Summer 2005

On the design for flexibility of manufacturing systems : a stochiastic approach

Nathapol Areeratchakul
New Jersey Institute of Technology

ABSTRACT

ON THE DESIGN FOR FLEXIBILITY OF MANUFACTURING SYSTEMS; A STOCHASTIC APPROACH

by
Nathapol Areeratchakul

Flexibility has emerged as one of the most strategic imperatives for company viability in today's fast paced economy. This realization has stimulated extensive research efforts in this area most of which have focused mainly on defining flexibility and its attributes, the need for flexibility and how to measure it. Nevertheless, despite the considerable amount of publications regarding flexibility and its related subjects, insufficient attention has been given to the optimality of the design for flexibility and the inherent needs to meet uncertainty. Bridging this gap is the intent of this work.

In this dissertation, developed analytical models are for the optimum design of flexible systems. The models introduced are based on extensions of the single period stochastic inventory model and real option theory to determine the optimum level of the various flexibility attributes that are required to meet the needs of a concern in an uncertain environment. Our premise stems from the fact that flexibility does not come at "no cost." That is, when designing a system, the more flexibility built in it, the more the cost that will be incurred to maintain it. On the other hand, if the system is designed with low levels of flexibility, it may not be able to meet the uncertain demand, therefore causing loss of future revenue. The developed models, then, are applied to examples where data are obtained from machine tool manufacturers to show how to strike a balance between the two conflicting scenarios of over and under-flexible designs.

ON THE DESIGN FOR FLEXIBILITY OF MANUFACTURING SYSTEMS; A STOCHASTIC APPROACH

by
Nathapol Areeratchakul

Dissertation
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy in Industrial Engineering

Department of Industrial and Manufacturing Engineering

August 2005

APPROVAL PAGE

ON THE DESIGN FOR FLEXIBILITY OF MANUFACTURING SYSTEMS; A STOCHASTIC APPROACH

Nathapol Areeratchakul

Dr. Layek Abdel-Malek, Dissertation Advisor — Date
Professor of Industrial and Manufacturing Engineering, NJIT

Dr. Athanassios K. Bladikas, Committee Member — Date
Associate Professor of Industrial and Manufacturing Engineering, NJIT

Dr. Carl Wolf, Committee Member — Date
Professor of Industrial and Manufacturing Engineering, NJIT

Dr. Reggie J. Caudill, Committee Member — Date
Professor of Industrial and Manufacturing Engineering, NJIT

Dr. Rene Cordero, Committee Member — Date
Associate Professor of School of Management, NJIT

BIOGRAPHICAL SKETCH

Author: Nathapol Areeratchakul

Degree: Doctor of Philosophy

Date: August 2005

Undergraduate and Graduate Education:

- Doctor of Philosophy in Industrial Engineering,
  New Jersey Institute of Technology, Newark, NJ, 2005

- Master of Science in Industrial Engineering,
  New Jersey Institute of Technology, Newark, NJ, 1998

- Bachelor of Science in Industrial Engineering,
  Kasetsart University, Bangkok, Thailand, 1988

Major: Industrial Engineering

Presentations and Publications:

Layek Abdel-Malek and Nathapol Areeratchakul,
"An analytical approach for evaluating and selecting vendors with interdependent performance in a supply chain"
International Journal of Integrated Supply Management 2004 –Vol. 1, No.1 pp. 64-78.

This dissertation is dedicated to the memory of my parents Watana and Wanida Areeratchakul

ACKNOWLEDGMENT

I would like to express my deepest and most heartfelt appreciation to Dr. Layek Abdel-Malek, who not only served as my mentor, but also gave me constant support, encouragement, and reassurance. I owe a great debt of gratitude not only to Dr. Layek Abdel-Malek, but also to Dr. Athanassios Bladikas, Dr. Carl Wolf, Dr. Reggie Caudill, and Dr. Rene Cordero for actively participating in my committee. I would like to also thank the Department of Industrial Engineering for their financial support throughout my studies at NJIT.

In particular, I would like to acknowledge Otegbeye Kike Mojisola, who went above and beyond the call of duty in her effort to assist me in writing and formatting of this dissertation. Without her support, the dissertation itself may have proceeded along a different path.

I would like to also thank Dr. Ronald Kane for patiently editing the dissertation and assisting me with all the format corrections.

I would like to thank my family. My father Watana and mother Wanida have provided me with wonderful support throughout. Also, my brother Nathaporn has shared kind words of encouragement at several opportune moments. And finally, a special thanks to my girlfriend, Kanya, for her continued love and support.

TABLE OF CONTENTS

TABLE OF CONTENTS
(Continued)

TABLE OF CONTENTS
(Continued)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES
(Continued)

LIST OF SYMBOLS

Symbol	*Description*
B_g	The firm's available budget
c_τ	Purchasing unit costs of flexibility class τ
D_τ	Random variable, represents the need for flexibility *i* and $0 \le D_\tau \le 100$
$f(D_\tau)$	The probability density function of D_τ
K_τ	The truncated constant value for flexibility class τ (the formulae for K_τ are shown in Table 3.1)
L_τ	The lower bound degree of the need for flexibility
N	Total number of considered flexibility classes
U_τ	The upper bound degree of the need for flexibility
x_τ	Degree of flexibility i to be invested and $L_\tau \le x_\tau \le U_\tau$
Z	The expected cost to be minimized
α_τ	The over-designed cost multiplier and
β_τ	The under-designed cost multiplier and β_τ has to be greater than 1 since if value of the parameter less than one the investment will not occur
τ	Flexibility classes' index

CHAPTER 1

INTRODUCTION

1.1 Background

Since the end of the cold war, industrial globalization has been growing rapidly. In his book, "The Challenge of Global Capitalism…", Robert Gilpin (2000) [1] opines that this growth has largely been propelled by the fast paced advancements in information technology, the increased affordability of high computing power, the declining costs of transportation and the signing of treaties and various trade agreements that have served to greatly reduce barriers. To facilitate quick response to the increasing expectation levels of customers, ahead of the competition, companies now tend to focus more on their core business activities while outsourcing those functions that could be better served by others.

One of the common arguments put forth in favor of outsourcing, in addition to cost concerns, is the flexibility it affords the firm in responding quickly to the ever changing customer demands/requirements. To attempt to meet all aspects of demand strictly within an organizational domain would invariably entail huge and sometimes, unjustifiable tie up of a company's capital in fixed investments. Therefore, by outsourcing some of the production functions, the company could leverage some level of agility the practice would provide. Nevertheless, while outsourcing has the potential of affording increased flexibility; it cannot be used as a substitute for faulty internal operations. Thus we could, for instance, have an organization having a flexible supply network, but non-flexible internal production operations. For such an organization, delivering value to the market place ahead of competition could still be a tall ambition given that its production systems are not designed to confer this advantage, thus putting at risk the relevance of such a company in its market. Besides, the emerging practice in most of today's supply chains is the selection of partners only among the best of breeds; that is, those with excellent internal processes that ensure that a seamless transfer of value among channel members down to the final customer is achieved effectively and efficiently.

In light of these realizations, manufacturing companies are now beginning to pay closer attention to their production systems for the purpose of leveraging them as competitive advantage. This has in turn spurned a significant amount of research initiatives in the arena of manufacturing flexibility and the topics covered thus far have been widespread. These vary from the various angles that have been proposed from which manufacturing flexibility could be viewed, the various definitions of flexibility classes, its measurement, and its deployment across the various echelons of the manufacturing enterprise and down to the analysis of the supply chain flexibility attributes.

In this research, our focus is on the flexibility of the factory floor; more so, since it could be used to depict the behavior of a supply chain. As would be seen in sections to follow, there are various classes of manufacturing flexibility. Examples of which include volume, product, process, expansion, and routing. Therefore, in practice, it is difficult to design economically and without loss of functionalities a single manufacturing system that provides all of these flexibilities. Rather than designing such a system, manufacturers might be better off focusing on designing one which incorporates a combination of the most suitable flexibility types and at the needed levels. For instance, for high-technology manufacturers such as semiconductors and pharmaceuticals, their products generally have short life cycles and they are constantly faced with the need to quickly introduce new products to the market. For such a manufacturer to survive, product flexibility must be inherent in their operations. In this case therefore, placing emphasis on other flexibility types over product flexibility, which is more in dire need could constitute an improper move.

The issues arising here are how much of the needed flexibility types should be acquired and what manufacturing system designs would yield this level of flexibility effectively and efficiently. The reasons for these are not far fetched when we consider that flexibility comes at a high price; to over-invest translates to unnecessary loss of capital to the manufacturer as the excess capacity is non-value yielding. On the reverse side, flexibility confers competitive advantage to manufacturers; hence under-investment may lead to the loss of market share. There are numerous system design options currently available to provide manufacturers with the ability to be flexible in their production operations. However, designing a manufacturing system to equip the user with the right flexibility level is a rather challenging task.

On the basis of the aforementioned, and as a contribution in this area, our research aims at developing models that address some of the limitations of existing solutions such that when implemented, should be useful to manufacturers in their flexibility investment decisions.

1.2 Manufacturing Flexibility Uncertainty

The basic competitive priorities first recognized by Skinner (1969) are quality, price, delivery performance and flexibility. In 1990 Ferdows and De Meyer used empirical research to support their argument that tradeoffs among these competitive priorities exist. They introduced a sand cone model to suggest a natural sequence of improvement, which they referred to as cumulative capabilities. The first priority was quality, then dependability, then flexibility and finally cost. Thus based on the sand cone model, flexibility should be the second to the last competitive priority that firms should adopt. However, according to Lau (1996), Hayes and Wheelwright (1984), flexibility is the most important competitive strategy and a key competitive weapon for firms in today's fast paced environment.

The significance of manufacturing flexibility has been acknowledged in several industries, Michael Wall (2003). For instance, in the automotive industry, certain manufacturers have made significant efforts in creating flexible manufacturing facilities enabling the production of a wide range of vehicles based upon diverse and robust platforms. Examples of such manufacturers include Toyota, Honda, and GM. Toyota is presently overhauling its own facilities to improve flexibility. The new facility will allow Toyota to increase production in the popular and profitable full-size pickup segment with an all-new Tundra and other variants such as the Sequoia and LX470 among other full-frame offerings. Honda has made various investments and plant infrastructure changes, which ensure its major North American production facilities can assemble nearly any vehicle sold in the market. GM's mid-size SUV facilities (Oklahoma City, Okla., & Moraine, Ohio) utilize the company's new Lansing Grand River operation. The Lansing

Grand River assembly plant is noteworthy because it has the ability to produce five different vehicles at a time. Other industries, such as food processing, mechanical devices, pharmaceuticals, plastic, and personal products are also in dire need for flexibility because their inherent nature requires that a large variety of products be simultaneously offered to the consumer, Abdel-Malek et al (2000).

In the literature, flexibility is usually viewed in the context of manufacturing, workforce, organization, and the supply chain flexibility. However, as mentioned before, manufacturing flexibility is the core focus of this research. By definition, manufacturing flexibility means the capability to respond quickly to shifts in market requirements. In particular, each flexibility type generates results differently and they can be explained based on Sethi and Sethi (1990) as follows:

- Process flexibility allows manufacturers to vary the product mix as demand changes.
- Volume flexibility allows manufacturers to increase or reduce the production rate without costly efforts.
- Product flexibility allows manufacturers to introduce new products to market with shorter introduction time and lower costs.
- Expansion flexibility allows manufacturers to be able to augment existing capacity to meet significant increases in the level of total demand.
- Routing flexibility allows manufacturers to alternate the routes that each product can take through the production facility.
- Operation flexibility allows a manufacturer to alternate the different process plans and processing sequences that can be used to manufacture a part.

- Machine flexibility allows manufacturers to be able to adjust to perform quickly, more that one operation.
- Material handling flexibility allows manufacturers to transport varying item and unit loads, reduce the frequencies at which loads must be moved, and provide the ability to vary pick-up and delivery points.

Although, there is a weath of literature encompassing a wide range of aspects of manufacturing flexibility, many researchers have addressed several issues that require further investigation. These can be viewed in two perspectives; issues related to management, and those related to valuation. For the former, Abdel-Malek et al (2000) argue that insufficient analytical methodologies has been developed in literature so that it becomes difficult for management to implement most of these methodologies in determining what type and to what degree of flexibility they should invest in. In addition, they also point out that the existing decision models do not incorporate the link between the specific types of flexibility and related specific types of the uncertainty environment. Skinner (1996) also argues that the most serious problem and the main weakness in manufacturing strategy is the lack of maps of "how to's", i.e. maps depicting appropriate routes from the manufacturing task to the design of the manufacturing system. From latter works, it is now widely recognized that established capital budgeting techniques such as the traditional discounted cash flow (DCF) approach, often lead to incorrect valuation of flexibility, Dixit and Pindyck (1994) and Trigeorgis (1996).

Therefore, it can be started that despite of the existing wealth of literature in this arena, the design of manufacturing systems to obtain the optimal value for different flexibility classes with respect to the degree of uncertainty that each firm has to be contend with needs more investigation. The problem is further compounded by the fact that at the time of planning, it is difficult to pinpoint the suitable kind of flexibility and even when this is known, the level of flexibility to allow become another matter. For example, a manufacturer may decide that a certain level of volume flexibility is required and subsequently invest in equipment designed for this type and level of flexibility. However, future demand may show that the lot sizes have the same order of magnitude. Therefore, the manufacturer did not need to invest this much to acquire this piece of equipment. Yet, the opposite also could happen. That is the variation in lot size may be greater than what was expected leading to an insufficient level of flexibility to accommodate demand changes. These scenarios cited here reveal that designing a flexible system is challenging, requiring the anticipation of future needs for the various categories of flexibility.

1.3 Flexibility versus Uncertainty

The relationship between flexibility and uncertainty has been the subject of interest for several researchers who raised several important issues. A number of researchers have suggested that while flexibility is frequently used as a tool to hedge against uncertainty, the manifestation of uncertainty is dependent upon the operational level from which it is viewed. Further, specific types of flexibility are required to accommodate the effects of each type of uncertainty. Gerwin (1987) attempts to associate types of uncertainty with types of flexibility, see Table 1.1. Correa (1994) has suggested that environmental uncertainty and variability in outputs are the two main reasons that manufacturing flexibility is sought. These two factors, in whatever form they may materialize, can be translated into types of operational change which can be further categorized according to whether the need for change is planned or unplanned. Unplanned changes, either originating internally or externally, are referred to as stimuli, i.e. the cause of the

Table 1.1 Association of Flexibility Types and Uncertainty, Gerwin (1987)

Flexibility type	Uncertainty
Mix	Uncertainty as to which products will be accepted by customers creates the need for mix flexibility
Changeover	Uncertainty as to the length of product life cycle leads to changeover flexibility
Modification	Uncertainty as to which particular attributes customers want....leads to modification flexibility
Rerouting	Uncertainty with respect to machine downtime makes for rerouting flexibility
Volume	Uncertainty with regard to the amount of customer demand for the products offered leads to volume flexibility
Material	Uncertainty as to whether the material inputs to a manufacturing process meet standards gives rise to the need for material flexibility
Sequence	Sequence flexibility arises from the need to deal with uncertain delivery time of raw materials

requirement for flexibility. The sources of stimuli, Correa suggests, can be categorized as process, labor, suppliers, customers, society, corporate and other functions and competitors. Moreover, unplanned change has five main dimensions: size, novelty, frequency, certainty and rate. In response, management attempts to impose forms of control and as a consequence, flexibility is required to handle those elements that remain. Hyun and Ahn (1992) cite four strategies for using flexibility, namely, “reactive internal uncertainty”, “reactive external uncertainty”, “proactive internal uncertainty” and “proactive external uncertainty”. Upton (1994) suggests that modular design, inventory and dedicated plants are all ways of reducing the need for flexibility. Newman et al

(1993) suggests that the method used to reduce the effects of both external and internal uncertainty can be related to product and process characteristics. Companies mass producing a narrow range of products may work on reducing internal uncertainty and limit the amount of external uncertainty they need to accommodate by using dedicated technology, centralized infrastructures and buffers before and after the process. Conversely, companies processing a wide range of products and volume types can use flexible technology and a decentralized infrastructure to accommodate the effects of external uncertainty and internal buffers to limit internal uncertainty. By the summarized fact that flexibility and uncertainty are obviously linked and the value of flexibility vary accordingly to uncertainty; sound models for determining the optimal level of flexibility must take into account this relationship. In this research, uncertainty is considered to include the need for flexibility and the demand of products. It should be noted that the need for flexibility arises from the uncertainty which is derived from the aggregation of the internal and external uncertain factors, as shown in Figure 1.1.

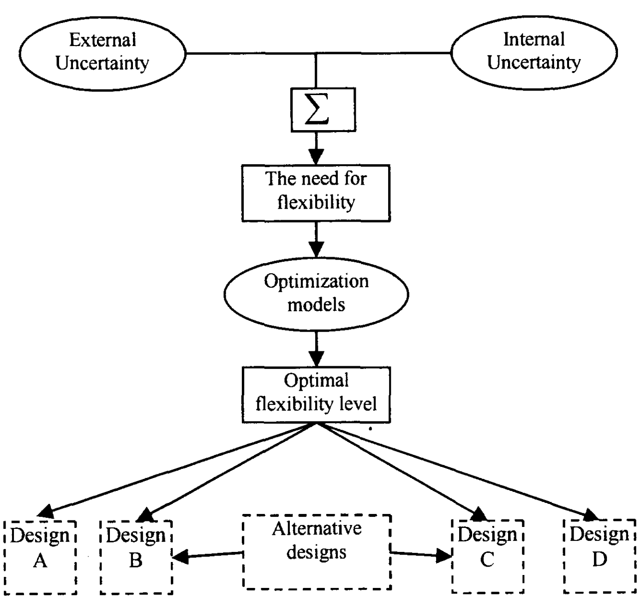

Figure 1.1 Uncertainty Considered in This Research

1.4 Methodology Overview and Research Objectives

The main objective of this research is to develop models and a methodology that will guide managers in optimally planning for flexibility. Two stochastic models are introduced. Both models are derived to aid in the decision between going for the more flexible system design or the more dedicated one. Based on the fact that flexibility comes at a price, an over-flexible system design would result in incurring unnecessary costs that could rub off negatively on the product's pricing such that it might eventually become unattractive to customers while at the same time, making the company lose the opportunity to invest in other potential investment alternatives. On the other hand, an under-flexible system design may lead to the firm losing market share or even going out of business due to lack of sufficient capacity to respond swiftly to market dynamics. Therefore, it is crucial for a manager to be able to map out a production strategy that minimizes the possibility of over/under designing for manufacturing flexibility.

The first model extends a stochastic model which takes into account the uncertainty associated with the need for flexibility, and the costs of the related investment. The single period model, known as the newsboy model is suitable here because like the single period ordering situation, the expenditures of investment for flexibility are largely irreversible. In other words, the expenditures are mostly sunk costs that cannot be recovered, Pindyck (1988). These costs are flexibility unit purchasing cost, under-design system for flexibility cost, and over-design system for flexibility cost. In this research, the under-design and over design costs are assumed to be the functions of unit purchasing cost where the value of the unit purchasing cost is known. Uncertainty as regards the need for flexibility is represented by a truncated probability distribution. A methodology for solving the problem is developed based on: 1) the triangular approximation of the area under the cumulative distribution function (CDF) which represents the need for each flexibility class and 2) Quadratic programming for obtaining the optimal solution.

The second model is developed taking into account the ability of flexibility to allow managers to take control over output in the bid to address products' demand uncertainty. As previously mentioned, to value such abilities, which are intangible, the financial tools such as discounted-cash-flow (DCF) valuation; could prove inappropriate. A better approach to valuation would incorporate both the uncertainty inherent in business and the active decision making required for a strategy to succeed. Many researchers have introduced real options as a better approach for such valuations. The main advantages of using option pricing theory are that the complex risk structure of a flexible project is handled more appropriately than in the traditional method mentioned above and that the problem of estimating a risk-adjusted rate is avoided in most cases. Many researchers implement real options theory in the valuation of an individual manufacturing flexibility type such as process, product, volume, and expansion. Nevertheless, when designing a manufacturing system, it can inherit several classes of flexibility. Therefore, the available models, which consider individual flexibility classes, might not be appropriate for the valuation of a manufacturing system design. To tackle this challenge, the second model aggregates more than one class of flexibility. This should give managers a more applicable model that helps them make more accurate and informed decisions.

1.5 Organization of the work

This dissertation is organized as follows. This introductory chapter, chapter 1, gave a definition of the problem and enumerated the objectives of the thesis. Chapter 2 conducts a detailed literature review to provide the motivation of the work highlighting the contribution of the existing solutions and the needed extensions in this arena. Chapter 3 formulates the first mathematical model that considers where the need for flexibility is the source of underlying uncertainty. This chapter details the model's objective, underlying assumptions, and developed algorithms to obtaining the optimal solutions. Chapter 4 formulates the second mathematical model that takes into account, the underlying uncertainty specific to the needs of each flexibility class and focuses on addressing the determination of the optimal level of the desired flexibility for two pertinent flexibility classes namely volume and product flexibility. It also details the model's objective, underlying assumptions, and developed algorithms to obtaining the optimal solutions. In chapter 5, case studies are presented on the design of suitable manufacturing systems and specification attributes with respect to the determined optimal level of flexibility for each class. Chapter 6 presents the conclusions of this dissertation and recommends directions for further research.

CHAPTER 2

LITERATURE REVIEW

This chapter explores past work on flexibility and uncertainty that has been documented in the literature, and is considered to be closely related to this research. A review of the literature reveals that past work on manufacturing flexibility has mainly focused on the search for its definitions, concepts, measures, and its relationship with uncertainty. These are explained in brief in the sections 2.1-5 respectively. In section 2.6, the chapter concludes with a summary of the literature review and the contribution of this research.

2.1 What is Flexibility?

There is no universally accepted definition for the term *flexibility*. Kumar (1987) attributed the cause of this to the many dimensions from which flexibility can be viewed. Swamidass (1988) stated more reasons which include the overlap in scope of terms used by different authors in defining flexibility, the fact that some terms used to define flexibility aggregate others, and the fact that even when different researchers use the same term to define flexibility, they may attach different meaning to the term. Several views on flexibility from the literature are discussed here.

Gerwin (1987) defines flexibility as an adaptive response to environmental uncertainty. A conceptual model is proposed that places flexibility within a broad context. The model includes five variables: environmental uncertainty, strategy, required manufacturing flexibility, methods for delivering flexibility, and performance measurement. However, Gerwin expands this definition by arguing that an enterprise could leverage flexibility to anticipate and prepare for environmental uncertainties through redefinition. For example, a firm can encourage customers to see the benefits of shorter lead times or more frequent new product introductions, and then provide higher levels of service in these dimensions through superior manufacturing flexibility.

De Groote (1994) defines flexibility as a hedge against environmental diversity and proposed a general framework for analyzing flexibility. The framework consists of three elements which are, the set of technologies whose flexibility is to be evaluated, the set of environments in which those technologies operate, and a performance criterion for evaluating different technologies in different environments.

2.2 Flexibility Dimensions

Browne et al (1984) defined eight classes of flexibility. They are machine flexibility, process flexibility, product flexibility, routing flexibility, volume flexibility, expansion flexibility, operational flexibility, and production flexibility. Later, Sethi and Sethi (1990) identified the existence of at least 50 different terms for various types of flexibility which were referred to in the literature where their definitions are "not always precise and not in agreement with one another, even for the identical terms". Based on Brown et al, Sethi & Sethi also extended the flexibility types to eleven categories while Brown's et al original eight remain the same. Recently, Vokurka and O' Leary-Kelly (2000) have identified four additional flexibility dimensions which are automation, labor, new design, and delivery. The definition and origin of each flexibility dimension is described in Table 2.1.

