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A bi-level programming approach for the shipper-carrier network problem

Ya Wang
New Jersey Institute of Technology

ABSTRACT

A BI-LEVEL PROGRAMMING APPROACH FOR THE SHIPPER – CARRIER NETWORK PROBLEM

by
Ya Wang

The Stackelberg game between shippers and carriers in an intermodal network is formulated as a bi-level program. In this network, shippers make production, consumption, and routing decisions while carriers make pricing and routing decisions. The oligopolistic carrier pricing and routing problem, which comprises the upper level of the bi-level program, is formulated either as a nonlinear constrained optimization problem or as a variational inequality problem, depending on whether the oligopolistic carriers choose to collude or compete with each other in their pricing decision. The shippers' decision behavior is defined by the spatial price equilibrium principle. For the spatial price equilibrium problem, which is the lower level of the bi-level program, a variational inequality formulation is used to account for the asymmetric interactions between flows of different commodity types. A sensitivity analysis-based heuristic algorithm is proposed to solve the program. An example application of the bi-level programming approach analyzes the behavior of two marine terminal operators. The terminal operators are considered to be under the same Port Authority. The bi-level programming approach is then used to evaluate the Port Authority's alternative investment strategies.

A BI-LEVEL PROGRAMMING APPROACH FOR THE SHIPPER – CARRIER NETWORK PROBLEM

by
Ya Wang

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APPROVAL PAGE

A BI-LEVEL PROGRAMMING APPROACH FOR THE SHIPPER – CARRIER NETWORK PROBLEM

Ya Wang

Dr. Maria P. Boilé, Dissertation Co-Advisor — Date
Assistant Professor of Transportation Engineering
Department of Civil and Environmental Engineering, Rutgers University

Dr. Lazar N. Spasovic, Dissertation Co-Advisor — Date
Professor of Transportation and Management
School of Management, NJIT

Dr. Athanassios K. Bladikas, Committee Member — Date
Associate Professor of Industrial Engineering
Director, Interdisciplinary Program in Transportation, NJIT

Dr. John Tavantzis, Committee Member — Date
Professor of Mathematics
Department of Mathematical Sciences, NJIT

Dr. Steven I-Jy Chien, Committee Member — Date
Associate Professor of Civil and Environmental Engineering
Department of Civil and Environmental Engineering, NJIT

BIOGRAPHICAL SKETCH

Author: Ya Wang

Degree: Doctor of Philosophy in Transportation

Date: January 2002

Undergraduate and Graduate Education:

- Doctor of Philosophy in Transportation,
  New Jersey Institute of Technology, Newark, NJ, 2002
- Master of Science in Transportation,
  New Jersey Institute of Technology, Newark, NJ, 2001
- Bachelor of Art in Economics and Business,
  Nankai University, Tianjin, P. R. China, 1996

Major: Transportation Engineering

Presentations and Publications:

Boilé, M. P., Spasovic, L. N., Hausman, K., Wang, Y., and Rowinski, J. (2000). A Generalized Cost User Equilibrium Model for Assignment of Multi-Commodity, Multi-Class Truck Trips. Final Report Submitted to the New Jersey Department of Transportation.

Rowinski, J., Boilé, M. P., Spasovic, L. N., Wang, Y. (2001). A Multicommodity, Multi-Class Generalized Cost User Equilibrium Assignment Model. TRB ID 01-2596, Accepted for Presentation at the 80th Annual Meeting of the Transportation Research Board.

Spasovic, L. N., Chien, S., Feeley, K. C., Wang, Y., Hu, Q. (2001). A Methodology for Evaluating of School Bus Routing A Case Study of Riverdale, New Jersey. TRB Paper No. 01-2088, Transportation Research Board 80th Annual Meeting.

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LIST OF MATHEMATICAL NOTATION

This is a glossary of mathematical notation used throughout this dissertation.

S: Set.

|S|: Number of elements in a set.

Notation on the Shipper Network

Sets

N: Set of nodes on the shipper network, $n \in N$.

CN: Set of centroid nodes, $CN \subset N$, b, $b1$, $b2 \in$CN.

IN: Set of intermediate nodes, $IN \subset N$.

P: Set of paths on the shipper network, $p \in P$.

$P(b1, b2)$: Set of paths connecting origin $b1$ to destination $b2$.

L: Set of links on the shipper network, $l \in L$.

$L(p)$: Set of links within path p, $l \in L(p)$.

CL: Set of centroid connectors, $CL \subset L$.

LP: Set of links representing port terminals, $LP \subset L$, $lp \in LP$.

C: Set of commodity types, $c \in C$, $ci \in C$. ci is the i_{th} commodity type in C.

Parameters and Variables

$\Lambda = \left[\delta_{l,n}\right]_{L|*|N|}$: Link-Node incidence matrix,

$$\delta_{l,n} = \begin{cases} 1 & \text{if link } l \text{ originates from node } n \\ -1 & \text{if link } l \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} .$$

$\Lambda = \left[\Lambda_{CN}, \Lambda_{IN}\right]$, with

$\Lambda_{CN} = \left[\delta_{l,n\in CN}\right]_{L|*|CN|}$ being the Link-Centroid Node incidence matrix and

$\Lambda_{IN} = \left[\delta_{l,n\in IN}\right]_{L|*|IN|}$ being the Link-Intermediate Node incidence matrix.

$\left[\lambda_{p,l}\right]_{|P|*|L|}$: Path-Link incidence matrix,
$$\lambda_{p,l} = \begin{cases} 1 & \text{if link } l \text{ is contained in path } p \text{, i.e. } l \in L(p) \\ 0 & \text{otherwise} \end{cases}.$$

vot_c: Value-of-time implicit for commodity c.

$S_{b,c}$: Supply of commodity c at centroid b.

$S_{b,c'}$: Supply of commodity c' at centroid b.

S_b: Vector of supplies at centroid b, $S_b = \left(\cdots, S_{b,c\in C}, \cdots\right)^T_{|C|}$.

S: Vector of supplies, $S = \left(\cdots, S_{b\in CN}, \cdots\right)^T_{|CN|}$.

$S_{b,-c}$: Vector of supplies of the commodity types other than c at centroid b, $S_{b,-c} = \left(\cdots, S_{b,c'\in C,c'\neq c}, \cdots\right)^T_{|C|-1}$.

S_{-c}: Vector of supplies of commodity types other than c, $S_{-c} = \left(\cdots, S_{b\in CN,-c}, \cdots\right)^T_{|CN|}$.

$\pi_{b,c}$: Supply price of commodity c at centroid b.

π_b: Vector of supply prices at centroid b, $\pi_b = \left(\cdots, \pi_{b,c\in C}, \cdots\right)^T_{|C|}$.

π: Vector of supply prices, $\pi = \left(\cdots, \pi_{b\in CN}, \cdots\right)^T_{|CN|}$.

$D_{b,c}$: Demand of commodity c at centroid b.

$D_{b,c'}$: Demand of commodity c' at centroid b.

D_b: Vector of demands at centroid b, $D_b = \left(\cdots, D_{b,c\in C}, \cdots\right)^T_{|C|}$.

D: Vector of demands, $D = \left(\cdots, D_{b\in CN}, \cdots\right)^T_{|CN|}$.

$D_{b,-c}$: Vector of demands of the commodity types other than c at node centroid b, $D_{b,-c} = \left(\cdots, D_{b,c'\in C,c'\neq c}, \cdots\right)^T_{|C|-1}$.

D_{-c}: Vector of demands of commodity types other than c, $D_{-c} = \left(\cdots, D_{b\in CN,-c}, \cdots\right)^T_{|CN|}$.

$\rho_{b,c}$: Market (demand) price of commodity c at centroid b.

ρ_b: Vector of demand prices at centroid b, $\rho_b = \left(\cdots, \rho_{b,c\in C}, \cdots\right)^T_{|C|}$.

ρ: Vector of demand prices, $\rho = \left(\cdots, \rho_{b\in CN}, \cdots\right)^T_{|CN|}$.

$Q_{b1,b2,c}$: Demand of commodity c from centroid $b1$ to centroid $b2$.

$\mu_{b_1,b_2,c}$: Minimum generalized average cost for commodity c between centroid b_1 and centroid b_2.

$f_{l,c}$: Flow of commodity c on link l.

f_l: Vector of flows on link l, $f_l = \left(\cdots, f_{l,c\in C}, \cdots\right)^T_{|C|}$.

f: Vector of flows on all shipper links, $f = \left(\cdots, f_{l\in L}, \cdots\right)^T_{|L|}$.

$f_{l,-c}$: Vector of flows of the commodity types other than c on link l, $f_{l,-c} = \left(\cdots, f_{l,c'\in C,c'\neq c}, \cdots\right)^T_{|C|-1}$.

f_{-c}: Vector of link flows of the commodity types other than c, $f_{-c} = \left(\cdots, f_{l\in L,-c}, \cdots\right)^T_{|L|}$.

$R_{l,c}$: Service charge for transporting commodity c on link l.

R_l: Vector of commodity specific service charges on link l, $R_l = (\cdots, R_{l,c\in C}, \cdots)^T_{|C|}$.

R_L: Vector of service charges on all shipper links, $R_L = (\cdots, R_{l\in L}, \cdots)^T_{|L|}$.

Cap_l: Capacity on link l.

tt_l: Free flow travel time on link l.

$t_{l,c}$: Average travel time of commodity c on link l.

$GC_{l,c}$: Average generalized travel cost of commodity c on link l.

$h_{p,c}$: Flow of commodity c on path p.

$GC_{p,c}$: Average generalized path travel cost, which is the sum of the average generalized cost on the links within path p, $GC_{p,c} = \sum_{l \in L(p)} GC_{l,c} = \sum_{l \in L} \lambda_{p,l} * GC_{l,c}$.

Functions

$t_{l,c}(f_l)$: Average travel time of commodity c on link l.

$ro_{l,c',c}, \tau 0_{l,c}, \tau 1_{l,c}, \tau$: Constants in the average travel time function.

$GC_{l,c}(f_l)$: Average generalized travel cost of commodity c on link l.

$S_{b,c}(\pi_b)$: Supply function of commodity c at centroid b.

$\sigma 0_{b,c}, \sigma 1_{b,c',c}$: Constants in the supply function.

$D_{b,c}(\pi_b)$: Demand function of commodity c at centroid b.

$\varsigma 0_{b,c}, \varsigma 1_{b,c',c}$: Constants in the demand function.

$\pi_{b,c}(S_b)$: Inverse supply function of commodity c at centroid b.

$\gamma_{b,c}, \lambda_{b,c',c}$: Constants in the inverse supply function.

$\rho_{b,c}(D_b)$: Inverse demand function of commodity c at centroid b.

$\alpha_{b,c}, \beta_{b,c',c}$: Constants in the inverse demand function.

Notation on the Carriers' Network

Sets

T: Set of carriers, $t \in T$, $ti \in T$. ti is the i_{th} carrier in T.

TM: Set of a special type of carriers: the terminal operators, $TM \subset T$.

V: Set of origin and destination (O-D) pairs on carriers' network, $v \in V$.

V^t: Set of O-D pairs on the carrier t's sub-network.

PH: Set of paths on the carriers' network, $p \in PH$.

$PH(v)$:	Set of paths connecting carrier O-D pair v.
A:	Set of links on the carriers' network, $a \in A$.
A^t:	Set of links on the carrier t's sub-network.
X:	Set of nodes in the carriers' network, $x \in X$.
X^t:	Set of nodes in the carrier t's sub-network.

Parameters and Variables

$e_{a,c}$:	Flow of commodity c on the link a.
e_a:	Vector of flows on link a, $e_a = \left(\cdots, e_{a,c \in C}, \cdots\right)_{\lvert C \rvert}^T$.
e_t:	Vector of link flows on the carrier t's sub-network, $e_t = \left(\cdots, e_{a \in A^t}, \cdots\right)_{\lvert A^t \rvert}^T$.
e:	Vector of link flows on the carriers' network, $e = \left(\cdots, e_{a \in A}, \cdots\right)_{\lvert A \rvert}^T$.
$fp_{p,c}$:	Flow of commodity c on carrier path p, $\forall p \in PH$.
$R_{v,c}$:	Service charge per unit of commodity c between O-D pair v.
R_v:	Vector of service charges between O-D pair v, $R_v = \left(\cdots, R_{v,c \in C}, \cdots\right)_{\lvert C \rvert}^T$.
R_t:	Vector of service charges between O-D pairs on the carrier t's sub-network, $R_t = \left(\cdots, R_{v \in V^t}, \cdots\right)_{\lvert V^t \rvert}^T$.
R_{-t}:	Vector of service charges between O-D pairs on all other carriers' sub-networks except the carrier t's, $R_{-t} = \left(\cdots, R_{t' \in T, t' \neq t}, \cdots\right)_{\lvert T \rvert - 1}^T$.
R_V:	Vector of service charges between all carrier O-D pairs, $R_V = \left(\cdots, R_{v \in V}, \cdots\right)_{\lvert V \rvert}^T$ and $R_V = \left(\cdots, R_{t \in T}, \cdots\right)_{\lvert T \rvert}^T$.
$g_{v,c}$:	Service demand of commodity c between O-D pair v.
g_v:	Vector of service demands between O-D pair v, $g_v = \left(\cdots, g_{v,c \in C}, \cdots\right)_{\lvert C \rvert}^T$.
g_V:	Vector of service demands between all carrier O-D pairs,

$$g_V = (\cdots, g_{v \in V}, \cdots)^T_{|V|}.$$

$[\tau_{v,x}]_{|V|*|X|}$: Incidence matrix between O-D pair v and node x,

$$\tau_{v,x} = \begin{cases} 1 & \text{if O-D pair } v \text{ starts at node } x \\ -1 & \text{if O-D pair } v \text{ ends at node } x. \\ 0 & \text{otherwise} \end{cases}$$

$[\chi_{a,x}]_{|A|*|X|}$: Incidence matrix between link a and node x,

$$\chi_{a,x} = \begin{cases} 1 & \text{if link } a \text{ starts from node } x \\ -1 & \text{if link } a \text{ ends at node } x \\ 0 & \text{otherwise} \end{cases}.$$

$[\delta_{p,v}]_{|PH|*|V|}$: Incidence matrix between carrier path p and O-D pair v,

$$\delta_{p,v} = \begin{cases} 1 & \text{if path } p \text{ connects } v, \text{i.e. } p \in PH(v). \\ 0 & \text{otherwise} \end{cases}.$$

$[\delta_{p,a}]_{|PH|*|A|}$: Incidence matrix between carrier path p and link a,

$$\delta_{p,a} = \begin{cases} 1 & \text{if link } a \text{ is in path } p \in PH \\ 0 & \text{otherwise} \end{cases}.$$

$\overline{E}_a$: Capacity on link a.

Functions

$Z_t(g_t(R_t, R_{-t}), R_t, e_t)$: Profit of carrier t as a function of the vector of service charges at this carrier's sub-network (R_t) and the vector of service charges at the other carriers' sub-networks (R_{-t}) and the vector of link flows at this carrier's sub-network (e_t),

$g_{v,c}(R_V)$: Demand function of commodity c between O-D pair v as a function of the vector of service charges, R_V.

$g_{v \in V',c}(R_t, R_{-t})$: Demand function of commodity c between O-D pair v on carrier t's sub-network as a function of the vector of service charges on this sub-network (R_t) and the vector of service charges on the sub-networks of the carriers other than t (R_{-t}).

$g_{v\in V^t}(R_t, R_{-t})$: Vector of the demand functions between O-D pair v on carrier t's sub-network, $g_{v\in V^t}(R_t, R_{-t}) = (\cdots, g_{v,c\in C}(R_t, R_{-t}), \cdots)_{|C|}$.

$g_t(R_t, R_{-t})$: Vector of the demand functions between all O-D pairs on carrier t's sub-network, $g_t(R_t, R_{-t}) = (\cdots, g_{v\in V^t}(R_t, R_{-t}), \cdots)$.

$AC_{a,c}(e_{a,c})$: Average operating cost function for commodity c on link a.

$MC_{a,c}(e_{a,c})$: Marginal operating cost function for commodity c on link a.

$r_{a,c}, r'_{a,c}, r''_{a,c}$: Constants in the average/marginal operating cost function.

<u>Relationship between the Shipper Network and the Carriers' Network</u>

$[\xi_{l,v}]_{|L|*|V|}$: Incidence matrix between links on the shipper network and O-D pairs on the carriers' network, with $\xi_{l,v}$=1 if link l corresponds to O-D pair v and $\xi_{l,v}$=0 otherwise.

Other Notation

U: Set of investment strategies available to the Port Authority, $u \in U$.

$\overline{E}^u_a$: Capacity on terminal link a under investment strategy u.

$\Delta\overline{E}^u_a$: Capacity improvement on terminal link a under investment strategy u.

(S^u, f^u, D^u): Spatial price equilibrium solution under investment strategy u.

(R^u, e^u): Equilibrium service charge and link flow under investment strategy u.

ϑ: Convergence parameter.

ε: Perturbation parameter.

υ_c: Percentage of passing through freight of commodity c for the local region the port authority is located.

ζ: Economic multiplier.

TNB^u: The terminal operators' net benefit under investment strategy u.

$CS^u_{b,c}$: The consumer surplus at centroid b from the consumption of commodity c under investment strategy u.

$PS^u_{b,c}$: The producer surplus at centroid b from the production of commodity c under investment strategy u.

$SNB^u_{b,c}$: The shippers' net benefit at centroid b from the consumption and the production of commodity c under investment strategy u.

SNB^u: The shippers' net benefit under investment strategy u.

$ASNB^u$: The shippers' net benefit adjusted to account for the passing through traffic and the external economy under investment strategy u.

NSB^u: Net social benefit under investment strategy u,
$NSB^u = TNB^u + ASNB^u \quad \forall u \in U$.

I^u_a: Capital expense for the capacity improvement on link a under investment strategy u.

PIC^u_a: Present value of all capital expenses on link a under investment strategy u in the analysis period.

AIC^u_a: Annual investment cost of all capital expenses on link a under investment strategy u in the analysis period

IC^u_a: Hourly investment cost of all capital expenses on link a under investment strategy u in the analysis period,
$IC^u_a = p2^u_a * (p1^u_a * \Delta \overline{E}^u_a) \qquad \forall a \in A, u \in U$.

$p1^u_a$, $p2^u_a$: Constants.

IC^u: Total hourly investment cost under investment strategy u,
$IC^u = \sum_{a \in A} IC^u_a$.

CHAPTER 1

INTRODUCTION

1.1 Background

Advances in transportation and information technology and the recent expansion of the global economy are two major factors driving the fundamental changes in the shipping industry and the role of United States ports. The advances in technology include: containerization of cargo, new ship designs, and information technology (IT) driven freight logistics.

Containerization begun in 1966 with the maiden voyage of a Sea-Land container ship from Newark, New Jersey to Rotterdam, Holland (Talley, 2000). In the container ship operation, freight loaded in steel boxes or containers is moved directly between an origin and a final destination. The reduction and, in some cases, full elimination of the freight loading, unloading, and repackaging at the intermediate points (as it was done in the pre-container era) resulted in faster service at a lower cost. The storage of cargo in a sturdy steel box reduced the damage cost as well. Since the introduction of containerization, the volume of freight moving via containers has grown steadily. Figure 1.1 shows that from 1994 to 1999, the US container traffic measured in TEUs (twenty-foot equivalent units) has increased by 36 percent, from 19.58 million to 26.67 million TEUs (The American Association of Port Authorities, 2001).

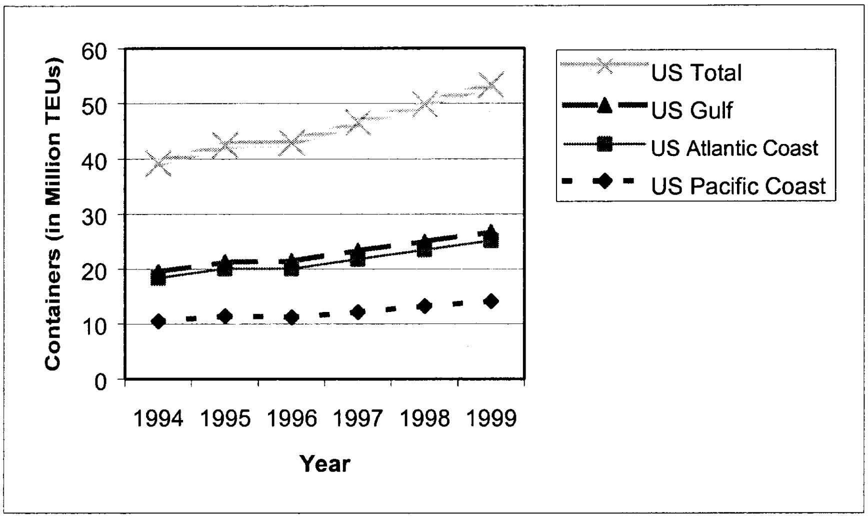

Figure 1.1 US Container Traffic 1994 to 1999 (in Million TEUs) (The American Association of Port Authorities, 2001).

Another trend accompanying the advent of containerization is the increased size of container ships. The standard ship size has increased from less than 1000 TEUs in the late 1960s to 6000 TEUs in the late 1990s (Airriess, 2000). The driving factor behind this trend is the fact that the average cost of transporting a container decreases with an increase in the ship size. For example, a 6000 TEU ship has approximately 20 percent lower average unit cost compared to a 4000 TEU ship. However, this cost advantage can be realized only if the ship sails fully loaded. Unused ship capacity can quickly translate into high opportunity cost, which is very detrimental to profit. In addition, the larger ship has a higher operating cost (e.g., fuel, crew, and maintenance costs). It also needs larger cranes with longer reach from the berth to the ship, more sophisticated equipment, and a technology-skilled labor force, which translates into a higher terminal handling cost.

Collectively, the issues discussed above have resulted in a higher cost per port call. To ensure that the capacity of their ships is fully utilized, the ocean carrier lines share vessels by forming various strategic alliances and partnerships as well as outright mergers. During this process, many routes have been reconfigured and some of the ports are no longer visited.

Now turning to the change in the manufacturing process, the manufacturing industry adopted the just-in-time (JIT) principle in order to reduce the inventory cost and to increase flexibility of production. JIT is a demand driven process. All inventories is kept at the suppliers, and brought in via a reliable transportation system only when and where it is needed. The recent advance in information technology, such as Electronic Data Interchange (EDI), Automatic Equipment Identification (AEI) and Global Equipment Positioning System (GPS), have greatly facilitated the communication between producers and consumers of goods and providers of transportation services, and enabled the real time tracking of the containers (Helling, 2000). The uncertainty and the inventory costs and requirements are reduced, and as long as the JIT system has a highly reliable transportation component, it is no longer necessary to locate the distribution centers close to the production facilities and the markets. Ports have now become an important part of the logistics link in the JIT production and delivery chain, since use of information driven logistics has opened up more potential markets. In other words, the market reach of the ports is no longer restricted to their neighboring region. For example, ten years ago, 90 percent of the port traffic at the Port of New York and New Jersey was tied to its regional market. Now, more than 15 percent is destined to the Midwest and Canada (Cottrill, 1999).

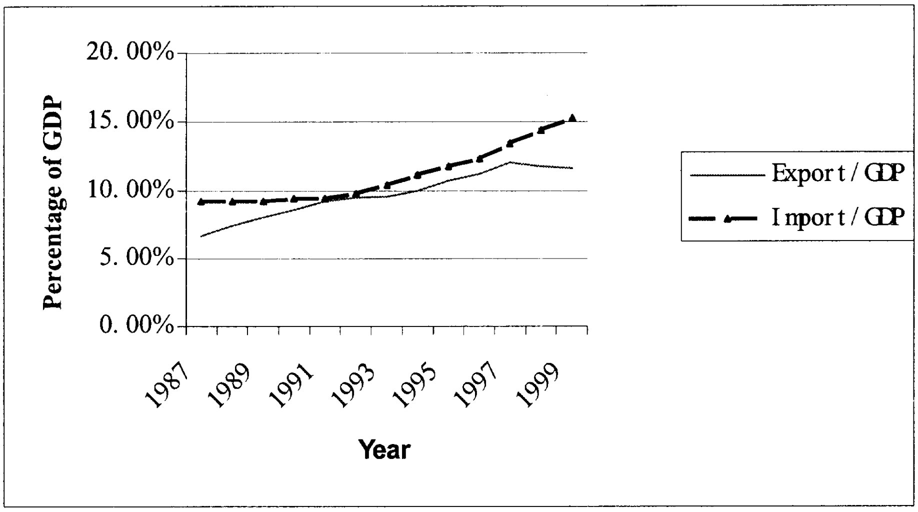

Figure 1.2 Share of International Trade in Goods and Services in the US Gross Domestic Product (International Trade Administration (ITA) of the United States Department of Commerce, 2000)

The international flow of goods, capital and labor have increased rapidly, driven by the formation of trade agreements such as the North American Free Trade Agreement (NAFTA), the economic and political organizations such as the European Union (EU) and the Asia-Pacific Economic Cooperation (APEC). With this trend of globalization, the United States has become deeply immersed in the international trade. Figure 1.2 shows that from 1987 to 1999, the import and export as a percentage of Gross Domestic Product (GDP) (based on the 1996-dollar) in America grew from 15 percent to 26 percent. The rapid growth of trade with the Asia Pacific region, especially China, contributed greatly to this trend. The trade with China, including Hong Kong, as a percentage of the total US trade, grew from 5.54 percent in 1992 to 7.11 percent in 1999 (International Trade Administration (ITA) of the United States Department of Commerce, 2000).

The trends discussed above brought both opportunities and challenges to the ports. Fueled by the boom of international trade, container cargo at the ports is expected to grow correspondingly. Attracting the international market with strong growth potential is critical to the development of the ports. The strong growth of trade between the US and the Asian Pacific region has prompted several ports both in the East and the West Coast to identify this traffic as the strategic area in their development plans. The Port Authority of New York and New Jersey, for example, plans to form an agreement with the Suez Canal Authority to capture a larger share of the US bound Asia cargo. According to its forecast, by shifting just seven percent of the current West Coast-bound traffic to the East Coast via the Suez Canal, the port volumes could double by 2020 (The Port Authority of New York and New Jersey, 1999). Clearly, there will be a competition for this traffic among the US ports.

Competition among seaports is an old issue dating back to the colonial times. Up to 1980s, only a few large ports along the East Coast such as the Port of New York, the Port of Boston, the Port of Philadelphia, and the Port of Baltimore were competing with each other for international trade. The shift to larger container ships will intensify this competition since the consolidated shipping lines are likely to redesign their routes and call only at a few ports. Equipped with advanced information and communication technologies, the shipping lines that are not restrained by a contract can evaluate the service charge and congestion levels at any port in almost real time and change their selection of port calls accordingly.

> *"Corporate managers can, if they want, evaluate options and switch ports as easily as they change television channels. When Steven Shyne, manager of international transportation at Pfizer Inc. became dissatisfied with port service in NY he began shifting more specialty chemical shipments to the Big Apples archrival in Montreal-and he saved time and money in the process.*
>
> *Herbert Ovida, manager of regional export development program at the PA of NY-NJ is acutely aware of the new competition. "We used to think that the customer was*

> *captive, that they had no choice", admits Mr. Ovida. "But today the small ports have grown up, and NY has a hell of a lot of competition." "*[1]

Besides the increased intensity, the scope of competition has been expanded to that between the East and the West ports and even to the national level.

With this increase in competition, the ports are rethinking ways to bolster capacity and improve service quality in order to maintain current and attract new business. To accommodate larger vessels, the ports need to deepen their navigation channels and berths, acquire new larger cranes, implement state-of-the-art information and communication technologies, and provide seamless connections between the modes such as on-dock rail and truck. The increased competition adds to the pressure of infrastructure investment. According to a report by the National Highway System (Department of Transportation, 2000), most US port terminals have various deficiencies, among which the most important is insufficient intermodal connectors that enable the seamless movement of containers between modes. Hence, it can be expected that without prompt infrastructure improvements, our ports may become a bottleneck for the international trade between the United States and the rest of the world. This eventually may impair the competitiveness of the export industry and hinder the growth of the US economy. In response to the above opportunities and challenges, most ports have started to or plan to redesign and reorganize their operations. Faced with the investment pressure, various construction projects such as dredging and on-dock rail are being carried out at many ports. In addition, most ports have come up with a long-term investment plan. The federal government is a major source of funds for these projects. To receive the funds, the ports need to conduct a comprehensive cost and benefit analysis. This gives rise to the

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[1] Miles, G. L. (1994). The War of the Ports. International Business. 7. 30-36.

investment issue of port authorities' decisions, which is one of the issues discussed in this dissertation.

The benefits of a port expansion are accrued not only by the immediate region in which the port is located, but also extend beyond this region. The first focus of this analysis is on economic benefits directly related to the operation at ports. Operations at the ports are carried out by private terminal operators. Their activities include setting the service charge and making the routing decisions. In the face of the intensified competition, lowering service charges used to be the frequently preferred strategy by the private terminal operators to increase business. However, severe service charge competition and high operating costs have become detrimental for profit. Hence, it is important to find an appropriate set of service charges, which are both competitive and profitable, and make the optimal routing decision, which will allocate the limited resources to the most efficient use, thus reducing the operating cost. Here arises the pricing and routing issue of private terminal operators, which is addressed in this dissertation.

The investment issues of the Port Authority and the pricing and routing issue of the private terminal operators give rise to the realistic importance of this dissertation. The dissertation aims to develop a model, which can derive the optimal service charge and routing pattern at port terminals as applied to the Port Authority's investment decision in the framework of a multimodal freight network. More specifically, the model can aid the transportation planner to forecast the production, consumption, and link flow pattern on a freight network, evaluate the performance of the port terminal operation, and determine the best investment strategies. Even though the primary motivation of this dissertation was to develop a model for evaluating port terminal operations, the resulting methodology is general and directly applicable to different types of carriers, such as competing truck, rail, or intermodal service providers.

The rest of this chapter is organized as follows. Section 1.2 describes the role of the seaports in the multimodal freight transportation system. Section 1.3 presents the research problem to be solved in this dissertation. Section 1.4 gives the organization of the dissertation.

1.2 Role of Seaports in the Multimodal Freight Transportation System

Ports in the U.S. are usually publicly owned facilities, consisting of channels, berths, docks, and land, managed by a Port Authority, a public (or quasi-public) agency operating in the public interest. A Port Authority may have several terminals within its port complex and lease the land and facilities to private operators. The terminal operators together with the other public and private transportation carriers that own and operate transportation facilities constitute a multimodal freight transportation system. Through this system, vehicles and containers carrying commodities from the shippers to the receivers are located in spatially separated markets. Due to the existence of different transportation modes and related complicated interactions between components of the freight system within the port terminal, the analysis of port operations must be considered within this framework of a multimodal freight system. In this section, the behavioral principles of three major types of players involved in the port operation and the interaction between these players will be described. Following this, the factors influencing each player's behavior will be discussed.

1.2.1 Players Involved in Port Operation

A set of players involved in the port operation include the steamship lines, railroads and motor carriers, brokers, shippers, forwarders, port terminal operators, a Port Authority, and the other regulatory agencies. Following the generalized definition of Harker et al. (1986-a, 1986-b), these players could be combined into three major groups: the shippers, the carriers, and the regulatory agency. It is the interaction of these players, which results in an equilibrium of the spatially separated markets. At the same time, their individual rent-seeking behaviors may disturb the market away from the equilibrium from time to time. Understanding the behavior of each player and its rationale is the key to modeling a freight network system.

1.2.1.1 The Shipper. As defined by Harker (1986-a, 1986-b), the shippers are economic agents who engage in moving commodity over the spatial network to explore the potential economic benefit arising from the difference in commodity price between different regions. The rent seeking behavior of the shippers serves as an "invisible hand", which drives the spatial market to an equilibrium point where all the potential benefits have been exhausted. Here, the shippers are a generalized notion in the respect that any one among the producer, consumer, forwarder or broker could be classified as shipper, as long as the above definition is satisfied.

The shipper's decision variables consist of the amount of commodity supplied and demanded at each market, and the freight flow between these markets. These flows are assigned to a carrier or a sequence of carriers for a movement over their transportation networks. The selection of the sequence of carriers is equivalent to the selection of the transshipment locations. The transshipment locations are the points where the intermodal transfer of the goods (or the interlining practice--the exchange of goods between different carriers) takes place. The objective of the shipper is profit maximization, which is accomplished by shipping the commodity to market via the path with the cheapest generalized cost, but with a high enough price to cover the sum of the production and the transportation cost.

1.2.1.2 The Carriers. According to Harker (1986-a, 1986-b), the carriers are the economic agents who operate the transportation facility and provide the transportation service. The transportation facilities operated by the carriers may include the vessel, the railroad, the berth, the warehouse, and so on. Facilities under different carriers could be represented as non-overlapping sub-networks with various origins, destinations, and intersection nodes. The union of all carriers' sub-networks is called the carriers' network. The transportation service here is a generalized concept, which includes all those processes needed in transporting the freight from an origin to a destination within a carrier's sub-network. In this dissertation, it is assumed that the links between each O-D pair on a carrier's sub-network represent not only the physical location, but the type of service process taking place there as well. Different service processes taking place in the same physical location are defined as different links. In another word, a link on a carrier's sub-network is both geographical and service orientated. Based on the above definition, the shipping lines, the rail companies and motor carriers, and the port terminal operators all belong to the same group: the carriers. They all satisfy the behavior principles described below.

The carrier's decision variables consist of link flow and service charge on the routes on the sub-network under its control. The carrier's objective is profit maximization. It is accomplished by setting the service charge and routing pattern in such a way that, for the flow of each commodity between each O-D pair, two conditions are satisfied: 1. The marginal revenue (i.e. the gain in revenue by providing an additional unit of service) equals the lowest marginal cost (i.e. the increase in the operating cost by providing an additional unit of service). 2. The marginal cost on any used path between this O-D pair equals the lowest marginal cost on all the paths between this O-D pair.

The next issue is modeling the carriers' behavior. First, with the merger of the carriers' industry (Luberoff, 2000), the competition between different carriers exhibits an oligopolistic nature. A carrier has certain power in setting the price instead of being simply a price taker. A carrier adjusts its decision frequently according to the environment and changes in the demand level, capacity of the facility, and service charges offered by other carriers. Thus, the marginal cost pricing principle of the carriers is not applicable here. Second, besides the competition, there exists certain level of pricing collusion among the carriers. For example, before the Ocean Shipping Reform Act (OSRA) was passed in 1998, the annual conference of ocean carriers (which also serve the port) attempted to set some price agreement, though the agreement is not always diligently observed (Pei, 2000). The above features are usually not captured by the carrier level models presented in previous studies such as in Harker (1986-a, 1986-b). This dissertation aims to fill these gaps by formulating the oligopolistic pricing behavior of the carriers under either the non-collusive or collusive pricing schemes subject to the shippers' equilibrium.

1.2.1.3 Port Authority. Besides the terminal operators as a type of carriers, the Port Authority is another player involved in the supply side of port operations. The Port

Authority is a public agency, responsible for major infrastructure investments at a port; it decides where and how much to invest. Its objective is the maximization of the net social benefit to the region it serves. Unlike the service charge and the routing pattern of the carriers, the Port Authority's investment plan is carried out during a longer time period. In this respect, the investment decision is not as flexible as the carriers' pricing and routing decisions. Usually, its benefits are returned in a long run, unless the claim of one Port Authority to increase the capacity seems so credible to other competing ports that it effectively hinders the investment at those ports. The latter case happens when the demand is not great enough for every port to expand its capacity and the Port Authority taking the initial step to invest has an edge over other competing Port Authorities, which react slowly.

1.2.1.4 Interactions among Players. The way the players interact with each other is influenced by whether the decision they are making is a long-term or a short-term one, and whether the market in which they operate can be described as a monopoly, oligopoly or perfect competition. The long-term decision, once committed, is difficult to change in the short term. Hence, the interaction between the long-term decision-maker and the short-term decision-maker has a sequential nature. The market conditions also influence the interaction by bestowing the decision-maker with different levels of market power under different market conditions. For example, the monopoly supplier or the monopoly consumer has strong control over the market price. On the contrary, under the market condition of perfect competition, the supplier or the consumer acts as the price taker. Understanding the short-term or long-term nature of each player's decision and the market conditions is very important to the understanding of the interaction between and among the players. The nature of the decision and the market condition in which each player acts are introduced below.

A shipper's decision is assumed to be a short-term decision. It is assumed that there are numerous shippers in the system, which indicates that the market condition for the shippers is perfect competition. The profit-seeking behavior of all shippers determines the market price and the flow pattern in the transportation market, although none of them alone has the power to determine price individually.

The pricing and routing decision of the carriers is assumed to be a short-term decision. This assumption is justified by the fact that both the service charge and the routing pattern can be easily adjusted according to the level of demand. It is assumed that there are only a few carriers providing service at each port. The carriers can observe each other's behavior and react accordingly. The market condition of the carriers' industry is assumed to be oligopoly.

The investment decision by a Port Authority is a long-term decision. The number of Port Authorities in the market is even smaller than the number of carriers. The Port Authorities can observe each other's behavior and react accordingly as well. Unlike the carrier, the Port Authority is a public agency. Besides the economic factors, its behavior is considerably influenced by social and political factors, which render the economic approach ineffective in formulating the competition between various Port Authorities. Hence, the dissertation will not deal with this problem. Instead, the dissertation focuses on developing a model, which can be used to evaluate the economic impact of several alternative investment strategies and to identify the best strategy for a Port Authority.

Based on the above discussion, the interaction between the three levels of players can be summarized as follows:

Each shipper makes its commodity production and consumption decision based on its knowledge of the pattern of the market prices in the spatially separated markets. It makes its routing decision based on its knowledge of the pattern of the service charges set by the carriers and the travel time function between O-D pairs on the carriers' network. This is determined by the purely competitive assumption of shippers' market, which indicates that each individual shipper is a price taker.

Each carrier makes its pricing and routing decision based on its knowledge of the Port Authority's investment decision and its forecast of the shippers' and competing carriers' reaction. The sequential nature of the interaction between the Port Authority and the carriers is determined by the fact that the Port Authority's investment decision is a long-term decision while the carriers' pricing and routing decision is a short-term decision. The interaction between different carriers is characteristic of the behavior in an oligopoly market.

The shippers' and carriers' behaviors in turn influence the Port Authority's decisions. How the shippers and the carriers react to the Port Authority's investment strategy determines the effectiveness of this strategy. The Port Authority makes its investment decision based on its forecast of the effects on carriers' pricing and routing decision and the shift of the production, consumption and link flow pattern under different investment strategies.

1.2.2 Factors Influencing the Players' Behavior

Various factors influence each player's behavior and, consequently, the throughput of a port terminal. In general, these factors can be classified into three categories: 1) commodity supply and demand functions at a market, 2) transportation technology used by modes and which is usually characterized by the operating costs and service characteristics (e.g., travel time), and 3) how does a shipper perceives service characteristics and derives a generalized cost (e.g., travel time plus related freight charges). A small variation in the above factors may shift the production, consumption and flow pattern in the system tremendously, and consequently change the profit of the port operation significantly. Hence, a systematic analysis of the port operation should take into account all of the above factors. These factors are discussed in this section.

