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# Optimization of headway, stops, and time points considering stochastic bus arrivals

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## **ABSTRACT**

### **OPTIMIZATION OF HEADWAY, STOPS, AND TIME POINTS CONSIDERING STOCHASTIC BUS ARRIVALS**

**by  
Liuhui Zhao**

With the capability to transport a large number of passengers, public transit acts as an important role in congestion reduction and energy conservation. However, the quality of transit service, in terms of accessibility and reliability, significantly affects model choices of transit users. Unreliable service will cause extra wait time to passengers because of headway irregularity at stops, as well as extra recovery time built into schedule and additional cost to operators because of ineffective utilization of allocated resources.

This study aims to optimize service planning and improve reliability for a fixed bus route, yielding maximum operator's profit. Three models are developed to deal with different systems. Model I focuses on a feeder transit route with many-to-one demand patterns, which serves to prove the concept that headway variance has a significant influence on the operator profit and optimal stop/headway configuration. It optimizes stop spacing and headway for maximum operator's profit under the consideration of demand elasticity. With a discrete modelling approach, Model II optimizes actual stop locations and dispatching headway for a conventional transit route with many-to-many demand patterns. It is applied for maximizing operator profit and improving service reliability considering elasticity of demand with respect to travel time. In the second model, the headway variance is formulated to take into account the interrelationship of link travel time variation and demand fluctuation over space and time. Model III is developed to optimize the number and locations of time points with a headway-based vehicle controlling approach.

It integrates a simulation model and an optimization model with two objectives - minimizing average user cost and minimizing average operator cost. With the optimal result generated by Model II, the final model further enhances system performance in terms of headway regularity.

Three case studies are conducted to test the applicability of the developed models in a real world bus route, whose demand distribution is adjusted to fit the data needs for each model. It is found that ignoring the impact of headway variance in service planning optimization leads to poor decision making (i.e., not cost-effective). The results show that the optimized headway and stops effectively improve operator's profit and elevate system level of service in terms of reduced headway coefficient of variation at stops. Moreover, the developed models are flexible for both planning of a new bus route and modifying an existing bus route for better performance.

**OPTIMIZATION OF HEADWAY, STOPS, AND TIME POINTS  
CONSIDERING STOCHASTIC BUS ARRIVALS**

**by  
Liuhui Zhao**

**A Dissertation  
Submitted to the Faculty of  
New Jersey Institute of Technology  
in Partial Fulfillment of the Requirements for the Degree of  
Doctor of Philosophy in Transportation**

**John A. Reif, Jr. Department of Civil and Environmental Engineering**

**August 2016**

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**APPROVAL PAGE**

**OPTIMIZATION OF HEADWAY, STOPS, AND TIME POINTS  
CONSIDERING STOCHASTIC BUS ARRIVALS**

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This dissertation is dedicated to my beloved family:

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My Mother, Xiulan Zhao,

My Sister, Danqing Liu,

for all their love, patience, and support.

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## TABLE OF CONTENTS

Chapter	Page
1 INTRODUCTION .....	1
1.1 Problem Statement .....	3
1.2 Objectives and Work Scope .....	5
1.3 Research Approach .....	8
1.4 Dissertation Organization .....	8
2 LITERATURE REVIEW .....	10
2.1 Bus Transit Service Reliability .....	11
2.1.1 Influencing Factors .....	13
2.1.2 Improvement Strategies .....	15
2.2 Bus Route Planning .....	17
2.3 Bus Control Study .....	24
2.4 Optimization Algorithms .....	30
2.4.1 Optimization Modelling Approaches .....	32
2.4.2 Solution Algorithms .....	33
2.5 Summary .....	39
3 METHODOLOGY .....	40
3.1 Model I – The Basic Model .....	41
3.1.1 Route Configuration .....	41
3.1.2 Assumptions .....	42
3.2.3 Model Formulation .....	52

**TABLE OF CONTENTS**  
**(Continued)**

<b>Chapter</b>	<b>Page</b>
3.2 Model II – The Enhanced Model .....	50
3.2.1 Route Configuration .....	51
3.2.2 Assumptions .....	51
3.2.3 Model Formulation .....	52
3.3 Model III – The Extended Model .....	58
3.2.1 Route Configuration .....	59
3.2.2 Assumptions .....	59
3.2.3 Model Formulation .....	60
3.4 Summary .....	63
4 SOLUTION ALGORITHMS .....	66
4.1 Single-Objective GA .....	66
4.2 Multi-Objective GA .....	69
4.3 Simulation-based GA .....	72
4.4 Summary .....	74
5 CASE STUDIES .....	76
5.1 Case Study I .....	76
5.1.1 Optimization Results .....	77
5.1.2 Sensitivity Analysis .....	80
5.2 Case Study II .....	90

**TABLE OF CONTENTS**  
**(Continued)**

<b>Chapter</b>	<b>Page</b>
5.2.1 Optimization Results .....	91
5.2.2 Sensitivity Analysis .....	95
5.3 Case Study III .....	104
5.3.1 Optimization Results .....	108
5.3.2 Comparative Analysis .....	115
5.4 Summary .....	121
6 CONCLUSIONS .....	123
6.1 Findings .....	123
6.2 Future Studies .....	126
APPENDIX A .....	128
APPENDIX B .....	132
REFERENCES .....	135

## LIST OF TABLES

<b>Table</b>		<b>Page</b>
1.1	Characteristics of the Proposed Models .....	7
2.1	Selected Studies on Optimal Bus Service Planning .....	26
2.2	Selected Studies on Time Points and Slack Time Optimization .....	31
5.1	Model Parameters and Baseline Values .....	77
5.2	Optimization Results under Different Scenarios .....	80
5.3	Model Parameters of the Case Study .....	92
5.4	Results for Optimized and Existing Operations .....	94
5.5	Fixed-Route Headway Adherence and Level of Service .....	97
5.6	Simulation Inputs - Stops, Distances and Intersections .....	105

## LIST OF FIGURES

<b>Figure</b>	<b>Page</b>
1.1 Study approach .....	8
3.1 Model I – A general feeder bus route .....	42
3.2 Headway variance vs. stop .....	46
3.3 Model II – A general conventional bus route .....	51
3.4 Model III route configuration .....	59
3.5 Model descriptions and applications .....	64
4.1 GA binary encoding .....	68
4.2 GA selection .....	68
4.3 Mutation operation .....	69
4.4 Crossover operation .....	70
4.5 The Pareto front in a multi-objective solution pool .....	71
4.6 Simulation-based genetic algorithm framework .....	73
4.7 Simulation flow chart .....	75
5.1 Profit vs. stop spacing and headway under different scenarios.....	78
5.2 Average operator cost vs. average user cost under different scenarios .....	83
5.3 Profit under different scenarios .....	84
5.4 Boxplot of stop spacing and headway for scenarios 1 and 3 .....	85
5.5 Average user and operator costs vs. demand level .....	86
5.6 Boxplot for optimized headways vs. demand level .....	89
5.7 Boxplot for optimized stop spacings vs. demand level .....	89



**LIST OF FIGURES**  
(Continued)

<b>Figure</b>	<b>Page</b>
5.8 Optimized stop spacing, headway, and costs vs. $\alpha$ .....	90
5.9 User cost components vs. $\alpha$ .....	90
5.10 Cumulative outbound and inbound demand distribution .....	93
5.11 Optimized and existing stop locations vs. average load .....	96
5.12 Headway coefficient of variation under different scenarios .....	98
5.13 Boxplots of stop level $c_{vh}$ under different scenarios .....	99
5.14 Average operator and user costs vs. optimized headway .....	101
5.15 Average operator and user costs vs. travel time elasticity .....	102
5.16 Boxplot of optimized headways vs. travel time variance .....	103
5.17 Average user and operator costs, operator profit vs. $v_t$ .....	114
5.18 Stop locations along the route .....	105
5.19 Outbound hourly boarding and alighting demand .....	106
5.20 Comparison of headways and $c_{vh}$ from planning and simulation models .....	107
5.21 Simulated bus trajectories with no control .....	108
5.22 Average user and operator costs vs. time points .....	109
5.23 Optimized time points vs. outbound boarding/alighting profile .....	110
5.24 Comparison of simulated headways and planned headway .....	111
5.25 Headway coefficient of variation vs. time points .....	112
5.26 Simulated bus trajectories with control .....	113

**LIST OF FIGURES**  
**(Continued)**

<b>Figure</b>	<b>Page</b>
5.27 Average passenger wait and in-vehicle time, outbound bus travel times under controlled and uncontrolled operations .....	115
5.28 Headway coefficient of variation under different scenarios .....	116
5.29 Simulated bus trajectories under scenario 2 .....	117
5.30 Optimized time points vs. outbound boarding/alighting profile .....	118
5.31 Headway coefficient of variation vs. control points under different scenarios .....	120
5.32 Controlled bus trajectories under scenario 2 .....	121

## **CHAPTER 1**

### **INTRODUCTION**

To transport a large number of passengers within a given time period, public transit acts as an important role in congestion reduction and energy conservation. In urban areas with high population density, high market shares of public transit especially during peak periods significantly improves urban mobility. The Texas Transportation Institute's 2012 Annual Urban Mobility Report indicated that public transportation reduced travel delay by 865 million hours, equivalently a 21-billion-dollar congestion cost savings, based on the statistics of 498 urban areas in 2011. Additionally, public transportation saved more than 4 billion gallons of gasoline consumption (equivalent to 10 million dollars) and reduced 37 million metric tons of carbon dioxide emissions annually, according to American Public Transportation Association (2015). Besides all its savings, the return on investment in public transportation is high – 4 dollars in economic returns are generated for every 1 dollar invested in public transportation, and 1 billion U.S. dollars investment in transit infrastructure could create as many as 36 thousand jobs, according to American Public Transportation Association (2012). With its role in increasing mobility, reducing environmental impacts, and improving social equity status, an efficient and attractive transit system is critical for the physical structure and long-term socioeconomic development of a city and its surrounding area.

Despite reduced ridership and declining service quality in public transit, there is a growing realization that more attention should be given to efficient transit systems. Aging

population, rising fuel prices, increasing traffic congestion – the problems associated with continuous urbanization and the increasing sizes of cities – justify the need for more reliance on transit systems (Litman, 2014). Therefore, research has been conducted to investigate the determinant factors of transit ridership. Many factors were found contributing to bus ridership decline, including internal factors (e.g., service quantity, pricing, and service quality factors) and external factors (e.g., socio-economic, spatial, and transit subsidy factors) (Taylor and Fink, 2003).

Among the internal factors, service reliability, which has enormous impact on passengers and operators, was found more influential to transit ridership than service frequency and price (Cervero, 1990; Abdel-Aty and Jovanis, 1995; Syed and Khan, 2000; Krizek and El-Geneidy, 2007; Daraio et al., 2016). Unreliable service has great negative impacts on both passengers and operators. For passengers, extra time needs to be added to their trip planning to account for possible delays and ensure on-time arrival due to travel time variation (Furth and Muller, 2006). For operators, a certain amount of recovery time built into the schedules is necessary to absorb the variation of vehicle travel times, resulting in longer round-trip travel time and increased fleet size requirement.

However, conventional surface transit systems (e.g., buses), sharing the right-of-way with other vehicles, are inevitably suffering from service irregularity. The bus arrival/departure time deviating from a posted schedule is sometimes unavoidable because of various factors, such as temporal and spatial boarding/alighting demand fluctuation, traffic conditions, and irregular departure headways at the terminals/upstream stops. Especially under congested traffic conditions, it is difficult for buses to return to the driving lane after picking-up/dropping-off passengers at stops, leading to longer dwell time.

The vehicle travel time variation dominated by traffic congestion levels often leads to transit service uncertainty, and growing congestion further raises a burden to both transit agencies and users. Although extensive research attention has been given to vehicle control strategies for improving service reliability performance (e.g., Barnett, 1974; Wirasinghe, 1993; O'Dell and Wilson, 1999; van Oort et al., 2010; Cats et al., 2011; Delgado et al., 2012; van Oort et al., 2012), the fact that a majority of transit networks were planned without consideration of stochasticity limits the efficiency of these countermeasures.

Recent studies pointed out that well-located stops have the potential to alleviate the impact of traffic congestion (El-Geneidy et al., 2006; Delmelle et al., 2012; Ibarra-Rojas et al., 2015). However, thorough investigation of the influence of service planning on system performance is needed, especially under the situation where passengers are sensitive to service accessibility and reliability. Considering the potential of a cost-efficient bus system in maintaining service reliability and attracting patronage, it is critical to design a bus route under congestion condition in order to achieve a high level of service.

## **1.1 Problem Statement**

Due to inherent stochastic nature, buses tend to travel in pairs in spite of evenly scheduled headways. Even starting from a small upstream disturbance, headway deviation could be magnified due to stochastic link travel times and passenger boarding/alighting activities at downstream stops. Although it is recognized that temporal demand fluctuation, roadway geometry, and traffic congestion affect service reliability (Woodhull, 1987; Strathman and Hopper, 1993; Chien et al., 2007; Chen et al., 2009; Lin and Ruan, 2009; Islam and

Vandebona, 2010; El-Geneidy et al., 2011), the investigation of the impact of stochastic bus arrivals on the optimal service planning for a given bus route has not been carried out.

Previous studies on developing strategies to improve system reliability were often on the operational level via adjusting operations to promote schedule adherence, whereas the research on the planning level has been rarely conducted (Guihaire and Hao, 2008; Kepaptsoglou and Karlaftis, 2009; van Oort et al., 2012; Ibarra-Rojas et al., 2015). In fact, optimal stops, headway and time points (i.e., control points) could offset small disturbances and mitigate headway variations, without imposing additional financial burdens.

To optimize service planning, the trade-off between service accessibility and efficiency always needs to be considered. In general, shorter stop spacing and headway provide greater level of accessibility, whereas larger stop spacing and longer headway lower the operating cost. Under the circumstances of stochastic vehicle arrivals, the research problem becomes even more complicated, since the interactions of the decision variables (i.e., stops and headway), traffic conditions, and passenger boarding/alighting activities also need to be considered for a proper planning.

Hence, for optimal service planning for a given bus route under stochastic vehicle arrivals, a sound model that can handle the interrelationship between multiple decision variables and model parameters is necessary. Since traditional exact algorithms are not capable of solving such a complicated problem, heuristic/metaheuristic algorithms should be applied to search for the optimal solutions.

Due to consideration of the interactions of decision variables and travel time variability, as well as model applicability in a real world bus route, traditional mathematic

algorithms are not capable of solving such problems. Thus, heuristic/Metaheuristic algorithms could be adapted to the models.

## **1.2 Objectives and Work Scope**

The objective of this study is to develop optimization models for the planning of a fixed bus route considering the impact of stochastic bus arrivals, which could improve service reliability with maximized operator's profit considering demand elasticity. Considering the impact of headway variation, the proposed models will determine the optimal number and locations of bus stops, headway, and time points. The discrete approach for stops and time points and the continuous variable of headway under consideration of travel time elasticity of demand increase the complexity of the problem. Therefore, metaheuristic algorithms need to be applied for problem solving.

The contributions of this study compared to previous studies are as follows:

- 1) incorporating the headway variance in the optimization model reflecting more realistic bus operation conditions,
- 2) providing the guidelines for profit maximum service planning considering travel time variation under different scenarios,
- 3) analyzing the trade-off between users (i.e., passengers) and the operator with multi-objective optimization models, which offers a broader view of the service planning problem,
- 4) optimizing time point locations with a headway-based control strategy and developing a dedicated simulation-based optimization algorithm, which can be easily implemented in the advance of ITS technology,
- 5) integrating strategic and tactic of strategies for tackling the service variability issue, which provides a basis for greater reliability improvement at the operational level.

It is expected that the proposed models are capable of maximizing operator profit and achieving high level of reliability with optimized stop, headway, and time points. Also, the proposed models should outperform the traditional models neglecting service variability, in terms of system efficiency and cost effectiveness. Therefore, to prove the concept that consideration of stochastic vehicle arrivals will impose great influence on bus service planning, a basic model with simplified network is developed. Two advanced models are proposed based on the first model to conduct further analysis. In particular, the first model (Model I – the basic model) deals with a many-to-one/one-to-many uniform distributed demand pattern along a feeder bus route. To incorporate the concept of stochastic vehicle arrivals, the headway variance at stops is integrated in the model. Without detailed analysis of the determinant factors, the functional form of headway variance is assumed dependent on stop sequence based on the results from previous simulation studies. With continuum modelling approach, the analysis is conducted to show the comparison between Model I and previous models.

To enhance Model I and deal with a general transit route with many-to-many heterogeneous demand attributes, the second model (Model II) is proposed, in which the influencing factors of headway variance (i.e., both at-stop and en-route variation factors) are analyzed and formulated. Although the continuum approach in Model I could efficiently explore the relationship between decision variables and the objective function, converting stop spacing to actual locations may be a problem, especially under heterogeneous demand, traffic and geometric condition. Therefore, instead of finding the optimal stop spacing of a route, Model II considers feasible stop locations as decision variables and determines the optimal set of stops and headway to maximize operator profit.



Finally, Model III (the extended model) optimizes time points with a headway-based control strategy, and investigates the impact of time points on the service reliability. It is acknowledged that time points could prevent small variation from propagating to greater variation; however, where and how many time points should be selected for a bus route remains a problem. The discrete variables for time points exponentially enlarge the body of feasible solutions, which increases the complexity of the problem. With the headway-based control strategy, Model III is solved with a simulation-based metaheuristic solution algorithm. For evaluating the potential change in system performance with proposed models, the comparison between the new model and other traditional models is also presented in this dissertation.

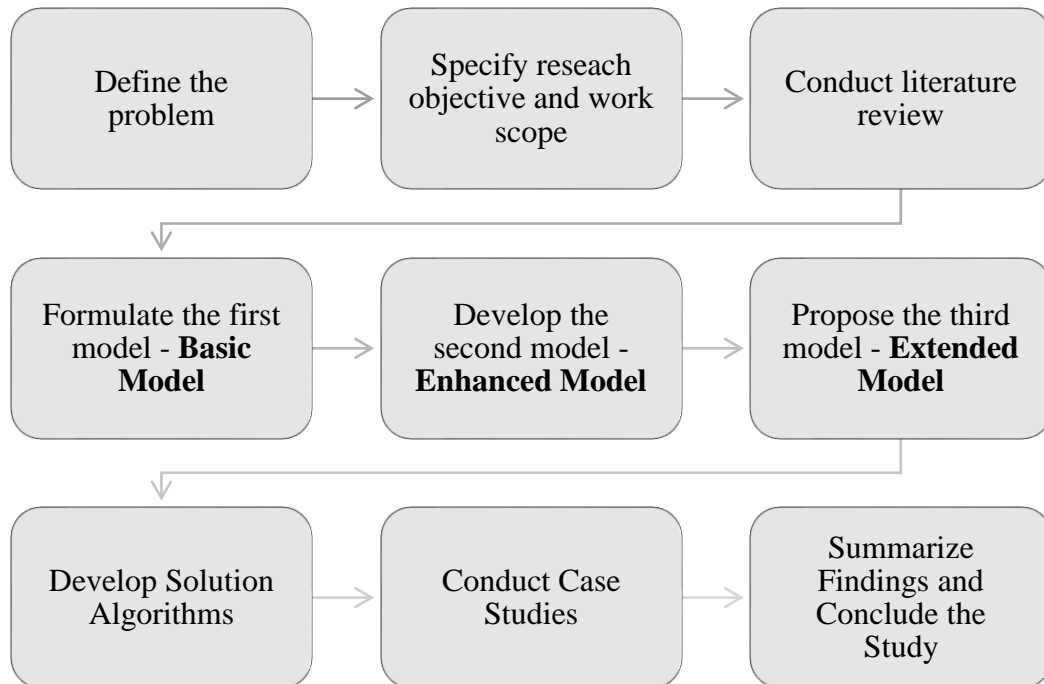
The differences among the three models are illustrated in Table 1.1, where the planning parameters are listed in the left panel of the table, and the model capability of handling these parameters is marked in the right panel of the table.

**Table 1.1** Characteristics of the Proposed Models

PARAMETERS		MODEL		
		I	II	III
Demand Pattern	Uniform (Many-to-one)	√	√	√
	Heterogeneous (Many-to-many)		√	√
Decision Variables	Headway	√	√	
	Stop Spacing	√		
	Stop Locations		√	
	Time Points			√
Stochastic Factors	At-stop Factors	√	√	√
	En-route Factors		√	√

### 1.3 Research Approach

After defining the research objective and work scope, a comprehensive literature review on related topics is conducted for the dissertation. Three optimization models with different emphases are formulated. Due to the characteristics of model formulation approaches and the complexity of the problems, dedicated solution algorithms are developed for solving the developed models. With the developed models and solution algorithms, three case studies are conducted to test the model capability and applicability. Finally, all findings are summarized with a discussion of future research following the dissertation. The study approach is illustrated in Figure 1.1.



**Figure 1.1** Study approach.

### 1.4 Dissertation Organization

This dissertation is organized into six chapters. Chapter 1 introduces the research background, identifies the research problem, as well as defines the research objective and

work scope. The proposed study approach is also represented in Chapter 1. Chapter 2 summarizes a comprehensive review of related studies and solution algorithms applied to solve the developed models. Chapter 3 presents three models developed for a single bus route under consideration of travel time variability, each of which has its own emphasis and serves specific planning purposes. Chapter 4 describes the solution algorithms applied to solve the models developed in Chapter 3. Chapter 5 presents the case studies in Chengdu, China with the model discussed in Chapter 3 and solution algorithms presented in Chapter 4. The optimal results are compared with those generated from the traditional models without considering travel time variability, and the influence of model parameters on the objective values are investigated with sensitivity analyses. Finally, Chapter 6 summarizes the findings from the case studies and proposes future research directions.

## CHAPTER 2

### LITERATURE REVIEW

As discussed earlier in the introduction, ridership is influenced by both external (e.g. social economic characteristics) and internal (e.g., service quality) factors of a bus transit system. Under the circumstances where a fixed-route bus transit is given, the population and social economic profiles in the service area will not change drastically within a given time period. Therefore, considering the decision-making process of bus users, the quality of service, including service availability (e.g., frequency, service coverage, and access) and comfort /convenience (e.g., passenger load, reliability, and travel time), is the major influencing internal attribute, which reflects the passenger's perception of service performance (Kittelsohn & Associates et al., 2013).

Regarding passengers' attitudes towards the service quality of transit systems, For Transit Cooperative Research Program (TCRP) project B-11, dedicated surveying techniques were developed and pilot studies were conducted at three transit agencies (Morpace International, Inc., 1999). Among nine categories (i.e., comfort, nuisances, scheduling, fares, cleanliness, in-person information, passive information, safety, and transfers) and 46 attributes identified and surveyed for the transit systems, it was found that the attributes relating to scheduling were among the top area of both existing and potential concerns. For another project of National Cooperative Highway Research Program (NCHRP), a survey was conducted among customers of five different transit agencies around US about the satisfaction of these transit systems (Dowling, 2008). It was identified

that passengers consistently considered frequency the most important factor, with reliability and waiting time (which relates to frequency and reliability) consistently stated as the major contributors to passengers' satisfaction.

Besides the analysis of passengers' reception of transit service quality through descriptive surveys, other studies also quantified the value of passengers' transit travel time. TCRP Report 95 showed that the value of walking/initial waiting time (waiting time under regular headways) was about double the value of in-vehicle time (Evans and Pratt, 2004). Moreover, the unreliable transit service increased the average waiting time (i.e., additional waiting time because of stochastic bus arrivals), which could be converted to a monetary valuation of service variability. It was found that such value of excess waiting time under service variability was typically 2 to 3 times higher than normal value of waiting time (Bly, 1976). Another study in Auckland (Vincent, 2008) also found the value of excess waiting time was 3 to 5 times in-vehicle time. The TCRP Report 165 also indicated that ridership elasticity as respect to travel time is second to the highest: just lower than facility expanding and improvement (Kittelson & Associates, Inc., 2013). These previous studies indicated that unreliable service leads to increased waiting and in-vehicle time, which significantly reduces system attractiveness to passengers. Therefore, the system reliability should be considered to retain current patron and further stimulate ridership.

## **2.1 Bus Transit Service Reliability**

Service reliability has been referred as one of key indicators of transit system performance (Evans and Pratt, 2004; Dowling, 2008; Kittelson & Associates, Inc., 2013). Several stochastic factors contribute to the uncertainty of transit services, including dispatching

time from the terminal, en-route travel time, and dwell time at stops, all of which are correlated with each other: a late bus will pick up more passengers at a certain stop, leading to much longer headway with its preceding bus, whereas a following bus may have less passengers to pick up and catch up with the late bus easily, causing bus bunching.

Stochastic traffic conditions and spatially/temporally fluctuated demand cause variations in vehicle travel times, which lead to increased waiting times and delays for the passengers as well as inefficient vehicle and personnel utilization for the operators. Under such stochastic conditions, additional buffer time needs to be planned in average passenger travel time to ensure on-time arrivals. Such buffer time is considered as an important portion of passenger travel cost, which is highly sensitive to service reliability (Turnquist and Bowman, 1980; Furth and Muller, 2006) and will ultimately affect mode choice decisions. Moreover, passengers boarding at a downstream stop, in general, would experience longer wait time and planned travel time than those boarding at upstream stops, since minor upstream variations may easily propagate to downstream locations, especially under congestion conditions.

On the other hand, from the perspective of the operators, unreliable service means more recovery time built into schedules and more resources needed to satisfy the demand. As improved reliability helps the operator optimize resource usage and maximize production, considering the reliability in the design phase is critical to ensuring a successful service planning.

### **2.1.1 Influencing Factors**

A number of endogenous and exogenous factors (Woodhull, 1987) cause unreliable bus services. Endogenous factors include passenger boarding profiles, route configuration, stop spacing, and driver behavior. Exogenous factors mainly include traffic congestion and accidents, traffic signalization, on-street parking, and weather conditions. Considering bus running and dwelling along the journey, these factors can be categorized into two groups: the factors related to roadway geometry and traffic condition along the route – en-route factors, and the factors related to boarding profile – at-stop factors (Levinson, 1983; Strathman et al., 2000; Bertini & El-Geneidy, 2004; Lin and Bertini, 2004; Dueker et al., 2004).

To model the variation along the route, Adebisi (1986) formulated headway variance in terms of boarding demand and travel time variation caused by traffic conditions. The model was effective to describe the service disturbance along the route and yet simplified by neglecting the detailed roadway geometry. As indicated in the study, the travel time and its variance on a link between two adjacent stops are influenced by the traffic conditions as well as the frequency of delay-producing elements, such as intersections and narrow bridges. Later, Adamski (1991) analyzed dwell time variability at bus stops due to different passenger handling types. Stochastic boarding and alighting times were assumed, and different types of distributions were tested to represent the parallel and series passenger handling processes at stops.

Investigating service reliability at urban bus stops, Chien et al. (2000) found that headway variance increased when the stop location was further away from the beginning of a route. Lin and Ruan (2009) proposed a probability-based headway regularity measure

to investigate the factors influencing service performance. In their study, service reliability was defined as the probability that buses arrive at a stop within a tolerable interval, which was a function of bus dwell time, stop sequence, maximum anticipated headway, and numbers of boarding/alighting passengers.

Besides the above mathematical formulations proposed by various studies (i.e., Adebisi, 1986; Adamski, 1991; Chien et al., 2000; Lin and Ruan, 2009), the widespread implementation of Automatic Vehicle Location systems (AVL) and Automatic Passenger Counters (APC) in the transit industry has enhanced the ability of system monitoring and reliability analysis. Several studies have employed collected data from AVL/APC to evaluate different aspects of system performance and to investigate the causes for service variability (Strathman et al., 1999, 2000, 2002; Furth et al., 2003; Hammerle et al., 2005; Furth, 2006; Mazloumi et al., 2008). Based on these studies, distance between time points, route length, number of stops, and boarding/alighting profiles were found significantly related to service reliability.

Although the studies revealed that many factors could affect service reliability (Woodhull, 1987; Strathman and Hopper, 1993; Chien et al., 2007; Chen et al., 2009; Lin and Ruan, 2009; Islam and Vandebona, 2010; El-Geneidy et al., 2011), the relationship between unstable services and stop/headway optimization have not been thoroughly investigated in the previous research (Wirasinghe and Ghoneim, 1981; Kuah and Perl, 1988; Furth and Rahbee, 2000; Chien and Qin, 2004).



### **2.1.2 Improvement Strategies**

Considering the en-route and at-stop factors of bus service disturbance, there are two major categories of countermeasures to improve bus performance (Adebisi, 1986). When the en-route factors predominate the cause of variability, the redesign of bus routes, such as reducing route length, modifying bus stops or introducing bus transit priority scheme, could improve service reliability. If the passenger loading factors are the major reason for the variability, bus control strategies, such as introducing holding strategies and bus monitoring schemes, are effective for better system performance.

Similarly, as discussed by van Oort and van Nes (2008), service reliability can be improved strategically (e.g., via network design), tactically (e.g., via timetable planning), and operationally (e.g., via vehicle controlling). Although the most popular approach to elevating schedule/headway adherence is at the operational level, greater reliability can be achieved at the tactical and strategic levels.

To fill in the research gap, this study optimizes the service planning variables including bus stops, dispatching headway, and time points for a given bus route for better service reliability, where the en-route and at-stop factors are considered for modelling headway variance. Therefore, the following section briefly introduces two categories of countermeasures, with detailed related studies reviewed in Sections 2.2 and 2.3.

#### ***Bus Route Planning***

Turnquist (1981) analyzed different scenarios with the combinations of headways, travel time variations, and route densities. The simulation results revealed that two interactions, namely frequency-demand and demand-travel time variations, took vital parts in the

network reliability. van Oort and van Nes (2008) investigated the impacts of timetable planning and network design on service reliability, and identified driving ahead of schedule as a source of increased waiting times. With the field data collected in the Netherlands, their study showed that the route length, line coordination, and stop spacing contributed to the deviation of travel times. The authors suggested that possible route design strategies to improving service reliability included splitting route into two separate routes, enhancing route coordination, or determining stop spacings under the consideration of dwell time variation caused by demand fluctuation.

Later, with a newly designed transit route, van Oort and van Nes (2009b) analyzed the impacts of infrastructure improvements and vehicle control strategies on the service performance in the route with enhanced right of way, improved vehicle and station design, real-time information, and well-planned timetables. Significant improvements on quality of service were observed after the introducing of new route, including reduced dwell time variation, improved schedule adherence, and shorter passenger waiting time.

Recently, El-Geneidy et al. (2011) assessed the quality of service in a bus route in Minneapolis, Minnesota, and identified that many bus stops were underutilized, while stop consolidation could possibly lead to substantial improvement of performance (El-Geneidy et al., 2006). Through analyzing the empirical data collected from modified bus routes, their studies confirmed that stop redundancy and inefficient resource allocation were common issues in existing bus systems, and that proper changes in the route configuration could lead to service performance improvement in terms of reliability.

### ***Bus Operational Control***

A number of strategies aiming at controlling headway variability have been proposed and evaluated. Among vehicle control approaches to improving service reliability, holding strategies are widely applied, which reduce service disturbance by regulating departure time from stops according to predefined criteria.

Holding strategies can be classified into two categories: schedule-based and headway-based. Schedule-based strategies define bus departure time based on the scheduled departure time, while a headway-based strategy regulates the departure time based on the headways between consecutive buses. Since both passenger boarding profile and traffic condition factors affect service variability, the interactions between passenger activity, transit operations, and traffic dynamics need to be modeled for impact analysis of holding strategies on bus performance (Cats et al., 2011).

## **2.2 Bus Route Planning**

The evaluation of transit network is always related to the vehicle requirements on each route, such that the problem of network design and frequency setting are mostly addressed at the same time (Ceder and Wilson, 1986). In terms of service planning for a given bus route, stop spacing and headway were usually jointly optimized in previous studies. The following sections briefly describe other major components in the planning process, including objectives, network settings, and demand patterns.

### ***Objectives***

Different objectives could be set in bus service planning process. As a result, significant differences in the attractiveness and performance of an optimized network can be observed depending on the objectives (van Nes and Bovy, 2000). Considering different stakeholders in the process of bus route planning, major objectives could be grouped into three categories. From the passengers' point of view, a good bus route is featured with high accessibility/low in-vehicle time and commonly used objectives favoring passengers include maximizing passenger surplus or minimizing total user cost. From the operator's point of view, however, a good bus route should be profitable or featured by low operating cost/high ridership and level of service, with the objectives such as maximizing operator profit and minimizing operator cost in favor of the transit operators. Considering the passengers and the transit operators in the entire system, objectives such as maximizing social welfare or minimizing total system cost are mostly commonly investigated.

With different perspectives, previous studies optimized bus route planning either focusing on single objective (e.g., maximizing profit, minimizing total passenger travel time) or balance the benefits of passengers and operators with the objective of maximizing social welfare or minimizing system cost.

### ***Network Settings***

Early studies optimized stop spacing and headway with simplified topographic structures, and analyzed relationships among decision variables, model parameters and objective functions. Recent research focused more on model applicability to a real world system, considering realistic conditions and practical constraints.

### ***Demand Patterns***

The demand patterns for a bus route could be generally categorized into many-to-one and many-to-many patterns. A many-to-one demand pattern involves multiple origins and one destination, which is likely the demand of a feeder bus route connecting a residential area and a CBD (or a major terminal). A many-to-many demand pattern represents travel flows from multiple origins to multiple destinations. Considering demand sensitivity to service quality and quantity, fixed demand (i.e., demand is assumed to be stable) and elastic demand (i.e., demand is sensitive to fare and/or quality of service) are usually studied in bus route planning.

### ***Previous Studies***

Early research often focused on simplified network to investigate the relationship between decision variables, model parameters and objective values in the bus route planning. Most of the earlier studies applied analytic approaches for optimizing simplified bus networks, where demand was assumed equally distributed over study area and bus lines/stops were aligned with equal length/spacing. Usually, fixed many-to-one demand pattern without spatial or temporal changes was analyzed. Among them, Vuchic and Newell (1968) developed an analytic model to optimize the stop spacing for a rapid transit system, which minimized total travel time. Mohring (1972) developed an analytic model to optimize service frequency for a given bus route based on the minimization of total system cost (i.e., sum of user waiting cost and operator cost). With the assumption of fixed demand, it was found that service frequency provided on a route should be proportional to the square root of demand density (i.e., ridership per unit distance or time).

Later, Kuah and Perl (1988) presented a mathematic model for jointly optimizing route spacing, headway and stop spacing for a feeder bus system in a rectangular network, where both constant and variable stop spacings along the routes were analyzed. Ceder et al. (1983) proposed a mathematical model to find smallest number and location of bus stops so that no passenger was further away than the maximum allowable walking distance. In their study, the network was represented by arc and node, with nodes representing community locations and stops to be located along the arcs or on nodes. Ghoneim and Wirasinghe (1987) developed a mathematical model in order to determine the optimum zone configuration for a commuter rail line for minimizing total system cost, in which many-to-one/one-to-many demand pattern was considered. By simplifying the demand pattern, the investigations could be emphasized on the other model parameters to be studied. However, such fixed and many-to-one/one-to-many demand assumption has its limits and does not fit in a network with many-to-many or elastic demand patterns.

Some studies optimized bus route planning taking into account temporal demand variations and demand elasticity. For instance, Furth and Wilson (1981) optimized headways over time and route for maximizing net social benefit (i.e., sum of operator's benefit and user wait time savings), considering demand elasticity with respect to wait time. Considering time-dependency and fare elasticity of demand, Chang and Schonfeld (1991) optimized route spacing, headways and fares for a feeder bus system (i.e., many-to-one or one-to-many demand pattern). Later, Spasovic et al. (1994) optimized route length and fare considering travel time and fare elasticity of demand for a feeder bus system. Although temporal variation was taken into consideration, these studies still dealt with many-to-one demand patterns that were only suitable for feeder bus systems.

