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Bus arrival time prediction using stochastic time series and Markov chains

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ABSTRACT

BUS ARRIVAL TIME PREDICTION USING STOCHASTIC TIME SERIES AND MARKOV CHAINS

by
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Public transit agencies rely on disseminating accurate and reliable information to transit users to achieve higher service quality and attract more users. With the development of new technologies, the concept of providing users with reliable information about bus arrival times at bus stops has become increasingly attractive. Due to the fact that bus operation parameters and variables are highly stochastic, modeling prediction of bus travel and arrival times has become one of the many challenging tasks.

Stochastic time series and delay propagation models to predict bus arrival times using historical information were developed. Markov models were developed to predict propagation of bus delay to downstream bus stops based on heterogeneous conditions. The bus arrival times were predicted using a Markov model only and performance measures were obtained and a combined arrival time prediction model consisting of delay propagation and full autoregressive model was also developed. The inclusion of bus delay propagation into the bus arrival time prediction algorithm is an important contribution to the research efforts to predict bus arrival times. The research showed that Markov models can provide accurate bus arrival time predictions without increasing the need for a large number of bus operation variables, simulations and high polling frequency of the geographical bus location as used by other modeling approaches.

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by
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CHAPTER 1

INTRODUCTION

1.1 Introduction

In recent years, Intelligent Transportation Systems (ITS) have been extensively used in planning, control and management of surface transportation systems. Traffic, highway and transit management extensively use ITS based systems, and this trend is growing at an increasing rate, gaining popularity amongst agencies funding and implementing these systems. Public transit agencies have either successfully implemented, or are in the process of implementing various ITS applications in public transit planning, bus maintenance and operation, light rail and high speed commuter rail.

Public transit agencies rely on disseminating accurate and reliable information to transit users to achieve higher service quality and attract more users. With the development of ITS, the concept of providing users with reliable information about bus arrival times at bus stops has emerged as Advanced Traveler Information Systems (ATIS) and Advanced Public Transportation Systems (APTS). Transit agencies obtain information in real-time on bus travel time, bus location, speed, passengers on board and dwell time. With the help of this data, transit agencies can provide information to users such as, arrival time and anticipated delays in advance. The information collected in real time becomes historical information and assists transit agencies with planning, management, and control of the system as well as the improvement of service.

1.2 Problem Statement

As part of a larger ATIS program, numerous research projects have been performed towards developing algorithms to predict bus arrival times at bus stops. Due to the fact that bus operation parameters and variables are highly stochastic, modeling bus travel and arrival times has become one of the many challenging tasks among researchers. A number of deterministic models based on regression analysis have been proposed to estimate bus operational parameters including models to predict bus arrival time. Such models, however, are oversimplified and may not be representative of real bus conditions as such models do not account for the changes in bus travel time with changing traffic and transit conditions.

A number of deterministic and stochastic models have been proposed by researchers to predict bus operations on freeways. However, limited models exist to estimate bus conditions on urban arterials and local streets. Hence, this research develops models to predict bus arrival time along urban arterials and local streets based on stochastic time series modeling of travel time and the inclusion of delay propagation in the prediction of the bus arrival time. Delay propagation is the process where delay incurred at an upstream bus stop is carried forward to downstream stops.

Bus arrival time prediction models based on linear and non-linear regression, Kalman filtering and artificial neural networks do not account for the propagation of bus delay to downstream time-points (i.e. locations long the route where the bus has to be at predetermined times as scheduled). The existence of propagation of bus delay is not questionable as bus drivers are constantly aware of the delay incurred in the previous time-point and the possibility of these delays propagating to downstream time-points.

Hence, drivers tend to adjust their speed to reach the downstream time-points on-time and adhere to the schedule. Some transit agencies do provide schedule adherence status to the passengers waiting at the bus stops. However, limited research has been performed to model bus delay and its propagation to downstream bus stops, including the possibility of including bus delay information in the prediction of bus travel and arrival time.

1.3 Research Objectives

The primary objective of the research is to develop models to predict the bus arrival time at stops using historical bus travel time information. The research uses a stochastic approach to predict bus travel time and delay propagation based on historical information of bus travel time and delay. Before any prediction models are proposed, existing bus travel and arrival time prediction models were studied to determine their limitations. In addition, the research established the potential for using the stochastic behavior of bus travel time and the propagation of delay at bus stops in the prediction of the bus arrival time.

The research focused on stochastic time dependent prediction models assuming that bus travel times can be treated as random variables and distributed over time. In this research, the appropriate stochastic time series models were identified. The ability of the models to capture the temporal variations of bus travel time was also determined. Since existing stochastic time series models do not consider the propagation of bus delays to downstream stops, this research is also focused on modeling the propagation of delay of buses to downstream stops. In general, the objectives of the research are outlined as follows:

- Develop bus arrival time prediction models based on stochastic time series processes and bus delay propagation.

- Compare the performance of a number of bus travel time and delay propagation models based on various measures of effectiveness and evaluate the performance of the models.

- Analyze the suitability and transferability of the proposed bus arrival time models to different locations.

1.4 Organization of the Dissertation

The dissertation has been organized into eight chapters. Chapter 1 consists of a brief introduction on bus arrival prediction and its importance in ATIS. This is followed by a discussion of the specific problem and objectives of this research. A comprehensive literature review is presented in Chapter 2, covering past research pertaining to the prediction of bus arrival time using various methods. In Chapter 3, algorithms to predict bus arrival times are proposed, including a discussion of the necessary assumptions and constraints of the proposed model. The chapter also includes a discussion on the theoretical background of a number of time series models and stochastic Markov processes. Chapter 4 presents a case study for which bus travel time analysis and delay propagation were performed. Chapter 5 consists of results from the proposed bus travel time prediction mode using autoregressive models. Chapter 6 consists of an analysis of the bus delay prediction models using a Markov process. Bus arrival time predictions are made in Chapter 7 using both time series methods and delay propagation methods. Finally, conclusions about the performance of the time series models and delay propagation models are drawn in Chapter 8. The contributions of the research are outlined and recommendations for future research are made.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This literature review focused on a number of studies performed by researchers to predict bus travel and arrival time. Through the literature review, the importance of models for the prediction of bus arrival times for operational, management and control of bus transit is well established. Although, the usage of bus arrival time prediction models in ATIS is well studied, few studies have been performed on bus arrival prediction models using stochastic time series and models that incorporate delay propagation in the estimate of bus arrival times. In this literature review, the models and proceedings required for a bus arrival prediction model, as proposed by various researchers, are discussed. Hence, the literature review led to the motivation for a stochastic time series and delay propagation model for bus arrival prediction.

2.2 Application of Bus Arrival Prediction Models

The importance of bus arrival time prediction models as a component of ATIS has been well argued by numerous researchers (Park et al., 1995; Dailey et al., 2002; Abdelfattah and Khan, 1998). In addition to ATIS, the Route Guidance System (RGS) and Advanced Traveler Management System (ATMS) have also relied on accurate short-term prediction of bus arrival times. Hence, bus arrival prediction has become a critical component of these systems (Chien and Kuchipudi, 2002).

The purpose of both APTS and ATIS systems should be to benefit travelers on freeways, as well as, bus passengers on urban roads by reducing their travel time and waiting time at bus stops. The primary objective of APTS, relating to transit passenger, is to improve the distribution of information about the public transit system such as, travel time, delay, vehicle position (Dailey et al., 2000). Hence, the prediction of bus arrival time at bus stops is found to be valuable information for passengers to help reduce their waiting time.

The importance of bus arrival time prediction is more significant for trips if the expected arrival time is relatively far into the future and the headway of the bus service along the bus route is very long. In such cases, bus passengers are more eager to know the bus arrival time than if the bus service is very frequent. Also, in transit operations, the stochastic variation of bus arrival times due to other roadway conditions and vehicle ridership could be significant. This variation can deteriorate the headway adherence of the bus and lengthen the passenger waiting time at bus stops. Hence, providing accurate information on bus arrival at stops includes accounting for this variation and ultimately becomes critical for providing quality service to passengers and improving the attractiveness of bus transit (Ding and Chien, 2002).

2.3 Data Source and Errors

Automatic Passenger Counting (APC) systems can provide real time information about passenger and bus location (Eisele, 1997). This real time information becomes a source of historical information for future use. Many transit agencies have incorporated an on-board APC system into a smart bus concept. APC systems have become attractive among bus transit planners and managers for bus transit management and control because of the large amount of information provided by the APC system. As of 2000, 88 transit agencies in the United States had operational AVL and APC systems and 142 were planning to install them. With growing interest in using AVL systems for bus arrival information prediction, a number of researchers have focused on using the AVL/APC system for the prediction of bus arrival time. However, most of these bus arrivals time prediction models used by transit agencies are proprietary and literature on the topic is not available (TCRP, 2003).

The amount of data obtained from APC/AVL systems can be quite large in terms of the number of records. In addition, such data often contain errors, which are generally outliers. Chien and Kuchipudi (2002) stated that using real time data, which have a higher standard deviation (outliers) of travel time data, can adversely affect the accuracy of the prediction model. Hence, numerous techniques have been developed by researchers to eliminate outliers and erroneous data (Dailey and Cathey, 2002; Dion and Rakha, 2003; Shalaby and Farhan, 2003). However, a higher standard deviation of observed travel time data may be inherent to the data due to the nature of traffic and transit characteristics. It is also possible that higher standard deviations of bus travel time are due to non-recurring events. Hence, it is apparent that before developing suitable prediction models, a statistical analysis of trip samples should be performed to determine the standard deviation of the bus travel time and outliers should be identified.

2.4 Bus Arrival Time Prediction Models

The application of bus travel and arrival time prediction models has emerged as a critical component for many ITS and ATMS systems. Due to the inherent stochastic nature of bus travel time, prediction and estimation of arrival time has been a challenging task. At the same time, there is also increasing demand to predict arrival time in real time for use in adaptive control strategies.

Private vendors have developed a number of proprietary models to predict bus arrival times. These models have been used by a number of public transit agencies to implement advanced public transportation information systems. Most of these algorithms and even entire information systems are patented and are not available to the general public (TCRP, 2003).

2.4.1 Time-Distance Model

Lin and Zeng (1999) developed multiple algorithms to predict the arrival time of buses. These algorithms used the current location and the arrival and departure times of the bus provided by AVL equipped buses. Using time-distance diagrams, the trajectory of the bus is constructed using the bus position obtained from the on-board Global Positioning System (GPS). The model consists of four different types of algorithms. Algorithm 1 used the bus location data and the pre-defined travel time matrix to determine the arrival time at the next bus stop. The arrival time at the next bus stop is determined as the recorded arrival time of the bus at the previous stop plus the travel time from the pre-defined matrix. Algorithm 2 used bus schedule data in addition to bus location data. Both algorithms assumed that the bus travels at a constant speed regardless of delay. Algorithm 3 used the bus location, bus schedule and bus delay data, to predict bus arrival time based on an assumption that drivers adjust their travel speed depending on delay. Algorithm 4 was identical to Algorithm 3 but differentiated between links containing at least one time-point and links containing no time-points. The distinction was made to introduce the dwell time into the prediction algorithm, since the dwell times at the time-points were found to be very high along the bus route used in the case study. The overall precision measures of the algorithms were determined by using the average deviation of the predicted arrival from the actual arrival time. The results showed that Algorithm 4 has the best performance. The results obtained a minimum average deviation of predicted arrival time from observed arrival time of 2.0 minutes and a maximum of 3.6 minutes.

Translink (Texas Transportation Institute, 2000) used a time-based algorithm to predict bus travel times along a fixed bus route in the Texas A&M University campus. The time-based algorithm was based on a fixed value of the travel time on the route, determined as a simple average travel time and an average dwell time at individual bus stops. The entire bus route was divided into one-minute zones, with the total number of zones equal to the average bus travel time. The length of each zone is the estimated average distance traveled by a bus in one minute. The algorithm works by geographically locating the bus in the zone by an AVL system. The travel time to a bus stop was then estimated by determining the number of one-minute zones the bus has to travel in order to reach that bus stop. At bus stops that had an average dwell time value of more than one minute, time zones would overlap and the value of the predicted travel time was then decreased by one minute after the bus dwelled in the same time zone for more than one minute.

The assumption used in the algorithm is that the simple average travel time of the bus is sufficient to account for any variation in the bus speed due to change in roadway conditions by time of day and pedestrian movements along the bus route. The major limitation of the algorithm is that the bus is operated inside the campus with very limited traffic impedance. Hence, variation in travel time is not a significant factor in the algorithm.

2.4.2 Regression Models

The prediction of bus travel and arrival time on urban corridors is a difficult task because of the stochastic nature of transit operations and the impact of factors such as passenger demand at stops, traffic control and interaction with other vehicles. The estimation of bus travel time can be obtained using classical statistical analysis such as regression models.

Frechette and Khan (1997) used Bayesian regression analysis to estimate vehicular travel times between links in CBD locations. Data for the independent variables were obtained using a video camera. The independent variables included volume of through, left and right turning vehicles, number of signalized intersections, percentage of stopped vehicles on each link, and the percentage of heavy vehicles. Though these parameters can be obtained using a video camera, this method has two major limitations. Firstly, installation of a video camera would be required in each and every intersection along the bus route, which would be expensive. Also, data extraction from the video camera would be a difficult process and not a real time process compared to AVL technology. The regression models were developed for both one-way and two-way street configurations. Although the results were quite appreciable in terms of their high R^2 values, the implementation of the algorithm to predict bus travel times requires collection of a large number of independent variables. The authors recommended that developed models could be incorporated into an ATIS to provide average vehicular travel times.

Abdelfattah and Khan (1998) developed non-linear regression models of bus delays. The independent variables that influence bus delays in mixed traffic lanes were collected using a video camera. This field information was used as input into the micro-simulation package NETSIM and various measures of effectiveness were obtained. The regression models used a number of independent variables such as link length, number of bus stops per link, number of buses per link, vehicle turning movements, traffic density of heavy vehicles, density of traffic signals and number of passengers riding the bus. These variables are not easily available for the prediction of bus travel time, considering the amount of data and method of data collection required. The R^2 obtained from the regression models were above 0.8. Bus arrival times were determined from average bus speed plus expected delays estimated by the developed regression models. A constant bus speed was used to determine the bus travel time without any delays. The average difference between the estimated and actual bus arrival times of 70 buses over a 30-minute period was determined. For two streets under study, the average difference was found to be less than 30 seconds.

Patnaik et al. (2004) developed a multivariate linear regression model to estimate bus arrival time between time-points along a bus route. The data were obtained from the APC system installed on buses. The travel time of the bus between time-points was estimated using linear regression of the historical travel times between the time-points. The linear regression model consisted of independent binary variables to represent time of day of the bus trip, dwell time, number of stops and distance between time-points. The study used cumulative values of dwell time and number of stops between time-points as independent variables, since individual values of dwell time at stops would not be significant compared to the total travel time between time-points. The models showed significant results in terms of higher R^2 values, but did not present comparisons between observed and predicted bus arrival times.

2.4.3 Artificial Neural Network Models

Artificial neural networks (ANN) and in particular multilayer neural networks that utilize a back propagation algorithm have been used to predict link travel time for buses as well as for other vehicles. The majority of the neural network applications used in the field of transportation are conventional ANN models.

Ding and Chien (2002) developed link-based and stop-based artificial neural networks (ANN) for predicting bus arrival times in real time. The link-based ANN is designed to predict bus arrival times by accumulating bus travel times on all traversed links between pairs of stops. Unlike the link-based ANN, the stop-based ANN estimates the bus arrival time to a downstream stop using traffic conditions instead of bus travel times accumulated on all traversed links. For training the ANN models, back propagation learning algorithms were used.

The factors affecting bus arrival time (e.g., volumes, average speed, delays) were inputs for link-based and stop-based ANN models. The bus arrival times predicted by ANN models were compared with results from the micro-simulation model CORSIM, which was calibrated with field data. The Root Mean Square Error (RMSE) between simulated bus arrival and predicted arrival time increased as the number of downstream stops increased. The RMSE values increased from 1 minute to 4 minutes showing that the model performed well for the study bus route (Ding and Chien, 2002).

2.4.4 Kalman Filter Models

Kalman filters have also been used for prediction of bus travel time as well as vehicle travel times on freeways for incident detection models. Bae and Kachroo (1995) developed a Kalman filter model to estimate arterial travel time for buses by using AVL equipped buses as probe vehicles. A prototype bus arrival time estimation model was developed based on an online parameter adaptation algorithm. Based on the dynamics of both single and multiple stops, a prediction model for bus arrival time was developed. The model relied on an extended autoregressive model and considered time varying passenger boarding and alighting rates.

Chien and Kuchipudi (2002) developed a model to predict travel time using real-time and historical data based on Kalman filters. In the research, a Kalman filtering algorithm was used to update the state variable (travel time) continuously as new observations became available. In addition to the real time information, the research considered the use of aggregate data from previous time intervals and days as historical seeds to evaluate the prediction accuracy.

Dailey et al. (2000) used time series data obtained from an AVL system to predict bus arrival time. The data consisted of time and location pairs and was used with historical data in an optimal filtering framework to predict bus arrival times. The filtering model continuously predicts the arrival time of the bus as a function of both time and space. The filtering model assumed that vehicle locations are available irregularly, typically on a one to five minute basis. The vehicles are assumed to move with constant speed over a limited distance. The variability of the modeling process is normally distributed and includes the starting and stopping motions of the vehicles. The algorithm provided predicted bus arrival times up to an hour in advance. Using Kalman filters as an optimal filtering technique, the bus arrival time was determined. The model, however, did not explicitly use the variability of the bus travel time during different times of day to include temporal variability of bus travel time and assumed that such variability is included in the modeling process. The model is entirely based on the trajectory motion of the bus, which relies on a high polling frequency, so that the bus position is updated within minutes.

Shalaby and Farhan (2003) proposed two Kalman filter algorithms for the prediction of bus running time and passenger dwell time alternately in an integrated framework. The "Link Running Time Prediction Algorithm" made use of the last three days of historical data of the bus link running time for the prediction. The bus link running time for the previous bus on the current day at the current time is also used to predict the bus running time one period ahead. The "Passenger Arrival Rate Prediction Algorithm" employed similar historical data on passenger arrival rates to predict the dwell time. The predicted arrival rate is multiplied by the predicted headway and by the passenger boarding time. The bus arrival time prediction results obtained from the Kalman filter model were compared with the results obtained from historical average, regression and artificial neural network models. The Kalman filter model produced a Mean Relative Error (MRE) between 0.044 and 0.087, while predicting travel time using four previous days of data and comparing these data with the data observed on the fifth day. The results showed that the Kalman filter model performed similar to regression and

ANN models in the normal condition scenario without any congestion. However, the Kalman filter model outperformed other models in special event and lane closure scenarios. The performance of algorithms was tested using data from microscopic simulation.

2.5 Bus Delay Propagation

2.5.1 Delay Propagation Models

Few bus arrival time prediction models have included the propagation of bus delay at downstream stops, though the importance of delay propagation has been stated in previous studies (Dailey et al., 2000). For the prediction of bus arrival times, it is important to incorporate into prediction algorithm information on bus delays, specifically, how these delays propagate as a bus travels over a route. In this context, bus delay is defined as the deviation of bus arrival time from the scheduled arrival time. Very often, bus stops are not equidistant and links connecting the stops are not homogeneous in traffic characteristics. The correlation of delay between any two stops is hence different, requiring an approach to determine bus delays by link. However, the degree of correlation is related to the amount of built-in slack time embedded in the bus schedule (Lin and Bertini, 2002). The prediction of bus arrival times at subsequent bus stops requires that the delay incurred at an upstream stop should be considered to propagate to downstream stops. Hence, considerations of bus delay propagation to downstream stops due to delay at upstream bus stops have to be made while predicting bus arrival times at the downstream bus stops.

Many bus transit systems provide real-time schedule adherence status to their operators on an in-vehicle display terminal (Strathman et al., 2001). Using the schedule adherence status, skilled bus operators constantly adjust their speed to keep their buses on schedule and try to avoid being too late or early. When a bus is delayed at a current bus stop, the bus operator's ability to reach the downstream stop in time depends on the amount of slack time built into the bus schedule and the characteristics of traffic along the link.

Limited studies have been performed regarding the application of delay propagation in modeling bus arrival times. Bus delay propagation in the form of simple linear relationships with travel distance has been used (Lin and Zeng, 1999). The model used a dimensionless input factor $\lambda \in \{0,1\}$ in the prediction model, where $\lambda = {}^{d_{ij}}\!/_{d}$, d_{ij} is the distance between stops i and j and d is the total length of the trip.

Lin and Bertini (2002) formulated a Markov chain model to capture the propagation of bus delay at downstream bus stops. It was assumed that the bus stops were uniformly spaced with equal distance (homogenous condition) and data used to demonstrate the delay propagation model were not a real-world data.

The research described in this dissertation proposes the use of Markov chains to model the propagation of bus delay to downstream stops. Previous studies have demonstrated the use of finite state Markov chain models to simulate and predict traffic conditions on freeways and arterials. These models were used in conjunction with microscopic simulation to describe driver behavior in situations such as, traffic assignment and estimation of vehicle delay between signalized intersections (Evans et al., 2001; Lin et al., 2003).

Lin et al. (2003) developed a simple approach to predict delay of vehicles along an arterial based on Markov chains. The approach used a one-step transition matrix that related the delay of through vehicles at an intersection to the delay status at the adjacent upstream intersection. The parameters of the transition matrix were determined based on three key factors including, flow conditions at the intersection, the proportion of net inflows into the arterial from the cross streets, and the signal coordination level. The research also assumed homogeneous conditions between adjacent intersections. Numerical results show that the model can yield delay predictions with a reasonable degree of accuracy under various traffic conditions and signal coordination levels. The model was not validated using field data and the results from the model were compared with the simulated results. The models were not used to predict bus arrival times.

2.5.2 Finite State Markov Chains

Numerous studies are available regarding estimation using Markov chains (Anderson and Goodman, 1957; Meshkani and Billard, 1992, Robertson, 1990). The simplest method to estimate transition probabilities is to determine the ratio of individual transitions observed to the total number of transitions. Numerous studies have been performed to improve the determination of transition probabilities. Anderson and Goodman (1957) developed a maximum likelihood method to determine transition probabilities of a finite Markov chain for large sample data. Anderson and Goodman (1957) and Lee et al. (1968) compared the maximum likelihood method and Bayesian estimators to estimate a transition probability matrix when aggregate proportion data were used. Bayesian estimators improved the overall estimation of the process over least square and maximum likelihood methods. The Empirical Bayes method is able to produce non-zero transition probabilities, which is common while using a maximum likelihood estimation method for smaller sample size. However, empirical Bayes methods require more than one data set that is independent and identically distributed, which may not be always available.

2.6 Stochastic Time Series Models

2.6.1 Stochastic Models for Traffic Prediction

Time series prediction methods have been established as a prominent statistical method for long term prediction of social and economical variables in planning and management areas. Due to recent developments in ITS, the same statistical prediction methods used for social and economic variables have been used in the prediction of traffic parameters. However, predicting traffic variables are mostly limited to short term prediction, in terms of minutes or hours rather than days or months, which is more common in long term prediction of economic variables. Hence, researchers are more focused on developing time series methods for use in short term traffic prediction.

A number of travel time prediction models have been developed for freeway conditions mainly for the purpose of incident detection. These prediction models rely on data obtained from detectors placed at strategic locations along freeway segments. Distinctions between travel time prediction models in freeway conditions and urban arterial or downtown conditions are mainly related to statistical uncertainties in prediction. Due to the fact that fluctuations of traffic conditions exist on arterials and freeways, travel time is considered time dependent and stochastic. Compared to urban roadways where a stochastic process is prominent, it is more common among researchers to consider travel time to be deterministic on freeways. Compared to freeways, a limited number of stochastic time dependent models have been proposed to predict bus travel and arrival time along urban arterials (Hamed et al., 1995; Park et al., 1995).

Short term prediction of traffic variables can be distinguished as either empirical or based on traffic flow theory. Empirical methods are based on fairly standard statistical methodology of linear and non-linear regression and adaptive filtering methods. In traffic process theory, fairly complicated origin-destination flows, traffic assignment and distribution along paths and links are used for traffic prediction (Arem et al., 1997). Due to the simplicity of empirical methods in comparison to methods based on traffic processes, a number of researchers have used time series prediction methods to predict traffic parameters (Lee and Fambro, 1999; William et al., 1998; Ross, 1982; Ahmed and Cook, 1979).

Lee and Fambro (1999) developed a Subset Autoregressive Integrated Moving Average Model (S-ARIMA) for short-term prediction of freeway traffic volume at two sites in San Antonio using data obtained from the TransGuide project. The study concluded that time series models are effective for short-term prediction of vehicle travel time along a freeway. Williams et al. (1998) used seasonal ARIMA models to predict urban freeway traffic flow assuming that time series of traffic flow data has characteristics of being periodically cyclic. The 15-min flow rate was obtained from loop detector data from locations at Capital Beltway in northern Virginia. The missing data were replaced by using a Kalman filter method and simple average of historical observations.

Ross (1982) analyzed 5-minute volumes obtained from 24h/day count from loop detectors along a freeway and an urban arterial. Different smoothing constant (α) values for exponential smoothing models were compared at different days for urban arterial time series data to obtain an optimal value of α. The optimal value of α was obtained by differentiating the mean square error function of the model for different values of α. The study also suggested the use of varying α values for different times of the day instead of using the fixed value of α for the entire week.

Ahmed and Cook (1979) used a Box-Jenkins approach to determine ARIMA models for predicting traffic volume and occupancy. The data used to develop the models were obtained from detectors along freeway sections and aggregated to 30 and 60 seconds for the duration of one peak period. The ARIMA models were compared with a moving average model and double exponential smoothing models using the mean absolute error and mean square error. The ARIMA models outperformed the moving average and double exponential smoothing models

Smith et al. (2002) developed seasonal ARIMA models to predict traffic flow along London Orbital Motorway using 15-min traffic flow obtained from detectors on the freeway. Instead of using a visual inspection method to determine the parameters of the ARIMA model, which is the foundation of the Box-Jenkins approach, the research used Akaike Information Criterion (AIC) to search for a suitable ARIMA model. Box-Jenkin's method is often termed as subjective because of the subjective method of choosing the model order using visual interpretations. By using the appropriate AIC, the subjectivity of selecting the appropriate model can be removed.

2.6.2 Comparison of ARIMA Models

It is certain from the literature review that one category of time series models cannot guarantee that it will always give higher prediction accuracy than another category of time series model. Hence, researchers often use a number of time series models and compare the performance measure of the model to determine the best model. Lee and Fambro (1999) developed a full ARIMA model (F-ARIMA), an exponential smoothing model and a subset ARIMA model for comparison purposes. Exponential smoothing models gave slightly better result than other time series models, except for subset ARIMA model which gave the best results. The researchers concluded that exponential smoothing models should be considered for short-term prediction because these models were also determined to be easier to implement than ARIMA models.

Williams et al. (1998) also compared predicted freeway traffic flow developed using seasonal time series, seasonal exponential smoothing, neural network and historical average models. The seasonal ARIMA model produced the smallest Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE), followed by the seasonal exponential smoothing model then the neural network model and finally the historical average model. Ahmed and Cook (1979) also found better results from ARIMA model compared to exponential smoothing and historical average models.

Exponential smoothing and historical average models are non-parametric methods of time series trend estimation and not entirely used for prediction purposes. However, these models are still used for prediction purposes due to their simplicity in computation. Hence, these models are popular for one-step-ahead univariate real time prediction (Makridakis and Wheelwright, 1978).

AR, MA and ARIMA models are not adaptive such as, Kalman filtering and are not widely used for real time prediction. However, modified forms of AR, MA and ARMA models are available to work as an adaptive model although implementation of such models is computationally intensive (Brockwell and Davis, 2002; Chatfield, 2000).

2.7 Summary

The literature review revealed that bus arrival time prediction models take into account historical vehicle operations including the bus arrival time for the last several buses on the same route over the previous several months, and traffic patterns. These prediction algorithms use historical data as a basis to state the existing traffic conditions at the time of prediction. The historical profile of travel times is the best knowledge of the past for use as a basis for the prediction of travel times in the future (Franco and Taranto, 1995).

Time-Distance models (Lin and Zheng, 1999) were developed for rural settings, where traffic congestion is not a major issue. Hence, these algorithms may not be applicable to urban areas where congestion is a major issue. These algorithms are also not able to explain fluctuations in arrival time of the bus due to changes in traffic volume and other traffic parameters. The model assumed that the delay at a downstream stop is directly proportion to the travel distance between the bus stops and is propagated to downstream stops in the same magnitude if the distances between the links are equal. This assumption may be reasonable in a rural environment where traffic characteristics between the bus stops may not have a major influence on frequency and amount of bus delays. However, in an urban environment, equidistant links may have plenty of variations in traffic characteristics, which in turn could affect bus delays.

The regression models were developed to determine the vehicular travel time and bus delays. Even though the obtained R^2 values were statistically significant, the studies did not include the prediction of bus arrival times (Frechette and Khan, 1997; Abdelfattah and Khan, 1998). The models took into account independent factors that affect the travel time of the bus along a link such as, distance, speed, number of intersections, and traffic volume. The models do not take into account the presence of temporal variations of bus travel times and propagation of delays to downstream stops. In addition, due to the large number of data required to develop regression models, both studies used output from a micro-simulation package. The field data were collected as an input to the micro-simulation package.

