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Decision models for fast-fashion supply and stocking problems in internet fulfillment warehouses

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ABSTRACT

DECISION MODELS FOR FAST-FASHION SUPPLY AND STOCKING PROBLEMS IN INTERNET FULFILLMENT WAREHOUSES

by
Jingran Zhang

Internet technology is being widely used to transform all aspects of the modern supply chain. Specifically, accelerated product flows and wide spread information sharing across the supply chain have generated new sets of decision problems. This research addresses two such problems. The first focuses on fast fashion supply chains in which inventory and price are managed in real time to maximize retail cycle revenue. The second is concerned with explosive storage policies in Internet Fulfillment Warehouses (IFW).

Fashion products are characterized by short product life cycles and market success uncertainty. An unsuccessful product will often require multiple price discounts to clear the inventory. The first topic proposes a switching solution for fast-fashion retailers who have preordered an initial or block inventory, and plan to use channel switching as opposed to multiple discounting steps. The FFS Multi-Channel Switching (MCS) problem then is to monitor real-time demand and store inventory, such that at the optimal period the remaining store inventory is sold at clearance, and the warehouse inventory is switched to the outlet channel. The objective is to maximize the total revenue. With a linear projection of the moving average demand trend, an estimation of the remaining cycle revenue at any time in the cycle is shown to be a concave function of the switching time. Using a set of conditions the objective is further simplified into cases. The Linear Moving Average Trend (LMAT) heuristic then prescribes whether a channel switch should be made in the next period. The LMAT is compared with the optimal policy and the No-Switch and Beta-

Switch rules. The LMAT performs very well and the majority of test problems provide a solution within 0.4% of the optimal. This confirms that LMAT can readily and effectively be applied to real time decision making in a FFS.

An IFW is a facility built and operated exclusively for online retail, and a key differentiator is the explosive storage policy. Breaking the single stocking location tradition, in an IFW small batches of the same stock keeping unit (SKU) are dispersed across the warehouse. Order fulfillment time performance is then closely related to the storage location decision, that is, for every incoming bulk, what is the specific storage location for each batch. Faster fulfillment is possible when SKUs are clustered such that narrow band picklists can be efficiently generated. Stock location decisions are therefore a function of the demand arrival behavior and correlations with other SKUs. Faster fulfillment is possible when SKUs are clustered such that narrow band picklists can be efficiently generated. Stock location decisions are therefore a function of the demand behavior and correlations with other SKUs. A Joint Item Correlation and Density Oriented (JICDO) Stocking Algorithm is developed and tested. JICDO is formulated to increase the probability that M pick able order items are stocked in a δ band of storage locations. It scans the current inventory dispersion to identify location bands with low SKU density and combines the storage affinity with correlated items. In small problem testing against a MIP formulation and large scale testing in a simulator the JICDO performance is confirmed.

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This dissertation is dedicated to my beloved parents:
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CHAPTER 1

INTRODUCTION

1.1 Research Background

Rapid evolution of consumer buying behavior and options, has motivated retailers to adopt a variety of new inventory management and logistics control strategies. These include Omni channel retailing which combines outlet and online stores with regular stores (Melis, Campo, Breugelmans, and Lamey, 2015) and purely online retail where orders are shipped immediately. In this context this research addresses two related problems. The first focuses on fast fashion supply chains in which inventory and price are managed in real time to maximize retail cycle revenue. The second is concerned with explosive storage policies in Internet Fulfillment Warehouses (IFW).

For the first problem the focus is specifically on fashion goods, which are characterized by a short life cycle, high customer demand uncertainty, long supply lead times, and high price discounting after the regular selling period (Huang, Hsu, and Ho, 2014). A new generation of retailers (e.g., Zara and H&M) has successfully developed and implemented a fast-fashion supply (FFS) chain, which involves frequent in-season assortment changes, quick response sourcing of products (Iyer, 1997), and/or data driven placement of products in the appropriate retail channel. Here focus on the last strategy, whereby the retailer is able to use real-time demand information to switch product inventory to alternate channels.

In cases where the retailer is unable to achieve quick response sourcing, then a large quantity is ordered to meet the projected demand for the selling season. The question comes out: how to maximize the benefit with the “large quantity” in the planned period. To forecast fashion product demand is a tough but critical topic, where prediction for whole season, generally three to six months, is effort-consuming and highly risky.

Price markdown is an effective strategy to reduce inventory piling up or lost sales, where most of the researches from this aspect are focusing on markdown prices. Lower prices would motivate customers but also increase the potential postpone for purchase since customers would prefer to anticipate future markdowns and intentionally delay purchasing until a sale occurs, particularly in the fashion industry. This issue is relieved in Fast Fashion Supply. Some key features of FFS as indicated, short selling cycle, frequent collection turnover and quick response strategy, are effective to combat such “strategic” customer behavior (Cachon and Swinney, 2011). Shorter selling cycle and more collections weaken the enticement to postpone purchase to the clearance sales since it is risk waiting if a dress might stock out next week and new collection are displaying. Meanwhile, the effect of quick response is significant. With real-time inventory and demand monitoring, the chance that store will have inventory left for clearance price is reduced efficiently. Thus, the instance to start price markdown, rather than the setting up levels of sales prices, is more emphasized in this research.

On the other hand, outlet as another alternate selling channel, facing different group of customers, are grasping more attention of retailers, customers, and researchers. Promotion, outdated collection or factory made are keywords of outlet malls, which call for different operational strategy with regular retailers. Some of fashion appeal companies open outlet stores in outlet mall, where there is stable customer resources, e.g., tourisms, dealing with abandoned inventory when regular ones are ready to launch a new collection.

Pricing and timing, as the authors demonstrate in clearance sales, are crucial topics for researchers in such area.

The second research problem investigates operational control methods for fulfillment warehouses in online retailing systems. Internet retail, driven by its biggest champion Amazon, is growing rapidly and becoming a disruptive force in retail supply chains. Internet retailers compete with brick and mortar retailers on both the marketing side, where the goal is to sell a product virtually, and on the fulfillment side, where the goal is to provide delivery within a few days. US online retail sales as a percent of total retail sales have risen from 2.8% in 2006 to 8.2% in 2016 (Commerce, 2017), confirming that strategies adopted by many internet retailers have been successful. The published literature is primarily focused on the retailing side (Brynjolfsson, Hu, and Rahman (2013), Verhoef, Kannan, and Inman (2015), Chen and Leteney (2000)), and with only limited reported work on the fulfillment side. Onal et al. (2017) were one of the first to report on IFWs and demonstrate the fulfillment time performance advantages.

The key infrastructure components of internet retail are a network of internet fulfillment warehouses (IFWs) and a parcel delivery network. Some IFWs are simply adapted from traditional warehouses and similar in structure to the more classical mail order fulfillment facilities. Our research finds that successful Internet Fulfillment Warehouses (IFWs) are operating with design and control paradigms that are quite different from traditional fulfillment centers. IFWs present a new operational model in the design and control of warehouses. Structurally different, they are a key entity in transforming the global retail economy. Specifically the use of an explosive storage policy combined with commingled storage are shown to be key features in achieving fast fulfillment.

Traditional warehousing methods view a warehouse as a connection between a supplier and a retailer, which means that it is used as a place to receive bulk from producers and move them to stores. In an IFW there are no stores and the warehouse integrates all functions making it possible to achieve direct delivery given a diversity of customer orders. For example, the same order can combine, pens, shirts, and pasta. A traditional warehouse would require a lot of effort to fulfill thousands of orders like above, however, online retailers can respond to them in hours, even minutes.

Specific research problems that the authors propose are: (i) Formulate an MCS decision model which maximize the revenue from three different selling channels in a fixed selling horizon, then do validation in simulator to demonstrate that the total revenue is a convex function of T to reach optimal; (ii) Identify the optimization objective in IFW stocking process and the dependencies between inbound (stocking) and outbound (picking) phases; set up storage dispersion matrix by involving storage density as the basis of modified stocking algorithm; (iii) Develop established stocking algorithms combining heuristics and mixed-integer programs that leverage the explosive storage to improve the picking efficiency and consequently reduce the fulfillment time, in both stationary and dynamic way; and (iv) Extend the stocking policy by optimizing the inventory structure with involving item correlation. A dynamic stocking algorithms for optimization of the search bandwidth in storage density and stocking list size leading to higher picking probability and stocking efficiency would be considered as a further research.

1.2 Zara Fast Fashion Model

Fashion products, unlike ordinary goods, have short life cycle, highly uncertain customer demand, long lead time for manufacturing and delivering, and promotion strategy to clean up stocks is often executed after selling period (Huang et al., 2014). Due to the uncertainty of demand before the selling period, retailers would prefer to purchase a large amount of products to reduce the risk of lost sales, especially in the case where lack of historical data or for new products with no trend to be launched on. With fast fashion introduced into industries, Quick Response (QR), a movement in the apparel industry to shorten lead time (Iyer, 1997). Local sourcing instead of outsourcing to chase lower labor cost and material cost from other countries, offers a faster delivery environment to guarantee quick response in fashion market.

Zara launches a higher variety of products per season than its competitors and sells them with fewer markdowns (Caro and Gallien, 2010). Figure 1.1 illustrates the life cycle of a typical Zara article, which can be divided into four distinct phases (Gallien, Mersereau, Garro, Mora, and Vidal, 2015).

The first phase is established as a design, purchase, and production phase before introducing the new article to store as well as market. In this phase, articles are designed and manufactured by Zara or sourced from suppliers. A new season of products in average two weeks leads to a situation that designers are targeting to direct rather than capture customer tastes while manufacturing location is either close to majority of the market or within a quick delivery distance.

Following design and manufacturing, a series of initial shipments is shipped to stores, which originate from centralized warehouse stocks in Spain. Initial shipments arrived in approximately three days before articles begin selling in stores, and subsequent replenishments occur weekly thereafter, where the second shipments are determined after observing the first three or four days sales.

The third phase is a replenishment phase as mentioned above. Weekly shipments to stores are delivered until the end of the four-six week life cycle. This phase is addressed in Caro and Gallien (2010), which established a sales forecasting stochastic model during the replenishment period. Limited transshipments among stores and returns to the warehouse may occur toward the end of this phase.

The last phase is clearance phase at the end of the selling season in which products are aggressively and maybe multiple-times discounted to clear stores and warehouses for the subsequent selling season. Caro and Gallien (2012) has proposed a pricing model in clearance phase with multi-stage discounted prices.

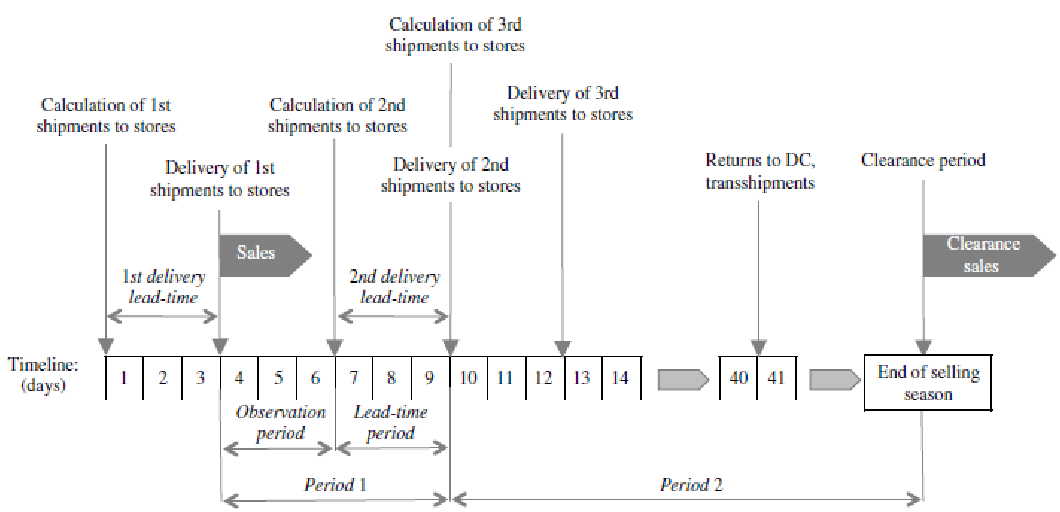

Figure 1.1 Life cycle of a typical Zara article.