Some authors argue that flexibility is a multi-dimensional variable. Slack (1983) defines two basic dimensions: range and response. Range flexibility would be the ability

Table 2.1 Flexibility Taxonomies, Dimensions, and Definitions

Flexibility type	Definitions
Machine[a]	The various types of operations that the machine can perform without requiring prohibitive effort
Process[a]	The ability to change between production of different products with minimal delay
Product[a]	The ability to change the mix of products in current production
Routing[a]	The ability to vary the path, that a part may take through the manufacturing system
Volume[a]	The ability to operate profitably at different production volumes
Expansion[a]	The ability to expand the capacity of the system as needed, easily and modularly
Operation[a]	The ability to interchange the sequence of manufacturing operations for a given part
Production[a]	The universe of part types that the manufacturing system is able to make. This flexibility type requires the attainment of the previous seven flexibility types
Material handling[a]	The ability to move different part types efficiently for proper positioning and processing through the manufacturing facility it serves
Program[a]	The ability of the system to run virtually unattended for a long enough period
Market[a]	The ease with which the manufacturing system can adapt to a changing market environment
Labor[c]	Range of tasks that an operator can perform within the manufacturing system
New Design[d]	Speed at which products can be designed and introduced into the system
Delivery[c]	Ability of the system to respond to changes in delivery requests
Automation[b]	Extent to which flexibility is housed in the automation (computerization) of manufacturing technologies

[a]Definitions adapted from Sethi and Sethi (1990) and Gupta and Somers (1992)
[b]Definitions adapted from Parthasarthy and Sethi (1993)
[c]Definitions adapted from Slack (1983)
[d]Definitions adapted from Dixon (1990) and Suarez et al(1995, 1996)

of the system to adopt different states. One production system will be more flexible than another in a particular aspect if it can handle a wider range of states, for instance, to manufacture a greater variety of products or to produce at different aggregate levels of output.

However, Slack (1989) adds that the range of states a manufacturing system can adopt does not totally describe its flexibility. The ease with which it moves from one state to the other in terms of costs, time and organizational disruption is also considered important. A production system which moves quickly, smoothly and cheaply from one state to the other should be considered more flexible than another system which can only cope with the same change at greater cost and/or organizational disruption. The way the system moves from one state to another would define Slack's other flexibility dimension, namely; response flexibility.

While range and response are clearly two different dimensions of flexibility, it is important to notice that they are not independent. Manufacturing systems tend to be more responsive to small changes and less responsive to big changes.

Time is another dimension which, to some authors, is important for the understanding of flexibility. Carter (1986) believes that different kinds of flexibility have an impact on the production system in different time frames: very short term, short term, medium term and long term; as a consequence, different kinds of flexibility should be sought in order to achieve the different time frame objectives.

Stecke and Raman (1986) also considered "time" in their analysis regarding the relationship between flexibility and productivity and proposed that, in the short term, production flexibility enables the system to maintain its production level in face of unforeseen events, such as machine breakdowns. With regard to the long term, Stecke and Raman propose that production flexibility would be related to the inter-dependence between the process and product life cycles. Flexible systems in the long term would tend to cause a relaxation in the one-to-one relationship which the conventional production systems would present. This relationship is discussed in detail in Hayes and Wheelwright (1984).

Another dimension identified by Gerwin (1987) as a basic issue in defining manufacturing flexibility is the level at which it is to be considered:

i) the individual machine or manufacturing system;

ii) the manufacturing function such as forming, cutting or assembling;

iii) the manufacturing process for a single product or group of related ones;

iv) the factory or the company's entire factory system

At each level, says Gerwin, the domain of flexibility concept may be different and alternative means of achieving flexibility would therefore be available.

A company which intends to be flexible in the introduction of new products in the market place (at the highest level, that of the company's entire factory system) should take actions different from those of a company which plans to make a machine more flexible by developing jigs and fixtures in order to shorten its set-up time (the lowest level, of the individual machine). In the former for instance, it is essential that the flexibility of the product design team is developed. In the latter the flexibility of this team is possibly less important.

Gupta and Buzacott (1989) define three dimensions of manufacturing flexibility: sensitivity, stability, and effort. With respect to each change, sensitivity relates to the magnitude of change tolerated before there is a corrective response. Stability relates to the size of each disturbance or change for which the system can meet expected performance targets. Whereas sensitivity and stability determine whether a system responds to a change or not, effort relates to how well a system responds to a change. Effort depends on such factors as the time to respond to change and the cost of response.

Donner and De Silva (1990) propose dimensions which are similar to those of Slack's. According to these authors, flexibility would have three dimensions: range, switch ability, and modifiability. Range, similarly to Slack's range, relates to a set of states a machine or a set of machines can adopt to do useful work. Within a given set, transitions can be made between states. The general approach relates to taking up a new set of states, which may or may not include those individual states belonging to the set of states prior to the modification.

Mandelbaum (1978) defines two basic dimensions of manufacturing flexibility: action flexibility and state flexibility. Action flexibility would be the capacity for taking new actions to meet new circumstances, that is, leaving options open so that it is possible to respond to change by taking appropriate action. State flexibility would be the capacity to continue functioning effectively despite the change, i.e. the system's robustness or tolerance to change.

Table 2.2 summarizes the different dimensions of manufacturing flexibility based on these authors.

Table 2.2 Summary of the Different Dimensions of Manufacturing Flexibility according to Selected Authors

Mandelbaum (1978)	Slack (1983)	Gupta & Buzacott (1986)	Stecke & Raman (1986)	Carter (1986)	Gerwin (1986)	Dooner & De Silva (1990)
Action	Range	Sensitivity	Time	Time	Organization Level	Range
State	Response	Stability				Switch- ability
		Effort				Modify-ability

2.3 Flexibility Measures

One of the difficulties found by authors who study flexibility of manufacturing systems is how to measure it. And the reasons are not far fetched as flexibility translates to the ability or potential to realize a set of goals rather than something measurable with hindsight, such as performance. Various research approaches have been employed to measure flexibility and these can be broadly categorized as either qualitative or quantitative, Beach et al (2000). Nevertheless, the emphasis here will be on those of a quantitative nature.

Qualitative research in the field of flexibility tends to deal with issues focusing on concepts such as those related to process technology and business strategy. Conversely, quantitative research tends to address specific manufacturing issues and are operational nature. A wide range of techniques for the quantitative measures of flexibility has been employed. Examples of frequently cited techniques are the path analytical model, Petri net modeling, information theoric, decision theoric, financial analysis, value based, and empirical data analysis. As a result, many different measurement schemes have been created because of the lack of universal acceptance for any one scheme. This supports the fact that any measurement of flexibility must be user or situation specific, Gupta (1993).

Sarker et al(1994) introduced a more robust classification of approaches to research in this area, namely; aggregate and attribute.

Aggregate – the unification or integration of flexibility measures of individual subsystems into quantification of the manufacturing systems flexibility; and *Attribute* - the construction of a measure of manufacturing flexibility based on parameters selected from a cross section of those functions of the manufacturing system which contribute to flexibility.

Cited examples of the aggregated approach are: Abdel-Malek and Wolf (1991) who in their work, integrate an individual measure of each component by constructing a parameterized average flexibility representing an index of manufacturing flexibility. Similarly, Hutchinson and Sinha (1989) are cited as providing an example of the attribute approach by using "decision theory measure" to derive an economic value of flexibility under demand uncertainty.

Several observations can be made from Sarker's evaluation of these two approaches. Concerning the aggregated approach, it is noted that a fundamental prerequisite is the identification of the relationships between flexibility types; and the difficulty of achieving this has been previously discussed. Measuring flexibility in monetary terms as proposed in the integrated measures of Son and Park (1987) and (Ramasesh and Jayakumar, 1991) has an immediate appeal to management and hence has a practical value. Conversely, the attribute approach is dependent on the identification of parameters and factors other than flexibility and types. A variety of approaches, which have been used for identifying these factors, are cited. However, many are of a theoretical nature and hence of questionable relevance in real world applications. Exceptions are: Lim (1986) whose use of an organizational survey attempts to identify how management defines flexibility; Gerwin (1987) who proposes the use of a methodology for systematically identifying factors based on their use to counter uncertainty; and, the factor analysis approach adopted by Gupta and Somers (1992).

In their attempt to develop an "instrument for measuring and analyzing manufacturing flexibility", Gupta and Somers identified from the literature 34 items affecting manufacturing flexibility and a preliminary instrument was created to measure them, e.g. "Time required to introduce new products", "Time required to add a unit of production capacity", "Number of new parts introduced per year", etc. The results of a survey of 269 companies were tabulated using factor analysis techniques to create a construct of 9 principal types of flexibility based on 21 lower order items: volume, programming, process, product and production, market, machine, routing, material handling, expansion and market. The construct, which is built on the taxonomy proposed by Sethi and Sethi (1990), was tested further on 113 companies and was found to exhibit "adequate reliability and validity".

Ramasesh and Jayakumar (1991) refer to the notion of aggregated flexibility as "the joint effect of all types of flexibilities that exist in the manufacturing system under consideration". The rationale for considering the measurement of flexibility at this level is the unsuitability of lower order measurements of flexibility in the strategic decision making process. The lower order measures are seen as "on the whole non-financial", "local measures which look at one or a few dimensions and ignore possible interactions and trade-offs which may exist between the different flexibility types", and "the measurements are isolated in that they are derived independently of the manufacturing environment".

The quantitative model subsequently developed is constructed around the view that a value of flexibility could be used as a surrogate measure (Gupta and Buzacott, 1989) and that a stochastic mathematical programming model could be used to measure objects of managerial control (Jaikumar, 1984). The resulting model uses machine, material, labor and volume flexibility to construct and aggregate flexibility. The resulting measure, involving the distribution of the net revenues of the flexibility measure was said to, "present performance-related benefits of decisions concerning the flexibility aspects of a manufacturing system".

The perspective of the research conducted to date has had a significant bearing on the development of so many measures of flexibility. However, a significant factor must be the absence of an agreement on the purpose of measuring flexibility, e.g. to compare the effectiveness of alternative types of technology, to measure the operational performance of a cell or manufacturing facility, or to assess the feasibility of developing particular business strategies. Another reason perhaps, is the absence of any agreement on the constituent types of flexibility. This situation is exacerbated by the fact that several important facilitators/enablers of manufacturing flexibility have been consistently ignored in the manufacturing literature specifically labor (Chen et al 1992) and information technology. Ramasesh and Jayakumar (1991) provide a review of flexibility types and suggested measures. Information technology, while acknowledged as an important facilitator of manufacturing flexibility by Sethi and Sethi (1990) is similarly ignored. A possible reason for this is that facilitators, like machine tools, labor and information technology, are thought to contribute to a system's flexibility rather than being types of it.

2.4 Uncertainty and Flexibility

A number of authors suggest that the uncertain environment and variability of outputs are the main reasons for an organization to seek manufacturing flexibility. For instance, Swamidass and Newell (1987) developed a model incorporating environmental uncertainty and manufacturing flexibility, tested it empirically, and based on the results, stated that an organization may find at least some help in coping with the high uncertainties that the environment imposes by increasing its manufacturing flexibility. Gerwin (1987) argues that social systems facing uncertainty utilized flexibility as an adaptive response; he further suggest that, since there are several kinds of uncertainty, there should be several kinds of corresponding flexibilities to cope with them. Gupta and Goyal (1989) suggest that flexible manufacturing systems can utilize flexibility as an adaptive response to unpredictable situations. Slack(1990a) also suggests that companies use flexibility to cope with short and long term uncertainties. Gerwin and Tarondeau (1989) take the analysis one step further, using Gerwin's (1987) classification, by suggesting links between particular types of flexibility and different types of uncertainty. Atkinson (1985) argues that companies seem to be trying to develop more flexible manpower structures to be able to cope more efficiently with uncertainty regarding the supply of labor. Flexibility may also be developed as insurance (Carter, 1986) against process short term uncertainty (Stecke and Raman, 1986). Flexible manufacturing systems are important, according to Muramatsu et al (1985), for companies to be able to adapt to severe changes in the market. Gerwin (1987), Kumar (1987), Chambers (1995), also argue that the competition, which, nowadays is based more than ever on the responsiveness of the companies to different customer requirements, shorter product life cycle and greater product proliferation. Slack (1990a) also analyzes the links between types of variability and types of flexibility. These show that flexibility and uncertainty indeed have a strong relationship which should not be overlooked in flexibility planning and management processes.

Surprisingly, in the increasingly turbulent environment of today's competitive market, the literature on flexibility management contains little research work which considers environmental uncertainty explicitly. Certainly, more research work is needed in the area. It was found that the Gifford et al(1979)'s idea of uncertainty to be most appropriate in this area. According to the authors, considered globally, uncertainty will be low if data is available at the time needed and if the decision maker discerns a pattern of regularity among the cues that make the data useful for the prediction of future events or trends. This idea of uncertainty, according to this view, is broadly associated with that of predictability. Predictability seems to be a concept which is less controversial than uncertainty and also closer to the jargon normally used in industrial environments and therefore probably more easily understood by decision makers.

Correa (1994) conducted a research linking flexibility to uncertainty of environment and variability. She conducted case studies in Brazilian and English companies by interviewing several managers from different departments of these firms. Her results showed that managers cope with uncertainties by trying to control them or by building flexibility in the system. She also defined the different types of uncertainty that are considered significant (this research will apply some of the same uncertainty types). These uncertainties are defined as follows.

- Parts and material supply: This type of uncertainty occurs when suppliers delay the shipment or delivery of parts.
- Machine breakdowns: This uncertainty is related to the machine breaking down.
- Labor absenteeism: This is uncertainty related to labor.
- Demand: This uncertainty can be divided into two kinds; demand mix and demand volume uncertainty.
- Labor Supply: As more sophisticated production processes are adopted, one can no longer find as many qualified people as the job requires. As a consequence the training program has to be intensified.
- Government intervention: Uncertainty that occurs because of government influences such as the exchange rate mechanisms.
- Union behavior: Unexpected events such as strikes can be influenced by unions.
- Product variety: The different types of products that a firm has to build to satisfy its customer. The more product variety there is, the more the uncertainty of this type.

- New product introduction: The uncertainties with new product introduction and product changes (regarding launch dates, specifications and so on).

Dreyer and Grønhaug (2004) showed that, it is possible to achieve sustained competitive advantage in highly uncertain environments. Their study showed that fish processing firms with a sustainable competitive advantage have developed types of flexibility to match different factors of uncertainty in their industry.

Pagell and Krause (2004) address the relationship between environmental uncertainty and operational flexibility through their research that utilized a mail survey of North American Manufacturers. The results from their efforts can be summarized as i) no relationship found between the measures of environmental uncertainty and operational flexibility, ii) no relationship found between a firm's performance and its effort to align the level of operational flexibility with its external environment, and iii) the sample of advance manufacturing users reported more certain external environments than the random survey sample of manufacturers.

2.5 Determining Appropriate Flexibility Level

In this section, the literature is presented regarding two possible approaches that will be utilized in this dissertation in finding the optimal level in flexibility investment. The first approach is the single period modeling, while the second one is real options modeling. Real options modeling has been widely used in flexibility valuation and other applications in investment under uncertainty. However, few attempts have been made to deploy it in the determination of the optimal level of flexibility investment of a manufacturing system, where more than one flexibility class is involved, Kulatilaka

(1995). In the area of single period modeling (SPP), also known as the newsboy or newsvendor modeling, a few articles have been published relating to the manufacturing flexibility issue. These include the work of Khouja (1995) where he adopts the SPP to obtain the optimal production rate for volume flexible manufacturing systems. Because of the scarce availability of SPP literature in the flexibility arena, this section therefore takes a more critical look at the rich literature addressing the subject of real options in flexibility valuation.

Usually, the merit of flexibility to a firm can be considered as both reactive and proactive in value. While the prior refers to the value that a company gains from protecting its loss from downside risk, the latter means value that the company gains from using their flexibility as a tool to win market share. Flexibility is not only relevant in manufacturing, but also in finance, human as well as organizational behavior. In financial terms, flexibility is commonly referred to as "options".

An option gives the holder an opportunity without the corresponding obligation to do something specific. Two basic options, on the financial markets, are the call option and the put option. The call option gives the holder the opportunity to buy the underlying asset at a predetermined price, the exercise price, up to a pre-set date. The holder has the opportunity to choose whether to exercise the option or not. It will only be exercised if the value of the underlying asset exceeds the value of the exercise price. Conversely, a put option, which gives the holder the right to sell at a predetermined price, up to a pre-set date, will only, be exercised if the value of the exercise price exceeds the value of the underlying asset. The value of a call option, C , and the value of a put option, P ,at the date of exercise are written as functions of the exercise price, X ,and the value of the underlying asset, *S* and are expressed as given below. (Hull (2002)).

C = Max [S - X, 0], (1)

P = Max [X- S, 0], (2)

The payoff at exercise can thereby be illustrated as in Fig.1.

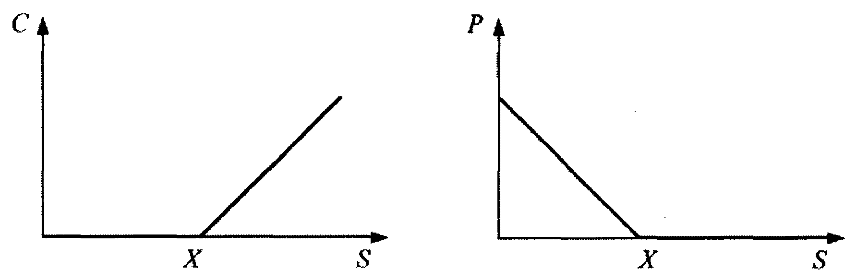

Figure 2.1 Payoff from Call and Put Options

In 1973, Black, Schole, and Merton introduced an option pricing model, and since then, the valuation of complex financial securities and options has been established. Myers (1987), then introduced the concept of real options which is usually utilized in the valuation of a project in the flexibility of the firm. An important factor differentiating real options from financial options is the underlying assets. In the case of financial options, the value of the underlying asset is often easily observed at the financial markets but in the case of real options whose value e.g. depends on revenues, is much harder to observe and gather data about it. This will also make it difficult to replicate the payoff of the option since revenues of a firm could not be seen as a traded security in many cases. Therefore, it is often assumed that markets are complete, i.e. in this case, that the revenues can be replicated by a portfolio of traded assets, a tracking portfolio, whose movements of value are identical to the movements of revenues.

Real options in valuation of manufacturing flexibility, based on Sethi & Sethi's framework, can be separated into three different levels, namely, basic, system, and aggregate level, Bentgson (2001). Only a few articles have been published for real options valuation of flexibility at the basic level. An article cited here is the work of Kulatilaka (1988). He introduces a real options model to value an operation's flexibility under price uncertainty. He considers only one machine where its operation modes can be switched.

More articles through have been published in the valuation of different flexibility types (mix, product, volume, and expansion) at the system level. The cited articles for the valuation of mix flexibility include the earlier work of Margrabe (1978), who considers the general option to exchange one risky asset for another. This paper, developed first for pricing financial securities, could also be applicable to value mix flexibility to facilitate the switch from one product line to another under uncertain profit margins. Triantis and Hodger (1990) develop a real options model, which evaluates investments in process flexible equipment where profit margins are uncertain, there is no switching cost, and production decisions are taken at pre-set points in time. Andreou (1990) evaluates a process flexible manufacturing system producing two products with no switching cost. His model considers capacity constraints and seven different scenarios. Process flexibility is then valued for each scenario where profit contribution of both products follows geometric Brownian motions. Bengtsson (1999) considers the value of having the option to hire personnel on short-term contracts when demand of a product or aggregated demand is uncertain. A contract, which lasts for three months is considered when production decisions are made every month. Bengtsson (2002) evaluates product mix flexibility under environments of demand uncertainty. His model considers multiple products, set-up cost, and capacity constraints. A Monte Carlo numerical method is utilized in the calculations.

For the issue of product flexibility, cited articles include the earlier work of Stulz (1982). He evaluates the European option on a maximum of two risky assets with a fixed exercise price and no capacity constraints. His model can be used to value product flexibility and also can be solved using Black and Scholes formula, the formula that developed to value financial call options. Johnson (1987) extends the work of Stulz to include several risky assets, i.e. where there are several mutually exclusive products that can be produced. Triantis (1988) develops a model to value product flexibility under uncertain profit contribution with no switching cost. His model includes capacity constraints and also allows temporarily shutdowns or reopening of an operation. Kamrad and Ernst (1995) consider the valuation of multi-product agreements where demand, delivery schedule, and output prices are known. The model assumes that only one product type can be produced and set-up cost is applied whenever switching among products occurs. A numerical lattice approach is used to obtain the estimated value of these production agreements.

For volume flexibility, Tannous (1996) developed a managerial tool based on options theory that can be used, along with other tools to determine the optimal budget for the purchase of volume flexible equipment under the assumption that the equipment should be quite similar in their performance of the same tasks. Here, volume flexibility is calculated by applying the Black and Schole's basis model. For expansion flexibility, Pindyck (1988) does not explicitly address the problem of valuing this flexibility type.

Instead, capacity choice and capacity expansion are examined to maximize the value of the firm when investments are irreversible and demand is uncertain. He and Pindyck (1992) extend the earlier work of Pindyck (1988) to include flexible capacity and compare this to the situation when only dedicated equipment is used. Trigeorgis (1996) considers the option to expand where the underlying value of the project is uncertain. These options are developed for the situation when e.g. a firm is able to increase the value of an ongoing project by an additional investment. Kumar (1995) presents a real options model to value expansion flexibility. His model focused on two period investment scenarios where a primary investment could result in an option to make a secondary investment.

One article that addresses the issue of product life cycle in the area of real options is the work of Bollen (1999). He developed an option valuation framework that incorporates a product life cycle. He also developed a model based on his framework to value the option to change a project's capacity. He demonstrated that using standard real options techniques without incorporating product life cycle can lead to a significant error in valuating capacity options.

Table 2.3, partially reproduced from Bentgson (2002), contains the summary of valuation of manufacturing flexibility based on Sethi &Sethi's framework using options theory. This table also summarizes the results of each research effort, the applications and contributions. As it is evident from the table, the available models consider only one flexibility class and mostly do not include the effect of life cycle on the value of flexibility.