The current geographic location of the bus was used to predict the future location of the bus and predict bus arrival times using Kalman filtering. The model relies on high frequency of availability of the GPS location, which is most difficult in dense cities with high rise structures (Dailey et. al, 2000). Another bus arrival time prediction model using Kalman filter model was based on historical bus travel time information and constant headway of the bus (Shalaby and Farhan, 2003). Contrary to Kalman filter and regression models, ANN model requires more intensive computation and data requirements.

The literature review provided a background on limited number of models to predict bus arrival times based on assumptions of stochastic variation of bus travel times and delay propagation. Also, literature about proprietary prediction models developed by private vendors is not available, which are actually being implemented by the transit agencies. The models described in the literature review are either deterministic models such as time-distance or regression models or too complicated such artificial neural network models. Except time-distance model, all the other models require a large number of independent variables. Many of these variables are not available to the transit agencies.

In comparison to numerous studies in bus travel time prediction, the literature review revealed that the application of delay propagation to predict bus arrival times has been limited. It is important to incorporate information on bus delay into the bus arrival time prediction algorithm.

CHAPTER 3

ALGORITHM AND METHODOLOGY

3.1 Introduction

This chapter describes the assumptions used for the identification and implementation of time series and Markov chains to predict bus arrival times. The algorithms to obtain the bus arrival time prediction is presented, which is followed by a theoretical background of the stochastic process of time series models and Markov chains, which forms a basis to obtain the bus arrival prediction models.

3.2 Assumptions and Constraints

Conceptually, time series models rely on data from historical time periods for the prediction of future time periods. The time series models presume that a pattern or combination of patterns occurs periodically over time and these patterns can be defined using mathematical functions. As a consequence, one can also assume that historical data can be used to identify the patterns or combination of patterns of bus travel time. The motivation for using time series models for predicting bus travel times is that it is not necessary to establish relationships between independent variables and the dependent variable under study in a time series model as it is needed in regression models.

Time series prediction models determine what would happen in successive time periods rather than why it would happen. Hence, the bus arrival prediction model is not necessarily concerned with why the bus arrived at a specific future time period, but is more concerned with when the bus would arrive. Unlike regression and artificial neural network models used for predicting bus travel times, time series models do not attempt to discover the factors affecting bus travel times.

Time series models assume that the variable under study is the function of past values in time and these past values are available in discrete and equispaced time intervals. The function can be characterized by some reoccurring pattern or combination of patterns. The following relationship demonstrates these assumptions:

$$z_t = f(z_{t-1}, z_{t-2}, \ldots\ldots\ldots\ldots, z_{t-p}) \qquad (3.1)$$

Where,

z_t = Predicted value at future time t

$z_{t-1}, z_{t-2}, \ldots$ = Historical values at equally spaced times t-1, t-2,...

p= Number of time periods used in the forecast

In Equation 3.1, the predicted value of the variable is determined by a function of historical values of the same variable. Unlike regression models, the explicit relationship between the historical values and the predicted value is not significant.

In this research, the prediction of bus travel time assumes that some recurring pattern of bus travel time exists during different times of the day due to recurring traffic conditions. Based on these patterns, future bus travel times can be predicted. The foundation of model building for discrete observations over time using time series methods is the assumption of a stochastic process, where the prediction model describes the probability structure of the sequence of observations. The future value is predicted by determining the probability distribution of the population using sample past values.

Most stochastic bus arrival time prediction models lack factors that account for bus delay propagation to downstream stops. Hence, a separate analysis of bus delay propagation is provided in this dissertation assuming that bus delay can be observed as a stochastic Markov process. The individual bus trips are assumed to be independent and identically distributed, which is a necessary condition to implement Markov chains. The predicted bus arrival time is presented in the dissertation using delay propagation only and also using a combination of both stochastic models to predict travel time and bus delay propagation to downstream stops. The bus arrival prediction model using only bus delay propagation requires existing bus schedule information. Bus schedule information is assumed to be the best estimation of future bus arrivals in the absence of any delay at an upstream stop or at the origin.

3.3 Bus Arrival Time Prediction Algorithms

3.3.1 Delay Propagation Model

The prediction of the bus arrival time using delay propagation at downstream stops is based on predicted delay and the scheduled arrival time at each time-point. It is assumed that the prediction of the bus arrival time is solely based on the prediction of the bus delay at downstream stops due to observed delay at the origin stop. The predicted arrival time of the bus at the downstream stop is determined by modifying the scheduled arrival time of the bus by the predicted delay of the bus. The algorithm to predict the bus arrival time using the predicted delay and scheduled arrival time is presented as follows:

At $t < (A_1)$,

$$(A_1) = (D_{0,1}) + (SAT_1) \tag{3.2}$$

$$(A_2) = (SAT_2) + (D_{0,2}) \tag{3.3}$$

$$\cdots\cdot$$

$$(A_d) = (SAT_d) + (D_{0,d}) \tag{3.4}$$

Where,

$(D_{0,1})$, ... , $(D_{0,d})$ = Predicted delay at time t at the origin (i=0)

(A_1),............., (A_d) = Predicted arrival time of bus at time-point 1 to destination d

(SAT_1),.............., (SAT_d) = Scheduled arrival time at time-points1 to destination d

At any time "t", when the bus has left the origin time-point and has not reached the next time-point, the arrival time of the bus at the subsequent downstream time-points can be predicted using Equations 3.2 through 3.4. The arrival time of the bus at any downstream time-point is predicted based on its scheduled arrival time at the time-point and the delay predicted at the time-point. This delay is a consequence of the delay observed at the origin time-point. In Equations 3.2, $(D_{0,1})$ is determined using a transition matrix of delay states between time-points 0 and 1. However, $(D_{0,2})$, ... , $(D_{0,d})$ are determined using the successive application of the Markov chain (Kemeny and Snell, 1976), based on the following relationship:

$$D_{0,i>0} = P_0 . (P(1))^i . s^T \tag{3.5}$$

Where,

$D_{0,i>0}$ = Predicted delay at i, given a delay state at 0

P_0 = Initial probability vector at $i=0$

$(P(1))^i$ = ith Power of transition matrix $P(1)$, between time-points 0 and $1 = P(1)$

s^T = Transpose of delay states at $i=0$

The delay at time-points 1 through d can be determined by using the integer multiple of the transition probability matrix between time-points 0 and 1, which is $P(1)$, as shown in Equation 3.5. The equation is used to predict delay at downstream time-points based on a specific time-point and it is assumed that the time-points have similar traffic and geometric characteristics. The power of the transition matrix in Equation 3.5 is based on the assumption that the delay states between two time-points are homogenous having similar traffic characteristics. However, time-points may not be similar in character. For example, the distance between two time-points can be different. To reflect the heterogeneity of delay states between two time-points, Equation 3.5 is changed as follows:

$$D_{0,i>0} = P_0.P(1).P(2)......P(i).s^T \tag{3.6}$$

Where,

$D_{0,i>0}$ = Predicted delay at i, given a delay state at 0

P_0 = Initial probability vector at $i=0$

$P(1), P(2), ... P(i)$ = Transition matrix between time-points *0 and 1, 1 and 2,..... and i-1 and i.*

s^T = Transpose of Delay States at $i=0$

The heterogeneity between the links is reflected by using individual probability matrices corresponding to each consecutive link instead of using the i^{th} power of the transition probability matrix $P(1)$. This is shown as Equation 3.6, which consists of individual transition matrices.

Similarly, when the bus has left time-point *1*, the origin time-point is now *1* instead of *0*. The transition matrices and the delay states are changed to reflect the delay at time-point *1* and the subsequent delay propagation at downstream time-points. Hence, Equations 3.5 and 3.6 are changed as follows to include the observed delay at time-point *1* and predict the delay at downstream time-points $(i>1)$.

$$D_{1,i>1} = P_0 . (P(2))^i . s^T \tag{3.7}$$

Where,

$D_{1,i>1}$ = Predicted delay at i, given a delay state at *1*

P_0 = Initial probability vector at $i=1$

$P(2)$ = Transition matrix between time-points *1 and 2*

s^T = Transpose of Delay States at $i=1$

$$D_{1,i>1} = P_0 . P(2) . P(3) ...... P(i) . s^T \tag{3.8}$$

Where,

$P(2), ... P(i)$ = Transition matrix between time-points *1 and 2, 2 and 3,.. i-1 and i*

Hence, the algorithm to predict bus arrival times at time-point *2* through *d* is updated as follows to reflect the new position of the bus:

At $(A_1) < t < (A_2)$,

$$(A_2) = (SAT_2) + (D_{1,2}) \tag{3.9}$$

....

$$(A_d) = (SAT_d) + (D_{1,d}) \tag{3.10}$$

Where,

$(D_{1,2})$, ... , $(D_{1,d})$ = Predicted delay at time-points *2* through *d* from the origin time-point $(i=1)$

At time-point $i=0$, the delay at the downstream time-point $(D_{0,2})$ is based on the transition probabilities of delay states between $i=0$ *and* $i=1$. As the bus traverses downstream when $i=1$, the prediction of delay at time-point $i=2$ $(D_{1,2})$ is based on transition probabilities of delay states between $i=1$ *and* $i=2$.

3.3.2 Travel Time and Delay Propagation Model

In this research, the arrival time of a bus at a time-point is determined by using a predicted value of the bus travel time and the dwell time at each time-point. In addition to travel and dwell time, a third component of the predicted delay at downstream time-points is introduced during the prediction process. Unlike the algorithm described in Section 3.3.1, this algorithm does not require the scheduled bus arrival time at time-points. Instead, the algorithm requires the observed arrival time of the bus at the previous time-point.

The bus arrival time at each time-point is determined by using a combined prediction of the stochastic travel time and the delay propagation. The historical values of bus travel times are used to predict the future bus travel time at individual time-points using an autoregressive (AR) model in combination with the predicted delay. This combined algorithm to predict bus arrival time is shown by the following equations:

At, $DEP_0 < t < (A_{i=1})$

$$(A_1) = (DEP_0) + (TT_{0,1}) + (D_{0,1}) \tag{3.11}$$

$$(A_2) = (DEP_0) + (TT_{0,2}) + (D_{0,2}) \tag{3.12}$$

....

$$(A_d) = (DEP_0) + (TT_{0,d}) + (D_{0,d}) \tag{3.13}$$

Where,

(DEP_0) = Observed departure time at the origin *(i=0)*

(A_1),............., (A_d) = Predicted arrival time of bus at time-point *1* to destination *d*

$(D_{0,1})$,............., $(D_{0,d})$ = Predicted delay at time-point *1* to *d* due to delay at the origin *(i=0)*

$(TT_{0,1})$, …. $(TT_{0,d})$ = Predicted travel time between time-points *0 to 1, ... 0 to d.*

In Equation 3.11, the arrival time of the bus at the first time-point *(i=1)* is determined by using the observed bus departure time at the origin *(DEP_0)* and the predicted travel time and delay due to observed delay at the origin time-point. The bus arrival time at subsequent time-points *(i>1)* is determined by using equations 3.12 and 3.13. Hence, when the bus is at the origin time-point *(i=0)*, the arrival time at all the downstream time-points can be predicted using Equations 3.11 through 3.13. The generalized expanded forms of Equations 3.11 through 3.13, where the travel time and delay are expressed using an autoregressive model and a Markov process, respectively are as follows:

$$(A_1) = (DEP_0) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots\ldots\ldots \varphi_p z_{t-p})_{0,1} + (P_0.P(1).S^T) \qquad (3.14)$$

$$(A_2) = (DEP_0) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots\ldots\ldots \varphi_p z_{t-p})_{0,2} + (P_0.P_{0,2}.S^T) \qquad (3.15)$$

Or,

$$(A_2) = (DEP_0) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots\ldots\ldots \varphi_p z_{t-p})_{0,2} + (P_0.P(1).P(2).S^T) \qquad (3.16)$$

....

....

$$(A_d) = (DEP_0) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots\ldots\ldots \varphi_p z_{t-p})_{0,d} + +(P_0.P_{0,d}.S^T) \qquad (3.17)$$

Or,

$$(A_d)=(DEP_0)+ (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots\ldots\ldots \varphi_p z_{t-p})_{0,d} +(P_0.P(1).P(2)\ldots\ldots\ldots P(d).S^T) \qquad (3.17)$$

Equations 3.14 through 3.17 represent the expanded form of Equations 3.11 through 3.13. In these new equations, the travel time on a link is defined by the autoregressive model and the delay incurred at the downstream time-point due to the upstream time-point is defined by the Markov model. The difference between Equations 3.15 and 3.16 is that Equation 3.15 requires inclusion of a transition matrix between time-point *0* and *2* ($P_{0,2}$) and Equation 3.16 requires the inclusion of transition matrices between time-points *0* and *1* *(P(1))* and *1* and *2* *(P(2))*.

As the bus traverses towards the downstream time-points, the observed value of the departure time at the previous stop is used in addition to the observed delay at the previous stop and the arrival times of the bus at the remaining time-points are updated. Hence, if the bus has an unexpectedly longer observed delay at a previous time-point, the arrival time at the following time-point is predicted accurately. For time-points located downstream of the origin *(i=0)* time-point, the following equations are used to predict the arrival times:

At, $(DEP_1) < t < (A_2)$

$$(A_2) = (DEP_1) + (TT_{1,2}) + (D_{1,2}) \tag{3.19}$$

$$(A_3) = (DEP_1) + (TT_{1,3}) + (D_{1,3}) \tag{3.20}$$

$$\cdots\cdot$$

$$(A_d) = (DEP_1) + (TT_{1,d}) + (D_{1,d}) \tag{3.21}$$

At, $(DEP_2) < t < (A_3)$

$$(A_3) = (DEP_2) + (TT_{2,3}) + (D_{2,3}) \tag{3.22}$$

$$\cdots\cdot$$

$$(A_d) = (DEP_2) + (TT_{2,d}) + (D_{2,d}) \tag{3.23}$$

Where,

(DEP_1), …, (DEP_2) = Observed departure time of bus at time-point *1* and *2*.

(A_2),............, (A_d) = Predicted arrival time of bus at time-point *2* to destination *d*.

$(D_{1,2})$,.............., $(D_{1,d})$ = Predicted delay at time-point *2* through *d* due to delay at time-point *1*.

$(D_{2,3})$,.............., $(D_{2,d})$ = Predicted delay at time-point *3* through *d* due to delay at time-point *2*.

$(TT_{1,2})$, ... $(TT_{d-1,d})$ = Predicted travel time between two consecutive time-points.

The predicted values of bus arrival time are determined by using Equations 3.19 through 3.23, for time-points other than the origin $(1<i<d)$. In these equations, time of prediction is after the departure time of the previous time-point and before the arrival time of the time-point $(DEP_{i-1} < t < A_i)$. The expanded forms of Equations 3.19 through 3.21 are as follows:

$$(A_2) = (DEP_1) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \dots\dots\varphi_p z_{t-p})_{1,2} + (P_0.P(2).S^T) \quad (3.24)$$

$$(A_3) = (DEP_1) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \dots\dots\varphi_p z_{t-p})_{1,3} + (P_0.P(2).P(3).S^T) \quad (3.25)$$

....

....

$$(A_d) = (DEP_1) + (\varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \dots\dots\varphi_p z_{t-p})_{1,d} + (P_0.P(2)\dots.P(d).S^T) \quad (3.26)$$

3.4 Selection of Travel Time Prediction Models

3.4.1 Box-Jenkins Approach

Box and Jenkins (1970) developed a systematic procedure to analyze and forecast time series using ARIMA models. Hence, this procedure is often called the Box-Jenkins methodology. The Box-Jenkins methodology to identify an appropriate ARIMA model is an iterative process and consists of four basic steps. The first step consists of identifying autoregressive, difference and moving average orders of the model, which are represented by *p, d and q,* respectively. This is done based on autocorrelation and partial autocorrelation plots of the time series. In the estimation process, the parameters of the model are estimated using the maximum likelihood method. Diagnostic tests are then performed to determine the adequacy of the model. If the model is considered adequate, it is used to forecast future values. If not, then the iterative cycle of identification, estimation and diagnostic checking is repeated until an adequate model is found.

3.4.2 Autocorrelation and Partial Autocorrelation Functions

The autocorrelation and partial autocorrelation functions of the differenced time series provide clues about the choice of the orders of p and q for the autoregressive and moving average models (Box and Jenkins, 1970). The autocorrelation function of a time series is similar to correlation in regression models, except that the autocorrelation function defines the relationship between values of the same variable at different time periods or lags (k). For a time series ($z_1, z_2, z_3, \ldots\ldots$), the sample autocorrelation at lag k is given by the following relationship (Bowerman and O'Connell, 1987):

$$r_k = \frac{\sum_{t=k+1}^{n}(z_t - \bar{z})(z_{t-k} - \bar{z})}{\sum_{t=1}^{n}(z_t - \bar{z})^2} \tag{3.27}$$

The partial autocorrelation function can be described in terms of p non-zero functions of the autocorrelations (Box and Jenkins, 1970). The partial autocorrelation coefficient of the observed time series at lag k is given by the following relationship (Bowerman and O'Connell, 1987):

$$r_{kk} = \{r_1\} \text{ if } k = 1 \tag{3.28}$$

$$r_{kk} = \frac{r_k - \sum_{j=1}^{k-1} r_{k-1,j} \cdot r_{k-j}}{1 - \sum_{j=1}^{k-1} r_{k-1,j} \cdot r_j} \text{ if } k = 2, 3, \ldots. \tag{3.29}$$

The graph of r_k against lag k is called the sample autocorrelation function (ACF) or in general a correlogram and the graph of r_{kk} against lag k is called the partial autocorrelation function (PACF). Correlograms can be used to describe the behavior of the time series in terms of seasonality, amount of differencing performed on the time series and appropriate orders of p and q. The correlograms also help determine if the model is purely an AR or a MA or an ARMA. This process forms a basis for the Box-Jenkins approach of determining the suitable AR, MA and ARMA model for the given time series.

Table 3.1 Identification of AR, MA and ARMA Model Based on ACF and PACF Plot (Bowerman and O'Connell, 1987)

Model	Autocorrelation Function	Partial Autocorrelation Function
Autoregressive process of order p	Exponential Decay	Spikes at lag 1 to p and 0 after lag p
Moving average process of order q	Spikes at lag 1 to q and 0 after lag q	Exponential Decay
Mixed AR and MA process	Exponential Decay	Exponential Decay

The statistical significance of r_k at any lag k is determined by the t-statistic given by the following relationship:

$$t_{rk} = \frac{r_k}{s_{rk}} \tag{3.30}$$

Where,

$$s_{rk} = \frac{1}{(n-a+1)^{1/2}}\left(1+2\sum_{j=1}^{k-1} r_j^2\right)^{1/2}$$

The statistical significance of r_{kk} at any k is determined by the t-statistic given by the following relationship:

$$t_{rkk} = \frac{r_{kk}}{\dfrac{1}{(n-a+1)^{1/2}}} \tag{3.31}$$

As a rule of thumb, it can be concluded that $r_{kk} = 0$ if $|t_{rk}| \leq 2$. In addition, the ACF and PACF plot can also indicate the seasonality in the time series by showing large positive values of r_k at the seasonal period. If the seasonal time series is not stationary, a seasonal differencing is done to create stationary time series. The autocorrelation and partial autocorrelation plots indicate the amount of seasonal differencing required by showing significant peaks at lags corresponding to the seasonal time periods.

3.4.3 Optimal Order of AR Model

The appropriate order of an autoregressive model is indicated by the autocorrelation and partial autocorrelation function of a time series, by using visual interpretations and is often subjective. Most often, the time series can be fitted with different orders of autoregressive model. However, only one model order is found to be optimum. This optimum model order is found to have the smallest Akaike's Information Criterion (AIC) (Jones, 1974). For this criterion, the mean square error for each model order has a minimum point. Akaike's Information Criterion (Akaike 1974) is given by the following relationship:

$$AIC_p = ln(\sigma_p^2) + \frac{2(p+1)}{N} \tag{3.32}$$

Where,

N =Number of data samples

σ^2 = Estimate of the Prediction Error Power for model of order p

Using the iterative process, the AIC value corresponding to each model order p is determined for each link. The optimum order p for the autoregressive model of the link is the one having the smallest value of AIC.

3.5 Stochastic Travel Time Models

3.5.1 Historical Average and Exponential Filter Models

Historical average and exponential filter models are considered as base models due to their simplicity in computations and are used by many researchers to compare more complex models. The historical average model uses historical data to obtain smoothed values of the time series. The historical average model can be used to predict a single period or multiple periods in advance. The historical average for the period t is given by the following relationship (Makridakis and Wheelwright, 1978):

$$y_t = (z_{t-1} + z_{t-2} + \dots\dots\dots + z_{t-n})/n \tag{3.33}$$

Where,

y_t = Predicted value for period t

$z_{t-1} + z_{t-2} + \dots.$ = Observed values at period t-1, t-2, ...

n = Number of values or periods used in the average

The historical average model essentially consists of determining the average of a set of past values and using that average value as the future value. The decision about how many time values should be included in the historical average is based on subjective judgment of the user and a comparison of the error between the observed and predicted values. However, using a smaller or a larger number of periods in determining the historical average depends on the amount of variability in the underlying pattern of data. In the historical average model, equal weights are given to past values.

In an exponential smoothing model unequal weights are given to past values, where recent observations are given larger weight compared to older observations (Chatfield, 1984). Exponential smoothing is based on the argument that the most recent observation carries more relevant information about the future value and should be given a larger weight. The general form of the exponential smoothing method is given by the following relationship (Makridakis and Wheelwright, 1978):

$$y_{t+1} = y_t + \alpha(z_t - y_t) \tag{3.34}$$

or,

$$y_{t+1} = \alpha z_t + \alpha(1-\alpha) z_{t-1} + \alpha(1-\alpha)^2 z_{t-2} + \alpha(1-\alpha)^3 z_{t-3} + \ldots\ldots\ldots\ldots \tag{3.45}$$

Where,

y_t, y_{t+1} = Predicted values for period t *and* $t+1$

$z_t, z_{t-1}, \ldots$ = Observed values for period $t, t-1, \ldots$

$\alpha \sim [0, 1]$ = Smoothing Factor

The appropriate value of the smoothing factor (α) is chosen by the subjective judgment of the user and is based on the variability of the underlying data. One of the ways to choose the appropriate value of α is by trial and error. For different values of α, the root mean square error (RMSE) values are then obtained. The smoothing factor (α) with the smallest RMSE is chosen as the most suitable model. When α has a value close to 1, the new predicted value will include a large portion of the error from the previously predicted values. Conversely, when α is close to 0, the new predicted value will include a smaller portion of the error from previously predicted values. The effect of choosing a large or small value of α is analogous to choosing longer or shorter periods in the historical average method. Due to the simplicity of the historical average and exponential models, they are used as base models to compare other more complicated time series models.

3.5.2 Autoregressive and Moving Average Models

Autoregressive and moving average models form a basis for more complicated ARIMA models which is a combination of autoregressive and moving average models. Autoregressive models are similar in form to multiple variable regression models. Regression models use multiple independent variables (regressors) to explain a single dependent variable using a linear or non-linear relationship. However, in autoregressive time series models, regressors are the previous observations in time, hence the name autoregression. In regression models, the regressors are required to be independent with a smaller correlation among the regressors. However, in autoregressive models the regressors (previous observations in time) are not required to be independent and errors or residuals are not assumed to be independent.

The autoregressive process of order p is given by the following equation (Makridakis and Wheelwright, 1978):

$$z_t = \phi_1 z_{t-1} + \phi_2 \; z_{t-2} + \ldots + \phi_p \; z_{t-p} + a_t \tag{3.36}$$

Where,

$z_t, z_{t-1}, z_{t-2}, \ldots, z_{t-p}$ = Values of a observed process at equally spaced times *t,t-1, t-2,* ...

$\varphi_1, \varphi_2, \ldots, \varphi_p$ = Finite set of weight parameters.

a_t = White Noise$(0,\sigma^2)$ with 0 mean and σ^2 variance.

In Equation 3.36, the mean of the time series is assumed to be zero. The introduction of a constant mean does not affect the autocorrelation function of the time series (Chatfield, 1984). The autoregressive model with a constant mean can be obtained by modifying Equation 3.36 as follows:

$$(z_t - \mu) = \varphi_1(z_{t-1} - \mu) + \varphi_2(z_{t-2} - \mu) + ... + \varphi_p(z_{t-p} - \mu) + a_t \tag{3.37}$$

Where,

μ = A constant mean

Unlike autoregressive models, moving average models use a linear combination of past errors, instead of the past values of the variable itself. The moving average process of order q is given by the following relationship (Makridakis and Wheelwright, 1978):

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - .... - \theta_q a_{t-q} \tag{3.38}$$

Where,

$a_{t-1}, a_{t-2}, ..., a_{t-q}$ = Previous values of errors at t-1, t-2,.....

$\theta_1, \theta_2, ..., \theta_p$ = Finite set of weight parameters.

Unlike moving average models, which are based on past errors, autoregressive models are based on past observations. This can be further explained by the correlation structure of these two types of models. The moving average model of q^{th} order has the following correlation structure:

$$Corr(z_t, z_{t-j}) \neq 0 \ (j<q), \tag{3.39}$$

$$Corr(z_t, z_{t-j}) = 0 \ (j>=q) \tag{3.40}$$

Where,

z_t, z_{t-1}, z_{t-j} = Observed time series at t, t-1,t-j

In a moving average model, there is no correlation between the present value (z_t) and all past values except the most recent one (z_{t-j}, where $j<q$). Hence, in a moving average model the autocorrelation function cuts off after lag q. The correlation structure

of an autoregressive model of q^{th} order is different from the moving average model and is shown below:

$$Corr(z_t, z_{t-1}) = \frac{covar(z_t, z_{t-1})}{var(z_t)} = \alpha, \tag{3.41}$$

$$Corr(z_t, z_{t-j}) = \alpha^j \quad (j>q) \tag{3.42}$$

Where,

α = Constant parameter

Equation 3.42 shows that there is some correlation between the current value (z_t) and all the past values (z_{t-j}, where $j \geq q$), which is different from the moving average model.

In addition to autoregressive and moving average models, it is possible to combine both models. The combined model provides the most general class of models called as ARIMA process of order p and q. The ARIMA process is given by the following relationship (Makridakis and Wheelwright, 1978):

$$z_t = \phi_1 z_{t-1} + \phi_2\ z_{t-2} + \ldots + \phi_p\ z_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots\ldots - \theta_q a_{t-q} \tag{3.43}$$

Hence, ARIMA models use a combination of past observations and past errors.

3.5.3 ARIMA Models

The ARIMA model, which can represent homogenous nonstationary behavior, is called an autoregressive integrated moving average (ARIMA) process of order (p,d,q). ARIMA models require the input time series to be stationary. The time series is differenced by d times to obtain the stationary time series. The ARIMA model is described by the following relationship (Makridakis and Wheelwright, 1978):

$$w_t = \phi_1 w_{t-1} + \phi_2\ w_{t-2} + \ldots + \phi_p\ w_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \tag{3.44}$$

Where,

$w_t = \nabla^d z_t$ = Differenced series

$\nabla z_t = z_t - z_{t-1}$ = (1-B) z_t

$\nabla^d z_t = (1 - \mathrm{B})^d \mathrm{z}_t$

The backward shift operator B is defined by $Bz_t = z_{t-1}$ or $B^m z_t = z_{t-m}$. When *d=0,* the model is autoregressive and a pure moving average model.

The general ARIMA model can be represented by the following relationship (Box and Jenkins, 1970):

$$\phi(B)w_t = \theta(B)a_t \tag{3.45}$$

In Equation 3.45, $\phi(B)$ signifies the autoregressive part, $\theta(B)$ the moving average part and a_t as white noise.

3.5.4 Subset ARIMA Models

An ARIMA model is specified by determining the order of the autoregressive and moving average terms. The model uses parameters of AR and MA for all lags up to the specified order. For example, if lags *1* and *n* are significant, the full ARIMA model uses parameters from *1* to *n.* If no moving average parameters are used, then the model is called full autoregressive model (F-AR). In some cases some lags between *1* and *n* may not be significant. Hence, the subset ARIMA models consist of parameters associated with lags that are statistically significant. Similarly, if there are no moving average parameters then the model is called subset autoregressive model.

The seasonal ARIMA model is also considered as a type of a subset model because seasonal models use significant lags associated with periods of seasonality.

Seasonal models can be AR, MA or ARMA. The general notation of a seasonal ARIMA model with both seasonal and non seasonal factors is ARIMA *(p,d,q)* x $(P,D,Q)_s$. The term *(p,d,q)* gives the order of the non-seasonal part and the term *(P,D,Q)* gives the order of the seasonal part. The value of *s* represents the number of observations in a seasonal cycle and *D* represents the degree of seasonal differencing used. The seasonal differencing combined with non-seasonal differencing is represented by the following relationship (Box and Jenkins, 1970):

$$w_t = (1-B)^d (1-B^s)^D z_t \tag{3.46}$$

Where,

s = Number of observations in a seasonal cycle

D = Degree of seasonal differencing

Hence, the general form of a seasonal ARIMA model is represented by the following relationship (Box and Jenkins, 1970):

$$\phi(B)\Phi(B^s)w_t = \theta(B)\Theta(B^s)a_t \tag{3.47}$$

Where,

$\Phi(B^s)$ = Seasonal autoregressive part

$\Theta(B^s)$ = Seasonal moving average part

The value of *D* can vary based on the seasonality and is suggested by the significance of the lags in the autocorrelation and partial autocorrelation diagram. In some cases, both seasonal and non-seasonal parameters may be present, where significant lags may be present due to seasonal data and some non recurring non-seasonal data.

