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Optimizing feeder bus network based on access mode shifts

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ABSTRACT

**OPTIMIZING FEEDER BUS NETWORK BASED ON
ACCESS MODE SHIFTS**

by
Zhaodong Huang

The methodology introduced in this dissertation is to optimally find a feeder bus network in a suburban area for an existing rail system that connects the suburban area with the Central Business District (CBD). The objective is to minimize the total cost, including user and supplier costs. Three major access modes (walk, feeder bus, and auto) for the rail station are considered and the cost for all modes makes up the user cost. The supplier cost comes from the operating cost of the feeder bus network. The decision variables include the structure of the feeder bus network, service frequencies, and bus stop locations.

The developed methodology consists of four components, including a Preparation Procedure (PP), Initial Solution Generation Procedure (ISGP), Network Features Determination Procedure (NFDP) and Solution Search Procedure (SSP). PP is used to perform a preliminary processing on the input data set. An initial solution that will be used in SSP is found in ISGP. The NFDP is a module to determine the network related features such as service frequency, mode split, stop selections and locations. A logit-based Multinomial Logit-Proportional Model (MNL-PM) model is proposed to estimate the mode shares of walk, bus and auto. A metaheuristic Tabu Search (TS) method is developed to find the optimal solution for the methodology.

In the computational experiments, an Exhaustive Search (ES) method is designed and tested to validate the effectiveness of the proposed methodology. The results of

networks of different sizes are presented and sensitivity analyses are performed to investigate the impacts of various model parameters (e.g., fleet size, parking fee, bus fare, etc.).

**OPTIMIZING FEEDER BUS NETWORK BASED ON
ACCESS MODE SHIFTS**

**by
Zhaodong Huang**

**A Dissertation
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy in Transportation**

Department of Civil and Environmental Engineering

August 2012

APPROVAL PAGE

**OPTIMIZING FEEDER BUS NETWORK BASED ON
ACCESS MODE SHIFTS**

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This dissertation is dedicated to
my beloved family: my wife, Tian Zhang;
my mother, Xingxia Chen;
my father, Zhuming Huang;
for their love and support.

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CHAPTER 1

INTRODUCTION

The field of public transit planning is a vast research area. Generally, the generation of a public transportation system can be addressed from different perspectives, including network design, roster of crews, demand evaluation, trip assignment, and exploration of different kinds of mathematical solution methods. The constantly increasing car usage leads to various negative impacts, such as air pollution, traffic congestion and urban parking problems. Developing public transportation becomes a crucial topic for modern society (Guihaire and Hao, 2008). Public transit agencies always try to provide efficient transit network and maximize level of service and minimize cost. From the users' perspective, the criteria of an efficient public transit system include demand coverage, total travel time, service frequency, number of transfers, etc. In terms of the service provider, the usual goal is to maximize profit. However, since transit is a critical public facility, the maximization of social benefit becomes its objective. In this case, trade-offs between user cost and operator cost are involved. As a major part in developing public transportation system, there are two different types of the transit network design problem. The first one is to design a regular transit network. The purpose of this network is to meet short-distance public transport demand in a given area. The second type of problem is the feeder bus network design problem. As defined by Kuah (1989), it is a problem of designing a feeder-bus network that connects pre-determined areas to an existing rail system.

As a high-capacity public transportation system, rail or rapid transit plays a vital role in moving large number of passengers in a multimodal system. Due to the high cost associated with the construction of a rapid transit line, it is unrealistic to distribute rapid transit stations to reach every scattered demand node of an area. On the other hand, to encourage transit use, avoid road congestion, and reduce air pollution and especially that associated with short trips and starting and idling times, feeder buses, as one of the essential elements of an intermodal transportation network, play a critical role of bringing riders from their dispersed origins to diverse destinations.

1.1 Problem Identification

Although many researchers have contributed to the feeder bus network design problem, most focus narrowly on exploring various route development strategies, developing metaheuristic approaches, or optimizing schedule coordination, the applications of such kinds of methodologies are still restricted.

The first limitation is the assumption of fixed demand. It is no denying that in some situations, the feeder bus demand is fixed. However, in most cases, especially when there are more than one alternative available for choosing, the demand will be affected by the level of service that the existing transit network delivered. It is noted that few of the recent studies in the field of regular transit network development do take variable demand into account, when mode split is considered. However, the mode alternatives are very limited (Fan and Machemehl, 2004; Lee, 2005; Liu et al, 2011; Cipriani et al, 2012). For example, in Fan's work, they only referred to auto and transit modes. Due to the intrinsic complexity in considering variable demand, most of these studies used a loop to determine bus service frequencies. Since the outputs from the loop are very sensitive to

the initial bus service frequencies, the determination of the initial inputs becomes a big problem especially for those users with limited experience. Moreover, in calculating the total cost, the objective function did not include the access cost of the users by other modes such as automobile and walking. In more realistic situations, walking always makes up the majority portion of access modes especially when the trip distance is within a walking distance threshold. In the absence of these modes, the developed methodology is problematic.

The second limitation is the structure of the bus routes and the street network. Most early models of feeder bus network development are based on simplified networks, such as parallel routes. More realistic route structures are introduced in some recent studies such as grid street network. However, there are still many assumptions which hinder the application of the methodology in the real-world. For example, the configuration of the feeder bus route developed by Lownes and Machemehl (2007) was solved as a travel salesman problem (TSP). Although the authors incorporated the stop selection strategy in the procedure, the formulation developed by them can only develop one feeder bus route at a time. Some multi-route development methodologies based on grid street network are also noticed, but they are always together with other additional assumptions, and more importantly, they are not developed for feeder bus systems.

The third one is about the route linkages between demand and bus access points. Some previous methods in developing a bus transit network simply aggregated the zonal travel demand into one single node to represent a bus stop. This assumption can make the problem easier to be solved but less practical. To be more realistic, demand is separated by grid streets into small zones, while the bus stops are located on the streets. To address

this problem, some studies simply assumed that the bus can stop anywhere along a street, and some literatures used pre-determined distribution nodes to represent the candidate bus stops on a street. These studies first calculated centroids for small zones. Based on some criteria, the distribution nodes were determined as access points from these centroids to the closest streets. However, these studies combined the adjacent distribution nodes prior to the optimization process. This is problematic since the criteria used in combining the distribution nodes may not be held after the final optimization process.

To overcome those limitations in the preceding studies and tackle the Feeder Bus Network Design Problem (FBNDP) in a more realistic situation, where the street is irregular grid network, demand has multiple choices to access the rail station, the bus stop locations are not pre-determined, an improved FBNDP model is introduced. An algorithmic solution is developed to help find a solution to minimize the total cost, defined by the combination of user cost and transit operator cost.

1.2 Research Scope

The purpose of this new FBNDP model is to develop a feeder bus system in a suburban area for an existing rail system that connects the suburban area and CBD. For convenience, the shape of the feeder bus service area in this study is assumed to be rectangular. Considering the distance from the suburban area to the CBD, the rail system is assumed to be the only mode for the trips in this area to access the destination work places in the CBD.

CBD Station

▲ Rail Station
• Demand centroid
— Grid street network
--- Boundary of feeder bus service area
═ Rail line
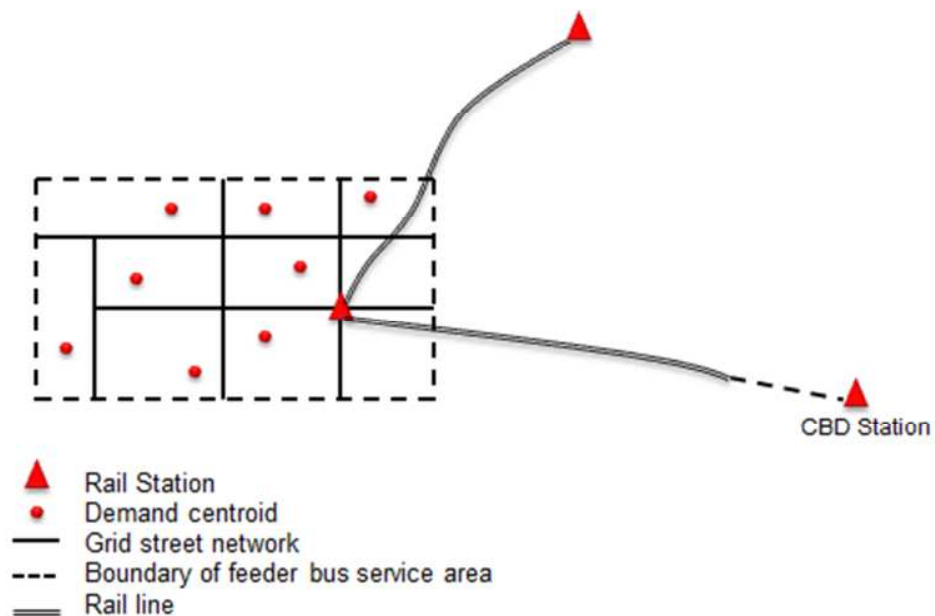
Figure 1.1 An illustration of the feeder bus service area.

The demand that this new model serves consists of peak period work trips to and from the CBD with a many-to-one demand pattern. It means the demand characterized by multiple origins in the suburban area and only one destination- suburban rail station, via which the CBD or transfer stations can be reached, as shown in Figure 1.1. Due to the characteristics of the working trips, the feeder bus services to be developed in this study only serve in the morning and afternoon peak hours during weekdays. During the

morning peak hours, the feeder bus system collect commuters from different areas and brings them to the rail station while in the afternoon peak hour, the system deliver these passengers from the rail station to their destinations. For the non-peak hour demands, if needed, a reduced service feeder bus network can also be developed based on the related demand data. The distribution of these work trips in the service area is inhomogeneous. A physical irregular street network divided the demand into several small zones. In each zone, the demand is aggregated as a centroid based on its distribution. There are three alternatives available for the work trips to access the rail station: walk, feeder bus and auto. The total number of these working trips is fixed, while the portion that use the feeder bus system is variable and sensitive to the level of service of transit system and the out-of-pocket expenses. The mode shares among these three alternatives are determined according to a two stage Multinomial Logit- Proportional Model (MNL-PM).
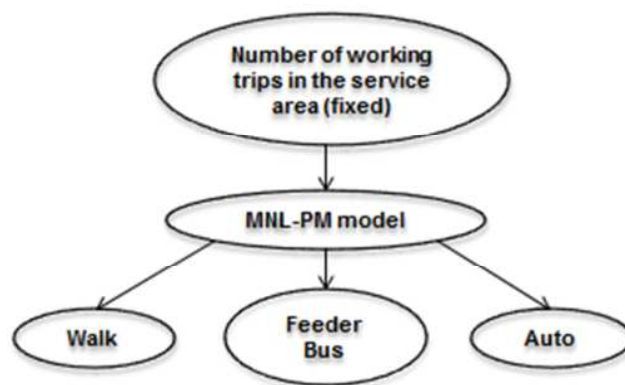
Figure 1.2 Access modes considered in the feeder bus route optimization.

1.3 Organization of the Dissertation

The organization of this dissertation is as follows: Chapter 2 is the literature review on the topic of transit network design. Firstly, the methodologies in transit network development used in the previous studies are discussed and the shortcomings in these

methods are presented. Then, two types of solution approaches are presented, the analytic method and the mathematical programming method. For complicated problems, especially when considering of various variables such as elastic demand, mode split, the mathematical programming methods are preferred, and metaheuristic algorithms are discussed to solve the problem when the second method is used.

In Chapter 3, a new mathematical nonlinear mixed integer programming model for the feeder bus network development problem is formulated. The related constraints and assumptions for the introduced model are also discussed.

Chapter 4 describes a solution method for the proposed methodology. This method consists of four major components, including Preparation Procedure (PP), Initial Solution Generation Procedure (ISGP), Network Feature Determination Procedure (NFDP) and Solution Search Procedure (SSP).

Chapter 5 focuses on computational experiments and algorithm evaluation. To begin with, three networks with different sizes are given. The performance of the Tabu Search based on tabu length, fixed/ dynamic tabu tenure are discussed and compared. A validation of the SSP is presented in section 5.4. Then, compared with a random selection method, the effectiveness of the ISGP is discussed. The computational results for all three networks are presented in section 5.6. The sensitivity analyses on various network parameters followes in section 5.7.

Chapter 6 provides a summary for this study. Section 6.1 gives a conclusion of the proposed methodology and research results obtained from the sensitivity analyses. The primary contributions of this study are presented in section 6.2. Finally, future possible extensions are presented. Such future improvements include the incorporation of a

regular bus network and feeder bus network, the consideration of more access modes or mode combinations; the consideration of traffic conditions; a more sophisticated route selection strategy; and the development of quality evaluation method.

CHAPTER 2

LITERATURE REVIEW

In this chapter, previous studies related to this work is reviewed and summarized in two sections. In section 2.1, tate-of-the-art transit network design problem models are discussed. Section 2.2 explains various solution methodologies applied in current network design problem models. Finally, a summary of the literature review is given in section 2.3.

2.1 Transit Network Design Problem (TNDP)

Many references address the TNDP in the context of the transit planning process. A conceptual model which includes a complete transit planning process was firstly introduced by Ceder and Wilson (1986). In that model, the transit planning process was divided into five steps: the design of routes; the setting of frequencies; the timetabling; the vehicle scheduling; and the crew scheduling and rostering, there are several inputs and outputs associated with each step as presented in Table 2.1.

Table 2.1 The Inputs and Outputs of Transit Planning Process

Independent inputs	Planning activity	Output
Demand data Supply data Route performance indicators	Network design	Route changes New routes Operating strategies
Subsidy available Buses available Service policies Current patronage	Frequencies setting	Service frequencies
Demand by time of day Times for first and last trips Running times	Timetable development	Trip departure times Trip arrival times
Deadhead times Recovery times Schedule constraints Cost structure	Bus scheduling	Bus schedules
Driver work rules Run cost structure	Driver scheduling	Driver schedules

Source: Ceder and Wilson, 1986

By incorporating the inputs and outputs at each step, Fan and Machemehl (2004) developed a systematic flow chart of a bus planning process as shown in Figure 2.1. The five process steps are represented as levels from Level A to Level E. The outputs from a higher level are inputs for the lower level. Although there is only one direction from higher level to lower level, these levels are not independent of each other. Mostly, the outputs of a lower level will also act and affect as feedback the inputs from a higher level. Actually, the feedbacks and loops exit in the whole process. For example, for elastic demand, the change of bus service frequency in Level B for a specific bus route will simultaneously affect the input of passenger demand of that line in Level A. Consequently, the network configuration is also changed. For this reason, a new flow chart of the transit planning process is provided in Figure 2.2.

Besides, it is noticed that although there are five steps in all, most existing studies on the topic of bus network design problem mainly focus on the first three steps. As

explained by Fan and Machemehl, the last two steps largely affect the operator's cost. However, the critical determinants of system performance from both the operator and user perspectives are bus route network configuration and its service frequencies.
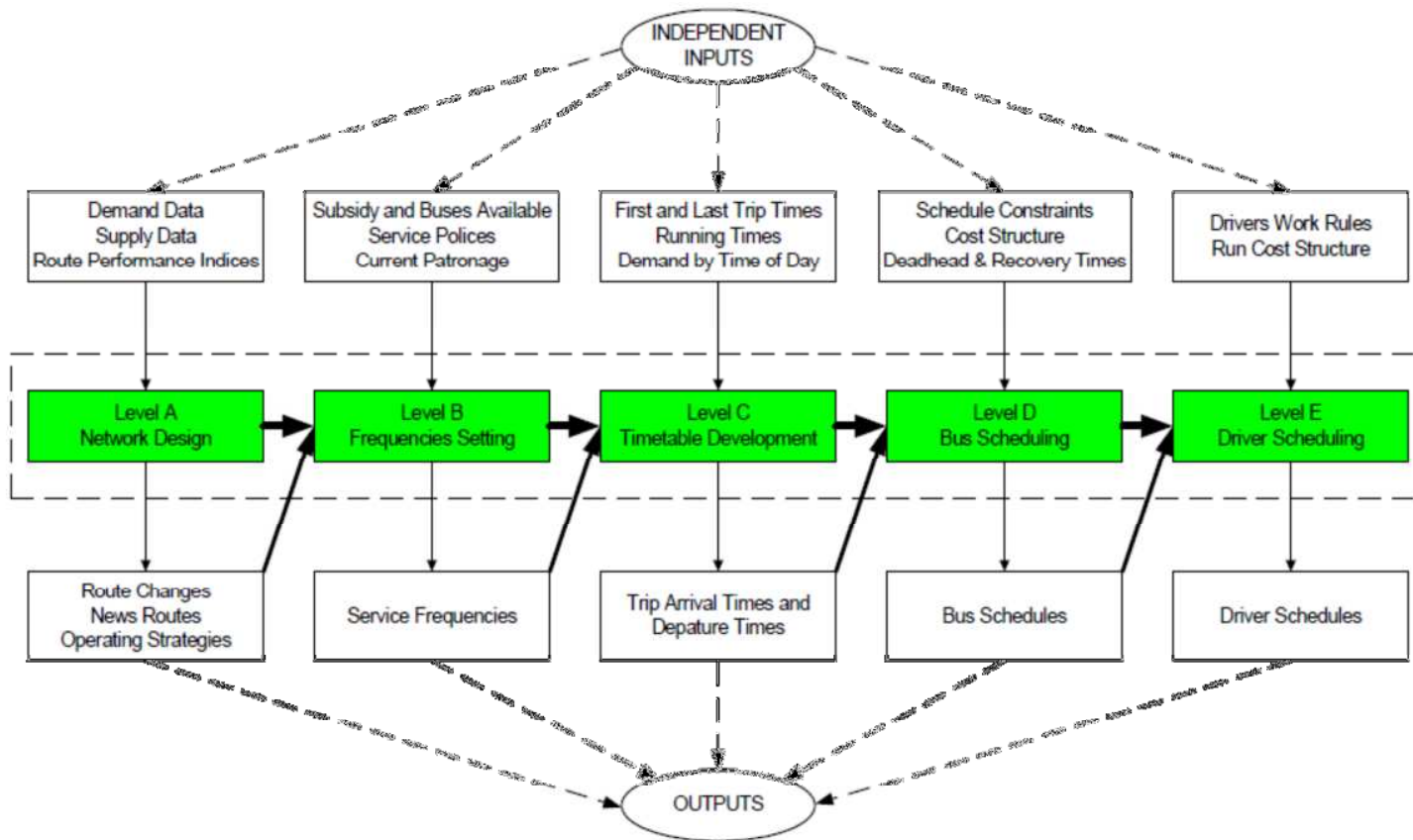
Figure 2.1 Bus planning process.

Source: Fan and Machemehl, 2004
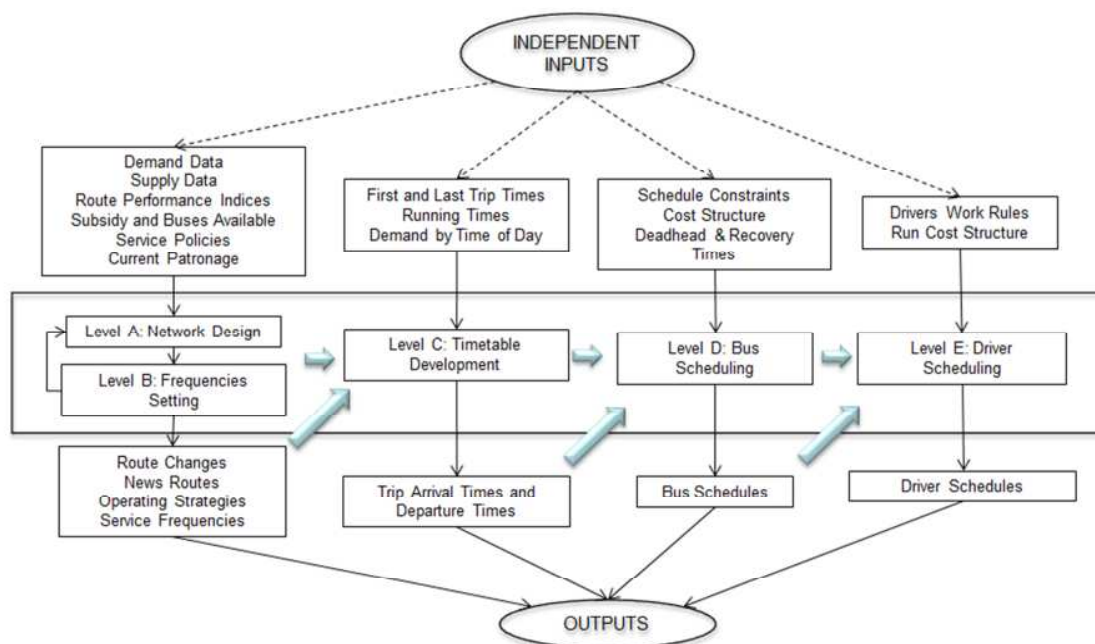
Figure 2.2 Modified bus planning process.

The main purpose of TNDP is to find an optimized bus route configuration. Generally, such optimization is achieved by minimizing various costs such as user cost, operator cost, and social cost. Since a lot of service operation related variables such as service frequency and schedule are associated with the optimization process, these optimized variables are always obtained together with the final optimal configuration.

The earliest TNDP can be traced back to Patz's work in 1925. In his work, the objective function is to minimize the number of empty seats under the constraints of bus capacity and demand. Lampkin and Saalman (1967) and Holroyd (1967)'s research were very early studies on the topic of bus network design problem. Lampkin and Saalman proposed a model to determine the bus route configuration and service frequencies separately. The authors acknowledged that the problems of designing routes and frequency determination should be considered simultaneously. Due to the problem size, these variables were determined separately in the paper. In the first step, a bus network

was composed based on a fixed O-D demand for the purpose of maximizing the number of passengers. In the second step, with the objective of minimizing total travel time, the service frequencies and timetables were determined. Holroyd's work considered a given matrix of origin-destination demand, which was uniformly distributed. He proposed a methodology to analyze a grid bus network.

As for Silman's work (Silman, et al, 1974), the number of buses and O-D pairs between zones were known as input data. The purpose was to find a set of bus service routes and the service frequencies in each route to minimize the sum of the journey time (including waiting, travel and transfer time) plus discomfort penalties proportional to bus occupancy. A two-phase planning method was offered. The first phase is route generation. Instead of a sequential route generation used by Lampkin and Saalman's work, the authors used a route addition and deletion technique by considering, the interaction between various bus lines. In the second phase, based on a set of generated routes, the service frequencies were determined under the constraints of the total number of buses and the finite capacity of the buses using a gradient method.

In Byrne's work (Byrne, 1976), the author assumed a rectangular area of length L and width D with a gridiron pattern of streets. The CBD area was located on the bottom of the area. The passenger demand to the CBD was represented by the number of passengers per unit area per unit time at point (x,y) of the area. The author used a model to locate parallel bus lines as well as headways to minimize the user and operating cost.

In Newell's work (Newell, 1979), the author discussed existing models and issues related to bus route design optimization. The author also pointed out that the main difficulty in optimizing a bus route is that the objective function is non-convex. To solve

the problem, a large computer and a vast amount of data were needed to determine even a nearly optimal route configuration.

In the 1970s, the importance of the integration of intermodal systems was recognized as one of the best solutions to improve the cost-effectiveness of public transportation systems.

Sullivan (1976) and Sullivan and Lovelock (1978) demonstrated the significance of intermodal integration with light-rail transit in an urban area. The authors concluded that a successful integration includes ease of transfer, schedule coordination, design of bus-rapid transit facilities and a fare structure. Moreover, the authors also indicated that the competition relationship between bus and rail in the U.S. needs to be turned into coordination and cooperation. Stanger and Vuchic (1979) also discussed the reason for integrating different modes of transit especially for the integration of bus and light rail. The authors proposed a methodology of physically designing rapid transit stations to improve the transfer efficiency between bus and light rail. Homburger (1972) and Dunn (1980) went over the operation of the Transport Federation to coordinate and integrate transport services. As a new form of association, it was to overcome the fragmentation of transport facilities in metropolitan areas.

An integration of bus and rail systems is thought to be one of the best intermodal integration types. The bus routes here are served as a feeder bus system, which picks up and delivers passengers to a rapid transit station.

Wirasinghe et al. (1977) proposed a methodology to optimize parameters for a coordinated rail and bus transit system that serves trips between a metropolitan area and its central business district (CBD). The author assumed that the highway network is

centered at the CBD and the railway lines are radial. The parameters that the author tried to optimize were the feeder bus boundary, the inter-station space, and the train headways. The objective function was to minimize the sum of operating costs and passenger time costs. Wirasinghe (1980) proposed an approximate analytical model of feeder bus routes, which serves a rail transit line during the peak-period with a many to one demand pattern. The model showed simple relationships between transit system parameters and demand, operating costs, etc., which could be used as "initial-solution" by transit planners.

The previous work on transit network design set down the route spacing, headway, and the stop spacing separately. Besides, the travel patterns assumed in those studies were many-to-one. To overcome such shortages, Kuah and Perl (1988) proposed an analytic model by incorporating all three variables, to optimize feeder bus routes. The objective function was to minimize the sum of bus user and operator costs. The model was based on the assumption of an idealized rectangular geometry area, where there was only a single rail line and the feeder bus routes were perpendicular to the rail line. The demand was fixed with a many-to-many travel pattern and the passengers walked to the nearest stop to access the rail station by bus. A diagram of this configuration is shown in Figure 2.3A.
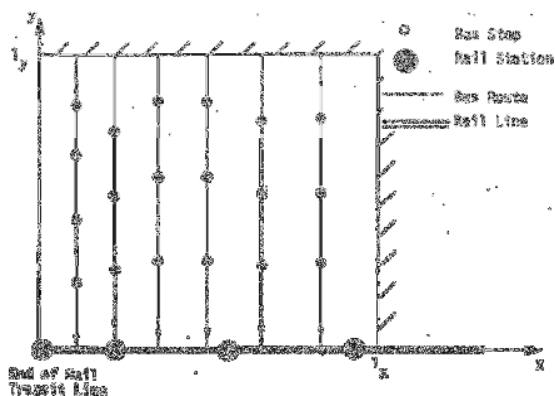
A Geometry of feeder bus service area.

Source: Kuah, 1988
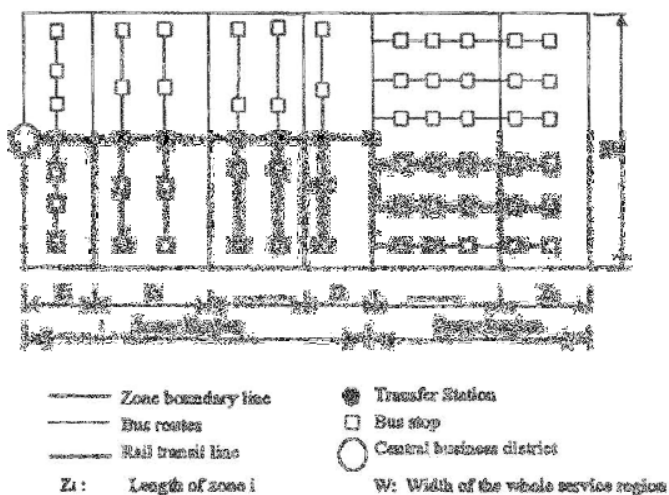
B Route configuration in transit corridor.

Source: Chien and Schonfeld, 1998
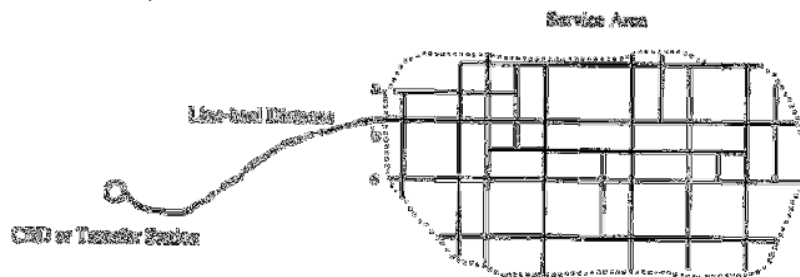
C Configuration and street pattern of the service.

Source: Chien and Yang, 2000

Figure 2.3 The geometry of route configurations.

In 1989, Kuah and Perl gave the first complete definition of Feeder Bus Network Design Problem (FBNDP). It is a problem of designing a feeder-bus network to access an existing rail system. Besides the definition, they also proposed the first mathematical programming model based on the network approach for designing a feeder-bus network in most general cases.

The network of this model includes two types of nodes: rail nodes and bus nodes. The rail nodes represent existing rail stations and the bus nodes stand for the pre-specified bus stop locations which need to be linked by several feeder bus routes. This model focused on the design of a set of feeder bus routes as well as the operating frequency on each route based on pre-specified bus-stop locations. In comparison with the previous work, there was no limitation on the configuration of bus routes. The objective function is to minimize the total cost including rail waiting and riding time cost, bus operating cost, bus waiting and riding time cost.

The key assumptions set by the author were as follows:

1. Pre-specified bus stops.
2. Demand is inelastic and concentrated on bus nodes.
3. Only one feeder bus route serve each bus node.
4. Each feeder bus route only connects one rail station.

Those initial models focused on the design of a set of bus routes and operating frequencies on each route to meet the O-D demand.

Chien and Schonfeld (1998) developed a mathematical model to jointly optimize a rail transit line and its feeder bus system on a predetermined hypothetical network with one rail corridor and perpendicular feeder routes. The objective function was to minimize the total cost including the supplier and user cost. The travel demand pattern was many-to-many and characterized by an irregular discrete distribution in the service area. As an

extension to Kuah and Perl's work, in addition to bus route spacing, bus stop spacing and bus headways, the authors introduced rail station spacing and rail line length as decision variables in the optimization process. The assumptions of service area and route configuration were similar to Kuah and Perl's, as shown in Figure 2.3B. To solve the problem, a successive substitution algorithm was developed to have a quick convergence towards to the minimum of the objective function.

Most of the previous work dealt with the transit route design problem by simplifying either the network structure, such as parallel and orthogonal bus routes; or the characteristics of demand, such as the demand distribution, trip pattern, and the elasticity of demand; or both. Chien and Yang (2000) came up with a more realistic methodology to optimize feeder bus routes that feeds a major intermodal transit transfer station. In their model, irregular service region and street pattern, an irregular and discrete demand distributions, and intersection delays were taken into account, as shown in Figure 2.3C. The objective function was to minimize the total cost, including user and supplier costs. The decision variables were route location represented by link and node incident matrices and feeder bus headway. The authors also provided an improved near-optimal algorithm to obtain the solution.

Although this model included many realistic situations, it did not consider bus stop selection related problems (e.g. stop location, stop spacing,, etc.). The authors assumed that the buses can stop anywhere along the bus route and the demand was also inelastic. Moreover, the bus capacity and size were neglected in the study as well.