Source: Gallien, J., Mersereau, A. J., Garro, A., Mora, A. D., and Vidal, M. N. (2015). Initial Shipment Decisions for New Products at Zara. *Operations Research*, *63*(2), 269-286. doi:10.1287/opre.2014.1343

1.3 Amazon Fulfillment Warehouse

Amazon is a well-known online retail company which is leading the development and operation strategy of e-commerce successfully. Their warehouses are dealing directly with individual customer orders, where the "dealing with" was named by fulfillment. This class of warehouse has been first called "fulfillment center". With total over a 110 Million square feet space of facilities and 250,000 employees, Amazon operates over 250 distribution facilities around the world including Internet Fulfillment Warehouses, returns centers, specialty centers, and redistribution centers. The first two fulfillment centers (FCs) were started in Seattle and Delaware. Both of them are relatively small compared with the warehouses newly built. The average size of Amazon Fulfillment Warehouse is over a million square feet.

The models presented here are the result of an observational study of IFWs, consisting of both facility visits and a review of published reports. The facility visits were to two Amazon fulfillment centers in the USA, one located in Indiana (1.2 Million sq. ft.) and the other in Delaware (0.9 Million sq. ft.). Both were of the Man-to-Part type and built in 2012, with approximately 2500 warehouse workers or associates. The product flows can be sequenced into three distinct process groups: (i) receiving and stocking (ii) order picking and consolidation and (iii) truck assignment and loading. The focus here is on the first group.

Amazon invested in robots made by Kiva Systems spending $775 million from 2012, to fulfillment customer orders more efficiently and labor-effectively. After introducing the Kiva robots, instead of routing around and searching for items, pickers are standing in a fixed station to complete the pick of customer orders from the shelfs moved by robots. The same process works for stocking. This significant innovation subverted the existing warehousing management strategies, bringing plenty of research opportunities. However, Kiva system is hardly to be popularized because of the high investment expense. Thus, in this research, the focus is still limited in IFWs which is operated manually.

1.4 Internet Fulfillment Warehouse

An observational study confirms that IFWs are operating under new paradigms, which are significantly distinguished from the traditional warehouses. The observational visits identified a variety of physical design and operational insights unique to IFWs. These insights were analyzed in the context of the existing knowledge base on warehouse operations. The physical flows from receiving (import) to shipping (output) are flowcharted from the insights. While schematically the flow appears to be identical to a traditional warehouse, the actual operations are quite differentiable. First of all, the overall timeline is much shorter, both the stocking and fulfillment times are measured in hours. Further, due to the large number of stocked SKUs and the high-volume throughput, inventory time is limited to better manage the warehouse size. The inventory turnover ratio of an IFW is estimated to be much higher than a traditional retail warehouse. The analysis indicates that an efficient IFW is differentiable from traditional warehouses by the following characteristics: The first objective of this work is to compare IFW's with the traditional warehouses. Specifically, the authors identify six key structural differentiations between traditional and IFW operations: (i) explosive storage policy (ii) very large number of beehive storage locations (iii) bins with commingled SKUs (iv) immediate order fulfillment (v) short picking routes with single unit picks and (vi) high transaction volumes with total digital control. In combination these have the effect of organizing the entire IFW warehouse like a forward picking area. Giving the observational view that it is operating in a chaotic mode with significantly high efficiency. Key differentiations will be explained in Chapter 4 in detail.

1.5 Research Objectives and Accomplishments

1.5.1 Dynamic Optimization of Price Differentiated Channel Switching in a Fixed Period Retail Supply

The fashion industry has perishable products, unpredicted demand. In contrast with traditional fashion industry who has long and inflexible supply and large profit margin by outsourcing, the fast-fashion supply chain focuses on avoid supply risks with sacrificing the benefit from low material and labor cost by monitoring store inventory as well as customer taste in a real-time level. As mentioned, since demand for fashion products is difficult to predict the authors assume that long term forecasting is highly unreliable. An FFS strategy then is to plan for a shorter products selling cycle, with a more frequent style turnover. The authors consider the case where the retailer operates a centralized warehouse from which product is supplied to multiple stores plus several outlet centers. At the start of the selling cycle a predetermined quantity of the product is ordered and delivered to the warehouse, from which small quantity shipments are made to the stores. Product is sold in three sequential channels with no overlap, regular store price, clearance store price, and outlet price. This is equivalent to a dynamic pricing model but limited to only two predetermined price steps. In the optimistic case demand remains strong through the season, and all the inventory is sold in the regular channel. In the pessimistic case demand weakens early on and the bulk of inventory is sold through retail. The FFS multi-channel switching (MCS) problem then is to monitor real-time demand and store inventory, such that at the optimal point the remaining warehouse inventory is switched to the outlet channel.

Accomplishments: A dynamic operational research on the real-time level decision-making problem. The authors show that there would be a solved switch time decision, in which the benefit is close to the actual optimal. This switch time decision is updating while product is right in selling period with known single or two steps markdown price. The performance validation is tested by replicated experiments in15 scenarios with different concave or convex declining demand behavior. A simulation model is built and used to capture the close-to-optimal switch solution and be compared with simple markdown plans, to confirm the advantages of multi-channel switch strategy.

1.5.2 Stocking Algorithm Development for Internet Fulfillment Warehouses

Online retailing is known as extremely large data transactions and fast response to customer orders. IFW as a combination of middle elements in traditional supply chain, is structural different in both facilities design and operation strategies. In IFW, the main objective is to optimize fulfillment performance for customer orders. Picking as a main procedure are required well-structured inventory environment to apply its batching or sequential strategies. Stocking as the supportive process, provides the potency to enhance warehouse operation efficiency by a well-defined stocking location assignment strategy. In IFW related stocking phase, to identify the performance driven objective rather than general space utilization and cost reduction is the predominant task before a feasible and efficiency storage policy can be applied.

1.5.2.1 Formulate a Stocking Objective. Traditional warehouses operate with a fixed storage location assigned to each SKU item, as such the stocking assignment problem is not applicable. When a random storage policy is used then the objective is primarily to maximize space utilization and secondarily to minimize picking routes. An IFW's explosive storage policy generates a new class of stocking problems, and the question then is what should be the assignment objective such that the overall fulfillment objective is minimized.

Accomplishments: Investigative research on the dependencies between the storage assignments and picking efficiency therefore the order fulfillment enhancement. The specific focus is how inbound movement can collaboratively improve the probability to complete pick lists while stocking effort is reduced at the same time. Two key features were identified and formulated: (i) the probability of creating a complete pick list with given number of stops and (ii) the storage density. This research emphasizes on the latter factor which is characterized in Chapter 4 detailed.

1.5.2.2 Stocking Algorithm to Optimize the Fulfillment Driven Objective. The problem of achieving a uniform inventory storage density can be formulated as a mixed integer program (MIP). But for large problems the solution time is very large, and efficient heuristics are needed, given that the problem is solved hundreds of time in an IFW day. Joint Order Frequency and Density Oriented (JOFDO) Stocking heuristics was developed to generate the stocking list for each incoming bulk batch. The heuristic solves the problem in two phases: 1st is to list the pending exploded packages and assign slots for them, 2nd is to group the packages with closest location assignments as a list to arrange to a free stocker.

Accomplishments: A static research on the behavior of storage location assignments and inventory allocation structure. The emphasis is to collaboratively improve the probability to complete pick lists with intuitive stocking assignment processing. Two key factors are identified and formulated: (i) the uniformity obtained by the distribution of inventory lots and (ii) the storage density influenced by the neighbor bin effect. This research focuses on problem formulation based on the above two factors, which is proposed in Chapter 4.2 in detail.

1.5.2.3 Item-Correlated Stocking Algorithm to Optimize Fulfillment Performance. Many items stocked in an IFW have correlated demand behaviors. Such correlations are usually defined one way, that is a demand for item A is linked to demand for item B, but the inverse is not necessarily true. Sticking location decisions must therefore be made so as to exploit this correlation during the picking process. The JOFDO heuristic is extended to integrate the correlation with the existing inventory state of other items. The Joint Item Correlation and Density Oriented (JICDO) heuristic adds an attractive force from the correlated items in making stocking decisions.

Accomplishments: A static and dynamic research on the item correlation and storage location assignment decisions. A reduced correlated-item storage location assignment model is presented with single-SKU processing assumption. Item correlation as another key factor is identified and formulated. Heuristics are proposed and evaluated by environmental simulation analysis. The results are shown in Chapter 4.

1.6 Research Significance

Internet economy brings distinguish effects on the design and operation of modern supply chain management. Innovations are coming out with new features of customer requirements and market behavior. Warehouses and retailers, as the intermediation between a customer and a producer, are meeting with great challenges but inestimable opportunities in front. Fashion industry, as well as e-retailing corporations are creating new strategies to satisfy unpredicted customer demand in which the real-time prediction and quick response system get most attention from researchers. This research develops these new models for diverse of fields, allowing extended work on the operation of fast fashion retailing and continuing research on the decision making models of internet fulfillment warehouses. These advanced models are needed by both traditional and internet retailers to survive in internet-based competition. Moreover, it also provide academic researchers ideas to formulate and optimize specific problems in such area.

CHAPTER 2

LITERATURE REVIEW

2.1 Fast Fashion Supply Chain

Fashion industry has such characterization as short product life cycles, volatile and unpredictable demand, and tremendous product variety, long and inflexible supply processes, and a complex supply chain (Sen, 2008). In such environment, every change in technology or customer preference, efficient supply chain management is studied based on different viewpoints, which gives the potential of success. Under the new critical factors influencing retailing and even market, current research literature on fast fashion retailer operation focuses on dynamic pricing, E-commerce or multi-channel retailing and Omni-channel retailing. Specifically, dynamic pricing, which indicates to multi-step non-increased pricing strategy, such as 10% to 25% to 75% off advertised in a specific store within two months, is essential issue for companies to attract more customers in order to lessen the inventory and improve sales. Moreover, retailers have to draw up the strategy to follow up the unknown demand in different period, with replenishment and pricing markdown. In our study, dynamic pricing is simplified to be single step, from retailer channel to outlet channel, with known discounted price and constant outlet demand. In the following subsections, the authors address several fast fashion features and a brief review of the background research is related.

2.1.1 Fast Fashion and Quick Response Supply

Sen (2008) provides an extensive review about the US fashion industry and the supply chain driving it. They note that a quick response retailer will track sales at the store-level on a real-time basis, and maintain minimal inventories at the store. Zara is the most prominent example of an FFS model and key aspects are reported by Ghemawat (2003). They observe that the FFS operations strategy combines two critical features: (i) quick response production capabilities and (ii) enhanced product design capabilities (Cachon and Swinney, 2011). Caro et al (2010) found the Zara supply chain incorporates a forecasting model which would prescribe the initial block inventory in or case, and an optimization model to control the retail strategy once actual sales data is tracked, the switching model in our case. Iyer (1997) discuss quick response manufacturing to retailer channels in general, while Cachon and Swinney (2009) give a detailed explanation about the strategic customer behavior under quick response. Huang (2013) derive a dynamic pricing model with partial backlogging to investigate the important factors that influence the replenishment cycle and profit. Caro and Gallien (2012) and Karakul (2008) show that regular demand behavior is a function of price and age of the product while clearance or discounted price is more difficult to manipulate. From discussions with leading fashion retailers, Choi (2007) found that many use a two-stage stocking policy, whereby an initial block inventory is supplemented with a second stocking order using actual demand data. Pricing decisions were also made similarly.