3.5.5 Residual Analysis and Performance of Models

A residual is defined as the difference between observed and predicted values of the time series. The residuals are plotted against time over the total period of fit and used to evaluate the overall fit of the time series model. The prediction errors are required to be random for the time series model to be adequate. If the mean residual is close to zero, it is concluded that there is no forecast bias in the model (Chatfield, 2000). A large individual residual suggests an outlier in the data.

In addition to the time series of residuals, an ACF plot of residuals describes the linear relationship between successive residuals. Hence, the ACF plot of residuals also provide an indication of whether the model has a good fit and indicates any remaining structure not explained by the model. For a good prediction model, the expected values of r_k should be zero or close to zero. The statistical significance of the individual r_k can be examined to determine if any exceed $2 / \sqrt{N}$ (where N is the number of sample observations) in magnitude.

The chi-square test statistic of the residuals indicates whether the residuals are uncorrelated or have additional information that could be explained by a more complex model. This is an approximate statistical test of the hypothesis that residuals are uncorrelated up to a given lag. The *p-values* of correlation between errors are also determined using the significance level of 95%. If the *p-value* is less than 0.005 then the hypothesis is rejected and concluded that the residuals are correlated and the fitted model is not adequate.

The performance evaluation of the bus arrival time prediction model is determined by evaluating Mean Absolute Relative Error (MARE) and Mean Absolute

Percentage Error (MAPE). MARE values are used to determine the performance of the arrival time prediction model, where the average of difference between the observed arrival time and the predicted arrival time is determined. However, MAPE values are used to determine the performance of the travel time prediction model. The following equations state the relationships for these measures:

$$MARE = \frac{\sum_{l=1}^{n} | ActualValue_l - ForecastValue_l |}{n} \tag{3.48}$$

$$MAPE = \frac{\sum_{l=1}^{n} \left| \frac{ActualValue_l - ForecastValue_l}{ActualValue_l} \times 100 \right|}{n} \tag{3.49}$$

Where,

l = Individual bus trips

n = Total number of bus trips

3.6 Discrete and Finite State Delay Propagation Model

3.6.1 Finite State Markov Chain

A Markov chain is a stochastic process where an outcome of an event depends only on the present state rather than past states. The process can be visualized using a simple pictorial representation as shown in Figure 3.1, where the process moves from state to state. If the state of the system is represented by s and m is the number of states, $(i, j = 1, \ldots.. m)$ then the n^{th} order transition probabilities for a Markov process is denoted by $p_{ij}(n)$ as defined by the following relationship:

$$p_{ij}(n) = P(x_{l+n} = s_j \mid x_l = s_i) \tag{3.50}$$

The transition probabilities $p_{ij}(n)$ denote the probability that the process goes from state i at the l^{th} step to state j after n steps or transitions. For a homogeneous Markov process, transition probabilities, $p_{ij}(n)$, do not depend on n and can be relaxed to p_{ij}. In a non-homogeneous Markov process, p_{ij} changes over n. With a slight modification in the probability structure used for a homogeneous Markov process, a non- homogeneous Markov process can be obtained. Quite often l and n are substituted as time periods instead of steps. In that case, when the probability relating the next period's state to the current state does not change over time, then the process is called a stationary Markov chain.

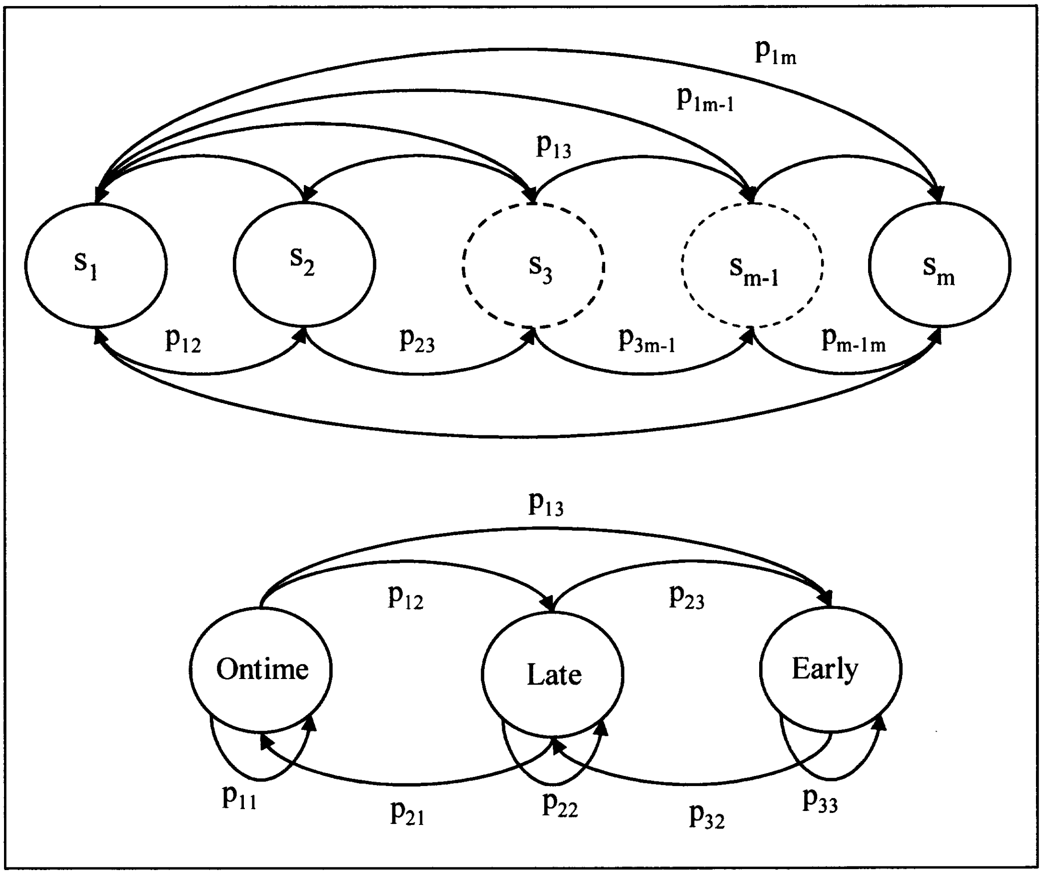


Figure 3.1 Discrete and finite state Markov chain with "*m*" delay states and delay states as "On-time, Late and Early".

The number of states is assumed to be finite. The transition probability matrix $\{p_{ij}\}$ is a square matrix of size *m x m* and is represented as follows:

$$P = \{ p_{ij} \} = \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix} \tag{3.51}$$

Where,

$P = \{ p_{ij} \}$ = Transition matrix

p_{11}, ….. p_{mm} = Transition probabilities from state 1 to m ($0 \leq p_{ij} \leq 1$)

m = Number of States.

In Equation 3.51, p_{11} represents the probability that given the system is in state *1*, it will move to state *1* after one step or one period of time. All the entries in the transition probability matrix are nonnegative and the entries in each row must sum to unity, which is represented as follows (Winston, 1993):

$$\sum_{j=1}^{m} P(x_n = s_j \mid P(x_{n-1} = s_i)) = 1 \tag{3.52}$$

$$\sum_{j=1}^{m} p_{ij} = 1(\forall i) \tag{3.53}$$

A Markov process is completely defined by its transition probability matrix and initial state or an initial probability vector (Taylor and Karlin, 1998). The initial probability vector gives the probabilities for the various possible starting states and is represented by P_0. Based on an initial probability vector, the probability of the process to be in state s_j after n steps is determined the by following relationship (Kemeny and Snell, 1976):

$$P_n = P_{n-1}.P(n) \tag{3.54}$$

Where,

P_n = Transition probabilities after n steps.

P_{n-1} = Initial probability vector.

$P(n)$ = Transition matrix between the steps $n\text{-}1$ and n.

By successive application of Equation 3.54, the probability of the process after n steps is given by the following relationship:

$$P_n = P_0 \cdot \prod_{i=1}^{n} P(i) \tag{3.55}$$

For a homogeneous n-step Markov process, where $P(n)$ are the same for each step or time-period, Equation 3.55 can be written as follows:

$$P_n = P_0 . P(1)^n \tag{3.56}$$

Similarly, for a non-homogeneous n-step Markov process, Equation 3.56 can be modified as follows:

$$P_n = P_0 . P(1) . P(2) \ldots P(n) \tag{3.57}$$

Where,

$P(1), P(2), \ldots P(n)$ = Transition matrix between steps 0 and 1, 1 and 2,...and steps $n\text{-}1$ and n.

P_n = Transition probabilities after n steps.

In Equation 3.57, instead of using the power of matrix $(P)^n$, individual matrices $P(1), \ldots P(n)$ are used to reflect the heterogeneous Markov process.

3.6.2 Estimation of Transition Probabilities

Transition probabilities are easily estimated if observations are available over time on individual transitions. Transition probabilities can then be determined as a ratio of the number of observed delay states to the total number of delay states. Anderson and Goodman (1957) developed a maximum likelihood estimation of stationary transition probabilities by the following method:

$$\text{Maximizing } P_n = P_0.P(1).....P(n) \tag{3.58}$$

Subject to,

$$p_{ij} \geq 0$$

$$\sum_{j=1}^{m} p_{ij} = 1$$

Hence,

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j} n_{ij}} (\forall i) \tag{3.52}$$

Where,

p_{ij} = Transition probabilities between states i and j of the matrix $P(n)$.

n_{ij} = Number of events where state changed from i to j.

The estimate of p_{ij} is the i^{th} *row and* j^{th} column of the table of n_{ij}*'s* divided by the sum of the i^{th} entry in the table (Anderson and Goodman, 1957). This method counts the frequency of an event (transition from state i to state j) that occurred in the past and determines the probability (p_{ij}) of that event divided by the total number of i events. The limitation of the maximum likelihood of estimation of transition probabilities is that, if any event is zero, then the p_{ij} for that particular event is zero.

3.6.3 Application of Markov Chain to Predict Delay

To apply a Markov chain model to predict bus delays at downstream stops, possible delay states at the upstream stop have to be determined. In this research, the term "Delay" is used generically to refer to the *"Early"*, *"On-time"*, and *"Late"* arrival of a bus at a time-point. Delay is defined as the difference between the observed and scheduled arrival time of a bus at a time-point. A constant value "*w*" is introduced, which is transit agency may use as a threshold value to define if the bus is "*Late*" or "*Early*". Since, the value of "*w*" may vary among transit agencies, in this research a value of "*w*" is assumed as *5* minutes. Bus delays are defined by the following relationships:

$$D_i = O_i - S_i \tag{3.60}$$

$$\begin{aligned} Late &= D_i > w, (w > 0) \\ Ontime &= (-w \leq D_i \leq +w) \\ Early &= D_i < -w \end{aligned}$$

Where,

D_i = Bus arrival delay at time-point *i*.

O_i = Observed arrival time of bus at time-point *i*

S_i = Scheduled arrival time of bus at time-point *i*

w = Constant in minutes (Based on a transit agency's policy to define Early, On-time and Late bus arrivals)

In Equation 3.60, the bus is *"Late"* if the delay (D_i) is greater than w=*5* minutes. If the delay is between -5 and +5 minutes, the bus is *"On-time"*. Similarly, if the delay is less than w=*-5* minutes, then the bus *"Early"*. For each trip, the delay (D_i) is determined using Equation 3.6. In this research, the delay states ($s_1, s_2, s_3, \ldots\ldots, s_m$) represent possible ranges of bus delay between any two time-points. The value of m depends on the number of delay states between each pair of time-points. In this research, the possible delay states are limited to *On-time, Early, Late*. More detailed delay states, such as {-10, -8,….., 0, ….,+8, +10) minutes can be used. However, to incorporate more detailed delay states requires a large number of samples corresponding to each delay state.

The transition probabilities between delay states are represented by a transition matrix of size $(m \times m)$ with elements $\{p_{i,j}\}$, where each element represents the conditional probability that given a current bus is delayed s_i at time-point i, the bus will be delayed s_j at time-point $i+1$. For each combination of time-points, separate transition matrices are determined represented by $(\{p_{i,j}\}_{0,1}, \ldots\ldots\ldots, \{p_{i,j}\}_{i-1,i})$. As the bus traverses downstream, the delay at the downstream time-point $(i+1)$ is predicted using the transition matrix $\{p_{i,j}\}_{i,i+1}$ and the initial probability vector at i. To determine the delay at the downstream time-points, Equation 3.55 is modified as follows:

$$d_n = P_0.P(1)....P(n).s^T \tag{3.61}$$

Where,

d_n = Delay at the nth time-point.

s^T = Transpose of delay states.

P_0 = Initial probability vector with the form {0, 1, 0, … 0} or $\sum_{i=1}^{m} p_i = 1$ at 0^{th} time-point.

The nature of the bus delay propagation to downstream time-points can be homogeneous or heterogeneous. Homogeneous propagation refers to an equal amount of delay propagated to downstream time-points from an upstream time-point. This is assumed to occur if the distances between downstream time-points are similar in length and traffic characteristics to the preceding time-points. In contrast to homogenous propagation, a heterogeneous propagation of delay occurs when the links connecting time-points differ in length and traffic characteristics. In homogeneous delay propagation, the transition matrix between time-points is assumed to be identical. However, for heterogeneous delay propagation, the transition matrix differs for each set of time-points as a result of differences in the length and traffic behavior on the links. Markov chain models can be used to model either homogeneity or heterogeneity of delay propagation between any combinations of time-points.

To determine the delay at the immediate downstream time-point 1 measured from time-point *0* (origin), Equation 3.61 is modified as follows:

$$d_1 = P_0.(P(1))^1.s^T \tag{3.62}$$

$$d_1 = P_0 \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}_{0,1}^{1} \begin{pmatrix} s_1 \\ \vdots \\ s_m \end{pmatrix} \tag{3.63}$$

Similarly, based on homogeneous propagation of delay between time-points *0* and *1* and *1* and *2*, the delay at time-point *2* measured from the time-point *0*, is predicted by using the following relationship:

$$d_2 = P_0.(P(1))^2.s^T \tag{3.64}$$

$$d_2 = P_0 \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}_{0,1}^{2} \begin{pmatrix} s_1 \\ \vdots \\ s_m \end{pmatrix} \tag{3.65}$$

Hence, by the successive application of Equations 3.62 and 3.63, the delay at any downstream time-point can be determined from any upstream time-point assuming a homogeneous propagation of delay.

However, assuming two corresponding links are homogeneous in characteristics may not be realistic. In most instances, links have variable lengths, traffic volumes and number of traffic signals. In such a situation, a heterogeneous Markov chain is used by obtaining different transition probability matrices between the corresponding time-points. For example, to predict the delay at time-point ($i=2$) from origin ($i=0$), Equation 3.61 is modified as follows:

$$d_2 = P_0.P(1).P(2).s^T \tag{3.66}$$

$$d_2 = P_0 \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}_{0,1} \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}_{1,2} \begin{pmatrix} s_1 \\ \vdots \\ s_m \end{pmatrix} \tag{3.67}$$

Where,

$\begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}_{0,1} \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}_{1,2}$ = Transition probabilities between time-points *0, 1* and *1, 2.*

By expanding Equation 3.61, the delay at the n^{th} time-point from any k^{th} $(k<n)$ time-point can be determined by the following relationships:

$$d_n = P_0.(P)^{n-k}.s^T \tag{3.68}$$

$$d_n = P_0.P(k)....P(n).s^T \tag{3.69}$$

Equation 3.68 assumes that links between time-points k and n have similar characteristics or are homogeneous and Equation 3.69 assumes that links between time-points k and n have dissimilar characteristics, or are heterogeneous. The difference between Equations 3.68 and 3.69 is that in Equation 3.68, the transition probability matrix is successively powered by an integer. In Equation 3.69, individual transition probabilities between any two time-points are multiplied.

CHAPTER 4

CASE STUDY ANALYSIS

4.1 Introduction

In this chapter, an explanation of the bus route selected as a case study is provided. The structure of the source data, which is gathered from an APC system installed on the buses running along the route, is also explained. The chapter provides a discussion of the variables required to successfully develop time series models for bus arrival time prediction. In addition, an analysis of the bus delay propagation pertaining to the case study is performed. The bus arrival time algorithm and the results obtained for the case study is presented in subsequent chapters.

4.2 Description of a Case Study

The data for this research was obtained from buses equipped with a GPS based APC system and operated in the State of New Jersey. Such buses run along a number of routes, one of which is route number 62, which runs from downtown Newark in Essex County to Woodbridge Center Mall in Middlesex County. The bus route map is shown in Figure 4.1.

The one-way trip time from Newark's Penn Station to Woodbridge Center Mall is about 1 hour 40 minutes. The bus travels through different land uses such as a downtown location in the city of Newark, industrial areas at Port Newark/Elizabeth, and suburban locations in towns of Metuchen, Edison and Woodbridge. The route is also comprised of different types of roadways such as, urban arterials, state highways and rural roads. The route comprises of roadways with a variety of traffic volumes and signal density. These conditions have a direct effect on bus travel time. The bus route consists of 135 stops with 14 designated time-points.

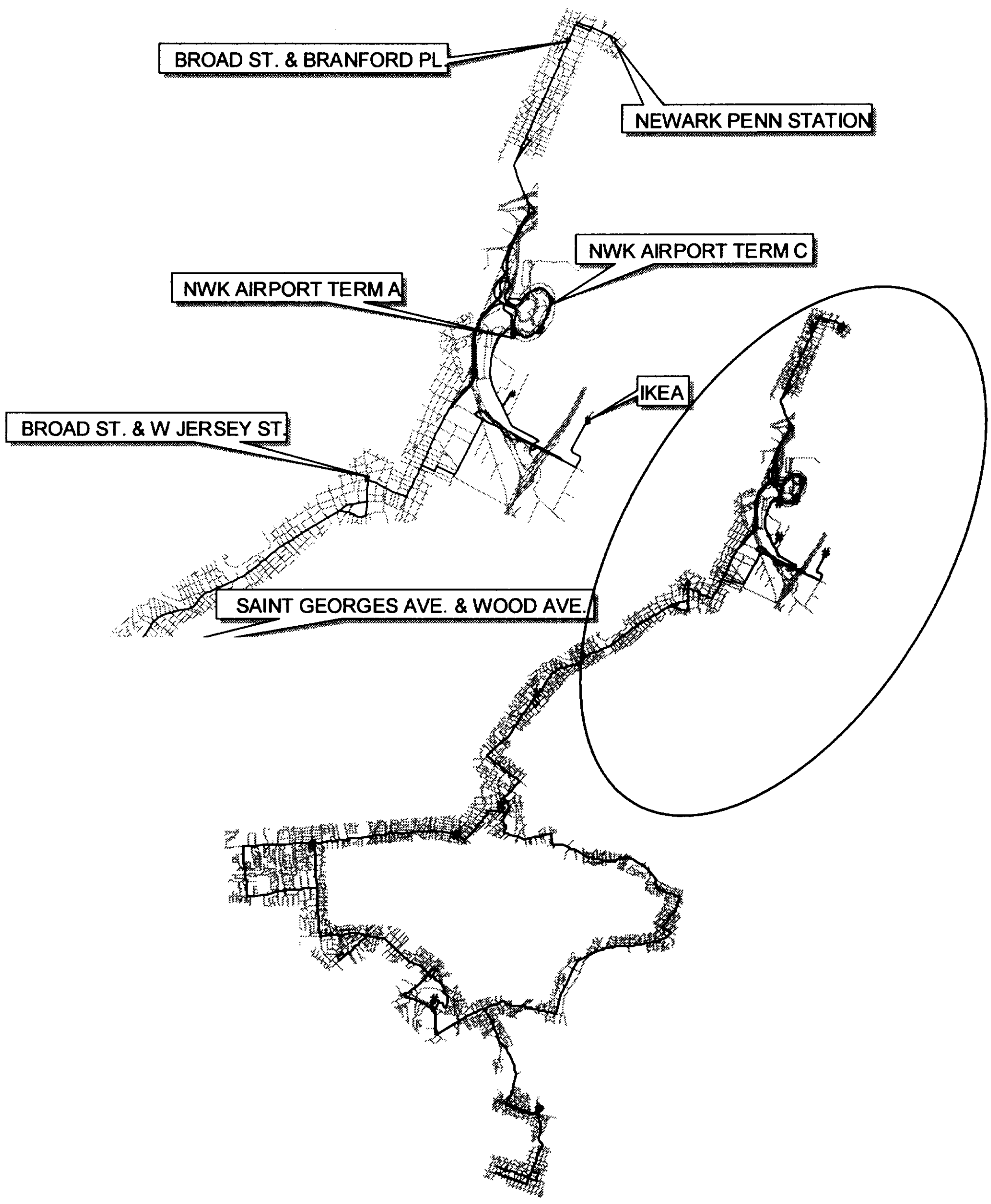


Figure 4.1 GIS diagram of bus route 62 with time-points.

4.3 APC Data Structure

The data collected from the APC system consists of 7 months of data from June to December 2002. In the database used in this research, information about the bus stop is available only if the bus actually stops there. If the bus did not stop at a bus stop because of the absence of boarding or alighting passengers, then no information is recorded. The APC system provides the following information about the existing condition of the bus:

- Location of bus in geographical coordinates (latitude and longitude);
- Scheduled arrival and departure time at time-points only;
- Observed arrival and departure time at time-points and bus stops;
- Time- points where the bus stopped;
- Dwell time at bus stops, door open and close time; and
- Number of passengers boarding and alighting.

The information about a bus trip provided in the data source is shown in Table 4.1, which consists of data for a single trip. In the table, "Stop ID" represents the identification given to each stop along the bus route. The door open time and close time at the corresponding stop is provided with the corresponding scheduled arrival time. Since Stop ID's 30, 40 and 80 are not time-points, the scheduled arrival times for these stops are not available. In addition, the bus trips are classified in the database into seven different time of day periods based on schedule start time of the trip. The classification of time of day is presented in Table 4.2.. These times indicate the scheduled trip start time.

Table 4.1 Sample AVL/APC Data of a Single Bus Trip

Scheduled Start Time	Open Time	Close Time	Stop ID	Scheduled Time	Ons	Offs
08:05:00	080118	080506	10	8:05 AM	15	0
08:05:00	080812	080814	40	-	0	0
08:05:00	080929	081006	50	8:09 AM	9	1
08:05:00	081121	081231	60	-	1	1
08:05:00	081313	081322	70	-	1	1
08:05:00	082052	082100	0	-	0	1
08:05:00	082350	082429	140	8:25 AM	3	4
08:05:00	082640	082647	0	-	0	2
08:05:00	082915	082930	150	8:28 AM	2	1
08:05:00	083140	083212	160	8:31 AM	1	6
08:05:00	084018	084023	190	-	0	1

Table 4.2 Classification of Time of Day

Time of Day	Hour of Day
Early Morning	4:00 - 7:00
Morning Peak	7:00 - 9:00
Late Morning	9:00 – 12:00
Mid-day	12:00 – 14:00
Early Afternoon	14:00 – 16:00
Afternoon Peak	16:00 – 18:00
Evening	18:00 - 21:00

4.4 Spatial Analysis of Link Travel Time

The preliminary analysis of the bus travel time data consists of obtaining travel time values from the system's database to arrange the data in a time series. This process consists of arranging the travel time data so that it corresponds to identified links and time periods. The observed travel time between two time-points is the difference between the door open time at the following stop and the door close time at the preceding stop. The door close and open times are obtained from the APC system installed in the bus.

In this research, the coefficient of variation of travel times for each link is determined to study the characteristics of the links along the bus route. It is presumed that the coefficient of variation of links along the bus route varies based on the land use of the link and not the link length. It is assumed that a higher coefficient of variation is observed at urban and downtown locations and a lower coefficient of variation is observed at rural and highway locations along the bus route.

For normal (Gaussian) distributions, the coefficient of variation (c_v) measures the relative scatter or variation in the data with respect to the mean and is expressed as the ratio of standard deviation to mean. A higher coefficient of variation is associated with a higher variability of travel times and a lower coefficient of variation is associated with a lower variability of observed travel times.

The coefficient of variation of travel time between links and time-points was found to vary depending on the length of the link and traffic characteristic along the link. These characteristics of the link are influenced by the link location (central business district (CBD), suburban or rural location). Table 4.3 provides the coefficient of variation of travel time at different links of the study bus route after averaging the travel times. For a link within the CBD (link 10-50), the coefficient of variation is higher than that of a link located outside the CBD (link 50-140), in spite of the fact that the length of the link 50-140 is longer than the link 10-50. This finding concurs with the fact that highway segments have less fluctuation in volume and speed than road segments located in the

CBD areas. It also shows that the coefficient of variation is independent of the link length and is the function of the link's land use and traffic characteristics.

Table 4.3 Coefficient of Variation of Observed Travel Times

Starting Time-point	Ending Time-point	Link Characteristics	Link Length (*miles*)	Coefficient of Variation
10	50	CBD-Local streets	0.59	0.316
50	140	Local streets and highway	1.32	0.104
140	180	Local streets	4.70	0.096
180	300	Local streets	3.70	0.098
300	500	Local streets	3.20	0.168
500	900	Highway and suburban streets	8.91	0.152
900	1000	Suburban streets	3.10	0.302

4.5 Bus Delay Propagation Analysis

4.5.1 Characteristics of Bus Delay

The propagation of delay between time-points depends on traffic characteristics and the length of the link. A longer link distance allows more time for the bus drivers to recover the schedule at the downstream time-point, if the bus departed late at the upstream time-point. In addition, the traffic characteristics and roadway type of the link also influences the schedule recovery process. Hence, bus delay characteristics at individual time-points were evaluated in this research prior to modeling the delay propagation from the origin time-point to downstream time-points.

The percentage of trips for each of the three delay states for all time-points was obtained. The delay states were determined assuming values of "*w*" from *1* to *5* minutes, which is the amount of flexible time included in the schedule. The percentage of trips for each delay state for all time-points with *w=4* and *5* minutes are shown as Figures 4.2 and 4.3, respectively.

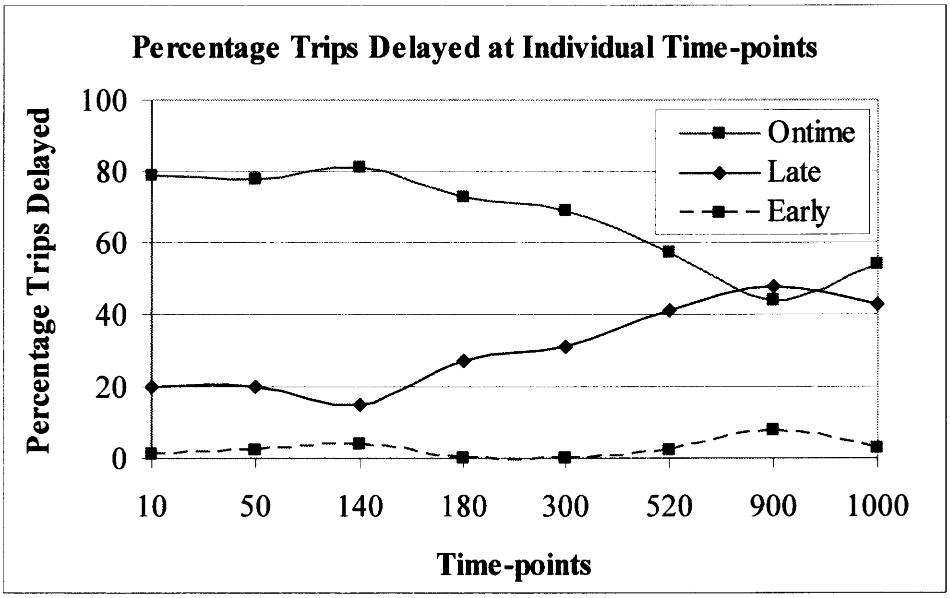


Figure 4.2 Percentage trips delayed at individual time-points with *w*=4 minutes.

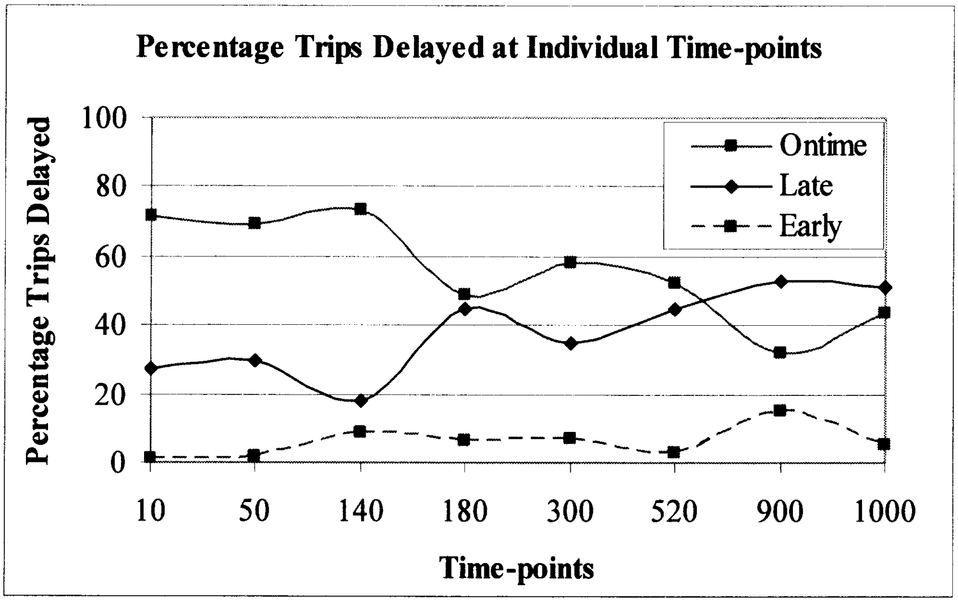


Figure 4.3 Percentage trips delayed at individual time-points with *w*=5 minutes.

