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## Modeling inventory and responsiveness costs in a supply chain

Robert Nearier  
*New Jersey Institute of Technology*

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## ABSTRACT

### MODELING INVENTORY AND RESPONSIVENESS COSTS IN A SUPPLY CHAIN

by  
**Robert Nearier**

Evaluation of supply chain performance is often complicated by the various interrelationships that exist within the network of suppliers. Currently many supply chain metrics cannot be analytically determined. Instead, metrics are derived from monitoring historical data, which is commonly referred to as Supply Chain Analytics. With these analytics it is possible to answer questions such as: *What is the inventory cost distribution across the chain? What is the actual inventory turnover ratio? What is the cost of demand changes to individual suppliers?* However, this approach requires a significant amount of historical data which must be continuously extracted from the associated Enterprise Resources Planning (ERP) system.

In this dissertation models are developed for evaluating two Supply Chain metrics, as an alternative to the use of Supply Chain Analytics. First, inventory costs are estimated by supplier in a deterministic  $(Q, R, \delta)^2$  supply chain. In this arrangement each part has two sequential reorder ( $R$ ) inventory locations: (i) on the output side of the seller and (ii) on the input side of the buyer. In most cases the inventory policies are not synchronized and as a result the inventory behavior is not easily characterized and tends to exhibit long cycles. This is primarily due to the difference in production rates ( $\delta$ ), production batch sizes, and the selection of supply order quantities ( $Q$ ) for logistics convenience. The  $(Q, R, \delta)^2$  model that is developed is an extension of the joint economic lot size (JELS) model first proposed by Banerjee (1986). JELS is derived as a

compromise between the seller's and the buyer's economic lot sizes and therefore attempts to synchronize the supply policy. The  $(Q, R, \delta)^2$  model is an approximation since it approximates the average inventory behavior across a range of supply cycles. Several supply relationships are considered by capturing the inventory behavior for each supplier in that relationship. For several case studies the joint inventory cost for a supply pair tends to be a stepped convex function.

Second, a measure is derived for responsiveness of a supply chain as a function of the expected annual cost of making inventory and production capacity adjustments to account for a series of significant demand change events. Modern supply chains are expected to use changes in production capacity (as opposed to inventory) to react to significant demand changes. Significant demand changes are defined as shifts in market conditions that cannot be buffered by finished product inventory alone and require adjustments in the supply policy. These changes could involve a  $\pm 25\%$  change in the uniform demand level. The research question is what these costs are and how they are being shared within the network of suppliers. The developed measure is applicable in a multi-product supply chain and considers both demand correlations and resource commonality.

Finally, the behavior of the two developed metrics is studied as a function of key supply chain parameters (e.g., reorder levels, batch sizes, and demand rate changes). A deterministic simulation model and program was developed for this purpose.

**MODELING INVENTORY AND RESPONSIVENESS COSTS  
IN A SUPPLY CHAIN**

**by  
Robert Nearier**

**A Dissertation  
Submitted to the Faculty of  
New Jersey Institute of Technology  
in Partial Fulfillment of the Requirements for the Degree of  
Doctor of Philosophy in Industrial Engineering**

**Department of Industrial and Management Systems Engineering**

**May 2008**

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**APPROVAL PAGE**

**MODELING INVENTORY AND RESPONSIVENESS COSTS  
IN A SUPPLY CHAIN**

**Robert Nearier**

3/31/08

---

Dr. Sanchoy Das, Dissertation Advisor  
Professor of Industrial and Manufacturing Engineering, NJIT

Date

3/31/08

---

Dr. Layek Abdel-Malek, Committee Member  
Professor of Industrial and Manufacturing Engineering, NJIT

Date

3/31/08

---

Dr. George Abdou, Committee Member  
Associate Professor of Industrial and Manufacturing Engineering, NJIT

Date

3/31/08

---

Dr. Athanassios Bladikas, Committee Member  
Associate Professor of Industrial and Manufacturing Engineering, NJIT

Date

4/3/08

---

Dr. Rene Cordero, Committee Member  
Associate Professor of Management, NJIT

Date



## **BIOGRAPHICAL SKETCH**

**Author:** Robert Nearier  
**Degree:** Doctor of Philosophy  
**Date:** May 2008

### **Undergraduate and Graduate Education:**

- Doctor of Philosophy in Industrial Engineering,  
New Jersey Institute of Technology, Newark, NJ, 2008
- Master of Science in Manufacturing Engineering,  
Polytechnic University, Brooklyn, NY, 1996
- Master of Business Administration in Accounting,  
Indiana University, Bloomington, IN, 1977
- Bachelor of Arts in History,  
State University of New York, Binghamton, NY, 1973

**Major:** Industrial Engineering

Dedicated with much love  
to my wife Lisa,  
who accepted and encouraged year after year of hard work and sacrifice.

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## CHAPTER 1

### INTRODUCTION

A supply chain is generally described as a group of interconnected industrial facilities. These facilities have a common objective of minimizing the costs associated with the inventory and logistics of a product. The primary motivation for “chaining” industrial facilities into a supply chain is to coordinate and integrate the movement of goods through the chain. Using a systems analysis model we can investigate and analyze a variety of integrative issues and coordination questions in the overall supply chain (e.g., risk sharing, inventory positions, distribution networks, and information flows). As a result, today there is considerable activity in both research and industry related to the design, management and operations of supply chains.

While there are several performance metrics in the study of supply chains, the primary metric continues to be inventory levels. Recently, the evaluation of inventory in supply chains has become even more complex for several reasons, including:

1. an increase in the number of points of sale, implying an increase in the number of inventory locations.
2. an increase in the product variety (color, size, model, etc.), implying an increase in the number of SKUs.
3. an increase in market competitiveness (supply > demand), implying more demand uncertainty.
4. a decrease in the product lifecycle, implying a more rapid price erosion of inventoried items.

Coupled with the more traditional inventory costs, these factors have led to a focus both on reducing the inventory at each point in the supply chain and on increasing the speed with which goods move through the supply chain (often referred to as the supply chain velocity). Consequently, supply chains are now one of the most researched areas in the related disciplines of industrial engineering and operations management. This research typically falls into one of two broad classes:

- (1) ***Micro models.*** These models investigate and analyze specific questions and phenomena within the supply chain (e.g., bullwhip effect, warehouse location, pipeline inventory levels). These questions are relevant to operational decisions.
- (2) ***Macro models.*** These models investigate and analyze the integrative issues and coordination questions in the overall supply chain (e.g., risk sharing, inventory positions, distribution networks, and information flows). These issues and questions are relevant primarily to tactical decisions, and to a lesser extent to strategic decisions.

The emphasis of this dissertation research is on macro models. Macro models will be developed for inventory costs and responsiveness costs in uncoordinated supply chains. The focus is specifically on two metrics: (i) inventory costs, and (ii) the cost of responding to demand rate changes. Evaluation of these two metrics is often complicated by the various interrelationships that exist within the network of suppliers. Currently many supply chain metrics cannot be analytically determined. Instead, metrics are derived from monitoring historical data, which is commonly referred to as Supply Chain Analytics. Through these analytics we are able to answer questions such as: *What is the inventory cost distribution across the chain? What is the actual inventory turnover*



*ratio?* or: *What is the cost of demand changes to individual suppliers?* However, this approach requires a significant amount of historical data which must be continuously extracted from the associated Enterprise Resources Planning (ERP) system. In this dissertation we develop models for estimating inventory and responsiveness costs, as an alternative to the use of Supply Chain Analytics.

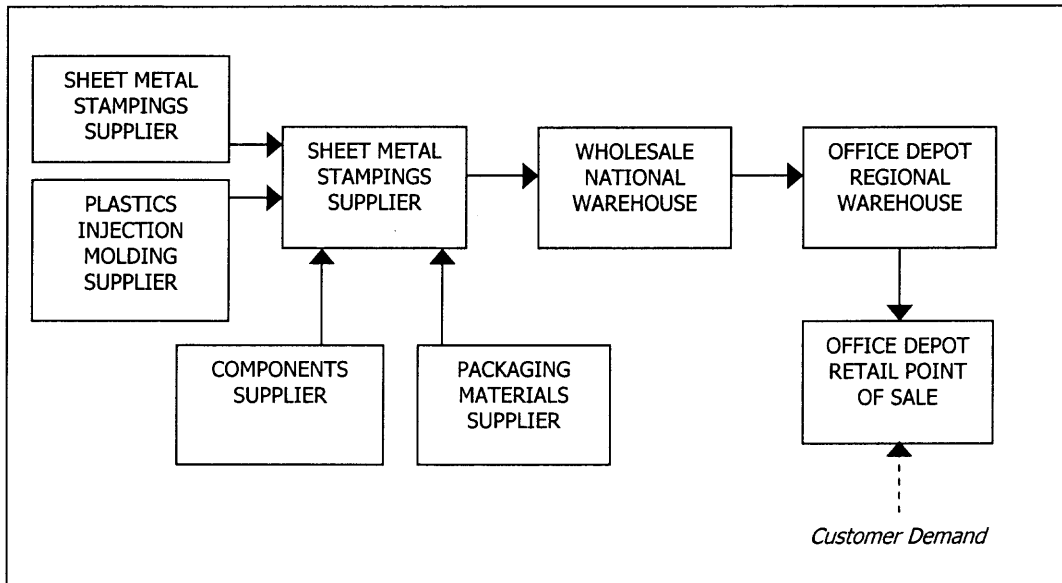
### 1.1 Inventory Costs in Uncoordinated Supply Chains

First, a model will be developed for estimating inventory costs by supplier in a deterministic  $(Q, R, \delta)^2$  supply chain. In this arrangement each part has two sequential reorder  $(R)$  inventory locations: (i) on the output side of the seller and (ii) on the input side of the buyer. In most cases the inventory policies are not synchronized. The result is that inventory behavior tends to exhibit long cycles, primarily due to differences in production rates  $(\delta)$ , production batch sizes, and the selection of supply order quantities  $(Q)$  for logistics convenience. The developed  $(Q, R, \delta)^2$  model is an extension of the joint economic lot size (JELS) model first proposed by Banerjee (1986). JELS is derived as a compromise between the seller's and the buyer's economic lot sizes and therefore attempts to synchronize the supply policy. The  $(Q, R, \delta)^2$  model is an approximation since it considers the average inventory behavior across a range of supply cycles. Five cases of supply relationships are considered by capturing the inventory behavior for each supplier in that relationship. For several case studies we find that the joint inventory cost for a supply pair tends to be a stepped convex function.

It is assumed that sequential facilities in a supply chain are differentiated by ownership and geography. For example, a family farm ice cream production facility

under the same ownership and/or in the same location does not constitute a supply chain. In such a case one would expect that all activities in this business (production, processing, storage and sales) are co-owned and co-located. Consequently there are no logistics or coordination issues. In contrast, consider the supply chain for a common desktop stapler as illustrated in figure 1.1. Clearly, each of the facilities are under separate ownership and have dispersed locations, but have a common objective of maximizing sales and their individual profits. There are three possible levels of inventory cooperation in such a chain:

1. UNCOORDINATED: Each supplier establishes its own inventory policies on both the input and output sides.
2. COORDINATED or INTEGRATED: Supply contracts are established between sequential suppliers in the network. These contracts establish the order or replenishment quantities and the reorder levels. There is a common understanding of the production rates at each supplier.
3. SYNCHRONIZED: The supply chain operates like a balanced assembly line in that order quantities, production batch sizes, and production rates are synchronized for perfect flow.



**Figure 1.1** A Stapler Supply Chain.

The supply chain research on uncoordinated supply chains is relatively limited. In this dissertation our focus is on this type of relationship.

## 1.2 Responsiveness Costs in Supply Chains with Shifting Demands

Second, a measure will be derived for responsiveness of a supply chain as a function of the expected annual cost of making inventory and production capacity adjustments to account for a series of significant demand change events. The premise here is that as supply chains are established each supplier expects a certain amount of business and accordingly allocates production resources or capacity. As the final product demand shifts each supplier is expected to use changes in production capacity (as opposed to inventory) to react to these significant demand changes. A significant change is defined as a shift in market conditions that cannot be buffered by finished product inventory alone and requires adjustments to the supply policy. Such a change could involve a

±25% change in the uniform demand level. The research question is what the resultant costs are and how they are being shared within the network of suppliers. A common complaint in many supply chains is that the retailer experiences less of this cost, and most of it is shifted to the assembly plants. An additional complexity is when a basket of products are being produced and there are issues of demand correlation and resource commonality. In this research estimates are made of the inventory adjustment cost and production rate change cost (responsiveness) in a multi-product supply chain with correlated demand and resource commonality.

### 1.3 Supply Chain Analytics

The two models developed in this dissertation can be integrated into supply chain analytics programs and reduce the dependence on post-ERP data. In general supply analytics consists of four basic analytical domains. These are: (i) Inventory Analysis, (ii) Procurement Analysis, (iii) Supplier Production Analysis, and (iv) Sales and Logistics Analysis. The first developed model is applicable in the inventory analysis category, while the second model is applicable in the supplier production analysis category. Brief definitions of each category (adapted from Logic Tools) are as follows:

1. *Inventory Analysis* - provides supply chain performance analytics centered around inventory issues, such as demand, ability to meet demand, inventory turns, inbound supplies, quantities on hand, and other key metrics. The business value of Inventory Management comes from both the ability to limit the direct costs of maintaining excess inventory as well as the direct and indirect costs of not meeting the just-in-time requirements of partners and OEMs.

2. *Supplier Production Analysis* - look at the critical issue of how well suppliers, individually and as a group, are performing in the supply chain. These analytics help maintain a critical component of the supply chain's value: supplier performance. This component can affect the bottom line – positively or negatively – of all stakeholders. Ensuring supplier performance helps maintain the overall value of the entire supply chain.
3. The common approach in supply chain analytics is to estimate performance metrics historical data. Many companies use ERP data to monitor, analyze, and report on their performance. This enables them to answer such questions as:
  - *What is the inventory distribution across the chain?*
  - *What is the inventory positioning strategy?*
  - *Which vendors are the most reliable? Are we carrying overstock?*
  - *Where are the bottlenecks in the procurement & distribution process?*
  - *What is the supply cost of each supplier?*
  - *What is the profit distribution in the chain?*

Supply Chain Analytics can answer all of these questions, and more. Supply Chain Analytics is an integrated set of reporting and analysis functionalities that draws critical insights from ERP data to give you better visibility into core supply chain processes. The problem though is that a significant amount of historical data is required and must be continuously extracted from the ERP system. This restricts the usability of this approach and in many cases is not amenable to prescriptive modeling. There is therefore a need for models that enables us to visualize the role and contribution of each

supplier in the chain, from the available system parameters. This will set the stage for forward analysis in supply chain design and analysis.

#### 1.4 Research Objectives

The overall objective of this research is to develop models for estimating (i) inventory costs and (ii) demand responsiveness costs in uncoordinated supply chains. The research was organized into the following four research objectives and their accompanying tasks. The first objective was undertaken as a prelude for establishing a modeling platform for the subsequent research, while the fourth objective focuses on numerical experiments with the two developed models.

*RO #1 – Develop a standard data model to capture the interrelationship between the production, replenishment, and transportation parameters in a supply chain.*

- TASK 1-1: Develop a standard view for defining and linking the required analytical data for supply chain analysis. This view is developed from a review of different data perspectives reported in the literature and the needs of the supply chain analytical models being developed in this research.
- TASK 1-2: Demonstrate the utility of the standard view in a tabular matrix format, so that data interrelationships between suppliers and within the same supplier are highlighted.

*RO #2 – Estimate inventory costs by supplier in a deterministic  $(Q, R, \delta)^2$  supply chain when the end demand is deterministic and uniform.*

- TASK 2-1: Determine and identify the number of significant supply cases in a  $(Q, R, \delta)^2$  supply chain. This is done by analyzing the inventory behavior under different supplier parametric relationships.
- TASK 2-2: Develop an inventory cost approximation for each case by capturing the inventory behavior for each supplier in that case. This will be an alternative to the current methods which are primarily dependent on historical inventory records.
- TASK 2-3: Develop a relationship between the approximated inventory costs, material costs and value-added costs and the contractual terms of the supply chain. These relationships are then used to determine when the inventory burden is not being distributed equitably among the suppliers. For example if  $R_{1A} \gg R_{2A}$  then supplier 1 is carrying most of the inventory burden associated with part A.

RO #3 – Estimate the inventory adjustment and production rate change costs (responsiveness) in a multi-product supply chain with correlated demand and resource commonality.

- TASK 3-1: Develop a measure for supply chain responsiveness as a function of (i) the inventory adjustment and (ii) capacity adjustment costs of each supplier.
- TASK 3-2: Extend the supply chain responsiveness measures to model demand correlations between products.
- TASK 3-3: Extend the supply chain responsiveness measures to model manufacturing resource commonality between products.

R0 #4 – Study the behavior of the  $(Q, R, \delta)^2$  inventory costs and responsiveness costs as a function of key supply chain parameters (e.g., reorder levels, batch sizes, and demand rate changes).

### 1.5 Significance of the Research

This research investigates inventory costs and responsiveness costs in coordinated supply chains. We propose a new model, labeled as  $(Q, R, \delta)^2$ , for describing the supplier relationships in such chains. In the  $(Q, R, \delta)^2$  extension we remove many of the restrictions in previous joint economic lot size (JELS) models. This permits us to represent many practical supply chain arrangements and makes it possible to estimate the true inventory costs of both the chain and each individual supplier. The model overcomes several problems that an exact model would experience. These include:

- The differing production rates of the suppliers become a key driver in the resulting inventory levels.
- While the demand for the chain may be continuous, the demand in the supply pairs is stop-and-go, that is, it is only positive while batch production is occurring.
- The demand at each supplier is dependent on the inventory policy of the downstream supplier and generates a lumpy demand.
- The inventory behavior tends to exhibit long cycles (intervals for the inventory pattern to repeat), except when the independent policies are perfectly synchronized.
- A large number of supply cases are possible based on the relationships between the  $(Q, R, \delta)^2$  parameters. Each of these shows a different inventory behavior.



The responsiveness cost model provides a reliable estimate of (i) the inventory adjustment and (ii) capacity adjustment costs of each supplier when demand conditions are shifting. Further, this model considers the issues of correlated demand and resource commonality. Classical supply chain analysis tends to consider a stochastic demand with uniform production, in which case models will attempt to focus on the inventory costs and service levels. In many case though the demand tends to be uniform at the production level with periodic shifts. Our approach then is on changing the production level so as to regularize the inventory level. This new model therefore provides details insights on the associate cost behavior.

## **1.6 Organization of the Dissertation**

This dissertation is organized into seven chapters. The organization is based partly on the four research objectives stated in section 1.4. Chapter Two provides a survey of the published literature about supply chain modeling. Chapter Three describes the visual structure and analytical possibilities of a nexus of supply chain entities. It then formulates a nexus problem for the case of stapler production. Chapter Four develops a methodology for performance analysis. Chapter Five covers the solution of the nexus problem that is formulated in Chapter Three. Chapter Six subjects nexus modeling to testing and analysis to evaluate its potential value to the field.

## **CHAPTER 2**

### **LITERATURE REVIEW**

Supply chain models have frequently focused upon five categories of information. The categories may be delineated as follows:

1. Productivity - concerns issues of resource usage and quality.
2. Inventory management – is important for asset management in general, and inventory position in particular.
3. Cost accumulation – is related to the question of profitability.
4. Lead time – affects the degree of responsiveness to customer requirements.
5. Demand – this determines value creation and bears upon both product mix and responsiveness.

The evolution of supply chain modeling reflects the influence of these factors. The survey that follows is a review of certain significant models that have been proposed. This survey reveals the progress that has been made so far and also the weaknesses in the approaches taken to date. In chapter 3 these five informational categories will be used as the basis for supply chain performance metrics.

#### **2.1 Supply Chain Research Areas**

A chain is a group of industrial facilities. What turns a mere group into a supply chain is that the facilities are interconnected. The purpose of connecting industrial facilities is to integrate the disparate operations of all facilities into a coherent, smoothly functioning whole.

There are ten areas of research in the supply chain field that have received extensive attention in the literature. They are as follows:

1. Risk and uncertainty in a supply chain
2. Supply chain optimization
3. Supply chain integration and communication
4. Inventory management policies
5. Design and implementation of a supply chain
6. Supply chain logistics
7. Use of queueing theory in supply chain analysis
8. Bullwhip effect
9. Supply chain flexibility
10. Supply chain system modeling.

The survey that follows is a brief review of the scholarly contributions that have been made in these ten areas. Each of the articles that is reviewed is devoted primarily to the subject area in which it has been placed. In some cases an article touches upon two or three of the ten categories. It should be noted that although the 10<sup>th</sup> area is the one labeled “modeling,” every article in all ten categories is concerned to some extent with what it refers to as a “model.” In section 2.2 an ambitious definition is asserted for the notion of a supply chain model. None of the proposed models in this section would meet the definition of section 2.2.

### **2.1.1 Risk and Uncertainty in a Supply Chain**

All supply chains have to make decisions about two inventory-related items: stock levels and order quantities. (This is also relevant to the discussion in section 2.1.10 about centralized vs. decentralized decisions).

The problem facing the chain is to provide acceptable delivery service at a reasonable cost. These decisions are made in an uncertain environment. There are two sources of uncertainty: customer demand and material supply.

The conventional way of representing uncertainty is with stochastic variables that are believed to follow a particular probability distribution (Beamon, 1998). The difficulty with this treatment of uncertainty is that it assumes the availability of probabilistic information. In case such information is unavailable an alternative method has been proposed. This alternative is based on the subjective judgment of experienced management (Petrovic, Roy and Petrovic, 1998). It involves fuzzy sets to represent customer demand and material supply. Stock levels and order quantities are calculated according to the rules of fuzzy arithmetic (Petrovic, Roy and Petrovic, 1999).

### **2.1.2 Supply Chain Optimization**

Optimization means deciding on the best utilization of the available resources (Escudero, Galindo, Garcia, Gomez, Sabau, 1999). It is important to optimize the supply chain structure under specific assumptions about production and transition. This is done in the hope that optimization will contribute to improvement in the chain's performance. Improvement in performance is measured according to stated criteria, such as cost

reduction, leadtime reduction, delivery promptness and waste elimination (Li and O'Brien, 1999).

A number of mathematical programming models have been proposed for this purpose (Escudero, Galindo, Garcia, Gomez and Sabau, 1999; Timpe and Kallrath, 2000; Lakhal, Martel, Kettani and Oral, 2001). Each of these models is based on a detailed method of representing the available resources. The uncertainties of resource availability are represented in deterministic form. Heuristics have been presented by which to obtain a solution for each model. The solution is calculated given specific assumptions about production, cost and value accumulation.

A dynamic program has been developed to minimize total supply chain cost (Graves and Willems, 2005). The dynamic approach is to choose suppliers, parts, processes and transportation modes from available options. Choices are made at each stage of the chain and from these choices a supply chain configuration is built. The program is applied specifically to supply chains configured as spanning trees.

An important aspect of supply chain cost minimization is the joint economic lot size (JELS). The JELS is the optimal choice at a specific stage of the chain, i.e. the order quantity that results in the lowest joint relevant cost for the supply pair at a specific stage. Hill (1997) tentatively proposes that the optimal order quantities for shipment between successive suppliers are found by applying a fixed factor of between 1 and the ratio of production rate to demand rate, with changes to that factor depending on the parametric relationship between each supply pair. Hill (1999) then derives a policy for successive shipment sizes that is optimal under certain conditions. This policy consists of a number of shipments that increase in size by the ratio of production rate to demand rate, followed

by a number of shipments that are equal in size. We will return to the concept of JELS in section 4.2.

### **2.1.3 Supply Chain Integration and Communication**

Integration means cooperation. Communication means exchange of information. In a supply chain integration involves cooperation of all facilities. This cooperation is achieved by the exchange of information between supply chain facilities.

Several issues related to integration and communication have been explored in the literature. First, there are certain questions relevant to the integration of a manufacturer's production schedule with its suppliers: (1) What are the cost implications of different levels of supply chain integration? (2) Does the increase in forecast effectiveness mitigate some of the shortcomings of a lower degree of supply chain integration? Wei and Krajewski (2000) discuss these questions.

Second, aggregate production decisions must be made to satisfy customer requirements. After assuring customer requirements, prices must be set so as to satisfy the dual objectives of profit realization and cost reduction. Barbarosoglu (2000) proposed the use of mathematical programming models to achieve these goals in accordance with the terms of customer contracts. The main emphasis is placed upon the conceptual and negotiation aspects of the models, and some solution procedures are cited from previous studies.

The use of contracts to achieve supply chain integration is also discussed by Cachon and Lariviere (2005). They demonstrate the benefit of using revenue-sharing contracts to allocate profit between any two supply chain levels (e.g., distributor-retailer).

The retailer decides optimal quantities and prices and the resulting profit is divided by the two contracting entities.

#### **2.1.4 Inventory Management Policies**

Practical inventory systems maintain stock records and order stock. This is the task of determining “when and how much.” The importance of inventory systems goes beyond this, however. They provide management with information on shortages, movements and financial considerations (Bonney, Head, Tien, Huang, Barson, 1996). These financial considerations are associated with inventory holding costs and inventory lead time. The length of the lead time affects investment in safety stock and customer service level. Pan and Yang (2002) show how separately owned entities can improve integration of inventory management by treating lead time as controllable.

A major issue in inventory management is the coordination of inventory policies adopted by suppliers, manufacturers and distributors. The purposes of this coordination are to smooth material flows and minimize costs while responsively meeting customer demand. Giannoccaro and Pontrandolfo (2002) present a mathematical approach to managing inventory decisions at all stages of the supply chain. Their approach allows an inventory order policy to be determined, the aim of which is to optimize the performance of the supply chain as a whole. Gjerdrum, Shah and Papageorgiou (2002) present a mathematical programming formulation that likewise seeks to determine inventory policies that optimize the performance of the supply chain as a whole. However, the aim of their model is to ensure adequate rewards for each supply chain participant.

### 2.1.5 Design and Implementation of Supply Chain

It has been said that supply chains are complex combinations of “man” and “machine” and are usually difficult to design (Hafeez, Griffiths, Griffiths and Naim, 1996). These four writers argue that systems engineering can be used as an effective tool for this purpose. Systems engineering takes into account intricacies associated with modeling the attitudinal, organizational and technological issues. In their paper they use an integrated system dynamics framework as an example of good total system design. The most important objective of the design is to move towards a minimum reasonable inventory scenario in the presence of capacity constraints, breakdowns and material supply lead-time bottlenecks.

The production-distribution system design problem involves decisions concerning the structure of a firm’s supply chain. Most of the literature uses mixed integer programming formulations to represent such facility design decisions. Dasci and Verter (2001) use an alternative modeling framework, one that uses continuous functions to represent spatial distributions of cost and customer demand. The proposed continuous model allows the derivation of a number of insights about the impact of problem parameters on facility design decisions.

Persson and Olhager (2002) have done a case study of supply chain simulation in the manufacture of mobile communication systems. The study has two objectives. First, they evaluate alternative supply chain designs with respect to three performance parameters: quality, lead-times and costs. Second, they enhance the understanding of the interrelationships among all parameters relevant to the design of the supply chain structure. The design alternatives differ in terms of the level of integration and



synchronization between supply chain stages. A model capturing the relationships among cost, quality and lead-times is presented based on the simulation study.

### **2.1.6 Supply Chain Logistics**

Jayaraman and Pirkul (2001) created an integrated model for locating production and distribution facilities in a multi-echelon environment. Designing such logistics systems requires two decisions, one strategic (location of plants and warehouses) and the other operational (distribution from plants to customer outlets through warehouses). Jayaraman and Pirkul provide a mixed integer programming formulation to the integrated model. They then present an efficient heuristic procedure that utilizes the solution generated from a Lagrangian relaxation of the problem. They use this heuristic procedure to evaluate the performance of the model.

A major cost element in the logistics of distributed warehousing is transportation cost. Vroblefski, Ramesh and Zionts (2000) consider two-level differential transportation cost structures and multilevel cost structures. Their objective is to determine the ordering lot size for each warehouse. Minner (2001) added to the traditional literature about the duty for manufacturers to take back used products from customers. They remind us that returned products might have a positive economic value. Minner's objective is to combine the problem of safety stock planning in a general supply chain with the integration of external and internal product return and reuse.

### **2.1.7 Use of Queueing Theory in Supply Chain Analysis**

Raghavan and Viswanadham (2001) present analytical models for evaluating the average lead times of make-to-order supply chains. In particular, they illustrate the use of generalized queueing networks to compute the mean and variance of the lead time. They give four examples and develop queueing models for each example. The first two examples compute the variance of lead time using queueing network approximations available in the literature. This analysis indicates that for the same percentage increase in variance, an increase at the downstream facility has a far more disastrous effect than the same increase at an upstream facility. In the third example they show that coordinated improvements at all the facilities is important, and improvements at individual facilities may not always lead to improvements in the supply chain performance. Their final example is an easy to use method to evaluate logistics decisions, e.g. who should own the logistics fleet – the manufacturer or the vendor?

### **2.1.8 Bullwhip Effect**

Two recent articles by the same four authors analyze the importance of supply chain replenishment rules. Dejonckheere, Disney, Lambrecht and Towill (2003) compare a traditional chain, in which only the first stage observes end consumer demand and upstream stages have to base their forecasts on incoming orders, with an information-enriched chain where customer demand data is shared throughout the chain. Two types of replenishment rules are analyzed: order-up-to policies (expressed as  $(s, S)$ ) and smoothing policies (policies used to reduce demand variability). For order-up-to policies information sharing helps to reduce the bullwhip effect significantly, especially at higher

levels of the chain. For smoothing policies information sharing is necessary to reduce order variance at higher levels of the chain. However, Chen and Samroengraja (2004) studied two different retail-level replenishment policies, and show that a policy that decreases order volatility does not necessarily decrease total supply chain cost.

A different approach to reducing the bullwhip effect is taken by Gilbert (2005). He uses ARIMA time-series models of demand and lead times to derive ARIMA time-series models of orders and inventory. Gilbert presents formulas for measuring the bullwhip effect upon orders and inventory. The effect is demonstrated to be large in the case of autocorrelated demand and long lead times.

Thonemann (2002) analyzes how the sharing of advance demand information (ADI) can improve supply chain performance but also increase the bullwhip effect. This means that both manufacturer and customers benefit from sharing ADI even though such sharing increases the variability of order quantities at upstream facilities. Thonemann considers two types of ADI. With aggregated ADI customers share with manufacturers information about whether they will place an order for some product in the next time period, but do not share information about which product they will order and which of several potential manufacturers will receive the order. With detailed ADI customers additionally share information about which product they will order, but which manufacturer will receive the order remains uncertain. Thonemann develops and solves mathematical models of supply chains where ADI is shared. He shows that under certain conditions it is optimal to collect ADI from either none or all of the customers.

Dejonckheere, Disney, Lambrecht and Towill (2003) introduce a general decision rule that avoids upstream variance amplification and generates smooth ordering patterns,

even when demand has to be forecasted. The magnitude of the bullwhip effect is found by Daganzo (2004) to be influenced by the inventory control policy much more than by the demand conditions. He derives a formula for the upper bound of the order variance in decentralized, multistage supply chains. For such chains he also derives a condition for the avoidance of the bullwhip effect that is independent of demand.

### **2.1.9 Supply Chain Flexibility**

Timely and accurate information flow within supply chains is a critical aspect of the success of the entities in a chain. D'Amours, Montreuil, Lefrancois and Soumis (1999) express information within a manufacturing network in terms of price-time alternatives. Order scheduling within the network is based on a price-time evaluation of the bids submitted by competing entities. Relationships between supplying firms and demanding firms are characterized by different levels of shared information about price and capacity. The impact of information sharing on networked manufacturing is illustrated using three different kinds of bidding protocol. These protocols reflect the flexibility with which entities can aggregate their information to conform to networking requirements.

Another aspect of supply chain success is volume-flexibility. The volume-flexibility of a manufacturing system is its ability to be operated profitably at different output levels. Both information flow and volume-flexibility can be improved by investing in relevant technologies. Volume-flexibility attempts to mitigate the effects of demand fluctuations and forecast errors by investing in manufacturing technology. The need for volume-flexibility is reduced by improving the information flow within the supply chain, which makes possible earlier detection of changes in demand, and this is

achieved by investing in information technology. Khouja and Kumar (2002) developed a production planning model that can be used to evaluate the effects of improvements in information flow resulting from investments in manufacturing technology and information technology.

Das and Abdel-Malek (2003) define flexibility as the willingness of a supplier to maintain the buyer-supplier relationship in the context of market uncertainty. They use the buyer-supplier relationship to model the durability of that relationship in the face of changing demand. Demand change leads to pressure to increase or decrease the unit supply price. Das and Abdel-Malek derive a measure of supply chain flexibility in terms of a supplier's willingness to adjust order quantities and lead times without insisting on adjustments in price.

Production flexibility and scheduling flexibility are studied by Milner and Kouvelis (2005). They studied the impact of three different demand characteristics upon the strategic value of these two kinds of flexibility. The value of each type of flexibility is shown to be greatest depending on the type of demand for one of three categories of products. Flexibility is discussed in this article explicitly in the context of supply chains.

Abdel-Malek, Areeratchakul and Otegbeye (2006) take an approach that recognizes fifteen separate classes of flexibility. Their explicit concern is with the flexibility of design of manufacturing systems, although they assert that their modeling strategy may be used to design flexibility for supply chain structures. This strategy is to design a manufacturing system based on the degree of flexibility appropriate to that design. They then apply the newsvendor type of model to minimize the costs incurred because of either excessive flexibility or insufficient flexibility.

### 2.1.10 Supply Chain Modeling

A wide variety of models have been proposed to describe the operations and design of a supply chain. The proposed models are based on differing perspectives about the management, structure and performance of a supply chain (Silver, Pyke and Peterson, 1998):

1. Centralized decision-making vs. decentralized decision-making. Centralized decision-making is known as the enterprise (or systems) approach to supply chain modeling. Decentralized decision-making is called the decomposition approach. The type of decision-making policy established in a supply chain is often referred to as inventory control. It involves decisions made in response to information about demand, lead time, quality, inventory status and cost.

There are two decisions to be made:

- Lot-sizing decisions – concerned with replenishment quantity
- Stocking decisions – concerned with relative order frequency

These decisions can be made from either a centralized or decentralized perspective. That is, the decisions can be controlled by managers responsible for either the supply chain enterprise taken as a whole, or by managers responsible only for echelon-level operations.

2. Local information vs. global information. The information that is important to a supply chain is about demand, costs and inventory status. Local information is that which is seen at a facility regarding its own costs, inventory status, and orders from the next downstream facility. The existence of local information implies that the

decision maker is decentralized. Global information is about the demand, costs and inventory status as seen at all facilities in the system. If information is global it may be seen by either a centralized or decentralized decision maker.

3. Discrete-time information vs. continuous-time information. Again, the information of interest is about demand, costs and inventory status. This information may be represented either with reference to a mathematically discrete number line or a continuous numerical continuum. This information is used to characterize lot-sizing decisions and stocking decisions.
4. Serial structure vs. network structure. The simplest supply chains are structured in the form of a series. They are pictured in a linear manner (see section 3.1.1). More complex supply chains are web-like in appearance. This is because of the interlocking nature of their supply relationships (see section 3.1.2). Both types of policies for making decisions and managing information, as discussed above, have each been illustrated in the literature by a serial structure and by a network structure.
5. Deterministic parameters vs. stochastic parameters. Deterministic models assume that the parameters of a model are known and can be specified with certainty. In stochastic models the parameters are taken to be unknown and the variables are assumed to follow a probability distribution. The parameters that are either deterministic or stochastic include demand, lead time, production rate and cost.
6. Arborescent type network structure vs. assembly type network structure. In an arborescent network each node in the network has at most one immediate predecessor but any number of immediate successors (Lee & Billington, 1993). An assembly type

network is where each node has at most one immediate successor but any number of immediate predecessors (Beamon, 1998).

7. Activity chain modeling vs. event process chain modeling. Activity chain modeling is concerned with optimizing the use of resources. The aim of resource optimization is cost minimization. Event process modeling is concerned with process improvement. Its aim is lead time reduction (Trienekens and Hvolby, 2001).

A supply chain has been described as a forward flow of materials and a backward flow of information (Min & Zhou, 2002; Beamon, 1998). A variety of models have been proposed to describe these flows. Again, the differences among these model proposals are due to the many choices of perspectives available to model developers. Each model is based on its developer's choices of which perspectives to emphasize. In the review that follows all models have in common the goal of describing the flows of materials and information in a supply chain. None of these models is broad enough to comprehensively describe all of the ten areas in this section. (This point is expanded in section 2.2).

The earliest models to be proposed in the relevant literature were based on centralized decision-making and concentrated on serial modeling. Following this perspective, Hanssmann (1959) was the first writer to propose a scheme to describe the three supply chain operations – procurement, transformation and distribution. He used sales revenue minus inventory costs as his performance measure. Hanssmann's focus is on management of inventory levels so as “to match supply and demand ... in the most economical way...” The emphasis is on minimizing the costs of holding excess inventory and the costs of an inventory stockout. Hanssmann's model is capable of “balancing” inventory holding costs and the revenues generated by those costs in a multi-



echelon system. It does not consider interdependencies between lead time realities and lot-sizing decisions.

The next major contribution to multi-echelon inventory theory was made in a seminal paper by Clark and Scarf (1960). They were interested in finding a way to describe the process of inventory valuation. The approach that Clark and Scarf followed used centralized control over a serial system for the case of stochastic demand and discrete time. They originated definitions for the concepts echelon stock and installation stock. With these concepts they were able to propose a method of value accumulation that avoided double-counting of inventory within a supply chain. The method consists of doing inventory valuation at a given echelon only in terms of the value added at that echelon.

The scenario studied by Clark and Scarf assumed only a single product. Schmidt and Nahmias (1985) extended the single product example to an assembly of two components. In this situation they analyzed a three-stage system: two production stages and one assembly stage. They succeeded in describing the optimal inventory policy for the two components and the end product.

Sherbrooke (1968) was concerned with several series of activities conducted in parallel. His method is called a decomposition approach. In this system supplies of goods are provided by a depot to a set of parallel bases. Sherbrooke assumed that an entity would follow an  $(S-1, S)$  inventory control policy at the base level. His model is used to find the optimal solution for  $S$ . Simon (1971) extended this method to the case of an  $(s, S)$  inventory policy.

Muckstadt and Thomas (1980) also restricted themselves to serial modeling. They applied centralized control to both global information and local information. Their model involves calculation of inventory position at each echelon and at each facility, rather than calculation of overall inventory position throughout the chain. This is another example of the decomposition approach. The performance measure used is the service level, expressed as the fill rate, and is calculated at each echelon and each facility.

DeBodt and Graves (1985) were also interested in multiechelon control procedures. Their model is based on global availability of information and centralized decision-making. They modeled control decisions at each echelon by a simple formula:  $Q_g = n Q_{g-1}$ . This formulation is used in making lot-sizing decisions and stocking decisions. It is based on echelon stock rather than installation stock, hence inventory information is global and control is centralized.

Cohen and Lee (1988), like Muckstadt and Thomas, followed a decomposition approach. They proposed a scheme for optimizing the supply chain as a series of four stochastic submodels: material control, production control, finished goods, and distribution. They also used fill rate as the measure of performance. In their model fill rates are local service targets that serve as linkages between the four submodels. The manufacturing lead time in the production submodule connects to the replenishment lead time in the distribution submodule.

Another interesting decomposition has been proposed by Svoronos and Zipkin (1988) that is applicable in the case of a supply chain with an arborescent structure. Such a structure is found, for example, where there is a warehouse that supplies inventory to several retailers. They suggested a procedure that requires each retailer to determine its

inventory policies independently. An order from retailer to wholesaler indicates that another such order is unlikely very soon. This then requires the wholesaler to determine its own inventory policy. From the warehouse's inventory policy it is possible to compute the retailer's lead time (delivery time plus delay time if the warehouse has no stock). The retailer's lead time becomes the basis for updating the retailers' inventory policies.

As noted above, Cohen and Lee used service targets as links between submodels (or stages). A similar method of establishing linkage, proposed by Lee and Billington (1993), is to treat supply and demand as stochastic quantities. They suggest computing the mean and variance of the replenishment lead time in a system with decentralized control. Then the mean and variance of demand for each SKU can be computed. This in theory makes possible the calculation of supply uncertainty. Finally, fill rates are used as the service targets of the upstream stage that replenishes the downstream stage.

A collection of deterministic models was offered by Williams (1981). In this article he gives seven heuristic algorithms to determine a production and/or distribution schedule that satisfies final product demand. The same objective function is employed for each of the seven algorithms and serves to minimize the sum of average inventory holding costs and average fixed processing costs. The results of empirical experiments are reported on the basis of several performance measures.

Zipkin (1995) showed further that it is possible for average warehouse inventory level to increase or decrease with little change in cost. This happens if retailers optimize their inventory policies based on the warehouse's average inventory level. Zipkin also

established that centralized decision-making requires less warehouse inventory than in the case of decentralized decision-making.

Extensions of serial models to network structures have proved to be challenging. Federgruen and Zipkin (1984) created a structure for supply chain optimization in the case of centralized control. Their results apply to a network that is of the arborescent type but not the assembly type. Thus, their model fits the transition activities of a supply chain but not the production activities.

The problem of extending a simple serial system to a more complicated network system was also addressed by Rosling (1989). Rosling's results are of limited applicability because he only considered the case of a single end product. With this restriction he was able to reduce a network structure to a serial structure. The modeling of a single end product subject to random demand is considered by Yang (2004) for the situation in which the raw material supply is also random. Yang describes optimal policies for raw material inventory and end product inventory for the separate cases of convex and linear purchase costs and selling prices.

## **2.2 Need for Comprehensive System Models**

Supply chains are not really new. There have been interconnected groups of productive facilities for hundreds of years. What distinguishes today's supply chains from past interconnected facilities is the emphasis on linkage. The connections between echelons in years past were often informal and poorly structured. The linkage between echelons in the supply chains of today is increasingly formal and well-structured.

This increased emphasis on linkage is due at least in part to the desire for a comprehensive model that represents all aspects of a supply chain. The models that have been proposed during the past four decades have tended to concentrate on only a few of the ten topical areas discussed in section 2.1. This is clear from the review of proposed models in subsection 2.1.10. It is obvious from the review that inventory management, in particular, has received a great deal of scholarly attention. However, after four decades of research and publishing there is still a need for a model that is descriptive enough and broad enough to encompass all aspects of a supply chain.

The approach that is needed may be called the systems view or enterprise view. The systems view is important because of the need within a supply chain to share information, technology and risk. To describe this view there is a need for a systems model. A systems model is defined to be a proposal that encompasses every area of supply chain operations and design.

The systems view helps all supply chain participants to appreciate the benefits of cooperation between facilities, as opposed to competition between independently owned entities. Cooperation leads to coordination of inter-echelon activities, whereas competition discourages cooperation in favor of self-interested actions. The potential benefits from cooperation include greater efficiency, enhanced productivity and improved quality.

### **2.3 Current Interest in Supply Chains**

There are two reasons for the current surge of interest in supply chain modeling:

1. The ability to transmit information instantaneously. This makes possible enhanced efficiency within the chain.
2. The ability to ship a fraction of a truckload, e.g. a box or pallet, efficiently. This mode of transportation has been called the Less than TruckLoad mode (Muriel, 1997). It permits greater flexibility within the chain than was possible when lot sizes were measured by a full truckload.

## CHAPTER 3

### A DESCRIPTIVE MODELING FRAMEWORK FOR SUPPLY CHAINS

The focus of this chapter is on our research objective #1: – Develop a standard data model to capture the interrelationship between the production, replenishment, and transportation parameters in a supply chain. The motivation for this objective stems from a need to have a model that enables us to visualize the role and contribution of each supplier in the chain, and how each of them is affecting the performance metrics associated with the chain. As an example say we query a supply chain specialist at Best Buy for the supply network of a DVD player. The expected response would be a flowchart chart which identifies: (i) a manufacturer in China, (ii) an import warehouse in Los Angeles, (iii) several regional warehouses, and (iv) the point of sale store. Clearly, this flowchart would be insufficient for conducting any analysis of the supply chain dynamics or behavior.

There is therefore a need for descriptive models that enable us to evaluate and analyze key performance metrics for each supplier in a multi-stage supply chain. Such a model would enable us to generate supply chain analytics results from the system parameters. This would be a significant improvement on the current approach of using post-ERP data to generate performance data. These performance metrics/analytics would include the profit/cost distribution, inventory positioning, production and inventory risk, and demand change responsiveness. The cost and performance dynamics in supply chains are dependent on a large number of parameters, and include a sequential relationship between these parameters. As a result it is necessary that any descriptive

model facilitate the analytical visualization of the supply chain, through matrices and/or stage wise interpretations. A descriptive model which combines these analytical and visualization capabilities will lead to more efficient and productive supply chains. Additionally, a common platform for the development of several prescriptive supply chain models will be available, and would replace the need for simulation based models to predict certain supply chain performance metrics.

### **3.1 Introduction to the Network Model of a Supply Chain**

NEXUS is a modeling format which captures and visually displays the supply chain dynamics and the interrelationship between suppliers. It is a descriptive model that quantitatively equates the relationships and interactions between sequential stages in a supply chain. Though the NEXUS model is intended to be descriptive in nature, it serves as a platform for a wide range of system-wide prescriptive analysis models. A nexus describes the connections between transformational facilities (suppliers). A nexus also indicates the transitional facilities that are needed to make those connections. One advantage of a nexus is the degree of detail it reveals about transactions within a supply chain. This detail is very helpful in the analysis of the chain's performance.

We develop here a 3-level modeling framework for the analysis of supply chains. The first is the "Macro Level" and this provides the network basis of the supply chain, while the second level is the "Micro Level" and this provides the supply chain-related details for each player in the supply chain. The third level is the "Analytics Level" which provides estimates of key performance metrics for the supply chain.



The interconnected facilities that define a supply chain (see beginning of chapter 1) may be described in mathematical terms as a network model or graph. Every node or unit within this network model corresponds to a facility (i.e., a supplier) in the supply chain. We assume each of the units within such a network model is a self-contained unit model of a single supplier or facility. In section 1.2 the unit model was introduced as being representative of a single supplier. The unit model was shown in that section to be the building block of the network model. As such the unit model is actually a sub-model. In this chapter the concept of a unit model is greatly enhanced. The unit model is employed as a means of subdividing a typical supply chain into manageable analytical components.

After creating a single unit model, relationships are established with other unit models to form a detailed network. The purposes of this network are to describe the operations of both individual suppliers and the supply chain in the aggregate. The unit models of the network that represent the facilities are mathematically interconnected, while the facilities are the stages of a supply chain and are of two types: (i) Transformational stages. (ii) Transitional stages. A transformational stage is a facility that converts input from a preceding stage into output for a succeeding stage. A transitional stage is a facility that connects two or more transformational stages. Where a transitional facility and transformational facility are directly connected they constitute a supply chain echelon.

The connections between transformational stages are necessary to allow transactions to occur between them. A mathematical model of interconnected stages is essentially a network of inter-stage transactions. These transactions are the “three

primary flows” mentioned at the beginning of chapter 1. The inter-stage transactional flows within a supply chain network may be defined more precisely as follows:

1. *Material flows* – transfers of inventory out of a transformational stage by a transitional stage to a succeeding transformational stage. Material flows are usually unidirectional (the exceptions being flows of materials returned, lost or destroyed). The direction of material flows is sometimes called “downstream.”
2. *Financial flows* – transfers of money out of a transformational stage to a preceding transitional stage or to a preceding transformational stage. Financial flows are usually unidirectional (the exceptions being flows of money to a succeeding transitional stage). The direction of financial flows is sometimes called “upstream.”
3. *Informational flows* – transfers of data about material flows and financial flows. Informational flows are bidirectional, because data about material flows are needed by succeeding stages, and data about financial flows are needed by preceding stages.

The NEXUS supply chain model we develop here focuses primarily on the first of these flows. Significant technological improvements have resulted in the availability of efficient information systems that meet the needs of the third flow. Every facility in the chain that contributes to the flow of inventory or information is motivated by an opportunity to benefit from the flow of money. The network model describes a facility’s contributions of inventory and information to other stages of the supply chain, and the monetary reward that is received from other stages in return for these contributions. The network model also describes the risk that is incurred as a result of these contributions.

Inventory, money and information all flow as required between the stages of any active supply chain. These flows are the tangible manifestations of the interdependencies

between suppliers. In fact it is the need to satisfy these interdependencies that cause suppliers to buy and sell from each other in the first place. These inter-stage transactions are modeled in terms of the interrelationships between suppliers. The interrelationships between suppliers within a supply chain are a crucial aspect of the modeling perspective that is proposed in this chapter. The inter-supplier perspective is developed in detail in section 3.4.

Interrelationships that are internal to a supplier are just as important. These interrelationships are the analytical components of each supplier. The interrelationships between the distinct parts of a facility are intra-stage transactions. The unit model displays the interrelationships between the parts of a facility in a compelling visual manner. The intra-supplier perspective is developed in detail in sections 3.2 and 3.3.