As noted, by the beginning of 2000, most bus route design problems still relied on an assumption of fixed demand. This assumption is valid when bus is the only mode

available to the demand to reach their destination. More often, the demands always face various alternatives in which the transit demand becomes sensitive to the level of service of buses. Actually, the importance of incorporating variable demand in the bus route development had been realized very early. Dubois et al. (1979) and Kocur and Hendrickson (1982) incorporated variable demand in the bus route optimization process. In Dubois et al.'s work, they proposed a set of procedures to modify the transportation network to accommodate the existing demand. Besides taking a public bus, walking, is an alternative in their work. Due to the feedback effect of the network upon the changes of demand, the authors used accessibility indices to avoid re-computing a trip matrix each time. As for Kocur and Hendrickson's work, they proposed a local bus service design model including route spacing, headway and fare. Their model applied an equilibrium framework with the demand sensitive to the level of service provided by the bus system.

Although variable demand is close to a real world situation, it did not draw enough attention and the fixed demand model was still preferable in many design processes. One reason was that in some cases with a limited number of alternatives, the assumption of fixed demand still holds. The second reason is the demand models at that time did not proved reliable enough for public transit network changes. In most cases, "those operators had little faith in demand models and were much more concerned about the impact of changes on existing riders than about the potential for generating new ridership." (Ceder and Wilson, 1986)

Oldfield and Bly (1988) proposed an analytical model to optimally find bus size based on the maximization of social benefit. In their work, the operating cost, level of demand, and demand elasticity were treated as decision variables. The demand variation

was based on a generalized cost of travel according to constant demand elasticity. Chang and Schonfeld (1991) introduced another analytical model to optimize bus routes with time dependent variables. The authors discussed four types of demand conditions, which include steady fixed demand, cyclical fixed demand, steady equilibrium demand and cyclical equilibrium demand. Different from previous work, the variation of demand in this model was based on time of the day rather than passenger travel time.

Lee and Vuchic (2005) presented an iterative approach based model to address the TNDP, which considered variable demand in the optimization process. The objective function was to minimize the user travel time under the constraints of the agency's operational factors such as the fare box recovery ratio. Three steps were proposed to achieve the goal. In the first step, based on O-D demand distribution, the shortest path between each O-D pair is determined. Then, the transit assignment procedure was performed under the criteria of shortest total travel time and those routes with low efficiency were eliminated. In the third step, the route improvement procedure was followed. In their work, the authors distinguished between two types of variable demand, variable transit demand and variable total demand. The first demand variation resulted from changes in the modal split while the second one was determined by the urban transportation planning process (FHWA/UMTA 1977). For simplicity purpose, only variable transit demand was considered. As discussed before, when it comes to mode split, previous studies either simplified it as a linearized approximation of a logit mode share model (Kocur and Hendrickson, 1982), or assumed a constant elasticity of demand to represent the variation of demand as a function of travel time. A more generalized logit

model was used in this paper, and two alternative modes were considered, which were transit mode and auto mode.

Although the variable demand was taken into account in bus route optimization, some underlying assumptions were still making it hard to implement in reality. In their work, the demands had been centralized at nodes and these nodes were assumed to be candidate bus stops. The access cost from origins to the bus stops was ignored. In a more realistic situation, walking mode is more common, especially for short distance trips. The consideration of this mode would affect the transit demand dramatically and therefore change the route structure and service frequencies on the network. Moreover, the street pattern that the model assumed is also problematic.

Fan and Machemehl (2004) proposed a more realistic bus network design model without aggregating the travel demand zone into a single node. The objective was to systematically find the optimal TNDP by using hybrid heuristic algorithms. As a multi-objective decision making problem, the objective function is to minimize the total cost, including transit user costs, transit operator costs, and unsatisfied demand costs. The model consists of three parts: an Initial Candidate Route Set Generation Procedure (ICRSGP); a Network Analysis Procedure (NAP); and a Heuristic Search Procedure (HSP).

Many contributions were made to the field of transit network development problems. The first improvement was the configurations of travel demand zone to bus routes. Chien and Yang (2001) discussed a type of configuration as shown in Figure 2.4A. The study area was divided into small zones that have identical width and length, and the demand is heterogeneously distributed in these zones. Chien and Yang developed

14 cases to analyze the fractions of passengers to access the bus routes in the context of a grid street network. Since they assumed that the bus could make stops everywhere on the street, the access distance from origin to the bus route is the vertical distance from the origin to its closest street. This assumption is valid when there is only one zone near the bus route. However, in the situation of more than one zone near the same route, there would be many passengers along even a short street waiting for the bus, and the bus has to make stops again and again. Fan and Machemehl introduced a new distribution pattern, which has been provided in Figure 2.4B. They aggregated the demand at each zone into centroids. If there is only one centroid near a street section, the access cost is still the vertical distance and the crossing point at the street is the candidate bus stop. If there are two centroids around a street section, the selection of the candidate bus stops is the trade-offs of access costs associated with different centroids. This procedure ensured that the number of bus stop is acceptable. One of the shortcomings of Fan's model is that the combination of distribution nodes from two demand centroids along each side of the street is performed prior the optimization process. There is no problem for such combination if both of the boarding stops for the two centroids are located on the same street segment. Otherwise, the shortest access point would be the best candidate bus stop location for a demand centroid.
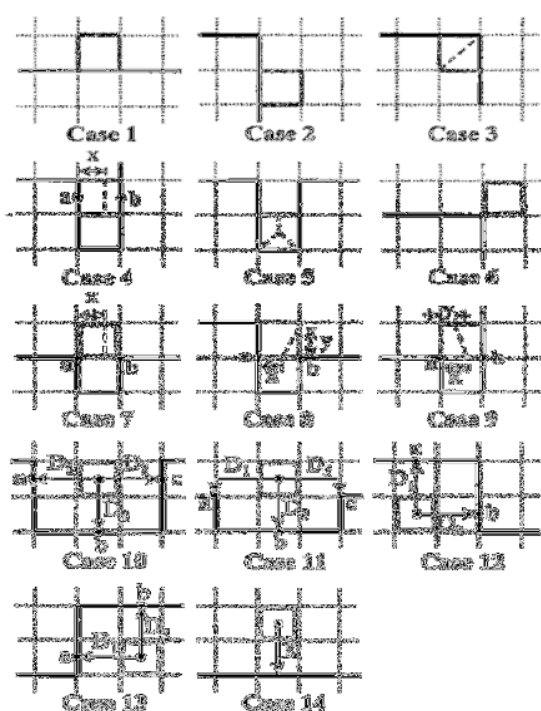
A Configurations of zones to bus route   **B** Distribution nodes illustration.
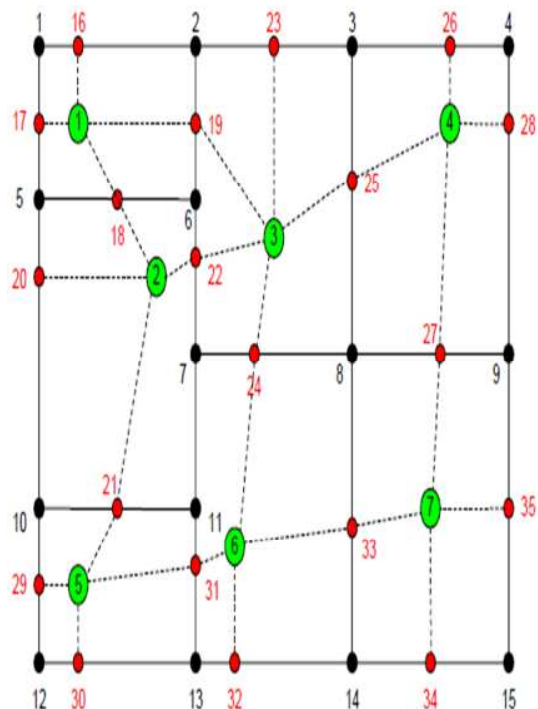
Source: Chien and Yang, 2001   Source Fan and Machemehl, 2004

Figure 2.4 Illustrations of zones and nodes.

The second improvement was the transit trip assignment model. In previous studies, the trip assignment models used in TNDP are very simple. The access cost is the only variable that affects the mode split, and the number of alternative modes is limited. A more complicated trip assignment model was proposed by Fan and Machemehl, which considered more characteristics that affect mode choice, such as the number of transfers, the walking distance, number of transfers, etc. In addition, this work considered unsatisfied demand. This cost is incurred when some traveler demand is not satisfied due to resource constraints. A constant value was assigned to the cost of unsatisfied demand. In many cases, the demand had their own O-D pair. The travel distance associate with each O-D pairs also varies. Simply assigning a constant to these distances is not the best choice. Moreover, if the distance between two centroids met some criterion, walking

would be the best access mode to directly access the destination. The auto mode was not considered in the trip assignment model.

Lownes and Machemehl (2007) proposed a methodology to design the commuter rail circulator network (CRCNDP). The CRCNDP is to optimize the route design of a circulator system which serves commuter rail stations. The optimization is performed on a grid street network. The demand at each zone is aggregated into a centroid. For simplicity purpose, these centroids are located at the center of each zone. The commuter rail stations are pre-allocated with candidate bus stops located at the mid-block and intersection locations. The optimization process consists of two components, feeder bus stop selection optimization and feeder bus route optimization. In their paper, the authors assumed that the commuter rail system begins at a park and ride facility in the suburban areas that surround a metropolitan area, which means the access mode for those stations located in suburban areas is the automobile. The feeder bus system that the author paid his attention to only serves one rail station. To make the problem size manageable, the study area of the feeder bus system is within a 2-mile area around the rail station. The size of this area on the basis of the 10 mph operating speed of buses on an urban route (Levinson, 1983).The coverage radius of suburban stations were 3 miles based on a net operating speed of park and ride at 15 miles/hour. Initially, according to a given commuter O-D table, those O-D pairs whose origins are located in suburban rail station coverage areas and the destinations are inside the rail station coverage area were considered as commuter rail users. Then, a commuter rail mode share was determined. Based on the determined and fixed commuter rail share and a pre-determined walking threshold, the commuter rail demand coverage for each candidate bus stop was

determined. Since not all the candidate bus stops would be selected, the demand not covered by the current feeder bus system was referred to as "unserved cost". The value of unserved cost was derived from Bailey's (2007) work, and set at $6000/household. Finally, the objective function consists of three components, which are user cost, operator cost, and unserved demand cost. In terms of bus stop selection, the author applied Murray (2003)'s methodology to select the optimum number of bus stop candidates. The method was a trade-off between the number of bus stops and the increment of population coverage. Then, the TSP route optimization process was followed. For a very small size network, the author implemented a full enumeration method using by GAMS. For a large size problem, this approach requires a lot of time and the authors employed a metaheuristic methodology--Tabu Search to reach the goal. Some limitations of this research have been concluded as follows:

1. The commuter rail demand obtained does not have to be met necessarily. The trade-off criterion is the value of the objective function. The author simply used "un-served cost" (Bailey, 2007) to represent the commuter rail demand that is not covered by feeder bus system.
2. The demand using the feeder bus system is fixed. Here, the only access mode to the rail station is walking from centroid to a bus stop and taking a bus to the final destination. Other access modes such as: walking to the rail station, driving to the rail station, parking and riding from a remote lot.

Mohaymany (2010) introduced multiple modes with various capacities and performances in FBNDP. The objective function is to minimize the user, operator, and social costs.

2.2 Techniques

In terms of the mathematical types of the models, the TNDP, which have been discussed in the previous section, can be grouped into two categories, the analytical models and the mathematical programming. The analytic model requires restrictive assumptions based on the geometry of streets and the spatial distribution of the demand. The application is restricted by its simple nature (Kuah, 1986). Unlike the analytical models, mathematical programming provides actual network design instead of approximate analytic relationships.

2.2.1 Analytic Methods

The analytic method used to solve the transit route optimization problem deals with finding first-order equations based on a continuous convex objective function to get the optimal transit related parameters including route length, route spacing, stop spacing, service frequencies, etc. The advantage of the analytic method is its rigorousness in theory. It can show clear relationships between decision variables and various transit planning parameters. However, due to the intrinsic complexities of TNDP, several assumptions are made to represent a simplified transit network to ensure the objective function is solvable. Moreover, the analytic method is also very sensitive to the size of network. As the number of streets increase, the feasible solutions will increase dramatically un-proportionally thus resulting in difficulty to obtain the solution.

The analytic method was used in the 1970s. Newell (1979) gave a very good review on the bus route design problem prior to 1979. Due to the limitation of the tools to solve combinatorial types of optimization problems, most of the algorithms at that time mainly delt with the service frequency determination on existing bus routes. Such

problems are generally convex optimization problems, and can be solved by the analytic method with rapid convergence. Wirasinghe et al. work (1977) usedan analytic method to solve the problem of parameter optimization for a coordinated rail and bus transit system. When taking the selection of route structure into account, the route design problem generally become non-convex and could not be solved by analytic method anymore.

The analytic method was also used to tackle several idealized problems when the route structure parameters were involved. One of the attempts is by Byrne and Vuchic (1972). The authors proposed a method to obtain optimal line spacings and frequencies on a parallel feeder bus routes which serve a linear shopping district. Hurdle (1973), Wirasinghe (1980) also made efforts to tackle the feeder bus route location problem by the analytic method. In their work, the feeder bus served a rapid transit line instead of the shopping district. Kocur and Hendrickon (1982) introduced variable demand into the analytic method. To ensure that the analytical approach find unconstrained optima be able to use Lagrange multipliers, the assumption of an infinitely fine rectangular grid network was made by Kuah and Perl (1988) who proposed a differentiation-based method to optimally determine stop spacing and bus route based on a parallel bus route assumption. Chang and Schonfeld (1991) used the analytic method with a time dependent and elastic demand. Chien and Schonfeld (1998) extended the application of analytic method to jointly optimize a rail transit line and its feeder bus system. In 2001, Chien et al. utilized the analytic method to make comparisons between a fixed route and a dial-a-ride bus systems.

Li and Lam (2009) proposed an analytical model to analyze the optimum service frequencies and fares of a multimodal transit system with feeder bus routes under

different market regimes. Passenger demand share for different modes (bus, rail, feeder bus-rail) was determined using a logit model considering the service variables of transit operator. Li's model considers elastic demand and passenger travel behavior. However, all the analyses were associated with a predetermined transit network.

The previous studies that used analytic methods to develop in transit networks made development process, several assumptions including simple network structure and regular service regions have to be made to make the problem simple enough to be solved analytically. However, these assumptions sacrifice the practical applicability of the models.

2.2.2 Mathematical Programming Methods

The application of the mathematical programming method in TNDP started 30 years ago along with the development of programming techniques, heuristic techniques and computational power. (Lownes and Machemehl, 2007)

The complexities revealed by past literatures in TNDP can be categorized into five folders (Baaj and Mahmassani, 1991):

1. Problem formulation: This problem consists of defining decision variables and objective functions. Those variables include service frequency, stop spacing, route spacing,, etc. Examples of objective functions are minimizing user's cost, operator's cost or total cost.
2. Non-linearities and non-convexities: Non-linearities mean that the relationship between decision variables and objective function is not linear. A non-convexity example can be illustrated by the fact that as the increase of bus services, the total travel time does not decrease in the absence of transit waiting time.
3. Combinatorial explosion: There are so many discrete variables involved in TNDP. As the size of the network increases, the solution space will increase exponentially, which makes the problem to be NP-hard.
4. Multiobjective nature: Trade-offs based on different perspectives are quite common in FNDP optimization process, such as minimizing travel time, maximizing profit, and maximizing social benefit, etc.

5. Spatial layout of routes: This difficulty is associated with the layout of transit routes.

Due to the complexities listed above, many proposed TNDP related models with simplified networks use the analytic method. When the network becomes very large and complicated network with multiobjective functions, the conventional analytic method loses its effectiveness. For such kind of problems, mathematical programming was introduced and various heuristic approaches were developed. Such heuristic approaches search the solution based on design guidelines, criteria established from past experiences, cost, and feasibility constraints. (Zhao, 2009)

An example of an early application of the mathematical programming method is demonstrated in Dubois et al.'s work in 1979. They developed a mathematical programming model to tackle the problem of modifying a transportation network to make it fit the existing demand. Considering the difficulty in solving the optimal network problem (ONP), the authors used a heuristic method to obtain an approximate solution.

Kuah (1989) developed an algorithm to solve his FBNDP model. His algorithm consists of two parts: constructive heuristic, which generates an initial solution; and an improvement procedure to further improve the initial solution. A sequential building procedure was used in the constructive stage and displacement and exchange procedures were employed in the second stage. All these procedures are inspired from an algorithm in multi-depot vehicle routing problem (MDVRP).

Shrivastav and Dhingra (2001) proposed a heuristic algorithm to develop feeder bus system from railway stations to various identified potential destinations in Mumbai, India. It was the first portion of a model that had been developed for integration of public buses and suburban railway system. The Shrivastava-Dhingra heuristic feeder route generation algorithm (HFRGA) was based on the demand matrix. In the beginning, those

potential destinations with relatively high demand were selected. Then, a shortest path algorithm was exercised to each of them to obtain initial feeder bus routes. In the second stage, the node insertion process was performed based on the criteria of maximum demand-deviated shorter time-path, which guaranteed to insert a node to the best route in the best possible way.

Lee and Vuchic (2005) pointed out the drawbacks of the previous combinatorial approaches. In the combinatorial approach, sample spaces are needed as candidate routes to help decide the optimality of the results. The number of the initial candidate routes is a key factor in the methodology, and the determination of which relies on the knowledge of the analyst. For this reason, an iterative approach was proposed. This approach started with the minimum in-vehicle travel time network generated by the shortest path algorithm (Dijkstra 1959; Whiting and Hillier 1960; Dantzig 1966)

To effectively find a superior solution for large size combinational optimization problems, various stochastic local search strategies have been developed such as Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithms (GA), Ant Colony Optimization (ACO), etc. All of these are known as metaheuristic approaches, that optimize a problem by iteratively trying to improve a candidate solution based on a given measurement of quality. It is also noted that such approaches are used to pursue reasonably good local optima but do not guarantee obtaining the global optimal solution.

Baaj and Mahmassani (1995) proposed a hybrid artificial intelligence/ operations research (AI/OR) method, which incorporates AI as efficient search techniques with conventional heuristics in TNDP.

To handle a much more complicated network, Chien et al. (2001) extended Chien and Yang's work (2000) by using a genetic algorithm (GA). The proposed GA consisted of two major operations: route generator and genetic operators. Initially, routes are randomly created by the route generator as an initial population. Then, route improvements were performed by GA to obtain a superior solution. A comparison between the exhaustive search algorithm (ES) and GA revealed that the solutions obtained by GA were acceptable but the CPU time decreased dramatically compared to ES. Chien et al. (2003) used GA for an improved methodology developed in their previous work to optimize a whole bus system in an area. The decision variables include the number of routes, route locations, headways, intersection delays, and realistic street patterns. Shrivastava and Mahony (2006) used the GA in a feeder bus route development model incorporating the coordination between feeder buses and rail service. In their work, the routes and coordinating frequencies of bus and rail were coded into a single string to represent decision variables in GA.

Martins (1998) extended Kuah's (1989) heuristic approach by introducing "two-phase building" approach prior to the constructive heuristics and introduced Tabu Search to improve the efficiency of the previous approach. The author compared the efficiency of four search heuristics, including displacement heuristic 1 and 2, exchange heuristic, and Tabu Search. The result revealed that the Tabu Search slightly outperform the three of other local search heuristics. Cordeau and Laporte (2003) used the Tabu Search heuristic in a multi-vehicle dial-a-ride problem to find a set of least cost vehicle routes which meet the demand. Fan and Machemehl (2004) studied the bus transit route

development problem with both fixed and variable demand and made comprehensive analyses on various metaheuristics algorithms.

Zhao and et al. (2005) developed an approach which integrated both a simulated annealing an a tabu search method to solve the problem of minimizing transfers and maximizing service coverage in transit network optimization. The simulated annealing search scheme helped the proposed approach avoid being trapped into poor local optima and the tabu remedy ensured that the approach visited a new solution to explore superior solution. The authors stated that this new search algorithm is much more effective than a plain simulated annealing method. Juan et al (2004) performed a comparison of between simulated annealing (SA) and tabu search (TS). The computational experiments showed that TS is more effective than SA in solving the FBNDP.

Fan and Machemehl (2008) used the Tabu Search (TS) method to solve the bus transit network development problem (BTRNDP). GA was used as a benchmark to measure the quality of TS. The results revealed that TS is at least as good as or even better than GA in solving the BTRNDP with variable transit demand.

Nee (2004) attempted to made a comprehensive comparison of different metaheuristic algorithms based on a modified feeder bus network development model proposed by Kuah (1989). The comparisons were performed among TS, GA, SA, and ACO. Based on the results, the author concluded that the Tabu Search is the most effective metaheuristic, became it obtained the best solutions.

2.3. Summary

As discussed in the introduction section, many researchers have contributed in transit network development processes by focusing on various route development strategies, metaheuristic approaches, or schedule coordination optimizations. The major shortcomings of the previous approaches are discussed in the remainder of this section.

As shown in Table 2.2, to make the network development problem easy to be solved, most of the previous work about FBNDP did not consider variable demand in route optimization process. However, in the real-world, for a given O-D matrix, the bus transit demand will be affected by the level of service that the existing transit network delivers. The most up to date systematic approach of feeder bus network development is provided by Lownes and Machemehl (2007). As mentioned previously, demand for a feeder bus system is fixed. In their methodology, there are only two situations for demand at a centroid, which are covered by feeder bus system or not. The authors simplified the un-served demand by assigning an un-served cost to it. For the covered demands, the access mode for them to the rail station is: walking + bus, which means walking to the nearest bus stop and taking the bus to the rail station. The authors did not take other modes or mode combinations into account, such as auto, and walk only. However, a recent study on commuter rail mode choice reveals that, walk, bus and auto modes make up of 75% of the commuter trips. (Bergman, et. al. 2011)

It was noted that the most recent work regular transit network development that considered variable demand is by Fan and Machemehl (2004). They proposed a model for the regular transit route network development problem (TRNDP), in which the demand mode split was considered. However, walk only mode was neglected again.

Moreover, the determination of service frequencies is of importance because the final results of bus service frequencies are very sensitive to the initial inputs of service frequencies for each bus route the authors do not pay much attention to it.

As for the route structures, most of early models of feeder bus network development are based on an over simplified network, such as parallel routes. Recent work started to incorporate more complicated routes. However, there are still a lot of restrictions. For example, the configuration of the feeder bus route developed by Lownes and Machemehl (2007) is solved as the travel salesman problem (TSP). Moreover, the methodology developed by the authors can only develop one route at a time. If we want to develop a set of feeder bus routes for a rail station, we have to run the method again and again. The methodology could provide one good route configuration each time, but it cannot guarantee that the combination of these routes is still good enough. Other methodologies for developing multiple bus routes do exist, but they are developed for regular bus networks.

As pointed out in Fan and Machemehl's (2008) work, most previous methods in developing bus transit networks did not consider the problem in the context of "distribution node". They simply aggregated the zonal travel demand into a single node to represent a bus stop. However, the existing "distribution node" method proposed by Fan and Machemehl combined two distribution nodes which are within a street segment prior to the optimization process. It would lead to ideal result if the access points of the centroids, that those two distribution nodes belong to, both located on the street segment. When there is only one access point on the street segment, the combination of multiple distribution nodes is questionable.

In terms of solving methods, previous references have made attempts to use various metaheuristic approaches to solve TNDP. For the preceding TNDP models that were solved by mathematical programming method, they can be classified into two categories according to the type of approach to find the neighborhood of a given solution. In the first category, neighbor solutions were obtained by exchanging or displacing nodes or linkages based on an initial route structure. (e.g.. Martins, 1998; Chien et al., 2001; Shrivastav and Dhingra; 2001; Lee and Vuchic, 2005; Juan, et. al., 2004;) In the second category, candidate solution route sets have been generated prior to the optimization process by using some criteria such as k-shortest paths algorithm. Any candidate route set can be a neighbor solution to the TNDP model and the metaheuristic method is used to effectively find an acceptable solution for the TNDP model from the solution sets. (e.g.. Fan and Machemehl, 2004, 2009;) Based on the comparisons made by previous references between different kinds of metaheuristics, the TS is thought to be one of the best stochastic local search strategies in solving TNDP in both categories. In consideration of the solution space involved in this study also belongs to the second category, the TS is selected in this study as a solution method to find an optimum set of route set from the solution space.

Table 2.2 Summary of Transit Network Design Models

Year	Author	Objectives	Decision Variables	Demand Pattern	Demand	Network Structure
1967	Lampkin & Saalmans	minimize total travel time	Candidate route choices, headway	many-to-many	fixed	straight-line connections between stops
1972	Byrne & Vuchic	minimize total cost	route spacing, headway	many-to-one	fixed	parallel routes feeding a rail line
1979	Newell	minimize total cost	route spacing, headway	many-to-one	fixed	grid street network
1982	Kocur & Hendrickson	operator profit or user benefit maximization or minimize total cost	route spacing, headway, fare	many-to-one	variable based on travel time (linear demand model)	parallel routes
1986	Vaughan	minimize travel time	route spacing, headway	many-to-many	fixed	ring and radial routes
1989	Kuah & Perl	minimize total cost	route network, headway	many-to-one & many-to-many	fixed	straight-line connections between stops feeding a rail line
1991	Chang & Schonfeld	minimize total cost	route spacing, stop spacing, headway	many-to-many	variable based on time of a day	parallel bus routes
1998	Chien & Schonfeld	minimize total cost	rail line length, station spacing, bus route spacing, headway	many-to-many	fixed	parallel bus routes feeding a rail line
2000	Chien & Schonfeld	minimize total cost	route network, headway	many-to-one	fixed	irregular grid street network
2004	Fan & Machemehl	minimize total cost	route network, selection of stops, headway	many-to-many	variable based on travel time; BLM-IPM model (auto, transit)	irregular grid street network
2005	Lee & Vuchic	minimize travel time, maximize profit, maximize social benefit	route network, headway	many-to-many	variable based on travel time; logit model (auto, transit)	straight-line connections between stops
2007	Lownes & Machemehl	minimize total cost	route network, selection of stops, headway	many-to-one	fixed	irregular grid street network; feeding a rail station
2009	Zhao	minimize transit transfers & total user cost &maximizing service coverage	route network, headway	many-to-many	fixed	regular grid street network

CHAPTER 3

MODEL FORMULATIONS

The objective of this study is to develop a new mathematical programming model for FBNDP. In this chapter, the model formulation of this new method is presented. The organizations of this chapter are as follows. In section 3.1, assumptions for the proposed model are firstly provided. The objective function, related constraints, and the explanations for those characteristics associated with these formulations are presented in section 3.2. Section 3.3 discusses constraints that are considered in the research. A conclusion of this chapter is followed in section 3.4.

3.1 Assumptions of the Feeder Bus Network Development Problem (FBNDP)

Following are the assumptions for the proposed FBNDP:

1. The demand that this study focuses on consists of work trips to and from the CBD via the suburban rail station. The origins of the commute are located in a pre-defined suburban area, where there is a rail station connecting to the destination CBD area. The characteristic of such trips is known as many-to-one demand pattern. Three modes are available to access the rail station, which are walk, bus and auto. Since the rail system is the only mode for these trips in this area to access the destination work places in CBD, the total demand for the rail system is constant. However, due to different access modes vary in cost, the bus, walking and auto shares are variable..

2. In calculating the stop delays, only one direction delays are counted during one peak hour period. For example, in the morning peak hour, when the feeder buses come back from the rail station, the stop delays occurred in this direction is ignored, since in the morning peak, the demand from the rail station is assumed to be zero. Of cause, if it is needed, the stop delays for both directions can be easily applied to the proposed FBNDP. The stop delay at each bus stop is assumed to be constant and independent of boarding demands.

3. No transfers between different feeder buses are considered in this study. Moreover, because the feeder bus network discussed in this study serves only one rail station, the transfers between feeder bus routes are also impractical in the real world.

4. The average bus travel speed, auto speed, walk speed are constant and independent of the condition of a network. The bus capacity and load factor are the same for all buses. In the real application, the average bus speed can also be obtained easily based on the traffic condition of a network.

3.2 Objective Function

The objective of the proposed new FBNDP model is to minimize the total cost of users and service suppliers. In this study, the total cost consists of total user's cost and total operator's cost as in the following Equation and Figure 3.1.

$$C_{total} = C_{users} + C_{operators} \tag{3.1}$$
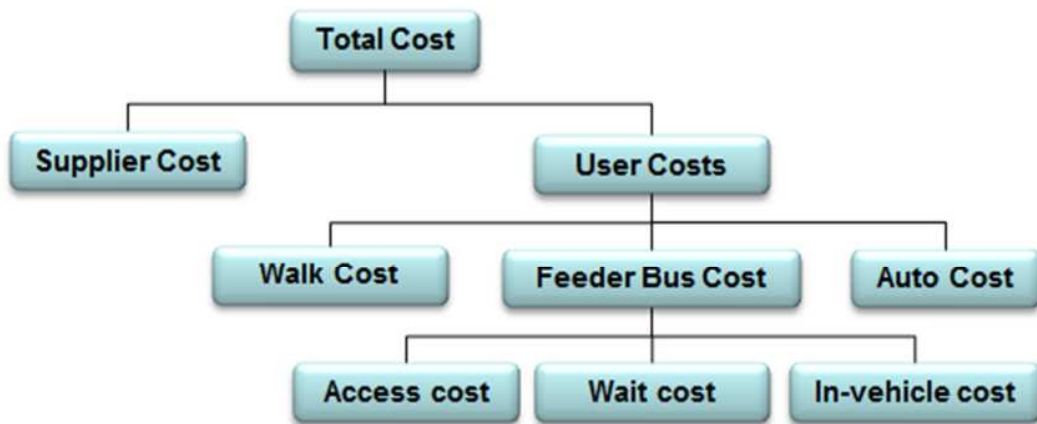
Figure 3.1 Total cost of the objective function.

The user costs may be defined by the costs of the access modes each traveler selects. As discussed previously, in this paper, three modes or mode combinations are considered. For example, if a traveler chooses to walk from his or her home to the rail station, the access cost for the traveler is only the value of walking time. The monetary cost is minimum or zero. This cost can be computed by Equation (3.2), where q_{wi} represents the demand at centroid i that chooses walking as the access mode to the rail station. R_{wi} is the length of the walking path from centroid i to the rail station. U_{wk} is the average walking speed, and λ_{wk} is the value of walking time.

$$C_w = \frac{\lambda_{wk}}{U_{wk}} \sum_{i=1}^{I} q_{wi} R_{wi} \qquad (3.2)$$

For those who take buses as access mode to the rail station, the access cost may be measured by the walking time to a bus stop, the waiting time for a bus, the riding time in the bus, and the bus delay at each bus stop along the route, which is presented in Equation (3.3).

$$C_b = \frac{\lambda_{wk}}{U_{wk}} \sum_{j=1}^{J} \sum_{i=1}^{I} q_{bi} X_{ij} L_{ij} + \lambda_{wait} \sum_{j=1}^{J} \sum_{i=1}^{I} q_{bi} X_{ij} \frac{H_{bj}}{2}$$
$$+ \sum_{i=1}^{I} \sum_{m=1}^{m} \sum_{j=1}^{J} (\frac{\lambda_{riding}}{U_{bus}} q_{bi}^{fmj} X_{ij} R_{bmj} + \lambda_{riding} q_{bi}^{fmj} X_{ij} \lambda_{stop} N_{smj}) \qquad (3.3)$$

Equation (3.3) consists of three terms. The first term indicates the total bus stop access cost, where q_{bi} stands for the bus demand at centroid i; X_{ij} is a 0-1 variable, if the demand at centroid i choose j as bus stop, X_{ij} =1, otherwise, 0; L_{ij} is the route length from centroid i to the distribution node j. The second term is the cost comes from waiting for the bus. H_{bj} is the bus head way at bus stop j. $H_{bj} = \frac{1}{F_{bj}}$, where F_{bj} is bus service frequency at distribution node j. q_{bi} represents the bus demands at centroid i Since there might be more than one bus route going through distribution node j, the service frequency F_{bj} can be expressed as $F_{bj} = \sum F_{bmj}$, where F_{bmj} is the bus frequency of route m. For this reason, the total demands at distribution node j will also be assigned to these routes based on some criteria. One of the methods is to assign the demands proportionally based on service frequencies (Ulusoy, et. al, 2010), which is $q_{bi}^{fmj} = q_{bi} * X_{ij} \frac{F_{bmj}}{\sum F_{bmj}}$, where q_{bi}^{fmj} is the bus demands from centroid i that choose bus route m at distribution node j to access the rail station. The third term is related to in-vehicle travel time. The in-vehicle related

travel time is made up of two components. One is in-vehicle travel time which is also converted into monetary cost by a parameter of in-vehicle value of time, λ_{riding}. The other is stop delays occurring at each bus stop. N_{smj} is the number of bus stops on route m after stop j.