Figures 4.2 and 4.3 show that the difference between the percentage of *"On-time"* and *"Late"* trips decreases as buses move downstream. For example, in Figure 4.2, the percentage of trips *"On-time"* at TP=140 is 80% and *"Late"* is 22%. At TP=900, the percentage trips *"On-time"* and *"Early"* are 50 percent each. In addition, as the value of "*w*" decreases, the percentage of *"Late"* trips increases and *"On-time"* trips decrease. At smaller "*w*" values, the percentage of *"Late"* trips is higher than *"On-time"* trips for all time-points.

The value of "*w*" to be adopted is subject to transit agencies. In this research, *w=5* minutes is assumed to be a reasonable time period to determine the bus delay status. The percentage of early trips is not significant except for smaller values of "*w*". This is because transit agencies discourage early arrivals. In reference to Figures 4.2 and 4.3, the bus arrival prediction models should be able to maximize the "*On-time*" and minimize the "*Late*" percentage of trips at each time-point. Figures 4.2 and 4.3 also show the "trouble" time-points, where buses have higher probability of arriving late and being unable to adequately recover the schedule.

4.5.2 Non-Homogenous Delay Propagation

Bus delay propagation to downstream time-points can be either homogeneous or heterogeneous. The type of delay propagation is important in modeling the delay propagation. Regardless of whether the delay propagation is homogeneous or heterogeneous, a Markov chain model can be devised. Homogeneous propagation refers to an equal amount of delay propagated to downstream time-points from an upstream time-point. This is assumed to occur if the downstream time-points are equidistant in length and traffic characteristics are similar along the route. In contrast to homogenous propagation, heterogeneous propagation of delay occurs when the links connecting time-points differ in length and traffic characteristics. In this research, the study bus route travels through links variable in length and traffic characteristics. Based on Table 4.4, which shows the links connecting time-points for the route under study are variable in length and land-use, it can be assumed that the links connecting individual time-points vary in traffic characteristics as well.

Table 4.4 Linear Regression Slopes Between TP=10 and Downstream Time-points

Downstream Time-points	Regression Slope (Intercept = 0)
50	0.966
140	0.312
180	0.454
300	0.413
500	0.896
900	0.651
1000	1.086

The homogeneity or heterogeneity of delay propagation along a bus route can be determined by comparing the slopes of a linear regression of delay at individual time-points. The linear regression of the delay at the origin time-point is the independent variable *(x)* and the delay at any downstream time-point *(y) is* the dependent variable observed during previous trips. Regression slopes less than *1* suggest that the delay at the downstream time-point *(y)* is reduced as the bus travels from one time-point to another. Hence, the slope of the resulting linear regression represents the schedule recovery by bus drivers and the delay propagation at a particular time-point along the link. The slopes are plotted for each link along the entire bus route. If the slopes are decreasing or increasing towards the destination time-point at a uniform rate, the links along the bus route can be termed homogeneous.

A linear regression was performed to determine the linear relationship of the bus delays at the origin time-point (TP=10) and each of the remaining downstream time-points. Figure 4.4 shows the slopes of the regression model relating the delay at the origin time-point to the delay at each downstream time-point. For example, the slope at TP=50 is greater than 1 and much higher than the slope at TP=140. This relationship shows that buses are less able to recover their schedule at TP=50 compared to TP=140. Figure 4.4 also shows that the slopes are not decreasing or increasing uniformly towards the destination time-point. Hence, it could suggest that the delay propagation is not homogeneous between individual links and the transition probabilities of each link are not the same. The existence of homogeneous delay propagation would have generated a uniformly decreasing or increasing curve.

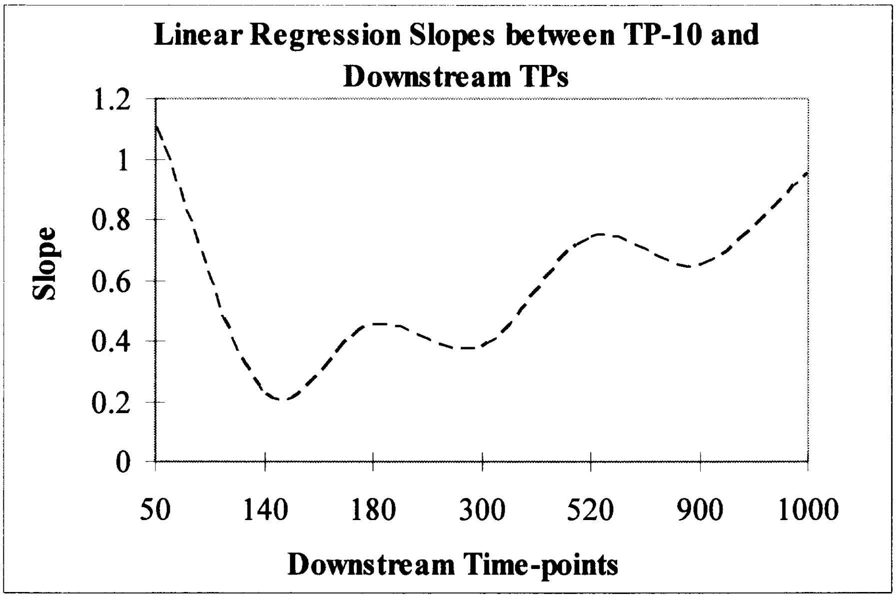


Figure 4.4 Linear regression slopes between TP=10 and downstream time-points.

Ideally, if there is a delay at the origin time-point, delays at the downstream time-points should gradually reduce and reach a negligible value at the destination time-point. This means bus drivers should be able to fully recover the schedule by the time the bus reaches the destination time-point. However, from Figure 4.4 it is observed that for the study route bus drivers are not able to fully recover the schedule at the destination time-point (TP=1000), though some amount of schedule recovery occurs at TP=140, as indicated by the decreasing slope from TP=50 to TP=140. An important observation from Figure 4.4 is that the schedule recovery process varies at each link between time-points because of the variable length of link and traffic characteristics. Hence, it would not be appropriate to assume that delay would propagate homogeneously towards downstream time-points.

The schedule recovery process shown in Figure 4.4 consists of all the samples (all available bus trips), and no temporal differentiation has been made. For example, a single slope is used to represent the schedule recovery process between the links. However, the amount of schedule recovery may vary during different time periods such as peak and off peak hours. Figure 4.4, suggests that the schedule recovery process varies at each link as shown by the varying amount of regression slopes.

CHAPTER 5

PREDICTION OF BUS TRAVEL TIME

5.1 Introduction

The bus travel times are predicted independent of the delay propagation model, which is based on Markov chains. The bus travel time prediction model is then included as a part of a combined model with delay propagation model to predict bus arrival times. In this chapter, the selection of appropriate autoregressive time series models to predict bus travel time is presented. In addition, the parameter estimates and performance measure of the models are also presented. Full autoregressive, seasonal and subset autoregressive models were then developed for individual links that would be used to predict bus travel times. Finally, comparisons between the autoregressive models and the more complex models were performed to select the most appropriate model.

5.2 Graphical Analysis

Bus travel time series were obtained for each bus stop along the bus route for different times of the day. The time series starts from Monday 5 AM and extends to Sunday 8 PM, as shown in Figures 5.1 and 5.2. Figure 5.1 shows bus travel time between TP=10 and TP=50 and Figure 5.2 shows bus travel time between TP=50 and TP=140. Each point in the graph is the mean bus travel time for different days at the same time of day. The vertical axes in the time series graphs represent the bus travel time between stops. The horizontal axis represents the day of week.

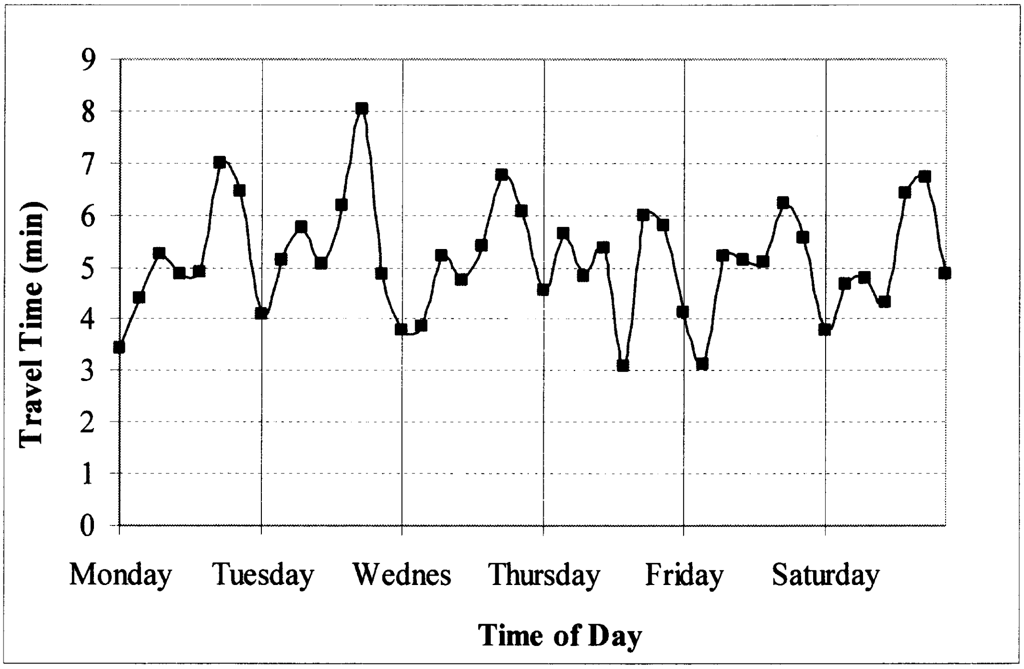


Figure 5.1 Time series plot of observed travel time for link 10-50.

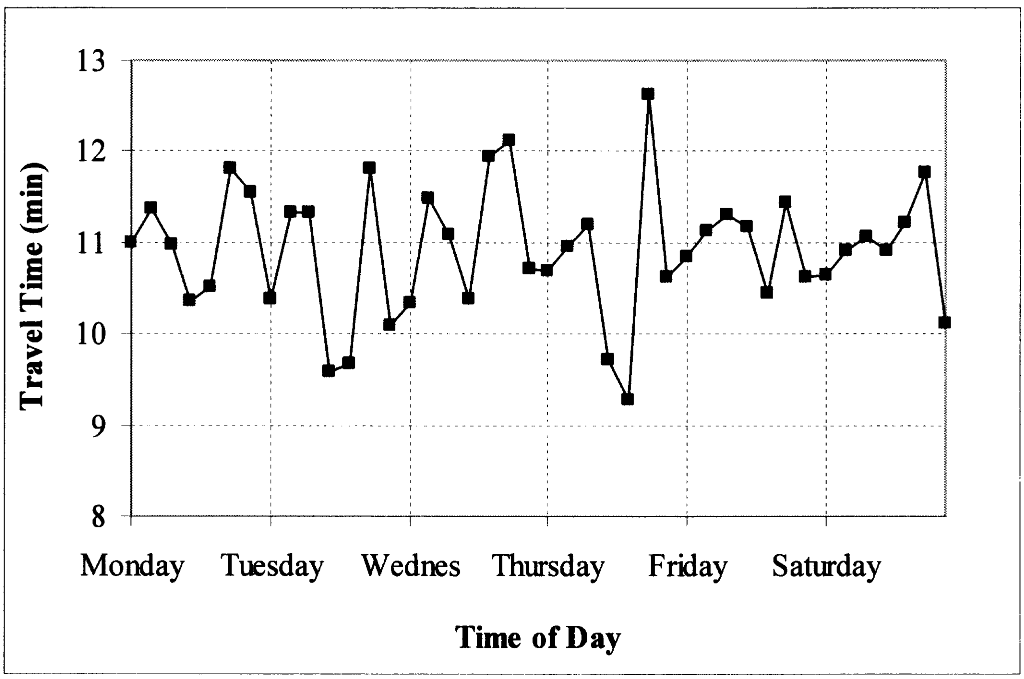


Figure 5.2 Time series plot of observed travel time for link 50-140.

The above time series provided some insight into the variability, trend and cyclic behavior of bus travel time. A distinct observation was that the time series graphs were very similar for all links in terms of trend and variability during time of day. On each day of the week, morning and evening peak hours showed higher travel times than other times of day. However, the peaks varied on different days. The graphs showed that the mean travel time increased from early morning towards the morning rush to afternoon peak and decreased thereafter, which is a very typical observation of traffic condition. The graphs showed higher travel time on Monday with slowly decreasing travel times for the remainder of the week.

The highest peaks in Figures 5.1 and 5.2 are primarily associated with the afternoon peak period. The afternoon peaks are usually followed by a smaller value of travel time in the evening. Similar trends in time series were obtained for both links 10-50 and 50-140. It was observed that the cyclic behavior is more prominent on link 10-50 than on link 50-140. This is because link 50-140 has a lower coefficient of variation and the link consists of highway sections with a lower probability of recurring congestion.

5.3 Autocorrelation Functions

The sample ACF/PACF plot and model fit plot were obtained for each time series. The ACF and PACF plots differ by link and by time of day. The ACF plot showed statistically significant lags at the peak periods, but also showed seasonality extending five days starting from Monday through Friday. Hence, in addition to full autoregressive models, subset/seasonal autoregressive models were also considered to determine bus travel time models using significant lags only and comparing these models with full autoregressive models.

The ACF and PACF plots indicated a full autoregressive model with lag 7 for most of the links. Hence, the time series fitted autoregressive parameters with p=7. The ACF plot of link 10-50 is shown in Figure 5.3. For link 50-140, the ACF plot is similar to the ACF plot of link 10-50. Lags are shown in the horizontal axis and the autocorrelation (r_k) is shown on the vertical axis. It can be seen from Figure 5.3 that all the parameters of the lags from 1 to 7 were not significant. The significant lags are 2, 5 and 7. These lags correspond to the morning peak, early afternoon and evening periods. The full autoregressive model was developed based on the above observations.

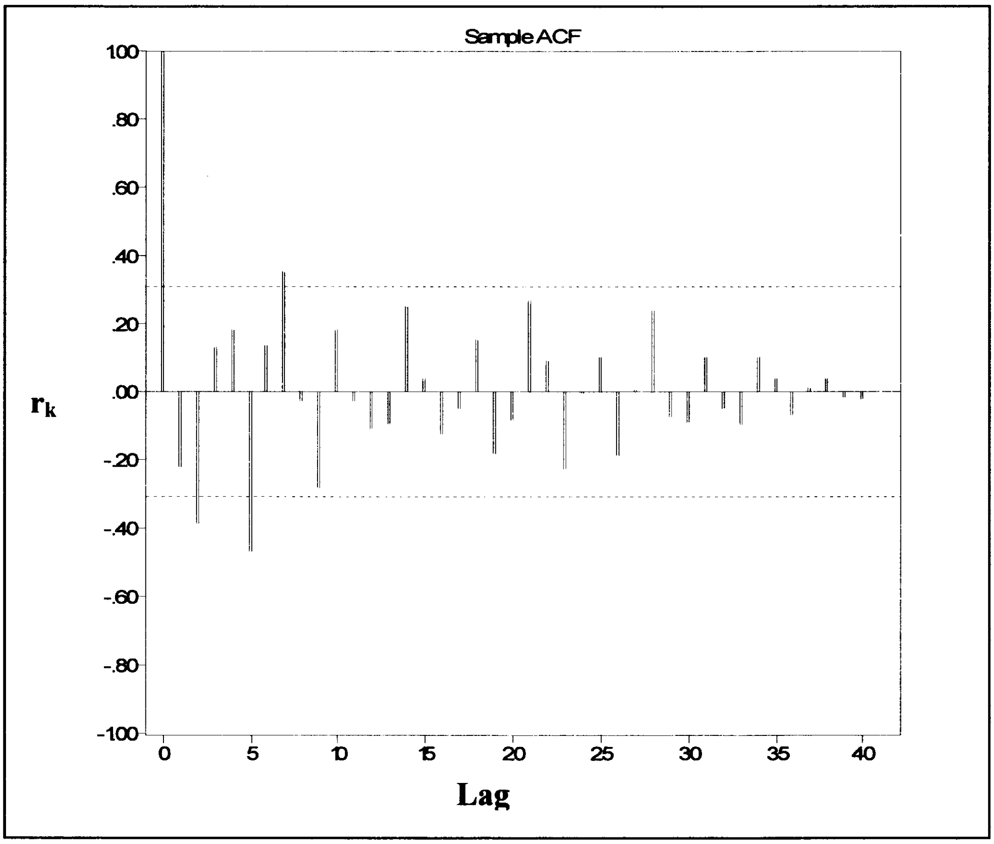


Figure 5.3 Autocorrelation plot of observed travel time for link 10-50.

The presence of seasonality in all time series was also checked with ACF and PACF plots. The time series of link 10-50 showed peaks at lags 7 and 14, but the peaks die down exponentially as shown in Figure 5.3. Hence, there is strong seasonality in this link. From the ACF and PACF plots, it is clear that the peaks at periods 2 and 7, which correspond to morning and afternoon peak, were more prominent. Using these observations for all links, the parameters were selected for full, subset and seasonal models.

5.4 Historical Average and Exponential Smoothing Models

The selection of the appropriate number of periods (n) used in the historical average model and the smoothing factor (α) in the exponential smoothing model was made iteratively. Mean absolute percent error (MAPE) was used as the performance measure to compare the performance of the model for different values of n and α. The higher the value of (n) in the historical average model, the smoother the fit of the curve resulting in larger reduction of the overall MAPE of the prediction model. In the exponential smoothing model, the lower value of α gives a smoother model and reduces the overall MAPE. The smoother curve means the variance in the original time series is not well described by the model.

From Table 5.1, the overall MAPE for link 10-50 is below 20% and 10% for the rest of the links except for link 300-500. The MAPE values observed for all the links are consistent with the coefficient of variation of travel times at these links. For example, the coefficient of variation for link 50-140 was lower than the coefficient of variation for link 10-50. Similarly, the MAPE of link 50-140 is lower than that of link 10-50.

Table 5.1 MAPE for Historical Average and Single Exponential Smoothing of Observed Bus Travel Time

Links	Historical Average		Exponential Smoothing	
	%	N	%	α
10-50	20.81	3	20.03	0.4
50-140	5.60	3	5.75	0.3
140-180	4.35	5	5.55	0.1
180-300	5.21	5	7.24	0.1
300-500	12.50	6	13.40	0.3
500-900	7.50	6	7.46	0.1
900-1000	7.55	6	8.00	0.1

Figures 5.4 and 5.5 show estimates of bus travel times using the historical average and exponential smoothing models and the observed travel time for links 10-50 and 50-140, respectively. The MAPE achieved for both models did not vary significantly. The estimated travel times obtained from both models are also similar. However, the smoothing models are not able to correctly estimate the peak values during certain time-periods. These peak values are the results of re-occurring peak conditions along the bus route. For this reason, historical average and exponential smoothing models are not suitable to predict bus travel times in links where there is substantial peaking of bus travel time data.

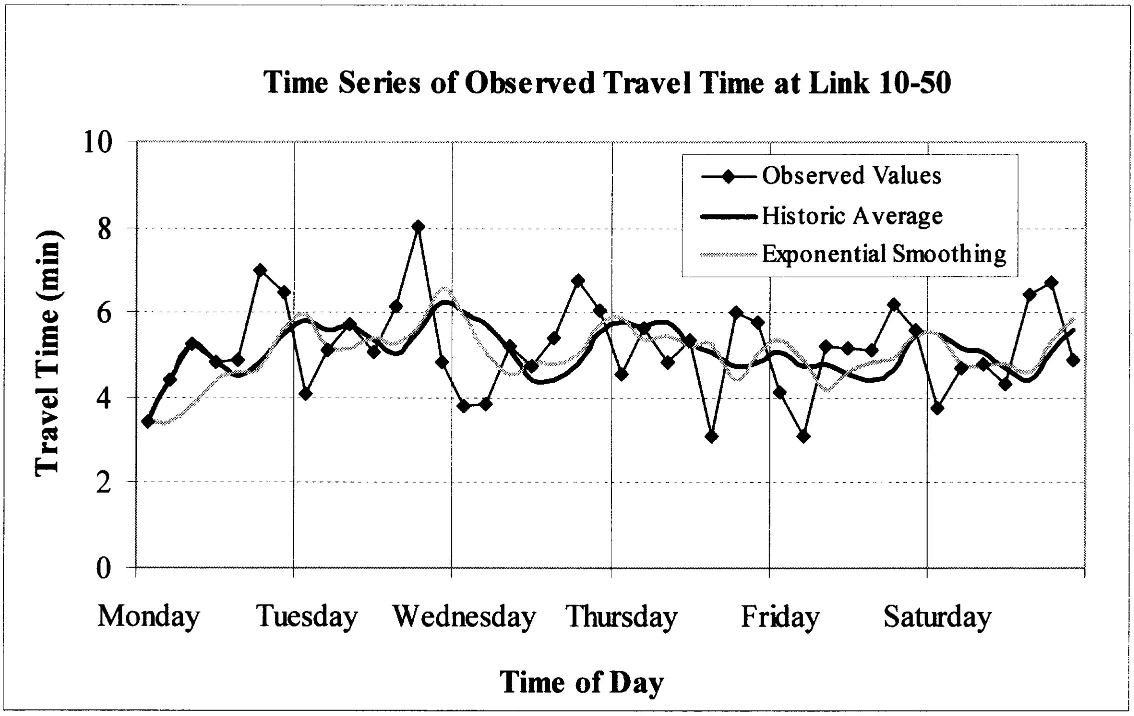


Figure 5.4 Smoothing model diagram for link 10-50.

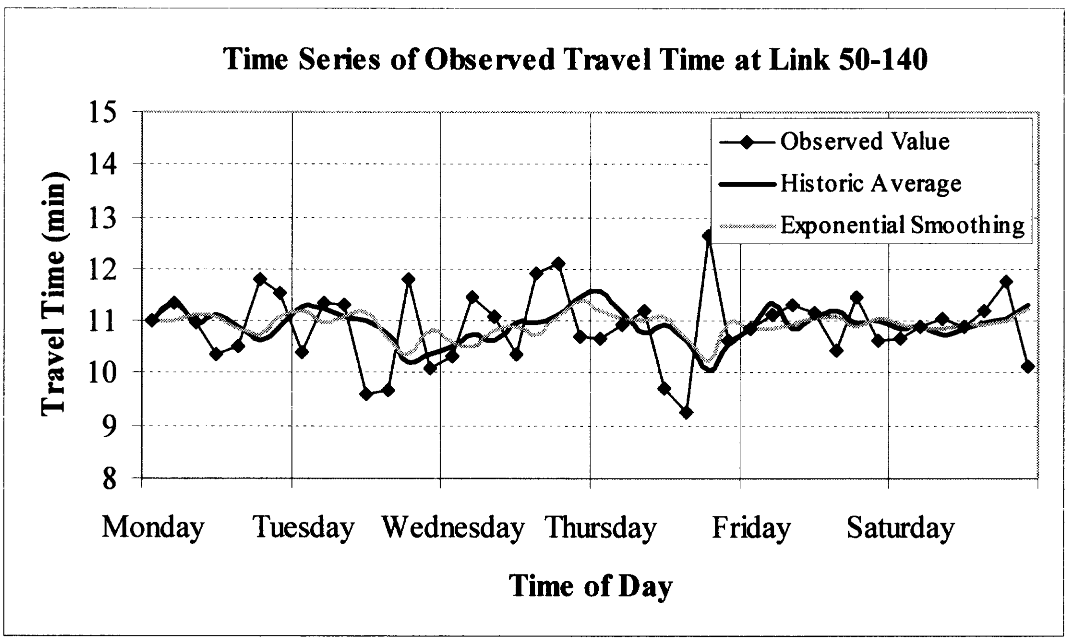
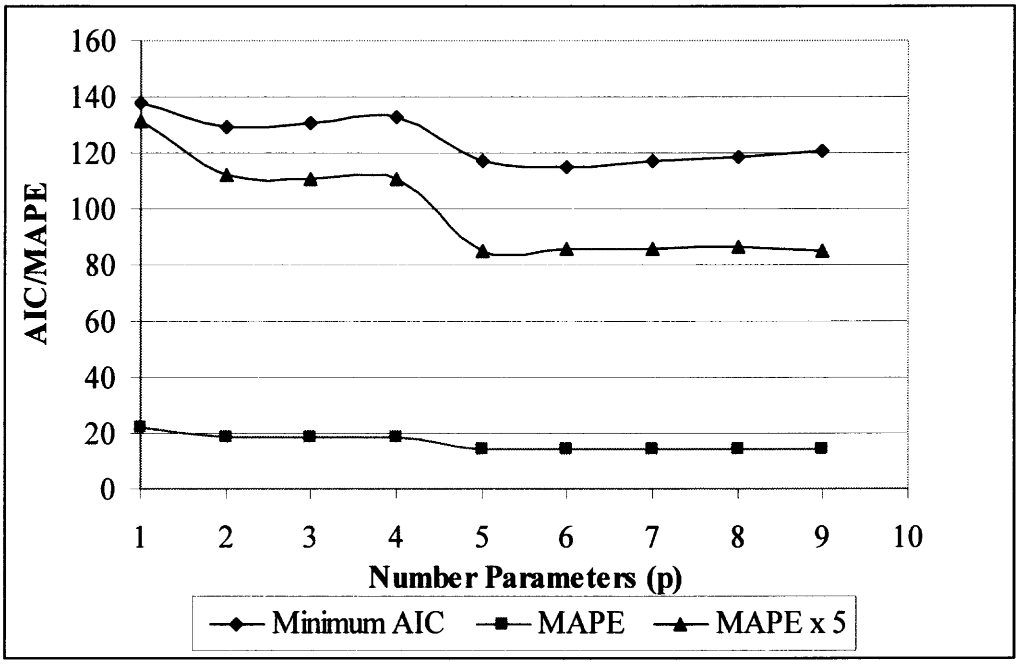


Figure 5.5 Smoothing model diagram for link 50-140.

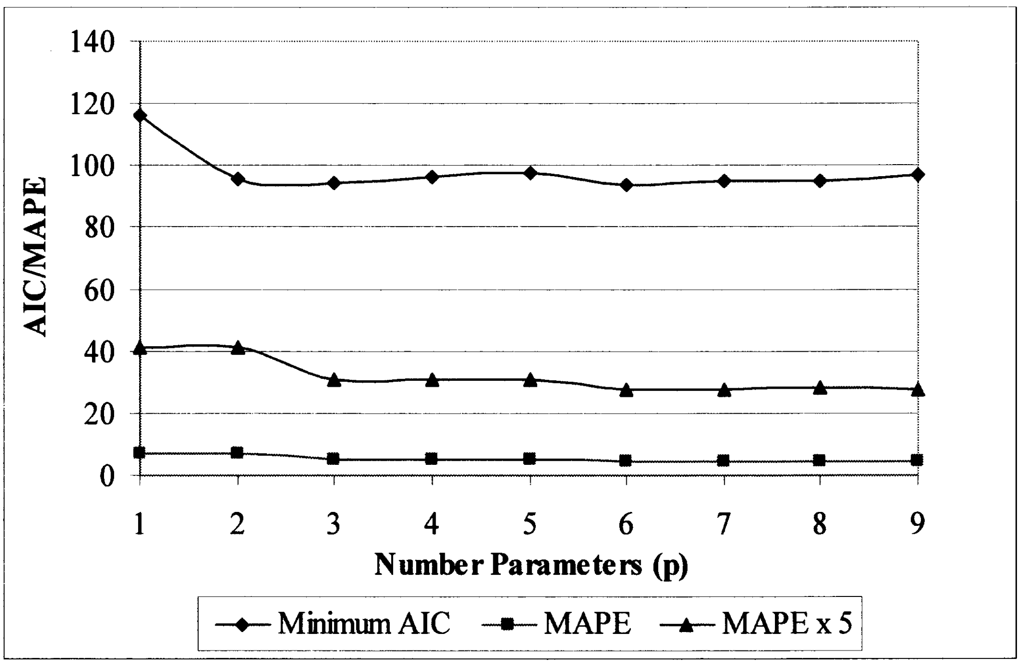
5.5 Minimum AIC and Parameter Estimates

The minimum value of the AIC is used to select the appropriate number of "p" parameters in the full autoregressive model. In addition, the Mean Absolute Percentage Error (MAPE) of the model is also used to evaluate the performance of the model. The MAPE and AIC were determined for different "p" parameters for two links, 10-50 and 50-140, as shown in Figures 5.6a and b. In both figures, there is a sudden drop in the AIC close to the optimum value of "p". The sudden drop in the AIC is also consistent with the drop in the MAPE. This confirms that the model selected based on minimum AIC consistently shows minimum MAPE as well. The process of selecting the final models using minimum values of MAPE and AIC were used for all links.

The number of parameters used in developing the full autoregressive model is always equal to the number of time-periods (time of day) in a day minus one time period. For example, in link 10-50, the minimum MAPE and AIC occurred at $p=6$. The number of time periods within any day for link 10-50 is 7 demonstrating the relationship between the optimum p value and the number of time periods within the day. Similarly, for links 140-180 and 180-300, the morning peak trips were not available, which resulted in one less time period and one less parameter in the full autoregressive model. Hence, the number of autoregressive parameters was 5 (p=5).