2.1.2 Multi-channel Distribution and Multi-Period Retailing

In today's retailing environment retailers are leveraging their supply chains to expand sales volume and profit beyond their traditional store channels (Chiang, 2003; Ding, Dong, and Pan, 2016). Several researchers have broadly studied customer behavior differences across channels and specifically looked at channel adoption, channel choice and usage (Verhoef et al., 2015). Innovations in retail promotions and expansion of outlet malls are providing new retail channels that are readily integrated into a multi-channel distribution strategy. Specifically dynamic pricing combined with targeted promotions can be used to effectively and quickly sell excess inventory (Grewal et al., 2011). Coughlan and Soberman (2005) present an analysis of two possible structures of dual-distribution through both regular retailer channel and outlet channel. One option is to sell in multiple channels simultaneously. Alternatively, the manufacturer or retailer can make sequential decisions in two or three channels. The identify possible decisions as (i) how much to distribute to a primary regular store channel, and (ii) whether or not to add an outlet into the distribution mix.

Two-period pricing models are widely studied in the literature, most of these consider the price to be the decision variable (J. Zhang, Shou, and Chen, 2013). Zhou et al (2015) consider a two period pricing model for launching fashion products. Three strategies are identified one of which is labelled the S-Strategy: that is the firm launches a new style and stops selling the previous one immediately. This operationally equivalent to the model developed here, in that the old design is shifted to another channel, so that the high value store channel is immediately focused on the new product. Similar to this research they observe that luxury retailers will sell then their discontinued styles in their outlet stores. Here the authors consider the price to fixed and decide on the switch time. Khouja et al (2010) analyze channel selection and price setting of a manufacturer or retailer with several channel options. Most of the research is focused on the consumer pricing behavior, and assume a known price demand relationship. Here the demand is assumed to unknown, and channels decisions are made in real-time using tracked demand data. Others have considered channel entry decision, most commonly an online or direct channel in addition to regular retail (Wang, Li, and Cheng, 2016). These, though, usually are not readily applicable to short life fashion products.

2.2 Warehouse Storage Location Assignment

2.2.1 E-retailing

E-commerce technology, differs and impacts widely on every walks of life from other technologies that the authors have seen in the past the century (Laudon, 2007). E-commerce technology is built with the development of Internet and customer start to change the way they can enjoy convenient life by using ecommerce (Yan, Li, and Sui, 2014). Particularly, industries dealing with daily human need have been challenged by the wave of Internet popularity. Numerous attempts from business companies have failed in transforming to be e-commerce platform, while several groups are struggling for economic survival but short of innovated features and logistics.

2.2.1.1 Amazon and E-retailer. Amazon, the leading e-retailer in the world, started the legend by selling books through the Internet and quickly extended the brand to various categories of products. With Barnes and Noble entered into online book retailing in 1997, the competition caused book prices to fall by 15% (Bailey, 1998). Similar to the book market, the advantages of online retailing attracted a lot of industries and companies from different category to join in the market which resulted in a price competition, therefore cutting down the inherent high profit margin. As the physical product flows increase, online retailers are facing unsustainable cost to maintain the shopping experience with the low benefit. In an extremely competitive market with low margins, the retailers surviving with a large amount of sales from the competition have presented two major approaches to market expansion: expanding across product lines and entering in foreign markets (Chakrabarti and Scholnick, 2002).

Motivated by online retailing competition, business and industry operations are integrated and automated to quickly response to customer requirements. Amazon's initial goal in regards to distribution was to eliminate the middlemen in the supply chain (Lang, 2012). Generally, product flows start from manufacturing in factory, by the way of stocking in warehouses and exhibiting in stores, finally to selling to customers. To reduce the processes, Amazon Fulfillment Warehouse is dealing with direct customer orders instead of the processes in warehouse-to-store-customer. With a combination of innovated methodologies in each specific operational phase, Amazon provides predominant performance in fulfilling customer orders.

Table 2.1 Survey Product Distribution (N=1000)

1. TYPE	Ratio	2. PRICE	Ratio	3. SIZE	Ratio
Electronics (Best Buy/Walmart)	25%	Less than $50	45%	Small	43%
Home Improvement (Home Depot)	20%	$50 to $100	20%	Medium	42%
Fashion (Multiple)	20%	$101 to $200	20%	Large	15%
Office Products (Staples/Office Depot)	20%	More than $200	15%		
Books (Barnes and Noble)	15%				

The authors has executed a survey method to evaluate the fulfillment time performance of Amazon and several competing online retailers and found that Amazon was able to deliver 46.2% of all orders within a day while for the competing retailers only

8.7% of the orders achieved this goal, among the total 1000 investigated orders in Table 2.1 and Figure 2.1. As benchmarks, the results provided are important for existing and new online retailers, allowing them to build a more target driven fulfillment strategy.

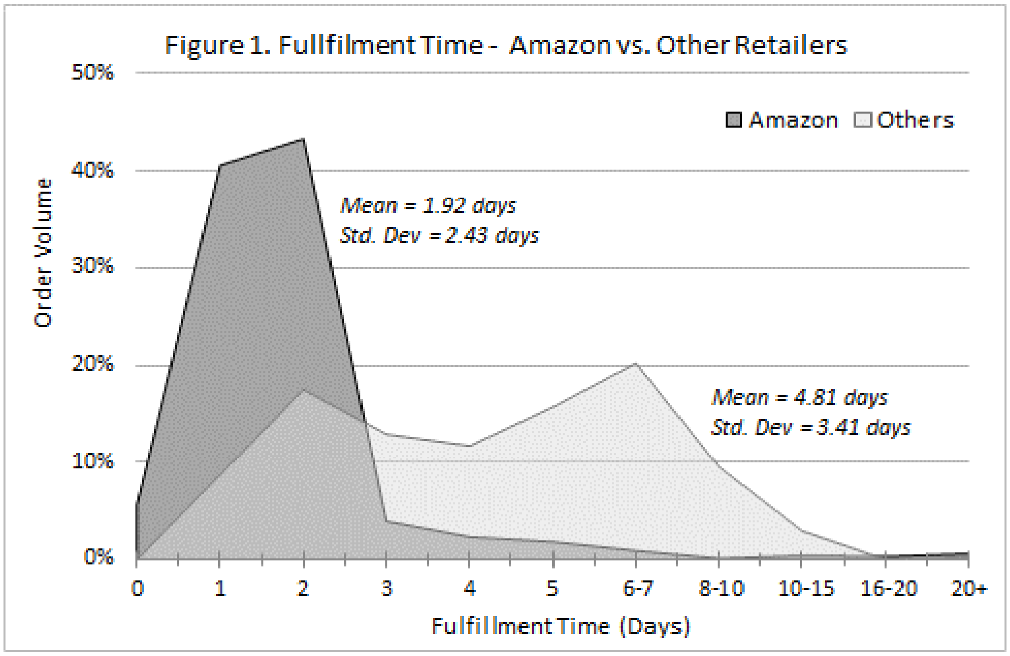

Figure 2.1 Fulfillment time comparison between Amazon and competing online retailers.

2.2.1.2 Internet Fulfillment Warehouses. This leading online retailing company has been constantly targeting improvements, although their current strategies has highlighted the online shopping performance among the competitors. As literatures or researches are more likely descriptive, the authors planned several visits to Amazon fulfillment centers and further analysis, revealed that the warehouses were actually highly efficient and at the frontlines of some new methods and operational strategies in warehouse design and control. With a relatively fast response, high and large transactions of small quantity units, this emerging element of supply chains is what the authors label as the Internet Fulfillment Warehouse (IFW). IFW has several differentiators, which make the key contributions to indicate the outstanding fulfillment behavior compared with traditional warehouses. These differentiators will be discussed in details in Chapter 4.

2.2.2 Storage Policies

2.2.2.1 Storage Policies Classification. Storage is a key activity of a warehouse. In warehouse design and control, storage assignment policies are decided to serve the most efficient way to operate the main function, fulfilling customer orders. What to stock, where to stock and how frequently a SKU should be replenished are three fundamental questions to indicate the purpose how the warehouse would like to perform. To minimize operating cost, improve the space utilization, therefore enhance the stocking and picking efficiency, optimization problems can be formulated from many aspects. What to stock and where to stock, are generally referred as storage assignment problem, on which plenty of literatures work on it and for which several common storage policies have been established and applied.

De Koster, De-Luc and Roodbergen (2008) has classified storage assignment policies as five types, including random storage, closest open location storage, dedicated storage, full turnover storage and class based storage. A lot of researchers have presented significant achievements on storage allocation (de Koster, Le-Duc, and Roodbergen (2007) and Gu et al. (2007)). Gu et al. (2007) establishes an extensive review on warehouse operation planning problems by presenting various decision support models and solution algorithms in each category with an emphasis on the characteristics of the process functions, explaining the availability of existing models and methods and guiding the direction to future research opportunities.

In Bozer et al. (1985), to split a pallet for more effective picking operations for forward-reserve problem is first proposed. In the isolated area, a small amount of SKUs randomly selected are stored in the forward area to speed up the fulfillment to the orders for these SKUs and reduce the material handling. Furthermore the forward-reserve stocking policy has been improved and established by Hackman and Rosenblatt (1990) to determine the characteristics of items assigned to forward area. Frazelle et al. (1994) has extended the problem by modelling the size of the forward and reserve areas to minimize material holding cost while approaching efficiency order picking and replenishment.

According to the improvement on picking process, new opportunity comes out along with the revealed characteristics from different picking strategy. Malmborg and Al-Tassan has extended the existing unit load warehousing systems to less-than-unit load pick systems and conducted it to dedicated storage, random storage, a combination of closest open location with randomized storage and Cube per Order Index. In Malmborg and Al-Tassan (2000), they have presented a mathematical model to estimate space requirements and order picking cycle times for a randomized storage with less than unit load order picking systems. Goetschalckx and Ratliff (1990) consider shared storage policy and illustrate that a duration-of-stay–based policy on behalf of shared storage is optimal with consistent Input / Output balance. Two shared storage assignment policies in an Automated Storage/Retrieval System (AS/RS) are compared in Kulturel et al. (1999), showing that the turnover-based policy outperforms the duration of stay-based policy in general cases. Turnover-based storage is another effective extended policy studied in plenty of literatures (Caron, Marchet, and Perego, 2000; Jarvis and McDowell, 1991; Petersen and Schmenner, 1999). In Pohl et al. (2011), turnover-based storage policies and warehouse designs are investigated with non-traditional aisles. De Koster et al., 2007; Gu et al., 2007 analyses class-based storage studies with a comprehensive survey and presents the features of class-based storage policy, as the most widely used and efficient strategy in general, to be the benchmark of the development on storage process in this study.

