parity blocks.

An RM2 data layout is defined by an RM, which can be constructed as follows:

1. Select the target redundancy ratio p and set $M = 1/p$.
2. Select the total number of disks N that satisfy: $N \geq 3M - 2$ if N is odd or $N \geq 4M - 5$ if N is even.
3. Construct a *seed placement vector* for M and N as (the prime here implies a transpose):

$$\underbrace{\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline -1 & M-1 & M-2 & \ldots & 2 & 1 & 1 & 2 & \ldots & M-2 & M-1 & 0 & \ldots & 0 \\ \hline \end{array}}_{\text{a total of } N \text{ elements}}{}'$$

4. Construct the $N \times N$ RM matrix column-by-column by rotating the seed placement vector, as shown below:

$$RM = \begin{bmatrix} -1 & 0 & \cdots & M-1 \\ M-1 & -1 & \cdots & M-2 \\ M-2 & M-1 & \cdots & M-3 \\ \vdots & \vdots & & \vdots \\ 1 & 2 & \cdots & 1 \\ 1 & 1 & \cdots & 2 \\ 2 & 1 & \cdots & 3 \\ \vdots & \vdots & & \vdots \\ M-1 & M-2 & \cdots & 0 \\ 0 & M-1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & -1 \end{bmatrix}_{N \times N}$$

In the data layout generated by this method, each parity group has $2M - 1$ elements, $2(M - 1)$ of which are data and one parity. **End quote.**

	D0	D1	D2	D3	D4	D5	D6
PG0	-1	0	0	2	1	1	2
PG1	2	-1	0	0	2	1	1
PG2	1	2	-1	0	0	2	1
PG3	1	1	2	-1	0	0	2
PG4	2	1	1	2	-1	0	0
PG5	0	2	1	1	2	-1	0
PG6	0	0	2	1	1	2	-1

(a)

D0	D1	D2	D3	D4	D5	D6
P_0	P_1	P_2	P_3	P_4	P_5	P_6
$D_{2,3}$	$D_{3,4}$	$D_{4,5}$	$D_{5,6}$	$D_{0,6}$	$D_{0,1}$	$D_{1,2}$
$D_{1,4}$	$D_{2,5}$	$D_{3,6}$	$D_{4,0}$	$D_{5,1}$	$D_{6,2}$	$D_{0,3}$

(b)

Figure 5.14 A Sample RM2 Layout with $M = 3$ and $N = 7$: (a)The redundancy matrix (RM), (b)Corresponding Disk Layout. D0 ... D6 are hard disks, PG0 ... PG6 are parity groups. The notation $d_{i,j}$ means this data block is protected by parity blocks p_i and p_j.

The load on each disk is $N - 1$ in the normal mode. When the disk array operates in degraded mode, the surviving disks process some of the read requests for the failed disks due to data block reconstructions. However, the load increase on each disk is not the same.

5.8.1 Load Increase on Each Disk with One Disk Failure For RM2

With one disk failure the blocks on the failed disk can be reconstructed by one parity group access (either one of the parity groups). The load on each disk is almost doubled when one disk is failed. Figure 5.15 shows the load increase on disk 2 ro 7 for RM2 with different distances when $N = 7$, $M = 3$ and disk 1 failed. The load is not balanced on all surviving disks due to the way RM2 allocates the parity groups.

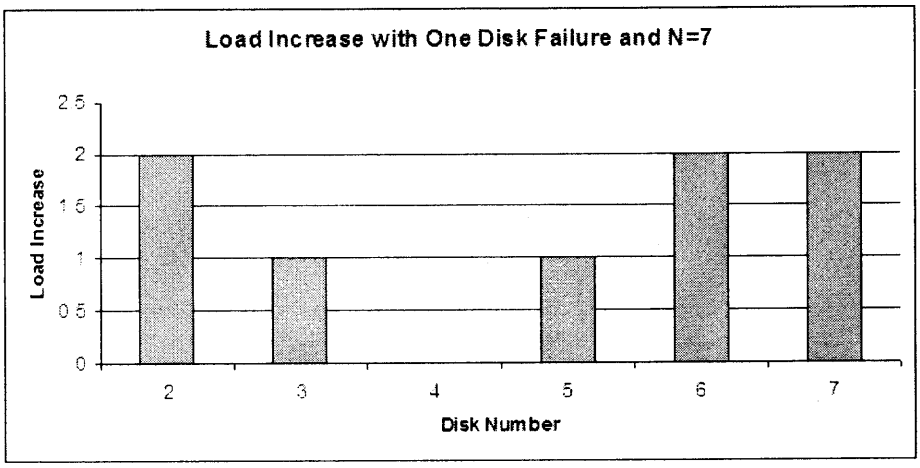

Figure 5.15 Load increase (times of the original load) on disk 2 to 7 for reads with disk 1 failed and $N = 7$, $M = 3$ for RM2.

5.8.2 Load Increase on Each Disk with Two Disk Failures For RM2

Figure 5.16 shows the load increase on each disk with different distance when N is 7, $M = 3$ and two disk failures. X axis is i^{th} disk, Y axis is the distance. Z axis is the times of load increase comparing that in the normal mode. Load increase is computed as follows. First compute the extra disk accesses on each surviving disk due to the failed disks. Then compute the total accesses on a surviving disk by adding itself normal disk accesses to the extra accesses. Finally Dividing this number by the normal disk accesses it is obtained how many times load increase is comparing to it's normal accesses.

Some observations from the result are as follows:

1. With two disk failures the load increase on each surviving disks is balanced when the distance of two failed disks is not one.
2. When the distance is one the load on the surviving disks is skewed.
3. The load increase on each disk is the largest when distance is one.

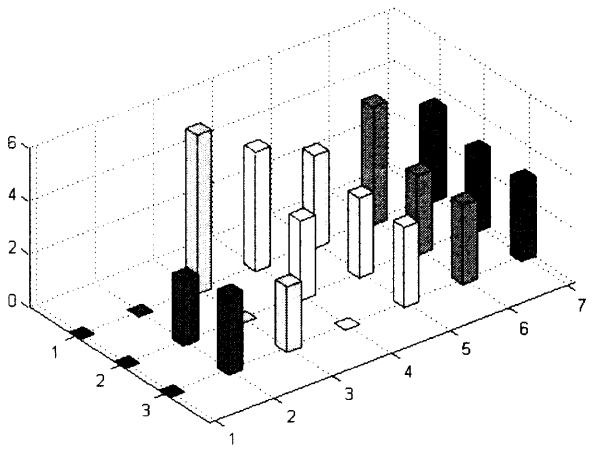

Figure 5.16 Load increase on each disk for reads with two disk failures and $N = 7$, $M = 3$ for RM2. X axis is i^{th} disk, Y axis is the distance. Z axis is the times of load increase comparing that in the normal mode.

Mean Load Increase on a Disk with Different Distances: Figures 5.17, 5.18 and 5.19 show the mean load increase on each disk with different distances when N is 7, 19, 39 and M is 3, 7, 13 and two disk failures. Y axis shows the mean load increase of a disk for a given distance. It is computed as follows. First obtain load increase on each surviving disk as discussed above. Then get the mean of the load increase on each surviving disk to get the mean load increase on a disk with a given distance.