1.2.2.1 Commodity Supply and Demand Functions. The producers and consumers at each market jointly determine commodity supply and demand. Each producer's behavior may be described by a supply function, which is mainly influenced by the technology and the cost of inputs (e.g., materials, salaries, and interest rates). Generally, supply is an increasing function in its own price and a decreasing function in the price of its input resources. If a product serves as an input to the production of another product, its price will have a major influence in the other product's production output. Alternatively, if two products are competitive, an increase in price of one may stimulate the production of the other. On the other hand, consumer behavior may be described by a demand function. Unlike supply, the demand is usually a decreasing function in its own price. The price of another product may influence the demand of the primary product either positively or negatively depending on whether these two products are complementary or competitive (i.e., they are substitutes for each other). The supply and demand functions at a market are obtained by aggregating the individual producer and consumer supply and demand functions.

1.2.2.2 Transportation Technology Used by Modes. Transportation technology used by modes is usually characterized by the operating costs and service characteristics (e.g., travel time). Operating cost is defined as the cost required to carry out a service process at a carrier's facility within certain time period. Travel time is defined as the time required to finish a service process (e.g. transporting, loading, unloading) at a carrier's facility. Besides the level of technology, the operating cost and the travel time also depend on the level of service demand, and the capacity of the carrier's facility or equipment. The technology and the capacity take effect by shifting the curve that represents the operating cost or travel time function upward or downward. On the contrary, the service demand takes effect by moving the operating cost or travel time along the curve that represents the operating cost function or the travel time function.

1.2.2.3 Service Characteristics Perceived by the Shippers. Shipper's behavior is characterized by how a shipper perceives service characteristics and derives a generalized cost. The service characteristics, which the shipper considers in its selection of the carriers or transportation modes, are quite diverse, including both qualitative and quantitative attributes. As identified by Evers (1996), there are at least six factors influencing the shipper's decision: 1) service charge, 2) reliability, 3) travel time, 4) over, short, and damage, 5) market considerations, and 6) carrier considerations. Among these, only travel time and service charge will be considered in formulating the shipper decision. As indicated in the Evers' study, travel time and service charge are the two of the most important determinants in the selection of carriers or transportation modes. The relative importance of these two attributes in affecting the shipper's decision is indicated by value-of-time. Value-of-time converts travel time into the same unit as service charge, so that they can be combined together to calculate the generalized cost. The flow of different commodity types has a distinct value-of-time. The inner value of a commodity is not the sole factor in determining it's value-of-time. The logistic opportunity cost, which is determined by the relative urgency to use certain commodity and the value-decreasing rate of the commodity with the passage of time, are some other important factors. In addition, the subjective perception of the players involved may also play a role in determining the value-of-time of the freight. The generalized cost perceived by a shipper is the linear combination of the service charge and the travel time that is converted into monetary value by the value-of-time.

1.3 Research Problem

The interactions discussed above can be formulated using a three-level model. The first level describes the behavior of the Port Authority in choosing the best investment strategy to maximize net social benefit. The second level describes the behavior of the carriers in choosing optimal service charge and routing pattern to maximize their profits. The third level formulates the behavior of the shippers, which is to determine the supply and demand of each commodity at each market (or centroid in transportation planning parlance) and the distribution pattern of each commodity on the network. The structure of the model is shown in Figure 1.3.

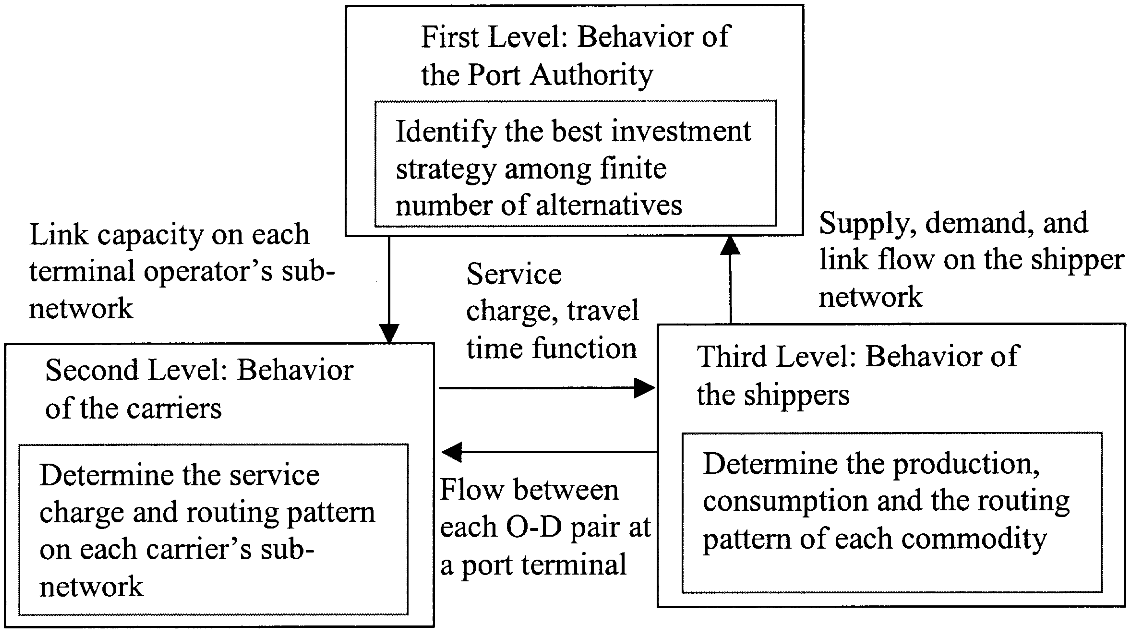

Figure 1.3 Interactions between Different Players

More specifically, the research questions to be answered by each level of the model in this dissertation are listed below:

1. Behavior of the Port Authority

Which investment strategy out of a finite set of alternatives should the Port Authority implement in order to maximize the net social benefit? What will be the impact of this strategy on the private terminal operators and consequently the shippers?

2. Behavior of the carriers

What is the equilibrium service charge and routing pattern on each carrier's sub-network given the competitive pricing game among the carriers? What is the optimal set of service charge and routing pattern on each terminal sub-network and the resulting profit if the carriers choose to price collusively? What will be the impact of the competitive or collusive pricing on the shippers' decisions?

3. Behavior of the shippers

What are the optimal locations at which goods are produced and consumed, their quantity and price? What is the equilibrium flow on the shipper network and the resulting cost?

1.4 Organization of the Dissertation

The dissertation is organized as follows. Chapter 2 reviews the models found in the literature to solve each of the above problems and discusses their advantages and limitations. Chapter 3 describes the structure of the freight network, including both the shipper perceived network and the carriers' network. The port facilities and the service processes that take place at a port terminal are viewed as a carrier's sub-network. Chapter 4 presents the basic assumptions, equilibrium conditions, and formulations for the carriers' pricing and routing problem with explicitly defined demand function. Chapter 5 extends the model from Chapter 4 by integrating the shipper level problem with the carriers' pricing and routing problem. A bi-level program is developed to formulate the Stackelberg game between the carriers and the shippers. A sensitivity analysis based heuristic algorithm is developed to solve the bi-level program. Chapter 6 formulates the Port Authority's investment behavior and uses the model from Chapter 5 to model the Port Authority's investment decisions. Chapter 7 presents the case study. A numerical example is developed and used to test the application of the model and the algorithms developed in this dissertation. The model is implemented in the GAMS software (Brooke, 1992). Finally, Chapter 8 examines possible extensions of the model and directions for future research.

CHAPTER 2

LITERATURE REVIEW

Chapter 1 defined the research problem of this dissertation. The problem consisted of modeling decisions of three types of players - the shippers, the carriers, and the Port Authority - and how they interacted in determining the production and consumption patterns, resulting commodity flows and related carrier operations plans and service charges. This chapter presents a review of the papers and studies from freight transportation planning and spatial economics that were deemed relevant for the formulation of a mathematical model of the research problem, and for designing a solution algorithm for solving the model. The chapter concludes with a discussion of how the approaches found in the literature relate to the models proposed in this dissertation.

2.1 Evolution of the Freight Transportation Planning Models

The freight transportation planning process in general needs to accomplish the following tasks:

1. Determine, based on the underlying economic principles, the amount of commodities to be produced and consumed at certain zones or regions under study.
2. Identify the trip matrix between the points of production and the points of consumption.
3. Allocate demand over the available transportation modes.
4. Provide the routing of the vehicles carrying the commodities over the modal networks.

Early freight models mimicked the traditional four-step urban transportation planning process for passengers. The first step, trip generation, represents the number of trips to be produced at origins and attracted at destinations. The second step, trip distribution derives the trip table that allocates the trips between origins and destinations. The third step of the process, modal split is the allocation of trip interchanges to available modes, while the fourth step, traffic assignment, disperses the modal trip interchanges to available routes (Manheim, 1979). The freight planning process is more complex than the passenger planning process because it involves interaction between at least two distinct interest groups: the shippers that send the commodity, and the carriers that provide the transportation system –(vehicles and the network) over which the commodity moves. The freight planning models have evolved from those focusing on one stage and one type of player/mode (e.g. the Railroads Routing Model (RRM) of Bronzini, 1981) to those solving the problems of all four stages jointly, and considering both the shippers and the carriers simultaneously (e.g. the Generalized Spatial Price Equilibrium Model (GSPEM) by Harker et al., 1986-a, 1986-b).

Two strains of models with distinct focuses have been used throughout the history to model the freight transportation planning process. The first strain is the pure spatial price equilibrium model (Samuelson, 1952; Takayama and Judge, 1964), the purpose of which is mainly to derive the competitive equilibrium of the commodity production, consumption and distribution pattern among the spatially separated markets in a simplified transportation network. On the simplified network, each Origin-Destination (O-D) pair is connected by a link, and the link-based transportation cost is fixed. The second strain is the freight network equilibrium model. It predicts the modal split and network assignment of freight flows over available links and paths on a general multimodal transportation network consisting of various modal networks. The general transportation network has the node-arc representation and flow-dependent transportation costs. Various O-D pairs are connected at intermediate nodes, and the connecting paths are made up of several links. Paths between different O-D pairs often share a link. Usually, these models include performance functions that relate the travel time and cost to traffic volumes. By assuming the amount of freight to be shipped as fixed, the models ignore the impact of network performance and shipping cost on the commodity production and consumption patterns in spatially separated markets.

With the recent methodological and theoretical advancements, these two strains have converged. New models are sound in economic and behavioral theory, and have been combined with the advancements in algorithmic developments and increased computational power. The following sections review briefly the progress of the spatial price equilibrium (SPE) model and freight transportation network equilibrium (FNE) model, including both formulations and algorithms.

2.1.1 Pure Spatial Price Equilibrium Model

The pure SPE model aims to find an equilibrium solution in terms of the commodity supplied and demanded at each market, and the inter-market network-based commodity flows. The solution satisfies the market equilibrium condition: the delivered price at the destination market (that is, the price of the product at the point of production plus the lowest transportation cost to the market) equals the market price that the customers are willing to pay. This means the appreciation of the supply price at the market has been exhausted by the transportation cost. Figure 2.1 illustrates this concept.

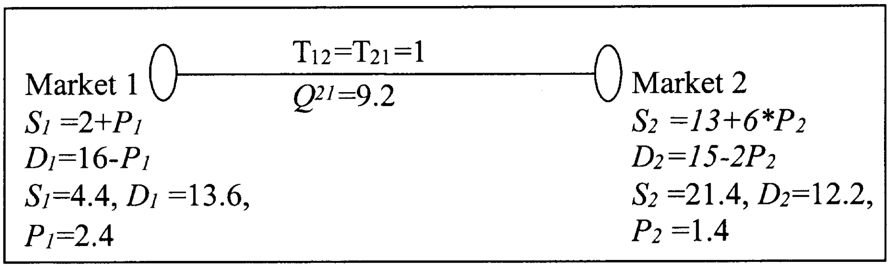

Figure 2.1 A Simple Pure SPE Model

The production and consumption on this simple network consisting of an origin and a destination are described by the following functions:

$$S_1=2+P_1 \quad (2.1)$$

$$D_1=16-P_1 \quad (2.2)$$

$$S_2 =13+6*P_2 \quad (2.3)$$

$$D_2=15-2P_2 \quad (2.4)$$

where:

S_i, D_i, P_i and T_{ij} ($\forall i,j$ = 1 or 2) denote the supply, the demand, the commodity price, and the transportation cost, respectively. The subscripts i or j indicate the markets, and Q_{ij} is the net commodity export from market i to market j. The spatial price equilibrium satisfies the following conditions:

$$\text{If } Q_{12}>0, \text{ then } P_1+T_{12}= P_2 \text{ and if } Q_{21}>0, P_2+T_{21}= P_1 \quad (2.5)$$

$$S_1-D_1= Q_{12}= -Q_{21} = D_2-S_2 \quad (2.6)$$

$$S_1, D_1, S_2, D_2\geq 0. \quad (2.7)$$

Solving Eqs. (2.5)-(2.7) yields the following:

$$-14+2*P_1 = 2-8*P_2 \tag{2.8}$$

$$P_2+1 = P_1 \tag{2.9}$$

with equilibrium supply, demand, price and commodity flows being:

S_1=4.4, D_1 =13.6, P_1=2.4, S_2 =21.4, D_2=12.2, P_2 =1.4, Q_{21}=9.2.

Samuelson's (1952) pure SPE problem, which assumed an over-simplified transportation network with fixed transportation cost between each O-D market and linear supply and demand functions at each market, was formulated as an optimization problem. Takayama et al. (1964) extended Samuelson's model to handle a multi-commodity case and formulated the pure multicommodity SPE problem as a quadratic programming problem. He assumed that the supply and demand functions are symmetric.

2.1.2 Freight Network Equilibrium Model

The FNE model aims to find the equilibrium solution in terms of link flows, which satisfies Wardrop's First and Second Principles (Sheffi, 1985). Wardrop's First Principle, also called the user equilibrium, states that at equilibrium all used paths between the same O-D pair for the same commodity have equalized lowest cost. This principle is used for modeling shippers' routing decisions. The modified statement implies that at equilibrium each shipper has no incentive to unilaterally change routes, paths or modes because it cannot further reduce its cost.

Wardrop's Second Principle, also called the system optimum, states that in order to minimize the total transportation cost, all used paths between the same O-D pair for the same commodity have the same lowest marginal cost. The principle applies to the carriers' optimal routing decisions. It is modified to state that at the optimum a carrier has no incentive to change its routing plan on the sub-network under its control because it cannot further reduce its cost.

A detailed review of the FNE models is given by Bronzini (1980) and Friesz (1983-b, 1985). Bronzini (1978) used a national freight transportation network model (i.e. a FNE model) to estimate the energy conservation impact as well as the modal traffic shares, transportation costs and service levels on a general multimodal transportation network. The model assumed that the commodity trip distribution matrix of various commodities between selected O-D regions was known, based on information provided by the U.S. Department of Commerce. The freight network was described by a set of performance functions: a cost function, a capacity function, and an energy function. The service attributes, such as cost, time, and energy were functions of freight tonnage. The assumed linear combination of these three attributes was used to represent the total disutility the shippers would perceive when choosing the bundles of modes and routes. The shippers' equilibrium was obtained by the standard labeling algorithm (Sheffi, 1985). In this approach, the disutility combines the carriers' cost (including the operating cost and the energy cost) with the time delay and is used to generate the shippers' shortest path. By mixing the carriers' concern (i.e. the operating cost and the energy cost) with the shippers' concern (i.e. the time delay) and ignoring the impact of service rate on shippers' choice in calculating the disutility, the model did not recognize a distinction between the carriers' and the shippers' behavior.

In a latter paper, Bronzini (1981) ran the models with both the average operating costs and the unit coal train rates to show that no major divergence existed between the average cost-based approach and the price-based approach in determining the modal split.

In the price-based approach, the unit coal train rates were determined using rate-cost equations developed via the linear regression method. The transportation prices for all other commodities except the unit coal train were simulated using marginal operating costs. Bronzini (1981) is among the earliest efforts to relate the carriers' pricing behavior to the carriers' cost. The limitation of the paper is that it only compared the divergences in modal splits determined from the price-based and the cost-based approaches. The potential divergence in the assignment patterns was ignored, thus the impact of using the average carrier cost instead of actual rates in analyzing the shippers' assignment cannot be ascertained.

Friesz et al. (1986) extended the freight network equilibrium models of Bronzini, 1978, 1981 by combining the trip distribution problem with the modal split and traffic assignment problem and treating the shippers' and the carriers' decisions separately. He divided the problem into two sub-problems: the shipper network and the carriers' network. The two networks are related through an incidence matrix between the arcs on the shipper network and the O-D pairs on the carriers' network. The trip distribution, modal split and traffic assignment problem on the shipper network, and the assignment problem on the carriers' network were solved sequentially. The solution satisfies the Wardrop's First and Second principles. The trip distribution problem was solved using the Gravity-type model (Manheim, 1979). There are two deficiencies in this approach. First, it does not consider the trip generation problem, and uses the Gravity model for trip distribution. Second, similar to Bronzini (1978), it assumes that the shippers' cost is a function of the link flow on the shipper network instead of being derived from the carrier's behavior. Furthermore, it makes a strong assumption that the cost or delay functions on both the shipper and the carriers' networks are separable.

2.1.3 The General Spatial Price Equilibrium Model

In the last two decades, the distinction between the SPE and the FNE models became less prominent. The recent SPE models deal with the modal split and traffic assignment problems as well as the traffic generation and distribution problems simultaneously via the use of a general multimodal transportation network. The recent SPE models that use the general multimodal transportation network are called the "general SPE model". For these models, the definition of the pure SPE condition in section 2.1.1 needs to be modified to include the Wardrop User Equilibrium condition of Section 2.2.2. The revised definition of an equilibrium called the modified Wardrop User Equilibrium in this dissertation states that for each commodity and a given O-D pair, the transportation cost on any used path is equivalent to the difference between the supply price at the origin and the demand price at the destination and is not higher than the costs on any unused paths.

Besides the use of the general multimodal transportation network, another advance of the general SPE model lies in its mathematical formulation. The recent studies adopt the nonextremal formulation, which uses either the Variational Inequality (VI) (see Florian et al., 1982; Friesz et al., 1984; Harker et al., 1986-a, 1986-b; Pang 1984) or the complementarity formulations (see Friesz et al., 1983-a), instead of the extremal formulation from Section 2.1.1, which use the mathematical programming approach. Florian (1982) pointed out that the application of the extremal formulation is restricted to the situation where the inverse supply, inverse demand and cost functions are continuous and have symmetric Jacobian matrix or separable functions. The nonextremal formulation has fewer restrictions, and can be used to formulate the problem with asymmetric Jacobian matrix and non-separable functions, which frequently arises in the problem with the general transportation network or dealing with multiple commodities.

Common to Florian (1982), Friesz (1983-a), (1984), Harker (1986-a, 1986-b), and Pang (1984) is that they all cast the multicommodity SPE problem in a general multimodal transportation network using a nonextremal formulation. However, these papers have distinct features. Florian (1982) proposed the VI formulation with only the nonnegativity constraints of the path flow variables. The flow conservation constraints were incorporated in the formulation via the use of an indicator representing the incidence relationship between path, link, and O-D pair. Friesz (1983-a) provided two nonlinear complementarity formulations: one using the path flow and the other using the arc flow formulation. Both Friesz (1984) and Pang (1984) gave the VI formulation using the node- arc incidence relationship. Pang's formulation differed from Friesz (1983-a) in that the commodity price is not restricted to be nonnegative. In spite of this difference, the two formulations are equivalent, as proven by Pang, under the condition that a unique solution exists for both formulations. Harker (1986-a, 1986-b) also employed the VI formulation using path-arc incidence relationship. As distinct from the general SPE models in Florian (1982), Friesz (1983-a), (1984), and Pang (1984), which didn't distinguish the shippers' and the carriers' behaviors, the Generalized Spatial Price Equilibrium Model (GSPEM) developed in Harker (1986-a, 1986-b) provided an explicit treatment of the shippers' and the carriers' behaviors. In this respect, GSPEM model is the most sophisticated SPE model.

GSPEM model in Harker (1986-a, 1986-b) described the shippers' behavior by using the modified Wardrop User Equilibrium condition presented at the beginning of this section. It described the carriers' behavior by using the Wardrop's System Optimal condition from Section 2.1.2. Similar to the shippers, the carriers were portrayed as economic agents with their own objective functions and a set of constraints, instead of being represented by a set of performance functions as in Bronzini (1978, 1981). Accordingly, in addition to being dependent on the shippers' service demands, the service attributes were influenced by carriers' decisions on the rates to be charged and routes offered. GSPEM model provided the simultaneous solution of the shippers' and carriers' problems by assuming the marginal cost pricing principle. Harker et al. (1985) used the model to analyze the U.S. coal industry.

The mathematical formulation (both extremal and nonextremal) of the general SPE model can be categorized into the quantity formulation (Nagurney, 1999), that uses the inverse supply and inverse demand functions; and the price formulation (Nagurney, 1999), that uses the supply and demand functions. Based on the assumption that the supply and demand functions are invertible, Florian (1982), Friesz (1984), Harker (1986-a, 1986-b) and Pang (1984) provided the quantity formulation of the general SPE model. For the non-invertible case, the reader is referred to Friesz (1983-a) and Nagurney (1999).

From above, it is apparent that various types of formulations can be used for the general SPE model. Accordingly, the solution algorithms are very diverse. For the extremal formulation, various algorithms in the linear and nonlinear programming theory such as the Frank-Wolfe algorithm (Sheffi, 1985) can be used. For the nonextremal formulation, various algorithms including the linearization algorithm, the relaxation algorithm, the projection algorithm, and various decomposition algorithms presented in Nagurney (1999) can be used. More specifically, the linearization algorithm has been used to solve the nonlinear complementarity problem. The decomposition algorithms are applied to the VI problem defined over a separable set or a Cartesian product of sets. Unlike the decomposition algorithms, the relaxation algorithm and the projection algorithm have broader applicability in that they can be applied to a problem defined over a nonseparable set as well. However, the decomposition algorithm is more suitable for the large scale VI problem since the algorithm decomposes the problem into a sequence of subproblems with much less dimensions and thus realizes efficiency of the algorithm. The decomposition algorithms are categorized into parallel or serial versions. These versions differ in that the serial version uses the information as soon as it becomes available. Depending on whether the subproblems in the iteration of the algorithm are linear or not, the decomposition algorithms are also categorized into the linear or the nonlinear decomposition algorithms in Nagurney (1999).

Florian (1982) proposed the linear approximation method (LAP) for the nonlinear programming formulation of a single commodity general SPE problem. He proposed the nonlinear parallel version decomposition method for the multicommodity general SPE problem with symmetric or separable functions, which was also formulated as a nonlinear programming problem. For the multicommodity general SPE problem with asymmetric and nonseparable functions (which was formulated as a VI problem) he proposed to use the Jacobian diagonalization method. As he mentioned but did not prove in the paper, the diagonalization method is the relaxation-type algorithm (Nagurney, 1999) that will converge if the Jacobian matrix is both row and column diagonal. Friesz (1983-a) proposed the successive linearization method, which was used to solve the nonlinear complementarity formulation of the general SPE problem and established the convergence condition for this method. Friesz (1984) compared the efficiency and accuracy of the successive linearization method shown in Friesz (1983-a) to the diagonalization method for the problems with various degrees of asymmetry. For the subproblems in the iteration of the diagonalization method, two versions of Frank-Wolfe algorithm were proposed depending on whether the network structure was employed. Through various numerical examples, he showed that the diagonalization method together with the version of Frank-Wolfe algorithm that employed the network structure is fast but not accurate. On the contrary, the diagonalization method together with the other version of Frank-Wolfe algorithm is accurate but slow. The successive linearization approach is both fast and accurate. Pang (1984) proposed the Gauss-Seidel-linearization method (i.e. the linearized serial version decomposition algorithm presented in Nagurney (1999)). This method decomposed the VI formulation of the multicommodity general SPE problem into a sequence of single-commodity SPE subproblems with the linear inverse supply, inverse demand and transportation cost functions, which were then solved by a special version of the parametric principal pivoting algorithm. Nagurney (1987) also suggested the use of Gauss-Seidel-type decomposition method to solve the VI formulation of the general SPE problem with alternative structures according to how Cartesian product of sets was defined. She compared the performance of the Gauss-Seidel method to the projection method and showed its superiority in terms of speed.

Based on the above discussion, Table 2.1, similar to that of Friesz (1983-b), was developed to categorize the models found in the literature and discussed above.

Table 2.1 Categorization of the models in the freight planning according to various criteria

	Criteria					
	Multiple commod-ities	General Multimodal transportat-ion network	Explicit treatment of shippers and carriers	Simultan-eous solution	Extremal formulation	Nonextremal formulation
Samuelson (1952)	No	No	No	No	Yes	No
Takayama (1964)	Yes	No	No	No	Yes	No
Bronzini (1978)	Yes	Yes	No	No	Yes	No
Friesz (1986)	Yes	Yes	Yes	No	Yes	No
Florian (1982), Friesz (1983-a, 1984), Pang (1984), and Nagurney (1999)	Yes	Yes	No	Yes	No	Yes
Harker et al. (1986-a, 1986-b)	Yes	Yes	Yes	Yes	No	Yes

2.2 Game Models

The previous section reviewed freight transportation planning models ranging from the most basic one (Samuelson, 1952) to the most comprehensive ones (Harker, 1986-a, 1986-b). However, even the most comprehensive model does not consider the complexity in the carrier-carrier or shipper-carrier interactions that arises from the oligopolistic behavior of the carriers in setting their service charges and routes. Some models in the game theory and the oligopoly theory can be applied to analyze these interactions. The basic concepts and applications of these game models are reviewed next.

2.2.1 Nash Equilibrium and Variational Inequality Formulation

In the oligopoly theory defined in Chapter 1, a Nash game has been frequently used to formulate the non-cooperative behavior of the oligopolistic players. In a Nash game, each player is conscious of its rivals' decisions; and, makes a decision in the belief that the rivals would maintain their current decisions. To illustrate this mathematically, assume there are m players. Each player i selects a strategy vector $x_i = \{x_{i1},\cdots,x_{in}\} \quad \forall x_i \in K_i$ to maximize its utility u_i denoted as a function of the strategy vector of all players: $u_i = u_i(x_1,\ldots,x_m)$ in the belief that any other player's decision $x_{j\neq i}$ would not change. Here, $K_i \subset R^n$ is a convex set. Based on the above notation, Nash equilibrium is a strategy vector $x^* = (x_1^*,\ldots,x_m^*) \in K$, such as that it yields:

"$u_i(x_i^*,\hat{x}_i^*) \geq u_i(x_i,\hat{x}_i^*) \qquad \forall x_i \in K_i, \forall i,$ *where* $\hat{x}_i^* = (x_1^*,\cdots,x_{i-1}^*,x_{i+1}^*,\cdots,x_m^*)$."[1]

Nash equilibrium defined above can be classified as either the Cournot or the Bertrand equilibrium, depending on whether the decision variable is quantity or price (Tirole, 1988).

Under the assumption that each u_i is continuously differentiable on K and concave in x_i, Nash equilibrium can be formulated as a VI problem (Nagurney, 1999).

> *"Theorem 6.1:(Variational Inequality Formulation of Nash Equilibrium)*
>
> *Under the previous assumptions,* x^* *is a Nash equilibrium if and only if* $x^* \in K$ *is a solution of the variational inequality* $\langle F(x^*), x - x^* \rangle \geq 0, \forall x \in K$ *where* $F(x) \equiv (-\nabla_{x1}u_1(x),\cdots,-\nabla_{xm}u_m(x))$ *is a row vector and where*

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[1] Nagurney, A. (1999). Oligopolistic Market Equilibrium. In Network Economics: A Variational Inequality Approach (pp. 211). Boston/Dordrecht/London: Kluwer Academic Publishers.

$$\nabla_{xi}u_i(x) = (\frac{\partial u_i(x_i)}{\partial x_{i1}},...,\frac{\partial u_i(x_i)}{\partial x_{in}}).\text{"}^2$$

This definition assumes that the strategy of each player is continuous. The oligopolies may be faced with discrete strategies as well: whether to collude or to compete. The "compensation principle" of Hicks and Kalder (Tirole, 1988) is used as a criterion to evaluate whether the collusion could be established and sustained. The principle states that the collusion should achieve better collective outcome than the sum of the outcomes achieved individually without any cooperation. This principle also requires that the winner can easily compensate the loser in the Nash game so that everyone is better off. The compensation principle is expressed as follows:

$$\sum_i u_i^n \le \sum_i u_i^c \quad \forall i: \text{Index for participant in the cooperation} \qquad (2.10)$$

where: u_i^c is the utility for the i_{th} participant under the collusive game and u_i^n is the utility for the i_{th} participant under the competitive game.

Weskamp (1985) and Dafermos (1987) formulated the production and distribution behavior of oligopolistic shippers in spatially separated markets as a Cournot game. These models are called the spatial oligopoly models, and the equilibrium is the spatial Cournot-Nash oligopolistic equilibrium. They employed the VI formulation and assumed an oversimplified transportation network. As an extension of Weskamp's paper, Dafermos established that the spatial price equilibrium model is an extreme and limiting case of the spatial oligopoly model. Proceeding from these works, Nagurney (1999) formulated the spatial oligopoly model using the general transportation network.

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[2] Nagurney, A. (1999). Oligopolistic Market Equilibrium. In Network Economics: A Variational Inequality Approach (pp. 212). Boston/Dordrecht/London: Kluwer Academic Publishers.

The oligopolistic competitive behavior of the supply-side of the transportation service and facility is rarely analyzed. Yang and Woo (2000-b) was the pioneer to approach this problem to study the interactions of two oligopolistic private toll collectors, each in charge of a single link on a general transportation network. The collectors' toll pricing behaviors are formulated as a Nash game. The authors also formulated the interaction between the two toll collectors and the road users, which fits well into the framework of a Stackelberg game.

2.2.2 Stackelberg Equilibrium, Bi-level Programming and Sensitivity Analysis Method

The Stackelberg game, also called a-leader-and-a-follower game, is characterized by the sequential nature of the decisions made by players, (i.e., the game has a hierarchical structure). In the game, the leader has the authority to make a first move based on its forecast of the follower's rational reaction. The follower makes its decision in the knowledge of the leader's decision. Both the leader and the follower aim to maximize their own utility. To illustrate this concept mathematically, the notation from the pervious section is used. The leader is designated by the subscript *l* and the follower, by *f*. The best strategy vector of the follower ($x_f^* = \{x_{f1}^*, \cdots, x_{fn}^*\} \quad \forall x_f \in K_f$) is associated with the strategy vector of the leader ($x_l = \{x_{l1}, \cdots, x_{ln}\} \quad \forall x_l \in K_l$) by a vector of the reaction functions: $x_f^* = R(x_l)$. The Stackelberg equilibrium is then defined as follows:

Definition: Stackelberg equilibrium is a strategy vector $x^* = (x_l^*, x_f^*) \in K$, such that $u_l(x_l^*, R(x_l^*)) \geq u_l(x_l, R(x_l)) \;\; \forall x_l \in K_l$ with $R(x_l^*) = x_f^*$ and $u_f(x_l^*, x_f^*) \geq u_f(x_l^*, x_f)$ $\forall x_f \in K_f$. (Tirole, 1988)

The above concept can be easily extended to define the equilibrium for the Stackelberg game between multiple leaders and followers.

Mathematically, the bi-level programming problem is used to present the static version of the Stackelberg game. Stackelberg game arises in various situations of the transportation industry. Accordingly, the bi-level programming technique has wide application in the analysis of the transportation problem. Typical examples include: the spatial Stackelberg-Nash-Cournot competitive network equilibrium problem (Miller, 1991), the toll pricing problem (Yang and Lam, 1996; Yang and Bell, 1997-a), the signal optimization problem (Yang and Yagar, 1995), the network design problem (Friesz et al., 1992), the facility location problem (Miller, 1995; Taniguchi et al., 1999), and Stakelberg equilibrium problem in the freight system (Fisk, 1983).

Different solution methodologies have been proposed to solve the bi-level programming problem as reviewed by Yang and Bell (2001), such as the gap penalty function approach (Meng and Yang 2001); the sensitivity analysis method based heuristic algorithm (Yang et al., 1995, 1996, 1997-a, 2000-a, 2000-b); and the simulated annealing algorithm (Friesz et al., 1992). Since the sensitivity analysis method is going to be used in this dissertation, only the papers expounding on this method and the studies using this method are reviewed.

In the literature, abundant studies have been performed on the methodology of the sensitivity analysis for the nonlinear optimization problem or the VI problem. The sensitivity analysis for the nonlinear optimization problem or VI problem is to compute the derivatives of primal variables and dual variables with respect to perturbation parameters. The general framework and rigorous approach for the sensitivity analysis of the network equilibrium problem formulated as the VI problem were provided in Tobin (1988) and Yang (1997-b). For the general SPE problem, Dafermos (1984) derived the direction of the change in the price and shipment pattern resulting from the change in the inverse supply, inverse demand or transportation cost functions. Chao et al. (1984) and Tobin (1987) provided the quantity sensitivity analysis method for the SPE problem with perturbation in the parameters of the functions. Nagurney (1999) gave the sensitivity analysis method for the parameter perturbation, which influences both the feasible set and the functions. The sensitivity analysis method by Chao (1984) was applied to the extremal formulation of the general SPE problem with separable nonlinear supply, demand and transportation cost functions. The method involves the inversion of a matrix with the rank being the number of regions, instead of the number of arcs. The number of arcs is usually much larger than the number of regions. One of the assumptions underling this method is that there is positive supply and demand at every node on the network. Tobin (1987) advanced the work by Chao (1984) by developing the sensitivity analysis methods for the nonextremal formulations (both VI and complementarity formulations) of the general SPE problem. He reduced the size of the system of equations to be solved in the sensitivity analysis by taking out the equations associated with the nonnegativity constraints. Unlike Chao (1984), the method in Tobin (1987) is not restricted to the SPE problem with separable functions and the assumption of this method requires only one of the supply and demand at a node to be positive.

The sensitivity analysis method has been frequently applied to construct a heuristic algorithm to solve the bi-level programming problem. For instance, Yang (1996, 1997-a) employed the sensitivity analysis method for the analysis of optimal pricing on a transportation network. Yang (2000-a) demonstrated the use of this method to determine the optimal toll and capacity improvement for privately operated roads on the network.

There are also examples in the literature, which explore this method to solve the competitive equilibrium between the oligopolies, among which there are a leader and a number of followers. For instance, Miller (1995) employed this method to solve the location, production and distribution problem of the Stackelberg leader firm subject to the spatial Cournot-Nash oligopolistic equilibrium or the spatial price equilibrium of its competitors on the freight network. In Miller's paper, the quantity decisions such as production and the distribution decisions of the shipper both the Stackelberg firm and its rivals are analyzed on a general transportation network while the role of the carriers in the distribution of the commodity between spatially separated markets is not considered. Yang and Woo (2000-b) combined this method with the quasi-Newton method to solve the pricing competition between two private road providers subject to the Wardrop's User Equilibrium (Sheffi, 1985) of the road users.

2.3 Features of the Study in this Dissertation

This dissertation aims to set up a framework to analyze the interaction between the oligopolistic carriers and the shippers as well as to answer the research questions formulated in the shipper level problem and the carrier level problem presented in Section 1.3. Based on the solution to these two problems, the questions associated with the Port Authority level investment problem are answered by calculating the economic impact of the alternative investment strategies and identifying the best strategy. To accomplish this, it is crucial to develop a model that can solve the shipper and the carrier level problems in a systematic way. The various aspects of the models reviewed in the previous sections, mainly the GSPEM model by Harker (1986-a, 1986-b) and the bi-level model by Yang and Woo (2000-b), are adopted and modified for this purpose.

The bi-level model in this dissertation is similar to the GSPEM model by Harker (1986-a, 1986-b) in two aspects. First, it treats the shippers and the carriers distinctively and integrates the shipper and the carrier level problems. Second, it formulates the shippers' behavior using the general SPE model. The general SPE model was chosen over the FNE model because it provides more generality and contains more economic interpretation by accounting for the economic factors in solving its trip generation and distribution problem.

The GSPEM model focuses on the shipper's and carrier's routing behaviors. It simplifies the pricing behavior of the carriers by assuming the marginal cost pricing principle. Proceeding from the work by Harker (1986-a, 1986-b), this dissertation considers the oligopolistic pricing and routing behavior of the carriers. The carriers no longer act simply as the price takers. Instead, they can use the pricing decision as a device to manipulate the demand in order to maximize their profits. In addition, the carriers can choose either to compete or collude in their pricing and routing decisions. Hence, the equilibrium of the carriers' pricing behavior follows the Nash equilibrium or the Compensation principle presented in Section 2.2. The other feature distinct from the GSPEM model is that the interaction between the carriers and the shippers is cast into Stackelberg game and formulated as a bi-level programming problem. The GSPEM model treats both the carriers and the shippers as price takers, with both reacting to the service charges in the market. No direct interaction between the carriers and the shippers are considered. In contrast, the model in this dissertation assumes that the carriers and the shippers react to each other's decision. A heuristic algorithm based on the sensitivity analysis method is developed, which solves the Stackelberg equilibrium in terms of the carriers' pricing and routing decisions and the shippers' commodity production, consumption and shipping decisions.

The bi-level model of this dissertation is similar to the bi-level model in Yang and Woo (2000-b). First, both models formulate the Stackelberg game between the supply-side (i.e. the leaders) and the demand-side (i.e. the followers) of the transportation service. Second, both models cast the non-cooperative behavior of the leaders in a Nash game and formulate it as a VI problem. Nevertheless, the model in this dissertation contrasts with Yang's model in three aspects. First, the behavior of the shippers, which are the followers, are stated using the general spatial price equilibrium condition instead of the Wardrop User Equilibrium condition that was used in Yang and Woo's study (2000-b). Instead of using the nonlinear programming formulation as shown in Yang's model, the VI formulation shown in Florian (1982), Friesz (1984) or Nagurney (1999) is used in this dissertation for the follower level problem that is the general SPE problem. The VI formulation provides more flexibility and generality in dealing with the case when the inverse supply, inverse demand or cost functions are nonseparable and asymmetric. Second, each carrier, which is the leader of the Stackelberg game in this dissertation, is in charge of a carrier's sub-network. The carrier's subnetwork is distinct from the shipper network. On a carrier's sub-network, both the pricing and routing problems are entailed. On the contrary, each toll collector in Yang and Woo's model (2000-b) operates only a single link in the transportation network of the road user. The toll collector has only one decision variable that is the toll on this single link. This difference gives the model in this dissertation more complexity. Third, various commodities characterized by distinct value-of-time are considered in this dissertation. On the shipper side, the interaction between various commodities in the commodity supply, demand, and transportation cost functions is accounted for in the general SPE model. On the carrier side, the model provides a solution to the problem concerning how to excise the price discrimination according to the commodity type in order to maximize profit.