Table 2.3 Summary of Past Works on the Real Options Approach to Manufacturing Flexibility

No.	Authors	Year	Manufacturing System Physical Characteristic	Product Life Cycle	Uncertainty*	Constraints	Application/Results
1	Margrabe [+]	1978			OP		Value an option to exchange one asset for another within the stated period.
2	Stulz [+]	1982			Pr		Value an option on maximum of two exchangeable risk assets with a fixed exercise price.
3	MacDonald and Seigel [+]	1985			OP		Value an option for temporal and costless shut down.
4	Johnson [+]	1987			Pr		Extends Stulz to include several assets.
5	Triantis	1988			OP		Develop two models, which evaluate a product flexible manufacturing system.
6	Pindyck	1988			D		Examine capacity choice and expansion flexibility to maximize the value of the firm.
7	Kulatilaka	1988			OP		Evaluate the special case of operation flexibility when, a flexible manufacturing system, provides different modes to produce one product.
8	Triantis and Hodder	1990			Pr	Capacity	Develop a model which evaluates process flexible equipment producing more than one product with costless switch cost.
9	Andreou	1990			Pr	Capacity, no back-order	Develop a model which evaluates process flexible equipment producing two products with costless switch cost.
10	He and Pindyck	1992			D		Extend from Pendyck to include flexible capacity and compare when only dedicated equipment is used.
11	Azzone and Bertele	1992			Pr		Study the impact of flexibility on the economic evaluation of flexible manufacturing system.
12	Kulatilaka	1993			IP		Value the flexibility of the Duel Fuel Steam Boiler which can be switch used gas or oil.
13	Kumar	1995			Pr		Value the expansion flexibility based on Magrabe's model.
14	Kulatilaka	1995			Pr		Develop a general model allow to value different kind of flexibility, however the underlying valuation model is essentially the same.
15	Kamrad & Ernst	1995			Y	Capacity of production &inventory	Valuation of the multi-product agreements to find the optimal production and inventory guideline.
16	Tannous	1996			D		Develop a model to find the optimal level of investment in volume flexibility.
17	Trigeorgis	1996			Pr		Consider the option to expand when an investment decision is made where the underlying value of the project is uncertain.
18	Bollen	1999		Yes	D		Develop the framework that incorporates a product life cycle and then used to value option to expand and contract capacity.
19	Bengtsson	1999			D		Consider the value of having the option to hire personnel on short contracts.
20	Bengtsson	2002			D		Value the mix flexibility of multiple products combined with real case study in manufacturing system.
21	Proposing		Yes	Yes	D	Capacity	Consider three types of flexibility and develop a framework that incorporates flexible manufacturing system physical characteristic to select the appropriate flexible manufacturing system design.

2.6 Summary of Literature and Contribution of the Dissertation

As can be seen from the available literature, though flexibility research has been widespread, it could be argued that flexibility taxonomies and flexibility measures have been the predominant aspects of flexibility that most of the documented work has been based on, Sarker (1994). Insufficient attention has been given to developing models that minimize the cost of over/under flexibility. This has been one of the key motivations for which our study has been undertaken. This research differs from the existing literature in that it takes into account the need for flexibility, during the design process of a manufacturing system. This need for flexibility arises from the stochastic nature of the environment in which a firm operates.

- The first set of models proposed in the research aggregates more than one flexibility class where the interrelationship among the various classes is incorporated into one of the models. In addition, the models provide the link to the design of the appropriate physical manufacturing systems that yield optimum flexibility levels suitable for each manufacturer's uncertain environment.
- The second set of models adapted from the real options framework considers two system flexibilities namely; product-mix and volume flexibility and incorporates the product life cycle.

CHAPTER 3

TE MODELS

3.1 Preliminary

The term "Flexibility" is not new to US manufacturers as it is a concept quite a number of them have already adopted as a way of doing business for almost two decades. Nevertheless, its impact had not been fully appreciated until the early 1990s, when the US started to lose global market share in its two most important industrial sectors, which are automobiles and electronics, Clark and Fujimoto (1990). Since then, manufacturing flexibility has become more and more important because it provides manufacturers with the ability to deliver to markets cost efficiently many varieties of products at batch sizes within a short time. This in turn enables them to compete and survive in the marketplace. Therefore, designing a flexible system that enables a company to meet the market's uncertainty is a very important endeavor.

Designing a system for flexibility is a challenging task. Since the value attached to each type of flexibility varies according to its underlying degree of uncertainty. In other words, flexibility has very little value in static environments, thus resulting in no need for its deployment in such cases. To invest in flexibility when its need is not present would result in the manufacturer investing a fortune in an unnecessary venture and lose the opportunity to invest the tied up capital in some other value adding activity or flexibility category. On the other hand, under investment in manufacturing flexibility could prevent the manufacturer from exploiting the full capabilities of the flexibility type in providing swift responses to market dynamics; and in some extreme cases, nullify the whole flexibility exercise. It can therefore be seen that flexibility investment decisions can be exceptionally complex with the level of complexity driven by the two aforementioned facts which can be summarized as: 1) the need for flexibility is induced by the aggregation of its underlying uncertainties mainly created by customers' demands and 2) the value of flexibility varies according to the levels of these uncertainties faced by firms. Hence, before making costly and irreversible investment decisions, it is imperative that management gains insights in all aspects pertaining to these uncertainties. The two most significant of these which form the focus of this thesis are: "what flexibility types would be more value adding given the nature of the firm's business operations" and "what are the optimal levels of investment for this flexibility given that their needs and values are uncertain in nature at the time of decision making." For the prior issue, management might find the important work of West (2003) very useful for flexibility deployment across the organization. The second issue however, still has a lot of areas that existing solutions cannot sufficiently address. The available models in the literature are more skewed towards the direction of presenting values of various flexibilities by mostly considering individual flexibility at a time. In addition, these models do not have the ability to inform management what suitable physical manufacturing system requires investment under uncertainty. The consequences of these shortfalls have been that managements are yet to be equipped with the right tools for flexibility investment decisions. This leads to the need for a new direction that aims to provide management guidelines for designing a flexible system optimally. This is the motivation for this dissertation.

This work focuses on the latter issue of determining the degree of flexibility that management should invest in by also taking into account the underlying uncertainty and thus facilitate the selection of the right manufacturing system design. To achieve these aims, two stochastic models are presented. The first model, which forms the core of this chapter, is adapted from that of the single period inventory problem known as the newsvendor problem. Chapter 4 focuses on the second model which is adapted from the real options theory. The details of the first model are explained in the following subsections.

3.2 Concepts and Assumptions of the 1st model

The first model takes into consideration two important facts, which are that: the needed level of flexibility is uncertain in nature and the optimal flexibility degree to be invested in should be the neutral point between over flexibility and under flexibility.

3.2.1 The Need for Flexibility

Abdel-Malek et al (2000) introduced of methodology to measure the need for machine and product flexibility. In their paper, machine flexibility is defined as a function of the process change frequency, and product flexibility as a function of the number of new product introductions per year and the number of model changes per year. Nevertheless, the need for flexibility is uncertain in nature and it is based on the underlying uncertainty degree that firms face. For instance, if there is a high uncertainty of the quantity of product demand, the need for volume flexibility is expected to be high.

For reasons aforementioned, to represent the need for flexibility as a predetermined value might not be a suitable approach. Therefore, in this research, the needs for flexibility levels are denoted distinctively as probability random variables, which assume the possible range of values from zero to one hundred percent (0 – 100%). To be more specific, zero percent means that there is no need for flexibility or in other words, firms need only a dedicated manufacturing system. On the other extreme, one hundred percent level means that firms need a maximum flexibility degree. Or in other words, firms need state of the art manufacturing systems which provide the most available flexibility to hedge against an extremely high degree of uncertainty. This range is divided into five sub-intervals to match management's expectation for flexibility level as follows.

1) 0 - 20% represents that management expects very low level of a particular flexibility class.

2) 20 - 40% represents that management expects low level of a particular flexibility class.

3) 40 – 60% represents that management expects intermediate level of a particular flexibility class.

4) 60 – 80% represents that management expects high level of a particular flexibility class.

5) 80 – 100% represents that management expects very high level of a particular flexibility class.

The purpose of dividing this range is to aid management in narrowing down their options and to enable the determination of the expectation of their needs for the respective flexibility classes.

3.2.2 Assumptions of Related Costs

In this analysis, three main pertinent costs are considered. They are flexibility unit purchasing cost, under flexibility cost, and over flexibility cost. Their definitions are explained in the following.

A) Flexibility unit purchasing cost is the investment cost to acquire a flexibility level of degree x_i, where x_i denotes the level of flexibility, represented by a function $c(x_i)$. Hence, this cost is denoted as $c_i\, x_i$.

B) Cost of under-design for flexibility is incurred when the degree of the required flexibility attributes exceeds the firms' available flexibility resources. The costs included in this category are (a) cost of outsourcing, and (b) opportunity cost. It is assumed that this cost is a function of purchasing cost for a unit of flexibility, denoted as $c_i\, \beta_i$, where c_i is the unit purchasing cost and β_i is an under design cost multiplier.

C) Cost of over-design for flexibility is incurred when firms spend too much money in designing a system for flexibility without utilizing it to full capacity. The costs included in this category are (a) the cost of maintaining the flexibility resources which include cost of used space, insurance, consumed power, and salvage value of manufacturing equipment, (b) operating cost, and (c) the capital tied up cost - the cost of tying up capital in flexibility investments, and not using this money for alternative purposes. We assume that this cost is a function of purchasing cost for a unit of flexibility, denoted as $c_i\, \alpha_i$, where c_i is the unit purchasing cost and α_i is an over design cost multiplier.

To be able to measure the flexibility of alternative system designs, the flexibility measure framework presented in Abdel-Malek and Wolf (1991) is implemented. In their work, the relative flexibility index is measured based on the physical attributes of a system. They also defined a set of attributes for measuring each flexibility class. The following is a brief explanation of these attributes. The complete details can be found in the aforementioned paper.

3.2.3 Flexibility Attributes

Attributes are defined as qualities or characteristics inherent in or ascribed to someone or something. For a manufacturing system, these attributes which govern several flexibility classes are engine horse power, machine envelop, machine capacity, number of axis. Some of the pertinent attributes, considered in the numerical examples treated in Chapter 5, need to be defined for the general reader and are explained as follows:

- Repeatability

  Repeatability indicates how precisely an equipment can repeatedly return to a certain point.

- Accuracy

  Accuracy is the minimum tolerances the equipment is capable of handling while processing an assigned job.

- Payload

  Payload concerns the maximum weight of material that equipment is capable of handling on a continuous basis

- Envelop

  Envelop is the area in which the equipment can move to perform an assigned task.

3.2.4 The Model

This section addresses the core of this chapter which is the development of the first model. This model is built on the assumption that the needs among flexibility classes are independent. Based on the classical Newsboy model, the modified model can be implemented to determine the optimum flexibility level to be invested for different flexibility classes. The nomenclature of symbols used here is as follows:

Symbol	*Description*
Z	The expected cost to be minimized
τ	Flexibility class index
N	Total number of considered flexibility classes
c_τ	Purchasing unit costs of flexibility class τ
α_τ	The over-designed cost multiplier
β_τ	The under-designed cost multiplier β_τ has to be greater than 1. If the value is less than one, the investment will not occur
D_τ	Random variable, represents the need for flexibility *i* and $0 \le D_\tau \le 100$
x_τ	Degree of investment in flexibility *i* , $L_\tau \le x_\tau \le U_\tau$
$f(D_\tau)$	The probability density function of D_τ
L_τ	The lower bound degree of the need for flexibility
U_τ	The upper bound degree of the need for flexibility
B_g	The firm's available budget
K_τ	The truncated constant value for flexibility class τ (the formulae for K_τ are shown in Table 3.1)

Table 3.1 Table Shows Formula of K_τ

Type of distribution	K_τ
Exponential	$\dfrac{1}{\left(1-e^{\frac{-b_\tau}{\mu_\tau}}\right)}$
Normal	$\dfrac{1}{\Phi\left(\frac{b_\tau-\mu_\tau}{\sigma_\tau}\right)-\Phi\left(\frac{a_\tau-\mu_\tau}{\sigma_\tau}\right)}$

In Table 3.1, $\Phi(z)$ denotes the distribution function of a standard normal variable, with a_τ representing its minimum possible value, and b_τ representing its maximum possible value. For convenience and without loss of generality, we set $a_\tau = 0$ and $b_\tau = 100$. Notice that, for bounded probability distributions such as the Uniform, Triangular, Beta, etc., the value of K_τ is constant and equal to one. This section demonstrates an algorithm developed for solving this model. (It is worth noting that the developed algorithm can also be used to solve the classical Newsboy problem with side constraints.)

The following Figure 3.1 shows the model's concept and its mechanism. Equation (3.1) shows the complete formulated model for the independent case.

$$Min(Z)=\sum_{\tau=1}^{N}\left[c_\tau K_\tau\left(x_\tau+\alpha_\tau\int_0^{x_\tau}(x_\tau-D_\tau)f(D_\tau)\,dD_\tau+\beta_\tau\int_{x_\tau}^{100}(D_\tau-x_\tau)f(D_\tau)dD_\tau\right)\right] \quad (3.1)$$

Subject to

$$\sum_{\tau=1}^{N}(c_\tau x_\tau)\le B_g, \qquad \tau=1,2,\ldots,N$$

$$\overset{N}{\underset{\tau=1}{\forall}} L_\tau \le x_\tau \le U_\tau$$

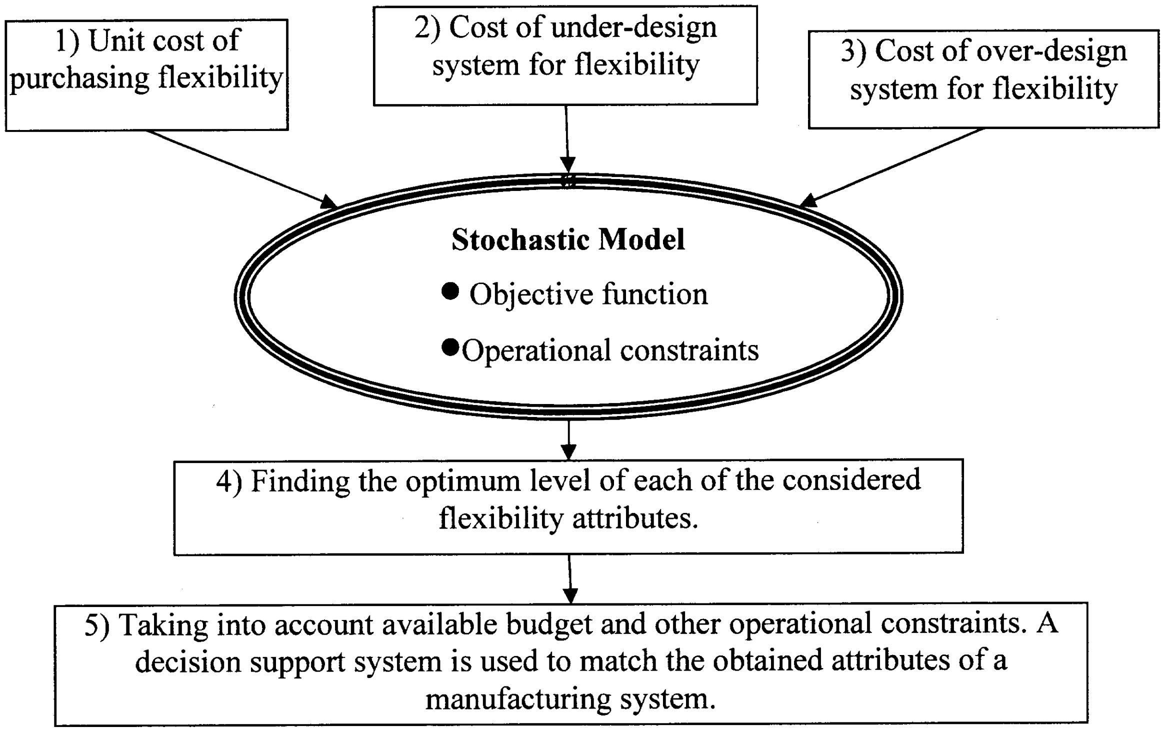

Figure 3.1 Schematic Diagram Shows Mechanism of Present Model

The following subsection shows relevant probability distributions that could be appropriate to describe the uncertain needs in flexibility levels.

3.2.5 The Characteristic of Distribution Functions

The triangular approach is used for estimating the area under the curve of the need for flexibility τ 's CDF (cumulative distribution function). Generally, we can divide these curve shapes into three major silhouettes; the ramp shapes, the parabola shapes with zero lower bounds, and the S shapes with non zero lower bounds. The ramp shape CDF is the shape of a distribution function such as the uniform distribution. The figure of this shape is shown in Section 3.3.1.1. The parabola shapes with zero lower bounds are the shapes of the CDF of functions such as the Exponential, Weibull, and Lognormal distributions, see Section 3.3.1.2. Finally, the last shape belongs to distributions such as the Normal, Student (t), and Beta distributions, see Section 3.3.1.3.

After, the short introduction of these important probability distributions above, the details of the characteristics of these distribution functions have to be explained in the following section since the developed algorithm rely on them.

3.3 Development of Solution Methodology

There are existing methodologies which, if modified, might be applied to solve the present flexibility problem. Nevertheless, several inherent disadvantages including their complex implementation, very limited number of constraints that can be applied, and their neglect of the lower bound which could sometimes lead to a negative optimal solution might make them unattractive options to consider. Therefore, the developed triangular approach takes all those disadvantages into consideration to yield a more user friendly approach and greater accuracy. In addition, it can be used to conduct necessary post-evaluation analyses such as sensitivity analysis.

The objective function of the model as shown in Equation (3.1) can be simplified and expressed alternatively in the form shown in Equation (3.2) and subsequently in the quadratic form shown in Equation (3.3)

$$Max \quad Z = \sum_{\tau=1}^{N} \left\{ c_\tau K_\tau \left[(\beta_\tau - 1)x_\tau - (\beta_\tau + \alpha_\tau) \int_0^{x_\tau} F(D_\tau)dD_\tau - \beta_\tau \, E[D_\tau] \right] \right\} \tag{3.2}$$

{See Appendix A for proof of Equation (3.2)}

$$Max \quad Z = \sum_{\tau=1}^{N} \left(A_\tau^{()} x_\tau^2 + B_\tau^{()} x_\tau + C_\tau^{()} \right) \tag{3.3}$$

Where, $A_\tau^{()}$, $B_\tau^{()}$, and $C_\tau^{()}$ are constants to be determined for each flexibility τ according to the probability distribution for its need; $(.)$.

The following, we demonstrates how to apply the triangular approach to obtain these constants. First note that the second term of equation (3.2) includes the integral of the cumulative distribution function. This area can be either expressed or approximated as that of a triangle using the following equation:

$$\int_0^{x_\tau} F(D_\tau)dD_\tau \approx \frac{1}{2}(x_\tau - x_{l,\tau})(\Delta_\tau (x_\tau - x_{l,\tau})) \qquad (3.4)$$

Where, x_τ - $x_{l,\tau}$ is the length of the triangle base, $F(x_\tau) = \Delta_\tau (x_\tau - x_{l,\tau})$ is the height of the triangle with respect to x_τ, and $\Delta_\tau = \left[\frac{F(x_{u,\tau})}{x_{u,\tau} - x_{l,\tau}}\right]$ represents the slope of the triangle. (More details about these parameters and how to obtain them for each probability density function are given in section 3.3.1).

The percentage error of the approximated area can be calculated using the following Equation;

$$error = \frac{\int_0^{x_\tau} F(D_\tau)dD_\tau - \left[\frac{1}{2}(x_\tau - x_{l,\tau})(\Delta_\tau (x_\tau - x_{l,\tau}))\right]}{\int_0^{x_\tau} F(D_\tau)dD_\tau} \times 100 \qquad (3.5)$$

Substituting Equation (3.4) into Equation (3.2), one can arrange its terms to obtain the quadratic form of Equation (3.3). Hence, the values of the coefficients of the objective function can be expressed as follows:

$$A_\tau = -c_\tau K_\tau \left(\frac{\beta_\tau + \alpha_\tau}{2}\right)\Delta_\tau,$$

$$B_\tau = c_\tau K_\tau (-(1-\beta_\tau) + (\beta_\tau + \alpha_\tau)\Delta_\tau x_{l,\tau}),$$

$$C_\tau = c_\tau K_\tau \left(-\left(\frac{\beta_\tau + \alpha_\tau}{2}\right)\Delta_\tau (x_{l,\tau})^2 - \beta_\tau E[D_\tau]\right) \qquad (3.6)$$

It should be noted that the shape of the cumulative distribution function, $F(D_\tau)$, plays a significant role in determining the values of ($x_{l,\tau}$, $x_{u,\tau}$, $F(x_{u,\tau})$). The next section lays out the procedures of obtaining the quadratic form of the objective function for different demand cumulative distribution functions ($F(D_\tau)$).

3.3.1 Modeling the Objective Function for Different Demand Distributions

As mentioned before, the triangular approach is used for estimating the area under the curve of a demand distribution function τ. Generally, these curves can be divided into three major silhouettes; ramp shapes, parabola shapes with zero lower bounds, and S shapes with non zero lower bounds. For each silhouette, the values of the parameters ($x_{l,\tau}$, $x_{u,\tau}$, $F(x_{u,\tau})$) have to be first appropriately defined. The following sub-sections present the necessary explanations. Table 3.2 summarizes the coefficients' formulae ($A_\tau^{(\cdot)}$, $B_\tau^{(\cdot)}$, and $C_\tau^{(\cdot)}$) for three major probability distributions: the uniform, the exponential, and the normal. In addition, based on these formulae, the application of a general distribution case can also be implemented.

3.3.1.1 Silhouette I: The Ramp Shape Distribution Function

The first silhouette describes the characteristics of the uniform distribution. As can be seen, the area under the curve is a right-angle triangle that yields exact solutions.