Maximum Load Increase: The maximum load increase on one of the surviving disks is a function of number of disks N and parity size M. It happens when the two failed disks are next to each other. In RM2 load increase on the surviving disks are skewed.

Figure 5.20 shows the max load increase as a function of number of disks. In order to make comparison with the results for Xcode, the redundancy ratio of RM2 is chosen as close as that of Xcode. First the prime number N_{Xcode} for Xcode is used

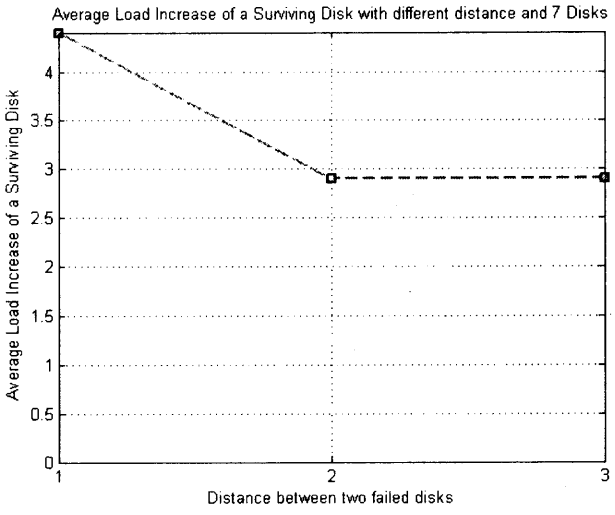

Figure 5.17 Mean load increase on a disk for reads with two disk failures and $N = 7$, $M = 3$ for RM2.

to compute the redundancy ratio for Xcode ($2/N_{Xcode}$). Taking the redundancy ratio for Xcode as guide M is computed by taking the floor of $N_{Xcode}/2$. Finally calculate the total number of disks N_{RM2} for RM2 by using the formula $3M - 2$ (for odd N). If N_{RM2} is even just add 1 to N_{RM2}.

The line at the top shows the maximum load increase on a disk in worst case and the line below it shows the mean load increase in worst case in the Figure 5.13. There are three flat lines at the bottom of the figure. The top line is the mean load increase, followed by the maximum load increase on a disk in the best case and the mean load increase in the best case . So the mean load of a RM2 array does not increase very much as N increases.

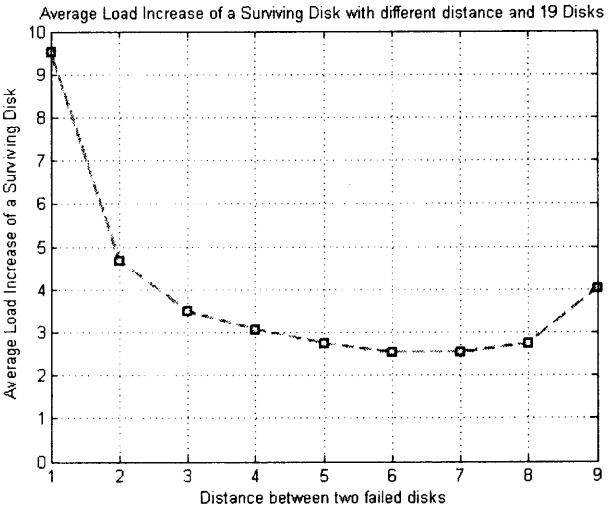

Figure 5.18 Mean load increase on a disk for reads with two disk failures and $N = 19$, $M = 7$ for RM2.

5.9 Comparison between Xcode and RM2

The following items are same in both systems.

1. The load on the surviving disks is skewed except for Xcode with $d = 1$.
2. The load increase on each disk is the largest with $d = 1$.

The following items are different.

1. Both the maximum load increase and mean load increase are larger in Xcode than RM2 due to the slightly higher redundant ratio in RM2.
2. The load increase on each disk is less skewed among surviving disks in Xcode than that in RM2.

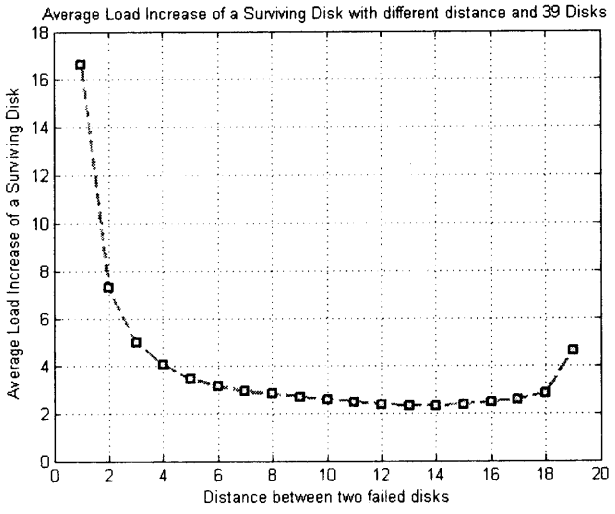

Figure 5.19 Mean load increase on a disk for reads with two disk failures and $N = 39$, $M = 13$ for RM2.

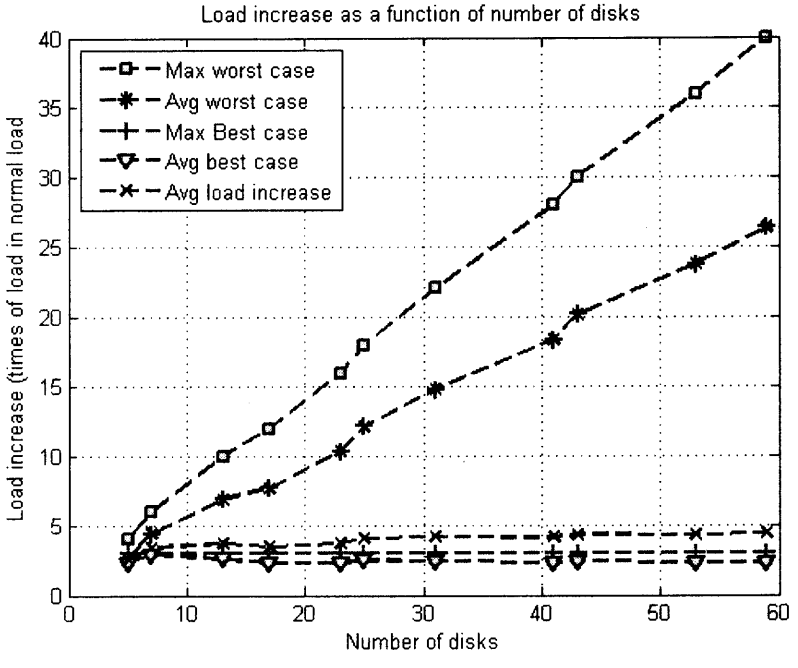

Figure 5.20 Maximum load increase on one disk as a function of N with two disk failures for RM2.

CHAPTER 6

CONCLUSIONS

In this dissertation a brief discussion of the viability for HDA - Heterogenous Disk Array is given: explosion of the amount of data generated, high data availability requirement, rapid cost drop in storage cost, the high cost of storage management and the need to automate it. A review of RAID levels builds the foundation for HDA.