Recognizing that many-to-one/one-to-many demand patterns were not applicable for service areas with heterogeneous demand distributions, some other studies have incorporated spatial variation of demand into the bus route planning models. Wirasinghe and Ghoneim (1981) optimized stop spacing for a single bus route considering many-to-many demand pattern, where the objective function was to minimize total system cost in a simplified local street network. In order to incorporate spatial characteristics, Chien and Schonfeld (1997) investigated a grid bus transit system in a heterogeneous urban environment. Their study assumed varying demand distribution over the irregular service area, and optimized the route, stop locations and operating headways for total cost minimization. Later, Chien and Spasovic (2002) introduced fare elasticity of demand into model development for bus route planning. Considering many-to-many demand patterns, zonal demand variation and route costs, and vehicle capacity constraints, route and stop locations, headways, and fare were optimized which maximized operator profit and social welfare.

The majority of the above studies typically assumed that bus stops could be allocated anywhere along the routes, and therefore treated stop spacing as a continuous variable. This continuum modelling approach yields optimal stop spacing that could be converted to actual stop locations later. For instance, Li and Bertini (2009) optimized the bus stop spacing with archived stop-level demand data, where travel demand was considered uniformly distributed over the bus route. The authors converted the optimal stop spacing into stop locations according to the actual street grid. Although the continuum approach could effectively demonstrate the sensitivity of optimal stop spacing to various route design parameters (e.g., demand distribution, vehicle capacity), it does have its

disadvantages. One of the major shortcomings is the difficulty to apply the stop spacing to a realistic street network, within which stops are usually located at intersections and restricted to geographical conditions. Another concern is that the continuum approach was often applied with the assumption of smooth and continuous distributed demand, which were not able to represent heterogeneous demand distributions.

Therefore, considering passenger boarding/alighting entry points, Chien and Qin (2004) optimized number and locations of bus stops for improving transit accessibility. In their study, the demand was assumed concentrated at several entry points on a segment of bus route. Chien et al. (2003) determined the locations of bus route and stops considering realistically geographic variations and heterogeneous demand distributions, where the irregular shaped area was cut into small rectangular zones and the corners of each zone were treated as candidate stop locations. Furth and Rahbee (2000) applied the discrete approach to optimize bus stop spacings for a given bus route. The intersections along the bus route were treated as candidate stops and a simple geographic model was applied to distribute collected demand data to the route service area. Later with a parcel-level geographic database, Furth et al. (2007) investigated the impact of stops to access distance, riding time, and operating cost considering various sets of stop locations, where the demand was estimated based on land use type and development intensity. Recently, DiJoseph and Chien (2013) optimized the number and locations of bus stops, headway and fare for a feeder bus route to maximize total operator's profit considering realistic networks, where the demand elasticity with respect to fare and service quality were considered.

The aforementioned studies, however, did not consider the variance of bus travel times, and thus the impacts of such variation on the design of stop spacing and headways



were unable to assess (Orloff and Ma, 1975; Chang and Hsu, 2001). They typically assumed that buses travel at a constant speed along the route, which is not influenced by the number of bus stops and traffic conditions. However, as the study by Levinson (1983) suggested, bus travel times and speed were derived as a function of stop frequency, stop duration, as well as bus dwell times. The survey conducted in his study showed that reducing the number of bus stops and dwell times lead to greater travel time saving than that achieved by eliminating traffic congestion. Based on these findings, Saka (2001) addressed the significance of bus stop spacing as an operational parameter under interrupted traffic conditions, where the delay time caused by traffic signals was captured. The sensitivity analysis with different stop spacing scenarios demonstrated that proper stop spacing could significantly improve the service quality, decrease travel time, and reduce the fleet size.

Not until recently, few studies have focused on the interaction of congestion and bus stop spacing. Ibeas et al. (2010) and Moura et al. (2012) optimized bus stop locations with a bi-level model where a mode choice model for passengers was applied. Although their work took into account the possible variations in demand due to different bus stop locations in the network, headway variation was not studied. Tirachini and Hensher (2011) investigated the impact of bus bunching on the optimal design of bus stop spacing, fare collection system, bus operating speed and headway, with emphasis on bus congestions at high demand bus stops. Without considering heterogeneous demand distribution along the route, two geographic areas were applied based on the level of demand. Within each of the areas, demand was assumed uniformly distributed. The objective of the optimization model was to minimize total cost without considering demand elasticity. With same network

settings, Tirachini (2014) optimized stop spacing for minimum total cost operation of an urban route considering the impacts of bus stop size on bus congestion at stops. Their study suggested that with the consideration of bus bunching, if the operating speed could be optimized, there was a range in which the number of stops decreased with demand to reduce the acceleration and deceleration delay, as opposed to what suggested by the traditional models.

Table 2.1 summarizes major studies conducted on the optimization of bus service planning.

### **2.3 Bus Control Study**

The procedure of timetable design mainly includes three steps: first is to decide the proper headway to satisfy route demand and level of service, which is usually optimized together with stop locations. Then, the number and locations of time points are determined where operational control strategies are to be applied. Finally, bus departure times for each time point need to be scheduled based on various control strategies (Liu and Wirasinghe, 1995a).

The major questions in the timetable design are to determine the number and locations of time points, as well as the amount of slack time allocated to each time point, since these factors have significant effects on the system cost as well as service reliability. Excessive time points reduce bus operating speed and increase the waiting time of through passengers, while inadequate time points are not able to mitigate the variation of travel time. Similarly, upstream time points may not be effective to prevent disturbance propagating to the rest of the route, while only a few passengers will benefit from downstream time points although most of the large headway deviations occur there.

Therefore, it is critical to select proper objective functions in the planning of time points to incorporate the aforementioned trade-offs and improve system performance.

There are mainly two types of holding strategies: schedule-based and headway-based strategies. The schedule-based strategies regulate bus departure time relative to the posted schedule, whereas the basis of the headway-based strategies is headways between consecutive vehicles. Since it is difficult to mathematically optimize time points and associated slack time simultaneously with schedule-based strategies, most of the previous studies focused on the determination of slack times associated with predefined time points (e.g., Liu and Wirasinghe, 2001; Mazloumi et al., 2012). With real-time information often required in a headway-based control strategy, the majority of previous studies on headway-based strategies applied simulation models to study the impact of control strategies without optimization of number and locations of time points.

### ***Schedule-based Strategies***

Focusing on schedule-based strategies, an early study conducted by Newell (1977) proposed a lower boundary of the slack time for a many-to-one/one-to-many bus route based on a set of simplified assumptions. It was found that the slack time was related to the standard deviation of bus travel time between time points, average passenger arrival/boarding rates, and the dispatching headway.

**Table 2.1** Selected Studies on Optimal Bus Service Planning

<b>Year</b>	<b>Author</b>	<b>Objectives</b>	<b>Decision Variables</b>	<b>Demand</b>	<b>Solution Algorithm</b>
<b>1968</b>	Vuchic & Newell	User Cost Minimization	Stop Spacing	Many-to-One	Dynamic Programming
<b>1981</b>	Wirasinghe & Ghoneim	Total Cost Minimization	Stop Spacing	Many-to-Many	Traditional Derivative
<b>1986</b>	Ceder & Wilson	Total Cost Minimization	Route, Headway	Many-to-Many	Heuristic
<b>1988</b>	Kuah & Perl	Total Cost Minimization	Route, Stop Spacing, Headway	Many-to-One	Traditional Derivative
<b>1997</b>	Chien & Schonfeld	Total Cost Minimization	Route, Headway	Many-to-Many	Traditional Derivative
<b>2000</b>	van Nes & Bovy	Multiple Objectives	Route and Stop Spacing	Many-to-One	Traditional Derivative
<b>2000</b>	Furth & Rahbee	Total Cost Minimization	Stop Spacing	Many-to-Many	Dynamic Programming
<b>2001</b>	Saka	Total Travel Time Minimization	Stop Spacing	Many-to-Many	Traditional Derivative
<b>2003</b>	Chien et al.	Total Cost Minimization	Route, Headway	Many-to-One	Exhaustive Search
<b>2004</b>	Chien & Qin	Total Cost Minimization	Stop Spacing	Many-to-Many	Exhaustive Search
<b>2006</b>	Dell'Olio et al.	Total Cost Minimization	Stop Spacing, headway	User Equilibrium	Heuristic
<b>2009</b>	Li & Bertini	User Cost Minimization	Stop Spacing	Many-to-Many	Traditional Derivative
<b>2010</b>	Ibeas et al.	Total Cost Minimization	Stop Spacing	Many-to-Many	Pattern Search
<b>2011</b>	Tirachini & Hensher	Total Cost Minimization	Five Decision Variables Including Stop Spacing	Many-to-Many	Heuristic
<b>2012</b>	Moura et al.	Total Cost Minimization	Stop Spacing	Many-to-Many	Heuristic
<b>2013</b>	DiJoseph & Chien	Total Profit Maximization	Stop Spacing, Headway, Fare	Many-to-One	Heuristic

Later, considerable research has been conducted on optimizing slack times at predefined time points for reliability improvement and total cost minimization (Barnett, 1974; Wirasinghe, 1993; O'Dell and Wilson, 1999; Eberlein et al., 2001; Hickman, 2001; Zhao et al., 2006; Furth, 1995; Furth and Muller, 2007, 2009; van Oort et al., 2010; Cats et al., 2011; Delgado et al., 2012; van Oort et al., 2012). Most of these studies investigated the impacts of holding strategies on bus on-time performance with predefined time points, which were mainly determined by several practical rules, such as terminals, stops with high boarding demand and low through passengers, etc. Although some studies stated that the best location for a time point was dependent on the demand distribution and network configuration, the optimal number and locations of time points were not theoretically determined.

Lesley (1975) related the locations of time points to headway coefficient of variation, and suggested that time points should be placed at bus stops with coefficient of variation greater than twice route-level average coefficient of variation. Abkowitz and Engelstein (1984) suggested that time point locations should be determined based on the standard deviation of bus travel times to a stop and the ratio of boarding passengers to through passengers. Later, Abkowitz et al. (1986) developed an algorithm to optimize the threshold headway for high frequency bus control. Their study revealed that the optimal locations were sensitive to the passenger boarding profile along the route.

Furth and Muller (2007) analyzed the cost impacts of number of time points and showed that increasing the time points led to higher benefit for the bus system with diminishing returns. However, the locations of time points were not optimized in their study. Considering the trade-off among different cost components in bus operation by

introducing time points, Wirasinghe and Liu (1995b) developed an analytical model to determine the number and locations of time points as well as the amount of slack times for a bus route with a single bus run considered. A number of assumptions were applied, including long bus headway, no missing passengers, and independent successive bus runs.

Later, Liu and Wirasinghe (2001) applied an optimization model in a simulation model to optimize time points and slack times simultaneously. The alternative schedule designs were selected based on a set of practical rules and evaluated against the total cost associated with the schedule. The exhaustive enumeration method applied in this study consumed a massive time to generate a good solution considering a large feasible solution pool. Meanwhile, the rules applied to reduce the number of feasible solutions may lead to failure in finding the global optimal solution.

Due to the limitations of the above study, Mazloumi et al. (2012) applied two heuristics algorithms (i.e., Ant Colony and Genetic algorithms) to optimize the number and locations of time points and associated slack times, with same cost components considered in the objective function. The alternative schedules with combinations of time points and predefined slack times were evaluated against the generalized total cost. In their study, the ant colony and genetic algorithms were compared in terms of efficiency. The time points as well as their corresponding slack times were all assumed as integer, with predefined set of slack time to be optimized.

Most of the abovementioned studies focused on the schedule-based strategies, which was suggested to fit long-headway services better. Although comparing to long-headway services, maintaining bus schedules with short headways is considered more difficult, due to the complexity of optimization problems with headway-based strategies

(which is generally considered to be effective in short-headway services), few studies optimized time points for short-headway services.

### *Headway-based Strategies*

Dealing with headway-based strategies, simulation is commonly applied to analyze vehicle controlling, where a random pattern of passenger arrivals and a binomial distribution of number of alighting passenger at each stop were typically assumed (Koffman, 1978; Andersson et al., 1979, Vandebona and Richardson, 1986; Senevirante, 1990; Lin et al., 1995; Fu and Yang, 2002). An early study conducted by Senevirante (1990) analyzed the impact of different operating strategies using simulation and showed a second-degree polynomial relationship between the standard deviation of headway and the number of time points, indicating that either too many or too few time points would have a negative effect on headway adherence and on-time performance.

Recently, Fu and Yang (2002) simulated the scenarios of one-stop control, two-stop control, and all-stop control with predefined time points. Two different control strategies, namely one-headway-based and two-headway-based, were examined with selected performance measures (i.e., user waiting and in-vehicle times, and bus travel time). They found that two-stop control appeared to be the best among others in terms of system performance. The results from these simulation models showed that the more boarding demand a stop experienced, the higher possibility it served as a time point.

To analyze the impact of bus control strategy on short-headway services, Daganzo (2009) proposed a headway-based approach to eliminate bus bunching, where an adaptive control scheme was developed. The approach dynamically determined bus holding times

at the control points based on real-time headway information, assuming buses can be controlled everywhere along the route.

The study conducted by Cats et al. (2011) investigated the impacts of holding control strategies on transit performance with a simulation approach, where one scheduled-based strategy and two headway-based strategies were chosen for control effectiveness comparison. The system evaluation focused on the effect of time points on service regularity in terms of headway coefficient of variation. The results showed that headway-based strategies led to shorter passenger waiting times at the cost of longer in-vehicle times, compared with schedule-based holding strategies.

Table 2.2 summarizes major previous studies on optimization of time points and slack time.

## **2.4 Optimization Algorithms**

Based on the number of objectives to be optimized, the optimization models can be categorized into two groups: single-objective and multi-objective models. The single-objective optimization model involves only one objective, whereas in the context of multi-objective optimization, two or more conflicting objectives are optimized. Dealing with different optimization models, a variety of algorithms has been developed and applied in the previous studies. In this section, the multi-objective models are reviewed since the majority of transit network design problems fall in this category, followed by the common solving algorithms.



**Table 2.2** Selected Studies on Time Points and Slack Time Optimization

Year	Author	Approach	Analysis of Time Points	
			Number	Location
1972	Osuna and Newell	Analytical	Single	Not Analyzed
1978	Koffman	Simulation	Single	Terminal
1986	Abkowitz et al.	Analytical & Simulation	Single	Related to Boarding Profile
1990	Senevirante	Simulation	User-Defined	Related to the Standard Deviation of Headway
1995	Wirasinghe and Liu	Analytical	Optimized (for a single bus run)	
1995	Lin et al.	Simulation	All-Stop	Not Analyzed
2001	Eberlein et al.	Analytical	Single	Terminal
2001	Hickman	Analytical	Single	Related to Boarding Profile
2001	Liu and Wirasinghe	Simulation	Optimized (within limited feasible solutions)	
2002	Fu and Yang	Simulation	Single, Two, All-Stop	Related to Boarding Profile
2006	Zhao et al.	Analytical & Simulation	Single	Not Analyzed
2009	Furth and Muller	Analytical	Multiple	Related to Boarding Profile
2010	van Oort et al.	Simulation	Single	Not Analyzed
2011	Cats et al.	Simulation	Multiple	Not Analyzed
2012	Mazloumi et al.	Analytical	Optimized (with predefined set of slack times)	
2012	van Oort et al.	Analytical	Single, Two	Related to Boarding Profile
2012	Lee et al.	Simulation	Single	Not Analyzed
2014	Lee et al.	Analytical	Single	Not Analyzed

### **2.4.1 Optimization Modelling Approach**

The goal of multi-objective optimization is to strike a balance among conflicting objectives. However, it is not trivial to find an optimal solution for the multi-objective problems without iterative interaction with the decision maker, since the optimal solution for one objective is not necessarily optimal for another (Miettinen, 1999). To tackle the issue, modelling approaches including weighted sum and Pareto-optimality as discussed below are commonly applied.

The weighted sum approach formulates all the conflicting objectives into a weighted function. As such, the multiple objective functions are converted into a single objective function. Many single-objective optimization problems listed in Section 2.2 can be categorized into this group, where two conflicting objectives (e.g., minimizing user cost, minimizing operator cost) are combined through weight factors (e.g., Ghoneim and Wirasinghe, 1987; Furth and Wilson, 1981; Chang and Schonfeld, 1991; Spasovic et al., 1994; Chien and Schonfeld, 1997; Li and Bertini, 2009; Furth and Rahbee, 2000; Furth and Muller, 2007, 2009; van Oort et al., 2010; Cats et al., 2011). The weighted sum modelling approach makes the optimization models easier to solve, however, it only reflects one certain relationship between two conflict objectives with a set of weight factors. Moreover, although critical, it is often difficult to assign appropriate weight factors for the conflicting objectives due to different scales, units and importance involved in these objectives.

The Pareto optimality approach tries to find a set of non-dominated solutions (Miettinen, 1999; Deb, 2001), which achieves a state that any objective cannot be further improved without degrading others. In bi-objective problems, the curve formed by this

solution set is called Pareto front (or trade-off curve), where the trade-off between the conflicting objectives is clearly represented.

### **2.4.2 Solution Algorithms**

A variety of algorithms have been developed and applied in previous studies for solving bus route planning problem with a weighted sum approach, including exact algorithms, heuristic and metaheuristic algorithms.

#### ***Exact Algorithms***

Through continuum approximation, Wirasinghe and Ghoneim (1981) optimized the stop spacing along a local bus route with a non-uniform many-to-many travel demand pattern. By separating the total cost function into segments based on transfer point, the stop spacing between any two transfer points can be calculated through setting the derivative of each cost function with respect to stop spacing equal to zero. Hurdle and Wirasinghe (1980), Kuah and Perl (1988) also applied traditional derivative methods to solve the developed optimization models for bus route planning. These methods are effective in solving bus route optimization but not applicable to the models with discrete variables. Formulating the objective function based on discrete stop locations, several algorithms were applied in bus stop optimization. The study conducted by Chien and Qin (2004) applied an Exhaustive Search algorithm (ES) to determine the optimal number and locations of stops numerically, which computed all the possible combinations for any number of stops and found the least-cost solution.

### *Heuristic Algorithms*

In the studied by Vuchic and Newell (1968) and Furth and Rahbee (2000), Dynamic Programming (DP) was applied for the optimization model, which broke down the discrete optimization problem into a set of simplified sub-problems and found the solution by tracing back through the optimal decisions made in each sub-problem. DP decomposes the n-dimensional optimization problem into a set of optimization sub-problems and solves the sub-problems starting from the smaller in size, which examines all possible ways to find the best solution and may be effective for the problem with high sub-problem overlapping rate.

Chien and Schonfeld (1998) applied the Gradient Descent algorithm (GD) to find an optimal/near-optimal solution for the joint optimization problem, where the decision variables included rail route length, station spacing and headway, as well as feeder bus route and stop spacing, and headway. The gradient vector was derived by setting the first order partial derivatives of the total cost function equal to zero. GD allows changes of all decision variables in one step to seek a new gradient vector, and thus provides an efficient way to find a descent direction in searching optimal solutions by computing the components in the gradient vector sequentially and iteratively.

LeBlanc (1975), Poorzahedy and Turnquist (1982), respectively, developed branch-and-bound based heuristic algorithms to solve the proposed bi-level optimization model, where the upper level was to minimize total cost and the lower level involved a user equilibrium assignment problem. Gao et al. (2004, 2005) designed heuristic solution algorithms for the proposed bi-level programming models to solve transit network design problems. Ibeas et al. (2010) optimized bus stop spacing with a passenger mode choice-

assignment model, where the stop spacing varies over zones in the entire network. In their study, the Pattern Search algorithm (PS) was applied to solve the bi-level optimization model. PS starts from generating an initial feasible stop spacing solution vector and iterates to find an optimal/near-optimal solution. It does not require the derivatives of the problem to be optimized, and could be applied to the functions that are not continuous or differentiable. Since there is a stage to find a good direction of descent within PS, a pre-established amount of change in one of the variable is critical to ensure a good local direction of movement in the solution space.

### ***Metaheuristic Algorithms***

In large-scale realistic transit network design and scheduling, metaheuristic algorithms including Simulated Annealing (SA), Tabu Search (TS), Ant Colony Optimization algorithm (ACO), and Genetic Algorithm (GA) are commonly applied to find a good solution efficiently (Fan and Machemehl, 2004). GA is an adaptive heuristic search algorithm that is based on the evolutionary idea of natural selection and genetics. It is applicable widely because of no constraints on the continuity, derivative existence, and unimodality of the problems to be solved. SA is a Monte Carlo simulation-based search algorithm, derived from the process of heating and then cooling a substance slowly to finally arrive at the solid state. The overall concept of TS is to avoid entrapment in cycles by forbidding or penalizing moves taking the solution in the next iteration to points in the solution space previously visited. ACO is inspired by the behavior of real ants seeking food between their colony and a source of food, and the main idea of ACO is the indirect communication of a colony of ants based on the pheromone trail.

To find a good local (possibly global) solution in a reasonable time domain, these algorithms generate a set of local optimums through efficiently reformulating the problem and selecting the best solution from the set as the optimal solution based on pre-defined stopping criteria. The mechanism behind the above algorithms could avoid being entrapped at the local optimum, such as mutation procedure in GA, heating stage in SA, Memory function in TS, and pheromone evaporation in ACO. The simplicity of the operations and the ability to find good solutions make these metaheuristic algorithms attractive for solving bus network design problems.

A variety of studies were conducted applying different metaheuristic solution algorithms. Similar to the solution methods for the discrete stop optimization problem, Mazloumi et al. (2012) applied ACO and GA for developing optimal schedules (i.e., identifying the optimal number and locations of time points) for a fixed bus route, where these two algorithms were compared in terms of accuracy and efficiency in providing the optimal solution. Through searching a ‘good’ result from a large set of potential schedule designs, it is found that both algorithms were able to find the optimal solution, although ACA demonstrated a higher efficiency than GA by evaluating less designs.

Pattnaik et al. (1998), Bielli et al. (2002), Chakroborty and Wivedi (2002), Chakroborty (2003), and Tom and Mohan (2003) applied GA to minimize total system cost with fixed demand, which generated a set of optimal routes and associated frequencies. Fan and Machemehl (2006a, 2006b, and 2008), Fan and Mumford (2010), Szeto and Wu (2011) and Nayeem et al. (2014) proposed different solution methods (e.g., SA, GA, TS) to optimize transit network design problem with various objectives. Zhao et al. (2005),

Zhao (2006), Zhao and Zeng (2006, 2008) applied different algorithms (e.g., SA, TS) for optimizing route configuration and associated frequencies. Yang et al. (2007) applied the ACO to optimize transit network design. Szeto and Jiang (2014) proposed a bi-level model for transit route design and frequency setting, considering passenger transfers and congestion effects, a bee colony metaheuristic algorithm (BCO) was applied to solve the problem and obtain robust solutions. Apply GA, Arbex and Cunha (2015) solved a multi-objective transit network design and frequency setting problem with two conflicting objectives (i.e., minimizing user cost and minimizing operator cost) to find a set of routes and associated frequencies for an urban transit system. With TS, Giesen et al. (2015) solved the multi-objective optimization problem for transit frequency setting, minimizing total user travel time and fleet size simultaneously, and suggested different solutions considering the trade-off between the two conflicting objectives.

In addition to mathematical programming techniques and simulation-based analysis methods in transit network optimization, few studies integrated simulation and optimization to find optimal values of decision variables. A simulation-based optimization approach integrates an optimizer guiding the search direction and a simulator for performance evaluation. With this approach, the analytical objective function is replaced with a simulation model. The simulation input is the decision variables and output is usually fitness value used by the optimizer in the process of searching for an optimal solution.

Although simulation-based optimization has been documented in other areas (Gen et al., 1996; Liu, 2001), a little has been done in the field of transit planning. Mazloumi et al. (2012) applied ACO and GA to solve a schedule-based optimization model

to determine the optimal time points and slack time, with the objective of minimizing total cost incurred with the schedule (i.e., passenger waiting time, delay penalty, total operation time). In their study, possible time points are predefined among the entire set of stops, and the slack times at each time points were only chosen from three possible values (i.e., 0, 1, 2 minutes). The system was simulated with VISSIM and the outputs of the simulation model were sent back to the optimization algorithms to calculate the objective value.

Li et al. (2013) developed an expected value model for optimizing the multiple bus headways for a single bus route, where stochastic simulation and genetic algorithm were integrated to solve bi-level objective functions. Two objectives were considered in their study, which included maximizing operator profit as the upper level model, and minimizing expected waiting time as the lower level model. Sun et al. (2014) developed a multi-objective optimization model for train routing and scheduling on a high-speed railway network. The model integrated route selection and train control optimization module with simulation module for scheduling.

As shown in these studies, the simulation-based optimization enables searching for an optimal solution for many complicated problems that may not be easily mathematically formulated and solved. The system dynamics could also be well represented with this approach. However, due to the inherent characteristics of simulation, a global optimal solution is sometimes difficult to find. Considering the planning for time points with headway-based strategies, since real-time headway information is often necessary in the optimization process, the simulation-based optimization could be applied to solve the problem with carefully calibrated simulation models mimicking real world vehicle operations.



## 2.5 Summary

It is revealed that traffic congestion and dwell time fluctuation at stops could result in additional travel time, which would affect both passengers and operators. Neglecting the impact of travel time variability in stop and headway optimization may lead to poor decision-making due to underestimated operating cost and passenger travel time. However, the influence of traffic flow and demand fluctuation on headway variance, as well as the impact of such variation on the optimal service planning, has not been thoroughly investigated.

Regarding setting time points for improving service reliability, it is recognized that the number and locations of time points for holding early-arrived vehicles have significant effects on both operator and passenger costs. Although previous studies stated that the best location for a time point depends on the demand distribution and network configuration, the optimal number and locations of time points were not theoretically determined.

Various solution methods were applied to solve the optimization problems for bus route planning, each with its advantages and disadvantages. Traditional derivative methods are effective in solving the optimization of decision variables (e.g., bus route, stops, and headway), yet they could only be applied for the models with continuous decision variables. The exhaustive search algorithms ensure that an optimal solution could be found, however, the computation time will increase exponentially with the expansion of problem scale. Heuristic/metaheuristic algorithms, on the other hand, could provide an optimal/near-optimal solution within a reasonable time span, which are commonly applied for large-scale complicated problems. Although, parameters involved in the heuristic/metaheuristic algorithms should be fine-tuned to avoid converging towards local optima.

## CHAPTER 3

### METHODOLOGY

This chapter presents three models applied for planning for a given bus route to achieve optimal pre-defined objectives. Model I determines optimal stop spacing and headway for a given feeder bus route with many-to-one/one-to-many demand. To investigate the impact of headway variance on the objective of profit maximization and planning of stops and headway, Model I is developed based on some simplified conditions, such as the limitation of bus stop locations and passenger demand distribution. The decision variables (i.e., headway and stop spacing) are optimized and their relationship with headway variance is explored.

It is expected that with Model I considering headway variance, a more cost-efficient and profitable system could be yielded, compared to the traditional model without such consideration. Also, the responses of objective values to model parameters (e.g., demand level and traffic congestion level) are expected to be different in the proposed model since it reflects a more realistic situation than the traditional model. With such concept being proved with Model I, Model II is developed to optimize stop locations and headway for a given conventional bus route. It intends to optimize stop locations and headway considering many-to-many demand to maximize operator profit. To take into account stochastic nature of bus operations, headway variance at each stop is modeled as a function of travel time variance between stops and demand fluctuations at stops. In this way, the

interactions between traffic conditions, passenger activities, and bus operations can be analyzed.

To further enhance system performance in terms of reliability and investigate the effects of control points in a bus route with optimal setting suggested by Model II, Model III is developed to optimize time points (if necessary) based on a headway-based controlling approach. Model III takes the input headway, fleet size and stop locations from the optimization results of Model II, and simultaneously minimizes average user cost and average operator cost for the study bus route with optimal time points.

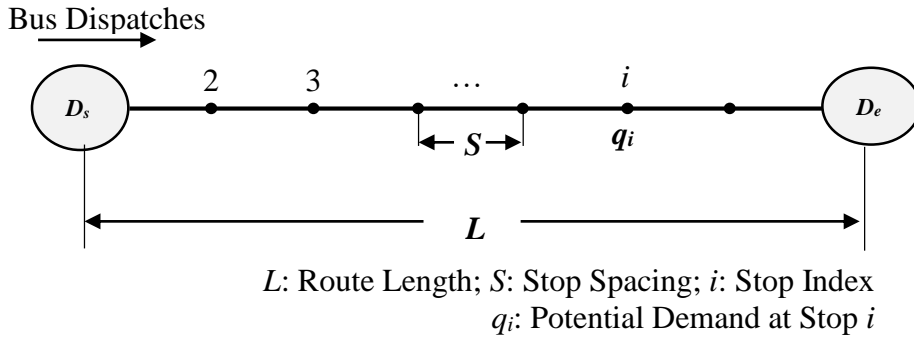
The derivations of the three models are presented in the following sections. The variables and their definitions involved in the model development are summarized in Appendix A.

### **3.1 Model I – The Basic Model**

The objective of this model is to optimize stop spacing and dispatching headway of a given feeder bus route considering the impact of stochastic vehicle arrivals and many-to-one/one-to-many travel patterns, which maximizes operator's profit considering demand elasticity.

#### **3.1.1 Route Configuration**

A general feeder bus route is illustrated in Figure 3.1, where  $D_s$  represents an origin terminal located in a residential area, and  $D_e$  represents the destination, which may be located in a Central Business District or at a major transit terminal. Let  $I$  be the set of stops and  $i$  is an index of stop. The route length is  $L$ , and  $S$  stands for the stop spacing.



**Figure 3.1** Model I – A general feeder bus route.

### 3.1.2 Assumptions

The assumptions considered to formulate Model I are discussed below:

1. For a general feeder bus route shown in Figure 3.1, the travel demand pattern is many-to-one/one-to-many. Buses pick up passengers from the origin (denoted as  $D_s$ ) and all intermediate stops in the residential area, and then buses drop them off at the destination (denoted as  $D_e$ ). The number of passengers travelling on the reverse direction is negligible.
2. The potential demand is uniformly distributed along the route over a period of time, which is sensitive to travel time.
3. Passenger arrives at the stops randomly.
4. Bus stops may be located any places along the study route.
5. Bus acceleration and deceleration delay per stop is constant.
6. The design service capacity is always greater than the demand, and the vehicle size is always sufficient to pick up all waiting passengers.
7. Bus ticket price is fixed and flat per passenger trip.

### 3.1.3 Model Formulation

In this section, the objective operator's profit is formulated, in which the impact of headway variance to vehicle and passenger travel times are considered.

### ***Operator's Profit***

The objective total profit denoted as  $P_b$  is defined as total revenue ( $R_b$ ) minus operator cost ( $C_b$ ). Thus,

$$P_b = R_b - C_b \quad (3.1)$$

### ***Revenue***

The total revenue is the sum of fare paid by all passengers who are sensitive to travel time will be introduced later. Thus,

$$R_b = f \sum_{i \in I} b_i \quad (3.2)$$

where  $f$  represents fare and  $b_i$  is the actual demand at stop  $i$ .

### ***Operator Cost***

The operator cost is the product of fleet size, denoted as  $F$ , and the average bus operating cost, denoted as  $u_b$ . The fleet size can be determined by the ratio of bus round-trip travel time ( $T_r$ ) to headway  $H$ . Thus,

$$C_b = u_b \frac{T_r}{H} \quad (3.3)$$

Note that  $T_r$  consists of vehicle running time, stop delay (i.e., acceleration/deceleration delay and dwell time) and recovery time due to travel time fluctuation. Let  $Q$  be the actual demand, and  $t_l$  is the recovery time at terminal. For the study feeder bus route,  $T_r$  can be formulated as:

$$T_r = \frac{2L}{V_b} + nd_s + t_l + Q\rho H \quad (3.4)$$

where  $d_s$  is acceleration/deceleration per stop, and  $\rho$  is the average boarding time per passenger.

The first component on the right hand side of Equation 3.4 is the round-trip link travel time from terminal  $D_s$  to terminal  $D_e$  and then back to terminal  $D_s$ . The second component is acceleration/deceleration delay at all stops, where  $n$  is the number of stops per direction. The third component is the recovery time at the destination terminal. The last component is dwell time to serve boarding passengers. Note that since only one-way demand is considered, acceleration/deceleration delay and dwell time only appear on one direction.

### 1. Demand Function

Considering elasticity of demand (Kocur and Hendrickson, 1982; Chang and Schonfeld, 1993; Spasovic et al, 1994), actual demand is a function of potential demand, expected travel time, and fare. In this study, fare is given and fixed, the main factor influencing passenger's mode choice is the expected travel time. Based on the previous study (van Nes and Bovy, 2000), the actual demand is formulated as a function of expected travel time and reference travel time. Thus,

$$b_i = q_i \frac{\exp(-e_i T_i)}{\exp(-e_i \bar{T}_i)}, \forall i \quad (3.5)$$

where  $q_i$  is the potential demand at stop  $i$  and identical for all  $i$ ,  $e_i$  is the coefficient factor of travel time.  $T_i$  and  $\bar{T}_i$  represent the expected and reference travel times from stop  $i$  to the destination terminal, respectively.  $T_i$  consists of  $t_{w_i}$  as the expected wait time,  $t_{v_i}$  as the expected in-vehicle travel time, and  $t_{a_i}$  as the average access time for passengers to stop  $i$ .

Note that, the reference travel time defined here is the expected travel time without variance, such that if travel time is deterministic,  $\bar{T}_i$  is equal to  $T_i$  and the actual demand at stop  $i$  equals the potential demand.

## 2. Wait Time

The users' wait time depends on the service frequency of the bus route, headway variance at stops and passengers' arrival behavior. For short headway services, passengers' arrivals at stops are random. Therefore, the average wait time at stop  $i$ , denoted as  $t_{w_i}$ , is a function of mean and variance of headway (Welding, 1957):

$$t_{w_i} = \frac{H}{2} \left( 1 + \frac{v_i}{H^2} \right), \quad \forall i \quad (3.6)$$

where  $H$  and  $v_i$  represent average headway and headway variance at stop  $i$ , respectively.