2.2.2.2 The Storage Location Assignment Problem. The existing storage location assignment (SLA) problem is to assign incoming supplies to storage locations in order to improve space utilization and reduce material holding cost (Gu et al., 2007). Frazelle (1989) lists three main stock location assignment strategies as dedicated storage, randomized storage and class-based storage. The definition is extended by introducing three criteria of SKU's popularity, maximum inventory and Cube-Per-Order Index (COI, defined as the ratio of the maximum allocated storage space to the number of storage/retrieval operations per unit time). Turnover-based, Class-based and COI-based location assignment problem becomes the emphasis of researches. With these established method, inventory allocation and dispersion along with warehouse design has shown different features, by which space utilization and picking efficiency are achieved.

Literature in the area is very rich and randomized storage policy has been applied commonly for its predominant performance on storage utilization and accuracy on travel time estimation. Randomized storage strategy is possible to assign any empty location to any SKU over different time periods to reduce the average idle time of all bins. Along with the advantage of randomized storage established above, disadvantages of splitting storage assignments of a single SKU into many different locations in the warehouse makes inventory control and picking operations complicated which requires using computerized systems heavily (Ross, 2015).

Another storage policy widely considered in recent researches is item associated /

correlated storage location assignment strategy. It introduces order similarity or item correlation into location assignment decision-making process, generally grouping highly correlated SKUs as a family and assigning location ranges to a grouped family instead of single SKU. Plenty of literatures and researches are working on item correlated storage policy. Order oriented or item correlated storage polices, closely connected to this research, will be described in Chapter 4.

CHAPTER 3

FAST-FASHION SUPPLY CHANNEL SWITCHING DECISION MODEL

The authors consider a retailer selling a single fast fashion product through stores which are restocked from a central warehouse. Excess inventory is sold through an outlet channel which is also supplied from the same warehouse. The authors assume a single retail store and a single outlet store without loss of generality. The FFS strategy of the retailer is described by two attributes:

T The selling cycle, after which the product will no longer be sold

Π The initial product inventory or block quantity available for sale in period T

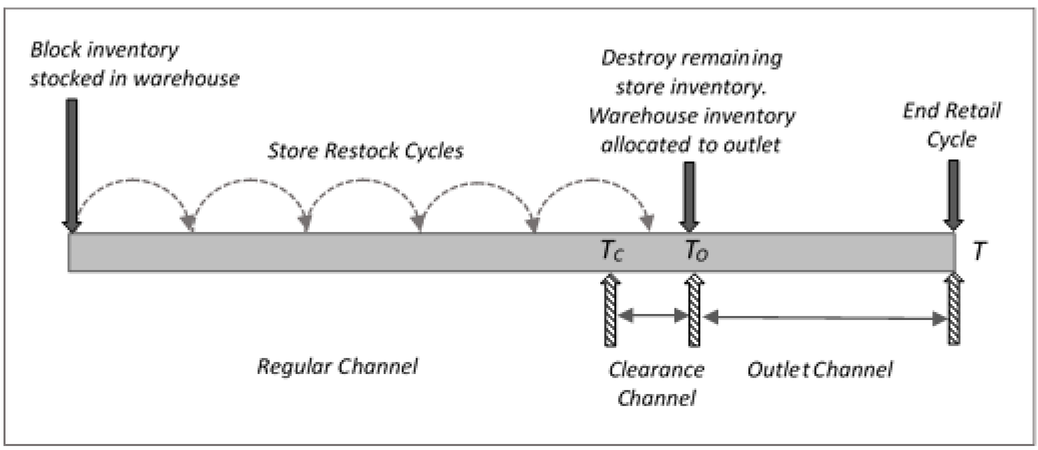

Figure 3.1 Life cycle of a fast fashion product in MCS problem, which consists of an initial shipment of inventory from supplier to warehouse, followed by several store restock cycles in regular channel, and switch to clearance channel at some moment with all rest inventory in warehouse delivered to outlet channel.

The inventory movements during the selling cycle are described in Figure 3.1. The authors assume the store is restocked using a classical base stock policy. The block inventory is sold through three sequential retails channels with no overlap. Any residual inventory after *T* is assumed to be unsold and have no revenue value. The first two channels are collocated at the store, while the third channel could be either a physical or online outlet channel. Since Π is fixed, the product sourcing cost is fixed and not effected by any subsequent decisions. The internet has enabled price transparency and fast fashion retailers are aware that customers are immediately alerted if the product is available at lower prices at a simultaneous channel. This motivates the exclusive channel distribution policy at any time t. For each item sold through the three channels the revenue price is assumed to be known and specified by the product merchandiser as follows:

P_R Regular unit retail price for items sold at the store

P_C Clearance unit price for items sold at a store promotion

P_O Outlet unit retail price for items sold through the outlet

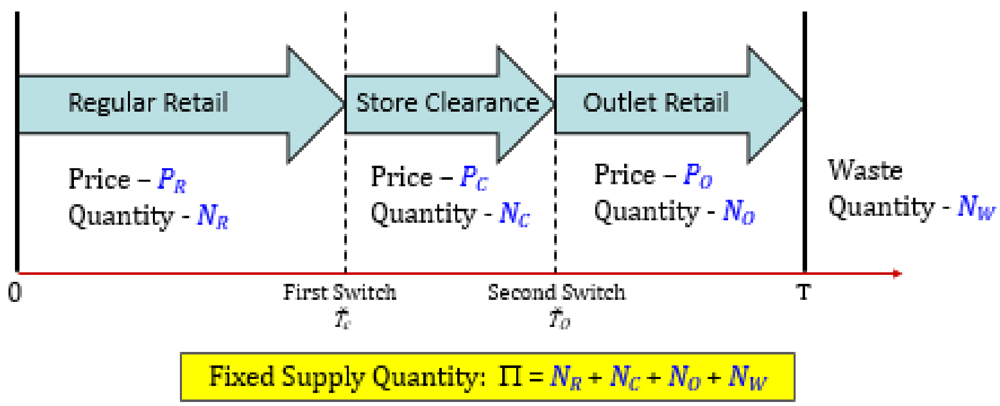

Figure 3.2 Three selling sequential channels.

The regular channel has the highest price and in the best case scenario the entire block inventory is sold in this channel. The authors assume the pricing relationship $P_R > P_O > P_C$ holds. Clearance sales are intended to clear out the store inventory when demand drops. Since $P_O > P_C$, outlet sales provide an attractive FFS option when compared to clearance sales. Outlet channels are known to attract price-sensitive, non-service-sensitive consumers compared with regular retailer channels (Coughlan and Soberman, 2005). By making small and frequent replenishments to the store, the clearance inventory can be minimized.

Figure 3.2 illustrates the product flow and the associated switching points in the FFS retail cycle. The objective of the MCS problem then is to maximize the revenue by making the following two switching decisions:

T_C Time at which the store switches from regular to clearance price

T_O Time at which store stops selling and all warehouse inventory is assigned to the outlet for immediate sale.

At T_O any remaining store inventory that could not be sold at price P_C is destroyed, that is there no back shipment to the warehouse. Since success for fashion products is unpredictable, switching decisions must leverage real-time market demand information.

3.1 Problem Formulation

3.1.1 The Demand Behavior

The primary uncertainty in the FFS problem is product demand, first whether the product will be successful or not and then the rate at which the demand will fade. Projecting demand for fashion products is in general a difficult task, and the behavior is best predicted from the actual sales data. Increasingly, customers are becoming forward looking, and when products are continuously discounted they are able to predict a future price from experience data. Customers arrive at the store at the beginning of the selling cycle, observe the selling price P_R and decide to whether purchase it immediately or delay the purchase anticipating future discounts. Caro and Gallien (2010) observe that a FFS strategy can disrupt this behavior by limiting the discount steps and percentages. This allows the retailer to limit the demand uncertainty caused by multiple pricing discounts.

Here the demand behavior is not restricted to any pre specified distribution, and the underlying demand behavior is unknown. Rather all decisions are based on the actual trailing demand as recorded at the store. It is assumed, though, that demand in the regular price channel starts with a period of rising trend which is followed by a period of decreasing trend. The model does not allow for a trend reversal once a declining trend is confirmed. Figure 3.3 illustrates the demand behavior for different rates of decline starting from the same initial demand. For a successful product the demand rises steeply and then declines at a very slow rate. At the end of *T* demand is still strong, and likely the entire stock *Π* is depleted, indicating no need to make channel switches. For an unsuccessful product, the demand rises slowly and then starts to drop quickly, such that demand is zero long before T. Clearly, at some point sales should have shifted to clearance and then outlet sales.

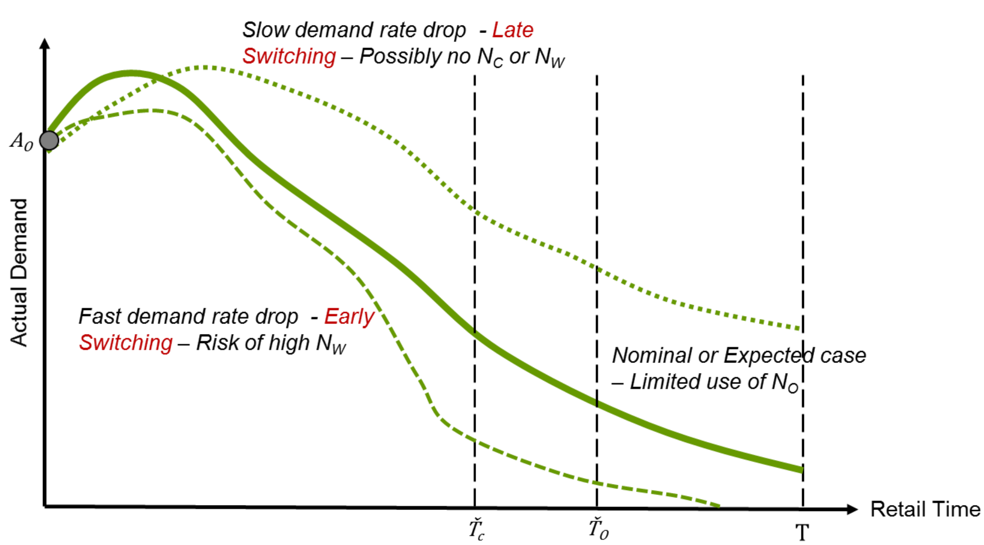

Figure 3.3 Demand behavior scenarios for a fast fashion product.

The literature on the direct relationship between demand under regular and clearance or outlet pricing for fashion products is somewhat limited. Smith (1994) and Caro and Gallien (2012) establish a forecasting model for the fast fashion industry. They studied the case of Zara, where the dependent variable is the demand rate of a specific product over a finite period and the regression includes multiple parameters such as product introduction time, current inventory levels, and competing products. They find customers are more sensitive to the relative markdown than to the absolute price cut. By using the non-changed coefficients of each term from the regression data of regular price sales, the authors can estimate customer behavior during the clearance period. Similarly, demand behavior in outlets, is influenced by many factors including relative price discounting, self-satisfaction of the shopping experience and brand image. It is well documented that outlet malls provide a shopping arena in which deals are available constantly (Sierra and Hyman, 2011). Therefore, for a specific product, the authors expect that in a finite selling cycle the outlet channel operates with constant demand from a stabilized customer group, at a fixed discount level.

Figure 3.4 shows the demand behavior when switching decisions are made. For modeling purposes the authors assume the clearance demand follows the same pattern as that exhibited by the regular demand. Let A_t be the actual demand at time t, then the initial clearance demand is estimated as $(1+\alpha)A_{Tc}$, where α is the estimated increase in demand as the price is discounted from P_R to P_C. There are a wide range of pricing-demand models and our approach is that these will determine α. For example consider Choi (2007) model demand as a linear function of the consumers' price sensitivity, and the regular or ''normal'' price. Outlet demand is more stable and here it is assumed to be constant. The outlet demand is then constant and given by βA_0. Note that both α and β are upper banded at 1.