There is the load balancing effect that RAID1 disk arrays have a higher access rate than RAID5 disk arrays per GB. Given the Virtual Array (VA) size and access rate its width or the number of its Virtual Disks (VDs) constituting the VA is determined first. Several online single-pass data allocation methods are processed and evaluated. An allocation is acceptable if it does not exceed the disk capacity and overload disk bandwidth especially in the presence of disk failures. Results show methods considering both bandwidth and capacity utilization of disks in the system outperform the others.

When disk bandwidth rather than capacity is the bottleneck, the clustered RAID paradigm is applied, which offers a tradeoff between disk space and bandwidth. Experiments show clustered RAID can increase the number of allocations dramatically when RAID5 requests dominate the allocations requests.

Four mirrored disk organizations are described in this study: basic mirroring - BM, group rotate declustering - GRD, interleaved declustering - ID, and chained declustering - CD. The last three organizations provide a more balanced disk load than BM when a single disk fails, but are more susceptible to data loss than BM when additional disks fail. The four organizations are compared from the viewpoint of: (a) reliability (results are quoted from [63]), (b) performability, (c) performance.

In (b) and (c) discrete requests to small randomly placed blocks are postulated. For (b) the mean number of disk requests processed is computed to the point where data loss occurs. For the sake of tractability in (c) the response time is obtained assuming Poisson arrivals and a FCFS policy. The ranking from the viewpoint of reliability and performability is: BM, CD, GRD, ID (with two clusters). BM and CD provide the worst performance, ID has a better performance than BM and CD, but is outperformed by GRD. These results are also shown using an asymptotic expansion method.

The load increase and imbalance of the X-code method is analyzed in this study. The X-codes method exhibits the same cost as RAID6 when operating in normal mode. Similarly to RAID5 and RAID6 the read load in normal mode is almost doubled with a single disk failure. With two disk failures the read cost for unavailable blocks increases quadratically with the number of disks (N), while the overall cost increases linearly with N. A general expression for disk loads is derived and graphs to quantify the load imbalance are presented.

APPENDIX A

M/G/1 QUEUING FORMULAS

Most of the equations in this section can be found in [56]. Let b_i be the ith moment of service time, λ be the arrival rate, $\rho = \lambda b_1$ be the utilization factor, the mean waiting time

$$\overline{W} = \frac{\lambda b_2}{2(1-\rho)} = \frac{\lambda(b_1^2 + \sigma_B^2)}{2(1-\rho)} = \frac{\lambda b_1^2(1+c_B^2)}{2(1-\rho)}$$

where $c_B^2 = var[B]/b_1^2 = (b_2 - b_1^2)/b_1^2 = b_2/b_1^2 - 1$, is the coefficient of variation squared.

The second moment of waiting time is:

$$\overline{W^2} = \frac{\lambda b_3}{3(1-\rho)} + \frac{(\lambda b_2)^2}{2(1-\rho)^2}$$

The moments of response time

$$\overline{R^i} = E[(W+T)^i]$$

Given the waiting time and service time are independent, the first two moments are as follows:

$$\overline{R} = b_1 + \overline{W}$$

$$\overline{R^2} = b_2 + \overline{W^2} + 2\overline{W}b_1$$

APPENDIX B

EXPECTED NUMBER OF PROCESSED REQUESTS

The analysis for $N = 8$ disks is given for the various RAID1 organizations separately. At state S_i with i failed disks the failure rate is $(N - i)\lambda$ and the throughput varies depending on whether the arrival rates to disks are from a single or multiple sources. Equation (4.7) is applied in the calculations that follow.

Basic Mirroring

The mean number of requests processed at S_i, $0 \leq i \leq 4$ when the arrival rates are from multiple sources, i.e., the maximum throughput is $(N - i)\mu$, is given as follows:

$N_0 = (8\mu)(1)/(8\lambda) = \mu/\lambda.$

$N_1 = (7\mu)(1)/(7\lambda) = \mu/\lambda.$

$N_2 = (6\mu)(6/7)/(6\lambda) = 6\mu/(7\lambda).$

$N_3 = (5\mu)(4/7)/(5\lambda) = 4\mu/(7\lambda).$

$N_4 = (4\mu)(8/35)/(4\lambda) = 8\mu/(35\lambda).$

The total number of requests processed is:

$N = \sum_{i=0}^{4} N_i = 128\mu/(35\lambda).$

If disk requests are from one source, then they need to remain proportional, the maximum throughout is halved from 8μ to 4μ when a single disk fails. It follows

$N'_0 = (8\mu)(1)/(8\lambda) = \mu/\lambda,$

$N'_1 = (4\mu)(1)/(7\lambda) = 4\mu/(7\lambda),$

$N'_2 = (4\mu)(6/7)/(6\lambda) = 4\mu/(7\lambda)$,

$N'_3 = (4\mu)(4/7)/(5\lambda) = 16\mu/(35\lambda)$, and

$N'_4 = (4\mu)(8/35)/(4\lambda) = 8\mu/(35\lambda)$.

$N' = 99\mu/(35\lambda)$.

Group Rotate Declustering

The case when requests are independent is discussed first. The maximum throughput in GRD is the same as that in BM when there is no disk failure or one disk failure.

$N_0 = (8\mu)(1)/(8\lambda) = \mu/\lambda$.

$N_1 = (7\mu)(1)/(7\lambda) = \mu/\lambda$.

$N_2 = (6\mu)(3/7)/(6\lambda) = 3\mu/(7\lambda)$.

$N_3 = (5\mu)(1/7)/(5\lambda) = \mu/(7\lambda)$.

$N_4 = (4\mu)(1/35)/(4\lambda) = \mu/(35\lambda)$.

$N = \sum_{i=0}^{4} N_i = 13\mu/(5\lambda)$.

The results are same when requests are dependent, because there is no bottleneck and requests can be routed to balance disk loads.

Interleaved Declustering

The case when requests are from multiple sources is considered first. For a single cluster regardless of the number of disks N, $N_0 = N_1 = \mu/\lambda$ and

$N = \sum_{i=0}^{1} N_i = (2\mu)/\lambda$.

With $N = 8$ and $c = 2$ clusters with $n = N/2 = 4$ disks per cluster, it can tolerate at most two disk failures.

$N_0 = (8\mu)(1)/(8\lambda) = \mu/\lambda.$

$N_1 = (7\mu)(1)/(7\lambda) = \mu/\lambda.$

$N_2 = (6\mu)(4/7)/(6\lambda) = 4\mu/(7\lambda).$

$N = \sum_{i=0}^{2} N_i = 18\mu/(7\lambda).$

When requests are from a single source, a single disk failure will result in a drop in the maximum throughput at other clusters with nonfailed disks by a factor $(n-1)/n$. The computation is otherwise straightforward.

