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## Virtual transshipments and revenue-sharing contracts in supply chain management

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## **ABSTRACT**

### **VIRTUAL TRANSSHIPMENTS AND REVENUE-SHARING CONTRACTS IN SUPPLY CHAIN MANAGEMENT**

**by  
Zhaoqiong Qin**

This dissertation presents the use of virtual transshipments and revenue-sharing contracts for inventory control in a small scale supply chain. The main objective is to maximize the total profit in a centralized supply chain or maximize the supply chain's profit while keeping the individual components' incentives in a decentralized supply chain.

First, a centralized supply chain with two capacitated manufacturing plants situated in two distinct geographical regions is considered. Normally, demand in each region is mostly satisfied by the local plant. However, if the local plant is understocked while the remote one is overstocked, some of the newly generated demand can be assigned to be served by the more remote plant. The sources of the above virtual lateral transshipments, unlike the ones involved in real lateral transshipments, do not need to have nonnegative inventory levels throughout the transshipment process. Besides the theoretical analysis for this centralized supply chain, a computational study is conducted in detail to illustrate the ability of virtual lateral transshipments to reduce the total cost. The impacts of the parameters (unit holding cost, production cost, goodwill cost, etc.) on the cost savings that can be achieved by using the transshipment option are also assessed.

Then, a supply chain with one supplier and one retailer is considered where a revenue-sharing contract is adopted. In this revenue-sharing contract, the retailer may obtain the product from the supplier at a less-than-production-cost price, but in exchange, the retailer must share the revenue with the supplier at a pre-set revenue-sharing rate. The objective is to maximize the overall supply chain's total profit while

upholding the individual components' incentives. A two-stage Stackelberg game is used for the analysis. In this game, one player is the leader and the other one is the follower. The analysis reveals that the party who keeps more than half of the revenue should also be the leader of the Stackelberg game.

Furthermore, the adoption of a revenue-sharing contract in a supply chain with two suppliers and one retailer under a limited amount of available funds is analyzed. Using the revenue-sharing contract, the retailer pays a transfer cost rate of the production cost per unit when he obtains the items from the suppliers, and shares the revenue with the suppliers at a pre-set revenue-sharing rate. The two suppliers have different transfer cost rates and revenue-sharing rates. The retailer will earn more profit per unit with a higher transfer cost rate. How the retailer orders items from the two suppliers to maximize his expected profit under limited available funds is analyzed next. Conditions are shown under which the optimal way the retailer orders items from the two suppliers exists.

**VIRTUAL TRANSSHIPMENTS AND REVENUE-SHARING  
CONTRACTS IN SUPPLY CHAIN MANAGEMENT**

by  
**Zhaoqiong Qin**

**A Dissertation  
Submitted to the Faculty of  
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Doctor of Philosophy in Transportation**

**Interdisciplinary Program in Transportation**

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## APPROVAL PAGE

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*To my dear parents*

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## CHAPTER 1

### INTRODUCTION

The importance of supply chain management has been receiving increasing recognition in the recent past. The rapid growth of supply chain consultants and software companies indicates that businesses emphasize the efficient management of supply chains. At the same time, research in this area has been growing as a key focus of the academic community in operations management.

A supply chain can be thought of as a network of facilities and distribution nodes that perform the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to consumers.

A supply chain includes the marketing, distributing, planning, manufacturing, and purchasing functions of an organization. These entities traditionally operate independently. They have their own objectives and these objectives are sometimes conflicting. Marketing's objective of good customer service and maximum sales dollars may conflict with manufacturing and distribution's goals. Manufacturing operations are designed to maximize throughput and lower costs with little consideration for the impact on inventory levels and distribution capabilities. Purchasing contracts are often negotiated with little information beyond historical buying patterns. A supply chain also has to be tactically balanced and operationally streamlined to cope with a number of uncertainties and variabilities arising from sources such as supplier delivery performance and lead times, manufacturing process times and yields, transit times, and demand. Clearly, there is a need for a mechanism or method through which these different objectives or functions can be integrated together by managing the supply chain. This dissertation aims to take a step in this direction.

There are four major decision areas in supply chain management: 1) location, 2) production, 3) inventory, and 4) transportation. There are both strategic and operational elements in each of these decision areas. As the term implies, strategic decisions are made typically over a longer time horizon. These are closely linked to the corporate strategy, and guide a supply chain's policies from a design perspective. On the other hand, operational decisions are short-term, and focus on activities on a day-to-day basis.

Location decisions of production facilities, stocking points, and sourcing points are the natural first step in creating a supply chain and involve a commitment of resources to long-term planning. These decisions are of great significance since they represent the basic strategy for accessing customer markets, and will have a considerable impact on revenues, costs, and levels of service. Production decisions include what products to produce, which plants to produce them in, allocation of suppliers to plants, plants to distribution centers, and distribution centers to customer markets. As before, these decisions have a big impact on the revenues, costs and customer service levels of the firm. These decisions assume the existence of the facilities, and determine the exact paths through which a product flows to and from these facilities. Inventory decisions refer to means by which inventories are managed. Inventories of either raw materials, semi-finished or finished goods exist at various points of a supply chain. Since inventories of raw material, semi-finished or finished goods incur costs, their efficient management is critical in supply chain operations. Inventory management has a strategic component since top management sets goals. However, most researchers have approached the management of inventory from an operational perspective that includes the determination of the optimal levels of order quantities and reorder points, and setting safety stock levels at each stocking location. Transportation decisions focus on delivery means, delivery frequencies, delivery dependency and costs (Ganeshan and Harrison [24]).

Inventories are necessary in a supply chain because they:

- 1) improve customer service and actually increase sales;
- 2) reduce production setup costs by allowing longer production runs;
- 3) reduce purchasing and transportation costs because of economies of scale or forward buying;
- 4) provide protection and continuity of operations in case of labor strikes, natural disasters, jammed transportation and surges in demand.

However, inventories also incur carrying costs arising from opportunity, obsolescence, space needs, overhead, insurance, etc..

There are tradeoffs in inventory management. For example, in a supply chain with stochastic demand, the major tradeoff is as follows: Too high a level of inventory leads to inefficient capital investment, expensive markdowns and needless handling costs, while too low a level leads to lost sales and loss of goodwill. Reasonable inventory levels lead to an efficient level of service, and minimized costs. One of the main objectives of this dissertation is to minimize the total cost or maximize the channel profit by using mechanisms and methods that control inventories for specific supply chains.

The inventory control problem is complicated by the fact that demand is uncertain, and this uncertainty can cause stockouts and the inability to fill orders. To minimize supply and demand imbalances in a supply chain, vendors utilize various methods of inventory management. As an example, consider a supply chain with two retailers located across the country and one supplier located outside the country. The two retailers must plan their replenishment strategy from the supplier, according to their own demand distributions for the product. When actual demand occurs at each store, it may be beneficial for the two retailers to make transshipments among themselves, as their reaction time is considerably shorter than that of the supplier.

In this context, an appropriate transshipment strategy can result in substantial cost savings as well as service improvements.

Contracts provide a mechanism to manage inventory in supply chains. These contracts include wholesale-price contracts, buy-back contracts, quantity-flexibility contracts, sales-rebate contracts, price-discount contracts, etc.. These contracts are used in a supply chain for specific objectives. For example, wholesale-price contracts are the simplest, but they can not achieve coordination, i.e., they can not induce the retailer to order the optimal amount from the total channel point of view. As an incentive for the retailer to order more and move towards channel coordination, buy-back contracts are offered in a supply chain. As auctions between suppliers and retailers have become a more important and complex part of supply chains, there is increasing demand for the contract design.

In recent years, revenue-sharing contracts are popular in supply chains with short-life-cycle products such as video-cassettes where the peak popularity of a rental title lasts only a few weeks but the cost of a tape has traditionally been high relative to the price of a rental. In a conventional sales agreement, the retailer purchases each tape from his supplier for about \$65 and collects about \$3 per rental. Hence, a tape earns a profit only after 22 rentals. However, because the demand for a tape typically starts high and tapers quickly, a retailer cannot justify purchasing enough tapes to cover the initial peak demand entirely. Blockbuster Inc., a large video retailer had poor availability of newly released videos indicated by customer complains (McCollum [57] and Shapiro [70]). Seeking a solution to this problem, in 1998 Blockbuster entered into a revenue-sharing contract, agreeing to pay its suppliers a portion (probably in the range of 30-45%) of its rental income in exchange for a reduction in the initial price per tape from \$65 to \$8. The break-even point for a tape dropped to approximate six rentals, thereby allowing Blockbuster to purchase many more tapes. The adoption of revenue-sharing coincided with a significant

improvement in performance at Blockbuster: Warren and Peers [88] reported that Blockbuster's market share of video rentals increased from 24% in 1997 to 40% in 2002.

A consignment contract with revenue-sharing is another case. Amazon.com has an online "marketplace" where anyone can list for sale new, used, or refurbished items (books, CDs, electronics, tools and hardware, kitchen and housewares, etc.). With a few minor restrictions, sellers decide on how many units to list and the items' selling price. The listing itself is free. Amazon.com charges sellers according to the following policy, which is essentially a consignment contract with revenue-sharing: Amazon.com collects a fee only when an item is sold. At that time, Amazon.com collects the sales price from the buyer, deducts a commission of \$0.99 plus 15% of the sales price (10% for Electronics and Camera & Photo items), and deposits the rest in the seller's account. If the seller's item is not sold within 60 days, the listing is closed and the seller pays nothing. (For more details on Amazon.com's marketplace, go to: <http://www.amazon.com>.) A consignment arrangement with revenue-sharing naturally favors the retailer. Since no payment to the supplier is made until the item is sold, the retailer has no money tied up in inventory and bears no risk associated with demand uncertainty.

It is necessary to establish appropriate performance measures in order to manage a supply chain. A performance measure, or a set of performance measures, is used to determine the efficiency and/or effectiveness of an existing mechanism or method, or to compare competing alternative mechanisms or methods. The measures evaluating a mechanism or method's effectiveness and/or efficiency in supply chain management can be categorized as either qualitative or quantitative. Qualitative performance measures are those for which there is no single direct numerical measurement, although some aspects of them may be quantified. They include customer satisfaction ( the degree to which customers are satisfied with the product and/or

service received), flexibility (the degree to which the supply chain can respond to random fluctuations in the demand pattern), information and material flow integration (the extent to which all functions within a supply chain communicate information and transport materials), effective risk management (the degree to which the effects of the risks contained in all the relationships within a supply chain are minimized), supplier performance (with what consistency suppliers deliver raw materials to production facilities on time and in good condition), etc.. Quantitative performance measures are those that may be directly described numerically. They may be categorized by 1) objectives that are based directly on costs or profits (cost minimization, sales maximization, inventory investment minimization and return on investment maximization); and 2) objectives that are based on some measures of customer responsiveness (fill rate maximization, product lateness minimization, customer response time minimization, lead time minimization and function minimization which is to minimize the number of business functions that are provided by more than one business entity). As mentioned above, an important element in a supply chain's modeling is the establishment of appropriate performance measures. The performance measures are expressed as functions of one or more decision variables. These decision variables are then chosen in such a way as to optimize one or more performance measures (Beamon [8]).

What follows in this dissertation includes a literature review of supply chain management is presented in Chapter 2. A study of how the parameters (unit holding cost, production cost, goodwill cost, etc.) affect cost savings in a supply chain with two capacitated manufacturing plants using virtual lateral transshipment compared with no lateral transshipment is presented in Chapter 3. Chapter 4 considers how the revenue-sharing contract is adopted in a supply chain with one supplier and one retailer to maximize the channel profit. Chapter 5 analyzes the adoption of a revenue-sharing contract in a supply chain with two different suppliers and one retailer

under a limited amount of available funds. Chapter 6 summarizes this dissertation, including its contributions and possible future extensions.



## CHAPTER 2

### LITERATURE SURVEY

Research in inventory control is mainly concerned with deciding the ordering quantity or production quantity so as to minimize the total ordering, production, holding and stockout costs. The newsvendor problem is a classic and typical inventory control problem involving one single period (see Wang and Gerchak [86] and Henig and Gerchak [35]). Ouyang and Chang [60] analyzed the newsboy problem under more realistic circumstances with defective items, which have different salvage values from that of the perfect items left over at the end of the planning horizon. They presented models that maximize the expected profit under fixed and random defective rates and showed that the optimal order quantity exists. Furthermore, to avoid the difficulty of estimating the goodwill cost of shortage, they used the service level (the percentage of demand to be served directly from stock) as a constraint for the objective function. They also developed a solution to the optimal order quantity to maximize the expected profit. Numerical examples proved that the models perform well.

Khouja [48] made a reievw of some 90 publications about the extensions to the newsvendor problem and classified them into eleven categories based on the types of extensions. These extensions focus on the problems involving multiple periods, multiple locations with transshipment, contract adoptions, and capacitated constraints.

Production/inventory control with lateral transshipments has also received quite some attention. Here, transshipments are “real” in the sense that items are transferred from one location’s inventory to that of another location. All studies were conducted in the non-capacitated context. Allen [2], Das [18], Gross [30], Hoadley and Heyman [38], Karmarkar [43], Karmarkar and Patel [46], Krishnan and Rao

[49] studied structural properties of the optimal policies for single-period problems. Karmarkar [44], Robinson [66], and Showers [72] studied the structures of optimal policies for multi-period problems. Karmarkar [44] actually treated the following problem: When being confined to the transshipment setting, his model assumed that transshipment decisions are made before demand arrivals, and on the other hand did not insist on the nonnegative-level requirement. Karmarkar [45] also extended the “newsboy” problem into a multi-location, multi-period system with transshipments being possible between different locations. He showed the upper- and lower-bounds for the general multi-period problem and gave the computational methods to the optimal solution. Robinson [66] assumed that transshipment decisions are made upon demand arrivals. Indeed, Robinson did not make hard constraints out of the nonnegative-level requirement. But he limited problem parameters into ranges that ensure the validity of the real transshipment interpretation of his results. Showers [72] allowed ordering only at regular intervals, with transshipments occurring in the intervening periods. Herer and Tzur [36][37] treated a transshipment problem in a multi-period deterministic-demand setting. Rudi, Kapur and Pyke [69] studied a two-location inventory system with transshipment and presented models for inter-firm and intra-firm transshipments. They showed that the optimal inventory orders increase with the transshipment prices and concluded that in general, the optimal inventory order choice that maximizes each location’s own profit does not maximize the joint profit.

A capacitated supply system is common in practice and has received quite some attention. Federgruen and Zipkin [22][23] first dealt with the capacitated inventory control problem. When a firm is constrained by a fixed capacity, they found the optimal policy to be of the modified base stock type whereas the firm should produce as much as what is allowed by the capacity to reach as close as possible the base stock point. Other studies in this vein included Glasserman [26], Glasserman and Tayur [27][28], Kapuscinski and Tayur [42], and Tayur [77]. Ciarallo, Akella, and

Morton [14] considered random capacity. They found the optimal policy to be still of the base stock type. Güllü [32] proposed to use a G/G/1 queue analogy to compute such a base stock level. Henig and Gerchak [35] and Wang and Gerchak [86] derived structural results of the optimal policies in the presence of random production yields.

Vericourt, Karaesmen and Dallery [84] analyzed a capacitated supply system with multi-classes of customers for a single item with backorder and no lost sales. Because of limited resources, it was necessary to ration the items among the customers in different classes. They modeled the supply system as a multi-customer make-to-stock queue for a certain number of customer classes. They showed an optimal stock allocation policy that minimizes average inventory holding and backorder costs. Furthermore, they gave an exact algorithm to compute the optimal parameters.

Güllü, Onol and Erkip [33] considered a single-item periodic-review inventory system over a finite planning horizon under an uncertain supply. To model supply uncertainty, a three-point (completely available, partially available, or completely unavailable) probability mass function is used and a model to minimize expected holding and backorder costs over the planning horizon is developed. The authors demonstrated the optimality of a non-stationary order-up-to policy, provided computational results and observed that the optimal order-up-to levels decrease as the unit cost ratio  $h/b$  increases and increase as the probability of partial availability (for a fixed unavailability probability) increases. Specifically they provided a simple newsboy-like formula for computing the optimal order-up-to levels for the case of two-point stationary supply availability (completely available or completely unavailable).

Lau and Lau [52] considered a system with a vendor stocking several items under a set of resource constraints. Using a Lagrangian relaxation approach, they developed a solution and presented case studies.

For capacitated inventory control, some authors concentrated on optimizing parameters for given policy shapes and deriving heuristics under the continuous-

review and mostly Poisson demand settings. Among those who considered the problem are Alfredsson and Verrijdt [1], Axsäter [6][7], Cohen, Kleindorfer, and Lee [15], Dada [16], Grahovac and Chakravarty [29], Lee [53], Sherbrooke [71], and Slay [73]. Archibald, Sassen, and Thomas [4] characterized the optimal policy for a two-location continuous-review problem. Tagaras and Cohen [74] compared different rules for lateral transshipments by simulation.

Supply chain contracts have grown more prominent in their roles in supply chain management during the past few years. Supply contracts such as buy-back (Pasternack [62]), quantity flexibility (QF)(see Tsay [82]) and penalty scheme (see Lariviere [50]) have been well analyzed when the demand is stochastic with a given retail price. The research based on the returns contract included retailer competitions (Padmanabhan and Png [61]), two echelon inventory systems (Cachon and Zipkin [11]), and risk-free returns to the supplier (Webster and Weng [89]). There are many other forms of contracts with a given retail price such as wholesale-price contracts (Lariviere and Porteus [51]) and revenue-sharing contracts (Wang, Li and Shen [87]).