The automobile mode is the third alternative. When a centroid is is not covered by any feeder bus services, the automobile becomes the only mode to access the rail station. The automobile cost is composed of in-vehicle cost ($\frac{\lambda_{auto}}{U_{auto}}\sum_{i=1}^{I} q_{ai}R_{ai}$), parking cost

($\lambda_{park}\sum_{i=1}^{I} q_{ai}$) and auto operating cost ($\lambda_{ac}\sum_{i=1}^{I} q_{ai}R_{ai}$).

$$C_a = \frac{\lambda_{auto}}{U_{auto}}\sum_{i=1}^{I} q_{ai}R_{ai} + \lambda_{park}\sum_{i=1}^{I} q_{ai} + \lambda_{ac}\sum_{i=1}^{I} q_{ai}R_{ai} \qquad (3.4)$$

Finally, the total user cost C_{user} is:

$$C_{user} = C_w + C_b + C_a \qquad (3.5)$$

As discussed in Chapter 2, most previous references either assumed all the existing demand has to be met by a bus service network or considered the demand that cannot be covered by bus network as unsatisfied demand. To determine the cost associated with the un-covered demand, a constant value to these demands is assigned to get a cost. However, in many cases, all the demands have their own O-D pairs. The travel distance associate with each O-D pair also varies. Simply assigning a constant to them is problematic. For this reason, an assumption, that automobiles would be the only access mode for those unsatisfied demands, is made in this dissertation and the auto costs are also considered to replace the un-satisfied cost. The reason for including all the users' access costs in the objective function is that, basically the FBNDP is trade-offs between

service provider and users. If some portions of users' costs are not included in the optimization process, the result would also be unreasonable.

In terms of the operator's cost, the feeder bus system is the only mode served by an agency. The operator's cost from bus service is measured by \$/hour as shown in the Equation (3.6). The total round trip travel time for a feeder bus consists of round trip time ($\frac{2R_m}{U_b}$), and stop delays along this trip ($\lambda_{stop} * J_m$). The value of average bus speed here already includes the delays at intersections. One can note that only stop delays in one direction are considered. This is due to the characteristics of the working trips. In this study, the feeder bus system only serves one rail station. Most of the demands using the feeder bus system are working trips during the morning and evening peak hour, which means there is only one peak direction at a time. For example, in the morning peak hour, the peak direction would be the inbound (to the rail station) trips. There are few passengers on the outbound direction trips, so in calculating the round trip time, these stop delays in this direction are ignored. If needed, the non-peak direction bus route can also be replaced by a shortest path to further reduce the operating cost.

$$\lambda_{bc} \sum_{m=1}^{M} F_{bm} * (\frac{2R_m}{U_b} + \lambda_{stop} * J_m) \tag{3.6}$$

The parameters of q_{ai}, q_{bi}, and q_{wi} in the above equations represent demand of auto, bus and walking at centroid i respectively, which are determined by the following equations.

$$q_{ai} = Q_i * \frac{e^{Uauto}}{e^{Uauto} + e^{Ubus} + e^{Uwalk}} \tag{3.7}$$

$$q_{bi} = Q_i * \frac{e^{Ubus}}{e^{Uauto} + e^{Ubus} + e^{Uwalk}} \tag{3.8}$$

$$q_{wi} = Q_i * \frac{e^{Uwalk}}{e^{Uauto} + e^{Ubus} + e^{Uwalk}} \quad (3.9)$$

Q_i is the total demand at centroid i to the rail station. U_{auto}, U_{bus}, and U_{walk} are utilities of

auto bus and walking modes. The determinations of U_{auto}, U_{bus} and U_{walk} are discussed in

section 4.5.2.

Now, the final objective function can be expressed as:

$$
\begin{aligned}
Min\_Z = &\left(\begin{array}{l} \dfrac{\lambda_{wk}}{U_{wk}} \sum_{j=1}^{J}\sum_{i=1}^{I} q_{bi} X_{ij} L_{ij} + \lambda_{wait} \sum_{j=1}^{J}\sum_{i=1}^{I} q_{bi} X_{ij} \dfrac{H_{bj}}{2} + \\ \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J} (\dfrac{\lambda_{riding}}{U_{bus}} q_{bi}^{fmj} R_{bmj} + \lambda_{riding} q_{bi}^{fmj} \lambda_{stop} N_{smj}) \end{array} \right)_{BusUser} \\
&+ \left(\lambda_{bc} \sum_{m=1}^{M} F_{bm} * (\dfrac{2R_m}{U_b} + \lambda_{stop} * J_m) \right)_{BusOperating} + \left(\dfrac{\lambda_{wk}}{U_{wk}} \sum_{i=1}^{I} q_{wi} R_{wi} \right)_{WalkOnly} \\
&+ \left(\dfrac{\lambda_{auto}}{U_{auto}} \sum_{i=1}^{I} q_{ai} R_{ai} + \lambda_{park} \sum_{i=1}^{I} q_{ai} + \lambda_{ac} \sum_{i=1}^{I} q_{ai} R_{ai} \right)_{AutoOnly}
\end{aligned}
\quad (3.10)
$$

Table 3.1 contains parameters, their units and descriptions as used in Equation

3.1-3.10 and subsequent sections.

Table 3.1 Notation of Parameters

Parameter	Units	Description
i:	---	Centroid node; i= 1, 2, ···,I;
j:	---	Distribution node/ candidate bus stop; j=1, 2, ···, J;
m:	---	Feeder bus route id; m=1, 2,...,M;
J_m	---	Total number of stops on route m;
λ_{wait}:	\$/pass-hr	Value of waiting time;
λ_{wk}:	\$/pass-hr	Value of walking time;
λ_{riding}:	\$/pass-hr	Value of riding time;
λ_{auto}:	\$/pass-hr	Value of private mode driving time;
λ_{park}:	\$/day	Parking cost;
λ_{ac}:	\$/veh-mile	Unit auto operating-cost;
λ_{bc}:	\$/veh-hour	Unit bus operating-cost;
λ_{stop}:	hr/stop	Delays for each bus stop;
ρ	---	Bus load factor;
C	Seats/veh	Bus capacity;
U_{wk}:	miles/hr	Average walking speed;
U_{bus}:	miles/hr	Average bus speed;
U_{auto}:	miles/hr	Average auto speed;
BusLMax	miles	Maximum bus route length;
BusLMin	miles	Minimum bus route length;
Mmax	routes	Maximum allowable bus routes;
R_{wmax}	miles	Maximum walking access distance to the rail station;
R_{bwmax}	miles	Maximum walking access distance to the bus stop;
R_{bmin}	miles	Minimum inter-stop spacing
F_{max}	vehs/ hr	Maximum service frequency
F_{min}	vehs/hr	Minimum service frequency
q_{bi}:	pass/hr	Bus demands at centroid i; $q_{bi} = q_i * P_b$, P_b: percentage of demand choosing bus service;
q_{wi}:	persons/hr	Walk only demands at centroid i; $q_{wi} = q_i * P_w$; P_w: percentage of demand choosing walk;
q_{ai}:	persons/hr	Auto only demands at centroid i; $q_{ai} = q_i * P_a$; P_a: percentage of demand choosing auto;
Q_{bm}	people/hr	Total transit demand on feeder bus route m;
q_{bi}^{fmj}:	persons/hr	Bus demand from centroid i and board on bus route m at distribution node j; $q_{bi}^{fmj} = q_{bi} * X_{ij} \frac{F_{bmj}}{\sum F_{bmj}}$; F_{bmj}: service frequency (veh/hr) of bus route m which go through distribution node j; (Ulusoy, et. al, 2010)
L_{ij}:	miles	The access distance* from centroid i to distribution node j;
R_m:	miles	The total route length of bus route m;
R_{bmj}:	miles	The length of bus route m from distribution node j to the rail station;
R_{wi}:	miles	Walk access distance* from centroid i to the rail station;
R_{ai}:	miles	Auto access distance* from centroid i to the rail station; $R_{wi}=R_{ai}$;
N_{smj}:	stops	Number of stops that bus route m has after stop j;
F_{bm}:	vehs/hr	Service frequency of bus route m;

*All the distance used in this study indicates the Manhattan distance.

Table 3.1 Notation of Parameters (continued)

F_{bj}:	vehs/hr	Service frequency at the distribution node j; if there are multiple routes go through this distribution node, it is the total frequencies of all the routes; $F_{bj} = \sum F_{bmj}$; F_{bmj}: the frequency of bus route m which goes through the distribution node j (vehs/hr);
H_{bj}:	hr/vehicle	Head way at the distribution node j; $H_{bj} = \dfrac{1}{F_{bj}}$;
f_m:	buses/hr	Service frequency of feeder bus route m;
H_{bm}:	hr/veh	Head way of bus route m; $H_{bm} = \dfrac{1}{F_{bm}}$;
W:	buses	Maximum available fleet size;
X_{ij}:	---	1, if the demand at centroid I choose j as bus stop; 0, otherwise;
$isRj$:	---	1, if there is at least one route go through node j; 0, otherwise;

*All the distance used in this study indicates the Manhattan distance.

3.3 Constraints of the FBNDP

In this section, the realistic limitations in developing feeder bus network is presented and discussed. These constraints include bus stop selection, load factor, headway, route length, and fleet size.

3.3.1 Bus Stop Selection Constraints

$$0 \le \sum_{j=1}^{J} X_{ij} \le 1 \text{ for i=1, 2,...., I;} \tag{3.11}$$

$$X_{ij} L_{ij} \le R_{bw\max} \tag{3.12}$$

$$X_{ij} \le isR_j \tag{3.13}$$

There are three constraints in the category of bus stop selection in all. The first constraint is to ensure each centroid i at most choose one bus stop. If no distribution nodes available within R_{bwmax} miles around the centroid, $\sum_{j=1}^{J} X_{ij} = 0$; The second one prevents walking to

bus stop if the distance from a centroid to the stop is further than the threshold R_{bwmax}. For example, when $X_{ij}=1$, which means the demand at centroid i chooses distribution node j as boarding bus stop, the access distance L_{ij} must be equal or less than R_{bwmax}. In consideration of the cases that a distribution node might be selected even though no route goes through it, inequality (3.13) is introduced as the third biding element. isR_j is a 0-1 variable. If at least one route goes through node j, $isR_j = 1$, otherwise $isR_j = 0$.

3.3.2 Load Factor Constraint

The bus load factor ρ is the occupancy rate of a bus, and it ranges from 0 to 1. If a bus is fully occupied, ρ=1. For the bus route m, if the occupancy of the bus is C, the maximum allowable load factor is ρ, and the service frequency is F_{bm}, the total route capacity can be expressed as $F_{bm}*C*\rho$. q_{bi}^{fmj} is the demand of route m at centroid node j that makes $\sum_{i=1}^{I}\sum_{j=1}^{Jr} q_{bi}^{fmj}$ the total demand of route m. The route capacity needs to be equal or larger than the demand, so

$$F_{bm}*C*\rho \geq \sum_{i=1}^{I}\sum_{j=1}^{Jm} q_{bi}^{fmj} \quad m \in M \tag{3.14}$$

3.3.3 Headway Feasibility Constraint

$$H_{min} \leq H_{bm} \leq H_{max} \qquad m \in M \tag{3.15}$$

As addressed by most literatures, three different kinds of headways, supply frequency, demand frequency and policy frequency, are frequently mentioned in transit industry. The bus supply frequency is generally determined by the fleet size, and other economic resource essentials. The resource limitations which come from the service supplier set a

threshold of a minimum headway. Since public transportation is a customer oriented service, the purpose of such a system is to meet the demand of society. As implied by the name, the demand frequency is determined directly by the demand and its formulation has been presented in the previous load factor constraint section. The policy frequency is generally up to operators. It is used when the supply capacity is much higher or lower than the actual demand.

In this study, the frequency needs to be determined is the demand frequency. In some cases, when the demand frequency fluctuates too much, the policy frequency will be considered. The minimum and maximum policy frequencies used are 1 veh/hr and 12 vehs/hr respectively.

3.3.4 Route and Fleet Constraints

$$R_{\min} \leq R_m \leq R_{\max} \qquad m \in M \tag{3.16}$$

$$M \leq M_{\max} \tag{3.17}$$

$$J_m > 1 \quad m \in M \tag{3.18}$$

$$\sum_{m=1}^{M} F_{bm} * (\frac{2R_m}{U_b} + \lambda_{stop} * J_m) \leq W \tag{3.19}$$

Inequalities (3.16) and (3.17) provide the limitations on route length and number of routes. Inequality (3.18) requires all the feeder bus routes have at least two or more stops. Inequality (3.19) limits the fleet size which is available for using in FBNDP.

3.4 Summary

In this chapter, the formulation for the FBNDP is provided. The purpose of this model is to develop a feeder bus system that serves a suburban area. The decision variables are the layout of the bus routes and service frequencies. The objective is to minimize the total cost that is incurred by the users and theservice supplier. Three access modes including walking, automobile, and feeder bus system are considered in calculating the total user cost. In the next chapter, a solution methodology for the FBNDP is presented.

CHAPTER 4

SOLUTION METHODOLOGY FOR PROPOSED FORMULATIONS

4.1 Introduction

Early approaches in FBNDP have been widely discussed in Chapter 2. The major shortcomings of these methods were:

1. Failure to include the costs of some major access modes such as walk only or automobile only. In some studies, an un-satisfied cost was considered to simplify those costs which are ignored by a public transit network.

2. Failure to develop multiple feeder bus routes at a time on an irregular grid street network in considering variable demand circumstances.

3. Failure to take stop delays into account in developing the feeder bus network when the transit demand is variable.

4. Failure to develop an efficient method to determine service frequency when the transit demand is sensitive to the level of service.

5. Failure to develop an effective method to determine bus stop locations. It was either assumed the stop could be everywhere on the route or pre-assigned the stop to a location which may not be the shortest street access point for the nearest centroid.

To overcome the above disadvantages, a new FBNDP is developed. In Chapter 3, a formulation of this new model as well as the assumptions associated with the model were presented. In this chapter, a complete solution methodology for this new model is proposed. Section 4.2 presents a framework of this methodology. The methodology consists of four key modules, which are "Preparation Procedure", "Initial Solution Generation Procedure (ISGP)", "Network Features Determination Procedure (NFDP)", and "Solution Search Procedure (SSP)". Sections 4.3, 4.4, 4.5 and 4.6 discuss each of them respectively. The last section is a summary which makes a conclusion for this chapter.

4.2 An Overview of the Solution Methodology

There are totally six modules in the solution methodology. Besides the four components that are discussed separately in the following sections, there are two additional modules, they are "User Input", and "Output". Figure 4.1 shows a whole framework of this solution methodology.
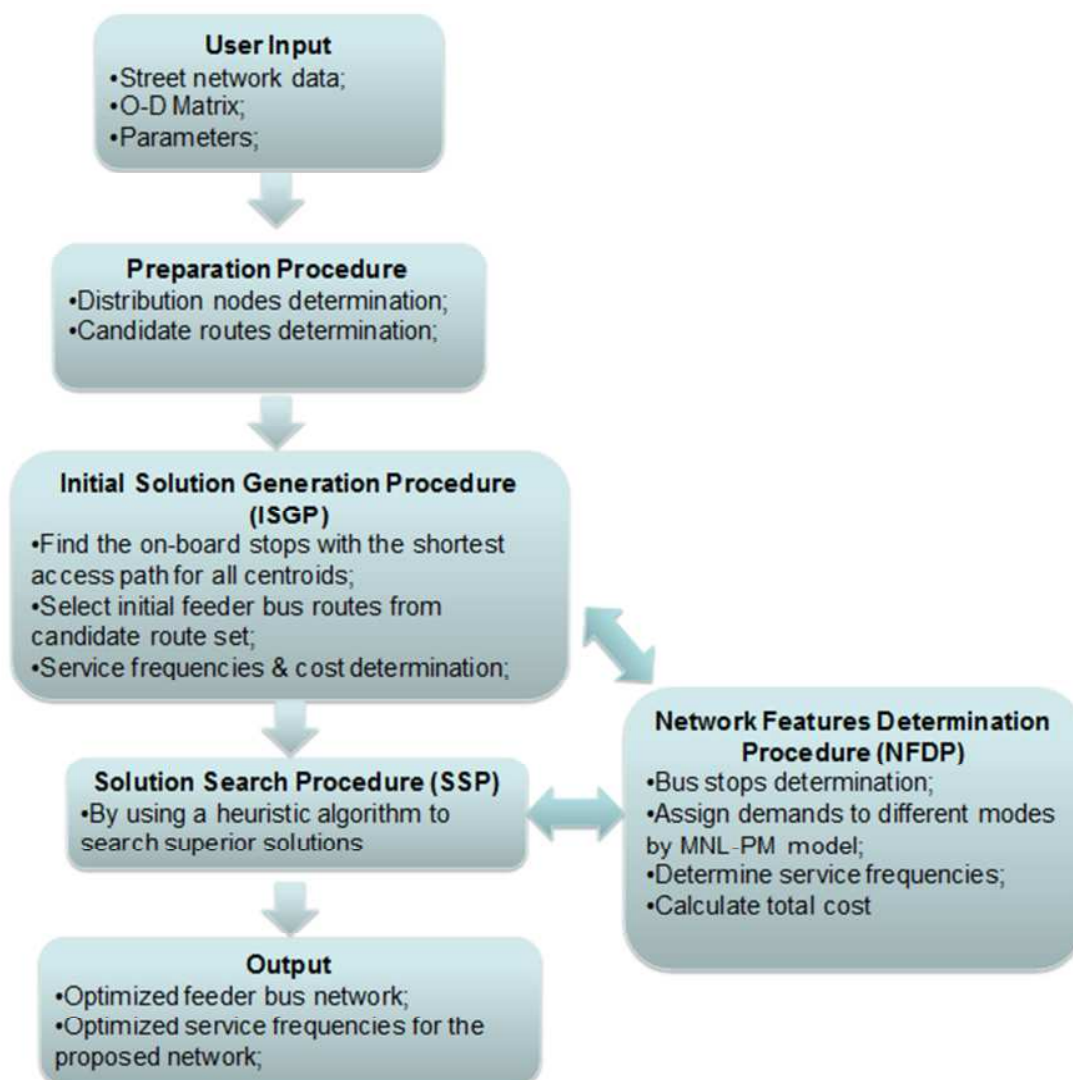
Figure 4.1 A framework of proposed solution methodology.

To begin with, the proposed methodology needs three kinds of data as initial inputs. The first one is the street network data. The network data consists of the coordinates of nodes, links, and centroids. Those nodes represent street intersections in the real world and the links are street segment connecting between different nodes as street segments. The traffic zones in this study are those areas divided by the grid street network. The demand distributed on these areas is aggregated into centroids. The locations of these centroids can be calculated based on the distribution or be pre-determined.

Since the purpose of this methodology is to develop a feeder bus system, the targets of the transit users are those whose demand pattern is characterized as many-to-one, such as peak-period working trips to and from the CBD. The input of O-D matrix also consists with such a demand pattern. The parameter input set is about all the parameters in this model. The outputs of the methodology include an optimized feeder bus network system, and service frequencies together with these routes.

4.3 Preparation Procedure

The preparation procedure is an intermediate procedure, which makes a connection between original data set and ISGP. Two tasks need to be done in this step: candidate stop set determination and candidate route set determination.

4.3.1 Candidate Stop Set Determination

In this work, the location and number of bus stops are also optimized. Two types of nodes are defined as candidate bus stops. The first is intersection node. As implied by the name, such nodes are represented as street intersections.

The second type of nodes is the distribution node, which comes from the centroid. The distribution nodes are those nodes distributed from various centroids. These nodes stand for the shortest access points from centroids to their neighbor streets. These nodes can be potential bus stops since they are the shortest points from centroids to the street. In this study, the geometric factors are not included, so the shortest path is the vertical distance between the centroid and its closest street. As shown in Figure 4.2, for each demand centroid, there are four shortest routes to its closest streets. It is obvious that if there is a bus route goes through the street, these nodes can be the ideal candidate stop locations.
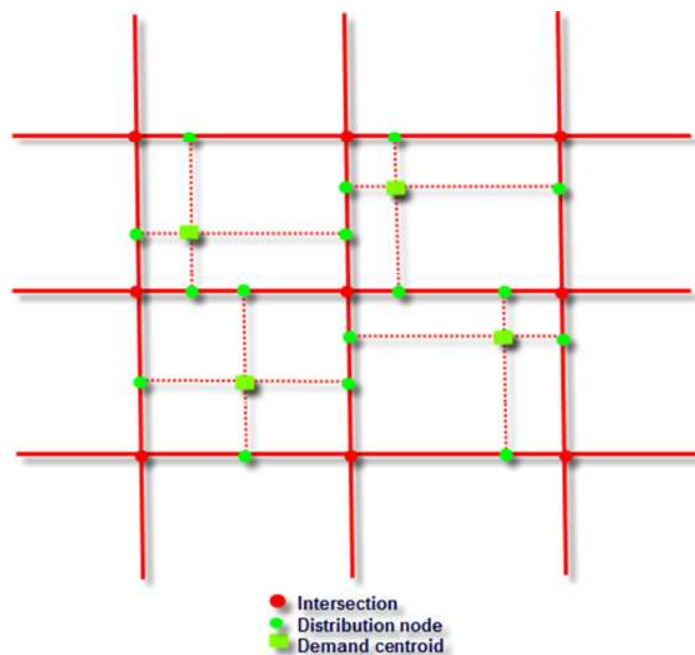
Figure 4.2 An illustration of intersections, distribution nodes and demand centroids.

The concept of distribution nodes were initially proposed by Fan and Machemehl in 2004. From Figure 4.2, it can be noted that in each street segment, there are usually two distribution nodes. In Fan and Machemehl's work, they merged these two nodes into one prior to the route optimization process based on some criterion. Though this

procedure might be right sometimes, in other cases, it may be problematic. The underlying problems have been presented in Figure 4.3. In Figure 4.3A, a bus route goes through four traffic zones, which have been centralized as centroids. The access points for these centroids are all located in the same street segment. In this case, to avoid too many bus stops in a short distance, the combination of some distribution nodes is necessary. However, when it comes to the case in Figure 4.3B, several issues rise. In this situation, node 1 is not an optimal stop for bus route 2 anymore. For centroid B, node 3 would be the best access point to the bus line, the combination of the distribution nodes from centroid A and B is not necessary. For this reason, the original distribution node 1' needs to be kept. The other issue is from distribution node 2. It is obvious that no route goes through the distribution node 2, only if distribution nodes are included in the candidate stop set, the demand from upper right centroid will be ignored.
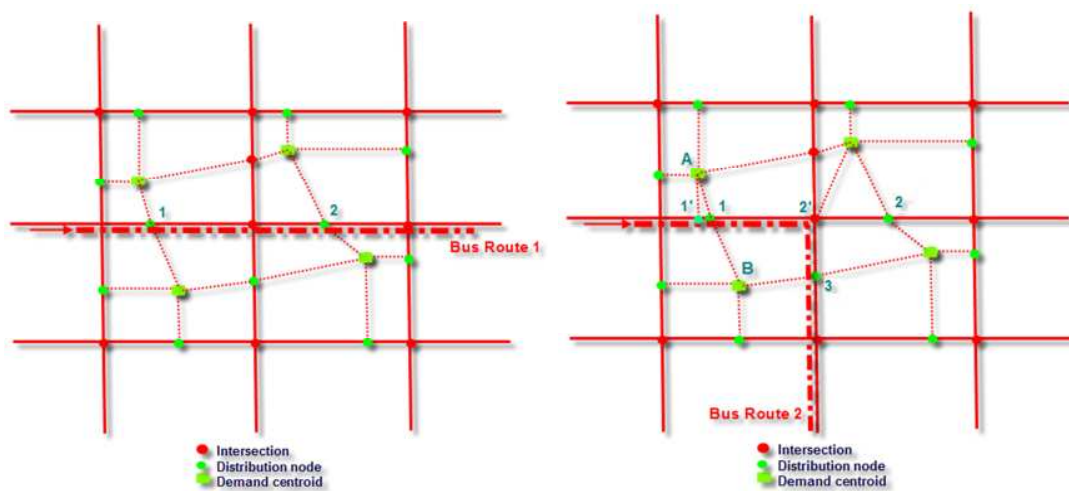
A Combined nodes in case 1.   **B** Combined nodes in case 2.
Figure 4.3 Combined distribution nodes and the underlying problems.

To solve these problems, all the distribution nodes will be kept and considered as candidate bus stops in this study. The initial candidate route stops are the combination of all the distribution nodes and intersection nodes. If the bus stop spacing is less than a

threshold, an optimization of bus stop locations is used after the primary route development process.

4.3.2 Candidate Route Set Determination

After the candidate bus stops are determined, the next procedure is to develop the candidate bus routes.

The candidate route set contains feasible routes connecting each candidate bus stop and the rail station. The final feeder bus network would be a combination of several routes chosen from this set. The candidate bus routes development procedure is to generate K shortest paths for each pair between candidate bus stops and the destination rail station. It is noted that the shortest paths from distribution nodes (candidate bus stops) that origin from a street segment are always included by the shortest path set whose origination is the intersection node of the segment. To eliminate the size of the candidate route set and improve the efficiency in SSP, the start point of the K-shortest paths is the street intersection. In the optimization process, if one of the shortest paths is selected as feeder bus route, the stop on this route with the longest distance from the rail station is used as the actual start point of the route.

The problems of the shortest path are unavoidable in most of the network flow optimization especially in the field of transportation. Since the end of the 1950s, there are large number of related studies published in various journals and proceedings. These works focused on different kinds of transportation problems, including those classical problems of determining the shortest path between two nodes based on various criteria (such as route length, travel time, etc.); and those problems with their own peculiarities (constraints, particular network structures). During the past decades, several shortest

paths related methods have been developed based on Dijkstra's work, e.g. label-marking method, modified label-marking method, dynamic programming method, fluid neural network algorithm, etc. Since no "best" algorithm exists for all kinds of transportation problems, the selection of a suitable shortest path methodology depends on the characteristics of the problem and its efficiency. (Pallottino and Scutella, 1998) Compared to the shortest path problem, the k-shortest path problem obtains less attention from researchers. There are two types of k-shortest path problems. The first is to find paths in which loops are allowed and the second is to find the loop-less paths.

Due to the simplicity of implementation, Yen's (1971) loop less algorithm of k-shortest path is used in this dissertation. The shortest algorithm used in Yen's method is Dijkstra algorithm. As a label-setting shortest path algorithm, the Dijkstra method is much more efficient than other label-correctings algorithm such as Bellman-Ford or Floyd-Warshall method. Although the flexibility is limited, such as a non-negative arc length assumption, it is powerful enough to be implemented in this study. In the following paragraphs, a basic idea of Dijkstra method will be introduced and then the procedures of Yen's k shortest paths algorithm is followed.

The purpose of the Dijkstra is to find the shortest path from origin node 1 to the destination node N. There are two sets in the Dijkstra method, set "S" and set "U". Set S is for recording the shortest path routing and set U is the candidate node set. In the beginning, all nodes in the network except for the origin node 1 in set S, are in set U. For each node in set U, a distance matrix will be calculated between these nodes to the first node in set S. If no link exists between them, the distance is set to be infinite. In every iteration, the distance matrix will be updated and the node with the smallest distance will

be selected and moved to set S. Iteration will continue until node N is moved to set S. Set S is the shortest path from node 1 to node N and the distance associated with N is the path length.

Yen's method is used to generate K shortest paths based on a given network. These K shortest paths are built sequentially at the iteration k based on the previous (k-1)th shortest path that has been determined in the (k-1)th iteration. To begin with, two lists, list A and list B, are created. List A is used to store k shortest paths, and list B is for candidate routes. When k equals to 1, the Dijkstra algorithm is used to find the shortest path from node 1 to node N. When k>1, for each iteration k, a deviation route search strategy is performed in set A^{k-1} from Q_1^{k-1}, …, Q_{n-1}^{k-1}. As shown in Figure 4.4, for each i, where i=1, 2,…, n-1, those deviation nodes that connect node i but not yet included in set A^k are found. Then, the Dijkstra algorithm is applied again to find the shortest route from each deviation node to the sink node N (destination node). Note that these routes do not go through the nodes of Q_1^{k-1}, …, Q_i^{k-1}. Now, a set of deviation routes, A_i^k is created and added to list B. These routes have the same subpath between node 1 to node i but vary from node i+1 to N. After all the deviation routes are generated for iteration k, a route is selected in list B with the shortest route length as A^k and then moved to list A. If there is more than one route with the same route length, arbitrarily choose one of them as the shortest path. The rest of the routes in list B will be kept until K iterations finished. When the size of routes in list A is larger than K, the algorithm will terminate as well. A diagram of the deviation route generation has been shown in Figure 4.4.
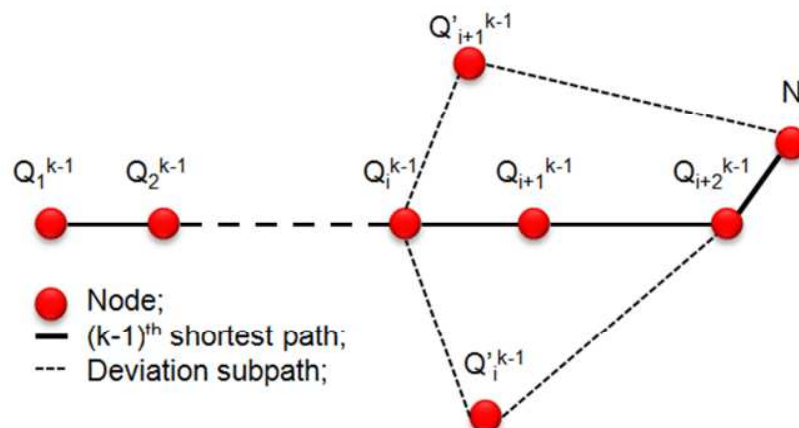
Figure 4.4 A diagram of the deviation nodes and related routes.