The need for a unit model arises from the conception of a supply chain as consisting of three physical entities:

1. Parts and/or products – may be categorized as follows:
  - (a) Procured parts – are obtained from outside the supply chain.
  - (b) Intermediate parts and/or products – are shipped as outputs by an upstream supplier and obtained as inputs by a downstream supplier.
  - (c) Final parts – are sold as outputs to a customer, an end user or another supply chain.
2. Transformational suppliers – converts procured parts and/or intermediate parts into intermediate products or final products. The procured and/or intermediate part(s) are the inputs for this type of supplier. The intermediate products and/or final products

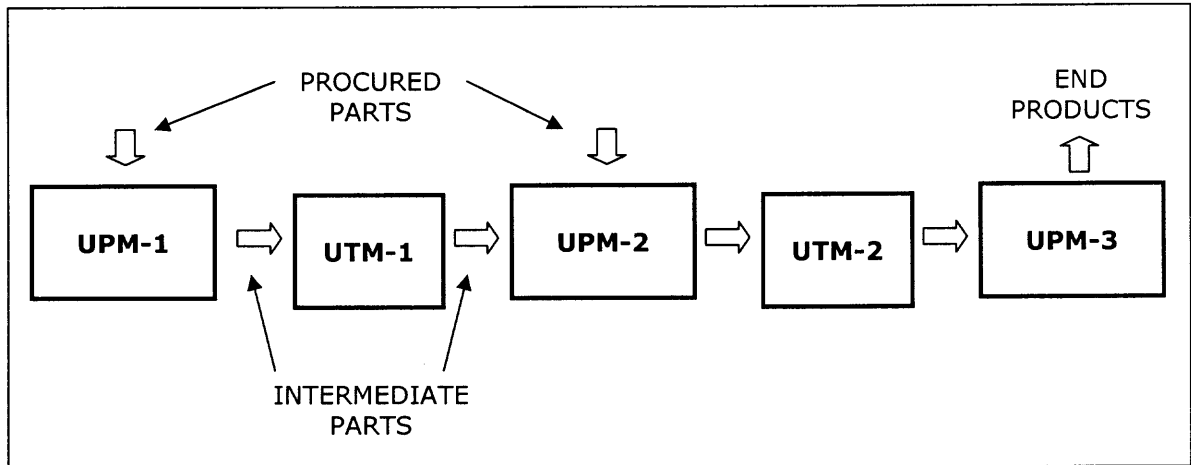
are the supplier's output(s). The conversion of the inputs into outputs represent the value-adding activity of the supplier.

3. Logistics suppliers – transports the outputs of an upstream supplier and delivers them as inputs to a downstream supplier.

NEXUS decomposes a supply chain into a series of Unit Process Models (UPMs) and Unit Transfer Models (UTMs), each of which are described as follows:

- The UPM model type will be used to represent the facilities at transformational stages. A UPM facility converts input from a preceding stage into output for a succeeding stage. UPM's typically involve multiple value-adding activities. Products can be inventoried in UPM facilities. Examples of UPM facilities are factories, warehouses, cross-docking facilities and retail stores.
- The UTM model type will be used to represent facilities at transitional stages. A UTM facility is concerned with movement of products between UPM facilities. As such it is engaged only in product transfer rather than value-adding activities.

Figure 3.1 shows the elementary NEXUS model for a supply chain. In this we identify the suppliers (UPMs), and the sequential relationship between them. For each supply pair we identify the associated logistics supplier (UTM). A supply pair is defined as 2 suppliers for whom a part is shipped between two UPMs. Figure 3.1 shows an elementary case in which there are 3 UPMs and 2 UTMs. Often we see that the number of UTMs is greater than the number of UPMs.



**Figure 3.1** Relationship Between UPMs and UTMs.

In our analysis we assume that a group of suppliers are members of a supply chain. According to the definition of a supply chain, the production and supply behavior of these members is governed by one or more supply contracts (Das and Abdel-Malek, 2003). These members share capital risk and market risk. Where the supply is basically a regular purchasing relationship then that supplier is not modeled as a member.

To build the macro level NEXUS model the following data is required:

- List of transformational suppliers who are members of the chain
- List of logistics suppliers who are members of the chain
- List of final or end products from the chain that are delivered to the customer
- List of intermediate parts that are shipped between suppliers
- List of procured parts and materials that are sourced from non-member suppliers

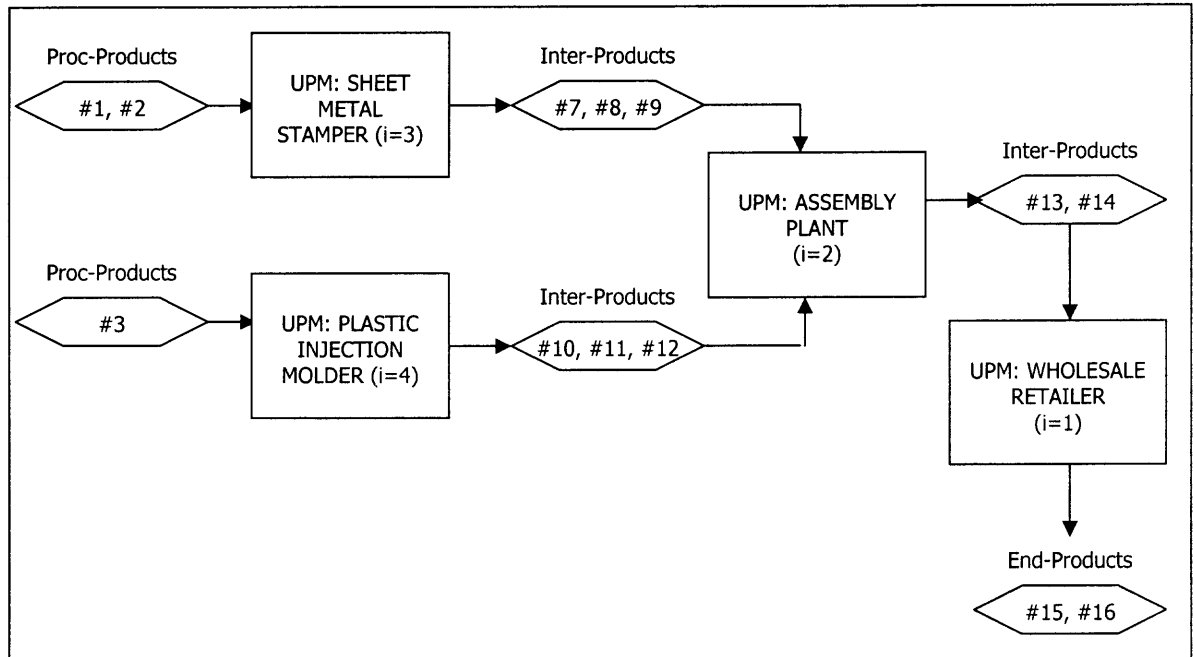
The data requirements are represented by notation as follows:

- $i$        $1, \dots, N$ , suppliers who are chain members
- $j$        $1, \dots, M$ , all parts and materials that are included in the analysis
- $O_{ij}$     Set equal to 1 if product  $j$  is an output from supplier  $i$  else is 0
- $I_{ij}$     Set equal to 1 if product  $j$  is an input to supplier  $i$  else is 0

$\beta_j$  Product type: 1 if end, 2 if intermediate, and 3 if procured product

In selecting the part set M, only primary elements that determine the dynamics of the supply chain need to be considered. For instance, common BOM items such as fasteners and washers are not to be included. Note that all parts, except the final products, will be an input to at least one supplier. This data is sufficient to generate the macro level NEXUS supply chain model.

Figure 3.2 shows the macro level model for the stapler example introduced later in section 6.3. The diamonds identify the flow of different part/products in the chain, and help us visualize the transformation process. Note that a UPM could have both procured and intermediate products as inputs, and could also have both intermediate and end products as outputs. Later, as we develop micro level descriptive and prescriptive models, a variety of macro level metrics will be added to this model. For example each UPM box will include the cost and profit assigned to that supplier for a unit of end product sale.



**Figure 3.2** An Example Macro Level NEXUS Model.

The macro level model is a visualization of the multi-echelon supplier system that makes up the chain. The model provides us with a structure to build the micro level UPM and UTM models. In the next two sections the concept of a UPM and UTM are further developed. In section 3.4 the integration of both types of entities will be discussed. This will involve the proposal of performance measures for the supply chain as a whole.

### 3.2 Unit Process Model (UPM)

We develop the micro level UPMs and UTMs in a matrix format, with assigned standard inputs and outputs. This makes them amenable to network analysis. These representations are then used to analyze system metrics such as (i) profit and cost (ii) inventory risk (iii) capital risk and (iv) demand responsiveness. In this section the unit

model concept is applied to transformational stages. The activities of a transformational stage may be categorized as follows:

1. Value-adding processes (manufacturing)
2. Storage of material for a significant time (warehousing)
3. Change in transport carrier (cross-docking)
4. Sale of goods (retail point-of-sale)

A unit model for a transformational stage will be referred to as a unit process model (UPM) because it is concerned with describing one of these four processes: manufacturing, storage, transporter change or sale. The UPM is defined by a set of inputs. Inputs represent all trackable items which enter a particular facility. Outputs represent all trackable items which leave that stage and go to another stage or to the final customer. The UPM is structured to describe input-output behavior within the context of overall supply chain requirements. This input-output behavior is the essence of a supplier's contributions to its chain. With this description of supplier behavior it is possible to evaluate the supplier's contribution to the chain's overall performance.

### **3.2.1 UPM Structure and Content**

A unit process model consists of three distinct but interrelated sections. Each section contains its own data set. The three sections of a UPM are:

1. Obligations established by a supply contract. A specific supplier's contractual obligations are stipulated by supply chain executives who are responsible for the overall performance of the supply chain, not the supplier's performance.



2. Operations controlled by a specific supplier. The supplier's internal operating parameters are controlled by the supplier's own executives.
3. Metrics by which to evaluate a specific supplier's contribution to overall supply chain performance.

The data set for the contractual section is presented in tabular form. The data set for the specific supplier section is presented as an input-output matrix. The elements of these three data sets are the numerical attributes of a transformational supplier. The attributes of each supplier's input-output behavior are represented by parameters that are selected to model supply chain performance. The assignment of parameters in a UPM type of model makes possible the mathematical evaluation of each supplier's input-output behavior.

### **3.2.2 Transformational Supplier Attributes**

There are three categories of supplier attributes that are modeled in the NEXUS supply chain system: *(i)* input part/material attributes, *(ii)* production attributes, and *(iii)* output part attributes.

Input part/material attributes include the following:

- Supply batch size – number of units of materials ordered per shipment from a supplier.
- Supply reorder level – number of units of materials maintained as safety stock in case of inventory shortage.
- Supply order cost – cost of processing an order to replenish a depleted supply batch.

- Unit purchase cost – number of dollars charged per unit by supplier for a specific supply material.
- Order variation – change per year in number of units of input ordered by a supply chain entity from a supplier.

Production attributes include the following:

- Unit holding cost
- Unit backorder cost
- Assembly resource cost – number of dollars needed to pay for activity to produce a single unit of output per unit time.
- Assembly time – number of hours needed to produce a single unit of output.
- Labor cost per unit time
- Labor utilization rate
- Production batch size - number of units of output produced in one production run.
- Bill of materials relationships – how many units of each type of material input are needed at a given stage to produce one unit of output.
- Response time – number of minutes needed to respond to demand change by customer.
- Lead time to change production rate
- Response cost – cost to adjust production rate in response to demand change.

Output part attributes include the following:

- Bill of materials explosion quantity – how many units of each type of material input are needed at all stages to produce one unit of final product.

- Demand reorder level – number of units of output maintained as safety stock in case production lead time decreases.
- Demand rate for output
- Unit sale price – number of dollars charged for one unit of output by a succeeding supplier.
- Production rate – number of units of output produced per unit time.

In chapters 4 and 5 these attributes will be treated as parameter values and will be used to develop performance metrics by which that supplier's operations will be evaluated. Most of the notation for the two data sets will be presented in chapters 4 and 5 as needed. Additional notation is defined in subsection 3.2.4 and will be used in chapter 4.

### **3.2.3 UPM Performance Metrics**

The following performance measures are proposed:

- Total inventory – purchase cost per unit times number of units of material, or work in process, in demand by the next supplier (but not yet transferred).
- Target inventory (see figure 5.4 and subsection 5.3.1)
- Inventory risk – number of units of inventory on hand above target level (surplus), or number of units of inventory on hand below target level (shortage).
- Profit share (see table 6.3)
- Profit ratio equilibrium (developed in subsection 4.5.4)
- Profit ratio – number of dollars of profit earned by a supplier in one year divided by number of dollars of sales earned in the same year.

- Demand change response efficiency (developed in chapter 5)
- Capital risk – number of dollars of profit earned by a facility in one year divided by cost of investment in inventory costs and value added cost.

#### **3.2.4 Standard UPM Format**

To develop the UPM format we first reviewed the data requirements of various supply chain analytical models. These are for the most part referenced and reviewed in chapter 2. We also reviewed the available data sources for commercial supply chain analytics tools (e.g., LogicTools, Cognos). From these reviews and the planned descriptive models in chapters 4 and 5, we developed a format or schematic for the UPM data model. The standardized UPM format developed here consists of the data groups: (i) Input Parameters, (ii) Transformation Arrays, and (iii) Output Parameters. Each data group can be represented by a three dimensional matrix array. Each UPM is identified by a “Title Bar” which provides the supplier name and supplier ID, and links the macro and micro level models.

UPM TITLE: (i) Supplier Name (ii) Supplier ID					
INPUT PARAMETERS	OUTPUTS (j)	7	8	9	10
	INPUTS (j)	TRANSFORMATION ARRAYS			
X1. Supply Reorder Policy	1	Z1. Bill of Materials			
X2. Supply Contract (price, vendor, etc.)	2	Z2. Value Adding Times/Costs			
X3. Inspection and Receiving	3	Z3. Production Batch/Setup			
X4. Resource/Labor Costs	4	Z4. Manufacturing Productivity			
	5	Z5. Quality Attributes			
	6				
		OUTPUT PARAMETERS			
		Y1. Production Reorder Policy			
		Y2. Supply Contract (price, buyer, etc.)			
		Y3. Annual Demand Behavior			

**Figure 3.3** Schematic of NEXUS Format for a UPM.

Figure 3.3 shows the proposed schematic for the UPM. The figure identifies the data layers for each data group. For instance we propose 4 layers of input data parameters. Later in chapters 4 and 5 we introduce specifics for each layer. The two dimensions of each data layer correspond to the input ( $I_{ij}$ ) and output ( $O_{ij}$ ) parts of the UPM. Again, the inputs and outputs link the macro and micro level models. Note that many data elements will be common between the input and output groups of sequential suppliers. For instance if we consider X2 and Y2 in figure 3.3, then the selling price of the seller is the same as the buying price of the buyer. Much of this data can be extracted from a company's ERP system. For example in the case of an SAP implementation, the Net Weaver function provides the capability to extract the needed data. Following are examples of data elements that might be included in the UPM:

$Z_{ij}$  Bill of materials quantity of input part  $j$  needed to produce one unit of output part  $\hat{j}$  – belongs to data group Z1

$\delta_{ij}$  Production rate of product  $j$  at supplier  $i$  – belongs to data group Z4

$P_{jJ}$  Number of output part  $j$  used in final product  $J$  – belongs to data group Y3

### 3.3 Unit Transition Model (UTM)

In this section the unit model concept is applied to transitional stages. The activities of a transitional stage are as follows:

- (1) Movement of goods (shipment)
- (2) Financial protection (insurance)

A unit model for a transitional stage will be referred to as a unit transition model (UTM) because it is concerned with describing both of these activities. Like the UPM, the UTM is defined by a set of inputs. Unlike the UPM, the outputs of a UTM are the same as the inputs, because in a transitional stage there are no value-adding activities. Therefore a UTM does not require an input-output matrix. A transporter's contribution to its supply chain consists of accepting (as its inputs) the goods of an upstream supplier, and moving them (as its outputs) to a downstream supplier. The transporter is expected to maintain the essential equality between inputs and outputs. With this description of transporter behavior it is possible to evaluate the transporter's contribution to its chain.

#### 3.3.1 UTM Structure and Content

A unit transition model consists of the same three distinct but interrelated sections as a UPM. Each section contains its own data set. The three sections of a UTM are:

1. Obligations established by supply contract. A specific transporter's contractual obligations are stipulated by supply chain executives who are responsible for overall supply chain performance rather than the transporter's performance.

2. Operations controlled by a specific transporter. A transporter's internal operating parameters are controlled by the transporter's own executives.
3. Metrics by which to evaluate a specific transporter's contribution to overall supply chain performance.

### **3.3.2 UTM Performance Attributes**

This part of a unit transition model contains the measures by which a transitional facility's performance is evaluated. The following performance measures are proposed:

- Total transportation cost – number of dollars spent to transport material, work-in-process or product (wage costs, fuel costs, repair costs).
- Uninsured inventory risk – number of units of material, work-in-process or product transported without insurance coverage.

## **3.4 The NEXUS Analytics Level Models**

The NEXUS analytics level is expected to have several models that use the upper level network and data to project expected values for key supply chain performance metrics. In traditional analytics these metrics are derived from the actual performance history of the suppliers. For instance, consider the metric Supplier Average Daily Inventory. Using the ERP transaction history an analytical tool can derive the mean variance of this metric. In NEXUS we expect instead to develop models that project these values from system parameters. Supply chain performance metrics can be divided into the following five categories:

- (1) Distribution Productivity

- (2) Demand Characterization
- (3) Asset Management and Capital Risk
- (4) Profit and Cost
- (5) Demand Change Responsiveness

These performance categories serve as motivation for formulation of a series of analytical level NEXUS models. Example models will provide information about all five of the above-mentioned areas, including:

- (1) The ratios of outputs to inputs at each stage of the supply chain, and within the chain as a whole. This data will include the material usage rate, the labor usage rate, and the ratios of product outputs to assembly time inputs.
- (2) The demand from the chain's external customers. We will assume that the demand rate is known.
- (3) The pipeline inventory at each stage of the supply chain, and within the chain as a whole. We will assume a continuous review inventory management policy, i.e. the inventory level is known at all times and replenishment is ordered at varying time intervals.
- (4) The cost of production of each output type at each supplier, and within the chain as a whole. Production cost is defined here as the cost of adding value to the inputs. Value-added costs include assembly resource cost and labor cost. Unit production cost will be calculated in chapter 4 and then used to calculate unit profit.
- (5) The cost efficiency in responding to customer demand.



In chapters 4 and 5 we will develop models for the 4<sup>th</sup> and 5<sup>th</sup> of these performance categories.

### **3.5 Chapter Summary**

The UPM and UTM are structured to describe the activities of suppliers that belong to a sequentially designed supply chain. These activities are performed within the context of overall supply chain requirements. The utility of the unit model structures is that they facilitate the visualization and analysis of supplier performance. The unit models are flexible enough to allow inclusion of many operating parameters, which is conducive to evaluation of supplier performance metrics. The use of the unit models at the NEXUS analytics level will enable us to project expected values for these performance metrics on the basis of the system parameters.

**CHAPTER 4**  
**PROFIT AND COST DISTRIBUTION ANALYSIS**  
**ACROSS THE SUPPLY CHAIN**

**4.1 The  $(Q, R, \delta)^2$  Supply Chain**

Many supply chains operate with a  $(Q, R, \delta)^2$  relationship between each pair of sequential suppliers. In this arrangement each part  $j$  has two inventory locations: (1) on the output side of source  $i$ , and (2) on the input side of the consumer  $\hat{i}$ . Estimating the inventory costs in a  $(Q, R, \delta)^2$  supply chain is a difficult problem. There is little mention of this problem in the literature, although it is a common arrangement in many supply chains. We find there are several reasons why the problem is difficult to capture in an exact model even when the end demand is deterministic and uniform:

- There are two linked storage locations each with its own independent inventory policy.
- The differing production rates of the suppliers become a key driver in the resulting inventory levels.
- While the demand for the chain may be continuous, the demand in the supply pairs is stop-and-go, that is demand is only positive while batch production is occurring.
- The demand at each supplier is dependent on the inventory policy of the downstream supplier and generates a lumpy demand.

- The inventory behavior tends to exhibit long cycles (i.e., intervals for the inventory pattern to repeat), except when the independent policies are perfectly synchronized.
- A large number of supply cases are possible based on the relationships between the  $(Q, R, \delta)^2$  parameters. Each of these shows a different inventory behavior.

## 4.2 The JELS Inventory Model

The  $(Q, R, \delta)^2$  inventory model may be considered as an extension to the joint economic lot size (JELS) model proposed by Banerjee (1986). As described in this section, Banerjee's model is based on a single vendor selling a single product to a single purchaser (or buyer). The relevant notation is:

$P$  Annual production rate for the product

$D$  Annual demand rate for the product

$S$  Vendor's setup cost per setup

$A$  Purchaser's order cost per order

$r$  Annual inventory carrying charge, for vendor and purchaser, as a fraction of inventory value

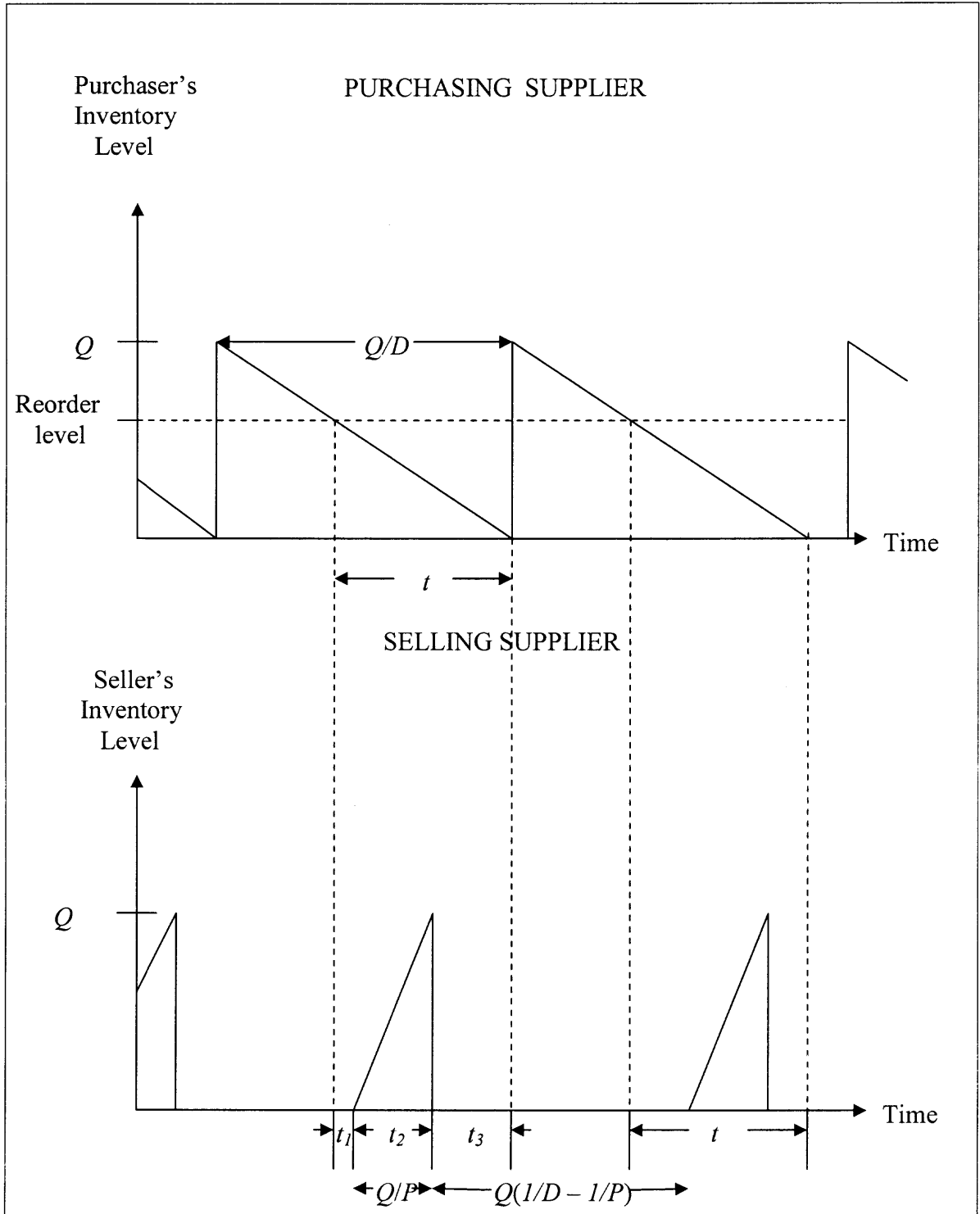
$C_v$  Unit production cost incurred by the vendor

$C_p$  Unit purchase cost paid by the purchaser

$Q$  Vendor's production lot size and purchaser's order lot size in units

The joint economic lot size (JELS) proposed by Banerjee is derived as a compromise between the vendor's economic lot size (ELS) and the purchaser's ELS. In the simple scenario considered by Banerjee, a purchaser transmits an order to buy

quantity  $Q$  of a single inventory item to a vendor. When the vendor receives the order it produces  $Q$  units of the item. After completing production the vendor ships the entire production batch in a single lot to the buyer. This transaction is based on a lot-for-lot replenishment policy. The inventory patterns employed in Banerjee's model are depicted in Figure 4.1 on the next page, embellished with symbolic indications of the purchaser's supply cycle  $Q/D$ , the vendor's production cycle  $Q/P$ , and the latter's interval of no production  $Q(1/D - 1/P)$ :



**Figure 4.1** Basic Joint Economic Lot Size Model.

The well-established derivation of ELS is given in terms of two costs. Each of these costs is expressed algebraically for both the vendor and the purchaser: (1) setup cost; (2) inventory carrying cost. The vendor's total relevant cost for lot size  $Q$  is given as the sum of setup cost and inventory carrying cost:  $TRC_v(Q) = (DS/Q) + (DQ/2P) rC_v$ . The first derivative of the vendor's cost function with respect to  $Q$  is set equal to 0 to find the vendor's optimal lot size:  $Q_v^* = \sqrt{2PS/rC_v}$ . The purchaser's total relevant cost for lot size  $Q$  is given as the sum of its own setup cost and inventory carrying cost:  $TRC_p(Q) = (DA/Q) + (Q/2) rC_p$ . The first derivative of the purchaser's cost function with respect to  $Q$  is set equal to 0 to find the purchaser's optimal lot size:  $Q_p^* = \sqrt{2DA/rC_p}$ .

Banerjee derives the joint total relevant cost for any lot size  $Q$  ( $JTRC(Q)$ ) by adding  $TRC_v(Q)$  and  $TRC_p(Q)$  as follows:

$$JTRC(Q) = (DS/Q) + (DQ/2P) rC_v + (DA/Q) + (Q/2) rC_p \quad (4.1)$$

Factoring terms leads to the following:

$$JTRC(Q) = (D/Q) (S+A) + (Q/2) r ((D/P) C_v + C_p) \quad (4.2)$$

By setting the first derivative of (4.2) with respect to  $Q$  equal to zero and solving for  $Q$  the optimal JELS of  $Q = Q_J^*$  is obtained:

$$Q_J^* = \sqrt{2D(S+A)/r((D/P)C_v + C_p)} \quad (4.3)$$

Let  $\alpha = S/A$  and  $\beta = DC_v/PC_p$ . Manipulation of terms in the ratio  $Q_v^*/Q_p^*$  reveals that the relationship between  $Q_v^*$  and  $Q_p^*$  may be written as:

$$Q_v^* = \sqrt{\alpha/\beta} Q_p^* \quad (4.4)$$

Therefore (4.3) may be simplified in two different ways as follows:

$$Q_J^* = \sqrt{(1+\alpha)/(1+\beta)} Q_p^* = \sqrt{(1+(1/\alpha))/(1-(1/\beta))} Q_v^* \quad (4.5)$$

Another variation of the JELS model is proposed by Goyal (1988). This model is derived under less restrictive assumptions than a lot-for-lot replenishment policy. Goyal assumes instead that to satisfy a purchase order of quantity  $Q$  the vendor can produce a lot of size  $nQ$ , where  $n$  is an integer multiple of the order quantity. The production lot size  $nQ$  may serve to satisfy an integral number of purchase orders. The joint total relevant cost for a purchaser's order quantity of  $Q$  and a vendor's lot size of  $nQ$  is stated as:

$$JTRC(Q,n) = (D/Q) (A + (S/n)) + (Q/2) r (C_p - C_v + nC_v (1 + (D/P))) \quad (4.6)$$

The optimal value of  $n = n^*$  is found by an iterative process. (At  $n = 1$  the total relevant costs calculated by (4.2) and (4.6) are the same). For a given value of  $n^*$  the ELS of the purchaser is found to be as follows:

$$Q(n^*) = \sqrt{2D(A + (S/n)) / r(C_p - C_v + nC_v(1 + (D/P)))} \quad (4.7)$$

The ELS for the vendor is equal to  $n^*Q(n^*)$ .

A more detailed solution to the single manufacturer-single buyer problem is now available from Lee (2005). Lee's model takes the following six costs into account: (1) buyer's ordering cost; (2) buyer's inventory holding cost; (3) manufacturer's production setup cost; (4) manufacturer's finished goods holding cost; (5) raw material ordering cost; (6) raw material holding cost. The notation employed by Lee is the following:

$D$  Annual demand of the buyer, units/year

$P$  Manufacturer annual rate of production, units/year

$D_R$  Annual demand for materials, units/year

$f$  Conversion factor of the raw materials to finished goods,  $f = D/D_R \leq 1$

$A$  Buyer's ordering cost per order, \$/order

$S$  Manufacturer's production setup cost per batch, \$/batch

$G$  Manufacturer's raw material ordering cost per order, \$/order

$C_Q$  Unit inventory value of buyer's incoming inventory, \$/unit

$C_V$  Unit inventory value of manufacturer's finished goods, \$/unit

$C_R$  Unit inventory value of raw material, \$/unit

$r$  Annual capital cost per dollar invested in inventory

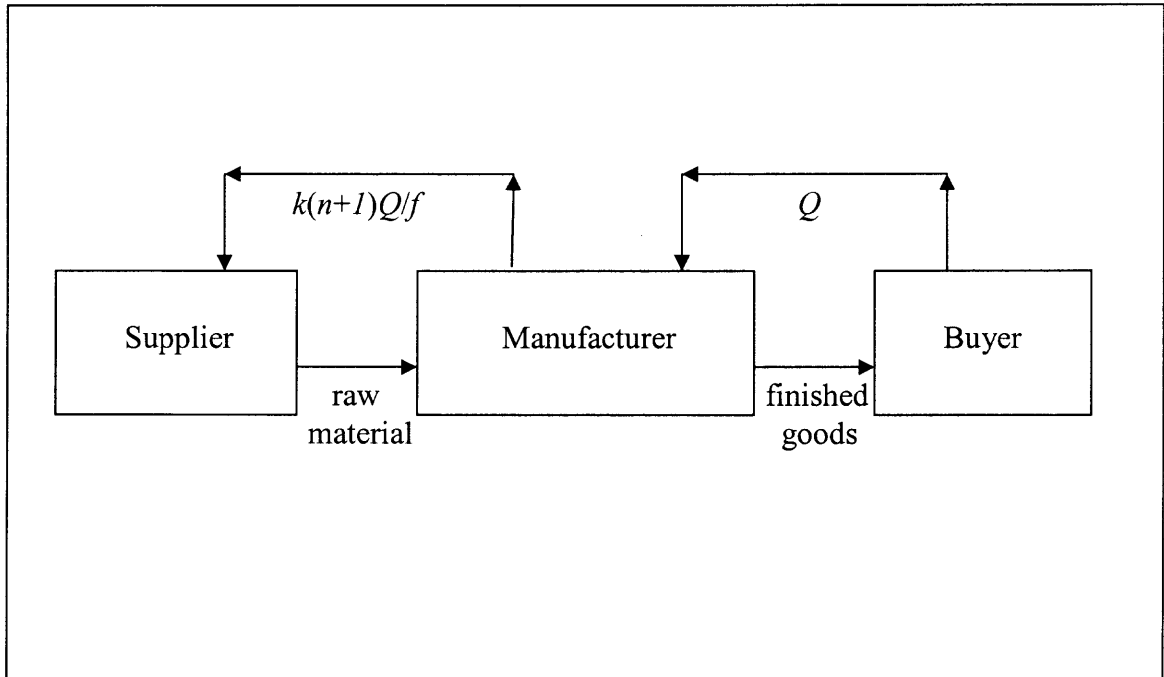
$Q$  Ordering lot size of buyer, units/order

$Q_M$  Production lot size per production run of manufacturer, units/cycle

$Q_R$  Raw material ordering lot size, units/order

Lee considers a supply chain of a raw materials supplier, finished product manufacturer and commercial buyer, although the integrated system that he proposes is intended to minimize the mean total cost of only the manufacturer and buyer. The diagram in figure 4.2 is taken from Lee's article and shows the essence of a contract diagram.



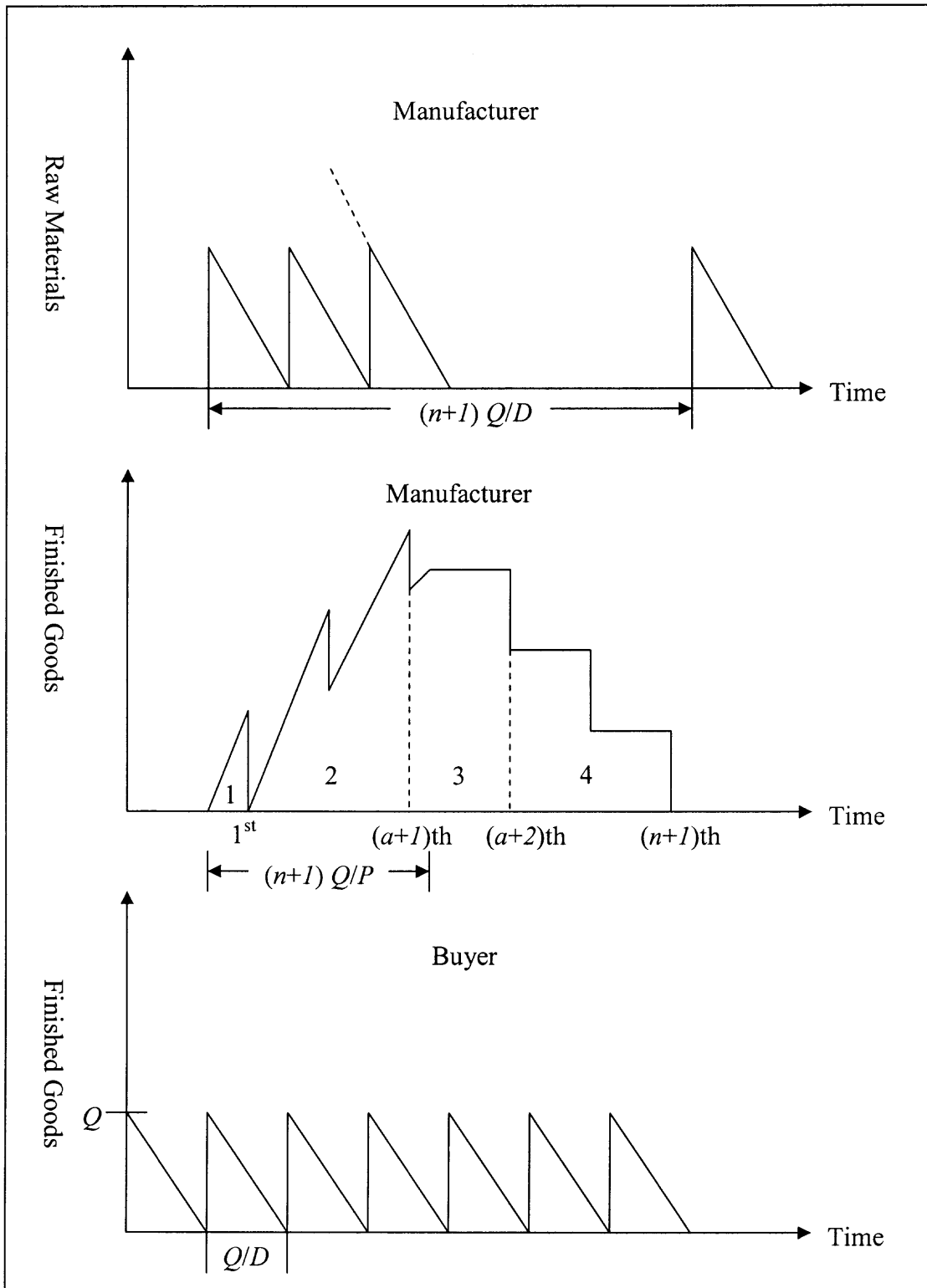


**Figure 4.2** The Integrated Inventory Control Model.

Lee derives a formula that minimizes the replenishment quantity in supply transactions between the manufacturer and the buyer. The derivation of this optimal replenishment quantity is based on the three relevant lot sizes:

1. The manufacturer's raw materials order lot size ( $Q_R$ )
2. The manufacturer's production lot size ( $Q_M$ )
3. The buyer's finished product order lot size ( $Q$ )

Figure 4.3 represents the patterns of inventory on hand for raw materials held by the manufacturer, finished goods held by the manufacturer, and finished goods held by the buyer.



**Figure 4.3** On Hand Inventories of Raw Materials and Finished Foods.

The relationship between the three above-mentioned lot sizes is as follows:  $Q_R = kQ_M/f = k(n+1)Q/f$ , where  $Q_M = (n+1)Q$ . Here  $n$  represents the number of shipments of finished products delivered by the manufacturer, and  $k$  represents the number of replenishments of raw materials ordered by the manufacturer. There are two possible cases for  $k$ , summarized by  $k = \{1, 2, 3, \dots, m\} \cup \{1, 1/2, 1/3, \dots, 1/m\}$  and  $m$  is an integer. Substituting either  $k = m$  or  $k = 1/m$  yields one of the following:

$$Q_R = m(n+1)Q/f \quad (4.8)$$

$$Q_R = (n+1)Q/mf \quad (4.9)$$

The joint total relevant cost in Lee's system is the sum of the six aforementioned costs. These are modeled as follows:

1. Buyer's cost of ordering finished goods:

$$(D/Q)A \quad (4.10)$$

2. Buyer's inventory holding cost:

$$(Q/2)rC_Q \quad (4.11)$$

3. Manufacturer's production setup cost:

$$(D/(n+1)Q)S \quad (4.12)$$

4. Manufacturer's finished goods holding cost:

Finished goods inventory is quantified for the manufacturer as the quotient of the time-weighted finished goods inventory and the production cycle length. The manufacturer's time-weighted inventory is expressed as the sum of the four areas indicated in Fig. 4.3. These areas represent the manufacturer's time-weighted finished goods inventory from the beginning of production to the 1<sup>st</sup> shipment, the 1<sup>st</sup> shipment to

the  $(a+1)$ th shipment, the  $(a+1)$ th shipment to the  $(a+2)$ th shipment, and the  $(a+2)$ th shipment to the  $(n+1)$ th shipment. The four areas are expressed as follows:

$$\text{Area 1} = Q(Q/P)2 = Q(Q/D)(D/P)/2$$

$$\text{Area 2} = (Q/2D) \sum_{i=1}^a [(2i-1)PQ/D - (2i-2)Q] = (Q^2/2D) [a^2 P/D - a(a-1)]$$

$$\begin{aligned} \text{Area 3} &= Q[a(P/D - 1) + (n-a)] (nQ/P - aQ/D)/2 + (n-a)Q[(a+1)Q/D - nQ/P] \\ &= (Q^2/2D) [2n(a+1) - Dn^2/P - 2a - a^2P/D] \end{aligned}$$

$$\begin{aligned} \text{Area 4} &= (n-a-1)Q^2/D + (n-a-2)Q^2/D + \dots + 2Q^2/D + Q^2/D \\ &= (Q^2/D) \sum_{i=1}^{n-a-1} i = (Q^2/2D) (n-a) (n-a-1) \end{aligned}$$

The sum of the four areas is expressed below:

$$\text{Time-weighted inventory} = (Q^2/2D) [n^2 (1 - D/P) + n + D/P] \quad (4.13)$$

The production cycle length is  $(n+1)Q/D$ . The quotient of the time-weighted inventory divided by the cycle length gives the manufacturer's average finished goods inventory:

$$(Q/2) [n(1 - D/P) + D/P] \quad (4.14)$$

The manufacturer's finished goods holding cost is then expressed in this manner:

$$(Q/2) rC_v [n(1 - D/P) + D/P] \quad (4.15)$$

5. Raw material ordering cost is calculated in two different ways depending upon the value of  $k$ . For  $k = \{1, 2, 3, \dots, m\}$  the raw material ordering cost per year is:

$$(D/Q) [Gf/m(n+1)] \quad (4.16)$$

For  $k = \{1, 1/2, 1/3, \dots, 1/m\}$  the raw material ordering cost per year is:

$$(D/Q) [Gmf/(n+1)] \quad (4.17)$$

6. As with the manufacturer's average finished goods inventory (see above), the

manufacturer's average raw material inventory is the quotient of the time-weighted inventory divided by the cycle length. For  $k = \{2,3,4, \dots, m\}$  there are two categories of time-weighted raw material inventories: (i) raw materials during production uptime, and (ii) raw materials during production downtime. During production uptime the time-weighted raw material inventory is:

$$(Q^2/2fP) (n+1)^2 [(2m-1) + (2m-3) + \dots + 3 + 1] \quad (4.18)$$

During production downtime the time-weighted raw material inventory is:

$$((n+1)^2/f) Q^2 (1/D - 1/P) [(m-1) + (m-2) + \dots + 2 + 1] \quad (4.19)$$

The total time-weighted raw material inventory is the sum of (4.18) and (4.19):

$$(1/2f) (Q^2/P) (n+1)^2 m^2 + (n+1)^2 Q^2 (1/D - 1/P) (m(m-1)/2f) \quad (4.20)$$

The corresponding cycle length is:

$$m(n+1)Q/D \quad (4.21)$$

The manufacturer's average raw material inventory for  $k = \{2,3,4, \dots, m\}$  is the quotient of (4.20) and (4.21):

$$(Q/2f) (n+1) [(D/P)m + (m-1) (1 - D/P)] \quad (4.22)$$

For  $k = \{1, 1/2, 1/3, 1/4, \dots, 1/m\}$  the time-weighted raw material inventory is:

$$((n+1)Q/mf) ((n+1)Q/2P) \quad (4.23)$$

The corresponding cycle length is:

$$(n+1) Q/D \quad (4.24)$$

The average raw material is the quotient of (4.23) and (4.24), which is:

$$(Q/2) ((n+1)/mf) (D/P) \quad (4.25)$$

Raw material holding cost is calculated in two different ways, depending upon the value of  $k$ . For  $k = \{1,2,3,\dots,m\}$  the raw material holding cost is:

$$(Q/2) rC_R ((n+1)/f) [(D/P)m + (m-1) (1 - D/P)] \quad (4.26)$$

For  $k = \{1, 1/2, 1/3, \dots, 1/m\}$  the raw material holding cost is:

$$(Q/2) rC_R ((n+1)/mf) (D/P) \quad (4.27)$$

In Lee's system the total relevant cost can be expressed for the case where  $k = \{1, 2, 3, \dots, m\}$  and for the case where  $k = \{1, 1/2, 1/3, \dots, 1/m\}$ . We will consider first the case of  $k = \{1, 2, 3, \dots, m\}$ . By factoring out  $D/Q$  from (4.10), (4.12) and (4.16) we obtain:

$$\Omega = A + (S/(n+1)) + (Gfm(n+1)) \quad (4.28)$$

By factoring out  $(Q/2)r$  from (4.11), (4.15) and (4.26) we obtain:

$$\Psi = C_Q + C_V [n(1 - D/P) + D/P] + C_R ((n+1)/f) [(D/P)m + (m-1) (1 - D/P)] \quad (4.29)$$

The total relevant cost is then stated as:

$$TC(m,n,Q) = (D/Q) \Omega + (Q/2)r \Psi \quad (4.30)$$

For particular values of  $m$  and  $n$  the optimal order quantity  $Q^*$  is found by differentiating  $TC(m,n,Q)$  with respect to  $Q$  and setting it equal to 0. The result is:

$$Q^* = \sqrt{2D\Omega / r\Psi} \quad (4.31)$$

For the case of  $k = \{1, 1/2, 1/3, \dots, 1/m\}$  we factor out  $D/Q$  from (4.10), (4.12) and (4.17) to obtain:

$$\Gamma = A + (S/(n+1)) + (Gmf/(n+1)) \quad (4.32)$$

By factoring out  $(Q/2)r$  from (4.11), (4.15) and (4.27) we obtain:

$$\Phi = C_Q + C_V [n(1 - D/P) + D/P] + C_R ((n+1)/mf) (D/P) \quad (4.33)$$

The total relevant cost is then stated as:

$$TC(m,n,Q) = (D/Q) \Gamma + (Q/2)r \Phi \quad (4.34)$$

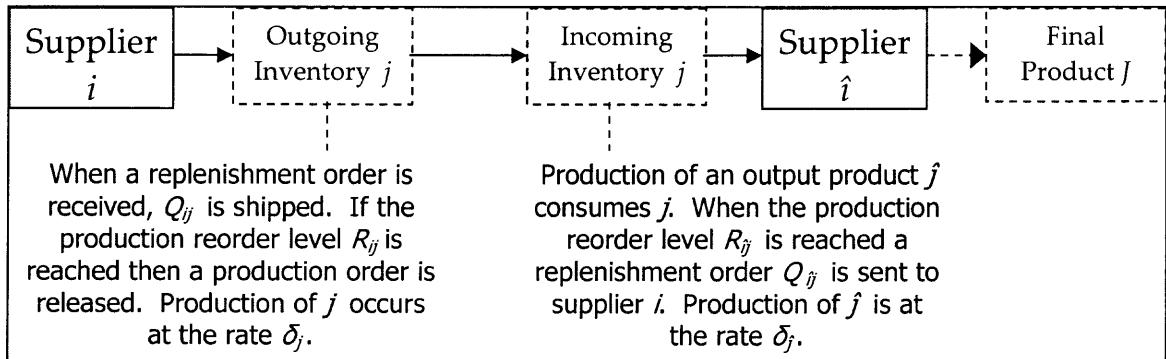
For particular values of  $m$  and  $n$  the optimal order quantity  $Q^*$  is found by differentiating  $TC(m,n,Q)$  with respect to  $Q$  and setting it equal to 0. The result is:

$$Q^* = \sqrt{2D\Gamma/r\Phi} \quad (4.35)$$

### 4.3 The $(Q, R, \delta)^2$ Inventory Behavior

As noted in section 4.1, in a  $(Q, R, \delta)^2$  relationship each part  $j$  has two inventory locations: (1) on the output side of source  $i$ , and (2) on the input side of the consumer  $\hat{i}$ .

The replenishment behavior in this scenario is depicted in Figure 4.4:



**Figure 4.4** Replenishment Behavior of Part  $j$  Between Two Inventory Locations.

In a  $(Q, R, \delta)^2$  supply chain the following four parameters govern the production-inventory-replenishment behavior and are defined by contract. Note that there are two reorder levels for part  $j$ , one on the production (output) side, and one on the consumption (input) side, and these are denoted by  $R_{ij}$  and  $r_{ij}$ .

$R_{ij}$  Production reorder level for part  $j$  at supplier  $i$

$r_{ij}$  Supply reorder level for incoming part  $j$  at supplier  $\hat{i}$

$Q_{ij}$  Replenishment order quantity of part  $j$  for supplier  $i$

(this is the production batch size for supplier  $i$ )

$\delta_j$  Production rate of part  $j$

There are two key questions in modeling the inventory behavior in a  $(Q, R, \delta)^2$  supply chain: (1) what is the inventory cycle length for the cost analysis, and (2) what are

the changes between cycles. In response to (1) we find that the production or supply cycle of the downstream supplier provides an effective solution. In response to (2) we find the ending inventory for each cycle is best represented by a uniform distribution.

In addition to the notation introduced in chapter 3, and the supply contract notation introduced in this section (see above), the notation given below is used to develop the cost-profit models in this chapter. The cost parameters and production parameters are under the control of individual suppliers and are not established by contractual agreement across the entire supply chain.

Cost Parameters:

- $Cs_{ij}$  Unit supply or selling price for part  $j$  from supplier  $i$
- $Co_{ij}$  Order cost for a batch of part  $j$  shipped out from supplier  $i$
- $Ch_j$  Unit inventory holding cost of part  $j$
- $Ca_j$  Assembly cost of part  $j$  per unit time
- $Cw_i$  Worker or labor cost rate at supplier  $i$  per unit time

Cost Components:

- $I'_{ij}$  Input side inventory costs of supplier  $i$  for a unit of final product  $J$
- $I^o_{ij}$  Output side inventory costs of supplier  $i$  for a unit of final product  $J$
- $M_{ij}$  Material costs of supplier  $i$  per unit of final product  $J$
- $A_{ij}$  Value added or production costs of supplier  $i$  per unit of final product  $J$

Production Parameters:

- $Lu_j$  Labor utilization rate in the production of part  $j$
- $B_j$  Production batch size of part  $j$
- $D_j$  Demand rate for part  $j$  when used in final product  $J$



$U_{ij}$  Residual inventory at the start of the no-activity period for part  $j$  at supplier  $i$

$\tau_j$  Unit production time of part  $j$

$T_{jj}$  Transportation lead time of part  $j$  needed to produce part  $\hat{j}$

Inventory Cycle Variables:

$X1_{ij}$  Number of complete order cycles in the supply of part  $j$  to supplier  $\hat{i}$

$Y1_{ij}$  Length of the partial order cycle in the supply of part  $j$  to supplier  $\hat{i}$

$X2_{ij}$  Number of consecutive supply cycles with no replenishments when  $Q_{ij} > Z_{jj} B_j$

$Y2_{ij}$  Length of the partial production cycle with no replenishments when  $Q_{ij} > Z_{jj} B_j$

$N_{jj}$  Number of production batches of  $j$  to meet the supply cycle demand of  $\hat{j}$

$X3_{ij}$  Number of replenishments during the seller production cycle

$X4_{ij}$  Number of seller production cycles in a 10 supply cycle window

#### 4.3.1 Model Assumptions

In the development of the  $(Q, R, \delta)^2$  model we make the following assumptions:

- Final product demand is deterministic and uniform.
- Supply quantities between suppliers and production batch sizes at each supplier are fixed. No partials or multiples are allowed.
- Demand at each supplier follows a classical BOM explosion, but is lumpy since downstream suppliers produce in batches.
- Production rates increase upstream, that is  $\delta_B > \delta_A$  where part B is used to make A.