Taylor [79] studied a retailer's profit under linear rebates and target rebates. He showed that a linear rebate can not achieve coordination (maximization of the channel profit) in an implementable way, and if the retailer sales effort does not influence demand, a properly designed target rebate can achieve coordination. But, if the retailer sales effort influences demand, such contracts as linear rebate or target rebate alone can not achieve coordination in an implementable way. Coordination must be achieved by a properly designed target rebate and linear rebate contract together. This paper presented the view that the provision of rebate strengthens incentives for a retailer sales effort, which is contrary to the view expressed by Emmons and Gilbert [20].

Wang, Li and Shen [87] studied supply chain management under a consignment contract with revenue-sharing. In their paper, the supplier decides on the retail price

and delivery quantity to the retailer and the retailer decides the percentage from the selling price to keep for himself and remits the balance to the supplier. Demand for the product is uncertain and price-sensitive during a single selling period or season. They presented models for centralized and decentralized supply chain systems. They showed that the optimal quantity and price are critically dependent on the price elasticity in the centralized system, while the optimal quantity in the decentralized system is the same as that in the centralized one and the optimal price depends on the retailer's revenue share, the channel's cost and the retailer's cost share in addition to the price elasticity. They also found that the retailer's optimal revenue share is increasing with the retailer's cost share.

Dana and Spier [17] studied a revenue-sharing contract in a perfectly (head to head) competitive market faced by downstream retailers. In their paper, they considered an upstream firm who decides the transfer cost per unit and the revenue-sharing rate on a one-unit sale to the downstream retailer, while the downstream retailer decides the optimal quantity to be ordered from this supplier. They closely adopted Deneckere and Peck [19]'s version of Carlton [13]'s model, but assumed "perfect" rather than imperfect competition. They obtained the relationship of the optimal quantity with the transfer cost per unit and the revenue-sharing rate on one unit sale. The paper showed that the supplier must simultaneously lower the transfer price below production cost and raise the revenue-sharing rate above zero to keep vertical-integration control, i.e., to maintain the incentives for the competitive downstream market to hold the same capacity as that in the overall channel. This conclusion is in accordance with that of Wang, Li and Shen [87] — retailer's optimal revenue-share increases with the retailer's cost share.

Cachon and Lariviere [10] presented a model to analyze the case that a single retailer chooses the optimal price and quantity to maximize the expected profit. They demonstrated that using a revenue-sharing contract, a supplier can coordinate

a single retailer channel. That is to say, when the revenue-sharing contract is under the condition that the transfer cost is equal to the revenue-sharing rate times the production cost, arbitrarily allocating the revenue-sharing rate can maintain the same optimal quantity as that in the overall channel. However, this contract does not coordinate a supply chain with demand that depends on costly retail effort. Neither buy-back, nor quantity flexibility nor sales rebate is able to do so in the price-setting supply chain. Furthermore, the authors explained why most contracts fail to coordinate the supply chain in the price-setting system. They also showed that revenue-sharing and buy-back contracts are equivalent in the strongest sense: for any buy-back contract there exists a revenue-sharing contract that generates the same cash flows for any demand realization.

Lariviere [50] analyzed supply chain contracting and coordination in stochastic demand. While he determined the best the manufacturer and the retailer can do for themselves, respectively, the author found out that it will not be the best outcome for the supply chain as a whole.

Numerous papers considered supply chain contracts over short horizons (see Khouja [47] and Tayur, Ganeshan and Magazine [78]). These papers are generally based on the newsvendor model. Pasternack [62] investigated a model with a single supplier and a single retailer and showed that the retailer tends to purchase too little inventory with a simple wholesale-price contract. With similar models, Lariviere and Porteus [51] analyzed the performance of a wholesale-price contract and Tsay [82] studied the performance of a quantity flexibility contract. Cachon [9] considered a model based primarily on a wholesale-price contract.

In supply chain contracting, some papers considered the impacts of the ways of the parameter-settings on supply chain management (Plambeck and Zenios [65] and Caldentey and Wein [12]). Anupindi and Bassok [3] considered a supply chain where a supplier sets a wholesale price using an approximation of the normal distribution.

Van Mieghem [83] analyzed how an exogenously-set transfer price can influence the capacity decisions of a manufacturer and an upstream supplier. Considering different supplier pricing policies, Jucker and Rosenblatt [40] and Lin and Kroll [55] focused on quantity discounts while Kabak and Weinberg [41] focused on multiple suppliers.

In this dissertation, “virtual transshipment” is used to solve the finite- and infinite-horizon inventory control problem for two capacitated plants in a stochastic-demand setting. The literature survey showed that this problem was not studied before in this vein.

A revenue-sharing contract is also studied here for a supply chain where the supplier and retailer try to maximize their own expected profit with an exogenously-set revenue-sharing rate. The objective is to show how the exogenously-set revenue-sharing rate influences the decision about who should be the leader of the game to maximize the supply chain’s expected profit. In the literature, the revenue-sharing rate is a decision variable.

The optimal order structure policy of a retailer with a limited amount of available funds, when there are two suppliers with different revenue-sharing contracts, is also analyzed.

## CHAPTER 3

### A COMPUTATIONAL STUDY ON LATERAL TRANSSHIPMENTS

#### 3.1 Background

Many factors constrain a manufacturing plant's capacity. These factors include the number of machines and their capabilities, the number of workers, the arrival rates of raw materials, the supply of power, water, and other essentials, etc.. It is obvious that a higher-than-needed capacity will lead to a waste of capital investment, overhead costs and processing fee (holding costs) while a less-than-needed capacity will imply lost revenue and unhappy customers (goodwill costs). Fortunately, recourse opportunities like outsourcing, lateral transshipments, and product substitutions exist that can help a plant to mitigate the complications caused by the limited capacities. When the plant's paternal firm owns several plants in different geographical regions at the same time, products may be transshipped among these plants in times of uneven demand. It is also possible that demand emerging in one region may be designated to be served by a more remote plant. We call both of these practices lateral transshipments, the former real and the latter virtual. Though lateral transshipments incur positive costs, using them cleverly can lead to cost savings.

In this chapter, virtual lateral transshipments between two capacitated plants within one firm are studied. The two plants are located in two distinct regions, and demand emerging from each region is normally served by the local plant. However, demand emerging from one region can always be designated to the other remote plant upon its arrival. The assignment of one order generated in region 2 to plant 1 is considered as (virtually) transshipping one item from plant 1 to plant 2. Unlike the real transshipment setting, the above is doable even when plant 1 has a negative inventory level, though its desirability should be decided by cost considerations. Once



made, it is assumed that the transshipment decisions can not be altered later. In the current setting, the unit transshipment cost from plant 1 to plant 2 stands for the difference in the average unit delivery costs from the two plants to a random order from region 2, rather than the unit delivery cost from plant 1 to plant 2. The corresponding problem involving real transshipments is most likely significantly harder for the obvious requirement that plants at the giving ends of the transshipment process maintain nonnegative inventory levels.

The problem to be studied here can also be used to model the situation in which differences between product designs, rather than geographical distances between production facilities, prevent the firm from fully utilizing its total production capacity. In such a situation, product substitution can be modeled as transshipments, where the transshipment cost merely represents the compensation paid to a customer who is persuaded to accept a different product than he/she initially desired.

### 3.2 Formulation

Yang and Qin [91] formulated the optimal control of a two-plant version of the aforementioned problem as a stochastic dynamic programming problem. The symbols used in the formulation are as follows:

$\bar{L}_i$ : plant  $i$ 's production leadtime which can be either 0 or 1;

$\bar{c}_{it}$ : plant  $i$ 's nonnegative capacity in period  $t$ ;

$\bar{p}_{it}$ : plant  $i$ 's nonnegative unit production cost in period  $t$ ;

$\bar{q}_{ijt}$ : nonnegative unit transshipment cost from plant  $i$  to plant  $j$  in period  $t$

(it is not necessary that  $\bar{q}_{ijt} = \bar{q}_{jit}$ );

$H_{it}(I)$ : plant  $i$ 's nonnegative convex per-period inventory handling cost in period  $t$  which assumes its minimum at  $H_{it}(0) = 0$ ;

$\alpha$ : the discount factor per period;

$D_{it}$ : random demand level that arises in region  $i$  in period  $t$ —demand levels

across different periods are assumed to be independent, while the levels across different regions need not be so;

$I_{it}$ : plant  $i$ 's inventory position at the beginning of period  $t$ ;

$x_{it}$ : production level at plant  $i$  in period  $t$ ;

$u_{ijt}$ : transshipment level from plant  $i$  to plant  $j$  in period  $t$ .

The details of the model formulation in the paper were as follows:

The convention of numbering periods  $t$  in the backward fashion, with the terminal period being denoted period 0 is adopted. The model was based on the assumption that results after period 0 were inconsequential. For  $i = 1, 2, t = 0, 1, 2, \dots$ , let  $\bar{h}_{it}^\infty = d_I H_{it}(+\infty)$  and  $\bar{b}_{it}^\infty = -d_I H_{it}(-\infty)$ . For  $t = 0, 1, 2, \dots$ , let

$$M_t = \max\{\bar{p}_{1t}, \bar{p}_{2t}, \bar{q}_{12t}, \bar{q}_{21t}, \bar{h}_{1t}^\infty, \bar{b}_{1t}^\infty, \bar{h}_{2t}^\infty, \bar{b}_{2t}^\infty\}.$$

It is assumed that  $M_t < +\infty$  for every  $t$  and  $E[D_{1t}] < +\infty$  and  $E[D_{2t}] < +\infty$ . These assumptions make the definitions of cost functions feasible.

In each period  $t$ , first, for each plant, items whose production is initiated in period  $t + 1$  arrive to the plant, if its production leadtime is 1. Then, after observing the starting inventory positions  $I_{1t}$  and  $I_{2t}$  at the two plants, the firm decides the production levels  $x_{1t} \in [0, \bar{c}_{1t}]$  and  $x_{2t} \in [0, \bar{c}_{2t}]$  at the two plants. For each plant, the items being produced are immediately available, if its production leadtime is 0. Then, demand levels  $D_{1t}$  and  $D_{2t}$  are realized at the two plants, so that the inventory positions at the two plants would become respectively  $I_{1t} + x_{1t} - D_{1t}$  and  $I_{2t} + x_{2t} - D_{2t}$ , if nothing else were to be done. Now the firm decides the virtual transshipment levels:  $u_{12t} \in [0, D_{2t}]$  for the relief of plant-2 demand and  $u_{21t} \in [0, D_{1t}]$  for the relief of plant-1 demand, so that the firm's starting inventory positions at the two plants in period  $t - 1$  will actually turn out to be  $I_{1,t-1} = I_{1t} + x_{1t} - D_{1t} + u_{21t} - u_{12t}$  and  $I_{2,t-1} = I_{2t} + x_{2t} - D_{2t} + u_{12t} - u_{21t}$ , respectively.

Plant  $i$  will mingle together its own undiverted demand and demand being virtually assigned to it from region  $j$ , and use it in a first-come-first-served fashion, with unserved demand being backlogged. Items demanded by region  $j$  and retrieved from plant  $i$ 's inventory will travel a certain nonnegative leadtime  $\bar{l}_{ij}$  to reach their common destination.

Let  $\tilde{g}_t(I_1, I_2)$  be the least possible total discounted expected cost that the firm has to face if at the beginning of period  $t$ , its inventory status is at  $(I_1, I_2)$ . Then, the following recursive relationship when  $\bar{L}_1 = \bar{L}_2 = 0$  can be obtained:

$$\begin{aligned} \tilde{g}_t(I_1, I_2) = & \inf\{\bar{p}_{1t} \cdot x_1 + \bar{p}_{2t} \cdot x_2 + E[\inf\{+\bar{q}_{12t} \cdot u_{12} + \bar{q}_{21t} \cdot u_{21} \\ & + H_{1t}(I_1 + x_1 - D_{1t} + u_{21} - u_{12}) + H_{2t}(I_2 + x_2 - D_{2t} + u_{12} - u_{21}) \\ & + \alpha \tilde{g}_{t-1}(I_1 + x_1 - D_{1t} + u_{21} - u_{12}, I_2 + x_2 - D_{2t} + u_{12} - u_{21}) \\ & | 0 \leq u_{12} \leq D_{2t}, 0 \leq u_{21} \leq D_{1t}\}] | 0 \leq x_1 \leq \bar{c}_{1t}, 0 \leq x_2 \leq \bar{c}_{2t}\}, \end{aligned} \quad (3.1)$$

while the only change needed for the relationship when either  $\bar{L}_i = 1$  is for the corresponding term “ $H_{it}(I_i + x_i - D_{it} + u_{ji} - u_{ij})$ ” in the above to be replaced by “ $H_{it}(I_i - D_{it} + u_{ji} - u_{ij})$ ”. For the terminal condition, it may be assumed that

$$\tilde{g}_0(I_1, I_2) = -\bar{p}_{10} \cdot I_1 - \bar{p}_{20} \cdot I_2, \quad (3.2)$$

which reflects values of the end-of-horizon inventory holdings.

While future derivations hinge upon the assumption that production leadtimes are 0 or 1, the two transshipment leadtimes need not be so, nor do they even have to be integral or equal to each other, since they do not explicitly enter the above formulation. Note that at any moment, items that are being transshipped between the two regions are related to already-executed decisions and hence are out of the model's sight. On the other hand, the transshipment costs should implicitly take into account not only the physical transportation costs between the regions, but also the backlogging costs felt by the receiving ends over the transshipment leadtimes. For

instance, when costs are stationary, and the  $t$ -subscripts can be dropped, the discount factor is almost 1, and region  $j$ 's backlogging cost rate is  $\bar{b}_j$ , it is expected that  $\bar{q}_{ij}$  is larger than  $\bar{l}_{ij} \cdot \bar{b}_j$  so that their difference can still represent the positive cost of transporting an item from region  $i$  to region  $j$ .

For the case where  $\bar{L}_1 = \bar{L}_2 = 0$ , let  $g_t(I_1, I_2) = \tilde{g}_t(I_1, I_2) + \bar{p}_{1t} \cdot I_1 + \bar{p}_{2t} \cdot I_2$ ,  $y_1 = I_1 + x_1$ ,  $y_2 = I_2 + x_2$ ,  $v_{12} = u_{12} + u_{21}$ , and  $w_{12} = u_{12} - u_{21}$ . The newly defined  $g_t(I_1, I_2)$  can be viewed as the present-value cost of having starting inventory levels  $I_1$  and  $I_2$  at the beginning of period  $t$  and including the inventory build-up cost;  $y_1$  and  $y_2$  are the post-production inventory levels at the two plants,  $v_{12}$  is the total absolute virtual transshipment level between the two plants; and  $w_{12}$  is the net virtual transshipment level from plant 1 to plant 2.

Now the following equations can be obtained:

$$\begin{aligned}
g_t(I_1, I_2) = & \alpha \bar{p}_{1,t-1} \cdot E[D_{1t}] + \alpha \bar{p}_{2,t-1} \cdot E[D_{2t}] + \inf\{(\bar{p}_{1t} - \alpha \bar{p}_{1,t-1}) \cdot y_1 \\
& + (\bar{p}_{2t} - \alpha \bar{p}_{2,t-1}) \cdot y_2 + E[\inf\{(\bar{q}_{12t} + \bar{q}_{21t})/2\} \cdot (v_{12} - |w_{12}|)] \\
& + ((\bar{q}_{12t} + \bar{q}_{21t})/2) \cdot |w_{12}| + ((\bar{q}_{12t} - \bar{q}_{21t})/2 + \alpha(\bar{p}_{1,t-1} - \bar{p}_{2,t-1})) \cdot w_{12} + \\
& H_{1t}(y_1 - D_{1t} - w_{12}) + H_{2t}(y_2 - D_{2t} + w_{12}) + \alpha g_{t-1}(y_1 - D_{1t} - w_{12}, y_2 - D_{2t} + w_{12}) \\
& | -D_{1t} \leq w_{12} \leq D_{2t}, v_{12} \geq |w_{12}| \} | I_1 \leq y_1 \leq I_1 + \bar{c}_{1t}, I_2 \leq y_2 \leq I_2 + \bar{c}_{2t} \}.
\end{aligned} \tag{3.3}$$

and

$$g_0(I_1, I_2) = 0. \tag{3.4}$$

Because of the nonnegativity of  $\bar{q}_{12t}$  and  $\bar{q}_{21t}$ , obviously  $v_{12} = |w_{12}|$  is always a solution for (3.3). In other words, the transshipment activity between plants 1 and 2 needs to occur only in one direction; and therefore one variable  $w_{12}$  denotes the entire transshipment activity: a  $w_{12}$  quantity is being transshipped from plant 1 to plant 2 when  $w_{12} \geq 0$  and a  $-w_{12}$  quantity is being transshipped from plant 2 to plant 1

when  $w_{12} \leq 0$ . Moreover, we have

$$\begin{aligned} g_t(I_1, I_2) &= \alpha \bar{p}_{1,t-1} \cdot E[D_{1t}] + \alpha \bar{p}_{2,t-1} \cdot E[D_{2t}] \\ &+ \inf\{g'_t(y_1, y_2) \mid I_1 \leq y_1 \leq I_1 + \bar{c}_{1t}, I_2 \leq y_2 \leq I_2 + \bar{c}_{2t}\}, \end{aligned} \quad (3.5)$$

$$g'_t(y_1, y_2) = (\bar{p}_{1t} - \alpha \bar{p}_{1,t-1}) \cdot y_1 + (\bar{p}_{2t} - \alpha \bar{p}_{2,t-1}) \cdot y_2 + E[g''_t(y_1, y_2, D_{1t}, D_{2t})], \quad (3.6)$$

$$\begin{aligned} g''_t(y_1, y_2, d_1, d_2) &= \inf\{((\bar{q}_{12t} + \bar{q}_{21t})/2) \cdot |w_{12}| + ((\bar{q}_{12t} - \bar{q}_{21t})/2 \\ &+ \alpha(\bar{p}_{1,t-1} - \bar{p}_{2,t-1})) \cdot w_{12} + H_{1t}(y_1 - d_1 - w_{12}) + H_{2t}(y_2 - d_2 + w_{12}) \\ &+ \alpha g_{t-1}(y_1 - d_1 - w_{12}, y_2 - d_2 + w_{12}) \mid -d_1 \leq w_{12} \leq d_2\}. \end{aligned} \quad (3.7)$$

### 3.3 Theoretical Results

Let function  $f(x_1, x_2)$  be an arbitrary real function defined on  $R^2$ . The following definitions concerning supermodularity, diagonal dominance, convexity, mild monotonicity, etc. will be used in Theorems 1, 2, 3, 4 and 5.