As discussed earlier,  $v_i$  is assumed to increase as stop  $i$  further away from the departing terminal, due to unexpected delays en-route (e.g., link length, traffic conditions, number of intersections) and at stops (e.g., stop locations, boarding/alighting passengers, and fare collection methods). Thus,

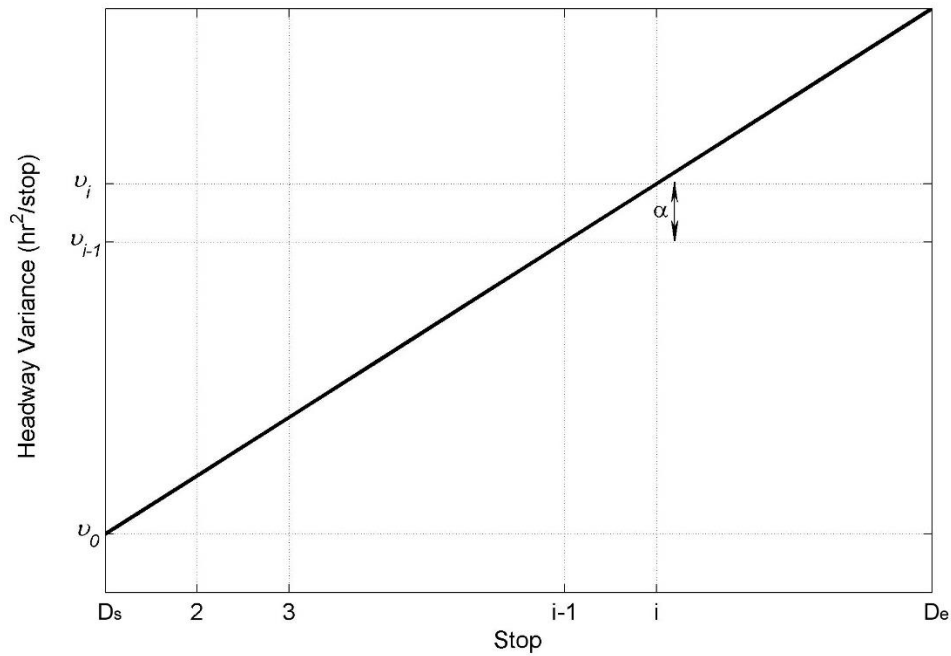
$$v_i = v_{i-1} + v_{ei}, \quad \forall i \quad (3.7)$$

where  $v_{ei}$  is the variation caused by en-route factors when buses travelling from stops  $i-1$  to  $i$ . In this study, the link length between each pair of consecutive stops is identical. For planning purposes, the travel time variation between each pair of stops is assumed to be identical along the route without further analysis of traffic condition and geographic variations. However, with more data collected for the study route, the empirical model of

$v_{ei}$  along the route can be incorporated in Equation 3.7. For now, let  $\alpha$  represent the increment of headway variance per stop. Thus,

$$v_i = v_{i-1} + \alpha, \quad \forall i \quad (3.8)$$

With Equation 3.8, the relationship between headway variance and stop location is illustrated in Figure 3.2. Note that the proposed model is able to adapt any form of Equation 3.7 (e.g., developed from empirical data) to compute the impact of headway variance to the components in the objective function.



**Figure 3.2** Headway variance vs. stop.

### 3. In-Vehicle Time

The expected in-vehicle travel time for passengers boarding at stop  $i$  is the sum of expected running time  $t_{v_i}$  and stop delay. The stop delay consists of acceleration/deceleration delay ( $d_s$ ) and dwell time at downstream stops. The expected in-vehicle running time  $t_r$  is



calculated through dividing the distance from stop  $i$  to the destination by the average speed  $V_b$ . Thus,

$$t_{v_i} = \frac{L-i \cdot S}{V_b} + \delta_i + (n-i)d_s + \sum_{k=i+1}^{n-1} \rho b_k H, \forall i \quad (3.9)$$

where  $i \in [1, n]$  is stop index, with the original terminal ( $i = 1$ ) and the destination terminal ( $i = n$ ).

In Equation 3.9, the expected link travel time is calculated as  $\frac{L-i \cdot S}{V_b} + \delta_i$ , where  $\delta_i$  is the extra time added to the mean travel time as a buffer time to ensure on-time arrivals due to service unreliability. In the case study,  $\delta_i$  is set as one standard deviation of travel time from stop  $i$  to the destination terminal. The acceleration/deceleration delay is represented as  $(n-i)d_s$ , the number of downstream stops from stop  $i$  multiplied by acceleration/deceleration delay per stop. The term of  $\sum_{k=i+1}^{n-1} \rho b_k H$  stands for the sum of expected dwell times at the downstream stops of stop  $i$ .

#### 4. Access Time

With the consideration that passengers are distributed along the bus route, the access path of each passenger is a segment on the route from passenger location to the stop. Hence, the average access time is the average access distance (i.e., a quarter of stop spacing  $S$ ) divided by access speed, denoted as  $V_p$ . Thus,

$$t_{a_i} = \frac{S}{4V_p} \quad (3.10)$$

### ***Objective Function***

Therefore, the objective profit (i.e., Equation 3.1) is revenue (i.e., Equation 3.2) minus operator cost (i.e., Equation 3.3), which is reformulated as:

$$\max. P_b = f \sum_{i \in I} b_i - u_b \frac{\frac{2L}{V_b} + nd_s + Q\rho H + t_l}{H} \quad (3.11)$$

The decision variables in the model are headway and stop spacing. Due to the interrelationship between stop spacing and headway as well as the consideration of demand elasticity with respect to travel time, it is tedious to solve the problem with traditional derivative methods. As summarized in the literature review, heuristic/metaheuristic algorithms are most commonly used in similar problems.

The capacity constraint (Equation 3.12) is applied to ensure that the service capacity is greater than or equal to the demand  $Q$ .

$$Q \leq \frac{C}{H} \quad (3.12)$$

where  $C$  is bus capacity and the right hand side in Equation 3.12 represents the hourly service capacity. Therefore, the maximum headway, denoted as  $H_M$ , can be derived as:

$$H_M = \frac{C}{Q} \quad (3.13)$$

### ***System Performance***

From the perspectives of transit users (i.e., passengers) and operator, the definitions of a ‘good’ system may be different. For passengers, bus services with short headways and high accessibility is favorable because of less travel time. On the other hand, the operator prefers a system with low operator cost while maintaining certain level of service to satisfy the

demand. To investigate system performance, average user cost and the average operator cost are applied to assess system attractiveness (i.e., users' perspective) and cost effectiveness (i.e., operators' perspective).

### ***Average Operator Cost***

As defined earlier, the total demand of the route is denoted as  $Q$ , and the total operator cost is represented by  $C_b$ . Therefore, the average operator cost, denoted as  $c_b$ , is the operator cost divided by demand. Thus,

$$c_b = \frac{C_b}{Q} \quad (3.14)$$

### ***Average User Cost***

Similar to the average operator cost, the average user cost is total user cost divided by demand. Let  $C_u$  be the total user cost. Then, the average user cost denoted as  $c_u$ , is formulated as follows.

$$c_u = \frac{C_u}{Q} \quad (3.15)$$

Total user cost consists of three major components: access cost, wait cost, and in-vehicle cost, which are the products of total access time, wait time, in-vehicle time and the corresponding values of time, respectively. Thus, the user cost associated with these time segments can be formulated in Equation 3.16:

$$C_u = C_a + C_v + C_w \quad (3.16)$$

where  $C_a$ ,  $C_v$ , **and**  $C_w$  represent access cost, in-vehicle cost, and wait cost respectively.

The wait cost at stop  $i$  is the product of boarding demand, average wait time, and the value of passenger wait time. Thus, total wait cost is the sum of wait costs at all stops:

$$C_w = u_w \sum_{i \in I} b_i \frac{H}{2} \left(1 + \frac{v_i}{H^2}\right) \quad (3.17)$$

where  $u_w$  is the value of passenger wait time.

The in-vehicle cost ( $C_v$ ) is the product of the total in-vehicle time for all the passengers (i.e., running time and in-vehicle delay) and the value of users' in-vehicle time,  $u_v$ . Thus,

$$C_v = u_v t_v \quad (3.18)$$

where  $t_v$  is total in-vehicle time experienced by passengers boarding from all stops along the route, where the in-vehicle time at stop  $i$  is explained in Equation 3.9. Therefore, the in-vehicle cost is:

$$C_v = u_v \sum_{i \in I} b_i \left( \frac{L-i \cdot S}{V_b} + \delta_i + (n-i)d_s + \sum_{k=i+1}^{n-1} \rho b_k H \right) \quad (3.19)$$

The access cost ( $C_a$ ) is defined as the product of total demand, the average access time ( $t_a$ ), and the value of users' access time ( $u_a$ ). Thus,

$$C_a = u_a Q \frac{S}{4V_p} \quad (3.20)$$

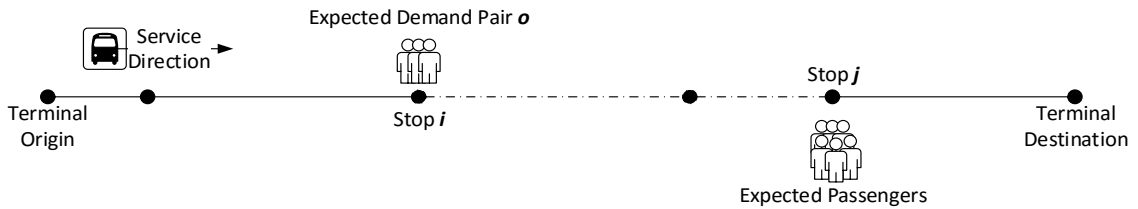
### 3.2 Model II – The Enhanced Model

The objective of Model II is to optimize stop locations and dispatching headway for a conventional bus route with many-to-many travel demand considering headway variation at stops induced by stochastic bus travel time and dwell time fluctuation. Model II is enhanced from Model I for dealing with specific many-to-one demand pattern to a more generalized many-to-many demand pattern considering feasible locations for bus stops.

Considering demand elasticity, it is expected that the outcome of the model could improve transit service reliability while achieving maximum operator profit.

### 3.2.1 Route Configuration

Unlike the basic model, Model II optimizes dispatching headway and discrete stop locations along a general bus route as shown in Figure 3.3. The study bus route length is  $L$ , and a set of potential stop locations is denoted as  $I$ . A set of origin-destination demand pairs is denoted as  $O$  and demand pair  $o$  ( $o \in O$ ) is denoted as  $q_o$ . For passengers travelling from one community  $x$  to another  $y$ , let  $D_{K_o}$  and  $D_{L_o}$  represent the access distance from origin  $x$  to the nearest stop  $K_o$  and the distance from the destination  $y$  to the nearest stop  $L_o$ , respectively.



**Figure 3.3** Model II – A general conventional bus route.

### 3.2.2 Assumptions

To formulate the objective profit function, the following assumptions are made:

1. The potential demand distribution is heterogeneous along the route, and the actual demand at stops is influenced by the expected travel time.
2. Passengers always choose the nearest stops and will arrive at the stops randomly.
3. Eligible stop locations, including potential and existing stops, are given. Compare to boarding, alighting activities move quick enough such that the dwell time is proportion to the number of boarding passengers (Daganzo, 2009).

4. Service capacity is always greater than demand and the vehicle size is large enough to pick up all waiting passengers.
5. Fare is fixed and flat.

### 3.2.3 Model Formulation

The objective of Model II is to maximize operator's profit, which is total revenue minus the operator cost.

#### *Operator's Profit*

As formulated in Equation 3.21, the objective total profit denoted as  $P_b$  is defined as total revenue ( $R_b$ ) minus the operator cost ( $C_b$ ), Thus,

$$P_b = R_b - C_b \quad (3.21)$$

#### *Revenue*

Revenue considered here is total fare revenue paid by the passenger, which is the fare  $f$  per passenger trip multiplied by the actual demand. Similar to what is formulated in Section 3.1, the actual boarding demand at stop  $i$  is a function of the potential demand, expected travel time, and reference travel time. Thus,

$$b_i = \sum_{o \in B_i} q_o \frac{\exp(-e_i T_o)}{\exp(-e_i \bar{T}_o)}, \forall i \quad (3.22)$$

where  $q_o$  is the expected demand of the demand pair  $o$ ,  $b_i$  is the actual demand at stop  $i$ ,  $B_i$  a set of demand pairs originating at stop  $i$  ( $B_i \subset O$ ),  $e_i$  is the coefficient factor of travel time.  $T_o$  and  $\bar{T}_o$  represent the expected and reference travel times for the demand pair  $o$ , respectively. The formulation of  $T_o$  is explained later in this section. Therefore, the revenue, product of actual demand and fare, is formulated as Equation 3.23:

$$R_b = f \sum_{i \in I} \sum_{o \in B_i} q_o \frac{\exp(-e_i T_o)}{\exp(-e_i \bar{T}_o)} \quad (3.23)$$

### *Operator Cost*

The operator cost is the product of fleet size ( $F$ ) and the average bus operating cost ( $u_b$ ).

The required fleet size for the bus route is defined as the round-trip bus travel time divided by headway. Thus,

$$C_b = u_b \frac{T_r}{H} \quad (3.24)$$

where  $T_r$  is vehicle round-trip travel time, consisting of vehicle running time between stops, intersection delay, acceleration/deceleration delay, dwell time at stops, and recovery time at the terminal. Thus,

$$T_r = 2(t_b + X \cdot d_x + N \cdot d_s + \rho \sum_{i \in I} b_i) + t_l \quad (3.25)$$

where  $X$  is number of intersections over the route,  $N$  is number of bus stops over the route,  $d_x$  is the average intersection delay,  $d_s$  is the acceleration/deceleration delay per stop,  $t_b$  is the total vehicle running time (i.e., route length divided by average speed), and  $t_l$  is the recovery time at the destination terminal due to travel time variance.

### *Expected Travel Time*

The expected travel time for any demand pair  $o$  includes three components: access/egress time, wait time, and in-vehicle time, which are discussed as follows.

### 1. Access/Egress Time

For any demand pair  $\mathbf{o}$ , they will choose a pair of stops with minimum access/egress time (denoted as  $K_{\mathbf{o}}$  and  $L_{\mathbf{o}}$ , respectively) to board and alight. After determining the access stops, the average access/egress time  $t_{a_{\mathbf{o}}}$  for this demand pair is equal to the average access/egress distance divided by the average passenger accessing speed:

$$t_{a_{\mathbf{o}}} = \frac{D_{K_{\mathbf{o}}} + D_{L_{\mathbf{o}}}}{V_p}, \forall \mathbf{o} \in \mathbf{O} \quad (3.26)$$

where  $V_p$  is average access speed.

### 2. Wait Time

The users' wait time depends on bus service headway, headway variance, and passenger arrival pattern. For short headways, passengers tend to arrive at stops randomly. Therefore, the average wait time at stop  $i$ , denoted as  $t_{w_i}$ , is formulated as (Welding, 1957):

$$t_{w_i} = \frac{H}{2} + \frac{\nu_i}{2H}, \forall i \in I \quad (3.27)$$

where  $\nu_i$  is the headway variance at stop  $i$ , and  $H$  is the average headway.

The headway variance at stops significantly affects the user wait time, which tends to increase as the stop approaching the end of the route due to unexpected delay en-route and at stops (Adebisi, 1986). The headway variance at stop  $i$ , denoted as  $\nu_i$ , is a function of the headway variance of stop  $i-1$ , dwell time at stop  $i-1$ , distance between stops  $i-1$  and  $i$ , passenger arrival rate at stop  $i-1$ , and as formulated as Equation 3.28 (Derivation shown in Appendix B). Thus,

$$\nu_i = (1 + 4\rho^2 b_{i-1}^2 + 2\rho b_i) \nu_{i-1} + 2l_{i-1} \nu_i \quad (3.28)$$



where,  $l_{i-1}$  represents the distance between stops  $i-1$  and  $i$ , and  $\rho$  is the average passenger boarding time. For simplicity, travel time variance between a pair of consecutive stops is treated as the product of link length and the unit travel time variance ( $v_l$ ). However, if sufficient travel time data between stops could be collected, a more realistic segment-level travel time variance function could be obtained and applied in the model to replace Equation 3.28.

### 3. In-vehicle Time

The in-vehicle travel time consists of four components: vehicle running time from an origin to a destination, delay at intersections, dwell time at stops, and acceleration/deceleration delay while buses approaching and exiting stops.

For demand  $q_o$  travelling from stop  $K_o$  to stop  $L_o$ , the in-vehicle time is affected by travel distance, number of stops and intersections between  $K_o$  and  $L_o$  and average vehicle speed. Thus, the in-vehicle travel time for the demand pair  $o$ , denoted as  $t_{v_o}$ , is the sum of cruise time (the link distance divided by the average cruising speed, as  $l_o/V_b$ ), extra planning time due to travel time variation (represented as  $\delta_o$ ), intersection delay (the product of number of intersections and the average intersection delay, as  $X_o d_x$ ), expected dwell time (the expected boarding demand at all intermediate stops between  $K_o$  and  $L_o$  multiplied by average boarding time, as  $\sum_{j=K_o+1}^{L_o-1} \rho b_j$ ), and finally the total stop delay (the product of number of intermediate stops and average stop delay, represented as  $N_o d_s$ ).

Thus,

$$t_{v_o} = \frac{l_o}{V_b} + \delta_o + X_o d_x + \sum_{j=K_o+1}^{L_o-1} \rho b_j + N_o d_s, \forall o \in O \quad (3.29)$$

where  $l_o$  is the distance between stops  $K_o$  and  $L_o$  for demand pair  $o$ ,  $V_b$  is average bus speed,  $X_o$  is the number of intersections between stops  $K_o$  and  $L_o$ ,  $N_o$  is the number of stops between stops  $K_o$  and  $L_o$ .

The average intersection delay  $d_x$  consists of non-random delay induced by signal timing (i.e., the ratio of green time to the cycle time), overflow delay induced by random arrivals and oversaturation, and acceleration and deceleration delay (Tirachini and Hensher, 2011). In this study,  $d_x$  is treated as an exogenous variable of the proposed model and set as constant.

### ***Objective Function***

To summarize, the objective profit is a function of dispatching headway and stop locations. The headway  $H$  is a continuous variable and  $I$  is a set of stops to be optimized. To maximize the total profit, the optimal headway and stop locations under the influence of travel time variance and the service capacity constraint must be found. Thus,

$$\begin{aligned} \text{Max. } & P_b(H, I) = R_b(H, I) - C_b(H, I) \\ \text{s.t. } & H \leq H_M \end{aligned} \quad (3.30)$$

where  $H_M$  is the maximum headway such that the service capacity is always greater than or equal to the demand.

The discrete modelling approach of stop locations and headway as well as the consideration of travel time elasticity of demand increase the complexity of the problem, which is categorized as mixed-integer non-linear optimization problem. While a heuristic

algorithm is able to handle this type of problems, it is not efficient if the solution pool is large. Therefore, the single-objective GA is applied to solve Model II.

### *System Performance*

The average user cost and the average operator cost are also used as indices of attractiveness and effectiveness of a system to users and the operator, respectively.

### *User Cost*

The user cost considered in Model II consists of access/egress cost ( $C_a$ ), wait cost ( $C_w$ ), in-vehicle travel cost ( $C_v$ ), as discussed below.

#### *1. Access/Egress Cost*

The access/egress cost  $C_a$  is the product of the value of passenger access time (denoted as  $u_a$ ) and total access/egress time (i.e., sum-product of actual demand multiplied by the associated average access/egress time for all demand pairs). Thus,

$$C_a = u_a \sum_{o \in O} q_o \frac{\exp(-e_t T_o)}{\exp(-e_t \bar{T}_o)} \cdot t_{a_o} \quad (3.31)$$

where  $t_{a_o}$  is the average access/egress time for demand pair  $o$ .

#### *2. Wait Cost*

The wait cost is the product of the value of passenger wait time  $u_w$ , and the total wait time.

The total wait time is the boarding demand at stop  $i$ , denoted as  $b_i$ , multiplied by the associated wait time for all  $i$ . Thus,

$$C_w = u_w \sum_{i \in I} b_i \cdot t_{w_i} \quad (3.32)$$

where  $b_i$  is determined by Equation 3.22 and  $t_{w_i}$  is determined using Equation 3.27.

### 3. In-vehicle Cost

The in-vehicle cost  $C_v$  is the product of the total in-vehicle time and the value of passenger in-vehicle time  $u_v$ . The total in-vehicle time is equal to the actual demand multiplied by the associated in-vehicle time for all demand pairs. Thus,

$$C_v = u_v \sum_{o \in O} q_o \frac{\exp(-e_t T_o)}{\exp(-e_t \bar{T}_o)} t_{v_o} \quad (3.33)$$

where  $t_{v_o}$  is determined by Equation 3.29.

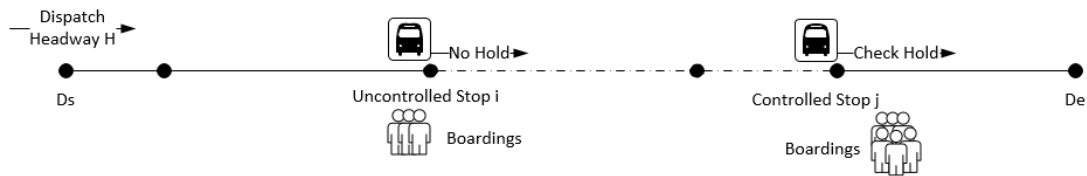
### 3.3 Model III – The Extended Model

With Models I and II, the headway and stop (spacing) of a feeder bus route and a conventional bus system can be optimized to maximize the operator's profit considering travel time variance. However, if the travel time variance over the route is too high to maintain reliable bus operation even with the optimal setting of stop (spacing) and headway, setting a number of control time points would be necessary to improve the service reliability.

Considering stochastic vehicle arrivals, Model III optimizes time points (or control points) based on a headway-based vehicle control strategy for improving system reliability. It could be integrated with Model II to optimize stop, headway and time points for a new bus route. With the availability of real world bus operation data, Model III can also be implemented in a given bus route to find optimal control points for better system performance.

### 3.3.1 Route Configuration

For any bus route with many-to-many demand pattern, assumes buses operate from the origin terminal  $D_s$  to the destination terminal  $D_e$  as shown in Figure 3.4. The length of the route is  $L$ , the set of stops is defined as  $I$  which consists of two sets of stops  $I_c$  and  $I_{uc}$  for controlled and uncontrolled stops, respectively. Other parameters are inherited from Model II, including  $b_i$  as the boarding demand at stop  $i$ .



**Figure 3.4** Model III route configuration.

### 3.3.2 Assumptions

Since this model applies a headway-based control strategy, which assumes that bus headways are available where needed, to incorporate the headway-based control strategy into the time point optimization model, a simulation model is developed to mimic real world bus operations. To set up the simulation, basic assumptions are made and listed below.

1. Passengers' arrival follows a Poisson distribution, where the arrival rate is estimated from given boarding distribution dictated by the stop locations.
2. Passengers' alighting follows a binomial distribution, where the possibility of alighting at each stop is calculated using given boarding/alighting data.
3. Vehicle dwell time is dependent on boarding demand as alighting happens quickly enough to be neglected (Daganzo, 2009).
4. Passengers are not prohibited from boarding into a crowded bus.
5. Bus overtaking is not allowed.

6. Link travel time between any pair of consecutive stops follows a Gamma distribution, where the shape parameter  $k$  and scale  $\theta$  are determined based on mean and variance of link travel time between the pair of stops.

### 3.3.3 Model Formulation

The objective functions considered in Model III include minimizing average operator cost and minimizing average user cost. The bus control strategy applied in the model is described below.

#### *Bus Control Strategy*

Assume that the headways at stops are available when needed. Consider a bus  $m$ , arrives at a control point  $i$  at  $A_i^m$ , with awaiting boarding demand  $b_i^m$ . Its preceding bus departed at  $D_i^{m-1}$ , and the estimated departure time of its following bus is  $D_i^{m+1}$ . If no holding occurs, the headways between bus  $m$  and its preceding/following buses are calculated in the following equations, respectively.

$$h_i^m = T_i^m - T_i^{m-1} \quad (3.34)$$

$$h_i^{m+1} = T_i^{m+1} - T_i^m \quad (3.35)$$

where  $h_i^m$  is the forward headway between bus  $m$  and its preceding bus  $m-1$  without holding,  $h_i^{m+1}$  is the estimated backward headway between bus  $m$  and its following bus  $m+1$ .

Let the headway control strength be  $\beta$ , and  $\beta H$  defines the minimum headway as a basis of a control threshold. Based on the previous studies (Fu and Yang, 2002; Cats et al., 2011, 2012), the bus control strategy applied in this study considers both forward and backward headways to keep even headways while restricting maximum allowable holding time with the minimum headway constraint.

Define the average headway of  $h_i^{m+1}$  and  $h_i^m$  as  $\overline{h_i^m}$ . With  $D_i^m$  representing the actual departure time of bus  $m$ , the control strategy is formulated in Equation 3.36. If stop  $i$  is not a controlled stop, the actual departure time of bus  $m$  equals the sum of bus arrival time and dwell time. Otherwise, the forward and backward headways will be checked for bus holding decision. The bus will be held within a maximum allowable holding range if the forward headway is shorter than backward headway.

$$D_i^m = \begin{cases} \max\{A_i^m + \rho b_i^m, \min(D_i^{m-1} + \overline{h_i^m}, D_i^{m-1} + \beta H)\} & \forall i \in I_c \\ A_i^m + \rho b_i^m & \forall i \in I_{uc} \end{cases} \quad (3.36)$$

Then, the holding time of bus  $m$  at stop  $i$ , denoted as  $\tau_i^m$ , is the actual departure time minus the sum of actual arrival time and the initial dwell time:

$$\tau_i^m = \begin{cases} \max\{0, \min(D_i^{m-1} + \overline{h_i^m}, D_i^{m-1} + \beta H) - (A_i^m + \rho b_i^m)\} & \forall i \in I_c \\ 0 & \forall i \in I_{uc} \end{cases} \quad (3.37)$$

### *Average User Cost*

The average user cost is equal to total user cost divided by total demand of the study period. The total user cost considered here consists of wait cost, in-vehicle cost considering bus holding. Note that the stop locations are given, the access cost is constant and will not affect the optimal solution. Thus, it is not included in the objective function.

#### *1. Wait Time*

For bus  $m$ , a number of boarding passengers,  $b_i^m$ , arrive at stop  $i$  between the departure time of the preceding bus  $m-1$  and the arrival time of bus  $m$ . The average wait time is half the headway, assuming random passenger arrivals. Let  $M$  be the set of buses travelling through

the route during the study period. The total wait time at stop  $i$  is the number of boarding passengers multiplied by the average wait time for all stop  $i$  and bus  $m$ :

$$T_w = \sum_{i \in I} \sum_{m \in M} b_i^m \frac{h_i^m}{2} \quad (3.38)$$

Additional passengers will board on bus  $m$  while the bus is hold at  $i$ , and their wait time is half of the holding time. Let the number of passengers arriving at stop  $i$  during  $\tau_i^m$  be  $x_i^m$ . The total wait time for these passengers is represented as follows.

$$T_x = \sum_{i \in I} \sum_{m \in M} x_i^m \frac{\tau_i^m}{2} \quad (3.39)$$

## 2. In-vehicle Time

Due to bus holding, there will be additional in-vehicle wait time for through passengers travelling from stop  $i$  to  $i+I$ . Let  $r_i^m$  be the through passengers in bus  $m$  at stop  $i$ , the total additional wait time during the study period is represented as follows.

$$T_{vw} = \sum_{i \in I} \sum_{m \in M} r_i^m \tau_i^m \quad (3.40)$$

Let  $t_i^m$  be the link travel time for bus  $m$  from stop  $i$  to stop  $i+I$ , since through passengers at stop  $i$  is  $r_i^m$  and passengers arriving during bus holding is  $x_i^m$ , the total link travel time is calculated as follows.

$$T_v = \sum_{i \in I} \sum_{m \in M} (r_i^m + x_i^m) t_i^m \quad (3.41)$$

Therefore, as defined earlier, the average user cost is the ratio between total user cost and total demand. Total user cost is the sum of user in-vehicle cost and wait cost. Total demand consists of initial boarding demand ( $b_i^m$ ) and additional boarding passengers ( $x_i^m$ ) during holding. The formulation is given in Equation 3.42.



$$c_u = \frac{\mathbf{u}_v \sum_{i \in I} \sum_{m \in M} \left( (r_i^m + x_i^m) t_i^m + r_i^m \tau_i^m \right) + \mathbf{u}_w \sum_{i \in I} \sum_{m \in M} \left( b_i^m \frac{h_i^m}{2} + x_i^m \frac{\tau_i^m}{2} \right)}{\sum_{i \in I} \sum_{m \in M} (b_i^m + x_i^m)} \quad (3.42)$$

### ***Average Operator Cost***

The total operator cost is the total bus operating hours multiplied by the unit operating cost ( $\mathbf{u}_b$ ). The time components involved in the controlled operation include link travel time, holding time at controlled stops, and recovery time at the destination terminal. Link travel time includes dwell time for passenger boarding/alighting, acceleration/deceleration delay, and intersection delay. Therefore, the average operator cost, which is the total operator cost divided by the total demand during the study period, is formulated as follows.

$$c_b = \frac{\mathbf{u}_b \left( \sum_{i \in I} \sum_{m \in M} (t_i^m + \tau_i^m) + t_i^m \right)}{\sum_{i \in I} \sum_{m \in M} (b_i^m + x_i^m)} \quad (3.43)$$

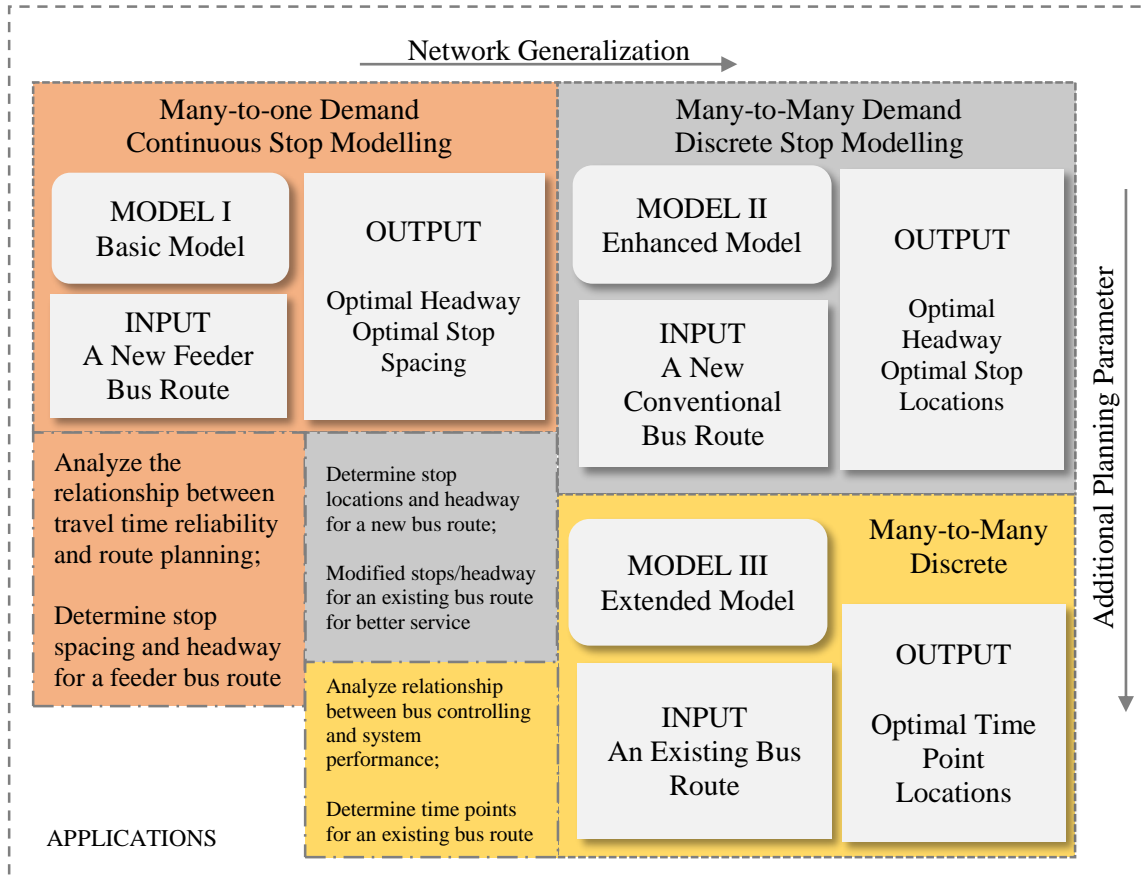
where  $t_i^m$  is the recovery time for bus  $m$ .

To summarize, the objective functions are minimizing average operator cost and minimizing user cost. The decision variables considered in the model are the number and locations of time points. A simulation model is integrated in the optimization model for mimicking real world bus operations and providing real-time headway information for applying the control strategy. A simulated-based multi-objective GA is applied to solve the model and strike a balance between two conflicting objectives.

## **3.4 Summary**

Three models are developed in this chapter, where the first proof-of-concept model is applied to investigate the influence of headway variance on the optimal service planning

in a simplified network. The second model applies to a more general bus route, and the final model optimizing time points for service reliability improvement. The relationships between the three models are represented in Figure 3.5.



**Figure 3.5** Model descriptions and applications.

For a feeder bus route with many-to-one/one-to-many demand pattern, Model I optimizes stop spacing and dispatching headway for profit maximization with the consideration of headway variance and travel time elasticity of demand. Since both headway and stop spacing are continuous decision variables, the problem is categorized as constrained nonlinear multivariable minimization problem. It is found that the optimal stop spacing is represented as a function of headway and the optimal headway is also related to

stop spacing. The consideration of demand elasticity with respect to travel time makes the problem more complicated to solve with traditional derivation methods. A single-objective Genetic Algorithm is applied to find an optimal/near-optimal solution of the problem. Model I applies to a simplified network, which emphasizes the analysis of the relationship between travel time variability and planning of stop spacing and headway.

For a conventional bus route with many-to-many heterogeneous demand pattern, Model II optimizes stop locations and dispatching headway for profit maximization with a more generalized headway variance model. The discrete modelling approach of stops and continuous variable of headway makes it a mixed-integer, non-linear programming problem, which is difficult to solve with traditional exact optimization methods. A single-objective Genetic Algorithm is applied to solve the problem. Model II can be used to determine optimal stop locations and headway for a new conventional bus route. It also applies in an existing bus route for modifying headway and stop location for better system performance.

For a given conventional bus route with predefined dispatching headway and stop locations, Model III optimizes the number of time points to strike a balance between two conflicting objectives: minimizing average user cost and minimizing average operator cost. Since it is a simulation-based optimization problem with multiple objective functions, a dedicated simulation model is integrated in a multi-objective Genetic Algorithm. Model III applies to an existing bus route for vehicle control point optimization to improve service reliability. Also, it is flexible to be integrated with Model II for simultaneously optimizing stop location, headway, and time points for a new bus route.

## CHAPTER 4

### SOLUTION ALGORITHMS

As discussed in Chapter 3, it is tedious to optimize the developed models with traditional exact algorithms. Heuristic/metaheuristic algorithms have been applied to solve similar transit network design problems, among which Genetic algorithm (GA) has been proved as an effective algorithm and commonly used to solve transit planning problems (e.g., Chien et al., 2001; Chakroborty, 2003; Fan and Machemehl, 2006). GA-based algorithms are applied to solve the developed models, which are discussed in this chapter.

#### 4.1 Single-Objective GA

GA is a search heuristic mimicking the process of natural selection, which is applied to find optimal or near optimal solutions of optimization and search problems (Mitchell, 1998). It adopts the concept of ‘survival of the fittest’, and involves mutation and reproduction in the selection process. In general, the proposed GA starts from generating an initial collection of possible solutions of an optimization problem, which is called a population. Each possible solution (or individual) is represented by a chromosome in the initial population. The fitness of each individual is evaluated in each iteration, which is usually represented by the objective value of the study optimization problem. Then, next generation population will be generated through a combination of genetic operators (i.e., selection, crossover, and mutation). The iteration process is repeated until a termination condition has been reached, i.e., no further improvement is observed, fixed

number of generations or maximum computation time has been reached, the search process with GA will be terminated and the final best individual will be recorded as the optimal solution. The below steps explain the procedures commonly involved in a GA.

1. Initialization

GA searches for the optimal solution directly from the solution pool, therefore, the very first step is generating initial population with a set of possible solutions. Each solution (or individual) is coded and represented by a chromosome.

2. Evaluation

To start the selection process, each chromosome in the population is evaluated against a fitness function, which is usually the objective function in the original problem.