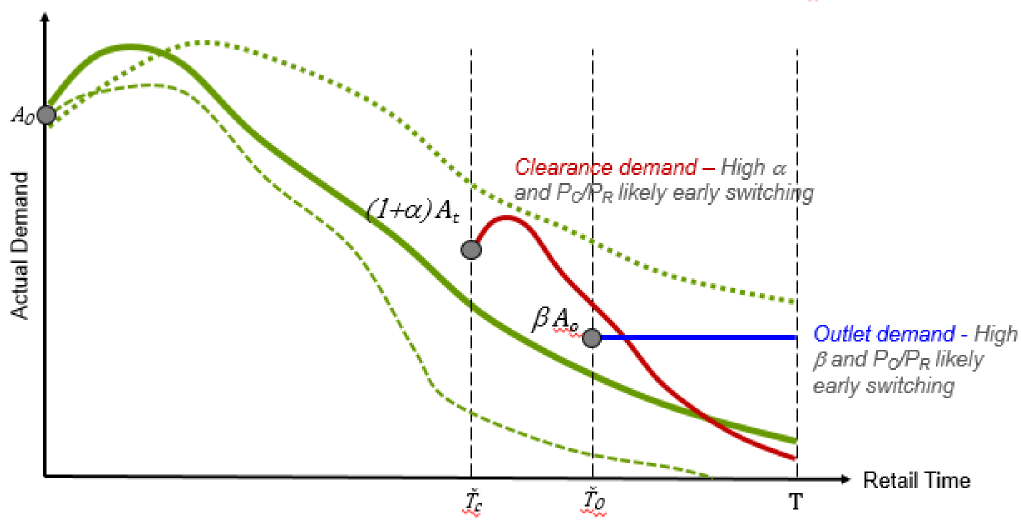

Figure 3.4 Demand behavior in clearance and outlet channels.

3.1.2 The MCS Objective Function

The block inventory is predetermined by marketing, and an input parameter to the MCS problem. Then, since the initial supply costs are fixed, the MCS objective is to maximize profit which is equivalent to maximizing revenues. As described above the product is sold in three channels, let N_R be the total sales in the regular price channel, N_C be the total sales in the clearance price channel, N_O be the total sales in the outlet price channel, and N_W be the unsold or salvage inventory at the end of the selling cycle. Then for a given $\{\Pi, T\}$ the MCS problem objective is:

Maximize Total Revenue: $$\phi = P_R N_R + P_C N_C + P_O N_O \qquad (3.1)$$

s.t. $N_R + N_C + N_O + N_W = \Pi$, where: $0 \leq T_C \leq T$, $0 \leq T_O \leq T$, and $T_C \leq T_O$

The store is restocked using a *(Q, R)* base stock policy. Ideally Q is not very large, so that the inventory risk at the store is minimized. The authors ignore the shipping cost of replenishments to the store and to the outlet. Further, the risk of lost sales is disregarded.

3.2 Solution Method

The nominal solution to the MCS problem is to do nothing, that is $T_C = T_O = T$, which implies the authors sell what the authors can at regular price and the remaining inventory is wasted. In an optimal solution to the MCS problem, though, $T_C \leq T_O \leq T$. A closed form solution to this problem is not feasible since the demand behavior is uncertain at any point in the selling cycle. Note that any time during the retail cycle the future sales are projections, and therefore ϕ is also a projection. The authors propose a heuristic solution to the MCS problem, and make the following assumptions:

1. At any time t, a linear trend model provides a reliable forecast of the regular demand. The slope of the future trend is estimated by an N-Period moving average slope of the training demand. Let A_t be the actual observed demand in period t, then the N-Period slope at time t is:

$$\delta_t = \frac{A_t - A_{t-N}}{N}$$

   Initially, δ_t will be positive, but the switching decision problem becomes relevant only after δ_t turns negative. By using the N-period moving average the authors dampen the effects of the demand change rate, similar to a classical moving average forecast. The forecasted regular demand for a future period τ then is:

$$F_\tau = A_t + \delta_t(\tau - t)$$

2. Clearance sales are used only to sellout the store inventory, no additional shipments are made to the store once a switch is made. The forecast for clearance sales is assumed to start off with an increase of α factor, such that, $F_{T_C+1}=A_{T_C}(1+\alpha)$. Demand then follows a linear trend similar to that observed during the regular sales period. The clearance demand parameters then are:

$$\hat{\delta}_t = \frac{A_0 - A_t}{t}$$

$$F_t = F_{T_C-1}(1+\alpha) - \hat{\delta}_t(t - T_C), where\ t > T_C$$

3. Outlet sales are uniform and constant at a discounted level such that $F_t=\beta A_0$, when $t>T_O$. The motivation being that outlet sales are more stable.
4. The solution strategy is to prefer outlet sales over clearance sales since $P_O>P_C$. The clearance period T_C to T_O is therefore limited only to any balance of the selling period after accounting for projected outlet sales.

Given the above assumptions, the problem reduces to a single decision τ with $\tau=T_C$, and in many cases $\tau=T_O=T_C$ implying no clearance sales. The Linear Moving Average Trend (LMAT) Heuristic is proposed as a solution to the MCS problem. The motivation for the LMAT heuristic, centers on the first assumption. Similar to a classical moving average forecasting method, the expectation is that the past N-Period trend is a reliable indicator of future sales in the regular channel. Note that switching is likely to occur in the latter part of the demand cycle when the primary demand drop has already occurred. This N-Period trend line then provides an estimate of the likely remaining revenues in the regular channel, allowing for comparison of revenue opportunities with the alternate outlet channel.

At any time t the system state is describe by $\{I_{s,t}, I_{w,t}\}$ where $I_{s,t}$ and $I_{w,t}$ are the store and warehouse inventory at time t. Assumption 4 above proposes a fixed relationship between T_C and T_O. The LMAT heuristic first determines the best switching time τ, and then decides whether $T_o= \tau$ or delayed to clear out some or all of the store inventory. As noted earlier, when a switch is made at τ, then the first priority is to sell through the outlet channel. Only if $T- \tau$ is sufficiently long will the clearance channel be activated.

3.2.1 The LMAT Objective

The LMAT heuristic is time iterative and uses a forward looking objective. At the current time t it estimates what would be the revenues, if a switch was made at a future time $t<\tau<T$. Equation (3.1) is then rewritten to project the sales in each channels, and therefore

described the expected revenues. Let $\phi_{t,\tau}$ be the revenue expectation generated from time t demand data, if a channel switch is made at τ. Then:

$$
\begin{aligned}
Max\text{:}\ \phi_{t,\tau} = {} & P_R\left(\Pi_0 - I_{w,t} - I_{s,t}\right) \\
& + P_R\left(\min\left\{\left(I_{w,t} + I_{s,t}\right), \frac{A_t + \max\{A_t + \delta_t(\tau - t), 0\}}{2}\right.\right. \\
& \left.\left.\cdot\, min\left\{(\tau - t), -\frac{A_t}{\delta_t}\right\}\right\}\right) \\
& + P_C\left(\min\left\{I_{s,\tau}, \min\left\{-\frac{F_\tau}{\hat{\delta}_t}, \max\left\{\mathrm{T} - \frac{I_{w,\tau}}{\beta A_0} - \tau, 0\right\}\right\}\right.\right. \\
& \left.\left.\cdot\, \frac{F_\tau + \max\left\{F_\tau + \hat{\delta}_t \cdot \left(\mathrm{T} - \frac{I_{w,\tau}}{\beta A_0} - \tau\right), 0\right\}}{2}\right\}\right) \\
& + P_O\left(\min\left\{\frac{I_{w,\tau}}{\beta A_0}, \mathrm{T} - \tau\right\} \cdot \beta A_0\right)
\end{aligned}
\tag{3.2}
$$

where,

$$
\begin{aligned}
I_{w,\tau} & \\
= I_{w,t} & \\
- \max & \left\{\frac{Q \cdot \left(\frac{A_t + max\{A_t + \delta_t(\tau - t), 0\}}{2} \cdot min\left\{(\tau - t), -\frac{A_t}{\delta_t}\right\} + R - I_{s,t}\right)}{\left|\frac{A_t + max\{A_t + \delta_t(\tau - t), 0\}}{2} \cdot min\left\{(\tau - t), -\frac{A_t}{\delta_t}\right\} + R - I_{s,t}\right|}, 0\right\}
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
I_{s,\tau} = \max & \left\{I_{s,\,t} + I_{w,\,t} - I_{w,\tau} - \frac{A_t + max\{A_t + \delta_t(\tau - t), 0\}}{2}\right. \\
& \left.\cdot\, min\left\{(\tau - t), -\frac{A_t}{\delta_t}\right\}, 0\right\}
\end{aligned}
\tag{3.4}
$$

All terms in Equations (3.2) to (3.4) incorporate the assumptions listed earlier. The first term in Equation (3.2) is the revenue already generated from regular channel sales, while the second term is the projected regular sales in the t to τ period. It considers the possibility of either selling out the block inventory before τ, or continuing sales through τ.

The third term in (3.2) is the projected clearance sales, and considers only the time remaining between regular and outlet channels. Given the strategy of preferring outlet sales to clearance, the remaining sales time allocated to the outlet and clearance and channels if a switch is made at τ is:

$$Outlet\ Sales\ Time = Min\left\{\frac{I_{w,\tau}}{\beta A_0}, T-\tau\right\}$$

$$Clearance\ Sales\ Time = Max\left\{T-\tau-\frac{I_{w,\tau}}{\beta A_0}, 0\right\}$$

Note that the allocated time to the clearance channel may not be fully utilized if the store inventory is not sufficient. The fourth terms projects the outlets sales. Equation (3.3) can be simplified as a possible predicted replenishment inventory subtracted from warehouse stock based on the sign of the term indicating store inventory sufficiency.

3.2.2 Conditional Optimization of the LMAT Objective

Using a simulation analysis, it can be shown that at any time t, Equation (3.2) is a concave function in the $t \leq \tau \leq T$ range. This indicates there is a switch time τ* that optimizes $\phi_{t,\tau}$. Since a closed-form solution for τ* is not possible, the authors use a conditional approach to analytically breakdown Equation (3.2) and derive an optimal solution. The following five conditions allow Equation (3.2) to be further analyzed and τ* derived.

$Condition$ 1:
$$\left(A_t + \frac{\delta_t}{2}\cdot \min\left\{(T-t), -\frac{A_t}{\delta_t}\right\}\right)\cdot \min\left\{(T-t), -\frac{A_t}{\delta_t}\right\} \geq I_{w,t} + I_{s,t}$$

$Condition$ 2:
$$\frac{\hat{I}_{w,\tau}}{\beta A_0} \geq (T-\tau)$$

$Condition$ 3: $$-\frac{A_t}{\delta_t} \geq (\tau - t)$$

$Condition$ 4: $$I_{s,\tau} \geq \min\left\{-\frac{F_\tau}{\hat{\delta}_t}, \max\left\{\mathrm{T} - \frac{I_{w,\tau}}{\beta A_0} - \tau, 0\right\}\right\} \cdot \frac{F_\tau + \max\left\{F_\tau + \hat{\delta}_t \cdot \left(\mathrm{T} - \frac{I_{w,\tau}}{\beta A_0} - \tau\right), 0\right\}}{2}$$

$Condition$ 5: $$\frac{I_{w,\tau}}{\beta A_0} \geq \frac{F_\tau}{\hat{\delta}_t} + \mathrm{T} - \tau$$

In combination the five conditions generate seven cases, as shown in Table 3.1 and described below.