**Definition 1** *Function  $f(x_1, x_2)$  is supermodular ( $SP[1, 2]$  or  $SP[2, 1]$ ) if for any  $(x_1, x_2)$ ,  $\Delta_1 \geq 0$ , and  $\Delta_2 \geq 0$ , it follows that*

$$f(x_1, x_2) + f(x_1 + \Delta_1, x_2 + \Delta_2) \geq f(x_1 + \Delta_1, x_2) + f(x_1, x_2 + \Delta_2).$$

**Definition 2** *Function  $f(x_1, x_2)$  is diagonally dominant in its 1st element ( $DD[1]$ ) if for any  $(x_1, x_2)$ ,  $\Delta_1 \geq 0$ , and  $\Delta \geq 0$ , it follows that*

$$f(x_1, x_2) + f(x_1 + \Delta + \Delta_1, x_2 - \Delta) \geq f(x_1 + \Delta, x_2 - \Delta) + f(x_1 + \Delta_1, x_2).$$

**Definition 3** *For any subset  $S$  of  $\{1, 2, \dots, n\}$ , we say function  $f(\mathbf{x})$ , defined on  $R^n$ , is  $CV[S]$  when it is jointly convex over the  $x_i$ 's for  $i \in S$ .*

**Definition 4** *Function  $y(\mathbf{x})$  mildly increases [decreases] in its  $i$ th element ( $MI[i]$  [ $MI[-i]$ ]) if for any  $\Delta \geq [\leq] 0$ , it follows that*

$$y(\mathbf{x}) \leq y(\mathbf{x} + \Delta \mathbf{e}^i) \leq y(\mathbf{x}) + |\Delta|.$$

**Definition 5** *Function  $y(\mathbf{x})$  is mildly shifting while increasing its  $i$ th element at the expense of its  $j$ th element ( $MS[i, -j]$  or  $MS[-j, i]$ ) if for any  $\Delta \geq 0$ , it follows that*

$$y(\mathbf{x} + \Delta \mathbf{e}^i - \Delta \mathbf{e}^j) \leq y(\mathbf{x}) + \Delta.$$

Let  $w_{12t}^*(y_1, y_2, d_1, d_2)$  be the optimal solution for the right-hand side of (3.7), and  $y_{1t}^*(I_1, I_2)$  and  $y_{2t}^*(I_1, I_2)$  a pair of optimal solutions for the right-hand side of (3.5). Structural properties for the optimal production and transshipment policies are established as shown in Theorem 1.

**Theorem 1** *For  $t = 0, 1, 2, \dots$ ,  $g_t(I_1, I_2)$  is  $SP[1, 2]$ ,  $DD[1]$ , and  $DD[2]$ . Consequently, the optimal policies can be selected so that, for  $t = 1, 2, \dots$ , the following are true. The optimal transshipment level  $w_{12t}^*(y_1, y_2, d_1, d_2)$  is  $MI[1]$ ,  $MI[-2]$ ,  $MI[-3]$ ,  $MI[4]$ ,  $MS[1, -2]$ , and  $MS[-3, 4]$ . Also, there are  $MI[-1]$  functions  $Y_{1t}^*(x)$  and  $Y_{2t}^*(x)$ ; the curves  $y_2 = Y_{2t}^*(y_1)$  and  $y_1 = Y_{1t}^*(y_2)$  intersect at  $(\bar{y}_{1t}^{**}, \bar{y}_{2t}^{**})$ ; and the optimal produce-up-to level  $y_{1t}^*(I_1, I_2)$  indicates that plant 1 should follow a modified base stock policy in observance of the capacity level  $\bar{c}_{1t}$ , while the base level is*

$$\begin{cases} Y_{1t}^*(I_2 + \bar{c}_{2t}) & \text{when } I_2 \leq \bar{y}_{2t}^{**} - \bar{c}_{2t}, \\ \bar{y}_{1t}^{**} & \text{when } \bar{y}_{2t}^{**} - \bar{c}_{2t} \leq I_2 \leq \bar{y}_{2t}^{**}, \\ Y_{1t}^*(I_2) & \text{when } I_2 \geq \bar{y}_{2t}^{**}; \end{cases}$$

*while the optimal produce-up-to level  $y_{2t}^*(I_1, I_2)$  is symmetrically determined. In particular,  $y_{1t}^*(I_1, I_2)$  is  $MI[1]$  and  $MI[-2]$ , while  $y_{2t}^*(I_1, I_2)$  is  $MI[-1]$  and  $MI[2]$ .*

From Theorem 1, it is clear that the actions are all mildly monotone in the inventory and demand levels that they are contingent upon; and the optimal production policy for each plant is of the modified base stock type in observance of its own capacity level, while the base level mildly decreases in the other plant's starting inventory level. In addition, the optimal transshipment policy can be implemented in an item-by-item fashion when demand is discrete.

All the results of Theorem 1 can be extended to the stationary infinite-horizon discounted-cost case. Theorems 2, 3, 4 and 5 give the details of the optimal structural policy for the stationary infinite-horizon discounted-cost case.

**Theorem 2** *For any fixed  $I_1$  and  $I_2$ , there exists some  $g_\infty(I_1, I_2)$  to which  $g_t(I_1, I_2)$  converges as  $t$  tends to  $+\infty$ . The convergence is also uniform in any bounded region of the  $(I_1, I_2)$ -space. In particular, there are  $\bar{A}(t)$  and  $\bar{B}(t)$  with  $\lim_{t \rightarrow +\infty} \bar{A}(t) = \lim_{t \rightarrow +\infty} \bar{B}(t) = 0$  such that*

$$|g_t(I_1, I_2) - g_\infty(I_1, I_2)| \leq \bar{A}(t) \cdot (|I_1| + |I_2|) + \bar{B}(t).$$

Next, it is showed that the limit function  $g_\infty(I_1, I_2)$  solves (3.3) with it being on both sides in the places of  $g_t(I_1, I_2)$  and  $g_{t-1}(I_1, I_2)$ , respectively. Define  $g_t''(y_1, y_2, d_1, d_2)$  so that it equals the right-hand side of (3.7) with  $g_{t-1}(I_1, I_2)$  being replaced by  $g_\infty(I_1, I_2)$ , and define  $g_\infty'(y_1, y_2)$  so that it equals the right-hand side of (3.6) with  $g_t''(y_1, y_2, d_1, d_2)$  being replaced by  $g_\infty''(y_1, y_2, d_1, d_2)$ .

**Theorem 3** *As  $t$  tends to  $+\infty$ ,  $g_t''(y_1, y_2, d_1, d_2)$  converges to  $g_\infty''(y_1, y_2, d_1, d_2)$ , and the convergence is uniform in any bounded  $(y_1, y_2, d_1, d_2)$ -region;  $g_t'(y_1, y_2)$  converges to  $g_\infty'(y_1, y_2)$ , and the convergence is uniform in any bounded  $(y_1, y_2)$ -region. Also,  $g_\infty(I_1, I_2)$  is equal to the right-hand side of (3.5) with  $g_t'(y_1, y_2)$  being replaced by  $g_\infty'(y_1, y_2)$ .*

According to Theorem 3, it is easy to show that

$$\begin{aligned} g_\infty(I_1, I_2) &= \alpha \bar{p}_1 \cdot E[D_1] + \alpha \bar{p}_2 \cdot E[D_2] \\ &+ \inf\{g_\infty'(y_1, y_2) \mid I_1 \leq y_1 \leq I_1 + \bar{c}_1, I_2 \leq y_2 \leq I_2 + \bar{c}_2\}, \end{aligned} \tag{3.8}$$

$$g_\infty'(y_1, y_2) = (1 - \alpha) \bar{p}_1 \cdot y_1 + (1 - \alpha) \bar{p}_2 \cdot y_2 + E[g_\infty''(y_1, y_2, D_1, D_2)], \tag{3.9}$$

$$\begin{aligned}
g''_{\infty}(y_1, y_2, d_1, d_2) = & \inf\{((\bar{q}_{12} + \bar{q}_{21})/2) \cdot |w_{12}| + ((\bar{q}_{12} - \bar{q}_{21})/2 + \alpha(\bar{p}_1 - \bar{p}_2)) \cdot w_{12} \\
& + H_1(y_1 - d_1 - w_{12}) + H_2(y_2 - d_2 + w_{12}) + \alpha g_{\infty}(y_1 - d_1 - w_{12}, y_2 - d_2 + w_{12}) \\
& | -d_1 \leq w_{12} \leq d_2 \}.
\end{aligned} \tag{3.10}$$

Let  $w_{12\infty}^*(y_1, y_2, d_1, d_2)$  be an optimal solution for the right-hand side of (3.10), and  $y_{1\infty}^*(I_1, I_2)$  and  $y_{2\infty}^*(I_1, I_2)$  a pair of optimal solutions for the right-hand side of (3.8). Theorem 4 shows that the preservation result for  $g_t(I_1, I_2)$  through a (3.3)-like operator as stated in Theorem 1 applies to  $g_{\infty}(I_1, I_2)$  as well and thus the results for the infinite-horizon optimal policies are similar to those for the finite-horizon policies.

**Theorem 4**  $g_{\infty}(I_1, I_2)$  is  $SP[1, 2]$ ,  $DD[1]$ , and  $DD[2]$ . Consequently, the optimal policies for the infinite-horizon problem can be selected so that the following are true. The optimal transshipment level  $w_{12\infty}^*(y_1, y_2, d_1, d_2)$  is  $MI[1]$ ,  $MI[-2]$ ,  $MI[-3]$ ,  $MI[4]$ ,  $MS[1, -2]$ , and  $MS[-3, 4]$ ; Also, there are  $MI[-1]$  functions  $Y_{1\infty}^*(x)$  and  $Y_{2\infty}^*(x)$ ; the curves  $y_2 = Y_{2\infty}^*(y_1)$  and  $y_1 = Y_{1\infty}^*(y_2)$  intersect at  $(\bar{y}_{1\infty}^{**}, \bar{y}_{2\infty}^{**})$ ; and the optimal produce-up-to level  $y_{1\infty}^*(I_1, I_2)$  indicates that plant 1 should follow a modified base stock policy in observance of the capacity level  $\bar{c}_1$ , while the base level is

$$\begin{cases} Y_{1\infty}^*(I_2 + \bar{c}_2) & \text{when } I_2 \leq \bar{y}_{2\infty}^{**} - \bar{c}_2, \\ \bar{y}_{1\infty}^{**} & \text{when } \bar{y}_{2\infty}^{**} - \bar{c}_2 \leq I_2 \leq \bar{y}_{2\infty}^{**}, \\ Y_{1\infty}^*(I_2) & \text{when } I_2 \geq \bar{y}_{2\infty}^{**}; \end{cases}$$

while the optimal produce-up-to level  $y_{2\infty}^*(I_1, I_2)$  is symmetrically determined. In particular,  $y_{1\infty}^*(I_1, I_2)$  is  $MI[1]$  and  $MI[-2]$ , while  $y_{2\infty}^*(I_1, I_2)$  is  $MI[-1]$  and  $MI[2]$ .

Theorem 5 shows that the infinite-horizon optimal policies can be obtained by taking limits of the finite-horizon optimal policies equally as well as obtaining the infinite-horizon cost function first and then finding the optimal solutions for the right-hand side of (3.3) with this cost function replacing its finite-horizon counterpart.



**Theorem 5** *Any limit point  $w_{12\infty}^*(y_1, y_2, d_1, d_2)$  of the sequence*

$$\{w_{12t}^*(y_1, y_2, d_1, d_2) \mid t = 1, 2, \dots\}$$

*is an optimal solution for the right-hand side of (3.10), and any limit pair of points  $(y_{1,\infty}^*(I_1, I_2), y_{2,\infty}^*(I_1, I_2))$  of the sequence  $\{(y_{1t}^*(I_1, I_2), y_{2t}^*(I_1, I_2)) \mid t = 1, 2, \dots\}$  constitutes a pair of optimal solutions for the right-hand side of (3.8).*

The detailed proofs can be found in Yang and Qin [91]. The above model and structural properties of the optimal policies will serve as the starting point and guidelines of the computational study that follows.

### 3.4 Computational Settings

A computational study is conducted that illustrates the benefit of virtual lateral transshipment. The study is based on the theoretical results obtained by Yang and Qin [91], and determines the impacts of the parameters on the cost savings that can be achieved by using virtual lateral transshipment.

For the cases that are studied here, demand is discrete and the parameters are stationary. Hence, the  $t$  signs will be suppressed whenever possible. Symmetry is also assumed, that is to say, the values of all the parameters at the two plants are equal. Therefore, the subscripts used for plant identification are also suppressed. In addition, a V-shaped inventory handling cost is assumed:  $H(I) = \bar{h}.I^+ + \bar{b}.I^-$ . As a default, the discount factor  $\alpha = 0.99$ , the capacity level  $\bar{c} = 7$ , the unit production cost  $\bar{p} = 30.0$ , the holding cost rate  $\bar{h} = 1.0$ , and the backlogging cost rate  $\bar{b} = 4.0$ .

Each discrete demand distribution used can be described by three parameters: the minimum demand level  $\bar{u}$ , the maximum demand level  $\bar{v}$ , and a parameter  $\beta$  which indicates the correlation between the demand levels in the two regions. Suppose  $D_1$  and  $D_2$  represent the stochastic demand in the two regions, respectively. Then for

any  $d_1, d_2 = \bar{u}, \bar{u} + 1, \dots, \bar{v} - 1, \bar{v}$ , it is assumed that

$$P(D_1 = d_1, D_2 = d_2) = \beta^{\bar{v}-\bar{u}-|d_1-d_2|}/\sigma, \quad (3.11)$$

where, as a normalizing factor,

$$\sigma = (\bar{v} - \bar{u} + 1)\beta^{\bar{v}-\bar{u}} + 2 \sum_{n=1}^{\bar{v}-\bar{u}} n\beta^{n-1}. \quad (3.12)$$

The correlation between the two demand levels increases with  $\beta$ . When  $\beta > 1$ , the two levels are positively correlated; when  $0 < \beta < 1$ , the two levels are negatively correlated; while when  $\beta = 1$ , the two levels are independent of each other. As a default, let  $\bar{u} = 0$ ,  $\bar{v} = 12$ , and  $\beta = 1$ . A representation of the demand distribution is shown in (3.13).

$$P(d_1, d_2) = \begin{pmatrix} & d_2 = \bar{u} & \bar{u} + 1 & \bar{u} + 2 & \dots & \bar{v} - 1 & \bar{v} \\ d_1 = \bar{u} & \beta^{\bar{v}-\bar{u}}/\sigma & \beta^{\bar{v}-\bar{u}-1}/\sigma & \beta^{\bar{v}-\bar{u}-2}/\sigma & \dots & \beta/\sigma & 1/\sigma \\ \bar{u} + 1 & \beta^{\bar{v}-\bar{u}-1}/\sigma & \beta^{\bar{v}-\bar{u}}/\sigma & \beta^{\bar{v}-\bar{u}-1}/\sigma & \dots & \beta^2/\sigma & \beta/\sigma \\ \bar{u} + 2 & \beta^{\bar{v}-\bar{u}-2}/\sigma & \beta^{\bar{v}-\bar{u}-1}/\sigma & \beta^{\bar{v}-\bar{u}}/\sigma & \dots & \beta^3/\sigma & \beta^2/\sigma \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \bar{v} - 1 & \beta/\sigma & \beta^2/\sigma & \beta^3/\sigma & \dots & \beta^{\bar{v}-\bar{u}}/\sigma & \beta^{\bar{v}-\bar{u}-1}/\sigma \\ \bar{v} & 1/\sigma & \beta/\sigma & \beta^2/\sigma & \dots & \beta^{\bar{v}-\bar{u}-1}/\sigma & \beta^{\bar{v}-\bar{u}}/\sigma \end{pmatrix} \quad (3.13)$$

All the cases that are studied are variants of the default case where all parameters are set at the default values. For every specific case, brute-force dynamic programming is used to find the optimal total expected discounted operational cost.