3. Evolution

- a. After evaluation of the population, some individuals with best fitness values are chosen as elite, which are preserved in the next generation.
- b. Other individuals of the next generation (i.e., children) are produced from a set of selected members from the population (i.e., parents) through two operators: mutation and crossover, which will be discussed later.

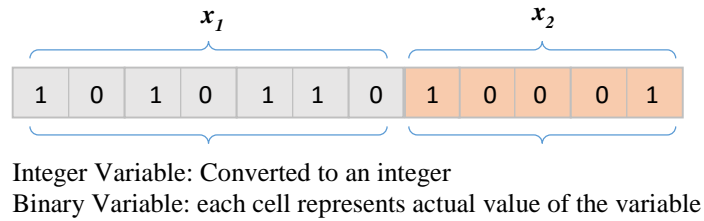
4. Termination Check

Stop GA if any of termination criteria is satisfied, and record the final best solution as the optimization result. Otherwise, go to Step 2 for further evolution. Common termination criteria include 1) solution being found 2) no further improvement being observed 3) fixed number of generations being reached 4) allocated computation time being reached.

### Initialization

The binary representation of the decision variables can be categorized into two groups, encoding integer variables such as number of stops (converted from stop spacing) and headway (in minutes), and encoding binary variables such as stop locations and time points.

The representation is illustrated in Figure 4.1.

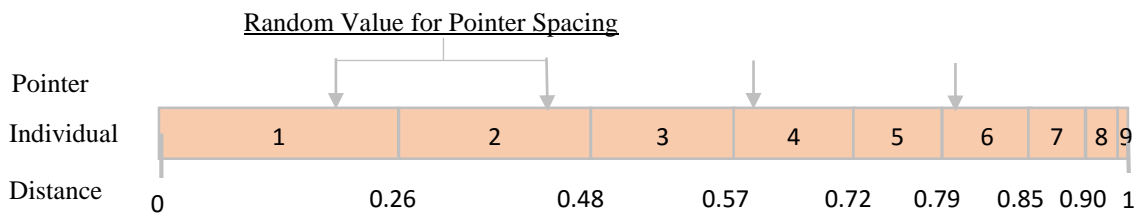


**Figure 4.1** GA binary encoding.

Evaluation and Selection

From one generation, some members of the population will be selected to produce offspring based on the fitness values. Commonly used selection functions including stochastic universal sampling, roulette wheel selection, and tournament selection. In this study, the stochastic universal sampling method is applied. With this method, the individuals are mapped to continuous segments of a line where the length of each segment is equivalent to the individual's fitness value. Uniformly distributed pointers are placed over the line, whose number equals to the total number of individuals to be selected.

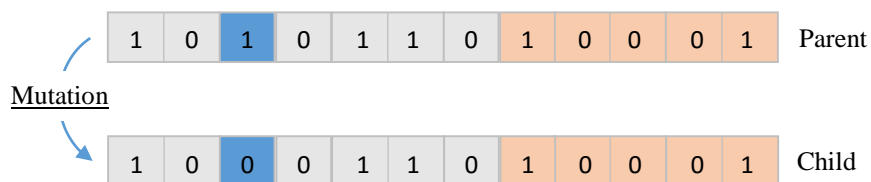
Assume that a total of 9 individuals in the population, 4 individuals are to be selected, which leads to the upper bound of  $\frac{1}{4}$  for a random number as selection spacing. If the number 0.2 is randomly generated from the range  $[0, 0.25]$  as the spacing, a pointer will be placed every 0.2 to select the individuals. The selection process is illustrated as in Figure 4.2.



**Figure 4.2** GA selection.

## Mutation

Mutation is one of two operators that produce offspring through operations of the selected parents. It takes one parent to generate one child. With the binary encoding, the mutation operates in one bit, whose location is determined by the algorithm used for mutation. By converting that one bit from 1 to 0 or vice versa, the value of variable in the offspring is changed. Figure 4.3 provides an example of mutation in the selected parent.



**Figure 4.3** Mutation operation.

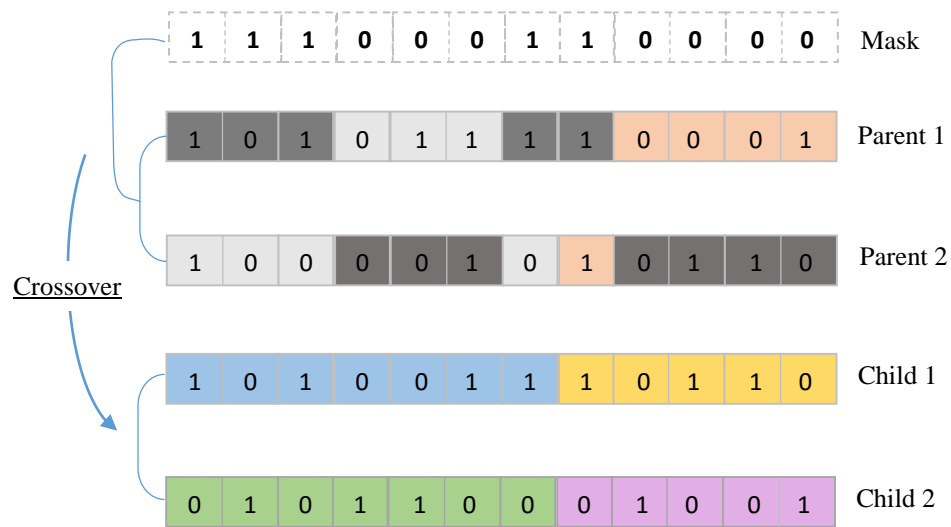
## Crossover

Crossover is another operator to produce offspring from the parents. Unlike mutation, it operates in two parents and creates two children. Commonly used methods include single-point, two-point, scattered crossover. The methods are similar in their logic but different in the points where the crossover happens. Figure 4.4 provides an example of scattered crossover.

## 4.2 Multi-Objective GA

In the context of multi-objective optimization, more objectives are involved which are often conflicting with each other. In this case, one extreme solution may not satisfy both objective functions, since the optimal solution for one objective may not necessary be the optimal for another. The concept of domination is used in the optimization, where a feasible solution is said to dominate another feasible solution (or non-dominated by another

solution) if this solution is no worse than the other solution with respect to all objective values and strictly better in at least one objective value. In a bi-objective optimization problem, the best solution would be a non-dominated set of solutions (i.e., Pareto front), where no improvement can be made in one objective without worsening another objective. Therefore, the ultimate goal of a solution algorithm for multi-objective problems is to find the Pareto Front.



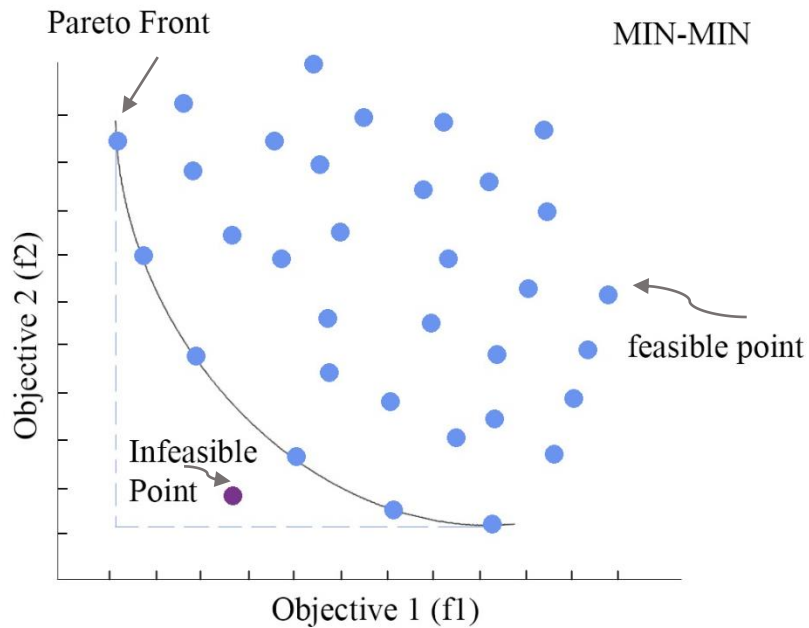
**Figure 4.4** Crossover operation.

The multi-objective solution algorithm applied in the dissertation is a controlled elitist GA based on the Non-Dominated Sorting GA II (NSGA-II) developed by Deb (2001), where detailed explanation of this algorithm is presented. Deb (2001) assumes two goals in the multi-objective optimization: to find a set of non-dominated solutions with least distance to a true Pareto-optimal set and to maximize the diversity in the non-dominated solution set. In the process of evolution, the controlled elitist GA favors individuals which could represent both lateral diversity and diversity along a Pareto front. In this case, some individuals with lower fitness values may also be preserved in the offspring if they could



help increase the diversity of the population. Through controlling the elitism, this algorithm reduces the probability of convergence to a suboptimal solution set and guarantees diversity and spread of the optimal solutions.

An illustration of Pareto front is shown in Figure 4.5, where two minimization objectives are involved in the optimization problem (i.e., a MIN-MIN problem). The blue dots represent feasible solutions for the problem, and the Pareto front is represented by the curved line. In this study, two objectives including minimizing average user cost and minimizing average operator cost are applied, and the solution algorithm tries to find a set of decision variables (e.g., stop locations or time points) which achieves Pareto optimality between users and the operator.



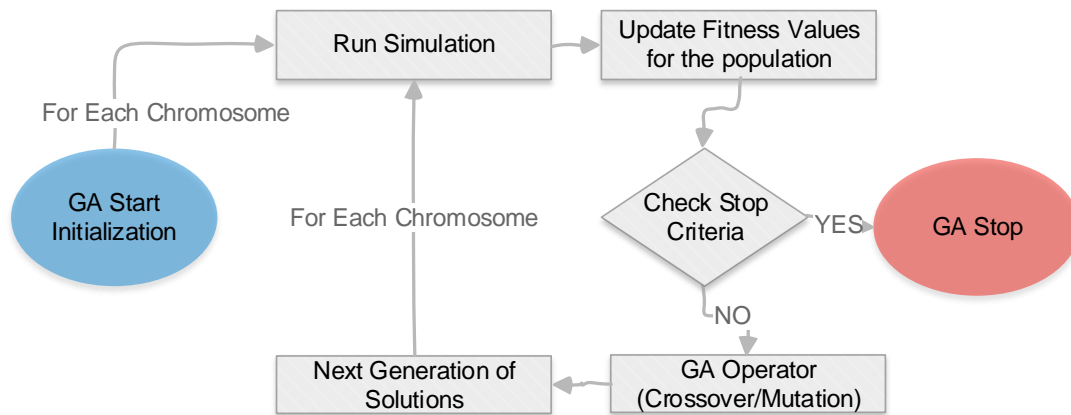
**Figure 4.5** The Pareto front in a multi-objective solution pool.

### 4.3 Simulation-based GA

The simulation-based multi-objective GA is applied to solve Model III, where the objective values are estimated with a simulation model. The framework is represented in Figure 4.6. The solution procedure starts with population initialization (i.e., the first generation) with GA, where a set of possible solutions is generated randomly. In the algorithm, the stop locations are coded in binary, where the location of control point is marked as '1', and all other stops are marked as '0'. For example, if the number of stops of a bus route is 10 and only one control point is designated at the fourth stop, then this solution will be coded as '0001000000'.

Each solution in the initialized population will be transmitted to the simulation model as an input. For each solution generated from GA, a set of simulation runs with different random seeds will be conducted to generate the expected value for each of the objectives. Then, the expected values are sent back to GA as the fitness value of that solution. That is, the simulation model takes a set of control points as the input, and provides the average user and operator costs as the outputs to be sent back to GA.

With the fitness values of the population estimated by the simulation model, the GA process continues with genetic operators (i.e., crossover and mutation) to generate the offspring as the next generation. The solutions keep evolving until any of the stop criteria is satisfied. Then, the GA stops, and the best solution in the last generation will be recorded as the optimal solution.



**Figure 4.6** Simulation-based genetic algorithm framework.

The flow chart of the simulation model is shown in Figure 4.7. It runs from time 0 till a pre-defined time with 1-second update interval, and includes three major functions for bus simulation, which are explained below. When the simulation ends, besides the average user and operator costs, it also provides vehicle trajectories and stop records for further evaluation.

### 1. *Bus dispatching check*

At each time step, the simulator will check whether it is time to dispatch an outbound bus according to the pre-defined headway. If the bus number is less than the fleet size, it means that the dispatching is able to maintain the scheduled headway, a new bus will be dispatched with an assigned ID. When the bus number exceeds the fleet size, headway checking will be skipped since the dispatching is dependent on the returning bus from inbound direction. Therefore, besides headway checking, bus arrival time at the outbound dispatching terminal will also be checked at each time step for new bus dispatching, and same check is conducted at the inbound dispatching terminal.

### 2. *Stop handling*

When a bus arrives at a stop (i.e., current time step equals to any bus arrival time at next stop), its arrival time, alighting passengers and current boarding passengers waiting at the stop are recorded immediately. If this stop is not a control point, the departure time of the bus will be current time plus dwell time due to passengers'

boarding/alighting activities, and the average waiting time for passengers at this stop will be half the headway.

If the stop is a control point, the holding time will be checked based on the pre-defined holding strategy, which will in turn lead to updated boarding passengers and average waiting time. The departure time will be current time plus dwell time and holding time.

After calculating the departure time, the link travel time to next stop will be generated based on the assumed travel time distribution. Then, an expected arrival time at next stop as well as average passenger in-vehicle time will be recorded.

### ***3. Terminal handling***

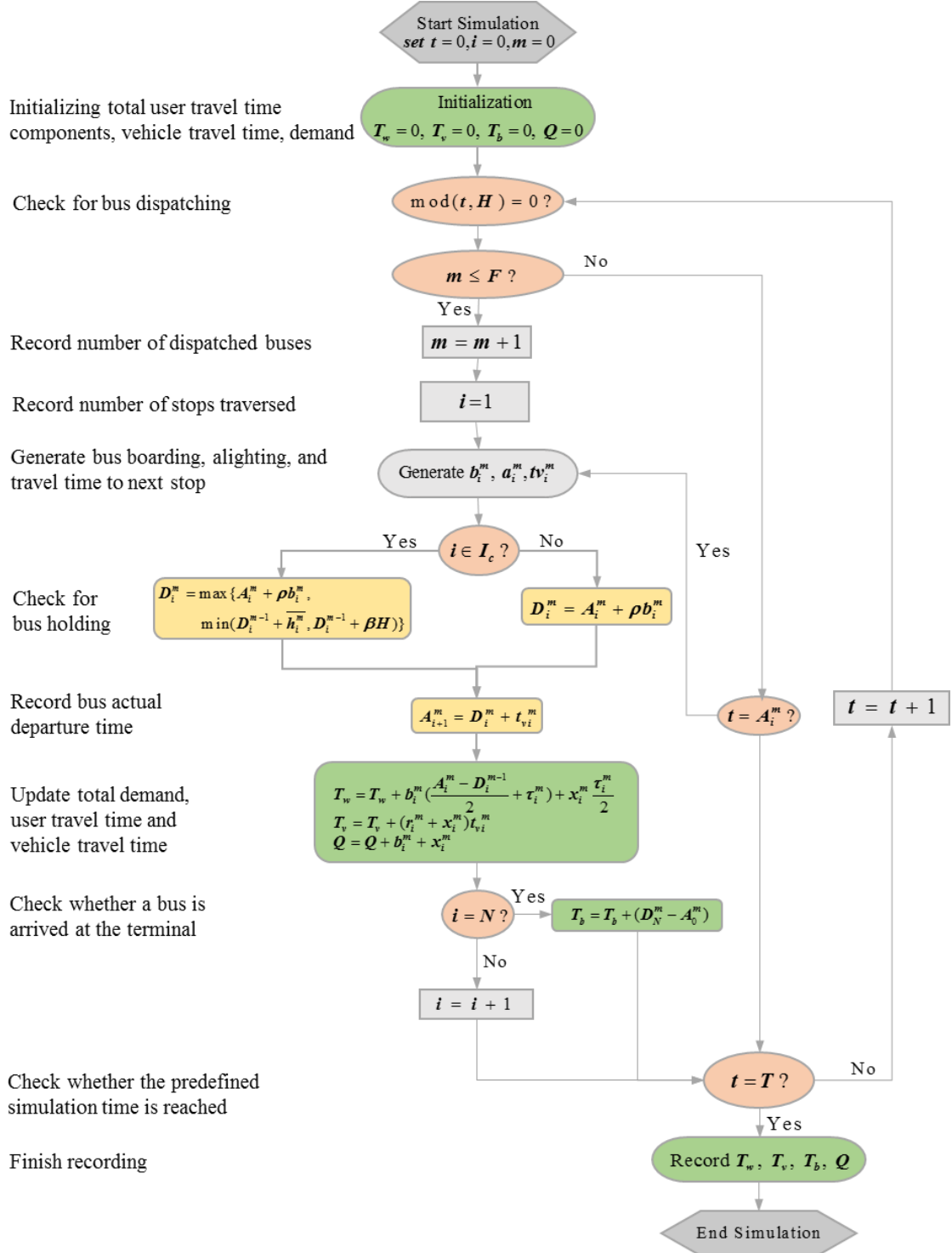
When the first outbound bus gets to the destination, a recovery time will be assigned, which is related to route travel time variability. After that, this bus will be dispatched for inbound service. For all the buses travelling in the network, if there is a record showing 'next stop' is the inbound destination terminal (i.e., outbound origin terminal), a new bus is ready to be added in the outbound bus pool, whose dispatching time is either the estimated arrival time of the corresponding inbound bus plus the dwell time for passenger handling, or one headway plus departure time of last bus, whichever is bigger.

## **4.4 Summary**

Three GA-based algorithms were explained in this chapter, including single-objective, multi-objective and simulation-based GA.

The single-objective GA fits the optimization problem developed in Models I and II, with the objective function of maximizing profit, whereas the multi-objective GA is applied to solve the problems in the sensitivity analysis for analyzing the trade-off between objectives of users and the operator. The simulation-based GA was developed to solve Model III for optimizing time points with a headway-based control strategy. The solution algorithms applied in the dissertation are coded in Matlab. With the developed models and

algorithms, case studies are conducted and presented in Chapter 5 for investigating model capability and effectiveness.



**Figure 4.7** Simulation flow chart.

## **CHAPTER 5**

### **CASE STUDIES**

This chapter is assessing the three models developed in Chapter 3 in three case studies, respectively. The single-objective GA is applied to solve Model I, which is presented in Section 5.1 – Case Study I. The trade-off between the objectives for the operator and users is analyzed in the sensitivity analysis. Similarly, case study II is discussed in Section 5.2, which investigates the applicability and capability of Model II. Case study III is presented in Section 5.3, where the optimization results of Model III solved by the simulation-based multi-objective GA are discussed. Finally, the findings of case studies are summarized.

#### **5.1 Case Study I**

A bus transit route of 5.15-mile-long in Chengdu, China is applied to demonstrate the applicability of the developed model. The actual many-to-many demand was collected for the study route during the morning peak period, with the directional demand of 125 pass/hr outbound and 23 pass/hr inbound. Since Model I applies to a feeder bus route with many-to-one/one-to-many demand pattern, the demand is assumed uniformly distributed along the route with hourly demand of 148 pass/hr.

The input parameters used in the analysis are given in Table 5.1. The average vehicle cruising speed is 25 mph and the average operating cost is 40 \$/bus-hr. The dispatching headway variance at the terminal is assumed to be zero. Average acceleration/deceleration delay and average passenger boarding time are 10 seconds and 6

seconds, respectively. The average access speed of passengers to the bus route is 2 mph. The values of access time, wait time, and in-vehicle time are 8 \$/pass-hr, 16 \$/pass-hr, and 8 \$/pass-hr, respectively. To evaluate the impact of headway variance to stop spacing and headway, three scenarios are defined:

*Scenario 1:* Headway variance is null, subject to deterministic bus arrivals at stops

*Scenario 2:* Actual profit for *Scenario 1* when implemented in the route with headway variance increment per stop of 1 min<sup>2</sup>/stop.

*Scenario 3:* Headway variance is a linear monotonic function which increases as the number of bus stops increases. The headway variance increment per stop is assumed as 1 min<sup>2</sup>/stop.

**Table 5.1** Model Parameters and Baseline Values

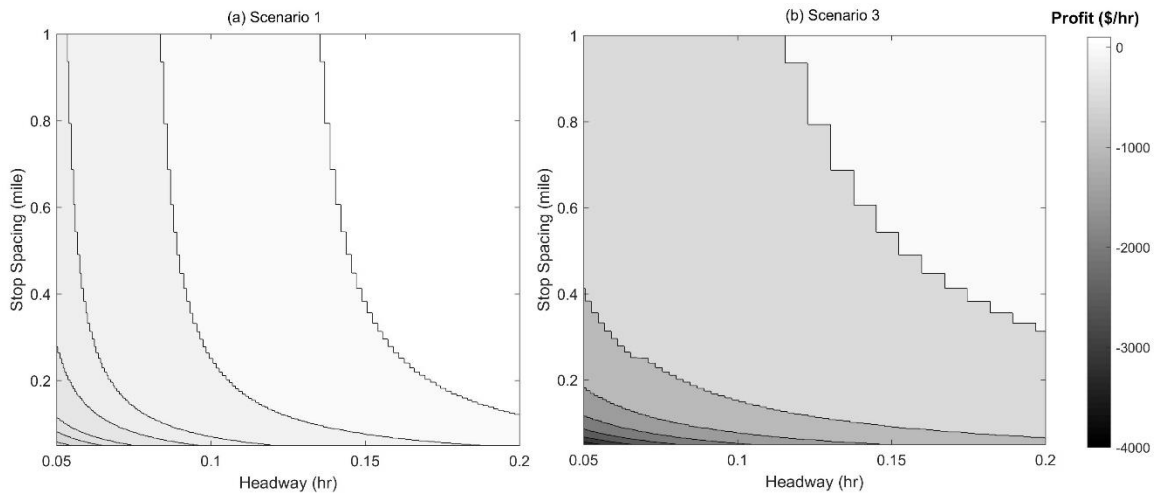
Parameter	Definition	Unit	Value
$C$	Bus capacity	spaces/bus	60
$d_s$	Average acceleration/deceleration delay per stop	sec	10
$L$	Route length	mi	5.15
$Q$	Potential route demand	pass/hr	148
$u_a$	Value of access time	\$/pass-hr	8
$u_v$	Value of in-vehicle time	\$/pass-hr	8
$u_b$	Average bus operating cost	\$/bus-hr	40
$u_w$	Value of wait time	\$/pass-hr	16
$V_b$	Average bus speed	mph	25
$V_p$	Average walking speed	mph	2
$c$	Dispatching headway variance	min <sup>2</sup>	0
$\alpha$	Increment of headway variance per stop	min <sup>2</sup> /stop	1
$\rho$	Average boarding time per passenger	sec	6
$f$	Fare	\$/trip	1.5
$e_t$	Coefficient of travel time		0.03

### 5.1.1 Optimization Results

The total profits with respect to stop spacing and headway under Scenarios 1 and 3 are illustrated in Figure 5.1 a and b, respectively. When headway and stop spacing increases,

the operator profit increases. It is noticed that when the headway variance is not considered (Scenario 1), the profit tends to be overestimated, and the changes of profit with stop spacing and headway is not significant. As shown in Figure 5.1 a, the range of profit is considerably small when the headway and stop spacing vary from their minimum values to the maxima.

In contrast, for Scenario 3 as shown in Figure 5.1 b, because of the influence of headway variance, the trend of profit with stop spacing and headway is obvious. For instance, under low stop spacing and low headway (i.e., high operator cost), although access time is short for the passengers, high variance in travel time lead to demand decrease due to travel time elasticity. Therefore, the profit is significantly small (always negative) when the headway and stop spacing are small, reflecting unrealistic configurations in terms of profit.



**Figure 5.1** Profit vs. stop spacing and headway under different scenarios.

The analysis of profit also indicates that the objective functions in both Scenarios 1 and 3 are concave. Many near-optimal solutions are available since the objective value (operator's profit) increases slowly near the optimized values of the decision variables. The



major implication is that minor justification of optimized decision variables to accommodate geographic and service constraints will not significantly affect the objective value. The result is similar to the findings in the previous study conducted by van Nes and Bovy (2000).

The optimization results under each scenario are listed in Table 5.2. Their average operator cost per passenger as well as average total cost per passenger are also calculated to reflect system cost effectiveness. As indicated in the table, the optimized headway and stop spacing are smaller under null headway variance scenario (Scenario 1). However, when the headway and stop spacing under Scenario 1 are implemented in a route where bus operations are easily interrupted by traffic, significantly increased user cost and operator cost due to underestimation of travel time variation can be observed (Scenario 2). Especially, the fleet size estimated in Scenario 1 is not enough to meet the demand requirement due to service fluctuation, which leads to fleet size increase and ultimately lower-than-estimated profit (-21 \$/hr under Scenario 2 vs. 62 \$/hr under Scenario 1).

Under Scenario 3, the optimized number of stops is less but the optimized headway is greater than those optimized under Scenario 1, respectively. Since the wait time and in-vehicle time significantly increase with the headway variance over the route, less stops could lead to shorter expected travel time (i.e., more demand). Compared with Scenario 2, the required fleet size is reduced through optimized configuration of stop spacing and headway with the consideration of variation along the route.

Therefore, although the initial results from a traditional model without considering headway variance (Scenario 1) seems appealing (e.g., high projected demand, low operating cost, high profit), it turns out to be a poor planning if implemented in the actual

route (Scenario 2). On the contrary, through taking into account the fact of headway variance, the proposed model reflects the actual route situation and provides a more reliable projection of both demand and profit. As a result, the system recommended from the proposed model (Scenario 3) perform better in terms of both average total cost and average operator cost compared with that under Scenario 2.

**Table 5.2** Optimization Results under Different Scenarios

Variable	Scenario 1	Scenario 2	Scenario 3
H (min)	10.4	10.4	12.0
S (mi)	0.37	0.37	0.47
n	14	14	11
Fleet Size	4	6	5
$C_w$ (\$/hr)	205	214	240
$C_A$ (\$/hr)	55	55	68
$C_I$ (\$/hr)	156	154	149
$C_U$ (\$/hr)	417	423	458
$C_O$ (\$/hr)	160	240	200
Demand (pass/hr)	148	146	146
Revenue (\$/hr)	222	219	219
Profit (\$/hr)	62	-21	19
Average Operator Cost per Passenger (\$/hr-pass)	1.1	1.6	1.4
Average Total Cost per Passenger (\$/hr-pass)	3.9	4.6	4.5

### 5.1.2 Sensitivity Analysis

Although in the previous section, a set of optimization solution is provided which achieves the maximum profit, such solution may not be the most favorable one for the passengers. Often, if there is no hard constraint on the operator cost, the exploration of other alternate solutions is desirable from the standpoint of transit users.

### *Trade-off between the Operator's and Users' Objectives*

To further investigate possible alternatives and the trade-off between the objectives of the operator and passengers under different scenarios, the average operator cost and average user cost as defined in Chapter 3, are applied for monitoring system efficiency and attractiveness. For each scenario to be analyzed (e.g., a different demand level), two objectives, namely minimizing average operator cost and minimizing average user cost, are applied to find the optimization solutions. As discussed in Chapter 4, the set of optimization solutions of this multi-objective optimization problem forms a Pareto front (i.e., Pareto optimality), which is also called a trade-off curve for the conflicting objectives. Such optimality is a state of resource allocation where it is impossible to make any single objective better off without making others worse off. Therefore, any solution in the curve satisfies the criteria that it is no worse than the other solution with respect to all objective values and strictly better in at least one objective value.

Shown in Figure 5.2 are the trade-off curves between these two objectives achieved by a set of optimized stop spacings and headways under different scenarios. Same as the definition in Section 5.1.1, let Scenario 1 represent the model results with null-headway-variance, Scenario 2 be the actual results for Scenario 1, and Scenario 3 represent the proposed model results with headway-variance.

The curves for Scenarios 1 and 3 are the Pareto fronts solved with respective models, whereas the curve for Scenario 2 consists of the actual costs if implementing the optimization results from Scenario 1. Similar to the findings discussed in Section 5.1.1, ignoring the impact of headway variance results in underestimated average user cost and

average operator costs with the optimized stop spacing and headway of Scenario 1 (refer to results of Scenario 2).

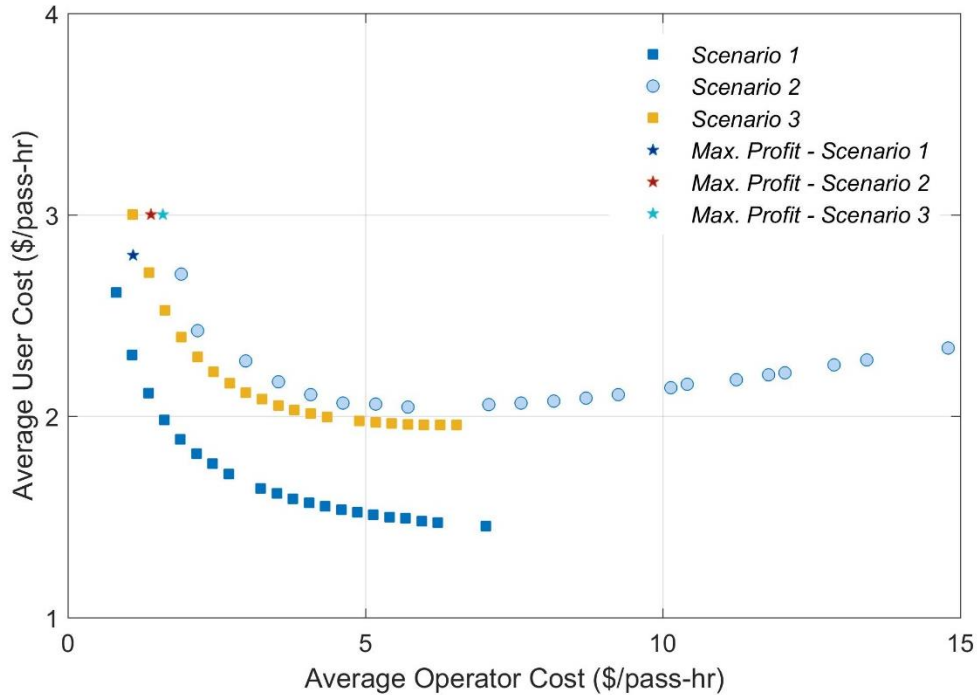
Considering the impact of headway variance (Scenario 3), both of the average user and operator costs yielded by optimized stop spacing and headway increase compared to those under Scenario 1. However, since the proposed model did consider the stochastic nature of bus operations, as shown in Figure 5.2, better planning of stops and headway effectively lower the average operator and user costs (as compared to Scenario 2).

Also included in Figure 5.2 are the optimization results with maximum profit objective functions for these scenarios as presented in Table 5.2. It should be noted that, comparing with other alternate solutions achieving Pareto optimality, the average user costs are higher for all scenarios with the optimization results yielding maximum profit. Therefore, if average operator cost could be slightly adjusted, other configurations of headway and stop spacings may be considered to lower the user cost, according to the trade-off curve.

The boxplots of corresponding profits are represented in Figure 5.3, which show the descriptive statistics of the profits calculated from the optimization solutions for each scenario. The upper bound and lower bound of the box are the third and first quartiles of the profits, with the box representing the likely range of variation. The black bars above and below the box define the full range of variation (i.e., the maximum and minimum values, respectively), and the red line inside the box indicates the median value of the profit.

Although revealed from Figure 5.2 that average user cost and average operator cost under Scenario 1 and Scenario 3 are significantly different, there is not much difference in terms of yielded profit. Thus, in the multi-objective optimization model, the operator's

profit is not as emphasized as in the single objective (i.e., profit maximization) model. However, comparing the profits under Scenario 1 and Scenario 2, the overestimation of profit with a traditional model ignoring the impact of headway variance can still be observed.

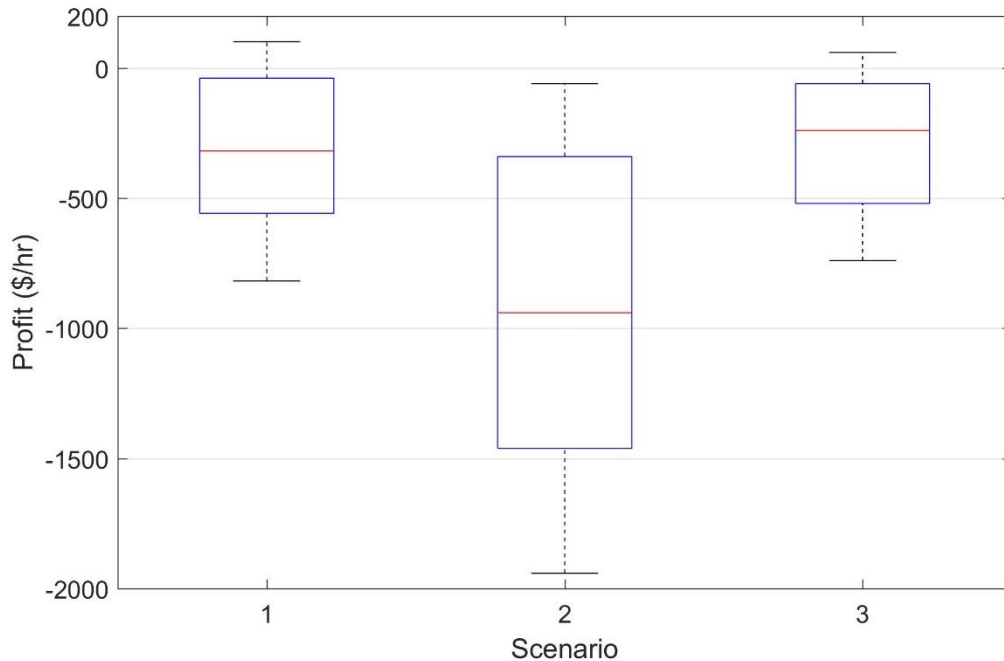


**Figure 5.2** Average operator cost vs. average user cost under different scenarios.

The boxplots of corresponding optimized stop spacings and headways for Scenarios 1 and 3 are plotted in Figure 5.4. In the figure, the box shows the descriptive statistics of the values for each variable (i.e., stop spacing, headway) for each scenario.