Table 3.2 Summary of the Coefficients of the Objective Function for Common Probability Distributions

Distribution Function	$A_\tau^{(\cdot)}$	$B_\tau^{(\cdot)}$	$C_\tau^{(\cdot)}$
Uniform Dist. (exact)	$-\left(\frac{c_\tau K_\tau(\beta_\tau+\alpha_\tau)}{2(b_\tau-a_\tau)}\right)$	$\left(\frac{c_\tau K_\tau\left(a_\tau(1+\alpha_\tau)+b_\tau(\beta_\tau-1)\right)}{(b_\tau-a_\tau)}\right)$	$-\left(\frac{c_\tau K_\tau\left(\alpha_\tau a_\tau^2+\beta_\tau b_\tau^2\right)}{2(b_\tau-a_\tau)}\right)$
Exponential Dist. (approximate)	$-\left(\frac{c_\tau K_\tau(\beta_\tau+\alpha_\tau)}{2(x_\tau^*)}\right)\left(1-e^{\frac{-x_\tau^*}{\mu_\tau}}\right)$	$-c_\tau K_\tau(1-\beta_\tau)$	$-c_\tau K_\tau \beta_\tau \mu_\tau$
Normal Dist. (approximate)	$-c_\tau K_\tau\left(\frac{\beta_\tau+\alpha_\tau}{2}\right)f(\mu_\tau)$	$c_\tau K_\tau\left((\beta_\tau-1)+(\beta_\tau+\alpha_\tau)K_{1,\tau}\right)$	$c_\tau K_\tau\left((\beta_\tau+\alpha_\tau)K_{2,\tau}-\beta_\tau\mu_\tau\right)$

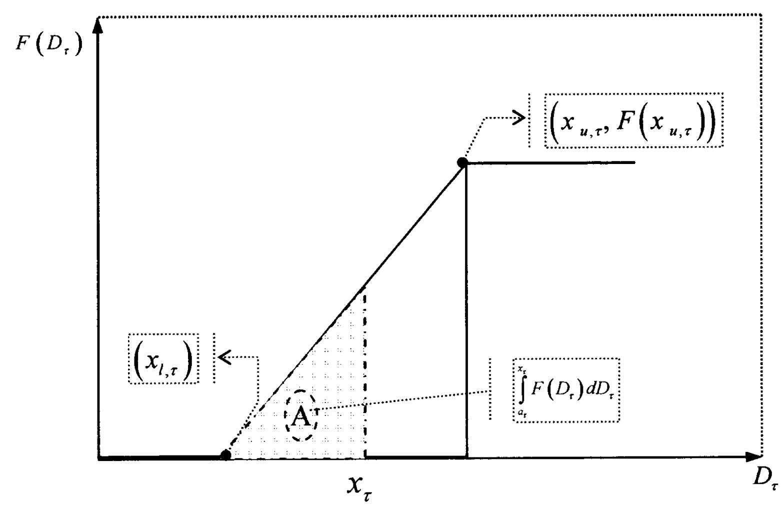

Figure 3.2 Triangular Presentation (shaded area) for Uniform distribution

Since the values of both a_τ and b_τ are known (i.e. $x_{l,\tau}=a_\tau, x_{u,\tau}=b_\tau$, thus, $F(x_{u,\tau})=1$), the parameters of the triangle (the value of the integral) can be determined in a straightforward manner as follows:

$$base = x_\tau - a_\tau,\ height\left(F(x_\tau)\right)=\Delta_\tau(x_\tau-a_\tau),\ slope(\Delta_\tau)=\left[\frac{1}{b_\tau-a_\tau}\right] \quad (3.7)$$

Then, the area of the triangle for this case is;

$$\int_{0}^{x_\tau} F(D_\tau)dD_\tau \approx \frac{(x_\tau - a_\tau)^2}{2(b_\tau - a_\tau)} \tag{3.8}$$

Consequently, substituting Equation (3.8) into Equation (3.6), the formulae for the coefficients of the objective function can be obtained.

$$A_\tau = -\left(\frac{c_\tau K_\tau(\beta_\tau + \alpha_\tau)}{2(b_\tau - a_\tau)}\right),\quad B_\tau = \left(\frac{c_\tau K_\tau\left(a_\tau(1+\alpha_\tau) + b_\tau(\beta_\tau - 1)\right)}{(b_\tau - a_\tau)}\right),\quad C_\tau = -\left(\frac{c_\tau K_\tau\left(\alpha_\tau a_\tau^2 + \beta_\tau b_\tau^2\right)}{2(b_\tau - a_\tau)}\right) \tag{3.9}$$

3.3.1.2 Silhouette II: The Parabola Shape Distribution Function with Zero Lower Bound

Among the probability distribution functions that belong to this family of shapes are the Exponential, the Weibull and the Lognormal distributions. The steps to determine the coefficients of the objective function for these types are as follows:

1) Calculate $F(x_\tau^*) = \theta_\tau = \left(\frac{\beta_\tau - 1}{\beta_\tau - \alpha_\tau}\right)$

2) Calculate $x_\tau^* = F^{-1}(\theta_\tau)$, where x_τ^* denotes the unconstrained optimal solution

   (If constraints are redundant or unbinding, the approach will give solutions equal to that obtained in step (2).)

3) Set the values of the parameters for the triangular area as follows:

   $x_{l,\tau} = 0,\ x_{u,\tau} = x_\tau^*,\ F(x_{u,\tau}) = F(x_\tau^*)$ (see Figure 3.3)

Then, one can proceed in a similar fashion as mentioned in the previous sub-section to obtain the triangle's parameters. The coefficients of the objective function are shown in Equation (3.11).

$$base = x_\tau,\ height\left(F(x_\tau)\right) = \Delta_\tau x_\tau,\ slope(\Delta_\tau) = \left[\frac{F(x_\tau^*)}{x_\tau^*}\right],\ area\left(\int_0^{x_\tau} F(D_\tau)dD_\tau\right) = \frac{x_\tau^2\left(F(x_\tau^*)\right)}{2(x_\tau^*)} \tag{3.10}$$

$$A_\tau = -\left(\frac{c_\tau K_\tau(\beta_\tau + \alpha_\tau)F(x_\tau^*)}{2(x_\tau^*)}\right), \quad B_\tau = -c_\tau K_\tau(1-\beta_\tau), \quad C_\tau = -c_\tau K_\tau \beta_\tau \mu_\tau \tag{3.11}$$

To illustrate further the application for this type of silhouette, consider the case of an exponentially distributed demand function. Its distribution function is given in Section 3.2.5.2. The coefficients of the objective function can be shown as follows:

$$\Delta_\tau = \left[\frac{1-e^{\frac{-x_\tau^*}{\mu_\tau}}}{x_\tau^*}\right], A_\tau = -\left(c_\tau K_\tau(\beta_\tau + \alpha_\tau)\left(1-e^{\frac{-x_\tau^*}{\mu_\tau}}\right)\right)\Big/2(x_\tau^*), \quad B_\tau = -c_\tau K_\tau(1-\beta_\tau), \quad C_\tau = -c_\tau K_\tau \beta_\tau \mu_\tau \tag{3.13}$$

and the percentage error of the approximated area in this case is

$$error = \frac{\left[x_\tau - \mu_\tau\left(1-e^{\frac{-x_\tau}{\mu_\tau}}\right) - \frac{1}{2}\frac{x_\tau^2}{x_\tau^*}\left(1-e^{\frac{-x_\tau^*}{\mu_\tau}}\right)\right]}{x_\tau - \mu_\tau\left(1-e^{\frac{-x_\tau}{\mu_\tau}}\right)} \times 100 \tag{3.14}$$

To be specific, in this case, the maximum error of Equation (3.14) occurs when x_τ approaches zero. Therefore, the maximum error for the exponential demand is

$$error_{\max} = \left(1 - \frac{\mu_\tau F(x^*)}{x^*}\right) \tag{3.15}$$

When constraints are binding, the unconstrained optimal solutions (x_τ^*) are the upper bound of optimal solution values of (x_τ), i.e. $x_\tau \le x_\tau^*$. The minimum error can be calculated by taking $\lim_{x_\tau \to x_\tau^*}$ of the error function. Hence, the formula for minimum error becomes:

$$error_{\min} = \left(1 - \frac{F(x_\tau^*)x_\tau^* e^{\frac{x_\tau^*}{\mu_\tau}}}{x^* e^{\frac{x_\tau^*}{\mu_\tau}} + \mu_\tau F(x_\tau^*)}\right) \tag{3.16}$$

Thus, the bounds of the error for the exponential distribution are given by

$$\left(1-\frac{F(x_\tau^*)\,x_\tau^*\,e^{\frac{x_\tau^*}{\mu_\tau}}}{x^*\,e^{\frac{x_\tau^*}{\mu_\tau}}+\mu_\tau F(x_\tau^*)}\right) \le error_{approx} \le \left(1-\frac{\mu_\tau F(x^*)}{x^*}\right) \tag{3.17}$$

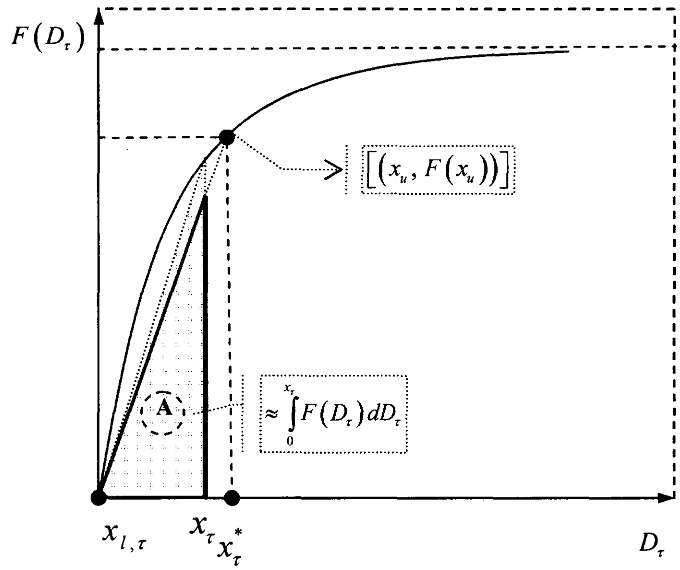

Figure 3.3 Triangular Approximation (shaded area) for Exponential Distribution

3.3.1.3 Silhouette III: The S Shape Distribution Function with Non Zero Lower Bound

Among the cumulative distribution functions (CDF) that belong in this category are the Normal, the Student (t), and the Beta distributions. Their silhouettes look similar to that which is shown in Figure 3.4. From that figure, one can see that setting the value of $x_{l,\tau} = 0$ is not suitable. Therefore, one has to find the appropriate value for $x_{l,\tau}$. It should be noted that by allowing $x_{l,\tau} > 0$, we are truncating the tail of the distribution function. Hence, the range of possible optimal solutions of item τ will be within $x_{l,\tau} \le x_\tau \le x^*$.

Because of the different nature of this type of distribution function, two approximate procedures are proposed. One can use both and then compare which of them produces a smaller cost. The first approach is based on a Taylor expansion of the demand distribution function, while the second is based on the calculation of the triangular area for the following specific values of $F(x)$; (0.001 and 0.9). Note that the triangular area, which is calculated for that range of $F(x)$ in essence covers the area under the CDF.

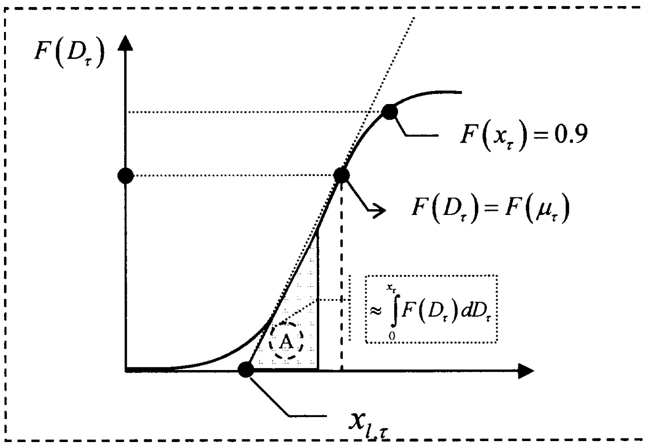

Figure 3.4 Triangular Approximation (shaded area) Obtained for Taylor's Expansion

3.3.1.3.1 The First approach (Taylor Series Expansion)

We expand the CDF of the demand using Taylor series around the expected value for item τ.

$$F(x_\tau)=F(\mu_\tau)+f(\mu_\tau)(x_\tau-\mu_\tau) \qquad (3.18)$$

Where, $F(x_\tau)$ represents the CDF, $F(\mu_\tau)$ represents the value of the CDF at μ_τ, $f(\mu_\tau)$ represents the value of the density function at μ_τ; where μ_τ is the mean of the demand for item τ.

Letting $F(x_\tau)=0$ in Equation (3.18), we obtain the formulae for $x_{l,\tau}$ and Δ_τ as shown

$$x_{l,\tau}=\mu_\tau-\frac{F(\mu_\tau)}{f(\mu_\tau)}, \quad slope(\Delta_\tau)=f(\mu_\tau) \qquad (3.19)$$

Hence, the parameters and the area of the triangle can be approximated as follows:

$$base = x_\tau - \mu_\tau - \frac{F(\mu_\tau)}{f(\mu_\tau)}, \quad height\left(F(x_\tau)\right) = f(\mu_\tau)\left(x_\tau - \mu_\tau - \frac{F(\mu_\tau)}{f(\mu_\tau)}\right), \text{ and the}$$

$$area\left(\int_0^{x_\tau} F(D_\tau)dD_\tau\right) = \frac{f(\mu_\tau)}{2}\left(x_\tau - \mu_\tau - \frac{F(\mu_\tau)}{f(\mu_\tau)}\right)^2 \qquad (3.20)$$

Substituting Equation (3.20) into Equation (3.2), we obtain the objective function coefficients as follows.

$$A_\tau = -c_\tau K_\tau\left(\frac{\beta_\tau + \alpha_\tau}{2}\right) f(\mu_\tau), \quad B_\tau = c_\tau K_\tau\left((\beta_\tau - 1) + (\beta_\tau + \alpha_\tau)K_{1,\tau}\right), \quad C_\tau = c_\tau K_\tau\left((\beta_\tau + \alpha_\tau)K_{2,\tau} - \beta_\tau \mu_\tau\right) \qquad (3.21)$$

Where,

$$K_{1,\tau} = \left(f(\mu_\tau)\mu_\tau - F(\mu_\tau)\right) \qquad (3.22)$$

$$K_{2,\tau} = \frac{-\left[\left(f(\mu_\tau)\mu_\tau - F(\mu_\tau)\right)^2\right]}{2f(\mu_\tau)} \qquad (3.23)$$

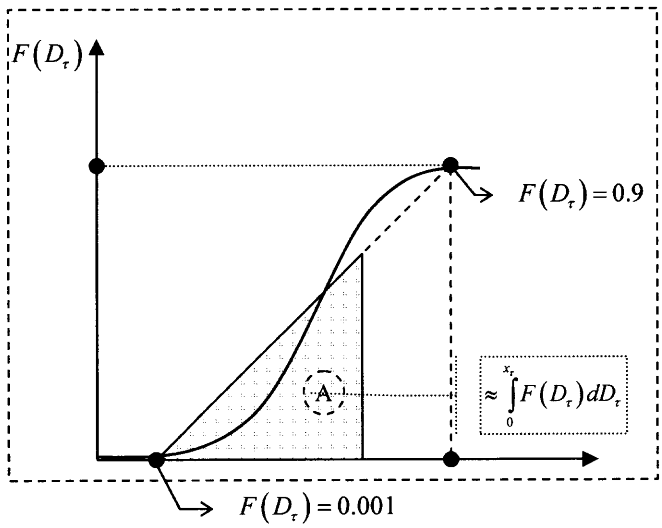

Figure 3.5 Triangular Approximation (shaded area) for Covering Range Approach

3.3.1.3.2 The Second Approach (Covering Range Approach)

For this approach, Δ_τ and $x_{l,\tau}$ are calculated by using the following Equations:

$$\Delta_\tau = \frac{(0.9 - 0.001)}{\left(F^{-1}(0.9) - F^{-1}(0.001)\right)} \quad \text{and} \quad x_{l,\tau} = F^{-1}(0.9) - \frac{0.9}{\Delta_\tau}$$

In a similar fashion, we can obtain the coefficients of the objective function.

3.3.2 Solving the Problem Using the Modified Simplex Method

After the objective function's constants have been obtained, the original Quadratic programming problem can be reduced into that of Linear programming. Then, the modified simplex method is implemented to obtain the optimal solutions. The modified simplex method is based on two steps which are; 1) reducing the nonlinear problem to obtain linear programming constraints using Karush-Kuhn-Tucker (KKT) conditions, and 2) solving the problem using phase 1 of the two-phase method to find a basic feasible solution for the quadratic problem; i.e., apply the simplex method (with modification) to the following linear programming problem

$$\text{Minimize} \qquad Z = \sum_j z_j$$

Subject to the linear programming constraints obtained from Step 1. The one modification in the simplex method is in choosing an entering basic variable which must follow the "Restricted-Entry Rule", see Hillier and Lieberman (7^{th} Edition). The detail of Step 1 is explained as follows.

The general matrix formulation of the QP model is as follows:

$$\text{Maximize} \qquad f(x) = \varpi x - \frac{1}{2} x^T Q x,$$

$$\text{Subject to:} \qquad \Lambda x \;\leq\; b \text{ and } x \geq 0,$$

where ϖ is a row vector, x and b are column vector, Q and Λ are matrices, and the superscript T denotes the transpose.

Noting that for a QP problem, its Karush-Kuhn-Tucker (KKT) conditions can be reduced to the convenient form that includes only linear programming constraints plus complementarities. Consequently, the general linear form for the QP is:

$$Qx + \Lambda^T u - y = \varpi^T ,$$

$$\Lambda x + \upsilon = b ,$$

$$x \geq 0,\ u \geq 0,\ y \geq 0,\ \upsilon \geq 0 ,$$

$$x^T y + u^T \upsilon = 0 ,$$

Thus, for the considered problem,

$$Q = \begin{bmatrix} -2A_1^{(.)} & 0 & \cdots & 0 \\ 0 & -2A_2^{(.)} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & -2A_i^{(.)} \end{bmatrix},\ \Lambda = \begin{bmatrix} c_1 & c_2 & \cdots & c_N \\ \xi_{1,1} & \cdots & \cdots & \xi_{1,N} \\ \vdots & \ddots & \ddots & \vdots \\ \xi_{M,1} & \cdots & \cdots & \xi_{M,N} \end{bmatrix}$$

$$\varpi = \left[B_1^{(.)} \quad \cdots \quad B_i^{(.)} \right],\quad b = \begin{bmatrix} B_g \\ R_1 \\ \vdots \\ R_N \end{bmatrix}$$

Next, using phase 1 of the two-phase method, the linear programming for the modified simplex method can be solved using the objective function as follows:

Minimize $\quad Z = z_1 + \ldots + z_i$

Subject to:

$$Qx + \Lambda^T u - y = \varpi^T , \tag{3.24}$$

$$\Lambda x + \upsilon = b ,$$

$$x \geq 0,\ u \geq 0,\ y \geq 0,\ \upsilon \geq 0 ,$$

$$x^T y + u^T \upsilon = 0 ,$$

3.4 Developed Methodology for the Interdependent Needs among Flexibilities

Based on Sethi and Sethi (1990), manufacturing flexibility can be categorized into three levels, which are basic, system, and aggregate level. Their framework depicted in Figure 3.6 can be viewed from two perspectives. First, the component flexibilities contribute to those of the system, which in turn influences the aggregate flexibilities. Alternatively, the manufacturing strategy dictates the extent of the system flexibilities that in turn dictate the component flexibilities required. In other words, basic flexibilities namely; machine, material handling, and operation are the foundation of higher level flexibilities such as volume, production flexibilities etc. As a result, the function of the need for aggregate and system flexibilities should be derived from that of basic flexibilities. In this research, the interdependency among different flexibilities is denoted as follows.

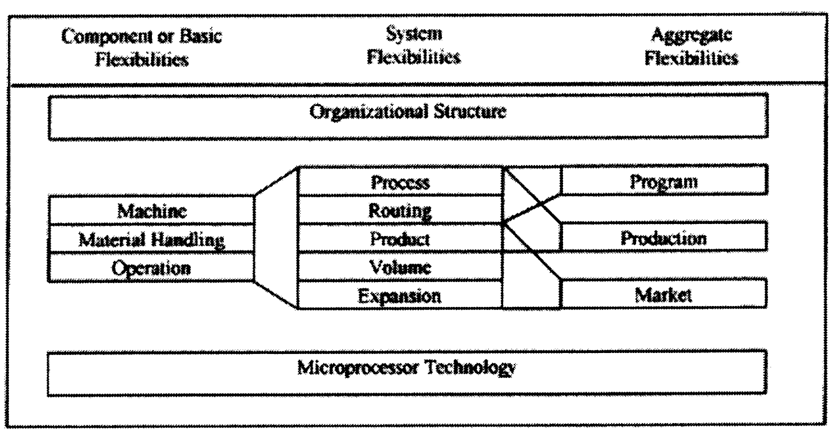

Figure 3.6 Linkages between The Various Flexibilities

$$D_{Ag} = f(D_S); \quad D_S = f(D_B) \tag{3.25}$$

where, D_{Ag} represents the need for aggregate flexibilities, D_S represents the need for system flexibilities, and D_B represents the need for basic flexibilities. As one can see from Equation (3.25), the probability density function of higher level flexibilities is now the joint density of lower level ones. For simple case such as that of the single relationship between one basic and one system flexibility, a simple transformation process can be implemented. But for complex relationships, the integral transformation, namely the Mellin transform, has to be utilized. The following sections 3.4.1 and 3.4.2 explain both transformation procedures in detail.

3.4.1 Simple Relationship: Functions of a Single Random Variable

This subsection explains the transformation process for the simple relationship of a system and a basic flexibility. The transformation is based on the implementation of the following Theorem 3.1.

Theorem 3.1 Suppose X is a continuous random variable with probability density function $f_X(x) > 0$ for $a \le x \le b$. Consider the random variable $Y = g(X)$ where $y = g(x)$ is a strictly increasing or decreasing differentiable function of x. Let the inverse of $y = g(x)$ be given by $x = v(y)$, then $Y = g(x)$ has a probability density function.

$$f_Y(y) = \begin{cases} f_X(v(y)).\left|\dfrac{d[v(y)]}{dy}\right| & g(a) \le y \le g(b) \quad \textit{if } g(x) \text{increasing} \\ f_X(v(y)).\left|\dfrac{d[v(y)]}{dy}\right| & g(b) \le y \le g(a) \quad \textit{if } g(x) \text{increasing} \end{cases} \tag{3.26}$$

To illustrate, assuming that management would like to define the optimal level of system flexibility and a basic flexibility where the need for that basic flexibility is as represented in Equation (3.27) and their relationship function represented in Equation (3.28).

$$f(D_B) = \frac{1}{(b-a)} \qquad , a \le D_B \le b \tag{3.27}$$

$$D_S = f(D_B) = p_S D_B^{p_B} \tag{3.28}$$

where, b, a, p_S, and p_B are constants.

The *PDF* of the need for system flexibility can be obtained using Theorem 3.1 as shown in Equation (3.29) and its *CDF* as shown in Equation (3.30).

$$f(D_S) = f_{D_B}\left(\left(\frac{D_S}{p_S}\right)^{\frac{1}{p_B}}\right) * \frac{d\left(\left(\frac{D_S}{p_S}\right)^{\frac{1}{p_B}}\right)}{dD_S} = \frac{1}{p_S p_B (b-a)} * \left(\frac{D_S}{p_S}\right)^{\frac{1}{p_B}-1}, \; p_S(a)^{p_B} \le D_S \le p_S(b)^{p_B} \tag{3.29}$$

$$F(D_S) = \begin{cases} 0 & (D_S/p_1) < a \\ \frac{1}{(b-a)}\left[(D_S/p_S)^{\frac{1}{p_B}} - a\right] & a \le (D_S/p_1) \le b \\ 1 & (D_S/p_1) \ge b \end{cases} \tag{3.30}$$

The *PDF* for basic and system flexibilities for the single relationship case is similar to those shown in Figures 3.7 and 3.8 respectively.