A quadratic extrapolation of the cost function is used to get around the problem of having an infinite number of states. Let  $I_1$  and  $I_2$  stand for the inventory positions at the two plants, respectively. In the computation, a constant  $\bar{I}$  is fixed and let the range of each  $I_i$  be from  $-\bar{I}$  to  $+\bar{I}$  although in theory, the range of the states' values is from  $-\infty$  to  $+\infty$ . Let  $g_0(I_1, I_2)$  stand for the initial value of the cost

function at the state of  $(I_1, I_2)$ . In each iteration of computation, only the values of the cost function at the state of  $(I_1, I_2)$  where both  $I_1$  and  $I_2 \in [-\bar{I}, \bar{I}]$  can be gotten. When it is no longer the case that both  $I_1$  and  $I_2 \in [-\bar{I}, \bar{I}]$ , the following rules are adopted to get the values of  $g_0(I_1, I_2)$  (see Figure 3.1).

1) When  $I_1 < -\bar{I}$  and  $-\bar{I} \leq I_2 \leq \bar{I}$ ,

$$g_0(I_1, I_2) = g_0(-\bar{I}, I_2) + (g_0(-\bar{I}, I_2) - g_0(-\bar{I} + 1, I_2))(-\bar{I} - I_1) \\ + 1/2(I_1 + \bar{I})(I_1 + \bar{I} - 1)(g_0(-\bar{I}, I_2) - 2g_0(1 - \bar{I}, I_2) + g_0(2 - \bar{I}, I_2)).$$

2) When  $I_1 > \bar{I}$  and  $-\bar{I} \leq I_2 \leq \bar{I}$ ,

$$g_0(I_1, I_2) = g_0(\bar{I}, I_2) + (g_0(\bar{I}, I_2) - g_0(\bar{I} - 1, I_2))(I_1 - 2\bar{I}) + 1/2(I_1 - \bar{I}) \\ (I_1 - \bar{I} + 1)(g_0(\bar{I}, I_2) - 2g_0(\bar{I} - 1, I_2) + g_0(\bar{I} - 2, I_2)).$$

3) When  $-\bar{I} \leq I_1 \leq \bar{I}$  and  $I_2 < -\bar{I}$ ,

$$g_0(I_1, I_2) = g_0(I_1, -\bar{I}) + (g_0(I_1, -\bar{I}) - g_0(I_1, 1 - \bar{I}))(-\bar{I} - I_2) + 1/2(I_2 + \bar{I}) \\ (I_2 + \bar{I} - 1)(g_0(I_1, -\bar{I}) - 2g_0(I_1, 1 - \bar{I}) + g_0(I_1, 2 - \bar{I})).$$

4) When  $-\bar{I} \leq I_1 \leq \bar{I}$  and  $I_2 > \bar{I}$ ,

$$g_0(I_1, I_2) = g_0(I_1, \bar{I}) + (g_0(I_1, \bar{I}) - g_0(I_1, \bar{I} - 1))(I_2 - \bar{I}) + 1/2(I_2 - \bar{I})(I_2 - \bar{I} + 1) \\ (g_0(I_1, \bar{I}) - 2g_0(I_1, \bar{I} - 1) + g_0(I_1, \bar{I} - 2)).$$

5) When  $I_1 < -\bar{I}$  and  $I_2 < -\bar{I}$ ,

$$g_0(I_1, I_2) = g_0(-\bar{I}, -\bar{I}) + (g_0(-\bar{I}, -\bar{I}) - g_0(1 - \bar{I}, -\bar{I}))(-I_1 - \bar{I}) \\ + (g_0(-\bar{I}, -\bar{I}) - g_0(-\bar{I}, 1 - \bar{I}))(-I_2 - \bar{I}) + 1/2(I_1 + \bar{I})(I_1 + \bar{I} - 1) \\ (g_0(-\bar{I}, -\bar{I}) - 2g_0(1 - \bar{I}, -\bar{I}) + g_0(2 - \bar{I}, -\bar{I})) + 1/2(I_2 + \bar{I}) \\ (I_2 + \bar{I} - 1)(g_0(-\bar{I}, -\bar{I}) - 2g_0(-\bar{I}, 1 - \bar{I}) + g_0(-\bar{I}, 2 - \bar{I})) + (I_1 + \bar{I}) \\ (I_2 + \bar{I})(g_0(-\bar{I}, -\bar{I}) + g_0(1 - \bar{I}, 1 - \bar{I}) - g_0(1 - \bar{I}, -\bar{I}) - g_0(-\bar{I}, 1 - \bar{I})).$$

6) When  $I_1 > \bar{I}$  and  $I_2 > \bar{I}$ ,

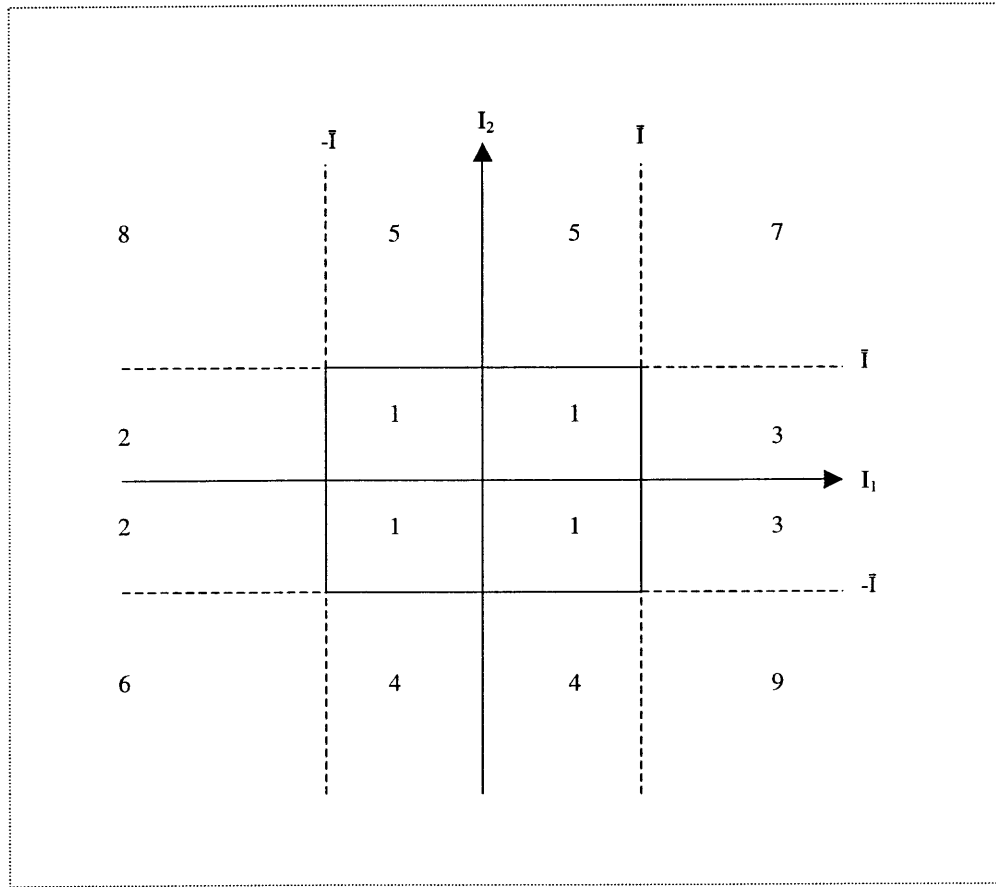
$$\begin{aligned}
g_0(I_1, I_2) = & g_0(\bar{I}, \bar{I}) + (g_0(\bar{I}, \bar{I}) - g_0(\bar{I}, \bar{I} - 1))(I_2 - \bar{I}) + (g_0(\bar{I}, \bar{I}) \\
& - g_0(\bar{I} - 1, \bar{I}))(I_1 - \bar{I}) + 1/2(I_1 - \bar{I})(I_1 - \bar{I} + 1)(g_0(\bar{I}, \bar{I}) \\
& - 2g_0(\bar{I} - 1, \bar{I}) + g_0(\bar{I} - 2, \bar{I})) + 1/2(I_2 - \bar{I})(I_2 - \bar{I} + 1)(g_0(\bar{I}, \bar{I}) \\
& - 2g_0(\bar{I}, \bar{I} - 1) + g_0(\bar{I}, \bar{I} - 2)) + (I_1 - \bar{I})(I_2 - \bar{I})(g_0(\bar{I}, \bar{I}) \\
& + g_0(\bar{I} - 1, \bar{I} - 1) - g_0(\bar{I} - 1, \bar{I}) - g_0(\bar{I}, \bar{I} - 1)).
\end{aligned}$$

7) When  $I_1 < -\bar{I}$  and  $I_2 > \bar{I}$ ,

$$\begin{aligned}
g_0(I_1, I_2) = & g_0(-\bar{I}, \bar{I}) + (g_0(-\bar{I}, \bar{I}) - g_0(-\bar{I}, \bar{I} - 1))(I_2 - \bar{I}) \\
& + (g_0(-\bar{I}, \bar{I}) - g_0(1 - \bar{I}, \bar{I}))(-I_1 - \bar{I}) \\
& + 1/2(I_1 + \bar{I})(I_1 + \bar{I} - 1)(g_0(-\bar{I}, \bar{I}) - 2g_0(1 - \bar{I}, \bar{I}) + g_0(2 - \bar{I}, \bar{I})) \\
& + 1/2(I_2 - \bar{I})(I_2 - \bar{I} + 1)(g_0(-\bar{I}, \bar{I}) - 2g_0(-\bar{I}, \bar{I} - 1) + g_0(-\bar{I}, \bar{I} - 2)) \\
& + (I_1 + \bar{I})(I_2 - \bar{I})(g_0(-\bar{I}, \bar{I}) + g_0(1 - \bar{I}, \bar{I} - 1) - g_0(1 - \bar{I}, \bar{I}) - g_0(-\bar{I}, \bar{I} - 1)).
\end{aligned}$$

8) When  $I_1 > \bar{I}$  and  $I_2 < -\bar{I}$ ,

$$\begin{aligned}
g_0(I_1, I_2) = & g_0(\bar{I}, -\bar{I}) + (g_0(\bar{I}, -\bar{I}) - g_0(\bar{I} - 1, -\bar{I}))(I_1 - \bar{I}) \\
& + (g_0(\bar{I}, -\bar{I}) - g_0(\bar{I}, 1 - \bar{I}))(-I_2 - \bar{I}) \\
& + 1/2(I_1 - \bar{I})(I_1 - \bar{I} + 1)(g_0(\bar{I}, -\bar{I}) - 2g_0(\bar{I} - 1, -\bar{I}) + g_0(\bar{I} - 2, -\bar{I})) \\
& + 1/2(I_2 + \bar{I})(I_2 + \bar{I} - 1)(g_0(\bar{I}, -\bar{I}) - 2g_0(\bar{I}, 1 - \bar{I}) + g_0(\bar{I}, 2 - \bar{I})) + (I_1 - \bar{I}) \\
& (I_2 + \bar{I})(g_0(\bar{I}, -\bar{I}) + g_0(\bar{I} - 1, 1 - \bar{I}) - g_0(\bar{I} - 1, -\bar{I}) - g_0(\bar{I}, 1 - \bar{I})).
\end{aligned}$$



**Figure 3.1** The distribution of  $I_1$  and  $I_2$  over  $(-\infty, +\infty)$

In the course of the computation, let the constant  $\bar{I} = 25$ . The computational results verify that the previous theoretical results are all correct.

At a specific unit transshipment cost  $\bar{q}$ , suppose the minimal cost at the state  $(0, 0)$  is  $c(\bar{q})$ . Then the term  $\eta\% = (c(+\infty) - c(\bar{q}_0))/c(+\infty)$  is defined as the percent cost saving that can be achieved by transshipment at a unit cost  $\bar{q}_0$ , since the case with  $\bar{q} = +\infty$  is effectively the one where transshipment is not allowed.

### 3.5 Computational Results

The computational results are presented in three tables. Table 3.1 contains the percent cost saving ( $\eta$ ) results for cases with varying  $\bar{u}$ ,  $\bar{v}$ , and  $\bar{q}$  values, while all other parameters are set at their default values; Table 3.2 contains the percent cost saving

results for cases with varying  $\bar{h}/\bar{p}$ ,  $\bar{b}/\bar{p}$ , and  $\bar{q}/\bar{p}$  values, while all other parameters are set at their default values; and Table 3.3 contains the percent cost saving results for cases with varying  $\beta$ 's and  $\bar{q}$ 's, while all other parameters are set at their default values.

**Table 3.1** The  $\eta$  Values under Different  $\bar{u}$ 's,  $\bar{v}$ 's and  $\bar{q}$ 's

$(\bar{u}, \bar{v})$	$\bar{q} = 0.0$	1.0	2.0	5.0	10.0	20.0	50.0	100.0	$+\infty$
(0, 3)	1.33	1.07	0.80	0.00	0.00	0.00	0.00	0.00	0.00
(0, 6)	1.07	0.83	0.58	0.00	0.00	0.00	0.00	0.00	0.00
(0, 9)	1.16	0.92	0.69	0.16	0.00	0.00	0.00	0.00	0.00
(0, 12)	2.96	2.76	2.58	2.04	1.38	0.73	0.13	0.00	0.00
(3, 3)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
(3, 6)	0.46	0.36	0.27	0.00	0.00	0.00	0.00	0.00	0.00
(3, 9)	0.88	0.77	0.65	0.37	0.09	0.02	0.00	0.00	0.00
(6, 6)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
(1,10)	1.32	1.14	0.97	0.50	0.15	0.02	0.00	0.00	0.00
(3,8)	0.55	0.43	0.31	0.07	0.00	0.00	0.00	0.00	0.00
(5,6)	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
(0, 13)	5.33	5.16	5.01	4.57	3.95	3.07	1.64	0.60	0.00
(3, 10)	2.06	1.95	1.85	1.55	1.14	0.70	0.20	0.02	0.00
(6, 7)	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

As expected, it is obvious from the tables that the percent cost saving  $\eta$  decreases to 0 as the unit transshipment cost  $\bar{q}$  increases to  $+\infty$ . When all parameters are at their default values, it is reasonable for  $\bar{q} = 5$ . In this case, it is shown that with transshipment the firm can achieve a saving of 2.04%, a substantial one in a competitive environment.

**Table 3.2** The  $\eta$  Values at Different  $\bar{h}/\bar{p}$ 's,  $\bar{b}/\bar{p}$ 's, and  $\bar{q}/\bar{p}$ 's

$(\bar{h}/\bar{p}, \bar{b}/\bar{p})$	$\bar{q}/\bar{p} = 0$	1/30	1/15	1/6	1/3	2/3	5/3	10/3	$+\infty$
(1/60, 1/60)	0.37	0.13	0.04	0.00	0.00	0.00	0.00	0.00	0.00
(1/60, 1/30)	0.91	0.64	0.40	0.15	0.04	0.00	0.00	0.00	0.00
(1/60, 1/15)	1.58	1.36	1.15	0.68	0.36	0.11	0.00	0.00	0.00
(1/60, 1/10)	2.06	1.87	1.70	1.22	0.74	0.34	0.03	0.00	0.00
(1/60, 2/15)	2.45	2.28	2.12	1.68	1.14	0.61	0.12	0.00	0.00
(1/30, 1/30)	1.07	0.79	0.52	0.19	0.04	0.00	0.00	0.00	0.00
(1/30, 1/15)	1.86	1.61	1.38	0.78	0.40	0.12	0.00	0.00	0.00
(1/30, 2/15)	2.96	2.76	2.58	2.04	1.38	0.73	0.13	0.00	0.00
(1/30, 1/5)	3.74	3.57	3.41	2.95	2.27	1.41	0.45	0.08	0.00
(1/30, 4/15)	4.38	4.22	4.08	3.67	3.03	2.10	0.88	0.23	0.00
(1/15, 1/15)	2.28	2.01	1.75	1.00	0.50	0.14	0.00	0.00	0.00
(1/15, 2/15)	3.60	3.37	3.15	2.51	1.64	0.88	0.16	0.00	0.00
(1/15, 4/15)	5.45	5.26	5.09	4.59	3.81	2.59	1.07	0.27	0.00
(1/15, 2/5)	6.71	6.56	6.41	5.99	5.31	4.11	2.14	0.87	0.00
(1/15, 8/15)	7.72	7.59	7.46	7.08	6.49	5.40	3.21	1.61	0.00

**Table 3.3** The  $\eta$  Values at Different  $\beta$ 's and  $\bar{q}$ 's

$\beta$	$\bar{q} = 0.0$	1.0	2.0	5.0	10.0	20.0	50.0	100.0	$+\infty$
0.05	13.87	12.45	11.04	8.77	6.46	4.44	1.68	0.20	0.00
0.2	13.14	11.80	10.52	8.36	6.12	4.21	1.61	0.14	0.00
0.333	12.18	10.99	9.81	7.75	5.61	3.84	1.41	0.10	0.00
0.5	10.50	9.49	8.53	6.68	4.74	3.15	1.06	0.07	0.00
0.667	7.98	7.30	6.66	5.18	3.60	2.24	0.66	0.04	0.00
0.8	5.67	5.25	4.85	3.81	2.60	1.51	0.36	0.02	0.00
1.0	2.96	2.76	2.58	2.04	1.38	0.73	0.13	0.00	0.00
1.25	1.35	1.25	1.16	0.92	0.59	0.28	0.04	0.00	0.00
1.5	0.72	0.67	0.62	0.47	0.28	0.12	0.01	0.00	0.00
2.0	0.31	0.29	0.26	0.19	0.10	0.04	0.00	0.00	0.00
3.0	0.13	0.12	0.11	0.07	0.03	0.00	0.00	0.00	0.00
5.0	0.06	0.05	0.05	0.03	0.01	0.00	0.00	0.00	0.00
20.0	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00



It can be seen in Table 3.1 that, at the same average demand level, the use of transshipment can result in larger cost savings when demand deviations are larger. It can also be observed that, when the total plant utilization level is close to 1, that is, when the sum of the average demand levels in the two regions is very close to the total capacity level at the two plants, the option of transshipment becomes extremely valuable. A way of explaining the above is that, when demand levels fluctuate very wildly or when the system is running close to its total capacity, there will be many opportunities for the two plants to help out each other.