$N'_0 = (8\mu)(1)/(8\lambda) = \mu/\lambda.$

$N'_1 = (4\mu)(1)/(7\lambda) = 4\mu/(7\lambda).$

$N'_2 = (4\mu)(4/7)/6\lambda) = 8\mu/(21\lambda).$

$N = \sum_{i=0}^{2} N_i = 41\mu/(21\lambda).$

Chained Declustering

The case when requests are from multiple sources is discussed first:

$N_0 = (8\mu)(1)/(8\lambda) = \mu/\lambda.$

$N_1 = (7\mu)(1)/(7\lambda) = \mu/\lambda.$

$N_2 = (6\mu)(5/7)/(6\lambda) = 5\mu/(7\lambda).$

$N_3 = (5\mu)(2/7)/(5\lambda) = 2\mu/(7\lambda).$

$N_4 = (4\mu)(1/35)/(4\lambda) = \mu/(35\lambda).$

$N = \sum_{i=0}^{4} S_i = 106\mu/(5\lambda).$

Next the case when requests are from a single source is considered. When there is only one disk failure, the rest of the disks in the array form a group and it can balance the load among them.

$N'_1 = (7\mu)(1)/(7\lambda) = \mu/\lambda.$

Two failed disks divide the surviving disks into two groups. The probability that the system tolerates two disk failures is $V_2 = 5/7$. There are three cases as shown in Figure B.1.

1	2	3	4	5	6	7	8	
1	0	1	0	0	0	0	0	y
1	0	0	1	0	0	0	0	o
1	0	0	0	1	0	0	0	x
1	0	0	0	0	1	0	0	o
1	0	0	0	0	0	1	0	y
0	1	0	1	0	0	0	0	y
0	1	0	0	1	0	0	0	o
0	1	0	0	0	1	0	0	x
0	1	0	0	0	0	1	0	o
0	1	0	0	0	0	0	1	y
0	0	1	0	1	0	0	0	y
0	0	1	0	0	1	0	0	o
0	0	1	0	0	0	1	0	x
0	0	1	0	0	0	0	1	o
0	0	0	1	0	1	0	0	y
0	0	0	1	0	0	1	0	o
0	0	0	1	0	0	0	1	x
0	0	0	0	1	0	1	0	y
0	0	0	0	1	0	0	1	o
0	0	0	0	0	1	0	1	y

Figure B.1 Analysis of two disk failures when N=8 with CD, first row is the disk number, 1 means the disk has failed and 0 means that the disk is functioning. The last column identifies different configurations.

1-Group 1 has one disk and Group 2 has five disks with weight $w_1 = 8/20$, marked as "y" in Figure B.1. The single disk in Group 1 is the bottleneck and has a throughput μ for processing two data blocks. It follows that the other five disks will have a throughput 3μ in processing six data blocks. The overall throughput is 4μ.

$N'^1_2 = (4\mu)(5/7 \times 8/20)/(6\lambda) = 4\mu/(21\lambda).$

2-Group 1 has two disks and Group 2 has four disks with weight $w_2 = 8/20$ marked as "o" in Figure B.1. Two disks in group one become the bottleneck with

a throughput 2μ for three data blocks. The throughput at four disks for five data blocks will equal $10\mu/3$, so that the total throughput is $16\mu/3$.

$$N'^2_2 = (16\mu/3)(5/7 \times 8/20)/(6\lambda) = 16\lambda/(63\mu).$$

3-Each group of three disks processes the load associated with four data blocks, so that the throughput is 6μ. This configuration has weight $w_3 = 4/20$ and is marked with "x" in Figure B.1.

$$N'^3_2 = (6\mu)(5/7 \times 4/20)/(6\lambda) = \mu/(7\lambda).$$

It follows $N'_2 = \sum_{j=0}^{2} N'^j_2 = 37\mu/(63\lambda)$.

When three disks fail there is always a single disk between two failed disks, which is a bottleneck. It is easy to show that the maximum throughput in this case is 4μ.

$$N'_3 = (4\mu)(2/7)/(5\lambda) = 8\mu/(35\lambda).$$

With four disk failures the throughputs at all surviving disks are the same and $V_4 = 1/35$.

$$N'_4 = (4\mu)(1/35)/(4\lambda) = \mu/(35\lambda).$$

$$N' = (581\mu)/(315\lambda).$$

APPENDIX C

ANALYSIS OF READ COST FOR AN UNAVAILABLE DATA BLOCK WITH TWO FAILED DISKS

Observation 1 There are always two blocks on each failed disk which can be reconstructed by using one parity group.

Proof: Because each parity group only contains $N-1$ blocks, it only uses $N-1$ disks. In other words each disk only has blocks of $N-1$ parities. So even though one disk is failed the block in the parity that is not on the disk can be repaired by using one parity group access. Since there are two parity groups (slope = 1 and slope = -1), there are two such blocks on each failed disks.

However the position of these two blocks are changing when the distance of two failed disks changes. The position has some impacts on the reconstructing cost of an unavailable data block.

Example: Figures C.1 and C.2 show the two parity groups with $N = 7$.

When two failed disks are neighbors those two blocks that can be repaired by one parity group access are the top and bottom blocks of the failed disks. Analysis of two failed disks is identical. So the focus is on one disk. For example disk 1 and disk 2 are failed. The two blocks are block (1,1) and block (7,1) from Figures C.1 and C.2. Block (1,1) is a data block and block (7,1) is a parity block. The data block can decide which way to go to reconstruct other data block. For example if block (2,2) needs to be reconstructed, either block (1,1) for q(5) or block (3,1) for p(4) needs to be reconstructed first. Obviously block (1,1) will be chosen for one parity group access. The parity block won't give any help to reconstruct other data blocks because all data blocks in that parity group is available. Had this block is not a parity block,

slope = 1	Disk Number						
Row #	1	2	3	4	5	6	7
1	2	3	4	5	6	0	1
2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	3
4	5	6	0	1	2	3	4
5	6	0	1	2	3	4	5
6	0	1	2	3	4	5	6
7							

Figure C.1 p parity groups with $N = 7$

slope = −1	Disk Number						
Row #	1	2	3	4	5	6	7
1	5	6	0	1	2	3	4
2	4	5	6	0	1	2	3
3	3	4	5	6	0	1	2
4	2	3	4	5	6	0	1
5	1	2	3	4	5	6	0
6							
7	0	1	2	3	4	5	6

Figure C.2 q parity groups with $N = 7$

it would help to reconstruct other unavailable data blocks. This will be discussed later.

When the distance of two failed disks increases the two blocks which can be reconstructed at once move toward the middle of the disks. They become the middle data blocks when the distance reaches the maximum. Figures C.1, C.2, C.3, C.4, C.5 and C.6 shows the results.

For the data block b(i,x) on failed disk i other than those two that can be reconstructed at once, there is an unavailable block b(j,y) in the same parity group which needs to be reconstructed first on failed disk j, $(1 \leq (i, j, x, y) \leq N)$. There are two ways to reconstruct block b(i,x), either by parity groups p or q. However if b(i,x) and b(j,y) in parity group p(i) and b(j,y) is a parity block, there is no way to reconstruct b(i,x) via p(i). b(i,x) has to be reconstructed via q(i). Then p(i) separates the data blocks on disk i into two groups. The data blocks in one group uses parity groups p to reconstruct themselves and those in the other group uses parity groups q. Each group contains a block that can be reconstructed via one parity group access. The rest blocks in the same group can be reconstructed via this block and the cost is multiple of one parity group access. These two blocks can be i) one data block and one parity block or ii) both data blocks. In case i the parity block won't help other data blocks's reconstruction. The data blocks on the disks can be reconstructed only via parity groups p or q. There is only one group. In case ii there are two groups and the average read cost of a data block is reduced.