In summary, the dissertation contributes to the body of transportation planning in that it provides a methodology to formulate the carrier-carrier and carrier-shipper interactions resulted from the carriers' oligopolistic behavior. In detail, a bi-level freight planning model formulating the oligopolistic pricing and routing problem of two or more carriers subject to the shippers' SPE problem is developed. A sensitivity analysis-based heuristic algorithm is proposed to solve this model. This model and algorithm are applied to solve the Stackelberg equilibrium between two terminal operators and shippers as well as to facilitate the investment decision of a Port Authority in a numerical example.

CHAPTER 3

DESCRIPTION OF THE NETWORK

This chapter presents a conceptual network structure of the multimodal freight transportation system and describes it from the perspective of different players, the shippers and the carriers. The organization of the chapter is as follows. Section 3.1 describes the roles of the shippers and the carriers in the freight network system, based on which the network will be decomposed to the shipper network and the carriers' network. Section 3.2 describes the structure of the shipper network, the carriers' network, and a sample carrier's sub-network graphically as three layers. Finally, Section 3.3 describes the attributes of the elements of each network layer and defines them mathematically.

3.1 Distinctive Roles of the Shippers and the Carriers

The shippers and the carriers have distinctive roles in the multimodal freight transportation system. These roles were discussed in detail in Section 1.2.3 and are briefly summarized here.

Each shipper is concerned with the choice of origin and destination markets for commodities, the choice of a carrier or a sequence of carriers (i.e. the choice of a transportation mode or a combination of modes) that move the goods between markets. It usually has no control over the detailed routing pattern within the system of an individual carrier. Corresponding to its role, the shipper perceives the freight system as an aggregated network. The basic components of this network include the centroid nodes that represent the origin and destination markets, the intermediate nodes that represent transshipment locations, and the links connecting these nodes.

In contrast to the shipper, the carrier's role entails the routing of freight on a detailed modal network. Each carrier conducts various transportation related service processes moving the freight over the physical transportation facilities between the point where it takes control of the goods and the point where goods leave this system. Corresponding to the carrier's role, the links on a carrier's sub-network represent the rail lines, the highway segments, the ocean lines, or various operations such as loading and unloading. The intermediate nodes on a carrier's sub-network represent the change of directions, the switching points, the stops, the change of facility class or the change of the service process. The origin and destination nodes on a carrier's sub-network represent the locations where the carriers take or release control of the goods.

The shippers and the carriers interact with each other. This is reflected in the network presentation, where a link on the shipper network corresponds to an origin-destination (O-D) pair on a carrier's sub-network. The incidence index between a carrier's O-D pair and a shipper's link is denoted as $\xi_{l,v}$. If link l on the shipper perceived network corresponds to O-D pair v on a carrier's sub-network, $\xi_{l,v}=1$. Otherwise, $\xi_{l,v}=0$.

3.2 Network Structure of the Multimodal Freight System

Figure 3.1 presents the multimodal freight system in a network structure with three layers: *Layer a* is a detailed multimodal freight carriers' network; *Layer b* is a sample carrier's sub-network: the terminal sub-network; *Layer c* is the shipper-perceived network. Through out this presentation, the relationship between the network layers is illustrated.

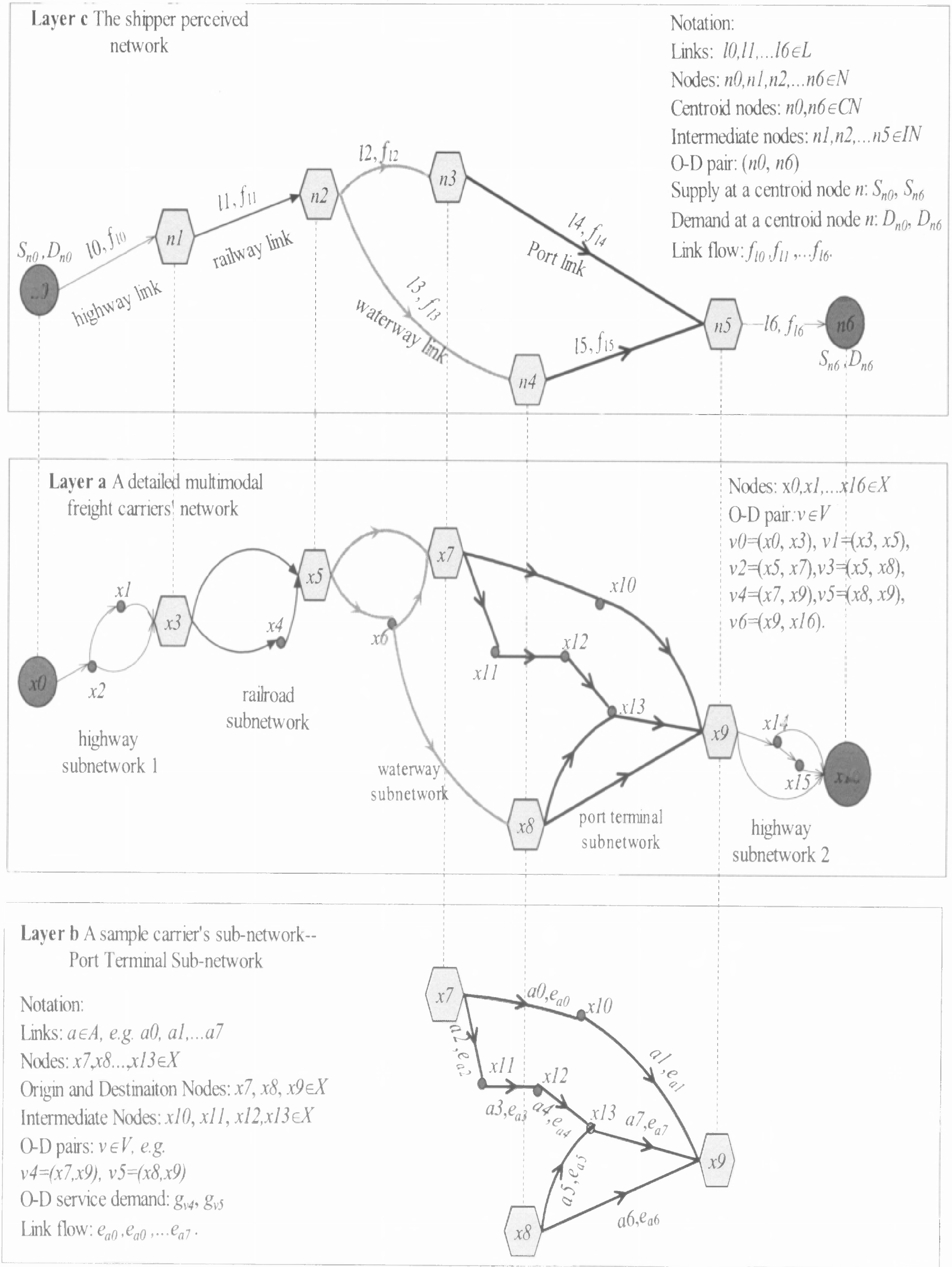

Figure 3.1 Schematic Representation of the Shipper Perceived Network, a Detailed Multimodal Freight Carriers' Network and a Sample Carrier's Sub-network

3.2.1 The Detailed Multimodal Freight Carriers' Network

The detailed multimodal freight carriers' network (or the carriers' network) shown in *Layer a* of Figure 3.1 is a union of five individual carriers' sub-networks, including two highway sub-networks, one railway sub-network, one waterway sub-network, and one port terminal sub-network. From this layer, it can be observed that the carriers' network satisfies two properties:

1. Different carriers' sub-networks are separable; and
2. Each carrier operates a specific transportation mode or a port terminal.

Nodes *x0* and *x16* in *Layer a* represent the commodity origin and destination markets respectively. Nodes *x3*, *x5*, *x7*, *x8*, and *x9* in *Layer a* represent the transshipment points. These nodes constitute the origin and destination nodes on each carrier's sub-network. Table 3.1 below shows all the possible O-D pairs on the carriers' network.

Table 3.1 Origin-Destination Pairs on the Carriers' Network

the individual carrier's sub-network	Carrier's O-D pair $v=(xi, xj)$ $\forall v \in V, xi, xj \in X$
Highway sub-network 1	$v0=(x0, x3)$
Railway sub-network	$v1=(x3, x5)$
Waterway sub-network	$v2=(x5, x7)$ and $v3=(x5, x8)$
Port terminal sub-network	$v4=(x7, x9)$ and $v5=(x8, x9)$
Highway sub-network 2	$v6=(x9, x16)$

3.2.2 Port Terminal Sub-network

Layer b in Figure 3.1 shows a sample carrier's sub-network decomposed from the carriers' network in *Layer a,* the port terminal sub-network. The port terminal operator is the carrier that will be analyzed in detail in the case study of this dissertation. Hence, more details concerning the port terminal sub-network will be discussed in the case study.

From a modeling perspective, however, the description of the carrier model is general, and any type of carrier may be considered in the analysis.

On a port terminal sub-network, a link presents a service process at some terminal facility. This section discusses the service process at a port terminal and provides an interpretation of the terminal sub-network shown in *Layer b*. The various complicated service processes at a port terminal can be grouped into four major systems: the marine side interface, the transfer system, the storage system, and the landside interface (Holguín-Veras, 1998). The different combinations of the processes belonging to the four systems may be used to move the cargo between an O-D pair on a terminal sub-network. These combinations can be represented as different paths between this O-D pair. For example, some cargo may be loaded into the railroad cars or trucks directly at the dock, hence skipping the storage system. Some other cargo may require service at each of the four systems (e.g., the cargo that is stored and processed at the terminal's warehouse before further distribution). Each system is represented by several parallel or sequential links. The links parallel to each other represent the alternatives to conduct the service process. For example, the process of lifting cargo can be conducted with mobile cranes, top loaders or straddle carriers. The links sequential to each other represent the sequential service processes.

The port terminal sub-network in *Layer b* can be interpreted based on the above discussion. For example, links *a2, a3, a4, a7,* connecting terminal O-D pair (*x7, x9*) may represent the service processes belonging to each of the four systems mentioned above. On the other hand, link *a6* can be interpreted as an operation process, which transfers container or cargo directly to the truck on the dock. Links *a0* and *a2* may be interpreted as service processes in the system of marine side interface, which may represent unloading goods on two alternative modes or processes. The sequential links *a3* and *a4* may represent the two service processes involved in moving goods from the apron to the storage system.

3.2.3 The Shipper Perceived Network

The shipper perceived network (or the shipper network) shown in *Layer c* of Figure 3.1 is aggregated from the carriers' network shown in *Layer a.* The aggregation procedure is demonstrated in this section. For example, assume that a shipper intends to ship a commodity from origin market *x0* to destination market *x16* shown in *Layer a.* The shipper may choose the following five steps to accomplish this goal. Each step is presented as a link on the shipper network in *Layer c.*

Step 1 A truck company ships goods from the origin market *x0* to a truck-rail transfer point *x3* on the highway sub-network 1 shown in *Layer a.* This step is presented as highway link *l0* connecting centroid node *n0* and intermediate node *n1* on the shipper network in *Layer c.*

Step 2 A rail company ships goods from *x3* to rail-water transfer point *x5* on the railway sub-network shown in *Layer a.* This step is presented as railway link *l1* connecting intermediate nodes *n1* and *n2* on the shipper network in *Layer c.*

Step 3 An ocean carrier ships goods from *x5* to an entry point *x8* to a port terminal on the waterway sub-network shown in *Layer a.* This step is presented as waterway link *l3* connecting intermediate nodes *n2* and *n4* on the shipper network in *Layer c.*

Step 4 A port terminal operator moves goods from entry node *x8* to exit node *x9* on the port terminal sub-network shown in *Layer a.* The goods are transferred to a truck company at node *x9.* This step is presented as a port link *l5* connecting intermediate nodes *n4* and *n5* on the shipper network in *Layer c.*

Step 5 A truck company ships goods from node *x9* to the destination market *x16* on the highway sub-network shown in *Layer a.* This step is presented as a highway link *l6* connecting intermediate node *n5* and centroid node *n6* on the shipper network in *Layer c.*

By connecting the above steps, a shipping path connecting markets (*x0, x16*) can be constructed on the shipper network *shown in Layer c,* which is *l0, l1, l3, l5, l6* or *n0→n1→n2→n4→n5→n6*. Similarly, the other path can be constructed in *Layer c,* which is Path 2: *n0→n1→n2→n3→n5→n6* or *l0, l1, l2, l4, l6.*

The above aggregation procedure indicates that: the centroid nodes (*n0*, *n6*) and the intermediate nodes (*n1*, *n2*, *n3*, *n4*, *n5*) in *Layer c* correspond to the origin and destination markets (*x0*, *x16*) and the transshipment points (*x3*, *x5*, *x7*, *x8*, *x9*) in *Layer a.* From this correspondence, the incidence matrix between the shipper links in *Layer c* and the carrier O-D pairs in *Layer a* can also be derived as shown in Table 3.2:

Table 3.2 The Incidence Matrix between the Carrier O-D pairs and the Shipper Links

$\zeta_{l,v=(xi,xj)}$	*l0*	*L1*	*l2*	*l3*	*l4*	*l5*	*l6*
v0=(x0, x3)	1	0	0	0	0	0	0
v1=(x3, x5)	0	1	0	0	0	0	0
v2=(x5, x7)	0	0	1	0	0	0	0
v3=(x5, x8)	0	0	0	1	0	0	0
v4=(x7, x9)	0	0	0	0	1	0	0
v5=(x8, x9)	0	0	0	0	0	1	0
v6=(x9, x16)	0	0	0	0	0	0	1

3.3 Attributes of the Network Elements

The attributes of the nodes and links on the network influence how the carriers and the shippers carry out their roles in the freight transportation system. This section will briefly describe these attributes and present their mathematical formulation.

3.3.1 Attributes on the Carriers' Network

As mentioned in Section 1.2.2, attributes on the carriers' network include operating cost, which is a function of the service demand and capacity. Eqs. (3.1) and (3.2) below give the mathematical form of the average operating cost function and the marginal operating cost function on the carriers' network.

$$AC_{a,c}(e_{a,c})= r_{a,c} + r'_{a,c} * \frac{e_{a,c}}{\overline{E_a}} + r''_{a,c} * (\frac{e_{a,c}}{\overline{E_a}})^2 \qquad \forall a \in A, c \in C \qquad (3.1)$$

$$MC_{a,c}(e_{a,c})= r_{a,c} + 2r'_{a,c} * \frac{e_{a,c}}{\overline{E_a}} + 3r''_{a,c} * (\frac{e_{a,c}}{\overline{E_a}})^2 \qquad \forall a \in A, c \in C \qquad (3.2)$$

In the above equations, $\overline{E_a}$, $\forall a \in A$ denotes the link capacity on the carriers' network, which is defined as the maximum flow of freight that the service process or facility presented by link *a* can efficiently handle in a unit of time. If the flow of freight exceeds the capacity, the level of service such as service time on this carrier link will deteriorate with the formation of a queue and congestion.

Attributes regarding each O-D pair on the carriers' network include the service demand ($g_{v,c}$), which is assumed to be a function of the vector of service fares on all carrier O-D pairs: $g_{v,c}(R_V)$. The exact form of service demand function will be discussed in a later chapter of this dissertation.

3.3.2 Attributes on the Shipper Network

As mentioned before in Section 1.2.1 and 1.2.2, the attributes associated with the shipper network include the commodity supply and demand functions at a centroid, the service fare and the travel time function on a link, and value-of-time. The mathematical form of these attributes will be defined here.

For each commodity at any market, the general form of the commodity supply and demand function is used, which assumes that the supply and demand of this commodity depend on the vector of the market prices of all commodities at this market. Equations (3.3-a), (3.3-b) present the supply and demand functions of commodity c at centroid b used in this dissertation.

$$S_{b,c}(\pi_b) = \sigma 0_{b,c} + \sum_{c' \in C} \sigma 1_{b,c',c} * \pi_{b,c'} \qquad \forall b \in CN, c \in C \qquad (3.3\text{-a})$$

$$D_{b,c}(\pi_b) = \varsigma 0_{b,c} - \sum_{c' \in C} \varsigma 1_{b,c',c} * \pi_{b,c'} \qquad \forall b \in CN, c \in C \qquad (3.3\text{-b})$$

The supply and demand functions are assumed to be invertible with the inverse supply and demand functions of commodity c at centroid b defined in Eqs. (3.4-a) and (3.4-b) below:

$$\pi_{b,c}(S_b) = \gamma_{b,c} + \sum_{c' \in C} \lambda_{b,c',c} * S_{b,c'} \qquad \forall b \in CN, c \in C \qquad (3.4\text{-a})$$

$$\rho_{b,c}(D_b) = \alpha_{b,c} - \sum_{c' \in C} \beta_{b,c',c} * D_{b,c'} \qquad \forall b \in CN, c \in C \qquad (3.4\text{-b})$$

The transit time on link l for flow of commodity c ($t_{l,c}$) is assumed to be a function of the vector of flows on link l as shown in Eq. (3.5). The service fare on a shipper link is derived from the service fare on the corresponding carrier's O-D pair ($R_{v,c}$) according to

Eq. (3.6). The generalized cost ($GC_{l,c}$) shown in Eq. (3.7) is the linear combination of these two attributes.

$$t_{l,c} = tt_l * (1 + \tau 0_{l,c} f_{l,c} + \tau 1_{l,c} * \left(\frac{\sum_{c' \in C} (ro_{l,c',c} * f_{l,c'})}{Cap_l} \right)^{\tau} \qquad \forall l \in L, c \in C \qquad (3.5)$$

$$R_{l,c} = \sum_{v \in V} \xi_{l,v} * R_{v,c} \qquad \forall l \in L, c \in C \qquad (3.6)$$

$$GC_{l,c} = R_{l,c} + vot_c * t_{l,c} \qquad \forall l \in L, c \in C \qquad (3.7)$$

Since a link on the shipper network is actually a path, or a sequence of links between an O-D pair on the carriers' network, the link capacity as perceived by a shipper needs to be derived based on the capacity of links in the carrier's network, as shown in P3.1. The reader is referred to Morlok et al. (1999) for related work in estimating system capacity. Alternatively, the capacity of the shipper link l is generated by the capacity of links on the carriers' network. It represents the maximum volume of the freight that can be handled in a unit of time, without exceeding the capacity of any link on the carriers' network. It is obtained by solving the following mathematical program:

Table 3.3 Derivation of the Link Capacity on the Shipper Network

P3.1

$$\underset{g_{v,c}}{Max}\, Cap_l = \sum_{v \in V} \xi_{l,v} * (\sum_{c \in C} g_{v,c}) \qquad \forall l \in L, c \in C \qquad (3.8)$$

s.t.

$$\sum_{a \in A, c \in C} e_{a,c} * \chi_{a,x} = g_{v,c} * \tau_{v,x} \qquad \forall x \in X \qquad (3.9)$$

$$e_a \leq \overline{E_a} \qquad \forall a \in A \qquad (3.10)$$

Equation (3.8) calculates the capacity of link l from the service demand on the carriers' network. This service demand is based on the incidence matrix between the shipper links and the carrier O-D pairs. Eq. (3.9) is the flow conservation constraint, which specifies that at an origin (or destination) node on a carrier's sub-network, the difference between the total inflow and the total outflow equals the total demand entering (or leaving) this node. At a intermediate node on a carrier's sub-network, the total inflow equals to the total outflow. Equation (3.10) is the capacity constraint on each carrier's link.

Problem P3.1 assumes that the shipper can estimate the perceived capacity on a shipper link accurately and it is equivalent to the capacity on the corresponding O-D pair on a carrier's sub-network. The capacity for a carrier O-D pair is defined as the maximum flow of freight that the carrier's sub-network can handle without exceeding the capacity on each link in this sub-network.

CHAPTER 4

PRICING AND ROUTING PROBLEM OF OLIGOPOLISTIC CARRIERS WITH EXPLICITLY DEFINED DEMAND FUNCTION

This chapter presents the pricing and routing decisions of two or more oligopolistic profit-maximizing carriers that face a set of commodity-specific demand functions explicitly defined for the service provided between each carrier O-D pair. The chapter is organized as follows. Section 4.1 states the assumptions of the analysis. The equilibrium conditions under both the competitive game and the cooperative (collusive) game are presented in Section 4.2. The objective function and the constraints of the carriers are introduced in Section 4.3. Section 4.4 presents the mathematical formulation. Section 4.5 establishes the existence and uniqueness of the equilibrium solution. Section 4.6 proposes a solution algorithm.

4.1 Assumptions

Each carrier makes its pricing and routing decisions based on its knowledge of the operating cost function and its forecast of the service demand function between the carrier O-D pairs under its control. The assumptions are:

A1: The average operating cost $AC_{a,c}(e_{a,c})$ for any $a \in A$ and $c \in C$ shown in Section 3.3.1 is a separable function of the flow for each commodity type. Both the $AC_{a,c}$ and $MC_{a,c}$ are continuous and strictly monotone increasing in the link flow $e_{a,c}$.

$$\frac{\partial AC_{a,c}}{\partial e_{a,c}} = r'_{a,c} * \frac{1}{\overline{E}_a} + 2r''_{a,c} * \frac{e_{a,c}}{(\overline{E}_a)^2} > 0 \tag{4.1}$$

$$\frac{\partial MC_{a,c}}{\partial e_{a,c}} = 2r'_{a,c} * \frac{1}{\overline{E}_a} + 6r''_{a,c} * \frac{e_{a,c}}{(\overline{E}_a)^2} > 0 \tag{4.2}$$

A2: The service demand function $g_{v,c} = g_{v,c}(R_V)$ for any $v \in V$ and $c \in C$ is linear and continuous. The vector of service demand functions is strictly monotone decreasing in $R_V = (\cdots, R_{v,c}, \cdots)_{|V|*|C|} \cdot [\nabla_{R_V} g_V(R_V)]$. The Jacobian matrix of the vector of demand functions $g_V(R_V) = (\cdots, g_{v,c}(R_V), \cdots)$ satisfies:

$$\frac{\partial g_{v,c}}{\partial R_{v,c}} < 0 \text{ and } \left|\frac{\partial g_{v,c}}{\partial R_{v,c}}\right| > \max\left(\sum_{v' \neq v \text{ or } c' \neq c} \left|\frac{\partial g_{v',c'}}{\partial R_{v,c}}\right|, \sum_{v' \neq v \text{ or } c' \neq c} \left|\frac{\partial g_{v,c}}{\partial R_{v',c'}}\right| \right). \tag{4.3}$$

A3: The carrier is a profit maximizer. Its profit is calculated as the total revenue obtained from providing service to the flow of all commodity types at all carrier O-D pairs minus the total operating cost on all carrier links in the carrier's sub-network under its control.

A4: The carriers can either compete or cooperate in setting the price for service and service parameters such as routing or link flow pattern.

4.2 Equilibrium Conditions

The Nash equilibrium condition and the Compensation Principle mentioned before in Section 2.3 are adopted to evaluate the equilibrium solutions to the competitive game and the collusive game among carriers respectively. Denoting the profit of carrier t's decision as $Z_t(g_t(R_t, R_{-t}), R_t, e_t)$, the equilibrium conditions are defined below:

E1: Nash Equilibrium

$$Z_t(g_t(R_t^*, R_{-t}^*), R_t^*, e_t^*) \geq Z_t(g_t(R_t, R_{-t}^*), R_t, e_t). \quad \forall t \in T \tag{4.4}$$

Condition **E1** represents a price and routing pattern $(R_V^*, e^*) = \left((R_t^*, e_t^*), (R_{-t}^*, e_{-t}^*)\right)$ at which no carrier can be better off by unilaterally changing its price and routing pattern (e.g. from (R_t^*, e_t^*) to any other feasible price and routing pattern (R_t, e_t)).

E2: Compensation Principle

$$\underset{R,e}{Max} \sum_{t \in T} Z_t(g_t(R_t, R_{-t}), R_t, e_t) \geq \sum_{t \in T} \underset{R_t, e_t}{Max}\ Z_t(g_t(R_t, R_{-t}), R_t, e_t) \quad (4.5)$$

Condition **E2** states that at equilibrium, the combined value of all carriers' profits under the collusive pricing must be greater than the sum of all individual carriers' profits under the competitive pricing.

4.3 The Carrier's Objective Functions and Feasible Region

This section defines the objective functions and the feasible region. Then, their respective properties are stated.

4.3.1 Objective Functions

In the competitive game, the objective of each carrier is to maximize its individual profit. The carrier does this by assuming that the other carriers will keep their current level of service charges ($\overline{R_{-t}}$) constant. The profit Z_t of carrier t is calculated as follows:

$$\begin{aligned} & Z_t(g_t(R_t, \overline{R}_{-t}), R_t, e_t) \\ & = \sum_{v \in V^t} \sum_{c \in C} g_{v,c}(R_t, \overline{R}_{-t}) * R_{v,c} - \sum_{a \in A^t} \sum_{c \in C} AC_{a,c}(e_{a,c}) * e_{a,c} \quad \forall t \in T \end{aligned} \quad (4.6)$$

The first part of Eq. (4.6) is the revenue for carrier t. This revenue depends on the carrier's own service charges R_t as well as the service charges of other carriers $\overline{R}_{-t}$.

In the cooperative (or collusive) game, the objective of the carriers is to maximize the total profit for all colluded carriers. The objective function Z for this game, given by Eq. (4.7), is the sum of all individual carriers' profits from Eq. (4.6).

$$\begin{aligned} Z &= \sum_{t\in T} Z_t(g_t(R_t, R_{-t}), R_t, e_t) \\ &= \sum_{t\in T}\Big(\sum_{v\in V^t}\sum_{c\in C} g_{v,c}(R_t, R_{-t}) * R_{v,c} - \sum_{a\in A^t}\sum_{c\in C} AC_{a,c}(e_{a,c}) * e_{a,c}\Big) \end{aligned} \tag{4.7}$$

In this game, as reflected by the notation in Eq. (4.7), the service charges at all carrier O-D pairs are decision variables.

4.3.2 Properties of the Objective Functions

According to assumptions **A1** and **A2**, both the demand function $g_{v,c}(R_V)$ and the average operating cost function $AC_{a,c}(e_{a,c})$ are continuous. Since the sum or the product of various continuous functions is also a continuous function, the objective functions shown in Eq. (4.6) and (4.7) are also continuous.

The first derivatives of the objective function with respect to $R_{v,c}$ and $e_{a,c}$, shown in Eqs. (4.8) and (4.9) respectively, are also continuous.

$$\frac{\partial Z_t}{\partial R_{v,c}} = g_{v,c}(R_t, \overline{R_{-t}}) + \sum_{v'\in V^t}\sum_{c'\in C} \frac{\partial g_{v',c'}(R_t, \overline{R_{-t}})}{\partial R_{v,c}} * R_{v',c'} \quad \forall t \in T, v \in V,\ c \in C \tag{4.8}$$

$$\frac{\partial Z_t}{\partial e_{a,c}} = MC_{a,c}(e_{a,c}) \qquad \forall t \in T, a \in A,\ c \in C \tag{4.9}$$

According to assumption **A2,** the demand function $g_{v',c'}(R_t, \overline{R_{-t}})$ is linear, hence the multiplier before $R_{v',c'}$ in the second element of Eq. (4.8) -- $\frac{\partial g_{v',c'}(R_t, \overline{R_{-t}})}{\partial R_{v,c}}$, is constant. Therefore, $\frac{\partial Z_t}{\partial R_{v,c}}$ in Eq. (4.8) is also a linear and continuous function.

Assumption **A1** states that the marginal operating cost function $MC_{a,c}(e_{a,c})$ is continuous. Therefore, $\frac{\partial Z_t}{\partial e_{a,c}}$ in Eq. (4.9) is also a continuous function.

Since both $\frac{\partial Z_t}{\partial R_{v,c}}$ and $\frac{\partial Z_t}{\partial e_{a,c}}$ are continuous functions, the vector of first derivatives of the objective function $\nabla Z_t(g_t(R_t, \overline{R}_{-t}), R_t, e_t)$ defined in Eq. (4.10) below is also continuous.

$$
\begin{aligned}
&\nabla Z_t(g(R_t, \overline{R}_{-t}), R_t, e_t) = \\
&= \left(\nabla_{R_t} Z_t(g(R_t, \overline{R}_{-t}), R_t, e_t), \nabla_{e_t} Z_t(g(R_t, \overline{R}_{-t}), R_t, e_t)\right) \qquad (4.10)\\
&= \left((\cdots, \frac{\partial Z_t}{\partial R_{v \in V^t, c \in C}}, \cdots), (\cdots, \frac{\partial Z_t}{\partial e_{a \in A^t, c \in C}}, \cdots) \right) \qquad \forall t \in T
\end{aligned}
$$

The objective functions Eqs. (4.6) and (4.7) are strictly concave in (R_V, e). This is shown in Appendix A.

4.3.3 Feasible Region

The profit maximization behavior of the carriers in either the competitive game or the collusive game is subject to the flow conservation constraints and the nonnegativity constraints in the carriers' network. The feasible region of this problem defined by these constraints is shown below:

$$KT = \{(R_V, e) | \sum_{a \in A} e_{a,c} * \chi_{a,x} = \sum_{v \in V} g_{v,c}(R_V) * \tau_{v,x} \quad \forall x \in X, c \in C \tag{4.11}$$

$$R_{v,c} \geq 0 \quad \forall v \in V, c \in C \tag{4.12}$$

$$e_{a,c} \geq 0 \quad \forall a \in A, c \in C \}. \tag{4.13}$$

4.3.4 Properties of the Feasible Region

According to assumption **A2,** $g_{v,c}(R_V)$ is linear. This implies that Eq. (4.11) is also linear. Therefore, the feasible region is polyhedral. The polyhedral set is defined as:

> *"A set S in E_n is called a polyhedral set if it is the intersection of a finite number of closed half-spaces; that is, S={x; $p_i^t x \leq \alpha_i$ for i=1,2...m}, where p_i is a nonzero vector and α_i is a scalar for each i."*[1]

Since a polyhedral set is closed and convex (Bazaraa, 1993), the feasible region of this problem *KT* is also closed and convex.

4.4 Mathematical Formulation

Based on the established properties of the objective function and the feasible region, the competitive game can be formulated as a Variational Inequality (VI) problem and the collusive pricing game can be formulated as a nonlinear optimization problem. The problem formulations are shown in Tables 4.1 and 4.2:

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[1] Bazaraa, M. S. et al, (1993). Convex Sets. In Nonlinear Programming Theory and Algorithms (pp. 55). New York: John Wiley & Sons, Inc.

Table 4.1 Variational Inequality Formulation for the Competitive Game

P4.1

Determine $(R_V^*, e^*) \in KT$, such that

$$\sum_{t \in T}(-\nabla_{R_t} Z_t(g_t(R_t^*, R_{-t}^*), R_t^*, e_t^*)(R_t - R_t^*) - \nabla_{e_t} Z_t(g_t(R_t^*, R_{-t}^*), R_t^*, e_t^*)(e_t - e_t^*)) \geq 0$$

$\forall\, (R_V, e) \in KT$ (4.14)

Table 4.2 Nonlinear Programming Formulation for the Collusive Game

P4.2

$$\underset{R_{v,c}, e_{a,c}}{Max}: Z = \sum_{t \in T} Z_t(g_t(R_t, R_{-t}), R_t, e_t)$$
$$= \sum_{t \in T}(\sum_{v \in V^t}\sum_{c \in C} g_{v,c}(R_t, R_{-t}) * R_{v,c} - \sum_{a \in A^t}\sum_{c \in C} AC_{a,c}(e_{a,c}) * e_{a,c}) \qquad (4.15)$$

s.t. $(R_V, e) \in KT$

4.5 Existence and Uniqueness of the Solution

The examination of the existence and uniqueness of the solution to problem **P4.1** in Table 4.1 is based on two theorems (Nagurney, 1999) that deal with the existence and uniqueness of the solution to a generic VI problem $F(x^*)(x\text{-}x^*)\geq 0$, $\forall x \in K$, where K is the feasible region for x. They are as follows:

> *"Let $B_R(0)$ denote a closed ball with radius R centered at 0 and let $K_R=K \cap B_R(0)$. K_R is then bounded. Let VI_R denote the variational inequality problem:*
>
> *Determine $x_R^* \in K_R$, such that*
>
> $$\left\langle F(x_R^*)^T, y - x_R^* \right\rangle \geq 0, \qquad \forall y \in K_R.$$
>
> *We now state:*

Theorem 1.5

VI(F,K) admits a solution if and only if there exists an R>0 and a solution of VI_R, x_R^*, *such that* $\|x_R^*\| < R$."

"Theorem 1.6 (Uniqueness under strict monotone)

Suppose that F(x) is strictly monotone on K, then the solution is unique if one exists."[2]

First, it will be shown that there is a solution to **P4.1**. This will be presented based on theorem 1.5.

Let M be a large positive constant and $KT_M = KT \cap \|(R_V, e)\| \le M$. Here, KT_M is compact and convex. **P4.3** shown in Table 4.3 is the restricted problem of **P4.1**.

Table 4.3 Restricted Problem of Problem **P4.1**

P4.3
Determines $(R_V^*, e^*) \in KT_M$, such that
$\sum_t (-\nabla_{R_t} Z_t(g_t(R_t^*, R_{-t}^*), R_t^*, e_t^*)(R_t - R_t^*) - \nabla_{e_t} Z_t(g_t(R_t^*, R_{-t}^*), R_t^*, e_t^*)(e_t - e_t^*)) \ge 0$
$\forall (R_V, e) \in KT_M$ (4.16)

Based on theorem 1.5, Problem **P4.1** admits a solution if and only if there exists a large constant M>0 and a solution (R_V^*, e^*) to problem **P4.3** such that $\|(R_V^*, e^*)\|$<M.

The existence of a solution to problem **P4.3** is guaranteed by the continuity of the function $\nabla Z_t(g(R_t, R_{-t}), R_t, e_t)$ and the compact convex property of the feasible region KT_M based on another theorem of Nagurney (1999).

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[2] Nagurney, A. (1999). Variational Inequality Theory. In Network Economics: A Variational Inequality Approach (pp. 15, pp. 18). Boston/Dordrecht/London: Kluwer Academic Publishers.

"Theorem 1.4 (Existence Under Compactness and Continuity

If K is a compact convex set and F(x) is continuous on K, then the variational inequality problem admits at least one solution x."*[3]

The solution to **P4.3** is established above. Next, a large constant M will be constructed, which will make the solution (R_V^*, e^*) to **P4.3** satisfy $\|(R_V^*, e^*)\|<M$.

First, the highest service charge $R_{v,c} = \max_{v' \in V, c' \in C} R_{v',c'}$ is shown to be bounded for any nonnegative $g_{v,c}$ below.

$$\begin{aligned} g_{v,c}(R_V) &= g_{v,c}(0) + \sum_{v',c'} \frac{\partial g_{v,c}}{\partial R_{v',c'}} * R_{v',c'} \\ &\leq g_{v,c}(0) + \frac{\partial g_{v,c}}{\partial R_{v,c}} * R_{v,c} + \sum_{v' \neq v \text{ or } c' \neq c} \left| \frac{\partial g_{v,c}}{\partial R_{v',c'}} \right| * R_{v,c} \end{aligned} \qquad \forall v \in V, c \in C \tag{4.17}$$

$$R_{v,c} = \max_{v',c'} R_{v',c'} \Rightarrow g_{v,c}(R_V) \leq g_{v,c}(0) + \left(\frac{\partial g_{v,c}}{\partial R_{v,c}} + \sum_{v' \neq v \text{ or } c' \neq c} \left| \frac{\partial g_{v,c}}{\partial R_{v',c'}} \right| \right) * R_{v,c} \tag{4.18}$$

$$g_{v,c}(R_V) \geq 0 \Rightarrow g_{v,c}(0) + \left(\frac{\partial g_{v,c}}{\partial R_{v,c}} + \sum_{v' \neq v \text{ or } c' \neq c} \left| \frac{\partial g_{v,c}}{\partial R_{v',c'}} \right| \right) * R_{v,c} \geq 0 \tag{4.19}$$

From assumption **A2**, $\left(\frac{\partial g_{v,c}}{\partial R_{v,c}} + \sum_{v' \neq v \text{ or } c' \neq c} \left| \frac{\partial g_{v,c}}{\partial R_{v',c'}} \right| \right) < 0$. This combined with Eq. (4.19) yields that:

$$\Rightarrow R_{v,c} \leq -\frac{g_{v,c}(0)}{\left(\frac{\partial g_{v,c}}{\partial R_{v,c}} + \sum_{v' \neq v \text{ or } c' \neq c} \left| \frac{\partial g_{v,c}}{\partial R_{v',c'}} \right| \right)} \tag{4.20}$$

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[3] Nagurney, A. (1999). Variational Inequality Theory. In Network Economics: A Variational Inequality Approach (pp. 14). Boston/Dordrecht/London: Kluwer Academic publishers.

Equation (4.20) indicates that there exists a constant $\overline{R}_{\max}$ such that $R_{v,c} < \overline{R}_{\max}$ $\forall g_{v,c} \geq 0, v \in V, c \in C$. Since $R_{v,c}$ is defined as the highest service charge, the bounded $R_{v,c}$ indicates that R_V is bounded. This, together with the continuity of the demand functions $g_{v,c}(R_V)$ $\forall v \in V, c \in C$, indicates that the demand $g_{v,c}$ $\forall v \in V, c \in C$ must also be bounded. With a bounded $g_{v,c}$, the link flow $e_{a,c}$ must also be bounded as specified by the flow conservation constraint in Eq. (4.11) (i.e., there exists a value $\overline{e}_{\max}$ such that $e_{a,c} < \overline{e}_{\max}$ $\forall a \in A, c \in C$). Thus, the large value M is constructed as $M=||(\ldots, \overline{R}_{\max}, \ldots)_{|V|*|C|}, (\ldots, \overline{e}_{\max}, \ldots)_{|A|*|C|}||$.

With M thus constructed, the solution (R_V^*, e^*) to problem **P4.3** satisfies $||(R_V^*, e^*)|| < M$. Based on theorem 1.5 (Nagurney, 1999), this implies that there exists a solution to problem **P4.1**. Based on theorem 1.6 (Nagurney, 1999), this solution to problem **P4.1** is unique, since Eq. (4.14) in **P4.1** was shown in Section 4.3.2 to be continuous and strictly monotone in (R_V, e).

For the collusive game, the nonlinear optimization problem **P4.2** has continuously differentiable and convex objective function and a closed and convex feasible region KT. With these properties, problem **P4.2** is equivalent to VI problem **P4.4** shown in Table 4.4 according to Nagurney (1999).