(a)



(b)

Figure 5.6 Minimum AIC and MAPE for links a) 10-50 and b) 50-140.

5.6 Full Autoregressive Model

The analysis using full autoregressive models showed that higher orders of p resulted in a MAPE around 15% for link 10-50 and less than 10% for the rest of the links as shown in Table 5.2, with the exception of link 140-180 which has a MAPE of 10.78%. These values are a significant improvement from the historical average and exponential smoothing models. The AIC for all links listed in Table 5.2 are minimum values. Hence, the order of autoregressive models is optimum. The results also showed that single order of differencing is appropriate for the full autoregressive models.

Table 5.2 MAPE for Full AR Model of Observed Bus Travel Time

Link	MAPE	Model	AIC
10-50	14.30	(6,1,0)	115.14
50-140	4.61	(6,1,0)	93.71
140-180	10.79	(5,1,0)	160.77
180-300	4.72	(5,1,0)	98.91
300-500	6.53	(6,1,0)	148.56
500-900	4.88	(6,1,0)	184.51
900-1000	7.48	(6,1,0)	134.05

Table 5.3 provides the estimated parameters for link 10-50 with the standard error, t-value and p-values. The p-values for all parameter estimates are significant at a 95% confidence level with one parameter having a p-value of 0.0540. The full autoregressive prediction model for link 10-50 can be written as follows:

$$z_t = -0.632 z_{t-1} - 0.751 z_{t-2} - 0.677 z_{t-3} - \ldots\ldots - 0.315 z_{t-6} + a_t \qquad (5.1)$$

Or,

$$(1+0.632B^{1}+0.751B^{2}+0.677B^{3}+........+0.315B^{6})z_t=a_t \tag{5.2}$$

Table 5.3 Parameter Estimates of Full AR Model for Link 10-50

Model = F-AR (6,1,0)				
Parameter	**Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
φ_1	-0.63262	0.15875	-3.99	<.0001
φ_2	-0.75123	0.14151	-5.31	<.0001
φ_3	-0.67767	0.17677	-3.83	0.0001
φ_4	-0.51271	0.17540	-2.92	0.0035
φ_5	-0.76909	0.14209	-5.41	<.0001
φ_6	-0.31504	0.16348	-1.93	0.0540
MAPE	14.30%			

The estimated and observed travel times for links 10-50 and 50-140 using full autoregressive models are shown in Figures 5.7 and 5.8, respectively. The peaks predicted by the full autoregressive model are similar to the observed peak values, unlike the estimate predicted by the historical average and smoothing models. These figures also include 95% upper and lower confidence limits of the predicted travel time. The observed travel times are well within the confidence limits.

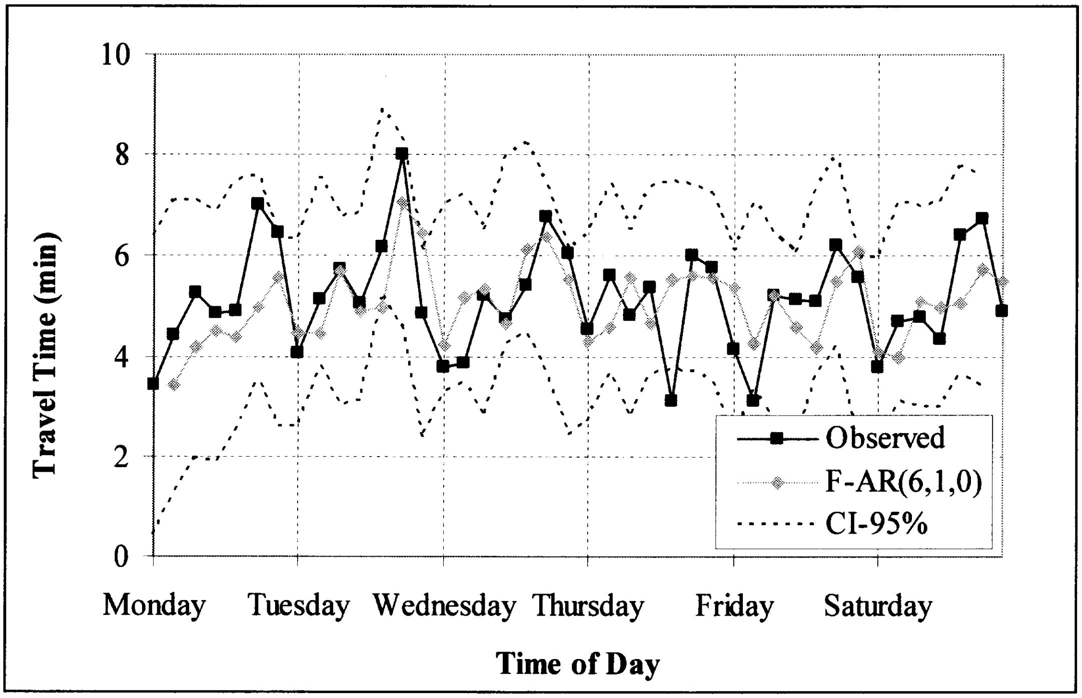


Figure 5.7 Fit diagram of full AR model for link 10-50.

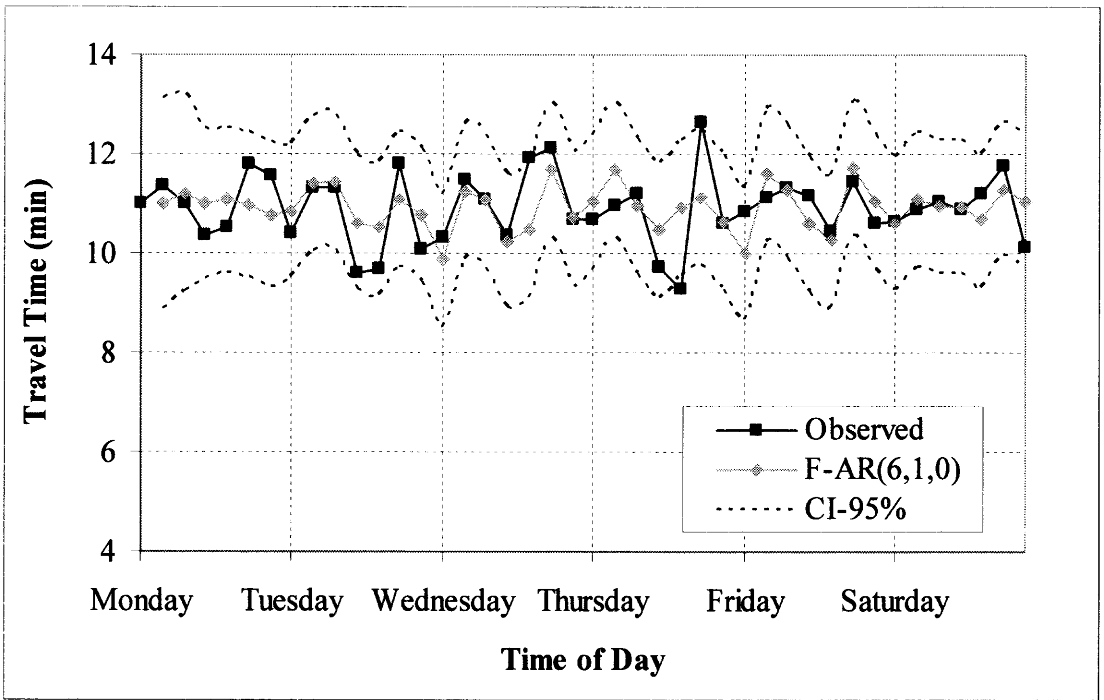


Figure 5.8 Fit diagram of full AR model for link 50-140.

5.7 Seasonal and Subset Autoregressive Model

The results of fitting a seasonal autoregressive model on the time series are shown in Table 5.4. For all links, the order of seasonal models used was 7 based on 7 time periods in a given day. It was assumed that the seasonality is repeated each day. However, the order of seasonality of links 140-180 and 180-300 was 6 because the number of time periods available was 6 in a day. For link 10-50, the MAPE obtained was similar to that of the full autoregressive model. However, the MAPE was higher for the seasonal model compared to the full autoregressive model for all other links. This demonstrates that a seasonal model may not be suitable for predicting bus travel times. Figures 5.9 and 5.10 show observed and predicted bus travel times for links 10-50 and 50-140, respectively.

Table 5.4 MAPE for Seasonal AR Model of the Observed Bus Travel Time

Links	MAPE	Model	AIC
10-50	18.40	$(1,1,0)(1,1,0)_7$	111.85
50-140	6.27	$(1,1,0)(1,1,0)_7$	88.21
140-180	14.15	$(1,1,0)(1,1,0)_6$	145.60
180-300	5.97	$(1,1,0)(1,1,0)_6$	88.63
300-500	6.68	$(1,1,0)(1,1,0)_7$	123.53
500-900	7.25	$(1,1,0)(1,1,0)_7$	168.44
900-1000	8.93	$(1,1,0)(1,1,0)_7$	118.48

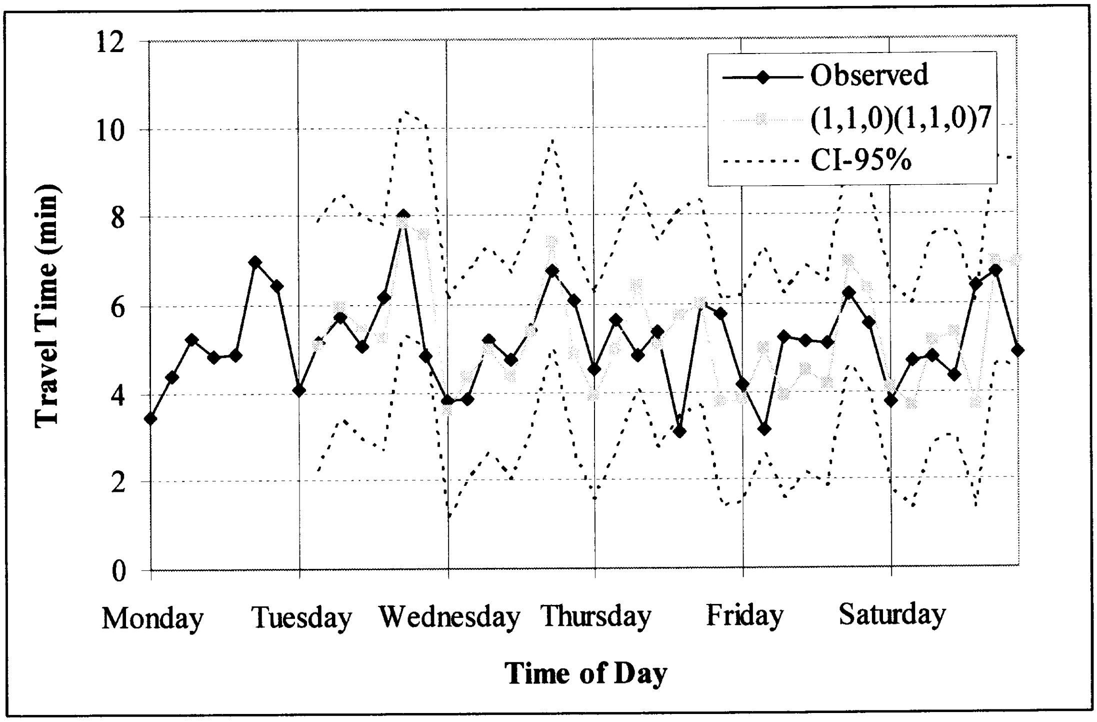


Figure 5.9 Fit diagram of seasonal AR model for link 10-50.

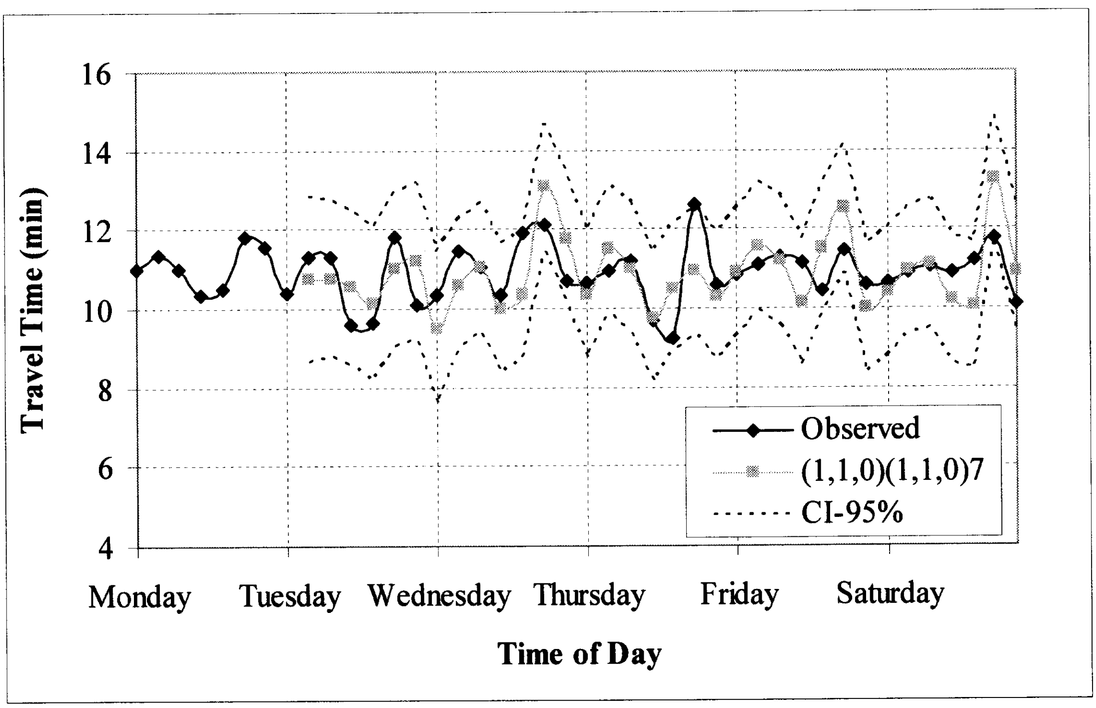


Figure 5.10 Fit diagram of seasonal AR model for link 50-140.

The subset model uses only those parameters that are significant rather than all the parameters as in the full autoregressive model. Subset models consist of parameters associated with the significant lags observed in ACF and PACF plots. For all links, parameters associated with statistically significant lags were identified from the results obtained from the full autoregressive model. Estimates of the statistically significant parameters have *p-value* less than 0.005. These significant lags are shown in Table 5.5. For example, for link 10-50, parameter estimates corresponding to lags 2 and 5 are statistically significant. Hence, the subset model for the link is represented by S-AR(2) at lags 2 and 5 in Table 5.5.

Table 5.5 MAPE for Subset AR Model of Observed Bus Travel Time

Link	MAPE	Model	AIC
10-50	17.81	S-AR (2) at lags 2, 5	126.72
50-140	5.914	S-AR (2) at lags 2, 7	109.73
140-180	14.04	S-AR (2) at lags 1 and 6	171.00
180-300	6.45	S-AR (2) at lags 1 and 6	108.96
300-500	8.09	S-AR (1) at lag 7	157.77
500-900	6.73	S-AR (2) at lags 2 and 7	196.55
900-1000	9.05	S-AR (2) at lags 1 and 7	141.18

Table 5.6 provides the estimated parameters for link 10-50 with the standard error, t-value and p-values. The p-values for all of the parameter estimates are significant at a 95% confidence level. The subset autoregressive model for link 10-50 can be written as follows:

$$z_t = -0.335 z_{t-2} - 0.464 z_{t-5} + a_t \tag{5.3}$$

Or,

$$(1+0.335B^2+0.464B^5)z_t=a_t \tag{5.4}$$

Table 5.6 Parameter Estimates of Subset AR Model at Lags 2 and 5 for Link 10-50

S-AR (2) at lags 2 and 5				
Parameter	**Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
φ_2	-0.33568	0.12556	-2.67	0.0075
φ_5	-0.46432	0.12821	-3.62	0.0003
MAPE	17.81%			

Figures 5.11 and 5.12 show estimated and observed bus travel times for links 10-50 and 50-140, respectively. For the time series, the peaks predicted by the model do not fit the observed pattern as the observed peaks seem to lag behind the predicted peaks as seen in the historical average and exponential smoothing model. Hence, subset autoregressive models may not appropriately model the observed bus travel time during peak periods.

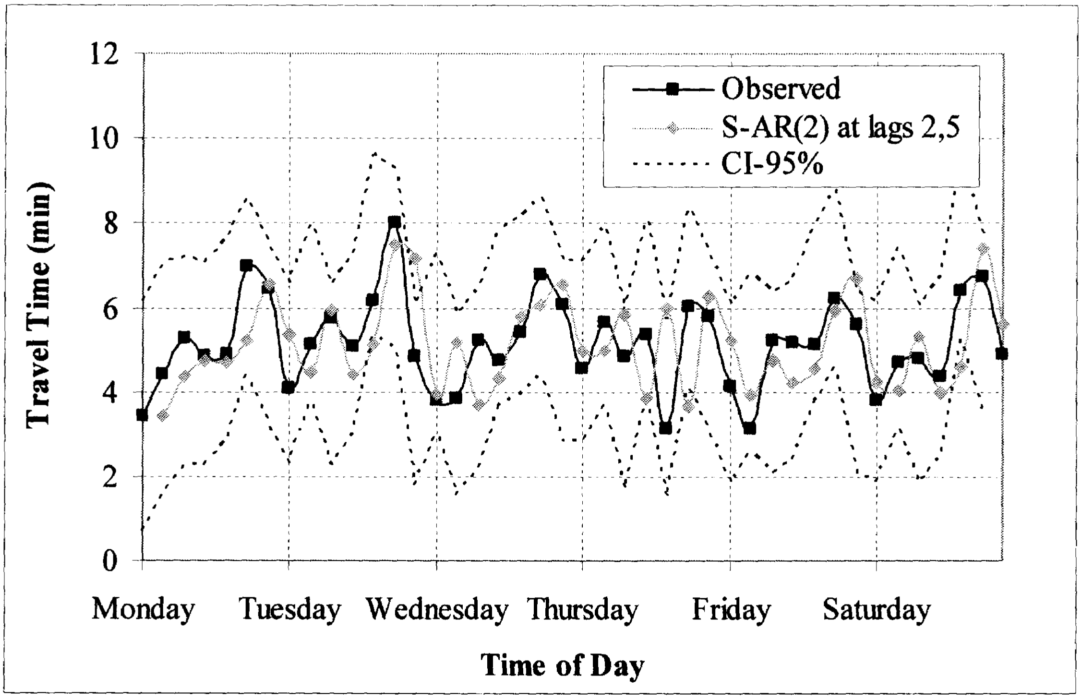


Figure 5.11 Fit diagram of subset AR model at lags 2 and 5 for link 10-50.

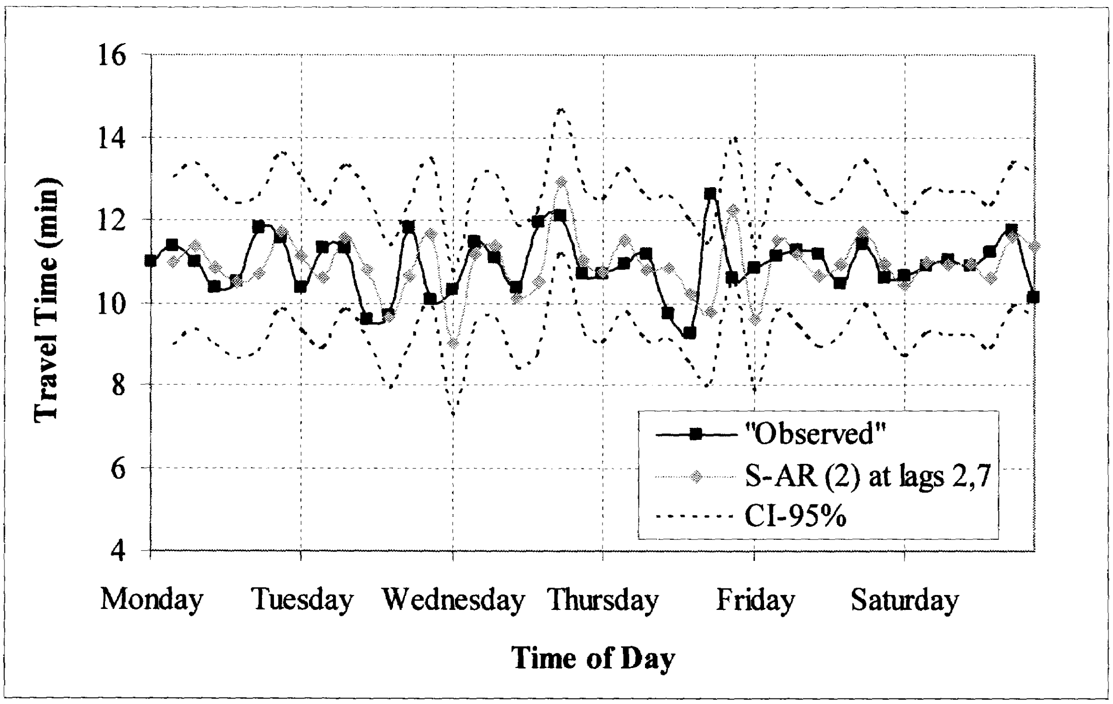


Figure 5.12 Fit diagram of subset AR model at lags 2 and 7 for link 50-140.

5.8 Comparison of Autoregressive Models

The analysis showed higher order full autoregressive models resulted in an MAPE of around 15% for link 10-50 and 4.5% for link 50-140. This is compared to the historical and smoothing exponential models which resulted in MAPE values of 20.81 and 5.61. The MAPE for the full autoregressive model are lower than the MAPE obtained from historical average, exponential smoothing models, seasonal and subset autoregressive models.

The results also showed that the appropriate model "*p*" parameter was 6 except for links 140-180 and 180-300, for which it was 5. In most cases, lags pertaining to peak hours were more significant than lags of other periods. The peaks predicted by the full autoregressive model are similar to the observed peak values.

For seasonal models there was a mixed response, since some links showed strong seasonality and some did not. For example, the time series for link 10-50 showed peaks at lags 7 and 14, but the peaks die down exponentially. For link 50-140, the peaks at lags 2 and 5 were statistically significant followed by statistically significant peaks at 10 and 14. However, there was not much increase in MAPE using seasonal models in comparison to full autoregressive models.

The subset model focuses on using individual significant parameter estimates rather than all the parameter estimates as in the full autoregressive model and consists of parameters associated with the significant lags only. The peaks predicted by the model do not fit the observed pattern as the observed peaks seem to lag behind the predicted peaks as in historical average and exponential smoothing model.

In conclusion, full autoregressive models perform better than seasonal and subset models. The presence of seasonality is not significant except in a few cases, because of the larger variation in observed travel times. Hence, seasonal and subset autoregressive models do not improve the overall MAPE above the performance of the full autoregressive model.

5.9 Cross Validation of Full AR Models

It is important to determine if full autoregressive models would perform differently when the time series is shortened. If the MAPE of the full autoregressive model is very different for the time series with a smaller amount of data, then the ability of the model to consistently give similar MAPE may be questionable. The ability of the full autoregressive model to predict bus travel time using variable lengths of time series was validated by splitting the time series data.

The full autoregressive model for the entire week *(Monday, ..., Saturday)* of time series data was split and evaluated using only the first four days *(Monday, ..., Thursday)*. Due to the fact that the number of samples was reduced, the parameter estimates for the model using the time series for the entire week and the parameter estimates for the model using a time series consisting of the first four days differed even though the full autoregressive model order was the same. In addition to the consistency of models, the analysis also showed the individual model's ability to provide reasonable forecast with a smaller number of time series data.

To compare the model's performance due to the reduced sample and to test the ability of the models to forecast the following two days of data, MAPE for individual links were obtained by using the reduced time series data consisting of four days only. Table 5.7 shows the comparisons of results obtained by using the full 7-day week, five days and four days of data to build full autoregressive models. The MAPE values using the entire week of data, five and four days of the data are shown in Table 5.7. The table shows that the MAPE increases slightly with a decrease in the number of days of time series data. However, there are no large changes in the MAPE as a result of the reduction in the number of samples in the time series.

Using time series consisting of five and four days, one day and two days lead forecasts were performed and the MAPE was obtained and shown in Table 5.7. The results showed that the MAPE for two days lead forecast was higher than MAPE for one day lead forecast. This showed that less than a full week of data can be used to adequately model full autoregressive given the resulting MAPE value do not vary significantly. However, in this research full week of data was used in the full autoregressive model.

Table 5.7 Cross Validation of Full AR Models

Links	Mean Absolute Percent Error (%)				
	Using Full Week Time Series	Using Five Days Time Series	Using Four Days Time Series	Two Days Lead Forecast	One Day Lead Forecast
10-50	14.30	15.55	15.34	20.51	13.42
50-140	4.61	4.94	5.32	4.96	2.87
140-180	10.79	11.10	11.71	9.89	11.59
180-300	4.72	4.93	4.26	7.18	4.66
300-500	6.53	6.28	6.45	5.22	8.81
500-900	4.88	4.85	4.45	7.13	4.75
900-1000	7.48	5.63	5.43	8.58	6.71

CHAPTER 6

PREDICTION OF BUS DELAY

6.1 Introduction

This chapter describes the use of a Markov process to determine the propagation of bus delay and provides a basis for using bus delay in the prediction of bus arrival times. The propagation of bus delay based on a Markov process has been explained in this chapter. Based on an assumption of either homogeneous or heterogeneous traffic conditions between the links, the transition probabilities are predicted and compared with observed transition probabilities. Predicted transition probabilities obtained using heterogeneous traffic conditions between links were close to the observed transition probabilities.

6.2 Estimation of Transition Probabilities

The formulation of a Markov process to determine bus delay propagation requires the estimation of transition probabilities of delay states between time-points. To determine transition probabilities of delay states between time-points, the number of observed delay states were obtained and classified as *"On-time", "Late" and "Early"* at each time-point. The transition probabilities were obtained using the following relationship:

$$\hat{p}_{ij}^{k} = \frac{n_i}{\sum_{j=1}^{m} n_{ij}}, m \in \{1,3\}, k \in \{1,K\} \tag{6.1}$$

Where,

m = Total number of delay states = 3

n_{ij} = Number of events where state changed from i to j

k = Number of time-points

The delay states and the representative transition matrix is shown as follows:

$$p_{ij} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} ontime-ontime & ontime-late & ontime-early \\ late-ontime & late-late & late-early \\ early-ontime & early-late & early-early \end{pmatrix} \quad (6.2)$$

Each p_{ij} represents the probability of being in a delay state *(j)* at the downstream time-point when the upstream time-point has a delay state of *(i)*. For example, p_{11} represents the probability that the delay state at the downstream time-point is *"On-time"*, given the delay state at the origin time-point (TP=10) is also *"On-time"*.

For each combination of origin time-point (TP=10) delay state and downstream time-point delay state, transition probabilities (p_{ij}) were determined using Equation 6.1 and are shown in Table 6.1. In addition, the transition probabilities of delay states between consecutive time-points were determined as shown in Table 6.2. The observed transition probabilities for the remaining links are provided in Appendix A. In Table 6.1, the probability of the bus being *"On-time"* at a downstream time-point is higher if the bus is *"On-time"* at the origin time-point. The "*Early*" delay state has lower frequency than other delay states. This is because transit agencies discourage drivers to reach time-points early.

The largest probability in any transition matrix occurs when the upstream time-point has "*On-time*" delay state and the corresponding delay state at the downstream time-point is also "*On-time*". This shows that most of the trips on the study route reach time-points "*On-time*" when buses reach the upstream time-points "*On-time*". However, this probability decreases as the bus moves downstream, as seen from the lower transition probabilities for time-points 180 to 1000 in Table 6.1.

Table 6.1 Observed Transition Probabilities Between TP=10 and Downstream Time-points Based on Maximum Likelihood Estimation

	On-time	Late	Early	Downstream TP	Total Sample Size
On-time	0.876	0.119	0.005	50	215
Late	0.107	0.893	0.000		
Early	0.000	0.500	0.500		
On-time	0.905	0.048	0.048	140	274
Late	0.595	0.380	0.025		
Early	0.500	0.000	0.500		
On-time	0.653	0.327	0.020	180	86
Late	0.622	0.378	0.000		
Early	0.000	0.000	0.000		
On-time	0.762	0.201	0.037	300	271
Late	0.596	0.385	0.019		
Early	0.333	0.000	0.667		
On-time	0.715	0.252	0.033	500	204
Late	0.436	0.526	0.038		
Early	0.333	0.333	0.333		
On-time	0.657	0.224	0.119	910	114
Late	0.383	0.553	0.064		
Early	0.000	0.000	0.000		
On-time	0.638	0.313	0.050	1000	266
Late	0.431	0.559	0.010		
Early	0.000	0.500	0.500		

Table 6.2 Observed Transition Probabilities Between Time-points Based on Maximum Likelihood Estimation

	On-time	Late	Early	Upstream Time-point	Downstream Time-point
On-time	0.876	0.119	0.005	10	50
Late	0.107	0.893	0.000		
Early	0.000	0.500	0.500		
On-time	0.922	0.026	0.052	50	140
Late	0.353	0.647	0.000		
Early	0.000	0.000	1.000		
On-time	0.777	0.215	0.008	140	180
Late	0.050	0.900	0.050		
Early	0.667	0.000	0.333		
On-time	0.865	0.125	0.010	180	300
Late	0.146	0.854	0.000		
Early	0.200	0.200	0.600		
On-time	0.790	0.189	0.021	300	500
Late	0.132	0.868	0.000		
Early	0.250	0.000	0.750		
On-time	0.898	0.020	0.082	500	910
Late	0.105	0.895	0.000		
Early	0.000	0.500	0.500		
On-time	0.887	0.113	0.000	910	1000
Late	0.000	1.000	0.000		
Early	0.545	0.182	0.273		

6.3 Prediction of Transition Probabilities

The transition probabilities for all combinations of time-points were predicted using Markov chains. This was done based on both homogeneous and heterogeneous propagation of delay between time-points. Transition probabilities of delay states were predicted at downstream time-points initially with reference to the origin time-point (TP=10). Thereafter, the transition probabilities of delay states were predicted with reference to the time-point (TP=50) and so on.