For example block (3,1) needs to be reconstructed when disk 1 and disk 2 are failed. Block (2,2) for p(4) or block (4,2) for q(3) needs to be reconstructed first. It is displayed in Figures C.7 and C.8. So the read cost for block (3,1) is

$$C_{b(3,1)} = min(C_{b(2,2)}, C_{b(4,2)}) + (N-2)D_{SR}.$$

slope = 1	Disk Number						
Row #	1	2	3	4	5	6	7
1	2		4	5	6	0	1
2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	
4	5	6	0	1	2		4
5	6	0	1	2		4	5
6	0	1	2		4	5	6
7							

Figure C.3 Block (2,1) can be reconstructed by *p(3)* when disk 1 and 3 are failed.

slope = −1	Disk Number						
Row #	1	2	3	4	5	6	7
1	5	6	0		2	3	4
2	4	5	6	0		2	3
3	3	4	5	6	0		2
4	2	3	4	5	6	0	
5	1	2	3	4	5	6	0
6							
7	0		2	3	4	5	6

Figure C.4 Block (5,1) can be reconstructed by *q(1)* when disk 1 and 3 are failed

slope = 1	Disk Number						
Row #	1	2	3	4	5	6	7
1	2	3		5	6	0	1
2	3		5	6	0	1	2
3		5	6	0	1	2	3
4	5	6	0	1	2	3	
5	6	0	1	2	3		5
6	0	1	2	3		5	6
7							

Figure C.5 Block (3,1) can be reconstructed by *p(4)* when disk 1 and 4 are failed.

slope = −1	Disk Number						
Row #	1	2	3	4	5	6	7
1	5	6	0	1		3	4
2	4	5	6	0	1		3
3	3	4	5	6	0	1	
4		3	4	5	6	0	1
5	1		3	4	5	6	0
6							
7	0	1		3	4	5	6

Figure C.6 Block (4,1) can be reconstructed by *q(2)* when disk 1 and 4 are failed.

Then the same analysis for block (2,2) and block (4,2) is carried out until the block that can be reconstructed by one parity access is reached. However it is impossible to reconstruct block (4,2) because it needs block (5,1) which has two unavailable blocks in parity group q(1). As it is discussed earlier, block (5,1) separates other data blocks on disk 1 into two groups and because block (5,1) is the last data block, it only has one group. So it is only possible to go with block (2,2). Block (2,2) needs block (1,1) which can be reconstructed by $(N-2)D_{SR}$ data block accesses. The total cost for block (3,1) is p(2), q(5) and p(4), i.e. $3(N-2)D_{SR}$. Because of the irrecoverableness of block (5,1) for blocks (2,1) to (N-2,1) they can only use the parity groups where slope is 1. They all go the same zig-zag way as for block (3,1).

Table C.1 displays the recovery path and read cost of all unavailable data blocks with $N = 7$ and $d = 1$. Block (5,1) can not be recovered. It separates other data blocks on disk 1 into two groups. However since block (5,1) is the last data block, it only has one group.

Table C.1 Read Cost for the Failed Disks 1 and 2 with N=7 and d=1

Block	Recovery path	Read cost $(N-2)D_{SR}$
(1,1)	p(2)	1
(2,1)	q(6) + p(3)	2
(3,1)	p(2) + q(5) +p(4)	3
(4,1)	q(6) + p(3) + q(4) + p(5)	4
(5,1)	p(2) + q(5) + p(4) + q(3) + p(6)	5
(1,2)	q(6)	1
(2,2)	p(2) + q(5)	2
(3,2)	q(6) + p(3) +q(4)	3
(4,2)	p(2) + q(5) +p(4) +q(3)	4
(5,2)	q(6) + p(3) + q(4) + p(5) + q(2)	5

slope = 1	Disk Number						
Row #	1	2	3	4	5	6	7
1	2	3	4	5	6	0	1
2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	3
4	5	6	0	1	2	3	4
5	6	0	1	2	3	4	5
6	0	1	2	3	4	5	6
7							

Figure C.7 Recovery path for block (3,1) with $N = 7$ and slope =1

slope = −1	Disk Number						
Row #	1	2	3	4	5	6	7
1	5	6	0	1	2	3	4
2	4	5	6	0	1	2	3
3	3	4	5	6	0	1	2
4	2	3	4	5	6	0	1
5	1	2	3	4	5	6	0
6							
7	0	1	2	3	4	5	6

Figure C.8 Recovery path for block (3,1) with $N = 7$ and slope =-1

Table C.2 displays the recovery path and read cost of all unavailable data blocks with $N = 7$ and $d = 2$. This time the recovery path is shown differently. It show the other unavailable data block needs to be reconstructed first in the same parity group. In this case block (2,1) is the one separates the data blocks on disk 1 into two groups. One group includes block (2,1), (4,1) and the other group includes the rest data blocks.

Table C.2 Read Cost for the Failed Disks 1 and 3 with N=7 and d=2

Block	Recovery path	Read cost $(N-2)D_{SR}$
(1,1)	(3,3) + q(5)	3
(2,1)	p(3)	1
(3,1)	(5,3) + q(3)	2
(4,1)	(2,3) + p(5)	2
(5,1)	q(1)	1
(1,3)	(3,1) +p(4)	3
(2,3)	q(6)	1
(3,3)	(5,1) + p(6)	2
(4,3)	(2,1) + q(4)	2
(5,3)	p(1)	1

Table C.3 displays the recovery path and read cost of all unavailable data blocks with $N = 7$ and $d = 3$. It is observed block (3,1) can not be used to reconstruct other unavailable blocks even through it only needs one party group access to be reconstructed. Block (3,1) is the one separates the data blocks on disk 1 into two groups. One group includes block (3,1) and the other group includes the rest data blocks.

Table C.3 Read Cost for the Failed Disks 1 and 4 with N=7 and d=3

Block	Recovery path	Read cost $(N-2)D_{SR}$
(1,1)	(4,4) + q(5)	2
(2,1)	(5,4) + q(4)	4
(3,1)	p(4)	1
(4,1)	q(2)	1
(5,1)	(1,4) + q(1)	3
(1,4)	(4,1) + p(5)	2
(2,4)	(5,1) + p(6)	4
(3,4)	q(6)	1
(4,4)	p(1)	1
(5,4)	(1,1) + p(2)	3

APPENDIX D

ALGORITHM TO COMPUTE THE LOAD INCREASE WITH TWO DISK FAILURES IN X-CODE

Algorithm 3 computes the load increase for each unavailable block. Recursion is carried out at Line 5 in the algorithm. If there is another data block unavailable it needs to be reconstructed first by using the parity group with the opposite slope. Other part of the algorithm is trivial.

Now a test drive is developed to call the recursive algorithm. Algorithm 4 shows it. One point worth mention here is that for a given unavailable block there are two possible ways to reconstruct it, by using either p or q parity group. The one with minimum cost will be chosen. Also notice at the beginning it is necessary to find the parity groups an unavailable block belongs to. This is due to the fact that for a given data block it is not known which parity group it belongs to because of the module operations in the above two formulas.