- Production will always outpace demand, that is  $\delta_B > D_A$ .
- The replenishment is instantaneous, though we do consider the transport inventory for costing purposes.
- There are no capacity resource restrictions and each batch is started immediately.
- Production output is available for order shipment immediately, and does not have to wait for the batch completion.
- No stockouts or backorders are allowed.
- Order and/or setup costs are fixed, and hence constant for a given demand rate.
- Per unit material and production are fixed and independent of quantity.

#### 4.3.2 The Inventory Cost Cycle

In classical inventory theory it has often been assumed that there is a repeating cycle of production and shipment. Our focus here is upon a pattern of inventory behavior that is non-repeating. We are interested in a pattern formed when production of a part  $j$  exceeds shipment of that part. When this occurs in a supply cycle the result is residual inventory. The presence of residual inventory within a supply cycle means that there will be a no-production sub-cycle. In the next subsection we consider the specific cases in which residual inventory is accumulated.

#### 4.3.3 Possible Supply Relationships

The analytical insights provided by the  $(Q, R, \delta)^2$  model are obtained from the four supplier parametric relationships which are enumerated below. These relationships are important for what they reveal about inventory behavior. We describe inventory

behavior in terms of the supply of part  $j$  between a pair of suppliers  $i$  and  $\hat{i}$  (see figure 4.4). Supplier interaction is modeled in the context of the input supply cycle. The supply cycle starts from the point at which production of a batch of part  $\hat{j}$  begins, using part  $j$  as input to the production process, and ends when production of the next batch of part  $\hat{j}$  starts:

$$\text{Length of Input Supply Cycle – Part } j \text{ for } \hat{j} = B_j/D_j \quad (4.39)$$

Observe that the supply cycle length is the same for all parts used in the production of  $\hat{j}$ . Further, since we assume that demand is invariant, the length of the supply cycle remains constant for the part.

We take  $\hat{i}$  to be the buying supplier and  $i$  to be the selling supplier. As soon as the buyer initiates production of a batch of  $\hat{j}$ , the buyer's inventory of  $j$  begins to be depleted. When the incoming reorder level is triggered a replenishment order of  $Q_{ij}$  is released and then immediately received. This continues until the batch production of  $\hat{j}$  is completed. It is clear that during the supply cycle the buyer inventory goes through a dynamic phase of depletion and replenishment, followed by a static phase of no inventory movement. From the buyer's perspective, therefore, the supply cycle may be divided into the following sub-cycles: the production cycle during which part  $\hat{j}$  is being manufactured, the replenishment cycle during which orders for part  $j$  are released and received, and the no-activity cycle during which the inventory remains static and there is no input or consumption.

At some point during this replenishment process the outgoing reorder level is triggered and the seller begins production of a batch of part  $j$ . This production process may continue beyond the replenishment process. In general there will be differences in

supplier batch sizes. This means that there will be residual inventory after the shipments cease if replenishment batch quantities ( $Q_{ij}$ ) have exceeded consumption batch sizes ( $Z_{ij}B_j$ ). It is clear that during the supply cycle the seller inventory goes through a production cycle, a replenishment phase, and a phase with no activity:

$$\text{Length of Production Cycle – Part } j = B_j/\delta_j \quad (4.40)$$

Since for part  $j$  the length of the supply cycle is given by equation (4.39) and that of the production cycle is  $B_j/\delta_j$ , the length of the no-activity cycle is  $B_j(1/D_j - 1/\delta_j)$ . From the seller's perspective, therefore, the supply cycle may be divided into the following sub-cycles: the production cycle during which part  $j$  is being manufactured, the replenishment cycle during which replenishment orders are being shipped, and the no-activity cycle during which the inventory remains static and there is no product output or shipments.

A variety of supply relationships are amenable to analysis within the  $(Q, R, \delta)^2$  model. We restrict our analysis to those cases for which  $B_j/Z_{ij}B_j < 2$ . Under this condition there will be a series of supply cycles with at least one production batch of part  $j$ , followed by a cycle when there is no production batch. The no-production cycle results from the sum of the residual inventories (production minus shipments) in each supply cycle. When the sum is greater than  $Z_{ij}B_j$  then no production is needed. Note that when  $B_j/Z_{ij}B_j < 1$  there might not be any no-production cycle. The selling supplier will need to produce multiple batches to meet the demand. When the condition  $B_j/Z_{ij}B_j < 2$  does not hold, then every supply cycle with production of part  $j$  will be followed by at least one no-production cycle. For now we will not consider this other condition.

Supply relationships are characterized by specific patterns of inventory behavior. Four of these cases are enumerated below. Cases #1 and #2 are from the buyer's perspective. Cases #3 and #4 are from the seller's perspective. At the start of the supply cycle, the existing buyer inventory feeds the production until the reorder level  $R_{ij}$  is reached, at which point the replenishment cycle begins and a series of replenishment orders will occur. The seller will initially satisfy the orders from the on-hand inventory until the reorder level  $R_{ij}$  is reached. At this point a production batch is initiated. Depending on the relationship between  $Z_{ij}B_j$  and  $Q_{ij}$  these replenishments may end before or after the production cycle, and in the extreme case could overlap several supply cycles.

CASE #1 -  $Z_{ij}B_j > Q_{ij}$

When  $Z_{ij}B_j > Q_{ij}$  the replenishment cycle ends before the production cycle. The replenishment cycle will consist of several complete order cycles, that is the order quantity  $Q_{ij}$  is fully consumed and the subsequent order triggered. In contrast for an incomplete or partial cycle the consumption process ends before reorder, and the inventory overlaps another supply cycle. The inventory level during the no-activity sub-cycle is the residual inventory at the end of the partial cycle. Since modern supply chains are intended to emphasize low inventory levels, we expect in general the condition  $Z_{ij}B_j > Q_{ij}$  to hold, implying a series of frequent small batch replenishments. In this case residual inventories are unlikely to occur.

CASE #2 -  $Q_{ij} > Z_{ij}B_j$

In this case each order replenishment of  $j$  will feed several production batches of  $\hat{j}$ , and hence there will be no replenishments for several supply cycles. Each replenishment satisfies one or more complete production cycles, and possibly a partial production cycle.

Following a series of replenishments these partial cycles will add up and there will be an extra supply cycle with no replenishment. In this case residual inventories commonly occur.

$$\text{CASE \#3 - } B_j/D_j > B_j/\delta_j$$

The selling inventory dynamics consist of receiving a sequence of replenishment orders from the buyer. In each instance the seller immediately ships  $Q_{ij}$  units. Since we assume that  $\delta_j > D_j$ , production will always outpace demand. Depending on the production rate, the replenishment cycle can be either longer or shorter than the production cycle.

$$\text{CASE \#4 - } B_j \gg B_j \text{ with no-production cycles}$$

This is the frequency of a non-repeating supply cycle that includes a no-production sub-cycle. The occurrence of a no-production sub-cycle follows an uneven pattern. Table 4.1 below shows the production and residual inventory pattern for the case when  $N_{ij}B_j = 1.8Z_{ij}B_j$  during a period of 10 supply cycles. The inventory numbers are in multiples of the supply cycle demand  $Z_{ij}B_j$ . From the table we see that the first no production cycle occurs after two production cycles, but the next three no production cycles occur in alternate cycles. Clearly there is an uneven pattern, though it will repeat. The end of the repeating cycle is indicated by a zero residual inventory, and in some cases the repeating cycle is short. For instance in table 4.1, it is 9 supply cycles long. On the other hand if  $N_{ij}B_j / Z_{ij}B_j = 1.7$  then the repeating cycle is 12 periods long.

Clearly, based on the ratio the repeating cycle could vary significantly. An exact estimate of the repeating cycle length will provide a precise estimate of the no-production cycle frequency.

**Table 4.1** The Residual Inventory Pattern

Supply Cycle	1	2	3	4	5	6	7	8	9	10
Production Quantity	1.8	1.8	0	1.8	0	1.8	0	1.8	0	1.8
Residual Inventory	0.8	1.6	0.6	1.4	0.4	1.2	0.2	1.0	0	0.8

In classical inventory theory the repeating cycle of production and shipment was usually based on the preceding production cycle. In our analysis, however, which is based upon a non-repeating cycle, we build our model upon the succeeding production cycle. Our approach is to approximate the numerical frequency of the no-production sub-cycles in the previous example of 10 supply cycle data.

CASE #5 - Output product  $\hat{j}$  is actually final product  $J$

For the case where part  $\hat{j}$  is a final product the inventory behavior is quite different because there is no subsequent supplier. We will elaborate upon this case at the end of subsection 4.4.2.

#### 4.3.4 Deterministic Simulation

An example supplier pair is subjected to deterministic simulation in this subsection. The parameters subjected to variation are production batch size ( $B_j$  and  $B_{\hat{j}}$ ), reorder level ( $R_{ij}$  and  $r_{ij}$ ), production rate ( $\delta_j$  and  $\delta_{\hat{j}}$ ), and replenishment quantity ( $Q_{ij}$ ). The value of the parameter for final product demand ( $D_j$ ) is fixed at 1 unit/hour. The inventory patterns that are formed during the supply cycle of a  $(Q, R, \delta)^2$  model are more volatile than the patterns found in the literature and illustrated in section 4.2 for a  $(Q, R, \delta)$  model. This volatility is due to the greater number of parameters in a  $(Q, R, \delta)^2$  model. The volatility

is seen in the results of a deterministic simulation that we have performed for 12 scenarios.

We begin with the base case, simulation #1, in which  $B_2$  is 250 units (see table 4.2). The buyer inventory exhibits a cycle of approximately 276 hours (see figure 4.5). During the cycle there are several peaks and valleys and also a plateau period. The seller inventory does not show a complete cycle in the simulation interval. Several sub-cycles are seen where the inventory shows a maximum-to-minimum behavior. Each of these sub-cycles are quite different. For instance, the sub-cycle from hours 828 to 1104 is quite different from the sub-cycle from hours 1104 to 1380. We can therefore conclude that the inventory behavior is quite complex and difficult to capture analytically.

In simulation #2 we increase  $B_2$  to 400 units (see table 4.3). Then in simulation #3 we increase  $\delta_2$  from 2 to 3 units per hour (see table 4.4). In both of these scenarios the buyer inventory cycle is approximately 414 hours (see figures 4.6 and 4.7). During this cycle there are three or four peaks and valleys as well as a plateau period. The seller inventory shows 4 complete cycles within the simulation interval. In both simulation #2 and simulation #3 we find that the first two of these complete seller cycles have a duration of approximately 345 hours and the second two complete seller cycles have a duration of approximately 414 hours. All four of these complete seller cycles contain a plateau period.

In simulation #4  $B_1$  is increased to 800 units (see table 4.5). This value for  $B_1$  is held constant for simulations #4 to #12. In simulation #4 the buyer inventory goes through 4 complete cycles of 414 hours (see figure 4.8). In each of these cycles there are again 3 or 4 peaks and valleys as well as a plateau period. The seller inventory goes

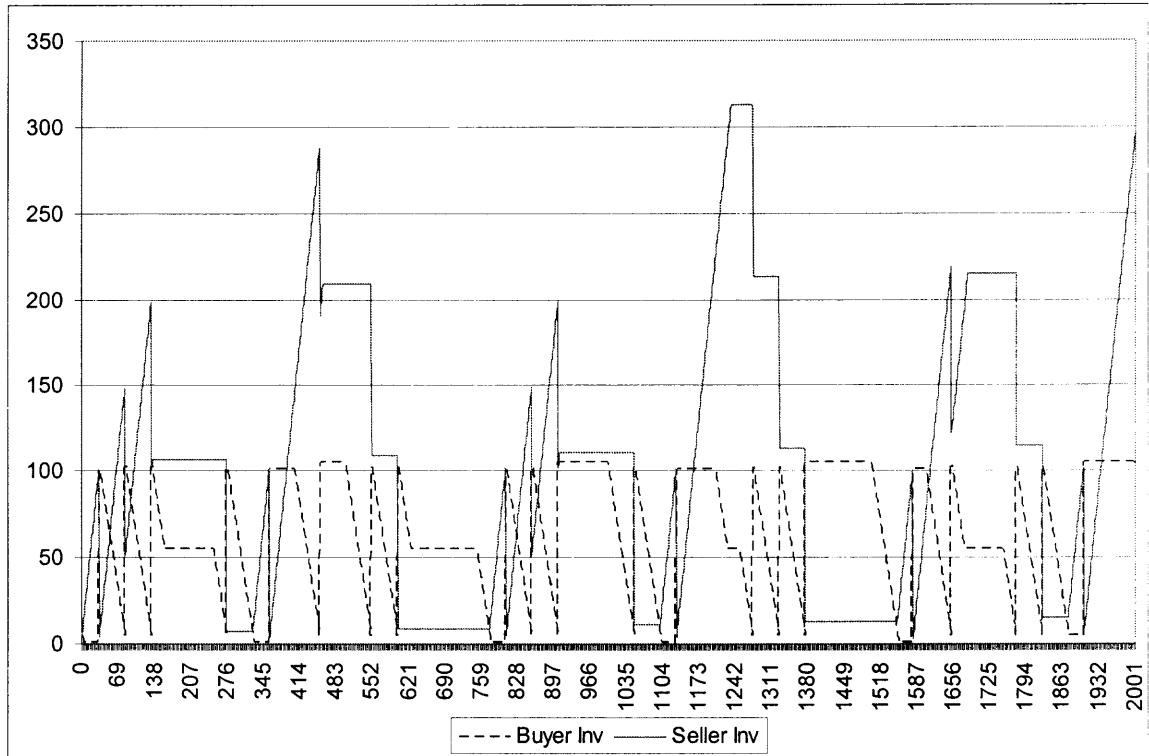


through 2 complete cycles. The first complete cycle has a length of about 345 hours, and the second one has a length of about 828 hours. A maximum-to-minimum behavior pattern is visible within both complete seller cycles.

In simulation #5  $B_2$  is increased to 800 units (see table 4.6). In simulation #6  $Q_{ij}$  is decreased to 65 units (see table 4.7). In simulation #7  $Q_{ij}$  is increased to 275 units (see table 4.8). In simulation #8  $R_1$  is increased to 10 units (see table 4.9). In simulation #9  $R_2$  is also increased to 10 units (see table 4.10). In simulation #10  $R_2$  is increased to 20 units (see table 4.11). In simulation #11  $\delta_1$  is increased to 5 units per hour (see table 4.12). In simulation #12  $\delta_2$  is increased to 10 units per hour (see table 4.13). The graphs of these 8 simulation scenarios are given in figures 4.9 to 4.16, where the x-axis represents time in hours, while the y-axis represents the inventory in units. The buyer inventory in these 8 figures exhibits complete cycles of varying lengths and inventory level maxima. The seller inventory exhibits 2 complete cycles in each figure with varying plateau maxima.

**Table 4.2** Simulation #1

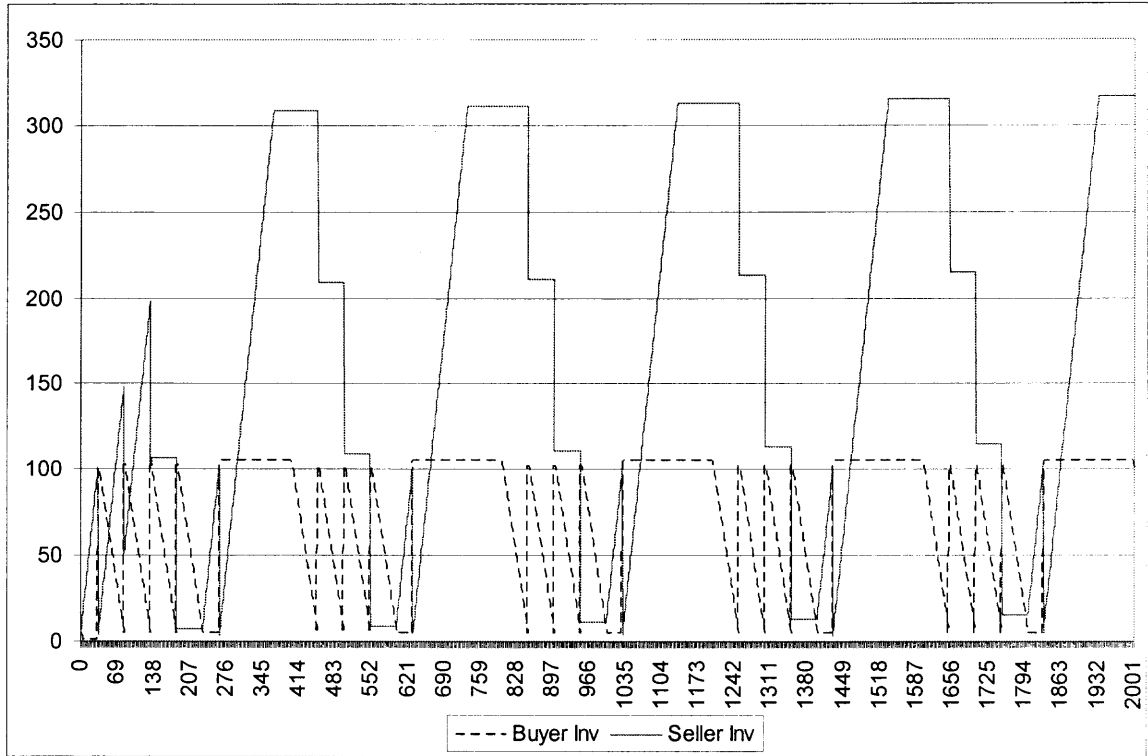
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	400	Replenishment quantity ( $Q_{ij}$ )	100	Production batch size ( $B_2$ )	250
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	2



**Figure 4.5** Simulation #1. Average Buyer Inv.: 59.63277 Average Seller Inv.: 95.78128

**Table 4.3** Simulation #2

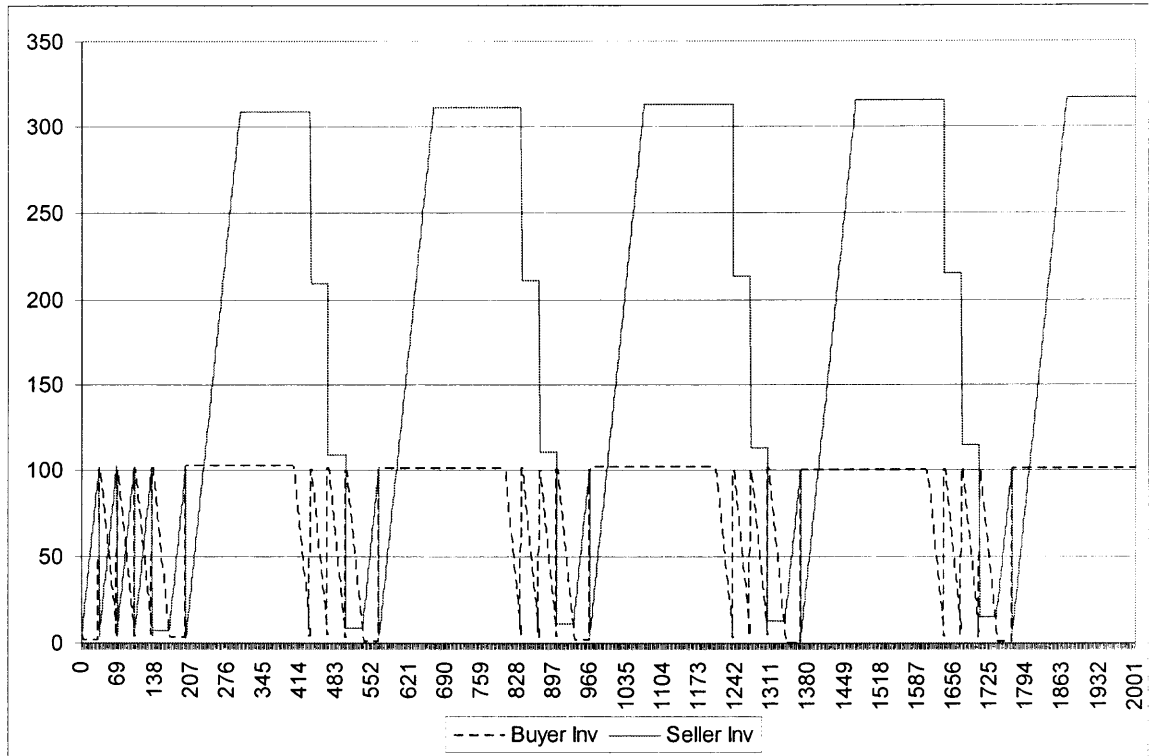
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	400	Replenishment quantity ( $Q_{ij}$ )	100	Production batch size ( $B_2$ )	400
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	2



**Figure 4.6** Simulation #2. Average Buyer Inv.: 68.85472 Average Seller Inv.: 160.8628

**Table 4.4** Simulation #3

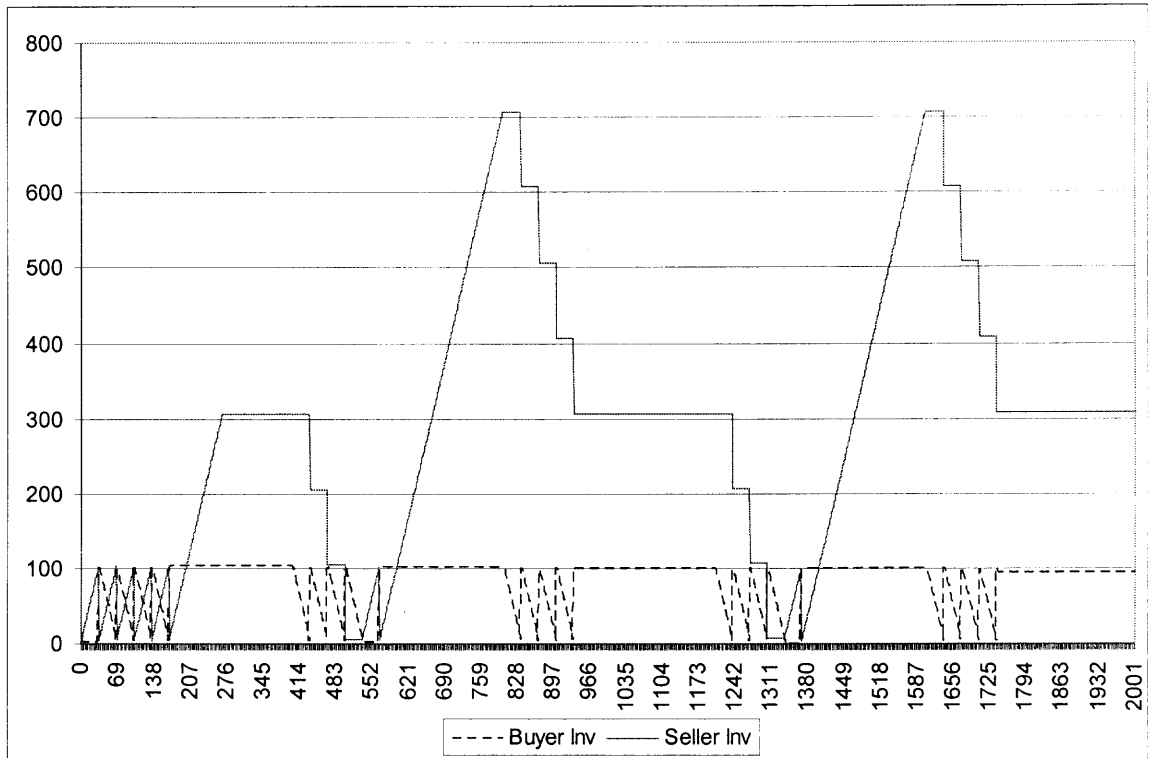
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	400	Replenishment quantity ( $Q_{ij}$ )	100	Production batch size ( $B_2$ )	400
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.7** Simulation #3. Average Buyer Inv.: 74.53672 Average Seller Inv.: 185.2308

**Table 4.5** Simulation #4

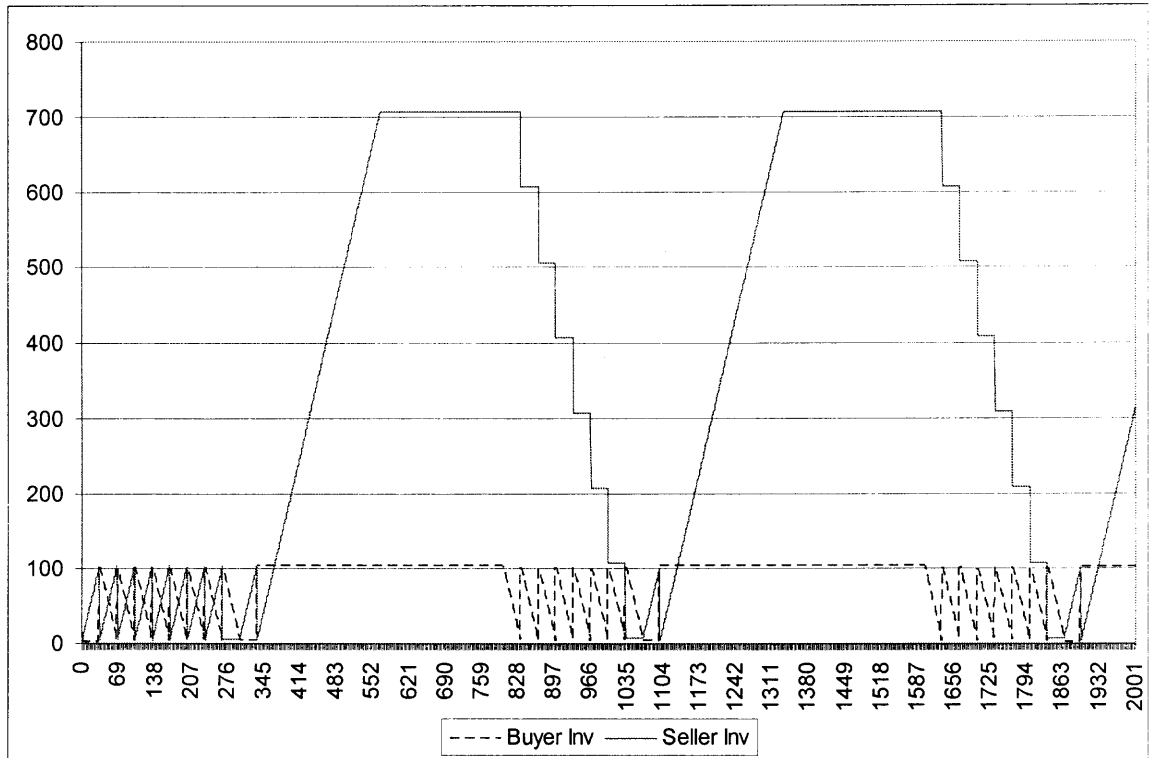
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	100	Production batch size ( $B_2$ )	400
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.8** Simulation #4. Average Buyer Inv.: 78.73527 Average Seller Inv.: 273.9887

**Table 4.6** Simulation #5

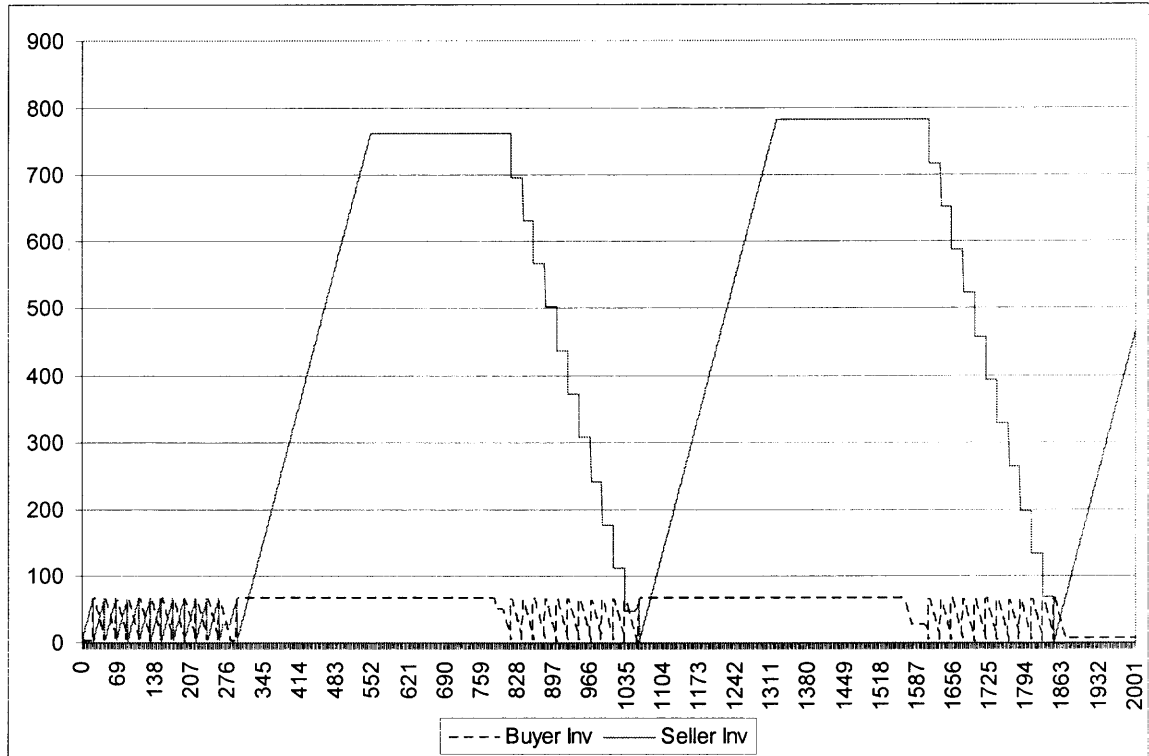
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	100	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.9** Simulation #5. Average Buyer Inv.: 73.86602 Average Seller Inv.: 316.0129

**Table 4.7** Simulation #6

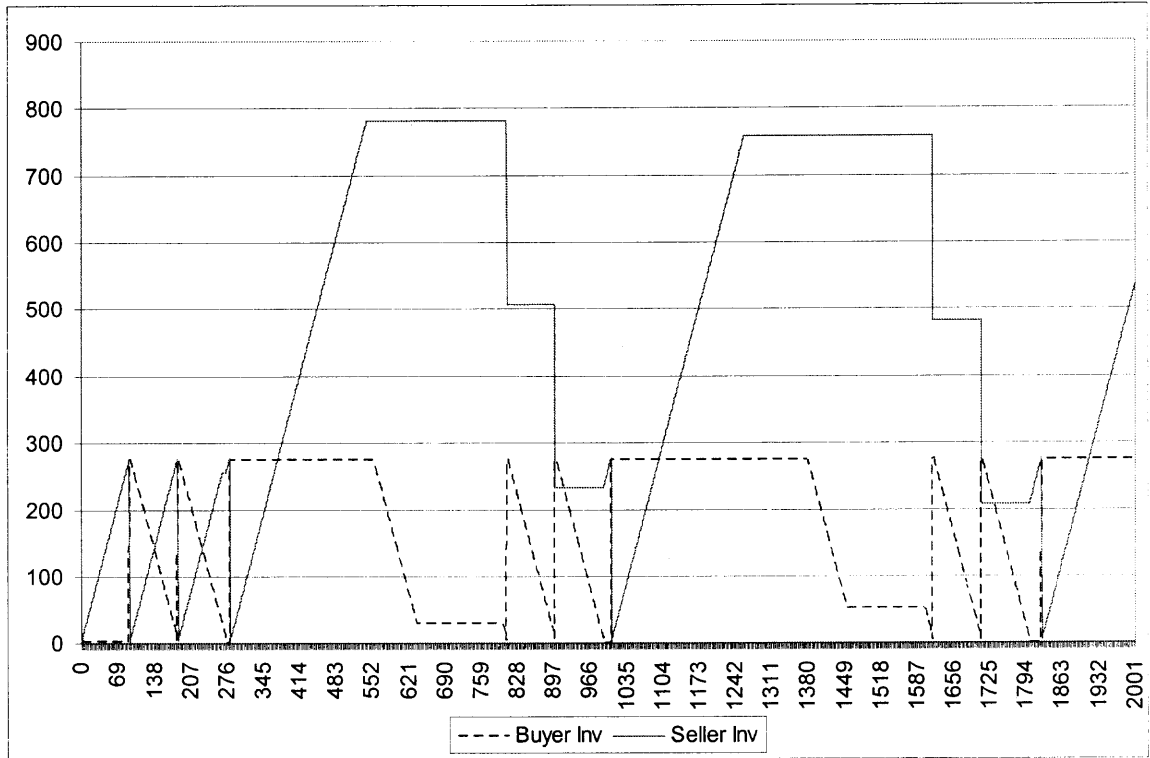
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	65	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.10** Simulation #6. Average Buyer Inv.: 51.29944 Average Seller Inv.: 362.117

**Table 4.8** Simulation #7

SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	275	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	5			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3

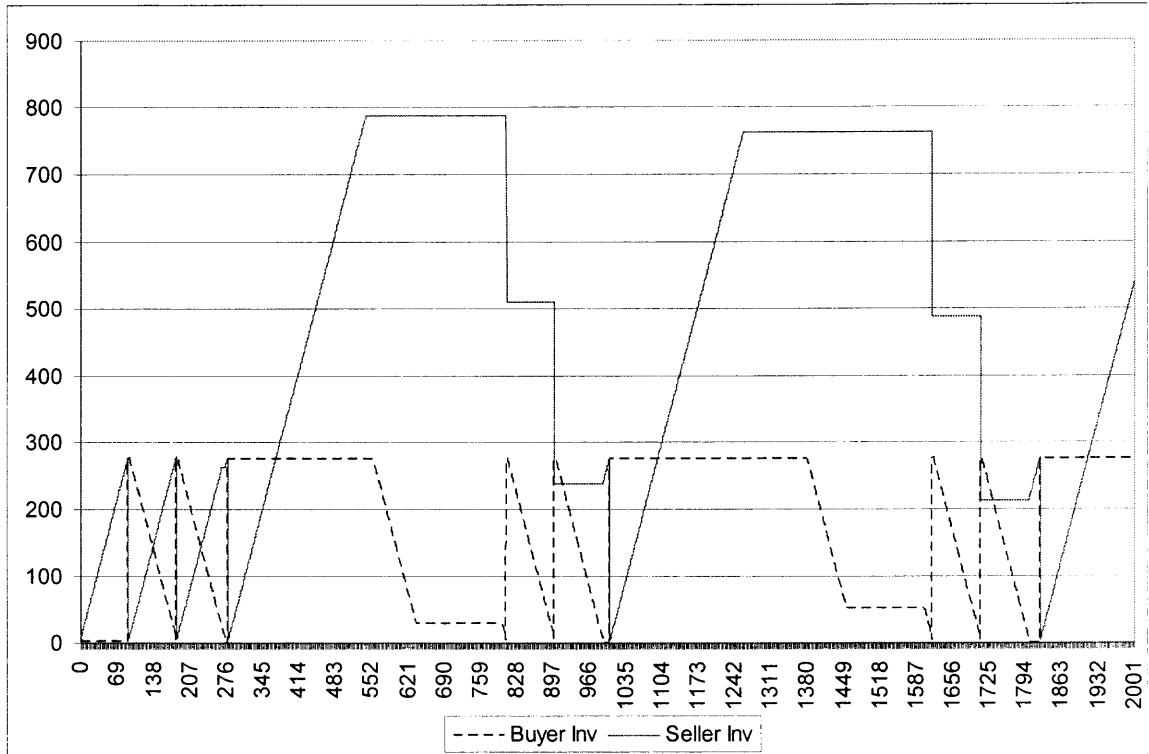


**Figure 4.11** Simulation #7. Average Buyer Inv.: 167.4948 Average Seller Inv.: 406.7716

**Table 4.9** Simulation #8

SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	275	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	10			Reorder level ( $R_2$ )	5
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3

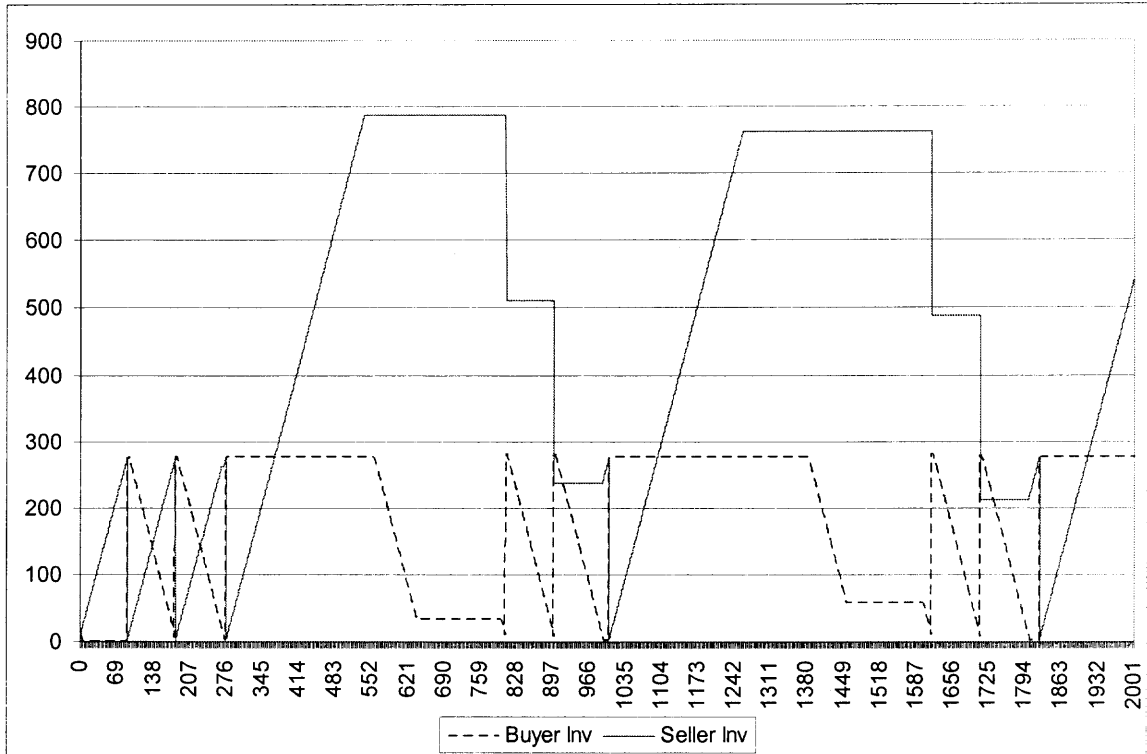




**Figure 4.12** Simulation #8. Average Buyer Inv.: 168.6029 Average Seller Inv.: 410.8644

**Table 4.10** Simulation #9

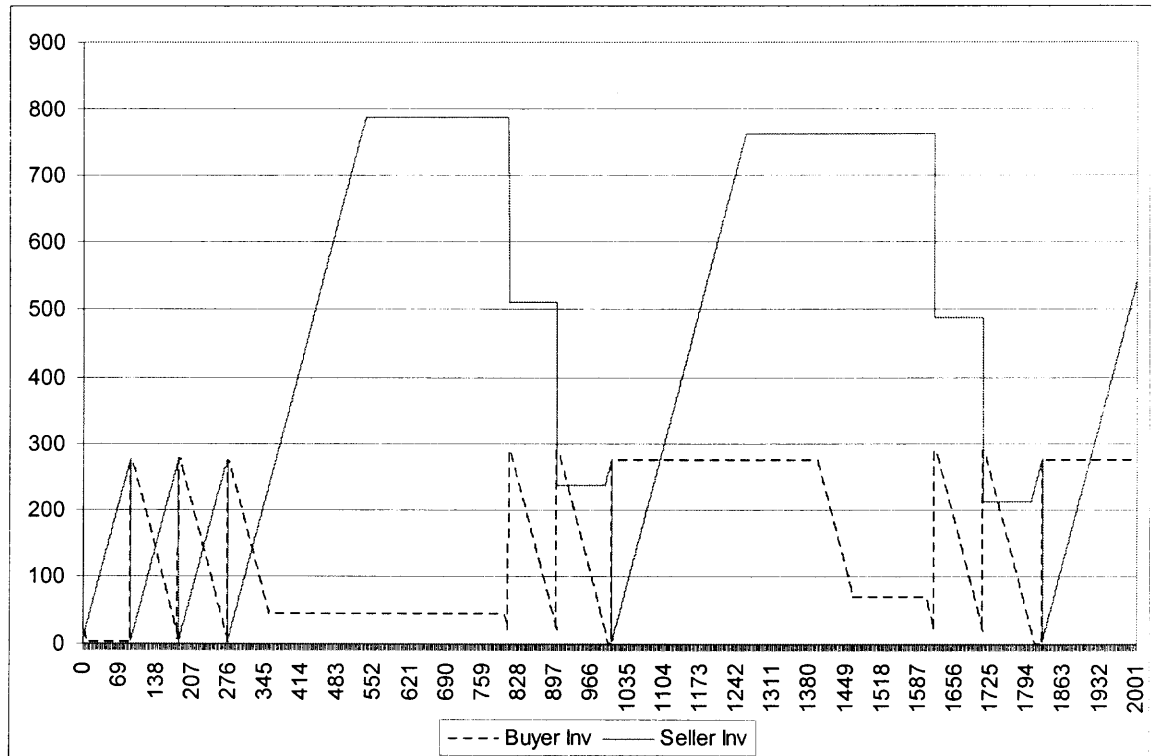
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	275	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	10			Reorder level ( $R_2$ )	10
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.13** Simulation #9. Average Buyer Inv.: 172.0726 Average Seller Inv.: 411.4915

**Table 4.11** Simulation #10

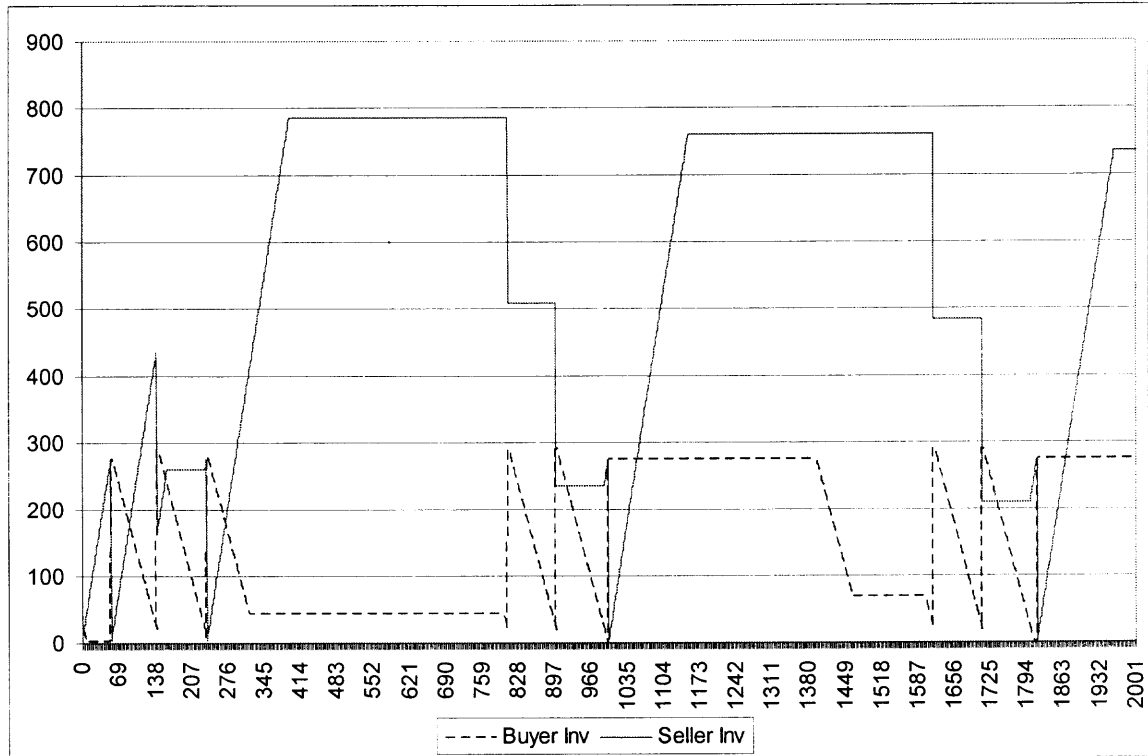
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	275	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	10			Reorder level ( $R_2$ )	20
Production rate/hr. ( $\delta_1$ )	3			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.14** Simulation #10. Average Buyer Inv.: 122.3487 Average Seller Inv.: 412.7651

**Table 4.12** Simulation #11

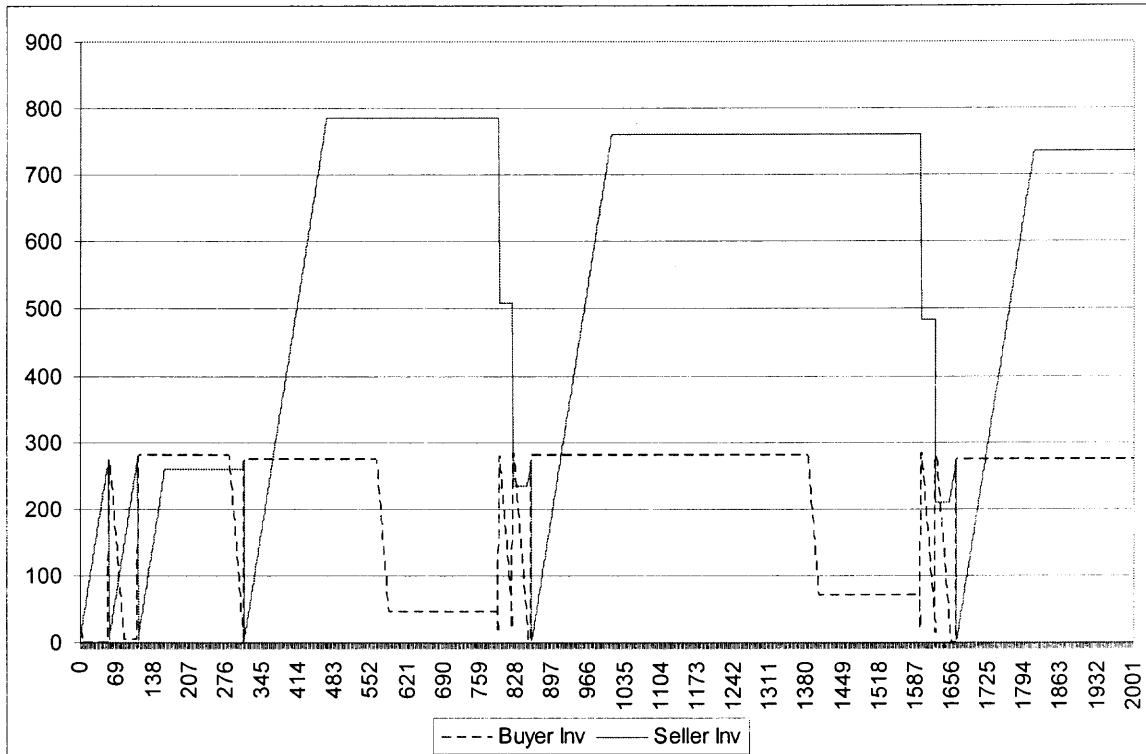
SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	275	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	10			Reorder level ( $R_2$ )	20
Production rate/hr. ( $\delta_1$ )	5			Production rate/hr. ( $\delta_2$ )	3



**Figure 4.15** Simulation #11. Average Buyer Inv.: 126.0105 Average Seller Inv.: 511.1098

**Table 4.13** Simulation #12

SELLER PARAMETERS		OTHER PARAMETERS		BUYER PARAMETERS	
Production batch size ( $B_1$ )	800	Replenishment quantity ( $Q_{ij}$ )	275	Production batch size ( $B_2$ )	800
Reorder level ( $R_1$ )	10			Reorder level ( $R_2$ )	20
Production rate/hr. ( $\delta_1$ )	5			Production rate/hr. ( $\delta_2$ )	10



**Figure 4.16** Simulation #12. Average Buyer Inv.: 204.4027 Average Seller Inv.: 512.1994

#### 4.4 The $(Q, R, \delta)^2$ Inventory Costs

In NEXUS the cost of supply is modeled for each part by each supplier as the sum of inventory-related costs, material costs and value-added or production costs. These are described as follows:

1. *Inventory Costs (VCOST)* - in the supply chain are classified into four categories:
  - i. Inventory holding cost of input parts for a buying supplier
  - ii. Inventory holding cost of output parts for a selling supplier
  - iii. Ordering and transport costs of input parts for a buying supplier
  - iv. In-transit inventory costs of output parts for a selling supplier
2. *Material Costs (MCOST)* - are the unit supply prices paid by the buyer to the seller as per the supply chain contract. Material costs are based on the acquisition

of raw materials needed for production of intermediate products and final products.