Based on homogeneous delay propagation between time-points, the transition probabilities between the downstream time-point (TP=1000) and time-point (TP=10) are determined as follows:

$$P_{10-1000} = P_{10-50} \cdot P_{50-140} \cdot P_{140-180} \cdot P_{180-300} \cdot P_{300-500} \cdot P_{500-900} \cdot P_{900-1000} \quad (6.3)$$

$$P_{10-50} = P_{50-140} = P_{140-180} = \dots\dots\dots = P_{900-1000} \quad (6.4)$$

$$P_{10-1000} = \{P_{10-50}\}^7 \quad (6.5)$$

$$P_{10-1000} = \begin{pmatrix} 0.876 & 0.119 & 0.005 \\ 0.107 & 0.893 & 0.000 \\ 0.000 & 0.500 & 0.500 \end{pmatrix}_{10-50}^{7} \quad (6.6)$$

Where,

$P_{10\text{-}50}, \dots\dots P_{10\text{-}1000}$ = Transition probabilities of delay states between TP=10 and remaining time-points.

Similarly, the transition probabilities between the downstream time-point (TP=900) and time-point (TP=10) are determined as follows:

$$P_{10-900} = P_{10-50} \cdot P_{50-140} \cdot P_{140-180} \cdot P_{180-300} \cdot P_{300-500} \cdot P_{500-900} \quad (6.7)$$

$$P_{10-900} = \{ P_{ij} \}^{6}_{10-50} \tag{6.8}$$

$$P_{10-900} = \begin{pmatrix} 0.876 & 0.119 & 0.005 \\ 0.107 & 0.893 & 0.000 \\ 0.000 & 0.500 & 0.500 \end{pmatrix}^{6}_{10-50} \tag{6.9}$$

Equation 6.3 is based on the assumption that the delay at TP=1000 due to the delay at TP=10 is a function of delay at individual links and these delays are homogeneously propagated as shown in Equation 6.4.

Unlike Equation 6.3, by assuming that propagation of delay is heterogeneous, the predicted delay state at downstream time-point (TP=1000) with reference to the origin time-point (TP=10) is obtained by following relationships:

$$P_{10-1000} = P_{10-50} \cdot P_{50-140} \cdot P_{140-180} \cdot P_{180-300} \cdots\cdots P_{900-1000} \tag{6.10}$$

$$P_{10-50} \neq P_{50-140} \neq \ldots\ldots\ldots\ldots \neq P_{900-1000} \tag{6.11}$$

$$P_{10-1000} = \begin{pmatrix} 0.876 & 0.119 & 0.005 \\ 0.107 & 0.893 & 0.000 \\ 0.000 & 0.500 & 0.500 \end{pmatrix}_{10-50} \cdots\cdots \begin{pmatrix} 0.887 & 0.113 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.545 & 0.182 & 0.273 \end{pmatrix}_{900-1000} \tag{6.12}$$

Using Equations 6.10 and 6.12, the p_{ij}'s for all the downstream time-points were determined with reference to time-point (TP=10). The results of the predicted transition probabilities at individual time-points with reference to the origin time-point (TP=10) are shown in Table 6.3. The predicted transition probabilities for all the links are presented in Appendix B.

Table 6.3 Predicted Transition Probabilities Between TP=10 and Downstream Time-points

	On-time	*Late*	*Early*	*Downstream TP*
On-time	0.876	0.119	0.005	50
Late	0.107	0.893	0.000	
Early	0.000	0.500	0.500	
On-time	0.849	0.099	0.050	140
Late	0.484	0.580	0.005	
Early	0.176	0.323	0.500	
On-time	0.698	0.272	0.028	180
Late	0.354	0.611	0.034	
Early	0.486	0.329	0.184	
On-time	0.650	0.325	0.024	300
Late	0.403	0.573	0.024	
Early	0.506	0.378	0.115	
On-time	0.563	0.405	0.032	500
Late	0.400	0.573	0.026	
Early	0.478	0.424	0.097	
On-time	0.548	0.390	0.062	910
Late	0.519	0.534	0.045	
Early	0.474	0.437	0.087	
On-time	0.520	0.463	0.017	1000
Late	0.397	0.590	0.012	
Early	0.468	0.507	0.023	

6.4 Comparison of Transition Probabilities

The transition probabilities of delay states between origin TP=10 and the remainder of the downstream time-points were predicted first by using Equation 6.9 and assuming a homogeneous propagation of delay and then by using Equation 6.10 and assuming a heterogeneous propagation of delay between time-points. The predicted transition probabilities were then compared with the observed values, which are shown in Figure 6.1, assuming a homogeneous propagation of delay, and Figure 6.2, assuming a heterogeneous propagation of delay.

Figure 6.1a consists of predicted and observed transition probabilities of "*On-time*" delay state between time-points TP=10 and 50, based on homogeneous propagation of delay. Figure 6.1b consists of predicted and observed transition probabilities of "*Late*" delay state between time-points TP=10 and 50. The comparison between predicted and observed transition probabilities of "*Early*" delay states are not shown because of their large prediction error. This is because the sample size of "Early" delay states is very small and in most cases zero.

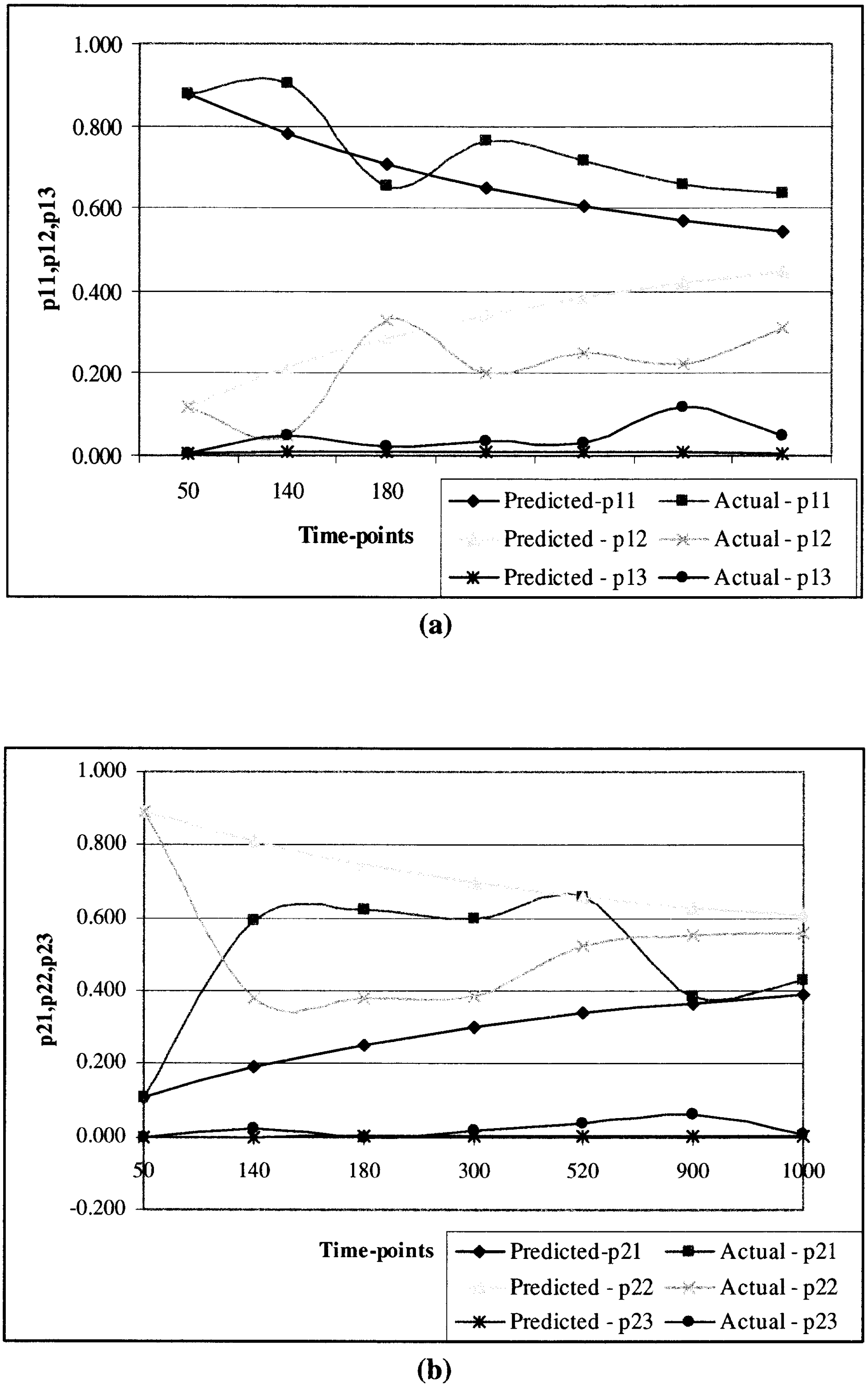
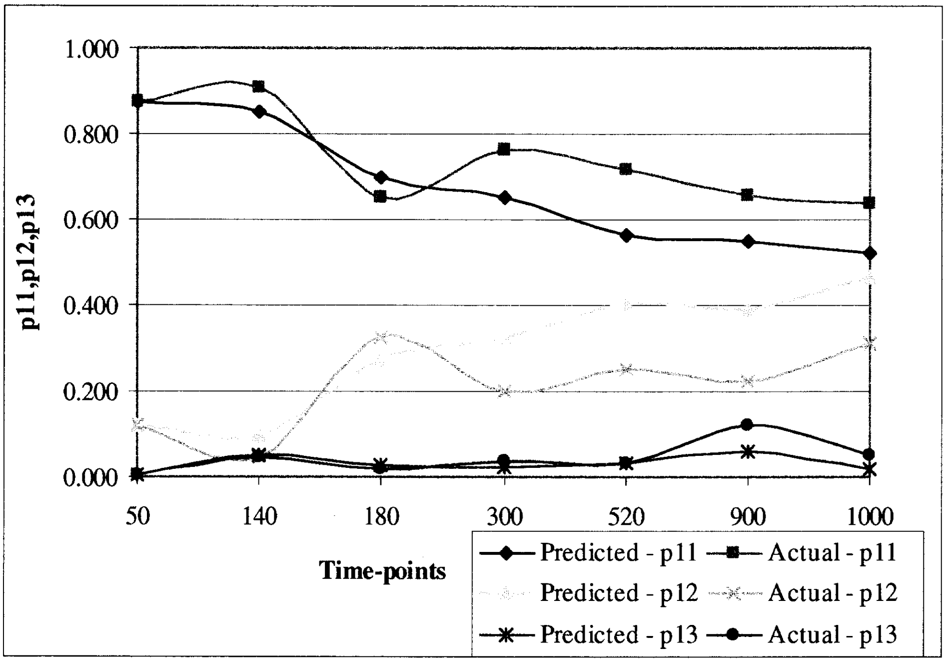


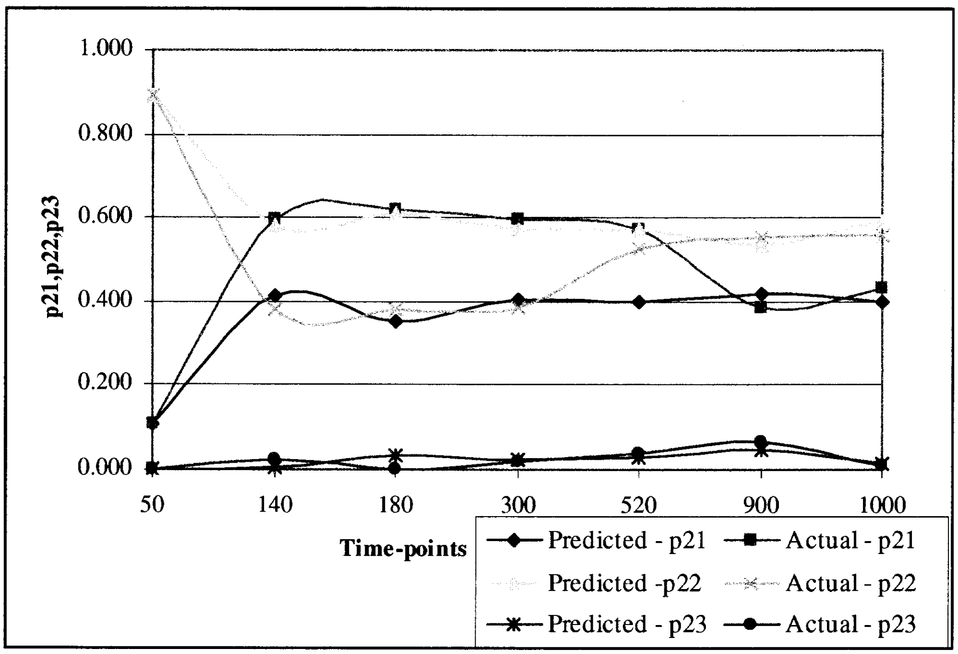
Figure 6.1 Comparison of predicted and observed transition probabilities a) p_{1j} and b) p_{2j} assuming homogeneous delay propagation.

In Figure 6.1a, three comparisons are made for transition probabilities p_{11}, p_{12} and p_{13}. The transition probability p_{11} refers to the probability of the delay state going from *"On-time"* to *"On-time"*. Transition probability p_{12} refers to the probability that the delay state goes from *"On-time"* to *"Late"*. For p_{11}, the predicted probability is a smooth curve similar to a negative exponential curve. This is because of the n^{th} power of a matrix for homogeneous propagation (Kemeny and Snell, 1979). The predicted probability is relatively close to the observed probability for this case. For p_{12}, the predicted probability that the bus goes from being *"On-time"* to being *"Late"* increases as the distance to the time-point increases.

Figure 6.1b compares the observed and predicted transition probabilities p_{21}, p_{22}, and p_{23}. The transition probability p_{21} refers to the probability that the delay state goes from *"Late"* to *"On-time"*. Transition probability p_{23} refers to the probability that the delay state goes from *"Late"* to *"Early"*. For p_{21}, the predicted probability that the delay state goes from *"Late"* to *"On-time"* increases as the distance to the time-point increases. This observation indicates that schedule recovery has a higher chance of occurring at further time-points. Differences between the predicted and observed probabilities seem to be larger for the first time-point and for the last two time-points. For p_{22}, the predicted probability that the delay state for the bus goes from *"Late"* to *"Late"* decreases as the distance to the time-point increases. This observation reflects the condition that schedule recovery is likely to occur as the distance increases, and therefore, the likelihood of the bus remaining in a *"Late"* state decreases as the distance to the time-point increases. For p_{23}, the predicted probability that the delay state for the bus goes from *"Late"* to *"Early"* is very similar to the observed probability.



(a)



(b)

Figure 6.2 Comparison of predicted and observed transition probabilities a) p_{1j} and b) p_{2j} assuming heterogeneous delay propagation.

Figure 6.2a compares the observed and predicted transition probabilities p_{11}, p_{12} and p_{13} assuming a heterogeneous propagation of delay. Figure 6.2b compares the observed and predicted transition probabilities p_{21}, p_{22}, and p_{23} also assuming a heterogeneous propagation of delay. In Figure 6.2, the predicted transition probabilities follow the observed probabilities closely and the differences appear to be much smaller than those for a heterogeneous propagation of delay. This shows that heterogeneous delay propagation is closer to the observed delay propagation.

The difference between the predicted and observed transition probabilities observed assuming a homogeneous propagation of delay for p_{21} and p_{22} is 0.406 and 0.430, respectively. This difference is reduced to 0.181 and 0.201 when a heterogeneous propagation of delay is assumed in Figure 6.2. Similarly, differences between other predicted and observed p_{ij} values are also significantly reduced when heterogeneous propagation of delay is assumed. Hence, it is observed that assuming heterogeneous delay propagation provides better prediction of the observed delay propagation process than homogeneous delay propagation.

In Figures 6.1a and 6.2a, the probability of buses reaching the downstream time-points *"On-time"* when buses are *"On-time"* at the origin time-point decreases , which is shown by the curve of p_{11}. Hence, it becomes more likely that the buses will go from being *"On-time"* to being *"Late"* at downstream time-points (p_{12} starts to increase). Similarly from Figures 6.1b and 6.2b, the probability to reach downstream time-points *"Late"* is somewhat constant when the bus is *"Late"* at the origin time-point, which is shown by the curve for p_{22}. However, at the destination time-point (TP=1000), the difference between the probability of the buses reaching *"Late"* and *"On-time"* when the bus is *"Late"* at the origin time-point, represented by p_{22} and p_{21}, respectively, is smaller than at other downstream time-points. This shows that bus drivers try to recover the *"Late"* bus to reach the destination time-point *"On-time"*.

6.5 Performance Evaluation of Markov Model

A chi-square goodness of fit test was employed to test whether the observed transition probabilities are close to the transition probabilities predicted from the Markov process. If the chi-square statistic is less than the table value, then the hypothesis that the observed data are the outcomes of the predicted Markov process is true. The chi-square model for *(r-1)* degrees of freedom is given by the following relationship (Lee and Judge, 1973):

$$\chi^2_{(r-1)} = \sum_{i}^{r} N(y_i - \hat{y}_i)^2 / \hat{y}_i \tag{6.13}$$

Where,

r = Number of delay states

N= Sample size

y_i = Observed proportion in state i

$\hat{y}_i$ = Predicted proportion in state I

The chi-square values obtained using Equation 6.13 determined for homogeneous and heterogeneous delay propagation are shown in Table 6.4. These values are compared against the chi-square values of 5.991 for a 95% confidence interval and 2 degrees of freedom. From Table 6.4, the calculated chi-square values for the transition probabilities predicted with reference to origin time-point (TP=10) assuming heterogeneous delay propagation are less than 5.991 except for links 10-180, 10-300 and 10-1000. This shows that the predicted transition probabilities are close to the observed transition probabilities and they are outcomes of the Markov process.

However, the calculated chi-square values for the transition probabilities predicted with reference to origin time-point (TP=10) assuming homogeneous delay propagation are higher than 5.991 for all links This shows that the predicted transition probabilities determined assuming homogeneous delay propagation are not close to the observed transition probabilities and they may not be an outcome of the Markov process.

Table 6.4 Comparison of Predicted and Observed Transition Probabilities

Upstream TP	Downstream TP	χ^2 (Calculated) (Homogeneous Delay)		
		$p_{11}...p_{13}$	$p_{21}...p_{23}$	$p_{31}...p_{33}$
10	50	0.000	0.000	0.000*
10	140	9.796	16.938	28.000
10	180	1.369	6.732	0.000*
10	300	13.984	9.328	19.862
10	500	18.852	6.699	9.031
10	900	8.687	15.427	0.000*
10	1000	5.002	1.993	93.667

(indicates zero sample size)*

Upstream TP	Downstream TP	χ^2 (Calculated) (Heterogeneous Delay)		
		$p_{11}...p_{13}$	$p_{21}...p_{23}$	$p_{31}...p_{33}$
10	50	0.000	0.000	0.000
10	140	2.356	3.677	0.916
10	180	2.032	5.513	9.000
10	300	3.685	5.366	15.394
10	500	5.203	4.482	5.110
10	900	3.150	0.413	2.000
10	1000	4.724	0.203	109.508

CHAPTER 7

BUS ARRIVAL TIME COMPUTATIONS

7.1 Introduction

This chapter presents the results of bus arrival time predictions using delay propagation only, autoregressive models only, and a combination of both methods. For comparison purposes, the MARE was obtained for the delay propagation algorithm and the combined algorithm as mentioned in Section 3.3.

7.2 Prediction of Bus Arrival Time Using Delay Propagation

The transition probabilities were predicted as explained in Chapter 3 and 6 assuming heterogeneous traffic conditions, as the assumption of heterogeneous transition probabilities produced results closer to the observed probabilities than the transition probabilities assuming homogeneous conditions. For individual trips, the predicted delay, obtained by using the Markov process at a time-point was compared with the observed delay and the MARE was calculated. As the difference between the observed and predicted delay is the same as the difference between the observed and predicted arrival time, the difference in delay was used to determine the reliability of the Markov process. The prediction of the bus arrival time uses scheduled arrival time and the predicted delay, which are shown by the following relationships:

$$\mathrm{A}_i = \mathrm{SAT}_i + \mathrm{PD}_i \tag{7.1}$$

$$\mathrm{OD}_i = \mathrm{AT}_i - \mathrm{SAT}_i \tag{7.2}$$

$$\text{Prediction Error} = \mathrm{AT}_i - \mathrm{A}_i = \mathrm{AT}_i - \mathrm{SAT}_i - \mathrm{PD}_i = \mathrm{OD}_i - \mathrm{PD}_i \tag{7.3}$$

$$\text{MARE} = \frac{\sum | OD_i - PD_i |}{k} \tag{7.4}$$

Where,

AT_i = Observed arrival time at time-point i = Door open time at i

A_i = Predicted arrival time at time-point i using Markov process

SAT_i = Scheduled arrival time at i

OD_i = Observed delay at i

PD_i = Predicted delay at i using Markov process

MARE values show the capability of the Markov delay model to reasonably predict the bus arrival time at various downstream time-points. MARE values were determined for each combination of time-points and are shown in Table 7.1.

Table 7.1 MARE of Bus Arrival Time at Time-points Using Markov Model Only

Origin	Destination						
	50	**140**	**180**	**300**	**500**	**900**	**1000**
10	3.38	3.22	3.3	3.24	3.24	3.27	3.27
50		2.71	2.43	2.04	3.15	3.33	2.82
140			2.94	2.25	2.77	3.17	2.43
180				1.48	2.67	4.02	4.14

Table 7.1 consists of MARE values obtained for selected route segments. The MARE values move decreases till time-point TP=500. After TP=500, the MARE values do not show a definite trend. This shows that the prediction capability of the Markov model increases when a time-point that is being used to predict the bus arrival time is located near the destination time-point. For example, while predicting delay at TP=300, the MARE is smaller when the origin is TP=140 compared to the MARE when the origin

TP=10. This is shown by the MARE value for TP=10 which is higher than that for TP=140. This result shows that the MARE decreases and the prediction accuracy increases as the bus arrival time is predicted closer to the destination time-point.

The MARE values are about 3 minutes when the origin TP=10, 50 and 140. However, the MARE values slightly exceeded 4 minutes for the origin time-points TP=180 and 300. The MARE value of 3.2 minute for link 10-50 is inadequate considering the average travel time of 5.0 minutes. However, considering the average travel time between time-points 10 and 1000 is 90 minutes, the 3 minute MARE is a highly desirable accuracy of predicting bus arrival times.

The benefits of using delay propagation to predict bus arrival time and relay delay information to passengers at downstream time-points are determined by comparing the probability distributions of predicted and observed delay at time-points. The probability distribution in the form of a histogram and normal curve of the observed and predicted delay for link 10-50 is shown as Figure 7.1. The comparison of probability distributions between predicted and observed delay for the rest of the links is presented in Appendix E.

In Figure 7.1, the left graph is the histogram and normal curve for the observed delay and the right graph is for the predicted delay. The graph for the predicted delay has a steeper curve than the curve for the observed delay. Also, the spread of absolute values of predicted delay is smaller than the spread for the observed delay, which indicates smaller mean and standard deviation for the predicted delay. Smaller mean and standard deviation of predicted delay indicates a lower probability of prediction error when providing the predicted delay information to passengers. Also, the prediction model did not produce very high prediction error, which shows that the prediction model is robust.

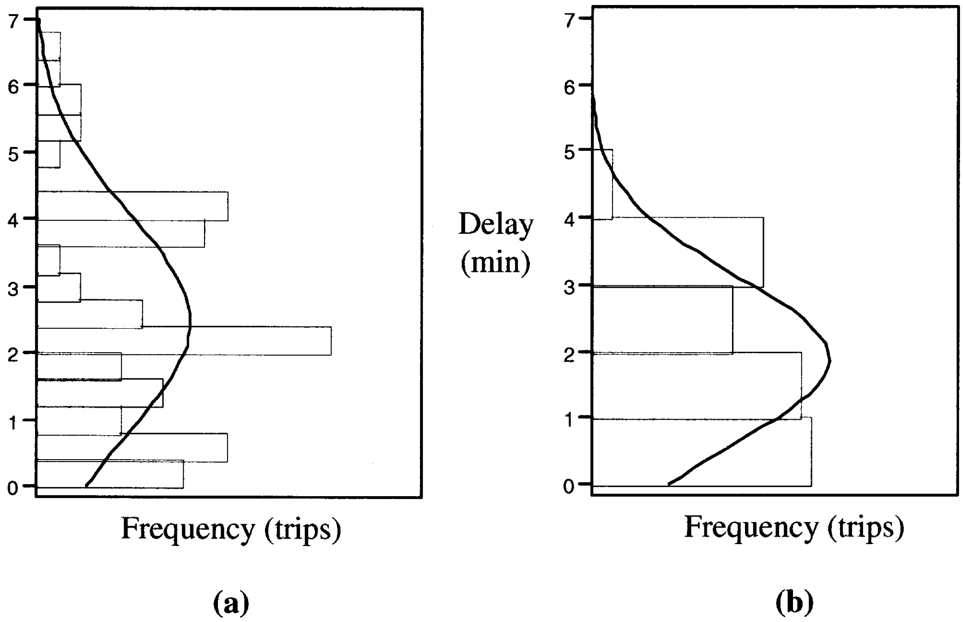


Figure 7.1 Histogram and normal curve of absolute values of (a) observed and (b) predicted delay at link 10-50.

7.3 Bus Travel Time Prediction Using Autoregressive Models

The appropriate full autoregressive models for each combination of time-points were determined to predict bus travel times using the optimum AIC method having the minimum MAPE. The full autoregressive models for all the links are presented in Table 7.2, including MAPE and AIC values. In addition to MAPE, MARE values were determined for each combination of time-points, which are presented in Figure 7.2.

Table 7.2 Final Full AR Models of Bus Travel Time

Link	MAPE	Model	AIC
10-50	14.30	(6,1,0)	115.14
10-140	9.70	(2,1,0)	181.13
10-180	7.40	(6,1,0)	221.64
10-300	7.39	(7,1,0)	255.65
10-500	6.58	(6,1,0)	270.03
10-900	4.59	(6,1,0)	270.66
10-1000	4.45	(7,1,0)	273.96
50-140	4.61	(6,1,0)	93.71
50-180	4.41	(6,1,0)	175.25
50-300	6.62	(6,1,0)	233.79
50-500	6.02	(6,1,0)	247.09
50-900	5.57	(6,1,0)	278.84
50-1000	10.61	(6,1,0)	279.59
140-180	10.79	(5,1,0)	160.77
140-300	9.39	(6,1,0)	226.54
140-500	9.39	(6,1,0)	226.55
140-900	4.38	(6,1,0)	261.22
140-1000	5.41	(6,1,0)	275.02
180-300	4.72	(5,1,0)	98.91
180-500	7.16	(3,1,0)	214.51
180-900	5.32	(7,1,0)	265.55
180-1000	5.70	(6,1,0)	280.28
300-500	6.53	(6,1,0)	148.56
300-900	5.42	(6,1,0)	227.55
300-1000	5.59	(6,1,0)	252.70
500-900	4.88	(6,1,0)	184.51
500-1000	10.90	(6,1,0)	299.59
900-1000	7.48	(6,1,0)	134.05

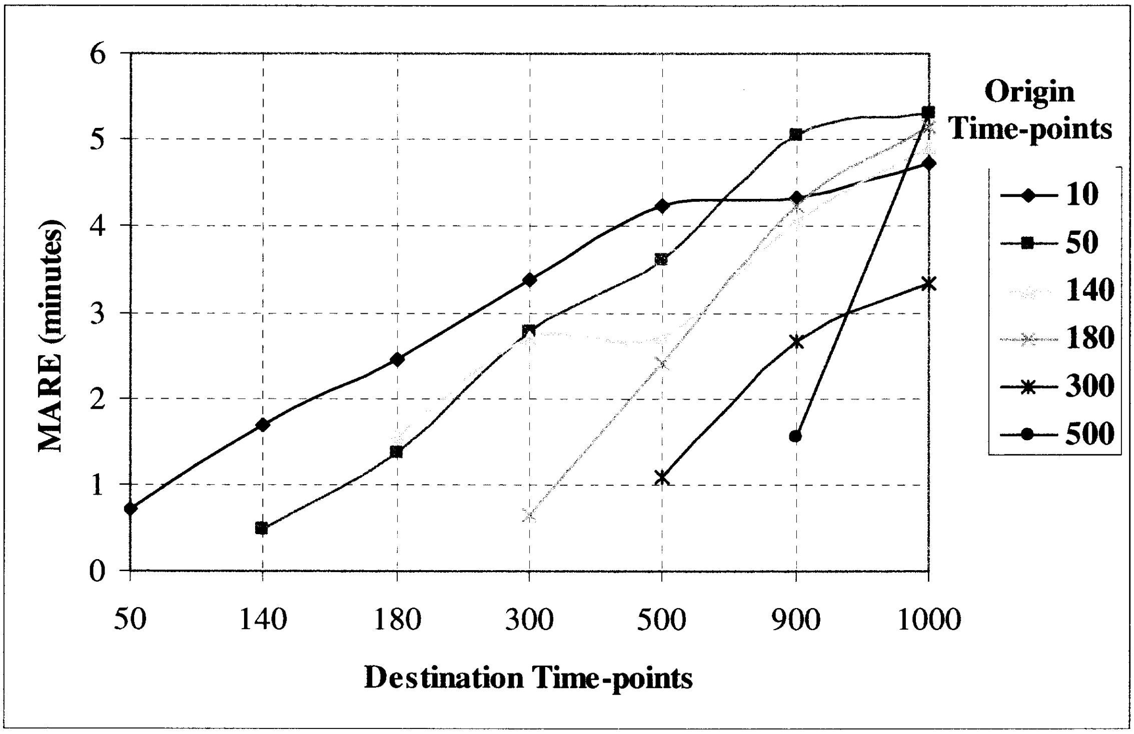


Figure 7.2 MARE values of bus travel time at time-points using full autoregressive model.