Notations in the K shortest path problem

i: i=1, 2, …, N; where 1 represent the origin node and N is the sink node (destination node);

A^k: a route set of the kth shortest path; the nodes in this set are denoted as Q_1^k, Q_2^k, …, Q_n^k; n is the number of nodes in set A^k;

A_i^k: deviation route set from A^k, which means A_i^k is coincident with A^k for the first portion of the path from node1 to node i. For the second portion of the route (last portion), node i+1 in A_i^k is not included in A^k, and the path from node i+1 to N is the shortest path between them. Note that those nodes that are included in the first portion in cannot be in the second part of route in A_i^k.

If the number of the total street intersections is J, the size of the candidate route set will be K*J. For each route that is in the candidate route set, except the rail station, all other nodes included in this route are taken as candidate bus stops, and will be determined in the next step.

4.4 Initial Solution Generation Procedure (ISGP)

The initial solution in many existing studies is obtained by randomly selecting routes from a candidate solution set. In this section, an Initial Solution Generation Procedure (ISGP) for a given street network is discussed. The advantages of the ISGP over the traditional selection method will be addressed in Chapter 5. This initial solution will be taken as a starting point in the Solution Search Procedure (SSP) section to search for asuperior solutions. The framework of this procedure is shown in Figure 4.5.
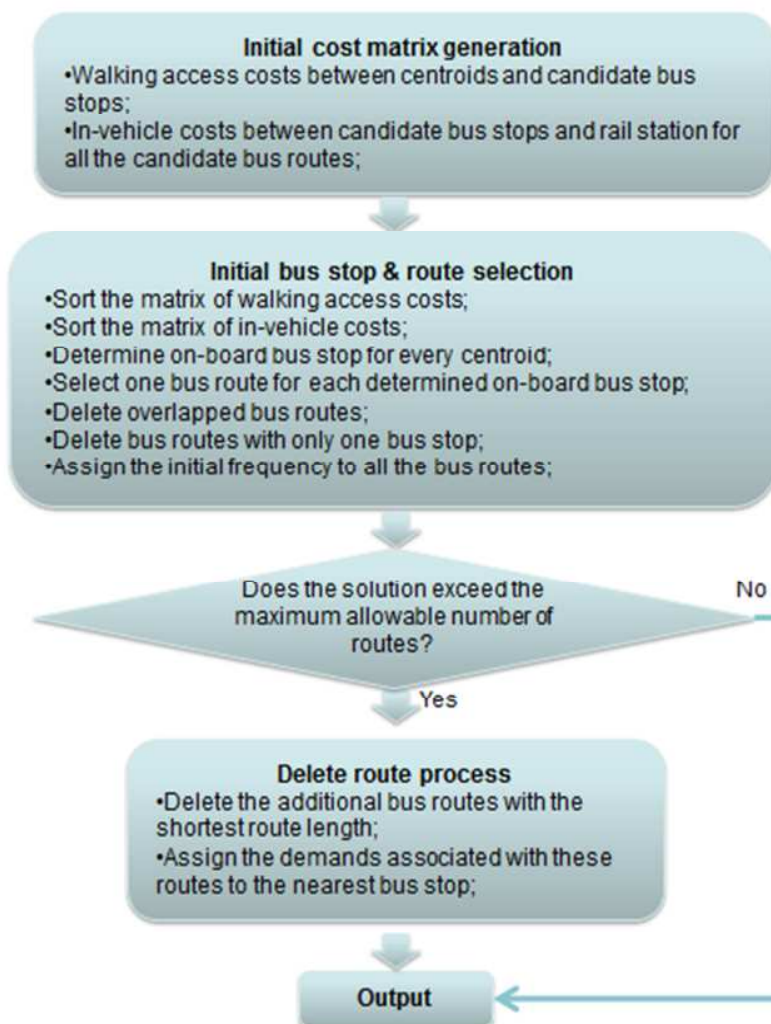
Figure 4.5 A framework of ISGP.

In the previous step, K-shortest paths have been determined. If all the paths are applied on the street network, there would always be several routes that go through each distribution nodes or intersection nodes around a centroid. As discussed previously, three access modes are available for the demand at each centroid, which are walk, bus, and auto. The factors that affect the bus stop selection are those costs including bus stop access cost, bus waiting cost and in-vehicle cost. Noted that all of the costs mentioned here are measured in $. The cost for in accessing a bus stop can be expressed as access time to the boarding bus stop multiplies by the value of walk time. Since a constant frequency is used as the initial bus service frequency for all the bus routes, the bus waiting cost for these bus route are the same. For this reason, the bus waiting cost does not included in the initial stop determination process. The in-vehicle cost here is the bus riding cost (in-vehicle travel time multiplies by the value of time on the bus).

4.4.1 Initial Cost Matrix Generation

The purpose of this first step is to generate cost matrices to use in stop selection. Two matrices are generated. The first one is the matrix of walking access cost. This data set includes the walking access cost from a centroid to every candidate bus stops. The in-vehicle cost matrix contains the in-transit cost from a candidate bus stop to the rail station via every available bus route.

4.4.2 Initial Bus Stop & Route Selection

Based on the cost matrices obtained in 4.4.1, an initial bus stop and route selection process is performed. As discussed before, this selection is made according to the cost. The stop selection priority in terms of cost is access cost>in-vehicle cost. It is assumed

that potential passengers If there is more than one candidate bus stop that near the centroid with the same walking access cost, the in-vehicle cost becomes the decision factor. The bus route selection process is followed as long as the boarding bus stops are determined. In rare cases, when both the access costs and in-vehicle costs are the same, an initial bus stop will be selected arbitrarily. The number of bus routes that go through each boarding bus stop can be more than one. It is noted that the in-vehicle cost associated with each candidate bus stop in the route selection process is different from which in the objective function. Here, it comes from one bus route which go through this stop with the shortest route length. Moreover, those routes which are overlapped by other routes or only have one bus stop will be deleted.

Other assumptions in initial bus stop and route selection:

1. The maximum number of bus stop that can be chosen by each centroid is one.
2. The service frequencies for all the routes in the candidate bus route set are constant.
3. The situation that the demands at a bus stop choosing multiple bus routes is skipped. But in calculating the total cost in the objective function, the split of the feeder bus demand over bus routes at a stop will be considered.

After the demands at each centroid have chosen their boarding bus stops and bus routes, the next step is to check if the total number of bus routes exceeds the maximum allowable number of routes.

4.4.3 Route Number Check

When the total number of routes in the initial solution set is more than the pre-defined maximum allowable routes, a route deletion process will be performed. The criteria of deleting additional routes are based on the route length. The process starts with calculating the route length for each bus route. Those routes with the shortest route length will be deleted. For the demands that associate to these deleted routes, they will be

assigned to the closest available stops. If no stops available within a pre-defined walking distance threshold, it means no bus service covers this centroid. The output of the initial bus route solution includes a set of bus routes, and boarding bus stops for each centroid. One can note that in the ISGP, only the constraint of maximum number of route is considered, which means the outcome of ISGP may violate some other constraints discussed in Chapter 3. Such violation(s) is (are) acceptable since the purpose of the ISGP is to find a relatively good start point for SSP. If the start point does not meet the constraints, a penalty will be applied and a feasible solution will be found by SSP in the next iteration.

4.5 Network Features Determination Procedure (NFDP)

The Network Features Determination Procedure (NFDP), which is called by the Solution Search Procedure (SSP) as discussed in section 4.6, is used to determine all the network related features. The input of this process is only a set of feeder bus routes and related data such as street network information, O-D Matrix, and other parameters. The outputs are a set of bus stops, mode split at every centroid, bus service frequencies, and the total cost for the given layout of the feeder bus network. The skeleton of this procedure has been shown in Figure 4.6. The NFDP includes three key components. The first one is boarding bus stop determination, the second one is mode split & trip assignment, and the third one is enhanced service frequency determination. Compared with those methodologies mentioned in previous references, a bunch of critical improvements have been made in this section:

1. Propose a MNL-PM mode choice model.

2. Add an initial service frequency selection module to optimally select initial input of frequencies to avoid manually input.

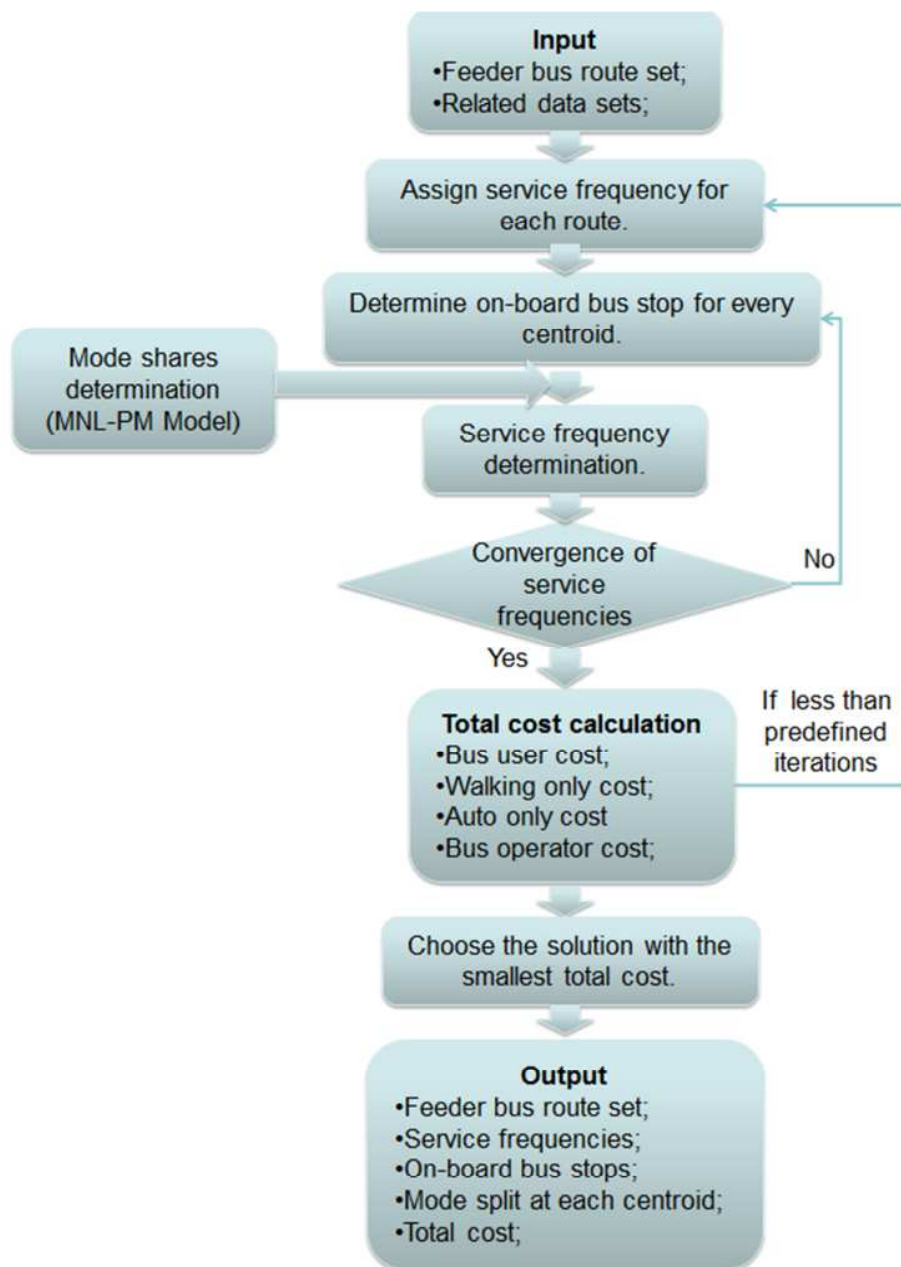3. Improve the service frequency calculation method to accelerate the convergence of the frequency.

Figure 4.6 The flow chart of NFDP.

4.5.1 Boarding Bus Stop Determination

The boarding bus stop determination process is developed based on the initial bus stop and route selection method which has been discussed in section 4.4. The cost priority assumption still holds in this process. Several modifications have been made to fit the method in the new context. Firstly, the input of route set for the previous method is a set of all available candidate feeder bus routes while the boarding bus stop determination in this section is only based on a given set of bus routes. Since the number of input bus routes is constrained by the maximum allowable bus routes, the module of route number check which exists in the previous method is not needed here. Secondly, the previous method is upon an assumption that all the service frequencies are constant and have the same value among different bus routes. Under this assumption, the bus waiting cost is not considered in stop selection. Here, the boarding bus stop selection is performed during the determination of service frequencies, which means at each iteration, the bus frequencies between different routes can be variable. For this reason, the cost of bus waiting time is also chosen as a factor that affects bus stop selection. The priority of this cost is the same as in-vehicle cost. Thirdly, regarding the situation that multiple bus routes go through a candidate bus stop, the actual bus frequency at that stop can be a sum of all the available bus route frequencies. The stop delay is included in both mode split process and total cost calculation but is not considered in the boarding stop selection. Finally, a bus stop inter-spacing check is done after the stop selection process. If the inter-space between two bus stops is less than pre-defined value, they are combined into one stop. The simplest criterion of the stop combination is to choose the middle point of these two stops as new bus stop. If necessary, the determination of the new stop can also

according to the bus demands from the centroids. For example, the distance between the new stop and centroids could be proportional to demand.

4.5.2 Mode Split and Trip Assignment

In this dissertation, the variable transit demand is considered for the development of thefeeder bus network. The elastic transit demand comes from the mode split among auto, transit, and walk. Basically, there are two kinds of variable demand, variable total demand and variable transit demand. A conventional urban planning process reveals the relationship between them. Figure 4.7 represents a classic 4-step demand forecasting model. While the total demand can be estimated depending on two steps of trip generation and trip distribution that are performed on a highway and transit network is actually obtained actually from a feedback process of the whole planning procedure. In FBNDP, since the structure of transit network is also a variable, theoretically, the determination of highway and transit networks, and demand estimation need to be obtained simultaneously. In other words, the FBNDP needs to be developed on the context of the whole urban planning process. The consideration of the entire process will involve a huge number of parameter calibrations, which will also bring additional "noises" to FBNDP. The emphasis of this dissertation is to propose a new FBNDP model and test its effectiveness. For simplicity, the total demand is assumed to be fixed and does not change along with the structure of the network.
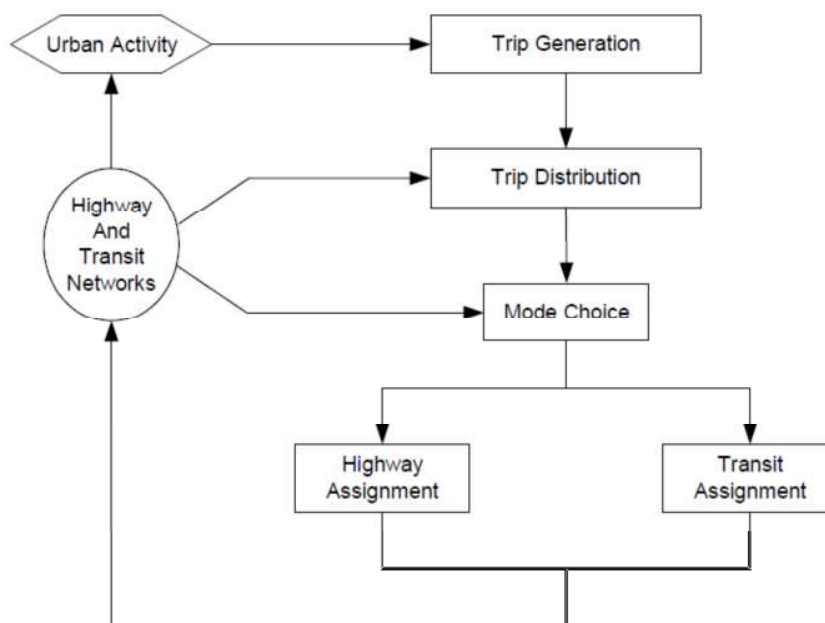
Figure 4.7 Conventional urban planning process.

Source: FHWA/ UMTA, 1977

When the total demand is given, the transit demand can be determined by the attributes and related decision rules. The attributes here are measured by the various mode costs, such as bus access cost, bus waiting cost, bus riding cost, bus fare cost, auto in-vehicle cost, auto operating cost, and walk only cost. Various discrete choice models can be used to split demand. These models are usually derived from an assumption of utility-maximizing behavior by the decision maker. (Train, 2009) The general form of the utility function can be expressed as:

$$U_j = \beta_j'x_j + \varepsilon_j \tag{4.1}$$

Where, U_j is the utility of alternative j; β_j is a vector of parameters for alternative j; x_j is a vector of attributes which make up the utility of alternative j; Since not all the utilities faced by the decision maker can be observed by the researcher, ε_j captures those

attributes which are not included as variables in x_j. The derivation of different discrete choice models are based on different assumptions of the unobserved density function $f(\varepsilon_j)$. As pointed out by Train, a closed form of probability that decision maker choose alternative j only exists in certain specifications of $f(\varepsilon_j)$. For example, logit model is derived under the assumption of Gumbel identically independently distribution (iid) extreme value, and the nested logit model is based on a type of generalized extreme value.

In this dissertation, there are three alternatives available for a trip decision maker to choose, which are walk only, auto only, and feeder bus service. The walk only alternative is available when the walking distance to the rail station is less than R_{wmax}. The bus alternative is in effect when there is at least one candidate bus stop within the bus stop access distance threshold R_{bwmax}. The utility functions for these alternatives can be expressed as follows:

$$U_{walk} = \beta^0_{walk} + \beta^1_{walk}TT_{walk} \tag{4.2}$$

$$U_{auto} = \beta^0_{auto} + \beta^1_{auto}TT_{auto} + \beta^2_{auto}TC_{auto} + \beta^3_{auto}PKC_{auto} \tag{4.3}$$

$$U_{bus} = \beta^0_{bus} + \beta^1_{bus}TT_{bus} + \beta^2_{bus}TC_{bus} + \beta^3_{bus}WT_{bus} \tag{4.4}$$

Where,

TT_{auto}: Travel Time by auto in minutes;

TC_{auto}: Travel Cost by auto ($);

PKC_{auto}:   Parking cost of auto ($/day);

TT_{bus}:   Travel Time by bus in minutes;

TC_{bus}:   Travel Cost by bus in $, ($/passenger-trip);

WT_{bus}: Waiting Time for bus in minutes;

TT_{walk}: Travel Time by walk in minutes;
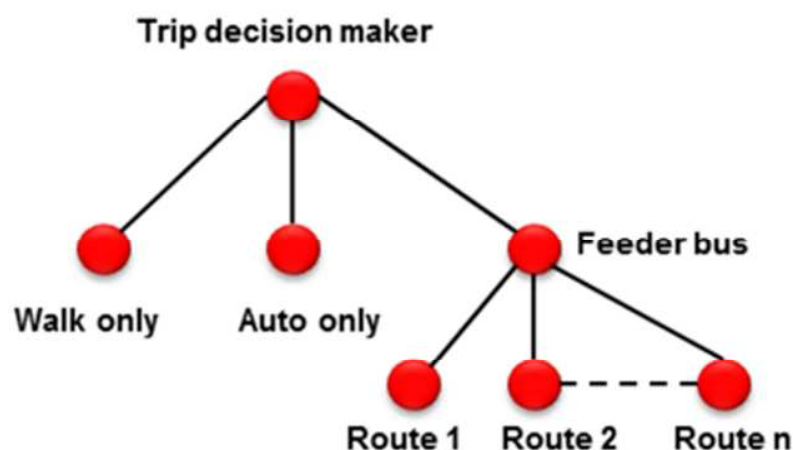
Figure 4.8 Structure of mode choices in this study.

Figure 4.8 illustrates the structure of mode choices in this dissertation. It is obvious that this is a nested structure. For the choice of feeder bus service, there might be multiple bus routes for selection. If these bus routes are considered identically as the walk and auto choices, a multinomial logit model (MNL) can be used. However, since those bus routes are not independent to each other, the iid assumption is violated, which results in the failure of MNL. The nested logit model (NL) can be applied in this situation. The disadvantage of NL is its complexity, which may dramatically affect the efficiency of the new FBNDP model. Inspired by BLM-IPM model (Fan, 2004), a two stage MNL-PM (Multinomial Logit-Proportional Model) is proposed as a decision rule to get mode shares for all the alternatives. The first stage of this new model uses a multinomial logit model to calculate the probabilities for choosing walking, auto and feeder bus service. This stage is based on an assumption that people choose bus service before they decide which bus route to select. If there are multiple routes going through a bus stop, the in-vehicle travel time used in calculating mode split is calculated as the shortest route length from this stop to the rail station divided by the average bus speed plus stop delays (total

number of stops after the boarding stop for the shortest route*stop delay coefficient). As for the bus fare, an estimation of fare calculation is made. The basic fare is $1.5 if the travel distance is within 2 miles. When the distance is larger than 2 miles the incremental charge is $0.25/mile. One should note that the bus fare is only considered in the determination of mode split. Since the fare is a cost to the user and a benefit to the service provider, it is excluded from the objective function. In the second stage, a proportional model (PM) is applied. As the name of this model implies, the PM model assigns the transit demand to different bus routes proportionally based on the service frequencies of these routes. Compared to the preceding BLM-IPM model, the first stage in MNL-PM is a multinomial logit model (MNL) instead of binary logit model (BLM) and the stop delays are considered in mode choice. Moreover, in the second stage, the transit demand assignment depends on service frequencies rather than trip time.

4.5.3 Enhanced Service Frequency Determination

In Chapter 3, three kinds of widely used transit service frequencies were discussed. In this study, the frequency needs to be determined is the demand frequency. The demand frequency for route m can be expressed as:

$$f_m = \frac{Q_{bm}}{\rho C} \tag{4.5}$$

Where

f_m: service frequency of feeder bus route m (buses/hr);

Q_{bm}:   total transit demand on feeder bus route m (people/hr);

ρ: occupancy rate;

C: bus capacity (seats/veh);

To make f_m within a practical range, the policy frequency is used as the lower and upper bounds of the demand frequency. In this study, the demand frequency cannot fluctuate under 1 bus/hr or over 12 buses/hr.

When it comes to variable transit demand, the service frequency of a route will affect the bus waiting time and consequently affect the mode share of bus. So a feedback loop is necessary to reach a pre-defined equilibrium state so that the bus frequencies can be obtained. Such feedback loop structure was shown in Figure 4.6.

The feedback loop starts from a set of determined service frequencies. Then, MNL-PM model is applied and the share of transit will be determined. The updated transit demand leads to the change of bus service frequencies again. The iterations will not stop until the convergence of the feeder bus service frequencies meets a pre-defined criterion and the final frequencies are obtained.

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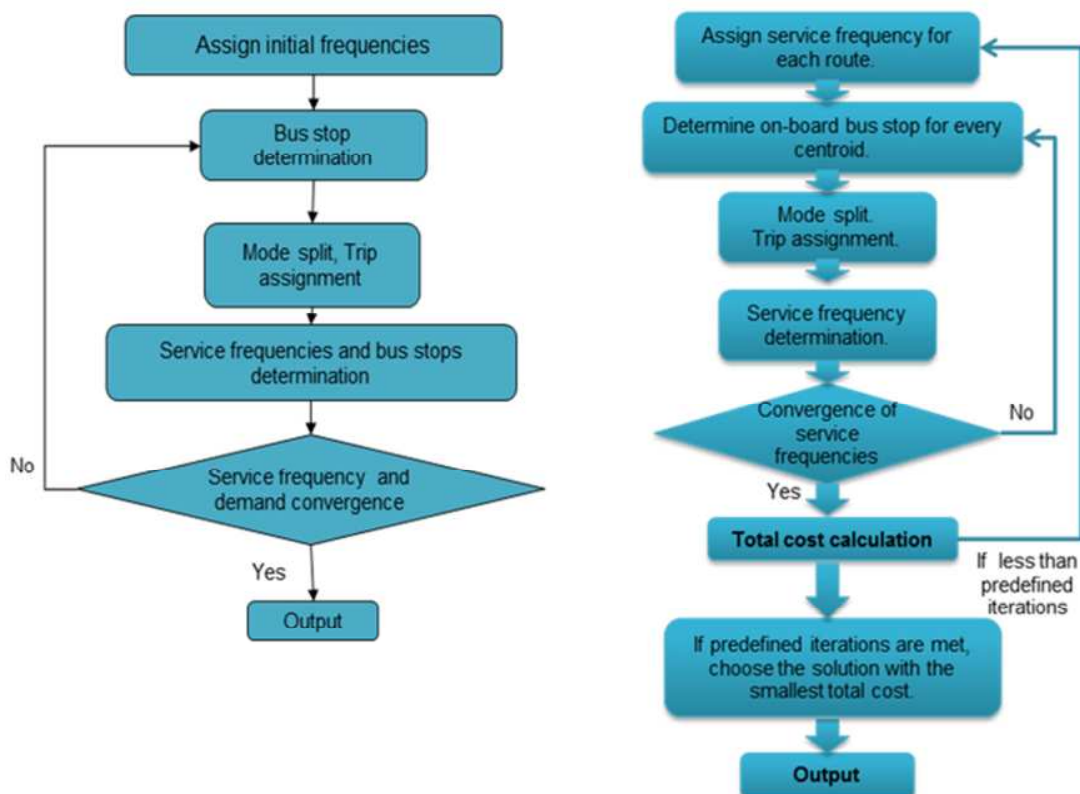
A. Conventional frequency determination process.

B. Enhanced frequency determination process.

Figure 4.9 Comparison between conventional and enhanced frequency determination processes.

The conventional bus frequency feedback loop has been provided in Figure 4.9A as a comparison to the proposed loop in Figure 4.9B. As described in the preceding Chapters, there are several places that can be further improved in this method. The first one is the assignment of the initial frequencies. When the transit demand is not very "strong", in other words, other modes such as auto or walk are competitive, a low initial frequency will lead to small transit demand. The small demand, as a feedback, will result in much lower demand frequencies. The new demand frequencies will attract much smaller transit demand than before, finally, the output of the frequencies will tend to be 0.

Possibly, such output is not an ideal solution for the given network. To overcome this shortcoming, additional iteration procedure is added after the total cost is calculated in the proposed method. In a given number of iterations, the new one will pick up f', $f' \in [1,12]$ as the initial frequency to run the frequency determination iterations. For efficiency considerations, the value of f' can be discrete integer. The output of the frequencies with the least total cost will be selected as final output. For simplicity, the initial value of frequencies for all the routes is the same, which is f'. The second improvement is called fast convergence strategy. Sometimes, the service frequencies will not converge quickly and oscillate between two positions. To expedite the convergence speed for those routes where the oscillation exists, the mean of the amplitude instead of equation (4.5) will be used as demand frequency after a predefined number of oscillations reached. When the difference between input frequencies and output frequencies is less than 10%, the iteration will terminate and the total cost will be calculated. If the output frequency of route m, f_m , is larger than the maximum policy frequency, 12 buses/hr will be used. Those bus demands that cannot be met by the new frequency are calculated. It is assumed that these unsatisfied demands will select auto as access mode and are proportionally assigned to those centroids that choosing the route m to go to the rail station. Equally, if the output is less than 1 bus/hr, the minimum policy frequency 1 bus/hr is used .

4.6 Solution Search Procedure

As concluded in chapter 2, the complexities of the TNDP can be categorized into several folders, including various decision variables, non-linearities and non-convexities,

combinatorial explosion, multi-objective nature and spatial layout of the route. Due to these intrinsic complexities, most transit network related problems are characterized as NP-hard problem. (Baaj and Mahmassani, 1991) The metaheuristic is believed to be one of the most efficient strategies in solving NP-hard problems. It is defined as an iterative generation process which guides a subordinate heuristic by integrating different concepts for exploring and exploiting the search space. Learning methods are employed in the whole process to structure information to efficiently find near-optimal solutions. (Osman and Laporte, 1996) As a metaheuristic, together with simulated annealing and genetic algorithms, TS was thought to be "extremely promising" for the future treatment of practical applications. (Glover and Laguna, 1997)

Various attempts have been made in the preceding studies to solve the transit network problem with different metaheuristics (Tabu Search (TS), Genetic Algorithm (GA), Simulated Annealing (SA), and Ant Colony Optimization algorithms (ACO)). (Cordeau and Laporte, 2003; Nee, 2004; Shrivastava and Mahony, 2006; Fan and Machemehl, 2008;) Based on the computational results, Fan and Machemehl draw the conclusion that the TS is at least as good as or even prevails over other methods in solving such problems. In consideration of the solution structure and the demand characteristics involved in this study, the proposed route optimization process will use TS as the heuristic search procedure to explore the superior solution beyond local optimality.

The TS is a memory-based iterative search approach, which was introduced by Glover (1989), and can be used for solving combinatorial optimization problems. It examines a trajectory sequence of solutions and movements to the best neighbor of the current solution. To access new solution instead of cycling in a local optima, those

movements that were recently visited are forbidden, or tabu, for a pre-defined number of iterations. The stop rules for the heuristic are that either the number of iterations reaches the pre-specified value or that a no more advanced solution generated after a pre-specified number of iterations. The major process of the TS that is used in this dissertation is described below and depicted in Figure 4.10.

Step 1. The TS starts from an initial solution which is obtained by ISGP.

Step 2. Parameters and variables that will be used in TS are initialized. For example, the objective function value for the initial solution is evaluated by the NFDP procedure. The initial solution is also converted as a matrix for use in TS. The solution representation and coding method are discussed in section 4.6.1.

Step 3. All feasible neighborhoods for the current solution are found in this step. A feasible neighborhood has to meet the criterion that the movement is not in the existing tabu list.

Step 4. If all the available movements are in the tabu list, an aspiration process will start to set the tabu tenure of the movement with the smallest objective function value to be zero so that it can be selected.

Step 5. According to the objective values, a neighborhood with the smallest objective function value that is not in the current tabu list is chosen.

Step 6. This step performs intensification and diversification strategies. An intensification is executed when iter_intensification>iter_intensificationMax; A diversification is called when iter_since_last_impv>iter_since_last_impvMax. The details of both methods are discussed in section 4.6.3.

Step 7. If the objective function value of the selected solution is better than the best solution of record, the record is updated with the new one. The memory of the last improvement iteration is reset.

Step 8. This step is to update all the parameters before the end of an iteration.

Step 9. When the number of iterations is larger than the pre-defined number, iterMax, go to the next stop; Otherwise, go back to step 3 and continue.