Now a main algorithm is deployed to loop through each failed data blocks. It is displayed in Algorithm 5. The results are the times of load increase of each disk comparing its load with no failed disk.

Algorithm 3 Load_increase_each_block_recursion

Input: Number of disks N, distance d, row number i of the block , disk number j of the block, slope s (1 or -1).

Output: Load increase $C_{i,j}$ to access $B_{i,j}$.

if s = 1 **then**

$PG \Leftarrow p$

else {s = -1}

$PG \Leftarrow q$

end if

for each data block and parity block in parity group PG **do**

if Block (i,j) can be reconstructed by one parity group access **then**

$C_{i,j} \Leftarrow C_{i,j} + 1$.

else {There is another unavailable parity block}

$C_{i,j} \Leftarrow \infty$.

else {There is another unavailable data block $B_{x,y}$}

$C_{i,j} \Leftarrow C_{i,j} + Load\_increase\_each\_block\_recursion(N, d, x, y, -slope)$.

end if

end for

Return $C_{i,j}$.

Algorithm 4 Load_increase_each_block

Input: Number of disks N, distance d, row number i of the block, disk number j of the block.

Output: Load increase of each disks for block $B_{i,j}$ $C_{i,j}$.

Find the parity groups p and q the block $B_{i,j}$ belongs to respectively.

for Each data block and parity block in parity group p (q) **do**

if $B_{i,j}$ can be reconstructed by one parity group access **then**

$C_{i,j}^{p(q)} \Leftarrow C_{i,j}^{p(q)} + 1$.

else {There is another unavailable parity block}

$C_{i,j}^{p(q)} \Leftarrow \infty$.

else {There is another unavailable data block $B_{x,y}$}

if The current parity group is p **then**

$slope \Leftarrow -1$

else {The current parity group is q}

$slope \Leftarrow 1$

end if

$C_{i,j}^{p(q)} \Leftarrow C_{i,j}^{p(q)} + Load\_increase\_each\_block\_recursion(N, d, x, y, slope)$.

end if

end for

$C_{i,j} \Leftarrow min(C_{i,j}^{p}, C_{i,j}^{q})$.

Algorithm 5 Load_increase_with_two_disk_failures

Input: Number of disks N.

Output: Load increase of each disks with different distance $C_{Increase}$.

$D \Leftarrow \frac{N-1}{2}$

for $d = 1$ to D **do**

  for Each broken block (i,j) **do**

    $C_{i,j} \Leftarrow Load\_increase\_each\_block(N, d, i, j)$.

    $C_{total} \Leftarrow C_{total} + C_{i,j}$.

  end for

end for

$C_{Increase} \Leftarrow \frac{C_{total}}{N-2} + 1$.

APPENDIX E

ALGORITHM TO COMPUTE THE LOAD INCREASE WITH TWO DISK FAILURES IN RM2

Algorithm 6, 7 and 8 compute the load increase of each disks with two disks failures in RM2.

Given parity blocks are stored in the first row (row index from 1 to M and column index from 0 to $N-1$), the formula for a parity group is $B_{1,i} = B_{2,<i+1+N-M>_N} \oplus B_{3,<i+2+N-M>_N} \oplus ... \oplus B_{M,<i+N-1>_N} \oplus B_{2,<i+N-M>_N} \oplus B_{3,<i-1+N-M>_N} \oplus ... \oplus B_{M,<i+N-2M+2>_N}$, where $0 \leq i \leq N-1$, $< X >_N = X$ module N. On the contrary if block (i,j) is given, column index of p and q parity groups the block belongs to is computed by the following formulas

$$x = j - (N - M + 2) + i$$

$$y = j - (N - M - 1) - i.$$

If x or y is less than zero

$$x = x + N$$

$$y = y + N.$$

Algorithm 6 RM2_Load_increase_each_block_recursion

Input: N, M ,d, **row number** i **of the block, disk number** j **of the block, parity** PQ**.**

Output: Load increase $C_{i,j}$ **to access** $B_{i,j}$**.**

if PQ = p **then**

$PG \Leftarrow q$

else {PQ = q}

$PG \Leftarrow p$

end if

for each data block and parity block in parity group PG **do**

if Block (i,j) can be reconstructed by one parity group access **then**

$C_{i,j} \Leftarrow C_{i,j} + 1$.

else {There is another unavailable parity block}

$C_{i,j} \Leftarrow \infty$.

else {There is another unavailable data block $B_{x,y}$}

$C_{i,j} \Leftarrow C_{i,j} + RM2\_Load\_increase\_each\_block\_recursion(N, M, d, x, y, PG)$.

end if

end for

Return $C_{i,j}$.

Algorithm 7 RM2_Load_increase_each_block

Input: N, M, d, **row number** i **of the block, disk number** j **of the block.**

Output: Load increase of each disks for block $B_{i,j}$ $C_{i,j}$.

Find the parity groups p and q the block $B_{i,j}$ belongs to respectively.

for Each data block and parity block in parity group p (q) **do**

if $B_{i,j}$ can be reconstructed by one parity group access **then**

$C_{i,j}^{p(q)} \Leftarrow C_{i,j}^{p(q)} + 1$.

else {There is another unavailable parity block}

$C_{i,j}^{p(q)} \Leftarrow \infty$.

else {There is another unavailable data block $B_{x,y}$}

if The current parity group is p **then**

$PG \Leftarrow p$

else {The current parity group is q}

$PG \Leftarrow q$

end if

$C_{i,j}^{p(q)} \Leftarrow C_{i,j}^{p(q)} + RM2\_Load\_increase\_each\_block\_recursion(N, M, d, x, y, PG)$.

end if

end for

$C_{i,j} \Leftarrow min(C_{i,j}^{p}, C_{i,j}^{q})$.

Algorithm 8 RM2_Load_increase_with_two_disk_failures

Input: N,M.

Output: Load increase of each disks with different distance $C_{Increase}$.

$D \Leftarrow \frac{N-1}{2}$

for $d = 1$ to D **do**

  for Each broken block (i,j) **do**

    $C_{i,j} \Leftarrow RM2\_Load\_increase\_each\_block(N, M, d, i, j)$.

    $C_{total} \Leftarrow C_{total} + C_{i,j}$.

  end for

end for

$C_{Increase} \Leftarrow \frac{C_{total}}{N-2} + 1$.

REFERENCES

[1] A. R. Calderbank, E. G. Coffman, Jr., and L. Flatto. Sequencing problems in two server systems. *Mathematics of Operations Research*, 10(4):585–598, 1985.

[2] A. R. Calderbank, E. G. Coffman, Jr., and L. Flatto. Optimum head separation in a disk system with two heads. *Journal of the ACM*, 31(4):826–838, September 1984.

[3] A. Thomasian. Access costs in clustered raid. *The Computer Journal*, 48(11):702–713, November 2005.

[4] A. Thomasian. Correction to "Clustered RAID Arrays and Their Access Costs". *The Computer Journal*, November 2005.