Table 3.1 Cases and the Conditional Relationships of the LMAT Objective

Case	**Condition Holds**				
	1	**2**	**3**	**4**	**5**
1	YES				
2	NO	YES	YES		
3	NO	YES	NO		
4	NO	NO	NO		
5	NO	NO	YES	YES	
6	NO	NO	YES	NO	YES
7	NO	NO	YES	NO	NO

Case #1 – The simplest case where demand for the fast fashion product is high and the forecasts indicate the current inventory can be sold out in the regular channel within *T*. If Condition 1 holds then this is the only likely case.

Case #2 and #3 - The case where if a switch occurs at τ then the projected warehouse inventory is less than the forecasted maximum outlet sales. This implies there will be no time allocated for clearance sales since the outlet channel will be active for the entire remaining time. This happens when both Conditions 1 and 2 hold. Further, there are two possible scenarios, Condition 3 holds implying at τ the regular demand is still positive (Case #2), alternatively demand has dropped to zero (Case #3).

Case #4 – This represents the case where none of the first three conditions holds, and indicates a situation where the demand has progressively become weaker. The supply chain is therefore pressed to make a switching decision in order to maximize the revenues.

Case #5, #6 and #7 – In the previous cases only two of the channels were active. When Condition 1 holds but Condition 2 does not hold, then the clearance channel will also be activated since the projected warehouse inventory at τ is not sufficient. When Condition 4 holds, that is the store inventory is large enough for clearance sales to continue through the available time (Case #5).

Table 3.2 Projected Total Revenue at *t* for the Conditional Cases

CASE	*TOTAL REVENUE*
1	$P_R \cdot \Pi_0$
2	$P_R \cdot (\Pi_0 - I_{w,t} - I_{s,t}) + P_R \cdot \frac{2A_t + \delta_t(\tau - t)}{2} \cdot (\tau - t) + P_O \cdot (\mathrm{T} - \tau) \cdot \beta A_0$
3	$P_R \cdot (\Pi_0 - I_{w,t} - I_{s,t}) - \frac{A_t}{2\delta_t} \cdot A_t \cdot P_R + P_O \cdot (\mathrm{T} - \tau) \cdot \beta A_0$
4	$P_R \cdot (\Pi_0 - I_{w,t} - I_{s,t}) + P_R \cdot (-\frac{A_t}{2\delta_t} \cdot A_t) + P_O \cdot I_{w,\tau}$
5	$P_R \cdot (\Pi_0 - I_{w,t} - I_{s,t}) + P_R \cdot \frac{2A_t + \delta_t(\tau - t)}{2} \cdot (t_s - t) + P_C \cdot I_{s,\tau} + P_O \cdot I_{w,\tau}$

6	$P_R \cdot (\Pi_0 - I_{w,t} - I_{s,t}) + P_R \cdot \frac{2A_t + \delta_t(\tau - t)}{2} \cdot (\tau - t) + P_C \cdot \left(\mathrm{T} - \frac{I_{w,\tau}}{\beta A_0} - \tau\right) \cdot \frac{2F_\tau + \hat{\delta}_t \left(\mathrm{T} - \frac{I_{w,\tau}}{\beta A_0} - \tau\right)}{2} + P_O \cdot I_{w,\tau}$
7	$P_R \cdot (\Pi_0 - I_{w,t} - I_{s,t}) + P_R \cdot \frac{2A_t+\delta_t(\tau-t)}{2} \cdot (\tau - \mathrm{t}) + P_C \cdot \left(-\frac{F_\tau}{\hat{\delta}_t} \cdot \frac{F_\tau}{2}\right) + P_O \cdot I_{w,\tau}$

For each of the above cases the conditions allow Equation (3.2) to be further simplified, and Table 3.2 describes the projected revenue as a function of τ.

3.2.3 LMAT Heuristic Solution

At the end of period t, the LMAT heuristic decides whether a switch from the regular channel to either the clearance or outlet channel will be made in the next period. The LMAT objective as described in Section 3.2, is to optimize the total revenue across all channels. The projected revenue at time t is described by Table 3.3. These functions are concave and the optimal $\tau*$ is analytically derived and shown in Table 3.2. Then if $\tau \le t+1$, the LMAT heuristic prescribes a switch in the next period, else regular channel sales will continue. The heuristic steps are then:

1. Starting from $t=1$ (end of period). Record the four state variables: $I_{w,t}$, $I_{s,t}$, A_t and δ_t.
2. If $\delta_t>0$ then there will no switch in the next period. Wait for $t+1$ demand data, and return to step 1.
3. Set $\tau=t+1$ and estimate $I_{w,\tau}$ and $I_{s,\tau}$ using Equations (3.3) and (3.4)
4. Determine which conditions are satisfied and then use table 1 to determine which case is currently applicable to Equation (3.2).
5. Using Table 3.3 determine $\tau*$ for the applicable case.
6. If $\tau*\le t+1$ then a switch is made in the next period. Else set $t=t+1$ return to step 1 and wait for an update to the state variables.

7. Set T_C=$t+1$ and T_O=Max{t+1, T- $I_{w,t}/\beta A_O$}

Table 3.3 is derived by taking the derivative of the Table 3.2 revenue equations for each of the listed cases. This decision policy is also summarized in Table 3.3. The authors see that for four of the cases no switch is prescribed for the next period, while for one case a switch is definite in the next period. For two other cases, the switch decision is predicated by a switch rule.

Table 3.3 τ* and the LMAT Decision Policy

CASE	τ*	SWITCH POLICY
1	Min{No switch (T+1) , t when stock out}	No Switch
2	$\dfrac{\frac{\beta A_0 P_O}{P_R} - A_t}{\delta_t} + t$	Switch If: $\frac{\beta A_0 P_O}{P_R} \geq A_t$
3	$t - \dfrac{A_t}{\delta_t}$	No Switch
4	$t - \dfrac{A_t}{\delta_t}$	No Switch
5	$t - \dfrac{A_t}{\delta_t}$	No Switch
6	$\{(\delta_t \mathrm{t} - A_t) \cdot (P_R - (1+\alpha) \cdot P_C) + [1 + (1+\alpha) \cdot \delta_t - \hat{\delta}_t/2] \cdot \left(T - \dfrac{I_{w,t}}{\beta A_0}\right) \cdot P_C\}/(\delta_t \cdot P_R - 2(1+\alpha) \cdot \delta_t \cdot P_C + P_C \cdot \hat{\delta}_t)$	Switch If: $\dfrac{R_p \cdot \mathrm{A_t} + R_\delta \cdot P_C \cdot \left(T - \frac{I_{w,t}}{\beta A_0} + t\right)}{R_p \delta_t - R_\delta P_C} \geq 0$
7	t	Switch Now

where: $R_p = P_R - P_C(1+\alpha)$ $\qquad$ $R_\delta = \delta_t(1+\alpha) - \hat{\delta}_t$

3.3 Evaluation of the LMAT Heuristic Solution

A key analytical question then is how well the LMAT heuristic performs in controlling the MCS fast fashion supply chain. The authors used a simulation model to compare the LMAT against the true optimal and two other baseline heuristics. A data driven simulation model was built on the MS-Excel/VBA platform. Key modelling parameters for the experimental problem are shown in Table 3.4. The problem is representative of a typical six month fashion retail cycle. The parameters were set such that at a constant demand decline rate, the demand would be exactly zero at $1.22T$, implying a selling cycle 22% longer than the planned cycle would be needed to sell out the block inventory. Then if the fashion product had average success or a mean demand of $0.5A_o$ over T, 80% of the starting inventory would be sold if no other channels are accessed. Similarly if the fashion product was not successful and mean demand *is* $0.33A_o$, only 50% of the inventory would be sold. The outlet and clearance prices discounts are also realistic at 65% and 80%.

Table 3.4 Key Parameters for the Experimental MCS Problem

T = 180 Periods	Π= 23000 Units	A_0 = 200 Units
P_R= \$100	P_C= \$20	P_O= \$35
α = 0.4	β = 0.5	N = 20
M = 30		

3.3.1 Real Time Demand Generator

Clearly, the fast fashion revenue projections are going to be closely related to the demand behavior. With this in mind the authors created a real time demand generator as an integral part of the simulation analysis. To evaluate a wide range of product success behaviors, the authors introduce *d* the demand profile factor to characterize this behavior.

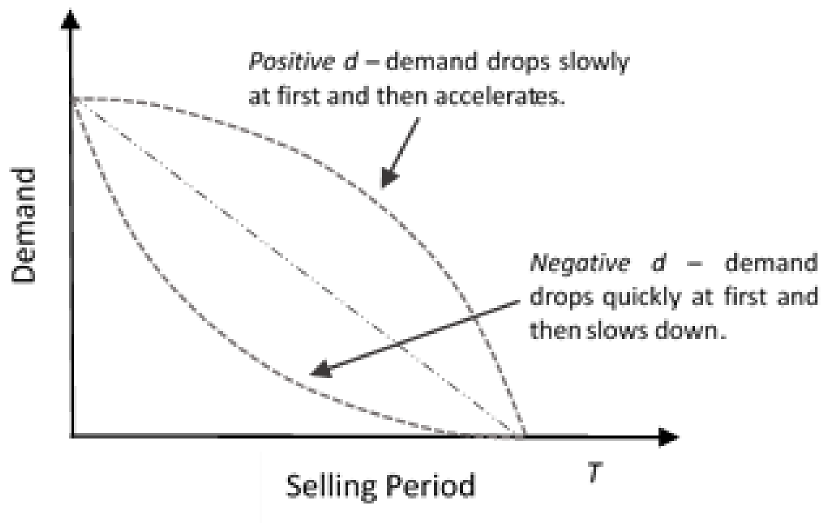

Figure 3.5 Range of generated demand profiles.

Consider a linear decline in demand as the nominal case, than as shown in Table 3.5, a variety of demand profiles can be generated around the nominal case. A positive *d* would be indicative of a successful product, with likely less need to use the clearance and outlet channels. In contrast, a negative *d* would be an unsuccessful product, and channel switching would be likely. Taking the nominal case where demand drops to zero at 0.9T, by changing *d* the authors were able to generate 15 problem sets. Within each set, the generator uses a random variable to specify the actual demand for the current period. This allows a number of different runs to be performed with each problem. Table 3.5 shows the *d* values for all problem sets. Over multiple product launches a fashion retailer can expect only a few products will be successful and have a $d>0.5$. Typically, the majority of products would have average success or $-0.5<d<0.5$, while many are expected to be unsuccessful $d<0.5$. The generator includes a random function which specifies the demand for each period t as a function of d, t and A_{t-1}. Each simulation run will therefore generate a unique demand sequence, with variance in the short term demand change rate. This variance will affect the four state variables $I_{w,t}$, $I_{s,t}$, A_t and δ_t and for each simulation the

LMAT heuristic run will, therefore, generate a different switching decision.

3.3.2 Simulation Results and Analysis

To benchmark the experimental results, two baseline switching rules were also evaluated in addition to the LMAT heuristic: (i) No Switch Rule – There is no switch and regular sales continue till T or when the demand drops to zero, whichever comes earlier, and (ii) Beta Switch Rule – If both of the conditions $I_{w,t} \geq A_t(T\text{-}t)/2$ and $A_t(P_R/P_O) \leq \beta A_O$ hold then a switch occurs. The Beta Switch is an intuitively smart logic rule, the first condition checks whether it is likely the warehouse inventory can be sold in the remain selling cycle. The second condition compares the price discounted demand rates in the regular and outlet channels. In addition, the optimal switching decision was determined by tracking the revenue ϕ if a switch was made at each of the time periods, and the highest revenue switch was assigned as the optimal decision. It is a hindsight solution since it is implementable only after the fact.