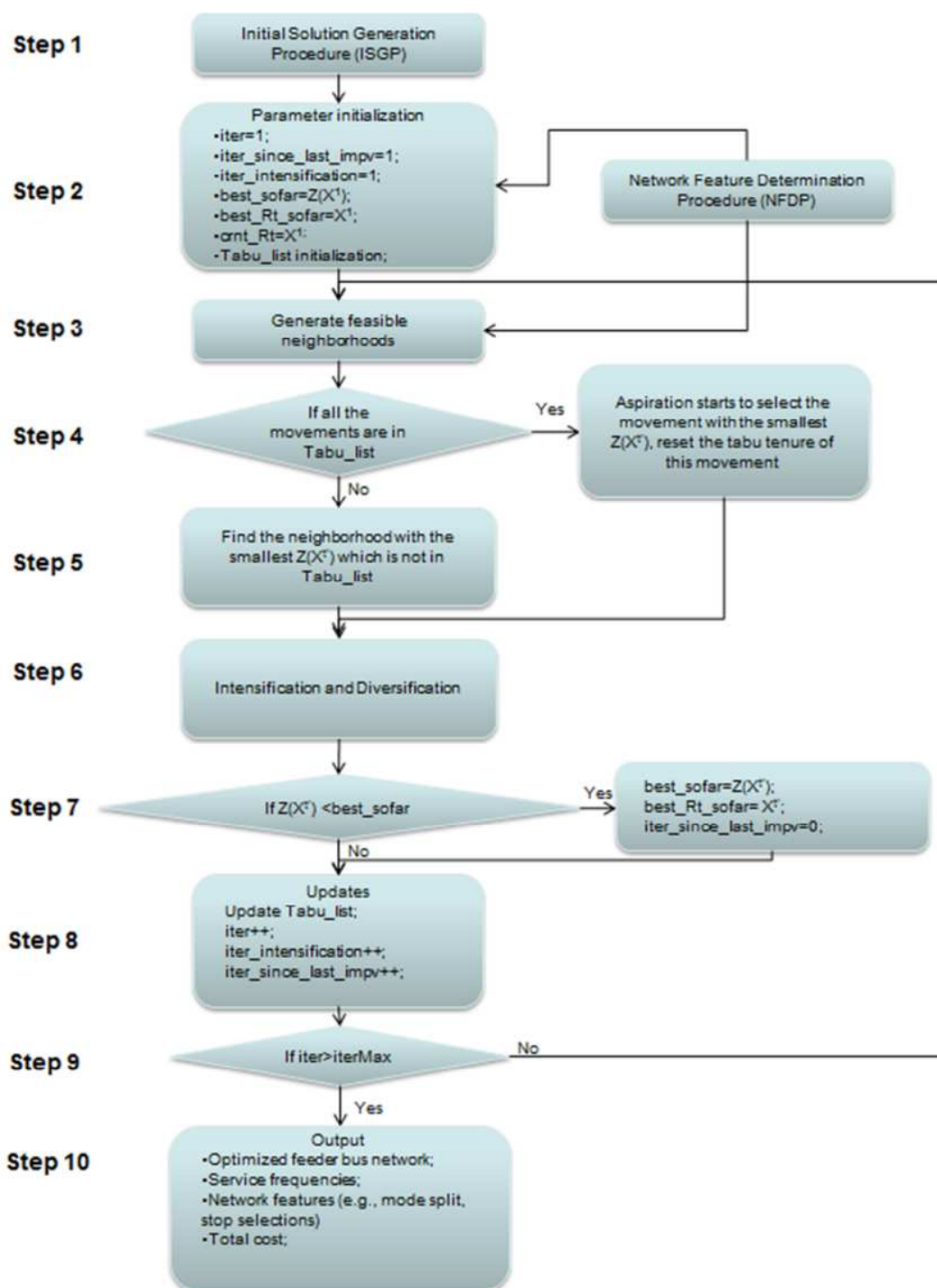Step 10. Terminate the TS process when the criteria are met and output the optimized solutions.

Figure 4.10 A flowchart of TS in FBNDP.

4.6.1 Representation of Solutions in TS

This section describes the representation of the solution in TS, including the search space, neighborhood structure and move mechanism. The decision variables of the proposed FBNDP are a set of feeder bus routes, and bus service frequencies. The bus service frequencies are determined by NFDP which is also based on given bus routes and has been discussed in section 4.5.

There are two route sets that will be used in the TS, a candidate route set R and a candidate solution set at the iteration t, X^t. The candidate route set R includes all the candidate bus routes generated in candidate route set determination process, $R = \{R_1, R_2, ...R_M\}$, where M is the total number of routes in the route set. In other words, the size of R is equal to M. The candidate solution set can be denoted as X^t, where $X^t \subset R$, the size of X^t cannot exceed the maximum allowable number of routes M_{max}. After X^t is identified, the value of the objective function at iteration t can be given as $Z(X^t)$. Obviously, the bus routes obtained in section 4.4 are initial solutions that connect each centroid with shortest path length can be denoted as X^1. Also the initial objective function is defined as $Z(X^1)$.

A neighborhood of a solution at iteration t is a set that is obtained by performing a movement on the previous solution set X^{t-1}. Three movements are considered. The first one is route deletion. It is performed when there is at least one bus route in the set X^{t-1}. This movement is to delete an existing route R_i from X^{t-1} and force the bus users from the impacted zones to choose other available routes to access rail station. The routes to be deleted can be any feasible one in X^{t-1}. The feasible movement indicates the one is not in the current tabu list. The deletion of a route will decrease the operating cost, if the

incremental user cost is less than the reduction of operating cost, thus leading to the total cost decrease. The second one is route addition. This movement is available when the number of routes in the solution set is less than M_{max}. By adding a new feasible route which is selected from candidate route set R, some centroid demand at centroids may have a new route choice with less bus user cost. If the increase of operating cost is lower than the decrease of the total user cost, the total cost will decrease as well. The last movement that is considered is route exchange. The exchange movement does not change the size of the solution set. It just moves one feasible route from solution set to candidate route set, while moving another feasible route from the candidate route set to the solution set. An exchange considered can be taken as a combination of route deletion and addition. After evaluating all the feasible movements and related value of objective function, a movement that is absent from the tabu list and has the least function value is selected and a new neighborhood solution X^t and its objective function value $Z(X^t)$ are obtained.
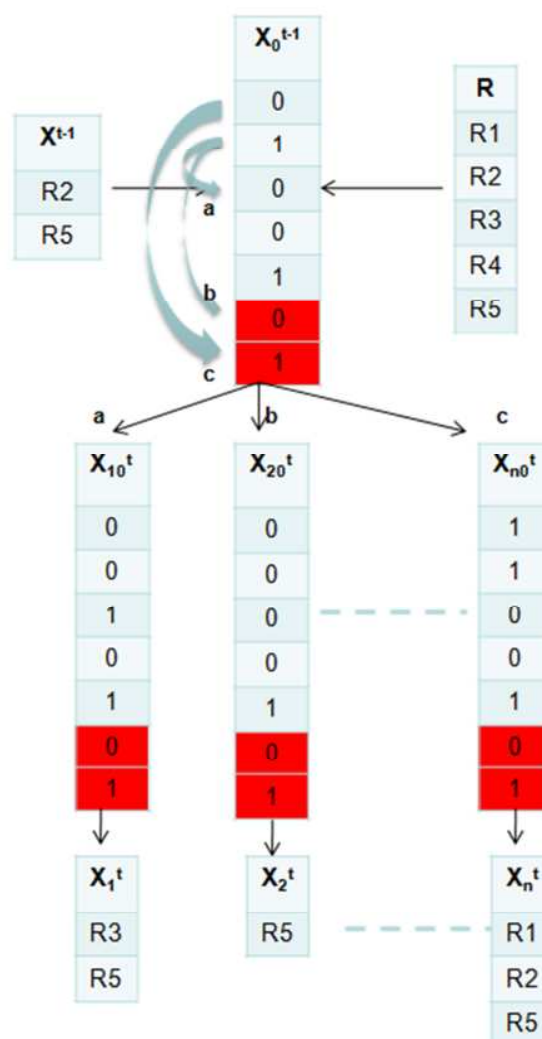
Figure 4.11 An illustration of solution coding and neighborhoods.

To incorporate all three movements in TS at iteration t, a data structure, X_0^{t-1}, is developed to stand for the solution set at iteration t-1. The X_0^{t-1} is an array in which the values can only be 1 or 0. This array consists of two parts, the basic part and the additive part. The basic part, which has the same size M as candidate bus route set R, represents the inclusion of the routes. 1 means the corresponding route is included, and 0, otherwise. Two additional rows, with the value of 0, and 1, as the second part, are attached to the first part. In set X_0^{t-1}, only one movement, the exchange, is performed. This movement in

X_0^{t-1} will acquire three types of outputs. For the first case, the movement is between two rows with different values, which are both located in the first part, such movement can be interpreted as exchange movement between candidate solution X^{t-1} and route set R. When one of the row is located in part one with the value of 1 and the other is in part two with the value of 0, this movement is classified as route deletion in X^{t-1}. On the other hand, if the value in part one is 0 and 1 in part two, the movement reflected in X^{t-1} is route addition. The exchange between two rows those both falling into part two is forbidden. One need note that no matter which case it is, the values in part two keeps constant all the time. To illustrate this, an example has been provided in Figure 4.11. If the total number of candidate routes is five, set R can be expressed as $R = \{R_1, R_2, R_3, R_4, R_5\}$. $X^{t-1} = \{R_2, R_5\}$ represents the candidate solution obtained at iteration t-1. X_0^{t-1} is the developed data structure for X^{t-1}. Three movements have been denoted as a, b and c respectively. The coded neighborhoods of X^{t-1} have been shown as $X_{10}^{t}, X_{20}^{t}, ..., X_{n0}^{t}$. After the movements, n new neighborhoods are obtained, which are $X_1^{t}, X_2^{t}, ..., X_n^{t}$.

4.6.2 Tabu Tenure

To avoid cycling in local optima, the recently visited neighbors are put in a tabu list to prevent being selected in a given number of iterations. The number of iterations is also known as the tabu tenure. The tabu tenure could be constant or dynamic. A constant tabu tenure is determined by user at the beginning of TS, and it does not change in future iterations. By contrast, the dynamic tabu tenure means the value of the tenure will change either randomly or systematically during the iterations. The random dynamic tenure is

randomly selected from a predetermined interval and usually follows a uniform distribution. As for the systematic dynamic tenure, a sequence of values are created before the iteration. These values are used directly without selection in uniform distribution in the following iterations. The effectiveness of constant and dynamic tabu tenure, as well as the tabu length depends on the problems under consideration. In Chapter 5, several comparisons are made and the tenures with the most efficient output are selected.

4.6.3 Diversification and Intensification

Diversification is an algorithmic mechanism that forces the search into areas which were not visited before. At iteration t, if the number of iterations since last improvement (iter_since_last_impv) is larger than iter_since_last_impvMax, the diversification procedure is applied. At the beginning of the diversification, a penalty value is added to every objective function value of feasible solutions at this iteration. The value of the penalty is dependent on the frequency that the solution is selected for the past iterations. The higher selection frequency, the higher penalty will be added. Then, based on the updated function values, the smallest one which is not in the tabu list is selected as current solution.

Intensification is a mechanism which explores the promising search spaces that have already been visited more thoroughly to ensure the best solutions in these areas are indeed found. At iteration t, if the intensification iteration (iter_intensification) reaches the pre-defined value (iter_intensificationMax), by intensification starts. It restarts the search from the best currently known solution and decreases the tabu tenures in the tabu list to encourage the consideration of additional neighborhoods.

4.6.4 Constraint Handling

The objective function presented in Chapter 3 is subjects to the constraints formulated in section 3.3. The bus stop selection and headway feasibility constraints are applied in the "Network Features Determination Procedure (NFDP) module as mentioned in section 4.5. The constraints that are considered in the TS are the load factor, route and fleet constraints. Violations of these restrictions will result in a penalty value to the objective function. This penalty value should be large enough to ensure to avoid the infeasible solution selections.

4.7 Summary

In this chapter, a solution method is proposed to solve the FBNDP introduced in Chapter 3. The solution consists of four major components, which are Preparation Procedure (PP), Initial Solution Generation Procedure (ISGP), Network Features Determination Procedure (NFDP) and Solution Search Procedure (SSP). The PP is a pretreatment prior to the whole process. The purpose of this procedure is to generate distribution nodes, candidate stops and a set of candidate bus routes based on the street network and demand distributions. The ISGP is a module to put forward an initial solution for FBNDP. This solution will be used in SSP as a starting point to search superior solution. The NFDP module is a key component both in ISGP and SSP to determine network related features including bus stop location, mode split and assignment, service frequency, and value of objective function. In SSP, TS is applied as a metaheuristic method to search superior FBNDP solutions beyond local optimality. In the next chapter, computational results and sensitivity analyses will be performed to exam the quality and effectiveness of the proposed model and solution methodology.

CHAPTER 5

COMPREHENSIVE EXPERIMENTS AND NUMERICAL RESULTS

5.1 Introduction

In this chapter, comprehensive experiments will be done and the related numerical results will be analyzed and discussed. Section 5.2 presents three experimental networks and the related parameters to be used in this chapter. Section 5.3 focuses on strategies and parameters of TS that can be applied in different sizes of networks to effectively obtain acceptable solutions. Section 5.4 introduces an Exhaustive Search method to validate the Solution Search Strategy (SSP) developed in Chapter 4. In section 5.5, a comparison is performed between different initial solution generation methods and the advantage of proposed ISGP is verified. The computational results of the three networks are followed in section 5.6. In section 5.7, the sensitivity analyses based on the proposed methodology of FBNDP are performed on a sophisticated network. A summary as a conclusion of this chapter is given in section 5.8.

5.2 Experimental Networks

The networks that are used in this chapter are classified into three different sizes are named, simple, sophisticated and more sophisticated. In computational results section, the results of the three networks are presented and discussed. Moreover, the simple network is used in the solution search strategy validation section. The sophisticated network is used in the sensitivity analyses and initial solution method comparisons. The structure of these three networks is similar to that discussed in Fan and Machemehl's study (2004). The distance has been converted into miles and the coordinates of some centroids have

been slightly adjusted. Because the purpose is to develop a feeder bus system, a rail station was added to each network. The configurations of the networks are shown in Figure 5.1, 5.2, and 5.3 respectively.
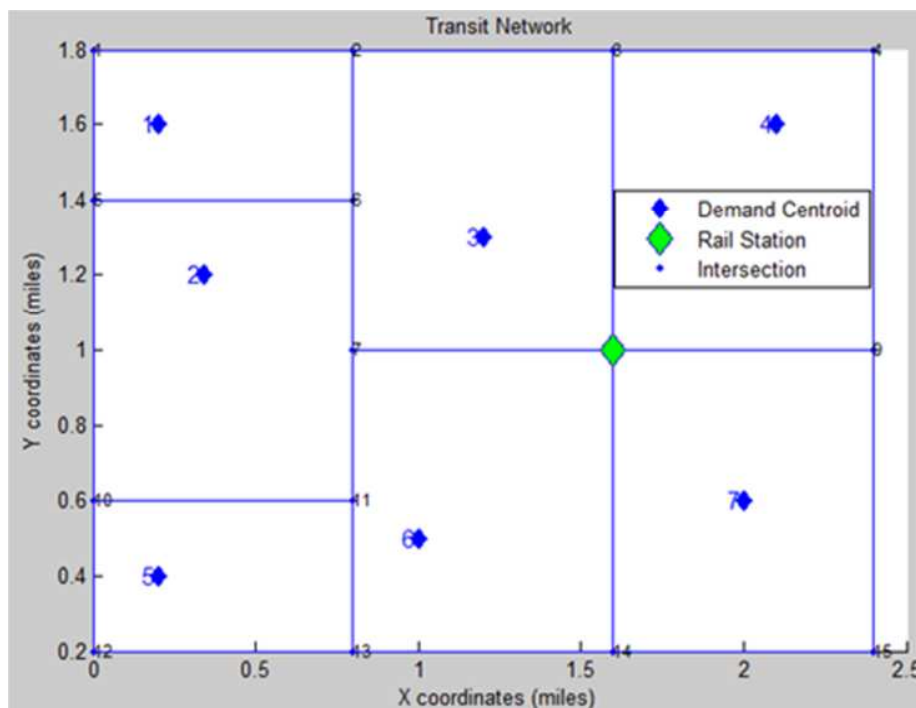
Figure 5.1 A simple street network.
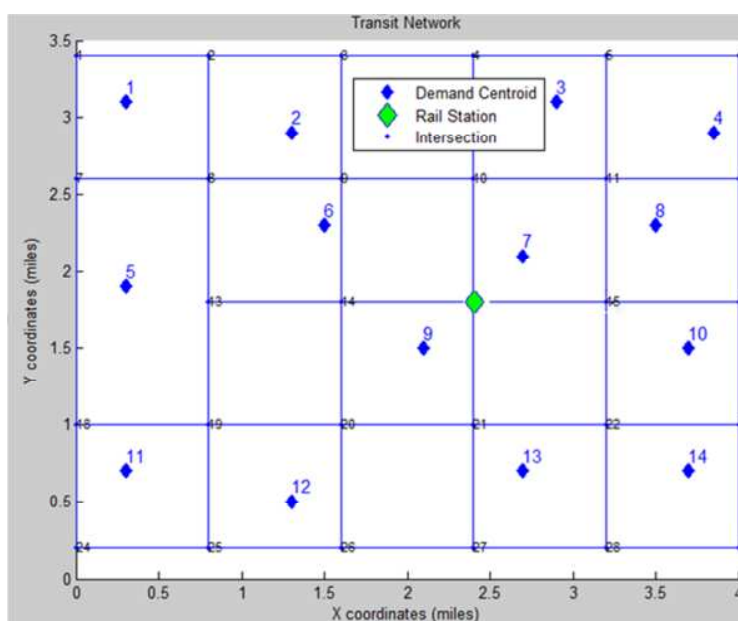
Figure 5.2 A sophisticated street network.
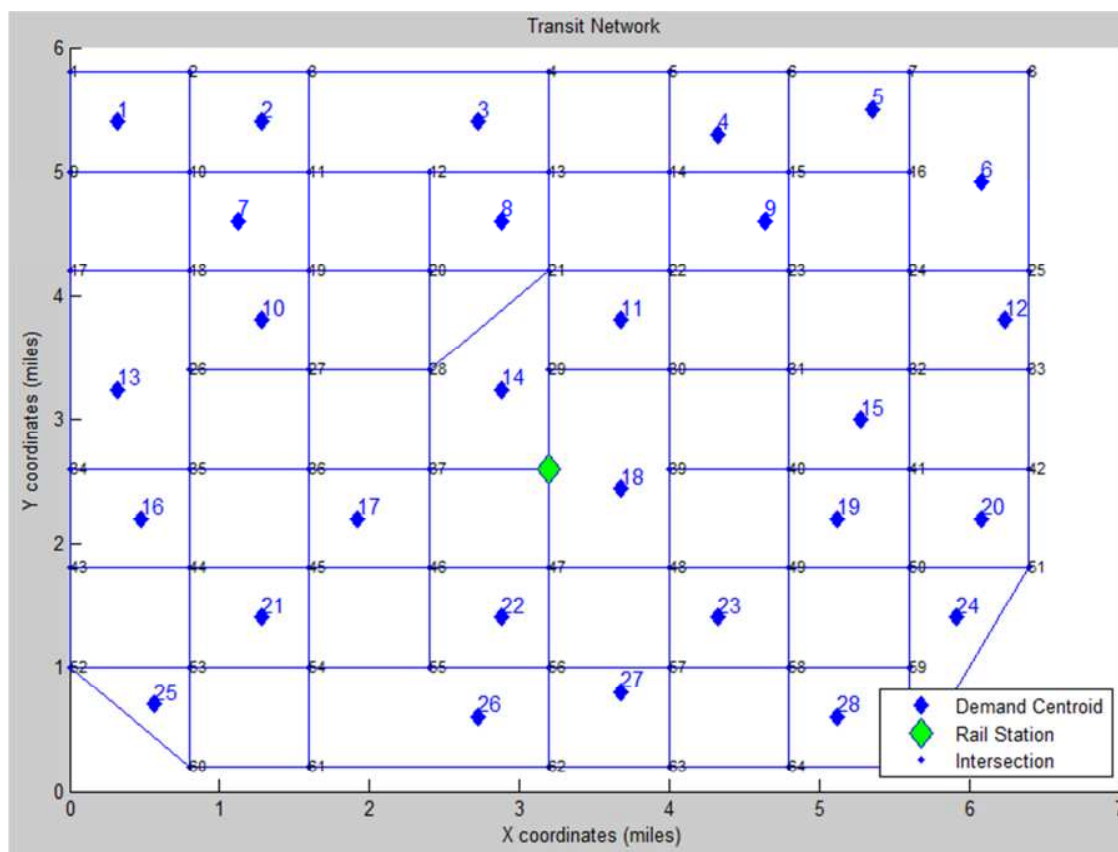
Figure 5.3 A more sophisticated street network.

There are two types of data in the given street network. The first is the basic network structure, which could be obtained from the physical street network. The structure is stored as nodes and links in two different files. The node file records all the street intersections and the corresponding coordinates. The link file describes which nodes are connected to each other. The second information is the demand centroids represented as blue diamond nodes. These centroids are aggregated from zonal demands depending on certain distributions and the terrain constraints. Generally, the location of each centroid can be determined according to the demand distribution. In this experimental section, for simplicity, these locations are pre-determined. If it is assumed that the demands of the afternoon peak hour is the same as that of the morning peak but

in the reverse direction, the demand matrix for one peak hour period is sufficient to develop the feeder bus network. Tables 5.1, 5.2 and 5.3 are the morning peak hour O-D matrices for the experimental networks. Other related parameters and mode choice coefficients that will be used in this chapter are shown in Table 5.4.

Table 5.1 Peak Hour O-D Matrix for Simple Network (morning peak)

	Centroids (Origin)						
	1	2	3	4	5	6	7
Rail Station (Destination)	200	400	600	200	800	300	500

Unit: passengers/hour

Table 5.2 Peak Hour O-D Matrix for Sophisticated Network (morning peak)

	Centroids (Origin)													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Rail station (Destination)	200	380	620	200	800	300	500	200	440	650	200	800	330	500

Unit: passengers/hour

Table 5.3 Peak Hour O-D Matrix for More Sophisticated Network (morning peak)

	Centroids (Origins)																											
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Rail Station (Destination	200	380	620	200	800	300	500	200	440	650	200	800	330	500	200	380	620	200	800	300	500	200	440	650	200	800	330	500

Unit: passengers/hour

Table 5.4 Parameters and Coefficients used in The Experimental Networks

Table 5.4A Values of Network Parameters

Parameter	Value	Description
λ_{wait} :	20 \$/pass-hr	Value of waiting time[1];
λ_{wk} :	20 \$/pass-hr	Value of walking time[1];
λ_{riding} :	10 \$/pass-hr	Value of riding time[1];
λ_{auto} :	10 \$/pass-hr	Value of private mode driving time[1];
λ_{park} :	5 \$/day	Parking cost;
λ_{ac} :	0.172 \$/veh-mile	Unit auto operating-cost[2];
λ_{bc} :	81.1 \$/veh-hour	Unit bus operating-cost[3];
λ_{stop} :	0.31 min/stop	Delays for each bus stop[4];
ρ	0.8	Bus load factor;
C	50 Seats/veh	Bus capacity;
U_{wk} :	3 miles/hr	Average walking speed[5];
U_{bus} :	18.7 miles/hr	Average bus speed;
U_{auto} :	32 miles/hr	Average auto speed[5];
F_{max}	12 vehs/ hr	Maximum service frequency
F_{min}	1 veh/hr	Minimum service frequency
BusLMax	15 miles	Maximum bus route length;
BusLMin	0.8 miles	Minimum bus route length;
Mmax	8 routes	Maximum allowable bus routes;
R_{wmax}	0.8 miles	Maximum walking access distance to the rail station;
R_{bwmax}	0.75 miles	Maximum walking access distance to the bus stop;
R_{bmin}	0.1 mile	Minimum inter-stop spacing[6]
W:	20 buses	Maximum available fleet size;
Source:		[1]: Bureau of Labor Statistics, 2010;
		[2]: Matthews, 2010;
		[3]: NTD, 2010;
		[4]: McKnight, et al., 2003;
		[5]: Litman, 2011;
		[6]: Texas Transportation Institute, 1996.

Table 5.4 Parameters and Coefficients used in The Experimental Networks (continued)
**Table 5.4B** Mode Choice Coefficients

Auto IVT (Min) β^1_{auto}	-0.025
Auto Operating Cost (\$) β^2_{auto}	-0.3
Parking Cost (\$) β^3_{auto}	-0.3
Bus IVT (Min) β^1_{bus}	-0.025
Bus Cost (\$) β^2_{bus}	-0.5
Bus Wait Time (min) β^3_{bus}	-0.075
Bus Fare (\$)	1.5, 0.25/mile if >2 miles*
Walk Time (Min) β^1_{walk}	-0.05
Constant coefficient for walk β^0_{walk}	-0.5
Constant coefficient for bus β^0_{bus}	0
Constant coefficient for auto β^0_{auto}	0

Source: Schultz, 1991.
* Estimated based on NJ Transit bus fare structure (NJT, 2012).

5.3 Sensitivity Analyses of Tabu Search (TS) Algorithm

The purpose of this section is to explore the best TS strategies that can be used in networks of different sizes. In this section, the parameters used in the TS are determined. The network structures, O-D demands, and related parameters used in these situations have been described in section 5.2.
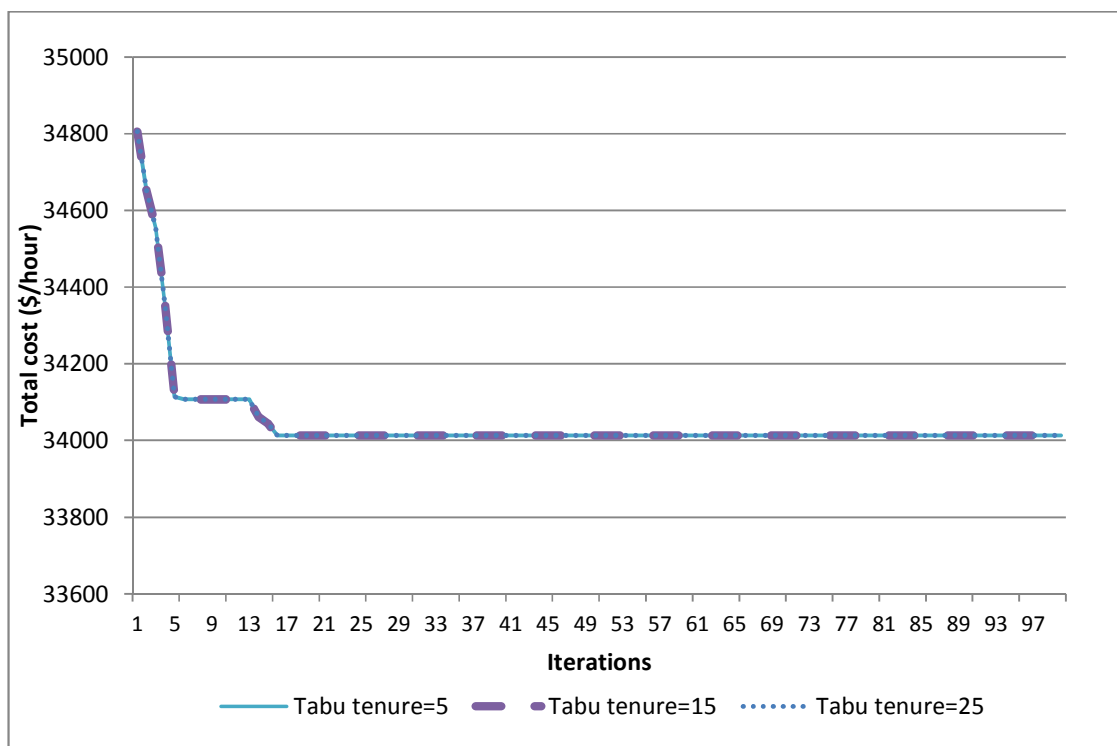
A. Sophisticated network.
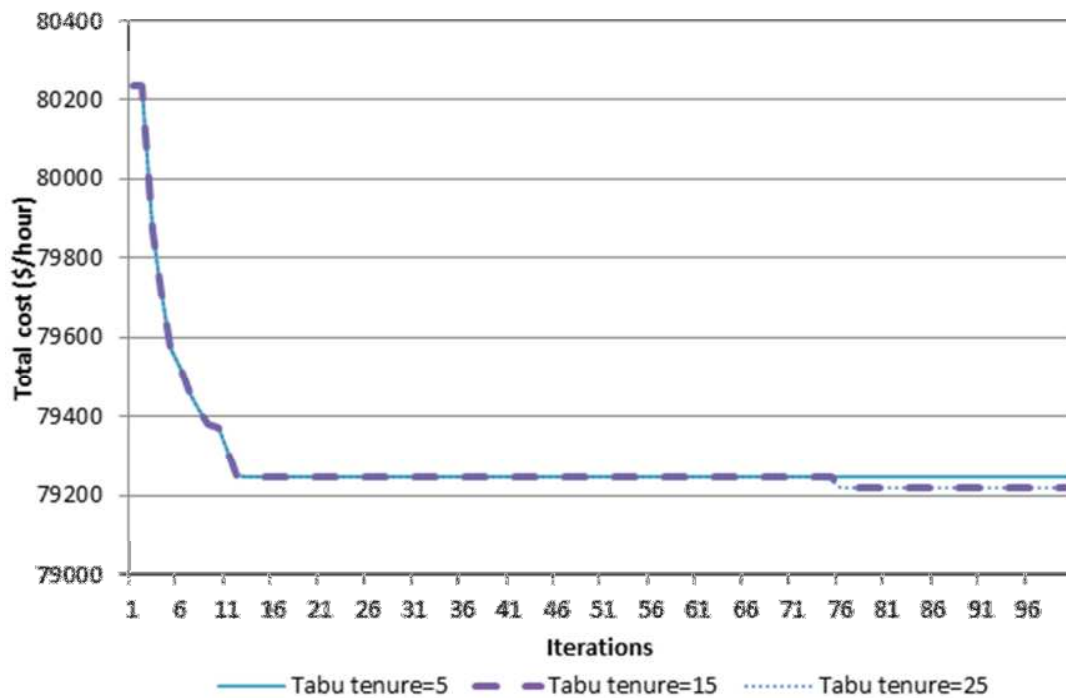
B. More sophisticated network.

Figure 5.4 Comparisons on the best solutions with different tabu tenure.

5.3.1 The Length of Tabu Tenure

To begin with the TS, the tabu tenure needs to be determined first. The following tests were performed on the sophisticated and more sophisticated networks to identify a suitable tabu move tenure. It is noted that the tabu length does affect the solution quality especially for larger sized networks. For example, as shown in Figure 5.4B, when the tenure is 5, the best solution was obtained at iteration 12 with the objective function value of 79249 $/hour. When the tenure is 15 or 25, the same solution is still found at iteration 12. However, a superior solution with the value of 79218 $/hour is discovered at iteration 76.

It is also noticed that when the tenure is 20, the convergence speed keeps stable. It is reasonably concluded that for smaller sized network, the tabu length of 5, 15, 25 are all acceptable. For larger sized network with 65 intersections and 28 demand centroids or more, 15 is the minimum value of tabu tenure and recommended to be used in the proposed computational networks.