It can be seen in Table 3.2 that, inventory handling costs that are larger in comparison to the production cost lead to larger savings from transshipment. This is because relatively larger inventory handling costs widen the relative gap in costs resulting from adopting different production/transshipment decisions.

The results of Table 3.3 indicate that, the more negatively correlated the demand levels at the two plants are, the larger the cost savings that transshipment can achieve. This is because there are more occasions for it to be profitable for the two plants to help out each other when they face more negatively correlated demand levels. It should be speculated that this trend will be more prominent when there are positive auto-correlations between demand levels in different periods at any single location, since then the pattern of the capacity utilization level at one plant being above while that at the other being below the average level will be sustained for even longer periods of time.

### **3.6 Comparison between Virtual and Real Transshipments**

Real transshipment, the transportation of items between inventories, can also be used to help out the plants when some plants are overstocked and others are understocked. In real transshipment, there are two cases:

- 1) Transshipment is made before demand happens;

For this case, the counterpart of (3.7) is

$$g_t''(y_1, y_2, d_1, d_2) = \inf\{((\bar{q}_{12t} + \bar{q}_{21t})/2) \cdot |w_{12}| + ((\bar{q}_{12t} - \bar{q}_{21t})/2 + \alpha(\bar{p}_{1,t-1} - \bar{p}_{2,t-1})) \cdot w_{12} + H_{1t}(y_1 - d_1 - w_{12}) + H_{2t}(y_2 - d_2 + w_{12}) + \alpha g_{t-1}(y_1 - d_1 - w_{12}, y_2 - d_2 + w_{12}) | - \max\{y_2, 0\} \leq w_{12} \leq \max\{y_1, 0\}\}.$$

2) Transshipment is made after demand happens.

For this case, the counterpart of (3.7) is:

$$g_t''(y_1, y_2, d_1, d_2) = \inf\{((\bar{q}_{12t} + \bar{q}_{21t})/2) \cdot |w_{12}| + ((\bar{q}_{12t} - \bar{q}_{21t})/2 + \alpha(\bar{p}_{1,t-1} - \bar{p}_{2,t-1})) \cdot w_{12} + H_{1t}(y_1 - d_1 - w_{12}) + H_{2t}(y_2 - d_2 + w_{12}) + \alpha g_{t-1}(y_1 - d_1 - w_{12}, y_2 - d_2 + w_{12}) | - \max\{(y_2 - d_2), 0\} \leq w_{12} \leq \max\{(y_1 - d_1), 0\}\}.$$

For both cases, the corresponding equations (3.5) and (3.6) are the same as the ones in virtual transshipment.

To compare virtual and real transshipments, it is assumed that real and virtual transshipments use the same transshipment costs of  $\bar{q}_{12t}$  and  $\bar{q}_{21t}$ . The ratios of  $100(\text{minimal cost})/(\text{minimal cost without transshipment})$  at the state (0,0) are computed for each case under the same environment, based on the corresponding equations. The results are presented in Tables 3.4 to 3.9, where  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  stand for the rates under virtual transshipment, real transshipment made before demand happens and real transshipment made after demand happens, respectively.

Table 3.4 shows the percent ratios under varying  $\beta$ 's with all other parameters being set at their default values; Table 3.5 shows the percent ratios under varying  $\beta$ 's and  $\bar{u}$ 's with all other parameters being set at their default values; Table 3.6 shows the percent ratios under varying  $\beta$ 's and  $\bar{v}$ 's with all other parameters being set at their default values; Table 3.7 shows the percent ratios under varying  $\beta$ 's and  $\alpha$ 's with all other parameters being set at their default values; Table 3.8 shows the percent ratios under varying  $\beta$ 's and  $\bar{h}$ 's with all other parameters being set at their default

values; and Table 3.9 shows the percent ratios under varying  $\beta$ 's and  $\bar{q}$ 's with all other parameters being set at their default values.

**Table 3.4** The  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  Values under Different  $\beta$ 's

$\beta$	$\alpha$	$\bar{q}$	$\bar{b}$	$\bar{h}$	$\bar{u}$	$\bar{v}$	$R_v$	$R_{1r}$	$R_{2r}$
0.05	0.99	5.0	4.0	1.0	0	12	91.23	91.23	91.23
0.2	0.99	5.0	4.0	1.0	0	12	91.64	91.64	91.64
0.333	0.99	5.0	4.0	1.0	0	12	92.25	92.25	92.25
0.5	0.99	5.0	4.0	1.0	0	12	93.32	93.32	93.32
0.667	0.99	5.0	4.0	1.0	0	12	94.82	94.82	94.82
0.8	0.99	5.0	4.0	1.0	0	12	96.19	96.19	96.19
1.0	0.99	5.0	4.0	1.0	0	12	97.96	97.95	97.95
1.25	0.99	5.0	4.0	1.0	0	12	99.08	99.08	99.08
1.5	0.99	5.0	4.0	1.0	0	12	99.53	99.52	99.52
2.0	0.99	5.0	4.0	1.0	0	12	99.81	99.80	99.80
3.0	0.99	5.0	4.0	1.0	0	12	99.93	99.92	99.92
5.0	0.99	5.0	4.0	1.0	0	12	99.97	99.97	99.97
20.0	0.99	5.0	4.0	1.0	0	12	100.00	99.99	99.99

**Table 3.5** The  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  Values under Different  $\beta$ 's and  $\bar{u}$ 's

$\beta$	$\alpha$	$\bar{q}$	$\bar{b}$	$\bar{h}$	$\bar{u}$	$\bar{v}$	$R_v$	$R_{1r}$	$R_{2r}$
0.5	0.99	5.0	4.0	1.0	0	12	93.32	93.32	93.32
0.5	0.99	5.0	4.0	1.0	4	12	92.50	92.50	92.50
0.5	0.99	5.0	4.0	1.0	6	12	100.00	100.00	100.00
1.0	0.99	5.0	4.0	1.0	0	12	97.96	97.95	97.95
1.0	0.99	5.0	4.0	1.0	4	12	99.54	99.54	99.54
1.0	0.99	5.0	4.0	1.0	6	12	100.00	100.00	100.00
1.5	0.99	5.0	4.0	1.0	0	12	99.53	99.52	99.52
1.5	0.99	5.0	4.0	1.0	4	12	99.84	99.84	99.84
1.5	0.99	5.0	4.0	1.0	6	12	100.00	100.00	100.00

**Table 3.6** The  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  Values under Different  $\beta$ 's and  $\bar{v}$ 's

$\beta$	$\alpha$	$\bar{q}$	$\bar{b}$	$\bar{h}$	$\bar{u}$	$\bar{v}$	$R_v$	$R_{1r}$	$R_{2r}$
0.5	0.99	5.0	4.0	1.0	0	8	99.91	99.91	99.91
0.5	0.99	5.0	4.0	1.0	0	10	98.63	98.63	98.63
0.5	0.99	5.0	4.0	1.0	0	12	93.32	93.32	93.32
1.0	0.99	5.0	4.0	1.0	0	8	99.96	99.96	99.96
1.0	0.99	5.0	4.0	1.0	0	10	99.61	99.61	99.61
1.0	0.99	5.0	4.0	1.0	0	12	97.96	97.95	97.95
1.5	0.99	5.0	4.0	1.0	0	8	99.99	99.99	99.99
1.5	0.99	5.0	4.0	1.0	0	10	99.87	99.87	99.87
1.5	0.99	5.0	4.0	1.0	0	12	99.53	99.52	99.52

**Table 3.7** The  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  Values under Different  $\beta$ 's and  $\alpha$ 's

$\beta$	$\alpha$	$\bar{q}$	$\bar{b}$	$\bar{h}$	$\bar{u}$	$\bar{v}$	$R_v$	$R_{1r}$	$R_{2r}$
0.5	0.97	5.0	4.0	1.0	0	12	94.90	94.90	94.90
0.5	0.98	5.0	4.0	1.0	0	12	94.04	94.04	94.04
0.5	0.99	5.0	4.0	1.0	0	12	93.32	93.32	93.32
1.0	0.97	5.0	4.0	1.0	0	12	98.38	98.37	98.37
1.0	0.98	5.0	4.0	1.0	0	12	98.15	98.15	98.15
1.0	0.99	5.0	4.0	1.0	0	12	97.96	97.95	97.95
1.5	0.97	5.0	4.0	1.0	0	12	99.63	99.63	99.63
1.5	0.98	5.0	4.0	1.0	0	12	99.58	99.57	99.57
1.5	0.99	5.0	4.0	1.0	0	12	99.53	99.52	99.52

**Table 3.8** The  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  Values under Different  $\beta$ 's and  $\bar{h}$ 's

$\beta$	$\alpha$	$\bar{q}$	$\bar{b}$	$\bar{h}$	$\bar{u}$	$\bar{v}$	$R_v$	$R_{1r}$	$R_{2r}$
0.5	0.99	5.0	4.0	1.0	0	12	93.32	93.32	93.32
0.5	0.99	5.0	4.0	2.0	0	12	92.05	92.05	92.05
0.5	0.99	5.0	4.0	3.0	0	12	90.74	90.74	90.74
1.0	0.99	5.0	4.0	1.0	0	12	97.96	97.95	97.95
1.0	0.99	5.0	4.0	2.0	0	12	97.49	97.48	97.48
1.0	0.99	5.0	4.0	3.0	0	12	97.12	97.10	97.10
1.5	0.99	5.0	4.0	1.0	0	12	99.53	99.52	99.52
1.5	0.99	5.0	4.0	2.0	0	12	99.44	99.43	99.43
1.5	0.99	5.0	4.0	3.0	0	12	99.34	99.33	99.33

Table 3.9 The  $R_v$ ,  $R_{1r}$  and  $R_{2r}$  Values under Different  $\beta$ 's and  $\bar{q}$ 's

$\beta$	$\alpha$	$\bar{q}$	$\bar{b}$	$\bar{h}$	$\bar{u}$	$\bar{v}$	$R_v$	$R_{1r}$	$R_{2r}$
0.5	0.99	3.0	4.0	1.0	0	12	92.25	92.25	92.25
0.5	0.99	5.0	4.0	1.0	0	12	93.32	93.32	93.32
0.5	0.99	7.0	4.0	1.0	0	12	94.23	94.23	94.23
1.0	0.99	3.0	4.0	1.0	0	12	97.61	97.60	97.60
1.0	0.99	5.0	4.0	1.0	0	12	97.96	97.95	97.95
1.0	0.99	7.0	4.0	1.0	0	12	98.27	98.26	98.26
1.5	0.99	3.0	4.0	1.0	0	12	99.43	99.42	99.42
1.5	0.99	5.0	4.0	1.0	0	12	99.53	99.52	99.52
1.5	0.99	7.0	4.0	1.0	0	12	99.62	99.61	99.61

The tables suggest the following results:

1) All ratios are increasing with  $\beta$  and  $\bar{q}$ , and decreasing in  $\bar{h}$ . This is expected and in accordance with the results in Tables 3.1 to 3.3. The ratios are also decreasing in  $\alpha$ .

2) All ratios are decreasing with  $\bar{u}$  and  $\bar{v}$  when the demands of the two plants are negatively correlated and the average demand is below capacity. However, they are increasing with  $\bar{u}$  and  $\bar{v}$  when the demands of the two plants are positively correlated and the average demand is above capacity. An explanation of this is that there are more opportunities for the two plants to help out each other when their demands are negatively correlated and the average demand is below capacity while transshipment is not useful when the demands are positively correlated and the average demand is above capacity.

3) All ratios are almost equal. This shows that virtual transshipment can absolutely supplant real transshipment, from the viewpoint of costs.

### 3.7 Conclusions

In this chapter, the goal is to find the benefit of virtual lateral transshipment, analyze the effects of the parameters such as unit holding cost, production cost, etc. on the cost savings under virtual lateral transshipment compared with that of no transshipment, and compare virtual transshipment with real transshipment.

A computational methodology is adopted, the expected percent cost savings are computed and compared with the no-transshipment case under the same system. The effects of different parameters on the expected percent cost savings is showed. These parameters include unit transshipment cost  $\bar{q}$ , the minimum demand level  $\bar{u}$ , the maximum demand level  $\bar{v}$ , holding cost rate  $\bar{h}$ , backlogging cost rate  $\bar{b}$ , and correlation coefficient  $\beta$ . The computational results confirm the intuition: The percent cost saving  $\eta$  decreases to 0 as the unit transshipment cost  $\bar{q}$  increases to  $+\infty$ ; the use of transshipment can result in larger cost savings when demand deviations are larger and the total plant utilization is closer to 1; the larger ratio of holding cost rate or backlogging cost rate to the production cost results in larger percent cost savings from transshipment; and the larger correlated coefficient leads to a smaller percent cost savings.

Based on the same computational methodology, the expected costs are computed and compared with the no-transshipment case under the same system and with real transshipments. The computational results show that virtual transshipment is preferred over real transshipment from the viewpoint of costs.

The computation is based on a system where there are two plants and the two capacitated plants are symmetric. Therefore a multi-plant non-symmetric system still remains un-studied. Future research will try to study these systems.

## CHAPTER 4

# STUDY OF A REVENUE-SHARING CONTRACT IN SUPPLY CHAINS

### 4.1 Background

Supply chain management involves matching supply with demand. If demand for a product in a supply chain is uncertain, the supply task is complex, because either supply may be in excess of demand, leading to overstock or supply may be short of demand, leading to lost sales. In the former case excessive inventory is referred to as inventory risk while in the latter case insufficient supply is referred to as supply risk. Although the firms in a supply chain may bear supply risk, supply chain management can let some firms have inventory risk, some others not have it, and thus lead to different total costs and profits. So, it is very important for a supply chain to distribute inventory risk to appropriate firms reasonably.

Consider a supply chain with one supplier and one retailer, and suppose the supply chain is involved with new products such as new films, videos or new style clothing with a short life cycle. At the beginning of these products' life cycle, they meet a tremendous demand, while after a short period the demand drops dramatically. In this case, if the wholesale-price contract is adopted in a supply chain, (the retailer obtains the product at a wholesale price larger than the production cost of the product) the retailer must have enough money to buy enough items of the product to meet the demand at the beginning of its life cycle. If the retailer does not have enough money to buy such a large quantity, he will lose market share for this product and furthermore lower the supply chain's profit and level of service. In such a supply chain, the supplier and the retailer can opt to trade with a revenue-sharing contract.



In a revenue-sharing contract, the retailer needs to pay a portion of the production cost (transfer fee) per unit to the supplier when he obtains the product from the supplier. After the retailer sells the product, he must share the revenue with the supplier. Transfer cost rate  $\alpha$  ( $0 < \alpha < 1$ ) is defined as the portion of the production cost per unit that the retailer needs to pay to the supplier and revenue-sharing rate  $r$  is defined as the portion of the revenue per unit kept by the retailer. Such a revenue-sharing contract has been widely adopted by the largest chains in industry since 1998 and has made great contribution to the increase of market share and total profit (Warren and Peers [88] and Mortimer [59]).

In this chapter the adoption of a revenue-sharing contract by a supply chain with one supplier and one retailer is presented and analyzed. The demand for the product is stochastic. The revenue-sharing rate  $r$  is fixed. At the end of the selling season, the unsold items will be sold at a salvage price  $s$  per unit, under two possible scenarios: salvage revenue not shared (SRNS) and salvage revenue shared (SRS). A two-stage Stackelberg game is used where one player is the game's leader and the other one is the game's follower. When the supplier is the Stackelberg game's leader (supplier's Stackelberg game), she decides on the transfer cost rate  $\alpha$  first and then the retailer decides on the quantity  $q$  to be ordered from the supplier based on the transfer cost rate. When the retailer is the Stackelberg game's leader (retailer's Stackelberg game), he decides on the transfer cost rate  $\alpha$  first and then the supplier decides on the quantity  $q$  to be provided to the retailer.

This chapter also contributes to the literature of the newsvendor model. In a standard newsvendor problem, the retailer decides on the optimal quantity to be ordered from the supplier. However, in this model the supplier also can decide on the optimal quantity to be provided to the retailer in the revenue-sharing contract based on the specific conditions.