3. *Value-added or Production Costs (ACOST)* - are incurred by the supplier in manufacturing the output part. These costs are aggregated into two terms: the labor cost per unit and the equipment or facility cost per unit. Note that in NEXUS we assume that the labor cost per unit time for a specific supplier is fixed for all of its output products.

There are certain costs that we do not consider in our model. These include production setup cost, transport cost between suppliers, and any overhead costs. Possibly at a later stage these costs can also be incorporated into the model.

In this section we will derive these costs specifically for each supplier. The derivation will focus on the costs per unit final (end) product sold. This will enable us to compare the cost profit behavior of the suppliers. The derivation process will be built around a triad of sequential suppliers  $\tilde{i}$ ,  $i$  and  $\hat{i}$  as shown in figure 4.17. Note that every part must be included in the BOM of at least one final product. All our derivations here are for a specific final product denoted by  $J$ . The demand for each part is therefore derived from, and is given by  $P_{jJ}D_J$ . When there are multiple final products, then we have to calculate the costs for each of them separately.



**Figure 4.17** Three Suppliers Feeding Parts in  $j$  Assembly.

We assume that each supplier maintains three types of inventory: input parts, output products, and output products in-transit. The inventory level for each of these is

determined both by the supply chain contract (or design) and the order behavior of the succeeding supplier. To accurately model the inventory cost, we describe the supply cycle for a part as the interval between two successive production batch starts at the buying supplier. The supply cycle starts when the buying supplier begins production of output part  $\hat{j}$  which uses input part  $j$ . When the reorder level is reached the buyer releases an order to the seller who then ships the batch. When the seller's output inventory reaches the reorder level then a batch of part  $j$  production begins.

To derive the inventory costs for the buying and selling sides of supplier  $i$ , we initially assume that all replenishment is instantaneous. Later we will add the seller's in-transit inventory cost to account for the replenishment lag. In-transit cost refers to the cost of maintaining and insuring materials while they are being moved, rather than the cost of transportation. Inventory-related costs also include the buyer's order cost. We therefore define the following four sub-elements which together define the supplier inventory costs for part  $j$ . For convenience the cost elements are all defined in the context of part  $j$ . The inventory costs associated with part  $\hat{j}$  can then be derived for a representative supply cycle, and then further detailed to the unit part level. Later we will reorient these cost elements in the context of supplier  $i$ , making it easier to derive the costs specific to a supplier.

$V1_{i\hat{j}j}$       **Input side inventory cost** - of supplier  $\hat{i}$  for part  $j$  from supplier  $i$  for production of part  $\hat{j}$ , per supply cycle of  $\hat{j}$

$V2_{i\hat{i}j}$       **Output side inventory cost** - of supplier  $i$  for part  $j$  shipped to supplier  $\hat{i}$  for production of part  $\hat{j}$ , per supply cycle of  $\hat{j}$

$V3_{i\hat{i}j}$       **Supply Order Cost** - of supplier  $\hat{i}$  for part  $j$  from supplier  $i$  for production

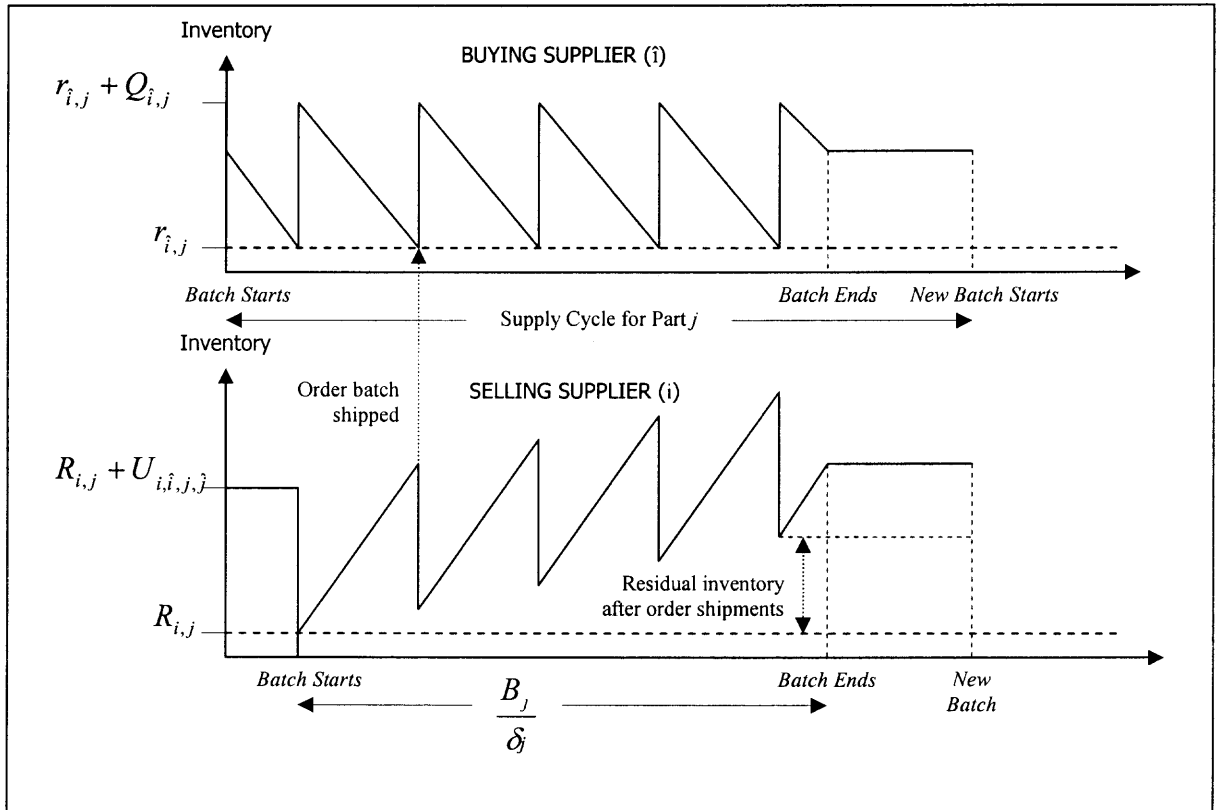
of part  $\hat{j}$ , per supply cycle of  $\hat{j}$

$V4_{i\hat{j}\hat{j}}$  **In-transit inventory cost** - of supplier  $i$  for part  $j$  shipped to supplier  $\hat{i}$  for production of part  $\hat{j}$ , per supply cycle of  $\hat{j}$

Note that our approach is that inventory costs  $V1$  and  $V3$  are borne by the buying supplier, while  $V2$  and  $V4$  are borne by the selling supplier. Also we relate all four inventory cost components to the supply cycle of part  $\hat{j}$  (defined in equation 4.39). This will enable us later (section 4.4.5) to track the inventory costs of each supplier in the context of a unit of final product. In the case where the assignments of  $V1$ ,  $V2$ ,  $V3$ , and  $V4$  are different from those assumed here, we will see later that the assumption is not restrictive and it is easy to reassign these costs to suit a particular application.

As mentioned earlier, traditional multi-echelon inventory models focus on the average and/or maximum inventory levels between supply pairs. In modern day supply chains the inventory levels tend to be low and fast moving. A true picture of the inventory costs can only be derived from modeling the specific dynamics between each pair of suppliers. Commonly in supply chain analytics, the inventory history of each supplier and part is studied in an attempt to track these true inventory costs. The NEXUS model provides a relatively accurate method for estimating these costs from the supply chain parametric data. The derivation of the actual inventory costs is based on the cyclical inventory level between a pair of suppliers. Clearly this inventory behavior has several different patterns based on the parametric relationships between the supply pair.





**Figure 4.18** Inventory Behavior for Part  $j$  in the Supply Cycle.

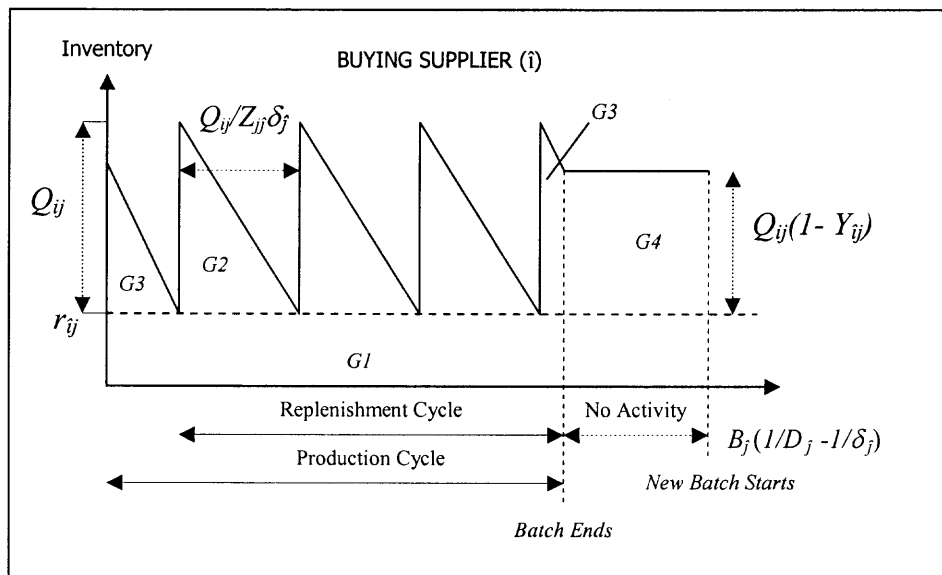
In figure 4.18 we illustrate this relationship for the nominal case of an intermediate product, for which the parametric relationships are:  $B_j > Q_{ij}$ ;  $\{B_j/D_j\} > \{B_j/\delta_j\}$  (buying supply cycle > selling supply cycle); and  $\delta_j > D_j$ . Figure 4.18 is an elaboration of figure 4.4. It plots the buying inventory dynamics for the supply of part  $j$  between the pair of suppliers  $i$  and  $\hat{i}$ . The selling inventory dynamics begins with the seller having no activity. At a future point it receives a sequence of replenishment orders from the buyer and on each instance it immediately ships  $Q_{ij}$  units.

While the supply cycle length will remain constant, the actual inventory dynamics will be slightly different in each supply iteration. For instance, the number of replenishments will increase and decrease by 1. Figure 4.18 provides a reliable estimate of the average behavior across a series of supply cycles, and we will use this to derive the

cost. Our derivation of the four inventory costs is derived from this nominal case and is described in the following sections. For each cost element there is some difference between intermediate, final, and procured parts, and the cost equations for each part type are presented.

#### 4.4.1 Input Side Inventory Costs (VI)

Figure 4.19 shows the details of the inventory behavior for an input part at the buying supplier ( $i$ ) for the length of the input supply cycle as defined by equation (4.39). As noted earlier this graph represents the behavior for an average cycle. Figure 4.19 illustrates the case when  $Z_{ij}B_j > Q_{ij}$  and the replenishment cycle ends before the production cycle.



**Figure 4.19** Inventory Behavior for Supply of Part  $j$  in Production of  $\hat{j}$  if  $Z_{ij}B_j > Q_{ij}$ .

To derive the total inventory cost during the buyer's input inventory cycle, we divide figure 4.19 into four areas: G1- the reorder or safety stock inventory; G2- the complete cycle replenishment inventory; G3- the partial cycle replenishment inventory;

and G4– the static or no activity period inventory. The G1 inventory is simply the product of the reorder level and the length of the supply cycle:

$$\text{G1 Inventory} = r_{ij} \{B_j/D_j\} \quad (4.41)$$

The replenishment cycle will consist of several complete order cycles, in each of which the order quantity  $Q_{ij}$  is fully consumed and the subsequent order triggered. In contrast, for an incomplete or partial cycle the consumption process ends before reorder, and the inventory overlaps another supply cycle. The number of complete order cycles ( $XI_{ij}$ ) in the supply of part  $j$  to supplier  $i$  is given by the integer portion of the ratio of the total part  $j$  demand per supply cycle and the order quantity. The length of each order cycle is given by  $Q_{ij}/\delta_j Z_{ij}$ . We therefore derive G2 as follows:

$$XI_{ij} = \text{Int}^+ \{Z_{ij}B_j / Q_{ij}\} \quad (4.42)$$

$$\text{G2 Inventory} = XI_{ij} (Q_{ij}/2) (Q_{ij}/\delta_j Z_{ij}) \quad (4.43)$$

Note that  $\text{Int}^+ \{ \}$  is a round-off function that selects only the integer portion of the number. In equation (4.43)  $Q_{ij}/2$  represents the average inventory, beyond the reorder level, during the replenishment cycle.

As shown in figure 4.19, it is possible that the supply cycle of the buyer will contain a partial replenishment cycle. Further, this partial cycle could be split into two parts, one at the start and one at the end of the production cycle. While the combined length of this partial cycle is constant, the average inventory level will vary between supply cycles, depending on the relationship between  $B_j$  and  $Q_{ij}$ . Since some cycles will have the early part of a partial cycle, others will have the later part, so we can approximate this average inventory by  $Q_{ij}/2$ . Let  $YI_{ij}$  be the relative length of the partial

order cycle in the supply of part  $j$  to supplier  $\hat{i}$ . This is derived as the fractional order lot used in the production of  $\hat{j}$ . We therefore derive G3 as follows:

$$YI_{ij} = \text{Int}^- \{Z_{ij}B_j / Q_{ij}\} \quad (4.44)$$

$$\text{G3 Inventory} = YI_{ij} (Q_{ij}/2) (Q_{ij}/\delta_j Z_{ij}) \quad (4.45)$$

Note that  $\text{Int}^- \{ \}$  is a function that selects only the fractional portion of the number.

Note that it is possible that  $Z_{ij}B_j / Q_{ij}$  is an integer and there are no partial cycles.

The inventory level during the no activity sub-cycle is the residual inventory at the end of the partial cycle. Again, this inventory will vary between cycles, but on average will be equal to the ending inventory at the end of the nominal partial cycle. Since the length of the production cycle for part  $\hat{j}$  is  $B_j/\delta_j$ , the length of the no activity cycle is  $B_j(1/D_j - 1/\delta_j)$ . We can thus derive G4 as follows:

$$\text{G4 Inventory} = Q_{ij} (1 - YI_{ij}) B_j (1/D_j - 1/\delta_j) \quad (4.46)$$

For the buyer's supply cycle the total inventory time product is given by G1+G2+G3+G4, and we therefore derive the inventory cost as:

$$VI_{ij} = Ch_j \left\{ r_{ij} B_j / D_j + (XI_{ij} + YI_{ij})(Q_{ij}/2)(Q_{ij}/\delta_j Z_{ij}) + Q_{ij} B_j (1 - YI_{ij})(1/D_j - 1/\delta_j) \right\}$$

$$\text{when } Q_{ij} < Z_{ij}B_j \quad (4.47)$$

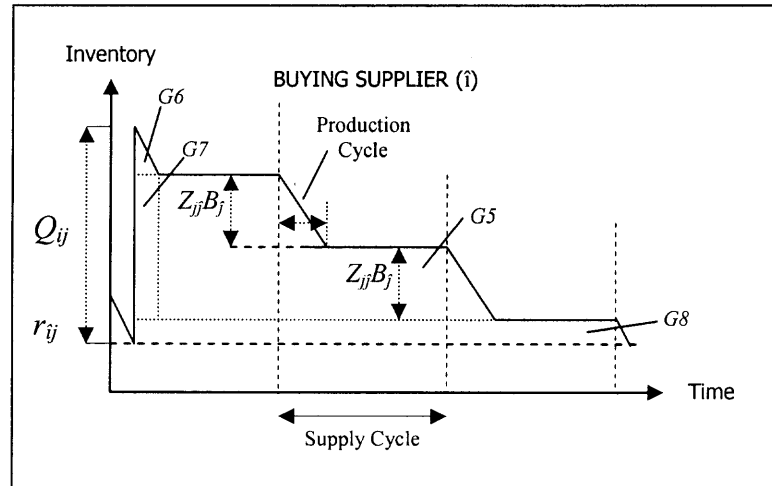
For the case when  $Q_{ij} > Z_{ij}B_j$  the inventory behavior is quite different as shown in figure 4.20. In this case each order replenishment of  $j$  will feed several production batches of  $\hat{j}$ , and hence there will be no replenishments for several supply cycles. Following each replenishment order the number of consecutive supply cycles with no replenishments is:

$$X2_{ij} = \text{Int}^+ \{Q_{ij} / Z_{ij}B_j\} - 1 \quad (4.48)$$

For example in figure 4.20, we illustrate the case when  $Q_{ij}/Z_{ij}B_j = 3.4$  and  $X2_{ij} = 2$ .

As the graph shows, there will be a replenishment in the first supply cycle, and none in the second and third supply cycles. Note that the replenishment satisfies  $X2_{ij}+1$  complete production cycles, and a partial production cycle. The relative length of this partial cycle is given by:

$$Y2_{ij} = \text{Int} \{ Q_{ij} / Z_{ij} B_j \} \quad (4.49)$$



**Figure 4.20** Inventory Behavior for Supply of Part  $j$  in Production of  $j$  When  $Q_{ij} > Z_{ij} B_j$ .

Following a series of replenishments, therefore, these partial cycles will add up and there will be an extra supply cycle with no replenishment. The total inventory associated with each replenishment of  $j$  must therefore be summed over multiple supply cycles and a partial cycle. For the supply cycles with no replenishment the inventory is:

$$\text{G5 Inventory} = \sum_{k=1 \text{ to } X2_{ij}} \{ (k-1)(B_j/D_j) Z_{ij} B_j + (B_j/\delta_j) Z_{ij} B_j / 2 \} \quad (4.50)$$

$$\text{G5 Inventory} = X2_{ij} (X2_{ij} - 1)(B_j^2 Z_{ij} / D_j) / 2 + X2_{ij} (B_j^2 Z_{ij} / \delta_j) / 2 \quad (4.51)$$

$$\text{G5 Inventory} = X2_{ij} (B_j^2 Z_{ij} / 2) (X2_{ij} / D_j - 1 / D_j + 1 / \delta_j) \quad (4.52)$$

In the supply cycle with replenishment, the replenishment will occur some time in the middle of the production cycle. On average this will occur at the production cycle

mid-point, but as noted earlier for a specific cycle this will vary. The G6 inventory sums the consumption inventory just before and after the replenishment, and is given by:

$$G6 \text{ Inventory} = (Z_{ij}B_j/4) (B_j/\delta_j) \quad (4.53)$$

G7 and G8 represent residual inventories during the supply cycle and are approximated by the average levels as follows:

$$G7 \text{ Inventory} = (Z_{ij}B_j) (B_j/2\delta_j) \quad (4.54)$$

$$G8 \text{ Inventory} = (Z_{ij}B_j/4) \{ (X2_{ij} + 1)(B_j/D_j) - (B_j/2\delta_j) \} \quad (4.55)$$

After every  $1/Y2_{ij}$  replenishments an extra supply cycle formed by the sum of the partial cycles will result. We label the inventory associated with this cycle as G9, which is given by:

$$G9 \text{ Inventory} = Y2_{ij} (Z_{ij}B_j/2) \quad (4.56)$$

In equation (4.56) the inventory burden of the extra cycle is distributed across the preceding supply cycles. For the buyer's supply cycle the total inventory time product is given by  $G1+G5+G6+G7+G8+G9$ , and we therefore derive the inventory cost as:

$$VI_{ijj} = Ch_j \{ r_{ij} B_j/D_j \} + \\ Ch_j \{ X2_{ij}(B_j^2 Z_{ij}^2/2) (X2_{ij}/D_j - 1/D_j + 1/\delta_j) + (Z_{ij}B_j/4)(B_j/\delta_j) + (Z_{ij}B_j) (B_j/2\delta_j) + \\ (Z_{ij}B_j/4) \{ (X2_{ij} + 1)(B_j/D_j) - (B_j/2\delta_j) \} + Y2_{ij}(Z_{ij}B_j/2) \} / (X2_{ij} + 1 + Y2_{ij})$$

$$\text{when } Q_{ij} > Z_{ij}B_j \quad (4.57)$$

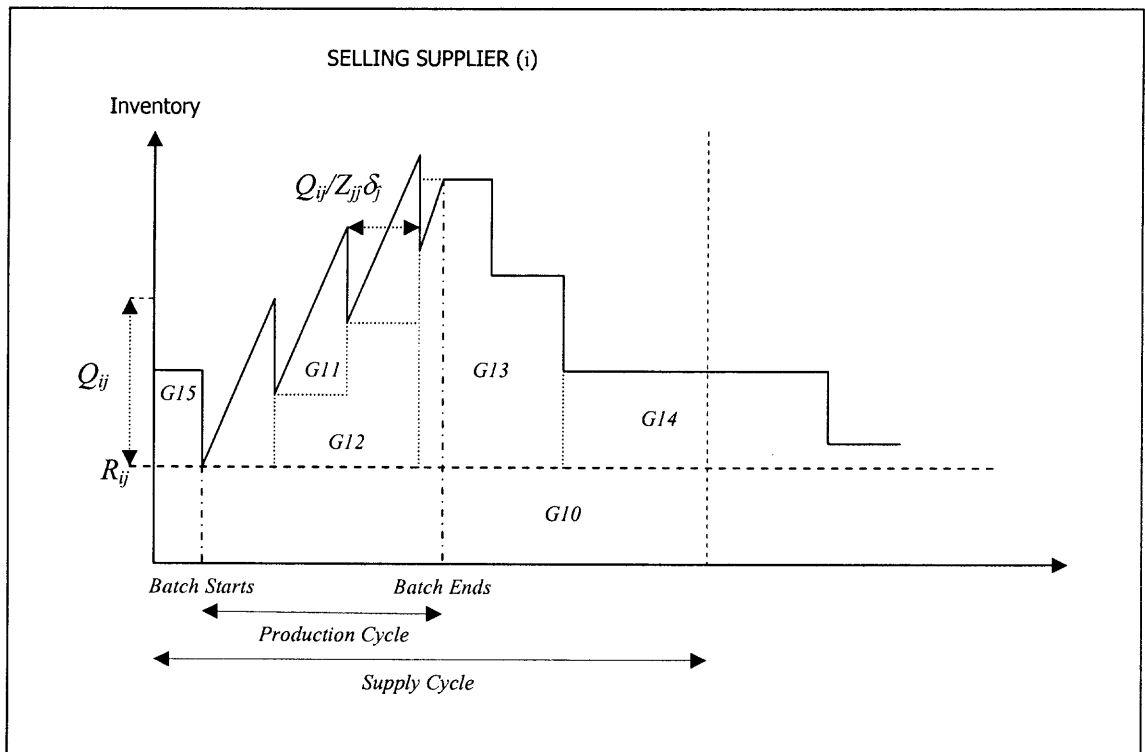
$$VI_{ijj} = Ch_j \{ r_{ij} B_j/D_j \} + Ch_j \{ Y2_{ij} B_j^2 Z_{ij}^2 / 2 Q_{ij} \} \\ + Ch_j (B_j^3 Z_{ij}^2 / 2 Q_{ij}) \{ (X2_{ij}^2 - (X2_{ij}/2) + 1/2)(1/D_j) + (X2_{ij} + 5/4)(1/\delta_j) \}$$

$$\text{when } Q_{ij} > Z_{ij}B_j \quad (4.58)$$

In equation (4.58) the  $G5+G6+G7+G8+G9$  costs are divided by the number of supply cycles associated with each replenishment, since  $Vl_{ijj}$  is the inventory cost per supply cycle.

#### 4.4.2 Output Side Inventory Costs ( $V2$ )

Figure 4.21 shows the details of the inventory behavior for output part  $j$  at the selling supplier  $i$  for the length of the input supply cycle as defined by equation (4.39). The inventory behavior for the seller output can take on several different patterns, based on the relationship between the seller's production batch and the buyer's production batch.



**Figure 4.21** Seller Side Inventory Behavior of Product  $j$  in the Supply Cycle.

As in the case of the buyer inventory graph, figure 4.21 also represents the behavior for an average seller cycle. Depending on the production rate, the

replenishment cycle may be shorter than the production cycle. In figure 4.21 we show the case where it is longer. To derive the total inventory cost during the seller's output inventory cycle, we break up figure 4.21 into the following areas: G10- the reorder or safety stock inventory; G11- the full cycle triangular inventory when the production and replenishment cycles overlap; G12- the full cycle step inventory when the production and replenishment cycles overlap; G13 – the partial production cycle step inventory plus the step inventory during the remainder of the replenishment cycle; and G14 – the no activity inventory.

The G10 or reorder inventory is simply the product of the reorder level and the length of the supply cycle:

$$\text{G10 Inventory} = R_{ij}(B_j/D_j) \quad (4.59)$$

The number of production batches required is given by:

$$N_{jj} = \text{Int}^+ \{ B_j Z_{jj} / B_j \} + 1 \quad (4.60)$$

The total production in a supply cycle of part  $j$  for  $\hat{j}$  is therefore  $N_{jj}B_j$ , and the length of the seller production cycle is given by equation  $N_{jj}B_j/\delta_j$ . An additional condition that we assume here is that the production cycle of part  $j$  is shorter than the supply cycle of part  $j$  for  $\hat{j}$ , that is  $N_{jj}B_j/\delta_j < (B_j/D_j)$ . We also make one important assumption about the seller production behavior, that is, in every production cycle the seller will manufacture  $N_{jj}$  batches of part  $j$ , regardless of the inventory levels. These assumptions are needed to complete our modeling of the seller's inventory costs. As an extension to this research the exception to this condition can be studied. During a supply cycle the number of part  $j$  replenishment shipments from supplier  $i$  is given by  $XI_{ij}+1$ ,



where  $XI_{ij}$  is defined by equation (4.42). Then the number of replenishments which occur during the seller production cycle is given by:

$$X3_{ij} = \text{Int}^+ \{ (N_{ij} B_j Z_{jj} / Q_{ij}) (\delta_j / \delta_j) \} \quad (4.61)$$

when  $X3_{ij} \leq XI_{ij}$

Since there is one replenishment which coincides with the production batch start, there will be  $X3_{ij}$  full cycle triangular inventories when the production and replenishment cycles overlap. We therefore derive G11 as follows:

$$\text{G11 Inventory} = X3_{ij} (\delta_j / 2) (Q_{ij} / \delta_j Z_{jj})^2 \quad (4.62)$$

$$\text{G11 Inventory} = X3_{ij} (Q_{ij}^2 / 2 \delta_j Z_{jj}^2) (\delta_j / \delta_j) \quad (4.63)$$

Observe that it is possible that  $X3_{ij} = 0$  and there is no G11 or G12 inventory. The step inventory during the production cycle will build up in a series of blocks. The block increments are given by the difference in production rate and shipment quantity of the part, which is  $(\delta_j Q_{ij} / \delta_j Z_{jj}) - Q_{ij}$ . The first replenishment block will have no block, while the subsequent replenishments will have 1,2,.. blocks. These can be summed as an arithmetic series. We therefore derive G12 as follows:

$$\text{G12 Inventory} = \{ X3_{ij} (X3_{ij} - 1) / 2 \} \{ (\delta_j Q_{ij} / \delta_j Z_{jj}) - Q_{ij} \} (Q_{ij} / \delta_j Z_{jj}) \quad (4.64)$$

$$\text{G12 Inventory} = \{ X3_{ij} (X3_{ij} - 1) / 2 \} (Q_{ij}^2 / \delta_j Z_{jj}) \{ (\delta_j / \delta_j Z_{jj}) - 1 \} \quad (4.65)$$

when  $\delta_j \geq \delta_j$

The number of replenishments shipped after the production cycle ends is given by the total shipments per supply cycle minus the number during production, that is  $XI_{ij} + 1 - X3_{ij}$ . Observe that it is possible that  $X3_{ij} = XI_{ij} + 1$  and there are no shipments during this period. The inventory during this period is also represented by a series of blocks. The block increments are the shipment quantity  $Q_{ij}$ . The G13 inventory is therefore given by

the sum of these blocks minus the triangular inventory during the last partial period of part  $j$  production (when production ends before the next shipment). The relative height of this partial period is given by  $N_{jj}B_j - X3_{ij}Q_{ij}$  and the length is  $\{N_{jj}B_j - X3_{ij}Q_{ij}\}/\delta_j$ . We therefore derive G13 as follows:

$$\begin{aligned} \text{G13 Inventory} = & \{(X1_{ij} - X3_{ij})(X1_{ij} - X3_{ij} + 1)/2\} Q_{ij} (Q_{ij}/\delta_j Z_{jj}) \\ & - \{N_{jj}B_j - X3_{ij}Q_{ij}\}^2/2\delta_j \end{aligned} \quad (4.66)$$

At the start of the no activity period the residual inventory is approximated by a classical inventory balance equation. We assume that at the start of the supply cycle we have enough inventory to supply at least one replenishment. Since the total number of shipments is  $X1_{ij} + 1$  then  $X1_{ij}$  shipments are from the current production. Therefore,  $U_{ij}$  the residual inventory at the start of the no-activity period is:

$$U_{ij} = N_{jj}B_j - X1_{ij}Q_{ij} \quad (4.67)$$

We have to account for this inventory from the end of the last full cycle shipment as shown in figure 4.21. We need to also consider the length of the starting inventory period, which ends with the first replenishment. Earlier in subsection 4.4.1 we said that on average the first replenishment will occur after a half order cycle. The G14 inventory is then given by:

$$\text{G14 Inventory} = U_{ij} \{B_j/D_j - (X3_{ij} + 0.5)(Q_{ij}/\delta_j Z_{jj})\} \quad (4.68)$$

In the case when production continues beyond the last shipment, then G13 as defined by equation 4.66 will be negative. This will ensure that the partial period inventory will be subtracted from the G14 inventory, since in this case the partial cycle will overlap the G14 space. The starting period assumes we will have enough stock to supply one shipment. The inventory burden is therefore:

$$G15 \text{ Inventory} = Q_{ij}^2/2\delta_j Z_{jj} \quad (4.69)$$

To derive the seller's inventory cost we distribute the inventory costs of the production and no-production supply cycles over a 10 cycle window. The number of seller production cycles in the 10 supply cycle window is approximated by:

$$X4_{ij} = \text{Int}^+ \{10(Z_{jj}B_j)/N_{jj}B_j\} + 1 \quad (4.70)$$

The number of no-production cycles is therefore  $10-X4_{ij}$ , and on average every 10 supply cycles there will be  $10/X4_{ij}$  production cycles between each no-production cycle. On average the burden of carrying forward the residual inventory from each production cycle to the no-production cycle is given by:

$$G16 \text{ Inventory} = (N_{jj}B_j - Z_{jj}B_j) ((10/X4_{ij}) - 1) \quad (4.71)$$

For the no-production cycle there will be  $X1_{ij}$  replenishments from the starting inventory, and has a pattern similar to the G13 inventory. The no-production inventory is then given by:

$$G17 \text{ Inventory} = X1_{ij} (X1_{ij}+1) (Q_{ij}/2) (Q_{ij}/\delta_j Z_{jj}) \quad (4.72)$$

For the buyer's supply cycle the total inventory in the production cycle must include  $G11+G12+G13+G14+G15$ , as well as the residual inventory carry-forward burden of  $G16$  and the no-production cycle inventory of  $G17$ . We therefore derive the inventory cost as:

$$\begin{aligned} V2_{ijj} = & Ch_j \{ R_{ij}(B_j/D_j) \} + \\ & Ch_j(X4_{ij}/10) \{ X3_{ij}(Q_{ij}^2/2\delta_j Z_{jj}^2)(\delta_j/\delta_j) + (X3_{ij}(X3_{ij}-1)/2) (Q_{ij}^2/\delta_j Z_{jj}) \{ (\delta_j/\delta_j Z_{jj})-1 \} + \\ & ((X1_{ij}- X3_{ij})(X1_{ij}- X3_{ij}+1)/2) Q_{ij} (Q_{ij}/\delta_j Z_{jj}) - \{ N_{jj}B_j - X3_{ij}Q_{ij} \}^2/2\delta_j \\ & + U_{ij} \{ B_j/D_j - (X3_{ij}+0.5)(Q_{ij}/\delta_j Z_{jj}) \} + Q_{ij}^2/2\delta_j Z_{jj} + (N_{jj}B_j - B_j Z_{jj})((10/X4_{ij})-1) \} + \end{aligned}$$

$$Ch_j \{1 - (X_{4ij}/10)\} \{ X_{1ij} (X_{1ij} + 1) (Q_{ij}/2) (Q_{ij}/\delta_j Z_{jj}) \}$$

$$\text{when } B_j/B_j Z_{jj} < 2, R_{ij} = Q_{ij} \text{ and } \hat{j} \text{ is not a final product} \quad (4.73)$$

Simplifying, we get:

$$\begin{aligned} V_{2ijj} = & Ch_j \{ R_{ij}(B_j/D_j) \} + \\ & Ch_j (X_{4ij}/10) \{ (Q_{ij}^2/\delta_j Z_{jj}) [ .5X_{3ij}^2 (\delta_j/\delta_j Z_{jj}) + \\ & (0.5X_{1ij}^2 + 0.5X_{1ij} - X_{1ij} X_{3ij} + .5) ] \\ - & \{ N_{jj}B_j - X_{3ij}Q_{ij} \}^2/2\delta_j + U_{ij} \{ B_j/D_j - (X_{3ij} + 0.5)(Q_{ij}/\delta_j Z_{jj}) \} + (N_{jj}B_j - B_j Z_{jj})((10/X_{4ij}) - 1) \} + \\ & Ch_j \{1 - (X_{4ij}/10)\} (Q_{ij}^2/\delta_j Z_{jj}) \{ 0.5X_{1ij}^2 + 0.5X_{1ij} \} \end{aligned}$$

$$\text{when } B_j/B_j Z_{jj} < 2, R_{ij} = Q_{ij} \text{ and } \hat{j} \text{ is not a final product} \quad (4.74)$$

$$\begin{aligned} V_{2ijj} = & Ch_j \{ R_{ij}(B_j/D_j) \} + Ch_j (Q_{ij}^2/\delta_j Z_{jj}) \{ 0.5X_{1ij}^2 + 0.5X_{1ij} \} + \\ & Ch_j (X_{4ij}/10) \{ (Q_{ij}^2/\delta_j Z_{jj}) [ .5X_{3ij}^2 (\delta_j/\delta_j Z_{jj}) - X_{1ij} X_{3ij} + .5 ] \\ - & \{ N_{jj}B_j - X_{3ij}Q_{ij} \}^2/2\delta_j + U_{ij} \{ B_j/D_j - (X_{3ij} + 0.5)(Q_{ij}/\delta_j Z_{jj}) \} + (N_{jj}B_j - B_j Z_{jj})((10/X_{4ij}) - 1) \} \end{aligned}$$

$$\text{when } B_j/B_j Z_{jj} < 2, R_{ij} = Q_{ij} \text{ and } \hat{j} \text{ is not a final product} \quad (4.75)$$

In the case where part  $j$  is used in the manufacture of several parts  $\hat{j}$ , then one could argue that the production batches may be shared and hence the seller's inventory cost would be less than  $V_{2ijj}$ .

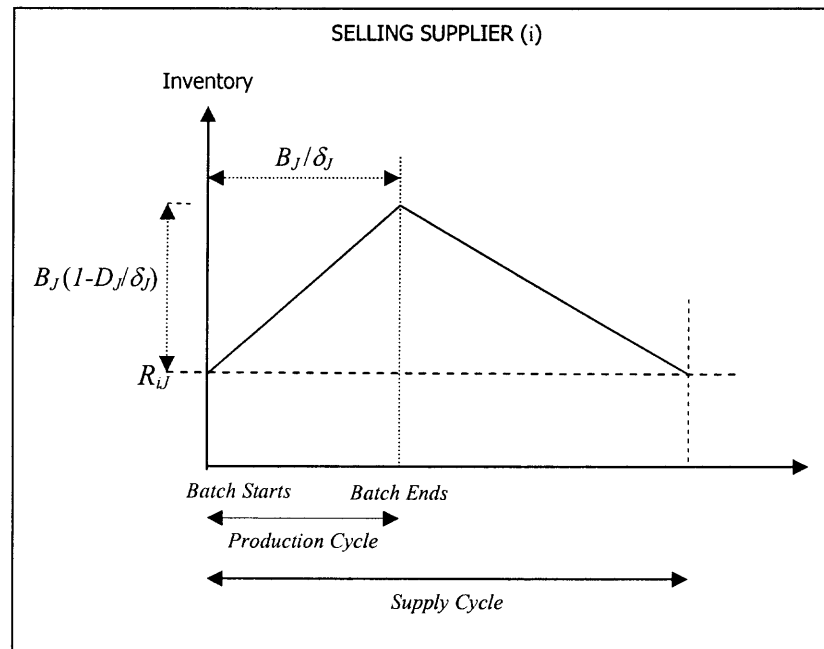
For the case where part  $\hat{j}$  is a final product the inventory behavior is quite different, as noted in subsection 4.3.3. In this case the inventory behavior cannot be approximated by equation (4.75) because there is no subsequent supplier. We assume

that the final product is delivered directly to the customer in a batch size of one. Further the delivery is at a uniform rate across the supply cycle. Figure 4.22 shows the inventory behavior. The supply cycle length is given by  $B_J/D_J$ . During the first part of the cycle both customer delivery and part production are occurring. The inventory will build up to the maximum point  $B_J(1-D_J/\delta_J)$ . We therefore derive the inventory cost as:

$$V2_{i0j0} = Ch_J \left\{ R_{iJ}(B_J/D_J) + 0.5B_J^2 (1/D_J - 1/\delta_J) \right\}$$

where  $J$  is a final product (4.76)

In the notation  $V2_{i0j0}$  of equation (4.76), '0' represents the end customer.



**Figure 4.22** Inventory Behavior of Final Product  $J$  in the Supply Cycle.

#### 4.4.3 Buyer's Supply Order Costs ( $V3$ )

We assume that the buyer has to bear a fixed cost associated with each replenishment order. This cost will be made of the components commonly considered in classical

inventory modeling: order processing, transportation, and receiving. In modern supply chains we can expect that the order processing costs should be quite small, given the efficiencies of ERP systems and the accompanying information technology. The physical costs of these replenishment orders, however, in particular the transportation costs, are much more difficult to reduce. Modern supply chains emphasize more frequent replenishments with smaller inventory levels, but a key determinant of the economics of this approach will be  $Co_{ij}$ . While we do not consider the demand uncertainty here, one benefit of this approach is that it does allow the ERP system to match supply SKUs more closely with demand SKUs.

Since on average the number of replenishment orders per supply cycle is  $B_j Z_{jj} / Q_{ij}$ , the order costs per supply cycle are:

$$V3_{ijj} = Co_{ij} \{ B_j Z_{jj} / Q_{ij} \} \quad (4.77)$$

From a cost modeling standpoint we expect an inverse relationship between  $Co_{ij}$  and  $Q_{ij}$ . In many cases the supply chain velocity is increased via the use of small package delivery services or LTL (less than truckload) shippers. These transportation vendors are able to ship smaller quantities of part  $j$  in shorter intervals. Typically these will incur a much higher  $Co_j$ . By using the NEXUS model we can evaluate in greater detail the economics of these transportation options.

#### 4.4.4 Seller's In-Transit Inventory Costs (V4)

In deriving the inventory costs in sections 4.4.1 and 4.4.2 it was assumed that inventory replenishments between the seller and buyer were instantaneous. While retaining this assumption, it is still possible to account for the inventory burden associated with the

transport of the inventory. If the reorder level is sufficiently padded to represent the transport or transit time, the instant replenishment assumption can still hold. Each replenishment shipment travels for a fixed transport time, hence the in-transit inventory costs per supply cycle are:

$$V4_{ijj} = Ch_j (B_j Z_{ij} / Q_{ij}) (T_{ij} Q_{ij}) \quad (4.78)$$

$$V4_{ijj} = Ch_j (B_j Z_{ij} T_{ij}) \quad (4.79)$$

From equation (4.79) we see that the in-transit inventory cost is independent of the order quantity.  $V4_{ijj}$  is a function only of the variable  $T_{ij}$ , and can be quite significant when  $T_{ij}$  is large. We find that the  $V4_{ijj}$  cost must be evaluated in combination with the  $V3_{ijj}$  cost. One reason for this is that we also expect an inverse relationship between  $Co_{ij}$  and  $T_{ij}$ . Typically, the V3 and V4 costs are assigned to different parties, and as a result an integrated view of the cost dynamics is missed. In NEXUS we are able to see this linked view and hence work towards reducing the net cost.

#### 4.4.5 Gross Inventory Costs Per Unit Final Product

In the preceding sections we derived the four inventory cost components in the context of the part  $j$  supply cycle. Here we extend these derivations to get the costs per unit of final product. Each supply cycle for part  $j$  produces the quantity  $B_j$ , and then this output is used in  $B_j/P_{jJ}$  units of end product  $J$ . Note that  $P_{jJ}$  is the bill of materials explosion quantity for part  $j$ , and was derived in chapter 3. We first derive the input side inventory costs of supplier  $i$  for a unit of final product  $J$ , as:

$$I'_{iJ} = \sum_j \sum_r \sum_{j'} \{ (V1_{rj'j} + V3_{rj'j}) / (B_j / P_{jJ}) \}$$

$$\text{where } Z_{j'j} > 0, P_{jJ} > 0, j \neq J \quad (4.80)$$

The output side inventory costs of supplier  $i$  for a unit of final product  $J$ , are then derived as follows:

$$I_{iJ}^O = \sum_j \sum_i \sum_j \{ (V2_{iij} + V4_{iij}) / (B_j / P_{jJ}) \} + \sum_i \sum_J \{ (V2_{i0J0} / B_J) \}$$

$$\text{where } Z_{ij} >, P_{jJ} > 0 \quad (4.81)$$

The gross inventory costs of supplier  $i$  per unit of final product  $J$  is then given by  $I_{iJ}^I + I_{iJ}^O$ . Differentiating between the input and output side costs provides the NEXUS model with the ability to further analyze the cost drivers of each supplier. Where multiple final products are being studied, then summing  $I_{iJ}^I + I_{iJ}^O$  over  $J$  will provide us with the costs across the portfolio of final products.

#### 4.5 Supplier Profits As a Function of Inventory Costs

In this section we derive functions for material costs and value-added (i.e., production) costs. These functions and the functions developed earlier for inventory costs (see section 4.4) are then used to generate predictive analytics for the supply chain. Specifically, we analyze the supply chain profitability and inventory using the following three metrics:

1. Profit earned by an individual supplier from a particular final product
2. The sharing of profit among all suppliers within a supply chain
3. Distribution of market values of all physical inventory within a supply chain

##### 4.5.1 Derivation of Material Costs

Material costs include all the procurement costs (excluding order costs) of raw materials needed for the production of intermediate products and final products. We consider only



the direct material input by a supplier to the final product. Indirect costs and other consumable supplies are not considered. The material cost is obtained from summing the supply costs of each part assembled into the output products by supplier  $i$ . For each unit final product  $J$  that includes output part  $j$ , the number of each input part is  $Z_{jJ}P_{jJ}$ . Then the material costs of supplier  $i$  per unit of final product  $J$  is:

$$M_{iJ} = \sum_j \sum_i \sum_j \{Cs_{ij}Z_{jJ}P_{jJ}\} \mid O_{ij}=1 \quad (4.82)$$

The condition in equation (4.82) ensures that  $M_{iJ}$  only includes the output parts of supplier  $i$ . Material costs will be a significant cost element for many suppliers. Observe that  $Cs_{ij}$  is the supply contract price, which is the primary determinant of material cost. The NEXUS model enables supply chain analysts to manipulate this cost so as to shift profits between suppliers.

#### 4.5.2 Derivation of Value-Added or Production Costs

We consider only the direct costs expended by the supplier in producing or processing the product. This is commonly referred to as the value-added or production costs. The value added to a product by a supplier is measured in terms of two factors: (1) labor cost, and (2) assembly resource cost. Both of these factors are measured per unit time. The assembly resource cost includes the cost of production equipment, tooling and any consumables. We introduce  $\tau_j$  as the unit production time for part  $j$ . Where the supplier is a warehouse or retail store then this would be the material handling time.

For the duration of  $\tau_j$  we assume that the associated assembly resources will be occupied at all time. The associated direct labor, however, may not be locked for the entire period. Let  $Lu_j$  be the labor utilization rate in the production of part  $j$ . This

represents the percent of unit assembly time  $\tau_j$  during which the labor resource will be dedicated to the production operation. For instance if  $\tau_j=2$  mins but a single worker is required for only 1 minute, then  $Lu_j=50\%$ . This notation also provides us with the flexibility to model the case when multiple workers are required. For instance when  $\tau_j=2$  mins and the required labor resource is 4 workers for 1 minute each then  $LU_j=200\%$ .

For each output part the cost per unit production time is then given by  $Cw_i Lu_j + Ca_j$ . The multiplier  $P_{jJ}$  then relates this cost to a unit of final product. The value added cost at supplier  $i$  is then given by:

$$A_{iJ} = \sum_j \sum_{i'} \sum_{j'} P_{jJ} (Cw_i Lu_j + Ca_j) \tau_j \quad | \quad O_{ij}=1 \quad (4.83)$$

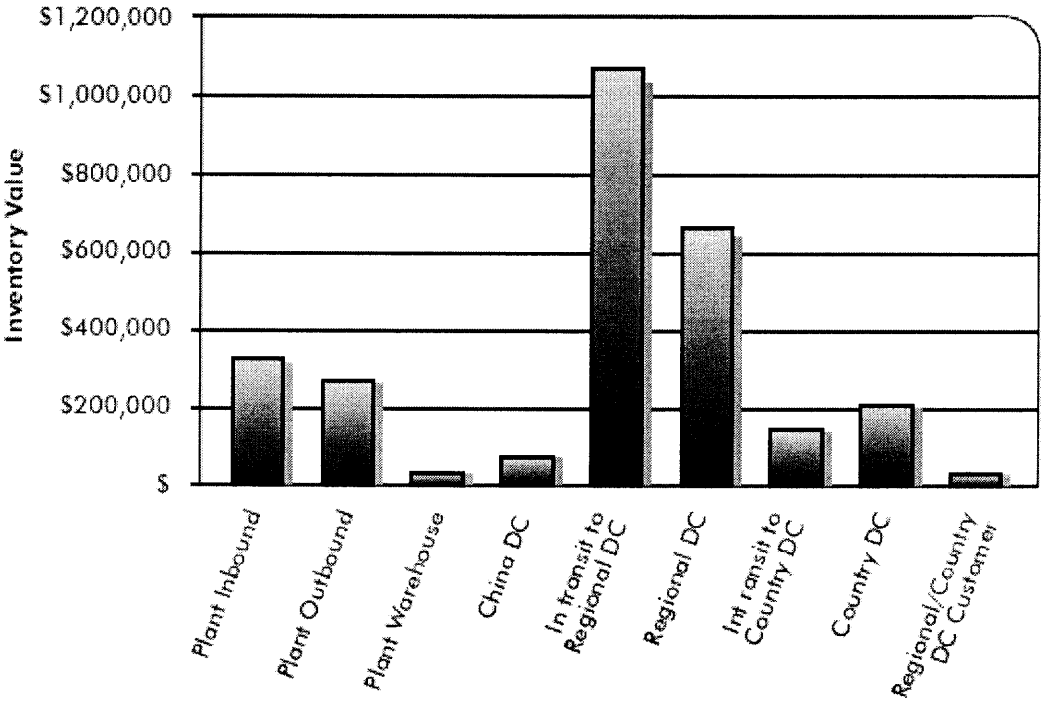
The condition in equation (4.83) ensures that  $A_{iJ}$  only includes the output parts of supplier  $i$ . In supply chain analysis we typically assume that the value adding or production costs are given, and are not the focus of supply chain cost reduction. The NEXUS model positions these costs in relation to all other costs in the supply chain. This helps to determine when the labor costs of a particular supplier are significantly affecting the profits of the supply chain. If the impact on profits is significant then analysts must consider whether production should be moved to a supplier with lower labor costs.

### 4.5.3 Inventory Cost Distribution

One of the very interesting questions in contemporary supply chain research is “how the inventory is distributed in the supply chain.” In supply chain analytics this is commonly referred to as *Inventory Positioning Analysis*, which provides supply chain performance analytics centered around inventory-related issues. Such issues include demand, ability

to meet demand, inventory turns, inbound supplies, quantities on hand, and other key metrics. The business value of inventory positioning analysis comes from the ability to limit the direct costs of maintaining excess inventory, as well as the direct and indirect costs of not meeting the just-in-time requirements of partners and OEMs.

An example output graph from inventory positioning analysis is shown in figure 4.23. This graph identifies the average inventory level at each point (or node) in the supply chain for an example product. Further analytics on this graph can be used to identify the actual inventory drivers in the supply chain. In the example we see that though there is less safety stock due to repositioning of inventory, there is more in-transit inventory and higher cycle stock levels. This implies that the factors affecting in-transit inventory and cycle stock would have a larger impact on overall inventory levels in the network configuration.



**Figure 4.23** An Example Inventory Positioning Analysis Report.

Traditionally inventory positioning analysis has been done by either studying the historical data or by conducting a simulation study. Our proposition is that the NEXUS model provides us with a reliable estimate of the inventory positioning data. Further, it provides a platform for the development of prescriptive models to optimize the inventory positioning strategy.