For each problem M=30 simulations runs were conducted, the revenue and switch time were tracked for the optimal decision, LMAT, No Switch and Beta Switch rules. This experimental set tests the LMAT robustness across the demand profiles and the randomness within each profile. The authors first examine the switching decision policy as a function of the demand profile factor. Table 3.5 gives the range of τ decisions and the average $\phi_{t,\tau}$ across the M runs for each problem set. For successful products ($d>0.5$) the store demand drops more slowly, and the authors observe that the switch occurs very late. The regular channel is active for more than 75% of the selling cycle and only a small portion of the block inventory is diverted to the other channels. For unsuccessful products ($d<-0.5$), the switch is much earlier as the retailer activates other channels to move the block inventory. In particular for market condition where $d<1.5$ the switching decision is quite aggressive and less than 50% of the selling cycle is in the regular channel. At the same time, the authors see that on average the revenue doubles between Problem 1 and 15. Clearly an early switch in Problem 15, or a late switch in Problem 1 would adversely affect ϕ.

Table 3.5 Problem Sets and Switching Behavior

Problem	*d*	Switching Range – τ^*		φ optimal
1	-2.5	55	60	$830,884
2	-2.20	68	72	$876,216
3	-1.80	83	87	$958,618
4	-1.50	94	99	$1,018,876
5	-1.25	101	105	$1,075,123
6	-0.65	112	117	$1,203,552
7	-0.35	115	119	$1,249,240
8	0	119	122	$1,320,609
9	0.15	124	127	$1,385,278
10	0.40	125	128	$1,429,999
11	0.80	127	130	$1,494,657
12	1.10	131	134	$1,568,814
13	1.50	134	135	$1,644,428
14	1.90	133	153	$1,692,815
15	2.50	147	163	$1,704,041

Table 3.5 also shows the range of switch time within the 30 simulations from each problem. Other than the last two problems, the τ^* range is within five periods. The results confirm that the problem represents a range of demand scenarios, providing a valid set of problems for testing the LMAT heuristic.

Table 3.6 Relative Performance of the LMAT and Other Rules

Problem	**LMAT Heuristic**		**No Switch Rule**		**Beta Switch Rule**	
	$\Delta\tau^*$	$\Delta\phi^*$	$\Delta\tau^*$	$\Delta\phi^*$	$\Delta\tau^*$	$\Delta\phi^*$
1	-1	0.1%	92	35.1%	32	7.8%
2	-1	0.2%	81	24.3%	26	7.5%
3	-2	0.2%	63	14.4%	24	5.9%
4	-2	0.2%	53	10.6%	23	4.8%
5	-2	0.3%	45	8.3%	21	4.9%
6	-1	0.3%	34	5.2%	21	4.4%
7	-1	0.1%	32	4.9%	20	4.1%
8	0	0.2%	28	4.0%	20	3.6%
9	0	0.3%	26	3.5%	19	3.3%
10	-1	0.3%	23	2.9%	16	2.8%
11	0	0.2%	21	2.5%	15	2.2%
12	-1	0.4%	19	2.2%	14	1.9%
13	-1	0.5%	16	1.7%	11	1.7%
14	-8	0.7%	8	0.9%	-3	1.7%
15	-13	0.9%	-12	0.8%	-8	3.4%

Table 3.6 compares the performance of the LMAT, No Switch and Beta Switch rules against the optimal solution. $\Delta\phi^*$ *d*enotes the average revenue loss relative to ϕ^* for each problem. The No-Switch rule is indicative of the overall utility of channel switching in a FFS. For successful products, the benefits are less than 2.5%, since most of the inventory is sold in the regular channel and N_w is relatively small. Depending on the gross margins for the product, even these small percentages could be significant. For products with average success the switching benefits are quite significant and found to be in the 3% to 5% range. For unsuccessful products, the benefits of channel switching are substantial in the 5%+ range. Problems 1 to 3 represent product that performed poorly in the market, and for these switching provides a 14% to 35% revenue opportunity.

For products with average and or high success, the Beta Switch rule matches the No Switch, so is not able to leverage the switching opportunity. But for unsuccessful products it does perform quite well and provides a solution within 4% to 8% of the optimal solution. The LMAT Heuristic performed very well and except for problems 13, 14 and 15, $\Delta\phi^*$ was less than 0.4%. This confirms that LMAT can readily and effectively be applied to real time decision making in a FFS situation. The authors also see that it performs best with unsuccessful products where $\Delta\phi^*$ was less than 0.2%. The performance strength relative to the Beta Switch was also greatest as d decreased. The LMAT heuristic was also found to be quite robust and performance matched the optimal solution closely across the 15 problems. Table 3.6 shows $\Delta\tau^*$ the average difference in switching times relative to the optimal solution in each run. The Beta Switch rule almost always prescribes a switch period late than the optimal. The LMAT heuristic though, almost always prescribe τ^* to be earlier than the optimal. For the majority of problems, $\Delta\tau^*$ was within a few periods of the optimal decision, and for three problems it matched the optimal solution, providing a projected revenue with a high accuracy.

3.4 Summary

Channel switching provides fast fashion retailers with an effective strategy to reduce the dependence on multiple discounting steps. Implementing this strategy requires the retailer to monitor market demand data in real-time, and make immediate switching decisions. This chapter formulated the Multi-Channel (regular, clearance and outlet) switch problem, with the objective of maximizing revenue from an initial block inventory. Following a peak demand the demand rate is assumed to be monotonic decreasing. For an unsuccessful product the overall demand drops quickly, while for a successful product the demand drops slowly and potentially the entire inventory can be sold in the regular channel. The objective is simplified into cases using a set of conditions, allowing for an analytical solution. The Linear Moving Average Trend (LMAT) heuristic is proposed, it decides whether a switch should be made from the regular channel in the next period.

Using a series of test problems, representing different levels of product success, the LMAT heuristic was compared with the optimal decisions and the No-Switch and Beta-Switch rules. The No-Switch rule is indicative of the overall utility of channel switching in a FFS. For products that performed poorly in the market, channel switching provides a 14% to 35% revenue opportunity. For products with average and or high success the Beta Switch rule matches the No Switch, and was unable to leverage the switching opportunity. But for unsuccessful products it does perform quite well and provides a solution within 4% to 8% of the optimal solution. The LMAT Heuristic performed very well and for the majority of test problems provided a solution within 0.4% of the optimal. This confirms that LMAT can readily and effectively be applied to real time decision making in a FFS situation.

The internet has made pricing history transparent and the managerial challenge for retailers is how to control pricing speculation. One solution is to use price differentiated sequential channels, and the LMAT solution allows a retailer to make the switching decision, using real time demand data. Many retailers are operating an internet store along with brick-and mortar stores. Often the internet store is equivalent to an outlet store, and with the right pricing differentiation a retailer can use this model to optimize the revenue across the channels. FFSs are characterized by a larger number of sequential product offerings, and a retail store can be choked by a slow moving product. In particular, smaller retailers with a single or just a few stores can mitigate the risk by a quick switch to an outlet channel as shown here. It is difficult for many retailers to match the ultra-fast supply chain of Zara, an alternative strategy then would be to launce multiple products with a fixed initial block inventory and selling cycle that matches their customer profiles and supply capabilities. The model here shows that this could be quite effective in mitigating fashion inventory risks.

CHAPTER 4

STOCKING ALGORITHMS IN INTERNET FULFILLMENT WAREHOUSES

Internet retail is generally described as the online marketing and sales of products directly to customers. Internet Fulfillment Warehouses are based on and differentiated from traditional ones to meet with the quick and large product flows and data transactions in online retailing. As an indicative characteristic, explosive storage policy establishes an impressive enhancement on effective picking and fulfillment process. It is abnormal to be explained with existing warehousing strategies, that this beehive and commingled storage do achieve respectably advanced performance with everything "messing up". IFWs provide rich analytical problems, depending on which powerful decision making models are implemented. Picking efficiency as a straight forwarded problem, has been investigated in our preceding researches. Several modified algorithms present a significant reduction on generating order pick lists in a narrow-band, resulting in less traveling distance and faster fulfillment. However, the improvement is limited by the structure of warehouse inventory or storage arrangement. To further indicate the effect of explosive storage and order picking algorithms, therefore, stocking policy is updated with explosive involved to optimize the influence to picking process then order fulfillment in this chapter.

4.1 Performance Evaluation of Explosive Storage Policies

4.1.1 Key IFW Structural Differentiators

The authors identified a variety of physical design and operational insights in several observational visits to IFWs. These insights are analyzed from the existing warehouse operations and unique to IFWs. The item flows from receiving port to shipping port were flowcharted from these insights and Figure 4.1 shows the item inventory flow timeline. While schematically the flow appears to be identical to a traditional warehouse the actual operations are quite differentiable. The overall timeline itself is much shorter and both the stocking and fulfillment times are measured in hours. Dealing with a large number of SKUs stocked and highly transacted, the warehouses are fully occupied in most of the operation time windows but every single inventory lot is stocked for a limited time in the warehouse. The authors estimate the inventory turnover ratio of an IFW is much higher than that of a traditional retail warehouse. The analysis indicates that an efficient IFW is differentiable from traditional warehouses by the following characteristics (Onal et al., 2017):

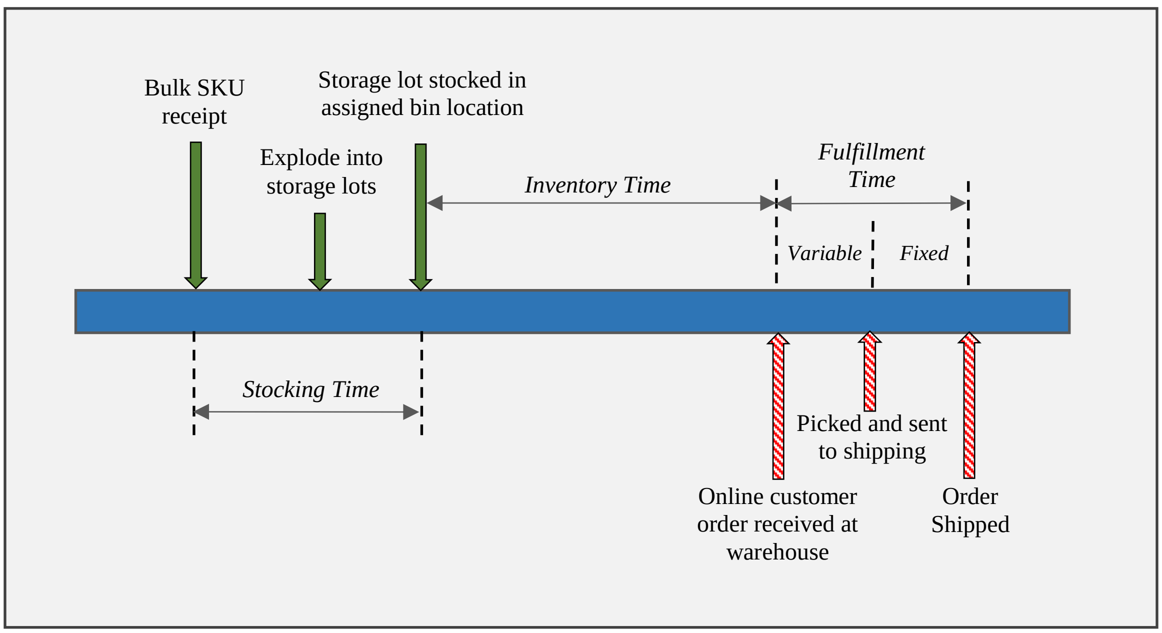

Figure 4.1 Inventory flow timeline in an IFW.