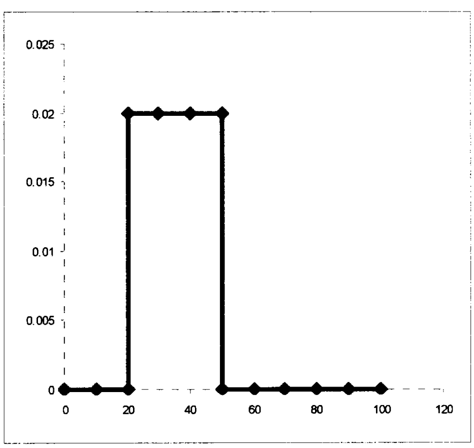

Figure 3.7 Probability Density Function of The Need for a Basic Flexibility $f(D_B), \{(a,b,p_S,p_B) = (20,50,2.5,0.8)\}$

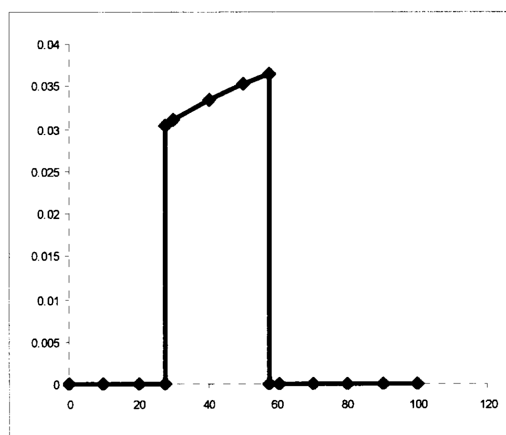

Figure 3.8 Probability Density Function of The need for System Flexibility $f(D_S), \{(a,b,p_S,p_B) = (20,50,2.5,0.8)\}$

To investigate how the total average cost will change, the total average cost for this case can be formulated as follows:

$$Min \quad E[TC] = \int_{p_S a^{p_B}}^{p_S b^{p_B}} c_S k_S\left[x_S + \alpha_S \max(x_S - D_S, 0) + \beta_S \max(D_S - x_S, 0)\right] f(D_S) dD_S + \int_a^b c_B k_B\left[x_B + \alpha_B \max(x_B - D_B, 0) + \beta_B \max(D_B - x_B, 0)\right] f(D_B) dD_B \tag{3.31}$$

where, its simplified version of Equation (3.31) can be expressed alternatively as follows.

$$Max \quad E[TC] = c_B K_B \left[(\beta_B - 1) x_B - (\alpha_B + \beta_B) \int_a^{x_B} F(D_B) dD_B - \beta_B E[D_B] \right]$$
$$+ c_S K_S \left[(\beta_S - 1) x_S - (\alpha_S + \beta_S) \int_{p_S a^{PB}}^{x_S} F(D_S) dD_S - \beta_S E(D_S) \right]$$

$$= c_B K_B \left[(\beta_B - 1) x_B - (\alpha_B + \beta_B) \int_a^{x_B} \left(\frac{D_B - a}{b - a} \right) dD_B - \beta_B \left(\frac{b + a}{2} \right) \right] +$$
$$c_S K_S \left[(\beta_S - 1) x_S - (\alpha_S + \beta_S) \int_{p_S a^{p_B}}^{x_S} \frac{1}{(b-a)} \left(\left(\frac{D_S}{p_S} \right)^{\frac{1}{p_B}} - a \right) dD_S - \beta_S \frac{p_S}{p_B + 1} \left(\frac{1}{b - a} \right) \left(b^{p_B + 1} - a^{p_B + 1} \right) \right] \quad (3.32)$$

After obtaining the objective function, the developed triangular approach, presented in section 3.3, can be implemented for solving the problem. The constant parameters necessary for triangular approach are

$$A_B = -c_B K_B \left(\frac{\alpha_B + \beta_B}{2(b - a)} \right), \quad B_B = c_B K_B \left(\frac{(\beta_B b + \alpha_B a)}{(b - a)} - 1 \right), C_B = -c_B K_B \left(\frac{\alpha_B a^2 + \beta_B b^2}{2(b - a)} \right)$$

$$A_S = -c_S K_S \left(\frac{\alpha_S + \beta_S}{2(g(b) - g(a))} \right), \quad B_S = c_S K_S \left(\frac{(\beta_S g(b) + \alpha_S g(a))}{(g(b) - g(a))} - 1 \right), C_S = -c_S K_S \left(\frac{\alpha_S g(a)^2 + \beta_S g(b)^2}{2(g(b) - g(a))} \right)$$

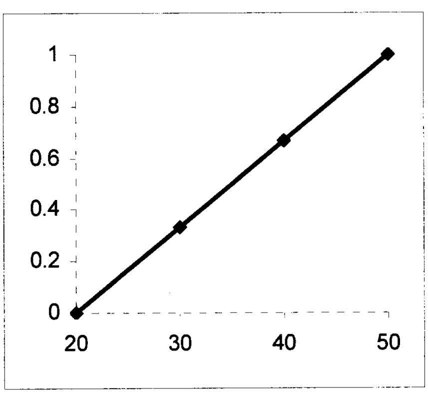

Figure 3.9 Probability Distribution Function of The Need for a Basic Flexibility $F(D_B)$

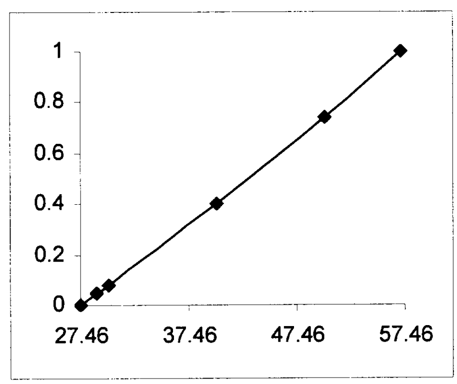

Figure 3.10 Probability Distribution Function of The Need for a System Flexibility $F(D_S)$

3.4.2 Functions of two or more random variables

Thus far, one system and one basic flexibility class were of interest. For a more complex scenario, the interrelationship among these flexibility classes can be portrayed as the network model shown in Figure 3.11. Their relationship functions are denoted as follows.

$$D_{VF} = p_{VF} D_{MF} D_{MH} D_O$$
$$D_{PR} = p_{PR} D_{MF} D_{MH} D_O$$
$$D_{PC} = p_{PC} D_{MF} D_{MH} D_O$$
$$D_{\mathrm{PRO}} = p_{PRO} D_{VF}^{\nu} D_{PR}^{\nu} D_{PC}^{\nu}$$

where, D_{MF}, D_{MH}, D_O, D_{VF}, D_{PR}, D_{PC}, D_{PRO} represent the need for machine, material handling, operation, volume, product, process and production flexibility, respectively, p_{VF}, p_{PR}, p_{PC}, p_{PRO} are normalized multipliers for these flexibilities, and ν is a positive integer.

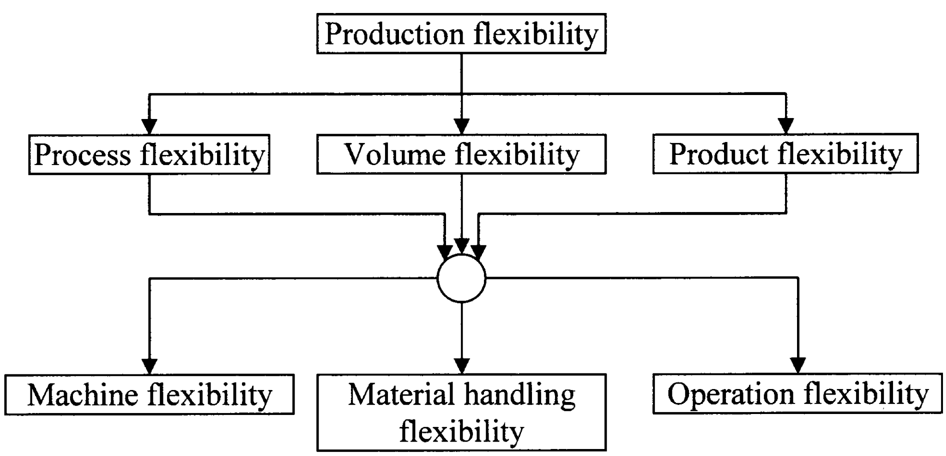

Figure 3.11 Network of Relationship for Production Flexibility

The transformation process is somewhat complex because of the combinations of several random variables. Therefore, to simplify, the integral transform such as the Mellin transform is useful in obtaining the probability distribution of the system and aggregate flexibilities.

If X is a nonnegative random variable, $0 \le X \le \infty$, the Mellin transform of its probability density function $f_X(x)$ is

$$\mathfrak{M}[f;s] = \mathbb{F}_X(s) = \int_0^\infty x^{s-1} f_X(x)\,dx \qquad (3.33)$$

and the inverse transform is

$$f(x) = \frac{1}{2\pi i} \lim_{\varsigma \to \infty} \int_{c-i\varsigma}^{c+i\varsigma} x^{-s}\, \mathbb{F}_X(s)\,ds \qquad (3.34)$$

Epstine (1948) has also extended this technique to random variables which are not elsewhere positive.

Theorem 3.2 Let $X_1, X_2, \ldots, X_n$ be independent random variables with probability density functions $f(x_1), f(x_2), \ldots, f(x_n)$. If $p_1, p_2, \ldots, p_n$ are constants and

$$Z = p_1 \prod_{i=1}^{n} X_i^{p_{i+1}} \qquad (3.35)$$

then,

$$\mathbb{F}_Z(s) = p_1^{s-1} \prod_{i=1}^{n} \mathbb{F}_{X_i}(p_{i+1}s - p_{i+1} + 1) \qquad (3.36)$$

A brief summary of the steps for the transformation process is given below:

- Implement Theorem 3.2 to obtain Equation (3.33)
- Conduct the Mellin inversion using Equation (3.34) to obtain the probability density function $f(z)$. It is essential to be able to perform explicitly the Mellin inversion. This is often the most difficult part of the computation and the different ways to proceed are explained in the next subsection (3.4.2.1).

- Based on the obtained density function $f(z)$, calculate $F(D_Z)$. Then using the suitable formula summarized in Table (3.2) to obtain $A_{\tau}^{(\cdot)}, B_{\tau}^{(\cdot)}$, and $C_{\tau}^{(\cdot)}$.

To perform Mellin inversion, the three effective ways can be implemented. Their details are explained as follows.

- Compute the inversion integral

  This is the direct approach which is not always the simplest. However, the integral (3.34) can be computed by the Cauchy's Residue Theorem stated as

  Theorem 3.3 (Cauchy's Residue Theorem) If Γ is a simple close positively oriented contour and f is holomorphic inside and on Γ except at the point $\psi_1, \psi_2, \ldots, \psi_n$ inside Γ, then

$$\int_{\Gamma} f(\psi)\,d\psi = 2\pi j \sum_{t=1}^{n} \operatorname{Res}(\psi_t) \tag{3.37}$$

  where the residue at an nth-order pole $s = a$ is given by

$$R_a = \frac{1}{(n-1)!}\frac{\partial^{n-1}}{\partial s^{n-1}}\left[(s-a)^n \mathbb{F}(s)x^{-s}\right] \tag{3.38}$$

- Use the tables

  In simple cases, using tables of the Mellin transforms such as those of Oberhettinger (1974) and the following properties are sufficient to obtain the result.

  - **Pertinent properties of the transformation**

    Let $\mathbb{F}_X(s) = \mathfrak{M}[f;s]$ be the Mellin transform of a distribution that is supposed to belong to $\mathrm{T}(\theta_1, \theta_2)$ and denoted by $S_f = \{s : \theta_1 < \operatorname{Re}(s) < \theta_2\}$

its strip of holomorphy (θ_1 is either finite or $-\infty$, θ_2 is finite or ∞). Then the following formulae hold with the regions of holomorphy as indicated.

- *Multiplication of the original function by some power of t*

$$\mathfrak{M}\left[(x)^z f(x);s\right] = \mathbb{F}_x(s+z), \quad s+z \in S_f, \qquad z \text{ complex}$$

- *Derivation of the original function*

$$\mathfrak{M}\left[\frac{d^k}{dx^k} f(x);s\right] = (-1)^k \frac{\Gamma(s)}{\Gamma(s-k)} \mathbb{F}_x(s-k), s-k \in S_f, k \textit{ positive integer}$$

- Use the Marichev (1982) approach

This approach is suitable for a problem with a large number of functions. Suppose we are given a function $\mathbb{F}(s)$, holomorphic in the strip $S(\theta_1, \theta_2)$, and we want to find its inverse Mellin transform. The first step is to try and cast $\mathbb{F}$ into the form of a fraction involving only products of a Γ-function. Thus $\mathbb{F}(s)$ is brought to the form

$$\mathbb{F}(s) = C \prod_{i,j,k,l} \frac{\Gamma(a_i + s)\Gamma(b_j - s)}{\Gamma(c_k + s)\Gamma(d_l - s)}$$

where, C, a_i, b_j, c_k, d_l are constants and where $\mathrm{Re}(s)$ is restricted to the strip $S(\theta_1, \theta_2)$ now defined in terms of these.

For such functions, the explicit computation of the inversion integral (3.34) can be performed by the theory of the residues and yield the precise formula given in Marichev (1982) as Slater's theorem. The result has the form of a function of hypergeometric type. The important point is that most special functions are included in this class. For a thorough description of the method, the reader is referred to Marichev's book, which contains simple explanations along with all the proofs and exhaustive tables.

To illustrate, first consider the volume flexibility, and assume the system flexibility to be dependent on three basic flexibilities; machine, material, and operation. Let the function of the need for volume flexibility be represented as follows.

$$D_{VF} = f\left(D_{MF}, D_{MH}, D_O\right) = p_{VF} D_{MF}^{p_{MF}} D_{MH}^{p_{MH}} D_O^{p_O} \tag{3.39}$$

$$D_\tau = \frac{1}{\left(b_\tau - a_\tau\right)}, \ \tau = MF, MH, O \tag{3.40}$$

Following the aforementioned transformation steps, starting by using Theorem 3.2, we obtain

$$\mathbb{F}_{VF}\left(s\right) = p_{VF}^{s-1} \mathbb{F}_{MF}\left(p_{MF}s - p_{MF} + 1\right) \mathbb{F}_{MH}\left(p_{MH}s - p_{MH} + 1\right) \mathbb{F}_O\left(p_O s - p_O + 1\right) \tag{3.41}$$

where, for the uniform distribution $\mathbb{F}\left(s\right) = \int_{a_\tau}^{b_\tau} x^{s-1} f\left(x\right) dx = \dfrac{b_\tau^s - a_\tau^s}{s(b_\tau - a_\tau)}$ (3.42)

Let p_{MF}, p_{MH}, p_O be equal to 1, and $p_1 \le \dfrac{1}{b_{MF} b_{MH} b_O}\%$, we obtain the following Equation

$$\mathbb{F}_{VF}\left(s\right) = \frac{K_1}{s^3} \begin{bmatrix} \left(p_{VF} b_{MF} b_{MH} b_O\right)^s - \left(p_{VF} b_{MF} b_{MH} a_O\right)^s - \left(p_{VF} b_{MF} a_{MH} b_O\right)^s + \\ \left(p_{VF} b_{MF} a_{MH} a_O\right)^s - \left(p_{VF} a_{MF} b_{MH} b_O\right)^s + \left(p_{VF} a_{MF} b_{MH} a_O\right)^s + \\ \left(p_{VF} a_{MF} a_{MH} b_O\right)^s - \left(p_{VF} a_{MF} a_{MH} a_O\right)^s \end{bmatrix} \tag{3.43}$$

where, $K_1 = \dfrac{1}{p_{VF}\left(b_{MF} - a_{MF}\right)\left(b_{MH} - a_{MH}\right)\left(b_O - a_O\right)}$.

Hence, the Mellin inversion is given by

$$f(x)=\frac{1}{2\pi i}\lim_{\varsigma\to\infty}\int_{c-i\varsigma}^{c+i\varsigma} x^{-s}\frac{K_1}{s^3}\begin{bmatrix}(p_{VF}b_{MF}b_{MH}b_O)^s-(p_{VF}b_{MF}b_{MH}a_O)^s-(p_{VF}b_{MF}a_{MH}b_O)^s+\\(p_{VF}b_{MF}a_{MH}a_O)^s-(p_{VF}a_{MF}b_{MH}b_O)^s+(p_{VF}a_{MF}b_{MH}a_O)^s+\\(p_{VF}a_{MF}a_{MH}b_O)^s-(p_{VF}a_{MF}a_{MH}a_O)^s\end{bmatrix}ds \tag{3.44}$$

Deploying the function developed by Oberhettinger (1974) given below:

$$(s)^{-v}a^s = \begin{cases}0 & x>a\\ \left[\left[\Gamma(v)\right]^{-1}\left[\ln(a/x)\right]^{v-1}\right] & x<a\end{cases} \tag{3.45}$$

the PDF of the need for volume flexibility can be calculated for each boundary as shown

$$f(D_{VF})=\begin{cases}\left[\left(\ln(\frac{b_O}{a_O})\right)\left(\ln(\frac{p_{VF}^2b_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})\right)+\ln\left(\frac{a_O}{b_O}\right)(\frac{p_{VF}^2a_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})+\ln\left(\frac{a_O}{b_O}\right)(\frac{p_{VF}^2b_{MF}^2a_{MH}^2a_O\,b_O}{D_{VF}^2})-\left(\ln(\frac{p_{VF}a_{MF}a_{MH}b_O}{D_{VF}})\right)^2\right] & ,p_{VF}a_{MF}a_{MH}a_O\le D_{VF}\le p_{VF}a_{MF}a_{MH}b_O\\
\left[\left(\ln(\frac{b_O}{a_O})\right)\left(\ln(\frac{p_{VF}^2b_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})\right)+\ln\left(\frac{a_O}{b_O}\right)(\frac{p_{VF}^2a_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})+\ln\left(\frac{a_O}{b_O}\right)(\frac{p_{VF}^2b_{MF}^2a_{MH}^2a_O\,b_O}{D_{VF}^2})\right] & ,p_{VF}a_{MF}a_{MH}b_O\le D_{VF}\le p_{VF}a_{MF}b_{MH}a_O\\
\left[\left(\ln(\frac{b_O}{a_O})\right)\left(\ln(\frac{p_{VF}^2b_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})\right)-\left(\ln(\frac{p_{VF}a_{MF}b_{MH}b_O}{D_{VF}})\right)^2+\ln\left(\frac{a_O}{b_O}\right)(\frac{p_{VF}^2b_{MF}^2a_{MH}^2a_O\,b_O}{D_{VF}^2})\right] & ,p_{VF}a_{MF}b_{MH}a_O\le D_{VF}\le p_{VF}b_{MF}a_{MH}a_O\\
\left[\left(\ln(\frac{b_O}{a_O})\right)\left(\ln(\frac{p_{VF}^2b_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})\right)-\left(\ln(\frac{p_{VF}b_{MF}a_{MH}b_O}{D_{VF}})\right)^2-\left(\ln(\frac{p_{VF}a_{MF}b_{MH}b_O}{D_{VF}})\right)^2\right] & ,p_{VF}b_{MF}a_{MH}a_O\le D_{VF}\le p_{VF}a_{MF}b_{MH}b_O\\
\left[\left(\ln(\frac{b_O}{a_O})\right)\left(\ln(\frac{p_{VF}^2b_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})\right)-\left(\ln(\frac{p_{VF}b_{MF}a_{MH}b_O}{D_{VF}})\right)^2\right] & ,p_{VF}a_{MF}b_{MH}b_O\le D_{VF}\le p_{VF}b_{MF}a_{MH}b_O\\
\left[\left(\ln(\frac{b_O}{a_O})\right)\left(\ln(\frac{p_{VF}^2b_{MF}^2b_{MH}^2a_O\,b_O}{D_{VF}^2})\right)\right] & ,p_{VF}b_{MF}a_{MH}b_O\le D_{VF}\le p_{VF}b_{MF}b_{MH}a_O\\
\left[\left(\ln\left(\frac{p_{VF}b_{MF}b_{MH}b_O}{D_{VF}}\right)\right)^2\right] & ,p_{VF}b_{MF}b_{MH}a_O\le D_{VF}\le p_{VF}b_{MF}b_{MH}b_O\end{cases}$$

For an example of the need for volume flexibility, consider the condition

where p_{MF}, p_{MH}, $p_O=1$, $p_{VF}\le\dfrac{1}{b_{MF}b_{MH}b_O}\%=\dfrac{1}{200}$, $f(D_{MF})=\dfrac{1}{10}, 5\le D_{MF}\le 15$,

$f(D_{MH})=\dfrac{1}{12}, 12\le D_{MH}\le 24$, and $f(D_O)=\dfrac{1}{10}, 10\le D_O\le 20$. The density function of the need for volume flexibility ($f\left(D_{VF}\right)$) is as follows.

$$f\left(D_{VF}\right)=\begin{cases}\frac{K_1}{2}\left[\left(\ln\left(\frac{36}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{12}{D_{VF}}\right)\right)^2+\left(\ln\left(\frac{9}{D_{VF}}\right)\right)^2+\left(\ln\left(\frac{6}{D_{VF}}\right)\right)^2+\left(\ln\left(\frac{6}{D_{VF}}\right)\right)^2\right]\\ \frac{K_1}{2}\left[\left(\ln\left(\frac{36}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{12}{D_{VF}}\right)\right)^2+\left(\ln\left(\frac{9}{D_{VF}}\right)\right)^2\right]\\ \frac{K_1}{2}\left[\left(\ln\left(\frac{36}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{12}{D_{VF}}\right)\right)^2\right]\\ \frac{K_1}{2}\left[\left(\ln\left(\frac{36}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2-\left(\ln\left(\frac{18}{D_{VF}}\right)\right)^2\right]\\ \frac{K_1}{2}\left[\left(\ln\left(\frac{36}{D_{VF}}\right)\right)^2\right]\end{cases}$$

The *PDF* and *CDF* of the need for volume flexibility are plotted shown in Figure 3.12 and 3.13, respectively.