[5] A. Thomasian and C. Han. Heterogeneous disk array architecture and its data allocation policies. In *Int'l Symp. on Performance Evaluation of Computer and Telecommunication Systems - SPECTS 2005*, pages 617–624, Cherry Hill, NJ, July 2005.

[6] A. Thomasian and C. Liu. Some new disk scheduling policies and their performance. In *Proc. ACM SIGMETRICS Conf. Measurement and Modeling of Computer Systems*, pages 266–267, Marina del Rey, CA, May 2002.

[7] A. Thomasian, B. A. Branzoi, and C. Han. Performance evaluation of a heterogeneous disk array architecture. In *13th Int'l Symp. on Modeling, Analysis, and Simulation of Computer and Telecommunication Systems - MASCOTS '05*, pages 517–520, Atlanta, GA, September 2005.

[8] A. Thomasian, C. Han and G. Fu. Performance evaluation of two-disk failure tolerant arrays. *IEEE Trans. Computers*, 56(6):799–814, June 2007.

[9] A. Thomasian, J. Spirollari, C. Liu, C. Han and G. Fu. Mirrored disk scheduling. In *Proc. Int'l Symp. Performance Evaluation of Computer and Telecommunication Systems -SPECTS'03*, pages 580–584, Montreal, Quebec, Canada, July 2003.

[10] A. Adya, W. Bolosky, M. Castro, R. Chaiken, G. Cermak, J. Douceur, J. Howell, J. Lorch, M. Theimer, and R. Wattenhofer. Farsite: Federated, available, and reliable storage for an incompletely trusted environment. In *In proc. of the 5th Symposium on Operating Systems Design and Implementation (OSDI).*, Boston, MA,2002.

[11] N. Allen. Don't waste your storage dollars: what you need to know. Research Note COM-13-1217, Gartner Group, March 2001.

[12] G. A. Alvarez, E. Borowsky, S. Go, T. H. Romer, R. Becker-Szendy, R. Golding, A. Merchant, M. Spasojevic, A. Veitch, and J. Wilkes. Minerva: An automated resource provisioning tool for large-scale storage systems. *ACM Transactions on Computer Systems*, 19(4):483–518, 2001.

[13] E. Anderson, M. Kallahalla, S. Spence, R. Swaminathan, and Q. Wang. Ergastulum: Quickly finding near-optimal storage system designs. HP Laboratories SSP Technical Report HPL-SSP-2001-05, June 2002.

[14] B. Jacob, S. W. Ng, and D. T. Wang. *Memory Systems: Cache, DRAM, Disk.* Morgan-Kaufmann Publisher, 2008.

[15] B. R. Haverkort, R. Marie, R. Rubino, and K. S. Trivedi. *Performability Modelling: Techniques and Tools.* Wiley, New York, NY, 2001.

[16] E. Bachmat and T. K. Lam. On the effect of a configuration choice on the performance of a mirrored storage system. *J. Parallel and Distributed Computing*, 65(3):382–395, March 2005.

[17] M. Blaum. An introduction to error-correcting codes, *Coding and Signal Processing for Magnetic Recording Systems*, B Vasic and E M Kurtas, editors, Chapter 9, CRC Press, Orlando, FL.

[18] E. Borowsky, R. Golding, A. Merchant, L. Schrier, E. Shriver, M. Spasojevic, and J. Wilkes. Using attribute-managed storage to achieve qos. In *Proc. of the 5th Int'l Conf. Workshop on Quality of Service*, Columbia University, New York, June 1997.

[19] B. A. Branzoi. *Performance Evaluation of Various Allocation Methods in a Heterogeneous Disk Array Architecture.* Masters thesis, Computer Science Dept., New Jersey Institute of Technology, Newark, NJ, December 2004.

[20] C. Orji and J. A. Solworth. Doubly distorted mirrors. In *Proc. ACM SIGMOD Int'l Conf. on Management of Data*, pages 307–316, Washington, D. C., May 1993.

[21] C. Polyzois, A. Bhide and D. M. Dias. Disk mirroring with alternating deferred updates. In *Proc. 19th Int'l Conf. Very Large Data Bases - VLDB*, pages 604–617, Dublin, Ireland, August 1993.

[22] S. Z. Chen and D. F. Towsley. A performance evaluation of raid architectures. *IEEE Trans. Computers*, 45(10):1116–1130, October 1996.

[23] C.R. Lumb, R. Golding and G. R. Ganger. D-SPTF: Decentralized request distribution in brick-based storage systems. In *Proc 11th Int'l Conf. Architectural Support for Programming Languages and Operating Systems - ASPLOS*, pages 37–47, Cambridge, MA, October 2004.

[24] D. F. Towsley, S. Z. Chen and S. P. Yu. Mirrored disk scheduling. In *Performance analysis of fault-tolerant mirrored disk systems*, pages 239–253, P. J. B. King, I. Mitrani, and R. J. Pooley, editors, North-Holland, Amsterdam, The Netherlands, 1990.

[25] E. Anderson, M. Hobbs K. Keeton, S. Spence, M. Uysal, and A. Veitch. Hippodrome: Running circles around storage administration. In *Conf. File and Storage Technologies - FAST'02*, Monterey, CA, January 2002.

[26] E. Anderson, R. Swaminathan, A. Veitch, G. A. Alvarez, and J. Wilkes. Selecting RAID levels for disk arrays. In *Conf. File and Storage Technologies - FAST'02*, pages 189–201, Monterey, CA, January 2002.

[27] E. G. Coffman, Jr. and M. Hofri. Queueing models of secondary storage devices. In *Stochastic Analysis of Computer and Communication Systems*, pages 549–588, H. Takagi, editor, Elsevier, Amsterdam, The Netherlands, 1990.

[28] G. A. Gibson. *Redundant Disk Arrays: Reliable, Parallel Secondary Storage.* MIT Press, 1992.

[29] R. Golding, E. Shriver, T. Sullivan, and J. Wilkes. Attribute-managed storage. In *Workshop on Modeling and Specification of I/O*, 1995.

[30] C. Han. *Studies of disk arrays tolerating two disk failures and a proposal for a heterogeneous disk array.* PhD thesis, New Jersey Institute of Technology, Newark, NJ, May 2004.

[31] M. Hofri. Should the two-headed disk be greedy? - yes it should. *Information Processing Letters - IPL*, 16(2):83–85, 1983.

[32] HP, An analysis of RAID 5DP. HP White Paper, http://www.hp.com.

[33] H. I. Hsiao and D. J. DeWitt. A performance study of three high availability data replication strategies. *J. Distributed and Parallel Databases*, 1(1):53–80, 1993.

[34] J. Wilkes, R. Golding, C. Staelin, and T. Sullivan. The HP autoraid hierarchical storage system. *ACM Trans. Computer Systems*, 14(1):108–136, 1996.

[35] H. H. Kari. *Latent Sector Faults and Reliability of Disk Arrays.* PhD thesis, Helsinki University of Technology, Espoo, Finland, 1997.

[36] C. Kenyon. Best-fit bin-packing with random order. In *SODA: ACM-SIAM Symp. on Discrete Algorithms (A Conference on Theoretical and Experimental Analysis of Discrete Algorithms)*, pages 359–364, 1996.

[37] L. Kleinrock. *Queueing Systems, Vol. I: Theory.* John-Wiley, New York, NY, 1975.