Table 4.4 Equivalent VI Problem of Problem **P4.2**

P4.4
Determines $(R_V^*, e^*) \in KT$, such that
$-\nabla Z\left(g_V(R_V^*), R_V^*, e^*\right)\begin{pmatrix}(R_V - R_V^*)\\(e - e^*)\end{pmatrix} \geq 0 \quad \forall\ (R_V, e) \in KT \qquad (4.21)$

The existence and uniqueness of solution to this problem **P4.4** and consequently problem **P4.2** can be similarly established as those shown above for problem **P4.1** since both the function $-\nabla Z\left(g_V(R_V^*), R_V^*, e^*\right)$ and feasible set KT for problem **P4.4** has the same properties as those shown in **P4.1**. The procedures of the examination are not repeated here.

4.6 Solution Algorithm

With the existence and uniqueness of the solution established, various algorithms found in the literature pertaining to variational inequalities and nonlinear programming can be used to solve the VI problem **P4.1** and the nonlinear optimization problem **P4.2**.

The relaxation algorithm reviewed in Section 2.1.3 is used to solve problem **P4.1**. Its advantage over the decomposition algorithm is that it is able to solve the problem with a non-separable feasible set. For problem **P4.1,** the feasible set KT is not separable due to the inclusion of a non-separable demand function $g_{v,c}(R_V)$ in the flow conservation constraint in Eq. (4.11). Therefore, the relaxation algorithm is chosen to solve the problem. The relaxation algorithm resolves problem **P4.1** as a sequence of VI subproblems, each of which will then be transformed into a nonlinear optimization problem of each individual carrier. More details regarding the application of the relaxation algorithm are described below.

First, the feasible region for the subproblem of each carrier $t \in T$ is defined by three constraint functions:

$$KT_t = \{(R_t, e_t) | \sum_{a \in A^t} e_{a,c} * \chi_{a,x} = \sum_{v \in V^t} g_{v,c}(R_t, \overline{R}_{-t}) * \tau_{v,x} \quad \forall x \in X, c \in C \tag{4.22}$$

$$R_{v,c} \geq 0 \quad \forall v \in V^t, c \in C \tag{4.23}$$

$$e_{a,c} \geq 0 \quad \forall a \in A^t, c \in C \}. \tag{4.24}$$

Equation (4.22) is the flow conservation constraint. In it, the vector of the service charges at the other carriers' sub-networks, denoted as $\overline{R}_{-t}$, is considered as constant. Similar to *KT*, KT_t is also a closed and convex set.

Second, the function for subproblem of carrier $t \in T$ is defined as follows:

$$\begin{aligned} & -\nabla Z_t\left(g_t(R_t, \overline{R}_{-t}), R_t, e_t\right) \\ & = \left(-\nabla_{R_t} Z_t\left(g_t(R_t, \overline{R}_{-t}), R_t, e_t\right), -\nabla_{e_t} Z_t\left(g_t(R_t, \overline{R}_{-t}), R_t, e_t\right)\right) \end{aligned} \tag{4.25}$$

Based on the above definition of the feasible set in Eqs. (4.22)-(4.24) and the function in Eq. (4.25) for the subproblem, VI subproblem of carrier $t \in T$ **P4.5** is defined below in Table 4.5.

Table 4.5 Variational Inequality Subproblem of Carrier $t \in T$

P4.5
Determine (R_t^* , e_t^*)$\in KT_t$, such that
$-\nabla Z_t\left(g_t(R_t,\overline{R}_{-t}),R_t,e_t\right)*\begin{pmatrix}(R_t - R_t^*)\\(e_t - e_t^*)\end{pmatrix} \geq 0 \qquad \forall(R_t, e_t)\in KT_t \qquad (4.26)$

In problem **P4.5** above, the Jacobian matrix of $-\nabla Z_t\left(g_t(R_t,\overline{R}_{-t}),R_t,e_t\right)$ is positive definite and symmetric as can be derived from the Section 4.3.2. Hence, problem **P4.5** is equivalent to a strictly convex nonlinear programming problem **P4.6** shown in Table 4.6.

Table 4.6 Nonlinear Programming Problem of Carrier $t \in T$

P4.6
$\underset{R_t,e_t}{Max} \; Z_t(g_t(R_t,\overline{R}_{-t}),R_t,e_t) \qquad (4.27)$
s.t. $(R_t, e_t)\in KT_t$.

Problem **P4.6** maximizes the individual carrier's profit in the belief that the vector of the service charges at the other carriers' sub-networks remains constant at $\overline{R}_{-t}$. The existence and uniqueness of a solution to problem **P4.6** can be similarly demonstrated using the logic from Section 4.5.

Based on the above discussion of the feasible region, the mathematical formulation for both the VI formulation in **P4.5** and the nonlinear programming formulation in **P4.6** for the subproblem of carrier t, the steps of the relaxation algorithm are as follows:

Step 0 Initialize the service charge and the link flow at each carrier's sub-network (R_V^0, e^0). Let i denote the order of the carrier. Let k denote the order of the iteration. Set i:=1, and k:=1. Let ϑ denote the convergence parameter that evaluates the relative improvement in results from two consecutive iterations. Set ϑ to a small value close to zero such as 10^{-6}.

Step 1 Denote ti as the carrier with its order in the set T being i. Solve problem **P4.6** for the i^{th} carrier with $\overline{R}_{-ti} = R_{-ti}^{k-1}$. Denote the solution as (R_{ti}^*, e_{ti}^*). Let $R_{ti}^k = R_{ti}^*$ and $e_{ti}^k = e_{ti}^*$. If $i<|T|$, solve the problem for the next carrier by setting i:=i+1. Otherwise, go to step 2.

Step 2 Calculate the relative improvements from R_V^{k-1} to R_V^k as: $\left(\sum_{v\in V}\sum_c \frac{\left|R_{v,c}^k - R_{v,c}^{k-1}\right|}{R_{v,c}^k + R_{v,c}^{k-1}}\right)$. For v and c with $R_{v,c}^k + R_{v,c}^{k-1}$=0, set $\frac{\left|R_{v,c}^k - R_{v,c}^{k-1}\right|}{R_{v,c}^k + R_{v,c}^{k-1}}$=0. If $\left(\sum_{v\in V}\sum_c \frac{\left|R_{v,c}^k - R_{v,c}^{k-1}\right|}{R_{v,c}^k + R_{v,c}^{k-1}}\right) < \vartheta$, stop. Otherwise, set k:=k+1, i:=1 and then go to step 1.

For the collusive game, the nonlinear programming problem **P4.2** as well as the subproblem **P4.6** shown in the relaxation algorithm above will be solved by the reduced gradient method (Bazaraa, 1993). This method linearizes the nonlinear programming problem using frequently updated derivative information and solves a sequence of linear programming problems.

CHAPTER 5

THE OLIGOPOLISTIC BEHAVIOR OF THE CARRIERS SUBJECT TO SHIPPERS' SPATIAL PRICE EQUILIBRIUM

Chapter 4 sets up the framework for solving the equilibrium pricing and routing problem of a single type of players: the oligopolistic carriers. This chapter extends this framework by taking into account the carrier-shipper interaction and solving jointly the oligopolistic equilibrium in the carriers' market and the spatial price equilibrium (SPE) in the shippers' market. As mentioned in Section 2.3, the carrier-shipper interaction is formulated as a Stackelberg game and modeled as a bi-level programming problem. In this bi-level game, the shippers' SPE model is the lower level problem, and the carriers' pricing and routing model from Section 4.4 is the higher-level problem. The linkage between the two levels is provided by the sensitivity analysis method (Tobin, 1987) of the SPE model.

This chapter is organized as follows. Section 5.1 introduces the SPE model. Section 5.2 presents the sensitivity analysis method used to generate the derivative of the equilibrium link flows with respect to the service charge in the neighborhood of the current service charge. This information is utilized to locally approximate the service demand function between each carrier O-D pair. Section 5.3 presents a bi-level program formulation for the Stackelberg game. A heuristic algorithm based on the sensitivity analysis method is developed in Section 5.4.

5.1 Spatial Price Equilibrium Model

In this section, the assumptions and the equilibrium conditions for the SPE problem are presented. Following these, a Variational Inequality (VI) formulation of the spatial price equilibrium is given. Finally, a decomposition algorithm is adopted to solve the problem.

5.1.1 Assumptions

The travel time functions, the inverse commodity supply functions, and the inverse commodity demand functions are the main inputs to the SPE problem. The following assumptions are made:

A1: The average travel time function $t_{l,c}(f_l)$ shown in Section 3.3.2 is continuous. The vector of travel time functions is strictly monotone increasing in the vector of flows (f) (i.e., the Jacobian matrix of the vector of the travel time functions, $[\nabla_f t(f)]$, is positive definite).

A2: The inverse commodity supply function $\pi_{b,c}(S_b)$ and the inverse commodity demand function $\rho_{b,c}(D_b)$, shown in Section 3.3.2, are continuous and linear. The vector of the inverse supply functions is strictly monotone increasing in the vector of supplies (S) (i.e., the Jacobian matrix of the vector of the inverse supply functions, $[\nabla_S \pi(S)]$, is positive definite). The vector of the inverse demand functions is strictly monotone decreasing in the vector of demands (D) (i.e., the Jacobian matrix of the vector of the inverse demand functions, $[\nabla_D \rho(D)]$, is negative definite).

A3: The inverse commodity supply and inverse commodity demand functions satisfy the following:

$$D_{b,c} = S_{b,c} = 0 \text{, then } \rho_{b,c}(D_b) > \pi_{b,c}(S_b) \quad \forall b \in CN, c \in C \tag{5.1}$$

Equation (5.1) states that for each commodity, when both its supply and its demand are zero, then the market price that the consumer is willing to pay is greater than the supply price.

A4: The commodity demand function $D_{b,c}(\rho_b)$ is linear and continuous. Its Jacobian matrix, $[\nabla_\rho D(\rho)]$, is negative definite and diagonal dominant, i.e.,

$$\frac{\partial D_{b,c}}{\partial \rho_{b,c}} < 0 \text{ and } \left|\frac{\partial D_{b,c}}{\partial \rho_{b,c}}\right| > \max\left(\sum_{c' \neq c}\left|\frac{\partial D_{b,c}}{\partial \rho_{b,c'}}\right|, \sum_{c' \neq c}\left|\frac{\partial D_{b,c'}}{\partial \rho_{b,c}}\right|\right) \tag{5.2}$$

A5: Each commodity flows through every node in the shipper network

$$\sum_{l} \delta_{l,n} * |f_{l,c}| > 0 \quad \forall n \in N, c \in C. \tag{5.3}$$

5.1.2 Spatial Price Equilibrium Conditions

Spatial price equilibrium conditions are defined in Section 2.1.3. Here, their mathematical presentation is given as follows:

E1: Wardrop's User Equilibrium Condition

$$h_{p,c}(GC_{p,c} - \mu_{b1,b2,c}) = 0 \quad \forall p \in P(b1,b2);\ b1,b2 \in CN;\ c \in C. \tag{5.4}$$

Condition **E1** states that for each commodity: 1. All used paths from origin *b1* to destination *b2* have the same minimum generalized shipping cost; and 2. If the cost on a path is greater than this minimum cost, the flow on this path is equal to zero.

E2: Market Equilibrium Condition between Two Spatially Separated Markets

$$Q_{b1,b2,c}(\pi_{b1,c} + \mu_{b1,b2,c} - \rho_{b2,c}) = 0 \quad \forall b1, b2 \in CN; c \in C. \tag{5.5}$$

Condition **E2** states that for each commodity, if there is a flow between $b1$ and $b2$, then the price at origin $b1$ plus the generalized cost of shipping from $b1$ to $b2$ must equal the destination price. At equilibrium, neither producer nor consumer has any motivation to change the production or consumption pattern.

Combining **E1** and **E2** yields the following:

$$h_{p,c} * (\pi_{b1,c} + GC_{p,c} - \rho_{b2,c}) = 0 \quad \forall p \in P(b1,b2); b1, b2 \in CN; c \in C \tag{5.6}$$

Equation (5.6) is the modified Wardrop User Equilibrium condition stated in Section 2.1.3.

5.1.3 Feasible Region

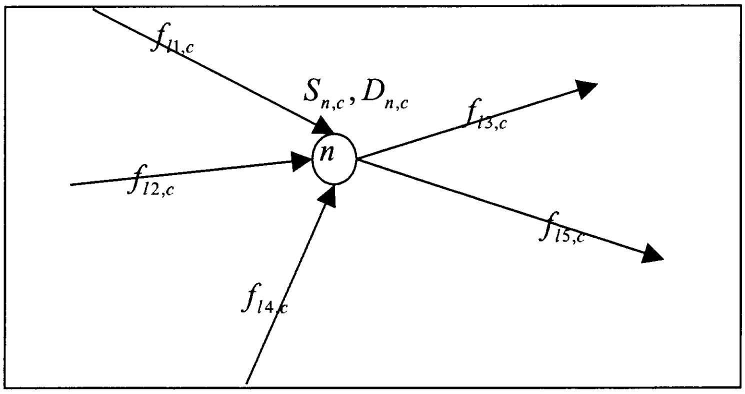
Feasible region KS is defined by the flow conservation constraints (Eqs. (5.7) and (5.8)) and the non-negativity constraints (Eq. (5.9)) as follows:

$$KS: \{(S, f, D) \mid -S_{b,c} + D_{b,c} + \sum_{l \in L} \delta_{l,b} * f_{l,c} = 0 \quad \forall b \in CN, c \in C \tag{5.7}$$

$$\sum_{l \in L} \delta_{l,n} * f_{l,c} = 0 \qquad \forall n \in IN, c \in C \tag{5.8}$$

$$(S, f, D) \geq 0\}. \tag{5.9}$$

Equations (5.7) and (5.8) state that for each commodity: 1. The supply at a market equals the sum of the local demand and the net outflow (e.g., the outflow minus the inflow); and 2. The inflow equals the outflow at an intermediate node.



.**Figure 5.1** Flow Conservation at Node $n \in N$

The concept of flow conservation is illustrated in Figure 5.1. Here, $f_{l3,c}, f_{l5,c}$ are the outbound flows and $f_{l1,c}, f_{l2,c}, f_{l4,c}$ are the inbound flows of commodity c at node n. If n is a centroid node, then the supply and the demand of commodity c are denoted as $S_{n,c}$ and $D_{n,c}$. By definition $\delta_{l1,n}, \delta_{l2,n}, \delta_{l4,n} = -1$ and $\delta_{l3,n}, \delta_{l5,n} = 1$. By substituting $\delta_{l1,n}, \delta_{l2,n}, \delta_{l3,n} \delta_{l4,n}, \delta_{l5,n}$ into Eqs. (5.7) and (5.8), the flow conservation constraints (Eqs. (5.10) and (5.11)) are obtained.

$$f_{l1,c} + f_{l2,c} + f_{l4,c} + S_{n,c} = D_{n,c} + f_{l3,c} + f_{l5,c} \quad \forall n \in CN \tag{5.10}$$

$$f_{l3,c} + f_{l5,c} - f_{l1,c} - f_{l2,c} - f_{l4,c} = 0 \;\; \forall n \in IN. \tag{5.11}$$

The feasible region KS defined by Eqs. (5.7)-(5.9) is a polyhedral set. As shown in Section 4.3.4, it is closed and convex.

5.1.4 Mathematical Formulation of the Spatial Price Equilibrium Problem

Table 5.1 shows the VI formulation of the multicommodity SPE problem by Nagurney (1999).

Table 5.1 Variational Inequality Formulation of the Multicommodity SPE Problem

P5.1

Determine $(S^*, f^*, D^*) \in KS$ such that

$$\left(\pi(S^*), GC(f^*), -\rho(D^*)\right) \bullet \begin{pmatrix} (S-S^*) \\ (f-f^*) \\ (D-D^*) \end{pmatrix} \geq 0 \qquad \forall (S, f, D) \in KS \tag{5.12}$$

5.1.5 Existence and Uniqueness of the Solution

The examination of the existence and uniqueness of the solution to problem **P5.1** in Table 5.1 is conducted similarly to that for problem **P4.1** in Table 4.1 of Chapter 4.

First, it will be shown that there is a solution to **P5.1**. This will be presented next based on theorem 1.5 from Section 4.5.

Let M' be a large constant and $KS_{M'} = KS \cap \|(S, f, D)\| \leq M'$. Here, $KS_{M'}$ is compact and convex. **P5.2** shown in Table 5.2 is the restricted problem of **P5.1**.

Table 5.2 Restricted Problem of **P5.1**

P5.2

Determine $(S^*, f^*, D^*) \in KS_{M'}$ such that

$$\left(\pi(S^*), GC(f^*), -\rho(D^*)\right) \bullet \begin{pmatrix} (S-S^*) \\ (f-f^*) \\ (D-D^*) \end{pmatrix} \geq 0 \qquad \forall (S, f, D) \in KS_{M'} \tag{5.13}$$

Based on theorem 1.5, problem **P5.1** admits a solution if and only if there exists a large constant $M'>0$ and a solution (S^*, f^*, D^*) to **P5.2** such that: $\|(S^*, f^*, D^*)\|<M'$.

Based on theorem 1.4 from Section 4.4, the existence of a solution to problem **P5.2** is guaranteed by the continuity of the vector of functions $\left(\pi(S^*), GC(f^*), -\rho(D^*)\right)$ and the compact convex property of the feasible region $KS_{M'}$.

The existence of a solution to **P5.2** is established. Next, a large enough constant M' will be constructed, which will make the solution (S^*, f^*, D^*) to **P5.2** satisfy $\|(S^*, f^*, D^*)\|<M'$.

First, the highest market price $\rho_{b,c} = \max_{b'\in CN, c'\in C} \rho_{b',c'}$ is shown to be bounded for any nonnegative $D_{b,c}$ in the following:

$$
\begin{aligned}
0 &\le D_{b,c}(\rho_b) = D_{b,c}(0) + \sum_{c'} \frac{\partial D_{b,c}}{\partial \rho_{b,c'}} * \rho_{b,c'} \\
&\le D_{b,c}(0) + \frac{\partial D_{b,c}}{\partial \rho_{b,c}} * \rho_{b,c} + \sum_{c'\neq c} \left| \frac{\partial D_{b,c}}{\partial \rho_{b,c'}} \right| * \rho_{b,c} \\
&\le D_{b,c}(0) + \left(\frac{\partial D_{b,c}}{\partial \rho_{b,c}} + \sum_{c'\neq c} \left| \frac{\partial D_{b,c}}{\partial \rho_{b,c'}} \right| \right) * \rho_{b,c}
\end{aligned}
\tag{5.14}
$$

From assumption **A2**, $\left(\frac{\partial D_{b,c}}{\partial \rho_{b,c}} + \sum_{c'\neq c} \left| \frac{\partial D_{b,c}}{\partial \rho_{b,c'}} \right| \right) < 0$. This combined with Eq. (5.14) yields that:

$$
\rho_{b,c} \le -\frac{D_{b,c}(0)}{\left(\frac{\partial D_{b,c}}{\partial \rho_{b,c}} + \sum_{c'\neq c} \left| \frac{\partial D_{b,c}}{\partial \rho_{b,c'}} \right| \right)}
\tag{5.15}
$$

Equation (5.15) indicates that the market price $\rho_{b,c}$ is bounded. Since $\rho_{b,c}$ is defined as the highest market price among all commodity types at all centroid nodes, the market price of any other commodity type at any other centroid node is also bounded.From assumption **A2**, the inverse commodity demand function is linear, continuous and strictly monotone. This implies that the demand function is also continuous in the market price. With market price bounded, the commodity demand is also bounded (i.e., there exists a positive value $\overline{D}_{\max}$, such that condition $D_{b,c}(\rho) < \overline{D}_{\max} \quad \forall b \in CN, c \in C$ is satisfied for any $\rho \geq 0$). This condition, in turn, leads to the conclusion that the supplies and the link flows are also bounded.

$$\sum_{b \in CN} (-S_{b,c} + D_{b,c} + \sum_{l} \delta_{l,b} * f_{l,c}) + \sum_{i \in IN} (\sum_{l} \delta_{l,i} * f_{l,c}) = 0$$

$$\Rightarrow \sum_{b \in CN} (-S_{b,c} + D_{b,c}) + \sum_{l} \left(f_{l,c} * (\sum_{b \in CN} \delta_{l,b} + \sum_{i \in IN} \delta_{l,i}) \right) = 0$$

$$\Rightarrow \sum_{b \in CN} (-S_{b,c} + D_{b,c}) + \sum_{l} \left(f_{l,c} * (\sum_{n \in N} \delta_{l,n}) \right) = 0$$

$$\sum_{l} \left(f_{l,c} * (\sum_{n \in N} \delta_{l,n}) \right) = 0 \Rightarrow \sum_{b \in CN} (-S_{b,c} + D_{b,c}) = 0$$

$$\Rightarrow S_{b,c} \leq \sum_{b \in CN} S_{b,c} \leq \sum_{b \in CN} D_{b,c} < \sum_{b \in CN} \overline{D}_{\max} \tag{5.16}$$

$$f_{h,c} \leq S_{b1,c}, f_{h,c} \leq D_{b2,c} \quad \forall h \in H(b1,b2);\ b1,b2 \in CN.$$

$$\Rightarrow f_{l,c} \leq \sum_{h} f_{h,c} < \sum_{h} \overline{D}_{\max} \quad \forall l \in L \tag{5.17}$$

Equations (5.16) and (5.17) prove that there exist $\overline{S}_{\max}$, $\overline{f}_{\max}$, such that $S_{b,c} < \overline{S}_{\max}$ $\forall b \in CN, c \in C$ and $f_{l,c} < \overline{f}_{\max}$ $\forall l \in L, c \in C$. Hence, M' can be constructed as $M' = \|(\ldots, \overline{S}_{\max}, \ldots)_{|CN|*|C|}, (\ldots, \overline{f}_{\max}, \ldots)_{|L|*|C|}, (\ldots, \overline{D}_{\max}, \ldots)_{|CN|*|C|}\|$.

With M' thus constructed, the solution (S^*, f^*, D^*) to **P5.2** satisfies $\|(S^*, f^*, D^*)\| < M'$. Based on theorem 1.5 from Section 4.5, this implies that there exists a solution to problem **P5.1.** Based on theorem 1.6 from Section 4.5, this solution is unique since functions $\left(\pi(S^*), GC(f^*), -\rho(D^*)\right)$ in Eq. (5.12) are strictly monotone in (S, f, D).

5.1.6 Solution Algorithm for the Spatial Price Equilibrium (SPE) Model

With the existence and uniqueness of the solution established, the serial nonlinear decomposition algorithm (Nagurney, 1999) reviewed in Section 2.1.3 is used to solve problem **P5.1**. This algorithm decomposes problem **P5.1** into a sequence of single commodity VI subproblems, which are then transformed into nonlinear optimization problems. The rationale for choosing this algorithm is that the feasible set KS for problem **P5.1** is a Cartesian product of the feasible sets for the variables associated with each commodity:

$$KS = \prod_{i=1}^{|C|} KS_{ci}. \ \forall ci \in C,$$

where ci denotes the i_{th} commodity in set C, and KS_{ci} is a feasible set for variables associated with commodity ci. KS_{ci} is defined as:

$$KS_{ci}: \{(S_{ci}, f_{ci}, D_{ci}) \mid -S_{b,ci} + D_{b,ci} + \sum_{l \in L} \delta_{l,b} * f_{l,ci} = 0 \quad \forall b \in CN \qquad (5.18)$$

$$\sum_{l \in L} \delta_{l,n} * f_{l,ci} = 0 \qquad \forall n \in NI \qquad (5.19)$$

$$(S_{ci}, f_{ci}, D_{ci}) \geq 0\}. \qquad (5.20)$$

P5.3 shown in Table 5.3 is the subproblem of **P5.1** for commodity *ci* defined over KS_{ci}.

Table 5.3 Variational Inequality Subproblem of **P5.1** for Commodity *ci*

P5.3
Determine $(S^*_{ci}, f^*_{ci}, D^*_{ci}) \in KS_{ci}$ such that
$\left(\pi_{ci}(S^*_{ci}, \overline{S}_{-ci}), GC_{ci}(f^*_{ci}, \overline{f}_{-ci}), -\rho_{ci}(D^*_{ci}, \overline{D}_{-ci})\right)^T \bullet \begin{pmatrix} (S_{ci} - S^*_{ci}) \\ (f_{ci} - f^*_{ci}) \\ (D_{ci} - D^*_{ci}) \end{pmatrix} \geq 0 \qquad (5.21)$
$\forall (S_{ci}, f_{ci}, D_{ci}) \in KS_{ci}$

In Eq. (5.21), the vector of commodity supplies, link flows and commodity demands of commodities other than *c*, denoted as ($\overline{S}_{-c}$, $\overline{f}_{-c}$, $\overline{D}_{-c}$), are considered constant. The functions $\left(\pi_{ci}(S^*_{ci}, \overline{S}_{-ci}), GC_{ci}(f^*_{ci}, \overline{f}_{-ci}), -\rho_{ci}(D^*_{ci}, \overline{D}_{-ci})\right)$ are separable since it is assumed that the interaction only exists between different commodities at the same centroid or on the same link. Therefore, problem **P5.3** is equivalent to a nonlinear optimization problem **P5.4** shown in Table 5.4.

Table 5.4 Equivalent Nonlinear Programming Problem to **P5.3**

P5.4	
$$\underset{S_{ci},f_{ci},D_{ci}}{Min} : \sum_b \int_0^{S_{b,ci}} \pi_{b,ci}(S_{b,ci},\overline{S}_{b,-ci})dS_{a,ci} + \sum_l \int_0^{f_{l,ci}} GC_{l,ci}(f_{l,ci},\overline{f}_{l,-ci})df_{l,ci} - \sum_b \int_0^{D_{b,ci}} \rho_{b,ci}(D_{b,ci},\overline{D}_{b,-ci})dD_{b,ci}$$	(5.22)
s.t. $(S_{ci},f_{ci},D_{ci}) \in KS_{ci}$	

Based on the above discussion of VI subproblem **P5.3** and its equivalent nonlinear optimization problem **P5.4**, the steps of the serial nonlinear decomposition algorithm are as follows:

<u>**Step 0**</u> Initialization

Initialize the vectors of the commodity supplies, link flows and commodity demands in the shipper network as (S^0, f^0, D^0). Let k denote the order of the iteration, and i denote the order of the commodity in set C. Set k:=1; i:=1. Let ϑ denote the convergence parameter that evaluates the relative improvement in the results between two consecutive iterations. Set ϑ to a value close to zero, e.g. 10^{-6}.

<u>**Step 1**</u> Relaxation and computation

Solve problem **P5.4** using the reduced gradient algorithm (Bazaraa, 1993) for ci with $\overline{S}_{-ci} = (S^k_{c1},\dots S^k_{ci-1}, S^{k-1}_{ci+1},\dots S^{k-1}_{cm})$, $\overline{D}_{-ci} = (D^k_{c1},\dots D^k_{ci-1}, D^{k-1}_{ci+1},\dots D^{k-1}_{cm})$ and $\overline{f}_{-ci} = (f^k_{c1},\dots f^k_{ci-1}, f^{k-1}_{ci+1},\dots f^{k-1}_{cm})$. Denote the optimal solution to **P5.4** as $(S^*_{ci}, f^*_{ci}, D^*_{ci})$.

Set $(S_{ci}^k, f_{ci}^k, D_{ci}^k) = (S_{ci}^*, f_{ci}^*, D_{ci}^*)$. If $i<m$, solve the problem for the next commodity type by setting $i:=i+1$. Otherwise, go to Step 2.

Step 2 Convergence verification

If $\sum_b \sum_c \frac{|S_{b,c}^k - S_{b,c}^{k-1}|}{S_{b,c}^k + S_{b,c}^{k-1}} + \sum_l \sum_c \frac{|f_{l,c}^k - f_{l,c}^{k-1}|}{f_{l,c}^k + f_{l,c}^{k-1}} + \sum_b \sum_c \frac{|D_{b,c}^k - D_{b,c}^{k-1}|}{D_{b,c}^k + D_{b,c}^{k-1}} \leq \vartheta$, stop; otherwise, set $k:=k+1$, $i:=1$ and go to Step 1.

The serial nonlinear decomposition algorithm shown above is illustrated by the flowchart in Figure 5.2:

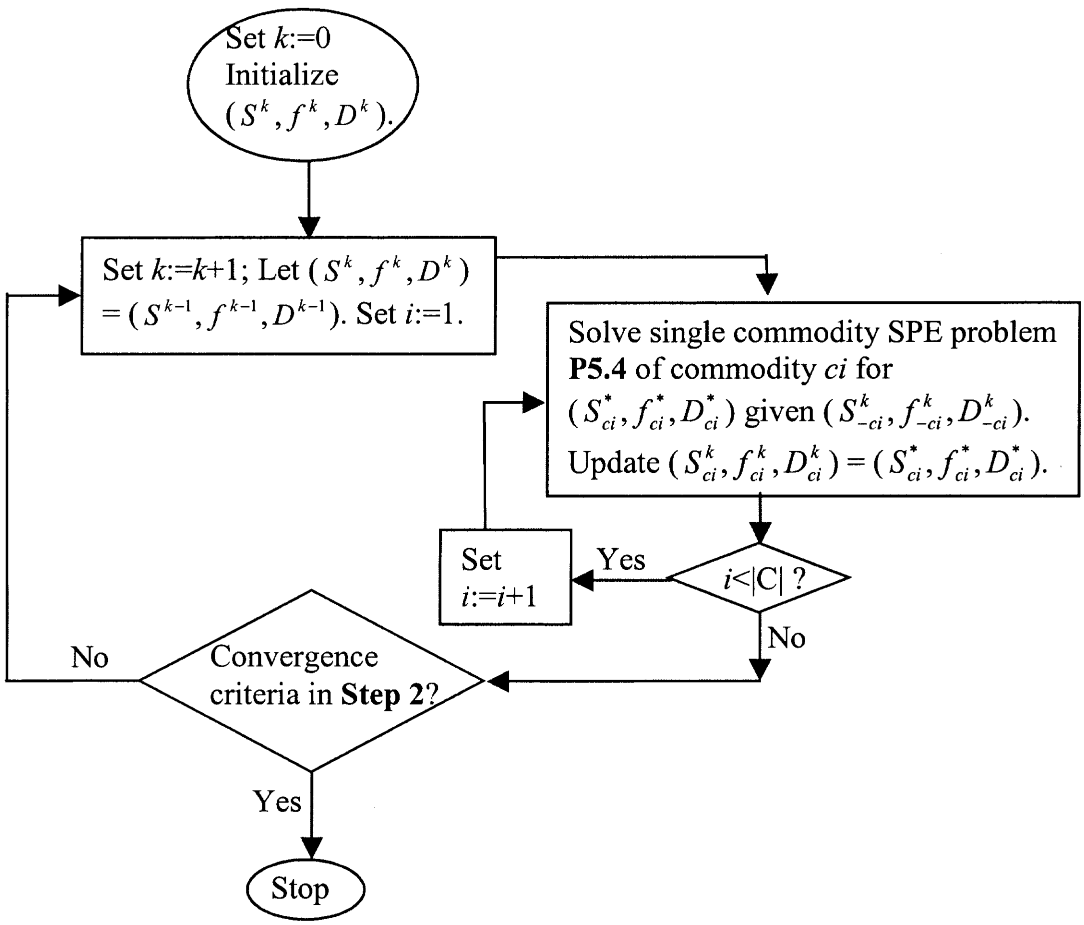


Figure 5.2 Flow Chart of the Serial Nonlinear Decomposition Algorithm

If the serial decomposition algorithm converges, the solution obtained must be an equilibrium solution. This is stated in the following proposition:

Proposition 5.1: The solution $(S^{k-1}, f^{k-1}, D^{k-1})$ is a spatial price equilibrium solution satisfying **E1** and **E2** if the nonlinear serial decomposition algorithm converges to (S^*, f^*, D^*) after k-1 iterations, which means $(S^{k-1}, f^{k-1}, D^{k-1})$ = (S^k, f^k, D^k) = (S^*, f^*, D^*).

To demonstrate this, the proof from Sheffi (1985) for the Jacobian diagonalization method is adopted.

Proof: Let $\eta_{n,c}$ denote the dual variable for the flow conservation constraints in Eqs. (5.18) and (5.19). If $(S^{k-1}, f^{k-1}, D^{k-1})$ = (S^k, f^k, D^k) = (S^*, f^*, D^*), then the Karush-Kuhn-Tuker (KKT) conditions (Bazaraa, 1993) of problem **P5.4** are:

$$S^*_{b,c}(\pi_{b,c}(S^*_b) - \eta_{b,c}) = 0 \qquad \forall b \in CN,\ c \in C \qquad (5.23)$$

$$D^*_{b,c}(-\rho_{b,c}(D^*_b) + \eta_{b,c}) = 0 \qquad \forall b \in CN,\ c \in C \qquad (5.24)$$

$$h^*_{p,c}(\eta_{b1,c} + \sum_{l \in L(p)} GC_{l,c}(f^*_l) - \eta_{b2,c}) = 0 \qquad \forall p \in P(b1,b2), c \in C \qquad (5.25)$$

$$\eta_{n,c} \geq 0 \quad \forall n \in N, c \in C \qquad (5.26)$$

or

$$h^*_{p,c}(\pi_{b1,c} + GC_{p,c} - \rho_{b2,c}) = 0 \qquad \forall p \in P(b1,b2), c \in C \qquad (5.27)$$

The KKT conditions (Eqs. (5.23)-(5.26) or Eq. (5.27)) correspond to the equilibrium conditions **E1** and **E2** (Eqs. (5.4) and (5.5)) or the adjusted Wardrop User Equilibrium condition (Eq. (5.6)). Thus, Proposition 5.1 is proven.

5.2 Local Approximation of the Demand Functions on the Carriers' Network

Section 5.1 describes the SPE problem and the solution method. Given the current level of service charges, the equilibrium link flow is solved from the spatial price equilibrium model. The service charge is the decision of the carrier. If a carrier changes the service charge, the shippers will react by adjusting their commodity supply, commodity demand and link flow pattern correspondingly. In a general Stackelberg game, the decisions of the lower level decision maker and the upper level decision maker are related using a reaction function. However, due to the large scale of the shipper network and the multidimensional nature of the spatial price equilibrium model, it is difficult to derive the explicit forms of the reaction functions of the shippers that would relate the shippers' flows to the carriers' service charges. An alternative approach is to locally approximate those reaction functions using the sensitivity analysis method.

5.2.1 Sensitivity Analysis Method of the Spatial Price Equilibrium Problem

In this section, the sensitivity analysis method for a general VI problem (Tobin, 1987) is applied to problem **P5.1** to generate the derivative of the equilibrium link flows with respect to the service charges in the neighborhood of the current service charges. This method is described in Appendix B. The conditions required for applying this method to problem **P5.1** are shown to be satisfied in Appendix C.

Given the vector of the current level of service charges on all shipper links $R_L^0 = (\cdots, R_{l,c}^0, \cdots)_{|L|*|C|}$, the equilibrium solution of the primal variables and dual variables $\left(S^*, f^*, D^*, \eta^*\right)$ to problem **P5.1** satisfies the KKT conditions defined in Table 5.5.

Table 5.5 Karush-Kuhn-Tuker Conditions of Problem **P5.1**

P5.5

$$\pi_{b,c}(S_b^*)\text{-}\eta_{b,c}^* = 0 \quad \text{if } S_{b,c}^* > 0 \quad \forall b \in CN, c \in C \qquad (5.28)$$

$$-\rho_{b,c}(D_b^*) + \eta_{b,c}^* = 0 \quad \text{if } D_{b,c}^* > 0 \quad \forall b \in CN, c \in C \qquad (5.29)$$

$$GC_{l=(n1,n2),c}(f_l^*, R_L^0) + \eta_{n1,c}^* - \eta_{n2,c}^* = 0 \quad \text{if } f_{l,c}^* > 0 \quad \forall l \in L, c \in C \qquad (5.30)$$

The first derivative of the flow conservation constraints (Eqs. (5.7), (5.8)) and Eqs. (5.28)-(5.30) in **P5.5** is taken with respect to the service charge of commodity *ci* on a shipper link *lj* ($R_{lj,ci}$), which is the perturbation parameter for problem **P5.1.** Eqs. (5.31), (5.32) and Eqs. (5.33)-(5.35) in Table 5.6 are obtained.

Table 5.6 Sensitivity Analysis Method of Spatial Price Equilibrium Problem

P5.6

$$\sum_{l \in L} \frac{\partial f_{l,c}}{\partial R_{lj,ci}} * \delta_{l,n} = 0 \qquad \forall n \in IN, c \in C \qquad (5.31)$$

$$\sum_{l \in L} \frac{\partial f_{l,c}}{\partial R_{lj,ci}} * \delta_{l,b} - \frac{\partial S_{b,c}}{\partial R_{lj,ci}} + \frac{\partial D_{b,c}}{\partial R_{lj,ci}} = 0 \qquad \forall b \in CN, c \in C \qquad (5.32)$$

$$\sum_{c' \in C} \frac{\partial \pi_{b,c}(S_b^*)}{\partial S_{b,c'}} * \frac{\partial S_{b,c'}}{\partial R_{lj,ci}} - \frac{\partial \eta_{b,c}}{\partial R_{lj,ci}} = 0 \qquad \text{if } S_{b,c}^* > 0 \quad \forall b \in CN, c \in C \qquad (5.33)$$

$$\sum_{c' \in C} -\frac{\partial \rho_{b,c}(D_b^*)}{\partial D_{b,c'}} * \frac{\partial D_{b,c'}}{\partial R_{lj,ci}} + \frac{\partial \eta_{b,c}}{\partial R_{lj,ci}} = 0 \qquad \text{if } D_{b,c}^* > 0 \quad \forall b \in CN, c \in C \qquad (5.34)$$

$$\frac{\partial \eta_{n1,c}}{\partial R_{lj,ci}} + \sum_{c'} \frac{\partial GC_{l,c}(f_l^*, R_{l,c}^0)}{\partial f_{l,c'}} * \frac{\partial f_{l,c'}}{\partial R_{lj,ci}} + \frac{\partial GC_{l,c}}{\partial R_{lj,ci}} - \frac{\partial \eta_{n2,c}}{\partial R_{lj,ci}} = 0$$

$$\text{if } f_{l,c}^* > 0 \qquad \forall l = (n1, n2), l \in L, c \in C \qquad (5.35)$$

Where $\frac{\partial GC_{l,c}}{\partial R_{lj,ci}} = \begin{cases} 1 & \text{for } l = lj \text{ and } c = ci \\ 0 & \text{for } l \neq lj \text{ or } c \neq ci \end{cases}$

P5.6 consists of a set of linear equations. By solving problem **P5.6**, the derivatives of the equilibrium solution $\left(S^*, f^*, D^*, \eta^*\right)$ to problem **P5.1** with respect to $R_{lj,ci}$ in the neighborhood of R_L^0 are derived.