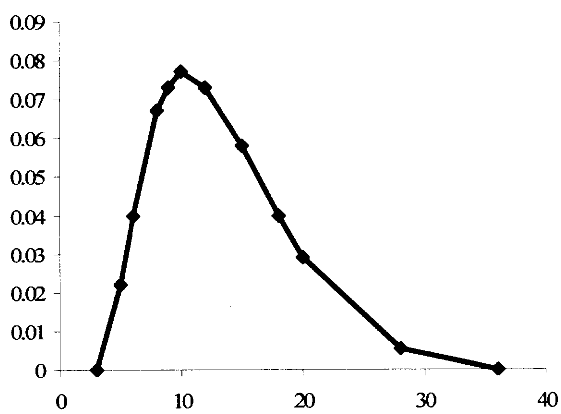

Figure 3.12 *PDF* of The Need for Volume Flexibility $F\left(D_{VF}\right)$

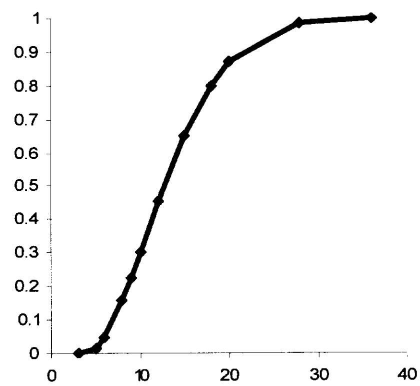

Figure 3.13 *CDF* of The Need for Volume Flexibility $F\left(D_{VF}\right)$

In a similar fashion, the density function of the need for product and process flexibility can be obtained by changing the normalized multipliers from p_{VF} to p_{PR} and p_{PC} respectively. The next step is to transform the probability distribution of the production flexibility. In this research, production flexibility has the belonging relationship function.

$$f\left(D_{\text{Pro}}\right)=p_{\text{Pro}}\left(D_{VF}D_{PR}D_{PC}\right)=p_{\text{Pro}}p_{VF}p_{PR}p_{PC}\left(D_{MF}^3D_{MH}^3D_O^3\right) \tag{3.46}$$

The above equation is similar to that of a Cobb-Douglas production function. Young (1995) suggested that the lognormal can approximate the probability distribution of the Cobb-Douglas production function. In addition, empirical studies by Garvey and Taub (1997) identify circumstances where the lognormal can approximate the combined (joint) distribution function of a program's total cost and schedule with a function similar to the Cobb-Douglas function. In this research, the probability distribution of the need for aggregated flexibility is approximated by a lognormal distribution. The parameters for the distribution of the need for production flexibility are calculated using the following formulae.

$$\mathbb{F}_{\text{Pro}} = \left(p_{\text{Pro}} p_{VF} p_{PR} p_{PC}\right)^{s-1} \prod_{\tau=MF}^{O} \mathbb{F}_{\tau}\left(3s-2\right)$$

$$\mu_{D_{\text{Pro}}} = \left(p_{\text{Pro}} p_{VF} p_{PR} p_{PC}\right) \prod_{\tau=MF}^{O} \mathbb{F}_{\tau}\left(4\right)$$

$$\sigma_{D_{\text{Pro}}} = \left(p_{\text{Pro}} p_{VF} p_{PR} p_{PC}\right)^{2} \left(\prod_{\tau=MF}^{O} \mathbb{F}_{\tau}\left(7\right) - \left(\prod_{\tau=MF}^{O} \mathbb{F}_{\tau}\left(4\right) \right)^{2} \right) \tag{3.47}$$

$$f\left(D_{\text{Pro}}\right) = \frac{1}{D_{\text{Pro}} \sqrt{2\pi} \sigma_{\text{Pro}}} e^{-\frac{1}{2}\left[\frac{\ln D_{\text{Pro}} - \mu_{\text{Pro}}}{\sigma_{D_{\text{Pro}}}}\right]^{2}}$$

$$F\left(D_{\text{Pro}}\right) = \int_{p_{\text{Pro}} p_{VF} p_{PR} p_{PC} a_{MF} a_{MH} a_{O}}^{D_{\text{Pro}}} \frac{1}{t\sqrt{2\pi}\sigma_{\text{Pro}}} e^{-\frac{1}{2}\left[\frac{\ln t - \mu_{\text{Pro}}}{\sigma_{D_{\text{Pro}}}}\right]^{2}} dt$$

The average total cost of the objective function can be obtained by including all flexibilities shown explicitly as follows.

$$E[TC] = \begin{cases} c_{MF}\left[\left(\beta_{MF}-1\right)x_{MF}-\left(\alpha_{MF}+\beta_{MF}\right)\int\limits_{a_{MF}}^{x_{MF}}\left(\frac{D_{MF}-a_{MF}}{b_{MF}-a_{MF}}\right)dD_{MF}-\beta_{MF}\left(\frac{b_{MF}+a_{MF}}{2}\right)\right]+ \\ c_{MH}\left[\left(\beta_{MH}-1\right)x_{MH}-\left(\alpha_{MH}+\beta_{MH}\right)\int\limits_{a_{MH}}^{x_{MH}}\left(\frac{D_{MH}-a_{MH}}{b_{MH}-a_{MH}}\right)dD_{MH}-\beta_{MH}\left(\frac{b_{MH}+a_{MH}}{2}\right)\right]+ \\ c_{O}\left[\left(\beta_{O}-1\right)x_{O}-\left(\alpha_{O}+\beta_{O}\right)\int\limits_{a_{O}}^{x_{O}}\left(\frac{D_{O}-a_{O}}{b_{O}-a_{O}}\right)dD_{O}-\beta_{O}\left(\frac{b_{O}+a_{O}}{2}\right)\right]+ \\ c_{VF}\left[\left(\beta_{VF}-1\right)x_{VF}-\left(\alpha_{VF}+\beta_{VF}\right)\int\limits_{p_{VF}a_{MF}a_{MH}a_{O}}^{x_{VF}}F_{VF}dD_{VF}-\beta_{VF}\mu_{VF}\right]+ \\ c_{PR}\left[\left(\beta_{PR}-1\right)x_{PR}-\left(\alpha_{PR}+\beta_{PR}\right)\int\limits_{p_{PR}a_{MF}a_{MH}a_{O}}^{x_{PR}}F_{PR}dD_{PR}-\beta_{PR}\mu_{PR}\right]+ \\ c_{PC}\left[\left(\beta_{PC}-1\right)x_{PC}-\left(\alpha_{PC}+\beta_{PC}\right)\int\limits_{p_{PC}a_{MF}a_{MH}a_{O}}^{x_{PC}}F_{PC}dD_{PC}-\beta_{PC}\mu_{PC}\right]+ \\ c_{\mathrm{Pro}}\left[\left(\beta_{\mathrm{Pro}}-1\right)x_{\mathrm{Pro}}-\left(\alpha_{\mathrm{Pro}}+\beta_{\mathrm{Pro}}\right)\int\limits_{p_{\mathrm{Pro}}p_{VF}p_{PR}p_{PC}a_{MF}a_{MH}a_{O}}^{x_{\mathrm{Pro}}}F_{\mathrm{Pro}}dD_{\mathrm{Pro}}-\beta_{\mathrm{Pro}}\mu_{\mathrm{Pro}}\right] \end{cases}$$

Finally, in a similar fashion, the developed triangular approach can be implemented to obtain the solutions.

CHAPTER 4

OPTIONS MODEL

This chapter presents the second model that is based on real options. This model takes into consideration what is argued to be the two most important flexibilities namely; product-mix and volume flexibility. The difference between the first model and the second is that while the former is concerned with the needed levels of flexibility on the basis of the sources of the underlying uncertainty, the latter considers the underlying uncertainty specific to the need for any given flexibility class. For instance, the key underlying uncertainty that directly relates to volume and product-mix flexibility is the demand for the products. While directly linking the underlying uncertainty to the governing flexibility might reduce the subjectivity of the decision making process, it would significantly increase the complexity of the modeling efforts. This is because the model must be formulated separately for any given pair of uncertainties and the governing flexibility. It is for this reason that the scope of this research is limited to considering only the volume and product-mix flexibility, and particularly since both are governed by the same underlying uncertainty - the demand for products. This chapter starts by exploring the limitations of the traditional discounted cash flow approach in aiding investment decisions, then the real options theory is discussed briefly and on the basis of these, the second model and its solution methodology are presented.

4.1 Limitations of Traditional Discounted Cash Flow (DCF) Approach

The traditional approach to valuing a potential capital investment project is known as the "net present value", or NPV, approach. The NPV of a project is the present value of its expected future incremental cash inflows and outflows. The discount rate used to calculate the present value is a "risk-adjusted" discount rate, chosen to reflect the risk level of the project. As the level of risk of the project increases, the discount rate also increases.

The example by Lenos and Scott (2001) will be referred to throughout this section as the limitations of DCF are shown. Let consider an opportunity to invest $104 to build a plant that a year later, will have a realizable value of either $180 or $60 with equal probability. For simplicity, assume that, once constructed, the plant will operate indefinitely and continuously, at a constant output rate and require no future follow-on investment. Following traditional practice, let S be the listed stock price of an identical plant. Recall that the exercise of such a "twin security" is implicitly assumed in traditional NPV analysis for the purpose of estimating the required rate of return on a project. The twin security is assumed to have a value of $36 if the realized value of the project is $180 and a value of $12 if the realized value of the project turns out to be $60. Finally, assume both the plant and its "twin security" have an expected rate of return (or discount rate) of 20 percent, while the risk-free rate is assumed to be 8 percent.

4.1.1 The DCF Approach

Traditional discounted cash flow (DCF) techniques, including net present value (NPV) analysis, would discount the plant's expected cash flows using the expected rate of return of the plant's twin security as the appropriate discount rate. The discount rate would be estimated by determining the project's beta (risk) coefficient from the prices of its "twin security" and applying the Capital Asset Pricing Model (CAPM). The gross value of project V would then be given by the expression:

$$V = \frac{E(V_1)}{(1+k)} = \frac{\left[pV^+ + (1-p)V^-\right]}{(1+k)} = \frac{[0.5\times 180 + 0.5\times 60]}{(1+.20)} = 100$$

Subtracting the present value of investment costs gives the project's NPV:

$$NPV = V - I = 100 - 104 = -4$$

Thus the value of this investment opportunity is a negative $4. In the absence of managerial flexibility, traditional DCF would expectedly reject this project. As will be explained shortly, if flexibility or various kinds of options are present, investment in the plant may actually become economically desirable despite its negative static NPV.

4.1.2 Real options approach

The uniqueness of real options technique lies in its ability to afford management avenues of correctly quantifying the additional value of a project's operating flexibility. In the absence of such flexibility it gives results identical to those of the traditional DCF. Its economic foundation rests with the explicit recognition of market opportunities to trade and create desired payoff patterns through securities transactions. Let's consider a simple example of managerial flexibility - flexibility to defer investment. The flexibility to defer project for a year gives management the right, but not the obligation, to make the investment by the following year as they could wait and make the investment if the project value in that year turns out to exceed the necessary investment at that time. Thus, with the flexibility to defer the investment, the payoff structure would be as follows.

$$E = \frac{\left[p'E^{+} + (1-p)E^{-}\right]}{(1+r)} = \frac{[.4 \times 67.68 + .6 \times 0]}{1.08} = 25.07$$

The above result reveals that though the project has a negative NPV of $4 if taken immediately, the investment proposal should not be rejected outright since opportunity to invest in the project after a year is worth a positive $25.07. The value of the flexibility to defer is equal to almost one third of the project's gross value.

4.2 The Model

This section outlineS a stochastic model that derives the value of product mix and volume flexibility stemming from the ability of a system to respond to products demand uncertainty. Associated with each product is a profit stream that is determined by the realization of uncertain variables (which in this case happens to be the products demand). Given the expected demand of a product ω denoted by D_{ω}, its dynamics are modeled by a mean reverting stochastic process:

$$d \ln D_{\omega,t} = \left[\theta_{\omega}(t) - a_{\omega} \ln D_{\omega,t}\right] dt + \sigma_{\omega} dz \qquad (4.2)$$

where, a_{ω} and σ_{ω} represents constant drift and diffusion rate, and $\theta_{\omega}(t)$ is a function of time chosen to ensure that the model fits the initial term structure. For $\theta_{\omega}(t)$, the trinomial tree methodology explained in Section 4.3.2 can be deployed to construct D_{ω} and determine the value of $\theta_{\omega}(t)$ as in real options pricing, and without loss of generality, the world can be assumed to be risk neutral (Cox J.C., Ross S.A., 1976). The demand dynamics model (Equation (4.2)) can be depicted as in Figure 4.1.

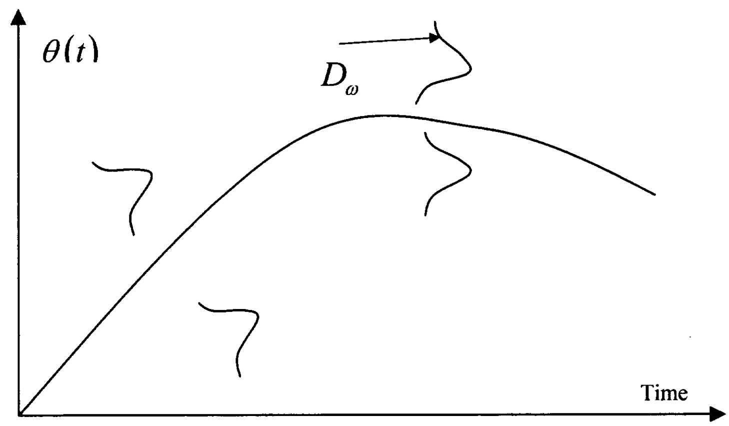

Figure 4.1 The Demand Dynamics

As one can see from Figure 4.1, this process aids in including the product life cycle concept since on average, D_ω follows the slope of the term structure denoted here as $\theta(t)$. When it deviates from that curve, it reverts back to it at rate a_ω. Hence one can represent the demand of product ω in different states such as the growth state when the slope of $\theta(t)$ has a positive value or the decline state when the slope of $\theta(t)$ is negative in value. Thus for this purpose $\theta(t)$ is modeled using the well known equation of the Product Life Cycle (PLC) curve given by:

$$\theta(t) = k_p e^{\frac{-(t-\mu_p)^2}{\sigma_p}}$$

where, k_p represents the demand peak. To proceed, a preliminary assumption has to be made. Recall that the project has some finite life of T years. A system which inherits product mix flexibility can switch to producing between products ω_i and ω_j while incurring a set up cost δ_{ij}. This set up cost originates from several sources such as retooling, retraining, inventory changes, lost time, and compensatory wages. We assume that the set up cost here is a nonlinear function of the product-mix flexibility index where this index has a possible range of values from zero (with highest set up cost) to one hundred (with the least set up cost) as shown in equation (4.5) below.

$$\delta_{ij} = \lambda_{ij}\left(\kappa_{ij} - PF\right)^2 \tag{4.5}$$

where, λ_{ij} represents the estimated set up cost in the absence of product-mix flexibility, κ_{ij} represents the set up cost reduction index, PF represents the product-mix flexibility index, and $\delta_{ij} = 0$ if there is no switching.

Thus, the value of product-mix flexibility is known with certainty at time step t and is given by the maximum profit for different products;

$$F(t,j) = \max_i\left[\pi^i(D_t) - \delta_{ij}\right] \tag{4.6}$$

where, $\pi^i(D_t)$ represents cash flow of product i at time t. The product i cash flow, $\pi^i(D_t)$, is represented by the following equation.

$$\pi(D_{\omega,t}) = R_\omega D_\omega - R_\omega MAX\left((D_{\omega,t} - \psi_\omega),0\right) - \xi_\omega C_\omega MAX\left((\psi_\omega - D_{\omega,t}),0\right) - f(\psi_\omega) \tag{4.7}$$

where, R_ω denotes the estimated unit revenue of product ω; ψ_ω is the threshold level which is initially set equal to expected demand over the evaluation period; $D_{\omega,t}$ is the scaled demand of product ω for time period t, and $t = \frac{T}{n}$; T, the evaluation time horizon; n is the number of time steps; ξ_ω is the fraction of variable costs that cannot be recovered by liquidating excess inventory; and $f(\psi_\omega)$ is the given fixed cost to produce at the threshold level. It should be noted that equation (4.7) is based on the assumption that the firm is risk averse and therefore sets its production system to be able to produce a product ω at maximum capacity equal to its long run expected demand. Thus, from this cash flow, one can see that the firm exactly matches scaled demand with the threshold variable, that is, when $D_{\omega,t} = \psi_{\omega}$. In this case the second and the third term of equation (4.7) will be null, while the first term of this equation measures the net revenue. In all other cases, scaled demand will be lower or higher than this threshold value and this leads to the loss of revenue from overestimated and underestimated net cash flow due to the absence of volume flexibility.

As defined by Sethi and Sethi (1990), volume flexibility is the ability of the system to be operated profitability at different output volumes. This means that a volume flexible production system should allow producing a product profitably within a specific range. Thus, as opposed to a production system with no volume flexibility, the threshold level for a volume flexible system should be increased and reduced within an allowance limit. Here, it is assumed that the maximum limit is $\psi_{\omega} \times (1 + \eta_1 \times VF_{\omega})$ and the minimum limit is $\psi_{\omega} \times (1 - \eta_2 \times VF_{\omega})$. Thus the cash flow of product ω with volume flexibility is represented as follows:

$$\pi(D_{\omega,t}) = R_{\omega}^{VF} - R_{\omega}^{VF} MAX\left(\left(D_{\omega,t} - \psi_{\omega} \times (1 + \eta_1 \times VF_{\omega})\right), 0\right) - \xi_{\omega} C_{\omega}^{VF} MAX\left(\left(\psi_{\omega} \times (1 - \eta_2 \times VF_{\omega}) - D_{\omega,t}\right), 0\right) - f(\psi_{\omega}) - f(VF_{\omega}) \quad (4.8)$$

where, η_1 and η_2 are non negative constants, C_{ω}^{VF} represents the reduced unit cost when volume flexibility is introduced, R_{ω}^{VF} represents the increased revenues, and $f(VF_{\omega})$ represent the additional fixed cost to produce at this level of volume flexibility, here assume; to be $MAX(\eta_1,\eta_2)\times VF\times f(\psi_\omega)$. Having introduced the details of the model, the following sub-sections explain the solution algorithms.

4.3 Solution Algorithms

In these sub-sections, the basic solving tool, known as the trinomial method, is introduced and the algorithms for solving the model are explained.

Product-mixed flexibility gives the ability to switch and produce different products at any period of time. For this reason, close form solutions cannot be realistically obtained so that the numerical procedure becomes the preferred option. The trinomial method developed by Hull and White (1994) gives the most suitable solution approach for this problem. The details of this method are explained in the following subsection.

4.3.1 A General Tree Building Procedure

Hull and White (1994) proposed a robust two-stage procedure for constructing trinomial trees to represent a wide range of one-factor models which follow the mean reverting process. The procedure is explained as follows:

- ***First stage***

The procedure for general models having the form given in Equation (4.2) has the property that it can fit any term structure, Ross (1985). It is assumed that the δt period rate, R, follows the same process as D_ω in Equation (4.2):

$$df(R)=\left[\theta(t)-af(R)\right]dt+\sigma dz$$

The procedure starts by setting $x=f(R)$, so that

$$dx = \left[\theta(t) - ax\right]dt + \sigma dz$$

The first stage entails building a tree for a variable x^* that follows the same process as x except that $\theta(t) = 0$ and the initial value of x is zero. The outline of the procedure here is identical to that of the Trinomial tree depicted in Figure 4.2.

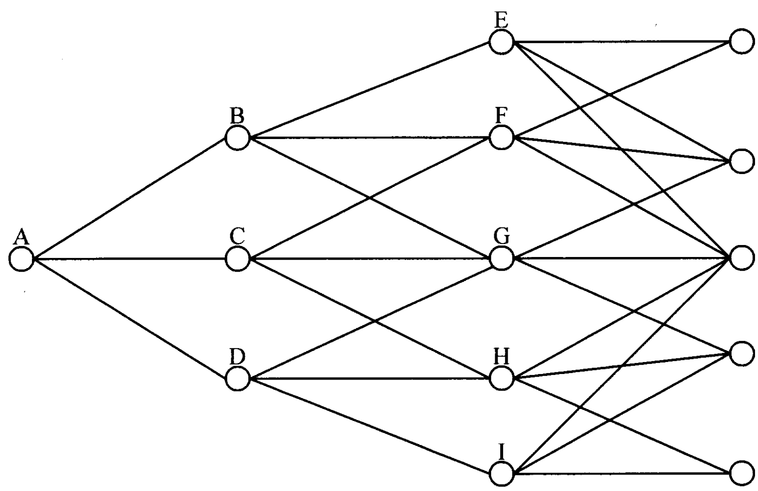

Figure 4.2 The Trinomial Tree

The necessary parameters for building such a tree can be summarized as follows.

$$\delta R = \sigma\sqrt{3\delta t}, \; j_{\max} = \frac{0.184}{(a\delta t)}, \; j_{\min} = -j_{\max}$$

And the associated probabilities for the alternative branching method (shown in Figure 4.3) are given by:

$$\begin{aligned} p_u &= \frac{1}{6} + \frac{a^2 j^2 \delta t^2 - aj\delta t}{2} \\ p_m &= \frac{2}{3} - a^2 j^2 \delta t^2 \qquad \text{(Figure 4.3(a))} \\ p_d &= \frac{1}{6} + \frac{a^2 j^2 \delta t^2 + aj\delta t}{2} \end{aligned}$$

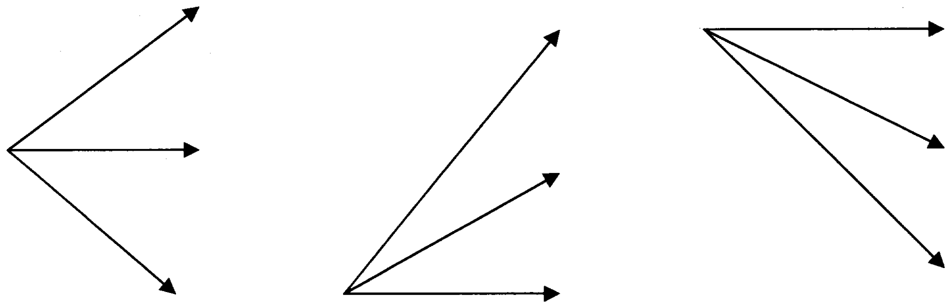

Figure 4.3 Alternative Branching Method

$$p_u = \frac{1}{6} + \frac{a^2 j^2 \delta t^2 + aj\delta t}{2}$$
$$p_m = -\frac{1}{3} - a^2 j^2 \delta t^2 - 2aj\delta t \qquad \text{(Figure 4.3(b))}$$
$$p_d = \frac{7}{6} + \frac{a^2 j^2 \delta t^2 + 3aj\delta t}{2}$$

$$p_u = \frac{7}{6} + \frac{a^2 j^2 \delta t^2 - 3aj\delta t}{2}$$
$$p_m = -\frac{1}{3} - a^2 j^2 \delta t^2 + 2aj\delta t \qquad \text{(Figure 4.3(c))}$$
$$p_d = \frac{1}{6} + \frac{a^2 j^2 \delta t^2 + aj\delta t}{2}$$

- ***Second stage***

Define α_i as $\alpha(i\delta t)$, the displacement values for R. Then, the second stage involves displacing the nodes at time $i\delta t$ by an amount α_i. This is done to provide an exact fit to the product life cycle curve. Suppose that the values of R have been determined for $i \le m(m \ge 0)$. The next step is to determine α_m so that the tree correctly fits a $(m+1)\delta t$ product demand. Define g as the inverse function of $f(R)$ so that the δt-period interest rate at the jth node at time $m\delta t$ is

$$g(\alpha_m + j\delta x)$$

The demand of product ω at time $(m+1)\delta t$ is given by

$$D_{\omega,m+1} = \sum_{j=-n_m}^{n_m} R_{m,j} \exp\left[-g(\alpha_m + j\delta x)\delta t\right]$$

This equation can be solved using a numerical procedure such as Newton-Raphson's. The value of α_0 of α when $m = 0$ is $f(R(0))$.

Once α_m has been determined, the $Q_{i,j}$ for $i = m+1$ can be calculated using

$$Q_{m+1,j} = \sum_k Q_{m,k} q(k,j) \exp\left[-g(\alpha_m + k\delta x)\delta t\right]$$

where, $q(k,j)$ is the probability of moving from node (m,k) to node $(m+1,j)$ and the summation is taken over all values of k where this is nonzero.

4.3.2 Steps for Solving the Model

In summary, the model's basic solving steps can be highlighted as follows:

- First define the time steps (δt).
- Scale the cumulative demand over the evaluation horizon using the equation of $\theta(t)$ as defined above.
- Build the trinomial tree representing the mean reverting demand for each (δt) of all considered products.

- Based on the calculated demand, calculate the cash flow of all products with different degrees of volume flexibility starting from 0 to 1, using equation (4.8). It should be noted that when volume flexibility is 0, equation (4.8) will be reduced to equation (4.7).
- Using these payoff trinomial trees, calculate the value of product mix flexibility.

Having introduced the two sets of models, one utilizing the newsboy method, and the second utilizing real options, applications of these approaches is presented in the next chapter. The applications are derived from data obtained from Hass Machinery, Mitutoyo, AGV USA, ABB robot.

CHAPTER 5

APPLICATIONS

In this chapter, the proposed models are used in a case study and the implementation steps are explained in detail.

Consider the scenario where an agricultural machinery company would like to invest in a manufacturing system for producing transmission parts such as the main frame clutch housings, pump housings, and transfer cases for tractors. Because of the high level of demand uncertainty, management would like to concentrate on optimizing volume, process and product flexibility. The basic preliminary system design which is suitable for handling the estimated required capacity is made to include eight machining centers, a coordinate measuring machine (CMM), a gantry robot and two track type shuttle carts. The rough layout of this manufacturing system is shown in the following schematic diagram.