Therefore, the contribution of this chapter is twofold. First, the adoption of the combination of the Stackelberg game with the revenue-sharing contract to the newsvendor problem has been proposed. Secondly, the model for the newsvendor problem is presented where the supplier decides on the optimal quantity to be provided to the retailer in the revenue-sharing contract.

The chapter is organized as follows. Section 4.2 elaborates on supply chain modeling and analyzes the adoption of the combination of the Stackelberg game with the revenue-sharing contract to the newsvendor problem. A computational analysis has been done in Section 4.3. Section 4.4 concludes the chapter.

## 4.2 Model

For ease of reference, the nomenclature used in this chapter is presented first.

$x$  : Demand variable;

$F(x)$  : Cumulative distribution function;

$f(x)$  : Probability density function;

$p$  : The per unit retail price;

$c$  : The per unit production cost;

$s$  : The per unit salvage price;

$r$  : The per unit revenue sharing rate;

$\alpha$  : The per unit transfer rate of production cost;

$q$  : The quantity of the product provided by the supply chain to the customers;

$q_0^*$  : The optimal quantity of the product provided by the centralized supply chain;

$q_S^*$  : The optimal quantity of the product provided by the decentralized supply chain when the supplier is the leader of the Stachelberg game and the salvage revenue is not shared;

$q_R^*$  : The optimal quantity of the product provided by the decentralized supply

chain when the retailer is the leader of the Stachelberg game and the salvage revenue is not shared;

$q_S^o$  : The optimal quantity of the product provided by the decentralized supply chain when the supplier is the leader of the Stachelberg game and the salvage revenue is shared;

$q_R^o$  : The optimal quantity of the product provided by the decentralized supply chain when the retailer is the leader of the Stachelberg game and the salvage revenue is shared;

$\alpha_R^*$  : The optimal transfer cost rate when the supplier is the leader of the Stachelberg game and the salvage revenue is not shared;

$\alpha_S^*$  : The optimal transfer cost rate when the retailer is the leader of the Stachelberg game and the salvage revenue is not shared;

$\alpha_R^o$  : The optimal transfer cost rate when the supplier is the leader of the Stachelberg game and the salvage revenue is shared;

$\alpha_S^o$  : The optimal transfer cost rate when the retailer is the leader of the Stachelberg game and the salvage revenue is shared;

$S$  : Supplier;

$R$  : Retailer;

$g(x)$  : The generalized failure rate;

$B$  : The expected supply chain profit obtained by the centralized channel.

In the setting,  $g(x) = xf(x)/(1 - F(x))$  and its support is  $[a, b]$ .  $g(x)$  is termed the generalized failure rate which gives (roughly) the percentage decrease in the probability of a stock out from increasing the stocking quantity by 1%. A distribution has an increasing generalized failure rate (IGFR) if  $g(x)$  is weakly increasing for all  $x$  such that  $F(x) < 1$  (Lariviere and Porteus [51]). Suppose

$$y = \max\{q \mid g(q) = 1 \text{ and } q \in [a, b]\}.$$

Because  $g(q)$  is weakly increasing in  $q$ ,  $g(q) > 1$  when  $q > y$ .

The following assumptions are used in the model.

- 1) The demand for the product is uncertain and has a distribution with an increasing generalized failure rate (IGFR);
- 2) the retail price  $p$  dollars/unit is fixed;
- 3) the revenue sharing  $r$  per unit is fixed;
- 4) the production cost is  $c$  dollars/unit;
- 5) the salvage is  $s$  dollars/unit;
- 6)  $rp > s$  and  $(1 - r)p > s$ ;
- 7)  $p - c < rp - s$  and  $p - c < (1 - r)p - s$ ;
- 8) the shortage cost is 0 dollars/unit.

#### 4.2.1 The Centralized Channel

In this chapter, first the centralized channel is analyzed where the supplier and the retailer belong to the same firm. Let  $B$  be the expected channel profit, which is a function of  $q$ .

$$B(q) = p \cdot \int_0^q x f(x) dx + pq \cdot \int_q^{+\infty} f(x) dx + s \cdot \int_0^q (q - x) f(x) dx - cq. \quad (4.1)$$

Let  $q_0^*$  denote the optimal quantity in the centralized channel for the function in (4.1).

It follows after some algebra that

$$F(q_0^*) = (p - c)/(p - s). \quad (4.2)$$

#### 4.2.2 The Decentralized Channel

Normally the supplier and the retailer do not belong to the same partner and thus they always maximize their own profits, respectively.

##### 1. Salvage Revenue not Shared—SRNS

In SRNS, a supply chain where the supplier and the retailer share the normal sales revenue but do not share the salvage revenue when there are leftover items is analyzed. In this case, the salvage revenue is kept in its entirety by the supplier or the retailer who decides the inventory quantity in the supply chain.

### 1.1 Supplier is the Leader of the Stackelberg Game

When the supplier is the leader of the Stackelberg game, she dictates the transfer cost rate per unit  $\alpha$  to the retailer to optimize her own benefit given the retailer's best response. The retailer decides the optimal order quantity  $q$  to maximize his own expected benefit.

At the end of the sales season, the retailer sells the leftover items of the product at the salvage price  $s$  dollars/unit. Let  $\Theta_R$  and  $\Theta_S$  denote the retailer and supplier's expected profits, respectively.

$$\Theta_R(\alpha, q) = rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx + s \cdot \int_0^q (q-x)f(x)dx - \alpha cq. \quad (4.3)$$

Given  $\alpha$ , the optimal quantity  $q_R^*(\alpha)$  the retailer chooses to maximize  $\Theta_R$  satisfies the following equation:

$$F(q_R^*(\alpha)) = (rp - \alpha c)/(rp - s). \quad (4.4)$$

From (4.4), the  $q_R^*(\alpha)$ 's inverse function can be obtained:

$$\alpha_R^*(q) = (rp(1 - F(q)) + sF(q))/c. \quad (4.5)$$

Combining (4.5) with (4.3), the expression  $\Theta_R(q) = \Theta_R(q, \alpha)$  can be gotten while  $\alpha = \alpha_S^*(q)$  as follows:

$$\begin{aligned} \Theta_R(q) &= rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx + s \cdot \int_0^q (q-x)f(x)dx \\ &\quad - (rp(1 - F(q)) + sF(q))q. \end{aligned} \quad (4.6)$$

Just because

$$d\Theta_R^*(q)/dq = (rp - s)f(q)q \geq 0, \quad (4.7)$$

the retailer's expected profit is nondecreasing in  $q$ .

Given  $\alpha$  and  $q$ , the supplier's expected profit  $\Theta_S(\alpha, q)$  can be expressed as

$$\begin{aligned} \Theta_S(\alpha, q) &= (1 - r)p \cdot \int_0^q xf(x)dx + (1 - r)pq \cdot \int_q^{+\infty} f(x)dx \\ &+ (\alpha c - c)q. \end{aligned} \quad (4.8)$$

Therefore, suppose that the supplier has chosen  $\alpha = \alpha_R^*(q)$  that induces the retailer to respond with a particular  $q$ , the expected profit  $\Theta_S^*(q) = \Theta_S(\alpha_R^*(q), q)$  that she can get can be determined by

$$\begin{aligned} \Theta_S^*(q) &= (1 - r)p \cdot \int_0^q xf(x)dx + (1 - r)pq \cdot \int_q^{+\infty} f(x)dx \\ &+ (rp(1 - F(q)) + sF(q) - c)q. \end{aligned} \quad (4.9)$$

The first- and second-order derivatives of the function  $\Theta_S^*(q)$  in  $q$  can be gotten as follows:

$$d\Theta_S^*(q)/dq = p - c + (s - p)F(q) + (s - rp)g(q)(1 - F(q)); \quad (4.10)$$

$$d^2\Theta_S^*(q)/dq^2 = (s - p)f(q) - (s - rp)g(q)f(q) + (s - rp)g'(q)(1 - F(q)). \quad (4.11)$$

The following Lemma 1 shows that the  $q_S^*$  that maximizes the supplier's profit  $\Theta_S^*(q)$  can be found by solving the first-order optimality condition.

**Lemma 1** *The supplier's profit  $\Theta_S^*(q)$  is quasi-concave in  $q$ .*

**Proof:** From Assumption 1) and the definition of IGFR, it is clear that  $g(q)$  is nondecreasing for all  $q$ . Hence,  $y$  can be defined as follows:

$$y = \max\{q \mid g(q) \leq 1 \text{ and } a \leq q < b\}.$$

When  $a \leq q \leq y$ , by (4.11) and Assumption 6), the following can be obtained:

$$d^2\Theta_S(q)/dq^2 \leq (s - rp)f(q)(1 - g(q)) + (s - rp)dg(q)/dq(1 - F(q)).$$

It is known that  $dg(q)/dq \geq 0$  because  $g(q)$  is nondecreasing in  $q$ . By the definition of  $y$ , it is also known that  $g(q) \leq 1$ . Therefore, the following is obtained:

$$d^2\Theta_S(q)/dq^2 \leq 0.$$

That is,  $\Theta_S(q)$  is concave in  $[a, y]$ .

When  $y \leq q < b$ , by (4.10) and Assumption 7), one can obtain

$$d\Theta_S(q)/dq \leq (rp - s)(1 - F(q))((1 - g(q))).$$

By Assumption 6) and the fact that  $q(q) \geq 1$  due to the definition of  $y$ , the following is derived:

$$d\Theta_S(q)/dq \leq (rp - s)(1 - F(q))((1 - g(q))) \leq 0.$$

That is,  $\Theta_S(q)$  is nonincreasing in  $[y, b]$ . Combining the above two facts, it is known that  $\Theta_S(q)$  is quasi-concave over  $[a, b]$ .  $\square$

According to Lemma 1 and (4.10),  $q_S^*$  satisfies the following equation:

$$p - c - (p - s)F(q_S^*) + (s - rp)q_S^*f(q_S^*) = 0. \quad (4.12)$$

The following proposition summarizes what has just been derived.

**Proposition 1** *When the supplier is the leader of the Stackelberg game, she would choose a transfer cost rate  $\alpha_R^*(q_S^*)$ , and the retailer would respond with  $q_S^* = q_R^*(\alpha_R^*(q_S^*))$ .*

## 1.2. Retailer is the Leader of the Stackelberg Game

When the retailer is the Stackelberg game's leader, he announces the transfer rate per unit  $\alpha$  to the supplier with the purpose of optimizing his own expected profit given the supplier's best response. The supplier decides the optimal quantity  $q$  to maximize her own expected profit. At the end of the sales season, the supplier sells the leftover items at the unit salvage value  $s$ .

Given  $\alpha$  and  $q$ , the supplier's expected profit  $\theta_S(\alpha, q)$  can be expressed as

$$\begin{aligned} \theta_S(\alpha, q) = & (1-r)p \cdot \int_0^q x f(x) dx + (1-r)pq \cdot \int_q^{+\infty} f(x) dx \\ & + s \cdot \int_0^q (q-x) f(x) dx - (1-\alpha)cq. \end{aligned} \quad (4.13)$$

Hence, given  $\alpha$ , the optimal quantity  $q_S^*(\alpha)$  that the supplier would provide to maximize her own profit  $\theta_S(\alpha, q)$  satisfies

$$F(q_S^*(\alpha)) = ((1-r)p - (1-\alpha)c) / ((1-r)p - s). \quad (4.14)$$

From (4.14), the inverse function of  $q_S^*(\alpha)$  can be obtained:

$$\alpha_S^*(q) = (c - (1-r)p(1 - F(q)) - sF(q)) / c. \quad (4.15)$$

Combining (4.15) with (4.13), the expression  $\theta_S^*(q) = \theta_S(\alpha_S^*(q), q)$  can be obtained:

$$\begin{aligned} \theta_S^*(q) = & (1-r)p \int_0^q x f(x) dx + (1-r)pq \cdot \int_q^{+\infty} f(x) dx \\ & + s \cdot \int_0^q (q-x) f(x) dx - ((1-r)p(1 - F(q)) + sF(q))q. \end{aligned} \quad (4.16)$$

Just because

$$d\theta_S^*(q)/dq = ((1-r)p - s)f(q)q \geq 0, \quad (4.17)$$

$\theta_S^*(q)$  is a nondecreasing function in  $q$ , and thus the supplier's expected profit is nondecreasing in  $q$ .

Given  $\alpha$  and  $q$ , the retailer's expected profit  $\theta_R(\alpha, q)$  can be expressed as

$$\theta_R(\alpha, q) = rp \cdot \int_0^q x f(x) dx + rpq \cdot \int_q^{+\infty} f(x) dx - \alpha cq. \quad (4.18)$$



Therefore, suppose that the retailer chooses  $\alpha = \alpha_S^*(q)$  that induces the supplier to respond with a particular  $q$ , the expected profit  $\theta_R^*(q) = \theta_R(\alpha_S^*(q), q)$  he can get can be determined by

$$\theta_R^*(q) = rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx - q(c - p(1 - r)(1 - F(q)) - sF(q)). \quad (4.19)$$

The first- and second-order derivatives of the function  $\theta_R^*(q)$  in  $q$  can be obtained as follows:

$$d\theta_R^*(q)/dq = p - c - (p - s)F(q) - g(q)(1 - F(q))[(1 - r)p - s]. \quad (4.20)$$

$$\begin{aligned} d^2\theta_R^*(q)/dq^2 &= -(p - s)f(q) + g(q)f(q)[(1 - r)p - s] \\ &\quad - g'(q)(1 - F(q))[(1 - r)p - s]. \end{aligned} \quad (4.21)$$

The following Lemma 2 shows that the  $q_R^*$  that maximize the retailer's profit  $\theta_R^*(q)$  can be found by solving the first-order optimality condition  $d\theta_R^*(q_R^*)/dq = 0$ .

**Lemma 2** *The retailer's profit  $\theta_R^*(q)$  is quasi-concave in  $q$ .*

**Proof:** As in the proof of Lemma 1,  $y$  is defined in

$$y = \max\{q \mid g(q) \leq 1 \text{ and } a \leq q < b\}.$$

When  $a \leq q \leq y$ , from (4.21) and Assumption 6), it is known that

$$d^2\theta_R(q)/dq^2 \leq -f(q)[(1 - r)p - s][1 - g(q)] - dg(q)/dq(1 - F(q))((1 - r)p - s).$$

It is known that  $dg(q)/dq \geq 0$  because  $g(q)$  is nondecreasing in  $q$ . It is also known that  $g(q) \leq 1$  by the definition of  $y$ . Hence, it is true that

$$d^2\theta_R(q)/dq^2 \leq 0.$$

That is,  $\theta_R(q)$  is concave in  $[a, y]$ .

When  $y \leq q < b$ , by (4.20) and Assumption 7), The following is obtained:

$$d\theta_R(q)/dq \leq (p - s)(1 - F(q))[1 - g(q)].$$

By the definition of  $y$ , we have  $q(q) \geq 1$ . This and Assumption 6) lead to

$$d\theta_R(q)/dq \leq (p - s)(1 - F(q))((1 - g(q)) \leq 0.$$

That is,  $\theta_R(q)$  is nonincreasing in  $[y, b)$ . Combining the above two facts, it is seen that  $\theta_R(q)$  is quasi-concave in  $[a, b)$ .  $\square$

By (4.20), it is seen that  $q_R^*$  satisfies the following equation:

$$p - c - (p - s)F(q_R^*) - q_R^*f(q_R^*)[(1 - r)p - s] = 0. \quad (4.22)$$

The following proposition summarizes what has just been derived.

**Proposition 2** *When the retailer is the leader of the Stackelberg game, the retailer would choose the transfer cost rate  $\alpha_S^*(q_R^*)$ , and the supplier would respond with  $q_R^* = q_S^*(\alpha_S^*(q_R^*))$ .*

### 1.3. Comparison between the Two Stackelberg Games

The supply chain's total profit is the sum of the supplier's and retailer's profits. When the supplier is the leader of the Stackelberg game, the supply chain's total profit is the sum of (4.3) and (4.8), while when the retailer is the leader of the Stackelberg game, the supply chain's total profit is the sum of (4.13) and (4.18). But since the profits earned from internal transactions offset each other, the total profit should be completely earned from the end customers, and hence is a function of only the quantity  $q$  of delivery from the supplier to the retailer: the total profit  $B(q)$  as expressed in (4.1) for the centralized case. The first- and second-order derivatives of

$B(q)$  over  $q$  are as follows:

$$\begin{cases} dB(q)/dq = p - c - (p - s)F(q), \\ d^2B(q)/dq^2 = (s - p)f(q). \end{cases} \quad (4.23)$$

By Assumption 6) (hence  $p \geq s$ ), it is known that  $B(q)$  is concave in  $q$ . As described in (4.2), the optimal quantity  $q_0^*$  satisfying  $F(q_0^*) = (p - c)/(p - s)$  maximizes the supply chain's total profit. Due to the concavity of  $B(q)$ , when  $q < q_0^*$ , the supply chain's total profit increases with the quantity of delivery  $q$ .