5.2.2 Locally Approximated Service Demand Function

With $\dfrac{\partial f_{l,c}(R_L^0)}{\partial R_{lj,ci}}$ $\forall l, lj \in L, c, ci \in C$ solved from problem **P5.6**, the reaction function of the link flow $f_{l,c}$ with respect to the vector of service charges on all shipper links ($R_L = (\cdots, R_{l,c}, \cdots)_{|L|*|C|}$) in the neighborhood of R_L^0, denoted as $f_{l,c}(R_L)$, can be approximated locally as follows:

$$f_{l,c}(R_L) = f_{l,c}(R_L^0) + \sum_{lj \in L} \sum_{ci \in C} \frac{\partial f_{l,c}(R_L^0)}{\partial R_{lj,ci}} (R_{lj,ci} - R_{lj,ci}^0) \qquad \forall l \in L, c \in C \qquad (5.36)$$

The property of the function in Eq. (5.36) is shown in the following proposition.

Proposition 5.2: The vector of the locally approximated flow functions on the shipper links as shown in Eq. (5.36) is monotone decreasing in the vector of the service charges if the vectors of the inverse commodity supply, inverse commodity demand and generalized cost functions are strictly monotone. For the sub-vector of the flow variables that have positive equilibrium solution at the current level of service charge, the sub-vector of the locally approximated functions is strictly monotone decreasing in the sub-vector of the service charges corresponding to these flow variables. The proof is shown in Appendix D.

Along with the reaction function $f_{l,c}(R_L)$ estimated by Eq. (5.36), the service demand function at each carrier O-D pair is estimated based on the incidence matrix between the links on the shipper network and the O-D pairs on the carriers' network $[\xi_{l,v}]_{|L|*|V|}$. The formula used for the estimation of the demand function is shown in Eq. (5.37).

$$g_{v,c}(R_V)=\sum_{l\in L}\xi_{l,v} * f_{l,c}(R_L)=\sum_{l\in L}\xi_{l,v} * (f_{l,c}(R_L^0)+\sum_{lj\in L}\sum_{ci\in C}\frac{\partial f_{l,c}(R_L^0)}{\partial R_{lj,ci}}(R_{lj,ci}-R_{lj,ci}^0)) \qquad (5.37)$$

Here, $R_V=(\cdots,R_{v,c},\cdots)_{|V|*|C|}$, $R_L=[\xi_{l,v}]_{|L|*|V|} * R_V$, and $R_L^0=[\xi_{l,v}]_{|L|*|V|} * R_V^0$.

The continuous and monotone property of $f_{l,c}(R_L)$, together with the nonnegativity of $\xi_{l,v}$, guarantees that $g_{v,c}(R_V)$ in Eq. (5.37) is also a continuous and monotone function. In Eq. (5.37), $g_{v,c}(R_V)$ representd the approximated service demand function in the neighborhood of R_V^0,

5.3 Bi-level Programing Problem

By combining the shippers' SPE model from Section 5.1 and the carriers' pricing and routing model from Section 4.4, the bi-level program formulating the Stackelberg game between the carriers and the shippers is constructed. The bi-level programs for the competitive game and the collusive game are shown in Table 5.7.

Table 5.7 Bi-level Programs

a) Competitive Game

P5.7

$$\sum_{t\in T}\left(-\nabla_{R_t} Z_t(g_t(R_V^*),R_t^*,e_t^*)(R_t - R_t^*) - \nabla_{e_t} Z_t(g_t(R_V^*),R_t^*,e_t^*)(e-e^*)\right)\geq 0$$

$$\forall (R_V, e)\in KT \tag{5.38}$$

where $g_{v,c}(R_V)=\sum_l \xi_{l,v} * f_{l,c}(R_L)$ as defined in Eq. (5.37).

s.t.

$$\pi(S^*)\bullet(S-S^*)+GC(f^*,R_L^0)\bullet(f-f^*)-\rho(D^*)\bullet(D-D^*)\geq 0$$

$$\forall (S,f,D)\in KS \tag{5.39}$$

where $R_L^0=\left[\xi_{l,v}\right]_{|L|*|V|} * R_V^*$.

b) Collusive game

P5.8

$$\underset{R_{vl,c},e_{a,c}}{Max}: \sum_t Z_t(g_t(R_V),R_t,e_t) \tag{5.40}$$

s.t. $(R_V, e)\in KT$,

where $g_{v,c}(R_V)=\sum_l \xi_{l,v} * f_{l,c}(R_L)$ as defined in Eq. (5.37).

s.t.

$$\pi(S^*)\bullet(S-S^*)+GC(f^*,R_L^0)\bullet(f-f^*)-\rho(D^*)\bullet(D-D^*)\geq 0$$

$$\forall (S,f,D)\in KS \tag{5.41}$$

where $R_L^0=\left[\xi_{l,v}\right]_{|L|*|V|} * R_V^*$.

5.4 Solution Algorithm for the Bi-level Programming Problem

This section presents the development of a sensitivity analysis method-based heuristic algorithm to solve the bi-level programming problems **P5.7** and **P5.8**. The detailed steps of this algorithm are as follows:

Step 0 Initialization

Initialize (S^0, f^0, D^0)=(0). Initialize the service charges on all carrier O-D pairs as R_V^0. Set k:=1. Set the convergence parameter ϑ defined in Section 5.1.6 to a value close to zero, such as 10^{-6}.

Step 1 Solve the shipper level problem at the current service charges on the shipper links ($R_L^{k-1} = \left[\xi_{l,v}\right]_{|L|*|V|} * R_V^{k-1}$).

Use the serial nonlinear decomposition algorithm from Section 5.1.6 to solve problem **P5.1**. Obtain the solution (S^*, f^*, D^*). Set (S^k, f^k, D^k)=(S^*, f^*, D^*). Go to Step 2.

Step 2 Sensitivity analysis

Solve **P5.5** for the dual variable (η^k) based on the current SPE solution (S^k, f^k, D^k). Solve **P5.6** for the derivative of (S^k, f^k, D^k, η^k) with respect to R_L in the neighborhood of the current service charges (R_L^{k-1}), based on which locally approximate the flow function $f_{l,c}(R_L)$ according to Eq. (5.36). Then, derive the local approximated demand function $g_{v,c}(R_V)$ according to Eq. (5.37). Go to Step 3.

Step 3 Solve the carrier level problem **P4.1** or **P4.2** from Section 4.4 based on the local approximated demand function $g_{v,c}(R_V)$ derived in Step 2.

Substitute $g_{v,c}(R_V)$ into problem **P4.1** or **P4.2**. Solve problem **P4.1** using the relaxation algorithm from Section 4.6 or solve **P4.2** using the reduced gradient algorithm (Bazaraa, 1993). Obtain the solution R_V^*. Set $R_V^k = R_V^*$. Calculate $\left(\sum_{v\in V}\sum_{c\in C}\frac{\left|R_{v,c}^k - R_{v,c}^{k-1}\right|}{R_{v,c}^k + R_{v,c}^{k-1}}\right)$. If this value is less than ϑ, stop. Otherwise, set k:=k+1 and go to step 1.

The sensitivity analysis method-based heuristic algorithm introduced above will be further illustrated in Figure 5.3 below.

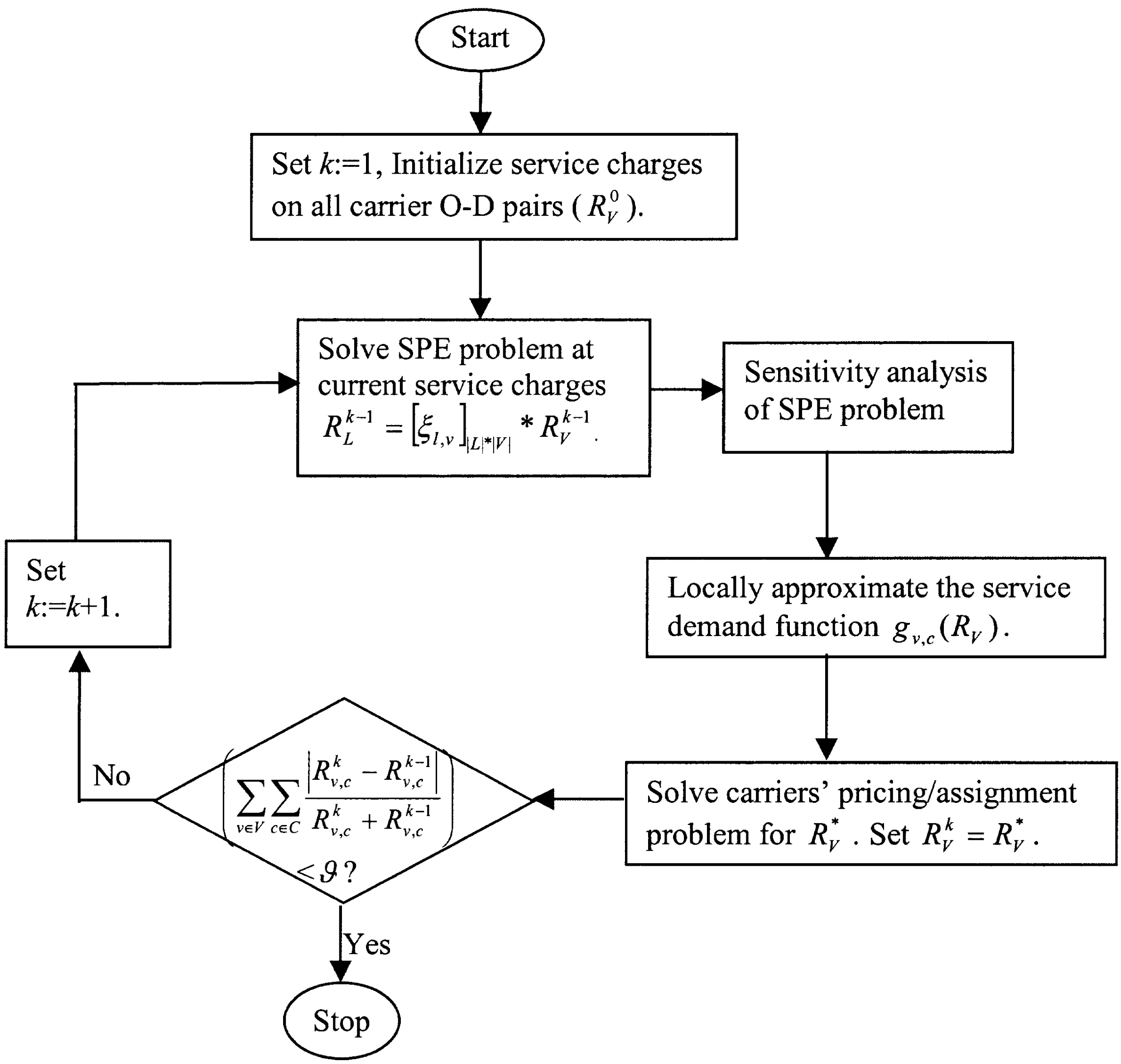


Figure 5.3 Flow Chart of the Sensitivity Analysis Method-Based Heuristic Algorithm

CHAPTER 6

PORT AUTHORITY'S INVESTMENT PROBLEM

Chapter 5 provided a bi-level programming approach to solve the Stackelberg equilibrium between carriers and shippers. This approach can be used to facilitate the Port Authority's investment decisions, as demonstrated in this chapter. The chapter is organized as follows. Section 6.1 introduces the criteria for comparing alternative investment strategies. Section 6.2 defines the net social benefit. Section 6.3 defines the investment cost. Section 6.4 presents the mathematical formulation of the Port Authority's investment problem.

6.1 Criteria Used in Comparing Alternative Investment Strategies

To facilitate investment decisions of the government and to help the Port Authority in evaluating alternative strategies, three criteria suggested in the literature (National Research Council, 1988) can be adopted. These criteria include:

Is the current infrastructure sufficient? Investing in the infrastructure of a port whose facilities are underutilized would be wasteful. However, when the facility operates at or over capacity, investment is warranted. The bi-level programming model in Section 5.3 can be used to identify critical links, meaning links that are highly utilized and may require future investment in capacity expansion.

Is there an external economy or market inefficiency? In the port operation, in addition to the economic gain occurring directly at the port, there is a substantial spillover of economic benefit to other sectors or industries in the region. This economic impact of the port on the local economy justifies the government's involvement in the port infrastructure investment.

Is the incremental net social benefit brought by the investment greater than the incremental investment cost? The net social benefit is defined as the sum of the net benefits of all players in the port vicinity affected by the Port Authority's investment in the port infrastructure. The incremental net social benefit accumulated through the investment strategy *u* is defined as the difference between the net social benefit under the investment strategy *u* and that under the do-nothing strategy. The investment cost is the capital expense associated with an investment strategy. Both the net social benefit and the investment cost in this dissertation are expressed in dollars per hour. For an investment strategy to be feasible, the incremental net social benefit should exceed the incremental investment cost. The investment strategy that yields the highest ratio of incremental net social benefit to the incremental investment cost is the most desirable alternative strategy.

6.2 Net Social Benefit

As indicated in Section 6.1, the net social benefit (*NSB*) is an important measure of the worthiness of an investment strategy. To make an accurate estimation of the net social benefit, the various players impacted by the Port Authority's investment in the port infrastructure need to be identified. The benefits to these players are estimated, and the sum of the benefits is the NSB.

The carriers and the shippers are two major types of players impacted by the Port Authority's investment decision. The investment improves the terminal operators' operating cost and the shippers' generalized cost. In response to the improvement in these costs, the terminal operators as well as the other carriers and the shippers will adjust their behavior until a new Stackelberg equilibrium is attained. The Bi-level program **P5.7** and **P5.8** from Section 5.3 can predict this new equilibrium, and is used to estimate the terminal operators' net benefit (*TNB*) and the shippers' net benefit (*SNB*) associated with the Port Authority investment decision. These benefits are defined in Section 6.2.1 and Section 6.2.2 below.

6.2.1 Terminal Operators' Net Benefit

Terminal operators are the producers of the port service. For the terminal operators, the monetary value of their net benefits is indicated by the total profits earned from their services. Let (R^u, e^u) denote the terminal operators' decision at the Stackelberg equilibrium under investment strategy u. Then, the terminal operators' net benefit under investment strategy u (TNB^u) is given as Eq. (6.1).

$$\begin{aligned} TNB^u &= \sum_{t\in TM} Z_t(g_t(R_t^u, R_{-t}^u), R_t^u, e_t^u) \\ &= \sum_{t\in TM}\Big(\sum_{v\in V^t}\sum_{c\in C} g_{v,c}(R_t^u, R_{-t}^u) * R_{v,c}^u - \sum_{a\in A^t}\sum_{c\in C} AC_{a,c}(e_{a,c}^u) * e_{a,c}^u\Big) \end{aligned} \qquad \forall u \in U \qquad (6.1)$$

The service demand $g_{v,c}(R_t^u, R_{-t}^u)$ and the link flow $e_{a,c}^u$ are in units of flow per hour; TNB^u is in dollars per hour. Assuming that all terminals' profits occur in the port vicinity, 100 percent of TNB^u are included in the calculation of the net social benefit. As to the carriers other than the terminal operators, their profits may or may not occur in the port vicinity. Here, for the sake of simplification, the profits of the carriers other than the port terminal operators are not included in the analysis.

6.2.2 Shippers' Net Benefit

Shippers are the users of the terminal service. According to the economic theory (Wohl, 1984), their net benefit is the monetary value of their total willingness to pay (i.e. the integral of the inverse demand function or the area under the inverse demand curve) minus the amount that they actually do pay. The shippers can be either the consumers or the producers of the transported commodities. The shippers' net benefit is segregated into two sources: 1. consumer surplus (*CS*) (i.e. the area of triangle AEC at $\rho^u_{b,c}$ in Figure 6.1) from the consumption of the transported commodities; and 2. producer surplus (*PS*) (the area of the triangle BCD at $\pi^u_{b,c}$ in Figure 6.1) from the production of the transported commodities. Consumer surplus is the consumer's total willingness to pay (i.e. the area of trapezoid AEHO in Figure 6.1) minus what the consumer actually pays for the transported commodities (i.e. the area of rectangle CEHO in Figure 6.1). Producer surplus is the total sales revenue (i.e. the area of rectangle OCDG in Figure 6.1) minus the total production cost (i.e. the area of trapezoid BDGO in Figure 6.1).

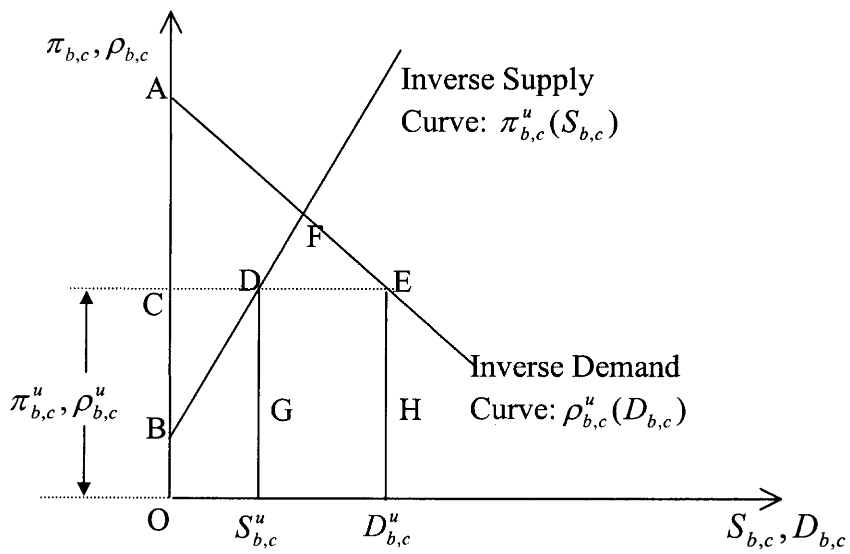


Figure 6.1 Shippers' Net Benefit

Figure 6.1 illustrates how to estimate various sources of the shippers' net benefit based on the spatial price equilibrium (SPE) solution under investment strategy u (S^u, f^u, D^u, π^u, ρ^u, GC^u). To illustrate this application, the inverse supply and inverse demand functions from Section 3.3.2 are restated in Eqs. (6.2) and (6.3) by fixing the cross effects of the other commodities.

$$\begin{aligned}\pi^u_{b,c}(S_{b,c}) &= (\gamma_{b,c} + \sum_{c'\in C, c'\neq c} \lambda_{b,c',c} * S^u_{b,c'}) + \lambda_{b,c,c} * S_{b,c} \\ &= \gamma^u_{b,c} + \lambda_{b,c,c} * S_{b,c} \qquad \forall b \in CN, c \in C, u \in U \qquad (6.2) \\ \text{where } \gamma^u_{b,c} &= (\gamma_{b,c} + \sum_{c'\in C, c'\neq c} \lambda_{b,c',c} * S^u_{b,c'})\end{aligned}$$

$$\begin{aligned}\rho^u_{b,c}(D_{b,c}) &= (\alpha_{b,c} - \sum_{c'\in C, c'\neq c} \beta_{b,c',c} * D^u_{b,c'}) - \beta_{b,c,c} * D_{b,c} \\ &= \alpha^u_{b,c} - \beta_{b,c,c} * D_{b,c} \qquad \forall b \in CN, c \in C, u \in U \quad (6.3) \\ \text{where } \alpha^u_{b,c} &= (\alpha_{b,c} - \sum_{c'\in C, c'\neq c} \beta_{b,c',c} * D^u_{b,c'})\end{aligned}$$

In Eqs. (6.2) and (6.3), $\pi^u_{b,c}(S_{b,c})$ is the inverse supply function of commodity c at centroid b, given that the supply vector for the other commodities is $S^u_{b,-c}$. The term $\rho^u_{b,c}(D_{b,c})$ is the inverse demand function of commodity c at centroid b, given that the demand vector for the other commodities is $D^u_{b,-c}$. The curves of $\pi^u_{b,c}(S_{b,c})$ and $\rho^u_{b,c}(D_{b,c})$ are plotted in Figure 6.1. Using these functions in Eqs. (6.2) and (6.3), the consumer surplus and the producer surplus at centroid b for commodity c can be calculated.

The consumer surplus at centroid b from the consumption of commodity c ($CS^u_{b,c}$) is calculated using the formula in Eq. (6.4).

$$CS^u_{b,c} = \int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \rho^u_{b,c}(D^u_{b,c}) * D^u_{b,c} \quad \forall b \in CN, c \in C, u \in U \tag{6.4}$$

The producer surplus at centroid b from the production of commodity c ($PS^u_{b,c}$) is calculated using the formula in Eq. (6.5).

$$PS^u_{b,c} = \pi^u_{b,c}(S^u_{b,c}) * S^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \quad \forall b \in CN, c \in C, u \in U \tag{6.5}$$

Combining Eqs. (6.4) and (6.5), the shippers' net benefit at centroid b from the consumption and the production of commodity c ($SNB^u_{b,c}$) is obtained as follows:

$$\begin{aligned} SNB^u_{b,c} &= CS^u_{b,c} + PS^u_{b,c} \\ &= \int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \rho^u_{b,c}(D^u_{b,c}) * D^u_{b,c} + \pi^u_{b,c}(S^u_{b,c}) * S^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \\ &\forall b \in CN, c \in C, u \in U \end{aligned} \tag{6.6}$$

The shippers' net benefit (SNB^u) is calculated as the sum of $SNB^u_{b,c}$ for each centroid and each commodity type as follows:

$$\begin{aligned} SNB^u &= \sum_{b \in CN, c \in C} SNB^u_{b,c} = \sum_{b,c}\left(CS^u_{b,c} + PS^u_{b,c}\right) \\ &= \sum_{b,c}\left(\int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \rho^u_{b,c}(D^u_{b,c}) * D^u_{b,c} + \pi^u_{b,c}(S^u_{b,c}) * S^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \right) \\ &= \sum_{b,c}\left(\int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \right) - \sum_{b,c}\left(\rho^u_{b,c}(D^u_{b,c}) * D^u_{b,c} - \pi^u_{b,c}(S^u_{b,c}) * S^u_{b,c}\right) \\ &= \sum_{b,c}\left(\int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \right) - \sum_{b1,b2 \in CN, c}\left(GC^u_{b1,b2,c} * Q^u_{b1,b2,c}\right) \\ &= \sum_{b,c}\left(\int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} \right) - \sum_{b,c}\left(\int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \right) - \sum_{l \in L, c}\left(GC^u_{l,c} * f^u_{f,c}\right), \quad \forall u \in U \end{aligned} \tag{6.7}$$

In Eq. (6.7), the first element is the sum of consumers' willingness to pay for each commodity at each market. The second element is the sum of production cost for each commodity at each market. The third element is the total generalized transportation cost. The supply $S^u_{b,c}$, the demand $D^u_{b,c}$, and the link flow $f^u_{l,c}$ are all in units of flow per hour; whereas SNB^u is in dollars per hour.

6.2.3 Adjustments to the Shippers' Net Benefit

It is important to note that two adjustments are required before including the shippers' net benefit in the calculation of the net social benefit. First, only part of the commodity transported via the port terminals is produced or consumed in the local region. The rest is bound for designations outside the region and as such, does not contribute to the region's net social benefit. The portion of the shippers' net benefit that directly contributes to the net social benefit of the local region is called the local shippers' net benefit. A ratio (υ_c) is used to denote the through-traffic as a percentage of the total freight of commodity c. Then, the local shippers' net benefit is expressed as a percentage of the total shippers' net benefit of commodity c by $1-\upsilon_c$. Second, in addition to the shippers' net benefit directly related to the local production and consumption of these traded commodities, other economic sections in the port vicinity are involved and benefit in a meaningful way. All associated manufacturing and services benefit from the traded commodities. To account for this external net benefit, a multiplier (ζ) is used to denote the ratio of external benefit to the localized shippers' net benefit.

Taking into account the passing through traffic and the external economy, the adjusted shippers' net benefit under investment strategy u ($ASNB^u$) is calculated in Eq. (6.8).

$$ASNB^u = (1+\zeta)*\sum_{c\in C}\left((1-\upsilon_c)*\sum_{b\in CN} SNB_{b,c}^u\right) = (1+\zeta)*\sum_{c\in C}\left((1-\upsilon_c)*\sum_{b\in CN}(CS_{b,c}^u + PS_{b,c}^u)\right) \quad \forall u \in U \tag{6.8}$$

Given the Stackelberg equilibrium (S^u, f^u, D^u, R^u, e^u), the net social benefit under investment strategy u (NSB^u) is calculated as:

$$NSB^u(S^u, f^u, D^u, R^u, e^u) = TNB^u(R^u, e^u) + ASNB^u(S^u, f^u, D^u) \quad \forall u \in U \tag{6.9}$$

The above discussion of the various sources of net social benefit is illustrated in Figure 6.2.

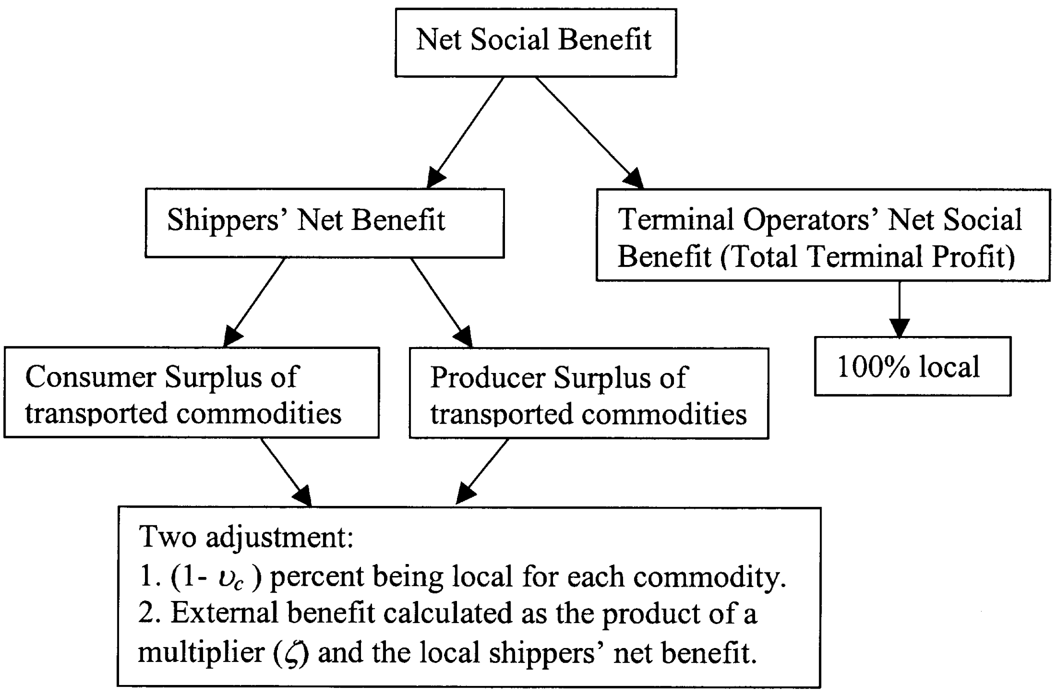


Figure 6.2 Net Social Benefit

6.3 Investment Cost

There are a finite number of alternative investment strategies. Associated with each investment strategy ($u \in U$) is a specific vector of capacity improvement pattern $\Delta\overline{E}^{u} = (\cdots, \Delta\overline{E}^{u}_{a}, \cdots)_{|A|}$. Under the do-nothing-strategy, $\Delta\overline{E}^{u} = 0$. The investment cost associated with an investment strategy is defined below.

To compare among different investment strategies, the capital expenses for the investment projects must be converted to common units. However, the facilities that are improved under different strategies may vary in their service lives. Even under the same strategy, different facilities may also vary in their service lives. In order to convert the capital expenses of different service lives into a commensurate unit, a common analysis period or planning horizon is designated. Then, the cost outlays for each year are estimated for the initial investment as well as replacements, if necessary, occurring over the planning horizon. Finally, the annual investment cost is converted into the same unit as the net social benefit, which is dollars per hour. More details on how to calculate the hourly investment cost are presented below.

The hourly investment cost on link a under investment strategy u is a function of the capacity improvement $\Delta\overline{E}^{u}_{a}$, the analysis period designated, the service life of the facility improved, and the discount rate. The total investment cost of the Port Authority is the sum of the investment costs on all improved links.

For the investment cost, a linear function similar to that shown in Yang and Meng (2000) is implemented. The flow dependent investment cost such as the maintenance cost is not considered. The investment cost on link a under investment strategy u (IC_a^u) is defined as follows:

$$IC_a^u = p2_a^u * (p1_a^u * \Delta\overline{E}_a^u) \qquad \forall a \in A, u \in U \qquad (6.10)$$

In Eq. (6.10), $p1_a^u$ is a parameter that represents the cost of one additional unit of capacity. The value of $p1_a^u$ is determined by the type of facility represented by link a and the resources such as technology used for investment strategy u. In Eq. (6.10), $p1_a^u * \Delta\overline{E}_a^u$ represents the capital expense for the capacity improvement of $\Delta\overline{E}_a^u$ on link a. The term $p2_a^u$ is a factor that converts the capital expense into an hourly investment cost. The value of $p2_a^u$ depends on the analysis period, the service life of this capital expense, and the discount rate. The method to calculate $p2_a^u$ is illustrated in Appendix E.

Let $p_a^u = p1_a^u * p2_a^u$. Then, Eq. (6.10) can be restated as: $IC_a^u = p_a^u * \Delta\overline{E}_a^u$. The total hourly investment cost under investment strategy u (IC^u) can be calculated as the sum of IC_a^u over all links. Thereby:

$$IC^u(\Delta\overline{E}^u) = \sum_a IC_a^u(\Delta\overline{E}_a^u) = \sum_a p_a^u * \Delta\overline{E}_a^u \qquad \forall a \in A, u \in U \qquad (6.11)$$

6.4 Mathematical Formulation of the Port Authority's Investment Problem

The objective of the Port Authority is to maximize the ratio between the incremental net social benefit brought about to the region through an investment, and the incremental investment cost. The incremental net social benefit through investment strategy u (ΔNSB^u) is calculated as follows:

$$\begin{aligned}&\Delta NSB^u(S^u,f^u,D^u,R^u,e^u)\\&= NSB^u(S^u,f^u,D^u,R^u,e^u) - NSB^0(S^0,f^0,D^0,R^0,e^0)\end{aligned} \qquad \forall u \in U \qquad (6.12)$$

where $NSB^u(S^u,f^u,D^u,R^u,e^u)$ is the net social benefit under investment strategy u. $NSB^0(S^0,f^0,D^0,R^0,e^0)$ is the net social benefit under the do-nothing strategy. The incremental investment cost is calculated as follows:

$$\Delta IC^u(\Delta\overline{E}^u) = IC^u(\Delta\overline{E}^u) - IC^0(\Delta\overline{E}^0) = IC^u(\Delta\overline{E}^u) \qquad \forall u \in U \qquad (6.13)$$

where $IC^u(\Delta\overline{E}^u)$ is the hourly investment cost under investment strategy u. The term $IC^0(\Delta\overline{E}^0)$ is the hourly investment cost under the do-nothing strategy, which equals to zero, since $\Delta\overline{E}^0$=0.

Combining Eq. (6.12) and Eq. (6.13), the investment problem for the Port Authority is defined and stated in Table 6.1.

Table 6.1 Port Authority's Investment Problem

P6.1

$$\frac{\Delta NSB^{u^*}(S^{u^*},f^{u^*},D^{u^*},R^{u^*},e^{u^*})}{IC^{u^*}(\Delta\overline{E}^{u^*})} = \underset{u\in U}{Max}\left[\frac{\Delta NSB^u(S^u,f^u,D^u,R^u,e^u)}{IC^u(\Delta\overline{E}^u)}\right] \qquad (6.14)$$

s.t. **P5.7** or **P5.8**

In problem P6.1, u^* denotes the most desirable investment strategy.

CHAPTER 7

CASE STUDY

This chapter focuses on solving the Stackelberg equilibrium between two oligopolistic, private port terminal operators that set service charges and routing patterns in the terminal sub-networks. At the same time shippers make commodity production, consumption and routing decisions on the shipper network by assuming the service charges of the carriers other than the terminal operators to be constant. Various strategies the Port Authority can use to invest in terminals are evaluated. A numerical example is developed to demonstrate: 1. the capability of the bi-level programming method and the sensitivity analysis method-based heuristic algorithm in solving the Stackelberg equilibrium; and 2. the applicability of the model in facilitating the Port Authority's investment decision. Through this numerical example, the power of the GAMS software package is demonstrated by implementing the bi-level program and the sensitivity based heuristic algorithm.

Section 7.1 presents the transportation networks used in the example. The attributes of the network elements, such as the link operating cost function on the terminal sub-networks, the inverse supply and demand functions, and the link generalized cost function on the shipper network, are defined. Section 7.2 presents the computational efficiency of the sensitivity analysis based-heuristic algorithm in solving the Stackelberg equilibrium of the terminal operators and the shippers. This is followed by the results from the GAMS model and the verification of the equilibrium conditions for both the shippers' SPE problem, and the terminal operators' oligopolistic pricing and routing problem. Finally, section 7.3 compares four proposed investment strategies by evaluating criteria from the perspective of various players.

7.1 Structure and Attributes of Network Elements

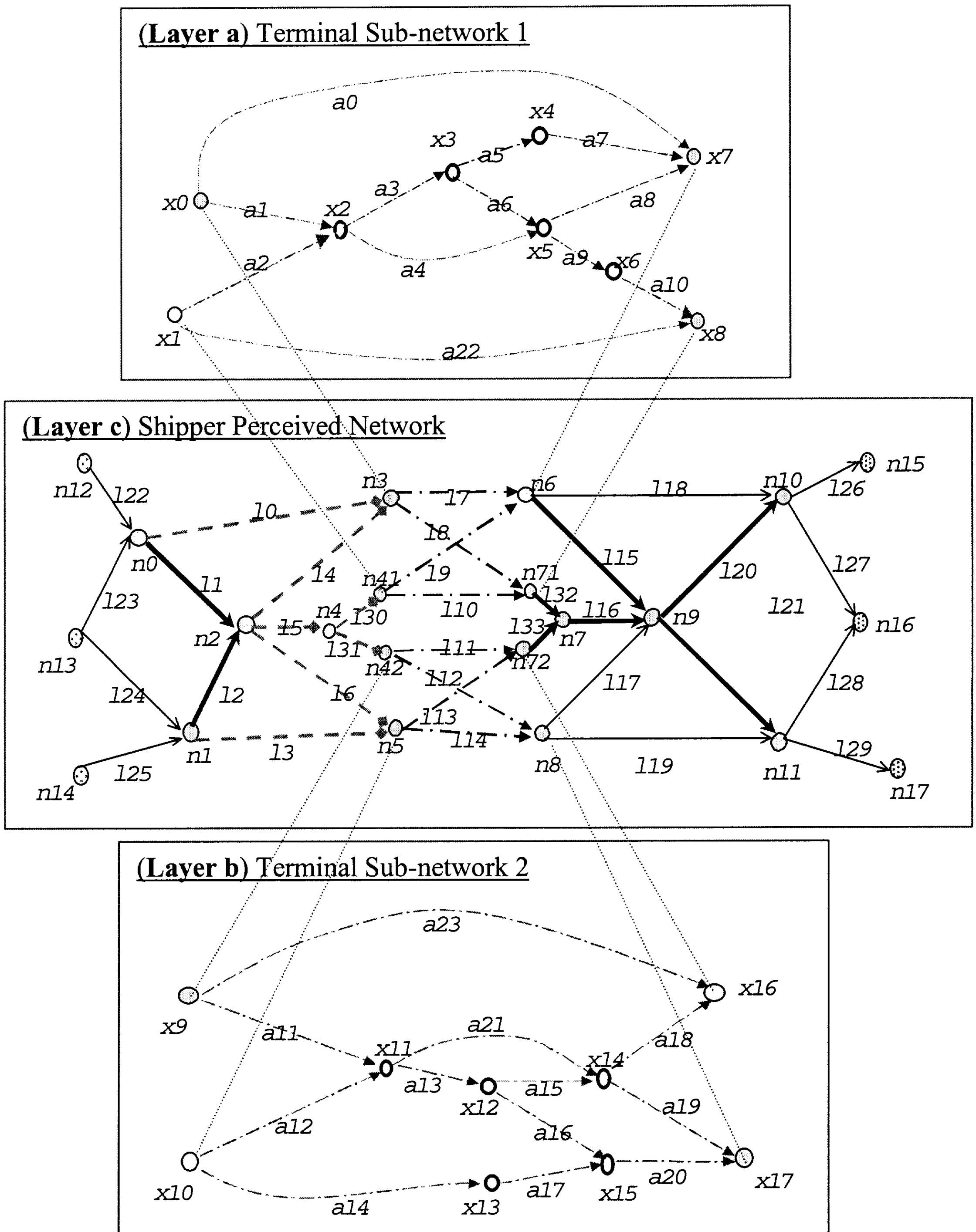
7.1.1 Terminal Operators' Sub-networks

The two terminal operators' sub-networks that are used in this numerical example are shown in Figure 7.1, *Layers a* and *b.* Terminal sub-network 1, shown in *Layer a* of Figure 7.1, consists of nine nodes (*x0-x8*), twelve links (*a0-a10*, *a22*) and four O-D pairs [*v0*=(*x0*, *x7*), *v1*=(*x0*, *x8*), *v2*=(*x1*, *x7*), *v3*=(*x1*, *x8*)]. Terminal sub-network 2, shown in *Layer b* of Figure 7.1, also consists of nine nodes (*x9-x17*), twelve links (*a11-a21*, *a23*) and four O-D pairs [*v4*=(*x9*, *x16*), *v5*=(*x9*, *x17*), *v6*=(*x10*, *x16*), *v7*=(*x10*, *x17*)].

7.1.2 The Shipper Network

The shipper network used in this numerical example is shown in *Layer c* of Figure 7.1. The network consists of 22 nodes (*n0-n17, n41, n42, n71, n72*) and 34 links (*l0-l33*). Among the 22 nodes, *n*12-*n*17 are centroid nodes, which represent nine O-D pairs ((*n12*, *n15*), (*n12*, *n16*), (*n12*, *n17*), (*n13*, *n15*), (*n13*, *n16*), (*n13*, *n17*), (*n14*, *n15*), (*n14*, *n16*), (*n14*, *n17*)).

The shipper network and the terminal sub-networks are related through the incidence matrix between the port links *l7-l14* in the shipper network and the O-D pairs *v0-v7* in the terminal sub-networks.



Legends:

⟶	Highway links	➞ (bold)	Railway links
— — ◆	Waterway links	— · ➞	Port links
-·-·-·➞	Links in terminal sub-networks	O	Intermediate nodes on the shipper network and origin/destination nodes on the terminal networks
⨀	Centroid nodes	**O**	Intermediate nodes at a terminal network

Figure 7.1 Transportation Networks for the Example

The incidence relationship between the port links and the terminal O-D pairs is:

$$\left[\xi_{l,v}\right]_{|L|*|V|} = \begin{array}{c} \\ l7 \\ \vdots \\ l14 \end{array} \begin{array}{c} \begin{array}{ccc} v0 & \cdots & v7 \end{array} \\ \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \end{array}$$

. This relationship is also manifested in Figure 7.1. For example, (*x0*, *x7*) are the origin and destination nodes of O-D pair *v0,* corresponding to (*n3*, *n6*), which are the starting and ending nodes of link *l7*.