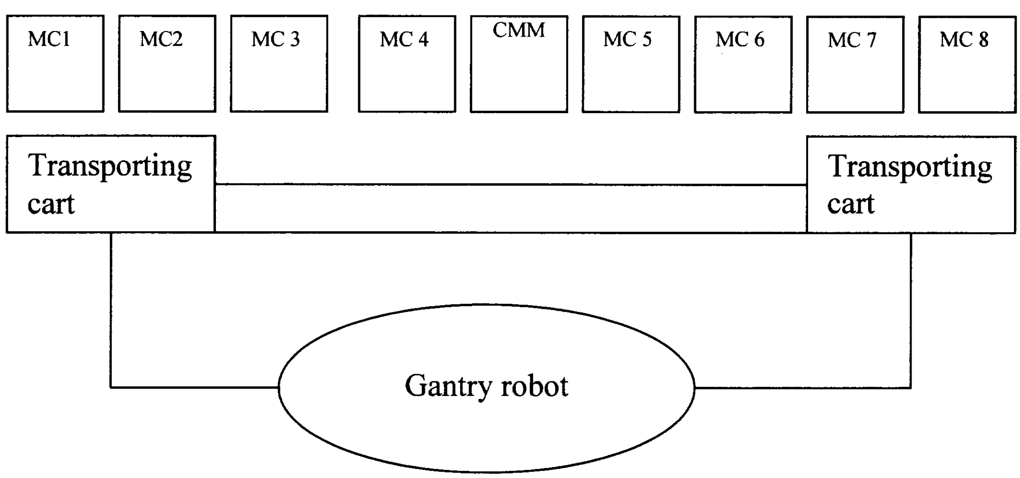

Figure 5.1 The Preliminary System Design

Due to the fact that the future requirements are uncertain, management defines system requirement as the minimum and maximum possible value relative to the best and worst value that available equipments in the market can provide. These requirements are shown in Table 5.1. One can see that these requirements stem from two important factors which arise from market uncertainty (such as range of lead time, number of product introduced) and the uncertainty of the required equipment attributes.

Table 5.1 Management Requirements

Required attributes	min	max
Number of required tools	63	90
Parts envelop (in^3)	1647.64	64000
Parts weight (lb)	70	1100
Number of axes	3	5
Range of variance of quantities ordered per year	4350	12000
Range of lead times (days)	20	45
Number of process plans	32	60
Number of path	25	53
Range of products	20	45
Batch sizes	2	40
Number of product introduced	12	20
Number of model change	20	30
Tool change time (sec.)	2	10
Tool capacity	75	90
CMM measuring speed (ft. /min.)	1	4

From these uncertain requirements combined together, the joint probability distributions representing the need for different flexibilities can be obtained. Their network of relationships is shown in Figure 5.2.

Since the need distributions of these flexibilities consist of several uncertain required attributes, a multi criteria decision model is required to combine and normalize them. Topsis *(Technique for Order Preference by Similarity to Ideal Solution)*, introduced by Hwang and Yoon (1981), is implemented to perform the transformation process. The details of this methodology are explained in Appendix B. In addition, one can implement the steps already mentioned in Section 3.4 to obtain the joint distributions. However, it is an exceptionally difficult task to integrate for the Mellin inversion transform which requires knowledge of advanced complex analysis. Therefore, an easier alternative, the Monte carlo numerical method is used in this example, which is a more effective way for general users.

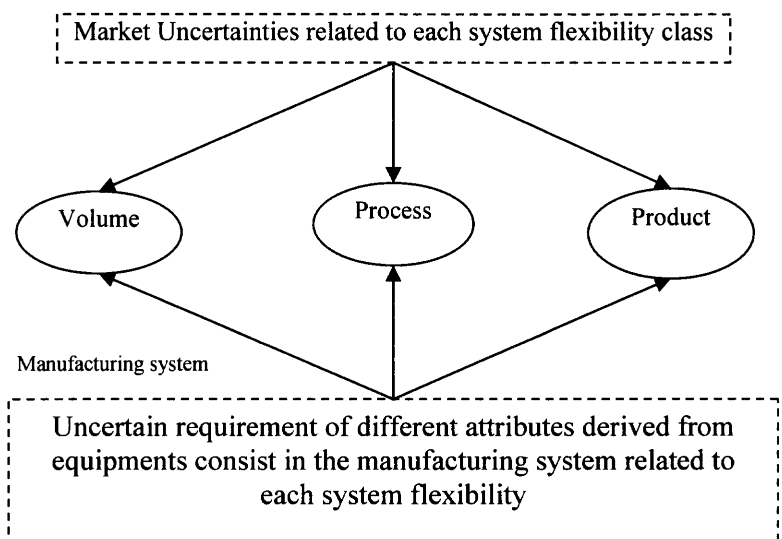

Figure 5.2 The Relationship Network of the Need for Flexibility

The distributions needed for different flexibilities are obtained by varying the iteration numbers from 100 to 10000 and are shown in Figures 5.3, 5.4 and 5.5 showing the volume, process, and product flexibility respectively.

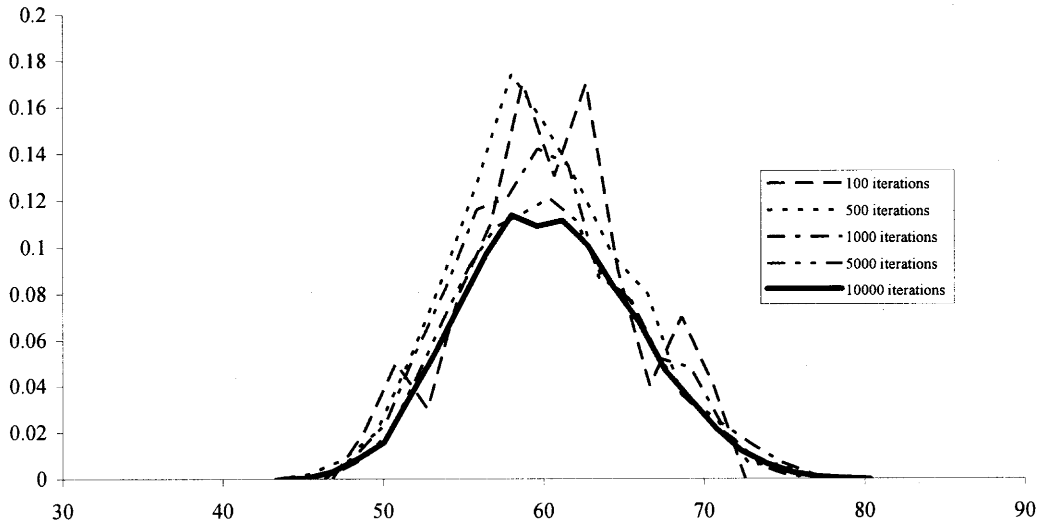

Figure 5.3 The Distribution of the Need for Volume Flexibility

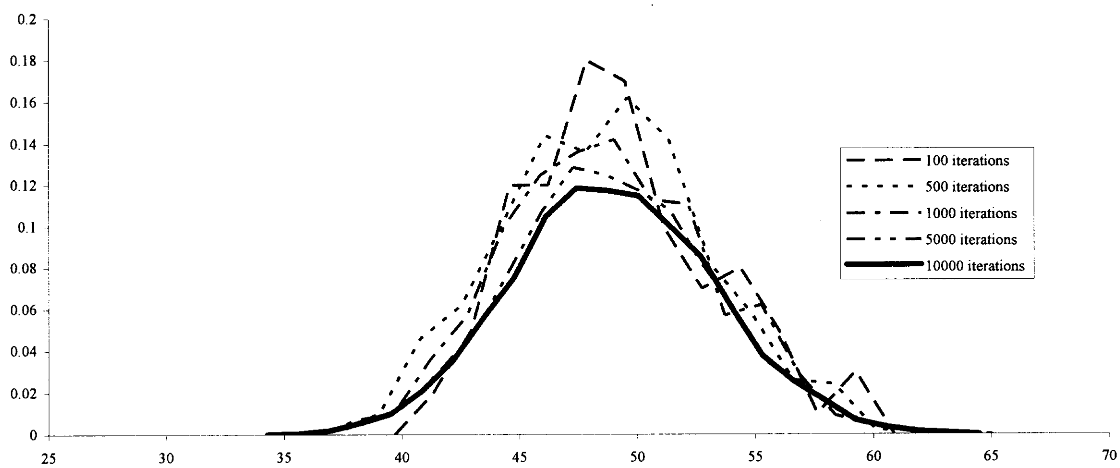

Figure 5.4 The Distribution of the Need for Process flexibility

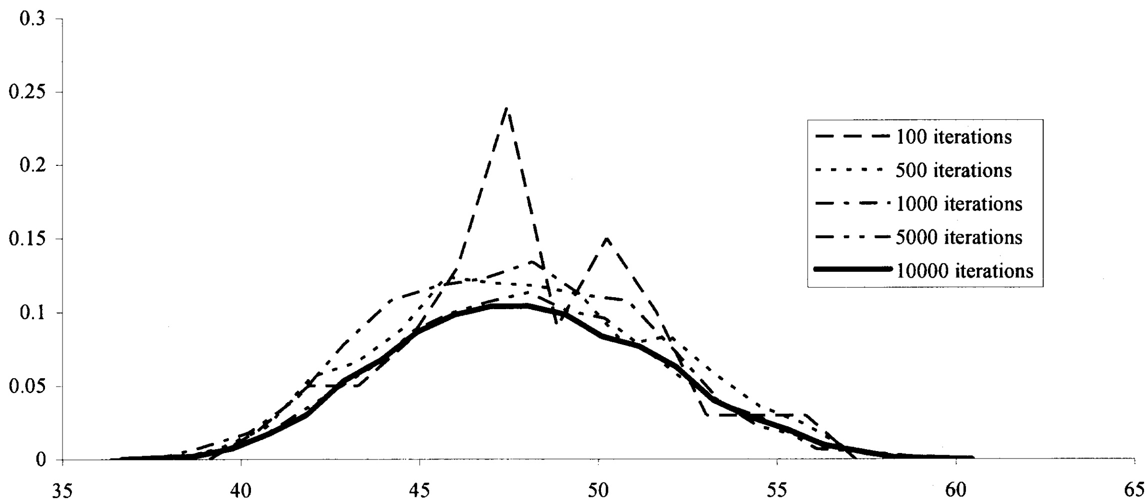

Figure 5.5 The Distribution of the Need of Product flexibility

The manufacturer is assumed to have existing suppliers that provide equipment. These suppliers are Hass, ABB robot, AGV USA, and Mitutoyo. The list of these equipment is shown in following Table 5.2.

It becomes obvious from Table 5.2 that though there is only a small set of equipment, their combinations can result into more than 10,000 system alternatives. For each alternative, the relative flexibility index is obtained, and on the basis of this the system alternatives are ranked in ascending order of flexibility level. The unit costs of volume, product, and process flexibility are approximated using least square linear regression analysis and shown in Appendix C.

Having obtained all the needed parameters, the first model can now be used to find the optimal level of system flexibility for each class. These optimal levels can be used to select the most suitable system alternative with the best attributes mix. In other words, the optimal basic flexibility can be obtained from the combination of these attributes. The implementation steps are detailed below.

Table 5.2 Lists of the Equipment Used in the First Model

	Machining Center	**VF-1**	**VF-5XT**	**VF-6**	**VF-8**	**VF-11**	**VS-1**	**VS-3**	**HS-3**	**HS-4R**
1	# of machining centers	8	8	8	8	8	8	8	8	8
2	memory	32	64	64	64	64	16000	16000	16000	16000
3	payload	3000	4000	4000	4000	4000	10000	10000	10000	10000
4	envelop	6400	39000	61440	76800	144000	210000	375000	450000	594000
5	tools capacity	20	31	31	31	31	31	31	40	75
6	tools change time	4.2	4.2	4.2	4.2	4.2	4.2	4.2	2.8	2.8
7	number of axes	3	3	3	3	3	5	5	5	5
8	HP	20	30	30	30	30	30	30	40	75
9	accuracy	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002
10	repeatability	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
	price	41900	106900	124900	149900	199900	261395	301395	371395	461395
	Robot (ABB)	**FP-3**	**FP-4**	**FP-5**	**FP-6**	**FP-6HD**	**FP-7HD**			
1	velocity (ft/min) (x)	492.125	492.125	492.125	574.1469825	656.16798	656.16798			
3	working range	21968547.88	39055196.23	48818995.28	85433241.75	85433241.75	85,433,241.75			
4	repeatability	0.0079	0.0079	0.0079	0.005748	0.005748	0.005748			
5	number of axes	2	4	4	4	4	6			
6	payload	132.27738	330.69345	661.3869	1322.7738	2204.623	5511.5575			
7	price	46780	54678	68866	87840	94320	250000			
	AGVs	**LVC 150**	**LVA 2200**	**FLB2600**	**LVQ3000**	**FLA4000**				
1	# of transporting carts	2	2	2	2	2				
3	max load area	1260	2016	651.9	3014	3180				
4	transfer height	18	30	35	45	140				
5	velocity (ft/min)	200	240	240	340	400				
6	payload	800	2200	2600	3000	4000				
	price	35000	44000	45000	50000	52450				
	CMM	**C544**	**C574**	**C776**	**C7106**	**C9108**	**C9168**	**C9208**	**C121210**	
1	measuring envelop	5051.181968	8793.6198	30463.6725	43428.5724	44667.77753	71348.75789	89121.02	89054.41	
3	accuracy	0.000004	0.000004	0.000004	0.000004	0.000004	0.000004	0.000004	0.000004	
4	payload	220	396	1760	2200	2640	3300	3960	4400	
5	measuring speed (ft/min)	1.5748032	1.5748032	1.5748032	1.5748032	0.5905512	1.968504	2.952756	2.952756	
6	price	40000	45400	63999	75264	84999	102053	174333	225000	

- Given the under-design, over-design cost multipliers as shown in Table 5.3 as well as the available budget, the first model is formulated as follows.

Table 5.3 Input Data for the First Model

Flexibility	Volume	Process	Product
Unit cost ($/ % flexibility index)	88002.13	46624.96	12629.62
α_τ	2	3	2
β_τ	3	2	3

$$E[TC]=\begin{cases} c_V\left[(\beta_V-1)x_V-(\alpha_V+\beta_V)\int_{a_V}^{x_V}F_V dD_V-\beta_V\mu_V\right]+c_{PR}\left[(\beta_{PR}-1)x_{PR}-(\alpha_{PR}+\beta_{PR})\int_{a_{PR}}^{x_{PR}}F_{PR}dD_{PR}-\beta_{PR}\mu_{PR}\right] \\ +c_P\left[(\beta_P-1)x_P-(\alpha_P+\beta_P)\int_{a_P}^{x_P}F_P dD_P-\beta_P\mu_P\right] \end{cases}$$

Subject to

$$\sum_{\tau=M}^{P} c_\tau x_\tau \le Bg$$

- Using the Triangular approach, the objective function of average total cost is transformed into the quadratic form and the objective function's coefficients are calculated.

  Since all flexibility need distributions can be closely fitted by the Beta distribution, the methodology explained in section 3.3.1.3.1 can be implemented. The objective coefficients are obtained and shown in Table 5.4.

Table 5.4 Table Shows the Objective Coefficients

Flexibility	A	B	C
Volume	-15170.85	1712058.04	-54203766.22
Process	-9171.35	905070.51	-25057809.36
Product	-3153.12	292158.25	-7444948.56

- Transform the quadratic programming problem into a linear programming problem by using KKT's conditions and compute the optimal solutions. The optimal solution is shown in Table 5.5.

Table 5.5 The Optimal Solutions for the First Model

Flexibility	The Optimal Level (%)
Volume	56.42590392
Process	49.34225958
Product	46.32849135

In the preceding example, the first model measured to determine the optimal flexibility levels based on the uncertain nature of the needs for different flexibility classes. The next example shows the application of the second model in establishing the optimal flexibility level based on the uncertainty associated with product demand. While the first example considered could be termed a semi-structured problem due to the fact that the subjective judgment of the decision maker comes to a large extent in play, the second example, as will be seen, is more structured in nature since it relates to the more objective product demand. The details of the implementation steps for the second model are explained below.

For the application of the second model, a scenario is considered where the manufacturer produces three products, namely: (1) pump housing, (2) transfer cases, and (3) main frame clutch housings. These parts are of tractor components. The parts' data are shown in Table 5.6.

Table 5.6 Considered Parts' Data

Part #	Unit revenue ($)	Unit cost ($)	$f(\psi_{\omega})$ ($)
1	190	100	124,500
2	162.5	150	130,000
3	210	135	120,000

Let the project life be 3 years and divided into 30 time steps, with a 10% risk free rate. The term structures of these products have a bell shape, governed by $\theta(t)$'s equation. Their parameters (μ,σ) are (2006, 0.066), (2006.1, 1), and (2006.3, 2) respectively.

Here, the trinomial tree approach is used to construct the mean reverting demand of these three products, and on its basis, the volume flexibility payoff tree is then constructed using equation (4.8).

Next, all the volume flexibility payoff trees are combined to obtain the product-mix flexibility payoff trees. The results from this application, in addition to those of the first example, are discussed in the next section.

5.1 The first Model

The optimal solutions obtained from the deployment of the first model using the first example problem in this chapter are given in Table 5.7.

Table 5.7 The Optimal Solution Alternatives

Alternatives #	System Price ($)	Payload (Lb.)	Number of axes	Envelop (Inch.^{3})	Accuracy (Inch.)	Repeatability (Inch.)	Horse power	Tool capacity	Tool change times (Sec.)	Memory (KB.)	Transfer Height (ft.)	Max load area (Inch.^{2})	Working range (Inch.^{3})	Measuring speed (ft./min.)
208	849533	2600	3	6400	0.0003	0.005748	20	20	4.2	32	35	651.9	85433241.75	2.953
1349	1648179	2640	3	6400	0.0003	0.005748	20	20	4.2	32	45	3014	85433241.75	1.772
6227	1800040	1322.7738	3	76800	0.0003	0.005748	30	31	4.2	64	30	2016	85433241.75	2.953
5484	2353833	2204.623	3	61440	0.0003	0.005748	30	31	2.8	64	45	3014	85433241.75	2.953
10516	3663333	800	4	89121.023	0.0002	0.005748	40	40	2.8	16000	18	1260	85433241.75	2.953
10577	3907000	220	4	5051.182	0.0002	0.005748	75	80	2.8	16000	30	2016	85433241.75	1.575
10754	4034359	661.387	4	89121.02	0.0002	0.007874	75	80	2.8	16000	45	3014	48818995.28	2.953

From the resulting solutions, the following findings are made:

- The optimal level of system flexibility can be obtained as a function of the different levels of basic flexibility. For example, let us make a comparison of alternatives 10754 and 208. While 10754 has the highest machine flexibility index of 79.8%, 208 has the lowest index of 46.64% in this category. However, a look at the material handling flexibility reveals that 10754 having the lowest index in contrast to 208 having the highest index for this flexibility type. Thus, given these two extremes, and faced with constrictive budgets, where the need for one flexibility type, say machine flexibility is paramount over the other, it becomes easy for the decision maker to rightly decide to go with the 10754 option over the 208.

The model facilitates the efficient selection of a manufacturing system equipped with the best preferred attributes. For example, using Table 5.7, the desired attribute happens to be horse power, then the decision maker could go for alternative 10577 rather than 10574 since for the same horse power of 75 the former incurs less costs than the later and the flexibility level of product, process, and volume is being the same for both.

5.2 The Second Model

Table (5.8-5.11) shows the impacts of demand volatility for the three products considered. Notices that, the higher the product demand volatility, the greater the volume and process flexibility required. This is as expected, since higher volatility implies larger variances, and hence, greater uncertainty. The results produced by the model provide a reference point for the manufacturer in deciding the optimal level of flexibility investments related to product demand uncertainty across all product types. For example, from Table , we see that product 1, with less variance of 0.666 in demand structure, requires less investment needs for volume flexibility, while products 2 and 3 with larger variations (each with variance of 1), hence greater volatility, require more volume flexibility.

Table 5.8 The Impact of Volatility to Volume Flexibility Value for Product 1

VF	0.1	0.2	0.3	0.4	0.5
0	4,735,443.54	4,652,119.20	4,500,254.79	4,274,812.84	3,979,327.67
10	5,185,696.28	5,099,375.42	4,941,479.25	4,699,690.69	4,385,273.89
20	5,608,742.47	5,516,209.97	5,351,936.40	5,096,494.20	4,760,521.25
30	5,900,782.41	5,877,382.41	5,721,759.15	5,452,902.97	5,097,587.10
40	**6,046,667.00**	**5,890,326.41**	**5,845,981.41**	5,793,636.41	5,409,272.99
50	5,954,773.41	5,840,206.41	5,827,861.41	**5,814,405.41**	5,694,658.43
60	5,769,215.41	5,756,561.41	5,743,105.41	5,730,738.41	**5,718,084.41**
70	5,507,039.41	5,491,616.41	5,357,049.41	5,344,702.41	5,333,468.41
80	5,204,010.41	5,186,356.41	5,169,782.41	5,158,437.41	5,146,092.41
90	4,877,831.41	4,857,955.41	4,839,679.41	4,828,692.41	4,816,146.41
100	4,581,051.41	4,560,175.41	4,540,368.41	4,531,281.41	4,518,738.41

Table 5.9 The Impact of Volatility to Volume Flexibility Value for Product 2

VF	0.1	0.2	0.3	0.4	0.5
0	1,734,539.54	1,582,096.50	1,324,404.25	1,005,661.08	649,549.42
10	1,985,870.63	1,824,813.19	1,556,313.98	1,232,177.68	869,083.45
20	2,210,165.84	2,037,793.15	1,763,374.79	1,427,669.76	1,067,199.65
30	2,407,911.64	2,221,721.99	1,942,819.39	1,602,379.26	1,242,943.41
40	2,576,884.06	2,377,873.99	2,092,084.43	1,754,877.63	1,394,639.50
50	**2,711,664.83**	2,505,754.43	2,216,672.86	1,886,095.57	1,521,654.73
60	2,711,649.83	**2,508,315.83**	**2,318,526.40**	1,989,166.58	1,620,275.32
70	2,711,164.83	2,491,288.83	2,286,597.83	**2,064,869.72**	1,699,506.99
80	2,646,227.83	2,411,674.83	2,199,447.83	1,903,015.83	**1,759,793.14**
90	2,625,121.83	2,379,450.83	2,159,779.83	1,860,801.83	1,684,258.83
100	2,604,015.83	2,314,362.83	2,127,492.83	1,817,562.83	1,637,686.83

Table 5.10 The Impact of Volatility to Volume Flexibility for Product 3

VF	0.1	0.2	0.3	0.4	0.5
0	7369304.367	7161688.095	6808145.804	6,367,364.74	5,862,151.96
10	7709284.282	7479917.946	7101559.252	6,646,826.35	6,124,850.91
20	8006593.378	7753112.104	7358423.211	6,875,483.52	6,361,891.03
30	8257981.872	7977337.316	7571018.555	7,080,707.86	6,565,850.48
40	8463288.704	8158569.825	7737266.006	7,253,159.54	6,738,323.69
50	8621707.832	8298169.481	7874021.333	7,398,431.79	6,876,159.40
60	**8723626.036**	**8378906.442**	8000000.334	7,506,248.69	6,972,433.03
70	8722281.036	8355450.442	**8047397.413**	**7,569,710.63**	7,047,705.25
80	8710170.036	8313228.442	8035052.413	7,557,365.63	**7,103,655.19**
90	8676873.036	8272363.442	7983941.413	7,518,254.63	7,090,421.19
100	8629302.036	8225450.442	7888743.413	7,463,962.63	7,079,310.19

Table 5.11 The Impact of Volatility to Process Flexibility for All Products

PF	0.1	0.2	0.3	0.4	0.5
0	3,885,048.96	3717156.622	3494758.488	3232193.177	2937636.893
10	5,125,048.96	4,957,156.62	4,603,946.62	4,279,370.62	3,944,803.62
20	6,217,048.96	6,039,158.96	5,818,488.96	5,583,921.96	5,288,251.96
30	7,328,718.96	7,070,830.96	6,746,263.96	6,510,589.96	6,186,022.96
40	8,539,368.96	8,294,579.96	7,862,445.96	7,593,680.96	7,248,008.96
50	9,563,948.96	9,239,381.96	8,696,162.96	8,676,508.96	8,308,966.96
60	9,885,126.96	9,431,907.96	8,876,586.96	8,875,797.96	8,663,452.96
70	**9,885,638.96**	**9,442,423.96**	8,991,991.96	8,988,455.96	8,864,995.96
80	9,785,417.96	9,328,704.96	**9,020,850.96**	**9,019,394.96**	**8,945,070.96**
90	9,385,148.96	8,919,359.96	8,531,705.96	8,496,027.96	8,381,460.96
100	8,865,148.96	8,669,490.96	8,074,923.96	8,029,250.96	7,706,905.96

Table 5.12 shows the different profitability levels that could be realized from different mixes of process and volume flexibility levels for the purpose of adjusting to product demand.