5.3.2 Fixed and Dynamic Tabu Tenure

This section tests the performance of the TS by using fixed and dynamic tabu tenures on the sophisticated network. The fixed tabu tenure is 15. The value of dynamic tabu tenure will be randomly selected between 0 and 15. To make a comparison to fixed tabu tenure, five attempts are made by using dynamic tabu tenure as shown in Figure 5.5. It can be seen that the results obtained by 4 out of 5 attempts are inferior to the one obtained by the TS with fixed tabu tenure. For this reason, fixed tabu tenure strategy is selected for the application of TS in this dissertation.
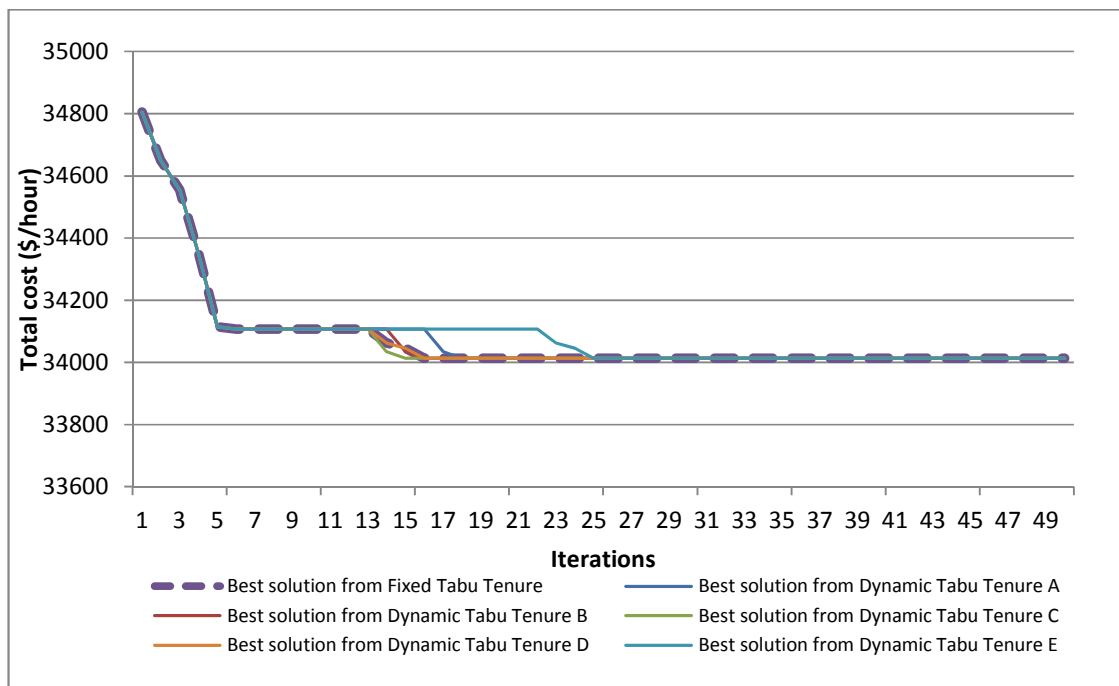
Figure 5.5 The performance of fixed and dynamic tabu tenure

5.4 Validation of the Solution Search Strategy

Since the objective function value (total cost) is determined by the configuration of the network (e.g., street pattern, feeder bus services) and the demand distributions, the global minimum total cost can be gained by comparing the total costs for all feasible route set. An Exhaustive Search (ES) algorithm is developed in this section to validate the effectiveness of the convergence of the TS, which is the solution search strategy used in this dissertation. The following flow chart demonstrates the procedures of the exhaustive search algorithm. The test is performed according to a simple network as presented in section 5.2. All the parameters except Mmax and K are taken from Table 5.4. To make the search space for the exhaustive search method in a reasonable size, the maximum allowable number of bus routes Mmax is set to be 2 in this validation section. Moreover,

the number of shortest paths, K, is set to be large enough so that all the available routes can be obtained between origin and destination.
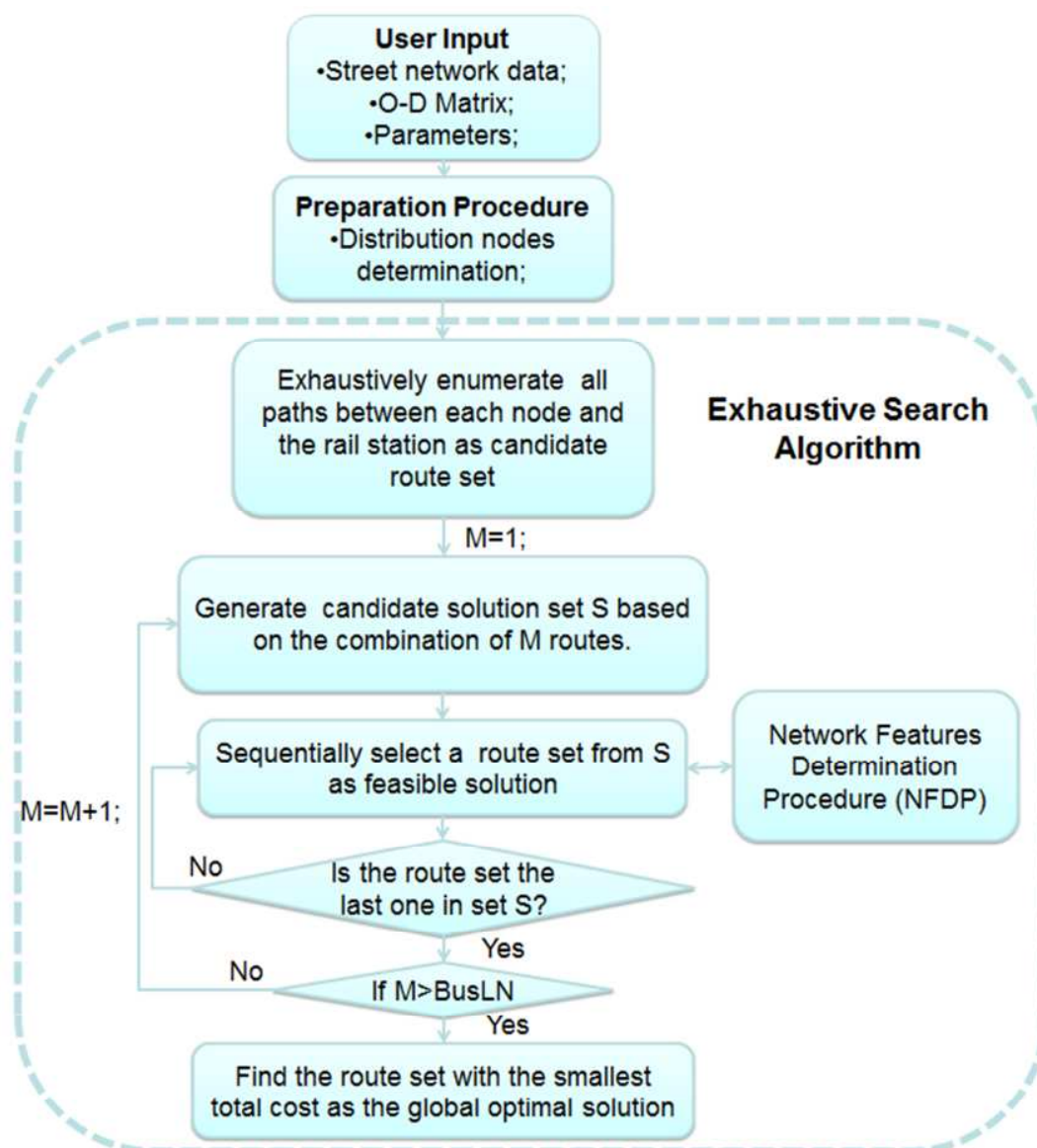

Figure 5.6 A flow chart of the exhaustive search algorithm.

It can be seen that two external modules, which have been discussed in Chapter 4 are called during the algorithm, including "Preparation Procedure (PP)" and "Network Features Determination Procedure (NFDP)". The first module is used to determine the

distribution nodes. The NFDP module is used to calculate the total cost as well as the network related features such as mode split at each centroid, bus stop selection, etc..

The ES starts with an exhaustive enumeration of all available paths between a distribution node and the rail station. The maximum allowable number of bus routes is M, and is set to be 1 initially. Then, a candidate solution set S is developed based on the different combinations of M route(s). Two loops are followed to sequentially select a combination of the routes during which the NFDP is called to obtain the total cost for the given route set. The nested loop will not terminate until all the selections are made. If the maximum number of bus routes Mmax >1, the outer loop will continue to find the total cost under a different number of bus routes. Finally, the global optimal solution is found by choosing the bus route set with the minimum total cost. The proposed ES algorithm can be used to find a global optimal solution on a given network since it is based on complete comparisons among all the feasible solutions. However, if the network is complicated, the candidate solution space would become extremely huge and lead the method to be computationally expensive to solve the network development problem.

Figure 5.7 and Figure 5.8 shows the results obtained. The total number of paths for the simple network is 540. The number of combinations for M=1 and M=2 are 540 and 145,530 respectively. So the total number of feasible solutions is 146070. After searching all the solutions by the ES, the global optimal solution was found with the time consumption of 5081 computing seconds and the minimum total cost is 15584 $/hour. By contrast, the best solution is also found by TS after executing 3 iterations with 120 seconds time consumption. Since the optimal solutions obtained by TS and ES are identical, the solution search strategy used in this dissertation.
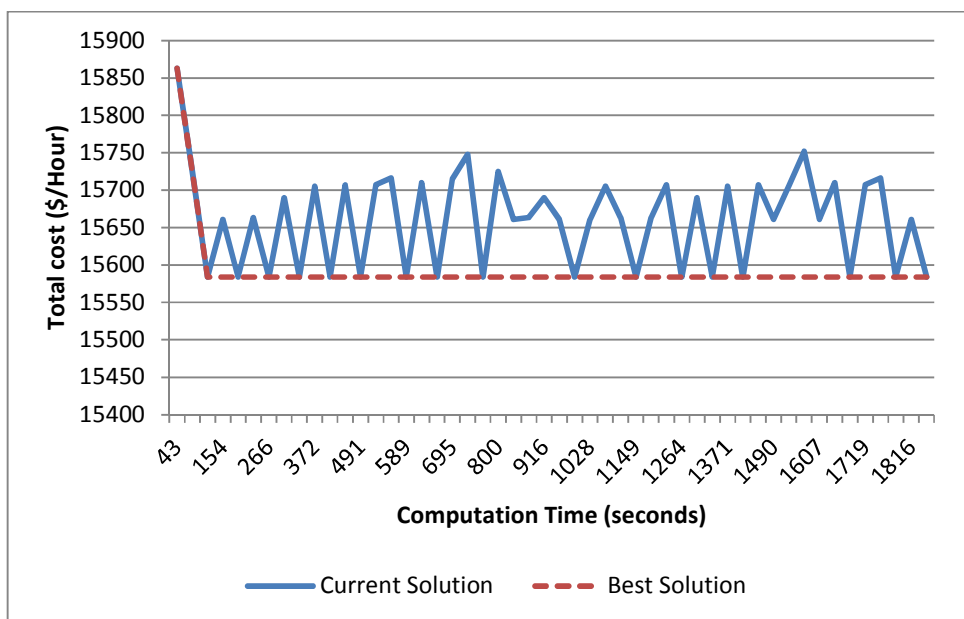
A. Total cost vs. Computation Time
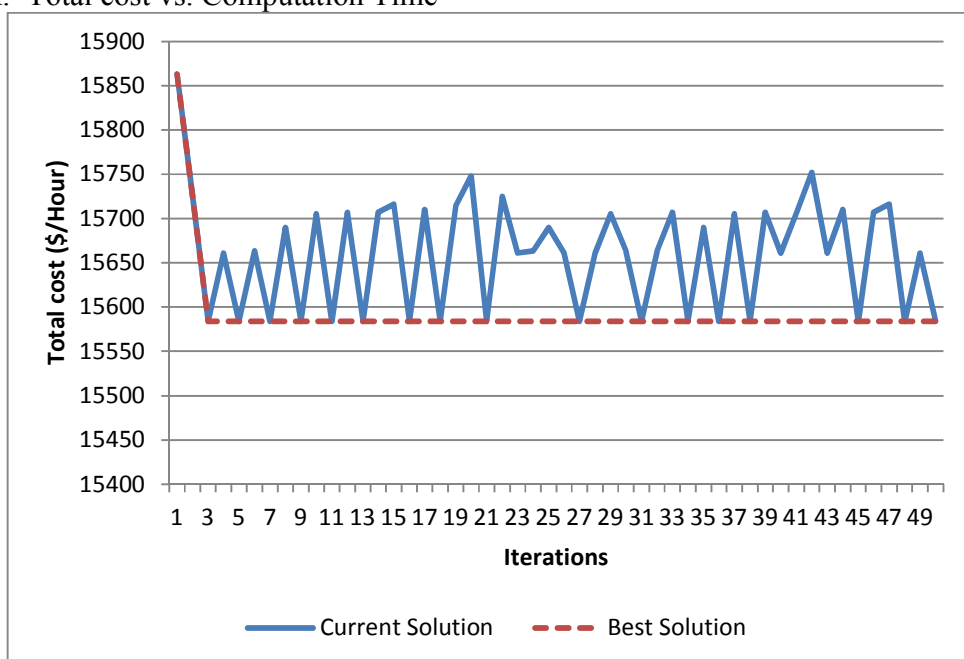
B. Total cost vs. Number of iterations

Figure 5.7 Total cost, computation time and iterations in TS.
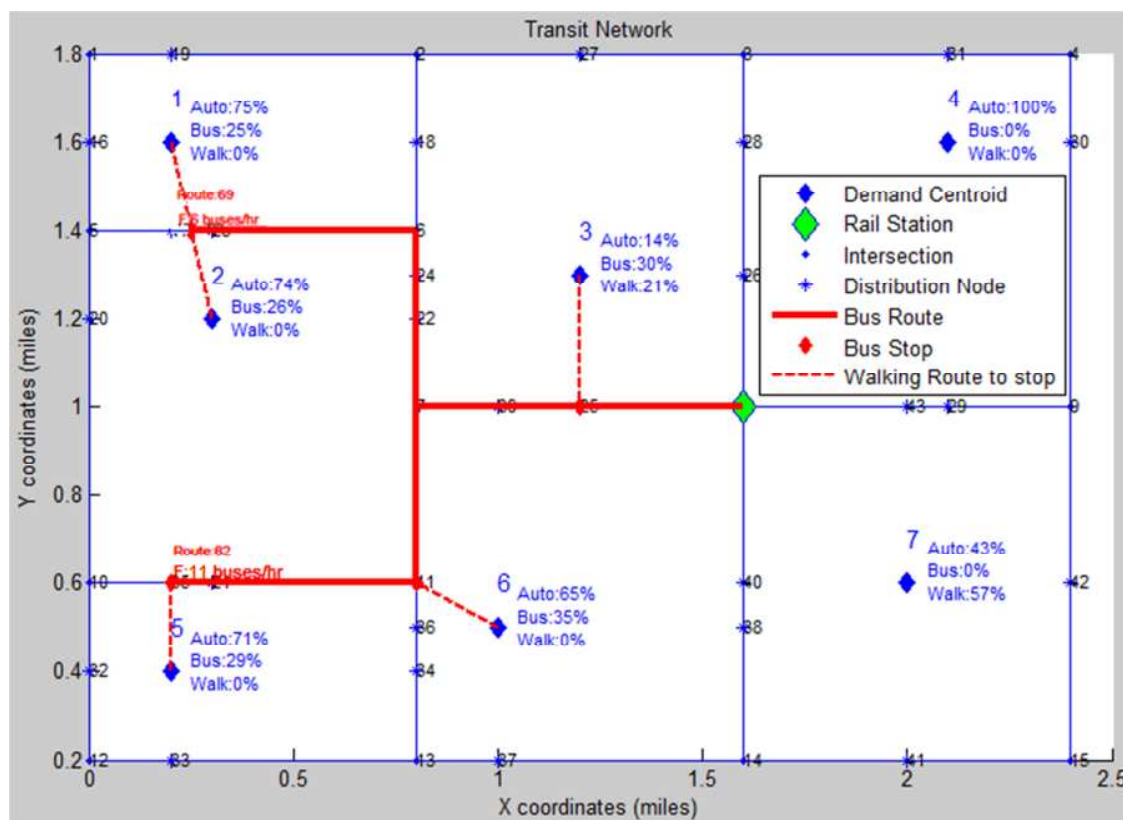
Figure 5.8 Configuration of the optimal solution for a simple network.

5.5 Comparisons between Initial Solution Generation Methods

In most previous references, the initial solution is usually selected randomly from a solution set. In Chapter 4 of this reference, an Initial Solution Generation Procedure (ISGP) was put forward. The purpose of this section is to verify the advantage of the ISGP by comparing it with the random selection method. To distinguish the differences easily, this comparison is carried out on a sophisticated network as it was shown in Figure 5.2 and Table 5.2. The parameters used in this section are from Table 5.4A and 5.4B.
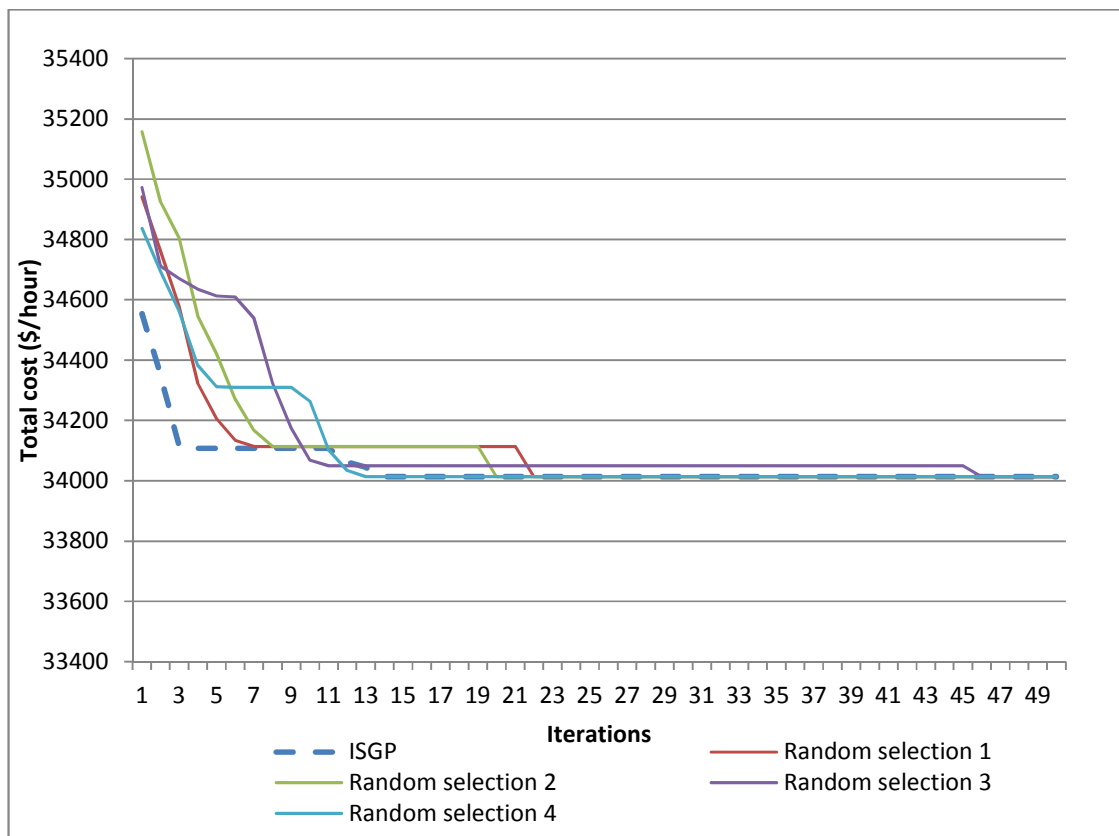
Figure 5.9 Comparison between initial solution generation methods.

Figure 5.9 exhibits the effectiveness of initial solution generation methods on the performance of TS. One can note that the convergence speed for the ISGP method is much faster than most of the attempts of the random selection methods. Among the four random selection attempts, only one case obtained the optimal solution a little bit earlier than ISGP. For all the other attempts, their convergence speeds are much slower. Moreover, the initial total cost got from ISGP is also superior to others. So, the ISGP provided in this study is the best method for using in TS to start with.

5.6 Computational Results

5.6.1 Outputs For the Computational Networks

It was shown in previous sections that TS with a fixed tabu tenure of 15, together with the strategies of intensification and diversification, can perform best in the FBNDP model. The followings show the results by applying the new FBNDP model to different networks. The route structures for the three street networks have been demonstrated in Figure 5.10. Needs to be pointed out is that, compared with the result in solution search strategy validation section in 5.4, the maximum allowable bus routes for the simple network are 8 routes instead of 2. The relationships between the size of the network and overall mode shares, variety of costs have been shown in Figure 5.11. It is obvious that as the size of the network gets larger, the advantage of automobile is outstanding. It is also observed that in the perspective of minimizing total cost, the feeder bus services do not have to cover all the centroids, especially for those centroids far away from the rail station. It can be explained that as the distance between a centroid and the rail station increases, the in-vehicle bus cost grows. Since auto does not have stop delays and its average travel speed is much higher than bus service, the advantage of automobile makes majority of people choose auto instead of bus even if there is bus services near it. So, in term of total cost, the expansion of a bus route in these areas would be costly and uneconomic. Table 5.5 shows the detailed network configurations including route path, route length and headway.
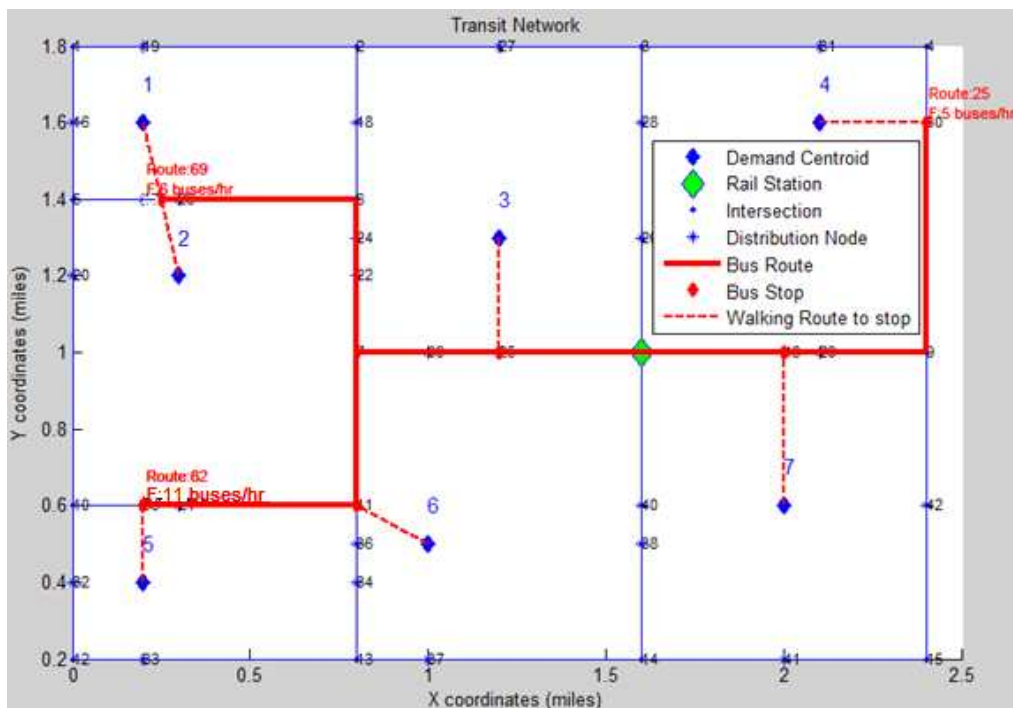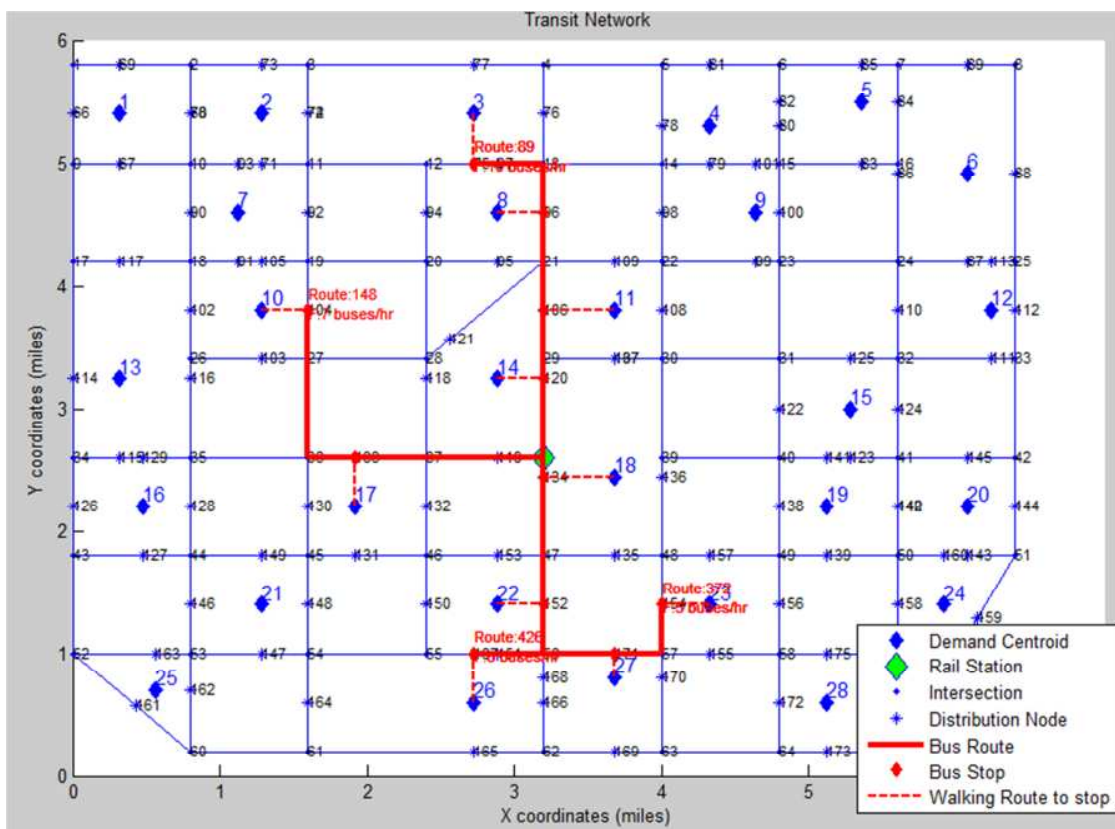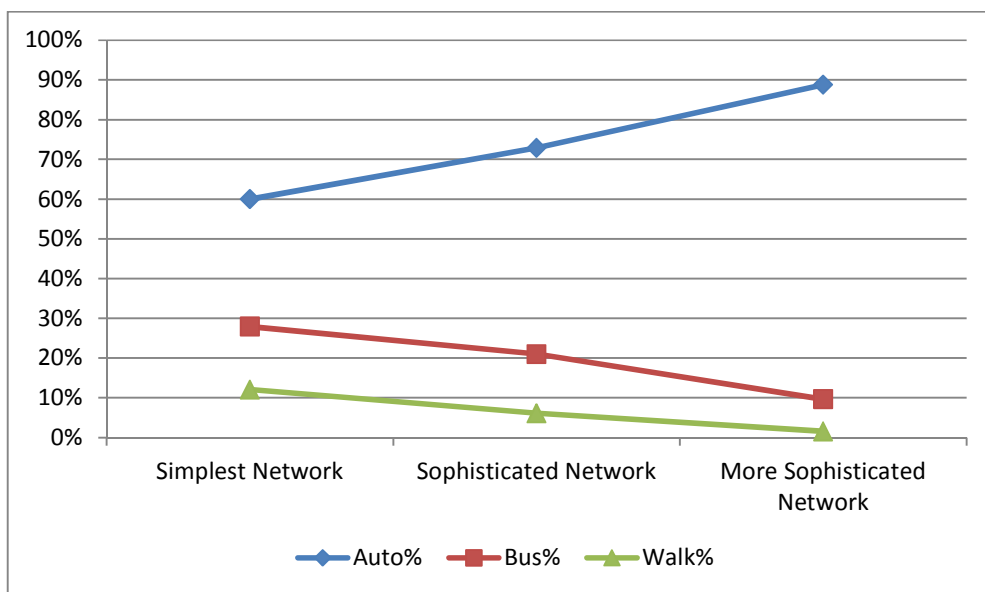
A. Simple network
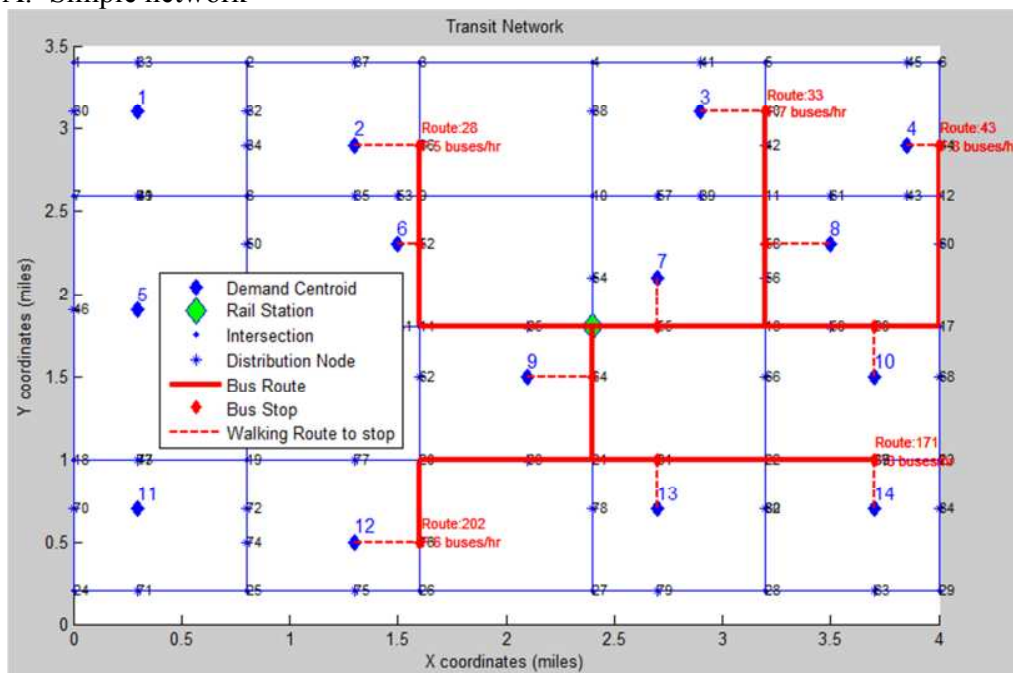
B. Sophisticated network

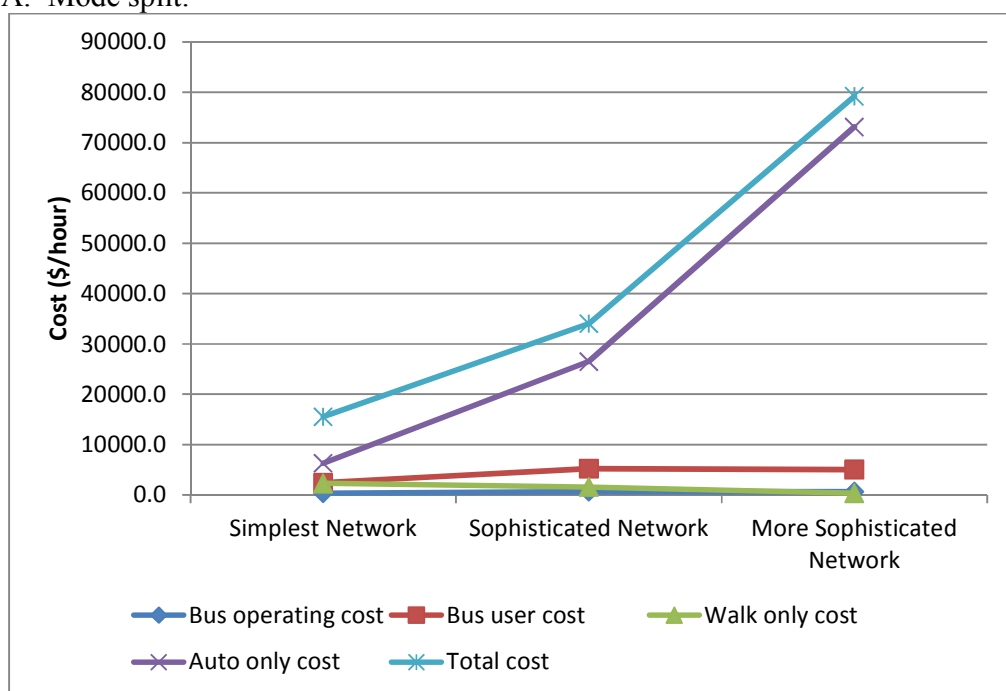**Figure 5.10** Results of network configuration.