To achieve the Pareto front for the average user and operator costs considering headway variation, both of the optimized headways and stop spacings are recommended to be increased (Scenario 3). Especially, under Scenario 3, short stop spacing is considered not appropriate in terms of profit maximization due to accumulated headway variance along the stops over the route, and all values are higher than 0.4 mile. A t-test is conducted

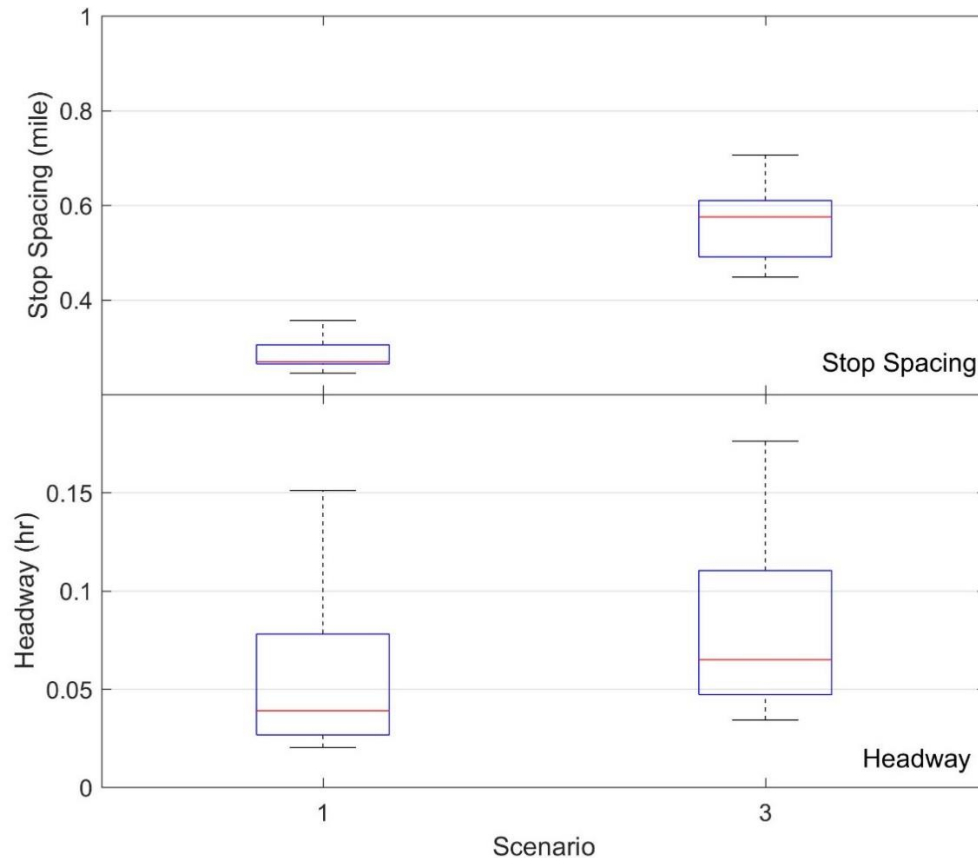
to evaluate the statistical significance of the difference between the optimized stop spacings for Scenarios 1 and 3. With null hypothesis of equal mean and a confidence level of 0.9, the result shows there is a statistically significant difference between two set of stop spacings.



**Figure 5.3** Profits under different scenarios.

Looking into the optimized headways under both scenarios, it is found that the median headway under Scenario 3 is higher than that under Scenario 1, and more headways are distributed near the upper bound (i.e., 0.2 hr). Due to wide spread of headways and close distance between two median values, T-test is conducted to investigate whether such difference is statistically significant. The same null hypothesis and confidence level discussed above is set, and the result suggests that null hypothesis can be rejected. Therefore, two set of headways under Scenarios 1 and 3 are considered as significantly different with 90% confidence level.

Therefore, it is proved that incorporating headway variance in the optimization model has significantly changed the configurations of stop spacing and headway for a given feeder bus route.



**Figure 5.4** Boxplots of stop spacing and headway for scenarios 1 and 3.

### *Influence of Model Parameters*

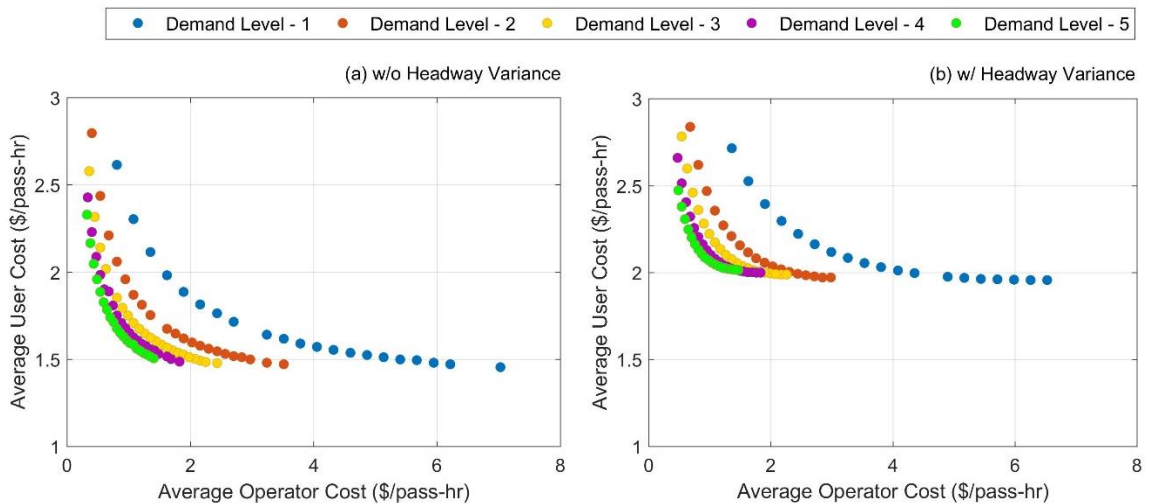
Concluded from the above analysis that headway variance does have an influence on the objective values as well as on the optimized stop spacing and headway. Furthermore, statistically, such significant influence cannot be neglected in the planning process. Since such findings come from only one specific set of model parameters, sensitivity analyses by varying route design parameters (i.e., demand level, headway variance increment) are also

conducted to assess the result differences from Model I and the traditional model without considering headway variance.

### *Demand Level*

The responses of objectives to demand level changes are analyzed for the models without and with headway variance, as shown in Figure 5.5 a and b, respectively. Demand level – 1 represents the base condition, while demand level 2 is two multiplied by the base demand and so on.

It was found that with both models, when the demand level increases, the average operator costs yielded by the Pareto-optimal set are reduced. The closer curves under high demand levels with headway variance reflect that there are lower bounds towards which the average user cost and operator cost could be reduced when the demand level increases. Comparing Figure 5.5 a and b under a same demand level, it was found that no significant difference is revealed in the range of average operator cost, while it is obvious that the average user costs are higher when the headway variance is considered.



**Figure 5.5** Average user and operator costs vs. demand level.



The corresponding optimized headways and stop spacings are plotted in Figures 5.6 and 5.7, respectively. Shown in Figure 5.6 are headways from the Pareto-optimal set for both the proposed model considering headway variance and the traditional model not considering headway variance under different demand levels. It is observed that without considering headway variance, the median value of headways is similar for different demand levels. In contrast, considering headway variance, the median of headways reduces while the range shrinks significantly, revealing that with the proposed model, the optimized headways are more sensitive to demand changes.

Similarly, the boxplot of optimized stop spacings with the traditional model without headway variance (Figure 5.7 a) as well as the results from one-way Analysis of Variance (ANOVA) indicate that no significant differences exist in the mean and variance of stop spacings among different demand levels. With the proposed model, the optimized stop spacings increase significantly, indicating that short spacing is not recommended due to stochastic vehicle arrivals. Also, the differences in mean and variance of stop spacings among demand levels are revealed based on the results of statistical analysis.

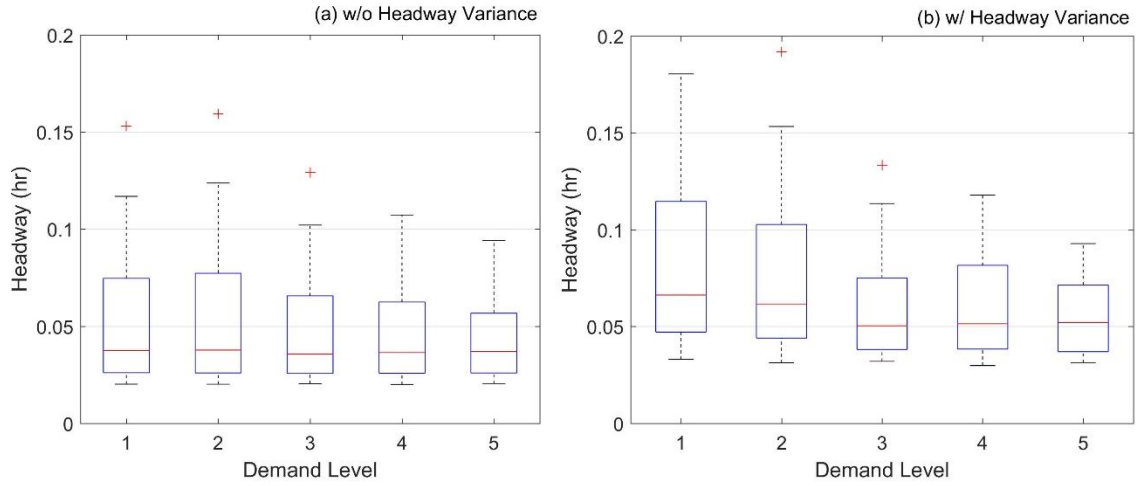
In summary, based on the analysis of trade-off curve and corresponding Pareto-optimal set, when the headway variance is considered in the optimization model, longer headways and stop spacings are recommended. Also, the proposed model with consideration of headway variance is more sensitive to demand changes. Considering the steep curves under high demand levels (Figure 5.5) and the shallow ranges of optimized headways and stop spacings (Figures 5.6 and 5.7), it is reflected that a small change in the headway could lead to a significant increase in the average user cost. Therefore, it is

especially critical to choose proper stop spacing and headway for highest cost effectiveness and system attractiveness under high-demand condition.

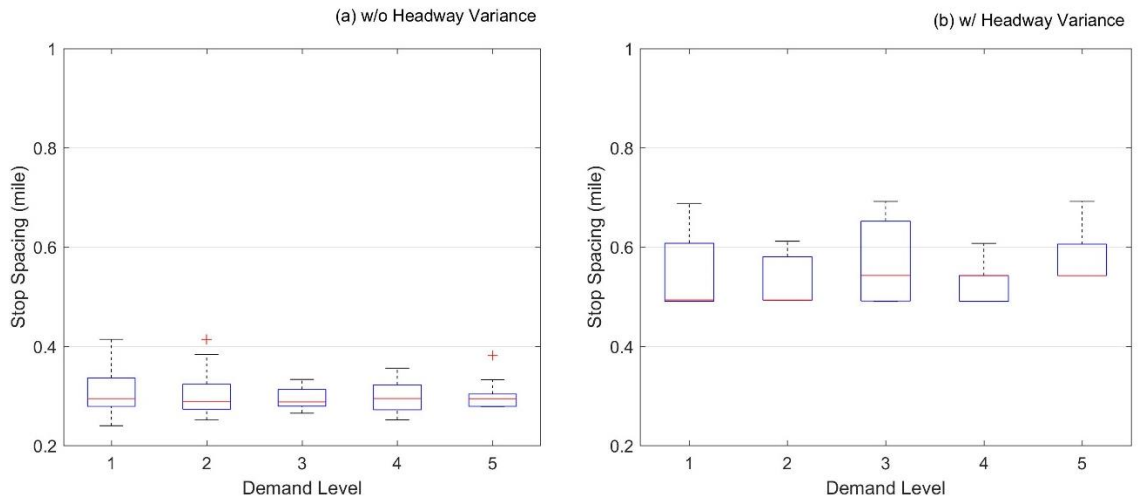
#### *Headway Variance Increment*

When traffic congestion gets worse, the headway variance of bus operations is expected to be higher due to such influence. Therefore, it is necessary to understand how the stop spacing and headway should be adjusted if the variance is increased. Figure 5.8 illustrates the optimized objective values vs. headway variance increment (i.e.,  $\alpha$ ). While the average user cost will inevitably increase due to large service interruption, the model suggests that stop spacing and headway should be increased in response to the change for more cost-effective and attractive operation.

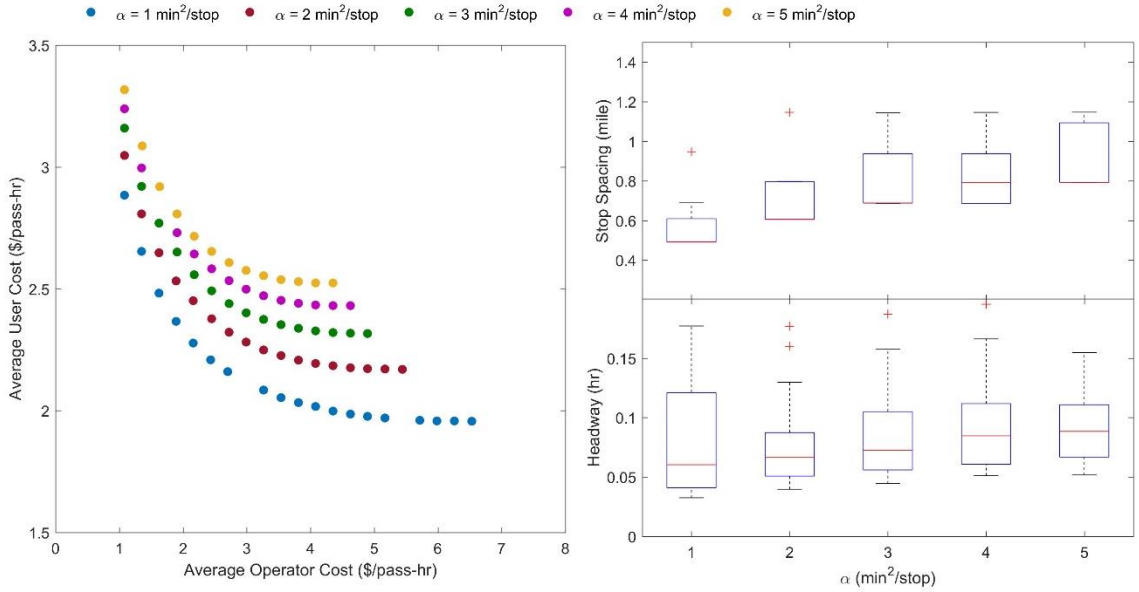
Decomposing the average user cost into average access cost, wait cost, and in-vehicle cost, the trend for each cost component is illustrated in Figure 5.9. It is found that when the headway variance is considered in the model, the impact of large traffic disturbances on the average in-vehicle costs could be neglected with rearranged stop spacings and headways (Figure 5.9 b). However, the average wait cost, majorly affected by headway variance, is inevitably increased due to increased headway variance (Figure 5.9 c). Due to widened stop spacings in response to increased headway variance, the average access cost also rises (Figure 5.9 a). As a result, as shown in Figure 5.9 d, the overall average user cost significantly increases with the increase of headway variance.



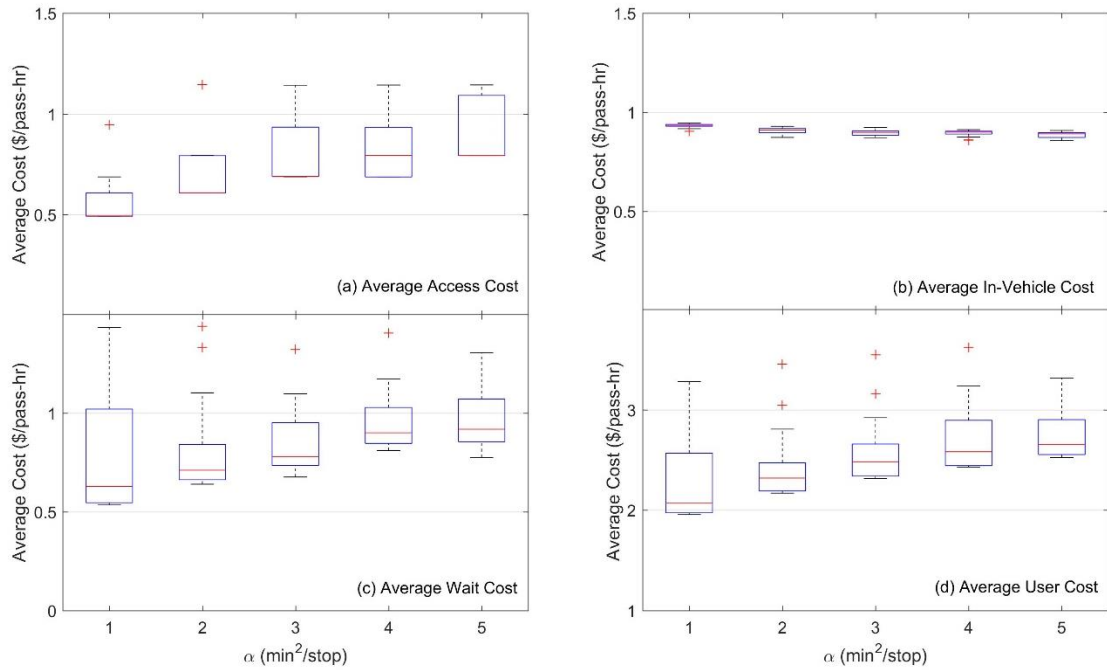
**Figure 5.6** Boxplot for optimized headways vs. demand level.



**Figure 5.7** Boxplot for optimized stop spacings vs. demand level.



**Figure 5.8** Optimized stop spacing, headway, and costs vs.  $\alpha$ .



**Figure 5.9** User cost components vs.  $\alpha$ .

## 5.2 Case Study II

The enhanced model is applied to optimize dispatching headway and stop locations for the same bus route as in case study I. A total of 30 intersections are counted along the route.

The existing operation serves 16 stops with a 5-minute dispatching headway. The average stop spacing is 0.36 miles, and the minimum spacing between two stops is 0.12 miles. The directional demands are 125 pass/hr and 23 pass/hr for outbound and inbound, respectively.

Considering the geographic condition and existing stops, there are 43 feasible stop locations for model inputs. The start and end stops of the route are fixed and serve as bus terminals. Pre-determined stop locations due to political, passenger demand, traffic, geometric and other practical concerns could also be taken care of in the optimization process. Following the assumptions in case study I, the average vehicle cruising speed is 25 mph, and the average passenger access speed to bus stops is 2 mph. All vehicles are dispatched on time from the terminals. The average delay per intersection and the average acceleration/deceleration delay per stop are both 10 seconds, and the average passenger boarding time is 6 seconds. The vehicle travel time over the route varies, whose variance is 4-min<sup>2</sup>/mile, which is an approximately equivalent assumption to 1 min<sup>2</sup>/stop headway variance in case study I. The model parameters are summarized in Table 5.3. Based on the survey, the directional cumulative boarding and alighting demand distributions are illustrated in Figures 5.10.

### **5.2.1 Optimization Results**

Considering travel time variation, the optimization results were found for the proposed model and compared with those under existing operation and the traditional model without considering variation, described as follows.

#### *Scenario 1: Existing operation*

The stop locations and headway configuration under existing operation is used to calculate the objective value

*Scenario 2:* Optimization results without considering travel time variation

Under this scenario, the traditional model without considering travel time variation is solved against the same objective – maximizing operator’s profit.

*Scenario 3:* Actual profit for Scenario 2 when implemented in the route with variance

Since the optimized solution under *Scenario 2* cannot reflect the actual situation where stochastic vehicle arrivals present, the operator’s profit is re-calculated taking into account 4-min<sup>2</sup>/mile travel time variance.

*Scenario 4:* Optimization results considering travel time variance

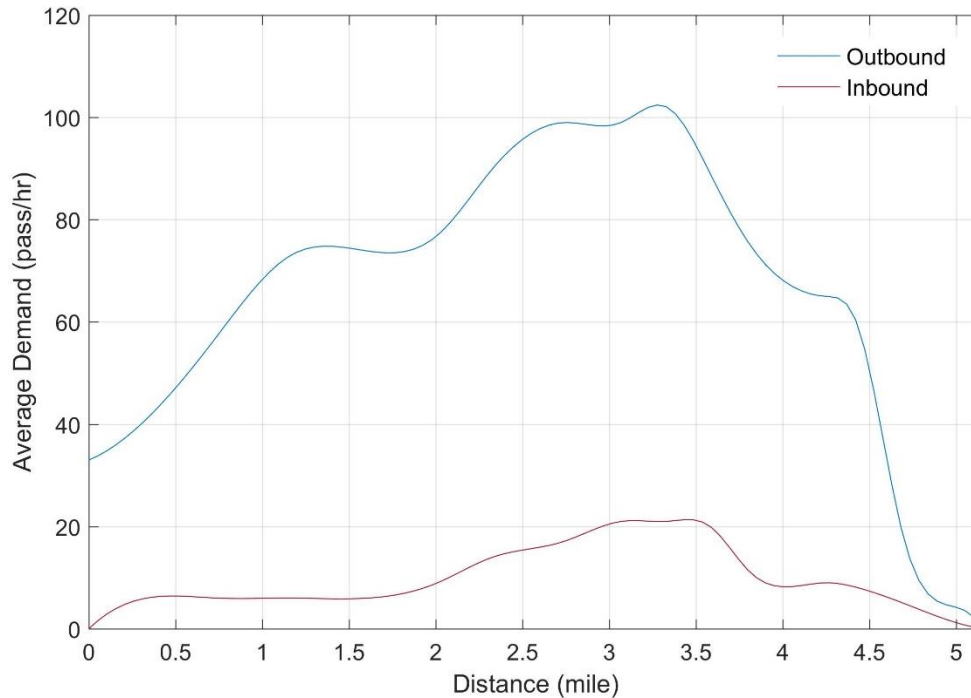
Under this scenario, the proposed model considering travel time variance of 4 min<sup>2</sup>/mile is solve to maximize the operator’s profit.

*Scenario 5:* Optimized headway with existing stops by the proposed model

The operation headway is re-optimized with existing stops using the proposed model considering 4-min<sup>2</sup>/mile travel time variance.

**Table 5.3** Model Parameters of the Case Study

<b>Variables</b>	<b>Descriptions</b>	<b>Values</b>	<b>Units</b>
$X$	Total number of intersections	30	
$c$	Dispatching headway variance	0	min <sup>2</sup>
$d_s$	Average delay per stop due to acceleration/deceleration	10	s
$L$	Route length	5.15	miles
$d_x$	Average delay per intersection	10	s
$u_a$	Value of passenger access time	8	\$/pass-hr
$u_o$	Unit operating cost	40	\$/bus-hr
$u_v$	Value of passenger in-vehicle time	8	\$/pass-hr
$u_w$	Value of passenger waiting time	16	\$/pass-hr
$v_t$	Travel time variance	4	min <sup>2</sup> /mile
$V_b$	Average bus cruising speed	25	mph
$V_p$	Average passenger walking speed	2	mph
$\rho$	Average passenger boarding time	6	s



**Figure 5.10** Cumulative outbound and inbound demand distribution.

Based on the parameters listed in Table 5.3, a total user cost of 981 \$/hr and operator cost of 480 \$/hr was estimated with existing stops and dispatching headway when headway variance exists, resulting in a negative profit (Scenario 1 in Table 5.4). Due to extremely short headway, the fleet size requirement is high enough to produce a high operator cost and negative profit.

Under Scenario 2, the maximum profit of 62 \$/hr was yielded by the optimized configuration of 12 stops and 10-min headway. The null variance assumption for Scenario 2 makes it possible to increase the number of stops in order to lower the access cost. Due to neglecting travel time variation, vehicle round-trip travel time is inevitably underestimated, thus, implementing such stop and headway configuration will lead to unexpected short of vehicles. Therefore, to satisfy the same level of demand, the fleet size needs to be enlarged which ultimately leads to a higher operator cost and in turn lower

profit (Scenario 3). Also, due to travel time elasticity of demand, underestimation of travel time also means overestimation of the actual demand (147 pass/hr under Scenario 3 vs. 148 pass/hr under Scenario 2).

**Table 5.4** Results for Optimized and Existing Operations

Variables	Scenarios				
	1	2	3	4	5
Headway (min)	5	10	10	12	12
Fleet Size (buses)	12	4	6	5	5
Number of Stops	16	12	12	9	16
Demand (pass/hr)	146	148	147	147	146
User In-Vehicle Cost (\$/hr)	731	271	576	441	731
User Access Cost (\$/hr)	109	146	145	206	110
User Wait Cost (\$/hr)	141	198	218	242	252
Total User Cost (\$/hr)	981	615	939	889	1,093
Operator Cost (\$/hr)	480	160	240	200	200
Revenue (\$/hr)	219	222	221	221	219
Profit (\$/hr)	-261	62	-19.5	21	19
Average Operating Cost per Passenger (\$/pass-hr)	3.3	1.1	1.6	1.4	1.4
Average User Cost (\$/pass-hr)	6.7	4.2	6.4	6.0	7.5
Average Total Cost per Passenger (\$/pass-hr)	10.0	5.3	8.0	7.4	8.9

The results from the proposed model is represented as Scenario 4, showing that compared to the existing configuration, the total profit was significantly improved with the optimized configuration of 12-min headway and 9 stops. The required fleet size is reduced to half of the existing one, leading to reduced operator cost. Meanwhile, proper arrangement of stops and headway results in increased demand and reduced user cost. Comparing the results under Scenario 3, because of less stops and longer headway under Scenario 4, the operator cost is well controlled which leads to an increase in the profit.



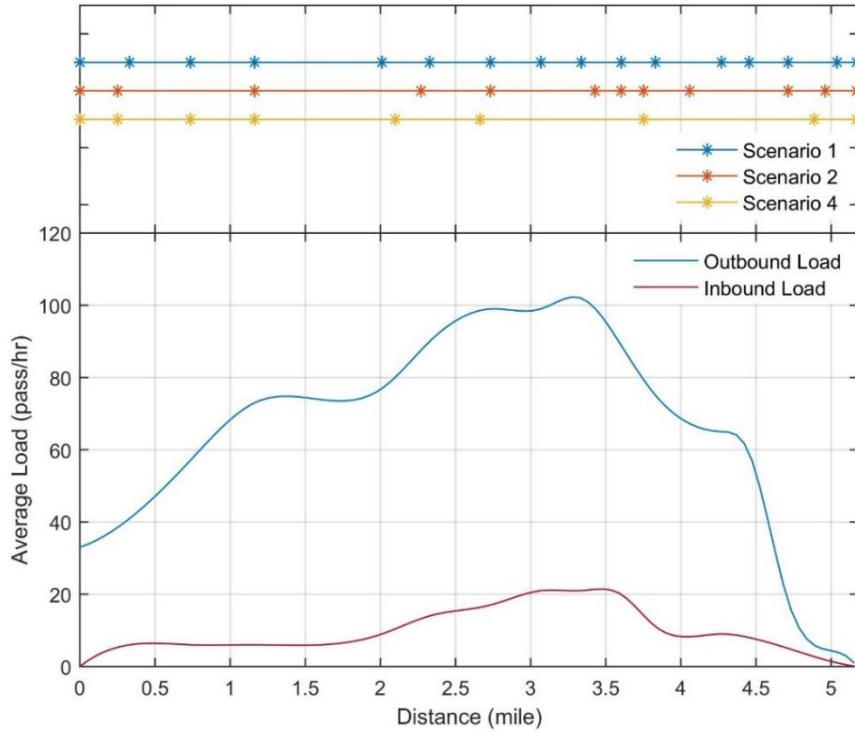
Therefore, the average operating cost and average total cost are lowered, confirming that a more cost-efficient system could be achieved with the proposed model.

To further investigate the influence of stop optimization with the developed model, Scenario 5 were created by keeping existing stop configuration and optimizing the headway with the proposed model. Although the configuration of existing stop and optimized headway seems to provide a more appealing result than the configuration of existing stop and headway, the cost-efficiency of such system is still lower than that yielded by the proposed model (similar profit but much lower average total cost per passenger).

Shown in the upper panel of Figure 5.11, the optimized stop locations in Scenario 4 are quite different from the existing ones (Scenario 1). It is understandable that due to dense intersections and travel time variance, fewer stops are suggested placed along the route. In the meantime, to accommodate the demand, stops are located in the high demand segments.

### **5.2.2 Sensitivity Analysis**

In this section, system performance in terms of reliability is examined for the scenarios created in the previous section. Then, the trade-off between average operator cost and average user cost is further examined.



**Figure 5.11** Optimized and existing stop locations vs. average load.

### *Service Reliability*

As discussed earlier, bus service reliability has been regarded as a key indicator of transit system performance. By definition in Transit Capacity and Quality of Service Manual (TCQSM) (Kittelson & Associates, 2003), the headway coefficient of variation ( $c_{vh}$ ) can be used to measure the bus bunching effect and indicate the level of service (LOS), which is the standardized measure of headway dispersions. Each category of LOS, corresponding range of  $c_{vh}$  and the description are presented Table 5.5. For instance, when  $c_{vh}$  is smaller than 0.21, bus service is provided like clockwork, yielding LOS A representing the best bus performance. In contrast, if  $c_{vh}$  is larger than 0.75, most vehicles bunch, the worst performance is observed and defined as LOS E. In this section, the definition of LOS and  $c_{vh}$  are applied to investigate service reliability under the influence of travel time variance.

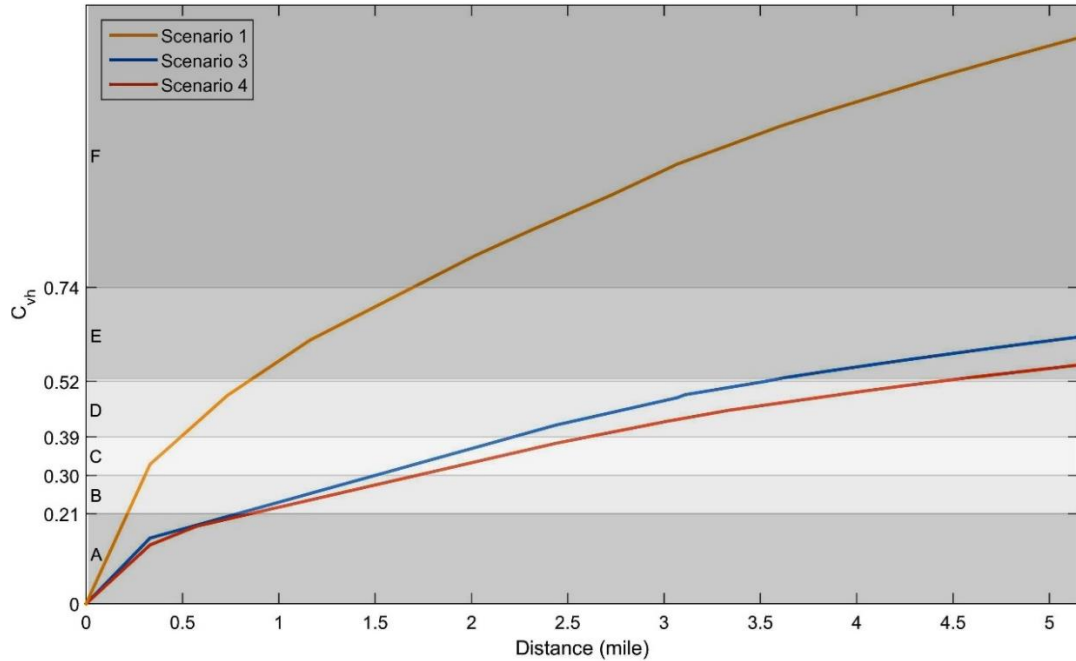
**Table 5.5** Fixed-Route Headway Adherence and Level of Service

<b>LOS</b>	$c_{vh}$	<b>Passenger and Operator Perspective</b>
A	0.00 - 0.21	Service provided like clockwork
B	0.22 - 0.30	Vehicles slightly off headway
C	0.31 - 0.39	Vehicles often off headway
D	0.40 - 0.52	Irregular headways, with some bunching
E	0.53 - 0.74	Frequent bunching
F	$\geq 0.75$	Most vehicles bunched

Source: Transit Capacity and Quality of Service Manual (Kittelson & Associates, 2003)

As a result, the stop-level headway coefficient of variation is calculated with the optimization result yielded under each scenario (i.e., existing operation as Scenario 1, optimized result without travel time variance as Scenario 3, and with variance as Scenario 4). The trends of  $c_{vh}$  and the categories of LOS are illustrated in Figure 5.11, and the orange, blue and red lines represent Scenarios 1, 3, and 4, respectively. As shown in the figure, the optimized configuration of headway and stops from both models significantly improve service reliability compared to existing operation. Although service reliability is inevitably deteriorated at downstream stops because of propagated headway variation, the LOSs at all stops are improved with optimized configurations of headways and stops.

By comparing the headway coefficient of variation under Scenarios 3 and 4, the proposed model considering travel time variance performs better in controlling service deterioration. Since  $c_{vh}$  at the downstream stops are lowered, the LOS has been improved with the proposed model.



**Figure 5.12** Headway coefficient of variation under different scenarios.

To determine the LOS on a route basis, the average headway coefficient of variation  $\overline{c_{vh}}$  for all stops was calculated (i.e., sum of stop level  $c_{vh}$  divided by the total number of stops). Basic descriptive statistics for stop level  $c_{vh}$  under each scenario is plotted in Figure 5.13 with hexagram point in each category indicating route level  $\overline{c_{vh}}$ .

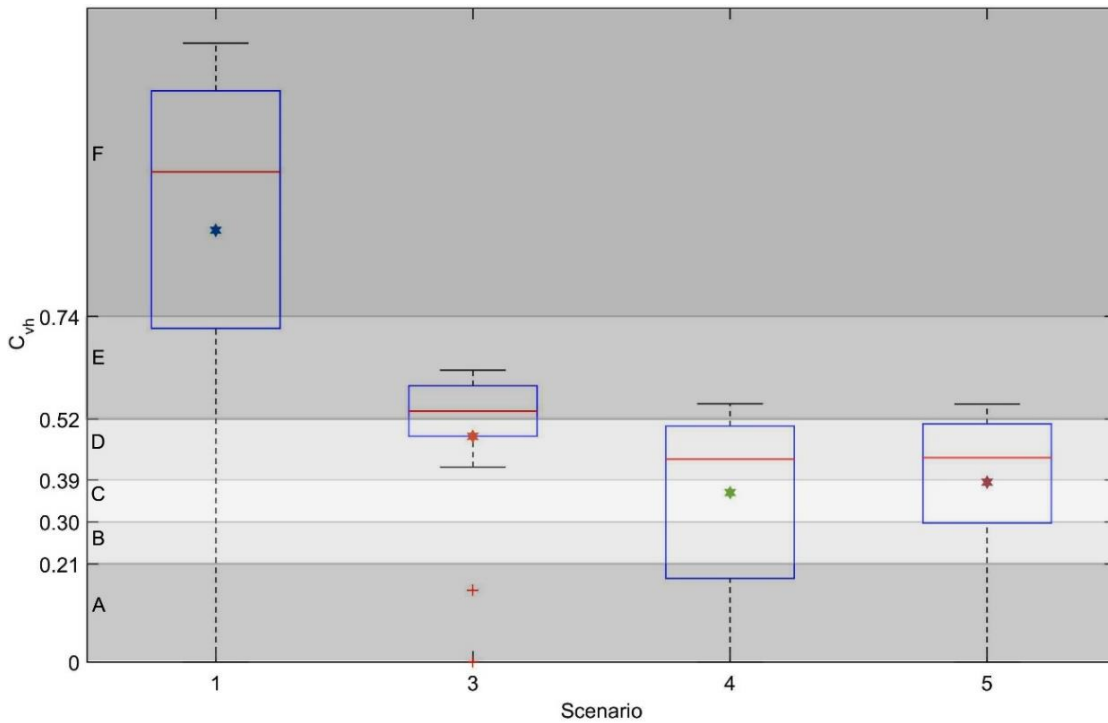
Under Scenario 1, the LOS at most stops with existing operation are worse than E, where most vehicles bunched together due to combined effects of traffic congestion and short headway. However, under Scenario 5, although the travel time variance remains the same, the values of  $c_{vh}$  are significantly reduced due to optimized headway with the proposed model taking into account travel time variance.

With the configurations recommended by the optimization model without consideration of travel time variance (Scenario 3), both of the stop-level  $c_{vh}$  and route-level  $\overline{c_{vh}}$  are improved, comparing with the existing operation. However, since half of the

stops experiencing LOS E, the route LOS falls in category D (i.e., irregular headways with some bunching).

Under Scenario 4 (optimization with consideration of variance), the LOS is further enhanced compared to those under Scenario 3. With more stops having lower  $c_{vh}$ , the route LOS is improved to C. Although due to inherent traffic condition, the optimization of headway and stops is not able to yield a higher route LOS, most of the stop LOS have been improved significantly.