Figure 7.2 shows the MARE values obtained from different upstream time-points to downstream time-points using multiple trips. The MARE of the predicted travel time using full autoregressive models varied linearly as the bus traversed downstream. The MARE increased from 1 minute while predicting bus travel time to time-point (TP=50) to 5 minutes for travel time at time-point (TP=1000). However, the MARE values at time-point (TP=1000) from all the other upstream time-points remained constant at around 5 minutes. Considering that the average travel time from the origin time-point (TP=10) to time-point (TP=1000), which is 1.5 hours, the MARE of 5 minutes can be considered an acceptable result.

7.4 Combined Model of Autoregressive and Delay Propagation Models

Using the combined algorithm, which includes a full autoregressive time series model to predict bus travel time and a Markov process to predict bus delay at time-points, the arrival times of bus at individual time-points were predicted. The bus travel times between time-points were predicted using a full autoregressive model based on historic bus travel time data. The predicted delay propagation at individual downstream time-points was determined using the Markov model based on heterogeneous delay propagation between time-points.

The difference between the predicted and the observed arrival times were compared using MARE values, as shown in Figure 7.3. The MARE values increased as the prediction horizon extended to downstream time-points, reaching close to 6 minutes at the destination time-point (TP=1000). This observation is valid for all origin time-points, as shown by each curve corresponding to origin time-points in Figure 7.3. In addition, MARE values decreased as the time-point from where the prediction is being performed is closer to destination time-points. For example, the curve for time-point (TP=50) is lower than the curve for the time-point (TP=10). This shows that there is reduction in MARE values when bus arrival time prediction is done from the time-point closer to the time-point for which the bus arrival time is being predicted. Hence, it can be concluded that the prediction accuracy increases as the reference time-point is moved closer to the time-point where bus arrival time prediction is being predicted.

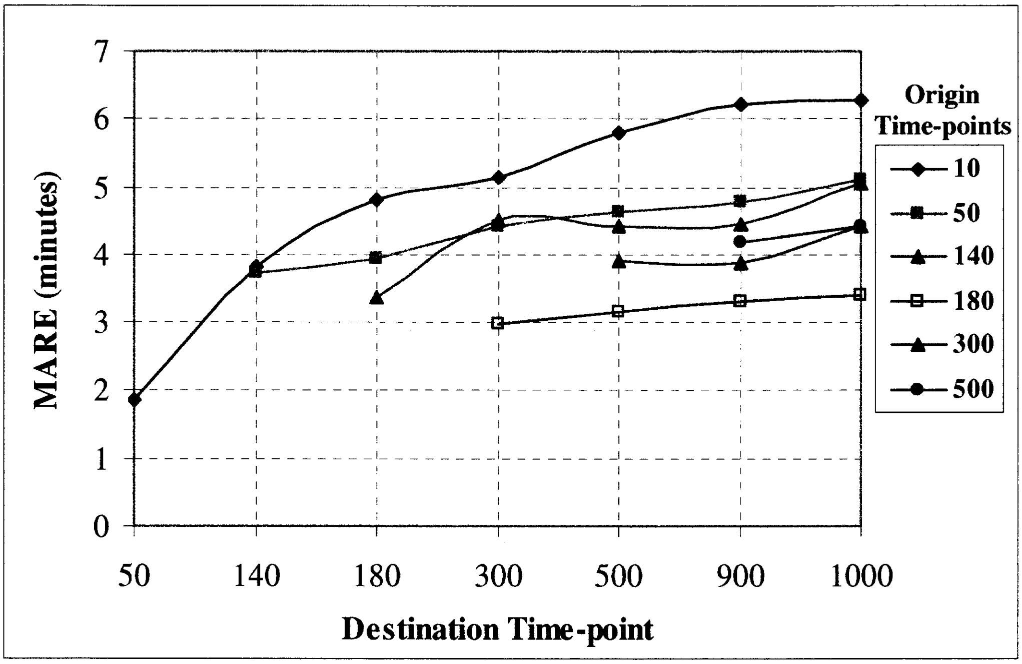


Figure 7.3 MARE values at time-points using combined model.

The effect of combining bus delay propagation with the full autoregressive model to predict bus arrival times is shown by comparing the MARE values of predicted and observed arrival times using only the full autoregressive models, as presented in Figure 7.4.

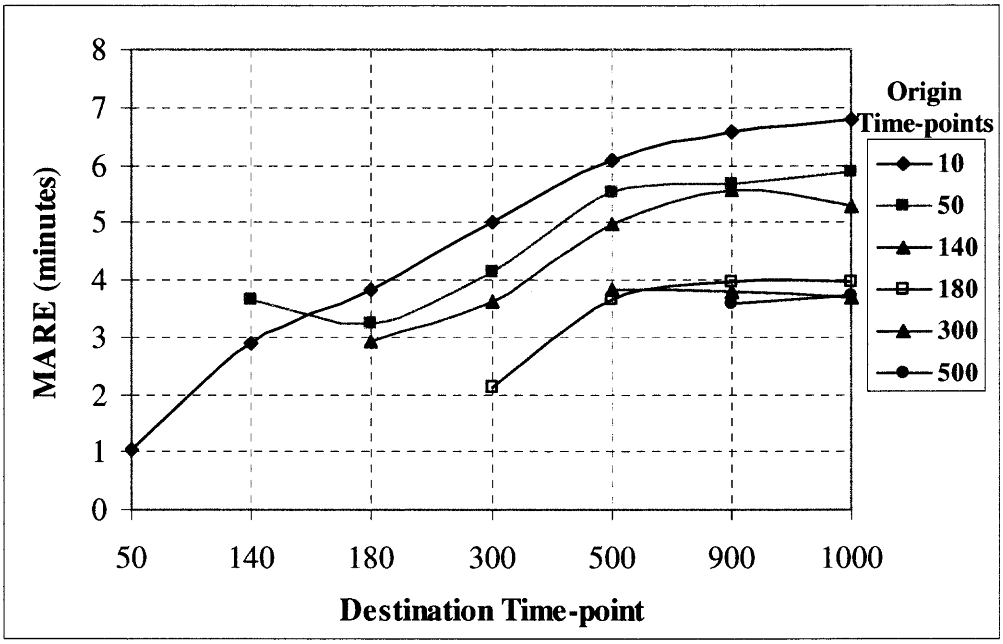
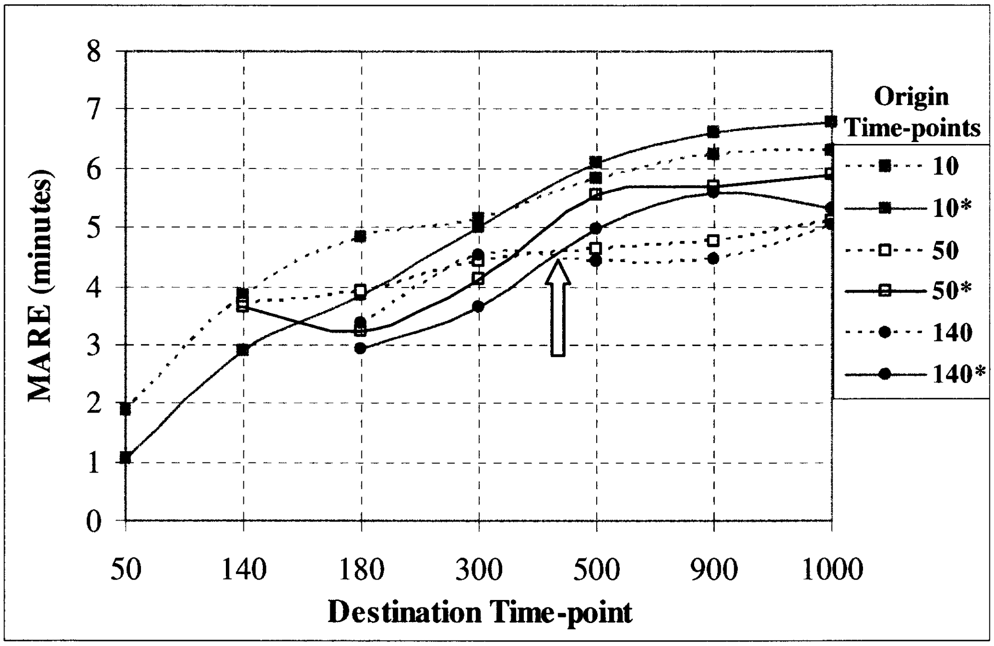
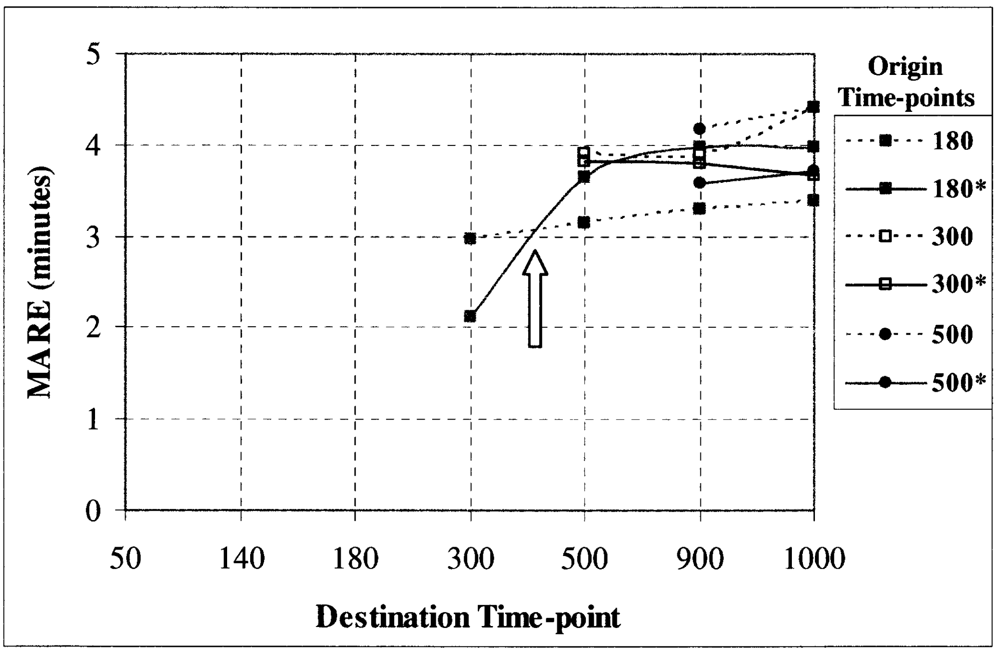


Figure 7.4 MARE values using full autoregressive model only.

To better compare Figures 7.3 and 7.4, the curves obtained in both figures were combined and are presented as Figure 7.5. In Figure 7.5, the curves for time-point (TP=10), 10* represents the MARE values determined by using full autoregressive model only and 10 represents the MARE values determined by using the combined model consisting of the full autoregressive and Markov models. Figure 7.5a consists of MARE values for time-points TP=10 to TP=140 and Figure 7.5b consists of MARE values for time-points TP=180 to TP=500.



(a)



(b)

Figure 7.5 MARE values using full autoregressive and combined model from time-points a) 10 through 140 and b) 180 through 500.

In Figure 7.5, the MARE values obtained using the combined model and the full autoregressive model only crossed each other between time-points TP=300 and TP=500, when the prediction was performed from time-points TP=10 through 300. For example, before the TP=300, the difference of MARE values using the combined model and full autoregressive model for the curve corresponding to origin TP=10 is positive. This is shown in Figure 7.5 by a block arrow. The difference of MARE values is negative after TP=500. This observation indicates that the benefit of including delay propagation with the autoregressive model to predict bus arrival times is not positive till TP=500.

This observation suggests the possibility of the existence of an optimal "location" before which, the benefit of including delay propagation in the combined model may not become beneficial in terms of a reduction in the arrival time prediction error. The reason behind the positive benefits of including bus delay propagation with the full autoregressive models after the "optimal location" is because of a significant portion of the arrival time constitutes the predicted amount of delay , which is determined by the Markov model.

CHAPTER 8

CONCLUSION

8.1 Results and Findings

In this research, bus arrival time prediction models were developed using historical bus travel time information. The research used a stochastic approach to predict bus travel time and delay propagation based on historical information of bus travel time and delay. The existing bus travel and arrival time prediction models were studied to determine their limitations. In this research, the appropriate stochastic time series models were developed. The ability of the models to capture the temporal variations of bus travel time was also determined. Since existing stochastic time series models do not consider the propagation of bus delays to downstream stops, this research also focused on modeling the propagation of delay of buses to downstream stops. The bus arrival time prediction models were developed using a case study route and historical information on bus travel times and delay were obtained.

The analysis of historical bus travel time showed that the coefficient of variation differed between time-points. The coefficient of variation is consistently related to the MAPE achieved using time series prediction models. Links with a higher coefficient of variation had a higher MAPE and links with a lower coefficient of variation had a lower MAPE. This is because of the presence of a larger proportion of a random component in the observed travel time.

The recurring patterns of bus travel times were modeled by using full autoregressive models, which produced better MAPE than other time series models. Full autoregressive models also closely predicted bus travel times and were able to accommodate variations in the observed travel time. The seasonal models and subset autoregressive models failed to significantly improve the MAPE when compared with full autoregressive models. Hence, only full autoregressive models were used to predict bus travel times for links in the final analysis. The full autoregressive models also performed better than historical average and smoothing models for all links in terms of the MAPE for these models. Full autoregressive models showed a better fit than historical average and smoothing models in terms of capturing the periodic variations during peak periods.

The comparison of predicted and observed transition probabilities between delay states results showed that the Markov process can adequately model the propagation of delay to downstream stops using the assumption of heterogeneous delay propagation. The chi-square comparisons between predicted and observed transition probabilities of links did not follow the Markov process when using the assumption of homogeneous delay propagation to downstream time-points.

The benefits of using bus delay propagation to predict bus arrival times at downstream time-points were shown by producing a probability curve of the deviation between predicted and observed arrival time. The normal curve of predicted delay is narrower and less spread that the curve for observed delay. This showed the probability of deviation from the scheduled arrival time is smaller if delay propagation is used to predict bus arrival times. Hence, it is beneficial for transit agencies to relay predicted bus arrival time to passengers by using only bus delay propagation at downstream stops than not providing any information.

A separate analysis of the benefits associated with including the delay propagation with the full autoregressive model to predict bus arrival times was also performed. The positive benefits of delay propagation and its benefit to predict bus arrival time at downstream stops were established. The inclusion of delay propagation into the arrival time prediction algorithm decreased the relative error when predictions were made at time-points with some distance from the origin time-point. These conditions occurred because the contribution of the predicted bus delay at destination time-points was significant in proportion to the predicted travel time. Finally, the comparisons were made between results of the model using delay propagation only and the combined model. In summary, the research results are as follows:

- The recurring patterns of bus travel times were modeled by using full autoregressive models, which produced better MAPE than other time series models analyzed in the research.

- The comparison of predicted and observed transition probabilities between delay states showed that the Markov process can adequately model delay propagation.

- After analyzing the benefits of the bus arrival time prediction model using the predicted delay only, it is beneficial to relay such arrival time information to passengers using the predicted delay only.

- A separate analysis of benefits from combining the delay propagation with the full autoregressive models to predict bus arrival times was also performed. However, an optimal location exists along the route, after which the inclusion of delay propagation may not be beneficial.

- MARE values do not remain constant while predicting bus arrival time at a time-point. In this research, MARE values increased from 1 minute to 5 minutes. The MARE values differed when predicting the bus arrival time at a time-point from two different upstream time-points.

8.2 Research Contributions

The objective of this research was to develop an arrival time prediction algorithm with focus on bus operation conditions on urban streets. The case study used in this research included suburban locations as well as downtown locations, so the research demonstrated differences in the prediction algorithm between urban and suburban locations.

The methodology proposed in this research consisted of full autoregressive models to predict bus travel times based on historical information and a Markov process to predict delay at downstream time-points. The inclusion of bus delay propagation into the bus arrival time prediction algorithm is an important contribution to the research efforts to predict bus arrival times. A significant attempt has been made in this research to explain the delay propagation phenomena into the arrival time prediction algorithm. It was shown that the proposed methodology was able to produce significant benefits to accurately predict bus arrival time by including delay propagation.

The research showed that the transit agencies can provide more accurate bus arrival times by using delay propagation and a Markov process instead of using the scheduled arrival time only to estimate the bus arrival time. Transit agencies can also choose to use historical travel time information and predicted delay to provide more accurate bus arrival time information.

The delay propagation model based on Markov chains is entirely based on historical bus information. The model does not require collection of a large number of variables. The literature review showed that bus arrival time prediction models were also developed based on Kalman filters models using the current geographic location of the bus and the predicted trajectory of the bus. However, this model requires a high polling frequency to determine the geographic location of the bus. Another Kalman filter model was based on historical bus travel time information and a constant headway of the bus. This research, however, does not require frequent observation of the geographic location of the bus and the assumption of constant headway is not made. The results of the research showed that by using the historical information of delay propagation, bus arrival time predictions can be done with accuracy of between 1 to 5 minutes for a total travel time of 1.5 hours. Hence, by using delay propagation model based on Markov chains, accurate bus arrival time predictions can be performed without collecting a large number of bus operation variables, simulations and additional hardware requirements.

8.3 Recommendations for Future Research

In this research, the transition probabilities defining the delay states between the time-points were determined based on heterogeneous conditions between time-points. Between any two time-points, a constant transition probability matrix is used to define the relationship between the delay states. However, the transition probabilities of delay states between any two time-points can vary depending on temporal conditions. For example, the transition probabilities may vary between peak and off-peak conditions and between weekday and weekends for the same delay states. Hence, there is a possibility of more accurately defining the transition probabilities between any two time-points based on temporal variations. However, this requires a larger sample size than that used in this research.

A larger sample size can obviously increase the accuracy of the transition probabilities. However, the literature review showed that using a larger sample size creates an opportunity to use more complex processes to obtain better estimates of the transition probabilities, such as a Bayesian approach and non linear programming.

The research showed that there exists an optimal location. Before or after this location, the inclusion of bus delay propagation into the arrival time prediction model produces benefits or disbenefits. Hence, there is a possibility to determine a relationship between the optimal location and the variables that influence bus arrival time, such as distance from the origin, travel time, and number of time-points.

APPENDIX A

OBSERVED TRANSITION PROBABILITIES

The observed transition probabilities for each combination of time-points are provided in the following tables.

Table A.1 Observed Transition Probabilities of Individual Links

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
10-50	0.876	0.119	0.005	0.107	0.893	0.000	0.000	0.500	0.500
50-140	0.922	0.026	0.052	0.353	0.647	0.000	0.000	0.000	1.000
140-180	0.777	0.215	0.008	0.050	0.900	0.050	0.667	0.000	0.333
180-300	0.865	0.125	0.010	0.146	0.854	0.000	0.200	0.200	0.600
300-500	0.790	0.189	0.021	0.132	0.868	0.000	0.250	0.000	0.750
500-900	0.898	0.020	0.082	0.105	0.895	0.000	0.000	0.500	0.500
900-1000	0.887	0.113	0.000	0.000	1.000	0.000	0.545	0.182	0.273

Table A.2 Observed Transition Probabilities of Links from TP=10

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
10-140	0.905	0.048	0.048	0.595	0.380	0.025	0.500	0.000	0.500
10-180	0.653	0.327	0.02	0.622	0.378	0.000	0.000	0.000	0.000
10-300	0.762	0.201	0.037	0.596	0.385	0.019	0.333	0.000	0.667
10-500	0.715	0.252	0.033	0.436	0.526	0.038	0.333	0.333	0.333
10-900	0.657	0.224	0.119	0.383	0.553	0.064	0.000	0.000	0.000
10-1000	0.637	0.312	0.050	0.431	0.558	0.009	0.000	0.500	0.500

Table A.3 Observed Transition Probabilities of Links from TP=50

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
50-180	0.591	0.386	0.023	0.222	0.778	0.000	0.000	0.000	1.000
50-300	0.770	0.194	0.036	0.256	0.744	0.000	0.111	0.000	0.889
50-500	0.714	0.241	0.045	0.227	0.773	0.000	0.500	0.333	0.167
50-900	0.607	0.262	0.131	0.300	0.700	0.000	0.500	0.500	0.000
50-1000	0.632	0.308	0.060	0.324	0.676	0.000	0.556	0.333	0.111

Table A.4 Observed Transition Probabilities of Links from TP=140

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
140-300	0.790	0.195	0.015	0.056	0.944	0.000	0.310	0.000	0.690
140-500	0.671	0.298	0.031	0.000	1.000	0.000	0.500	0.250	0.250
140-900	0.594	0.323	0.083	0.167	0.833	0.000	0.333	0.000	0.667
140-1000	0.607	0.353	0.040	0.118	0.882	0.000	0.621	0.276	0.103

Table A.5 Observed Transition Probabilities of Links from TP=180

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
180-500	0.574	0.368	0.059	0.206	0.794	0.000	0.636	0.182	0.182
180-900	0.421	0.447	0.132	0.360	0.520	0.120	0.667	0.000	0.333
180-1000	0.562	0.371	0.067	0.179	0.795	0.026	0.750	0.188	0.063

Table A.6 Observed Transition Probabilities of Links from TP=300

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
300-900	0.711	0.171	0.118	0.171	0.800	0.029	0.250	0.250	0.500
300-1000	0.708	0.257	0.035	0.143	0.857	0.000	0.292	0.542	0.167

APPENDIX B

PREDICTED TRANSITION PROBABILITIES

The predicted transition probabilities for each combination of time-points are provided in the following tables.

Table B.1 Predicted Transition Probabilities of Links from TP=10

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
10-140	0.849	0.100	0.051	0.414	0.581	0.006	0.176	0.324	0.500
10-180	0.699	0.272	0.029	0.354	0.611	0.034	0.487	0.329	0.184
10-300	0.650	0.326	0.024	0.403	0.573	0.024	0.506	0.379	0.115
10-500	0.563	0.405	0.032	0.400	0.573	0.026	0.479	0.424	0.097
10-900	0.548	0.390	0.062	0.420	0.534	0.046	0.475	0.438	0.088
10-1000	0.520	0.463	0.017	0.397	0.590	0.013	0.469	0.507	0.024

Table B.2 Predicted Transition Probabilities of Links from TP=50

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
50-180	0.752	0.222	0.026	0.307	0.658	0.035	0.667	0.000	0.333
50-300	0.689	0.288	0.023	0.369	0.607	0.024	0.644	0.150	0.206
50-500	0.588	0.380	0.032	0.378	0.597	0.026	0.580	0.252	0.168
50-900	0.568	0.368	0.064	0.402	0.554	0.044	0.547	0.321	0.132
50-1000	0.539	0.444	0.017	0.380	0.608	0.012	0.557	0.407	0.036

Table B.3 Predicted Transition Probabilities of Links from TP=140

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
140-300	0.705	0.282	0.012	0.185	0.785	0.030	0.644	0.150	0.206
140-500	0.598	0.378	0.024	0.257	0.716	0.027	0.580	0.252	0.168
140-900	0.577	0.363	0.061	0.306	0.659	0.034	0.547	0.321	0.132
140-1000	0.545	0.439	0.017	0.291	0.700	0.009	0.557	0.407	0.036

Table B.4 Predicted Transition Probabilities of Links from TP=180

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
180-500	0.703	0.272	0.025	0.228	0.769	0.003	0.334	0.211	0.454
180-900	0.660	0.270	0.070	0.286	0.694	0.020	0.323	0.423	0.254
180-1000	0.623	0.358	0.019	0.265	0.730	0.006	0.425	0.506	0.069

Table B.5 Predicted Transition Probabilities of Links from TP=300

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
300-900	0.729	0.196	0.075	0.210	0.779	0.011	0.224	0.380	0.395
300-1000	0.688	0.292	0.020	0.192	0.805	0.003	0.415	0.477	0.108

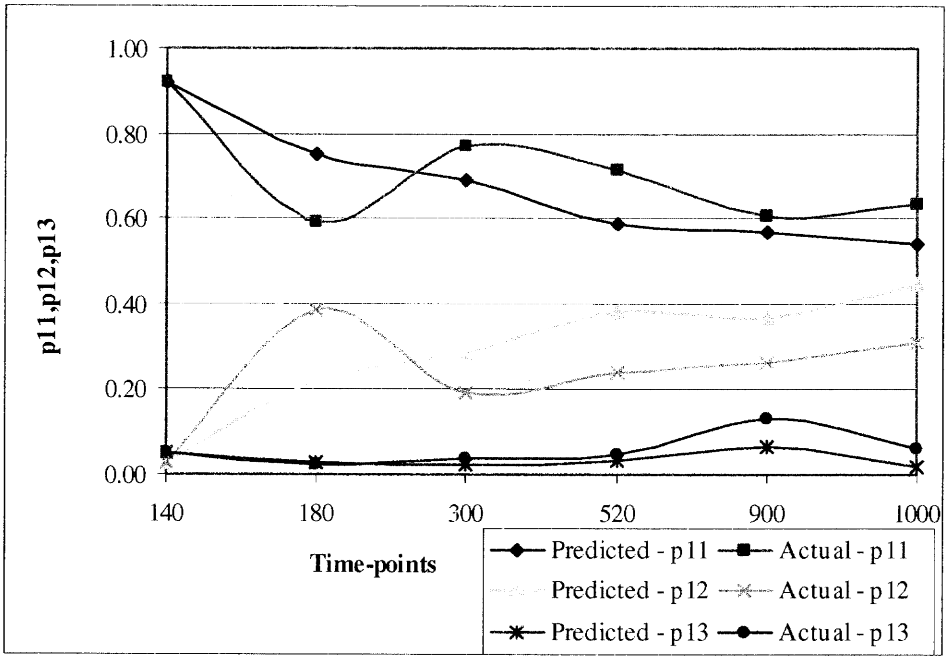
Table B.6 Predicted Transition Probabilities of Links from TP=500

Link	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	p_{31}	p_{32}	p_{33}
500-1000	0.841	0.137	0.022	0.093	0.907	0.000	0.273	0.591	0.136

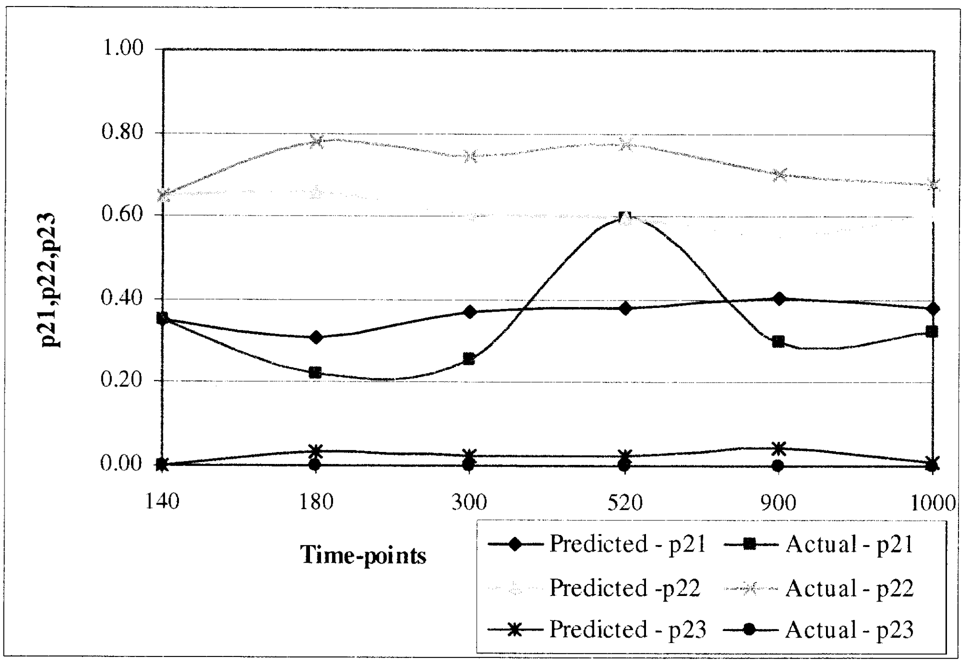
APPENDIX C

GRAPHICAL COMPARISONS OF TRANSITION PROBABILITIES

The graphical comparisons between observed and predicted transition probabilities at different origin time-points are shown in the following figures.