Recall that, when the supplier is the leader of the Stackelberg game, the optimal delivery quantity  $q_S^*$  is determined by (4.12), i.e.,

$$F(q_S^*) = (p - c - (rp - s)q_S^*f(q_S^*)) / (p - s).$$

So by Assumption 6), it is obvious that  $q_S^* \leq q_0^*$ . When the retailer is the leader of the Stackelberg game, the optimal delivery quantity  $q_R^*$  is determined by (4.22), i.e.,

$$F(q_R^*) = (p - c - ((1 - r)p - s)q_R^*f(q_R^*)) / (p - s).$$

Again by Assumption 6), it is true that  $q_R^* \leq q_0^*$ . So the optimal delivery quantity for each decentralized case is not larger than the optimal delivery quantity for the centralized case.

Based on the above analysis, the larger the optimal delivery quantity, the more profit the overall decentralized supply chain will gain. The following proposition gives the criterion of judging whether one of the two alternatives for the decentralized chain is better than the other from the prospective of the total supply chain's profit.

**Proposition 3** *The delivery quantity  $q$  in the supply chain is greater when the supplier is the leader of the Stackelberg game than when the retailer is the leader of the Stackelberg game if  $r < 1/2$ . Otherwise, it is greater when the retailer is the leader*

*of the Stackelberg game. Therefore, for the supply chain to gain more profit, the party who keeps more than half of the revenue should also serve as the leader of the Stackelberg game.*

**Proof:** It can be shown that

$$d\Theta_S^*(q)/dq - d\theta_R^*(q)/dq = pqf(q)(1 - 2r). \quad (4.24)$$

So the left-hand side of the above equation is greater than 0 if and only if  $r < 1/2$ . By the quasi-concavity of  $\Theta_S^*(q)$  and  $\theta_R^*(q)$ , it is known that  $q_S^* > q_R^*$  if  $r < 1/2$  and  $q_S^* < q_R^*$  if  $r > 1/2$ .  $\square$

## 2. Salvage Revenue Shared—SRS

In SRS, a supply chain where the supplier and the retailer share not only the normal sales revenue but also the salvage revenue when the leftover stock is sold at the salvage price  $s$  dollars/unit is studied.

### 2.1. Supplier is the Leader of the Stackelberg Game

First, the case where the supplier is the leader of the Stackelberg game is discussed. When the supplier dominates the supply chain, she first dictates the transfer cost rate per unit  $\alpha$  to the retailer. The retailer then orders the optimal quantity of items from the supplier to maximize his own expected profit. At the end of the sales season, the retailer sells the leftover inventory at the salvage price, but still shares it with the supplier at the same revenue-sharing rate  $r$ .

Based on the above description, the supplier decides the optimal transfer cost rate per unit  $\alpha$  and the retailer decides the optimal quantity  $q$  of the items to be ordered from the supplier.

Let  $L_R$  and  $L_S$  denote the retailer and supplier's expected profits, respectively. Then, the following expressions can be obtained.

$$L_R(\alpha, q) = rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx + rs \cdot \int_0^q (q-x)f(x)dx - \alpha cq. \quad (4.25)$$

From (4.25), the following equation relating to the optimal quantity  $q_R^o$  to maximize  $L_R(q, \alpha)$  can be gotten.

$$F(q_R^o) = (rp - \alpha c)/r(p - s). \quad (4.26)$$

Based on (4.26), it is known that  $\alpha$  is the function of  $q$  as follows.

$$\alpha_R^o(q) = (rp - r(p - s)F(q))/c. \quad (4.27)$$

Now the retailer's optimal profit denoted by only the variable  $q$  while  $\alpha_R^o(q) = (rp - r(p - s)F(q))/c$  can be gotten as follows.

$$\begin{aligned} L_R(q) = L_R(q, \alpha_R^o(q)) &= rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx \\ &+ rs \cdot \int_0^q (q-x)f(x)dx - (rp - r(p - s)F(q))q. \end{aligned} \quad (4.28)$$

Now the supplier's expected profit is denoted by the following equation:

$$\begin{aligned} L_S(q) = L_S(q, \alpha_S^o(q)) &= (1-r)p \cdot \int_0^q xf(x)dx + (1-r)pq \cdot \int_q^{+\infty} f(x)dx \\ &+ (1-r)s \cdot \int_0^q (q-x)f(x)dx - (c - rp + r(p - s)F(q))q. \end{aligned} \quad (4.29)$$

Using some algebra, the first-order and second-order derivatives of  $L_S(q)$  in  $q$  can be derived:

$$\begin{aligned} dL_S(q)/dq &= p - c - (p - s)F(q) - (p - s)rqf(q) = p - c - (p - s)F(q) \\ &- (p - s)rg(q)(1 - F(q)). \end{aligned} \quad (4.30)$$

$$d^2L_S(q)/dq^2 = -(p - s)f(q) - (p - s)rg'(q)(1 - F(q)) + (p - s)f(q)rg(q). \quad (4.31)$$

**Lemma 3** *The supplier's profit  $L_S(q)$  is quasi-concave in  $q$ .*

The proof is the same as that for Lemma 1. Lemma 3 shows that the optimal quantity  $q_S^o$  exists and satisfies the following equation.

$$p - c - (p - s)F(q_S^o) - (p - s)r q_S^o f(q_S^o) = 0. \quad (4.32)$$

**Proposition 4** *In the case where the salvage revenue is shared and the supplier is the leader of the Stackelberg game, the supplier would choose the transfer cost rate  $\alpha_R^o(q_S^o)$ , and the retailer would respond with  $q_S^o = q_R^o(\alpha_R^o(q_S^o))$ .*

## 2.2. Retailer is the Leader of the Stackelberg Game

When the retailer leads the supply chain, he claims the transfer cost rate per unit  $\alpha$  he would offer to the supplier. The supplier decides the optimal quantity  $q$  of the items based on the claimed transfer cost rate to optimize her own expected profit.

Let  $l_R$  and  $l_S$  denote the retailer and supplier's expected profits, respectively. Their expressions using  $\alpha$  and  $q$  can be gotten as follows.

$$\begin{aligned} l_S(\alpha, q) = & (1 - r)p \cdot \int_0^q x f(x) dx + (1 - r)pq \cdot \int_q^{+\infty} f(x) dx \\ & + (1 - r)s \cdot \int_0^q (q - x) f(x) dx - (1 - \alpha)cq. \end{aligned} \quad (4.33)$$

From (4.33), the following expression of the optimal quantity  $q_S^o(\alpha)$  to maximize  $l_S(q, \alpha)$  can be obtained:

$$F(q_S^o) = ((1 - r)p - (1 - \alpha)c) / (1 - r)(p - s). \quad (4.34)$$

Therefore it is easy to get the following expression for the inverse function  $\alpha_S^o(q)$ .

$$\alpha_S^o(q) = (c + F(q)(1 - r)(p - s) - (1 - r)p) / c. \quad (4.35)$$

Now  $q$  is used to express the supplier's maximized expected profit  $l_S(q) = l_S(q, \alpha_S^o(q))$  as follows:

$$\begin{aligned} l_S(q) = & (1-r)p \cdot \int_0^q xf(x)dx + (1-r)pq \cdot \int_q^{+\infty} f(x)dx \\ & + (1-r)s \cdot \int_0^q (q-x)f(x)dx - [(1-r)p - F(q)(1-r)(p-s)]q. \end{aligned} \quad (4.36)$$

It is easy to show the expression of the retailer's expected profit  $l_R(q) = l_R(q, \alpha_S^o(q))$  as follows:

$$\begin{aligned} l_R(q) = & rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx + rs \cdot \int_0^q (q-x)f(x)dx \\ & - [c - (1-r)p + F(q)(1-r)(p-s)]q. \end{aligned} \quad (4.37)$$

From the above, the first-order and second-order derivatives of  $l_R(q)$  can be shown as follows after some algebra:

$$\begin{aligned} dl_R(q)/dq = & p - c - (p-s)F(q) - qf(q)(1-r)(p-s) = p - c \\ & - (p-s)F(q) - g(q)(1-F(q))(1-r)(p-s). \end{aligned} \quad (4.38)$$

$$\begin{aligned} d^2l_R(q)/dq^2 = & -(p-s)f(q) - g'(q)(1-F(q))(1-r)(p-s) \\ & + g(q)f(q)(1-r)(p-s). \end{aligned} \quad (4.39)$$

**Lemma 4** *The retailer's profit  $l_R(q)$  is quasi-concave in  $q$ .*

The proof is the same as that for Lemma 2. From Lemma 4), it is clear that if  $q_R^o$  is the optimal quantity to maximize  $l_R(q)$ , then  $q_R^o$  exists and satisfies the following equation.

$$p - c - (p-s)F(q) - qf(q)(1-r)(p-s) = 0. \quad (4.40)$$

**Proposition 5** *In the case where the salvage revenue is shared and the retailer is the leader of the Stackelberg game, the retailer would choose the transfer cost rate  $\alpha_R^o(q_R^o)$ , and the supplier would respond with  $q_R^o = q_S^o(\alpha_S^o(q_R^o))$ .*

### 2.3. Comparison between the Two Stackelberg Games

When salvage revenue is shared between the two partners, it is still necessary to decide which Stackelberg game is preferred. Proposition 6 gives the answer.

**Proposition 6** *When salvage revenue is shared, the inventory quantity  $q$  in the supply chain is greater when the supplier is the leader of the Stackelberg game than when the retailer is the leader of the Stackelberg game if  $r < 1/2$ . Otherwise, it is greater when the retailer is the leader of the Stackelberg game. Therefore, when  $r < 1/2$ , the supply chain will gain more profit if we choose the supplier to be the leader of the Stackelberg game while otherwise it will gain more if we choose the retailer to be the leader.*

**Proof.** For any  $q$ ,

$$dL_S(q)/dq - dL_R(q)/dq = (p - s)qf(q)(1 - 2r). \quad (4.41)$$

When  $r < 1/2$ , (4.41)  $> 0$ , therefore the optimal results  $q_S^o > q_R^o$ . When  $r > 1/2$ , the optimal results  $q_S^o < q_R^o$ . Proposition (6) is proved.  $\square$

### 3. Comparison between SRNS and SRS Cases

In the revenue-sharing contract, salvage revenue is part of the total revenue. Especially when the demand is stochastic, salvage revenue could be substantial. Whether it is shared between the supplier and the retailer will affect their own decisions and further affect the total profit of the supply chain.

**Proposition 7** *For any  $r \in [0, 1]$ , the optimal order quantity is greater in SRNS than in SRS. That is to say, the supply chain profit is greater in SRNS than in SRS.*

**Proof.**

$$d\Theta_S(q)/dq - dL_S(q)/dq = qs(1 - r)f(q). \quad (4.42)$$



$$d\theta_R(q)/dq - dl_R(q)/dq = qsr f(q). \quad (4.43)$$

It is clear that both (4.42) and (4.43) are larger than 0 for any  $r \in [0, 1]$ . Therefore, Proposition 7 is proved.  $\square$

The cases where the follower of the game keeps all the salvage revenue and where the follower keeps only the same  $r$  portion of the salvage and the regular sales revenue have been tackled. The analysis arrives at the same conclusion as presented in Propositions 3 and 6 that the whole supply chain will be better off if the party that retains more than half of the revenue is also the leader of the Stackelberg game. It can also be proved that, under the same revenue sharing rate  $r$ , the delivery quantity in this case of SRS will be smaller, and hence the supply chain's profit will be less, than in the case of SRNS, regardless of the choice of the party to be designated as the Stackelberg game's leader. So, from the perspective of the total supply chain's profit, SRS is not as attractive as SRNS in supply chain management.

### 4.3 Computation

A computational study is presented, which helps to further prove the earlier propositions by computation, and to show the impacts of the parameters.

#### 4.3.1 Settings for the Computational Study

For the example, the demand distribution is assumed to be uniform  $U[0.0, 10000.0]$ . As a default, let  $p = 12.0$ ,  $s = 2.8$ ,  $c = 10.0$ . To pick a value of the revenue-sharing rate  $r$ , check all of the values of the parameters and guarantee to meet the assumptions  $rp \geq s$ ,  $(1 - r)p \geq s$ ,  $p - c \leq rp - s$  and  $p - c \leq (1 - r)p - s$ . Based on the above settings and description,  $r$  should be in  $[0.4, 0.6]$ .

### 4.3.2 Computational Results

First, the impact of the revenue-sharing rate  $r$  on the total profit in a supply chain is investigated. The results are showed in Table 4.1, and they indicate that when  $r < 1/2$ , the total profit of the supply chain is greater if the supplier is the leader of the Stackelberg game than if the retailer is the leader of the Stackelberg game, while when  $r > 1/2$ , the result is reversed. Also the total profit in the supply chain is greater when the salvage revenue is not shared than when it is shared in the same Stackelberg game. The results of the computation are in accordance with Propositions 3, 6 and 7.

**Table 4.1** The Impact of the Revenue-Sharing Rate  $r$  on the Total Profit in a Supply Chain

r	B	B11	B12	B21	B22
0.40	2173.91	2104.59	1946.37	1996.45	1868.21
0.42	2173.91	2090.57	1963.14	1983.73	1880.97
0.44	2173.91	2075.91	1979.85	1970.95	1893.78
0.46	2173.91	2060.72	1996.45	1958.11	1906.62
0.48	2173.91	2045.10	2012.90	1945.25	1919.49
0.50	2173.91	2029.14	2029.14	1932.37	1932.37
0.52	2173.91	2012.90	2045.10	1919.49	1945.25
0.54	2173.91	1996.45	2060.72	1906.62	1958.11
0.56	2173.91	1979.85	2075.91	1893.78	1970.95
0.58	2173.91	1963.14	2090.57	1880.97	1983.73
0.60	2173.91	1946.37	2104.59	1868.21	1996.45

Secondly, the impact of the revenue-sharing rate  $r$  on the transfer cost rate in a supply chain is investigated. The results are showed in Table 4.2, and they indicate that the transfer cost rate  $\alpha$  is increasing with the revenue-sharing rate  $r$  in all games.

When the retailer shares more revenue, he needs to pay a greater transfer cost rate  $\alpha$  to make the two-stage game completed successfully. This result is also in accordance with that of Wang, Li and Shen [87] who demonstrated that the retailer's optimal revenue-sharing rate was increasing with the retailer's cost rate.

**Table 4.2** The Impact of the Revenue-Sharing Rate  $r$  on the Optimal Transfer Cost Rate

$r$	$\alpha_R^*$	$\alpha_S^*$	$\alpha_R^o$	$\alpha_S^o$
0.40	0.44	0.34	0.42	0.36
0.42	0.46	0.37	0.44	0.38
0.44	0.49	0.39	0.47	0.40
0.46	0.51	0.41	0.49	0.42
0.48	0.53	0.43	0.51	0.44
0.50	0.55	0.45	0.53	0.47
0.52	0.57	0.47	0.56	0.49
0.54	0.59	0.49	0.58	0.51
0.56	0.61	0.51	0.60	0.53
0.58	0.63	0.54	0.62	0.56
0.60	0.66	0.56	0.65	0.58

#### 4.4 Conclusions

A supply chain under a revenue-sharing contract has been analyzed. A mild restriction assures that the leader's expected profit is unimodal. That means the optimal results can be derived using first-order conditions.

It has been shown that the parameter  $r$  is very important for the supply chain's overall profit. When  $r$  is less than one half, the case where the supplier is the leader is preferred, while when  $r$  is larger than one half, the case where the retailer is the

leader is preferred. In addition, whatever the value of  $r$  is, not sharing the salvage revenue is preferred because the supply chain can gain more profit.

The model neglects a number of factors such as supplier competition, retailer competition and the retailing price's change. Future research can consider extensions of what was presented in this chapter.

## CHAPTER 5

### ADOPTION OF REVENUE-SHARING UNDER LIMITED FUNDS

#### 5.1 Background

In today's highly competitive environment, retailers are faced with many challenges especially when they have a limited amount of available funds. A revenue-sharing contract offers an efficient way to buy enough items using the limited amount of available funds to meet the demand and thus increase the expected profit.

The purpose of this chapter is to show the structure of the optimal strategy for the retailer under a revenue-sharing contract. Especially it is desired to present how the limited amount of available funds affects the retailer's decisions to order items from two suppliers.

This chapter is organized as follows. First the model is presented to analyze the considered situation. After analyzing the formulation for the model and providing the structure of the optimal strategy, some managerial insights are provided.

#### 5.2 Model

There are two suppliers offering different transfer and revenue-sharing rates. It is assumed that the higher the transfer rate (what the retailer pays per unit), the higher profit the retailer can gain per unit. If there is no constraint on available funds, it is clear that the retailer would rather choose the supplier who has the higher transfer rate. However because of the shortage of funds the retailer would take items from different suppliers to optimize his own expected profit. The goal is to show under which conditions the retailer just needs to take the items from the supplier with higher transfer rate per unit, and under which conditions he only needs to take them from the supplier with the lower transfer rate, or take them from both suppliers.

### 5.2.1 Model Formulation

Because the retailer will gain higher profit per unit for items he obtains from the supplier with a higher transfer rate, it is assumed that the retailer will sell the items from the supplier with a higher transfer rate first and only sell items from the supplier with a lower transfer rate after all the items having a higher transfer rate have been sold. Let supplier 1 denote the one with a higher transfer rate while supplier 2 the one with a lower transfer rate.