C. More sophisticated network

Figure 5.10 Results of network configuration. (Continued)

A. Mode split.

B. Costs.

Figure 5.11 Relationships between network sizes, mode split and costs.

Table 5.5 The Results of Network Configurations for Simple, Sophisticated and More sophisticated Networks

A. Simple network.

Centroid	StopID	RouteID	Route Path	Route Length (miles)	Headway (min)	Auto%	Bus%	Walk%	Total Cost ($/hr)
1	17	69	[17,23,6,24,22,7,39,25,8]	1.8	10.2	76%	24%	-	1214.0
2	23	69	[17,23,6,24,22,7,39,25,8]	1.7	10.2	75%	25%	-	2162.7
3	25	69	[17,23,6,24,22,7,39,25,8]	0.4	10.2	28%	10%	41%	2780.2
		82	[35,21,11,7,39,25,8]	0.4	5.0		21%		
4	30	25	[30,9,29,43,8]	1.4	11.6	73%	27%	-	1126.7
5	35	82	[35,21,11,7,39,25,8]	1.8	5.0	71%	29%	-	4170.2
6	11	82	[35,21,11,7,39,25,8]	1.2	5.0	64%	36%	-	1458.3
7	43	25	[30,9,29,43,8]	0.4	11.6	33%	24%	43%	2626.0

B. Sophisticated network.

Centroid	StopID	RouteID	Route Path	Route Length (miles)	Headway (min)	Auto%	Bus%	Walk%	Total Cost ($/hr)
1	-	-	-	-	-	100%	-	-	1322.7
2	36	28	[36,9,52,14,65,15]	1.9	11.1	78%	22%	-	2296.7
3	40	33	[40,42,11,58,56,16,55,15]	2.1	8.4	80%	20%	-	3629.1
4	44	43	[44,12,60,17,69,59,16,55,15]	2.7	7.1	84%	16%	-	1373.4
5	-	-	-	-	-	100%	-	-	4835.1
6	52	28	[36,9,52,14,65,15]	1.3	11.1	66%	34%	-	1462.6
7	55	33	[40,42,11,58,56,16,55,15]	0.3	8.4	27%	14%	43%	2006.4
		43	[44,12,60,17,69,59,16,55,15]	0.3	7.1		17%		
8	58	33	[40,42,11,58,56,16,55,15]	1.3	8.4	68%	32%	-	1058.9
9	64	171	[67,22,81,21,64,15]	0.3	7.7	27%	17%	43%	2027.2
		202	[76,20,63,21,64,15]	0.3	9.5		14%		
10	69	43	[44,12,60,17,69,59,16,55,15]	1.3	7.1	67%	33%	-	3383.5
11	-	-	-	-	-	100%	-	-	1303.7
12	76	202	[76,20,63,21,64,15]	2.1	9.5	80%	20%	-	4711.7
13	81	171	[67,22,81,21,64,15]	1.1	7.7	64%	36%	-	1683.2
14	67	171	[67,22,81,21,64,15]	2.1	7.7	79%	21%	-	2918.8

C. More Sophisticated network

Centroid	StopID	RouteID	Route Path	Route Length (miles)	Headway (min)	Auto%	Bus%	Walk%	Total Cost ($/hr)
1	-	-	-	-	-	100%	-	-	1539.0
2	-	-	-	-	-	100%	-	-	2751.1
3	75	89	[75,97,13,96,21,106,29,120,38]	2.9	5.7	88%	12%	-	4265.3
4	-	-	-	-	-	100%	-	-	1362.5
5	-	-	-	-	-	100%	-	-	5920.8
6	-	-	-	-	-	100%	-	-	2240.2
7	-	-	-	-	-	100%	-	-	3468.0
8	96	89	[75,97,13,96,21,106,29,120,38]	2.0	5.7	76%	24%	-	1142.7
9	-	-	-	-	-	100%	-	-	2918.2
10	104	148	[104,27,36,133,37,119,38]	2.8	8.1	87%	13%	-	4276.1
11	106	89	[75,97,13,96,21,106,29,120,38]	1.2	5.7	68%	32%	-	1108.4
12	-	-	-	-	-	100%	-	-	5609.5
13	-	-	-	-	-	100%	-	-	2201.2
14	120	89	[75,97,13,96,21,106,29,120,38]	0.6	5.7	55%	45%	-	2296.7
15	-	-	-	-	-	100%	-	-	1235.4
16	-	-	-	-	-	100%	-	-	2462.6
17	133	148	[104,27,36,133,37,119,38]	1.3	8.1	69%	31%	-	3405.5
18	134	372	[154,57,171,56,152,47,134,38]	0.2	11.5	28%	13%	43%	1132.0
		426	[167,151,56,152,47,134,38]	0.2	9.9		15%		
19	-	-	-	-	-	100%	-	-	4880.7
20	-	-	-	-	-	100%	-	-	1966.9
21	-	-	-	-	-	100%	-	-	3240.2
22	152	372	[154,57,171,56,152,47,134,38]	1.2	11.5	64%	17%	-	1011.7
		426	[167,151,56,152,47,134,38]	1.2	9.9		19%		
23	154	372	[154,57,171,56,152,47,134,38]	2.8	11.5	89%	11%	-	2677.6
24	-	-	-	-	-	100%	-	-	4459.0
25	-	-	-	-	-	100%	-	-	1430.8
26	167	426	[167,151,56,152,47,134,38]	2.1	9.9	81%	19%	-	4875.2
27	171	372	[154,57,171,56,152,47,134,38]	2.1	11.5	79%	21%	-	1910.1
28	-	-	-	-	-	100%	-	-	3430.0

5.6.2 Network Size and Computing Speed

The size of a network dramatically affects the computation time. In this dissertation, three sized networks are tested. As demonstrated in Figure 5.1, the simple network consists of 15 intersections, and 7 centroids. Since each centroid has 4 distribution nodes, the total number of candidate stops is 43. The sophisticated network has 29 intersections, with 14 demand centroids. The number of candidate stops is 85. As for the more sophisticated network as shown in Figure 5.3, 65 street intersections and 28 demand centroids are included. There are totally 177 candidate stops. As discussed in Chapter 4, for each street intersection, K shortest paths are generated. The sizes of candidate route set for simple, sophisticated and more sophisticated networks are 15*K, 29*K and 65*K respectively. In the numerical experiments, 8 is used as the value of K. So the total number of candidate routes for each network is 120, 232, and 520 respectively. In Section 4.6 has shown the available movements for a given route set. For the routes in the route set, they can be exchanged with the routes in the candidate route set or deleted from the route set. When the route number is less than Mmax, new routes can be added to the route set. If the size of a given route set is n, where n<Mmax, the size of candidate route set is S, the number of available exchange movements will be (S-n)*n; the number of deletion and addition movements is S. So, the total neighbors for a route set with n routes are (S-n)*n+S. For the given networks, S_{simple}=120, $S_{sophisticated}$=232, $S_{moresophisticated}$=520, the neighbors for these networks are (120-n)*n, (232-n)*n, (520-n)*n respectively. Figure 5.12 presents the computational time for different size of networks. It can be seen that the computation time increases exponentially as the network size increases.
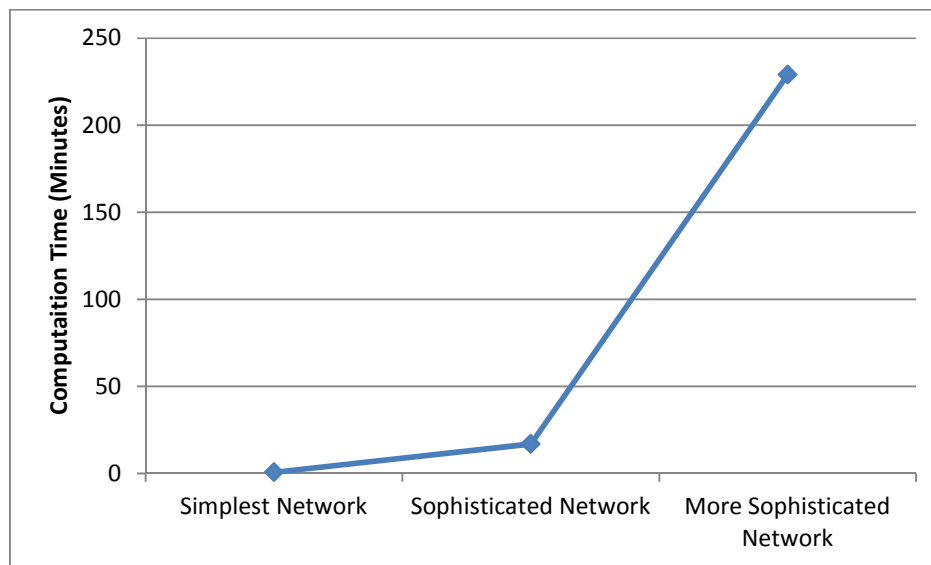
Figure 5.12 Computation time vs. network size.

5.7 Sensitivity Analyses on Proposed FBNDP Methodology

In this section, a series of sensitivity analyses for different parameters is performed. For the purpose of showing the relationships between the parameters and decision variables. The sophisticated network is used in this section. The base line of the parameters is the same as it was shown in Table 5.4.

The focus of the first part is to perform fleet size sensitivity analysis for the FBNDP. The default maximum available fleet size in this study is 20 buses. As shown in section 5.6, the optimized fleet size for a sophisticated network is 9 buses. The test is carried out using an available fleet size from 6 buses to 12 buses. Figure 5.13 illustrates the relationship between various costs and fleet size. In the beginning, due to the fact that the desired number of buses is more than the available fleet size, as the fleet size increases, the bus user cost expands accordingly; meanwhile, the decrease of walk and

auto costs leads to a reduction of total cost. It is obvious that as consistent with the previous result, for the given sized network and O-D demand, at least 9 buses are required to achieve minimum total cost.
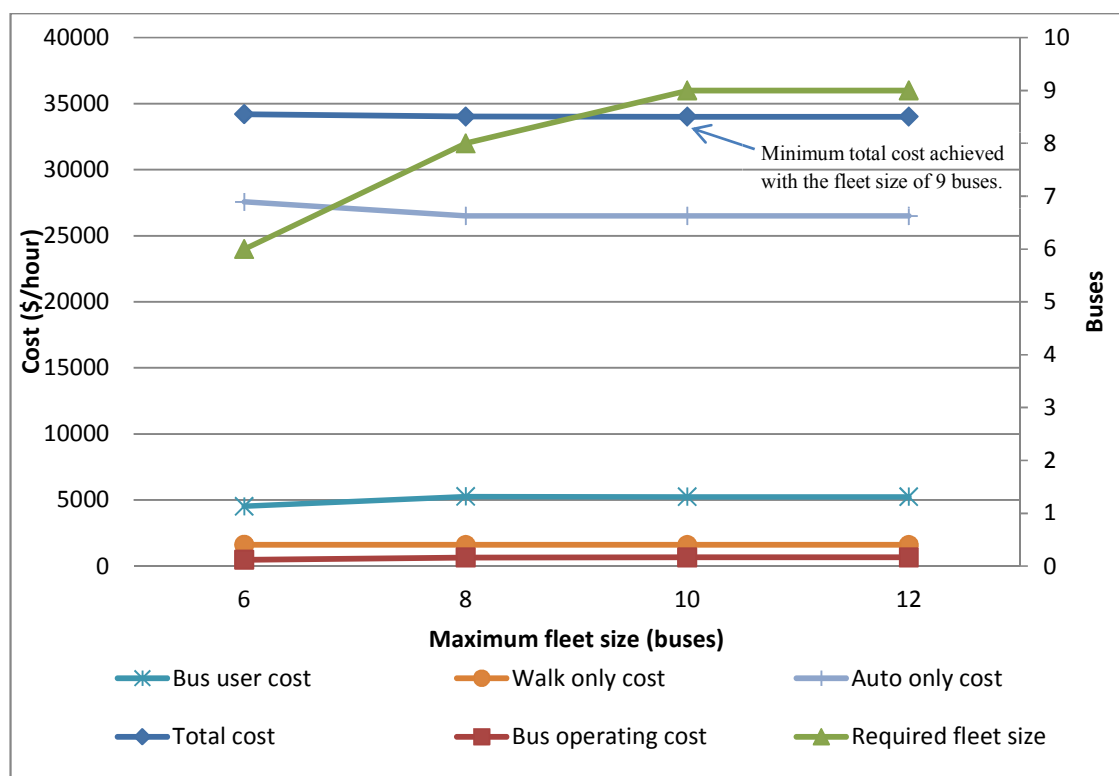

Figure 5.13 Costs vs. Fleet size

The unit auto cost includes two components: parking cost and unit millage cost. Here, the average parking fee is taken as a factor to investigate the impacts of auto cost on the feeder bus network configuration. To make comparisons, two scenarios are tested. In the first scenario, the maximum available fleet size is 20 buses while in the second scenario, the fleet size constraint is released. The costs (i.e., user, supplier and total costs) versus parking fee are presented in Figure 5.14. The solid lines represent the first scenario, and the dotted lines stand for the second scenario. It is noted that as the parking fee goes up from $7 /day to $13 /day, the total cost is in its ascending path all the time

regardless of the limitation of feeder bus fleet size. However, an adequate fleet size does slow down the growth of total cost. As depicted in Figure 5.14, when the parking fee is $13 /day, in the first scenario, the total cost is $48479 /hour. If 4 additional buses are added to the feeder bus fleet, compared to scenario A, although the bus operating and bus user costs increase by $288 /hour and $400 /hour, the auto cost decreases by $2495 /hour and the total cost falls to $46622 /hour. It is also noticed that when the parking fee is $11 /day, the 15% increase of fleet size (from 20 buses to 23 buses) only contributes to a 0.4% total cost decrease (from $43751 /hour to $43565 /hour). Figure 5.15 shows the difference of bus network between these two cases. One can note that in the second scenario, due to the adequate fleet size, some bus routes are extended to attract more people. Nevertheless, such expansions do not generate much ridership (Figure 5.16). This can be explained by the fact that the average bus speed is much lower than auto speed. When the distance between a centroid and the rail station is too far away, the demands at this centroid would prefer auto over bus even if there is bus service near by. In other words, when the fleet size is limited, an extension of existing bus routes to remote places is not efficient in decreasing the total cost.
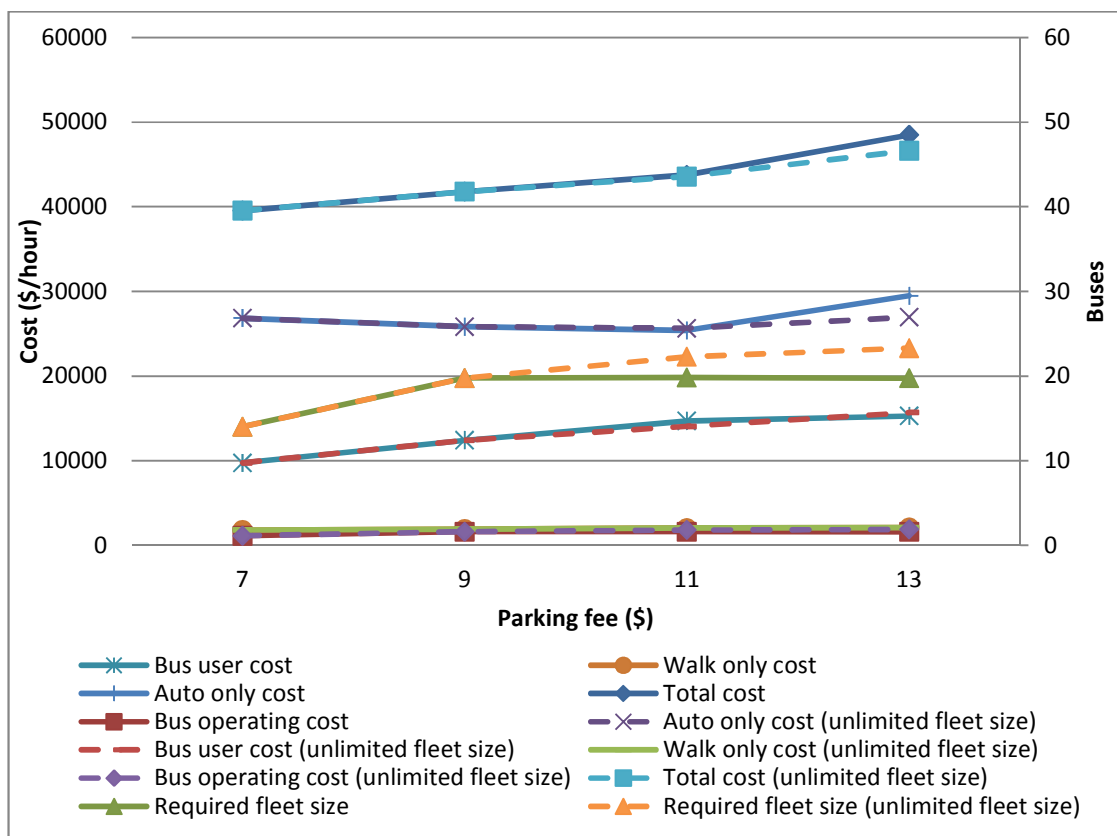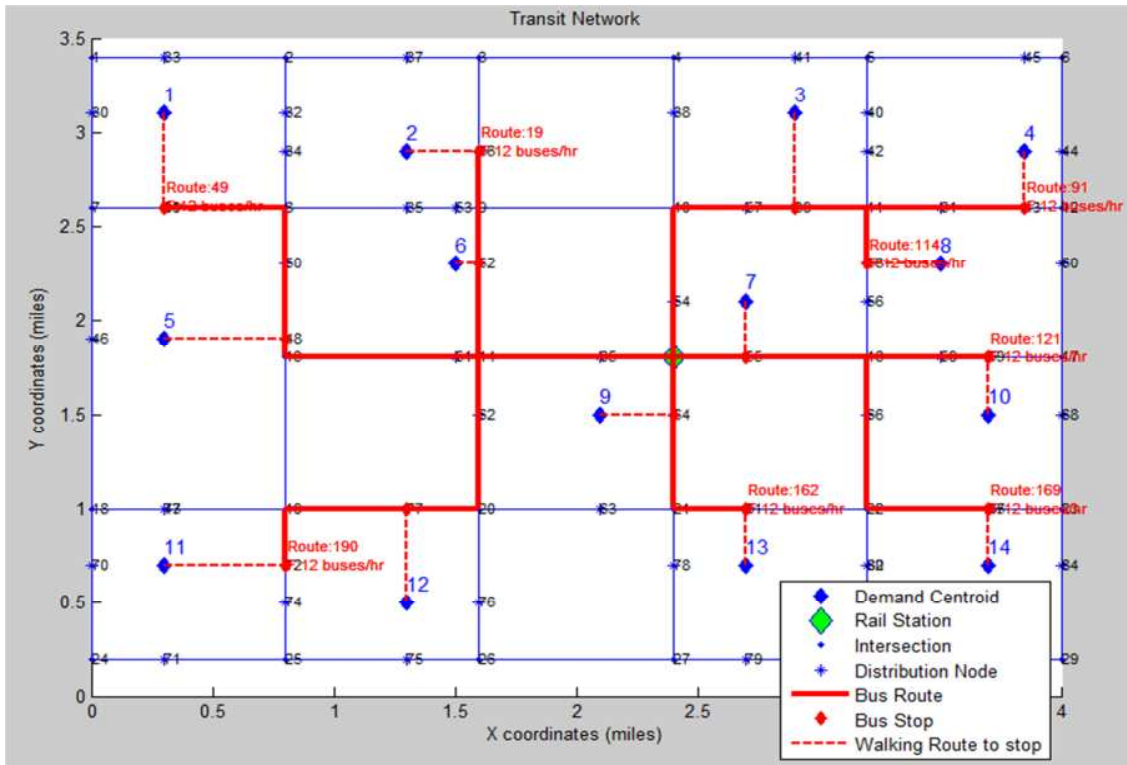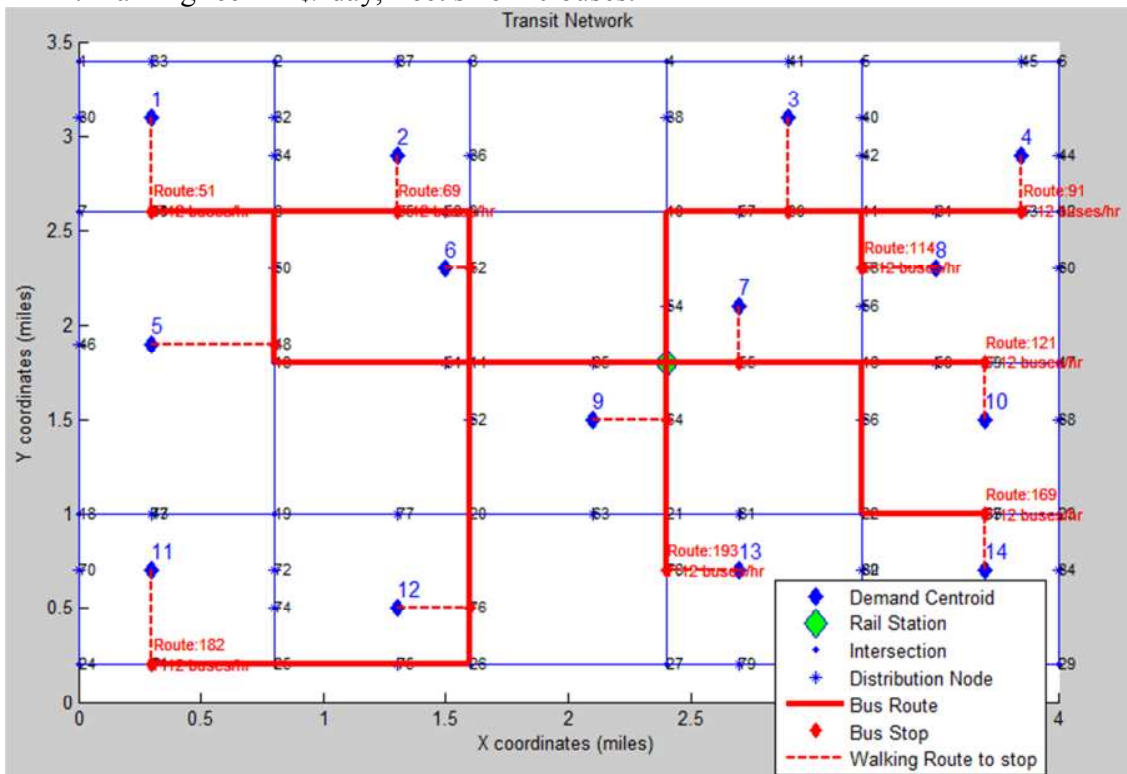
Figure 5.14 Costs vs. Parking fee

A. Parking fee=11 $/ day, fleet size=20 buses.

B. Parking fee=11 $/day, without fleet size constraint.

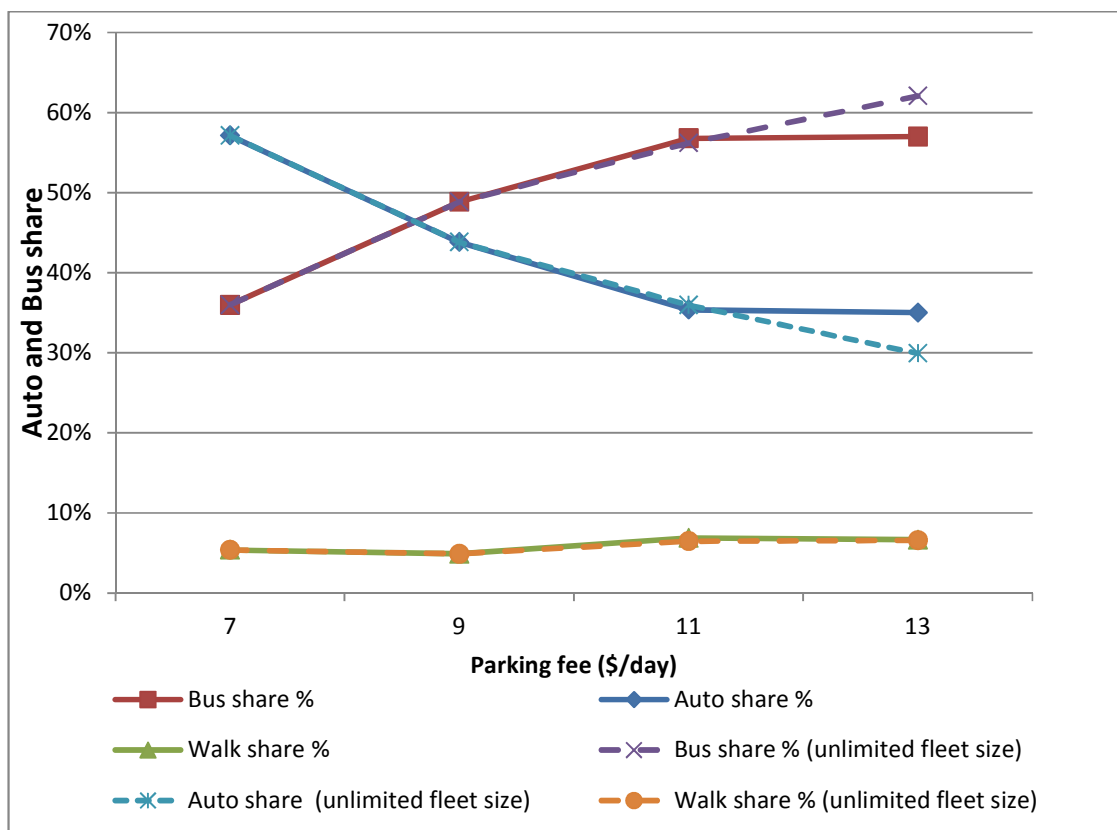**Figure 5.15** Feeder bus networks vs. Fleet size.

Figure 5.16 Mode shares vs. Parking fee.

Figure 5.16 presents the relationship between parking fee and average mode shares. As expected, a parking fee increase results in the decrease of auto share and the increase of bus share. It is also found that because of the limitation of fleet size, when the parking fee exceeds $11/day, the mode shares in the first scenario do not change apparently. By comparison, in the second scenario, the auto share continuously decreases as the parking fee increases. Yet the trend of the auto cost does not decrease all the way down, which means the auto cost and parking fee is not a linear relationship when the mode split and feeder bus network are taken into account. In terms of the walk share, it is dramatically influenced by the bus services nearby. As shown in Figure 5.16, it drops initially since the inflation of the parking fee which stimulates the bus demand and finally

the bus services. After that the walk share fluctuates as the number of bus routes changes near those centroids.

The impact of bus fare on the costs is shown in Figure 5.17, Figure 5.18 and Figure 5.19. As depicted in Figure 5.17, the upward adjustment of bus fare dramatically influences the bus share, which drops from 25% to 8%. By contrary, the walk share and auto share increase consequently. These impacts are also reflected in the costs. Along with the bus fare increase, the auto and walk costs increase; the bus user and bus operating costs keep decreasing. It is also noticed that the increase of total cost is not parallel with other costs. As the bus fare goes up by 150%, the total cost only increases by 5%. It has to be pointed out that in the real world, when the congestion factor is taken into account, the actual total cost increase may be higher.
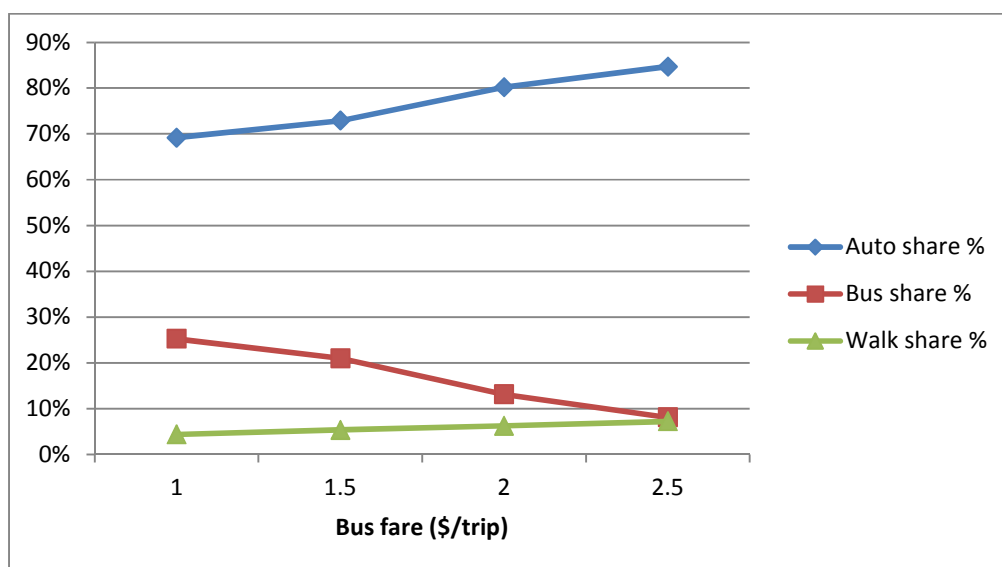
Figure 5.17 Mode shares vs. Bus fare.
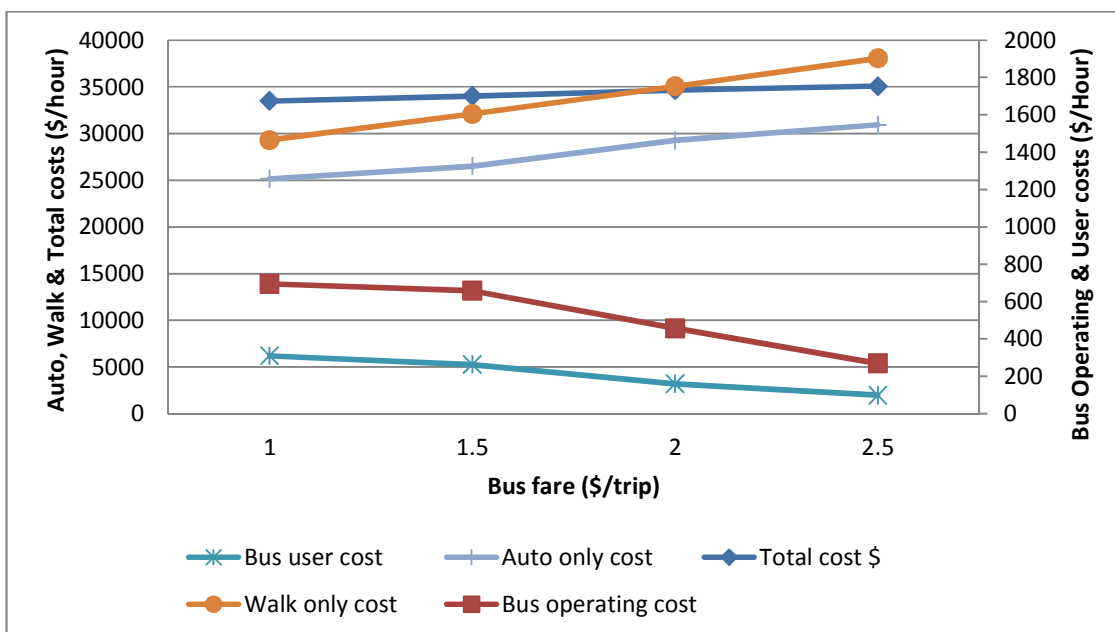
Figure 5.18 Costs vs. Bus fare.