[38] L. Kleinrock. *Queueing Theory, Vol. II: Applications.* John-Wiley, New York, NY, 1975.

[39] L. Golubchik, J. S. Liu, and R. R. Muntz. Chained declustering: Load balancing and robustness to skew and failures. In *Proc. 1st Int'l Workshop on Research Issues in Data Engineering - RIDE*, pages 88–95, Tempe, AZ, 1992.

[40] E. Lee and R. Katz. Performance consequences of parity placement in disk array. In *Int'l Conf. on Architectural Support for Programming Languages and Operating Systems - ASPLOS*, pages 190–199, Santa Clara, CA, USA, 1991.

[41] M. Blaum, J. Brady, J. Bruck, and J. Menon. Evenodd: An optimal scheme for tolerating double disk failures in raid architectures. *IEEE Trans. Computers*, TC-44(2):192–202, February 1995.

[42] J. Menon and D. Mattson. Distributed sparing in disk arrays. In *Proc. 37th Annual IEEE Computer Society Conf. - COMPCON*, pages 410–421, San Francisco, CA, February 1992.

[43] A. Merchant and P. Yu. Performance analysis of a dual striping strategy for replicated disk arrays. In *Proceedings of the Second International Conference on Parallel and Distributed Information Systems*, pages 148–157, San Diego, January 1993.

[44] R. Nelson and A. Tantawi. Approximate analysis of fork-join synchronization in parallel queues. *IEEE Transactions on Computers*, 37(6):739–743, 1988.

[45] S. W. Ng. Improving disk performance via latency reduction. *IEEE Trans. Computers*, 40(1):22–30, January 1991.

[46] P. Corbett, B. English, A. Goel, T. Grcanac, S. Kleiman, J. Leong, and S. Sankar. Row-diagonal parity for double disk failure correction. In *Proc. 3rd Conf. File and Storage Technologies - FAST'04*, pages 1–14, San Francisco, CA, March/April 2004.

[47] P. Lyman, H. R. Varian, P. Charles, N. Good, L. L. Jordan and J. Pal. How much informantion? http://www2.sims.berkeley.edu/research/projects/how-much-info-2003/, December 3, 2003.

[48] P. M. Chen, E. K. Lee, G. A. Gibson, R. H. Katz, and D. A. Patterson. RAID: High-performance, reliable secondary storage. *ACM Computing Surveys*, 26(2):145–185, June 1994.

[49] R. Paquet and M. Nicolett. The cost of storage management: A sanity check. Research note DF-14-6838, Gartner Group, November 2001.

[50] C.-I. Park. Efficient placement of parity and data to tolerate two disk failures in disk array systems. *IEEE Trans. Parallel and Distributed Systems*, 6(11):1177–1184, November 1995.

[51] D. A. Patterson, G. A. Gibson, and R. H. Katz. A case for redundant arrays of inexpensive disks (RAID). In *Proc. of ACM SIGMOD 1988 Int'l Conf. Management of Data*, pages 109–116, Chicago, IL, June 1988.

[52] R. A. Hill. System for managing data storage based on vector-summed size-frequency vectors for data sets, devices, and residual storage on devices. US Patent 5345584, 1994.

[53] R. Muntz and J. Lui. Performance analysis of disk arrays under failure. In *Proc. of the 16th Very Large Data Bases - VLDB Conference*, pages 162–173, Brisbane, Australia, August 1990.

[54] S. Rhea, P. Eaton, D. Geels, H. Weatherspoon, B. Zhao, and J. Kubiatowicz. Pond: the oceanstore prototype. In *Proc. of 2nd Conf. on File and Storage Technologies (FAST)*, pages 1-14, March 2003.

[55] E. Shriver. A formalization of the attribute mapping problem. HP Labs Technical Report HPL-SSP-95-10, HP Labs, July 1996.

[56] H. Takagi. *Queueing Analysis - A Foundation of Performance Evaluation.* North-Holland, 1991.

[57] H. Takagi. *Queueing Analysis-Vol. 1: Vacation and Priority Systems.* North-Holland, Amsterdam, The Netherlands, 1991.

[58] Tandem Database Group. Nonstop sql: A distributed, high-performance, high-reliability implementation of sql. In *High Performance Transaction Systems*, pages 60–103, D. Gawlick, M. N. Hayne, and A. Reuter, editors, Springer-Verlag, 1987.

[59] Teradata Corporation. Dbc/1012 database computer system manual. Release 2, http://www.teradata.com, 1985.

[60] A. Thomasian. Reconstruct versus read-modify writes in raid. *Information Processing Letters - IPL*, 93(4):163–168, February 2005.

[61] A. Thomasian. Comment on "raid performance with distributed sparing". *IEEE Trans. Parallel and Distributed Systems - TPDS*, 17(4):399–400, April 2005.

[62] A. Thomasian. Shortcut method for reliability comparisons in raid. *The Journal of Systems and Software*, 79(11):1599–1605, November 2006.

[63] A. Thomasian and M. Blaum. Mirrored disk organization reliability analysis. *IEEE Trans. Computers*, 55(12):1640–1644, December 2006.

[64] A. Thomasian and G. Fu. Anticipatory disk arm placement to reduce seek time. *Computer Systems: Science and Engineering - CSSE*, 21(3):173–182, May 2006.

[65] A. Thomasian and C. Han. Affinity based routing in zoned mirrored disks. *The Computer Journal*, 48(3):292–299, March 2005.

[66] A. Thomasian and C. Liu. Performance comparison of mirrored disk scheduling methods with a shared non-volatile cache. *J. Distributed and Parallel Databases*, 18(3):253–281, December 2005.

[67] A. Thomasian and J. Menon. RAID5 performance with distributed sparing. *IEEE Trans. Parallel and Distributed Systems*, 8(6):640–657, 1997.

[68] A. Thomasian and J. Menon. Performance analysis of RAID5 disk arrays with a vacationing server model for rebuild mode operation. In *Proc. 10th Int'l Conf. Data Engineering*, pages 111–119, Houston, TX, USA, February 1994.

[69] A. Thomasian and J. Xu. Data allocation in disk arrays with multiple raid levels. In *work in process, Fall 2007.*

[70] A. Thomasian and J. Xu. Reliability and performance of mirrored disk organizations. In *The Computer Journal, 2008, to appear.*

[71] K. S. Trivedi. *Probability and Statistics with Reliability, Queueing, and Computer Science Applications, Second Edition.* Wiley, New York, NY, 2002.

[72] W. W. Wilcke, et al. IBM intelligent bricks project - petabytes and beyond. *IBM Journal of Research and Development*, 50(2/3):181–197, March/May 2006.

[73] D. Whitehouse. Reworked images reveal hot Venus, January 2004.

[74] Q. Xin, E. Miller, and T. Schwars. Evaluation of distributed recovery in large-scale storage systems. In *Proc. of 13th IEEE Int'l Symp. on High Performance Distributed - HPDC Computing.*, Honolulu, HI, June 2004.

[75] L. Xu and J. Bruck. X-code: Mds array codes with optimal encoding. *IEEE Trans. Information Theory*, 45(1):272–276, 1999.