In section 4.4 we developed estimates for the different inventory levels at each supplier. These will now be used to create the inventory positioning data. We introduce the following notation:

$V_{ij}^I$  Average input side inventory value at supplier  $i$  for final product  $J$

$V_{ij}^O$  Average output side inventory value at supplier  $i$  for final product  $J$

Equations (4.57) and (4.58) define the input side inventory costs per supply cycle. Dividing this by the holding cost and the supply cycle length will give us the average input side inventory level. The input side inventory level for supplier  $i$  is therefore derived as follows:

$$V_{ij}^I = \sum_j \sum_i \sum_{j'} \{ C_{s_{ij'}} V_{1_{i'j'j}} / Ch_{j'} (B_j/D_j) \}$$

$$\text{where } Z_{jj'} >, P_{jj'} > 0, j \neq J \quad (4.84)$$

Equations (4.75) and (4.76) define the output side inventory costs per supply cycle. Dividing this by the holding cost and the supply cycle length will give us the average output side inventory level. The output side inventory level for supplier  $i$  is therefore derived as follows:

$$V_{ij}^O = \sum_j \sum_i \sum_j \{ C_{s_{ij}} V_{2_{iij}} / Ch_j (B_j/D_j) \} + \sum_i \sum_J \{ C_{s_{iJ}} V_{2_{i0J0}} / Ch_J (B_J/D_J) \}$$

$$\text{where } Z_{jj'} >, P_{jj'} > 0 \quad (4.85)$$

The total inventory at supplier  $i$  is then  $V_{ij}^I + V_{ij}^O$ . This data can then be used to generate a graph similar to that shown in figure 4.23.

#### 4.5.4 Supplier Profit Ratio Equilibrium

Our first objective in the NEXUS model is to estimate each supplier's contribution to the cost of each final product. This cost contribution is derived from summing the inventory, material, and value adding costs derived in this chapter, as follows:

$$\text{Supplier Cost per unit Final Product} - J = I'_{iJ} + I^O_{iJ} + M_{iJ} + A_{iJ} \quad (4.86)$$

Only direct costs are considered here, since most indirect costs cannot be controlled by the supply chain model. The net revenue for the supplier is determined by the total parts supplied per unit product, and the associated supply contract price. The net profit of supplier  $i$  per unit of final product  $J$  is then given by:

$$\Pi_{iJ} = \sum_j \{Cs_{ij}P_{jJ}\} - \{I'_{iJ} + I^O_{iJ} + M_{iJ} + A_{iJ}\} \mid O_{ij}=1 \quad (4.87)$$

The total profit in the chain for final product  $J$  is then:

$$\Psi_J = \sum_i \Pi_{iJ} \quad (4.88)$$

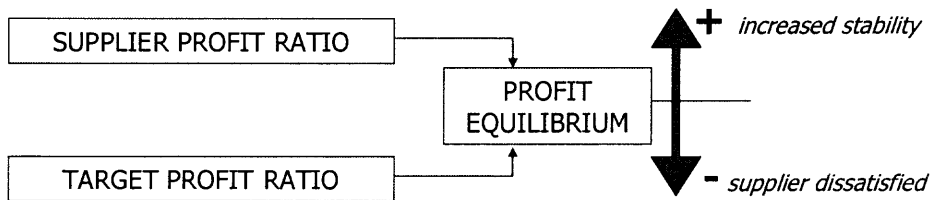
We now define “stability of the supply chain” as the likelihood that one or more suppliers will violate their contractual obligations (e.g., order quantities, delivery lead times, etc.) or terminate their participation in the chain. One reason for supply chain instability is that a supplier's net profit ( $\Pi_{iJ}$ ) is not attractive enough and the supplier becomes unstable. Since supply chains are designed to operate with low levels of inventory and frequent replenishments, the stability of the chain is dependent on all chain entities (suppliers) being equitably compensated for the goods or services they provide. We derive supply chain stability below as a function of the nominal and benchmark profit equilibrium. The predictive analytics of NEXUS enable us to (i) identify which suppliers,

if any, are approaching a profit disequilibrium, and (ii) evaluate possible solutions to this problem.

The profit ratio ( $\eta_{ij}$ ) of supplier  $i$  for final product  $J$  is derived as a function of the net unit profit ( $\Pi_{ij}$ ) and the value adding ( $A_{ij}$ ) and inventory ( $I^I_{ij}$  and  $I^O_{ij}$ ) costs of that supplier. These two costs represent the true capital outlay of the supplier, and the base on which any return on investment would be computed. We exclude the material cost ( $M_{ij}$ ) because we consider it to be a pass-through cost within a supply chain. Observe that in the case of a warehouse facility, if the material cost was included it would seem that it has a very high capital investment per unit, which is not really the case. The profit ratio is therefore given by:

$$\eta_{ij} = \Pi_{ij} / \{I^I_{ij} + I^O_{ij} + A_{ij}\} \tag{4.89}$$

Our proposition is that the profit ratio is a key determinant of the stability of the chain. Let  $\eta^T_{ij}$  be the target profit ratio for a supplier. The target profit ratio can be set equal to the industry average gross margin for that supplier category. For example, we may know that sheet metal manufacturers typically operate with a 25% gross margin so  $\eta^T_{ij}$  would be set to this percentage. Figure 4.24 illustrates our definition for estimating a supplier’s profit equilibrium for a specific final product.

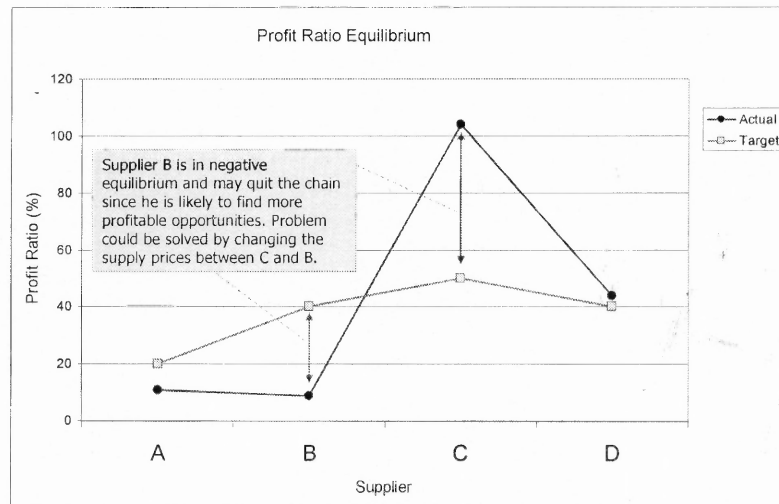


**Figure 4.24** The Supplier Profit Equilibrium.

The profit equilibrium  $\varepsilon_{iJ}$  for supplier  $i$  is the difference between the supplier profit ratio and the target profit ratio, that is  $\varepsilon_{iJ} = \eta_{iJ} - \eta_{iJ}^T$ . When  $\varepsilon_{iJ}$  is negative for a supplier, then that supplier is probably dissatisfied and may violate its obligations to the chain, and possibly leave the chain. The profit equilibrium is therefore an indicator of the supply chain's partnership stability.

Figure 4.25 shows an example plot of the profit equilibrium for a 4 supplier chain. In the nominal case we set  $\eta_{iJ}$  equal to the chain average, which is given by:

$$\text{Average Profit Ratio} = \left\{ \frac{\sum_i \sum_j C S_{ij} P_{jJ} \sum_i (I'_{iJ} + I^O_{iJ} + M_{iJ} + A_{iJ})}{\sum_i (I'_{iJ} + I^O_{iJ} + A_{iJ})} \right\} \Big|_{O_{ij}=1} \quad (4.90)$$

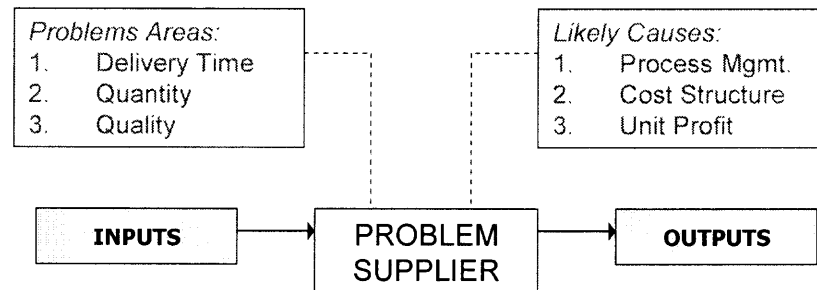


**Figure 4.25** An Example of the Profit Equilibrium.

The nominal case distributes  $\Psi_J$  at a uniform rate across the supply chain. In reality this is unlikely to occur, but the representation provides a snapshot of the vested interests of the suppliers. In the target case we set  $\eta_{iJ}^T$  equal to the industry average for that supplier category (e.g., sheet metal manufacturer). In figure 4.25 we see that for supplier B,  $\eta_{BJ}^T = 40\%$  while  $\eta_{BJ} = 9\%$ , indicating B has a negative  $\varepsilon_{BJ}$ .

The NEXUS profit equilibrium view identifies supplier instability, and hence should motivate some corrective action. In the past supply chain analysts were dependent on latent analytics data or supplier action before they would take such corrective action. Figure 4.26 identifies the problem areas that can result once a supplier becomes unstable, and latent analytics on which we would focus. These are metrics that serve as signals of supplier behavior. Figure 4.26 also identifies the likely causes and/or solutions to the problem of the negative  $\varepsilon_{BJ}$ .

If the process management at supplier B is weak then this might be corrected through intervention. If the cost structure of B is too high then this might not be correctable, and a new supplier might have to be found. This new supplier might have lower labor costs ( $Cw_i$ ) and/or newer equipment with lower costs ( $Ca_j$ ). Finally, the unit profit for supplier B might be too low, in which case profits from another supplier could be shifted into B. For instance in figure 4.25 we see that supplier C has a significant positive  $\varepsilon_{CJ}$ . By increasing the contracted supply costs between B and C we can therefore increase B's revenue and consequently  $\Pi_{BJ}$  and  $\eta_{BJ}$ .



**Figure 4.26** Analyzing the Problem Supplier.



## 4.6 Chapter Summary

We have treated a pair of suppliers as members of a supply chain nexus, with each member represented by its own unit model. Within the unit model we separated a supplier's operating parameters into those assigned by contract and those stipulated by the supplier. This separation was conducive to the modeling of unit supply costs for each part by each supplier. We emphasized the details of inventory-related supply costs because they are the focus of supply chain cost reduction.

Based on the unit supply costs we derived the unit profit for a supplier within the nexus. We used unit profit and unit supply costs to develop the supplier profit ratio. We have shown that the supplier profit ratios may be treated as a key determinant of supply chain stability. We have also shown that inventory-related costs are the basis for inventory positioning analysis.

## **CHAPTER 5**

### **ANALYSIS OF SUPPLY CHAIN RESPONSIVENESS TO DEMAND CHANGES**

In this chapter we utilize the NEXUS model to derive a metric for evaluating the responsiveness of the supply chain to demand changes. Supply chains are intended to have low levels of fast moving inventory. Such a situation makes them vulnerable to risk if there is a significant change in the final product demand level (either upwards or downwards). A good supply chain should be able to absorb these changes with little additional cost and limited capital risk. We begin by reviewing the relevant literature about responsiveness. We then introduce our approach to modeling the cost and risk elements in the supply chain associated with demand change.

#### **5.1 Modeling the Supply Chain Response Process**

It has been remarked by some writers that there have been few efforts to derive a quantitative measure of supply chain responsiveness. A review of the relevant literature shows that the very definition of responsiveness is still open to debate. Even the term responsiveness is not always used, as some writers refer instead to flexibility or agility. Bateman, Stockton and Lawrence (1999) define flexibility as “the ability to change the company economically to meet a competitive need.” They develop a model by which to calculate “mix response flexibility.” This is derived as the expected value of the setup time for all machines in a manufacturing system, i.e. the product of the probability of each setup and the duration of those setups. Since a production setup is required every time a new product is processed in the system, each setup represents a change in the

system that becomes necessary when demand changes. The expected value of all setup times is, in effect, the mean sensitivity to change (MSTC). The MSTC is a measure of the difficulty of using the same equipment to produce different products. It represents one of the few attempts that have been made to quantify a system's ability to respond to a change in external demand. We have adapted the idea of expressing responsiveness as a product in our own modeling efforts, by quantifying responsiveness as the product of the estimated number of demand changes and the cost of responding to each change.

A noteworthy step in the direction of defining responsiveness is taken by Holweg (2005). He finds that supply chain responsiveness is the result of a complex interaction of many variables. Holweg groups these variables into three categories – product, process and volume. Because of the large number of interdependent variables he dismisses the possibility of finding either a single approach to attaining responsiveness or a single metric by which to measure it. He does, however, assert that there is a need for quantification of responsiveness. As a step towards developing a quantitative model Holweg proposes a conceptual framework for responsiveness based on the three proposed categories of variables.

Khouja and Mehrez (2004) propose a linear model for the development of a flexible production plan. The model is utilized to decide the production levels for a product with a very short selling season. Khouja and Mehrez assert that system holding cost and production system flexibility are the crucial factors for development of the best production plan. They suggest that variation in production rate could increase production costs but could also decrease holding costs. By permitting changes in production rate the Khouja and Mehrez model allows the production plan to consider forecast revisions. In

their linear model the total cost of production flexibility is expressed in terms of: (i) holding cost as a percentage of inventory value, and (ii) production rate change cost to satisfy end-of-period demand. We build upon this approach in the following sections of this chapter, where the total cost of supply chain responsiveness is expressed in terms of (i) holding cost during the lead time needed to change the production rate in response to a percentage change in external demand, and (ii) capacity adjustment cost incurred in response to the percentage change in external demand.

Van Hoek, Harrison and Christopher (2001) address the growing importance of global supply chains, which is leading to a need for greater agility (their word) of the chain as a whole, as opposed to greater manufacturing agility. They introduce a tabular framework based on a wide range of viewpoints about how to characterize agility. Bhatnagar and Sohal (2005) attempt to provide an alternative to the ideal of quantifying supply chain responsiveness. Their model is concerned with how responsiveness is affected by qualitative factors such as labor, infrastructure, proximity to markets and political stability. They provide a large sample study to support their claim that responsiveness depends heavily upon non-quantitative factors.

### **5.1.1 Model Assumptions**

In the development of a model for responsiveness we make the following assumptions:

- Final product demand is deterministic and uniform, but the level of inventory and level of production capacity both change after a significant demand change event.
- The frequency and magnitude of significant demand changes can be projected from historical data.

- Backorders are allowed following a positive change in demand.
- The capacity adjustment period is comprised of a fixed and a variable component.
- The production rate at each supplier increases or decreases linearly over the adjustment period.
- The frequency of demand changes can be projected (forecasted) with some degree of reliability.
- A supplier will respond to a change in demand by adapting its production-inventory process.

### **5.1.2 Demand Change and Its Effects on the Production-Inventory Process**

For our purposes responsiveness is regarded as the ability of a supply chain to efficiently react to a change or changes in external customer demand for its output. It is the responsiveness of the chain as a whole that is important, because satisfaction of external demand depends on the combined performance of all suppliers. To describe combined performance an aggregate measure of supplier costs is needed, rather than a measure of individual supplier costs. We classify demand changes (final product) into three levels:

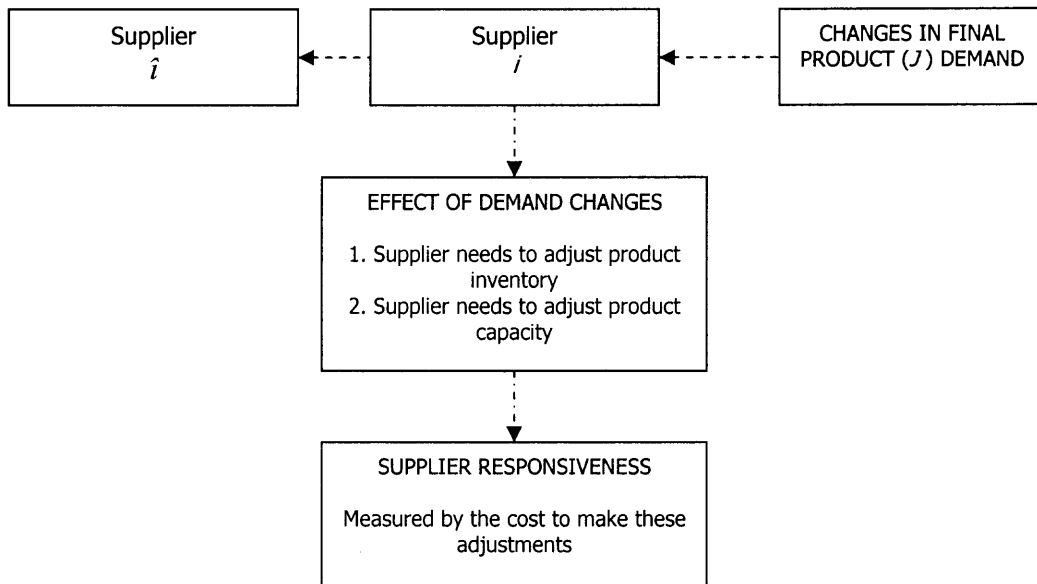
1. *Statistical Variation* - Typically, demand is characterized by a probability distribution and the variance is an indicator of statistical variation. The classical approach is to utilize the supply chain inventory to absorb this variation, and there is a vast literature on models for single echelon and some for two echelon solutions. Several of these are referenced in the chapter 2 literature review section.

2. *Significant Changes or Market Shifts* - We refer to level II changes or market shift changes as significant demand changes. These are changes associated with a variety of market conditions that could last for several supply cycles. Changes of this type are disruptive to supply chain performance. Often we read news articles about supply chain shortages or overstocks. Typically these are the results of a significant demand change to which the suppliers were not able to respond. Significant demand changes indicate a more than  $\pm 25\%$  change in the demand level and could be as high as 50%.
3. *Major Trends* – These are due to a major change in the demand behavior of a product. Changes above 50% are major trends and are too large to be modeled, and will usually require a structural redesign of the chain.

We introduce figure 5.1 to illustrate our conception of the response at each supplier when there is significant change in the demand for a final product ( $J$ ). Figure 5.1 identifies two effects of the change as the supplier responds by adapting its production-inventory process. If the demand slows then the supplier needs to work out the excess inventory. Alternatively, if the demand increases then there might be a short period of backorders and the supplier needs to bring the inventory back to target levels. In both scenarios the supplier will have to adjust production capacity to match demand levels. This could involve adding/eliminating production shifts or overtime production; adding/closing production equipment or lines; or adding/canceling subcontracted resources. A supplier's response to significant demand change will therefore consist of adjustments to one or both of the following: (i) inventory levels, and (ii) production capacity.

There are two situations in which a supply chain's response to significant demand change could include a capacity adjustment instead of only an inventory adjustment. First, an adjustment to production capacity is probably necessary when a change in final product demand is permanent. The existing capacity is likely to be sufficient to satisfy a temporary increase in demand and will certainly be adequate to meet a temporary decrease in demand. The response to a temporary change in demand would thus be limited to an adjustment in inventory levels.

Second, a chain's response to significant demand change could include a capacity adjustment depending upon the nature of the product that is in demand. Fisher (1997) proposes that the final products of supply chains be classified as either (i) functional products, or (ii) innovative products. Functional products are those with stable demand, while innovative products are subject to unpredictable demand. Products with stable demand should be produced by supply chains with efficient production facilities. For such "lean" operating structures there are few (if any) significant demand changes that require a major response. Adjustments to the production capacity of efficient facilities are therefore unnecessary. Products for which demand is unpredictable require flexible production facilities. For flexible facilities there are many demand changes that are expected to require a response. The very flexibility of such operations helps adjustments to be made to production capacity. These adjustments enable a chain to be flexible or "agile" in its response to changes in product demand (Naylor, Naim, Berry, 1999).



**Figure 5.1** The Supplier Response to Demand Changes.

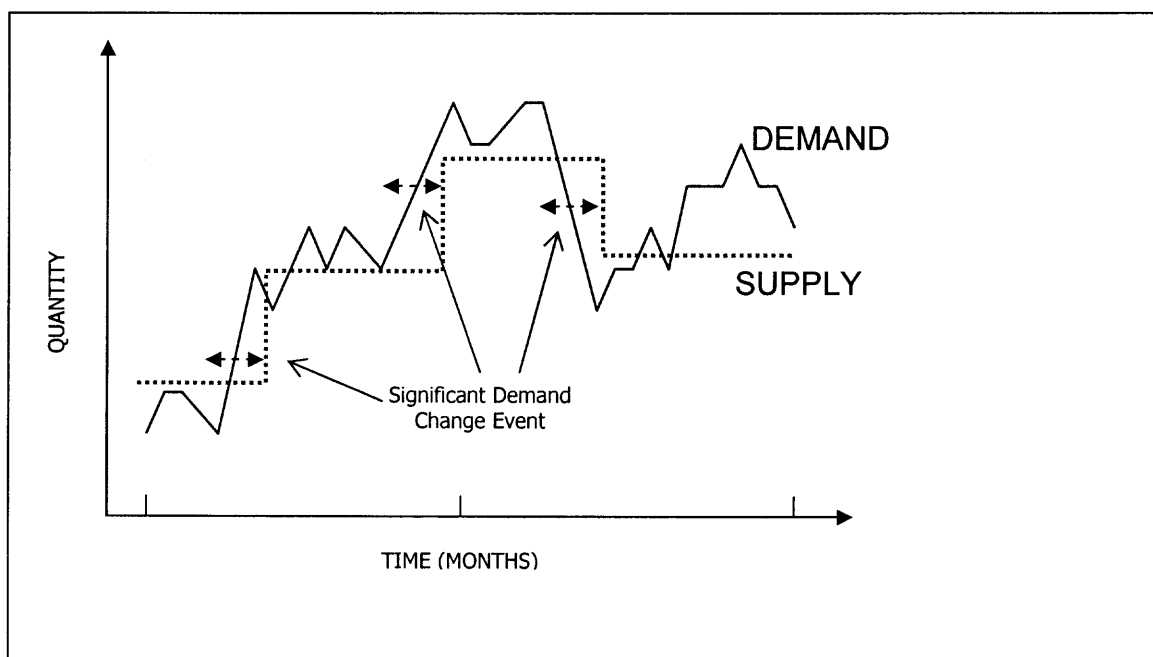
Modern supply chains are expected to use changes in production capacity (as opposed to inventory) to react to significant demand changes. Our focus is exclusively on significant demand changes, and the cost efficiency with which the suppliers in a chain are able to respond to these changes. Often these demand changes are monitored over short intervals (monthly or quarterly) leading to frequent changes at the production facilities. The greater the cost of making these changes, the less responsive is the chain. The more time is needed to make the changes, the less efficient is the chain.

For each instance of demand change an individual supplier could incur a cost to make adjustments in inventory levels and production capacity. We model these costs as the sum of a fixed cost per instance of change plus a linear cost which is a function of the degree of demand change. In the next subsection we introduce our characterization of the demand change process, and use this to develop a change cost function.



### 5.1.3 Characterizing Significant Demand Changes

Earlier we differentiated between statistical demand changes (or demand noise) and significant changes. Tsay (1999) discusses the relationship of supply chain responsiveness to customer demand changes. He argues that the flexibility of a supply chain should be measured by its ability to respond to changes in forecasted demand shifts (equivalent to significant demand changes) rather than actual short-term demand changes (equivalent to statistical variations). Here our focus is exclusively on significant demand changes, and the approximate cost with which the suppliers in the chain are able to respond to these changes.



**Figure 5.2** Final Product Demand Behavior.

Figure 5.2 shows the example demand behavior of a final product. Such a graph can be derived from historical data for the product in question. The most obvious way to characterize this demand is to determine the mean and standard deviation. If the standard deviation is small enough to be absorbed by the existing inventory practice, then one does

not need to evaluate the chain's responsiveness. Such chains have no level II demand changes, and examples include supply chains for beverages, basic food items, and many consumer staples. When the overall standard deviation is too great then the demand behavior can be fragmented into the two levels of changes. For example in figure 5.2 we can identify three significant demand change events. If we ignore these three events, then the demand variance in the remaining periods can be absorbed easily by the supply chain.

We therefore characterize the significant demand change behavior by the number of significant demand rate changes per year, and the average demand change per event. These are defined as follows:

$\Delta D_J$  Average demand change quantity for final product  $J$  in significant change event

$E_J$  Estimated number of significant demand changes for final product  $J$  per year

For a given supply chain we would use the historical data to plot the figure 5.2 graph, then analyze the data to identify the significant demand change events. The results of this will provide us with the above two parameters. These projections of average demand change and demand change frequency are used to estimate the supply chain responsiveness. In the case where these parameters cannot be estimated then a responsiveness analysis cannot be done.

We then propose the following definition: The responsiveness of a supply chain is the expected annual cost of making inventory and production capacity adjustments to account for the significant demand change events. The responsiveness cost is therefore incurred as a result of changes in inventory level and production rate. This cost is derived for each supplier and then summed for the chain. This can be represented as follows:

Supply Chain Responsiveness =

$$\sum_J \sum_i E_J \left\{ \text{Inventory Adjustment Cost} + \text{Capacity Adjustment Cost} \right\} \bigg| \Delta D_J \quad (5.1)$$

Equation 5.1 derives the net annual cost the supply chain will experience in responding to the demand changes. In the following sections we derive both the inventory and capacity adjustment costs. These are then used to complete equation (5.1).

#### 5.1.4 Product Correlation and Commonality

In deriving an accurate measure of responsiveness, we must consider (i) production resources commonality and (ii) any demand correlations between final products. These factors could either amplify or mitigate the impact of demand changes. In the context of supply chain modeling we define each of these as follows:

**Product Resource Commonality** -  $\psi_{JJ}^i$ : The commonality is specified for each pair of final products in reference to each supplier. Let  $\psi_{JJ}^i$  be the extent to which resources used by supplier  $i$  in the production of parts for product  $J$  is the same as the resources used in the production of parts for product  $\hat{J}$ . These resources include tools, labor, inputs of parts/materials and production equipment. For a value of  $\psi_{JJ}^i$  equal to 1.0 there is complete commonality in the resources used to produce  $J$  and  $\hat{J}$ . When  $\psi_{JJ}^i$  is between 0.0 and 1.0 there is partial commonality in these resources. For a value of  $\psi_{JJ}^i$  equal to 0.0 there are no resources shared in the production of both  $J$  and  $\hat{J}$ . If supplier  $i$  does not participate in the supply chain of either one or both products then by default  $\psi_{JJ}^i = 0$ .

One proposition for deriving  $\psi_{JJ}^i$  would be to assign a cost value to each resource used by the supplier. Then if  $A_J$  is the set of resources used by product  $J$  and  $A_{\hat{J}}$  the set of

resources used by  $\hat{J}$ , we could set  $\psi^{i_{JJ}} = A_J \cap A_{\hat{J}} / A_J \cup A_{\hat{J}}$ . We follow this approach to calculate  $\psi^{i_{JJ}}$  in the illustrative example below.

In this example there are 3 categories of resources: materials (M), labor (L), and tools (T). The two sets of product resources used in the example are as follows:  $A_J = \{M1, M2, L1, L2, T1, T3\}$  and  $A_{\hat{J}} = \{M1, M3, L2, L3, T2, T3\}$ .

**Table 5.1** Resource Commonality Data

COST VALUES OF MATERIAL RESOURCES USED BY SUPPLIER <i>i</i>					
MATERIALS	RESOURCE M1	RESOURCE M2	RESOURCE M3	$A_J \cup A_{\hat{J}}$	$A_J \cap A_{\hat{J}}$
Unit Cost Value	\$12	\$14	\$18		
Product <i>J</i> Requirement	3	8			
Product $\hat{J}$ Requirement	5		9		
Total Resource Cost - Product <i>J</i>	\$36	\$112		\$148	
Total Resource Cost - Product $\hat{J}$	\$60		\$162	\$222	
Total Resource Cost - Both Products	\$96	\$112	\$162	\$370	\$72
COST VALUES OF LABOR RESOURCES USED BY SUPPLIER <i>i</i>					
LABOR	RESOURCE L1	RESOURCE L2	RESOURCE L3		
Unit Cost Value	\$10	\$11	\$12		
Product <i>J</i> Requirement	5	7			
Product $\hat{J}$ Requirement		4	7		
Total Resource Cost - Product <i>J</i>	\$50	\$77		\$127	
Total Resource Cost - Product $\hat{J}$		\$44	\$84	\$128	
Total Resource Cost - Both Products	\$50	\$121	\$84	\$255	\$88
COST VALUES OF TOOL RESOURCES USED BY SUPPLIER <i>i</i>					
TOOLS	RESOURCE T1	RESOURCE T2	RESOURCE T3		
Unit Cost Value	\$11	\$13	\$17		
Product <i>J</i> Requirement	6		3		
Product $\hat{J}$ Requirement		5	9		
Total Resource Cost - Product <i>J</i>	\$66		\$51	\$117	
Total Resource Cost - Product $\hat{J}$		\$65	\$153	\$218	
Total Resource Cost - Both Products	\$66	\$65	\$204	\$335	\$102
<b>TOTAL COST VALUES</b>				\$960	\$262

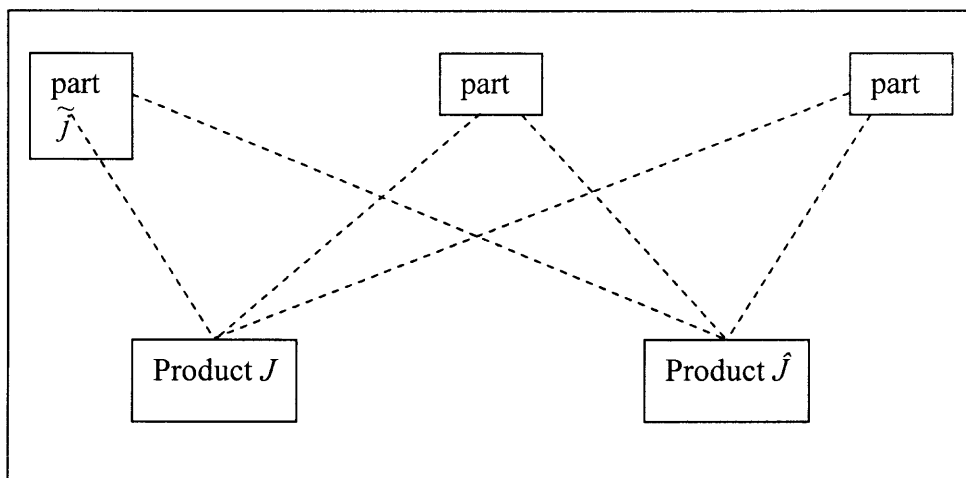
For this example  $\psi^{i_{JJ}} = A_J \cap A_{\hat{J}} / A_J \cup A_{\hat{J}} = \$262 / \$960 = .2729$ .

**Product Demand Correlation -  $\alpha_{JJ}$ .** The demand correlation  $\alpha_{JJ}$  expresses the extent to which demand for *J* is the same as demand for  $\hat{J}$ . For  $\alpha_{JJ}$  of 1.0 the variation in demand for *J* and the variation in demand for  $\hat{J}$  are in exactly the same direction. For  $\alpha_{JJ}$  less than 1.0 but greater than 0.0 the relationship between the demand variation of *J* and  $\hat{J}$  is in approximately the same direction. For  $\alpha_{JJ}$  of 0.0 there is no relationship between

demand for  $J$  and demand for  $\hat{J}$ . When  $\alpha_{J\hat{J}}$  is negative (but above -1.0) the demand variation of  $J$  and  $\hat{J}$  are in different (but not opposite) directions. For  $\alpha_{J\hat{J}}$  of -1.0 there is an inverse relationship between the variation in demand for  $J$  and the variation in demand for  $\hat{J}$ .

In our analysis here we assume that  $\alpha_{J\hat{J}}$  is given. In most cases the demand correlation could be determined by studying historical demand. Since we are focused on significant demand change events only, the history file may need to be filtered prior to correlations analysis.

Figure 5.3 depicts an example illustrating the importance of  $\psi^i_{J\hat{J}}$  and  $\alpha_{J\hat{J}}$ . Three component parts are shown:  $\tilde{j}$ ,  $j$  and  $\hat{j}$ . These three parts are each being used in the production of both  $J$  and  $\hat{J}$ . Thus three productive resources are common to  $J$  and  $\hat{J}$ . All three of these resources must be considered in the calculation of  $\psi^i_{J\hat{J}}$ . However, there are only two final products subject to external demand. The demand for these two products constitutes a single possible demand correlation  $\alpha_{J\hat{J}}$ . In summary, this example contains three resource commonalities and one demand correlation.



**Figure 5.3** Reference Chart for  $\psi^i_{J\hat{J}}$  and  $\alpha_{J\hat{J}}$ .

If  $\psi^i_{JJ}$  is 0 and there is a change in demand for  $\hat{J}$  ( $\Delta D_j$ ), then the chain's response to a change in demand for  $\hat{J}$  does not require the use of resources shared in the production of  $J$ . In this case there is no responsiveness cost due to  $\Delta D_j$ . The fractional adjustment to  $W^i_{JJ}$  (see page 115) is 0 and so the value of  $W^i_{JJ}$  is 1. If  $\psi^i_{JJ}$  is positive then the response to  $\Delta D_j$  involves production of  $\hat{J}$  with resources shared in the production of  $J$ , which does increase responsiveness cost. For positive  $\psi^i_{JJ}$  the fractional adjustment to  $W^i_{JJ}$  will be positive and the value of  $W^i_{JJ}$  will be between 1 and 2.  $W^i_{JJ}$  then potentially increases the basic cost ( $Cf_j + Cd_j((\Delta D_j/D_j)100)$ ) of adjusting to demand change.

If  $\alpha_{Jj}$  is 0 then  $\Delta D_j$  does not increase the cost of responding to a change in demand for  $J$ . In this case  $W^i_{JJ}$  is 1. If  $\alpha_{Jj}$  is positive then  $\Delta D_j$  does increase the responsiveness cost. This happens because the change in demand for  $\hat{J}$  is in a similar direction to the change in demand for  $J$ , and so the adjustment in production of  $\hat{J}$  (either an increase or decrease) is in a similar direction to the adjustment to the production of  $J$ . If  $\alpha_{Jj}$  is negative then  $\Delta D_j$  decreases responsiveness cost. In this case the change in demand for  $\hat{J}$  is in a different direction from the change in demand for  $J$ , and so the response to a change in demand for  $\hat{J}$  is in a different direction from the response to a change in demand for  $J$ .

## 5.2 Notation

In addition to the notation introduced earlier in chapter 3, and the demand change characterization data introduced in section 5.1, the following notation is used to develop the responsiveness models in this chapter.

*Cost Parameters:*

$Cf_j$  Fixed cost to change production rate of product  $j$  in response to a significant demand change event

$Cd_j$  Variable cost per unit percent change in the production rate of product  $j$  in response to a significant demand change event

$Cb_j$  Unit backorder cost for product  $j$

*Response Parameters:*

$L_j$  Fixed lead time to change production rate of product  $j$  in response to a significant demand change event

$\lambda_j$  Variable lead time to change production rate per unit percent change in demand for product  $j$  in response to a significant demand change event

$e_j$  Exponent of cost of adjusting to demand for final product  $J$

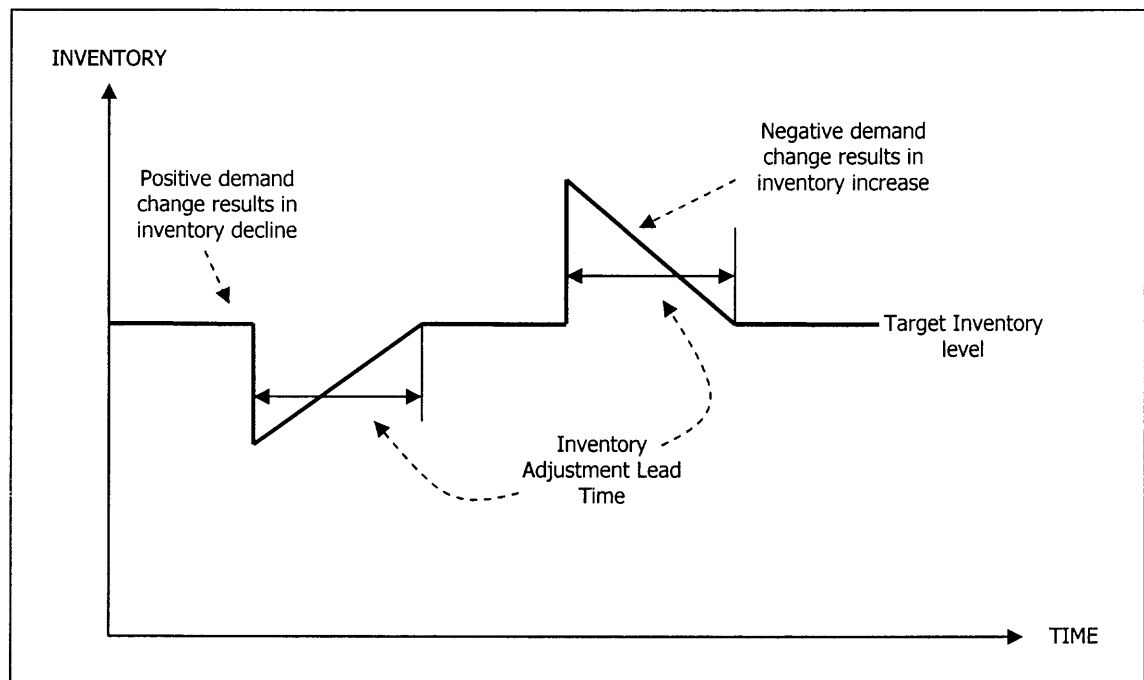
### 5.3 Responsiveness of a Single Supplier and Single Product

We begin by considering the case of one supplier selling a final product  $J$ . In this case responsiveness measures a supplier's ability to react to a change in demand by increasing or decreasing production capacity and inventory levels. We derive the cost for each of these next.

#### 5.3.1 Inventory Adjustment Cost

There are two inventory-related response costs: holding cost ( $Ch_j$ ) and backorder cost ( $Cb_j$ ). Both of these costs are assumed to be linear functions of time. Time is the key to supplier efficiency. To be efficient means to deliver ordered products on time in

response to a change in external demand. Mere delivery is not enough; promptness is also essential. Figure 5.4 depicts the inventory behavior following a significant demand change event. In the first instance there is an upward demand shift, as a consequence the inventory will drop with a risk of backorders or shortages. Over a period of time the supplier will ramp up production and get back to the target inventory level. In the second instance, there is a negative demand shift and the inventory will increase. Again, over a period of time the supplier will slow down production and bring the inventory back to the target level.



**Figure 5.4** Inventory Behavior During a Significant Demand Change.

We assume here that the inventory adjustment lead time is the same for both the upward and downward shifts in demand. We will also assume that in a year of demand changes the excess inventory will be balanced by the amount of inventory on backorder. Therefore, the average per unit inventory-related response cost for a supplier of final product  $J$  is given by  $(Ch_J + Cb_J)/2$ . This cost may be quantified in terms of the lead



time necessary to change the production rate in response to changes in demand. The lead time to respond to a single demand change is expressed as the sum of a fixed time and a variable time. The fixed time  $L_J$  is the number of weeks of lead time necessary to respond to a change in demand. The variable time is the product of the necessary number  $\lambda_J$  of weeks of response time per percentage point change in demand, and the number of percentage points by which demand changes. Therefore:

$$\text{Total Inventory Adjustment Lead Time} = L_J + \lambda_J((\Delta D_J/D_J)100) \quad (5.2)$$

During this interval the net inventory adjustment is equal to  $\Delta D_J$ , hence the average excess inventory or backorder is given by  $\Delta D_J/2$ . For each significant demand change event, the inventory adjustment cost is then given by:

$$\text{Inventory Adjustment Cost / Event} = \Delta D_J (Ch_J + Cb_J) \{L_J + \lambda_J((\Delta D_J/D_J)100)\} / 4 \quad (5.3)$$

Note that this is the cost for the last supplier who processes final product  $J$ .

### 5.3.2 Capacity Adjustment Cost

The capacity adjustment cost includes the expenses to the supplier in realigning its production resources in response to either an increase or decrease in the production rate. Capacity adjustment, on both the upside and downside, poses the greatest risk to supply chain efficiency. We characterize this cost into two components. The first is the immediate capital cost associated with the resource adjustment -  $Cf_J$ . For example if demand moves upwards then an additional shift may have to be trained and more space committed. In a highly responsive chain we would expect that  $Cf_J$  is very low. In manual production lines a surrogate for  $Cf_J$  is the classical hiring and firing costs. In many instances when the demand is unpredictable, companies will prefer to locate plants

in locations with low  $Cf_J$ . One reason for using a fixed cost, is that capacity adjustments commonly occur in fixed increments. For example if capacity adjustment requires a 100 units/day production rate change, then even if the actual change is only 10 units/day the supplier will need to make the full incremental adjustment.

The second component of capacity adjustment is a direct measure of the size of the change. This may represent adding workers to an existing shift, or shutting specific production equipments. The variable cost ( $Cd_J ((\Delta D_J/D_J)100)$ ) is the product of the response cost  $Cd_J$  per percentage point change in demand, and the number  $((\Delta D_J/D_J)100)$  of percentage points by which demand changes. For each change event, the capacity adjustment cost is then given by:

$$\text{Capacity Adjustment Cost / Event} = Cf_J + Cd_J \{(\Delta D_J/D_J)100\} \quad (5.4)$$

The total annual cost of responding to a change in external demand is expressed as the sum of inventory-related cost and capacity-related cost, or equations (5.3) + (5.4) multiplied by the expected number of changes. Then:

$$\Gamma_{i,J} = E_J \left\{ \Delta D_J (Ch_J + Cb_J) \{L_J + \lambda_J ((\Delta D_J/D_J)100)\} / 4 + Cf_J + Cd_J \{(\Delta D_J/D_J)100\} \right\}$$

$$\text{where } J \text{ is a output of } i \quad (5.5)$$

From equation (5.5) we see that a single supplier may pursue several strategies to reduce  $\Gamma_{i,J}$ . Where  $Ch_J$  and  $Cb_J$  are the dominant costs, then the emphasis should be on reducing  $\lambda_J$ . Often  $Cf_J$  will be a dominant cost, and the supplier may need to have capacity flexibility built in through either resource subcontracts or temporary labor pools.

#### 5.4 Responsiveness of Multiple Suppliers and Multiple Products

In this section we extend the responsiveness analysis to the multi-supplier multi-product case. The responsiveness of two or more suppliers is more important because, unlike the case of a single supplier, two or more suppliers may be interconnected to form a chain. There are three specific issues that need to be added to the multi-supplier multi-product case:

- The relative impact of the changes at each supplier must be evaluated against the total production activity at that supplier. A big change in a slow moving item might have little responsiveness impact on a supplier.
- The demand correlation between products must be considered when evaluating both the inventory and capacity adjustment costs.
- The product resource commonality at each supplier must be considered when evaluating only the capacity adjustment costs.

We introduce the following two factors to address the second and third issues from above:

*Demand Correlation Factor* -  $\theta_{jJ}$ : Using the demand correlations of all product pairs, and the demand change parameters, we determine the relative impact of the correlations on the demand changes for part  $j$ . The demand correlation factor is defined by 1 plus a ratio that measures this relative impact:

$$\theta_{jJ} = 1 + \left( \left( \sum_{\substack{j|\beta_j=1 \\ j \neq J}} \alpha_{Jj} E_j P_{jJ} \Delta D_j \right) / \left( E_J P_{jJ} \Delta D_J + \sum_{\substack{j|\beta_j=1 \\ j \neq J}} |\alpha_{Jj}| E_j P_{jJ} \Delta D_j \right) \right) \quad (5.6)$$

The denominator in the ratio gives the total annual demand change that is directly and indirectly related to part  $j$ . The direct annual unit change for part  $j$  that is attributed to final product  $J$  is given by  $E_J P_{jJ} \Delta D_J$ . The gross indirect change from other final

products is given by  $\sum_{\substack{j|\beta_j=1 \\ j \neq i}} \{|\alpha_{jj}| E_j P_{jj} \Delta D_j\}$ . Note that the absolute value is used since we wish to add the total demand change movement. The numerator in the ratio is the sum of the net indirect change, and differs in that the true correlation is used. The ratio therefore measures the net indirect changes in demand against the maximum gross changes in demand.

The value of  $\theta_{ij}$  will be between 0 and 2. In the nominal case when there is no correlated demand, then  $\theta_{ij}=1$  and there is no effect. In the example below we illustrate the behavior of equation (5.6).

**Table 5.2** Demand Correlation Factors of Final Products

PRODUCT	1	2	3	$\Delta D_j$
1		0.5	-0.2	20
2			-0.1	10
3				15

Consider a part ( $j=4$ ) that is used in the production of all three final products, and for cases  $P_{jj}=1$ . For the above data we get for  $\theta_{4,1}$  the annual direct change plus the gross indirect change is 28 units, while the net indirect change is 2 units, and  $\theta_{4,1}=1.071$ . This implies that the demand correlations could amplify the impact of the demand changes of product 1 on part 4 by as much as 7.1%. In contrast  $\theta_{4,3}=0.75$  which indicates that in this case the correlations compensate for some of the direct demand changes and the impact is dampened by 25%. Finally,  $\theta_{4,2}=1.39$ , which indicates that even though product 2 has the smallest  $\Delta D_j$ , any changes in its demand behavior is amplified by simultaneous demand changes from product 1 in particular.

*Resource Commonality Factor  $W_{jj}^i$* : Using the product resource commonality ( $\psi_{jj}^i$ ) in combination with demand correlation for all final product pairs, we determine

the combined effect on the production resources of each supplier.  $W_{JJ}^i$  is derived as the sum of 1 plus a ratio that measures this combined effect:

$$W_{JJ}^i = 1 + \left( \sum_{\substack{j|\beta_i=1 \\ j \neq J}} \psi_{JJ}^i \alpha_{Jj} E_j \Delta D_j \right) / (E_J \Delta D_J)$$

$$\text{if } W_{JJ}^i < 0 \text{ then } W_{JJ}^i = 0, \text{ else if } W_{JJ}^i > 2 \text{ then } W_{JJ}^i = 2 \quad (5.7)$$

In the nominal case when there is no correlation and/or commonality then the ratio is zero and  $W_{JJ}^i = 1$ . This ratio can be positive or negative and the denominator is  $E_J \Delta D_J$ , which is the annual change for product  $J$ . In figure 5.3 we showed how correlation and commonality for different final products, link up with the production resources of a supplier. The numerator in equation (5.7) is the total resource indexed demand change the supplier will experience from products that are correlated with  $J$ . Since  $\alpha_{Jj}$  could be negative it is possible the ratio could be negative. It is possible that  $E_j \Delta D_j \ll E_J \Delta D_J$  for the correlated products. In that case the ratio could be less than -1 or greater than 1. In general the responsiveness costs should not be modified by more than 100%, so we limit  $W_{JJ}^i$  to the range between 0 and 2.

#### 5.4.1 Inventory Adjustment Cost

The average inventory-related response cost for a supplier of intermediate product  $j$  is represented as  $(Ch_j + Cb_j)/2$ . As in the single supplier case, this cost may be quantified in terms of the lead time necessary to react to changes in demand. For the N-supplier case it is also important to adjust the lead time by the demand change input quantity. This adjustment to lead time, and the lead time itself, are both developed below. The adjustment to lead time is needed because of the change in production of  $j$  necessary to satisfy a change in demand for  $J$ . This change in production of  $j$  is  $P_{jJ} \Delta D_J$ . The change

in the input quantity of  $j$  needed to produce  $J$  as a result of estimated demand changes for  $J$  is  $E_J P_{jJ} \Delta D_J$ . The lead time to change production of  $j$ , in response to a change in demand for  $J$ , is  $L_j + \lambda_j(\Delta D_J/D_J)100$ .

The timeliness of a productive response to average demand change is formulated as the product of the adjusted average demand change input quantity and the lead time, which is  $(\theta_{jJ} E_J P_{jJ} (\Delta D_J/2)) (L_j + \lambda_j(\Delta D_J/D_J)100)$ . The inventory adjustment cost of part  $j$  due to changes in demand for  $J$ , is:

$$\begin{aligned} & \text{Inventory Adjustment Cost of } j \text{ for } J \text{ per Event} \\ & = \theta_{jJ} P_{jJ} \Delta D_J (Ch_j + Cb_j) \{L_j + \lambda_j((\Delta D_J/D_J)100)\} / 4 \end{aligned} \quad (5.8)$$

Note that this is the cost for only one final product.

#### 5.4.2 Capacity Adjustment Cost

The capacity adjustment cost includes the expenses to the supplier in realigning its production resources as it experiences demand changes in multiple products. Capacity-related cost in the multi-supplier case includes the fixed cost  $Cf_j$  which is the minimal cost of responding to a change in demand for  $j$ . The variable cost  $Cd_j ((P_{jJ} \Delta D_J / \sum_J P_{jJ} D_J)100)$  is the cost of production incurred in response to the percentage point change in demand for  $J$ . The fraction  $(P_{jJ} \Delta D_J / \sum_J P_{jJ} D_J)$  is the proportionate impact of the change on  $Cd_j$ .