The results reveal that, based on the demand structure, an optimal point at each level of these flexibilities is reached where the combination of both flexibility types in the system provides the maximum returns (profit) on the flexibility investment efforts. As can be seen from the table, increasing the flexibility level of either or both of the flexibility type(s), results in decreasing marginal utilities and hence, unnecessary tie-up of capital that could have been channeled to more value yielding ventures.

Table 5.12 The Optimal Combination of Process and Volume Flexibility

	PF					
VF	0	10	20	30	40	50
0	7155130.393	7506325.959	7810068.219	8066351.087	8275168.346	11131032.14
10	7559378.452	7910595.833	8214355.199	8470845.29	8679940.258	**11140329.38**
20	7937759.587	8,289,228.27	8593395.232	8850067.961	9059202.793	11129626.63
30	8293511.538	8645251.476	8949462.103	9206134.834	**9415269.667**	11,112,149.47
40	8618865.544	8970614.07	**9274824.699**	**9412721.082**	8364273.675	10041874.36
50	**8894619.033**	**9015589.174**	8846021.069	8628917.081	7535212.08	8971599.252
60	8308422.293	8186397.152	8016833.897	7799754.61	6668507.365	7901324.144
70	7442573.487	7319439.088	7150020.232	6933039.539	5783747.409	6831049.036
80	6,570,228.62	6436796.879	6265043.986	6048063.293	4876330.586	5760773.928
90	5681734.664	5542841.89	5,364,235.33	5142238.285	3934483.776	4690498.82
100	4751732.125	4611756.466	4428800.048	4204373.465	2930413.812	3620223.712

It should be mentioned that the solution approach here deviates from the standard techniques in real options valuation in that, while the latter is typically based on the assumption that there is a constant expected growth rate for product demand or price (Bollen, 1999), the methodology proposed here takes a more practical stochastic product life cycle approach which to a greater extent, captures market dynamics. The issue with the futuristic approach of real options is that by using the more theoretic geometric Brownian motion to predict an investment's future profitability, there is the likelihood of over/under valuing the options to invest in flexibility capacities owing to poor predictions of market peculiarities.

In summary, the examples considered reveal that the models of this dissertation are valuable in affording manufacturers avenues of selecting cost-effective, flexible design alternatives without loss of functionalities.

CHAPTER 6

CONCLUSION/FUTRUE RESEARCH DIRECTIONS

For a manufacturing organization to thrive in today's economy, prudent strategies for flexibility must be inextricably linked to all its production system designs. Establishing cost-effective, yet functional flexibility investment strategies in a production system is a highly challenging task, the complexities of which multiply with increasing market uncertainties. More so, flexibility investments are usually capital intensive and entail tie-up of capital over long horizons, so that unnecessary investments made in the bid to have a more flexible manufacturing system might result in lost opportunities from more value yielding ventures.

In this dissertation, models are developed that aim at addressing the stochastic optimization of flexibility investments in manufacturing systems. The models introduced are based on extensions of the single period stochastic inventory model and real option theory to determine the optimum level of the various flexibility attributes that are required to meet the needs of a concern in an uncertain environment. While the Newsboy approach is used to determine the optimal flexibility levels based on the uncertainty associated with the need for a particular flexibility class, the real options approach is used to address situations where variations in product demand constitutes the prime source of uncertainty.

The Newsboy model offers a systematic way of developing an objective function that adequately captures the uncertainty associated with such needs for flexibility. As it was shown in Chapter 3 of this dissertation, the needs of the flexibility distributions were considered in the extension of the classical Newsboy model with the objective of minimizing the costs of flexibility investments: and one of the contributions of this work is the reduction of the problem to a quadratic programming form, and developing a triangular approach in estimating the derived objective function's coefficients. This serves as an enabler to use an existing linear programming software to solve the problem as opposed to using the less structured and tedious nonlinear programming techniques.

In constructing the needs for flexibility distributions using Monte Carlo simulation and based on system attributes, two extremes are assumed in depicting management's expectations for flexibility levels, namely: 0% and 100% where 0% represents no level of need for a given flexibility class so that the firm needs only a dedicated manufacturing system. On the other hand, a 100% flexibility level represents the highest flexibility degree which affords management the highest potential of hedging against an extremely high degree of uncertainty. In narrowing down these distributions in the 0 to 100% range, a truncated constant value is introduced. The model developed here can be easily adopted for any given distribution type and different shapes of the distribution functions ranging from the ramp shape, parabola shape with zero lower bound, and the S shape with non zero lower bound.

Using the KKT conditions, the quadratic programming problem is reduced into linear programming as mentioned before, having the system flexibility levels across the various flexibility classes as decision variables and from which, the optimal levels of flexibility are then obtained by way of the modified simplex method.

Though the traditional capital budgeting techniques, predominant of which is the Net Present Value (NPV) approach are more commonly used today by most manufacturing firms in making capacity expansion and flexibility decisions, there are concerns regarding their static natures, and hence, their inability to adequately capture market dynamics which continuously assume stochastic behaviors. Real options have been shown to provide avenues of capturing future opportunities and increase managerial flexibility in making investment decisions that adjust well to market evolution. Hence, with real options, strategic decisions could be made more proactively.

This dissertation has extended the real options concepts by developing a model that would enable manufacturers ascertain the optimal level of a flexibility type, given variations in product demand. The model developed here, extends the mean reverting process in developing trinomial trees representative of a product's life cycle. The preference of this approach over standard techniques for real options based-approaches such as the binomial method which assume geometric Brownian motion representations of future needs, lies in their shortfall of assuming a futuristic outlook of needs by specifying a constant expected growth rate for such needs. Given today's realities, where product life cycles are constantly shrinking and demand patterns constantly evolving, stochastic representations should provide a more accurate picture.

In the options approach to flexibility investment strategies, the valuation framework is developed to incorporate more than one flexibility class as opposed to existing work in this area that only considers individual flexibility categories. The solution approach is initiated here by assuming that the demand for a product can be modeled by a mean reverting process where the demand term structure can be made to fit the product life cycle curve. Time steps are assumed in scaling the cumulative demand over evaluation horizons. Payoff trinomial trees are then constructed to calculate the optimal flexibility levels for the respective classes.

To show how the developed models can be used to achieve an optimal balance between the two conflicting scenarios of over and under flexible designs, numerical examples were presented based on applications in the production systems of an agricultural firm having Hass, ABB robot, AGV USA, and Mitutoyo as existing suppliers of production equipments. It was seen in the examples considered that even where only a few sets of equipment are involved, their combinations can result in more than 10,000 system alternatives. It was showed that it is possible to have two alternative systems having different cost structures, but the same flexibility index for a given flexibility class while having different flexibility indexes for other class types. Therefore, depending on the preferred flexibility class based on needs, and faced with constrictive budgets, the user can make efficient and effective trade-offs across the given alternatives in establishing the most cost effective strategy. For example, while in an example considered, a system alternative 10754 was shown to have the highest machine flexibility index, another alternative 208 had the lowest index in this category. A look at material handling flexibility on the other hand, revealed 208 to have the highest and 10754 having the lowest flexibility index. Therefore, where the need for material handling flexibility far outweighs that for machine flexibility, it would be more prudent to go for alternative 208 in favor of 10754. In all, the results reveal that the proposed models can be valuable in revealing cost-effective design alternatives, based on design specifications and market dynamics.

6.1 Future Research Directions:

Not much work has been done in the area of stochastic optimization of manufacturing flexibility investments using the Newsboy and options techniques. Some of the data collected for this research, like the costs of under flexibility, over flexibility, and set-up costs were assumed or obtained from internet sources. With the limited data availability, the applications of the models developed considered the production systems of one manufacturer, and this did not provide much opportunity for exploiting the models' potential. A more robust database of costs and flexibility attributes associated with manufacturing systems could be developed to provide more extensive grounds for the models' applications.

Although the models developed are focused on the optimum selection of flexibility attributes of manufacturing systems, they are extendable to cover other types of applications. Among these applications are those of design for supply chain flexibility, health care systems flexibility, and materials handling systems flexibility.

APPENDIX A

PROOF OF EQUATION 3.2

$$E[TC] = E[(c_\tau K_\tau)(x_\tau + \alpha_\tau \, max(x_\tau - D_\tau, 0) + \beta_\tau \, max(D_\tau - x_\tau, 0))]$$

$$= c_\tau K_\tau \left[x_\tau + \alpha_\tau \int_0^{x_\tau} (x_\tau - D_\tau) f(D_\tau) + \beta_\tau \int_{x_\tau}^{100} (D_\tau - x_\tau) f(D_\tau) dD_\tau \right]$$

$$= c_\tau K_\tau \left[x_\tau + \alpha_\tau x_\tau \int_0^{x_\tau} f(D_\tau) dD_\tau - \alpha_\tau \int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau + \beta_\tau \int_{x_\tau}^{100} D_\tau f(D_\tau) dD_\tau - \beta_\tau \int_{x_\tau}^{100} x_\tau f(D_\tau) dD_\tau \right]$$

$$= c_\tau K_\tau \left[x_\tau + \alpha_\tau x_\tau \int_0^{x_\tau} - \alpha_\tau \int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau + \beta_\tau \int_{x_\tau}^{100} D_\tau f(D_\tau) dD_\tau - \beta_\tau \int_{x_\tau}^{100} x_\tau f(D_\tau) dD_\tau \right]$$

$$= c_\tau K_\tau \left[x_\tau + \alpha_\tau x_\tau F(x_\tau) - (\alpha_\tau + \beta_\tau) \int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau + \beta_\tau E[D_\tau] - \beta_\tau x_\tau + \beta_\tau x_\tau F(x_\tau) \right]$$

$$\int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau \rightarrow \left. \begin{matrix} U = D; dV = f(D) dD \\ dU = dD; V = F(D) \end{matrix} \right\} \quad x_\tau F(x_\tau) - \int_0^{x_\tau} F(D_\tau) dD_\tau$$

$$= c_\tau K_\tau \left[(\beta_\tau - 1) x_\tau - (\alpha_\tau + \beta_\tau) \int_0^{x_\tau} F(D_\tau) dD_\tau - \beta_\tau E[D_\tau] \right]$$

APPENDIX B

TOPSIS APPROACH

This appendix explains the rationale of TOPSIS.

An ideal solution is defined as a collection of ideal levels (or ratings) in all attributes considered. However, the ideal solution is usually unattainable or infeasible. Then, to be as close as possible to such an ideal solution is the rationale of human choice. Since the ideal is dependent on the current limits and constraints of the economy and technology, a perceived ideal is utilized instead to implement the choice rationale, a normative decision process. Formally, the ideal solution is denoted as

$$A^* = (x_1^*, \ldots, x_j^*, \ldots, x_n^*),$$

where x_j^* is the best value of the *j*th attribute among all available alternatives.

The composite of all best attribute ratings attainable is the ideal solution, whereas the negative-ideal solution is composed of all worst attribute ratings attainable. The negative-ideal solution (A^-) is given as

$$A^- = (x_1^-, \ldots, x_j^-, \ldots, x_n^-),$$

where x_j^- is the worst value for the *j*th attribute among all alternatives. Then, the question should be asked whether the chosen alternative which is closest to the ideal solution concur with the chosen alternative which is farthest from the negative-ideal solution. Often they do not concur with each other.

TOPSIS defines an index called similarity (or relative closeness) to the ideal solution by a combination of the proximity to the ideal solution and the remoteness from the negative-ideal solution. Then, the method chooses an alternative with the maximum similarity to the ideal solution.

The algorithm of TOPSIS is presented as a series of successive steps:

Step 1: Calculate a normalized rating. The normalization vector is used for computing $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1,....,m; j = 1,....,n$

Step 2: Calculate a weighted normalized ratings. The weighted normalized value v_{ij} is calculated as

$$v_{ij} = w_j r_{ij}, i = 1,...,m; j = 1,...,n,$$

where w_j is the weight of the jth attribute.

Step 3: Identify the ideal and negative-ideal solutions. The A^* and A^- are defined in terms of the weighted normalized values:

$$A^* = \{v_1^*, v_2^*,....,v_j^*,.....,v_n^*\} = \{(\max_i v_{ij} \setminus j \in J_1)(\min_i v_{ij} \setminus j \in J_2) \setminus i = 1,.....,m\}$$

$$A^- = \{v_1^-, v_2^-,....,v_j^-,.....,v_n^-\} = \{(\min_i v_{ij} \setminus j \in J_1)(\max_i v_{ij} \setminus j \in J_2) \setminus i = 1,.....,m\}$$

where J_1 is a set of benefit attributes and J_2 is a set of cost attributes. A benefit attribute is defined in proportion with the value of a performance measure (e.g. quality). By contrast, a cost attribute is defined as being inversely proportional with the value of a performance measure (e.g. completion time).

Step 4: Calculate separation measures. The separation (distance) between alternatives can be measured by the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution, A^*, is then given by

$$S_i^* = \sqrt{\left(\sum_{j=1}^{n}\left(v_{ij} - v_j^*\right)^2\right)}, \quad i = 1,2,\ldots,m.$$

Similarly, the separation from the negative-ideal solution A^-, is given by

$$S_i^- = \sqrt{\left(\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2\right)}, \quad i = 1,2,\ldots.,m.$$

Step 5: Calculate similarities to ideal solution.

$$C_i^* = \frac{S_i^-}{\left(S_i^* + S_i^-\right)}, \quad i = 1,2,\ldots,m$$

Note that $0 \le C_i^* \le 1$; where $C_i^* = 0$ when $A_i = A_i^-$, and $C_i^* = 1$ when $A_i = A^*$

Step 6: Rank preference order. Choose an alternative with the maximum C_i^* or rank alternatives according to C_i^* in descending order.

APPENDIX C
FLEXIBILITY UNIT COST ESTIMATION

Regression analysis results for the flexibility unit cost estimation used in the model application of Chapter 5.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.652719
R Square	0.426042
Adjusted R Square	0.425883
Standard Error	581238.1
Observations	10800

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	2.71E+15	9.02E+14	2671.251	0
Residual	10796	3.65E+15	3.38E+11		
Total	10799	6.35E+15			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-4784709	119983.9	-39.8779	0	-5019900	-4549519	-5019900	-4549519
Product flexibility	12629.62	775.4091	16.28769	6.07E-59	11109.68	14149.57	11109.68	14149.57
Volume flexibility	88002.13	3134.916	28.07161	2E-167	81857.12	94147.14	81857.12	94147.14
Process flexibility	46624.96	1745.565	26.71053	2.9E-152	43203.34	50046.59	43203.34	50046.59

REFERENCES

[1] Abdel-Malek, L. (2000) Design and implementation of flexible manufacturing solutions in agile enterprises, International Journal of Agile Management Systems, 187-195.

[2] Andreou, S.A.(1990) A capital budgeting model for product-mix flexibility, Journal of Manufacturing and Operations Management 3, 5-23.

[3] Black, F., Scholes, M.(1973) The pricing of option and corporate liabilities, Journal of Political Economy81, 637-654.

[4] Bengtsson, J.(1999)The value of manufacturing flexibility: Real options in practice, working paper, WP 262, Department of Production Economics, Linkőping Institute of Technology.

[5] Bollen, N. (1999) Real Options and Product Life Cycles, Management Science 45 (5), 670-692.

[6] Brennan, M., Schwartz, E.(1985) Evaluating natural resource investments, Journal of Business 58 (2),135-157.

[7] Browne, J., Dubois, D., Rathmill, K., Sethi, S.P., Stecke, K.E (1984) Classification of flexible manufacturing systems, The FMS Magazine 2(2), 114-117.

[8] Carr, P. (1995) The valuation of American exchange options with application to real options, in: L. Trigeorgis (Ed.), real options in capital investment: models, strategies, and applications, Praeger, New York.

[9] Carter, M.F. (1986) Designing flexibility into automated manufacturing systems, proceedings of the second ORSA/TIMS conference on flexible manufacturing systems.

[10] Correa, H.L. (1994) Linking flexibility, uncertainty and variability in manufacturing systems, Avebury, Aldershot.

[11] De Groote, X. (1994) The flexibility of production processes: A general framework, Management Science 40 (7), 933-945.

[12] Dixit, A.K., Pindyck, R.S. (1994) Investment Under Uncertainty, Princeton University Press.

[13] Ferdows, K., De Meyer, A. (1990) Lasting improvements in manufacturing performance: In search of a new theory, Journal of Operations management 9, 168-184.

[14] Gerwin, D. (1987) An agenda for research on the flexibility of manufacturing processes, International Journal of Operations and Production Management 7, 38-49.

[15] Gilpin, R. (2000) The challenge of Global Capitalism: The world economy in the 21st Century, Princeton University Press.

[16] Gupta, Y.P., Goyal, G. (1989) Flexibility of manufacturing systems: concepts and measurements, European Journal of Operational Research 43,119-135.

[17] Hayes, R., Wheelwright, S. (1984) Restoring competitive edge: competing through manufacturing, John Wiley, New York.

[18] He, H., Pindyck, R.S. (1992) Investment in flexible production capacity, Journal of Dynamics and Control 16, 575-599.

[19] Hull, J.C. (2000) Options futures and other derivatives, 4th edition, Prentice Hall Inc., Englewood Cliffs, NJ.

[20] Hyun, J.H., Ahn, B.H. (1992) A unifying framework for manufacturing flexibility, Manufacturing Review 5(4), 251-260.

[21] Johnson, H. (1987) Options on the maximum or the minimum of several assets, Journal of Financial and Quantitative Analysis 22(3), 277-283.

[22] Kamrad, B., Ernst, R. (1995) Multi-product manufacturing with stochastic input prices and output yield uncertainty, in: L.Trigeorgis (Ed.), Real Options in Capital Investment: Models, Strategies, and Application, Praeger, New York.

[23] Kulatilaka, N. (1988) Valuing the flexibility of flexible manufacturing system, IEEE Transactions on Engineering Management 35 (4), 250-257.

[24] Kulatilaka, N. (1995) The value of flexibility: A general model of real options, in: L. Trigeorgis (Ed.), Real Options in Capital Investment: Models, Strategies, and Application, Praeger, New York.

[25] Kulatilaka, N. (1995) Operating flexibilities in capital budgeting: substitutability and complementarily in real options, in: L.Trigeorgis (Ed.), Real Options in Capital Investment: Models, Strategies, and Application, Praeger, New York.

[26] Kumar, V. (1987) Entropic measures of manufacturing flexibility, International Journal of Production Research 25 (7), 957-966.

[27] Lau, R.S.M. (1996) Strategic flexibility: A new reality for world-class manufacturing. S.M.A. Advanced Management Journal 61(2), 11-19.

[28] Majd, S., Pindyck, R.S. (1987) Time to build, option value, and investment decisions, Journal of Financial Economics 18 (1), 7-27.

[29] McDonald, R., Siegel, D. (1985) Investment and the valuation of firms when there is an option to shut down, International Economic Review 26 (2), 331-349.

[30] McDonald, R., Siegel, D.(1986) The value of waiting to invest, Quarterly Journal of Economics 101, 707-727.

[31] Myers, S.C., Majd, S. (1990) Abandonment value and project life, Advances in Futures and Options Research 4, 211- 224.

[32] Mandelbaum, M. (1978) Flexibility in decision making: an exploration and unification, Ph.D Thesis, Department of Industrial Engineering, The University of Toronto, Toronto, Canada.

[33] Margrabe, W. (1978) The value of an option to exchange one asset for another, Journal of Finance 33 (1), 177-186.

[34] Merton, R.C (1973) Theory of rational option pricing, Bell Journal of Economic and Management Science 4, 141-183.

[35] Newman, W.R., Hanna, M. M. (1993) Dealing with the uncertainties of manufacturing: Flexibility, buffer and integration, International Journal of Production Management 13, 19-34.

[36] Olhager, J., West, B.M.(1999) The house of flexibility: using the QFD approach to deploy manufacturing flexibility, Working Paper, WP-259, Department of Production Economics, Linkőping Institute of Technology.

[37] Pindyck, R.S. (1988) Irreversible investment, capacity choice, and the value of the firm, The American Economic Review 78 (5), 969-985,

[38] Sethi, A.K., Sethi, P.S., (1990) Flexibility in manufacturing: A survey, International Journal of Flexible Manufacturing Systems 2, 289-328

[39] Skinner, W. (1969) Manufacturing-missing link in corporate strategy, Harvard Business Review , May-June, 135-144.

[40] Skinner, W. (1996) Manufacturing of the 'S' curve, Production and Operations Management, 5(1), 3-14.

[41] Slack, N. (1983) Flexibility as a manufacturing objective, International Journal of Operations and Production Management 3, 5-13.

[42] Stecke, K.E. (1986) Production flexibility and their impact on manufacturing strategy, Working Paper 484, Gradate School of Business Administration, University of Michigan.

[43] Stulz, R.M. (1982) Options on the minimum or maximum of two risky assets: Analysis and application, Journal of Financial Economics 10 (2), 61-185.

[44] Suarez, F.F., Cusumano, M.A., Fine, C.H. (1995) An empirical study of flexibility in manufacturing, Sloan Management Review Fall, 25-32.

[45] Suarez, F.F., Cusumano, M.A. Fine, C.H. (1996) An empirical study of manufacturing flexibility in printed circuit board assembly, Operations Research 44(1), 223-240.

[46] Swamidass, P.M. (1988) Manufacturing Flexibility, Operations Management Association Monograph No.2, The Schneider Group, WACO, TX.

[47] Tannous, G.F. (1996) Capital budgeting for volume flexible equipment, Decision Sciences 27 (2), 157-184.

[48] Triantis, A.(1988) Contingent claims valuation of flexible production systems, Ph.D. Dissertation, Stanford.

[49] Trigeorgis, L. (1991) A log-transformed binomial numerical analysis method for valuing complex multi-option investment, Journal of Financial and Quantitative Analysis, 26, 309-326

[50] Trigeorgis, L. (1993)The nature of option interactions and the valuation of investments with multiple real options, Journal of Financial and Quantitative Analysis 28 (1), 1-20.

[51] Trigeorgis, L. (1996) Real Options: Managerial Flexibility and Strategy in Resource Allocation, The MIT Press, Cambridge, MA.

[52] Upton , D.M. (1994) The management of manufacturing flexibility. California Management Review 36 (2), 88-104.

[53] Wall, M. (2003) Manufacturing flexibility what constitutes the Holy Grail for some is just business as usual for others-supplier business, Automotive Industries.