Comparing Scenario 4 with Scenario 5, the values of  $c_{vh}$  at the end of route are similar (as indicated by similar maximum and 75<sup>th</sup> percentile values). However, the stop level LOS at the upstream segments of the route are significantly improved as indicated by much lower 25<sup>th</sup> percentile value under Scenario 4, due to optimized stop locations.



**Figure 5.13** Boxplots of stop-level  $c_{vh}$  under different scenarios.

### *Trade-off between the Operator's and Users' Objectives*

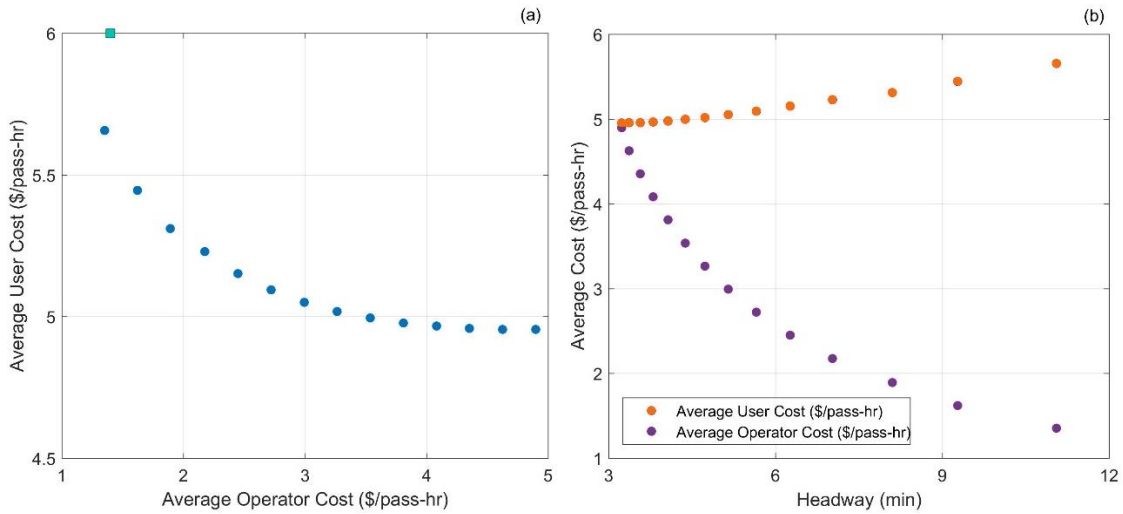
To further investigate the trade-off between the objectives of the users and the operator, similar to Case Study I, the average user and operator costs are applied to optimize the headway and stop spacing considering travel time variance.

Figure 5.14 a shows the Pareto front by solving the multi-objective optimization model with two conflicting objectives, achieved by a set of optimized configurations with similar stop locations but varying headways. The trade-off curve between average user cost and average operator cost suggests a relatively insensitive average user cost to headway. This is consistent with single profit-maximization objective: when the demand elasticity to the travel time variance is low, the users' benefit is prone to be sacrificed for higher profit.

Note that with the single-objective optimization model, the optimized stop and headway yield a pair of (1.4, 6.0) average operator cost and average user cost, as shown with the blue square in Figure 5.14 a. Obviously, the single-objective approach limits the interpretation of the model results with only one pre-defined relationship between the users and operator. Although it is helpful when the emphasis of the system is clear, sometimes such approach may fail to tell the whole story. Moreover, in this case study, even with consideration of demand elasticity, the optimization results with single objective still substantially favor the operator over the users. In this case, it is useful to illustrate the trade-off in a multi-objective approach, showing the relationship among objectives and also the sensitivity of each objective with decision variables.

Shown in Figure 5.14 b, the corresponding optimized headways range from 3 min to 12 min. With the same set of stop configuration, shorter headway is preferred to achieve a lower average user cost. If a lower operator cost is desired, however, higher headway

shall be selected. Figure 5.14 b is also helpful in the process of decision-making. For instance, if there is a range of average operator cost can be allocated (e.g., 1~3 \$/pass-hr), the range of feasible headways is between 5~12min. Therefore, depending on the goal (e.g., minimizing average user cost, or minimizing average operator cost, or balance these two costs), slight modification of headway is able to yield anticipated results.



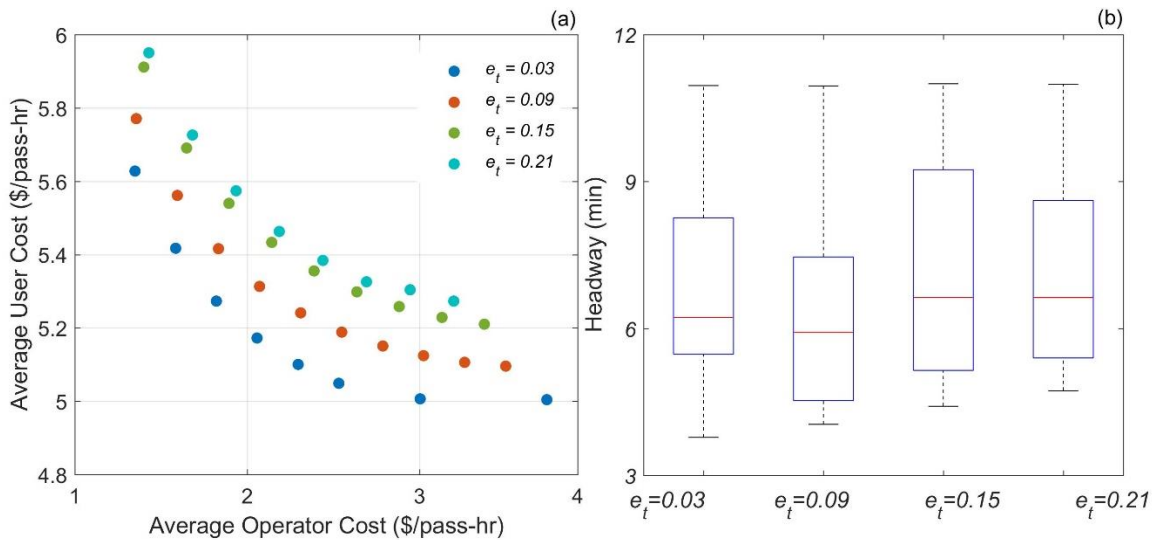
**Figure 5.14** Average operator and user costs vs. optimized headway.

### *Travel Time Elasticity of Demand*

Implied from the above discussion, the elasticity of demand may impact the optimization results due to the influenced sensitivity of average operator cost and average user cost to travel time variance. In light of this, sensitivity analysis is conducted to investigate how the elasticity affects the configurations of stops and headways. The travel time elasticity of demand ( $e_t$ ), changing from 0.03 to 0.21 with an interval of 0.06, is applied to solve the multi-objective optimization problem.

Shown in Figure 5.15 a, although the travel time variance is constant, due to higher travel time elasticity, the average user cost gets higher due to reduced demand. According

to the optimization solutions, whereas the stop locations should be rearranged when demand is highly sensitive to the travel time variance, the distributions of headways satisfying Pareto optimality, as shown in Figure 5.15 b, are not statistically different among these four scenarios according to the ANOVA result.

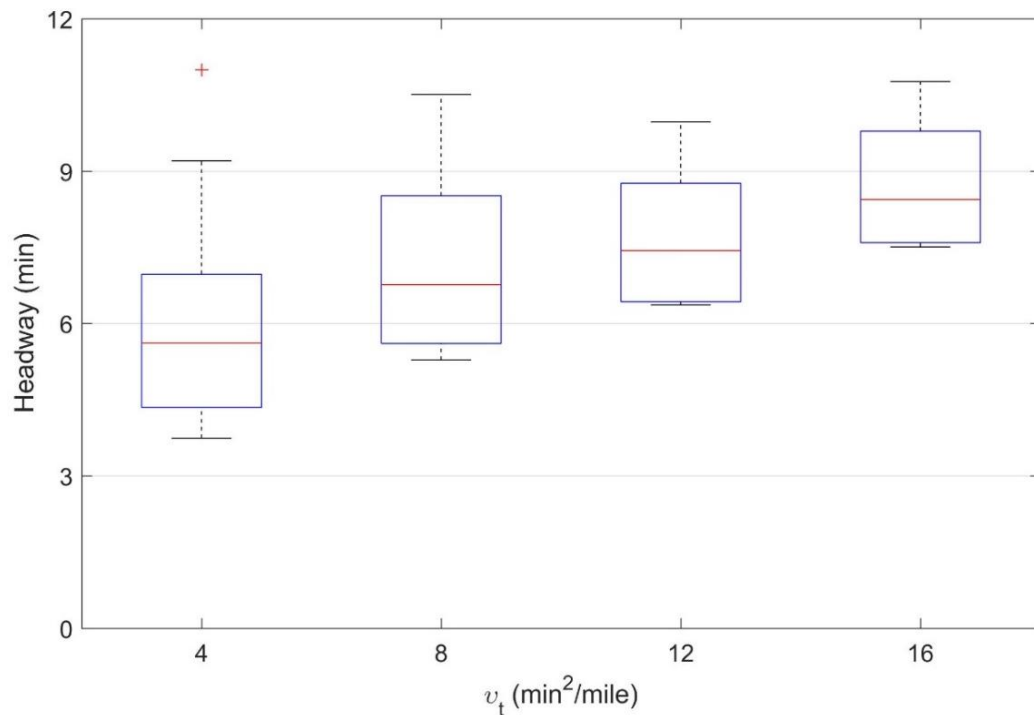


**Figure 5.15** Average operator and user costs vs. travel time elasticity.

### *Travel Time Variance*

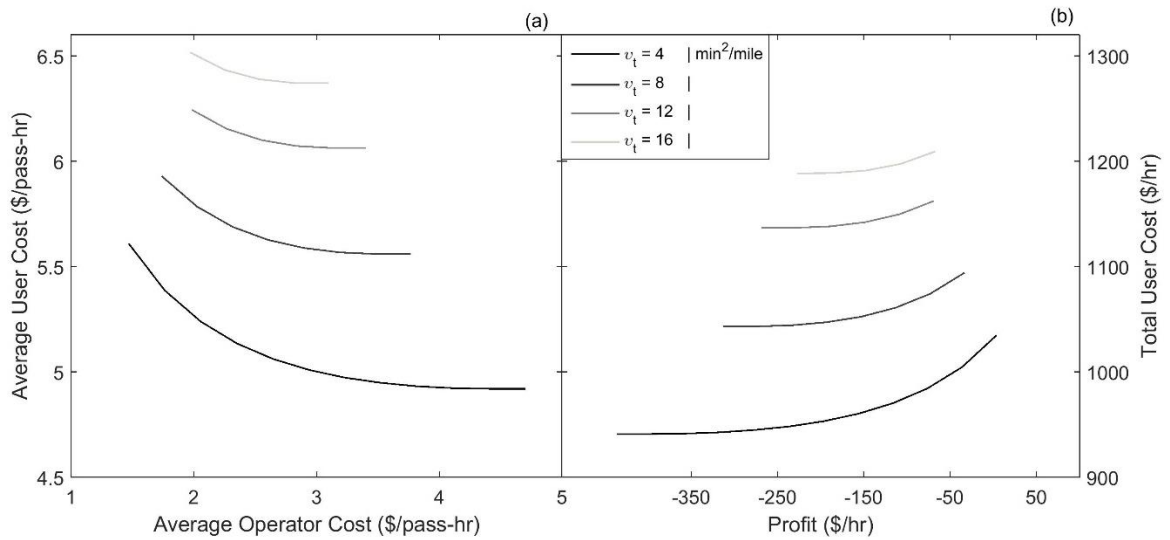
In addition, to understand how the model behaves when the travel time variance along the route is changed, sensitivity analysis is conducted. The travel time variance  $\nu_t$  varies from 4 to 16  $\text{min}^2/\text{mile}$  (i.e., the standard deviation of travel time varies from 2 to 4  $\text{min}/\text{mile}$ ) with an interval of 4  $\text{min}^2/\text{mile}$ . Instead of utilizing the single-objective function, the multi-objective optimization model is applied to better understand the changes in user and operator costs respectively. As a result of increased travel time variance, the optimized headways increase as shown in Figure 5.16. In addition, when the travel time variance is getting higher, not only the median of headways increases, but also the number of possible headways achieving Pareto optimality reduces.





**Figure 5.16** Boxplot of optimized headways vs. travel time variance.

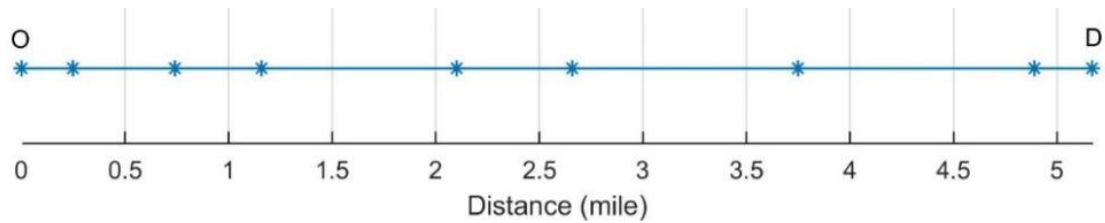
Looking into the trade-off curves for the average use and operator costs (Figure 5.17 a), it is found that as  $v_t$  increases, both of the average operator and user costs increase due to larger variation. Also observed from the shape of Pareto front, the feasible solution pool satisfying the Pareto optimality shrinks when the variance gets higher. It is notable that the increase of costs with travel time variance is not linear and the gaps are reducing. Correspondingly, the operator's profits and total user costs are shown in Figure 5.17 b. With the increase of travel time variance, it is not possible for the service to be profitable due to high operator cost as well as high travel time that leads to reduced demand and revenue. However, it should be noted that such conclusion is based on the short headway service. If the demand level is too low to maintain short headway, long headway service should be considered. Since passengers' behavior are different under short and long headway services, more investigations need to be conducted in the future research.



**Figure 5.17** Average user and operator costs, operator profit vs.  $v_t$ .

### 5.3 Case Study III

In this case study, Model III is applied to find proper number and locations of time points to improve bus service performance. The simulation-based optimization is applied to solve the problem. Bus stops, as the simulation inputs, are from the optimized solution generated by Model II (i.e., the planning model), consisting of 9 stops from the origin terminal to the destination terminal. The list of stops is shown in the Table 5.6 and the locations of the stops are shown in Figure 5.18. The headway is 12 minutes and fleet size is 5 according to the optimization result from Model II. The boarding/alighting profiles along the route are represented in Figure 5.19, showing that for the outbound direction, both of the boarding and alighting are higher downstream rather than upstream.



**Figure 5.18** Stop locations along the route.

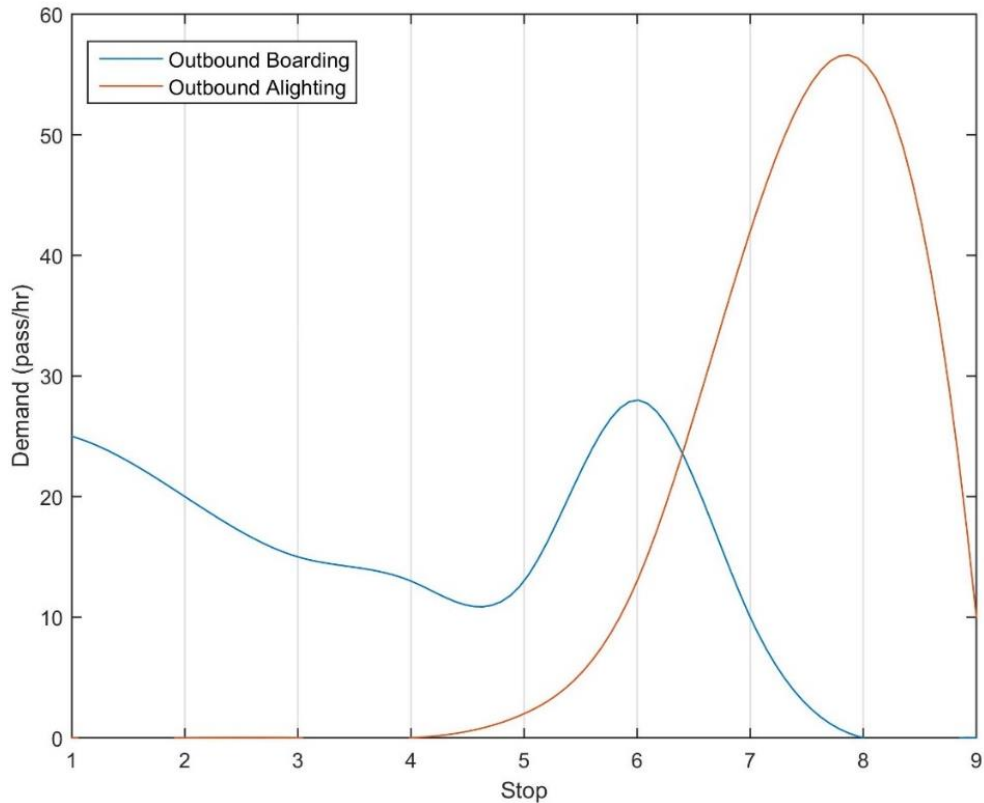
Same set of model parameters with the previous case study is applied, including an average of 10-second intersection delay, average of 6-second unit boarding time, an average of 25 mph cruising speed, 4 min<sup>2</sup>/mile travel time variance, \$40/bus-hr unit operator cost, \$8/pass-hr value of in-vehicle time, and \$16/pass-hr value of wait time.

**Table 5.6** Simulation Inputs - Stops, Distances and Intersections

Stop No	Distance	Intersections
1	0.00	0
2	0.25	0
3	0.74	3
4	1.16	5
5	2.10	8
6	2.66	11
7	3.75	18
8	4.89	27
9	5.15	30

30 repeats of 4-hour simulation with same parameters and different random seeds were conducted under no control scenario for model calibration. The average demand from the simulation model is 145 pass/hr, with average operator cost of 1.3\$/hr, and 4.6\$/hr of average user cost. The demand is slightly less than forecasted demand due to simulation variation, while the average headway at each stop is very close to the scheduled headway

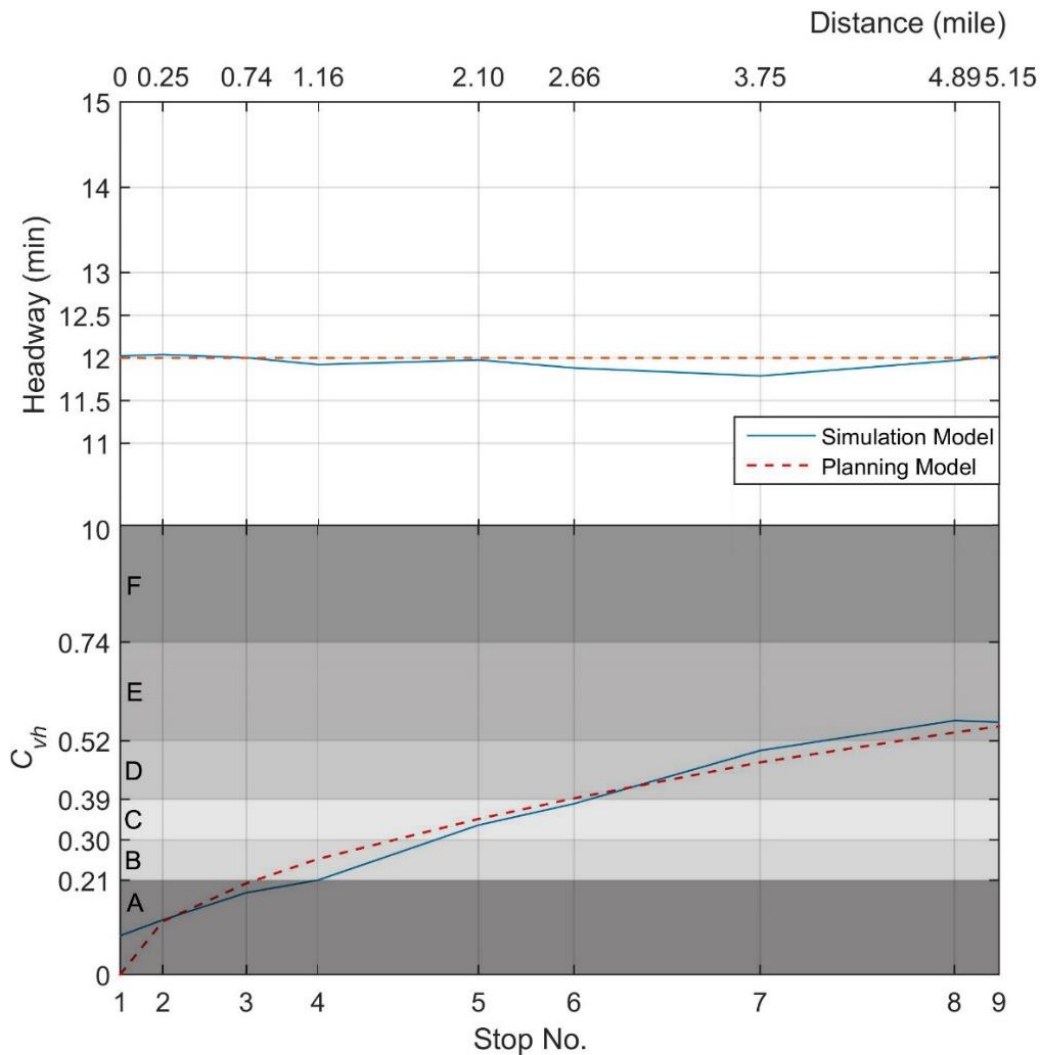
– within 30-second variation. Shown in Figure 5.20, the headway coefficient of variation for the simulation model well reflects the results from Model II. The overall trend and stop level of service are the same as the planning model, although small deviations occur along the route.



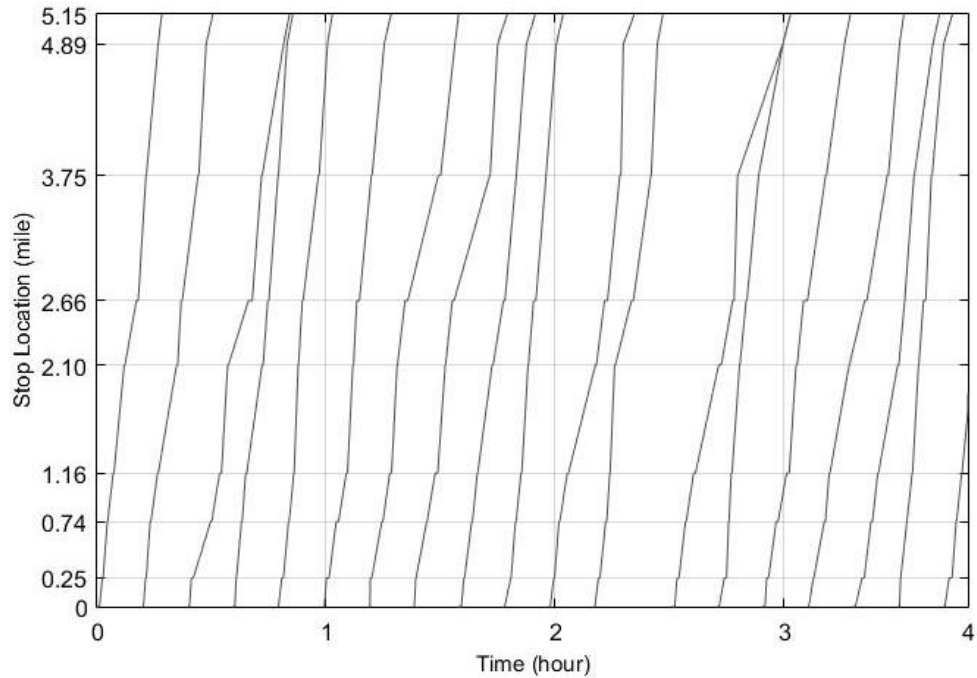
**Figure 5.19** Outbound hourly boarding and alighting demand.

Overall, the simulation model can be applied to reflect the planning model when there is no control along the route, and therefore is able to be used for further bus holding analysis and optimization. To present system performance under traffic congestion, the outbound bus trajectories from one simulation run are drawn in Figure 5.21. It is found that the headways at upstream stops are relatively stable because of fewer boarding/alighting demand (i.e., less disruption). In contrast, service deterioration appears and becomes more significant when the buses travel further downstream. In the study route, more demand is

concentrated in the middle and downstream of the route, meanwhile, intersections are more along the downstream segment of the route than the upstream. Therefore, it is reasonable that a small headway fluctuation from upstream turns into a huge deviation, leading to significantly increased headway coefficient of variation at downstream stops as shown in Figure 5.20.



**Figure 5.20** Comparison of headways and  $c_{vh}$  from planning and simulation models.

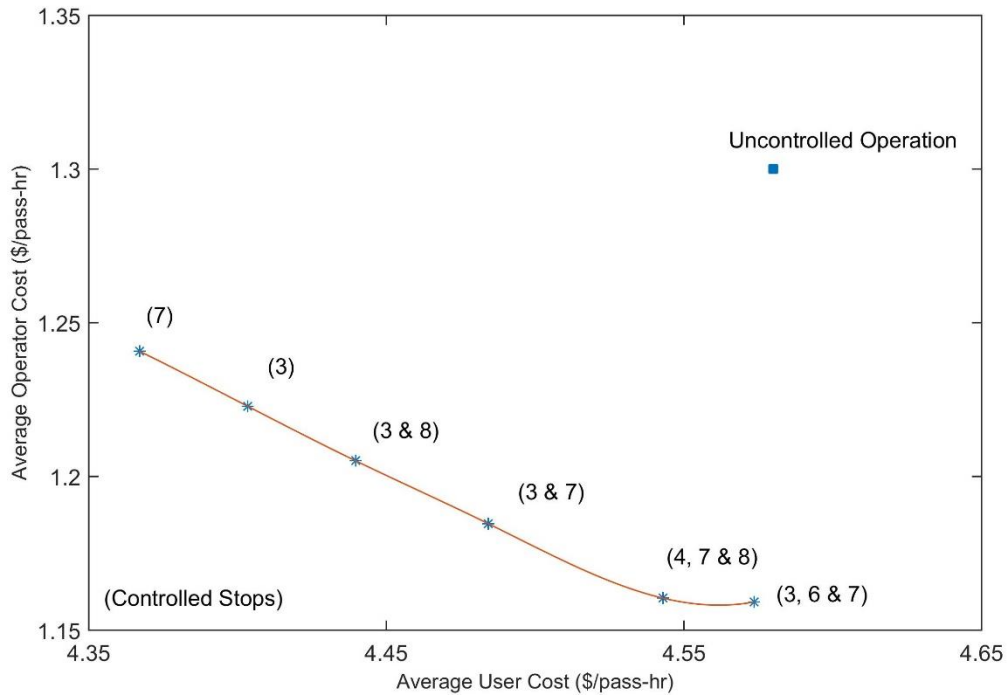


**Figure 5.21** Simulated bus trajectories with no control.

### 5.3.1 Optimization Results

Applying the control strategy into the system, the simulation model is embedded into the Genetic Algorithm, where the control points are set as decision variables, and minimizing average user cost and minimizing average operator cost are the two conflicting objectives. Figure 5.22 shows the trade-off between average operator cost and average user cost under different control point settings, where the controlled stops for each combination of average operator cost and average user cost in the Pareto front are presented. Also, as shown with blue square point, both of the average operator and user costs are higher under uncontrolled operations than controlled operations. It is reflected through the trend of trade-off curve and its solutions that more time points lead to higher average user cost but lower operator cost.

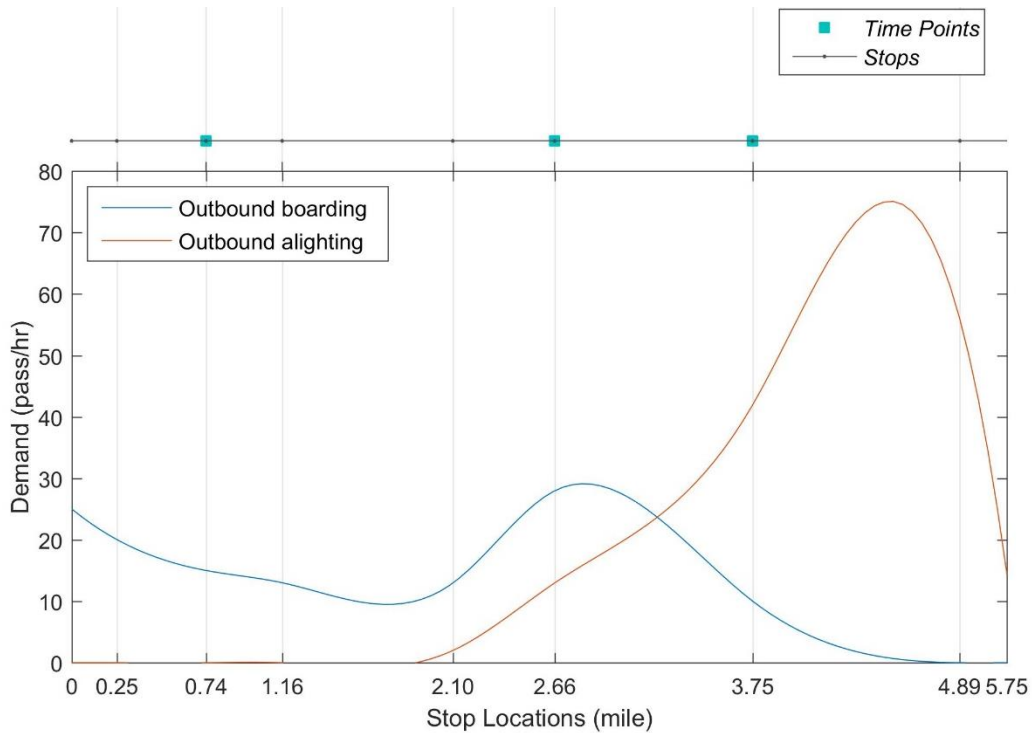
Since the headway-based strategy is applied in the optimization, it is reasonable that with more time points, service regularity is improved and the usage of vehicles are more cost efficient, so that the average operator cost could be reduced. However, setting more time points, on the other hand, means higher possibility of holding buses, which ultimately leads to more in-vehicle travel time. Although the waiting time may benefit from regular service, specific combinations of demand pattern and control point locations might result in higher average user cost, as illustrated in Figure 5.22.



**Figure 5.22** Average user and operator costs vs. time points.

To illustrate the impact of control points, the set of optimized control point that yields minimum operator cost is chosen to conduct system performance analysis. Figure 5.23 shows the locations of control points vs. the outbound boarding and alighting demand. The blue square points in the upper-level panel stand for the control points, whereas the blue and red lines in the lower-panel of the figure represent stop-level outbound boarding

and alighting, respectively. Similar to the findings from previous studies reviewed earlier in Chapter 2, one of the control points is placed in the upstream segment of the route to minimize fluctuation from upstream, and the other two are placed at the stops where the peak boarding demand are found to alleviate demand influence on headway deviation.



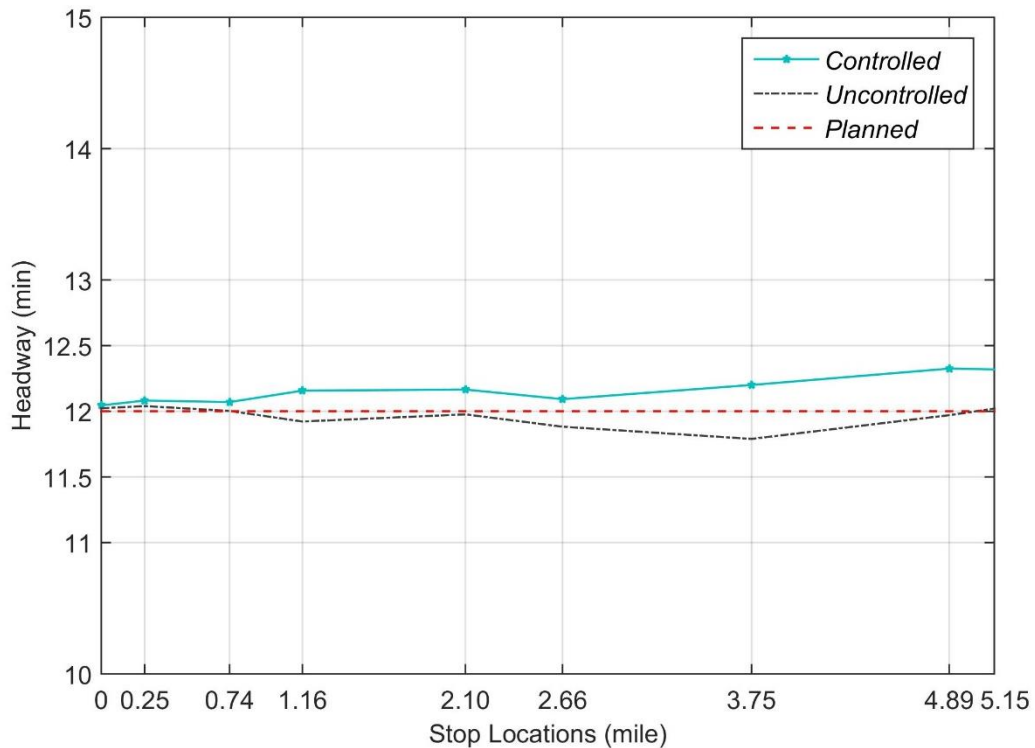
**Figure 5.23** Optimized time points vs. outbound boarding/alighting profile.

Summarized from the simulation results, Figure 5.24 shows the comparison among three stop-level headways, where in the Legend, ‘Controlled’ means average stop-level headway with optimized control points, ‘Uncontrolled’ means average headway without any control, and ‘planned’ headway is the optimized headway from Model II, which is also the input of the simulation model.

The inherent characteristics of headway-based control strategy imply the increase of average headway, which could be explained with two simple cases involved in the control procedure. First, if a headway is long, it may not meet the control criteria, and then



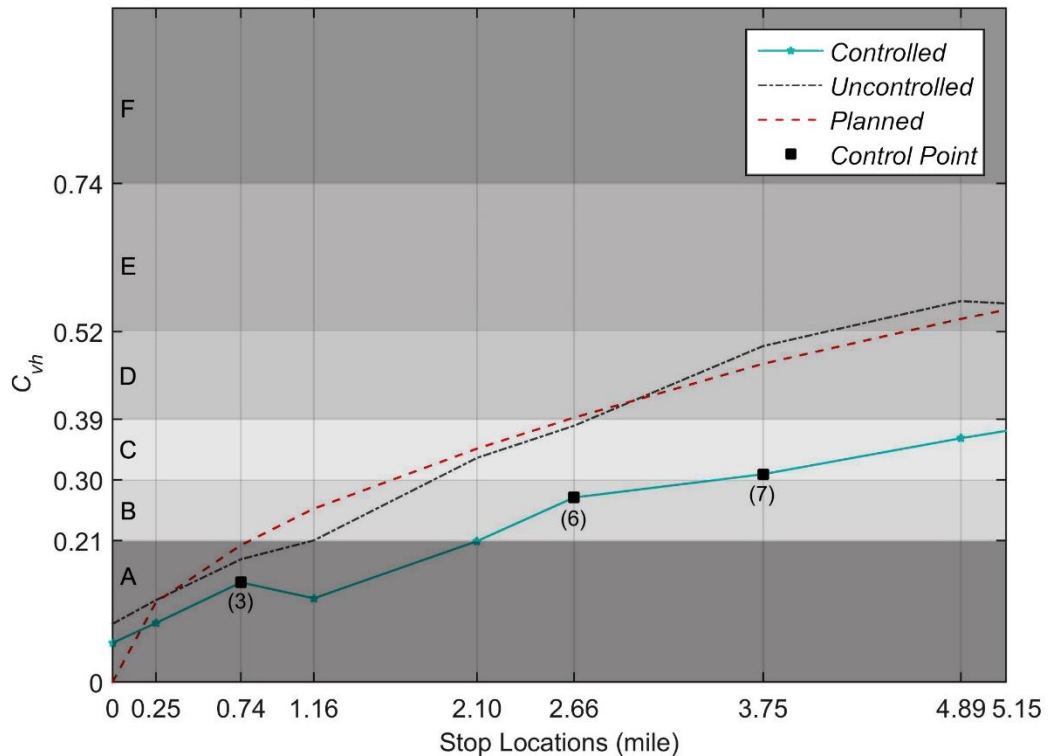
the bus will only dwell for passenger on/off activities. Second, if a headway is short, it probably meets the control criteria, and then the bus will be held for a certain time to ensure enough headways of this bus with the leading and following buses. As a result, such controlling strategy will lead to a little higher average headway than uncontrolled operation, as shown in Figure 5.24.