Expected capacity-related cost is the product of  $Cf_j + Cd_j ((P_{jJ} \Delta D_J / \sum_J P_{jJ} D_J)100)$  and  $E_J$ , which is  $(Cf_j + Cd_j ((P_{jJ} \Delta D_J / \sum_J P_{jJ} D_J)100)) E_J$ . Expected capacity-related cost is subject to the adjustment  $W_{jJ}^i$ . In the multi-supplier case  $W_{jJ}^i$  is derived as the sum of  $\theta_{jJ}^{e_j}$  (see page 114 for derivation of  $\theta_{jJ}$ ) and the fraction

$$\left( \sum_{\substack{j|\beta_j=1 \\ j \neq J}} \sum_{\substack{j|\beta_j=2 \\ j \neq j}} \psi_{jj}^i \alpha_{JJ} E_J P_{jJ} \Delta D_J / E_J P_{jJ} \Delta D_J \right). \quad \begin{array}{l} \text{Max} = 1 \\ \text{Min} = -1 \end{array}$$

The exponent  $e_j$  describes the shape of the curve for  $\theta_{jJ}$ . The value of  $\theta_{jJ}^{e_j}$  is between 0 and 2 (consistent with page 115), so the value of  $e_j$  should be between 0 and 1.

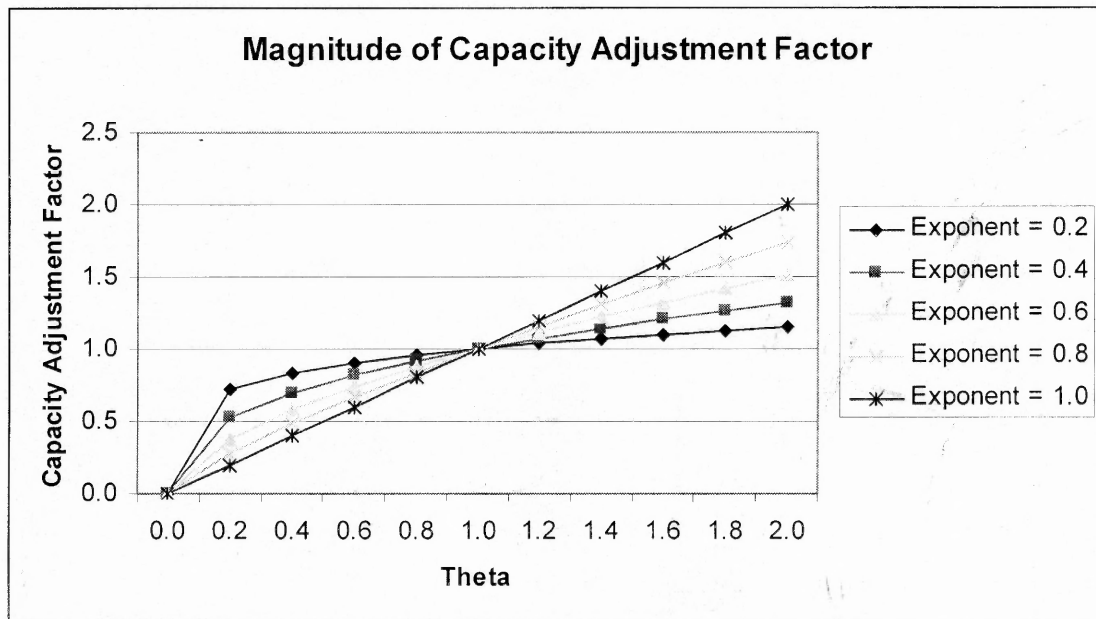
The factor  $W_{jJ}^i$  is then expressed as

$$W_{jJ}^i = \theta_{jJ}^{e_j} + \left( \sum_{\substack{j|\beta_j=1 \\ j \neq J}} \sum_{\substack{j|\beta_j=2 \\ j \neq j}} \psi_{jj}^i \alpha_{JJ} E_J P_{jJ} \Delta D_J / E_J P_{jJ} \Delta D_J \right). \quad \begin{array}{l} \text{Max} = 1 \\ \text{Min} = -1 \end{array}$$

The value of  $W_{jJ}^i$  must be between 0 and 2.

The capacity adjustment cost is subject to an adjustment  $W_{jJ}^i$ , which itself is determined in part by the magnitude of the adjustment factor  $\theta_{jJ}^{e_j}$ . The capacity adjustment factor  $\theta_{jJ}^{e_j}$  permits us to describe the impact of  $\theta_{jJ}$  on capacity adjustment cost. As already noted  $\theta_{jJ}$  is a measure of the combined strength of all demand correlations. The impact of  $\theta_{jJ}$  on capacity adjustment cost could be large or small, depending on whether the relationship between the demand correlations and capacity adjustment cost is linear or nonlinear. For product pairs with positively correlated demand the capacity adjustment cost tends to be high, because the capacity adjustments will be in the same direction in response to changes in product demand. In this case the magnitude of the capacity adjustment factor at a given level of  $\theta_{jJ}$  is large, because there is a strong relationship between demand correlations and capacity adjustment cost. For product pairs with negatively correlated demand the capacity adjustment cost tends to be low, because the capacity adjustments will be in opposite directions in response to changes in product demand. In this case the magnitude of the capacity adjustment factor at a given level of  $\theta_{jJ}$  is small, because there is a weak relationship between demand correlations and capacity adjustment cost.

Figure 5.5 shows the values of  $\theta_{jJ}^{e_J}$  as a function of different values of  $\theta_{jJ}$  for several possible values of  $e_J$ . For  $e_J$  of 1.0 the factor  $\theta_{jJ}^{e_J}$  increases linearly from 0, the lowest value of  $\theta_{jJ}$ , up to 2.0, the highest value of  $\theta_{jJ}$ . An  $e_J$  of 1.0 is likely when demand change is only a matter of statistical variation. In this situation  $\theta_{jJ}$  has a proportionate impact on the capacity adjustment cost. For  $e_J$  between 0 and 1.0 the factor  $\theta_{jJ}^{e_J}$  increases nonlinearly as  $\theta_{jJ}$  increases from 0 to 2.0. An  $e_J$  between 0 and 1.0 is a sign of a significant demand change. In this situation  $\theta_{jJ}$  has a disproportionate impact on the capacity adjustment cost. The capacity adjustment cost is affected by adding/eliminating production shifts, overtime, equipment or subcontracted resources.



**Figure 5.5** Magnitude of  $\theta_{jJ}^{e_J}$  at Different Values of  $e_J$ .

The advantage of using  $e_J$  to express nonlinearity, instead of some other nonlinear function, is that  $e_J$  expresses the relative linearity and nonlinearity of  $\theta_{jJ}^{e_J}$  at a given  $\theta_{jJ}$ . For small values of  $e_J$  the magnitude of  $\theta_{jJ}^{e_J}$  is insensitive to changes in the value of  $\theta_{jJ}$ ,



and so the impact of  $\theta_{jJ}^{e_j}$  on capacity adjustment cost is disproportionate. For large values of  $e_j$  the magnitude of  $\theta_{jJ}^{e_j}$  is very sensitive to changes in  $\theta_{jJ}$ .

The adjusted expected change-related cost for a single final product  $J$  is

$$(Cf_j + Cd_j((P_{jJ}\Delta D_J / \sum_J P_{jJ}D_J)100)) E_J W_{jJ}^i.$$

For each change event, the capacity adjustment cost is then given by:

$$\text{Capacity Adjustment Cost / Event} = W_{jJ}^i Cf_j + W_{jJ}^i Cd_j \{(P_{jJ}\Delta D_J / \sum_J P_{jJ}D_J)100\} \quad (5.9)$$

The total supplier  $i$  annual cost of responding to changes in demand for product  $J$  is expressed as the sum of inventory-related cost and capacity-related cost, or equations (5.8) + (5.9) multiplied by the expected number of changes. Then:

$$\begin{aligned} \Gamma_{i,J} = E_J \{ & \theta_{jJ} P_{jJ} \Delta D_J (Ch_j + Cb_j) \{L_j + \lambda_j ((\Delta D_J / D_J)100)\} / 4 \\ & + W_{jJ}^i Cf_j + W_{jJ}^i Cd_j \{(P_{jJ}\Delta D_J / \sum_J P_{jJ}D_J)100\} \} \end{aligned}$$

$$\text{where } J \text{ is a output of } i \quad (5.10)$$

The annual responsiveness cost of supplier  $i$  for all final products for which it provides input, is then given by:

$$\text{Supplier } i \text{ Responsiveness Cost} = \sum_J \Gamma_{i,J} \quad (5.11)$$

Summing equation (5.11) for all suppliers will give the responsiveness cost for the entire supply chain. This cost can then be benchmarked against the annual production cost by a ratio. An important question then is whether some suppliers are bearing a disproportionate share of the net responsiveness cost. This question will be addressed in chapter 6.

## 5.5 Chapter Summary

We have applied the concept of a supply chain nexus to the modeling of supply chain responsiveness. In doing so, we modeled the efficiency of the nexus response to demand change in terms of the cost of that response. All nexus members contribute to the overall response to demand change, and so the costs contributed by each to overall responsiveness have been included in our model.

The response to demand change requires supplier adjustments to internal operations. These adjustments affect either supplier inventories or supplier production capacity, or both. For this reason we have included two types of response costs in our model: inventory adjustment cost and capacity adjustment cost. The inventory adjustment has itself been subjected to a demand correlation factor. The capacity adjustment has been subjected to a factor that allows for product commonality and demand correlation, because capacity adjustment represents the greatest risk to supply chain responsiveness.

## CHAPTER 6

### Numerical Experiments with the NEXUS Model

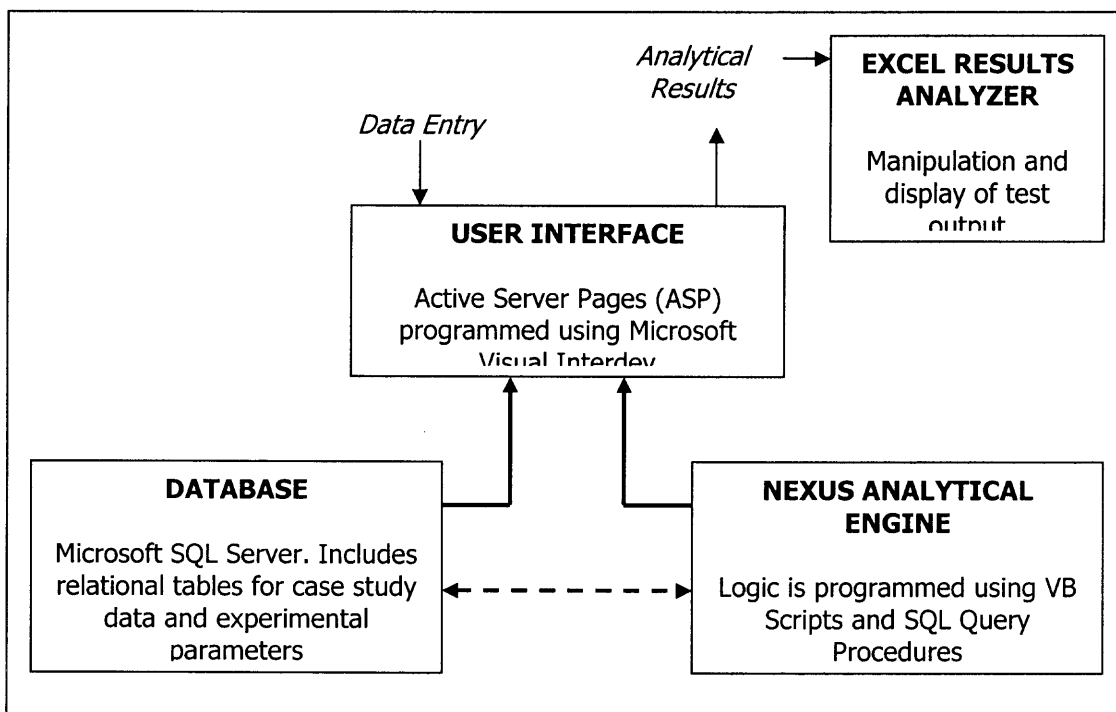
The NEXUS model provides a descriptive analysis of the supply chain behavior. We expect that subsequent research will utilize this descriptive analysis in combination with supply chain analytics to develop a variety of prescriptive models. Our goal in this chapter is to illustrate and study how NEXUS can be used to study the behavior of a supply chain, and what types of prescriptive questions can potentially be answered from the analysis.

First in section 6.1 we introduce the experimental platform that was developed for the numerical analysis. In sections 6.2 and 6.3 we focus mainly on inventory cost analysis using the equations developed in chapter 4. In section 6.2 an example supply relationship with 2 suppliers is described, the results of experiments that were conducted are presented, and an inventory cost analysis is provided. In section 6.3 the design of a more elaborate supply chain is described and an analytical discussion is provided. In section 6.4 we focus on the analysis of supply chain responsiveness using the equations developed in chapter 5. The analysis of responsiveness is based on experiments conducted upon the supply chain described in section 6.3.

#### 6.1 Development of the Experimental Platform

In order to conduct the planned experiments in an efficient manner it was necessary to develop an experimental platform. We wanted the ability to (i) create and store an example case study problem, (ii) quickly change one or more parameters in the problem,

and (iii) visualize and plot the analytical results. The nature of the NEXUS analysis implies that the case study data is best stored in a tabular format. This indicated that Microsoft Excel could potentially be used. But after evaluating the database size and the number of cross-linked data, we concluded that this approach would not work. Rather, a relational database such as Oracle or Microsoft SQL server would be the most suitable approach. Given this initial strategy, we planned to develop a platform with a web based user interface. This would not only meet our immediate numerical experimentation needs, but also be scalable and robust enough to handle future experimental requirements of the Supply Chain Research Group at NJIT. The software development of the NEXUS experimental platform is therefore a joint collaborative effort with other researchers in the group.



**Figure 6.1** Schematic of the Software Arrangement for the NEXUS Experimental Platform.

Figure 6.1 above shows the schematic arrangement of the software components in the NEXUS experimental platform. NJIT has a site license for two relational databases, Oracle and Microsoft SQL Server, and our selection was limited to these two. Considering the ease of programming and the known expertise of our research group the database SQL server was selected. In Appendix-I we list the tables and entities of the NEXUS database, plus the associated entity relationship diagram. The NEXUS analytical engine provides us two key functionalities: (i) manipulate and arrange the case study data, and (ii) implement the analytical equations developed in chapters 4 and 5 on the case study problem. The needed logic was programmed in two formats. The first of these were a set of Visual Basic (VB) Scripts. Each piece of VB Script is selectively triggered based on where we are in the NEXUS analytical process. The second, was a set of stored SQL query procedures. These powerful procedures are able to combine both extracted data with user input data, to execute a variety of analytical manipulations.

The user interface or front end of our platform was programmed in the web environment, and consisted of a series of pages that are accessed using a conventional web browser such as Internet Explorer. The pages themselves were programmed using the Active Server Pages (ASP) language. The pages were embedded with the corresponding VB Scripts and the call commands for the SQL Query Procedures. In Appendix-B we document the schematic flow of the web pages and include some sample screen shots. The majority of the case study data we entered into the database directly through this user interface. All experimental commands were also initiated from the user interface. The test results were displayed directly on the user interface. The entire

software arrangement was programmed and run on a Compaq Pentium II server running on Windows NT Server.

The tests results were only temporarily stored in the SQL database and then transferred to Microsoft Excel. This provided greater flexibility in manipulating the data and creating the needed visual outputs.

## 6.2 $(Q, R, \delta)^2$ Experiments for the Two-Supplier Case

### 6.2.1 Case Study Problem for $(Q, R, \delta)^2$ Inventory Cost Behavior Experiments

In this section we introduce the case study problem used to study the  $(Q, R, \delta)^2$  inventory cost models developed in chapter 4. The present case involves 2 suppliers, consistent with the deterministic simulation of subsection 4.3.4. We consider three perspectives: the input side (buyer's) inventory sub-component cost ( $V1$ ), the output side (seller's) inventory sub-component cost ( $V2$ ), and the total inventory cost ( $V1 + V2 + V3$ ). The  $V4$  cost is not included in the analysis because the seller's in-transit inventory cost is independent of  $Q_{ij}$ .

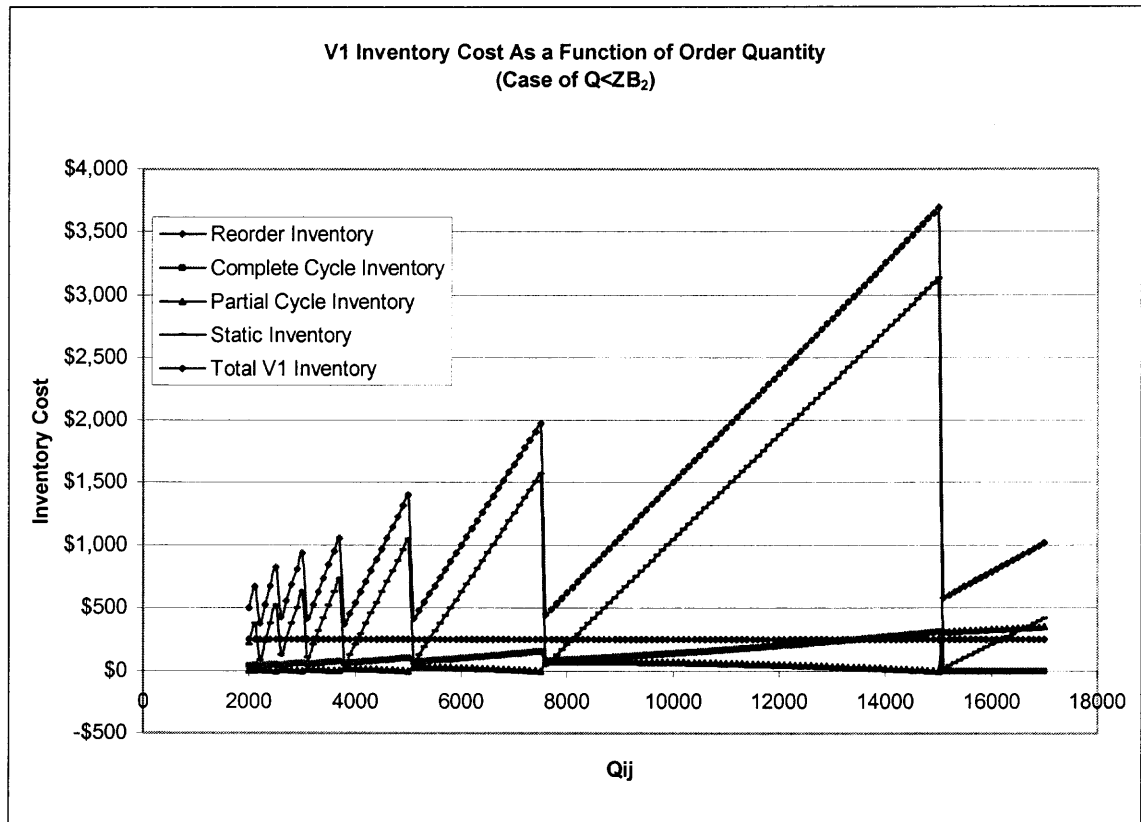
The problem is to estimate the actual inventory costs by approximating the cyclical inventory behavior between a pair of suppliers. We believe that a true picture of the inventory costs can only be derived by modeling the specific dynamics between the 2 suppliers. Many patterns of inventory behavior are possible, depending on the parametric relationship between the supply pair. Four cases of such behavior were discussed in subsection 4.3.3, and one other was discussed at the end of subsection 4.4.2. The supply relationship is parameterized as indicated in Table 6.1.

**Table 6.1** Case Study Parameters

$N$	2
$M$	2
$Z_{jj}$	1 (for all $j$ and $\hat{j}$ )
$Q_{11}$	2,000 (initially and then incremental increases of 100 to 17,000)
$R_{11} = Q_{11}$	at all times
$r_{21}$	1,000
$\delta_1$	1,000/hr.
$\delta_2$	200/hr. (initially and then incremental increases of 100/hr. to 600/hr.)
$Co_{21}$	\$1,450/order
$Ch_{21}$	\$4/item-year
$B_1$	24,000
$B_2$	13,000 (initially and then incremental increases of 1,000 to 17,000)
$D_1$	240,000/yr. or 65.75342/hr.

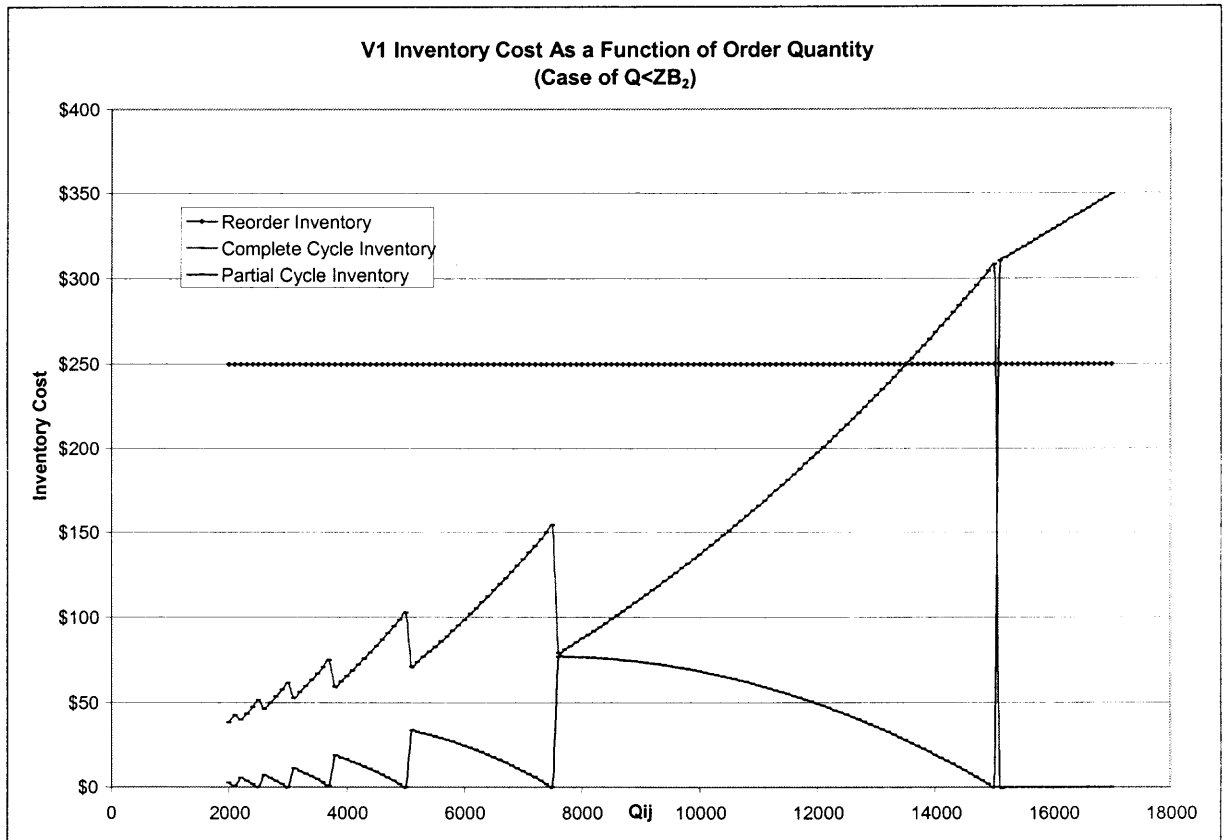
### 6.2.2 Analysis of Sensitivity of Input Side Inventory Cost ( $VI$ ) to $Q_{ij}$

A series of experiments were conducted on the case study problem. For this series the parameter  $Q_{ij}$  was varied from 2,000 to 17,000. The overall sawtooth pattern of the resulting cost is analyzed in terms of 4 sub-component inventories: reorder inventory, complete cycle inventory, partial cycle inventory and static inventory. The results are shown in figures 6.2 and 6.3. We make the following observations from these results:



**Figure 6.2** *V1* Inventory Cost Component Behavior At  $B_2 = 15,000$ .





**Figure 6.3** Inventory Cost Component Behavior At  $B_2 = 15,000$ .

- As expected, the reorder level inventory cost is constant, since the length of the supply cycle is constant. See figures 6.2 and 6.3.
- The complete cycle and partial cycle inventories are differentiated by concave sweeps and vertical jumps, but in opposite directions. See figure 6.3.
- The definitions of the complete cycle and partial cycle inventories may be rewritten, respectively, as  $XI_{ij} (Q_{ij}^2) (1/2\delta_j Z_{jj})$  and  $YI_{ij} (Q_{ij}^2) (1/2\delta_j Z_{jj})$ . The terms  $XI_{ij}$ ,  $YI_{ij}$  and  $Q_{ij}^2$  vary with  $Q_{ij}$ . For now we have held constant the term  $(1/2\delta_j Z_{jj})$  and so this does not affect the inventory behavior. The concavity of these two inventory patterns is caused by the exponential term  $Q_{ij}^2$ . The complete cycle inventory pattern is concave up, because the number of complete cycles is integral.

The partial cycle inventory pattern is concave down, because the length of the partial cycles fluctuates between  $[0,1)$ .

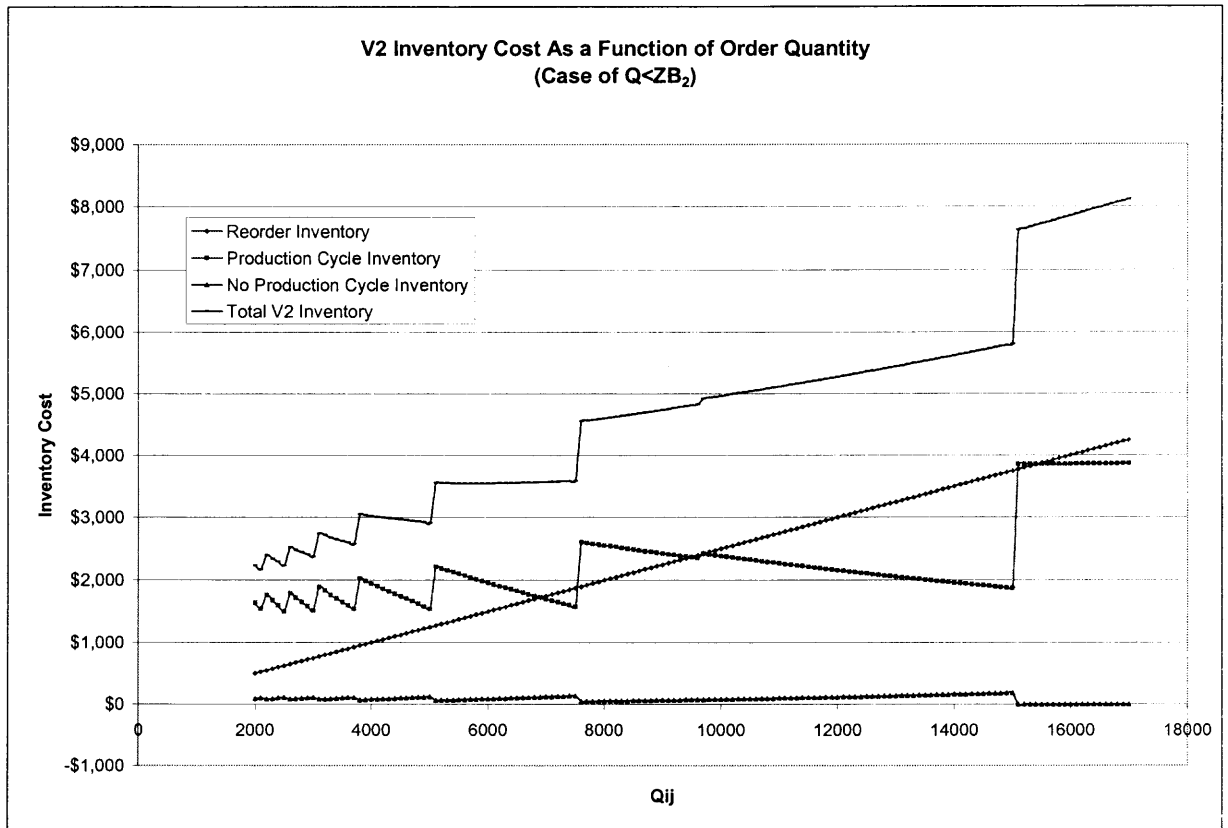
- The amplitude of both the complete order cycle and the partial order cycle steadily increases, because the term  $Q_{ij}^2$  becomes steadily larger for increasing experimental values of  $Q_{ij}$ . The integral number of complete orders magnifies  $Q_{ij}^2$  more than does the fractional length of any partial cycle.
- The frequency of the jumps in the complete cycle and partial cycle patterns is determined by differences in the number and length of the order cycles. The jumps downward in the complete cycle inventory occur at specific incremental values of  $Q_{ij}$ , at which the number of complete order cycles decreases by 1. The jumps upward in the partial cycle inventory occur at the same incremental values of  $Q_{ij}$ , at which the length of the partial cycle abruptly increases. At  $Q_{ij}$  of 2,000 there are 7 complete order cycles, whereas at  $Q_{ij}$  of 15,000 there is only 1 complete order cycle. As the frequency of complete cycles decreases, the frequency of abrupt increases in the length of the partial cycle also decreases. The cyclical frequency of the jumps decreases as the experimental values of  $Q_{ij}$  become larger.
- The sawtooth pattern of the  $VI$  inventory cost is clearly attributable to the behavior of the static inventory cost, which in the example constitutes the largest cost element.  $Q_{ij}$  and  $(1-YI_{ij})$  vary but  $B_f(1/D_f - 1/\delta_f)$  remains constant. The linearity of the pattern is produced by the product of two factors,  $Q_{ij}$  and  $(1-YI_{ij})$ .
- The length of a no-production cycle  $(1-YI_{ij})$  decreases where a fractional portion of the  $Z_{ij}B_f/Q_{ij}$  replenishment orders abruptly increases. The drop-offs in static

inventory occur precisely when the length of the no-production cycle decreases, because the length of a partial order cycle increases. The amplitude of the static inventory increases as the impact of  $(1-YI_{ij})$  is magnified by ever-larger values of  $Q_{ij}$ . When there is no partial order cycle, the static inventory is simply a function of  $Q_{ij}$ .

- The frequency of the static inventory cost drop-offs decreases steadily from the 1<sup>st</sup> to the 7<sup>th</sup> drop-off. The decreasing frequency is caused by the increasing distance between the values of  $Q_{ij}$  for which  $YI_{ij}$  increases.

### 6.2.3 Analysis of Sensitivity of Output Side Inventory Cost ( $V2$ ) to $Q_{ij}$

The general upward trend of the  $V2$  inventory cost is interrupted by local breakpoints. The  $V2$  inventory plateaus are slanted downward at small values of  $Q_{ij}$  and upward at large values of  $Q_{ij}$ . This behavior is analyzed in terms of 3 sub-cycle inventories: reorder inventory, production cycle inventory, and no-production cycle inventory. (i) The seller's reorder inventory level requires by contract that  $R_{ij} = Q_{ij}$  so as to avoid stockouts. (ii) The production cycle inventory is a combination of production area inventories: triangular inventories, block inventories, and residual inventories. These area inventories are enumerated by the number  $(XI_{ij})$  of shipments during the supply cycle, including number  $(X3_{ij})$  of shipments during the production cycle and number  $(XI_{ij} - X3_{ij})$  of shipments after the production cycle ends. (iii) The fraction of the cycles in a 10 supply cycle window that are no-production cycles is  $(1 - X4_{ij}/10)$ . Note that  $XI_{ij}$ ,  $X3_{ij}$  and  $X4_{ij}$  are defined to be integral (i.e., they are defined as round-off functions).



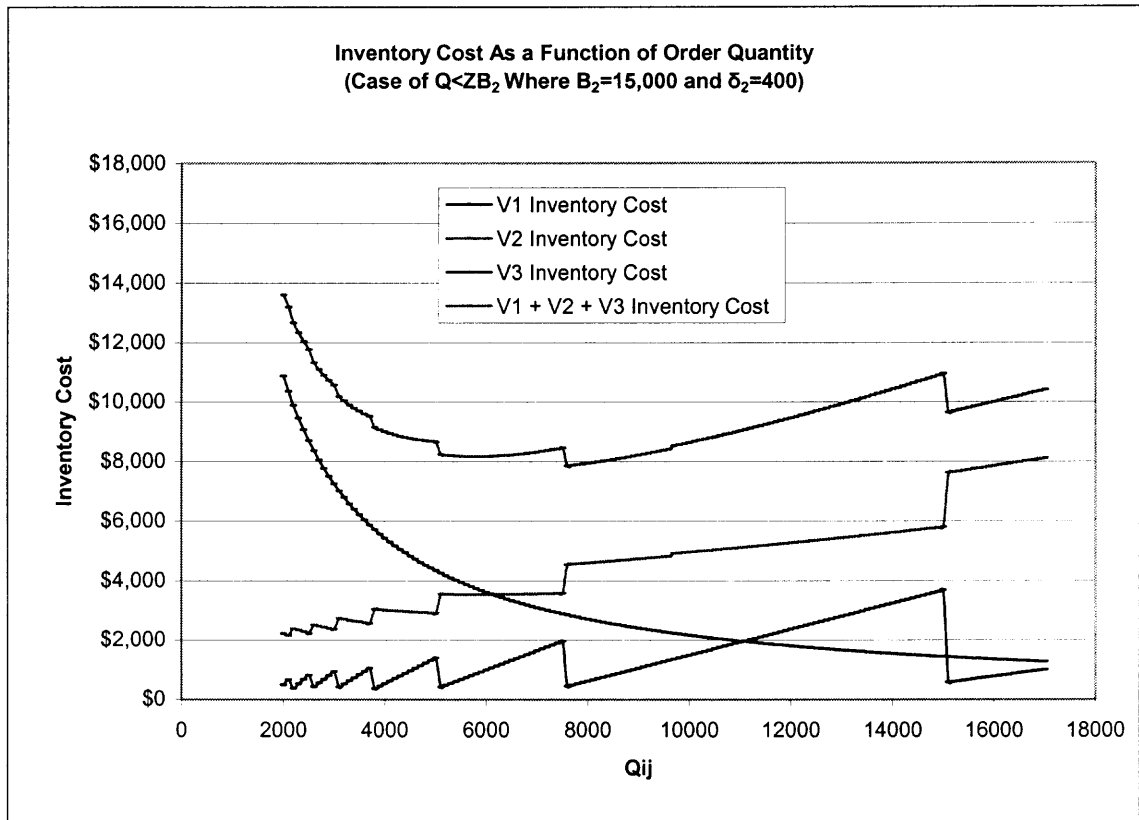
**Figure 6.4**  $V_2$  Inventory Cost Component Behavior At  $B_2 = 15,000$ .

- As  $Q_{ij}$  increases the reorder level inventory increases linearly as expected. This is the result of setting  $R_{ij} = Q_{ij}$ . The linear increase in the reorder level inventory is one reason why the  $V_2$  inventory plateaus are slanted downward at small values of  $Q_{ij}$  and upward at large values of  $Q_{ij}$  (see related comment below). See figure 6.4.
- The upward trend of the production cycle inventory is marked by vertical jumps upward. The frequency of these jumps is determined by  $X1_{ij}$  and  $X3_{ij}$ , which decrease at irregular intervals as  $Q_{ij}$  is increased. This accounts for the irregular occurrence of breakpoints in the production cycle pattern.

- The complex definition of production cycle inventory includes the terms  $Q_{ij}^2/\delta_j Z_{ij}$  and  $-(N_{jj}B_j - X3_{ij} Q_{ij})^2/2\delta_j$ . At low values of  $Q_{ij}$  the impact of the exponential term  $Q_{ij}^2$  is moderate compared with that of the negative term  $-(N_{jj}B_j - X3_{ij} Q_{ij})^2$ . As  $Q_{ij}$  is increased,  $Q_{ij}^2$  becomes relatively larger and eventually dominates the negative term. This explains why the downward movements of the production cycle inventory plateaus become progressively less steep as the values of  $Q_{ij}$  increase. For low values of  $Q_{ij}$  the downward movements are very sharp, while at  $Q_{ij}$  of 15,000 the downward movement almost levels off. The result is that the amplitude of the production cycle inventory plateaus is irregular. The gradual leveling of the production inventory plateaus is another reason why the slant of the  $V2$  inventory plateaus is downward at small values of  $Q_{ij}$  and upward at large values of  $Q_{ij}$ .
- The no-production cycle inventory is defined in terms of  $XI_{ij}$ , which decreases at irregular intervals as  $Q_{ij}$  increases. Overall this sub-component is a minor element.

#### 6.2.4 Analysis of Sensitivity of Total Inventory Costs to $Q_{ij}$

The overall inventory costs are seen in Figure 6.5. The curve is approximately convex with an optimal cost at  $Q_{ij}$  of 7,600.  $V1$ ,  $V2$  and  $V3$  are each sensitive to changes in  $Q_{ij}$ .



**Figure 6.5**  $V_1$ ,  $V_2$  and  $V_3$  Inventory Costs At  $B_2 = 15,000$ .

- The order cost  $V_3$  decreases asymptotically because of the impact of increasing values of  $Q_{ij}$  upon the number  $\{Z_{ij}B_j/Q_{ij}\}$  of replenishment orders. Note that this number is not defined to be integral. See figure 6.5.
- At  $Q_{ij}$  of 2,000 the order cost is \$10,875 and total inventory cost is \$13,606. At  $Q_{ij}$  of 7,600 the order cost is \$2,862 and total inventory cost is \$7,862. Thus order cost decreases by 73.7% while total inventory cost decreases by 42.2%. The result is that at  $Q_{ij}$  of 2,000 order cost comprises 79.9% of the total inventory cost, whereas at  $Q_{ij}$  of 7,600 order cost comprises 36.4% of the total inventory cost. As the values of  $Q_{ij}$  become larger, order component cost and total inventory cost both decrease steadily. The decrease in total inventory cost is more gradual because the  $V_3$  component cost is declining both absolutely and relatively.

- At  $Q_{ij}$  of 15,000 the order cost is \$1,450 and total inventory cost is \$10,952. Thus order cost decreases by 49.3% as  $Q_{ij}$  increases from 7,600 to 15,000. At  $Q_{ij}$  of 15,000 order cost is 13.2% of the total inventory cost. Thus as  $Q_{ij}$  increases from 7,600 to 15,000 the relative order cost declines, while  $V1$  and  $V2$  are both trending upward. This causes the total inventory cost curve to rise.
- As the values of  $Q_{ij}$  are increased, the  $V1$  inventory level jumps down vertically at irregular intervals, while the  $V2$  inventory level jumps up vertically at all but one of the same intervals (discussed below). The simultaneous occurrence of jumps down in the  $V1$  cost and jumps up in the  $V2$  cost is due to the presence of  $X1_{ij}$  in the definitions of  $V1$  and  $V2$ , and the presence of  $X3_{ij}$  in  $V2$  but not  $V1$ . The fraction  $Z_{jj}B_j/Q_{ij}$  is employed in defining  $X1_{ij}$  but not  $X3_{ij}$ .
- At  $Q_{ij}$  of 9,700 the  $V2$  inventory level jumps up vertically but the  $V1$  inventory level does not. This is caused by a change in  $X3_{ij}$ , defined as  $\text{Int}^+ \{(N_{jj}B_jZ_{jj}/Q_{ij})(\delta_j/\delta_j)\}$ , where  $N_{jj}$  is defined as  $\text{Int}^+ \{B_jZ_{jj}/B_j\} + 1$ .  $N_{jj}$  is calculated as  $\text{Int}^+ \{15,000/24,000\} + 1 = 1$ . The fraction  $\delta_j/\delta_j$  is calculated to be 400/1,000. For  $Q_{ij}$  of 9,600  $X3_{ij}$  is calculated at  $\text{Int}^+ \{(1 \times 24,000/9,600)(400/1,000)\} = 1$ . For  $Q_{ij}$  of 9,700  $X3_{ij}$  is calculated at  $\text{Int}^+ \{(1 \times 24,000/9,700)(400/1,000)\} = 0$ .  $X3_{ij}$  includes the mitigating fraction  $(\delta_j/\delta_j)$ , whereas  $X1_{ij}$  does not, and so  $X3_{ij}$  is smaller than  $X1_{ij}$ . Also  $X1_{ij}$  is defined by  $B_2$ , while  $X3_{ij}$  is defined by  $B_1$ , but  $N_{jj}$  is defined by both  $B_1$  and  $B_2$ . In subsection 6.2.5 the impact of the change in  $X3_{ij}$  is analyzed for 5 different experimental values of  $B_2$ .

### 6.2.5 Analysis of Sensitivity of $V1$ and $V2$ to $B_2$

We now experiment with  $B_2$  to study how the buying supplier's production batch size affects inventory cost. We increase  $B_2$  in increments of 1,000 from 13,000 to 17,000.

Our analysis of  $V1$  will involve the number of replenishment shipments ( $X1_{ij}$ ) and the length of the supply cycle ( $B_j/D_j$ ).  $B_2$  is in the numerator of both  $X1_{ij}$  and  $B_j/D_j$ . As  $B_2$  is increased, with  $Q_{ij}$  held constant, the number of replenishments and the length of the supply cycle both become larger. We analyze the impact of  $B_2$  on both unitized  $V1$  inventory cost and unitized total inventory cost.

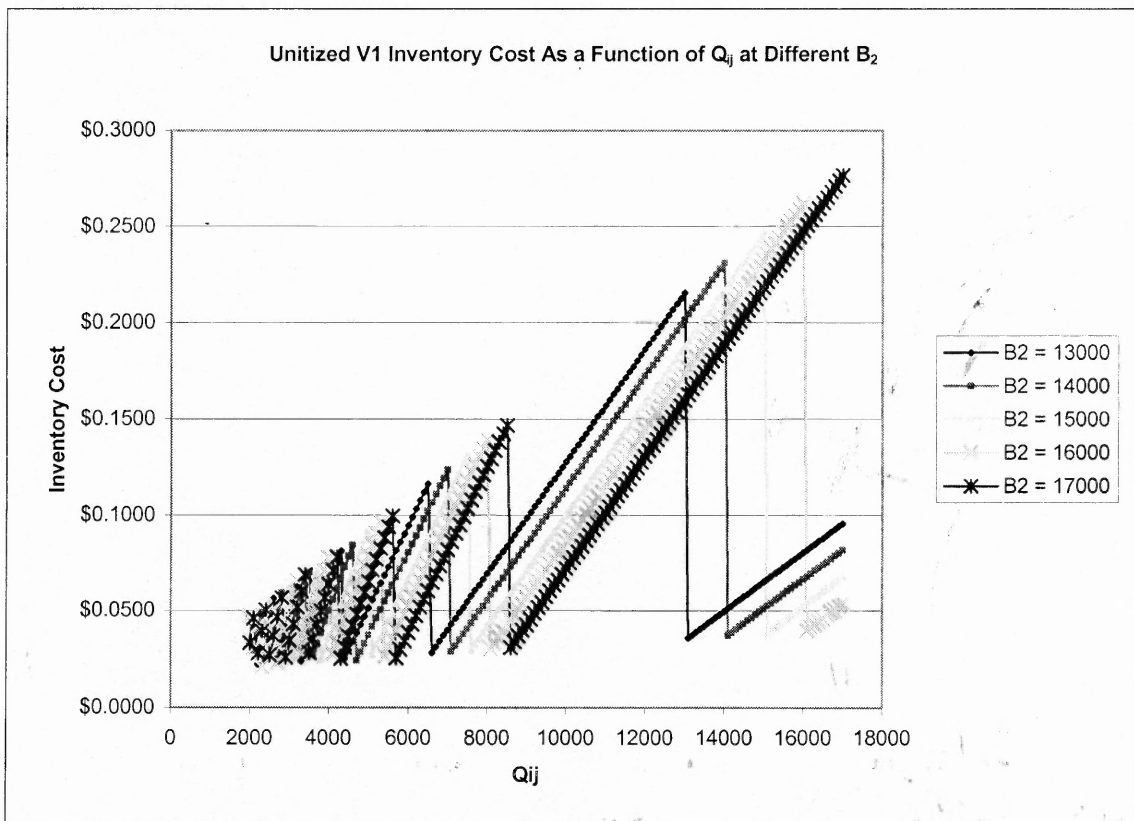
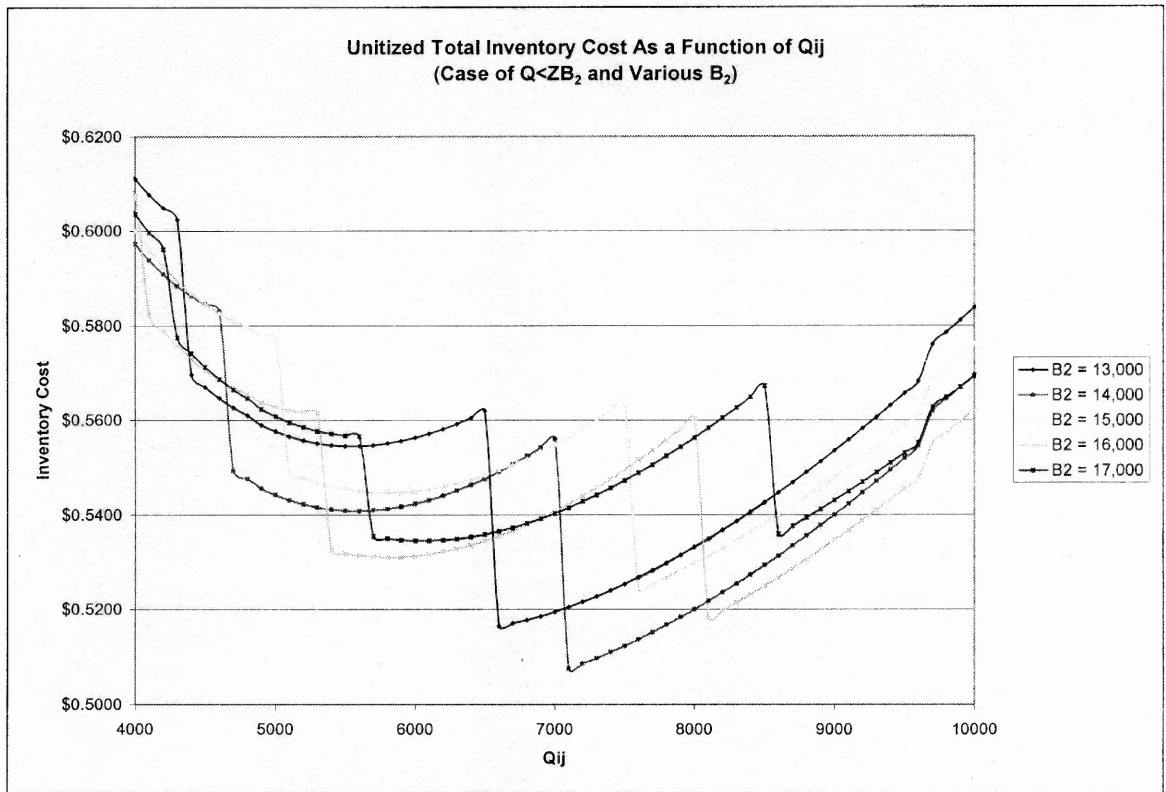
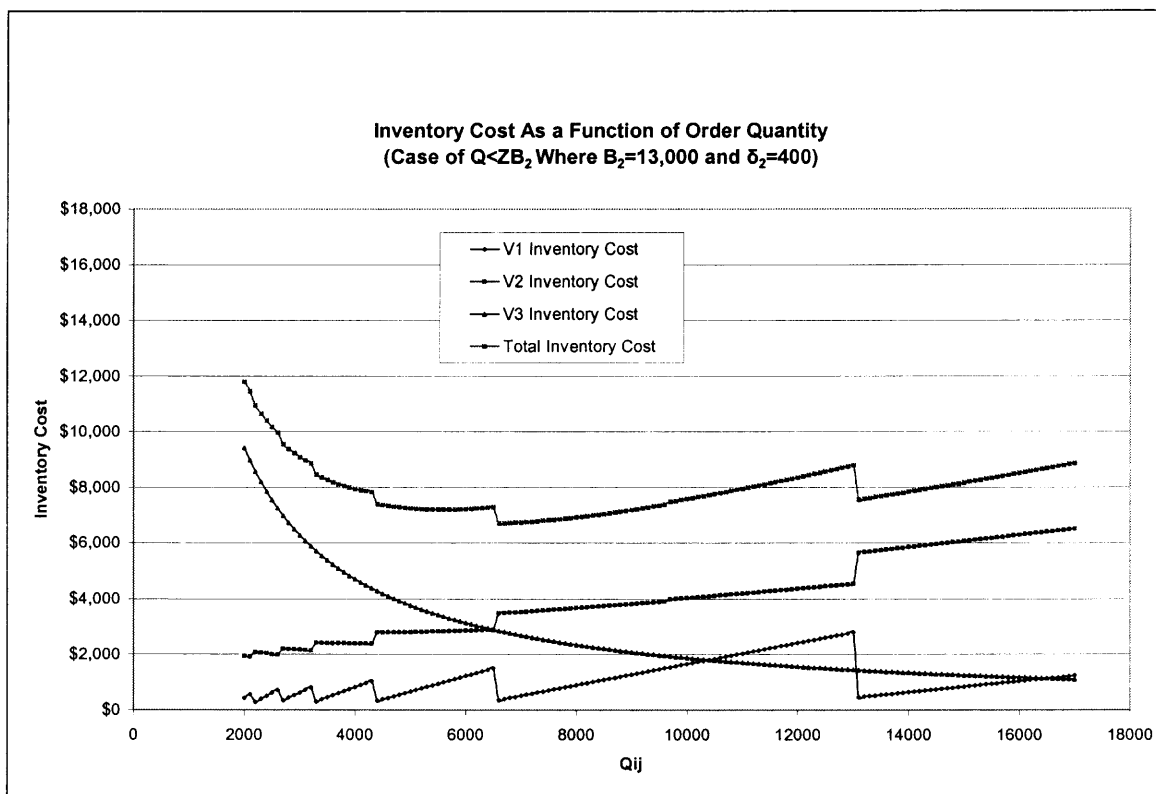


Figure 6.6 Unitized  $V1$  Inventory Cost.