Figure 5.19 illustrates the fare impact on the feeder bus networks. The situations reflected in the network structures are consistent with those in the bus mode share. As the bus demand decreases, to decrease the total cost, the bus service frequencies and the number of bus routes keep shrinking. When the bus fare equals $2.5, only two routes are left.
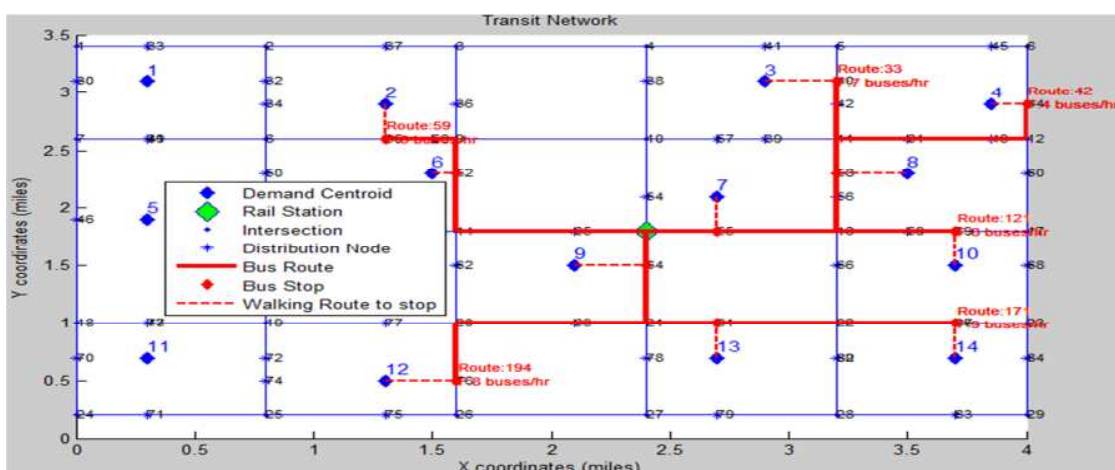
A. Bus fare=1 $/trip

Figure 5.19 Feeder bus networks vs. Bus fare.
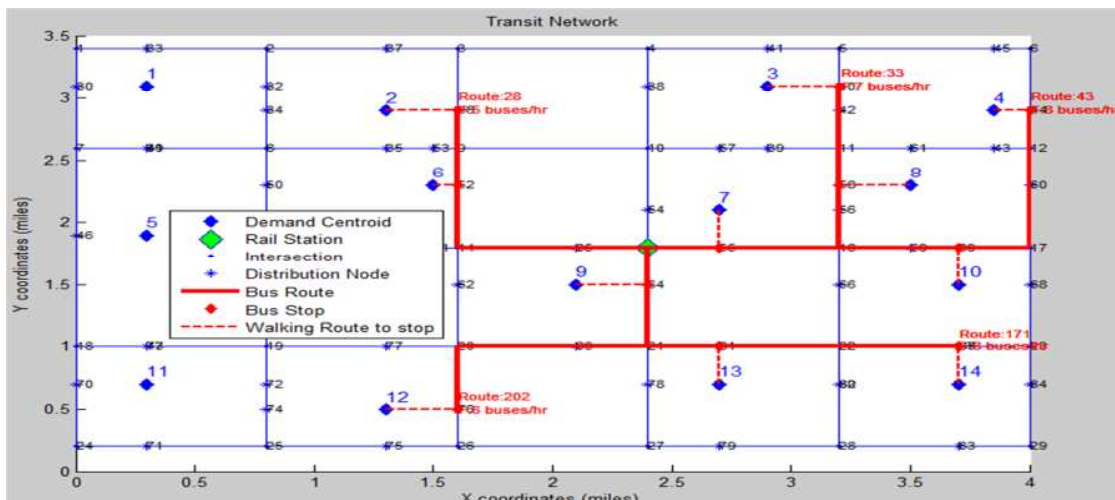
B. Bus fare=1.5 $/trip
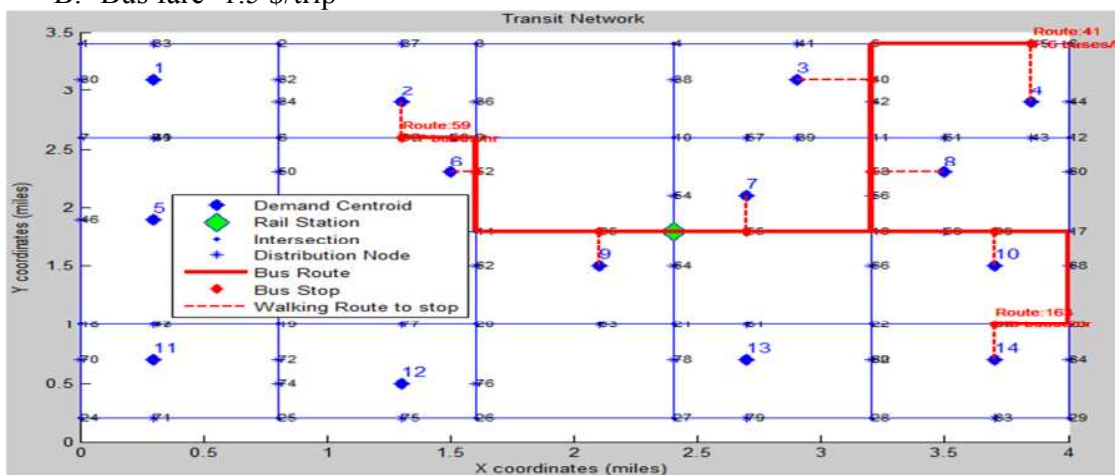
C. Bus fare=2 $/trip
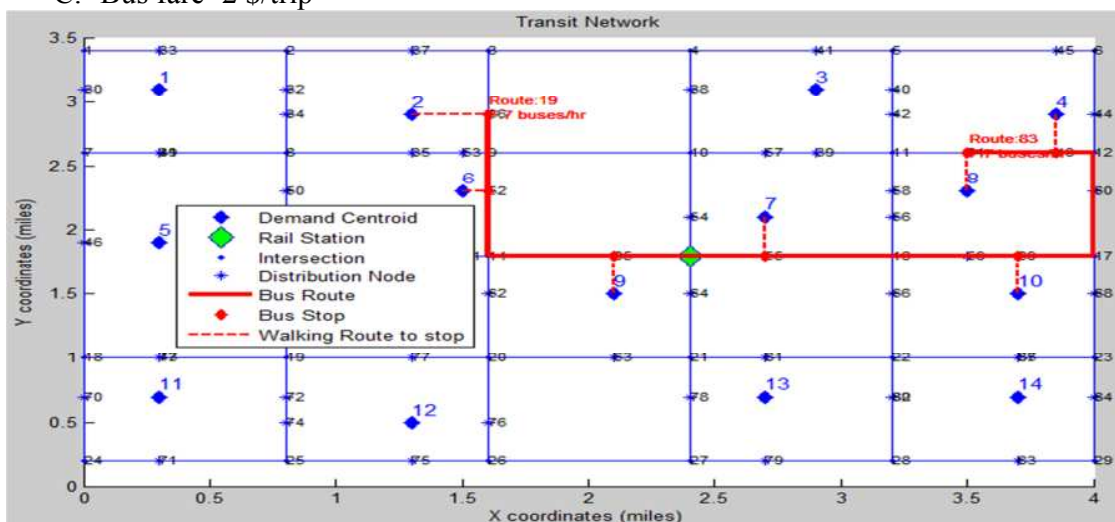
D. Bus fare=2.5 $/trip

Figure 5.19 Feeder bus networks vs. Bus fare. (Continued)

Figure 5.20 and Table 5.6 show the impact of taking walk mode on the feeder bus network into consideration. In scenario A, all the three access modes are included. In scenario B, only bus and auto modes are considered. Ignoring walk mode results in the overestimation of the feeder bus demand in those centroids within the maximum walking access distance (centroid 7, centroid 9) and the underestimation of total cost. The direct impact on bus services is the output of excessive service frequencies on relevant routes (Route 33, 43, 202, 171). Since the increase of service frequency will cause the rise of operating cost, to minimize the total cost, the expansion of a route with long distance is replaced by the one with shorter distance (Route 43 is replaced by Route 121; Route 33 is replaced by Route 41). The expense of such adjustments is the increase of total user cost at some centroids (e.g., centroid 4).
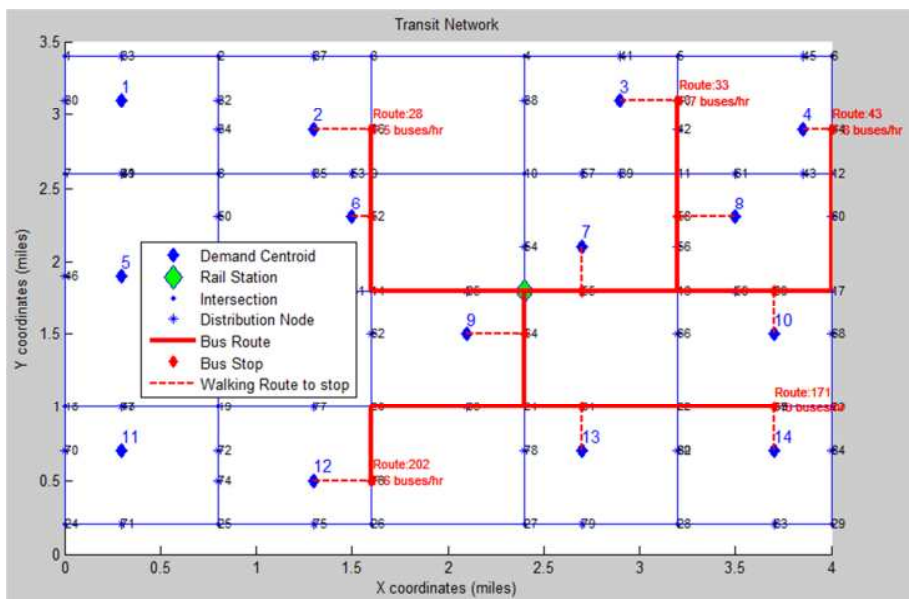
A. The feeder bus network (all walk, bus, auto modes are considered)
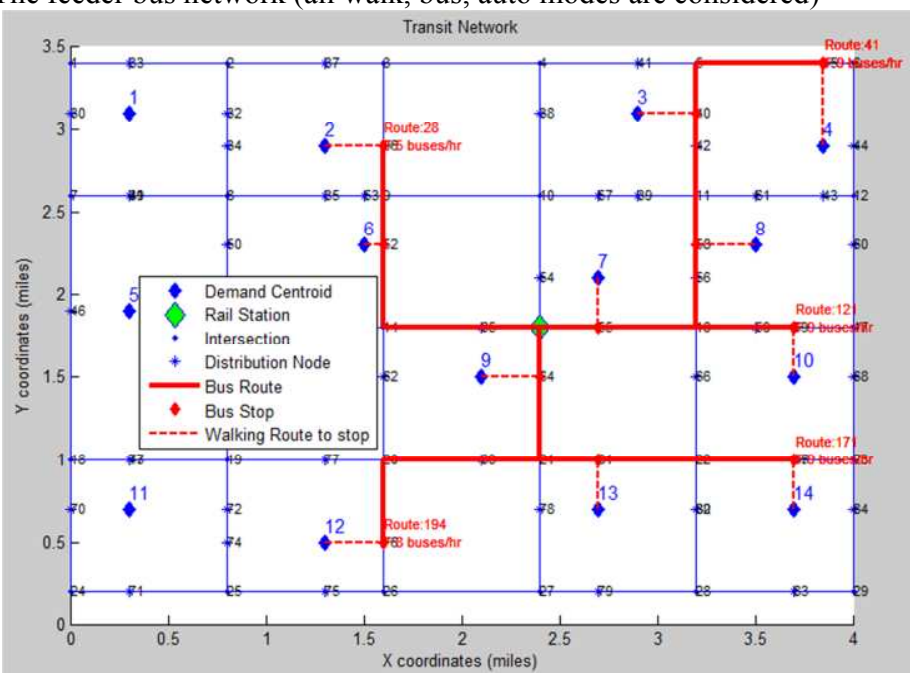
B. The feeder bus network (only bus, auto modes are considered)

Figure 5.20 The feeder bus network structure by considering different modes.

Table 5.6 The Feeder Bus Network Features by Considering Different Modes

A. The feeder bus network features (all walk, bus, auto modes are considered)

| Centroid	StopID	RouteID	Route Path	Route Length (miles)	Headway (min)	Auto%	Bus%	Walk%	Bus Opt. Cost ($/hr)	Bus User Cost ($/hr)	Walk Only Cost ($/hr)	Auto Only Cost ($/hr)	Total Cost ($/hr)
1	-	-	-	-	-	100%	-	-	-	-	-	1322.7	1322.7
2	36	28	[36,9,52,14,65,15]	1.9	11.1	78%	22%	-	79.5	419.4	-	1797.7	2296.7
3	40	33	[40,42,11,58,56,16,55,15]	2.1	8.4	80%	20%	-	125.2	597.7	-	2906.2	3629.1
4	44	43	[44,12,60,17,69,59,16,55,15]	2.7	7.1	84%	16%	-	203.1	124.2	-	1046.1	1373.4
5	-	-	-	-	-	100%	-	-	-	-	-	4835.1	4835.1
6	52	28	[36,9,52,14,65,15]	1.3	11.1	66%	34%	-	-	335.9	-	1126.7	1462.6
7	55	33	[40,42,11,58,56,16,55,15]	0.3	8.4	27%	14%	43%	-	442.6	851.6	712.2	2006.4
		43	[44,12,60,17,69,59,16,55,15]	0.3	7.1		17%		-				
8	58	33	[40,42,11,58,56,16,55,15]	1.3	8.4	68%	32%	-	-	275.2	-	783.7	1058.9
9	64	171	[67,22,81,21,64,15]	0.3	7.7	27%	17%	43%	144.7	394.3	752.9	629.7	2027.2
		202	[76,20,63,21,64,15]	0.3	9.5		14%		105.5				
10	69	43	[44,12,60,17,69,59,16,55,15]	1.3	7.1	67%	33%	-	-	877.5	-	2506.1	3383.5
11	-	-	-	-	-	100%	-	-	-	-	-	1303.7	1303.7
12	76	202	[76,20,63,21,64,15]	2.1	9.5	80%	20%	-	-	793.8	-	3917.9	4711.7
13	81	171	[67,22,81,21,64,15]	1.1	7.7	64%	36%	-	-	481.2	-	1202.0	1683.2
14	67	171	[67,22,81,21,64,15]	2.1	7.7	79%	21%	-	-	502.7	-	2416.1	2918.8

B. The feeder bus network features (only bus, auto modes are considered)

Centroid	StopID	RouteID	Route Path	Route Length (miles)	Headway (min)	Auto%	Bus%	Walk%	Bus Opt. Cost ($/hr)	Bus User Cost ($/hr)	Walk Only Cost ($/hr)	Auto Only Cost ($/hr)	Total Cost ($/hr)
1	-	-	-	-	-	100%	0%	-	-	-	-	1322.7	1322.7
2	36	28	[36,9,52,14,65,15]	1.9	11.1	78%	22%	-	79.5	419.4	-	1797.7	2296.7
3	40	41	[45,5,40,42,11,58,56,16,55,15]	2.1	6.7	79%	21%	-	245.1	591.7	-	2867.1	3704.0
4	45	41	[45,5,40,42,11,58,56,16,55,15]	3.0	6.7	91%	9%	-	-	114.7	-	1132.2	1246.9
5	-	-	-	-	-	100%	0%	-	-	-	-	4835.1	4835.1
6	52	28	[36,9,52,14,65,15]	1.3	11.1	66%	34%	-	-	335.9	-	1126.7	1462.6
7	55	41	[45,5,40,42,11,58,56,16,55,15]	0.3	6.7	46%	27%	-	-	754.4	-	1227.4	2088.3
		121	[69,59,16,55,15]	0.3	6.6		27%		106.5				
8	58	41	[45,5,40,42,11,58,56,16,55,15]	1.3	6.7	67%	33%	-	-	268.1	-	766.9	1035.1
9	64	171	[67,22,81,21,64,15]	0.3	6.7	47%	29%	-	174.3	671.9	-	1086.5	2064.6
		194	[76,20,63,21,64,15]	0.3	7.9		24%		131.8				
10	69	121	[69,59,16,55,15]	1.3	6.6	67%	33%	-	-	871.0	-	2491.9	3362.9
11	-	-	-	-	-	100%	-	-	-	-	-	1303.7	1303.7
12	76	194	[76,20,63,21,64,15]	2.1	7.9	79%	21%	-	-	787.5	-	3870.8	4658.3
13	81	171	[67,22,81,21,64,15]	1.1	6.7	63%	37%	-	-	472.1	-	1184.6	1656.7
14	67	171	[67,22,81,21,64,15]	2.1	6.7	78%	22%	-	-	499.2	-	2395.1	2894.3

The impact of stop delays is shown in Figure 5.21. The parking fee is set at $6/day for the both cases. To avoid the extra cost at bus stops, the consideration of stop delays makes the routes with fewer bus stops be selected. In Figure 5.21, when the stop delays are taken into account, the search strategy finds more effective routes (Route 91, Route 121) instead of Route 33, and Route 43. The replacement routes have 7 stops in total, which are one stop less than the previous ones.
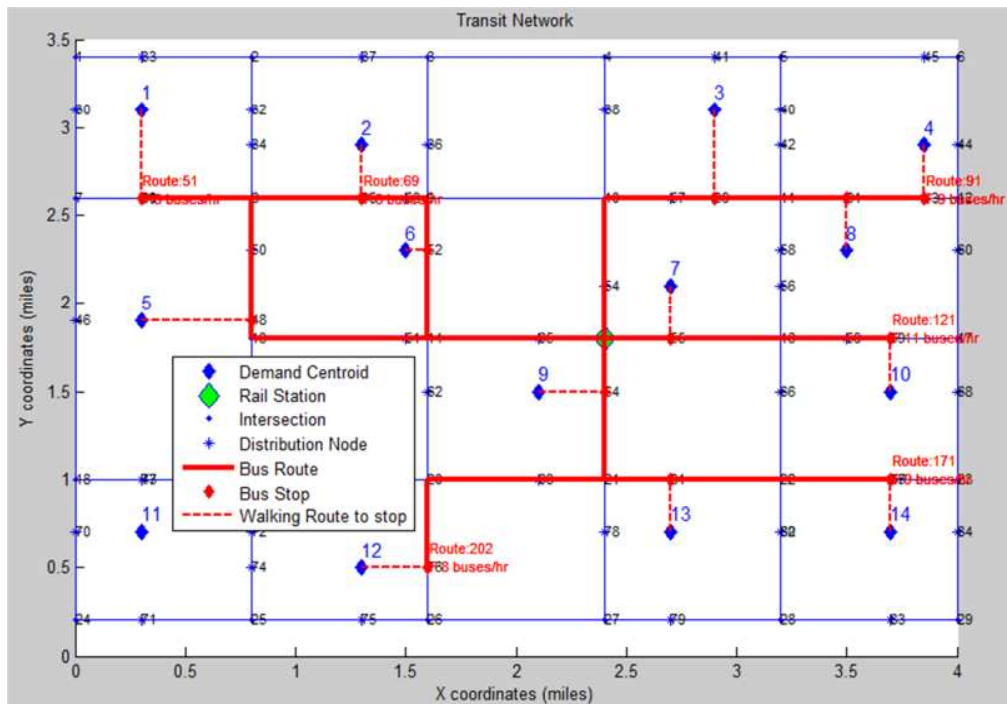
A. The feeder bus network (parking fee=$6/day, with stop delays)
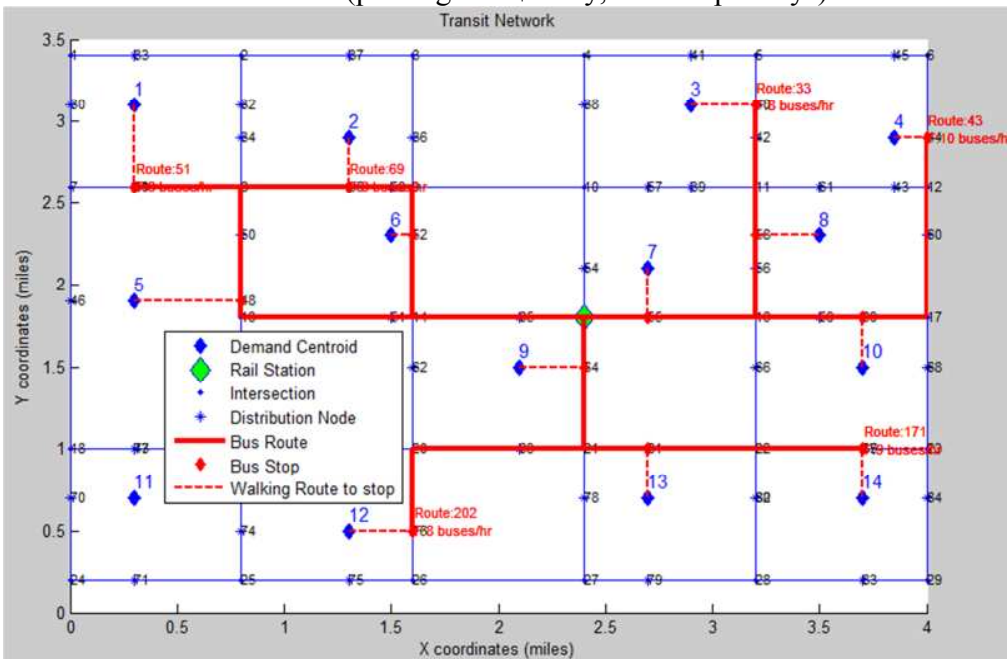
B. The feeder bus network (parking fee=$6/day, without stop delays)

Figure 5.21 The feeder bus network structure with and without stop delays.

5.8 Summary

This Chapter used the new FBNDP model on networks of different sizes. Based on the comparisons among various TS strategies, the appropriate parameters that can be used in TS were determined. Then, the validation of the proposed method as well as the evaluation of ISGP was performed. After that, the computational results for simple, sophisticated and more sophisticated networks are presented. Sensitivity analyses are followed to help reveal the relationships between different variables.

CHAPTER 6

SUMMARY AND FUTURE WORK

In this dissertation, an improved Feeder Bus Network Development Problem (FBNDP) model is presented. The solution approach consists of three main components, which are Initial Solution Generation Procedure (ISGP), Network Features Determination Procedure (NFDP), and Solution Search Procedure (SSP). The ISGP is to generate an initial feasible route set as a starting search point for SSP. The ISGP is called during the SSP process at every iteration to determine the stop locations, mode split and service frequencies of bus routes based on a given route set. Tabu Search (TS) is used as a metaheuristic method in SSP to search superior solution according to a given feasible solution. The inputs of the new FBNDP model are street network, demands from origins to the suburban rail station, and related parameters. The outputs are a feeder bus network system, and service frequencies for each route. The advantage of this proposed model over previous ones is the consideration of variable demand in much more sophisticated networks. The complexities include the consideration of various alternative modes, costs, a more effective and efficient frequency determination method, multiple feeder bus route development process, stop determination during the optimization procedure, and stop delays along the bus routes.

6.1 Conclusions

By using a TS algorithm presented in Chapter 4, the minimized total cost can be found and the related feeder bus network and service frequencies can be determined. In the numerical results discussion section, computational experiments were performed on a

simple, sophisticated and more sophisticated network to demonstrate the feasibility and efficiency of the methodology. The key findings and conclusions are listed in the following paragraphs.

In terms of the new FBNDP model, if the physical street network, O-D demand distribution and the related parameters are given, the developed methodology can be successfully used to help transit planners to develop a feeder bus system serving a rail station in a rural area. The efficiency of the TS in searching for the optimized bus network and finding the minimized cost is also confirmed. The performance of TS with fixed tabu tenure is much more stable than the one with a dynamic tenure. Moreover, as the increase of network size, to avoid the drop of efficiency, the tabu tenure needs to be increased accordingly. Compared to the traditional initial solution generation method, the ISGP is more powerful to help find a good starting point for the metaheuristic method. It is also noted that the computation time increases exponentially as the size of the network increases.

In terms of the sensitivity analyses results, several conclusions are made. Since the advantage of auto is outstanding especially when the distance between a centroid and the rail station is very long, a feeder bus network does not need to cover all the demands in a rural area. When the feeder bus demand is greater than the capacity of bus routes, the increase of fleet size can simultaneously decrease the auto, walk and total costs. For a sophisticated network with 29 intersections and demands of 6,120 persons/hour, the optimal solution can be achieved when at least 9 buses are provided.

The unit auto cost (parking fee) noticeably affects the mode shares, costs and bus network structures. As the parking fee increases, the total cost keeps increasing. One of

the methods to decrease the total cost is to reduce the unit auto cost. If unit auto costs increase, additional feeder buses can be added to meet the extra bus demands to slow down the growth of the total cost. As discussed previously, these buses are applied to the additions and extensions of bus routes. Increase of bus fare is not a good choice to decrease the total cost. Although it can considerably diminish the operating cost, it dramatically affects the bus share and finally increase the total cost. The decrease of bus share also results in the contraction of bus routes.

6.2 Contributions

The shortcomings of previous studies in the area of transit network develop problem have been pointed out in Chapter 2 and Chapter 4, such as simplifications on access modes and associated costs, simplifications on network structure, pre-determined bus stop locations, etc. The main contributions of this study can be concluded in five perspectives.

Here, all three primary access modes (walk, feeder bus and auto) are included in the feeder bus network development process. They are not only used to determine the total cost but are also contained in mode split process. Those centroids not covered by any feeder bus routes will use auto mode to access the rail station instead of simplifying as un-satisfied demand cost.

In the mode split process, a two stage MNL-PM model is proposed as a decision rule to get mode shares for all the alternatives. Compared to the preceding BLM-IPM model, the first stage in MNL-PM is a multinomial logit model (MNL) instead of binary logit model (BLM) and the stop delays are considered in the determination of mode choice. Moreover, in the second stage, the transit demand assignment depends on service frequencies rather than trip time.

In this disserttion, an enhanced service frequency determination process is developed. Compared to the conventional method, the enhanced approach can optimally select initial service frequency at each iteration based on the total cost from users and the operator. Moreover, to expedite the convergence speed and avoid the frequency oscillations in developing service frequencies, an amplitude based method is introduced.

Different from earlier studies where the bus stops are pre-determined or simplified, this dissertation embeds the bus stop determination process in the FBNDP model. The bus stops are determined according to distribution nodes and street intersections. When the inter-spacing of two candidate bus stops is less than a pre-defined value, they are combined as one stop; the location is in the middle of these two stops.

Few literatures take stop delays into account when developing a feeder bus network especially in consideration of variable demand. In this study, the stop delay is included in the FBNDP model. This factor helps the model avoid selecting the routes with abundant bus stops especially when the bus demand is huge.

6.3 Future Work

The FBNDP model discussed in this study has made many contributions to the existing feeder bus network development model. However, there is still more work that could be done in the future.

Firstly, the demand pattern in this model is many-to-one and is characterized as peak-period work trips to and from the CBD. In the future, a many-to-many demand pattern can be incorporated. In other words, the optimization can become appropriate for a whole transit network including regular bus routes, feeder bus routes or even shuttle bus

routes, etc. These different kinds of bus routes can compensate for each other's weaknesses and provide much better and efficient services for the whole suburban areas.

Secondly, more access modes or mode combinations can be included. Although walking, auto and bus account for about 75% share of access modes to a rail station, some mode combinations or methods also should not be ignored, such as "auto+feeder bus", "regular bus + feeder bus", and "carpool" . The consideration of these modes or methods can help the model be much closer to the real world.

Thirdly, for simplicity purpose, the traffic conditions at streets are not covered, which may allow the bus routes to make stops go through some street segments leading to very high traffic volume especially during the peak hour and cause additional delays. Moreover, the average bus speed used in this dissertation has included the delays occur at intersections. In future work, the intersection delays can be taken into account in the objective function.

Fourthly, it is assumed that one centroid can only choose one feeder bus stop is held. Future research can consider the move realistic situation where one centroid accesses multiple bus stops.

Lastly, a systematic solution quality evaluation method for the heuristic optimization algorithm in FBNDP needs to be developed in the future research. As known, the heuristic methods are used to seek sub-optimal solutions to optimization problems when the complexity of the problems restricts them to be solved and get exact solution. As mentioned in Chapter 2, when the network size increases and more decision variables are taken into account, the transit network development problem (TNDP) belongs to NP-hard category. To the best of author's knowledge, for such kind of TNDP

problems, there hasn't been any complete solution quality evaluation methods developed yet. Some researchers tried to find a bound for the optimal value. However, if the bound is loose, it is not easy to determine how much of any measured deviation from optimal is due to the poor performance of the heuristic and how much to the inadequacy of the bound. On the other side, computing tight bounds is also an intellectually complex and computationally intense especially for NP-hard problems. If sharp bounds can be obtained easily, there is no reason for the application of heuristic methods. (Rardin and Uzsoy, 2001)

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