In Figure 7.1, the shipper links (except *l7* to *l14)* correspond to certain O-D pairs on the sub-networks of the carriers other than the terminal operators. As mentioned before, the pricing and routing behavior of these other carriers is not the focus of this chapter. Their service charges are assumed to be constant, which indicates that the shippers will not change their perception of the generalized cost between the O-D pairs on the sub-networks of these carriers. Hence, the presentation of the sub-networks of these carriers as a set of links on the shipper network is sufficient for the demonstration purpose of this chapter.

7.1.3 Attributes of Network Elements

Tables F.1 through F.4 in Appendix F provide information on the values of various indexes associated with the case study network. The link operating cost function in the terminal sub-network, the inverse commodity supply, inverse commodity demand, link travel time and generalized cost functions are defined in Tables F.5-F.8 respectively. Parameters $\lambda_{b,c',c}$, $\beta_{b,c',c}$ in Table F.6 and $ro_{l,c',c}$ in Table F.7, which represent the interaction between commodity c and commodity c', are set in such a way that the strict monotone property of the inverse commodity supply, inverse commodity demand, and generalized cost functions is guaranteed. The capacity and the service charges on the shipper links are set at the values shown in Table F.8. The capacity (Cap_l) on a port link may be obtained as explained in Section 3.3.2, and the service charge on the port link is initialized at $R_{pl,c} = 0.2 * \underset{b1,b2 \in CN}{Max} (\alpha_{b2,c} - \lambda_{b1,c})$. The rationale is to initialize a service charge on the port link at a low level, so that, at equilibrium, the service charge is low enough to result in some service demand on this link. In order to have positive flow of commodity c on port link pl (i.e. $f_{pl \in L(p),c} > 0$), at least one path among the paths containing port link pl has positive path flow of commodity c. Let p connecting O-D pair $b1$ and $b2$ be such a path, that is $h_{p,c}$>0. The equilibrium condition $h_{p,c} * (\pi_{b1,c} + GC_{p,c} - \rho_{b2,c}) = 0$ from Section 5.1.2 indicates that: $R_{pl \in L(p),c} \le GC_{p,c} = (\rho_{b2,c} - \pi_{b1,c})$. Substituting $\pi_{b1,c}(S_{b1}) = \gamma_{b1,c} + \sum_{c'} \lambda_{b1,c',c} * S_{b1,c}$ and $\rho_{b2,c}(D_{b2}) = \alpha_{b2,c} - \sum_{c'} \beta_{b2,c',c} * D_{b2,c}$ into this inequality, it becomes: $R_{pl \in L(p),c} \le \left((\alpha_{b2,c} - \gamma_{b1,c}) - (\sum_{c'} \beta_{b2,c',c} * D_{b2,c} + \sum_{c'} \lambda_{b1,c',c} * S_{b1,c}) \right)$. Since $S_{b1,c}$ and $D_{b2,c}$ are also variables to be determined, it is natural to initialize the service charge ($R_{pl,c}$) based on $(\alpha_{b2,c} - \gamma_{b1,c})$. In this numerical example, the service charge is initialized at $0.2 * \underset{b1,b2 \in CN}{Max} (\alpha_{b2,c} - \lambda_{b1,c})$. According to the results of the GAMS program, this initial point results in convergence of the heuristic algorithm.

The value-of-time (vot_c) in the generalized cost function is set at \$2.50/day, \$10/day and \$5/day for commodity *c1*, *c2* and *c3* respectively. These values are arbitrarily selected; the accuracy of the value-of-time is not essential for the demonstration purpose of the numerical example in this chapter.

7.2. Computational Efficiency of the Heuristic Algorithm and Verification of Equilibrium Conditions of the Stackelberg Game

Based on the network structure and attributes information presented in Section 7.1, the specific form of the bilevel programming problems **P5.7** and **P5.8** from Section 5.3 is developed to formulate the Stackelberg game between the terminal operators and the shippers of this example, under either the competitive or collusive game. The sensitivity analysis method based heuristic algorithm is applied to solve these bilevel problems. The bilevel programming problem and the heuristic algorithm are implemented in a GAMS input file. The structure of the GAMS input file is not presented here. Interested readers can refer to Boilé et al. (1997, 1998) for a detailed discussion on the basic structure of a GAMS input file and the power of using the GAMS language to describe the network problem. However, the computational efficiency of this heuristic algorithm to solve the bilevel programming problems of this example is provided below.

7.2.1 Computational Efficiency

The heuristic algorithm from Section 5.3 iteratively solves the upper level terminal operators' pricing and routing problem and the lower level shippers' SPE problem via the linkage provided by the sensitivity analysis of the SPE problem. The upper level problem under the competitive game and the lower level problem are solved using the relaxation algorithms from Section 4.6 and the nonlinear serial decomposition algorithm from Section 5.1.6 respectively. In this example, all three algorithms are implemented in a GAMS input file by developing a set of GAMS models and using the iteration feature provided in GAMS language to solve them iteratively. These GAMS models correspond to the subproblems created in those three algorithms. Models are developed for the three single-commodity SPE problems created in the nonlinear serial decomposition algorithm. Each of these models has 85 equations and 139 variables.

Models are also developed for the two individual terminal operators' pricing and routing problems under the competitive game, which are created in the relaxation algorithm. Each of these models has 40 equations and 61 variables. In addition to the five models described above, another two GAMS models are developed for this example. One is the nonlinear optimization problem formulating the collusive game between two terminal operators, which has 79 equations and 131 variables. The other solves the set of equations in the linear program problem P5.6 from Section 5.2.1 for the sensitivity analysis of the lower level SPE problem; it has 55 equations and 73 variables. Since the problems corresponding to the seven models are either linear or nonlinear programming problems, all seven models are solved using a powerful solver implemented within GAMS, called CONOPT2 (Drud, 1996). CONOPT2 solves these models using the generalized reduced gradient algorithm (Bazaraa, 1993). These models were solved in 0.03 to 0.3 seconds on a PC with a 400-MHz Celeron processor.

The heuristic algorithm converges after 11 and 12 iterations for the collusive and competitive problems, respectively, when the stopping tolerance of 1E-8 is employed. The convergence pattern of the GAMS programs is shown in Figure 7.2. In this figure, the convergence ratio is calculated as $\sum_{v \in V} \sum_{c \in C} \frac{\left| R_{v,c}^{k} - R_{v,c}^{k-1} \right|}{R_{v,c}^{k} + R_{v,c}^{k-1}}$, with k denoting the number of iterations. For this sample problem, it is observed that the sensitivity analysis algorithm converges consistently for both the collusive game and the competitive game.

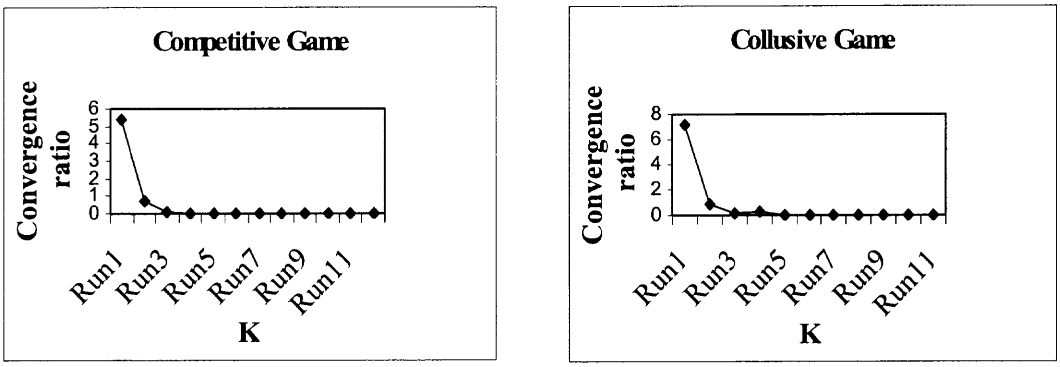


Figure 7.2 Convergence Pattern of the Heuristic Algorithm

With the convergence of the heuristic algorithm demonstrated above, the following two sections show that the solution that is obtained is, in fact, a Stackelberg equilibrium solution. First, the spatial price equilibrium will be demonstrated. Then, the optimality of the terminal operators' pricing and routing problem will be verified.

7.2.2 Verification of Spatial Price Equilibrium

Tables G.1 and G.2 in Appendix G show the solution of the shipper level problem including the commodity supply and demand, supply price at the origin, demand price at the destination, link flows and generalized costs. To show that the solution is a SPE solution, the conditions **E1-E2** specified in Section 5.1.2 must be verified. The verification of **E1** and **E2** involves the path enumeration and the path flow derivation. It would be an arduous task to enumerate all possible paths between each O-D pair; hence, only several paths between an O-D pair (*n13-n16*) are enumerated and **E1** and **E2** are verified for this O-D pair.

Figure 7.3 shows the generalized costs for commodity *c1* on the links between origin *n13* and destination *n16*.

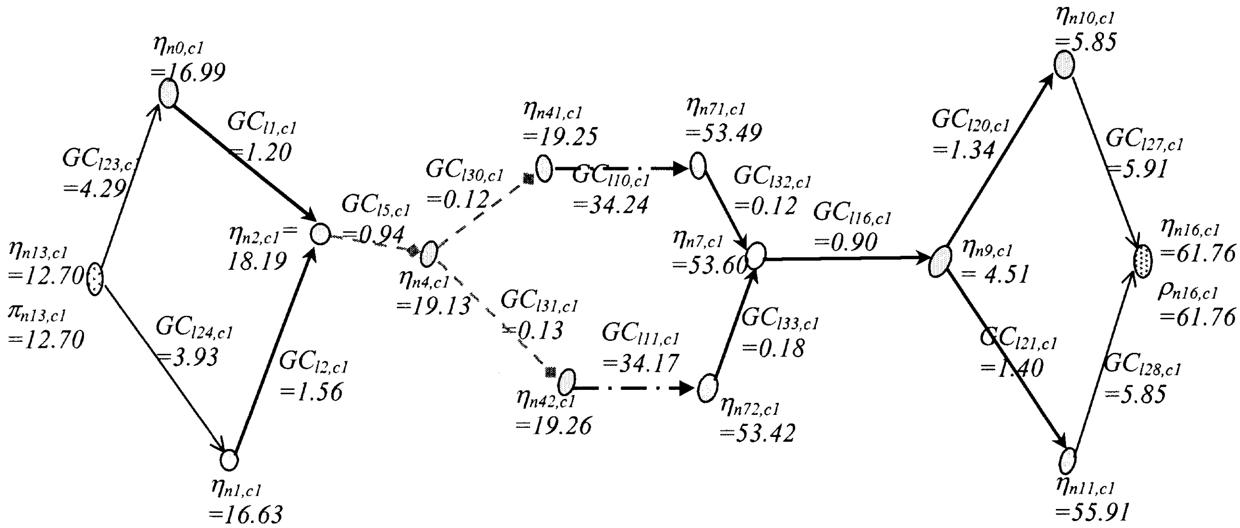


Figure 7.3 Solution for the Competitive Game

Between this O-D pair, there are 16 paths, among which four are enumerated below.

Path 1: l23-l1-l5-l30-l10-l32-l16-l20-l27

$GC_{p1,c1}$ = 4.29+1.20+0.94+0.12+34.24+0.12+0.90+1.34+5.91=$49.06/unit

Path 2: l23-l1-l5-l31-l11-l33-l16-l21-l28

$GC_{p2,c1}$ = 4.29+1.20+0.94+0.13+34.17+0.18+0.90+1.40+5.85=$49.06/unit

Path 3: l24-l2-l5-l31-l11-l33-l16-l21-l28

$GC_{p3,c1}$ = 3.93+1.56+0.94+0.13+34.17+0.18+0.90+1.40+5.85=$49.06/unit

Path 4: l24-l2-l5-l30-l10-l32-l16-l20-l27

$GC_{p4,c1}$ = 3.93+1.56+0.94+0.12+34.24+0.12+0.90+1.34+5.91=$49.06/unit

Similarly, the generalized costs on the other 12 paths can be calculated and it can be shown that they are all equal to $49.06/unit. Based on the generalized costs calculated above, the supply price and the market price shown in Figure 3.1, the following can be observed:

$GC_{p1,c1} = GC_{p2,c1} = GC_{p3,c1} = GC_{p4,c1} = Min_{pi(n13,n16)} GC_{pi,c1}$.

$\pi_{n13,c1} + Min_{pi(n13,n16)} GC_{pi,c1} - \rho_{n16,c1} = 12.70 + 49.06 - 61.76 = 0.$

Hence, **E1** and **E2** from Section 5.1.2 are satisfied for O-D pair (*n13-n16*).

For the other O-D pairs, similar verifications can be conducted that will not be repeated here. Instead, an alternative condition, which is in accord with the link and node incidence presentation, is provided in Proposition 7.1 and verified in place of **E1** and **E2**.

Proposition 7.1: A feasible solution $(S, f, D) \in KS$ is an equilibrium solution for the spatial price equilibrium problem, if and only if there exists a nonnegative vector $\eta = (\cdots, \eta_{i,c}, \cdots)\ \forall i \in N, c \in C$ satisfying Eqs. (7.1)-(7.3).

$$S_{a,c}(\pi_{a,c}(S_a) - \eta_{a,c}) = 0 \qquad \forall a \in CN,\ c \in C \tag{7.1}$$

$$D_{b,c}(-\rho_{b,c}(D_b) + \eta_{b,c}) = 0 \qquad \forall b \in CN,\ c \in C \tag{7.2}$$

$$f_{l,c}(\eta_{n1,c} + GC_{l=(n1,n2),c}(f_l) - \eta_{n2,c}) = 0 \quad \forall n1, n2 \in N,\ l = (n1,n2) \in L, c \in C \tag{7.3}$$

The proof of Proposition 7.1 is shown in Appendix H. Based on Proposition 7.1, the spatial price equilibrium can be verified by showing that there exists a nonnegative vector of the dual variables (η) that satisfies Eqs. (7.1)-(7.3). To demonstrate, verification for a few solutions of the collusive game is presented.

$S_{n12,c1}$=26.95 units/day>0 , and $\eta_{n12,c1} - \pi_{n12,c1}$=12.32-12.32=0.

→ $S_{n12,c1} * (\eta_{n12,c1} - \pi_{n12,c1}) = 0$.

$D_{n16,c3}$=8.52 units/day>0, and $\rho_{n16,c3}$- $\eta_{n16,c3}$=95.54-95.54=0.

→ $D_{n16,c3}$*($\rho_{n16,c3}$-$\eta_{n16,c3}$)=0.

$f_{l33,c1}$=25.026 units/day>0, *l33* connects *n72* and *n7*. $\eta_{n72,c2}$+$GC_{l33,c2}$-$\eta_{n7,c2}$ = 54.9162+0.1625-55.0787=0.→ $f_{l33,,c2}$*($\eta_{n72,,c1}$+$GC_{l33,,c2}$-$\eta_{n7,c2}$)=0.

Similar verifications can be conducted for all solutions. The conditions in Eqs. (7.1) and (7.2) are satisfied for each demand and supply market. This can be shown by following the above example and using the results from Table G.1, parts a and b. The condition in Eq. (7.3) is satisfied for all links as shown in Table G.2 parts a and b for the competitive and collusive games, respectively. Tables G.3 –G.5 present the Jacobian matrix of the locally approximated demand function for the competitive (G.3) and collusive (G.4) games and the verification of the diagonal dominance of the Jacobian matrix (G.5). Tables G.3 and G.4 also show that the data along the diagonal of the Jacobian matrix are negative. Therefore, diagonal dominance property indicates that the Jacobian matrix is negative definite (according to the proof in Appendix I). Hence, Proposition 5.2 proved in Appendix D is verified. More important is that the negative definite property guarantees that the upper level problem (i.e. the terminal operators' pricing and routing problem) is strictly convex. The strict convexity implies that the solution to the upper level problem is unique in the neighborhood of the locally approximated demand function. This demand function is a close guess of the relationship between the shippers' equilibrium link flow and the carriers' service charge. The verification of the optimality of this solution to the upper level problem is conducted below using a basic microeconomic principle; that is, at the profit maximization point where the marginal revenue equals the marginal cost (Tirole, 1988).

7.2.3 Verification of Optimality of the Terminal Operators' Pricing and Routing Problem

The solution of the terminal operators' pricing and routing problem that resulted from the GAMS model is shown in Tables G.6 and G.7 of Appendix G. Table G.6 presents the equilibrium travel demand, service charges and travel times, while Table G.7 presents the equilibrium link flows and operating costs. Link flows for all commodities are shown in Table G.8. To verify that the solution is optimal, the conditions shown in Eqs. (7.4) and (7.5) need to be verified.

$$g_{v,c} * (MR_{v,c} - MC_{v,c}) = 0 \qquad \forall v \in V, c \in C \tag{7.4}$$

$$fp_{p,c} * (MC_{p,c}(e) - MC_{v,c}) = 0 \qquad \forall p \in PH(v), c \in C \tag{7.5}$$

where $MC_{v,c} = \underset{p \in PH(v)}{Min} MC_{p,c}(e)$ and $MC_{p,c} = \sum_a \delta_{p,a} * MC_{a,c}(e_{a,c})$.

Marginal revenue on a terminal O-D pair v for commodity c ($MR_{v,c}$) is defined as the increase in revenue that results from providing one additional unit of service to the flow of commodity c on terminal O-D pair v. The marginal operating cost on a path $p \in PH(v)$ for commodity c ($MC_{p \in PH(v),c}$) is defined as the increase in the total operating cost of the terminal operator that results from providing one additional unit of service to the flow of commodity c on path p. The marginal cost on a path p is the summation of the marginal cost on all the links constituting this path. $MC_{v,c}$ is the minimum marginal cost on all paths connecting terminal O-D pair v for commodity c.

Marginal revenue ($MR_{v,c}$) is calculated based on information obtained from the sensitivity analysis. With $\nabla_{R_L} f(R_L^{k-1})$ derived at each iteration, both the locally approximated service demand function $g(R_V)$ and the inverse service demand function $R_V(g)$ can be estimated, and the marginal revenue can be calculated as shown below.

$$g(R_V) = g(R_V^{k-1}) + \nabla_{R_V} g(R_V^{k-1}) * (R_V - R_V^{k-1})$$

$$\text{with } \frac{\partial g_{v,c}}{\partial R_{v',c'}} = \sum_{l} \sum_{l'} \xi_{l,v} * \xi_{l',v'} * \frac{\partial f_{l,c}}{\partial R_{l',c'}}$$

$$R_V(g) = [\nabla_{R_V} g(R_V^{k-1})]^{-1} (\nabla_{R_V} g(R_V^{k-1}) * R_V^{k-1} - g(R_V^{k-1})) + [\nabla_{R_V} g(R_V^{k-1})]^{-1} * g$$

$$\text{let } \nabla_g R_V(g) = [\nabla_{R_V} g(R_V^{k-1})]^{-1}$$

$$MR = \frac{\partial (R_V(g)^T * g)}{\partial g} = R_V + \left[\frac{\partial R_V(g)}{\partial g} \right]^T * g = R_V + [\nabla_g R_V(g)]^T * g$$

The resulting marginal revenue and the minimum marginal cost for each terminal O-D pair for each commodity type are shown in Table G.9 of Appendix G. As shown in Table G.9, Eq. (7.4) is satisfied.

The results of path flow and the marginal path operating cost are shown in Table G.10 of Appendix G. From these data, two observations can be made for each O-D pair: 1. The marginal cost of any used path equals the lowest marginal cost among all the paths between this O-D pair; and 2. The marginal cost on any unused path is no less than the marginal cost on any used path between this O-D pair. For example, consider paths (*p0*, *p1*, *p2*, *p4*) connecting terminal O-D pair *v0*. Under the competitive game, all paths are used for commodity *c1*. Table G.10 shows that $MC_{p0,c1} = MC_{p1,c1} = MC_{p2,c1} = MC_{p4,c1}$ = \$9.678/unit. For commodity *c2*, paths *p0*, *p1* and *p4* are used, butpath *p2* is not used. Table G.10 shows that $MC_{p0,c1} = MC_{p1,c1} = MC_{p4,c1}$ = \$4.821/unit < $MC_{p2,c1}$ = \$4.880/unit. This indicates that Eq. (7.5) is satisfied. The routing pattern in the terminal sub-network minimizes the operating cost, while it satisfies the service demand generated by the current service charge.

The data of the terminal profits for both the collusive and the competitive game are shown in Table 7.1 below.

Table 7.1 The Terminal Profit under Collusive and Competitive Game

Pricing schemes	Profit at all terminals ($/hr)	Profit at terminal 1 ($/hr)	Profit at terminal 2 ($/hr)
Collusive game	7460.07	3278.97	4181.10
Competitive game	6810.59	3012.19	3798.40

As shown in Table 7.1, the total terminal profit obtained is $7460.07 per hour under the collusive game with $3278.97 from terminal 1's operation and $4181.10 from terminal 2's operation. This total profit is greater than the sum of the profits obtained by all individual terminal operators in the competitive game, that is $6810.59 per hour with $3012.19 from terminal 1's operation and $3798.40 from terminal 2's operation. Hence, the compensation principle is satisfied. A comparison of the profits of individual terminal operators shows that both terminal operators achieve better profits by cooperating. Therefore, collusion is likely to continue if the government does not interfere.

In summary, the pricing and routing pattern solved from the GAMS model is optimal for the terminal operators, and cooperation is beneficial to both terminal operators.

7.3 Using the Model to Evaluate Port Authority's Investment Decisions

The bi-level programming approach can be used by the Port Authority to identify the terminal links that are candidates for improvement, and evaluate the economic impact of various investment strategies. In this example, it is assumed that both terminal operators are under the same Port Authority. This application is demonstrated in this section.

7.3.1 Identifying the Candidate Terminal Links for Improvement

Denote the do-nothing strategy as *S0*. Using the results of the model, the candidate terminal links for improvement can be identified by comparing the current flow on a link (e_a^{S0}) with its capacity ($\overline{E}_a^{S0}$). The comparison is shown in Table G.8 of Appendix G. The links where flow exceeds capacity ($e_a^{S0} > \overline{E}_a^{S0}$) are candidates for improvement. These are *a0, a1, a4, a12, a13, a14, a17, a19, a20, a21, a22, and a23.*

The Port Authority can choose to invest in expanding the capacity of those candidate links to their current flow level, that is $\Delta\overline{E}_a = (e_a^{S0} - \overline{E}_a^{S0})$. Depending on the availability of funds, some or all candidate links may be improved. In this example, in addition to the do-nothing strategy *SO*, three investment strategies are envisioned. Their impact on the shippers and terminal operators are predicted using the bilevel model developed in this dissertation. The first investment strategy *S1* adds capacity on those links belonging to the first terminal: *a0 a1*, *a4*, and *a22*. The second investment strategy *S1* adds the capacity on those links belonging to the second terminal: *a12*, *a13*, *a14*, *a17*, *a19*, *a20*, *a21*, and *a23*. The third investment strategy *S3* adds the capacity on all of the candidate links. Table 7.2 presents the intended capacity improvement associated with investment strategies *S1*, *S2* and *S3*.

Table 7.2 shows that the competitive game has higher investment requirements than the collusive game. Under the competitive game, the terminal operator tends to charge a lower service fee, which results in more service demand. The induced demand results in capacity deficiencies.

Table 7.2 Capacity Improvement under Three Investment Strategies

	Collusive game			Competitive game		
Link	$\Delta\overline{E}_a^{S1}$ (units/hr)	$\Delta\overline{E}_a^{S2}$ (units/hr)	$\Delta\overline{E}_a^{S3}$ (units/hr)	$\Delta\overline{E}_a^{S1}$ (units/hr)	$\Delta\overline{E}_a^{S2}$ (units/hr)	$\Delta\overline{E}_a^{S3}$ (units/hr)
a0	10.8813	0	10.8813	15.7695	0	15.7695
a1	5.04124	0	5.04124	9.08145	0	9.08145
a4	0.27189	0	0.27189	4.61153	0	4.61153
a12	0	8.93173	8.93173	0	12.6596	12.6596
a13	0	7.02437	7.02437	0	11.0276	11.0276
a14	0	5.67637	5.67637	0	8.87876	8.87876
a17	0	10.1764	10.1764	0	13.3788	13.3788
a19	0	7.63156	7.63156	0	10.3923	10.3923
a20	0	9.929	9.929	0	14.3758	14.3758
a21	0	13.2731	13.2731	0	17.7631	17.7631
a22	15.1843	0	15.1843	23.4173	0	23.4173
a23	0	24.587	24.587	0	31.1064	31.1064

7.3.2 Comparison of Alternative Investment Strategies

For investment strategies *S1*, *S2* and *S3* proposed above, the equilibrium supply, demand and routing decision of the shippers as well as the equilibrium service charge and routing pattern in each terminal sub-network are forecast by solving the GAMS model with updated capacity information. Based on the equilibrium solution, various indexes regarding the port operation are calculated and compared for different investment strategies, as shown in Tables 7.3-7.4.

Table 7.3 shows that the investment of the Port Authority will generate additional service demand at the port terminal with capacity improvement, and may decrease the service demand at the port terminal without capacity improvement. For example, under the collusive game with the investment strategy *S1*, the service demand at port terminal 1 will increase from 102 units per hour to 108 units per hour. However, demand will decrease at port terminal 2 from 123 to 120 units per hour. Given the cost advantage resulting from the capacity improvement, terminal 1 is able to lower the service charge and attract some of terminal 2's business. The increase in average travel time that results from the increased demand at terminal 1 is greater than the travel time savings resulting from the increase in capacity. Consequently, an overall increase in the average travel time results. For example, for the competitive game, the investment strategy *S1* causes the average travel time at port terminal 1 to increase from 4.41 hours per unit to 4.69 hours per unit.

Table 7.3 Total Demand, Average Travel Time, Total Revenue and Total Profit under Four Investment Strategies

		Terminal 1				Terminal 2			
	Strategy	Total demand (units/hr)	Average Travel Time (hrs/unit)	Total Revenue ($/hr)	Total Profit ($/hr)	Total Demand (units/hr)	Average Travel Time (hrs/unit)	Total Revenue ($/hr)	Total Profit ($/hr)
Collusive Game	*S0*	102	3.87	3634	3279	123	2.81	4908	4181
	S1	108	4.05	3714	3429	120	2.77	4806	4104
	S2	96	3.68	3390	3075	139	3.14	5190	4623
	S3	102	3.85	3473	3219	137	3.09	5091	4542
Competitive Game	*S0*	123	4.41	3520	3012	141	3.14	4762	3798
	S1	132	4.69	3534	3171	139	3.10	4612	3682
	S2	118	4.23	3236	2771	163	3.59	4883	4199
	S3	127	4.51	3250	2918	160	3.53	4739	4078

Table 7.3 also shows that investment in one terminal can have the opposite effect on the other terminal. For example, the investment strategy *S2* increases the revenue and profit at port terminal 2 while it decreases the revenue and profit at port terminal 1 under both the collusive and competitive games. This result is attributed to the decrease of the service charge at terminal 1 in order to maintain business and the resulting decrease in demand for terminal 1 service. Table 7.3 also shows that the investment strategy *S3* may have different effects on the revenue and profit at the two terminals even though both terminals receive investment. For example, the investment strategy *S3* under the

competitive game decreases the profit at port terminal 1 while it increases the profit at port terminal 2. The investment intensifies the price competition between the two terminals, thus reducing the equilibrium service charges at both terminals as shown in Table 7.4.

Table 7.4 Equilibrium Service Charge of Commodity *c3*

O-D Pair		*R* ($/unit) for Collusive Game				*R* ($/unit) for Competitive Game			
		S0	*S1*	*S2*	*S3*	*S0*	*S1*	*S2*	*S3*
Terminal 1	*v0*	47	45	46	45	37	34	35	33
	v1	46	44	45	44	35	32	33	31
	v2	45	44	45	44	34	32	32	31
	v3	46	45	46	45	34	32	33	31
Terminal 2	*v4*	49	49	46	46	39	38	34	34
	v5	48	48	46	46	39	38	35	34
	v6	50	49	47	47	40	40	36	35
	v7	51	51	49	49	43	42	39	38

Depending on the elasticity of the service demand with respect to the service charge, revenue may increase or decrease with the reduction of the service charge. If the elasticity of the service demand is less than 1, the negative effect on the revenue from the decrease in service charge dominates the positive effect from the increase in demand. Hence, revenue will decrease. If the elasticity is greater than 1, revenue will increase. In this example, investment strategy *S3* decreases the revenue at port terminal 1 in spite of the increase in the service demand from 123 to 127 units per hour. This result indicates that, at the current equilibrium point, the demand elasticity at port terminal 1 is less than 1.

The Port Authority is also interested in the net social benefit (*NSB*) and the investment cost (*IC*) under different investment strategies. In calculating *NSB*, the economic multiplier ς is set to 4 to account for the external economy. To demonstrate the effect on *NSB* from the percentage of the through traffic, a set of different percentages of through traffic ($\upsilon_c = \upsilon \; \forall c \in C$) is used. In calculating *IC*, p_a^u is set to \$5 per unit for any terminal link *a* and any investment strategy *u*. Similar to the value-of-time, the values for these three parameters are also selected arbitrarily. Again, the accuracy of these arbitrary selected values does not affect the demonstration purpose of the numerical example. Based on these values of the parameters ς, p_a^u and υ_c and the Stackelberg equilibrium result of the GAMS model, *NSB* and *IC* for each investment strategy are calculated according to the formulas in Eqs. (6.9) and (6.11) from Sections 6.2.3 and 6.3 and are shown in Table 7.5.

Table 7.5 Net Social Benefit (*NSB*) and Investment Cost (*IC*)

	Collusive Game					Competitive Game				
	NSB (\$/hr)				*IC*	*NSB* (\$/hr)				*IC*
Strategy	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$	(\$/hr)	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$	(\$/hr)
S0	7460	9855	12249	15442	0	6811	9926	13042	17196	0
S1	7534	9992	12451	15729	157	6853	10116	13379	17729	264
S2	7698	10281	12863	16307	436	6970	10447	13923	18559	598
S3	7761	10401	13041	16560	593	6995	10607	14219	19034	862

Two observations can be made based on Table 7.5. First, if no freight is produced or consumed locally ($\upsilon=1$), the *NSB* is higher for the collusive game under all investment strategies. Collusion brings more profit to the terminal operation as a whole, thus increasing both operators' contribution in the *NSB* formula. Second, with the decrease in the percentage of through traffic, the *NSB* becomes higher for the competitive game. The competitive game generates more commodity production, consumption and shipment. With the percentage of through traffic low enough, the positive effect on *NSB* from the gain in local shippers' net benefit will more than offset the negative effect from the loss of total profit at port operation.

Based on the net social benefit (*NSB)* and the investment cost (*IC*) shown in Table 7.5, the incremental benefit-cost ratios under the various investment strategies are calculated according to the formulas in Eq. (6.14) from Section 6.5. These ratios are shown in Table 7.6

Table 7.6 The Ratio between Incremental Net Social Benefit (ΔNSB) and Incremental Investment Cost (ΔIC)

	Collusive Game				Competitive Game			
	$\Delta NSB/\Delta IC$				$\Delta NSB/\Delta IC$			
Strategy	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$
S0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S1	0.47	0.88	1.29	1.83	0.16	0.72	1.27	2.02
S2	0.55	0.98	1.41	1.98	0.27	0.87	1.47	2.28
S3	0.51	0.92	1.33	1.89	0.21	0.79	1.36	2.13

Table 7.6 shows that when through traffic is high, the do-nothing strategy may become the best choice, since the gain in *NSB* may not be enough to cover the increase in *IC*. For example, for both the competitive and collusive games, when the proportion of through traffic is 70 or 100 percent, the ratio between the incremental *NSB* and *IC* is less than one under all investment strategies. Table 7.6 also shows that with the decrease of through traffic, the investment strategy for the competitive game results in higher incremental benefit-cost ratios. When through traffic is high, there are lower incremental benefit-cost ratios.

Observations from Tables 7.5 and 7.6 together with the result in Table 7.3 indicate that, if a very high percentage of the freight is not produced or consumed locally, both the terminal operators and the Port Authority favor collusion of the terminal operators under the control of one Port Authority. In this case, the social benefit to the local region brought by the terminal operations is mainly contributed by the terminal profits, which are higher under the collusive game. With the decrease of this percentage and the investment, the weight of the shippers' contribution to the net social benefit becomes higher. The Port Authority prefers the competitive game, since it will bring about more shippers' net benefit and accordingly more net social benefit to the local region.

CHAPTER 8

FUTURE RESEARCH DIRECTIONS

The dissertation has proposed and developed a framework to analyze the pricing and routing behavior of freight carriers. It explored this application in facilitating the investment decisions of the Port Authority in the background of the general freight network that recognizes market-clearing forces. In particular, a bi-level programming approach has been introduced to solve the Stackelberg equilibrium between carriers and shippers, and the related investment issue is shown to be computationally tractable. The sensitivity analysis based heuristic algorithm that was used to solve the bi-level model was implemented within GAMS, a mathematical programming software package.

Possible extensions to the work presented in this dissertation are discussed below.

The model application presented in this dissertation analyzed the behavior of a special type of carrier, namely two port terminal operators. Pricing and routing of all other carriers were assumed to be fixed. Alternative investment strategies at the port terminals were also evaluated. The model and the algorithm is flexible, however, and can be adopted to analyze the pricing and routing decision of any or all other private carriers. In addition, the investment issues at other kinds of transportation terminal, such as a logistics terminal or airport terminal could be examined as well. For example, the public logistics terminal in Japan involves both the public and private sectors (Taniguchi et al, 1999), including various trucking companies. Issues that may be examined by using the methodology developed in this dissertation include the optimal location of the terminal, its optimal size, and how the investment decisions of a government interact with the routing decisions of a private company.

The hierarchical model can be extended to assess the impact of policies in various states, such as tax policy, investment policy and so on, on the location of the production and consumption pattern. The model can be used to analyze the interstate competition. To illustrate this point, suppose that one state invests in its transportation infrastructure. The investment is intended to increase the state's accessibility and attract more businesses. However, it may, at the same time, generate more traffic to the transportation facilities in the state, inducing higher congestion and land costs, and consequently, higher production cost. The end result could be that, despite an investment in transportation infrastructure, the state becomes a less desirable location for business than its neighboring states. The neighboring states may benefit more from the transportation infrastructure investment than the state that made the initial investment. To make sound decisions, planners need to balance these gains and losses carefully. The model introduced in this dissertation can be used to identify gains and losses resulting from the interactive relationship between traffic flow, production, consumption and the attributes of the transportation network such as capacity and cost, which provide criteria for decision making. Another application is to evaluate user, societal and environmental impacts of a particular public policy (e.g., introducing tolls on an interstate road) on a region.

A discrete value-of-time is assumed for each commodity. In the future, the value of time can be modified so that it follows a distribution (e.g., the normal distribution). The distribution pattern of the value-of-time differs according to the commodity type and the origin/destination of the commodity being shipped. In an application to passenger transportation, Dial (1979, 1996) develops a bicriteria traffic assignment model to analyze the assignment of traffic, the value of time of which has different distribution between different O-D pairs.

Numerous shippers are assumed in this dissertation and the shippers' market is considered to be purely competitive. A future study may relax these assumptions and consider, for example, that a finite number of shippers who compete for the limited resources for production and transportation exist in the network. The interaction of these shippers can be modeled using the spatial oligopolistic model by Nagurney (1999) as reviewed in Section 2.2.1. Accordingly, the bi-level model in this dissertation can be extended to analyze the interaction between the oligopolistic carriers and the oligopolistic shippers. Summarizing, the model has the potential to be a useful tool for transportation planning.

APPENDIX A PROOF OF THE STRICTLY CONCAVE PROPERTY OF THE OBJECTIVE FUNCTION IN EQS. (4.6) AND (4.7)

The objective function

$Z_t(g_t(R_t, R_{-t}), R_t, e_t) = \sum_{v\in V^t}\sum_{c} g_{v,c}(R_t, R_{-t}) * R_{v,c} - \sum_{a\in A^t}\sum_{c} AC_{a,c}(e_{a,c}) * e_{a,c}$ is strictly concave in (R_V, e). Its Hessian matrix is negative definite as shown below.

$$\frac{\partial^2 Z_t}{\partial R_{v,c}\partial R_{v',c'}} = \frac{\partial^2 Z_t}{\partial R_{v',c'}\partial R_{v,c}} = \frac{\partial g_{v,c}}{\partial R_{v',c'}} + \frac{\partial g_{v',c'}}{\partial R_{v,c}} \qquad \forall t \in T, v, v' \in V,\ c, c' \in C$$

$$\frac{\partial^2 Z_t}{\partial e_{a,c}\partial e_{a,c}} = -\frac{\partial MC_{a,c}}{\partial e_{a,c}} < 0,\ \frac{\partial^2 Z_t}{\partial e_{a,c}\partial e_{a'\neq a, c'\neq c}} = 0 \qquad \forall t \in T, a, a' \in A, c, c' \in C$$

$$\frac{\partial^2 Z_t}{\partial R_{v,c'}\partial e_{a,c}} = \frac{\partial^2 Z_t}{\partial e_{a,c}\partial R_{v,c'}} = 0 \qquad \forall t \in T, v \in V, a \in A, c, c' \in C$$

$$\Rightarrow H(Z_t) = \begin{bmatrix} [\nabla_{R_V} g_V(R_V)] + [\nabla_{R_V} g_V(R_V)]^T & 0 \\ 0 & -\nabla_e MC(e) \end{bmatrix} \qquad \forall t \in T \qquad \text{(A.1)}$$

According to assumption **A2** from Section 4.1, $[\nabla_{R_V} g_V(R_V)]$ is negative definite. Therefore, its transposed matrix $[\nabla_{R_V} g_V(R_V)]^T$ is also negative definite. Then, the sum of two negative definite matrices $[\nabla_{R_V} g_V(R_V)] + [\nabla_{R_V} g_V(R_V)]^T$ is also negative definite. This matrix is also symmetric. From assumption **A1** in Section 4.1, $-\nabla_e MC(e)$ is also negative definite and symmetric. Therefore, the Hessian matrix $H(Z_t)$, which as shown in Eq. (A.1) consists of $[\nabla_{R_V} g_V(R_V)] + [\nabla_{R_V} g_V(R_V)]^T$ and $-\nabla_e MC(e)$ along the diagonal and zero elsewhere, is also negative definite and symmetric.

The negative definite property of the Hessian matrix $H(Z_t)$ indicates that: 1. The part of the objective function Eq. (4.7) contributed by carrier t ($Z_t(g_t(R_t, R_{-t}), R_t, e_t)$) is strictly concave in (R_V, e). 2. The objective function for the collusive game (Z), as a summation of $Z_t(g_t(R_t, R_{-t}), R_t, e_t)$ is strictly concave in (R_V, e). 3. The vector of the first derivative of the objective function (Eq. (4.6) of individual carrier $\nabla Z_t(g_t(R_t, \overline{R}_{-t}), R_t, e_t)$ is strictly monotone decreasing in (R_t, e_t).