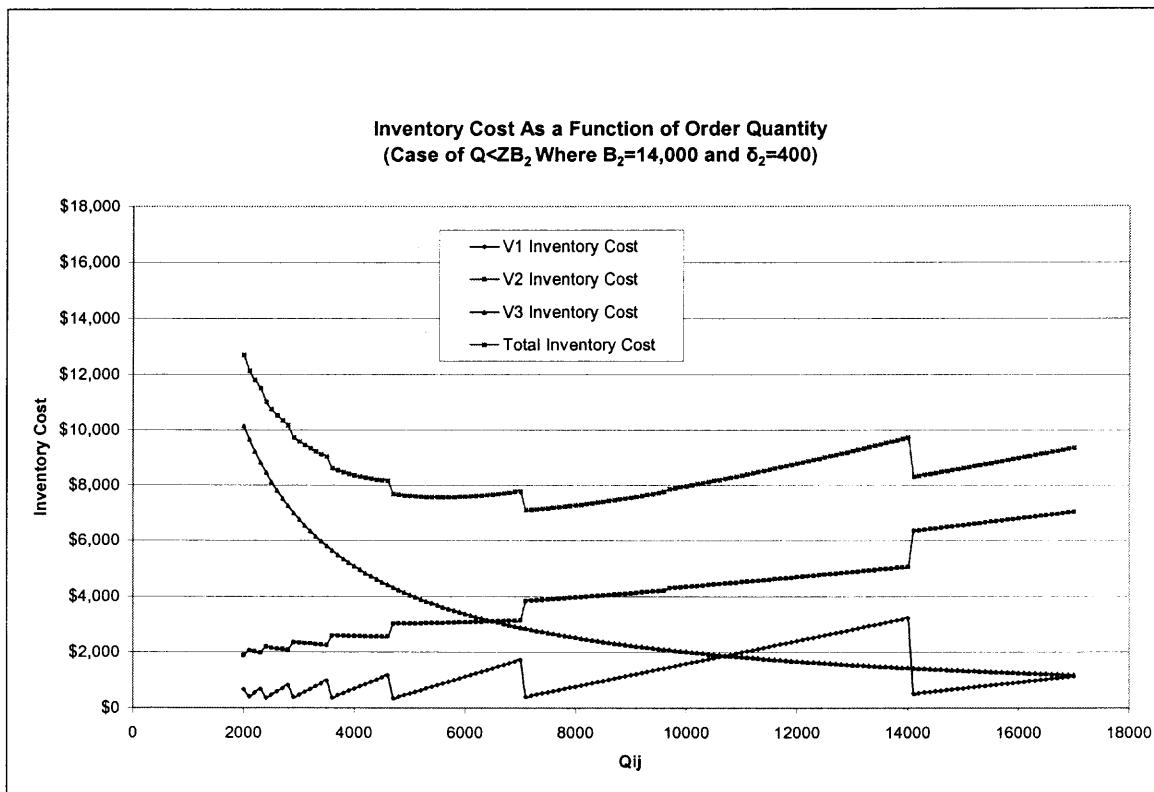




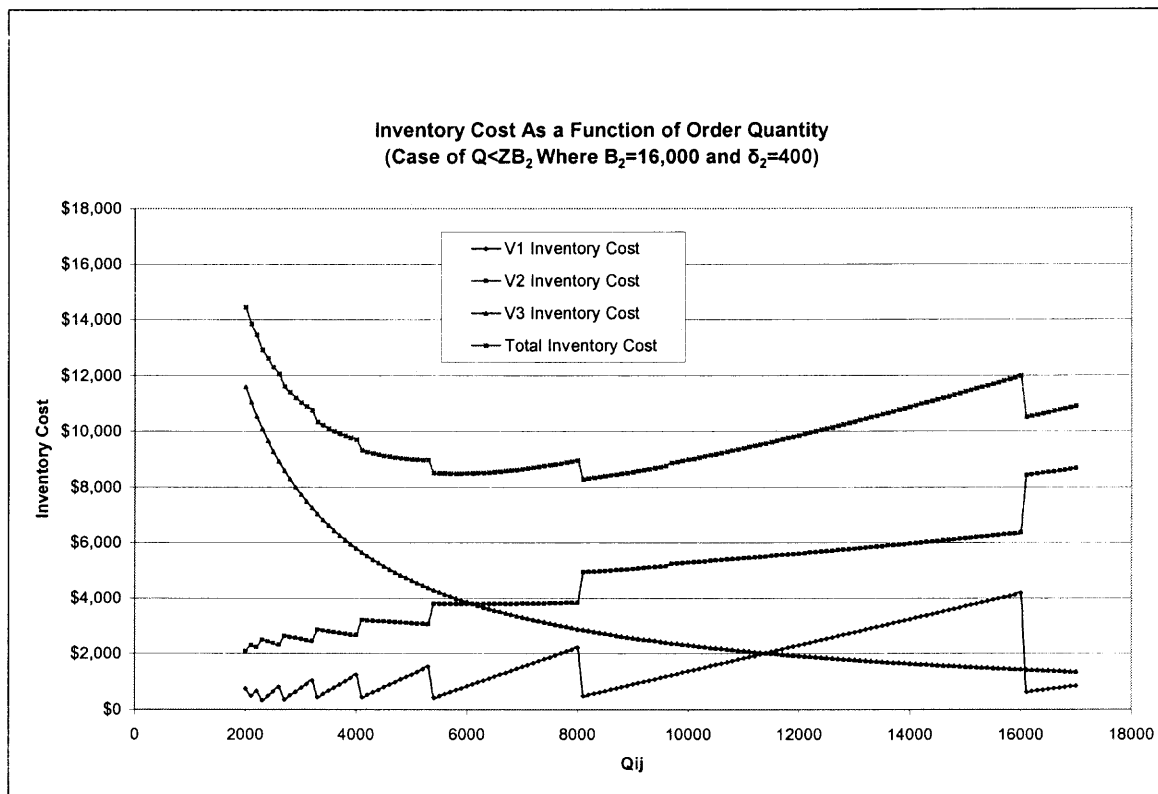
**Figure 6.7** Unitized Total Inventory Cost.



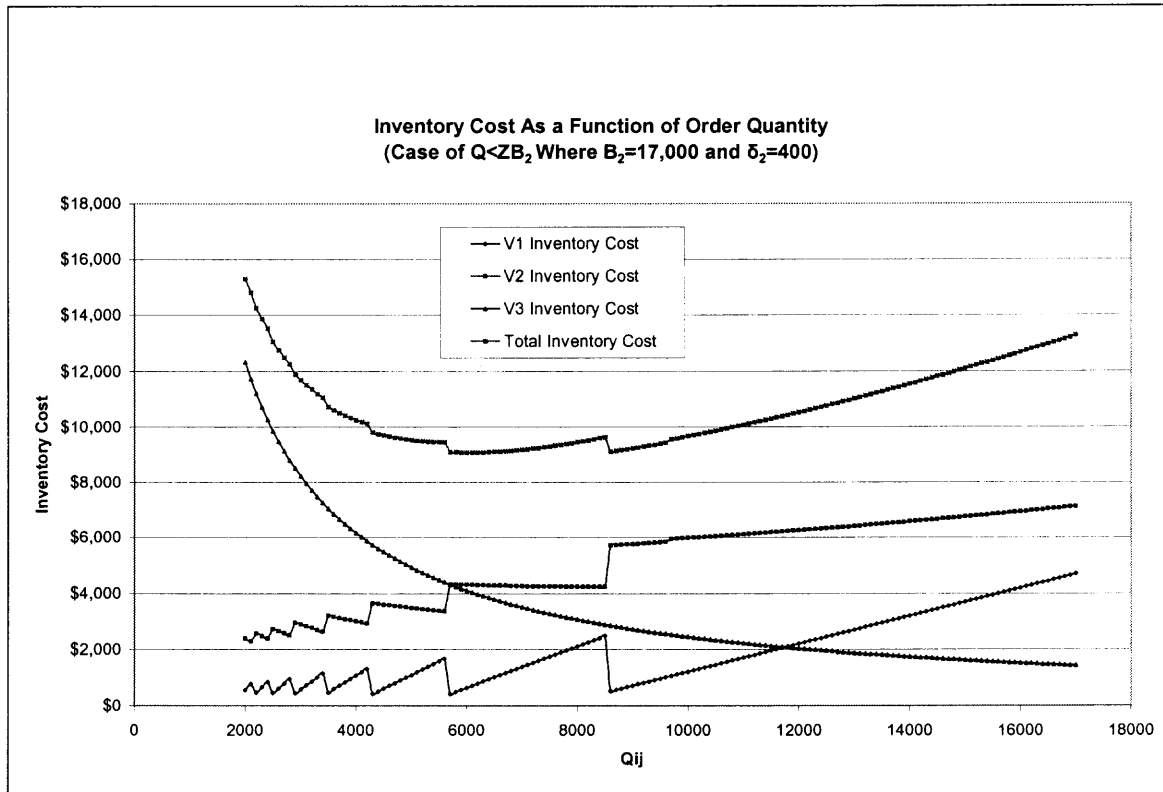
**Figure 6.8**  $V_1$ ,  $V_2$  and  $V_3$  Inventory Costs At  $B_2 = 13,000$ .



**Figure 6.9**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $B_2 = 14,000$ .



**Figure 6.10**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $B_2 = 16,000$ .



**Figure 6.11**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $B_2 = 17,000$ .

- In general the amplitude of the *total V1* inventory level is high when the number of replenishments is large and the supply cycle is long. As we increase  $B_2$  from 13,000 to 17,000, with  $Q_{ij}$  held constant, the *unitized V1* inventory level becomes steadily lower, reflecting larger production batch sizes. See figure 6.6.
- In general the frequency of the downward spikes in the *total V1* inventory level is high when the number of replenishments is small and the supply cycle is short. As we increase  $B_2$  from 13,000 to 17,000, with  $Q_{ij}$  held constant, the downward spikes in the *unitized V1* inventory become less frequent, reflecting longer supply cycles. See figure 6.6.
- $B_2$  is used in 2 of the production area inventories of  $V2$ . It is used as a positive term in  $B_j/D_j$  (to quantify residual inventory at the start of the no-activity period)

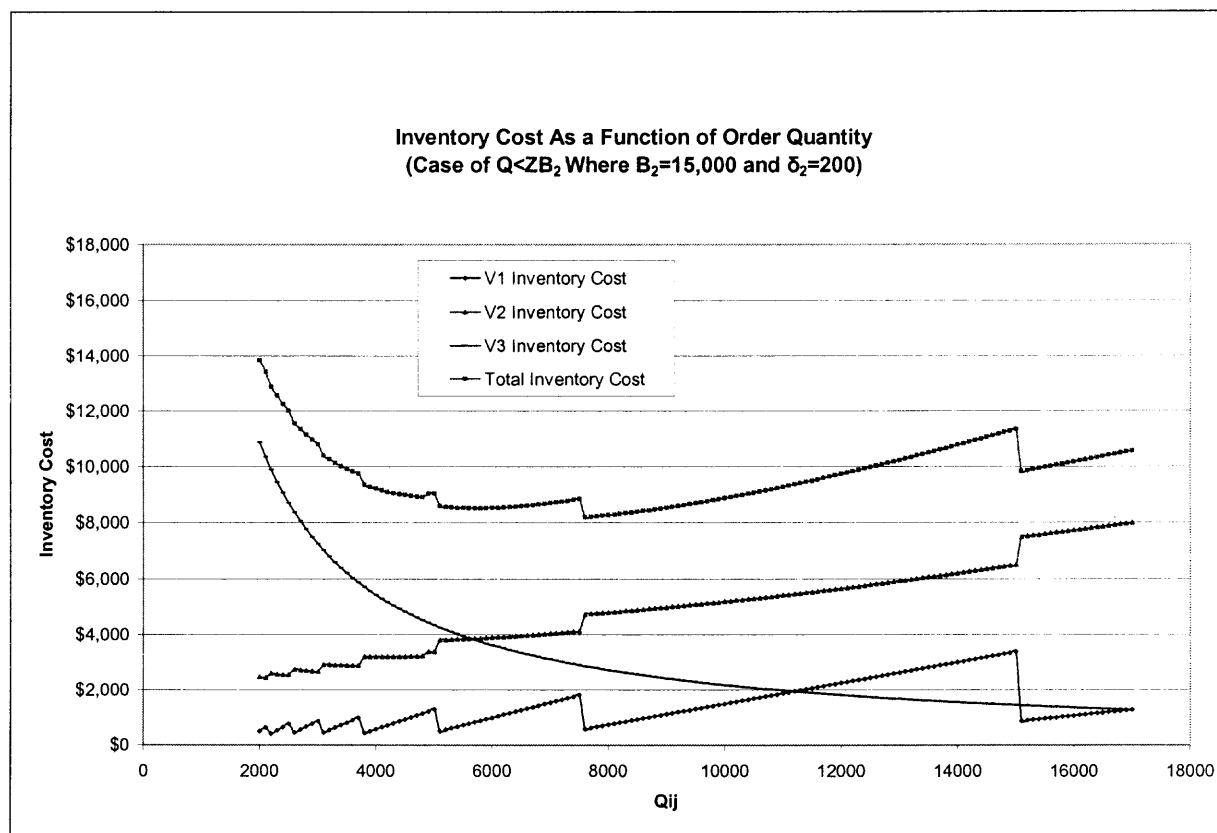
and as a negative term in  $(N_{jj}B_j - B_jZ_{jj})$  (to quantify the burden of carrying forward the residual inventory from each production cycle to the no-production cycle). The overall impact of  $B_2$  is more negative than positive, and so as  $B_2$  is increased from 13,000 to 17,000 the final upward spike in the  $V2$  inventory level is delayed further to the right. When  $B_2$  reaches 17,000 the final upward spike in  $V2$  is eliminated, and so the  $V2$  inventory level rises (from  $Q_{ij}$  of 8,600 to  $Q_{ij}$  of 17,000) without a spike, reflecting an overall decrease in residual inventory. See figures 6.5, 6.8 to 6.11.

- The curve of the total inventory cost (which includes  $V1$ ,  $V2$  and  $V3$  costs) is approximately convex for all 5 experimental values of  $B_2$  – 13,000 to 17,000. At  $Q_{ij}$  of 9,700 there is a small jump upward in the total inventory curves of all 5 values of  $B_2$ . This small jump is the result of our definition of  $X3_{ij}$ , which is  $\text{Int}^+ \{ (N_{jj} B_j Z_{jj} / Q_{ij}) ( \delta_j / \delta_j ) \}$ . Note that  $B_1$  is in the numerator, not  $B_2$ . As  $Q_{ij}$  increases from 9,600 to 9,700 the integral values of this function decrease from  $\text{Int}^+ \{ (24,000/9600)(400/1,000) \} = 1$  to  $\text{Int}^+ \{ (24,000/9700)(400/1,000) \} = 0$ .  $X3_{ij}$  appears within our  $V2$  equation in 4 different factors, 3 of which are negative. When the integral value of  $X3_{ij}$  decreases from 1 to 0 the weight of these negative factors becomes correspondingly smaller, and so the inventory level jumps up.
- The curve of the *unitized* total inventory cost is also approximately convex for all 5 experimental values of  $B_2$ . The convexity is contributed by the exponential terms in our definitions of  $V1$  and  $V2$ . The influence of the term  $Q_{ij}^2$  upon  $V1$  has already been noted in subsection 6.2.2. The term  $Q_{ij}^2$  is also included within three different factors of our definition of  $V2$ . See figure 6.7.

- At  $Q_{ij}$  of 4,000 the highest *unitized* total inventory cost is \$.6111, which is incurred at  $B_2$  of 13,000. At  $Q_{ij}$  of 7,100 the lowest *unitized* total inventory cost is \$.5075, which is incurred at  $B_2$  of 14,000. This is a decrease of 17% in the *unitized* total inventory cost. Then at  $Q_{ij}$  of 10,000 the highest *unitized* total inventory cost is \$.5839, which is again incurred at  $B_2$  of 13,000. This is an increase of 15% in the *unitized* total inventory cost. As described in chapter 4, the supply cycle ( $B_j/D_j$ ) for a part is the interval between 2 successive production batch starts of the buying supplier. Therefore the supply cycle wherever  $B_2$  is 13,000 is different than the supply cycle where  $B_2$  is 14,000. The contrast between the relatively high *unitized* total inventory costs at  $Q_{ij}$  of 4,000 and 10,000 with the intervening low *unitized* total inventory costs at  $Q_{ij}$  of 7,100 is really a comparison of unit costs for different supply cycles.
- The impact of  $B_2$  is greater upon  $V1$  than  $V2$ . This is because the decrease in frequency of the downward spikes in  $V1$  is more significant than the decrease in frequency of the upward spikes in  $V2$ . Therefore as  $B_2$  is increased from 13,000 to 17,000 the increments in  $V1$  cost are about twice as large as the decrements in the  $V2$  cost. This is seen in the increasing levels of the total inventory cost in figures 6.5 and 6.8 to 6.11. When  $B_2$  reaches 17,000 the final downward spike in total inventory cost is eliminated and the approximate convexity of the total inventory curve is most obvious.

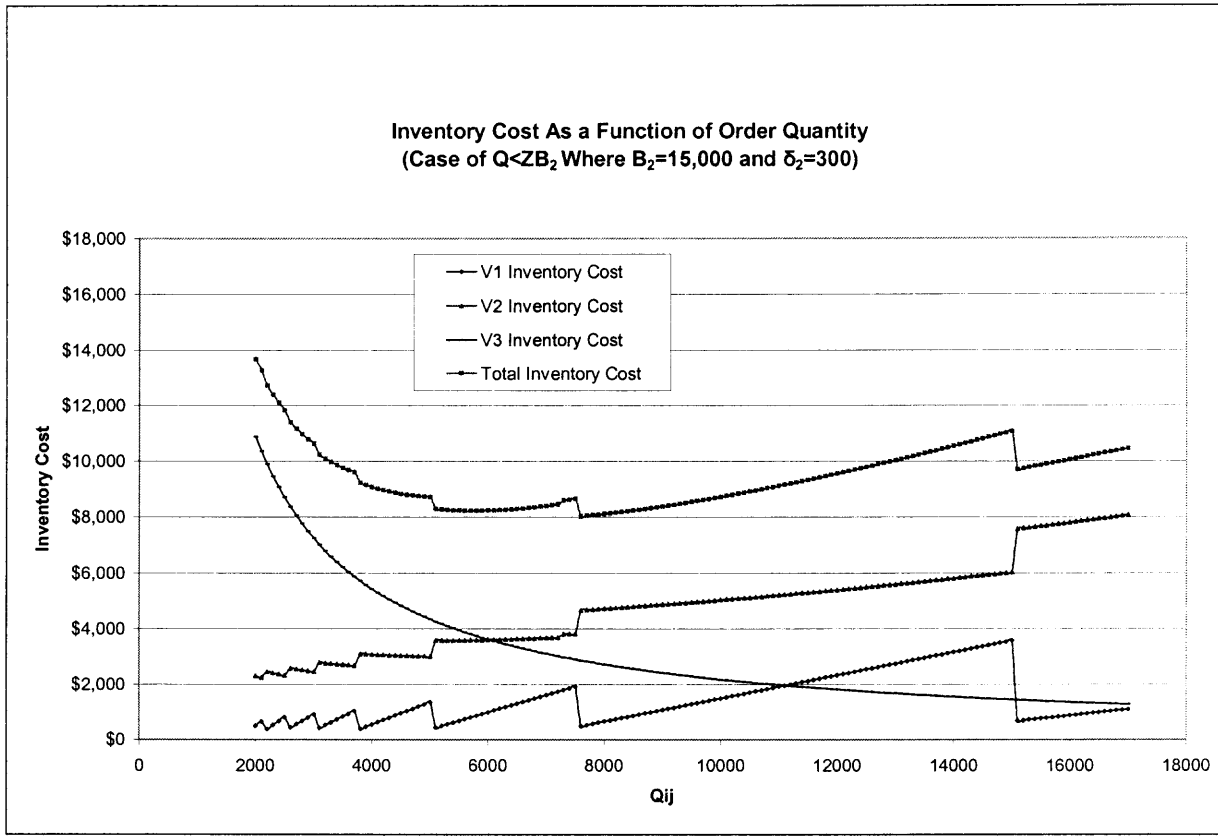
### 6.2.6 Analysis of Sensitivity of $V1$ and $V2$ to $\delta_2$

Finally, we experiment with  $\delta_2$  to study the impact of the buyer's production rate on inventory cost. We increase  $\delta_2$  in increments of 100 from 200 to 600. See figures 6.5 and 6.12 to 6.15.

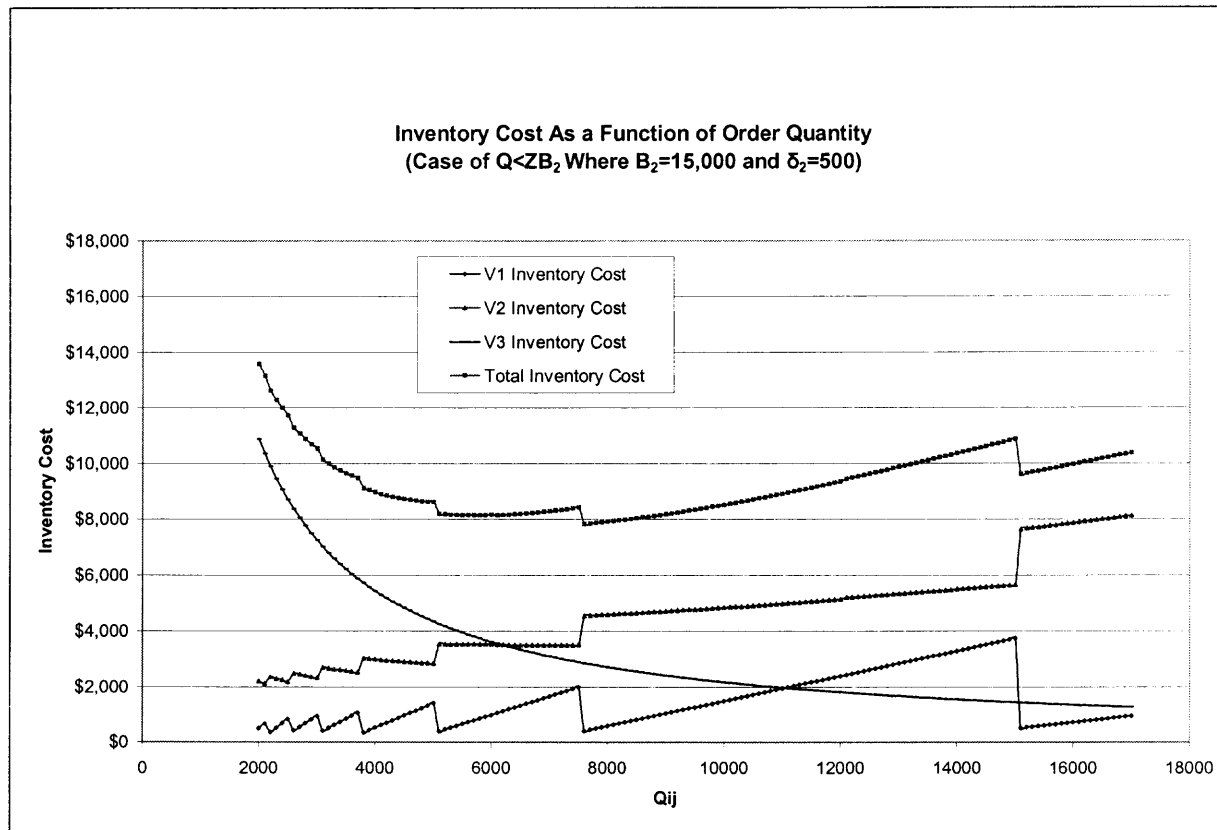


**Figure 6.12**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $\delta_2 = 200$ .

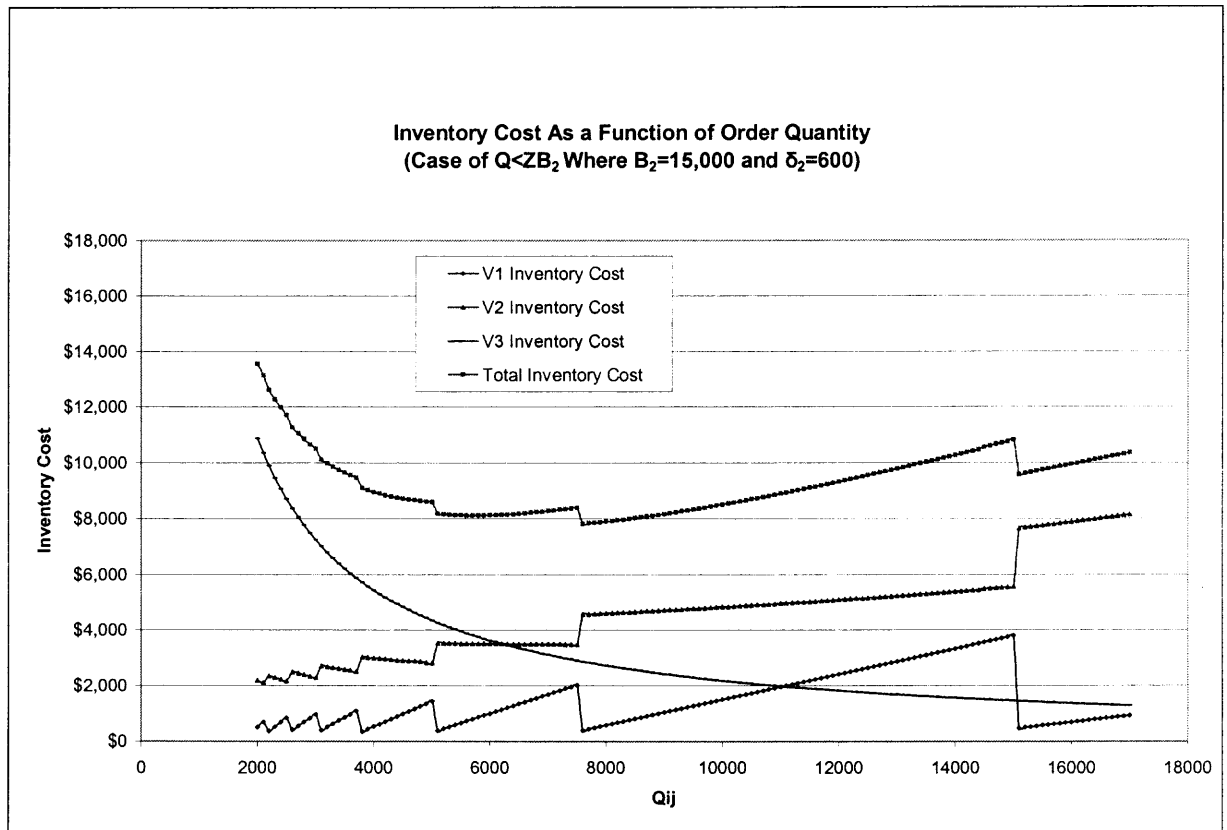




**Figure 6.13**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $\delta_2 = 300$ .



**Figure 6.14**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $\delta_2 = 500$ .



**Figure 6.15**  $V1$ ,  $V2$  and  $V3$  Inventory Costs At  $\delta_2 = 600$ .

- The  $V1$  inventory cost trends upward in staggered fashion. As we increase  $\delta_2$  the static inventory sub-component of  $V1$  becomes larger relative to the production cycle inventory sub-component. As the buyer's production rate increases, with  $B_2$  held constant, the production batch is completed more quickly, and so the length of the no-production cycle increases.
- As we increase  $\delta_2$  the gap between the  $V1$  inventory cost and the  $V2$  inventory cost becomes absolutely smaller, and the production cycle inventory sub-component of  $V2$  becomes relatively smaller. As the buyer's production rate increases, with  $B_2$  held constant, the seller's inventory stock is deployed more

quickly to satisfy the buyer's requirements, and so the seller's production cycle inventory decreases.

### 6.3 $(Q, R, \delta)^2$ Experiments for the N-Supplier Case

In this section we introduce a  $(Q, R, \delta)^2$  case study problem involving 4 suppliers. The inter-supplier relationships within this supply chain will allow us to study the inventory behavior and its impact on the costs and profits of a supply network. In order to conduct the planned experiments the supply chain of a sample product was studied. After reviewing several products a common desktop stapler was chosen. One reason for selecting this product was that our knowledge about the production process and the typical retail price enabled us to generate a fairly accurate set of parametric data. In our example we consider the case where there are two final products: Stapler Model 100 and Stapler Model 150. Table 6.2 lists all the parts and materials that make up the bill-of-materials for the two staplers. Note that parts/materials 1 to 6 are procured, while all others are outputs of the chain suppliers. The BOM is similar to products commonly found in supply stores such as OfficeMax or Staples.

**Table 6.2** Parts, Materials, and Final Products in the Case Study Supply Chain

PARTS LIST	
PART # ( <i>j</i> )	DESCRIPTION
1	1040 Steel Gauge 12
2	1080 Steel Gauge 9
3	HDPE 95% Pure
4	Spring
5	Rubber Gasket 1
6	Rubber Gasket 2
7	Metal Base Frame
8	Magazine Channel Frame
9	Drive Blade
10	Plastic Base Housing
11	Pusher
12	Main Body
13	Regular Stapler
14	Modern Stapler
15	Stapler Model 100
16	Stapler Model 150

From a review of current production and distribution practices for staplers we have organized the stapler supply chain into the following four suppliers:

1. Plastics Injection Molder
2. Sheet Metal Stamper
3. Assembly Plant
4. Wholesale Distributor

The wholesale retailer represents the import facility for a large office supply chain. In order to minimize costs the wholesale retailer has established the chain by identifying and teaming with suppliers located in Asia.

Figure 6.16 shows the NEXUS Supply Chain diagram for the case study. As introduced in chapter 3, this diagram lists the majority of the key parameters that describe the supply chain. From figure 6.16 we see that the final selling price of the staplers from the wholesaler is \$6.20 for Model 100 and \$6.25 for Model 150. These prices are reflective of the prices commonly seen in large office supply retailers in the US. We consider the baseline or nominal annual demand for the 2 stapler models to be  $D_{15} = 70,000$  and  $D_{16} = 40,000$  units/year.

In tables 6.3 and 6.4 and in figures 6.17 to 6.19 we provide some sample outputs that are generated for the baseline stapler supply chain problem. Note that in table 6.3 each row represents a unique  $(Q, R, \delta)^2$  relationship. These results are obtained from applying the  $(Q, R, \delta)^2$  model presented in chapter 4. Table 6.3 shows the results of the cost equations developed in chapter 4, and is the key document for understanding the cost structure of a supply chain. The data in table 6.3 are aggregated in table 6.4.

From table 6.3 we find that the total cost of inventory, materials and value added is \$5.633 (cost of last supplier), and that the net profit is \$ 2.217. The input side inventory cost is the sum of the V1 and V3 costs, while the output side inventory cost is the sum of the V2 and V4 costs. The material costs  $M_{i,j}$  are obtained by applying the formula  $Cs_{i,j} \cdot Z_{j,j}$  from equation 4.79 of chapter 4. The data necessary for the material cost calculations are presented in the NEXUS supply chain diagram of figure 6.16. The selling prices of each output part are given in the UPM's of that figure. For example the material cost of output part 7 is obtained as the product of the selling price ( $Cs_{i,j}$ ) and the BOM quantity ( $Z_{j,j}$ ). The value adding cost  $A_{i,j}$  is obtained by applying equation 4.80 from chapter 4. Total cost/unit is the sum of the input side inventory cost, output side

inventory cost, material cost and value adding cost. Net profit is the difference between total cost and selling price.

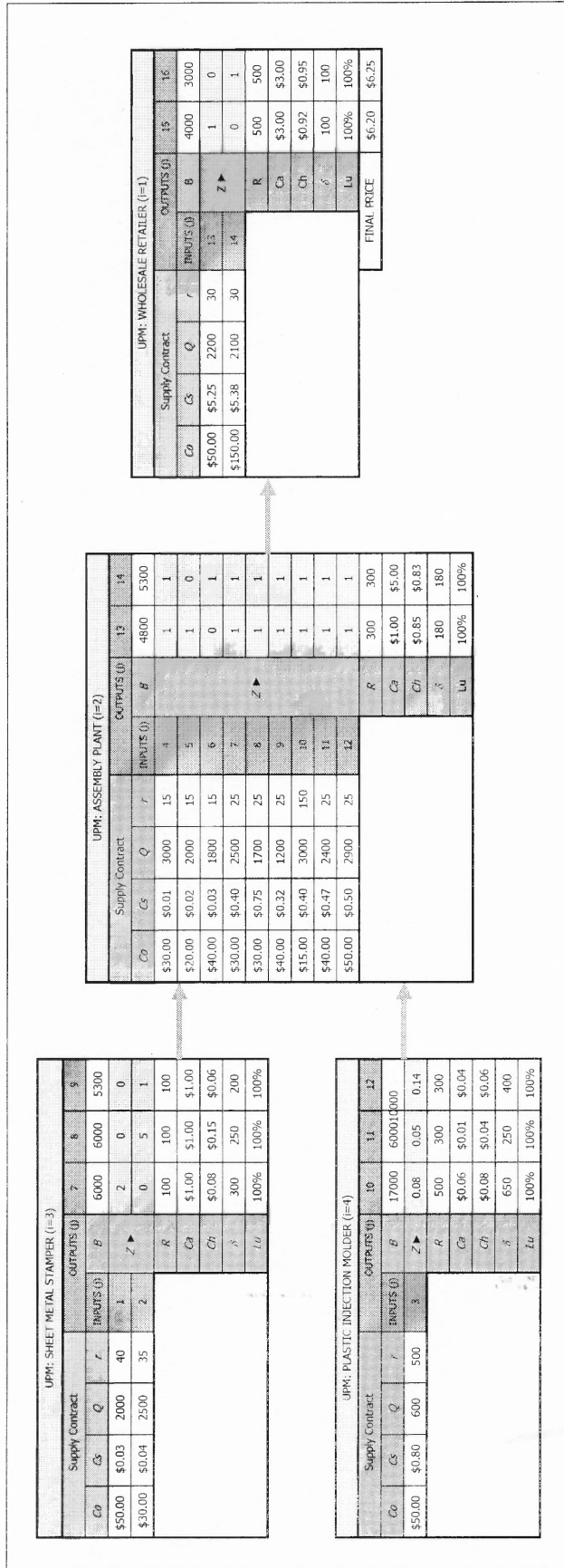


Figure 6.16 NEXUS Supply Chain Diagram for the Stapler Case Study.

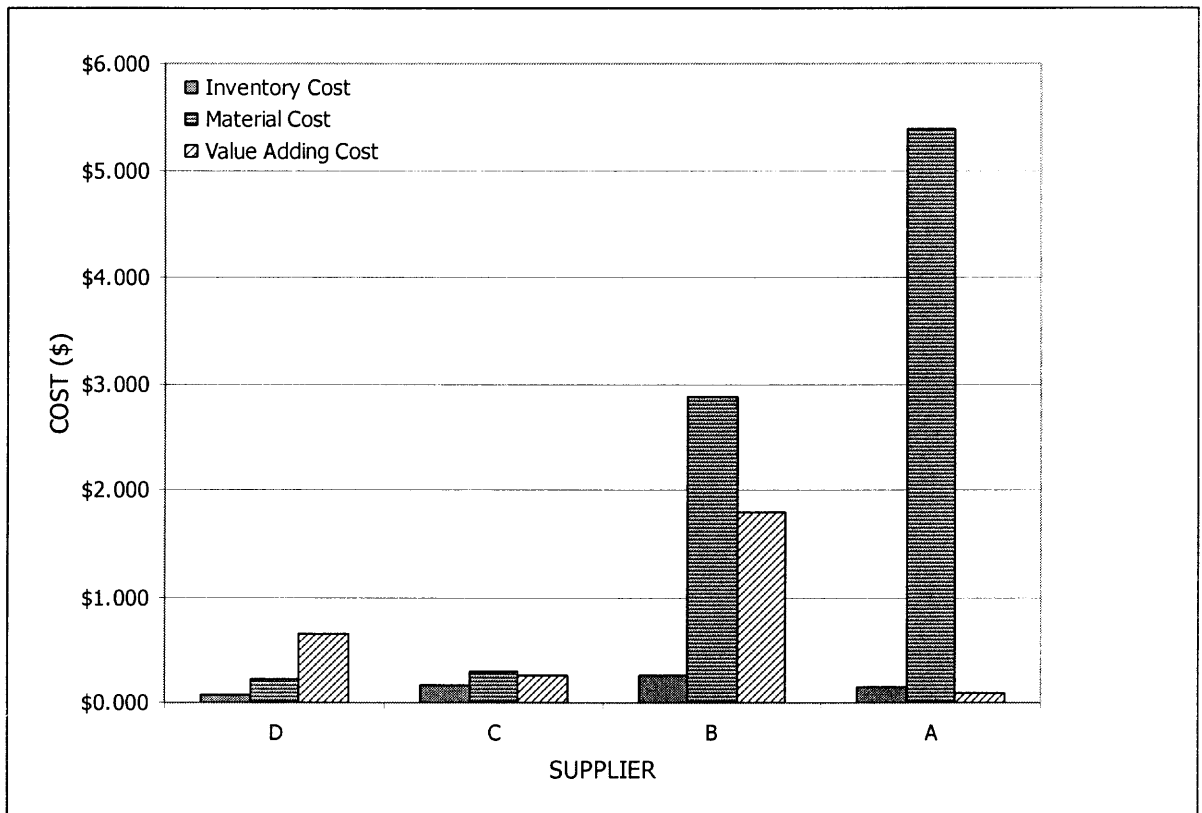


**Table 6.3 Profit/Cost Output for the Baseline Numerical Example**

SUPPLIER (i)	OUTPUT PART (j)	BOM (Z)	INVENTORY COSTS / UNIT J=16				INPUT SIDE INV COST (Ii)	OUTPUT SIDE INV COST (Io)	MATERIAL COST (Mij)	VALUE ADDING COST (Aij)	TOTAL COST/UN IT J=16	SELLING PRICE (Csj)	NET PROFIT (Piij)	PROFIT SHARE	PROFIT RATIO ( $\pi$ )	
			V1/(B/P)	V2/(B/P)	V3/(B/P)	V4/(B/P)										
C	7	1	\$0.003	\$0.002	\$0.050	\$0.002	\$0.053	\$0.005	\$0.084	\$0.201	\$0.400	\$0.199	9%	141%		
C	8	1	\$0.007	\$0.004	\$0.040	\$0.005	\$0.047	\$0.009	\$0.066	\$0.322	\$0.750	\$0.428	19%	351%		
C	9	1	\$0.007	\$0.001	\$0.040	\$0.002	\$0.047	\$0.003	\$0.108	\$0.198	\$0.320	\$0.122	6%	78%		
D	10	1	\$0.001	\$0.024	\$0.011	\$0.007	\$0.012	\$0.031	\$0.140	\$0.247	\$0.400	\$0.153	7%	84%		
D	11	1	\$0.001	\$0.001	\$0.011	\$0.002	\$0.012	\$0.003	\$0.191	\$0.246	\$0.470	\$0.224	10%	109%		
D	12	1	\$0.001	\$0.008	\$0.011	\$0.003	\$0.012	\$0.010	\$0.326	\$0.460	\$0.500	\$0.040	2%	11%		
B	14	1	\$0.023	\$0.054	\$0.151	\$0.038	\$0.174	\$0.092	\$1.800	\$4.946	\$5.380	\$0.434	20%	21%		
A	16	1	\$0.026	\$0.032	\$0.100	\$0.000	\$0.126	\$0.032	\$0.096	\$5.633	\$6.250	\$0.617	28%	243%		
TOTALS											\$2.811	\$0.185	\$0.482	\$2.217		

**Table 6.4** Profit/Cost Aggregated by Supplier for the Baseline Numerical Example

SUPPLIER (i)	INV COST (Ii+Io)	MATERIAL COST (Mij)	VALUE ADDING COST (Aij)	TOTAL COST/UNIT J=16	NET PROFIT (Πij)	PROFIT PERCENT	PROFIT RATIO (η)
D	\$0.080	\$0.216	\$0.657	\$0.953	\$0.417	19%	57%
C	\$0.163	\$0.300	\$0.258	\$0.721	\$0.749	34%	178%
B	\$0.266	\$2.880	\$1.800	\$4.946	\$0.434	20%	21%
A	\$0.157	\$5.380	\$0.096	\$5.633	\$0.617	28%	243%

**Figure 6.17** Cost Distribution Across Suppliers for the Baseline Numerical Example.

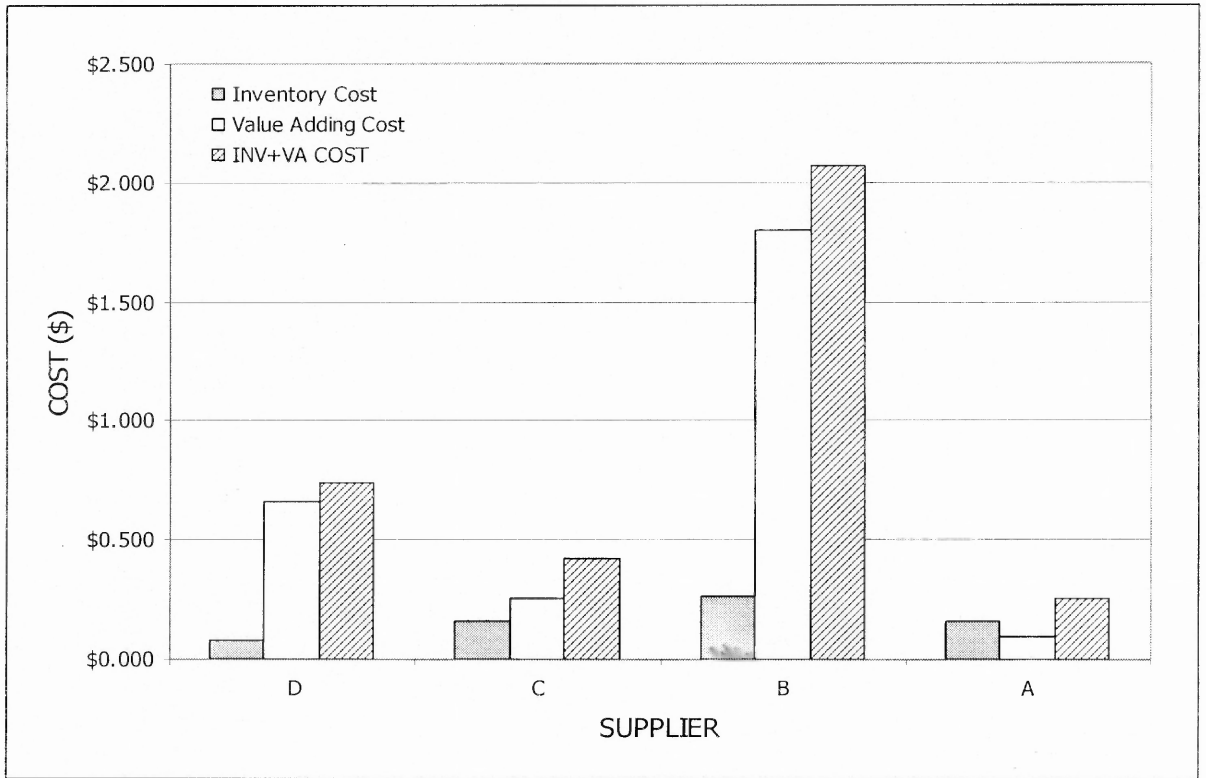


Figure 6.18 Non-Material Cost Distribution Across Suppliers.

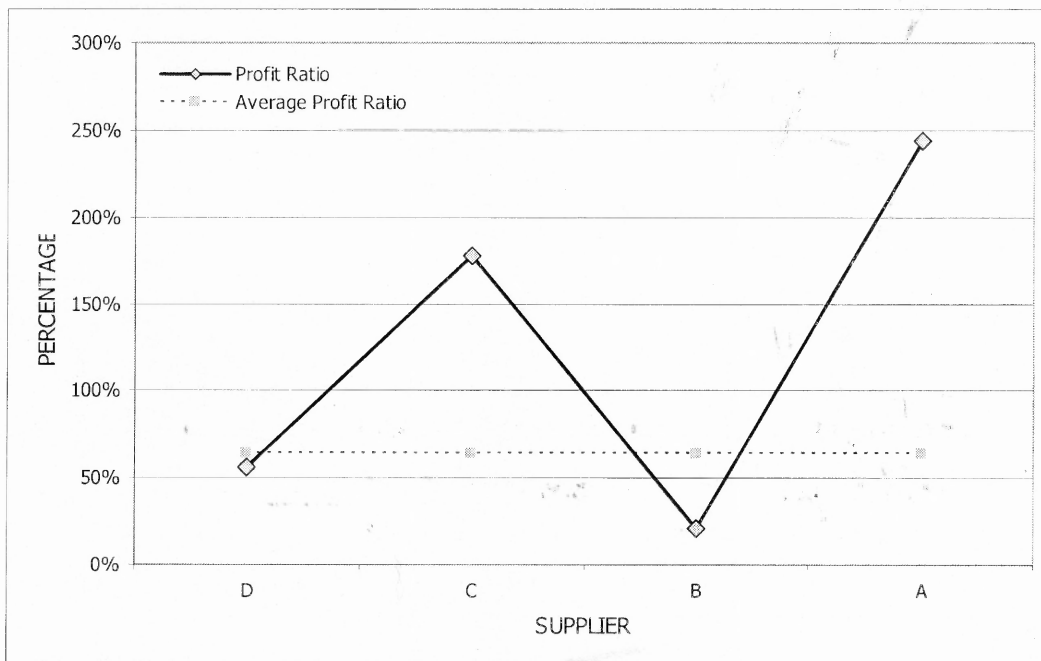


Figure 6.19 Profit Ratios for Each Supplier.

From reviewing the associated tables and figures we are able to derive the following analytics about this supply chain:

- Total inventory costs in the chain amount to \$ 0.67 which is 12% of the total product cost and 30% of the net profit.
- The inventory costs are primarily on the supplier input side with an average cost ratio of 2.6:1 between input and output side costs. Supplier B has the highest inventory cost burden of \$0.27 per unit final product, although this inventory cost is only 5% of supplier B's total cost. The inventory to total cost ratio for supplier C is 23%, for supplier D it is 8%, and for supplier A it is 2%.
- From figure 6.17 we see that the inventory costs relative to material and value adding costs are much larger for suppliers C and D than for suppliers B and A. Since supplier A has little value adding cost, it is the only supplier whose inventory costs exceed the value adding costs. Surprisingly, though, the supply chain design is favorable to supplier A since its inventory costs are only \$0.16 which is 24% of the total inventory cost. Supplier C also carries 24% of the inventory costs.
- The total value added in the supply chain is \$2.81 or 50% of the total cost. This metric is often a surrogate for the enterprise value of the chain. Clearly, as the value added ratio declines, it is more likely that competitive supply chains will be established.
- The sum of inventory and value added costs (also called non-material costs) is an indicator of the relative investment each supplier makes in the product. From figure 6.18 we see that supplier B has an investment of \$2.07 or 37% of the total

product cost. This indicates the relative importance from both an efficiency and cost perspective of supplier B. In contrast, the wholesale distributor (supplier A) has a non-material investment of only 4%.

- From table 6.4 we see that supplier C has the largest absolute profit (34%) in the chain, while suppliers D and B have the smallest (19% and 20%, respectively). Figure 6.19 plots the profit ratios and also shows the average profit ratio line (64%). As discussed in chapter 4, our proposition is that figure 6.19 is a key analytics indicator of supply chain stability. It is clear that in this case suppliers C and A are the primary beneficiaries in that they have very high profit ratios. An important question is whether the profit ratios of each supplier are proportionate to the responsiveness cost of each supplier. We address this question in section 6.4.

#### 6.4 Responsiveness Experiments

In this section we focus on the analysis of supply chain responsiveness using the equations developed in chapter 5. To test the utility of our responsiveness model we experiment with the parameters representing external demand, demand correlation, final product commonality and lead time to adjust to demand change. Demand is represented by  $\Delta D_J$  and  $E_J$ , correlation by  $\alpha_{JJ}$ , commonality by  $\psi^i_{JJ}$ , and lead time by  $L_J$  and  $\lambda_J$ . We use the same baseline experiment for the 4-supplier, 16-component case study of section 6.3. One advantage of doing so is that our responsiveness model includes the effects of final product correlation and resource commonality, which necessarily involve multiple suppliers and products.

Key parameters are shown in figure 6.16. Additional parameters relating specifically to the demand change behavior are shown in tables 6.5 and 6.6. Both final products have the same number of significant change events per year, but for product 15 the size of the change is much greater. The two products are negatively correlated so we expected the response costs to mitigate their joint impact. One of the key utilities of the NEXUS Model is that it will facilitate the design of more agile supply chains, that is, a network of suppliers that are able to more efficiently respond to major changes in the product demand. The experimental design for this section will focus on identifying what suppliers and supply conditions will lead to higher response costs.