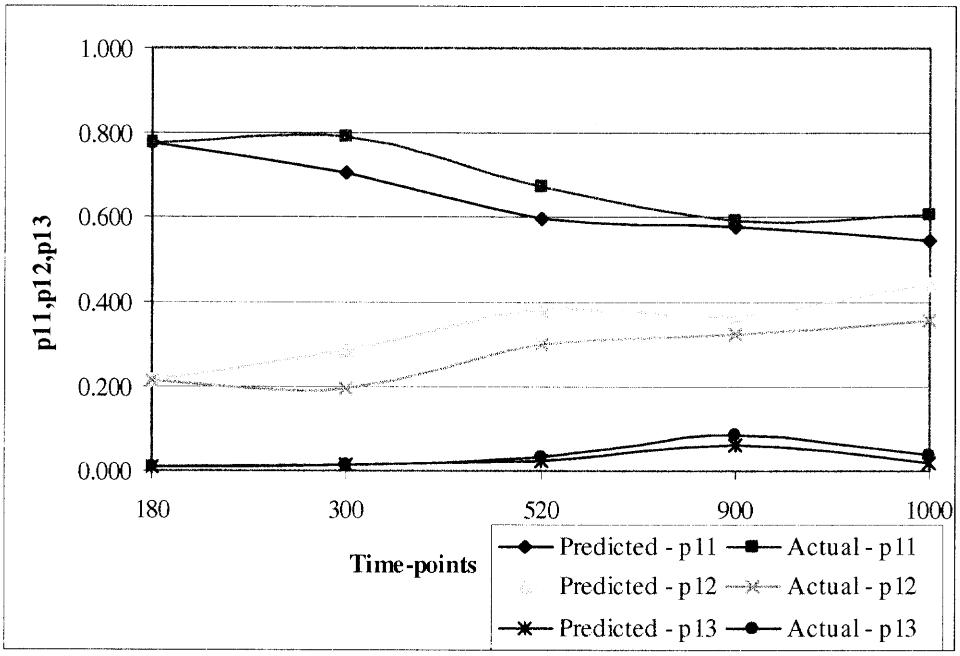


(a)

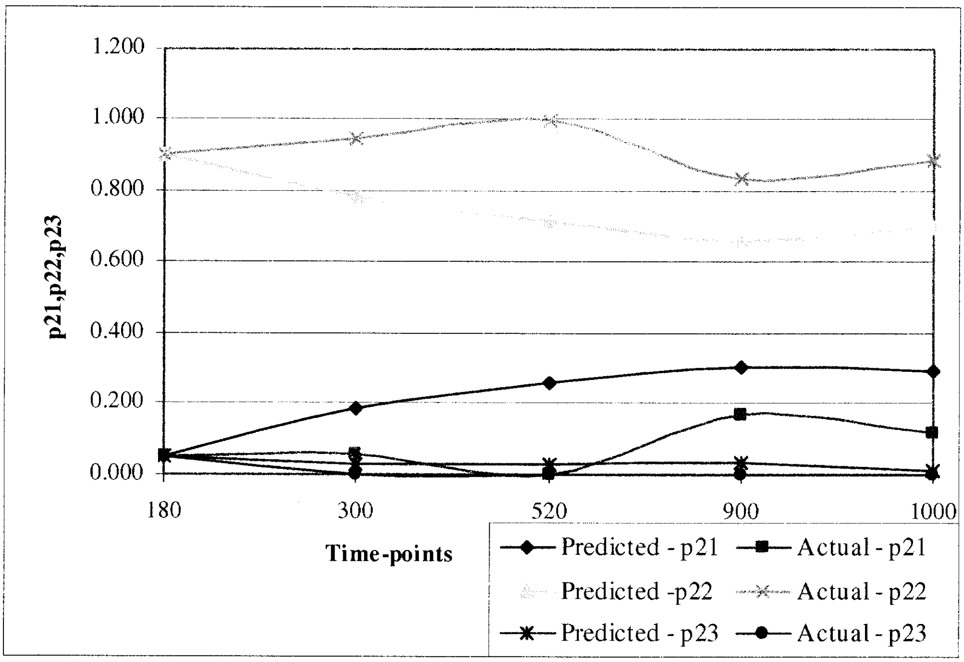


(b)

Figure C.1 Comparison of predicted and observed transition probabilities a) p_{1j} and b) p_{2j} assuming heterogeneous propagation of delay for TP=50.

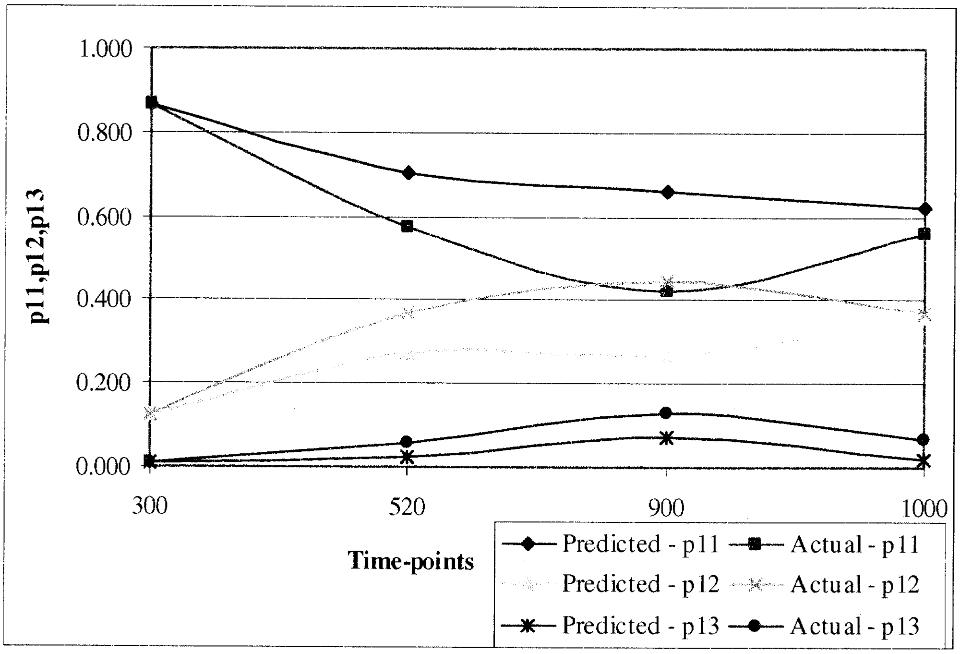


(a)

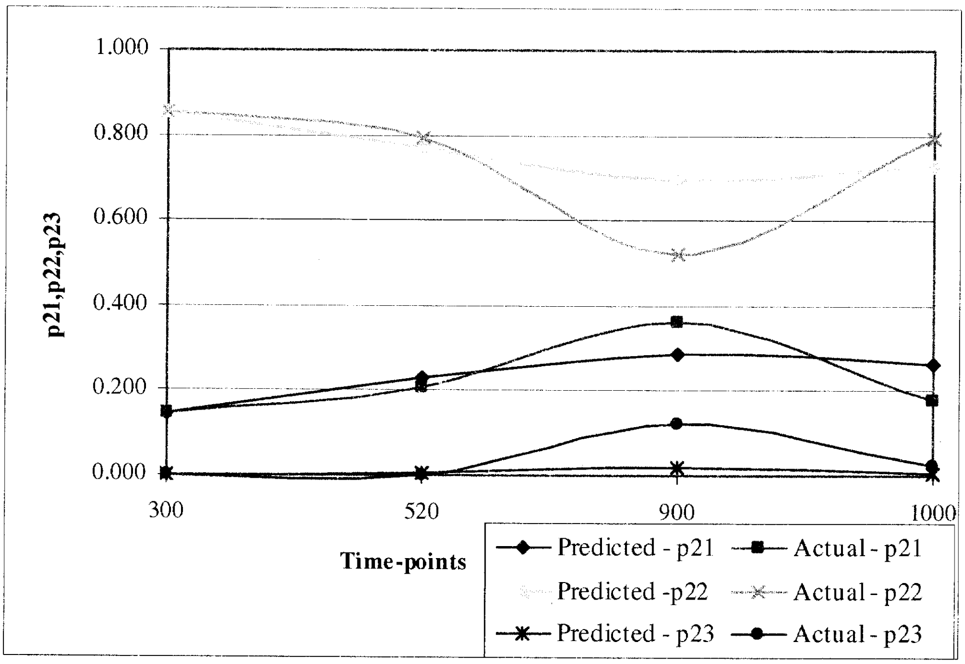


(b)

Figure C.2 Comparison of predicted and observed transition probabilities a) p_{1j} and b) p_{2j} assuming heterogeneous propagation of delay for TP=140.

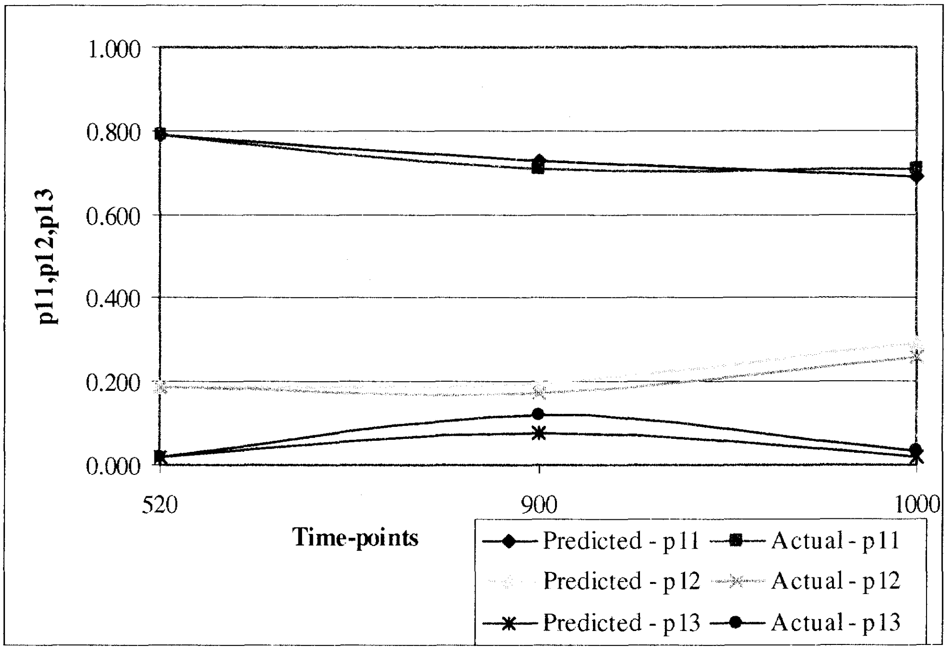


(a)

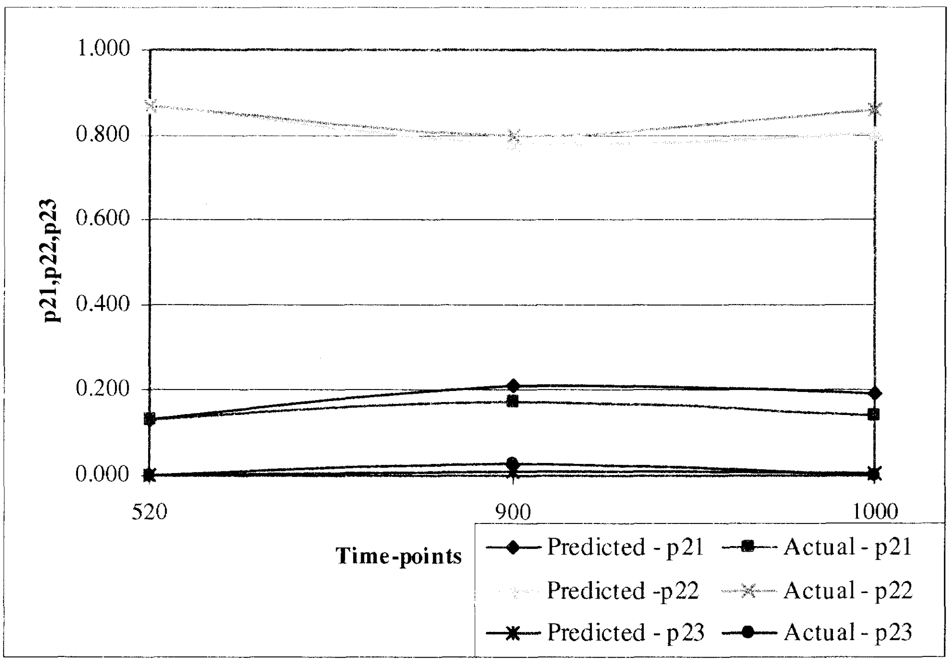


(b)

Figure C.3 Comparison of predicted and observed transition probabilities a) p_{1j} and b) p_{2j} assuming heterogeneous propagation of delay for TP=180.



(a)



(b)

Figure C.4 Comparison of predicted and observed transition probabilities a) p_{1j} and b) p_{2j} assuming heterogeneous propagation of delay for TP=300.

APPENDIX D

CHI-SQUARE COMPARISONS

The chi-square comparisons between observed and predicted transition probabilities for each combination of time-points are provided in the following tables.

Table D.1 Chi-Square Comparisons for Origin TP=50

Down-stream TP	p_{11}		p_{12}		p_{13}		Number of Samples	Chi Square
	Predicted	Observed	Predicted	Observed	Predicted	Observed		
140	0.922	0.922	0.026	0.026	0.052	0.052	77	0.000
180	0.752	0.591	0.222	0.386	0.026	0.023	44	6.942
300	0.689	0.770	0.288	0.194	0.023	0.036	139	6.621
500	0.588	0.714	0.380	0.241	0.032	0.045	112	9.341
900	0.568	0.607	0.368	0.262	0.064	0.131	61	6.344
1000	0.539	0.632	0.444	0.308	0.017	0.060	133	21.591
Down-stream TP	**p_{21}**		**p_{22}**		**p_{23}**		**Number of Samples**	**Chi Square**
	Predicted	**Observed**	**Predicted**	**Observed**	**Predicted**	**Observed**		
140	0.353	0.353	0.647	0.647	0.000	0.000	17	0.000
180	0.307	0.222	0.658	0.778	0.035	0.000	9	0.722
300	0.369	0.256	0.607	0.744	0.024	0.000	39	3.467
500	0.378	0.597	0.597	0.773	0.026	0.000	22	4.509
900	0.402	0.300	0.554	0.700	0.044	0.000	10	1.077
1000	0.380	0.324	0.608	0.676	0.012	0.000	37	1.027

Table D.2 Chi-Square Comparisons for Origin TP=140

Down-stream TP	p_{11} Predicted	p_{11} Observed	p_{12} Predicted	p_{12} Observed	p_{13} Predicted	p_{13} Observed	Number of Samples	Chi Square
180	0.777	0.777	0.215	0.215	0.008	0.008	121	0.000
300	0.705	0.790	0.282	0.195	0.012	0.015	205	7.681
500	0.598	0.671	0.378	0.298	0.024	0.031	161	4.481
900	0.577	0.594	0.363	0.323	0.061	0.083	96	1.262
1000	0.545	0.607	0.439	0.353	0.017	0.040	201	11.300
Down-stream TP	**p_{21} Predicted**	**p_{21} Observed**	**p_{22} Predicted**	**p_{22} Observed**	**p_{23} Predicted**	**p_{23} Observed**	**Number of Samples**	**Chi Square**
180	0.050	0.050	0.900	0.900	0.050	0.050	20	0.000
300	0.185	0.056	0.785	0.944	0.030	0.000	36	5.530
500	0.257	0.000	0.716	1.000	0.027	0.000	21	8.336
900	0.306	0.167	0.659	0.833	0.034	0.000	12	1.731
1000	0.291	0.118	0.700	0.882	0.009	0.000	34	5.435

Table D.3 Chi-Square Comparisons for Origin TP=180

Down-stream TP	p_{11} Predicted	p_{11} Observed	p_{12} Predicted	p_{12} Observed	p_{13} Predicted	p_{13} Observed	Number of Samples	Chi Square
300	0.865	0.865	0.125	0.125	0.010	0.010	104	0.000
500	0.703	0.574	0.272	0.368	0.025	0.059	68	6.910
900	0.660	0.421	0.270	0.447	0.070	0.132	38	9.742
1000	0.623	0.562	0.358	0.371	0.019	0.067	89	11.458
Down-stream TP	**p_{21} Predicted**	**p_{21} Observed**	**p_{22} Predicted**	**p_{22} Observed**	**p_{23} Predicted**	**p_{23} Observed**	**Number of Samples**	**Chi Square**
300	0.146	0.146	0.854	0.854	0.000	0.000	41	0.000
500	0.228	0.206	0.769	0.794	0.003	0.000	34	0.209
900	0.286	0.360	0.694	0.520	0.020	0.120	25	13.913
1000	0.265	0.179	0.730	0.795	0.006	0.026	39	4.170

Table D.4 Chi-Square Comparisons for Origin TP=300

Down-stream TP	p_{11}		p_{12}		p_{13}		Number of Samples	Chi Square
	Predicted	Observed	Predicted	Observed	Predicted	Observed		
500	0.790	0.790	0.189	0.189	0.021	0.021	143	0.000
900	0.729	0.711	0.196	0.171	0.075	0.118	76	2.182
1000	0.688	0.708	0.292	0.257	0.020	0.035	171	2.573
Down-stream TP	p_{21}		p_{22}		p_{23}		**Number of Samples**	**Chi Square**
	Predicted	**Observed**	**Predicted**	**Observed**	**Predicted**	**Observed**		
500	0.132	0.132	0.868	0.868	0.000	0.000	53	0.000
900	0.210	0.171	0.779	0.800	0.011	0.029	35	1.294
1000	0.192	0.143	0.805	0.857	0.003	0.000	70	1.328

APPENDIX E

PROBABILITY STRUCTURES OF OBSERVED AND PREDICTED DELAY

The probability distributions of observed and predicted delay determined based on Markov chain for different combinations of time-points are provided in the following figures. The left graph is the histogram and normal curve of the observed delay and the right graph is for the predicted delay. The vertical axis in the graph represents the delay in minutes and horizontal axis represents the frequency.

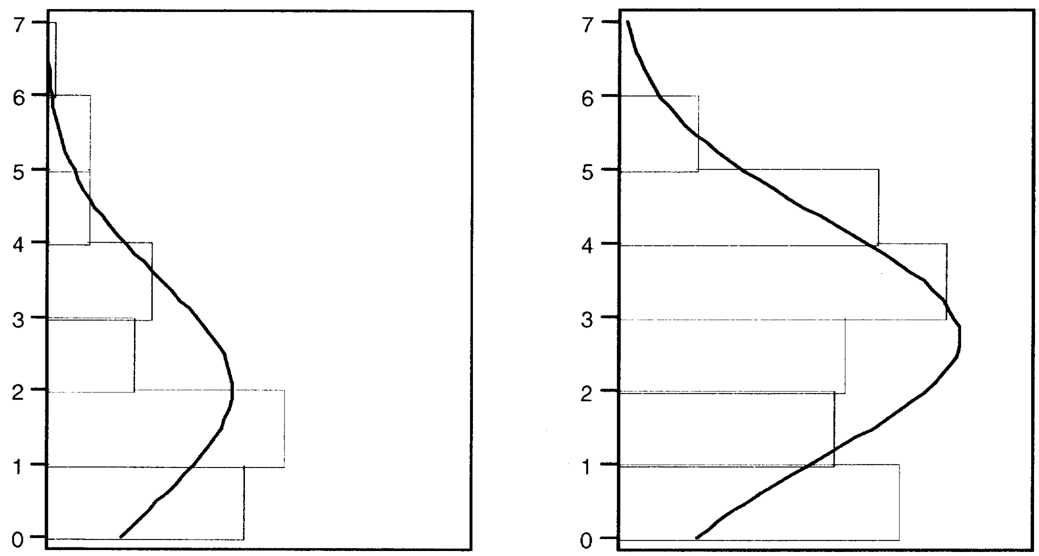


Figure E.1 Comparison of probability structures for link 10-140.

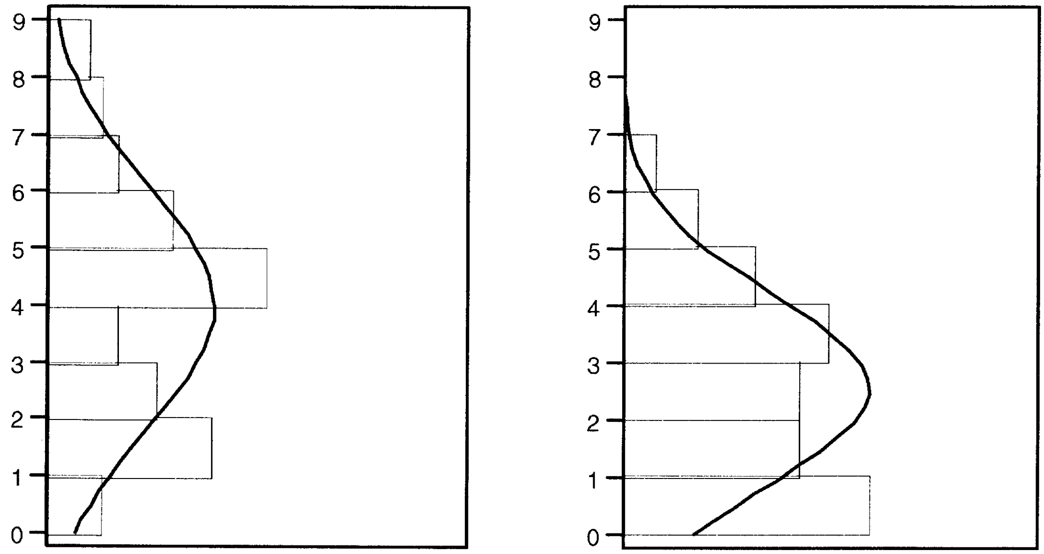


Figure E.2 Comparison of probability structures for link 10-180.

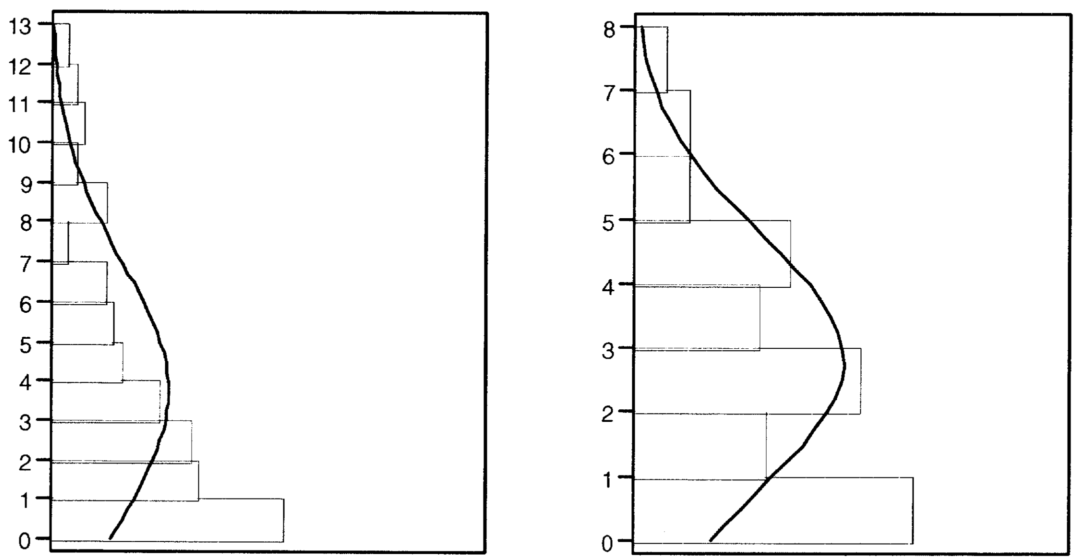


Figure E.3 Comparison of probability structures for link 10-300.

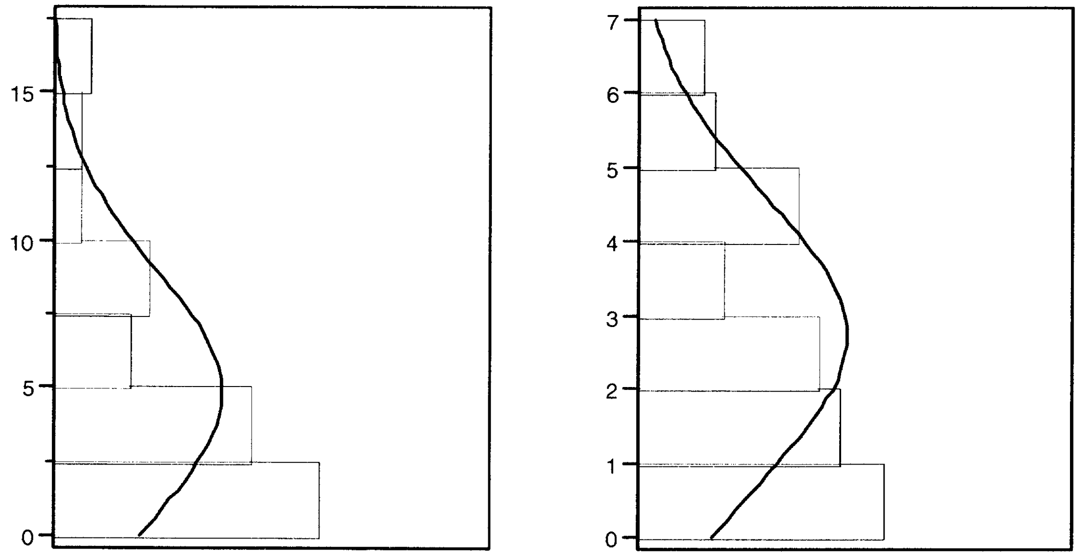


Figure E.4 Comparison of probability structures for link 10-500.

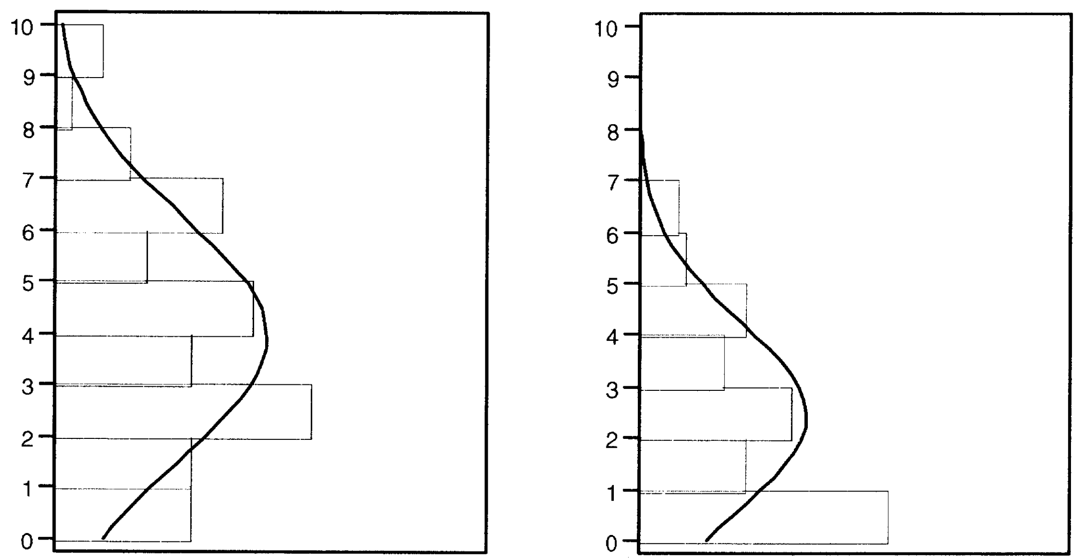


Figure E.5 Comparison of probability structures for link 10-900.

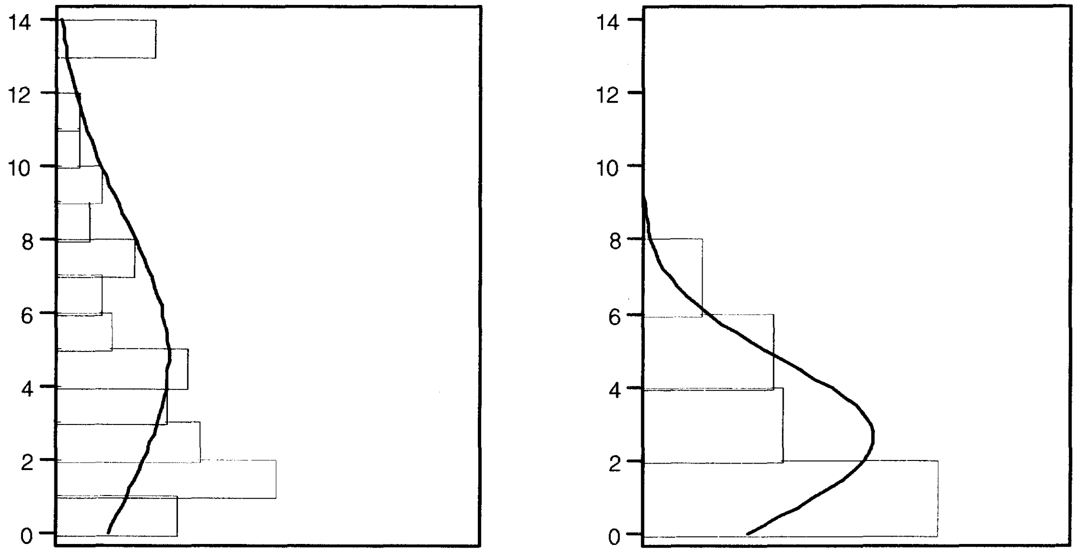


Figure E.6 Comparison of probability structures for link 10-1000.

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