APPENDIX B SENSITIVITY ANALYSIS FOR A PERTURBED VI PROBLEM

Let ε Denote the perturbation parameter. A perturbed VI problem **PB.1** can be defined in Table B.1:

Table B.1 A Perturbed VI Problem

PB.1
Determine $x^* = (\cdots, x_i^*, \cdots)_n^T \in K$
$F(x^*, \varepsilon) * (x - x^*) \geq 0$ $\forall x = (\cdots, x_i, \cdots)_n^T \in K$ $K: \left\{ x \middle\vert \begin{array}{ll} G(x) = 0 & (\eta) \\ x \geq 0 & (\varpi) \end{array} \right\}$

The sensitivity analysis method for problem **PB.1** estimates the derivative of the equilibrium solution of the primal and dual variables to problem **PB.1** (x^*, η^*) with respect to ε in the neighborhood of ε=0.

Four conditions are required for the sensitivity analysis of problem **PB.1**.

1. $F(x, \varepsilon)$ is once continuously differentiable in (x, ε). The equality constraints $G(x)$=0 are linear affine and the nonnegativity constraints $x \geq 0$ are concave.
2. Solution to **PB.1** at ε=0 is a local unique solution.
3. Solution to **PB.1** at ε=0 satisfies the strictly complementary slackness condition, that is if x_i=0 then ϖ>0.
4. The gradients of the binding nonnegativity constraints and the gradients of the equality constraints are independent.

With these four conditions satisfied, there exists a locally unique solution of the primal and dual variables $\left(x^*(\varepsilon),\eta^*(\varepsilon)\right)$ in the neighborhood of ε=0.

$x^*(\varepsilon=0)$ and $\eta^*(\varepsilon=0)$ satisfies the KKT conditions below:

$$\begin{cases} F(x^*,0)+\nabla_x G(x^*)^T\eta^* = 0 & \forall x^* > 0 \quad \text{(B.1)} \\ G(x^*)=0 & \forall x^* > 0 \quad \text{(B.2)} \end{cases}$$

The Jacobian matrix of Eqs. (B.1) and (B.2) with respect to (x, η) is:

$$J^*_{(x,\eta)} = \begin{bmatrix} \nabla_x F(x^*,0) & \nabla_x G(x^*)^T \\ \nabla_x G(x^*) & 0 \end{bmatrix}. \tag{B.3}$$

The Jacobian matrix of Eqs. (B.1) and (B.2) with respect to the perturbation variable ε is:

$$J^*_\varepsilon = \begin{pmatrix} -\nabla_\varepsilon F(x^*,\varepsilon) \\ 0 \end{pmatrix}.$$

Then, the first-order derivative of $x^*(\varepsilon)$ and $\eta^*(\varepsilon)$ with respect to ε can be calculated as follows:

$$\begin{pmatrix} \dfrac{\partial x^*}{\partial \varepsilon} \\ \dfrac{\partial \eta^*}{\partial \varepsilon} \end{pmatrix} = \begin{bmatrix} \nabla_x F(x^*,0) & \nabla_x G(x^*)^T \\ \nabla_x G(x^*) & 0 \end{bmatrix}^{-1} \bullet \begin{pmatrix} -\nabla_\varepsilon F(x^*,\varepsilon) \\ 0 \end{pmatrix}. \tag{B.4}$$

APPENDIX C CONDITIONS FOR THE SENSITIVITY ANALYSIS METHOD OF SPE MODEL P5.1

The four conditions from Appendix B specific for the sensitivity analysis of problem **P5.1** are shown to be satisfied below.

1. Functions $(\pi(S), GC(f, R_L), -\rho(D))$ are once continuously differentiable in (S, f, D, R_L). The flow conservation constraints (Eqs. (5.7) and (5.8)) are linear affine and the nonnegativity constraint (Eq. (5.9)) is concave.

$(\pi(S), GC(f, R_L), -\rho(D))$ are guaranteed to be continuously differentiable by assumptions **A1** and **A2** from Section 5.1.1. The linear affine property of Eqs. (5.7) and (5.8) and the concave property of Eq. (5.9) are obvious since all these equations are linear.

2. The equilibrium solution (S^*, f^*, D^*) to problem **P5.1** is locally unique.

This is proven in Section 5.1.5.

3. The strictly complementary slackness condition holds for the solution to problem **P5.1** (i.e., if a nonnegativity constraint is binding, the dual variable for this constraint is positive).

These conditions are shown in Eqs. (C.1)-(C.3) below.

$$\pi_{b,c}(S^*) > \eta_{b,c} \quad \text{if } S^*_{b,c} = 0 \quad \forall b \in CN, c \in C \tag{C.1}$$

$$\rho_{b,c}(D^*) < \eta_{b,c} \quad \text{if } D^*_{b,c} = 0 \quad \forall b \in CN, c \in C \tag{C.2}$$

$$GC_{l=(n1,n2),c}(f^*) + \eta_{n1,c} > \eta_{n2,c} \quad \text{if } f^*_{l,c} = 0 \quad \forall l \in L, c \in C \tag{C.3}$$

If Eqs. (C.1)-(C.3) are not satisfied, problem **P5.1** can be slightly perturbed until the satisfactory result is obtained.

4. The gradients of flow conservation constraints (Eqs. (5.7) and (5.8)) and the binding nonnegativity constraint (Eq. (5.9)) are independent.

To demonstrate this, the proof similar to that of Tobin (1987) is used here. In Tobin's paper, the commodity production or consumption is assumed to take place at any node on the shipper network. Unlike Tobin's approach, in this dissertation the nodes on the shipper network are categorized into intermediate nodes and centroid nodes depending on whether there is a production or consumption activity. Correspondingly, the flow conservation constraints are different for the centroid node and the intermediate node. This difference is taken into account in the proof below.

Proof:

Using the notation of Tobin (1987), $\dot{I}_f, \dot{I}_S, \dot{I}_D$ are the matrices remaining after the columns corresponding the nonbinding nonnegative constraints are deleted from the identity matrix of the order of the number of f, D and S. The conditions of the independence of gradients are equivalent to the full rank of the following matrix:

$$\begin{bmatrix} \Lambda_{CN}^T & I & -I \\ \Lambda_{IN}^T & 0 & 0 \\ 0 & \dot{I}_S & 0 \\ \dot{I}_f & 0 & 0 \\ 0 & 0 & \dot{I}_D \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{Gradients of the flow conservation constraints Eqs. (5.7) and (5.8).} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Gradients of the binding nonnegativity constraints Eq. (5.9).} \end{matrix}$$

Based on assumption **A5** from Section 5.1.1, which states that each commodity flows through each node, the rows in $[\Lambda_{IN}^T \quad 0 \quad 0]$ and $[\overset{\bullet}{I_f} \quad 0 \quad 0]$ are independent. According to assumption **A3** from Section 5.1.1 and the strictly complementary slackness conditions in Eqs. (5.30) and (5.31), the supply and the demand for each commodity type can not both be zero at any centroid node. This indicates that the rows in $[\Lambda_{CN}^T \quad I \quad -I]$ are independent from the rows in $[0 \quad \overset{\bullet}{I_S} \quad 0]$ and $[0 \quad 0 \quad \overset{\bullet}{I_D}]$. The independence between $[\overset{\bullet}{I_f} \quad 0 \quad 0]$, $[0 \quad \overset{\bullet}{I_S} \quad 0]$, and $[0 \quad 0 \quad \overset{\bullet}{I_D}]$ is obvious. Hence, it is shown that the above matrix is of full rank.

APPENDIX D PROOF OF PROPOSITION 5.2

Proposition 5.2: The vector of the locally approximated flow functions (i.e., $\left(\cdots, f_{l\in L,c\in C}(R_L),\cdots\right)_{|L|*|C|}$ with $f_{l,c}(R_L) = f_{l,c}(R_L^0) + \sum_{lj\in L}\sum_{ci\in C}\frac{\partial f_{l,c}(R_L^0)}{\partial R_{lj,ci}}(R_{lj,ci} - R_{lj,ci}^0)$ and R_L^0 being the vector of the current service charges) is monotone decreasing in the vector of the service charges (R_L) if the vectors of the inverse commodity supply, inverse commodity demand and generalized cost functions are strictly monotone. (For the sub-vector of the flow variables that have positive equilibrium solutions at the current level of service charges, the sub-vector of the locally approximated functions is strictly monotone decreasing in the sub-vector of the service charges corresponding to these flow variables.)

Proof:

The proof is conducted by verifying the Jacobian matrix of the local approximated functions: $J = \left[\frac{\partial f_{l\in L,c\in C}(R_L^0)}{\partial R_{lj\in L,ci\in C}}\right]_{(|L|*|C|)*|L|*|C|)}$ is negative semidefinite. This is accomplished by using the sensitivity analysis result from Tobin (1987) applied to the price formulation of spatial price equilibrium (SPE) model. The advantage of the price formulation over the quantity formulation is that the derivative of the link flow is separable from the derivative of the commodity supply and demand.

From assumption **A2** in Section 5.1.1, both the vector of the inverse supply functions and the vector of the inverse demand functions are linear and strictly monotone. This indicates that the vector of the supply functions and the vector of the demand functions are also linear and strictly monotone. Using the supply function and the demand function, the complementary formulation of the SPE problem similar to the one in Tobin (1987) is presented in Table D.1. In this formulation, the difference in the intermediate node and the centroid node is accounted for.

Table D.1 Complementary Formulation of SPE problem with Supply Function and Demand Function

PD.1

$$[GC(f)+\Lambda\pi]f+\left[S(\pi)-D(\pi)-\Lambda_{CN}^{T}f\right]\pi_{CN}+\left[-\Lambda_{IN}^{T}f\right]\pi_{IN}=0 \quad \text{(D.1)}$$

$$GC(f)+\Lambda\pi \geq 0 \quad \text{(D.2)}$$

$$S(\pi)-D(\pi)-\Lambda_{CN}^{T}f \geq 0 \quad \text{(D.3)}$$

$$-\Lambda_{IN}^{T}f \geq 0 \quad \text{(D.4)}$$

$$f \geq 0 \quad \text{(D.5)}$$

$$\pi \geq 0 \quad \text{(D.6)}$$

Applying the sensitivity analysis method for the complementary problem introduced in Tobin (1987) to problem **PD.1**, the following can be derived.

$$\begin{bmatrix}\nabla_{R'}f' \\ \nabla_{R'}\pi'\end{bmatrix}=\begin{bmatrix}\nabla_{f'}GC'(f,R) & \Lambda'_{IN} & \Lambda'_{CN} \\ -\Lambda'^{T}_{IN} & 0 & 0 \\ -\Lambda'^{T}_{CN} & 0 & \nabla_{\pi'}S'(\pi)-\nabla_{\pi'}D'(\pi)\end{bmatrix}^{-1}\begin{bmatrix}-\nabla_{R'}GC'(f,R) \\ 0\end{bmatrix} \quad \text{(D.7)}$$

Where, f', R', $GC'(f,R)$ and $\Lambda'=[\Lambda'_{IN} \quad \Lambda'_{CN}]$ denote for each commodity the vectors of flow variables, service charges, the generalized cost functions and the link node incidence matrix on the links with positive current flows. π', $S'(\pi)$ and $D'(\pi)$

denote for each commodity the vectors of the price variables, the supply functions and the demand functions for the nodes with positive current prices.

Let $\Psi = \begin{bmatrix} \nabla_{f'} GC'(f,R) & \Lambda'_{IN} \\ -\Lambda'^{T}_{IN} & 0 \end{bmatrix}$, $\Gamma = [\nabla_{\pi'} S'(\pi) - \nabla_{\pi'} D'(\pi)]$, $M = \begin{bmatrix} \Lambda'_{CN} \\ 0 \end{bmatrix}$ and $\begin{bmatrix} \Psi & M \\ -M^T & \Gamma \end{bmatrix}^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$.

Then, Eq. (D.7) can be restated as follows:

$$\begin{bmatrix} \nabla_{R'} f' \\ \nabla_{R'} \pi' \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} -\nabla_{R'} GC'(f,R) \\ 0 \end{bmatrix} \qquad \text{(D.8)}$$

Let $B_{11} = \begin{bmatrix} B_{111} & B_{112} \\ B_{113} & B_{114} \end{bmatrix}$ with B_{111} having dimension equals to $|f'|*|\,|R'|$. Here, both $|f'|$ and $|R'|$ equals to the summation of the number of links with positive current flows for each commodity (i.e., $|f'|=|R'|=\sum_{c\in C} \text{No. of links where } f_{l,c} > 0$). From Eq. (D.8), it is derived that:

$$\nabla_{R'} f' = -B_{111} * \nabla_{R'} GC'(f,R) \qquad \text{(D.9)}$$

In matrix $\nabla_{R'} GC'(f,R)$, the elements along the diagonal equal to 1 and all the elements off the diagonal are equal to 0. Hence, it follows:

$$\nabla_{R'} GC'(f,R) = I. \qquad \text{(D.10)}$$

Here, I is an identity matrix with dimension equals to $|f'|*|R'|$.

Equations (D.9) and (D.10) yield that:

$$\nabla_{R'} f' = -B_{111} \qquad \text{(D.11)}$$

From Eq. (D.11), it is easy to see that $\nabla_{R'} f'$ is negative definite if and only if B_{111} is positive definite. The sufficient condition for B_{111} to be positive definite is that B_{11} is positive definite. The latter is proved below.

As shown in Tobin (1987), $B_{11} = \left[\Psi + M\Gamma^{-1}M^T\right]^{-1}$. Hence, B_{11} is positive definite if $\left[\Psi + M\Gamma^{-1}M^T\right]$ is positive definite.

Ψ is shown to be positive definite below. According to the assumption **A4** from Section 5.1.1, $\nabla_{f'} GC'(f,R)$ is positive definite. It follows that:

$$x^T \Psi x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} \nabla_{f'} GC'(f,R) & \Lambda'_{IN} \\ -\Lambda'^T_{IN} & 0 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= x_1^T * \nabla_{f'} GC'(f,R) * x_1 > 0 \tag{D.12}$$

Hence, Ψ is positive definite.

As mentioned before, the vector of the supply functions and the vector of the demand functions are strictly monotone, which indicates that Γ is positive definite. Hence, Γ^{-1} is also positive definite. It follows that:

For any vector x, let $y = M^T x$, then
$$x^T M\Gamma^{-1}M^T x = \left(M^T x\right)^T \Gamma^{-1}\left(M^T x\right) = y^T \Gamma^{-1} y \geq 0$$

Hence, $M\Gamma^{-1}M^T$ is also positive definite.

With both Ψ and $M\Gamma^{-1}M^T$ proved to be positive definite, $B_{11} = \left[\Psi + M\Gamma^{-1}M^T\right]^{-1}$ is also a positive definite matrix. Consequently, B_{111} as a submatrix along the diagonal of B_{11} is positive definite, and $\nabla_{R'} f'$ is negative definite. This, based on the definition of

f', R', indicates that for each commodity the Jacobian matrix of the sub-vector of the locally approximated functions of the flow variables on the links with positives current flow is strictly monotone decreasing in the sub-vector of the service charges on those links.

$\nabla_{R'} f'$ is a submatrix along the diagonal of $\nabla_R f$ since $\nabla_{R'} f'$ only includes the derivative of flow variables with respect to the service charges on the links with positive current flows. In order to analyze the property of $\nabla_R f$, the derivative on the links with no current flows need to be considered as well, which is demonstrated below.

Under the conditions specified in Appendix *C*, the links with no flow will remain unused with small perturbation of the service charge (Tobin, 1987). Mathematically, $\frac{\partial f_{l,c}}{\partial R_{l',c}} = 0, \quad \forall f_{l,c}^0 = 0$. On the other hand, a small change of service charge on an unused link will not alter the assignment pattern in the network since this link will remain unused for a small change. This small change of the service charge on the link doesn't influence the cost structure of those used links. Hence, the derivative of link flow with respect to service charge on the current unused link is zero. Mathematically, $\frac{\partial f_{l',c}}{\partial R_{l,c}} = 0, \quad \forall f_{l,c}^0 = 0$.

With $\nabla_{R'} f'$ negative definite and all other elements zero, $\nabla_R f$ is negative semidefinite. This indicates that the vector of the locally approximated flow functions on the shipper link, that is $\left(\cdots, f_{l\in L, c\in C}(R_L), \cdots\right)_{|L|*|C|}$, is monotone decreasing in the vector of the service charges on the shipper links (R_L). Hence, Proposition 5.2 is proved.

APPENDIX E DERIVATION OF A FACTOR IN THE INVESTMENT COST FUNCTION

Consider that the investment project on link a has a 20 year analysis period and a 5% discount rate. The investment strategy u improves the capacity on link a by $\Delta\overline{E}_a^u$ units. The cost of additional unit of capacity is $p1_a^u$. The original capital expense (I_a^u) is calculated as $I_a^u = p1_a^u * \Delta\overline{E}_a^u$. Let the service life of the capital expense be 6 years. On every 6 years, a replacement investment is needed with the same capital expense as the original one (I_a^u). This investment project is illustrated in Figure E.1 below.

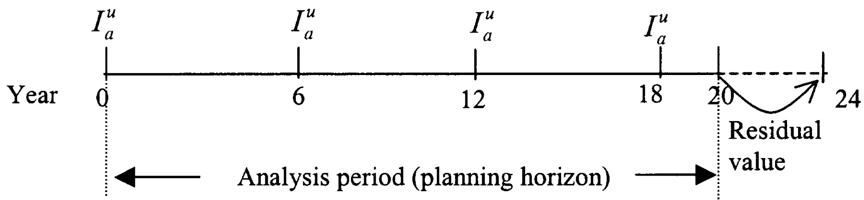


Figure E.1 A Sample Investment Project

Given the information of the analysis period, the service life, and the discount rate for this sample investment project, the following steps are used to derive $p2_a^u$.

1. Convert the initial capital expense and the replacement investment expense to present value:

   Present value of the initial investment: I_a^u

   Present value of the first replacement (6 years later): I_a^u *$PWF_{6,5\%}$

   Present value of the second replacement (12 years later): I_a^u *$PWF_{12,5\%}$

Present value of the third replacement (18 years later): I_a^u *$PWF_{18,5\%}$

Present value of the residual value: I_a^u *$CRF_{6,5\%}$*$SPW_{4,5\%}$* $PWF_{20,5\%}$

Present value of all capital expenses on link a under investment strategy u in the analysis period (PIC_a^u):

$PIC_a^u = I_a^u$ *(1+ $PWF_{6,5\%}$ +$PWF_{12,5\%}$ +$PWF_{18,5\%}$- $CRF_{6,5\%}$*$SPW_{4,5\%}$* $PWF_{20,5\%}$)

$= I_a^u$ *(1+0.7462+0.5568+0.4155-0.197*3.546*0.3769)

$= I_a^u$ *2.4552 (E.1)

Where, PWF is the single payment Present Worth Factor, CRF is the Capital Recovery Factor, SPW is the uniform series Present Worth Factor.

Using y to denote the number of years and $i\%$ to denote the discount rate, then

$PWF_{y,i\%} = \frac{1}{(1+i\%)^y}$, $CRF_{y,i\%} = \frac{i(1+i)^i}{(1+i\%)^y - 1}$, and $SPW_{y,i\%} = \frac{(1+i\%)^y - 1}{i(1+i\%)^y}$.

(Refer to the class notes of Transportation Economics by Pignataro).

2. Convert the present value of the capital expenses in Eq. (E.1) into the annual cost within the analysis period.

   Annual investment cost of all capital expenses on link a under investment strategy u in the analysis period (AIC_a^u):

   $AIC_a^u = PIC_a^u$ *$CRF_{20,5\%}$

   $= I_a^u$ *2.4552*0.080= I_a^u *(0.196416) (E.2)

3. Convert the annual investment cost in Eq. (E.2) into the hourly investment cost.

   Hourly investment cost of all capital expenses on link a under investment strategy u in the analysis period (IC_a^u):

$$IC_a^u = AIC_a^u * \frac{1}{365} * \frac{1}{24} = I_a^u *(0.196416)* \frac{1}{365} * \frac{1}{24} = I_a^u *2.24*10^{-5} \qquad \text{(E.3)}$$

From Eq. (E.3), $p2_a^u$=2.24*10^{-5} is derived, which are used to convert the capital expenses during the analysis period into the hourly investment cost. In the same way, $p2_a^u$ can be derived for the links and investment strategies with different service life or discount rate.

APPENDIX F INPUT DATA FOR THE NUMERICAL EXAMPLE

Table F.1 The Incidence Index Between Terminal O-D Pair and the Terminal Node $\tau_{v,x}$

$\tau_{v,x}$	*x0*	*x1*	*x7*	*X8*	*x9*	*x10*	*x16*	*x17*
v0	1	0	-1	0	0	0	0	0
v1	1	0	0	-1	0	0	0	0
v2	0	1	-1	0	0	0	0	0
v3	0	1	0	-1	0	0	0	0
v4	0	0	0	0	1	0	-1	0
v5	0	0	0	0	1	0	0	-1
v6	0	0	0	0	0	1	-1	0
v7	0	0	0	0	0	1	0	-1

Table F.2 The Incidence Index $\delta_{p,a}$ between Terminal Path $p \in PH$ and Link $a \in A$. $\delta_{p,a}=1$ Indicates that Link a is in Path p

$p.a,\ \forall p \in PH,\ a \in A$	$\delta_{p,a}$
p0.a0	*1*
p1.(a1,a3,a5,a7)	*1*
p2.(a1,a3,a6,a8)	*1*
p3.(a1,a3,a6,a9,a10)	*1*
p4.(a1,a4,a8)	*1*
p5.(a1,a4,a9,a10)	*1*
p6.(a2,a3,a5,a7)	*1*
p7.(a2,a3,a6,a8)	*1*
p8.(a2,a3,a6,a9,a10)	*1*
p9.(a2,a4,a8)	*1*
p10.(a2,a4,a9,a10)	*1*
p11.(a11,a13,a15,a18)	*1*
p12.(a11,a13,a15,a19)	*1*
p13.(a11,a13,a16,a20)	*1*
p14.(a12,a13,a15,a18)	*1*
p15.(a12,a13,a15,a19)	*1*
p16.(a12,a13,a16,a20)	*1*
p17.(a14,a17,a20)	*1*
p18.(a11,a21,a18)	*1*
p19.(a11,a21,a19)	*1*
p20.(a12,a21,a18)	*1*
p21.(a12,a21,a19)	*1*
p22.a22	*1*
p23.a23	*1*

Table F.3 The Incidence Index $\delta_{p,v}$ between Terminal Path $p \in PH$ and Terminal O-D Pair $v \in V$. $\delta_{p,v}=1$ Indicates that Path p Connects Terminal O-D Pair v

p.v, $\forall p \in PH$, $v \in V$	$\delta_{p,v}$
(p0,p1,p2,p4).v0	1
(p3,p5).v1	1
(p6,p7,p9).v2	1
(p8,p10,p22).v3	1
(p11,p18,p23).v4	1
(p12,p13,p19).v5	1
(p14,p20).v6	1
(p15, p16,p17,p21).v7	1

Table F.4 The Incidence Index $\xi_{l,v}$ Between Port link $l \in PL$ and the Terminal O-D Pair $v \in V$. If Port Link l Corresponding to Terminal O-D Pair v, $\xi_{l,v}=1$. Otherwise, $\xi_{l,v}=0$

$\xi_{l,v}$	*v0*	*v1*	*v2*	*v3*	*v4*	*v5*	*v6*	*v7*
l7	1	0	0	0	0	0	0	0
l8	0	1	0	0	0	0	0	0
l9	0	0	1	0	0	0	0	0
l10	0	0	0	1	0	0	0	0
l11	0	0	0	0	1	0	0	0
l12	0	0	0	0	0	1	0	0
l13	0	0	0	0	0	0	1	0
L14	0	0	0	0	0	0	0	1

Table F.5 Parameters in the Terminal Link Operating Cost Function

$$AC_{a,c}(e_{a,c}) = r_{a,c} + r'_{a,c} * \frac{e_{a,c}}{\bar{E}_a} + 0.5 * r'_{a,c} * (\frac{e_{a,c}}{\bar{E}_a})^2$$

Link	$\bar{E}_{a,c}$ (units*/hr)	$r_{a,c}$			$r'_{a,c}$		
		c1	*c2*	*c3*	*c1*	*c2*	*c3*
a0	18	0.4	0.7	0.6	4	4	4.5
a1	17.25	0.2	0.3	0.1	2	3	2.5
a2	20	0.3	0.34	0.2	2.5	2.5	2
a3	20	0.3	0.27	0.4	1.5	2.5	4
a4	17.25	0.7	0.6	0.8	2	1.5	2
a5	31	0.4	0.16	0.47	2	2	3.5
a6	25	0.3	0.17	0.45	2	3.5	3
a7	30	0.24	0.17	0.47	2	1.5	2
a8	15.5	0.27	0.18	0.47	2.5	4	2.5
a9	22.5	0.1	0.12	0.2	1	2	1
a10	28	0.12	0.1	0.2	4	3	3
a11	32	0.5	0.25	0.3	2.5	2	3
a12	24.7	0.4	0.27	0.29	3.5	4	3.5
a13	18	0.2	0.15	0.1	3.5	3	4
a14	19	0.7	0.4	0.45	2.5	2.5	4
a15	20.5	0.18	0.17	0.16	2.5	3	1.5
a16	25.5	0.2	0.27	0.17	2	2.5	1.5
a17	14.5	0.17	0.18	0.15	4	3	4
a18	28.5	0.6	0.4	0.4	2.5	1.5	2
a19	18	0.65	0.4	0.5	2	2	1.5
a20	25	0.6	0.38	0.4	2.5	2	2.5
a21	16	0.3	0.42	0.3	3	3	3.5
a22	24	0.6	0.7	0.5	2.5	2.5	4
a23	19	0.4	0.8	0.4	3.5	3	4

* units of flow

Table F.6 Parameters in Inverse Supply Function $\pi_{b,c}(S_b) = \gamma_{b,c} + \sum_{c'} \lambda_{b,c',c} * S_{b,c}$ and Inverse Demand Function $\rho_{b,c}(D_b) = \alpha_{b,c} - \sum_{c'} \beta_{b,c',c} * D_{b,c} \quad \forall b \in CN; c,c' \in C$

b.c, $\forall b \in CN, c \in C$	$\gamma_{b,c}$	$\alpha_{b,c}$	$\lambda_{b,c',c}$			$\beta_{b,c',c}$		
			c'=c1	*c'=c3*	*c'=c3*	*c'=c1*	*c'=c3*	*c'=c3*
n12.c1	7	11	0.2	0.003	-0.004	20	0	0
n12.c2	8	7	0.002	0.15	-0.004	0	20	0
n12.c3	8	7	-0.003	-0.006	0.2	0	0	20
n13.c1	6	11	0.15	0.006	-0.0012	20	0	0
n13.c2	10	7	0.001	0.25	-0.0015	0	20	0
n13.c3	14	7	-0.001	-0.009	0.15	0	0	20
n14.c1	8	11	0.15	0.002	-0.005	20	0	0
n14.c2	8	7	0.001	0.2	-0.006	0	20	0
n14.c3	7.5	7	-0.006	-0.006	0.25	0	0	20
n15.c1	145	70	10	0	0	0.24	0.002	-0.001
n15.c2	150	95	0	10	0	0.003	0.23	-0.008
n15.c3	95	118	0	0	10	-0.002	-0.009	0.26
n16.c1	145	72	10	0	0	0.15	0.001	-0.0015
n16.c2	150	108	0	10	0	0.0028	0.3	-0.008
n16.c3	95	97	0	0	10	-0.001	-0.008	0.2
n17.c1	145	60	10	0	0	0.3	0.0015	-0.002
n17.c2	150	110	0	10	0	0.001	0.2	-0.004
n17.c3	95	120	0	0	10	-0.0012	-0.007	0.4

Table F.7 Interaction Ratio $ro_{l,c',c}$ in Travel Time Function

$$t_{l,c}(f_l) = tt_l * \left[1 + 0.3 * f_{l,c} + 0.15 * \left(\frac{\sum_{c'} (ro_{l,c',c} * f_{l,c'})}{Cap_l} \right)^2 \right]$$

$ro_{l,c',c} = ro_{c',c}$ $\forall l \in L$		$c' \in C$		
		c1	*c2*	*c3*
$c \in C$	*c1*	1	0.004	0.005
	c2	0.003	1	0.004
	c3	0.004	0.003	1

Table F.8 Service Charge, Free Flow Travel Time and Capacity in Travel Time Function $t_{l,c}(f_l)$ and in Generalized Cost Function $GC_{l,c} = R_{l,c} + vot_c * t_{l,c}$

Link	$R_{l,c1}$ ($/unit*)	$R_{l,c2}$ ($/unit)	$R_{l,c3}$ ($/unit)	tt_l (hrs/unit)	Cap_l (units/hr)
l23,l26, l27	0.15	0.15	0.1	0.3	60
l22,l24,l25,l28,l29	0.25	0.15	0.2	0.15	52
l0,l18	0.12	0.05	0.075	0.09	106
l1,l6, l17,l20	0.07	0.11	0.05	0.05	60
l2,l4,l15,l21	0.07	0.08	0.06	0.045	45
l5,l16	0.04	0.03	0.03	0.03	52
l3,l19	0.15	0.125	0.1	0.1	60
*l30*l32*	0.005	0.005	0.005	0.005	52
l33	0.0025	0.0025	0.0025	0.0075	60
l7,	13.2	20.4	22.5	0.85	35.25
l8	13.2	20.4	22.5	1.25	35.25
l9	13.2	20.4	22.5	1.25	44
l10	13.2	20.4	22.5	0.85	44
l11	13.2	20.4	22.5	0.5	47.50
l12	13.2	20.4	22.5	1	43
l13	13.2	20.4	22.5	1	39.2
l14	13.2	20.4	22.5	0.5	39.2

* unit of flow

APPENDIX G SOLUTION OF THE NUMERICAL EXAMPLE

Table G.1 Quantities of Commodities Supplied and Demanded and the Production and Market Prices

a) Competitive Game

	c1					*c2*					*c3*				
Market	S (units[*]/hr)	π ($/unit[*])	D (units/hr)	ρ ($/unit)	η ($/unit)	S (units/hr)	π ($/unit)	D (units/hr)	ρ ($/unit)	η ($/unit)	S (units/hr)	π ($/unit)	D (units/hr)	ρ ($/unit)	η ($/unit)
n12	30.18	12.95	0.00	11.00	12.95	20.70	11.02	0.00	7.00	11.02	36.85	15.16	0.00	7.00	15.16
n13	44.23	12.70	0.00	11.00	12.70	17.55	14.39	0.00	7.00	14.39	30.96	18.44	0.00	7.00	18.44
n14	30.92	12.51	0.00	11.00	12.51	18.77	11.58	0.00	7.00	11.58	33.63	15.61	0.00	7.00	15.61
n15	0.00	145.00	28.52	63.19	63.19	0.00	150.00	3.61	94.39	94.39	1.30	108.00	38.79	108.00	108.00
n16	0.00	145.00	68.28	61.76	61.76	0.00	150.00	26.40	100.03	100.03	0.00	95.00	17.84	93.71	93.71
n17	0.00	145.00	8.54	57.49	57.49	0.00	150.00	27.00	104.78	104.78	0.65	101.49	46.77	101.49	101.49

b) Collusive Game

	c1					*c2*					*c3*				
Market	S (units/hr)	π ($/unit)	D (units/hr)	ρ ($/unit)	η ($/unit)	S (units/hr)	π ($/unit)	D (units/hr)	ρ ($/unit)	η ($/unit)	S (units/hr)	π ($/unit)	D (units/hr)	ρ ($/unit)	η ($/unit)
n12	26.95	12.32	0.00	11.00	12.32	17.78	10.60	0.00	7.00	10.60	30.92	14.00	0.00	7.00	14.00
n13	39.98	12.06	0.00	11.00	12.06	14.45	13.62	0.00	7.00	13.62	22.69	17.23	0.00	7.00	17.23
n14	27.23	11.98	0.00	11.00	11.98	16.16	11.09	0.00	7.00	11.09	28.47	14.36	0.00	7.00	14.36
n15	0.00	145.00	25.80	63.84	63.84	0.00	150.00	1.63	94.82	94.82	1.42	109.19	34.14	109.19	109.19
n16	0.00	145.00	62.78	62.57	62.57	0.00	150.00	22.85	101.04	101.04	0.05	95.54	8.52	95.54	95.54
n17	0.00	145.00	5.58	58.37	58.37	0.00	150.00	23.91	105.38	105.38	0.85	103.48	41.73	103.48	103.48

[*] units of flow

Table G.2 Spatial Price Equilibrium Flows and Generalized Costs

a) Competitive Game

$l=$ (ni,nj)	ni	nj	$f_{l,c1}$ (units/hr)	$\eta_{ni,c1}$ ($/unit)	$GC_{l,c1}$ ($/unit)	$\eta_{nj,c1}$ ($/unit)	$\eta_{ni,c1}+GC_{l,c1}-\eta_{nj,c1}$ ($/unit)	$f_{l,c2}$ (units/hr)	$\eta_{ni,c2}$ ($/unit)	$GC_{l,c2}$ ($/unit)	$\eta_{nj,c2}$ ($/unit)	$\eta_{ni,c2}+GC_{l,c2}-\eta_{nj,c2}$ ($/unit)	$f_{l,c3}$ (units/hr)	$\eta_{ni,c3}$ ($/unit)	$GC_{l,c3}$ ($/unit)	$\eta_{nj,c3}$ ($/unit)	$\eta_{ni,c3}+GC_{l,c3}-\eta_{nj,c3}$ ($/unit)
l0	*n0*	*n3*	18.56	16.99	1.60	18.59	0.00	10.23	22.02	3.71	25.73	0.00	18.44	24.46	3.02	27.47	0.00
l1	*n0*	*n2*	26.64	16.99	1.20	18.19	0.00	15.45	22.02	2.93	24.95	0.00	28.21	24.46	2.42	26.88	0.00
l2	*n1*	*n2*	40.27	16.63	1.56	18.19	0.00	20.00	21.71	3.24	24.95	0.00	35.56	24.17	2.71	26.88	0.00
l3	*n1*	*n5*	19.86	16.63	1.89	18.53	0.00	11.34	21.71	4.53	26.24	0.00	19.23	24.17	3.49	27.67	0.00
l4	*n2*	*n3*	6.47	18.19	0.40	18.59	0.00	1.86	24.95	0.78	25.73	0.00	4.55	26.88	0.59	27.47	0.00
l5	*n2*	*n4*	56.62	18.19	0.94	19.13	0.00	29.08	24.95	1.97	26.93	0.00	52.75	26.88	1.72	28.60	0.00
l6	*n2*	*n5*	3.82	18.19	0.34	18.53	0.00	4.51	24.95	1.29	26.24	0.00	6.47	26.88	0.79	27.67	0.00
l7	*n3*	*n6*	15.35	18.59	35.54	54.13	0.00	7.55	25.73	59.43	85.16	0.00	14.14	27.47	59.06	86.53	0.00
l8	*n3*	*n71*	9.68	18.59	34.90	53.49	0.00	4.54	25.73	58.49	84.22	0.00	8.85	27.47	57.86	85.33	0.00
l9	*n41*	*n6*	10.79	19.25	34.88	54.13	0.00	4.67	27.16	58.00	85.16	0.00	9.45	28.81	57.72	86.53	0.00
l10	*n41*	*n71*	16.22	19.25	34.24	53.49	0.00	7.54	27.16	57.06	84.22	0.00	14.26	28.81	56.53	85.33	0.00
l11	*n42*	*n72*	20.62	19.26	34.17	53.42	0.00	11.78	27.24	56.77	84.01	0.00	20.02	28.85	56.33	85.18	0.00
l12	*n42*	*n8*	9.00	19.26	34.83	54.08	0.00	5.10	27.24	58.27	85.50	0.00	9.03	28.85	57.59	86.43	0.00
l13	*n5*	*n72*	7.71	18.53	34.90	53.42	0.00	4.72	26.24	57.77	84.01	0.00	8.16	27.67	57.52	85.18	0.00
l14	*n5*	*n8*	15.98	18.53	35.56	54.08	0.00	11.13	26.24	59.26	85.50	0.00	17.54	27.67	58.77	86.43	0.00
l15	*n6*	*n9*	5.81	54.13	0.38	54.51	0.00	5.25	85.16	1.24	86.40	0.00	5.88	86.53	0.68	87.21	0.00
l16	*n7*	*n9*	54.22	53.60	0.90	54.51	0.00	28.58	84.46	1.94	86.40	0.00	51.29	85.54	1.67	87.21	0.00
l17	*n8*	*n9*	6.07	54.08	0.42	54.51	0.00	1.95	85.50	0.90	86.40	0.00	6.36	86.43	0.78	87.21	0.00
l18	*n6*	*n10*	20.33	54.13	1.72	55.85	0.00	6.96	85.16	2.83	87.99	0.00	17.70	86.53	2.92	89.45	0.00
l19	*n8*	*n11*	18.90	54.08	1.82	55.91	0.00	14.27	85.50	5.41	90.91	0.00	20.21	86.43	3.64	90.08	0.00
l20	*n9*	*n10*	30.39	54.51	1.34	55.85	0.00	6.52	86.40	1.59	87.99	0.00	25.70	87.21	2.23	89.45	0.00
l21	*n9*	*n11*	35.71	54.51	1.40	55.91	0.00	29.27	86.40	4.51	90.91	0.00	37.83	87.21	2.86	90.08	0.00
l22	*n12*	*n0*	30.18	12.95	4.04	16.99	0.00	20.70	11.02	11.00	22.02	0.00	36.85	15.16	9.30	24.46	0.00
l23	*n13*	*n0*	15.02	12.70	4.29	16.99	0.00	4.98	14.39	7.63	22.02	0.00	9.79	18.44	6.01	24.46	0.00
l24	*n13*	*n1*	29.21	12.70	3.93	16.63	0.00	12.57	14.39	7.32	21.71	0.00	21.17	18.44	5.73	24.17	0.00
l25	*n14*	*n1*	30.92	12.51	4.12	16.63	0.00	18.77	11.58	10.12	21.71	0.00	33.63	15.61	8.56	24.17	0.00