**Table 6.5** Baseline Parameters for Demand Change, Correlation and Commonality

<i>Demand Correlations</i>		<i>Resource Commonality</i>	
$\Delta D_{15}$	21,000	$\psi_{15,16}^i$	.90
$\Delta D_{16}$	12,000	$\psi_{13,14}^i$	.80
<i>Demand Parameters</i>		$\psi_{10,11}^i$	.60
$E_{15}$ and $E_{16}$	3.33	$\psi_{10,12}^i$	.50
$\alpha_{15,16}$	-0.60	$\psi_{11,12}^i$	.55
		$\psi_{8,9}^i$	.15
		$\psi_{7,9}^i$	.35
		$\psi_{7,8}^i$	.20

**Table 6.6** Baseline Parameters for Fixed Lead Time and Variable Lead Time

OUTPUT PART ( <i>j</i> )	Fixed Change Cost( <i>C<sub>fj</sub></i> )	Variable Change Cost( <i>C<sub>vj</sub></i> )	Unit Backorder Cost( <i>C<sub>bj</sub></i> )	Demand Exponent ( <i>e<sub>j</sub></i> )	Fixed Response Lead Time ( <i>L<sub>j</sub></i> ) Weeks	Variable Response Lead Time ( <i>λ<sub>j</sub></i> ) Weeks
1	\$ 100	\$ 200	\$ .35	.15	0	1
2	70	700	.40	.20	0	2
3	80	900	.40	.40	0	3
4	90	900	.80	1.30	0	4
5	60	250	.07	.70	0	5
6	50	270	.10	.90	0	5
7	30	20	.17	.40	2	.05
8	25	15	.35	.50	3	.05
9	100	30	.15	.60	2	.04
10	120	23	.20	.15	4	.06
11	30	0	.90	.95	3	.05
12	60	0	.13	.50	4	.04
13	70	25	2.00	.70	2	.05
14	80	25	1.98	.80	2	.05
15	95	8	2.15	.80	1	.01
16	90	8	2.20	.80	1	.01

In tables 6.7 and 6.8 we provide some sample responsiveness outputs that are generated for the baseline stapler supply chain problem. Table 6.7 shows the results of the response cost equations developed in chapter 5, and is the key document for understanding the response capability or agility of a supply chain. Table 6.8 aggregates the response cost data by supplier.

**Table 6.7** Response Cost Output for the Baseline Numerical Example

OUTPUT PART ( $j$ )	SUPPLIER ( $i$ )	INPUT SUPPLIER	INVENTORY ADJUSTMENT COST	CAPACITY ADJUSTMENT COST	RESPONSE COST $r_{i,15} + r_{i,16}$
07	C	B	\$ 520.14	\$ 1,696.25	\$ 2,216.39
08	C	B	\$ 1,337.49	\$ 1,206.98	\$ 2,544.48
09	C	B	\$ 399.46	\$ 2,513.39	\$ 2,912.85
10	D	B	\$ 965.37	\$ 2,817.87	\$ 3,783.24
11	D	B	\$ 2,514.49	\$ 125.10	\$ 2,639.58
12	D	B	\$ 587.31	\$ 309.94	\$ 897.25
13	B	A	\$ 2,497.37	\$ 233.10	\$ 2,730.47
14	B	A	\$ 921.69	\$ 266.40	\$ 1,188.09
15	A	None	\$ 1,341.78	\$ 771.32	\$ 2,113.11
16	A	None	\$ 786.71	\$ 60.44	\$ 847.15

**Table 6.8** Response Cost for Each Supplier

SUPPLIER ( $i$ )	PARTS	INVENTORY ADJUSTMENT COST	CAPACITY ADJUSTMENT COST	TOTAL RESPONSE COST
A	15, 16	\$ 2,128.49	\$ 831.76	\$ 2,960.26
B	13, 14	\$ 3,419.06	\$ 499.50	\$ 3,918.56
C	07, 08, 09	\$ 2,257.09	\$ 5,416.62	\$ 7,673.71
D	10, 11, 12	\$ 4,067.17	\$ 3,252.90	\$ 7,320.07

From the tables we find that the annual response cost for the supply chain is \$21,873. In section 6.3 we saw that the unit profit is \$2.217 with a mean annual demand of 110,000 units for the two stapler models. This gives an expected annual profit of



\$243,870. We therefore find that potentially 9% of the annual profits of the supply chain are lost in responding to significant demand changes. Chain designers can then evaluate this ratio to ascertain whether it is acceptable to the target profit structure. From reviewing the associated tables and figures we are able to derive the following responsiveness analytics about the case study supply chain:

- Expected annual responsiveness costs are \$21,873 and represent 9% of the annual supply chain profits.
- The responsiveness costs are almost evenly distributed between inventory change costs (54%) and capacity adjustment costs (46%).
- From table 6.7 we see that parts 12 and 16 have the lowest response costs, while part 10 has the highest response cost (\$3,783 or 17% of the total). The majority of the part 10 response cost (\$2,817) is for capacity adjustment. Part 9 also has a high capacity adjustment cost. Any efforts to improve the supply chain's responsiveness must focus on these two parts.
- From table 6.8 we see that suppliers C and D are both experiencing the majority of the responsiveness cost. In particular supplier D has a response cost to profit ratio of 16%, so we can expect this supplier to resist making adjustments in response to significant demand changes.
- Supplier B has a high inventory adjustment cost, but is clearly very efficient in making capacity adjustments to its assembly line. It has the lowest capacity adjustment cost (\$499).

We now present in graphical form the experimental results of tested changes in external demand, demand correlation, final product commonality and supplier lead time.

The two demand parameters are varied first while all other parameters are held constant. The parameter  $\alpha_{jj}$  is varied next, again while other parameters are held constant. The parameters  $\psi^i_{jj}$  and  $\psi^j_{jj}$  are varied next. The parameters  $L_j$  and  $\lambda_j$  are varied last. The remainder of this section provides a general review, makes specific observations, and suggests questions which can be answered from the NEXUS model.

#### 6.4.1 Responsiveness Cost Sensitivity to Demand Change Behavior

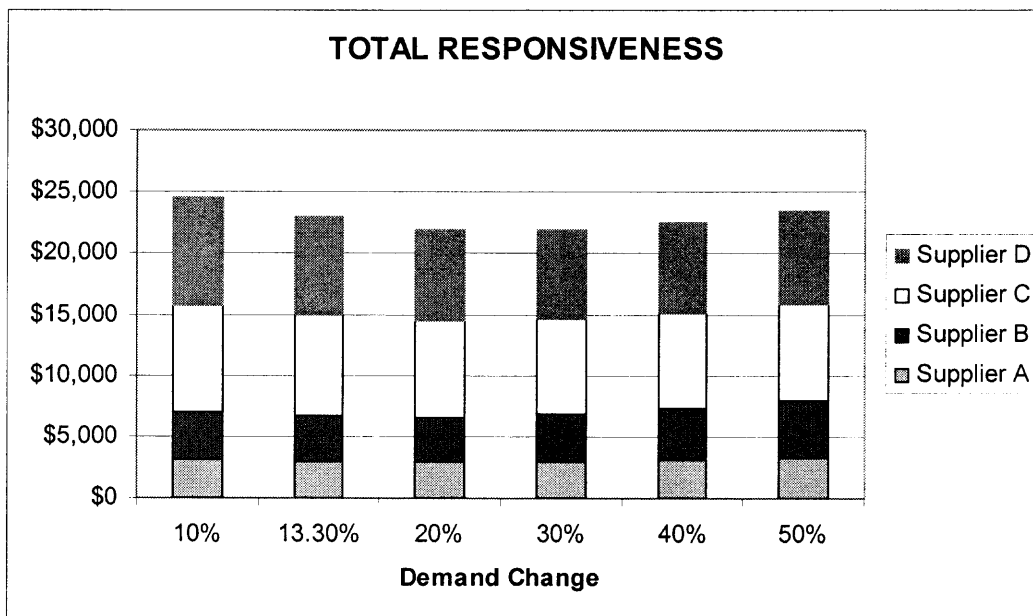
Our experiments in demand change focus on changes in external demand for the two final products. The pre-experimental demand is assumed to be: (1)  $D_{15} = 70,000$  and (2)  $D_{16} = 40,000$ . From this starting point six different percentage increases in demand are made, while six corresponding values of  $E_j$  are assumed. The six experimental conditions are given in table 6.9 as follows:

**Table 6.9** The Six Experimental Conditions

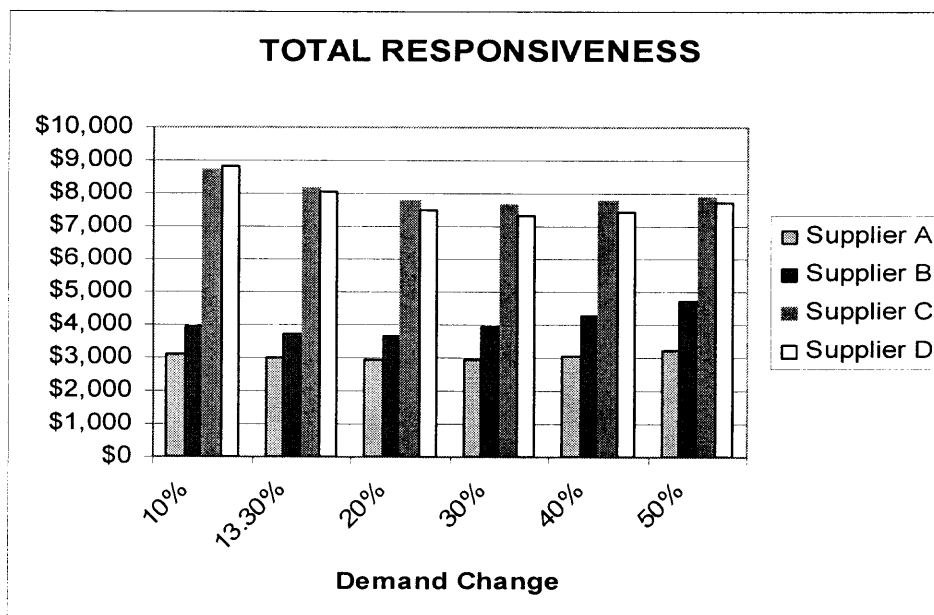
	10%	13.3%	20%	30%	40%	50%
$\Delta D_{15}$	7000	9310	14000	21000	28000	35000
$\Delta D_{16}$	4000	5320	8000	12000	16000	20000
$E_j$	10	7.5	5	3.33	2.5	2

The base case is for a demand change of 30%. When these six experiments are carried out the total cost of responsiveness is as seen in fig. 6.20. The highest responsiveness cost is \$24,584.11. Total responsiveness decreases until \$21,872.60 (the base case) and then increases to 23,517.84. The same basic pattern is found in fig. 6.21, which breaks down total responsiveness by supplier. For suppliers A, B, C and D total responsiveness trends downward from 10% to approximately 30%, then increases as demand change increases from 30% to 50%. The aggregated inventory-related cost of

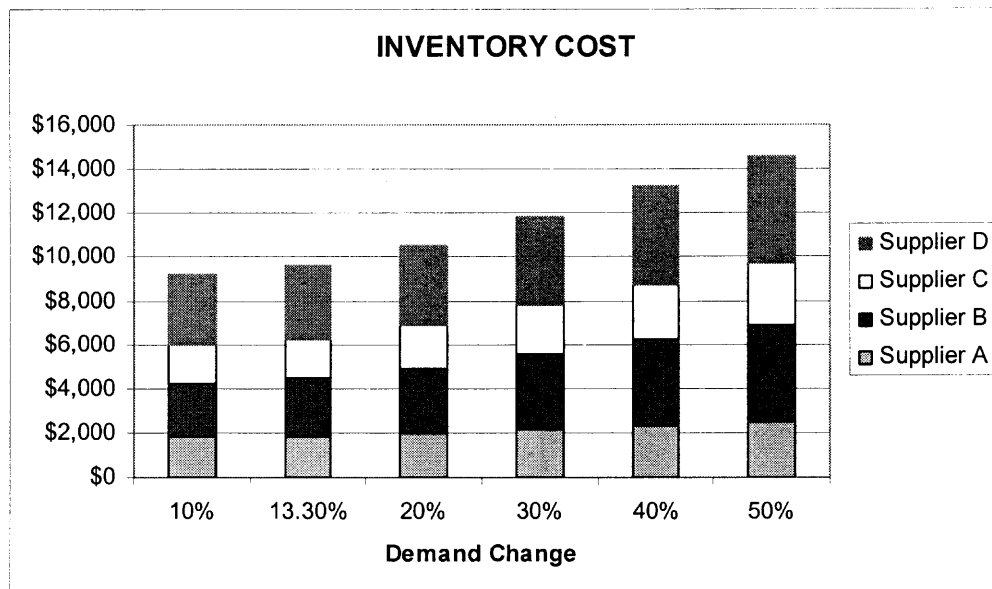
responsiveness increases steadily and almost linearly as demand change increases from 10% to 50%, as seen in fig. 6.22. This trend is seen for all four suppliers in fig. 6.23. The aggregated capacity-related cost of responsiveness decreases steadily and almost linearly as demand change increases from 10% to 50%, as in fig. 6.24. This trend is seen for all four suppliers in fig. 6.25.



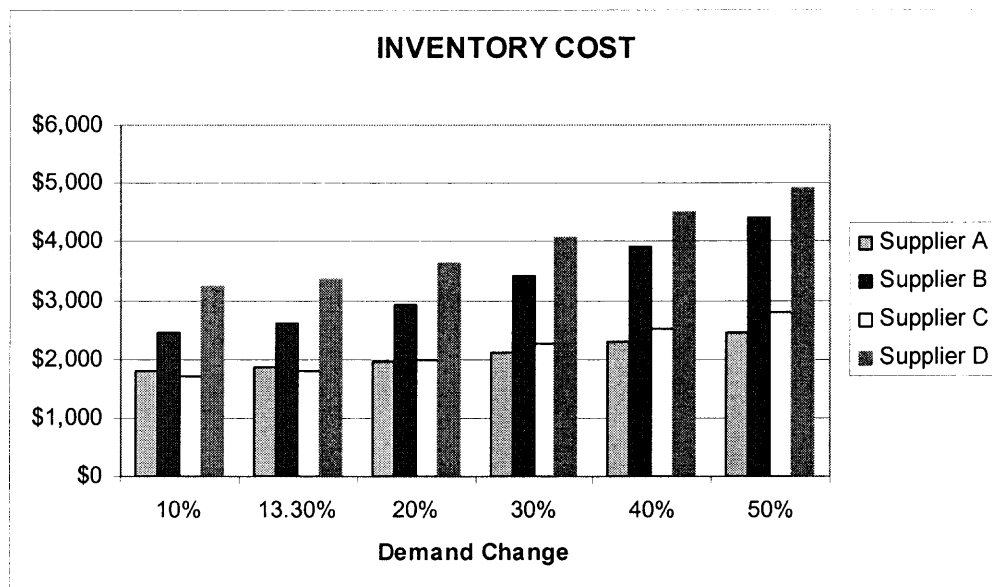
**Figure 6.20** Results of Six Experiments Involving Demand Change.



**Figure 6.21** Total Responsiveness Cost by Supplier.



**Figure 6.22** Inventory-Related Cost in the Aggregate.



**Figure 6.23** Inventory-Related Cost by Supplier.

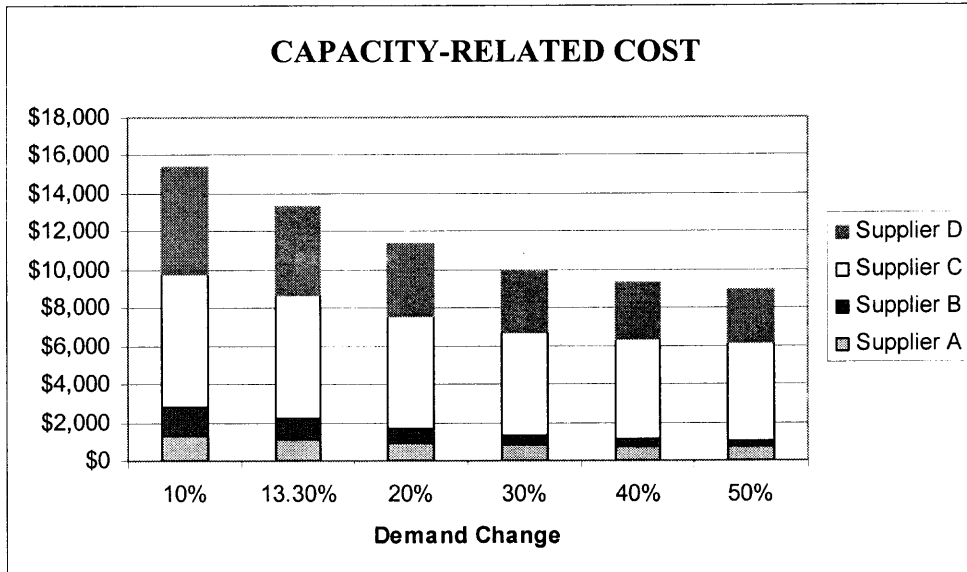


Figure 6.24 Capacity-Related Cost in the Aggregate.

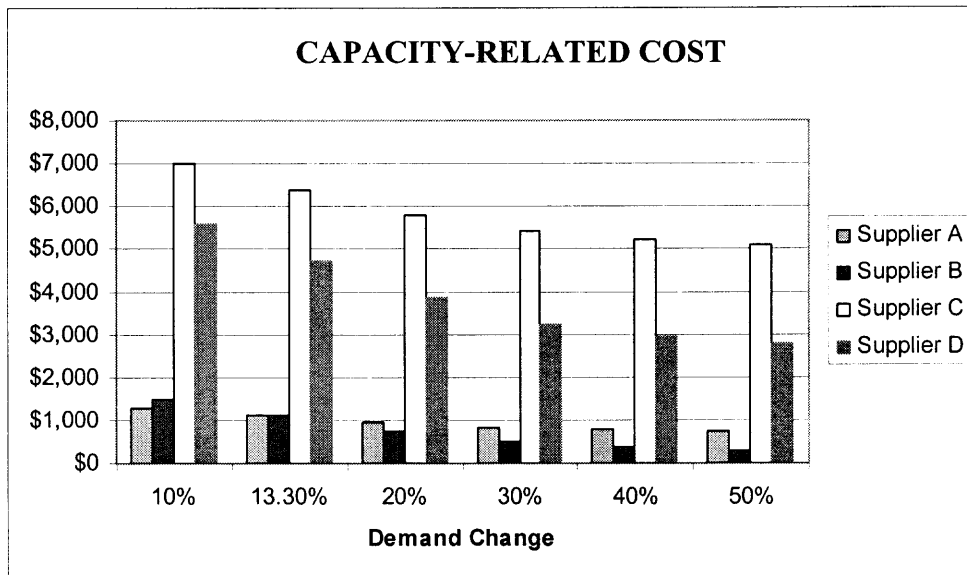


Figure 6.25 Capacity-Related Cost by Supplier.

**DEMAND CHANGE:**

*Observations*

1. The trends of inventory-related cost and capacity-related cost are in opposite directions and they offset each other.

2. Total responsiveness cost is lowest when inventory-related cost and capacity-related cost are both at their approximate midpoints.
3. The down-and-up pattern of total responsiveness is explained by simultaneously observing the behavior of inventory-related cost and capacity-related cost.

### ***Model Questions***

1. Which suppliers are the most responsive to changes in demand? Why?
2. At what point is the optimal total responsiveness?

### **6.4.2 Responsiveness Cost Sensitivity to Changes in Demand Correlation**

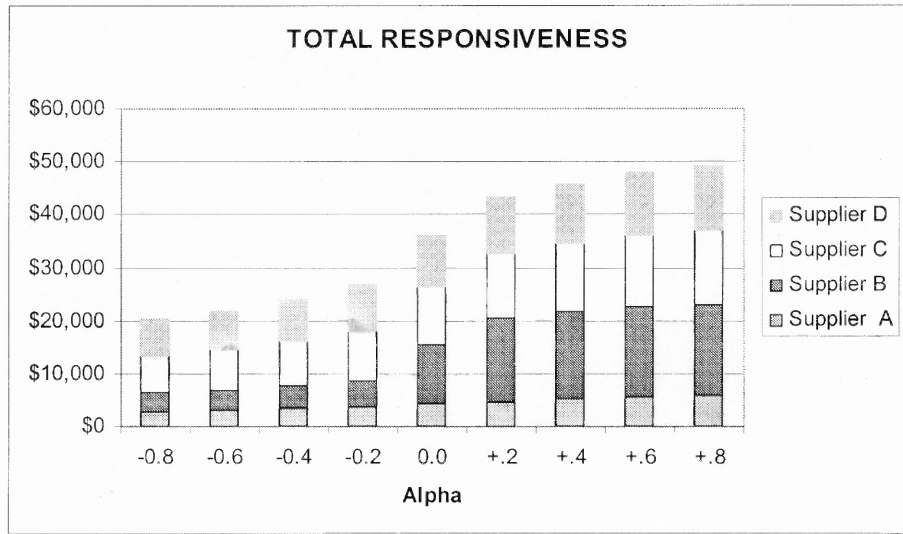
Our experiments concerning change in demand correlation make use of nine assumed values of  $\alpha_{JJ}$ , beginning with  $\alpha_{JJ}$  of -0.8 and varying in increments of 0.2 to  $\alpha_{JJ}$  of 0.8. In each experiment the value of  $\alpha_{JJ}$  is increased while all other parameters are held constant. The base case of \$21,872.60 is obtained for a value of  $\alpha_{JJ} = -0.6$ . The nine experimental results for total responsiveness obtained as a result of the changes in  $\alpha_{JJ}$  are seen in figure 6.26. Total responsiveness cost increases continuously as  $\alpha_{JJ}$  is increased. The lowest responsiveness is \$20,348.30 and the highest is \$49,117.82, so that the net change is \$28,769.52.

The overall trend of total responsiveness takes the shape of a concave function. The shape is concave up for  $\alpha_{JJ} < 0.0$  and concave down for  $\alpha_{JJ} > 0.0$ . An inflection point in the pattern is visible in figure 6.26 at  $\alpha_{JJ} = 0.0$ . An analysis of the changes in total responsiveness is given in table 6.10. All four suppliers contribute to the increase in total responsiveness, as revealed by figure 6.27. The increase in total responsiveness is attributable in part to inventory-related cost. Suppliers B, C and D each contribute

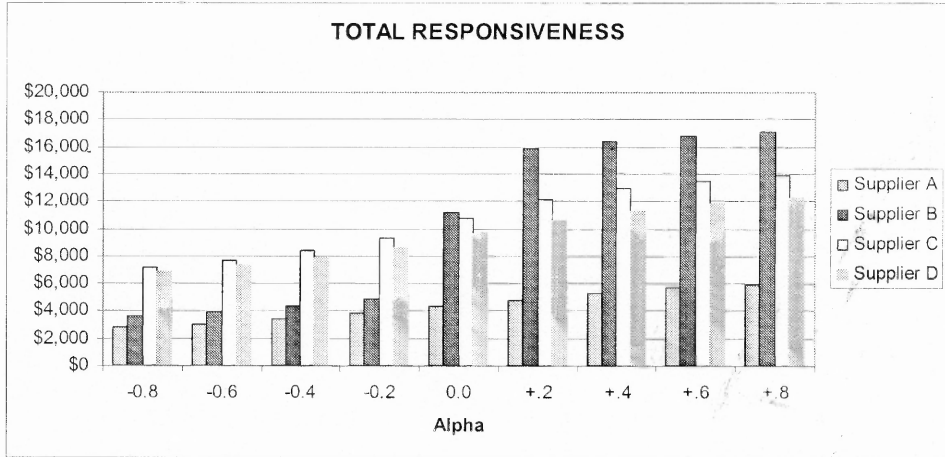
inventory-related cost to the total responsiveness, as seen in figure 6.28. The relative contributions of all suppliers to the inventory-related cost portion of total responsiveness is revealed by figure 6.29. The increase in total responsiveness is also attributable in part to capacity-related cost. All four suppliers contribute to the capacity-related cost portion of total responsiveness, as seen in figure 6.30. The relative contributions of all suppliers to the capacity-related cost portion of total responsiveness is revealed by figure 6.31.

**Table 6.10** Response Cost as a Function of Demand Correlation

DEMAND CORRELATION ( $\alpha_{ij}$ )	RESPONSE COST	INCREASE IN THE RESPONSE COST	RELATIVE INCREASE
-0.8	\$ 20,348.30	-	-
-0.6	\$ 21,872.60	\$ 1,524.30	5.30%
-0.4	\$ 24,038.68	\$ 3,690.38	12.83%
-0.2	\$ 26,766.26	\$ 6,417.96	22.31%
0.0	\$ 35,999.73	\$ 15,651.43	54.40%
+0.2	\$ 43,345.80	\$ 22,997.50	79.94%
+0.4	\$ 45,845.31	\$ 25,497.01	88.63%
+0.6	\$ 47,791.68	\$ 27,443.38	95.39%
+0.8	\$ 49,117.82	\$ 28,769.52	100.00%



**Figure 6.26** Results of Nine Experiments Involving Changes in Demand Correlation.



**Figure 6.27** Increases in Total Responsiveness by Supplier.



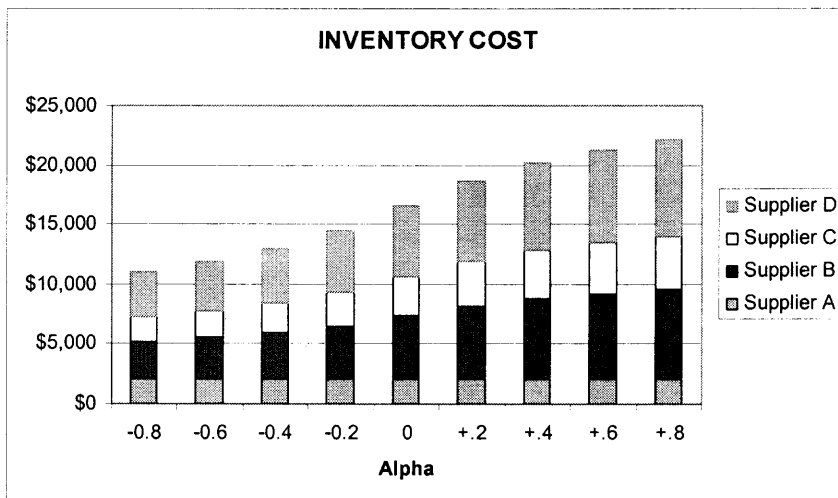


Figure 6.28 Inventory-Related Cost Portion of Total Responsiveness by Supplier.

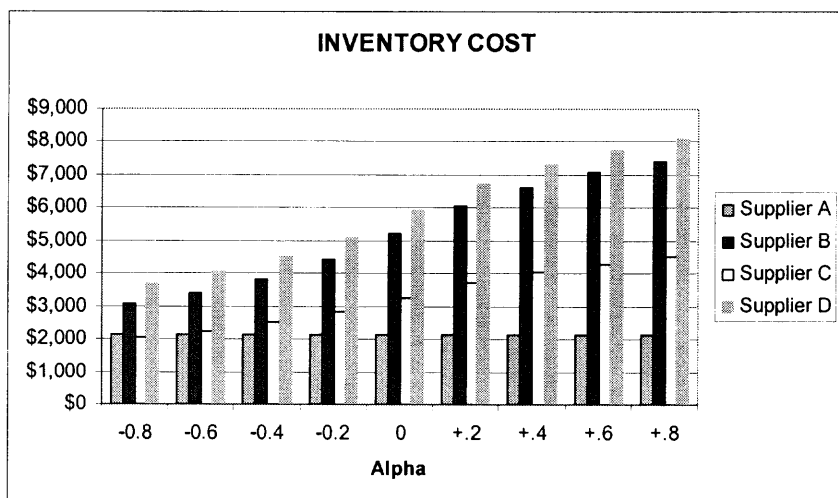
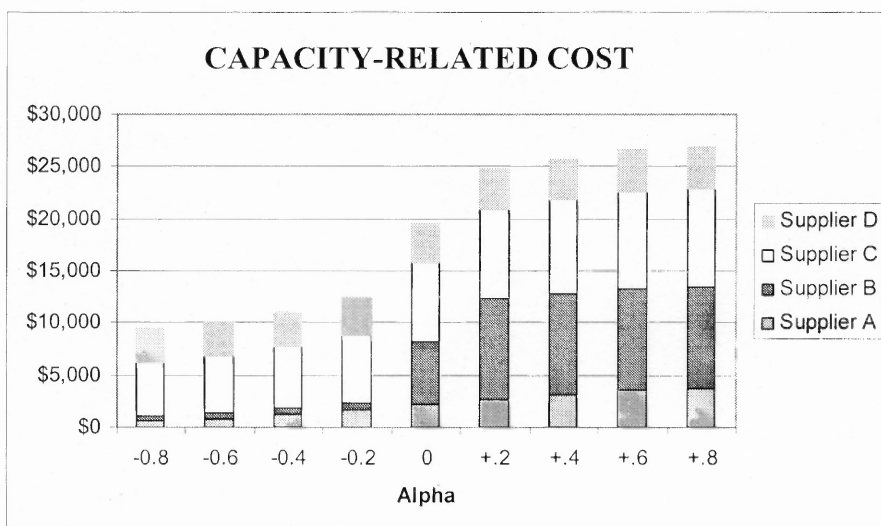
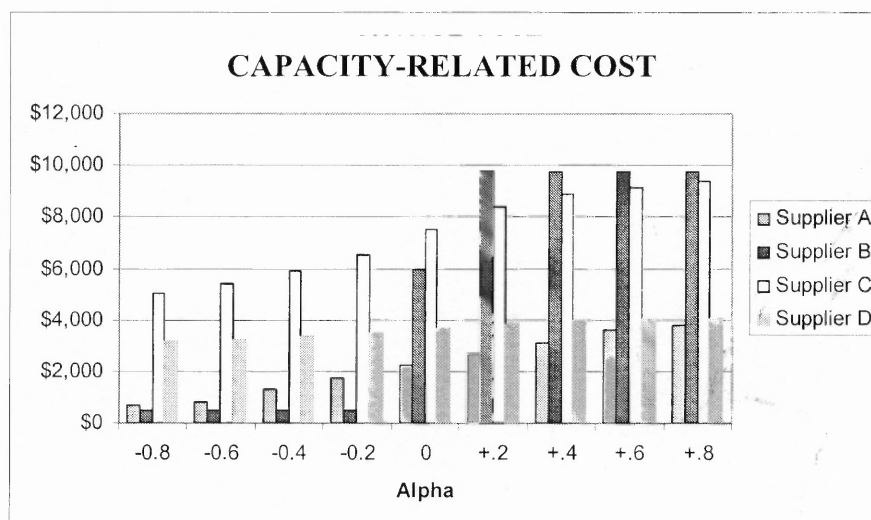


Figure 6.29 Relative Inventory-Related Cost by Supplier.



**Figure 6.30** Capacity-Related Cost Portion of Total Responsiveness by Supplier.



**Figure 6.31** Relative Capacity-Related Cost by Supplier.

## DEMAND CORRELATION:

### Observations

1. Total responsiveness increases monotonically as  $\alpha_{jj}$  is increased, with most of the change occurring when the value of  $\alpha_{jj}$  is between -0.2 and 0.2.
2. Chains with positively correlated product demand have higher total response cost.

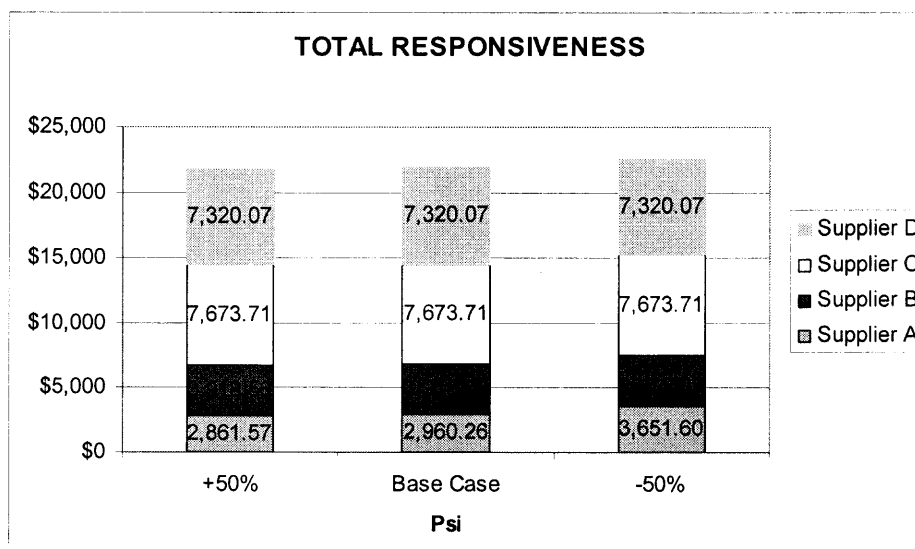
- The non-linearity in the trend of total responsiveness is due to the capacity-related cost.

### **Model questions**

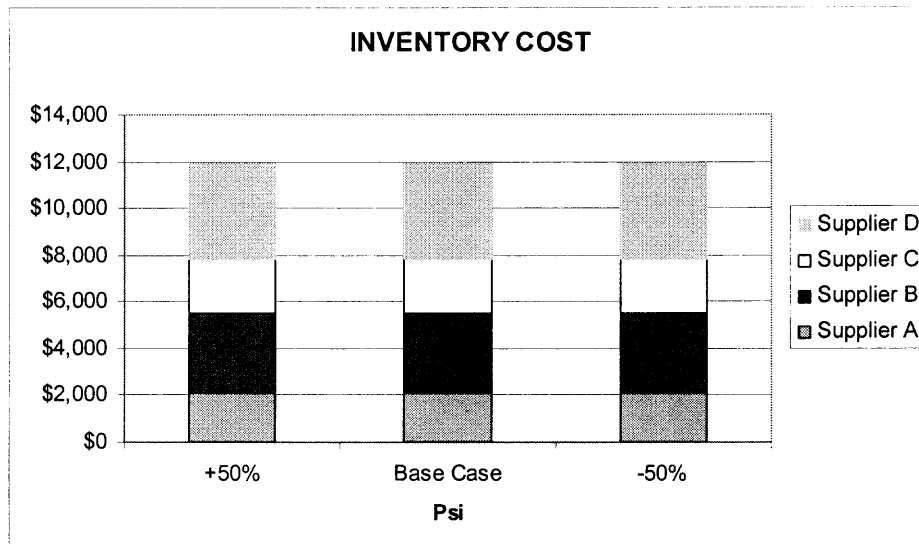
- To decrease total responsiveness cost should a supply chain focus on inventory-related cost or capacity-related cost?
- When there is a change in  $\alpha_{jj}$  due to marketing conditions, what is the impact on total responsiveness?

### **6.4.3 Responsiveness Cost Sensitivity to Changes in Product Commonality**

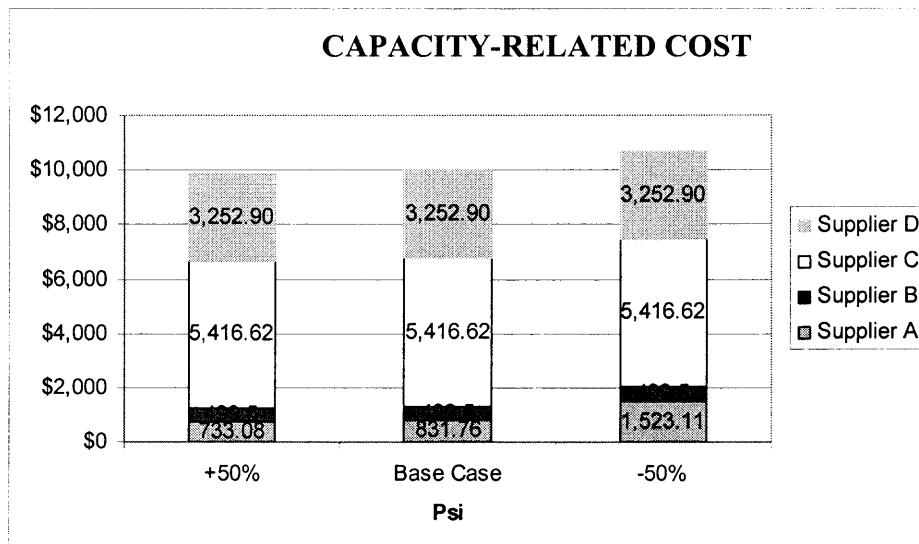
To test product commonality we increase the values of  $\psi^i_{jj}$  and  $\psi^i_{jj}$  by 50%, and then decrease those values by 50%. These changes have little effect on total responsiveness, as seen in figure 6.32. There is no impact on the inventory-related portion of total responsiveness, as seen in figure 6.33. The only impact on total responsiveness contributed by capacity-related cost is due to supplier A, as seen in figure 6.34.



**Figure 6.32** Total Responsiveness After Changes in Values of Product Commonality.



**Figure 6.33** Inventory-Related Cost Portion of Total Responsiveness.



**Figure 6.34** Capacity-Related Cost Portion of Total Responsiveness by Supplier.

#### 6.4.4 Responsiveness Cost Sensitivity to Changes in Response Lead Time

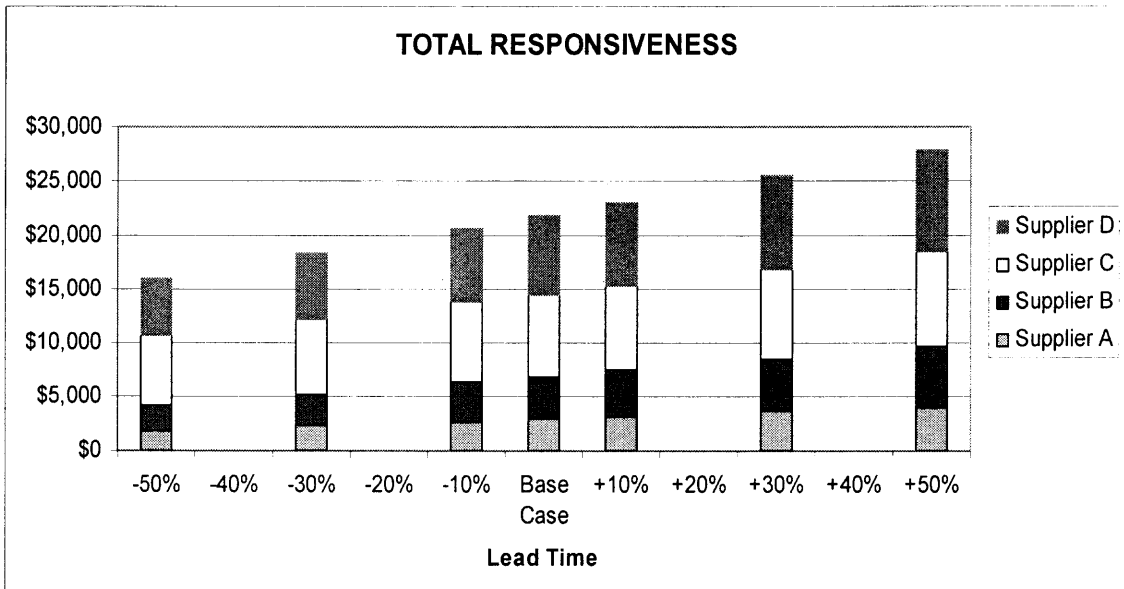
Our experiments with lead time involve simultaneous percentage increases and decreases in the parameters  $L_j$ ,  $\lambda_j$ ,  $L_j$  and  $\lambda_j$ . In each experiment the values of  $L_j$ ,  $\lambda_j$ ,  $L_j$  and  $\lambda_j$  are

changed while all other parameters are held constant. The base case from which percentage changes in  $L_j$ ,  $\lambda_j$ ,  $L_j$  and  $\lambda_j$  are made is given in table 6.11.

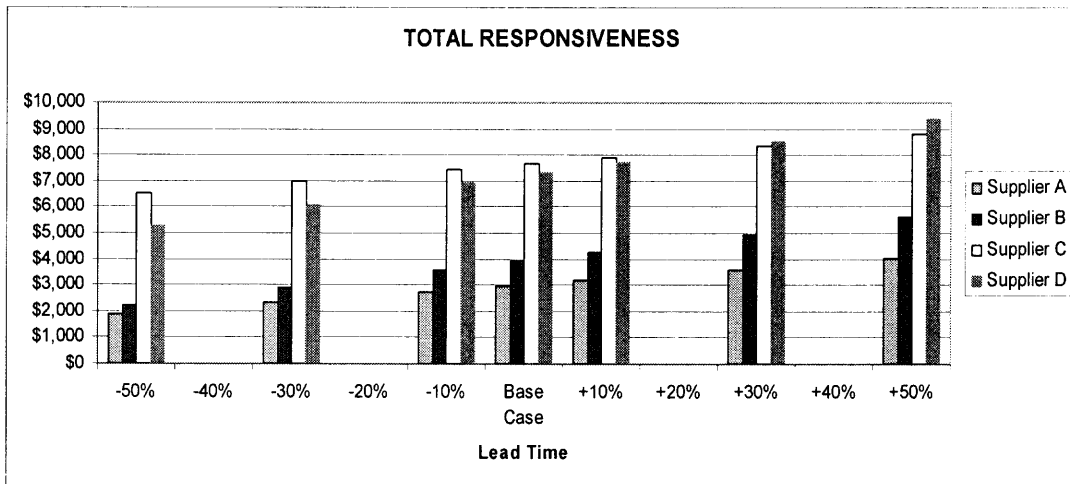
The experimental results for total responsiveness found as a result of the changes in lead time are seen in figure 6.35. The increase in total responsiveness is \$2,128.49 for supplier A, \$3,419.06 for supplier B, \$2,257.10 for supplier C and \$4,067.16 for supplier D. The relative increases in total responsiveness may be visualized by referring to figure 6.36. The increases in the inventory-related portion of responsiveness are exactly equal to the increases in total responsiveness. This is seen by comparing figure 6.37 and figure 6.38 to figure 6.35 and figure 6.36. The specific increases in responsiveness costs by supplier are given in table 6.12. The percentage increases and decreases in lead time had no impact at all on the capacity-related portion of total responsiveness. This is seen clearly in figure 6.39.

**Table 6.11** Base Case for Experiments Involving Lead Time

		BASE CASE	BASE CASE
		$L_j$ or $L_j$	$\lambda_j$ or $\lambda_j$
Supplier ID	Product ID	MinLeadTimetoChange	LeadTimetoChangePr
None	1	0	1
None	2	0	2
None	3	0	3
None	4	0	4
None	5	0	5
None	6	0	5
C	7	2	0.05
C	8	3	0.05
C	9	2	0.04
D	10	4	0.06
D	11	3	0.05
D	12	4	0.04
B	13	2	0.05
B	14	2	0.05
A	15	1	0.01
A	16	1	0.01



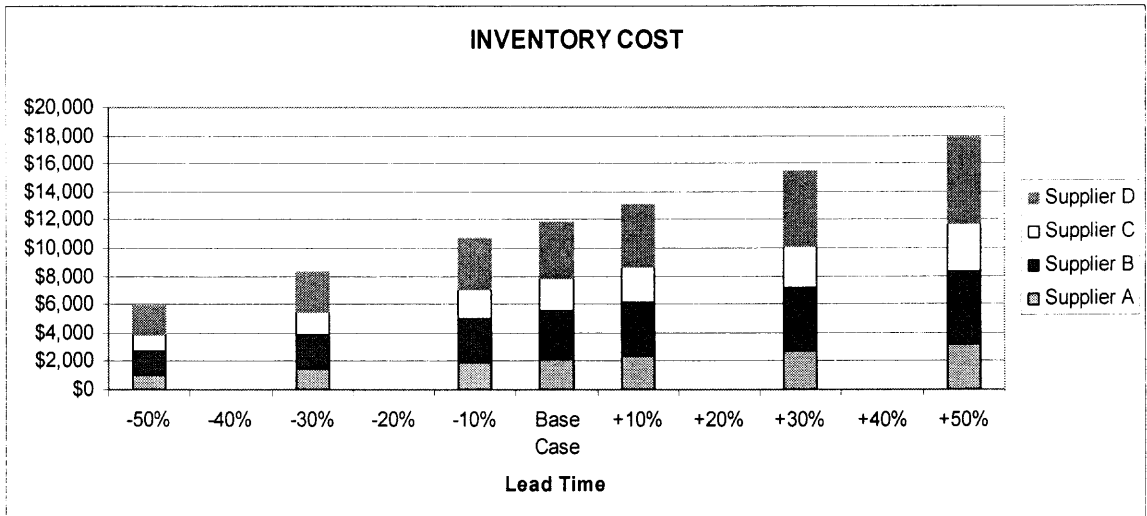
**Figure 6.35** Results of Experiments in Percentage Changes in Lead Time.



**Figure 6.36** Impact of Changes in Lead Time on Total Responsiveness by Supplier.

**Table 6.12** Increases in Total Responsiveness and in Inventory-Related Costs

SUPPLIER ID	RESPONSE COST INCREASE
A	\$2,128.49
B	\$3,419.06
C	\$2,257.10
D	\$4,067.16



**Figure 6.37** Inventory-Related Cost Portion of Total Responsiveness.



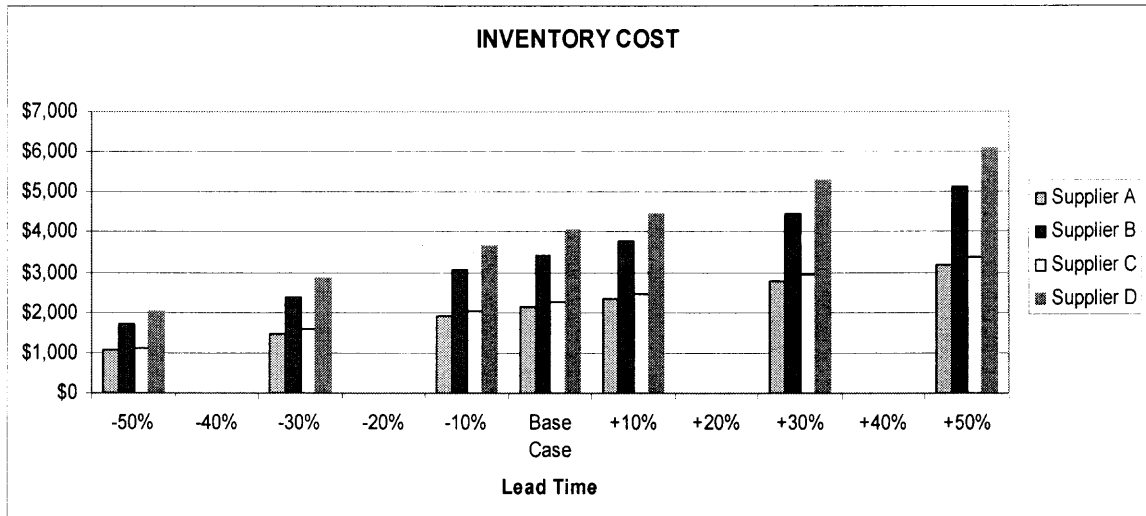


Figure 6.38 Relative Inventory-Related Cost of Total Responsiveness by Supplier.

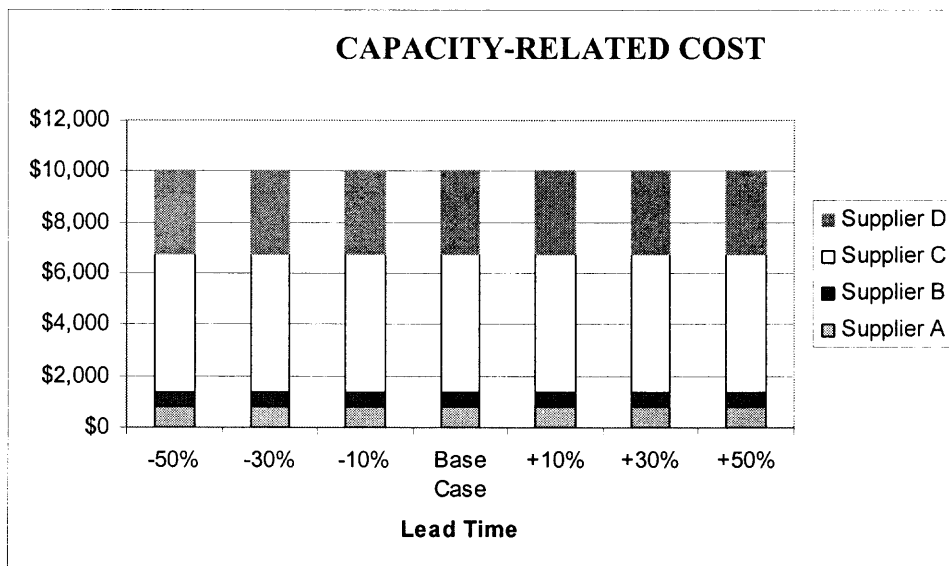


Figure 6.39 Capacity-Related Cost Portion of Total Responsiveness.

**RESPONSE LEAD TIME:**

*Observations*

1. For each supplier the relationship between the inventory-related cost and the capacity-related cost may be linearly approximated.

2. The inventory-related contribution to total responsiveness is significant for each supplier.

***Model Question***

1. Given that lead time can be reduced by means of an expenditure, what is the economic value of doing that?

### **6.5 Chapter Summary**

The impact of parametric change on supply chain performance is best understood by visualizing the experimental outcomes.

We have measured supplier performance in terms of three experimental outcomes: input side inventory cost, output side inventory cost, and supply order cost. We have seen that the replenishment order quantity has an impact on both of these inventory costs and on supply order cost. We have also measured supply chain performance in terms of responsiveness cost. We have seen that experimental changes to demand change, production correlation and response lead time have an impact on responsiveness cost.

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