The notation used is as follows:

$\alpha_1$  : the transfer rate per unit from supplier 1;

$\alpha_2$  : the transfer rate per unit from supplier 2;

$c$  : production cost per unit;

$r_1$  : revenue-sharing rate per unit from supplier 1;

$r_2$  : revenue-sharing rate per unit from supplier 2;

$p$  : the price per unit the vendor sells the items;

$Q_1$  : the number of items the retailer obtains from supplier 1;

$Q_2$  : the number of items the retailer obtains from supplier 2;

$E(Q_1, Q_2)$  : the retailer's expected profit if he gets  $Q_1$  and  $Q_2$  items from supplier 1 and 2, respectively;

$T$  : the total amount of funds the retailer has available to pay for transfer costs to obtain the items;

$Q_1^*$  : the optimal number of items the retailer obtains from supplier 1;

$Q_2^*$  : the optimal number of items the retailer obtains from supplier 2;

$f(x)$  : the probability density function of demand for the item;

$F(x)$  : the cumulative density function of demand for the item.

Following the above notation, it is known that if the retailer obtains the item from supplier 1, he can gain a profit of  $(r_1 \cdot p - \alpha_1 \cdot c)$  for each unit he sold. However,

if the retailer obtains the item from supplier 2, he can gain a profit of  $(r_2 \cdot p - \alpha_2 \cdot c)$  for each unit sold.

From the above description, the following assumptions can be made:

1.  $r_1 > r_2$ ;
2.  $\alpha_1 > \alpha_2$ ;
3.  $s < r_2 p$ ;
4.  $r_1 \cdot p - \alpha_1 \cdot c > r_2 \cdot p - \alpha_2 \cdot c$ .

In other words, the retailer gets more profit per unit from the items he sells from supplier 1 than from supplier 2.

The retailer will sell the unsold items at the salvage price  $s$  dollars/unit.

Based on the above notation and assumptions, the following can be obtained:

$$\begin{aligned} E(Q_1, Q_2) = & \int_0^{Q_1} r_1 p x f(x) dx + \int_{Q_1}^{Q_1+Q_2} (r_1 p Q_1 + r_2 p (x - Q_1)) f(x) dx \\ & + \int_{Q_1+Q_2}^{+\infty} (r_1 p Q_1 + r_2 p Q_2) f(x) dx + s \cdot \int_0^{Q_1+Q_2} (Q_1 + Q_2 - x) f(x) dx \\ & - \alpha_1 c Q_1 - \alpha_2 c Q_2. \end{aligned} \quad (5.1)$$

After some algebra the partial derivatives of  $E(Q_1, Q_2)$  are as follows:

$$\partial E(Q_1, Q_2) / \partial Q_1 = (r_2 - r_1) p F(Q_1) + (s - r_2 p) F(Q_1 + Q_2) + r_1 p - \alpha_1 c. \quad (5.2)$$

$$\partial E(Q_1, Q_2) / \partial Q_2 = (s - r_2 p) F(Q_1 + Q_2) + r_2 p - \alpha_2 c. \quad (5.3)$$

Based on the above partial derivatives for  $E(Q_1, Q_2)$ , the following partial second derivatives for it can be obtained:

$$\partial^2 E(Q_1, Q_2) / \partial Q_1^2 = -(r_1 - r_2) p f(Q_1) - (r_2 p - s) f(Q_1 + Q_2) \leq 0. \quad (5.4)$$

$$\partial^2 E(Q_1, Q_2) / \partial Q_2^2 = -(r_2 p - s) f(Q_1 + Q_2) \leq 0. \quad (5.5)$$

$$\partial^2 E(Q_1, Q_2) / \partial Q_1 \partial Q_2 = -(r_2 p - s) f(Q_1 + Q_2). \quad (5.6)$$

$$\begin{aligned} & \partial^2 E(Q_1, Q_2) / \partial Q_1^2 \cdot \partial^2 E(Q_1, Q_2) / \partial Q_2^2 - (\partial^2 E(Q_1, Q_2) / \partial Q_1 \partial Q_2)^2 \\ & = (r_2 - r_1)(s - r_2 p) p f(Q_1) f(Q_1 + Q_2) \geq 0. \end{aligned} \quad (5.7)$$

From (5.4), (5.5), (5.6) and (5.7), it is known that  $E(Q_1, Q_2)$  is concave in  $Q_1$  and  $Q_2$ .

The retailer needs to decide the optimal quantity  $Q_1^*$  and  $Q_2^*$  of items to be obtained from suppliers 1 and 2, respectively, under the available fund constraint. Therefore, the following formulation is appropriate:

$$\max E(Q_1, Q_2) \quad (5.8)$$

Subject to:

$$\begin{cases} \alpha_1 \cdot c \cdot Q_1 + \alpha_2 \cdot c \cdot Q_2 \leq T \\ Q_1 \geq 0 \\ Q_2 \geq 0. \end{cases} \quad (5.9)$$

Therefore, the following Kuhn-Tucker conditions are required for optimality:

- 1)  $Z_1(T - \alpha_1 c Q_1 - \alpha_2 c Q_2) = 0;$
- 2)  $Z_2 Q_1 = 0;$
- 3)  $Z_3 Q_2 = 0;$
- 4)  $r_1 p - \alpha_1 c - (r_1 - r_2) p F(Q_1) + (s - r_2 p) F(Q_1 + Q_2) - Z_1 \alpha_1 c + Z_2 = 0;$
- 5)  $r_2 p - \alpha_2 c + (s - r_2 p) F(Q_1 + Q_2) - Z_1 \alpha_2 c + Z_3 = 0;$
- 6)  $Z_1 \geq 0, Z_2 \geq 0, Z_3 \geq 0, Q_1 \geq 0, Q_2 \geq 0$  and  $T - (\alpha_1 c Q_1 + \alpha_2 c Q_2) \geq 0.$

### 5.2.2 The Structure of the Optimal Policy

Although it is difficult to get the closed forms of the optimal quantities  $(Q_1^*, Q_2^*)$  based on the above model, some structures of the optimal policy can still be found. Assuming that the limited fund  $T$  is fully expended, the following conditions that



make the retailer obtain the items only from supplier 1, only from supplier 2 or from both of them, respectively can be found out.

**Theorem 6** *When  $T \geq \alpha_1 c Q_1^* + \alpha_2 c Q_2^*$ , if  $\alpha_1/\alpha_2 \geq (r_1 p - s)/(r_2 p - s)$ , then it is impossible for the retailer to obtain items from supplier 1 while not obtaining anything from supplier 2. In other words, it is impossible for  $Q_1^* > 0$  and  $Q_2^* = 0$ .*

**Proof:**

If it is proved that when  $Q_1^* > 0$  and  $Q_2^* = 0$ ,  $\alpha_1/\alpha_2 < (r_1 p - s)/(r_2 p - s)$ , then Theorem 6 has been proven.

When  $Q_1^* > 0$ ,  $Q_2^* = 0$  and  $T \geq \alpha_1 c Q_1^* + \alpha_2 c Q_2^*$ , from the Kuhn-Tucker conditions we know that  $Z_1 \geq 0$ ,  $Z_2 = 0$  and  $Z_3 \geq 0$ .

From Kuhn-Tucker condition 4) the following equation can be obtained:

$$F(Q_1^*) = (r_1 p - (1 + Z_1)\alpha_1 c)/(r_1 p - s). \quad (5.10)$$

From Kuhn-Tucker condition 5) the following can be derived:

$$F(Q_1^*) = (r_2 p - (1 + Z_1)\alpha_2 c + Z_3)/(r_2 p - s). \quad (5.11)$$

Equating (5.10) and (5.11) for  $F(Q_1^*)$ , the following result can be obtained:

$$Z_3 = (c(1 + Z_1)(\alpha_2(r_1 p - s) - \alpha_1(r_2 p - s)) - sp(r_1 - r_2))/(r_1 p - s). \quad (5.12)$$

It is known that  $r_1 > r_2$ , and therefore,

$$Z_3 < (1 + Z_1)c(\alpha_2(r_1 p - s) - \alpha_1(r_2 p - s))/(r_1 p - s).$$

If  $\alpha_1/\alpha_2 \geq (r_1 p - s)/(r_2 p - s)$ , then

$$(\alpha_2(r_1 p - s) - \alpha_1(r_2 p - s))/(r_1 p - s) \leq 0,$$

and furthermore,  $Z_3 < 0$ . By now Theorem 6 has been proven.  $\square$

Theorem 6 shows that if the ratio of the transfer rates  $\alpha_1$  to  $\alpha_2$  is high enough, the structure of the optimal policy is that the retailer would never just obtain the items from supplier 1.

**Theorem 7** *When  $T \geq \alpha_1 c Q_1^* + \alpha_2 c Q_2^*$ , if  $r_2/\alpha_2 \leq (r_1 - r_2)/(\alpha_1 - \alpha_2)$ , then it is impossible for the retailer to obtain items from supplier 2 and not obtain anything from supplier 1. In other words, it is impossible for  $Q_1^* = 0$  and  $Q_2^* > 0$ .*

**Proof:**

In this condition we know that  $Z_1 \geq 0$ ,  $Z_3 = 0$  and  $Z_2 \geq 0$ .

From Kuhn-Tucker condition 4) the expression  $F(Q_2^*)$  is as follows:

$$F(Q_2^*) = (r_1 p - (1 + Z_1)\alpha_1 c + Z_2)/(r_2 p - s). \quad (5.13)$$

From Kuhn-Tucker condition 5) the expression  $F(Q_2^*)$  is as follows:

$$F(Q_2^*) = (r_2 p - (1 + Z_1)\alpha_2 c)/(r_2 p - s). \quad (5.14)$$

From (5.13) and (5.14) the following expression is obtained:

$$(1 + Z_1) = (Z_2 + (r_1 - r_2)p)/(c(\alpha_1 - \alpha_2)). \quad (5.15)$$

If (5.15) is used to replace  $(1 + Z_1)$  in (5.14), then the following result is gotten:

$$F(Q_2^*) = 1 - (\alpha_2(Z_2 + (r_1 - r_2)p)/(\alpha_1 - \alpha_2) - s)/(r_2 p - s). \quad (5.16)$$

Because  $0 < F(Q_2^*) < 1$ ,

$$Z_2 < r_2 p(\alpha_1 - \alpha_2)/\alpha_2 - (r_1 - r_2)p.$$

If

$$r_2/\alpha_2 \leq (r_1 - r_2)/(\alpha_1 - \alpha_2),$$

then

$$r_2 p(\alpha_1 - \alpha_2)/\alpha_2 - (r_1 - r_2)p \leq 0,$$

and furthermore  $Z_2 < 0$  which contradicts the condition  $Z_2 \geq 0$  for  $Q_1^* = 0$  and  $Q_2^* > 0$ . Theorem 7 is proved.  $\square$

Theorem 7 can be explained by intuition. If the profit margin per unit from supplier 1 is higher than the cost margin, then the retailer would never just get items from supplier 2.

**Theorem 8** *When  $T = \alpha_1 c Q_1^* + \alpha_2 c Q_2^*$ , if  $r_2/\alpha_2 > (r_1 - r_2)/(\alpha_1 - \alpha_2)$ , and  $\alpha_1/\alpha_2 \geq (r_1 p - s)/(r_2 p - s)$ , then how the retailer orders the items from the two suppliers is only restricted by the available funds constraint and it has the following rule:*

a)  $Q_1^* = 0$  and  $Q_2^* = T/(\alpha_2 c)$  if  $T$  is satisfying the following condition:

$$F(T/(c\alpha_2)) \leq 1 - ((r_1 - r_2)p\alpha_2 - s(\alpha_1 - \alpha_2))/((r_2 p - s)(\alpha_1 - \alpha_2));$$

b) If  $T$  is satisfying the following condition:

$$F(T/(c\alpha_2)) > 1 - ((r_1 - r_2)p\alpha_2 - s(\alpha_1 - \alpha_2))/((r_2 p - s)(\alpha_1 - \alpha_2)),$$

then  $Q_1^*$  and  $Q_2^*$  are both greater than 0 and satisfy the following expressions:

$$F(Q_1^*) = (p(r_1 - r_2) - c(\alpha_1 - \alpha_2)(1 + Z_1))/(p(r_1 - r_2)),$$

and

$$F(Q_1^* + Q_2^*) = (r_2 p - (1 + Z_1)\alpha_2 c)/(r_2 p - s),$$

where  $Z_1 \geq 0$ .

**Proof:**

First part a) is proved: If it can be shown that when

$$F(T/(c\alpha_2)) \leq 1 - ((r_1 - r_2)p\alpha_2 - s(\alpha_1 - \alpha_2))/((r_2p - s)(\alpha_1 - \alpha_2)), Q_1^* = 0,$$

and

$$Q_2^* = T/c\alpha_2,$$

satisfy the Kuhn-Tucker conditions, then part a) will be proved.

When  $Q_1^* = 0$  and  $Q_2^* = T/c\alpha_2$ , from Kuhn-Tucker condition 4) the expression  $F(Q_2^*)$  is as follows:

$$F(T/(c\alpha_2)) = (r_1p - (1 + Z_1)\alpha_1c + Z_2)/(r_2p - s). \quad (5.17)$$

From Kuhn-Tucker condition 5) the expression  $F(Q_2^*)$  is as follows:

$$F(T/(c\alpha_2)) = (r_2p - (1 + Z_1)\alpha_2c)/(r_2p - s). \quad (5.18)$$

From (5.17) and (5.18) the following expression is obtained:

$$1 + Z_1 = (Z_2 + (r_1 - r_2)p)/c(\alpha_1 - \alpha_2). \quad (5.19)$$

If (5.19) is used to replace  $(1 + Z_1)$  in (5.18), then the following result is gotten:

$$\begin{aligned} F(T/(c\alpha_2)) &= 1 - (\alpha_2((r_1 - r_2)p + Z_2)/(\alpha_1 - \alpha_2) - s)/(r_2p - s) \\ &\leq 1 - (\alpha_2((r_1 - r_2)p)/(\alpha_1 - \alpha_2) - s)/(r_2p - s) \\ &= 1 - ((r_1 - r_2)p\alpha_2 - s(\alpha_1 - \alpha_2))/((r_2p - s)(\alpha_1 - \alpha_2)). \end{aligned} \quad (5.20)$$

Therefore part a) is proved.

From Theorem 6, obviously it is impossible that  $Q_1^* > 0$  and  $Q_2^* = 0$  when  $\alpha_1/\alpha_2 \geq (r_1p - s)/(r_2p - s)$ . Furthermore when

$$F(T/(c\alpha_2)) > 1 - ((r_1 - r_2)p\alpha_2 - s(\alpha_1 - \alpha_2))/((r_2p - s)(\alpha_1 - \alpha_2)),$$

it is also impossible that  $Q_1^* = 0$  and  $Q_2^* > 0$ . Therefore  $Q_1^* > 0$  and  $Q_2^* > 0$  is the only option that can satisfy the given conditions. From the Kuhn-Tucker conditions, the two expressions can be gotten as follows:

$$F(Q_1^*) = (p(r_1 - r_2) - c(\alpha_1 - \alpha_2)(1 + Z_1))/(p(r_1 - r_2)),$$

and

$$F(Q_1^* + Q_2^*) = (r_2p - (1 + Z_1)\alpha_2c)/(r_2p - s),$$

where  $Z_1 \geq 0$ . Part b) is proved.

Theorem 8 is proved.  $\square$

### 5.3 Conclusion

This chapter has studied the optimal procurement strategy structure of a retailer under a limited amount of available funds. The conditions under which the retailer would never obtain the items only from the supplier with a higher transfer cost rate and the conditions under which the retailer would never obtain the items only from the supplier with a lower transfer cost rate have been obtained. Also the conditions under which the retailer would obtain the items just from the supplier with a higher transfer cost rate and the conditions under which the retailer would obtain the items from both suppliers have been given.

While the focus of this chapter has been on the retailer, further study of the effects on the channel and the supplier from a revenue-sharing contract are worth studying when the retailer faces a limitation of available funds.

## CHAPTER 6

### SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

This dissertation investigated the impact of virtual transshipments and revenue-sharing contracts on the operations and profitability of a limited supply chain.

#### 6.1 Summary

In Chapter 3, results from a computational study confirm the benefits of employing virtual lateral transshipment in a system with two capacitated manufacturing plants. Managerial insights on how to amplify such benefits are also explored.

The revenue-sharing contract is one popular mechanism in supply chain management. It was shown in Chapter 4 that the parameter  $r$  (the revenue-sharing rate) in the revenue-sharing contract is very important for the supply chain's outcome. The partner who keeps more than half of the revenue should be the leader of the Stackelberg game from the viewpoint of the supply chain's profit. In addition, whatever the value of  $r$  is, the supply chain will generate more profit when the salvage revenue is not shared between the two partners.

In Chapter 5, a revenue-sharing contract is adopted and the optimal order structure is analyzed when the retailer faces competition from the suppliers and available funding constraints. Managerial insights for the procurement methods were presented.

#### 6.2 Suggestions for Future Research

Future research in the subjects covered in this dissertation can take several directions by increasing the complexity of the problems or relaxing some of the assumptions used here.

Virtual transshipments were investigated using one retailer and two symmetric capacitated plants. A multi-plant, non-symmetric system would be an interesting and more realistic extension of this problem.

Revenue-sharing contracts were investigated using the retailer as the focus. Interesting extensions of this problem under limited availability of retailer funds, would be the study of the impacts a revenue-sharing contract has on the supplier and the entire channel. The problem can become even more complicated and realistic by introducing supplier competition, retailer competition, and removing the assumption that the retailer's selling price of the product does not change.

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