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l26	*n10*	*n15*	28.52	55.85	7.34	63.19	0.00	3.61	87.99	6.40	94.39	0.00	37.49	89.45	18.56	108.00	0.00
l27	*n10*	*n16*	22.21	55.85	5.91	61.76	0.00	9.86	87.99	12.04	100.03	0.00	5.92	89.45	4.27	93.71	0.00
l28	*n11*	*n16*	46.08	55.91	5.85	61.76	0.00	16.54	90.91	9.12	100.03	0.00	11.92	90.08	3.64	93.71	0.00
l29	*n11*	*n17*	8.54	55.91	1.59	57.49	0.00	27.00	90.91	13.86	104.78	0.00	46.12	90.08	11.42	101.49	0.00
l30	*n4*	*n41*	27.01	19.13	0.12	19.25	0.00	12.20	26.93	0.24	27.16	0.00	23.71	28.60	0.21	28.81	0.00
l31	*n4*	*n42*	29.61	19.13	0.13	19.26	0.00	16.88	26.93	0.31	27.24	0.00	29.04	28.60	0.25	28.85	0.00
l32	*n71*	*n7*	25.90	53.49	0.12	53.60	0.00	12.08	84.22	0.24	84.46	0.00	23.11	85.33	0.20	85.54	0.00
l33	*n72*	*n7*	28.32	53.42	0.18	53.60	0.00	16.51	84.01	0.45	84.46	0.00	28.18	85.18	0.36	85.54	0.00

b) Collusive Game

l= (*ni*,*nj*)	*ni*	*nj*	$f_{l,c1}$ (units/hr)	$\eta_{ni,c1}$ ($/unit)	$GC_{l,c1}$ ($/unit)	$\eta_{nj,c1}$ ($/unit)	$\eta_{ni,c1}+GC_{l,c1}-\eta_{nj,c1}$ ($/unit)	$f_{l,c2}$ (units/hr)	$\eta_{ni,c2}$ ($/unit)	$GC_{l,c2}$ ($/unit)	η_{njc2} ($/unit)	$\eta_{ni,c2}+GC_{l,c2}-\eta_{nj,c2}$ ($/unit)	$f_{l,c3}$ (units/hr)	$\eta_{ni,c3}$ ($/unit)	$GC_{l,c3}$ ($/unit)	$\eta_{nj,c3}$ ($/unit)	$\eta_{ni,c3}+GC_{l,c3}-\eta_{nj,c3}$ ($/unit)
l0	*n0*	*n3*	16.67	15.99	1.47	17.46	0.00	8.65	20.28	3.29	23.56	0.00	14.96	21.94	2.55	24.49	0.00
l1	*n0*	*n2*	23.74	15.99	1.09	17.08	0.00	13.02	20.28	2.57	22.84	0.00	22.86	21.94	2.02	23.96	0.00
l2	*n1*	*n2*	35.80	15.68	1.40	17.08	0.00	16.85	20.03	2.81	22.84	0.00	28.43	21.75	2.22	23.96	0.00
l3	*n1*	*n5*	17.95	15.68	1.75	17.43	0.00	9.87	20.03	4.09	24.12	0.00	15.82	21.75	2.98	24.72	0.00
l4	*n2*	*n3*	5.93	17.08	0.38	17.46	0.00	1.40	22.84	0.72	23.56	0.00	3.55	23.96	0.53	24.49	0.00
l5	*n2*	*n4*	49.53	17.08	0.83	17.91	0.00	24.03	22.84	1.67	24.51	0.00	41.59	23.96	1.38	25.34	0.00
l6	*n2*	*n5*	4.08	17.08	0.35	17.43	0.00	4.44	22.84	1.28	24.12	0.00	6.15	23.96	0.76	24.72	0.00
l7	*n3*	*n6*	14.07	17.46	38.06	55.52	0.00	6.44	23.56	64.20	87.76	0.00	11.62	24.49	65.84	90.33	0.00
l8	*n3*	*n71*	8.54	17.46	37.51	54.98	0.00	3.61	23.56	63.51	87.07	0.00	6.89	24.49	64.93	89.42	0.00
l9	*n41*	*n6*	9.45	18.02	37.51	55.52	0.00	3.69	24.71	63.05	87.76	0.00	7.29	25.51	64.82	90.33	0.00
l10	*n41*	*n71*	13.88	18.02	36.96	54.98	0.00	5.84	24.71	62.36	87.07	0.00	10.75	25.51	63.91	89.42	0.00
l11	*n42*	*n72*	18.11	18.03	36.89	54.92	0.00	10.07	24.79	62.08	86.87	0.00	16.13	25.55	63.73	89.28	0.00
l12	*n42*	*n8*	8.09	18.03	37.42	55.45	0.00	4.43	24.79	63.21	88.00	0.00	7.42	25.55	64.62	90.16	0.00
l13	*n5*	*n72*	6.92	17.43	37.49	54.92	0.00	4.08	24.12	62.75	86.87	0.00	6.69	24.72	64.56	89.28	0.00
l14	*n5*	*n8*	15.12	17.43	38.02	55.45	0.00	10.22	24.12	63.88	88.00	0.00	15.29	24.72	65.44	90.16	0.00
l15	*n6*	*n9*	5.10	55.52	0.35	55.88	0.00	4.58	87.76	1.15	88.91	0.00	4.55	90.33	0.59	90.92	0.00
l16	*n7*	*n9*	47.44	55.08	0.80	55.88	0.00	23.61	87.27	1.64	88.91	0.00	40.46	89.58	1.34	90.92	0.00

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l17	*n8*	*n9*	6.27	55.45	0.43	55.88	0.00	2.01	88.00	0.91	88.91	0.00	6.10	90.16	0.76	90.92	0.00
l18	*n6*	*n10*	18.41	55.52	1.59	57.11	0.00	5.55	87.76	2.45	90.21	0.00	14.36	90.33	2.47	92.80	0.00
l19	*n8*	*n11*	16.95	55.45	1.67	57.12	0.00	12.63	88.00	4.92	92.92	0.00	16.60	90.16	3.10	93.26	0.00
l20	*n9*	*n10*	27.61	55.88	1.23	57.11	0.00	4.60	88.91	1.30	90.21	0.00	20.91	90.92	1.87	92.80	0.00
l21	*n9*	*n11*	31.20	55.88	1.24	57.12	0.00	25.61	88.91	4.01	92.92	0.00	30.20	90.92	2.34	93.26	0.00
l22	*n12*	*n0*	26.95	12.32	3.67	15.99	0.00	17.78	10.60	9.68	20.28	0.00	30.92	14.00	7.95	21.94	0.00
l23	*n13*	*n0*	13.47	12.06	3.94	15.99	0.00	3.89	13.62	6.66	20.28	0.00	6.90	17.23	4.71	21.94	0.00
l24	*n13*	*n1*	26.51	12.06	3.62	15.68	0.00	10.56	13.62	6.41	20.03	0.00	15.79	17.23	4.51	21.75	0.00
l25	*n14*	*n1*	27.23	11.98	3.70	15.68	0.00	16.16	11.09	8.94	20.03	0.00	28.47	14.36	7.39	21.75	0.00
l26	*n10*	*n15*	25.80	57.11	6.73	63.84	0.00	1.63	90.21	4.61	94.82	0.00	32.73	92.80	16.39	109.19	0.00
l27	*n10*	*n16*	20.22	57.11	5.46	62.57	0.00	8.52	90.21	10.83	101.04	0.00	2.55	92.80	2.75	95.54	0.00
l28	*n11*	*n16*	42.57	57.12	5.45	62.57	0.00	14.33	92.92	8.12	101.04	0.00	5.91	93.26	2.28	95.54	0.00
l29	*n11*	*n17*	5.58	57.12	1.25	58.37	0.00	23.91	92.92	12.46	105.38	0.00	40.89	93.26	10.22	103.48	0.00
l30	*n4*	*n41*	23.33	17.91	0.11	18.02	0.00	9.53	24.51	0.20	24.71	0.00	18.04	25.34	0.17	25.51	0.00
l31	*n4*	*n42*	26.20	17.91	0.12	18.03	0.00	14.50	24.51	0.27	24.79	0.00	23.55	25.34	0.21	25.55	0.00
l32	*n71*	*n7*	22.42	54.98	0.10	55.08	0.00	9.46	87.07	0.20	87.27	0.00	17.64	89.42	0.16	89.58	0.00
l33	*n72*	*n7*	25.03	54.92	0.16	55.08	0.00	14.16	86.87	0.40	87.27	0.00	22.82	89.28	0.30	89.58	0.00

Table G.3 Jacobian Matrix of the Locally Approximated Demand Function for the Competitive Game

a) Terminal 1

$\partial f_{l,c} / \partial R_{l',c'}$	$R_{l7,c1}$	$R_{l8,c1}$	$R_{l9,c1}$	$R_{l10,c1}$	$R_{l7,c2}$	$R_{l8,c2}$	$R_{l9,c2}$	$R_{l10,c2}$	$R_{l7,c3}$	$R_{l8,c3}$	$R_{l9,c3}$	$R_{l10,c3}$
$f_{l7,c1}$	-1.3608	0.0907	0.0915	0.0786	0	0	0	0	0	0	0	0
$f_{l8,c1}$	0.0907	-0.9762	0.0354	0.0815	0	0	0	0	0	0	0	0
$f_{l9,c1}$	0.0915	0.0354	-0.9775	0.0821	0	0	0	0	0	0	0	0
$f_{l10,c1}$	0.0786	0.0815	0.0821	-1.3942	0	0	0	0	0	0	0	0
$f_{l7,c2}$	0	0	0	0	-0.3422	0.0228	0.023	0.0198	0	0	0	0
$f_{l8,c2}$	0	0	0	0	0.0228	-0.2451	0.0089	0.0205	0	0	0	0
$f_{l9,c2}$	0	0	0	0	0.023	0.0089	-0.2452	0.0206	0	0	0	0
$f_{l10,c2}$	0	0	0	0	0.0198	0.0205	0.0206	-0.3502	0	0	0	0
$f_{l7,c3}$	0.0001	0	0	0	0	0	0	0	-0.6781	0.0474	0.0478	0.042
$f_{l8,c3}$	0	0	0	0	0	0	0	0	0.0474	-0.4872	0.019	0.0425
$f_{l9,c3}$	0	0	0	0	0	0	0	0	0.0478	0.019	-0.4879	0.0428
$f_{l10,c3}$	0	0	0	0	0	0	0	0	0.042	0.0425	0.0428	-0.6955

b) Terminal 2

$\partial f_{l,c} / \partial R_{l',c'}$	$R_{l11,c1}$	$R_{l12,c1}$	$R_{l13,c1}$	$R_{l14,c1}$	$R_{l11,c2}$	$R_{l12,c2}$	$R_{l13,c2}$	$R_{l14,c2}$	$R_{l11,c3}$	$R_{l12,c3}$	$R_{l13,c3}$	$R_{l14,c3}$
$f_{l11,c1}$	-2.1778	0.1637	0.1692	0.1986	0	0	0	0	0	0	0	-0.0001
$f_{l12,c1}$	0.1637	-1.2024	0.0471	0.1773	0	0	0	0	0	0	0	0
$f_{l13,c1}$	0.1692	0.0471	-1.1994	0.1772	0	0	0	0	0	0	0	0
$f_{l14,c1}$	0.1986	0.1773	0.1772	-2.1312	0	0	0	0	-0.0001	0	0	0
$f_{l11,c2}$	0	0	0	0	-0.5468	0.041	0.0424	0.0499	0	0	0	-0.0001
$f_{l12,c2}$	0	0	0	0	0.041	-0.3013	0.0118	0.0445	0	0	0	0
$f_{l13,c2}$	0	0	0	0	0.0424	0.0118	-0.3005	0.0444	0	0	0	0
$f_{l14,c2}$	0	0	0	0	0.0499	0.0445	0.0444	-0.5346	-0.0001	0	0	0
$f_{l11,c3}$	0.0001	0	0	0	0	0	0	0	-1.0829	0.0855	0.0881	0.1069
$f_{l12,c3}$	0	0	0	0	0	0	0	0	0.0855	-0.5992	0.0255	0.093
$f_{l13,c3}$	0	0	0	0	0	0	0	0	0.0881	0.0255	-0.5978	0.0926
$f_{l14,c3}$	0	0	0	0.0001	0	0	0	0	0.1069	0.093	0.0926	-1.0559

Table G.4 Jacobian Matrix of the Locally Approximated Demand Function for the Collusive Game

$\partial f_{l,c} / \partial R_{l',c'}$	$R_{l7,c1}$	$R_{l8,c1}$	$R_{l9,c1}$	$R_{l10,c1}$	$R_{l11,c1}$	$R_{l12,c1}$	$R_{l13,c1}$	$R_{l14,c1}$	$R_{l7,c2}$	$R_{l8,c2}$	$R_{l9,c2}$	$R_{l10,c2}$	$R_{l11,c2}$	$R_{l12,c2}$	$R_{l13,c2}$	$R_{l14,c2}$	$R_{l7,c3}$	$R_{l8,c3}$	$R_{l9,c3}$	$R_{l10,c3}$	$R_{l11,c3}$	$R_{l12,c3}$	$R_{l13,c3}$	$R_{l14,c3}$
$f_{l7,c1}$	-1.36	0.09	0.09	0.08	0.13	0.06	0.06	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l8,c1}$	0.09	-0.98	0.04	0.08	0.13	0.04	0.06	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l9,c1}$	0.09	0.04	-0.98	0.08	0.13	0.06	0.04	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l10,c1}$	0.08	0.08	0.08	-1.40	0.24	0.09	0.09	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l11,c1}$	0.13	0.13	0.13	0.24	-2.18	0.16	0.17	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l12,c1}$	0.06	0.04	0.06	0.09	0.16	-1.20	0.05	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l13,c1}$	0.06	0.06	0.04	0.09	0.17	0.05	-1.20	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l14,c1}$	0.12	0.08	0.08	0.12	0.20	0.18	0.18	-2.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l7,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.34	0.02	0.02	0.02	0.03	0.02	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l8,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	-0.25	0.01	0.02	0.03	0.01	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l9,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	-0.25	0.02	0.03	0.02	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l10,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.02	-0.35	0.06	0.02	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l11,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.03	0.06	-0.55	0.04	0.04	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l12,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.02	0.02	0.04	-0.30	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l13,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.01	0.02	0.04	0.01	-0.30	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l14,c2}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.02	0.02	0.03	0.05	0.05	0.04	-0.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$f_{l7,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.67	0.05	0.05	0.05	0.08	0.04	0.04	0.08
$f_{l8,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	-0.49	0.02	0.05	0.07	0.03	0.04	0.05
$f_{l9,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.02	-0.49	0.05	0.07	0.04	0.03	0.05
$f_{l10,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.05	0.05	-0.69	0.13	0.05	0.05	0.07
$f_{l11,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.07	0.07	0.13	-1.07	0.09	0.10	0.12
$f_{l12,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.03	0.04	0.05	0.09	-0.60	0.03	0.10
$f_{l13,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.04	0.03	0.05	0.10	0.03	-0.60	0.10
$f_{l14,c3}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.05	0.05	0.07	0.12	0.10	0.10	-1.04

Table G.5 Verification of the Diagonal Dominance of the Jacobian matrix $\nabla_R g(R)$

L,c	Competitive Game			Collusive Game		
	$\sum_{l' \in L, c' \in C} \left| \partial f_{l,c} / \partial R_{l',c'} \right|$	$\sum_{l' \in L, c' \in C} \left| \partial f_{l',c'} / \partial R_{l,c} \right|$	$\left| \partial f_{l,c} / \partial R_{l,c} \right|$	$\sum_{l' \in L, c' \in C} \left| \partial f_{l,c} / \partial R_{l',c'} \right|$	$\sum_{l' \in L, c' \in C} \left| \partial f_{l',c'} / \partial R_{l,c} \right|$	$\left| \partial f_{l,c} / \partial R_{l,c} \right|$
l7,c1	0.2608	0.2609	1.3608	0.6445	0.6436	1.3623
l8,c1	0.2076	0.2076	0.9762	0.5219	0.5213	0.9771
l9,c1	0.209	0.209	0.9775	0.5260	0.5254	0.9782
l10,c1	0.2422	0.2422	1.3942	0.7839	0.7830	1.3960
l11,c1	0.5316	0.5316	2.1778	1.1584	1.1573	2.1805
l12,c1	0.3881	0.3881	1.2024	0.6488	0.6479	1.2032
l13,c1	0.3935	0.3935	1.1994	0.6540	0.6531	1.2003
l14,c1	0.5532	0.5532	2.1312	0.9630	0.9615	2.1330
l7,c2	0.0656	0.0656	0.3422	0.1629	0.1619	0.3425
l8,c2	0.0522	0.0522	0.2451	0.1316	0.1309	0.2453
l9,c2	0.0525	0.0525	0.2452	0.1325	0.1317	0.2453
l10,c2	0.0609	0.0609	0.3502	0.1975	0.1965	0.3505
l11,c1	0.5316	0.5316	2.1778	0.2917	0.2903	0.5472
l12,c1	0.3881	0.3881	1.2024	0.1632	0.1623	0.3014
l13,c1	0.3935	0.3935	1.1994	0.1645	0.1636	0.3007
l14,c1	0.5532	0.5532	2.1312	0.2428	0.2412	0.5349
l7,c3	0.0656	0.0656	0.3422	0.3847	0.3866	0.6729
l8,c3	0.0522	0.0522	0.2451	0.3034	0.3047	0.4852
l9,c3	0.0525	0.0525	0.2452	0.3042	0.3056	0.4859
l10,c3	0.0609	0.0609	0.3502	0.4472	0.4491	0.6920
l11,c3	0.1334	0.1333	0.5468	0.6636	0.6661	1.0718
l12,c3	0.0973	0.0973	0.3013	0.3755	0.3773	0.5958
l13,c3	0.0986	0.0986	0.3005	0.3765	0.3783	0.5947
l14,c3	0.1389	0.1388	0.5346	0.5754	0.5785	1.0401

Table G.6 Equilibrium Service Demands, Service Charges, and Travel Times

a) Competitive Game

Terminal O-D pair	$Q_{v,c1}$ (units/hr)	$R_{v,c1}$ (\$/unit),	$t_{v,c1}$ (hrs/unit)	$Q_{v,c2}$ (units/hr)	$R_{v,c2}$ (\$/unit),	$t_{v,c2}$ (hrs/unit)	$Q_{v,c3}$ (units/hr)	$R_{v,c3}$ (\$/unit),	$t_{v,c3}$ (hrs/unit)
v0	15.35	23.57	4.79	7.55	31.63	2.78	14.14	36.68	4.48
v1	9.68	22.67	4.89	4.54	28.93	2.96	8.85	34.96	4.58
v2	10.79	21.62	5.31	4.67	27.98	3.00	9.45	33.71	4.80
v3	16.22	21.74	5.00	7.54	29.31	2.78	14.26	34.03	4.50
v4	20.62	25.15	3.61	11.78	34.05	2.27	20.02	38.75	3.52
v5	9.00	25.56	3.71	5.10	32.95	2.53	9.03	39.01	3.72
v6	7.71	26.60	3.32	4.72	33.57	2.42	8.16	40.24	3.45
v7	15.98	28.28	2.91	11.13	37.51	2.18	17.54	43.04	3.15

b) Collusive Game

Terminal O-D pair	$Q_{v,c1}$ (units/hr)	$R_{v,c1}$ ($/unit),	$t_{v,c1}$ (hrs/unit)	$Q_{v,c2}$ (units/hr)	$R_{v,c2}$ ($/unit),	$t_{v,c2}$ (hrs/unit)	$Q_{v,c3}$ (units/hr)	$R_{v,c3}$ ($/unit),	$t_{v,c3}$ (hrs/unit)
v0	14.07	26.92	4.46	6.44	39.22	2.50	11.62	46.70	3.83
v1	8.54	26.35	4.46	3.61	37.43	2.61	6.89	45.73	3.84
v2	9.45	25.50	4.80	3.69	36.71	2.63	7.29	44.87	3.99
v3	13.88	25.96	4.40	5.84	38.93	2.34	10.75	45.92	3.60
v4	18.11	28.82	3.23	10.07	41.94	2.01	16.13	49.09	2.93
v5	8.09	28.84	3.43	4.43	39.92	2.33	7.42	48.47	3.23
v6	6.92	29.79	3.08	4.08	40.49	2.23	6.69	49.51	3.01
v7	15.12	31.07	2.78	10.22	43.49	2.04	15.29	51.42	2.80

Table G.7 Equilibrium Link Flows and Operating Costs

	Competitive Game						Collusive Game					
link	$e_{a,c1}$ (units/hr)	$AC_{a,c1}$ ($/unit)	$e_{a,c2}$ (units/hr)	$AC_{a,c2}$ ($/unit)	$e_{a,c3}$ (units/hr)	$AC_{a,c3}$ ($/unit)	$e_{a,c1}$ (units/hr)	$AC_{a,c1}$ ($/unit)	$e_{a,c2}$ (units/hr)	$AC_{a,c2}$ ($/unit)	$e_{a,c3}$ (units/hr)	$AC_{a,c3}$ ($/unit)
a0	13.40	4.49	7.15	2.60	13.23	5.12	12.14	4.01	5.99	2.25	10.75	4.09
a1	11.63	2.00	4.94	1.28	9.76	1.91	10.47	1.78	4.06	1.09	7.76	1.48
a2	6.23	1.20	2.26	0.64	7.02	1.03	5.14	1.02	1.59	0.55	4.99	0.76
a3	10.10	1.25	3.45	0.74	6.42	1.89	8.86	1.11	2.81	0.65	4.81	1.48
a4	7.76	1.80	3.74	0.96	10.36	2.36	6.74	1.63	2.85	0.87	7.93	1.93
a5	7.66	0.96	3.45	0.40	5.42	1.13	6.80	0.89	2.81	0.35	4.19	0.98
a6	2.44	0.50	0.00	0.17	1.00	0.57	2.06	0.47	0.00	0.17	0.62	0.53
a7	7.66	0.82	3.45	0.35	5.42	0.86	6.80	0.74	2.81	0.32	4.19	0.77
a8	5.08	1.22	1.61	0.62	4.94	1.39	4.58	1.12	1.33	0.54	3.97	1.19
a9	5.12	0.35	2.13	0.32	6.42	0.53	4.23	0.31	1.52	0.26	4.58	0.42
a10	5.12	0.92	2.13	0.34	6.42	0.97	4.23	0.77	1.52	0.27	4.58	0.73
a11	10.10	1.41	5.30	0.61	10.03	1.39	8.59	1.26	4.35	0.54	7.72	1.11
a12	13.59	2.86	8.41	1.86	15.36	3.14	12.75	2.67	7.62	1.70	13.26	2.67
a13	10.67	2.89	6.33	1.39	12.03	3.67	9.58	2.56	5.53	1.21	9.92	2.91
a14	10.09	2.38	7.44	1.57	10.35	3.22	9.28	2.22	6.68	1.43	8.71	2.71
a15	6.03	1.02	3.52	0.73	7.98	0.86	5.30	0.91	3.03	0.65	6.45	0.71
a16	4.64	0.60	2.81	0.56	4.05	0.43	4.29	0.56	2.50	0.53	3.47	0.39
a17	10.09	3.92	7.44	2.11	10.35	4.02	9.28	3.55	6.68	1.88	8.71	3.28
a18	8.81	1.49	4.93	0.68	9.16	1.15	7.41	1.33	4.01	0.63	6.99	0.95
a19	10.25	2.11	5.97	1.17	12.17	1.86	9.64	2.01	5.47	1.10	10.52	1.63
a20	14.73	2.51	10.25	1.37	14.40	2.25	13.57	2.33	9.18	1.25	12.18	1.92
a21	13.03	3.74	7.38	2.12	13.35	4.44	11.76	3.31	6.45	1.87	11.07	3.56
a22	20.78	3.70	9.95	1.95	16.69	4.25	18.19	3.21	7.94	1.66	13.06	3.27
a23	19.51	5.84	11.58	3.19	19.02	6.41	17.62	5.15	10.15	2.83	15.83	5.12

Table G.8 Flows of All Commodity Types vs. Capacities

Terminal link No:	$\sum_{c \in C} e_{a,c}$ (units/hr) Competitive Game	Collusive Game	$\overline{E}_a$ (units/hr)
a0	33.7695	28.8813	18
a1	26.3315	22.2912	17.25
a2	15.5015	11.7148	20
a3	19.9714	16.4842	20
a4	21.8616	17.522	17.25
a5	16.5311	13.8034	31
a6	3.4403	2.6808	25
a7	16.5311	13.8034	30
a8	11.6308	9.8753	15.5
a9	13.671	10.3274	22.5
a10	13.671	10.3274	28
a11	25.4311	20.6657	32
a12	37.3596	33.6317	24.7
a13	29.0275	25.0244	18
a14	27.8788	24.6763	19
a15	17.5305	14.7718	20.5
a16	11.497	10.2527	25.5
a17	27.8788	24.6763	14.5
a18	22.9015	18.4133	28.5
a19	28.3922	25.6316	18
a20	39.3759	34.929	25
a21	33.7632	29.2731	16
a22	47.4172	39.1843	24
a23	50.1063	43.587	19

Table G.9 Marginal Revenues and Minimum Marginal Costs

	Competitive game						Collusive game					
O-D pair	$MR_{v,c1}$ ($/unit)	$MC_{v,c1}$ ($/unit)	$MR_{v,c2}$ ($/unit)	$MC_{v,c2}$ ($/unit)	$MR_{v,c3}$ ($/unit)	$MC_{v,c3}$ ($/unit)	$MR_{v,c1}$ ($/unit)	$MC_{v,c1}$ ($/unit)	$MR_{v,c2}$ ($/unit)	$MC_{v,c2}$ ($/unit)	$MR_{v,c3}$ ($/unit)	$MC_{v,c3}$ ($/unit)
v0	9.68	9.68	4.82	4.82	10.85	10.86	8.52	8.52	4.01	4.03	8.39	8.38
v1	9.78	9.78	4.86	4.85	11.12	11.12	8.44	8.44	3.95	3.96	8.36	8.35
v2	7.64	7.64	3.39	3.38	8.69	8.70	6.62	6.62	2.82	2.83	6.68	6.66
v3	7.74	7.74	3.42	3.42	8.96	8.96	6.54	6.54	2.75	2.76	6.65	6.63
v4	13.12	13.12	6.13	6.13	14.41	14.42	11.40	11.40	5.28	5.29	11.23	11.23
v5	14.52	14.52	7.19	7.20	15.97	15.98	12.90	12.90	6.30	6.31	12.70	12.68
v6	16.51	16.51	8.82	8.82	18.45	18.46	14.71	14.70	7.75	7.75	14.79	14.77
v7	17.91	17.91	9.89	9.90	20.02	20.03	16.20	16.20	8.77	8.77	16.24	16.23

Table G.10 Path Flows and Marginal Path Costs

		Competitive Game						Collusive Game					
$v \in V$	$p \in PH(v)$	$Pf_{p,c1}$ (units/hr)	$MC_{p,c1}$ ($/unit)	$Pf_{p,c2}$ (units/hr)	$MC_{p,c2}$ ($/unit)	$Pf_{p,c3}$ (units/hr)	$MC_{p,c3}$ ($/unit)	$Pf_{p,c1}$ (units/hr)	$MC_{p,c1}$ ($/unit)	$Pf_{p,c2}$ (units/hr)	$MC_{p,c2}$ ($/unit)	$Pf_{p,c3}$ (units/hr)	$MC_{p,c3}$ ($/unit)
v0	p0	13.398	9.678	7.145	4.821	13.227	10.858	12.139	8.524	5.993	4.028	10.749	8.382
v0	p1	1.436	9.678	1.199	4.821	0.000	10.858	1.665	8.524	1.216	4.028	0.000	8.382
v0	p2	2.436	9.678	0.000	4.880	0.000	10.858	2.061	8.524	0.000	4.088	0.000	8.382
v0	p4	2.642	9.678	1.612	4.821	3.335	10.858	2.514	8.524	1.326	4.028	3.182	8.382
v1	p3	0.000	9.783	0.000	4.911	0.000	11.120	0.000	8.444	0.000	4.024	0.000	8.354
v1	p5	5.118	9.783	2.130	4.852	6.423	11.120	4.225	8.444	1.523	3.965	4.579	8.354
v2	p6	6.226	7.637	2.255	3.385	5.416	8.703	5.135	6.624	1.595	2.827	4.193	6.656
v2	p7	0.000	7.637	0.000	3.444	1.004	8.703	0.000	6.624	0.000	2.887	0.620	6.656
v2	p9	0.000	7.637	0.000	3.385	0.601	8.703	0.000	6.624	0.000	2.827	0.172	6.656
v3	p8	0.000	7.741	0.000	3.475	0.000	8.965	0.000	6.544	0.000	2.823	0.000	6.628
v3	p10	0.000	7.741	0.000	3.416	0.000	8.965	0.000	6.544	0.000	2.763	0.000	6.628
v3	p22	20.782	7.741	9.946	3.416	16.690	8.965	18.191	6.544	7.936	2.763	13.057	6.628
v4	p11	0.000	13.124	0.000	6.129	0.000	14.416	0.000	11.403	0.000	5.287	0.000	11.226
v4	p18	4.123	13.124	1.950	6.129	3.579	14.416	3.207	11.403	1.419	5.287	2.401	11.226
v4	p23	19.511	13.124	11.581	6.129	19.015	14.416	17.616	11.403	10.146	5.287	15.825	11.226
v5	p12	1.342	14.518	0.540	7.201	2.400	15.978	1.094	12.903	0.439	6.313	1.854	12.685
v5	p13	4.636	14.518	2.810	7.201	4.050	15.978	4.287	12.903	2.496	6.313	3.469	12.685
v5	p19	0.000	14.518	0.000	7.201	0.000	15.978	0.000	12.903	0.000	6.313	0.000	12.685
v6	p14	4.689	16.514	2.978	8.825	5.582	18.464	4.203	14.704	2.591	7.748	4.592	14.774
v6	p20	0.000	16.514	0.000	8.825	0.000	18.464	0.000	14.704	0.000	7.748	0.000	14.774
v7	p15	0.000	17.908	0.000	9.897	0.000	20.026	0.000	16.204	0.000	8.774	0.000	16.234
v7	p16	0.000	17.908	0.000	9.897	0.000	20.026	0.000	16.204	0.000	8.774	0.000	16.234
v7	p17	10.093	17.908	7.437	9.897	10.348	20.026	9.281	16.204	6.681	8.774	8.714	16.234
v7	p21	8.903	17.908	5.435	9.897	9.773	20.026	8.548	16.204	5.033	8.774	8.665	16.234

APPENDIX H PROOF OF PROPOSITION 7.1

Proposition 7.1: A feasible solution $(S, f, D) \in KS$ is an equilibrium solution for the spatial price equilibrium problem, if and only if there exists a nonnegative vector $\eta = (\cdots, \eta_{n,c}, \cdots)\ \forall n \in N, c \in C$ satisfying Eqs. (H.1)-(H.3).

$$S_{b,c}(\pi_{b,c}(S_b) - \eta_{b,c}) = 0 \qquad \forall b \in CN,\ c \in C \tag{H.1}$$

$$D_{b,c}(-\rho_{b,c}(D_b) + \eta_{b,c}) = 0 \qquad \forall b \in CN,\ c \in C \tag{H.2}$$

$$\begin{aligned} &f_{l,c}(\eta_{n1,c} + GC_{l=(n1,n2),c}(f_l) - \eta_{n2,c}) = 0 \\ &\forall n1, n2 \in N,\ l = (n1, n2) \in L, c \in C \end{aligned} \tag{H.3}$$

Proof:

First is to prove that **E1** from Section 5.1.2: $h_{p,c}$*($GC_{p,c}$- $\mu_{b1,b2,c}$)=0, $\forall p \in P(b1, b2),\ b1, b2 \in CN, c \in C$.

Define $L(p)$ as the sequence of links within path p. $L(p)$: $(l_0, l_1, l_2, l_3, \ldots l_m)$. l_i is the $(i+1)_{th}$ link in path p.

If $h_{p,c} > 0 \Rightarrow f_{l,c} > 0 \quad \forall l \in L(p)$. According to Eq. (H.3), it follows:

$$GC_{l=(n1,n2),c} + \eta_{n1,c} - \eta_{n2,c} = 0 \qquad \forall l \in L(p), c \in C \tag{H.4}$$

Summing Eq. (H.4) for all links in path p, the following can be obtained:

$$\begin{aligned} &(GC_{l_0=(b1,n0),c} + \eta_{b1,c} - \eta_{n0,c}) + (GC_{l1=(n0,n1),c} + \eta_{n0,c} - \eta_{n1,c}) \\ &\qquad + \cdots (GC_{lm=(nm-1,b2),c} + \eta_{nm-1,c} - \eta_{b2,c}) = 0 \\ &\Rightarrow \sum_{l \in L(p)} GC_{l,c} + \eta_{b1,c} - \eta_{b2,c} = 0 \end{aligned} \tag{H.5}$$

In the summation over link $l \in L(p)$, the dual variables for the intermediate nodes such as $\eta_{n0,c}, \eta_{n1,c}$ are cancelled out. Only those for the centroids $\eta_{b1,c}, \eta_{b2,c}$ remain in the final equation.

According to definition, the generalized cost for commodity c on path p is the summation of the generalized cost for commodity c on each link l in path p. That is $GC_{p,c} = \sum_{l \in L(p)} GC_{l,c}$. Substituting it into Eq. (H.5), it can be derived that:

If $h_{p,c}>0$, then $GC_{p,c} + \eta_{b1,c} - \eta_{b2,c} = 0 \quad \forall p \in P(b1,b2),\ b1,b2 \in CN, c \in C$. (H.6)

Based on the above proof, $GC_{p,c} + \eta_{b1,c} - \eta_{b2,c} = \sum_{l=(ni,nj) \in L(p)} (GC_{l,c} + \eta_{ni} - \eta_{nj})$. Hence, $GC_{p,c} - \eta_{b1,c} - \eta_{b2,c} > 0 \Rightarrow \sum_{l=(ni,nj) \in L(p)} (GC_{l,c} + \eta_{ni} - \eta_{nj}) > 0$. This together with Eq. (H.3) indicates that: for at least one $l \in L(p)$, $GC_{l,c} + \eta_{ni} - \eta_{nj} > 0$. According to Eq. (H.3) the flow on this link ($f_{l,c}$) equals zero. As long as one link on a path has no flow, the whole path has no flow. Hence, $h_{p,c} = 0$.

Therefore, it is proved that:

$$\text{if } GC_{p,c} + \eta_{b1,c} - \eta_{b2,c} > 0, \text{ then } h_{p,c} = 0. \quad \text{(H.7)}$$

Combining Eqs. (H.6) and (H.7), it follows that:

$$h_{p,c}*(GC_{p,c} + \eta_{b1,c} - \eta_{b2,c}) = 0,\ \forall p \in P(b1,b2),\ b1,b2 \in CN, c \in C. \quad \text{(H.8)}$$

$$\Rightarrow \mu_{b1,b2,c} = \min_{p \in P(b1,b2)} GC_{p,c} = \eta_{b2,c} - \eta_{b1,c}. \quad \text{(H.9)}$$

Substituting Eq. (H.9) into Eq. (H.8), it yields:

$h_{p,c}*(GC_{p,c} - \mu_{b1,b2,c}) = 0$, $\forall p \in P(b1,b2),\ b1,b2 \in CN, c \in C$.Thus, the proof of **E1** is complete.

Next, it is to show that **E2** from Section 5.1.2: $Q_{b1,b2,,c}*(\pi_{b1,c}+\mu_{b1,b2,c}-\rho_{b2,c})=0$ $\forall b1,b2 \in CN, c \in C$ can also be derived from Eqs. (H.1)-(H.3).

First is to prove that if $Q_{b1,b2,c}>0$ then $\pi_{b1,,c}+\mu_{b1,b2,c}-\rho_{b2,c}=0$.

$Q_{b1,b2,c}>0 \Rightarrow S_{b1,c}>0$, $D_{b2,c}>0$, according to Eqs. (H.1) and (H.2), $\Rightarrow$

$$\left.\begin{array}{l}(\pi_{b1,c}-\eta_{b1,c}) = 0 \\ (-\rho_{b2,c}+\eta_{b2,c}) = 0\end{array}\right\} \Rightarrow \pi_{b1,c} = \eta_{b1,c} \text{ and } \rho_{b2,c} = \eta_{b2,c}$$

$$\Rightarrow \pi_{b1,c} - \eta_{b1,c} - \rho_{b2,c} + \eta_{b2,c} = 0 \qquad \text{(H.10)}$$

$$\Rightarrow \pi_{b1,c} + (\eta_{b2,c} - \eta_{b1,c}) - \rho_{b2,c} = 0$$

As demonstrated in the proof of **E1**, $\mu_{b1,b2,c} = \eta_{b2,c} - \eta_{b1,c}$. Hence,

$$Q_{b1,b2,c}>0 \Rightarrow \pi_{b1,c} + \mu_{b1,b2,c} - \rho_{b2,c} = 0 \quad \forall b1,b2 \in CN, c \in C \qquad \text{(H.11)}$$

Second, prove that if $\pi_{b1,c} + \mu_{b1,b2,c} - \rho_{b2,c} > 0$ $\forall b1,b2 \in CN, c \in C$ then $Q_{b1,b2,,c}=0$.

Conversely, if $Q_{b1,b2,c} \neq 0$, it must be positive since $Q_{b1,b2,c}$ is assumed to be nonnegative. Then, $\pi_{b1,c} + \mu_{b1,b2,c} - \rho_{b2,c} = 0$ as proved above, which constitutes a contradiction to the assumption $\pi_{b1,c} + \mu_{b1,b2,c} - \rho_{b2,c} > 0$. Therefore,

$$\pi_{b1,c} + \mu_{b1,b2,c} - \rho_{b2,c} > 0 \Rightarrow Q_{b1,b2,c} = 0 \quad \forall b1,b2 \in CN, c \in C \qquad \text{(H.12)}$$

Combining Eqs. (H.11) and (H.12), the proof of **E2** is accomplished.

Thus, the proof of proposition 7.1 is completed.

APPENDIX I A SUFFICIENT CONDITION FOR A MATRIX TO BE POSITIVE DEFINITE

Matrix M, $M = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$, is positive definite if the following conditions are satisfied: $a_{ii} > \sum_j | a_{ij} |$ and $a_{ii} > \sum_j | a_{ji} |$. $\forall j \neq i$.

Proof:

For any nontrivial vector $X \in R^n$, the following could be derived:

$$
\begin{aligned}
X^T MX &= \sum_i a_{ii} x_i^2 + \sum_{i,j,\forall i \neq j} a_{ij} x_i x_j \\
&\geq \sum_i a_{ii} x_i^2 - 0.5 \sum_{i,j,\forall i \neq j} | a_{ij} | * (x_i^2 + x_j^2) \\
&= \sum_i a_{ii} x_i^2 - 0.5 \sum_i \sum_{j,\forall j \neq i} | a_{ij} | * x_i^2 - 0.5 \sum_j \sum_{i,\forall i \neq j} | a_{ij} | * x_j^2
\end{aligned}
$$

From the conditions, it follows $a_{ii} > \sum_j | a_{ij} |$ and $a_{jj} > \sum_i | a_{ij} |$. Hence,

$$
X^T MX > \sum_i a_{ii} x_i^2 - 0.5 \sum_i a_{ii} x_i^2 - 0.5 \sum_j a_{jj} x_j^2 = 0
$$

The proof is completed.

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