**Figure 5.24** Comparison of simulated headways and planned headway.

Although the average headway may be slightly increased, when one looks into service reliability in terms of headway coefficient of variation, significant improvement is achieved under controlled operation. In Figure 5.25, the headway coefficients of variation are compared among three scenarios, where the red dotted line represents the results from the optimization model II, the black dotted line is the performance with simulated uncontrolled operation, and the blue line shows the results under controlled operation. It is found that after the upstream control point, service reliability is immediately improved with

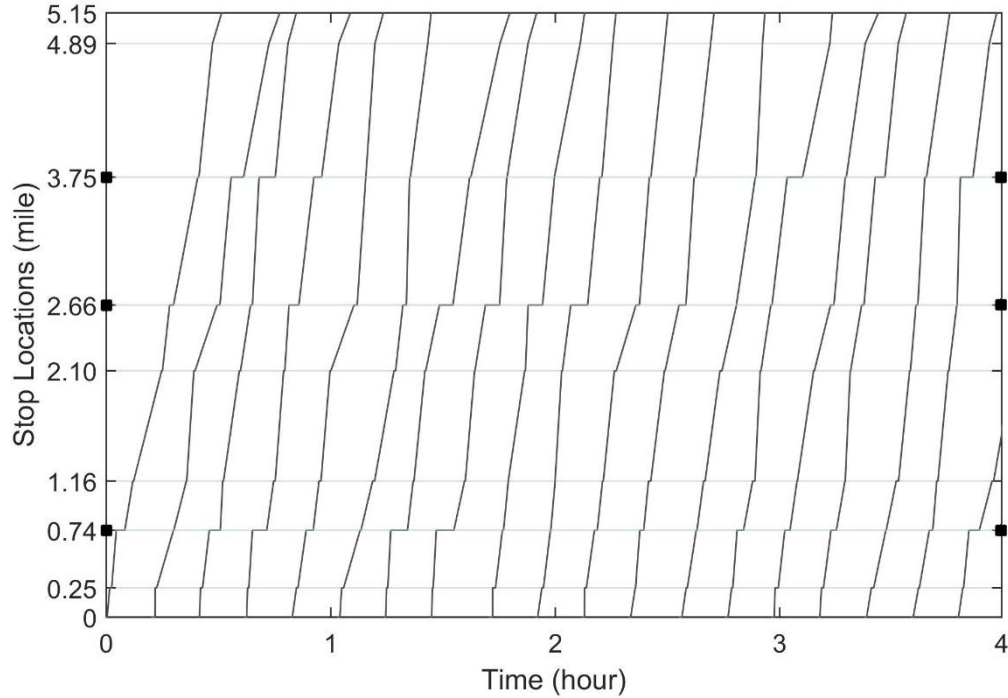
much lower  $c_{vh}$  value. Due to high boarding and alighting demand at downstream stops, the effects of control points are not significant compared to the upstream control point. However, two downstream control points are effective in curbing service deterioration, indicated through the slower increase of headway variation compared to the other two scenarios.



**Figure 5.25** Headway coefficient of variation vs. time points.

To visualize the operations with control points, bus trajectories are drawn within 4-hour simulation as shown Figure 5.26. The black dots in y-axis indicate the controlling locations along the route, whereas the x-axis represents the elapsed time during simulation. Compared with the uncontrolled bus trajectories shown in Figure 5.20, not only headways at upstream stops are regular, but also the headway regularity at downstream stops is

significantly improved. Although some buses may get closer after the last control point, no bunching occurs under controlled operation.



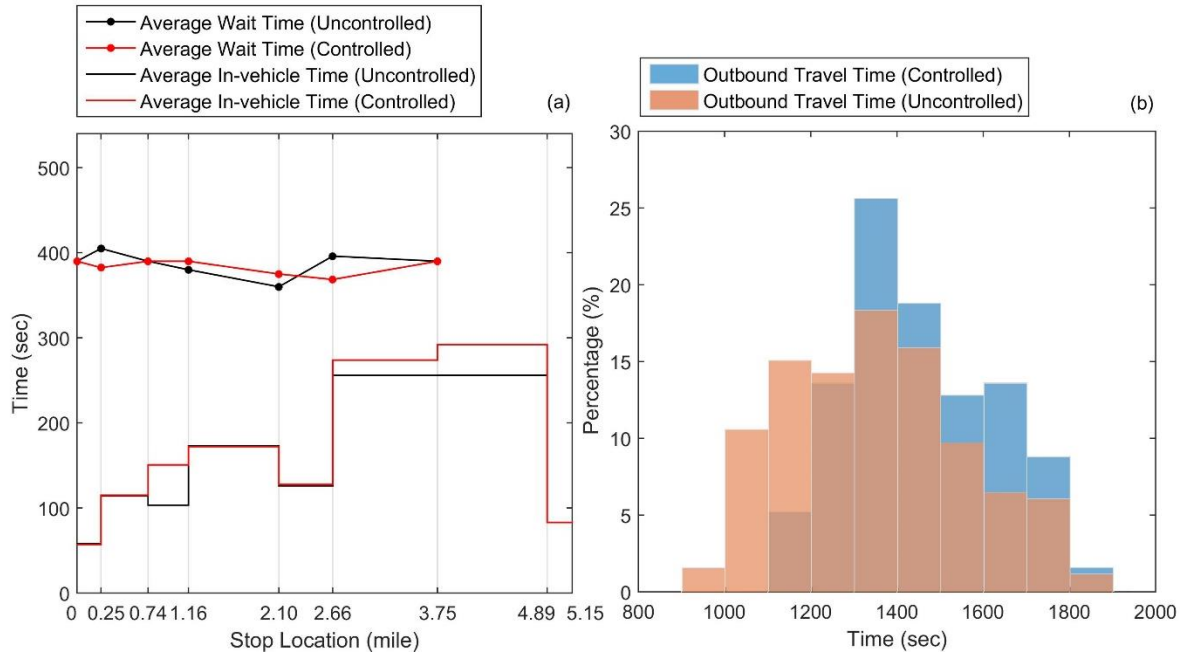
**Figure 5.26** Simulated bus trajectories with control.

As discussed before, the control points have different influences on passengers' wait time and in-vehicle travel time. With the simulation results, the average wait time and in-vehicle time per stop for outbound direction are calculated for both controlled and uncontrolled scenarios (Figure 5.27). For the uncontrolled operation, the average in-vehicle time only consists of dwell time and link travel time, whereas under controlled operation, the average in-vehicle time also include the extra holding time at stops.

In Figure 5.26 a, the black line series represent the uncontrolled operation and the red line series are for the controlled operation. For the average wait time, each point represents the average wait time at a stop. Since no outbound boarding demand at the last two stops (i.e., the stops at 4.89 mile and 5.15 mile), the average wait times for these stops

are not calculated. For any segment between a pair of consecutive stops, the average in-vehicle time is the sum of segment travel time, holding time, bus dwell time, and bus acceleration/deceleration delay. As shown in Figure 5.26 a, the stop-level average wait time evens out under controlled operation because of more regular headways after controlling, though passengers at some stops may experience a little long wait time due to slightly increased headways. On the other hand, due to additional holding time to ensure service regularity, the average in-vehicle travel times for the controlled segments are increased.

Represented in Figure 5.26 b are the outbound bus travel time distributions under controlled and uncontrolled operations. With the results of 30 replications of 4-hour simulation run, the orange bars stand for the percentage of outbound travel time of each range for uncontrolled operations, the blue bars are for the controlled operations. The overlays are shown in dark orange. Although longer travel times are expected in a system with control points, the impact of longer travel times on bus operation may be compensated by the improvement of service reliability, as reflected by reduction in the operator cost (i.e., higher cost efficiency).



**Figure 5.27** Average passenger wait and in-vehicle Time, outbound bus travel times under controlled and uncontrolled operations.

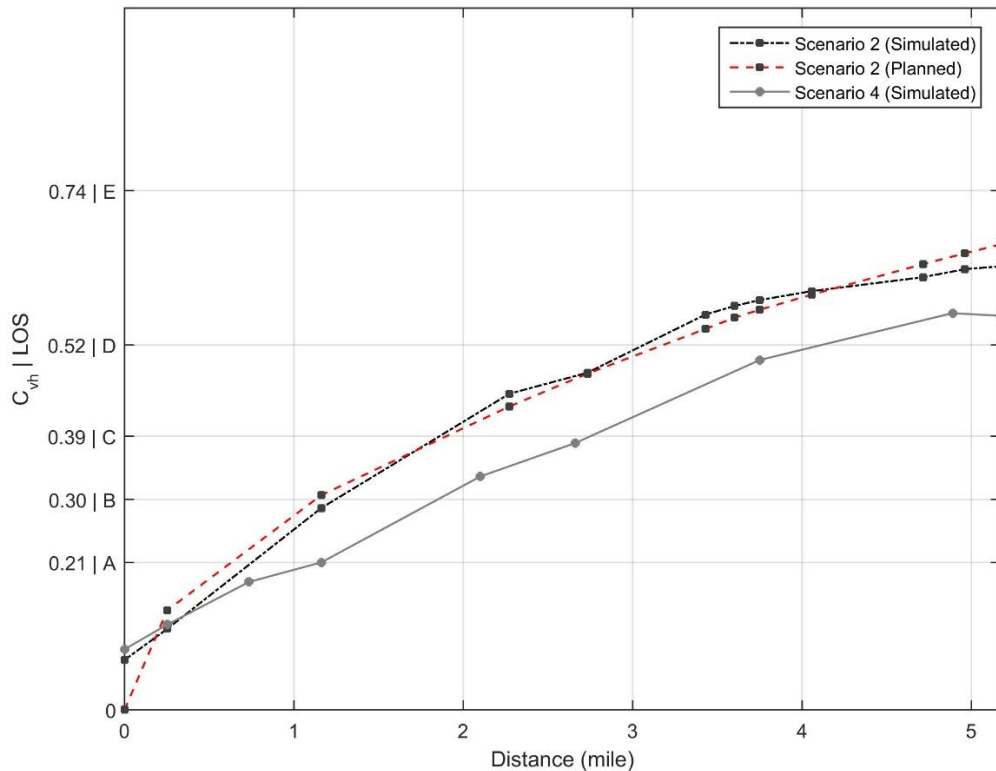
### 5.3.2 Comparative Analysis

For comparison, the simulation and optimization results from a traditional model without considering travel time variance are also analyzed. Same simulation settings are applied except for the scheduled headway and stop locations, which come from the optimal results of Scenario 2 (i.e., a traditional model without considering travel time variance) in Section 5.2.

It is found from the simulation that the average stop-level headway is 11 min instead of 10-min optimized headway, leading to an average user cost of 4.4 \$/pass-hr and 1.6 \$/pass-hr average operator cost. Therefore, for the planner using the traditional model to estimate resource requirement and profit, the situation exists that due to large (unexpected) fluctuation and underestimated fleet size, maintaining the suggested headway is impossible. Sequentially, the whole system needs to be modified in order to satisfy the

projected demand, or otherwise suffer from ridership decline. Either way, the result is clear: unexpected increase from the projected operator cost.

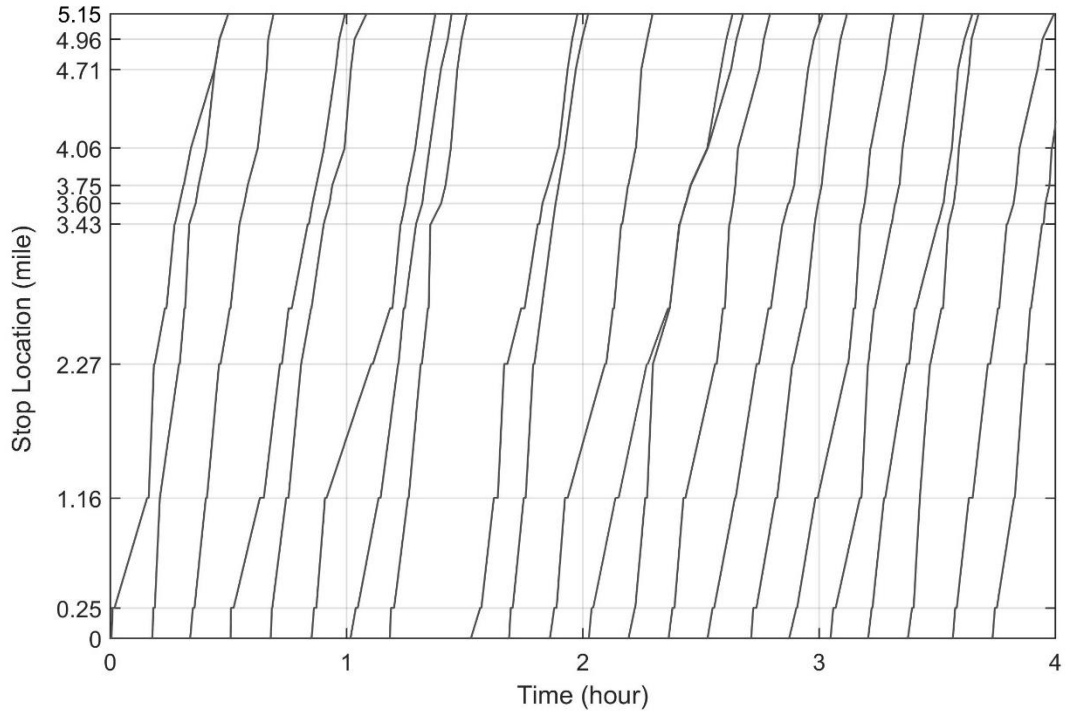
Shown in Figure 5.28, the headway coefficient of variation from simulation model is compared with that from planning model for calibration and further analysis. In addition, the stop level  $c_{vh}$  under Scenario 4 (i.e., the proposed Model II) is included for comparison. Consistent with the results from the planning models, the simulation results also reveal a significant difference of  $c_{vh}$  between the proposed and traditional models.



**Figure 5.28** Headway coefficient of variation under different scenarios.

A sample of bus trajectory from simulation model is illustrated in Figure 5.29, indicating that due to improper planning, buses are often off headways and bunching also occurs. The deterioration of service not only appears at downstream stops, but also exists in the upstream segments. On the contrary, with optimized stops and headway from the

proposed model, large headway deviation only occurs in the downstream segments as shown in Figure 5.20.

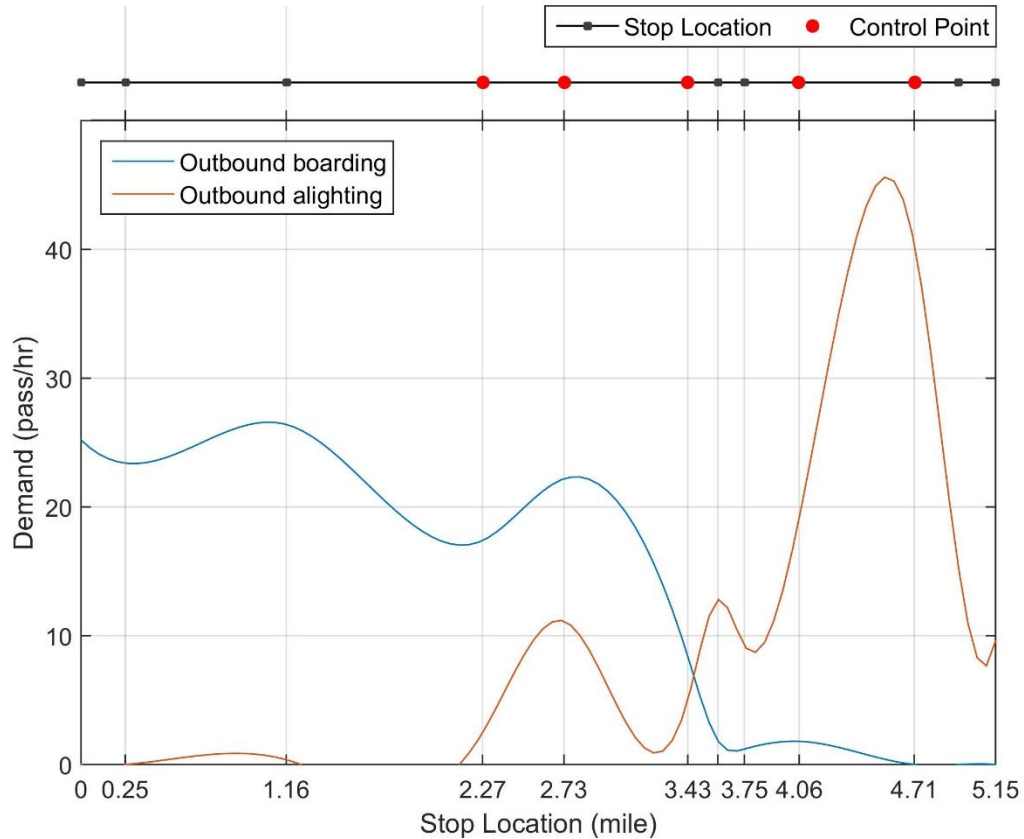


**Figure 5.29** Simulated bus trajectories under scenario 2.

Optimizing the number and locations of time points for the stop and headway configuration under Scenario 2, a similar Pareto front could be observed: less control points leads to lower average user cost but higher average operator cost, and higher user cost and lower operator cost if more control points are assigned along the route. Take the set of control points yielding the least operator cost for instance, the system performance is analyzed and presented as follows.

A set of five time points is selected which reduces the average operator cost by 0.3 \$/pass-hr and average user cost by 0.1 \$/pass-hr. The time points are shown in Figure 5.30 together with outbound passenger boarding/alighting profiles, which are recommended to be placed downstream due to dense stops (i.e., higher headway fluctuation is expected due

to the interactions of passenger activities, traffic conditions and bus operations, as shown in Figure 5.29) in order to lower the operator cost.



**Figure 5.30** Optimized time points vs. outbound boarding/alighting profile.

After applying the control points in the bus route, service reliability in terms of headway coefficient of variation is presented in Figure 5.31. The red lines represent simulation results for Scenario 2, with dark red for controlled and light red for uncontrolled operation. The gray dotted lines represent results for Scenario 4 as comparison, with dark gray for controlled and light for uncontrolled operations.

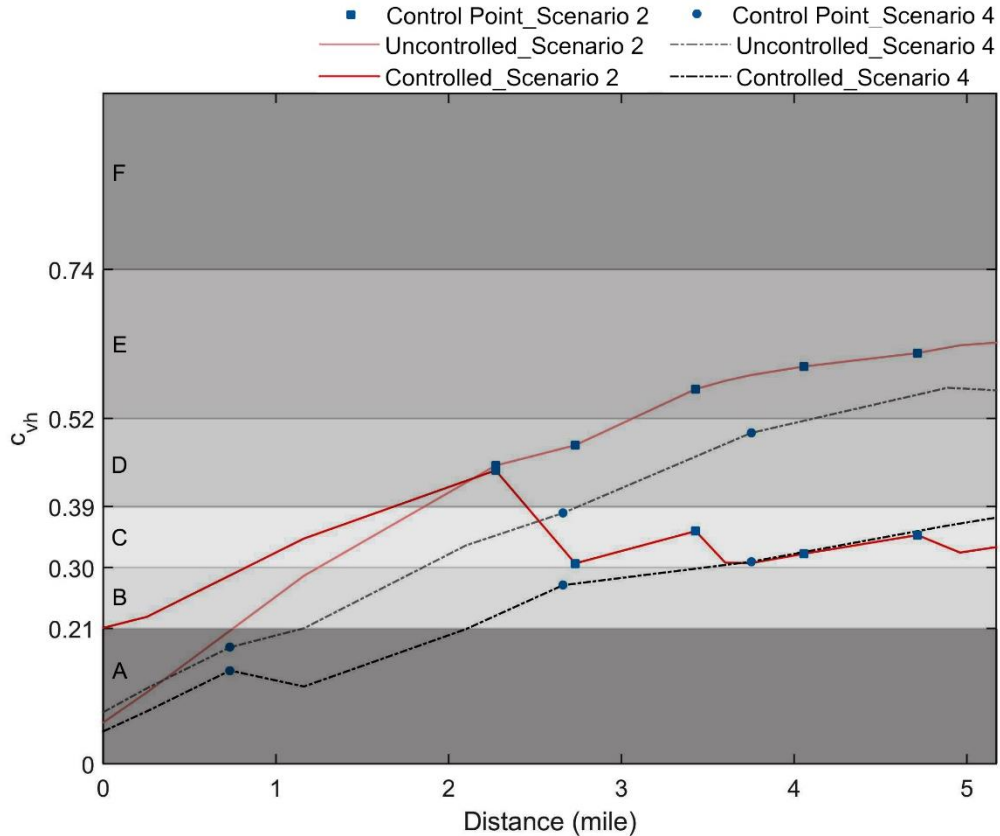
In general, after controlling, the stop level of service is improved for both scenarios. For instance, for Scenario 2, several downstream stops experiencing LOS D before controlling, whereas only one stop experiencing LOS D after introducing the control points.



Since the arrival headway is applied for calculating headway coefficient of variation, it is explainable that the variation is decreased at the immediate downstream stops of the control points.

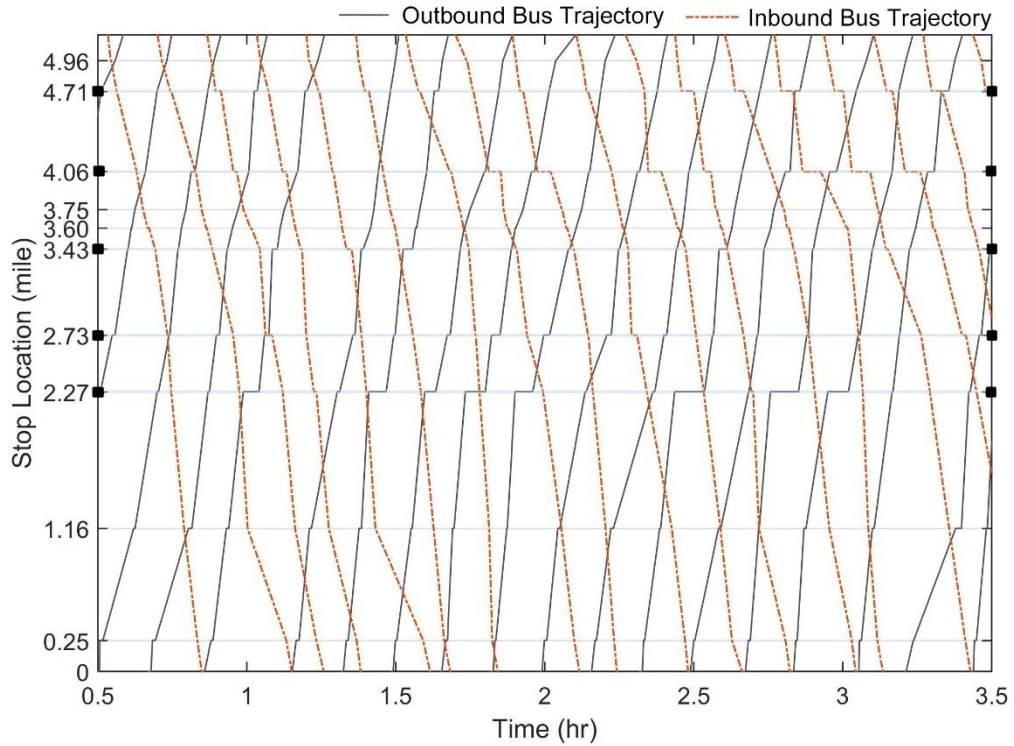
Looking into the reliability for the controlled operation under Scenario 2, although headway coefficients of variation at downstream stops are lowered due to densely allocated control points, LOS at upstream stops are not improved, if not worsen. It is obvious that compared to Scenario 4, the controlling effects with the optimal stop locations and headway under Scenario 2 is nothing better, although more time points (i.e., higher average operator cost) are placed along the route.

The reason that upstream stops suffer from severer service deterioration after controlling under Scenario 2 may be explained with vehicle trajectory data. As the inter-arrival time is used for headway calculation, when it comes to the outbound dispatching terminal, the headways and their variance are actually dependent on the inbound vehicle arrivals. Since no control points near the outbound dispatching terminal, it is understandable that inbound buses may suffer from large deviations, leading to high headway coefficient of variation.



**Figure 5.31** Headway coefficient of variation vs. control points under different scenarios.

An example from controlled bus trajectories is used for illustration as shown in Figure 5.32. Light grey lines are the outbound bus movements, whereas the orange dotted lines stand for inbound bus movements. The locations of time points are represented by the black dots located in y-axes. It is clear to see that with control points, the service regularity is significantly improved although there are still several irregular headways especially in the outbound direction. However, associated with the locations of time points, it is reasonable that the outbound headways are more regular at downstream stops (upper part in Figure 5.32), whereas the headway deviations at the stops near the outbound dispatching terminal are much larger than those stops near the inbound dispatching terminal.



**Figure 5.32** Controlled bus trajectories under scenario 2.

## 5.4 Summary

In this chapter, three case studies for the three proposed models were conducted to examine the capability and effectiveness of the developed models. Particularly, the input parameters were modified to fit model requirements. Sensitivity analysis were also conducted to investigate the relationship between input parameters (e.g., demand level, travel time variance level, etc.) and system performance.

The results not only presented maximized operator profit under demand elasticity with optimized decision variables, but illustrated service reliability in terms of headway coefficient of variation before and after optimization, with and without considering stochastic vehicle arrivals. The trade-offs between the objectives of users and operators

under different system settings were also analyzed against decision variables to provide a clearer picture for decision making. The findings are summarized in the following chapter.

## **CHAPTER 6**

### **CONCLUSIONS**

This chapter summarizes the findings from the previous chapters and concludes the dissertation with further research possibilities.

#### **6.1 Findings**

Stochastic bus arrivals caused by variable en-route travel time and dwell time at stops not only cost more wait and in-vehicle time but also suggest a greater fleet size managed by the transit supplier to maintain regular service. Previous planning models tended to overlook the influence of stochastic vehicle arrivals, which led to unrealistic results. Moreover, under congestion condition, implementing the planning model without considering variability could result in poor system performance and reduced transit attractiveness.

To solve the problem, this dissertation proposed new models to analyze the influence of travel time variability and optimize various decision variables (i.e., headway, stops, and time points) for maximum profit operation. A series of three models were developed and applied in the real world case study in Chengdu, China. With Model I as a proof-of-concept, the second model enhanced the first model through generalizing to fit a more realistic bus route. Finally, the third model extended Model II by optimizing another planning parameter (i.e., time point) to further improve system performance in terms of reliability.

In particular, Model I was applied to optimize stop spacing and headway, where the impacts of stochastic vehicle arrivals on both users and the operator were considered. As headway variance increased with the number of stops, the user wait times at downstream stops were found higher than those boarding at upstream stops. Therefore, the stop spacing, yielding the maximum profit suggested by the traditional model without considering headway variance, was shorter than that was obtained in this study. Furthermore, the total profit function was found relatively flat near the optimum, which implied that minor changes in the solution allowed transit operators considerable flexibility in fitting the stop locations to local circumstances without significant change in the profit.

Sensitivity analyses were conducted in terms of demand level and headway variance increment. Two additional indices (i.e., average operator cost per passenger for cost-effectiveness, and average user cost for system attractiveness) were also applied to further investigate the trade-offs between conflicting objectives for the users and the operator when the model parameters change. With increasing demand, the proposed model I was more capable to reflect the impact of demand changes on the average user and operator costs, with increased stop spacing and decreased headway. Also suggested by the proposed model I, when the headway variance increases, even though stop spacing and headway were increased responding to the change of congestion level, both of the average operator cost and average user cost were unavoidably increasing.

To further examine the influence of headway variance as well as enhance the model to be applied in more generalized network, Model II was developed to optimize stop locations and dispatching headway, which maximized operator profit considering demand elasticity. The headway variance was modelled to consider the joint effects of stop-to-stop

travel time, intersection delay, and the variation of dwell time over space and time. Finally, with the proposed model and solution algorithm, a case study in Chengdu CBD area in China was conducted. According to the results of the case study, it was found that the model was effective for a proper service planning so that the profit was significantly increased and the LOS of the study route was elevated, compared to both existing configuration and the configuration suggested by the traditional model without considering variance. The results also suggested that ignoring the impact of travel time variability in the service planning optimization led to poor planning decision and costed more to both transit users and supplier.

With the concern that the variance control through stop and headway optimization may not be efficient enough especially under high congestion condition, the third model - a simulation-based optimization model of time points was developed for further performance improvement, where a headway-based bus control strategy was applied. The developed model aimed at achieving the equilibrium of average user cost and average operator cost through finding the Pareto optimality between these two objectives.

To solve the problem, a multi-objective genetic algorithm was applied, in which the fitness value was estimated through simulation. A dedicated simulation model was developed and calibrated with the results from Model II. The decision variable of genetic algorithm was the input of the simulation model, and the output of the simulation model served as a basis for fitness evaluation in the genetic algorithm.

Taking the inputs (i.e., dispatching headway and stop locations) from the optimization results yielded by Model II, the developed model was examined against system performance. It was found that through control point optimization, service

reliability was significantly improved in terms of headway coefficient of variation. The side effect was, however, due to extra holding time at the controlling stops, the total travel time as well as stop-level headway was increased. On the other hand, passengers benefited from reduced waiting time at the cost of increased in-vehicle time.

The results showed that the headway-based holding strategy effectively improved system reliability. However, it should be noted that such effects were based on current configuration, where the deviation of headway was intermediate, with only downstream stops experiencing bus bunching. Situations may exist when most of the buses are late or demand level is very high, so that the effectiveness of adding control points should be further investigated. Although under extreme congestion conditions, it is hardly possible to control bus operation effectively, the proposed model provided the idea that how the planning and operation can be integrated to improve bus level of service under stochastic vehicle arrivals. Future studies are listed in the next section.

## **6.2 Future Studies**

This research attempted to provide mathematical models to deal with general feeder and conventional bus routes that are extendable for future applications. To enhance the developed models in adapting other real-world cases, calibration of input parameters should be conducted. To expand the proposed models for incorporating other influencing factors in the decision-making process, the following research is recommended.

1. Due to lack of link travel time information of the study route, the average travel time variance per mile was assumed. Actual link travel time information shall be collected, so that the functional form of travel time variance can be calibrated to match the actual traffic situation.



2. Random passenger arrivals assumed in this study is based on short-headway operations. Since passenger arrivals patterns are influenced by headway, they should be studied in the future research and incorporated into the proposed models.
3. Investigation of travel time and travel time variance elasticity of demand for Model II will lead to an enhanced model, which incorporates both mobility and reliability factors into passenger choices.
4. Introducing proper time points reduces headway variance. The bus route with reduced headway variance may allow additional stops to lower passenger access cost, which in turn leads to further modification of time points. Such interaction could be taken into account by integrating Model III with Model II, where time points, stops, and headway could be optimized simultaneously with a bi-level modelling approach.
5. Vehicle controlling strategies applied in the simulation model have an influence on system performance in terms of reliability. Therefore, Model III could be further enhanced by considering and comparing different controlling strategies.
6. Although the average dispatching headway is optimized by Model II, during bus operating, the actual headways could be adjusted based on operational and controlling needs. The investigation of such possibility could be integrated into Model III for enhancement.

**APPENDIX A**  
**VARIABLE DEFINITIONS**

The variables used in the dissertation and their definitions are summarized in Table A.1.

**Table A.1** Variables and Definitions

Variable	Definition	Unit	Value
$A_i$	a set of demand pairs ending at stop $i$	-	-
$AT_i^m$	arrival time of bus $m$ at stop $i$	-	-
$B_i$	a set of demand pairs originating at stop $i$	-	-
$C$	bus capacity	spaces/veh	60
$C_a$	access/egress cost	\$/hr	-
$C_b$	operator cost	\$/hr	-
$C_t$	total cost	\$/hr	-
$C_u$	user cost	\$/hr	-
$C_v$	in-vehicle cost	\$/hr	-
$C_w$	wait cost	\$/hr	-
$D_{K_o}$	access distance of $q_o$ from its origin to the nearest stop $K_o$	mile	-
$D_{L_o}$	access distance of $q_o$ its destination to the nearest stop $L_o$	mile	-
$DT_i^m$	departure time of bus $m$ at stop $i$	-	-
$F$	fleet size	-	-
$H$	average headway	hr	-
$H_m$	minimum headway	hr	-
$H_M$	maximum headway	hr	-
$I$	set of potential stop locations	-	-
$I_c$	set of controlled stops	-	-
$I_{uc}$	set of uncontrolled stops	-	-
$K_o$	boarding stop of $q_o$	-	-

**Table A.1** Variables and Definitions - Continued

Variable	Definition	Unit	Value
$L$	bus route length	mile	-
$L_o$	alighting stop of $q_o$	-	-
$M$	set of buses travelling through the route during the study time period	-	-
$N$	number of potential stops over the route	-	-
$N_o$	number of stops between stops $K_o$ and $L_o$	-	-
$O$	set of OD demand pairs	-	-
$Q$	demand along the bus route	pass/hr	-
$Q_i$	hourly served passengers at stop $i$	pass/hr	-
$S$	stop spacing	mile	-
$S_1$	outbound origin terminal	-	-
$S_n$	outbound destination terminal	-	-
$T_o$	expected average travel time for demand pair $o$	hr	-
$\bar{T}_o$	reference travel time for demand pair $o$	hr	-
$T_r$	round-trip bus travel time	hr	-
$T_v$	total in-vehicle travel time	hr	-
$T_{vw}$	total additional wait time for through passengers due to holding	hr	-
$V_b$	average bus cruising speed	mph	25
$V_p$	average passenger access speed	mph	2
$X$	total number of intersections over the route	-	-
$X_i$	number of intersections between stop $i$ and stop $i+1$	-	-
$X_o$	number of intersections between stops $K_o$ and $L_o$	-	-
$a_i$	number of alighting passengers at stop $i$	pass/hr	-
$b_i$	number of boarding passengers at stop $i$	pass/hr	-
$b_i^m$	number of boarding passengers at stop $i$ for bus $m$	-	-
$c$	dispatching headway variance	min <sup>2</sup> /mile	0
$c_b$	average operator cost	\$/pass-hr	-

**Table A.1** Variables and Definitions - Continued

Variable	Definition	Unit	Value
$c_u$	average user cost	\$/pass-hr	-
$c_{bh}$	stop level headway coefficient of variation	-	-
$\overline{c_{bh}}$	route level headway coefficient of variation	-	-
$d_s$	average acceleration/deceleration delay per stop	second	10
$d_x$	average intersection delay	second	10
$e_t$	coefficient of travel time	-	0.03
$f$	fare	\$/pass-trip	1.5
$h_i^m$	headway between bus $m$ and its preceding bus $m-1$	hr	-
$h_i^{m+1}$	estimated headway between bus $m$ and its following bus $m+1$	hr	-
$\overline{h_i^m}$	the average headway of and	hr	-
$l_{i-1}$	distance between stops $i-1$ and $i$	mile	-
$l_o$	distance between stops $K_o$ and $L_o$ for demand pair $o$	mile	-
$q_i$	potential demand at stop $i$	pass/hr	-
$q_o$	potential demand for OD pair $o$	pass/hr	-
$r_i^m$	through passengers in bus $m$ at stop $i$		-
$t_{a_i}$	average passenger access/egress time for passengers boarding at stop $i$	hr/pass	-
$t_{a_o}$	average passenger access/egress time for demand pair $o$	hr/pass	-
$t_b$	total vehicle running travel time	hr	-
$t_i^m$	link travel time for bus $m$ from stop $i$ to stop $i+1$	-	-
$t_l$	recovery time at terminal due to service unreliability	hr	-
$t_l^m$	recovery time for bus $m$	-	-
$t_{w_i}$	average passenger wait time at stop $i$	hr/pass	-
$t_{v_i}$	average passenger in-vehicle time for passengers boarding at stop $i$	hr/pass	-
$t_{v_o}$	average passenger in-vehicle time for demand pair $o$	hr/pass	-
$u_a$	value of passenger access time	\$/pass-hr	8

**Table A.1** Variables and Definitions - Continued

Variable	Definition	Unit	Value
$u_b$	unit operating cost	\$/ (veh-hr)	40
$u_v$	value of passenger in-vehicle time	\$/ (pass-hr)	8
$u_w$	value of passenger wait time	\$/ (pass-hr)	16
$v_{ei}$	variance caused by en-route factors	hr <sup>2</sup>	0
$v_i$	headway variance at stop $i$	hr <sup>2</sup> /mile	-
$v_t$	unit travel time variance	min <sup>2</sup> /mile	4
$x_i^m$	number of passengers arriving at stop $i$ during holding period of bus $m$	-	-
$\alpha$	increment of headway variance per stop	min <sup>2</sup> /stop	1
$\beta$	control strength	-	1
$\delta_i$	additional budgeted time due to service unreliability for demand boarding at stop $i$	hr	-
$\delta_o$	additional budgeted time due to service unreliability for demand pair $o$	hr	-
$\rho$	average passenger boarding/alighting time	second	6
$\tau_i^m$	holding time at stop $i$ for bus $m$	second	-

## APPENDIX B

### DERIVATION OF HEADWAY VARIANCE

This section explains the derivation of headway variance for Model II, which is based on the study conducted by Adebisi (1986). In the formulation of headway variance, the influences of intersection delay as well as link travel time variation are taken into account.

Let  $I$  be the set of stops, and  $i$  is an index of stop, as defined earlier. Assume that passenger arrival at each stop is uniformly distributed within a certain period, with arrival rate  $\alpha_i$  varying with stop and a standard deviation of zero. Assume the average travel time per mile is  $t$  and the variance is  $v_t$ . The intersections are independent, and for each intersection, there will be an average delay  $d_x$  without variance.

Therefore, the average travel time from stop  $i$  to stop  $i+1$ , denoted as  $E(t_i)$ , and the variance, denoted as  $v_{t_i}$ , could be represented as follows:

$$E(t_i) = l_i \cdot t + X_i \cdot d_x \quad (\text{B.1})$$

$$v_{t_i} = l_i v_t \quad (\text{B.2})$$

$l_i$ : the route length between stops  $i$  and  $i+1$

$X_i$ : the number of intersections between stops  $i$  and  $i+1$

Suppose that bus  $m$  arrives at stop  $i$  at  $T_i^m$ , the dwell time due to passenger boarding/alighting is  $d_i^m$ , the acceleration/deceleration delay time at stop  $i$  ( $d_s$ ) is fixed and identical for all stops. The travel time from stop  $i$  to stop  $i+1$  for bus  $m$  is denoted as  $t_i^m$ . Thus, bus arrival time at the immediate downstream stop, stop  $i+1$ , should be

$$T_{i+1}^m = T_i^m + d_i^m + d_s + t_i^m \quad (\text{B.3})$$

Let  $h_{i+1}^m$  be the headway between the bus  $m$  and the bus  $m-1$  at stop  $i+1$ , then

$$h_{i+1}^m = T_{i+1}^m - T_{i+1}^{m-1} \quad (\text{B.4})$$

Substitute Equation 5.17 with Equation 5.16, the headway  $h_{i+1}^m$  could be reformulated as

$$h_{i+1}^m = h_i^m + \rho \cdot \Delta q_i^m + \Delta t_i^m \quad (\text{B.5})$$

where  $\Delta q_i^m$  is the demand difference between bus  $m$  and bus  $m-1$ .

The average difference of link travel time between trips,  $\Delta t_i$  and the variance of travel time difference,  $v_{\Delta t_i}$ , are as follows:

$$\Delta t_i = 0, v_{\Delta t_i} = 2(1 - \rho_t) v_{t_i} \quad (\text{B.6})$$

where  $\rho_t$  is the correlation coefficient between  $t_i^m$  and  $t_i^{m-1}$  for all  $i$  and  $m$ . Especially, when the link travel times are independent under unstable traffic condition,  $\rho_t$  tends to be 0, when the traffic condition is relatively stable,  $\rho_t$  tends to be 1. Similarly, the average of boarding difference and the variance of such difference are as follows.

$$\Delta q_i = 0, v_{\Delta q_i} = 2(1 - \rho_q) v_{q_i} \quad (\text{B.7})$$

where  $\rho_q$  is the correlation coefficient between  $q_i^m$  and  $q_i^{m-1}$  for all  $i$  and  $m$ . Especially under congestion conditions, short headways are usually followed by long headways, making  $\rho_q$  close to -1. If the travel time variation is minor, the headway between vehicle arrivals at stops would be relatively regular,  $\rho_q$  tends to be 0. Therefore, the average headway at stop  $i$ , denoted as  $E(h_i)$ , will be identical for all stops, could be represented as follows:

$$E(h_i) = H, \forall i \in I \quad (\text{B.8})$$

The headway variance at stop  $i$ ,  $v_i$  can be formulated as:

$$v_i = v_{i-1} + v_{\rho \cdot \Delta q_{i-1}} + v_{\Delta t_{i-1,i}} + \rho \cdot \sigma_{(\Delta q_{i-1}, h_{i-1})} + \rho \cdot \sigma_{(\Delta q_{i-1}, \Delta t_{i-1})} + \sigma_{(h_{i-1}, \Delta t_{i-1})} \quad (\text{B.9})$$

$\sigma_{(\Delta q_{i-1}, h_{i-1})}$  : Covariance of the headway at stop  $i-1$  and the boarding demand difference at stop  $i-1$

$\sigma_{(\Delta q_{i-1}, \Delta t_{i-1})}$  : Covariance of travel time difference of stops  $i-1$  to  $i$  and the boarding demand difference at stop  $i-1$ . Since these two variables are independent, the covariance is zero

$\sigma_{(h_{i-1}, \Delta t_{i-1})}$  : Covariance of travel time difference of stops  $i-1$  to  $i$  and the headway at stop  $i-1$ . These two variables are independent, so the covariance is zero

The boarding demand for bus  $m$  at stop  $i$ ,  $q_i^m$ , could be estimated through the following formulation:

$$q_i^m = \int_{T_i^{m-1}}^{T_i^m} b_i dt \quad (\text{B.10})$$

From the above equation, the average boarding demand over a headway,  $q_i$ , and its variance  $v_{q_i}$  could be formulated.

$$q_i = b_i H, v_{q_i} = b_i^2 v_i \quad (\text{B.11})$$

Under the situation that long headways are followed by short headways, the covariance of the headway and the difference of boarding/alighting demand at stop  $i-1$ ,  $\sigma_{(\Delta q_{i-1}, h_{i-1})}$  could then be represented as  $\sigma_{(\Delta q_{i-1}, h_{i-1})} = 2b_{i-1} v_{i-1}$ . When the successive headways are independent,  $\sigma_{(\Delta q_{i-1}, h_{i-1})} = b_{i-1} v_{i-1}$ .

If congestion exists suggesting an unstable traffic condition,  $\rho_i$  is close to 0 and  $\rho_q$  tends to be -1. Therefore, the headway variance at stop  $i$  could be reformulated as:

$$v_i = (1 + 2\rho \cdot b_{i-1}) v_{i-1} + 4\rho^2 b_{i-1}^2 v_{i-1} + 2l_{i-1} v_i \quad (\text{B.12})$$



## REFERENCES

- A Handbook for measuring customer satisfaction and service quality.* (1999). Washington, D.C.: National Academy Press.
- Abdel-Aty, M. A., & Jovanis, P. P. (1995). The effect of ITS on transit ridership. *ITS QUARTERLY*, 3(2), 21-25.
- Abkowitz, M., & Engelstein, I. (1984). Methods for maintaining transit service regularity. *Transportation Research Record: Journal of the Transportation Research Board*, 961(1), 1-8.
- Abkowitz, M., Eiger, A., & Engelstein, I. (1986). Optimal control of headway variation on transit routes. *Journal of Advanced Transportation*, 20(1), 73-88.
- Adamski, A. (1992). Probabilistic models of passengers service processes at bus stops. *Transportation Research Part B: Methodological*, 26(4), 253-259.
- Adebisi, O. (1986). A mathematical model for headway variance of fixed-route buses. *Transportation Research Part B: Methodological*, 20(1), 59-70.
- Andersson, P. Å., Hermansson, Å., Tengvald, E., & Scalia-Tomba, G. P. (1979). Analysis and simulation of an urban bus route. *Transportation Research Part A: General*, 13(6), 439-466.
- Arbex, R. O., and Cunha C. B. (2015). Efficient transit network design and frequencies setting multi-objective optimization by alternating objective genetic algorithm. *Transportation Research Part B: Methodological*, 81, 355-376.
- Barnett, A. (1974). On controlling randomness in transit operations. *Transportation Science*, 8(2), 102-116.
- Benn, H. P. (1995). *Bus route evaluation standards*. Washington, D.C.: National Academy Press.
- Bertini, R. L., & El-Geneidy, A. M. (2004). Modeling transit trip time using archived bus dispatch system data. *Journal of Transportation Engineering*, 130(1), 56-67.
- Bielli, M., Caramia, M., & Carotenuto, P. (2002). Genetic algorithms in bus network optimization. *Transportation Research Part C: Emerging Technologies*, 10(1), 19-34.
- Bly, P. H. (1976). *Depleted bus services: the effect of rescheduling*, Crowthorne, UK: Transport and Road Research Laboratory.

- Cambridge Systematics & Economic Development Research Group. (1999). *Public transportation and the nation's economy: a quantitative analysis of public transportation's economic impact*. Washington, D.C.: American Public Transportation Association.
- Cats, O., Larijani, A., Koutsopoulos, H., & Burghout, W. (2011). Impacts of holding control strategies on transit performance: Bus simulation model analysis. *Transportation Research Record: Journal of the Transportation Research Board*, 2216(1), 51-58.
- Cats, O., Larijani, A., Ólafsdóttir, Á., Burghout, W., Andreasson, I., & Koutsopoulos, H. (2012). Bus-holding control strategies: Simulation-based evaluation and guidelines for implementation. *Transportation Research Record: Journal of the Transportation Research Board*, 2274(1), 100-108.
- Ceder, A., & Wilson, N. H. (1986). Bus network design. *Transportation Research Part B: Methodological*, 20(4), 331-344.
- Ceder, A., Prashker, J. N., & Stern, J. I. (1983). An algorithm to evaluate public transportation stops for minimizing passenger walking distance. *Applied Mathematical Modelling*, 7(1), 19-24.
- Cervero, R. (1990). Transit pricing research. *Transportation*, 17(2), 117-139.
- Chakroborty, P. (2003). Genetic algorithms for optimal urban transit network design. *Computer-Aided Civil and Infrastructure Engineering*, 18(3), 184-200.
- Chakroborty, P., & Wivedi, T. (2002). Optimal route network design for transit systems using genetic algorithms. *Engineering Optimization*, 34(1), 83-100.
- Chang, S., & Hsu, C. L. (2001). Modeling passenger waiting time for intermodal transit stations. *Transportation Research Record: Journal of the Transportation Research Board*, 1753(1), 69-75.
- Chang, S. K., & Schonfeld, P. M. (1991). Multiple period optimization of bus transit systems. *Transportation Research Part B: Methodological*, 25(6), 453-478.
- Chang, S. K., & Schonfeld, P. M. (1993). Welfare maximization with financial constraints for bus transit systems. *Transportation Research Record: Journal of the Transportation Research Board*, 1395(1), 48-57.
- Chen, X., Yu, L., Zhang, Y., & Guo, J. (2009). Analyzing urban bus service reliability at the stop, route, and network levels. *Transportation Research Part A: Policy and Practice*, 43(8), 722-734.
- Chien, S., & Spasovic, L. N. (2002). Optimization of grid bus transit systems with elastic demand. *Journal of Advanced Transportation*, 36(1), 63-91.

- Chien, S., & Schonfeld, P. (1997). Optimization of grid transit system in heterogeneous urban environment. *Journal of Transportation Engineering*, 123(1), 28-35.
- Chien, S., & Yang, Z. (2000). Optimal feeder bus routes on irregular street networks. *Journal of Advanced Transportation*, 34(2), 213-248.
- Chien, S., Dimitrijevic, B. V., & Spasovic, L. N. (2003). Optimization of bus route planning in urban commuter networks. *Journal of Public Transportation*, 6(1), 53-80.
- Chien, S., & Schonfeld, P. (1998). Joint optimization of a rail transit line and its feeder bus system. *Journal of Advanced Transportation*, 32(3), 253-284.
- Chien, S., & Qin, Z. (2004). Optimization of bus stop locations for improving transit accessibility. *Transportation Planning and Technology*, 27(3), 211-227.
- Chien, S., Daripally, S. K., & Kim, K. (2007). Development of a probabilistic model to optimize disseminated real-time bus arrival information for pre-trip passengers. *Journal of Advanced Transportation*, 41(2), 195-215.
- Chien, S., Chowdhury, S. M., Mouskos, K. C., & Ding, Y. (2000). Enhancements of CORSIM model in simulating transit operations. *Journal of Transportation Engineering*, 126(5), 396-404.
- Daraio, C., Diana, M., Di Costa, F., Leporelli, C., Matteucci, G., & Nastasi, A. (2016). Efficiency and effectiveness in the urban public transport sector: A critical review with directions for future research. *European Journal of Operational Research*, 248(1), 1-20.
- Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*. Chichester, UK: John Wiley & Sons.
- Deb, K., & Goel, T. (2001, March). Controlled elitist non-dominated sorting genetic algorithms for better convergence. In *Evolutionary Multi-criterion Optimization* (pp. 67-81). Berlin, DE: Springer Berlin Heidelberg.
- Delgado, F., Munoz, J. C., & Giesen, R. (2012). How much can holding and/or limiting boarding improve transit performance?. *Transportation Research Part B: Methodological*, 46(9), 1202-1217.
- Delmelle, E. M., Li, S., & Murray, A. T. (2012). Identifying bus stop redundancy: A gis-based spatial optimization approach. *Computers, Environment and Urban Systems*, 36(5), 445-455.
- DiJoseph, P., & Chien, S. I. J. (2013). Optimizing sustainable feeder bus operation considering realistic networks and heterogeneous demand. *Journal of Advanced Transportation*, 47(5), 483-497.

- Dowling, R., Flannery, A., Ryus, P., Vandehey, M., Petritsch, T., Landis, B., Roupail, N., and Bonneson, J. (2008). *Multimodal level of service analysis for urban streets*. Washington, DC: Transportation Research Board.
- Dueker, K. J., Kimpel, T. J., Strathman, J. G., & Callas, S. (2004). Determinants of bus dwell time. *Journal of Public Transportation*, 7(1), 21-40.
- Eberlein, X. J., Wilson, N. H., & Bernstein, D. (2001). The holding problem with real-time information available. *Transportation Science*, 35(1), 1-18.
- Economic recovery: Promoting growth*. (2012). Washington, D.C.: American Public Transportation Association.
- El-Geneidy, A. M., Horning, J., & Krizek, K. J. (2011). Analyzing transit service reliability using detailed data from automatic vehicular locator systems. *Journal of Advanced Transportation*, 45(1), 66-79.
- El-Geneidy, A., Strathman, J., Kimpel, T., & Crout, D. (2006). Effects of bus stop consolidation on passenger activity and transit operations. *Transportation Research Record: Journal of the Transportation Research Board*, 1971(1), 32-41.
- Fan, L., & Mumford, C. L. (2010). A metaheuristic approach to the urban transit routing problem. *Journal of Heuristics*, 16(3), 353-372.
- Fan, W., & Machemehl, R. B. (2004). *Optimal transit route network design problem: Algorithms, implementations, and numerical results*. Austin, TX: Southwest University Transportation Center, University of Texas at Austin.
- Fan, W., & Machemehl, R. B. (2006a). Optimal transit route network design problem with variable transit demand: genetic algorithm approach. *Journal of Transportation Engineering*, 132(1), 40-51.
- Fan, W., & Machemehl, R. B. (2006b). Using a simulated annealing algorithm to solve the transit route network design problem. *Journal of Transportation Engineering*, 132(2), 122-132.
- Fan, W., & Machemehl, R. B. (2008). Tabu search strategies for the public transportation network optimizations with variable transit demand. *Computer-Aided Civil and Infrastructure Engineering*, 23(7), 502-520.
- Fu, L., & Yang, X. (2002). Design and implementation of bus-holding control strategies with real-time information. *Transportation Research Record: Journal of the Transportation Research Board*, 1791(1), 6-12.
- Furth, P. G. (1995). A headway control strategy for recovering from transit vehicle delays. In *Transportation Congress: Civil Engineers - Key to the World Infrastructure*, 2, 2032-2039.

- Muller, T. H., & Furth, P. G. (2000). Integrating bus service planning with analysis, operational control, and performance monitoring. In *Proceedings of Intelligent Transportation Society of America Annual Meeting*.
- Furth, P., & Muller, T. (2007). Service reliability and optimal running time schedules. *Transportation Research Record: Journal of the Transportation Research Board*, 2034(1), 55-61.
- Furth, P. G., & Muller, T. H. (2009). Optimality conditions for public transport schedules with timepoint holding. *Public Transport*, 1(2), 87-102.
- Furth, P. G., & Rahbee, A. (2000). Optimal bus stop spacing through dynamic programming and geographic modeling. *Transportation Research Record: Journal of the Transportation Research Board*, 1731(1), 15-22.
- Furth, P. G., Mekuria, M. C., & SanClemente, J. L. (2007a). Parcel-level modeling to analyze transit stop location changes. *Journal of Public Transportation*, 10(2), 73-91.
- Furth, P. G., Mekuria, M., & SanClemente, J. (2007b). Stop spacing analysis using geographic information system tools with parcel and street network data. *Transportation Research Record: Journal of the Transportation Research Board*, 2034(1), 73-81.
- Furth, P. G., & Muller, T. (2006). Part 4: Capacity and quality of service: Service reliability and hidden waiting time: Insights from automatic vehicle location data. *Transportation Research Record: Journal of the Transportation Research Board*, 1955(1), 79-87.
- Furth, P. G., Hemily, B., Muller, T. H., & Strathman, J. G. (2006). *Using archived AVL-APC data to improve transit performance and management*. Washington, DC: Transportation Research Board.
- Gao, Z., Sun, H., & Shan, L. L. (2004). A continuous equilibrium network design model and algorithm for transit systems. *Transportation Research Part B: Methodological*, 38(3), 235-250.
- Gao, Z., Wu, J., & Sun, H. (2005). Solution algorithm for the bi-level discrete network design problem. *Transportation Research Part B: Methodological*, 39(6), 479-495.
- Gen, M., Liu, B., & Ida, K. (1996). Evolution program for deterministic and stochastic optimizations. *European Journal of Operational Research*, 94(3), 618-625.
- Ghoneim, N. S., & Wirasinghe, S. C. (1987). Optimum zone configuration for planned urban commuter rail lines. *Transportation Science*, 21(2), 106-114.
- Giesen, R., Martínez, H., Mauttone, A., & Urquhart, M. E. (2015). Multi-Objective Transit Frequency Optimization: Solution Method and Its Application to a Medium-Sized City. In *Proceedings of Transportation Research Board 94th Annual Meeting*.

- Guihaire, V., & Hao, J. K. (2008). Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice*, 42(10), 1251-1273.
- Hammerle, M., Haynes, M., & McNeil, S. (2005). Use of automatic vehicle location and passenger count data to evaluate bus operations. *Transportation Research Record: Journal of the Transportation Research Board*, 1903(1), 27-34.
- Hickman, M. D. (2001). An analytic stochastic model for the transit vehicle holding problem. *Transportation Science*, 35(3), 215-237.
- Hurole, V. F., & Wirasinghe, S. C. (1980). Location of rail stations for many to one travel demand and several feeder modes. *Journal of Advanced Transportation*, 14(1), 29-46.
- Ibarra-Rojas, O. J., Delgado, F., Giesen, R., & Muñoz, J. C. (2015). Planning, operation, and control of bus transport systems: A literature review. *Transportation Research Part B: Methodological*, 77, 38-75.
- Ibeas, Á., dell'Olio, L., Alonso, B., & Sainz, O. (2010). Optimizing bus stop spacing in urban areas. *Transportation Research Part E: Logistics and Transportation Review*, 46(3), 446-458.
- Islam, M. K., & Vandebona, U. (2010). Reliability analysis of public transit systems using stochastic simulation. In *Proceedings of the 33rd Australasian Transport Research Forum Conference*.
- John, M. P., Mumford, C. L., & Lewis, R. (2014). An improved multi-objective algorithm for the urban transit routing problem. In *Evolutionary Computation in Combinatorial Optimisation* (pp. 49-60). Berlin, DE: Springer Berlin Heidelberg.
- Kepaptsoglou, K., & Karlaftis, M. (2009). Transit route network design problem: review. *Journal of Transportation Engineering*, 135(8), 491-505.
- Koffman, D. (1978). A simulation study of alternative real-time bus headway control strategies. *Transportation Research Record: Journal of the Transportation Research Board*, 663(1), 41-46.
- Krizek, K. J., & El-Geneidy, A. (2007). Segmenting preferences and habits of transit users and non-users. *Journal of Public Transportation*, 10(3), 71-94.
- Kuah, G. K., & Perl, J. (1988). Optimization of feeder bus routes and bus-stop spacing. *Journal of Transportation Engineering*, 114(3), 341-354.
- LeBlanc, L. J. (1975). An algorithm for the discrete network design problem. *Transportation Science*, 9(3), 183-199.
- Lee, A., van Oort, N., & van Nes, R. (2014). Service reliability in a network context: Impacts of synchronizing schedules in long headway services. *Transportation Research Record: Journal of the Transportation Research Board*, 2417(1), 18-26.

- Lee, D. H., Sun, L., & Erath, A. (2012). Study of bus service reliability in Singapore using fare card data. In *Proceedings of 12th Asia-Pacific Intelligent Transportation Forum*.
- Lesley, L. J. S. (1975, April). The role of the timetable in maintaining bus service reliability. In *Proceedings of Operating Public Transport Symposium*.
- Levinson H.S. (1983). Analysing transit travel time performance. *Transportation Research Record: Journal of the Transportation Research Board*, 915(1), 1-6.
- Li, H., & Bertini, R. L. (2009). Assessing a model for optimal bus stop spacing with high-resolution archived stop-level data. *Transportation Research Record: Journal of the Transportation Research Board*, 2111(1), 24-32.
- Li, Y., Xu, W., & He, S. (2013). Expected value model for optimizing the multiple bus headways. *Applied Mathematics and Computation*, 219(11), 5849-5861.
- Lin, G. S. (1995). *Adaptive control of transit operations*. Washington, D.C.: Federal Transit Administration.
- Lin, J., & Ruan, M. (2009). Probability-based bus headway regularity measure. *Intelligent Transport Systems, IET*, 3(4), 400-408.
- Lin, W. H., & Bertini, R. L. (2004). Modeling schedule recovery processes in transit operations for bus arrival time prediction. *Journal of Advanced Transportation*, 38(3), 347-365.
- Litman, T. A. (2014). *Evaluating public transit benefits and costs – best practices guidebook*. Victoria, BC: Victoria Transport Policy Institute.
- Liu, B. (2001). Uncertain programming: A unifying optimization theory in various uncertain environments. *Applied Mathematics and Computation*, 120(1), 227-234.
- Liu, G., & Wirasinghe, S. C. (2001). A simulation model of reliable schedule design for a fixed transit route. *Journal of Advanced Transportation*, 35(2), 145-174.
- Mazloumi, E., Currie, G., & Sarvi, M. (2008). Assessing measures of transit travel time variability and reliability using AVL data. In *Transportation Research Board 87th Annual Meeting*.
- Mazloumi, E., Mesbah, M., Ceder, A., Moridpour, S., & Currie, G. (2012). Efficient transit schedule design of timing points: a comparison of ant colony and genetic algorithms. *Transportation Research Part B: Methodological*, 46(1), 217-234.
- Miettinen, K. (1999). *Nonlinear multiobjective optimization* (Vol. 12). Boston, MA: Springer Science & Business Media.
- Mitchell, M. (1998). *An introduction to genetic algorithms*. Cambridge, MA: MIT Press.
- Mohring, H. (1972). Optimization and scale economies in urban bus transportation. *The American Economic Review*, 62(4), 591-604.

- Moura, J. L., Alonso, B., Ibeas, Á., & Ruisánchez, F. J. (2012). A two-stage urban bus stop location model. *Networks and Spatial Economics*, 12(3), 403-420.
- Murray, A. T., & Wu, X. (2003). Accessibility tradeoffs in public transit planning. *Journal of Geographical Systems*, 5(1), 93-107.
- Nayeem, M. A., Rahman, M. K., & Rahman, M. S. (2014). Transit network design by genetic algorithm with elitism. *Transportation Research Part C: Emerging Technologies*, 46, 30-45.
- Newell, G. F. (1977). Unstable Brownian motion of a bus trip. In *Statistical Mechanics and Statistical Methods in Theory and Application* (pp. 645-667). Boston, MA: Springer US.
- O'Dell, S. W., & Wilson, N. H. (1999). Optimal real-time control strategies for rail transit operations during disruptions. In *Computer-Aided Transit Scheduling* (pp. 299-323). Berlin, DE: Springer Berlin Heidelberg.
- O'Neill, W. A., Ramsey, R. D., & Chou, J. (1992). Analysis of transit service areas using geographic information systems. *Transportation Research Record*, 1364(1), 131-138.
- Orloff, C. S., & Ma, Y. Y. (1975). *Analytic supply models for many-to-one transportation systems*. Princeton, NJ: Transportation Program, Princeton University.
- Pattnaik, S. B., Mohan, S., & Tom, V. M. (1998). Urban bus transit route network design using genetic algorithm. *Journal of Transportation Engineering*, 124(4), 368-375.
- Peng, Z., & Dueker, K. J. (1995). Spatial data integration in route-level transit demand modeling. *Journal of the Urban and Regional Information Systems Association*, 7(1), 26-37.
- Poorzahedy, H., & Turnquist, M. A. (1982). Approximate algorithms for the discrete network design problem. *Transportation Research Part B: Methodological*, 16(1), 45-55.
- Pratt, R. H., & Evans, J. IV. (2004). *Traveler response to transportation system changes. Chapter 10-Bus routing and coverage*. Washington, D.C.: Transportation Research Board.
- Public transportation: Benefits for the 21st century*. (2007). Washington, D.C.: American Public Transportation Association.
- Public transportation fact book*. (2015). Washington, D.C.: American Public Transportation Association.
- Public transportation ridership report*. (American Public Transportation Association). Retrieved June 10, 2016, from <http://www.apta.com/resources/statistics/Pages/ridershipreport.aspx>.



- Saka, A. A. (2001). Model for Determining Optimum Bus-Stop Spacing in Urban Areas. *Journal of Transportation Engineering*, 127(3), 195-199.
- Schrank, D., Eisele, B., & Lomax, T. (2012). *TTI's 2012 urban mobility report*. College Station, TX: Texas A&M Transportation Institute.
- Senevirante, P. N. (1990). Analysis of on-time performance of bus services using simulation. *Journal of Transportation Engineering*, 116(4), 517-531.
- Spasovic, L. N., & Schonfeld, P. M. (1993). Method for optimizing transit service coverage. *Transportation Research Record*, 1402(1), 28-39.
- Spasovic, L. N., Boile, M. P., & Bladikas, A. K. (1994). Bus transit service coverage for maximum profit and social welfare. *Transportation Research Record*, 1454(1), 12-22.
- Strathman, J. G., & Hopper, J. R. (1993). Empirical analysis of bus transit on-time performance. *Transportation Research Part A: Policy and Practice*, 27(2), 93-100.
- Strathman, J. G., Kimpel, T. J., Dueker, K. J., Gerhart, R. L., & Callas, S. (2002). Evaluation of transit operations: Data applications of Tri-Met's automated bus dispatching system. *Transportation*, 29(3), 321-345.
- Strathman, J.G., Dueker, K.J., Kimpel, T., Gerhart, R.L., Turner, K., Taylor, P., Callas, S. and Griffin, D. (2000). Service reliability impacts of computer-aided dispatching and automatic vehicle location technology: A Tri-Met case study. *Transportation Quarterly*, 54(3), 85-102.
- Strathman, J., Dueker, K., Kimpel, T., Gerhart, R., Turner, K., Taylor, P., Callas, S., Griffin, D. and Hopper, J. (1999). Automated bus dispatching, operations control, and service reliability: Baseline analysis. *Transportation Research Record: Journal of the Transportation Research Board*, 1666(1), 28-36.
- Sun, Y., Cao, C., & Wu, C. (2014). Multi-objective optimization of train routing problem combined with train scheduling on a high-speed railway network. *Transportation Research Part C: Emerging Technologies*, 44, 1-20.
- Syed, S. J., & Khan, A. M. (2000). Factor analysis for the study of determinants of public transit ridership. *Journal of Public Transportation*, 3(3), 1-17.
- Szeto, W. Y., & Wu, Y. (2011). A simultaneous bus route design and frequency setting problem for Tin Shui Wai, Hong Kong. *European Journal of Operational Research*, 209(2), 141-155.
- Szeto, W. Y., & Jiang, Y. (2014). Transit route and frequency design: Bi-level modeling and hybrid artificial bee colony algorithm approach. *Transportation Research Part B: Methodological*, 67, 235-263.

- Taylor, B. D., & Fink, C. N. (2003). *The factors influencing transit ridership: A review and analysis of the ridership literature*. Berkeley, CA: University of California Transportation Center.
- The changing face of transportation*. (2000). Washington, D.C.: U.S. Department of Transportation, Bureau of Transportation Statistics.
- Tirachini, A. (2014). The economics and engineering of bus stops: Spacing, design and congestion. *Transportation Research Part A: Policy and Practice*, 59, 37-57.
- Tirachini, A., & Hensher, D. A. (2011). Bus congestion, optimal infrastructure investment and the choice of a fare collection system in dedicated bus corridors. *Transportation Research Part B: Methodological*, 45(5), 828-844.
- Tom, V. M., & Mohan, S. (2003). Transit route network design using frequency coded genetic algorithm. *Journal of Transportation Engineering*, 129(2), 186-195.
- Transit capacity and quality of service manual*. (2013). Washington, D.C.: Transportation Research Board.
- Turnquist, M. A. (1981). Strategies for improving reliability of bus transit service. *Transportation Research Record*, 818(1), 7-13.
- Turnquist, M. A., & Bowman, L. A. (1980). The effects of network structure on reliability of transit service. *Transportation Research Part B: Methodological*, 14(1), 79-86.
- van Nes, R., & Bovy, P. (2000). Importance of objectives in urban transit-network design. *Transportation Research Record: Journal of the Transportation Research Board*, 1735(1), 25-34.
- van Oort, N., & van Nes, R. (2008). Improving reliability in urban public transport in strategic and tactical design. In *Proceedings of Transportation Research Boarding 87th Annual Meeting*.
- van Oort, N., & van Nes, R. (2009a). Regularity analysis for optimizing urban transit network design. *Public Transport*, 1(2), 155-168.
- van Oort, N., & van Nes, R. (2009b). Control of public transportation operations to improve reliability: Theory and practice. *Transportation Research Record: Journal of the Transportation Research Board*, 2112(1), 70-76.
- van Oort, N., & van Nes, R. (2009c). Line length versus operational reliability: network design dilemma in urban public transportation. *Transportation Research Record: Journal of the Transportation Research Board*, 2112(1), 104-110.
- van Oort, N., Boterman, J. W., & van Nes, R. (2012). The impact of scheduling on service reliability: trip-time determination and holding points in long-headway services. *Public Transport*, 4(1), 39-56.

- van Oort, N., Wilson, N., & van Nes, R. (2010). Reliability improvement in short headway transit services: Schedule-and headway-based holding strategies. *Transportation Research Record: Journal of the Transportation Research Board*, 2143(1), 67-76.
- Vandebona, U., & Richardson, A. J. (1986). Effect of checkpoint control strategies in a simulated transit operation. *Transportation Research Part A: General*, 20(6), 429-436.
- Vincent, M. (2008). *Measurement valuation of public transport reliability*. Wellington, N.Z.: Land Transport New Zealand.
- Vuchic, V. R., & Newell, G. F. (1968). Rapid transit interstation spacings for minimum travel time. *Transportation Science*, 2(4), 303-339.
- Weisbrod, G., & Reno, A. (2009). *Economic impact of public transportation investment*. Washington, D.C.: American Public Transportation Association.
- Welding, P. I. (1957). The instability of a close-interval service. *Journal of the Operational Research Society*, 8(3), 133-142.
- Wirasinghe, S. C. (1993). Cost based approach to scheduling travel time on a public transportation route. In *Proceedings of 12th International Symposium on the Theory of Traffic Flow and Transportation*.
- Wirasinghe, S. C., & Ghoneim, N. S. (1981). Spacing of bus-stops for many to many travel demand. *Transportation Science*, 15(3), 210-221.
- Wirasinghe, S. C., & Liu, G. (1995a). Optimal schedule design for a transit route with one intermediate time point. *Transportation Planning and Technology*, 19(2), 121-145.
- Wirasinghe, S. C., & Liu, G. (1995b). Determination of the number and locations of time points in transit schedule design—Case of a single run. *Annals of Operations Research*, 60(1), 161-191.
- Woodhull, J. (1987). *Issues in on-time performance of bus systems: A discussion paper*. Los Angeles, CA: Southern California Rapid Transit District.
- Yang, Z., Yu, B., & Cheng, C. (2007). A parallel ant colony algorithm for bus network optimization. *Computer-Aided Civil and Infrastructure Engineering*, 22(1), 44-55.
- Zhao, F., & Zeng, X. (2006). Simulated annealing—genetic algorithm for transit network optimization. *Journal of Computing in Civil Engineering*, 20(1), 57-68.
- Zhao, F., & Zeng, X. (2008). Optimization of transit route network, vehicle headways and timetables for large-scale transit networks. *European Journal of Operational Research*, 186(2), 841-855.

- Zhao, F., Ubaka, I., & Gan, A. (2005). Transit network optimization: Minimizing transfers and maximizing service coverage with an integrated simulated annealing and tabu search method. *Transportation Research Record: Journal of the Transportation Research Board*, 1923(1), 180-188.
- Zhao, J., Dessouky, M., & Bukkapatnam, S. (2006). Optimal slack time for schedule-based transit operations. *Transportation Science*, 40(4), 529-539.