Source: Onal, S., Zhang, J., and Das, S. (2017). Modelling and performance evaluation of explosive storage policies in internet fulfilment warehouses. *International Journal of Production Research*, 1-14. doi:10.1080/00207543.2017.1304663

Explosive Storage Policy - Traditional warehouses store a SKU either in a set of contiguous locations dedicated to the SKU or a random location for each arriving bulk. Locations are then selected using either a volume based or class based approach (Petersen and Aase, 2004). In these cases, at any time instance the actual number of locations where a specific SKU is stored is less than 10. In IFWs, the incoming bulk is immediately broken into units upon arrival. These exploded units are then aggregated into several storage lots with each having one or more units of the same SKU. The lots are then dispersed to bins throughout the warehouse as shown in Figure 4.2. Bin locations are determined by computer controlled inventory system and assigned to a specific worker to help collect them into the storage areas. These specific bin assignments could be decided by random or prescribed by either a fixed rule or a dynamic optimization algorithm. The authors describe this as an explosive storage policy and define it as: An incoming bulk SKU with large quantity of supplies is exploded into E storage lots such that no lot contains more than 10% of the received quantity; the lots are then stored in E locations anywhere in the warehouse randomly selected with no other restriction besides the available space limitation. In a traditional policy $E=1$, while in an explosive policy $E>10$.

Let $i \in N$ be the set of unique items or SKUs stored in the warehouse. Let E_i be the explosion factor and V_i the current total warehouse inventory for i, and L_i the number of unique bin locations where it is stocked. Then the authors introduce the following measures:

$$Explosion\ ratio\ for\ product\ i = \chi_i = L_i/V_i$$

$$Warehouse\ Explosion\ Ratio = \chi_0 = \frac{\sum_{i \in N} L_i}{\sum_{i \in N} V_i}$$

Note that L_i is not generally equal to E_i. Since batches of the bulk are arriving multiple times at different time instance, every explosion might send the lots to both existing and new locations. E_i is presented as a corresponding result from explosion process where L_i responses to all the relevant processes affecting inventory state change. The overall warehouse explosion ratio is then derived from inventory weighted function as above.

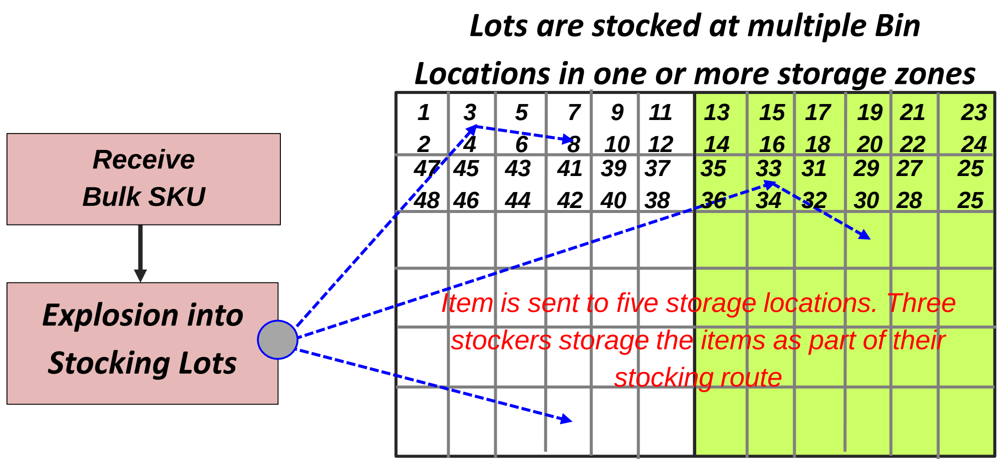

Figure 4.2 Explosive storage to multiple bin locations.

Source: Onal, S., Zhang, J., and Das, S. (2017). Modelling and performance evaluation of explosive storage policies in internet fulfilment warehouses. *International Journal of Production Research*, 1-14. doi:10.1080/00207543.2017.1304663

Since χ_i is time variant, value for measurement is usually referred to the mean. For the case where E_i is the same for all items then the mean χ_i is also the same and equal to the overall χ_0 ratio. This extreme case where each unit of V_i is stored in a different location results in that $\chi_i = 1$. In a traditional warehouse with random stocking, at most 3 to 4 storage locations can be expected with a low explosion ratio of $\chi_i < 0.01$, whereas in an IFW the likely range is $0.10 < \chi_i < 0.50$. In the design of the IFW storage policy, $\chi_0$and the associated χ_i are critical parameters. These in turn are related to the explosion factors E_i, which are therefore strategic decisions in the IFW design problem.

Very Large Number of Beehive Storage Locations - In traditional warehouses received items are stored in large volume locations which can be used for multiple bulk loads of a single SKU. Then the subsequent shipment of the bulk quantities has been shipped to retail points and unpacked there. In an IFW warehouse, however, the strategy is to store SKUs in small quantities but more places. This strategy requires a very large number of small storage location assignments, typically referred to as bins. Storage bins are commonly used in a forward picking area in a warehouse or for immediate fulfillment from a strategic retailer. In both cases the storage area is relatively small. In contrast the entire IFW warehouse is organized into racks that are divided into many small bins in a sort of beehive pattern. As a result millions of storage locations are built and set up in the million square foot warehouse, while compared with a similar sized traditional warehouse the number is only 10,000. This is the most apparent physical difference of an IFW warehouse.

Bins with Commingled SKUs - Shared storage policies have been widely used in traditional warehouses and have been studied in the literature. However, the term shared is described as using the same location for sequentially storing different SKU's over a planning horizon, but not always concurrently (Goetschalckx and Ratliff, 1990). One of the most radical differentiators of an IFW, is that multiple SKUs are simultaneously stored in the same bin. The authors propose this strategy as commingled storage since the more than one SKUs are arranged in an unorganized way within a bin. The picker takes effort to visually identify the SKU against others and match the barcode provided on a hand held tablet. It is not an inefficient stocking allocation from the classical warehousing viewpoint because they recommends easy and reliable identification of SKUs for efficient picking.

However, it is highly possible that multi-items ordered at the same time range can be fulfilled in the same bin within one pick trip by one picker with this commingled storage assignments. Clearly, commingled storage allows for higher explosion ratios.

Immediate Fulfillment Objective - Traditional warehouses deal with customer orders in a batch. The tactical objective for the batch of pending orders at the beginning of a day or a week is to fulfill them during the day or week. Operationally, the objective is to minimize the order pick routes then reduce the labor requirements. In IFWs, customer orders are received continuously throughout the day, which are then transmitted to the picking teams for immediate fulfillment. This strategy allows IFWs to be highly competitive against a physical retail store. Often the delivery date has already been promised to the customer when the online order was placed, implying little flexibility in fulfillment time delays.

The IFW predominant objective is order fulfillment time, measured generally as the mean for all orders. Time window for picking is much shorter with in IFWs and target fulfillment times are measured in hours, even minutes. Delivery trucks leave the warehouse at a constant frequency during the day. Let $\check{T}$ be the truck departure interval, then the real time planning window is a fraction of $\check{T}$ since ideally a customer order could ship out on the next truck. Our observations were that this focus on fulfillment time dominated the attitude of all workers at the IFW.

Short Picking Routes with Single Unit Picks - Order picking efficiency is a key decision problem in warehouse operations. In a traditional warehouse, current pick orders are likely to be dispersed throughout the warehouse. Given the relatively long planning windows, the pick list decision problem focuses primarily on picker travel time minimization. The structural differences described above significantly change the order picking behavior in IFWs. The authors observed that most customer orders are multiple items with small quantity or even single unit. The efficiency gains of batching multiple orders for the same SKU are not applicable in an IFW, except when orders arrive within a few hours of each other. Typically N is very large and the arrival time between orders for the same SKU is often longer than the order pick planning window. It was also observed that when customer orders include multiple SKUs, an IFW splits them into a separate small order for each unique SKU, by which the assumption that a customer order is for a single SKU is still holds. It was also observed that picked items for the same order are not necessarily aggregated into a common shipment.

The explosive storage strategy generates a stocking dispersion that results into an efficient picking solution whereby multiple customer orders are stored and able to be fulfilled in close proximity. As E_i is increased L_i also increases, and a customer order can be picked from any of the L_i locations. Given a list of active orders, the probability is high that a small number of orders can be picked from a tight picking area. As demonstrated, a very short pick route that walks by just one or two aisles can fulfill several orders, and potentially a set of multiple orders could be found in the same bin. Observe that the list of pending customer orders is dynamic in real time. This structural change in the picking behavior allows an IFW to achieve its same day shipment fulfillment objective. With given list of pending orders and current inventory state, a short and unique picking route is identified. It is possible that an IFW underperforms in terms of space utilization, but the fulfillment time objective is primarily optimized.

High Transactions and Total Digital Control - Information technology has brought great challenge to retail industries. Early information technologies adoption such as RFID have allowed warehouses to progressively improve operational efficiency. From our observational study, the level of digital activity control is much higher in an IFW. The explosive storage and single unit picks results in a higher rate of store/pick movement per shipment, and the number of corresponding data transactions is relevantly larger. Human level are followed controlled without any desired decision making and all movements are modelled and instructed by the central computer. Both stockers and pickers have only short term visibility, possibly for only 15 minutes ahead. As an example, only one stocking list is assigned to a stocker at one time, with a maximum of 15 or 20 items been assigned to the location close to the stocker. Possibly the controller help to update the stocking list in real time. There was also tight control on worker discretions, for example, workers must pick orders in the instructed sequence. In summary, IFWs integrate high levels of physical and data automation with high levels of labor, resulting in an efficient stocking strategy and picking efficiency, therefore the enhanced order fulfillment performance.

4.1.2 IFW Operation Process and Data/Decision Flows

Based on the observational visits to a leading internet retailer, the authors find that IFWs are introducing new process and decision flows which better leverage information technology to efficiently serve the internet driven supply chain economy. With all key differentiators demonstrated, new procedures has been involved into IFW operation. Figure 4.3 shows a detailed process and data flow in the leading internet retailing warehouse.

“Receiving” process begins at the moment trucks arriving at warehouse unload area and bulks loaded onto conveyor to enter unpack zone – or the authors call “explosion station”. Scanned and registered big boxes are opened and exploded into individual items or small packages, which are grouped by some strategy and placed into yellow totes. These totes will also be scanned to record both the items in that tote and the aiming locations they have been arranged.

Tote goes to different zone based on stocking assignments, starting the process of “Stocking”, or “Stow” process. At their destination, a free stocker is ready to locate these items from coming tote to their decided bins. Unlike the storage policies discussed in Chapter 2, items are scattered into the warehouse depending on order frequency or some other features. This “approximate random” storage algorithm contributes to diversity across the warehouse which increase the probability to quick fulfillment and reduce the potential of partial congestion.

After items are stored into specified locations, they are ready to be picked up for customer needs. Till now, inbound processes are completed.

“Picking”, as the connection of inbound and outbound phases, was motivated by customers’ click on the website. Picking lists are generated by algorithms and assigned to a zone and respective picker, with minimum picking time as primary objective and less walking distance as secondary target. Picked items in one list might come from different customers at different time, however, be located in the same narrow band and going to neighbor shipping areas.

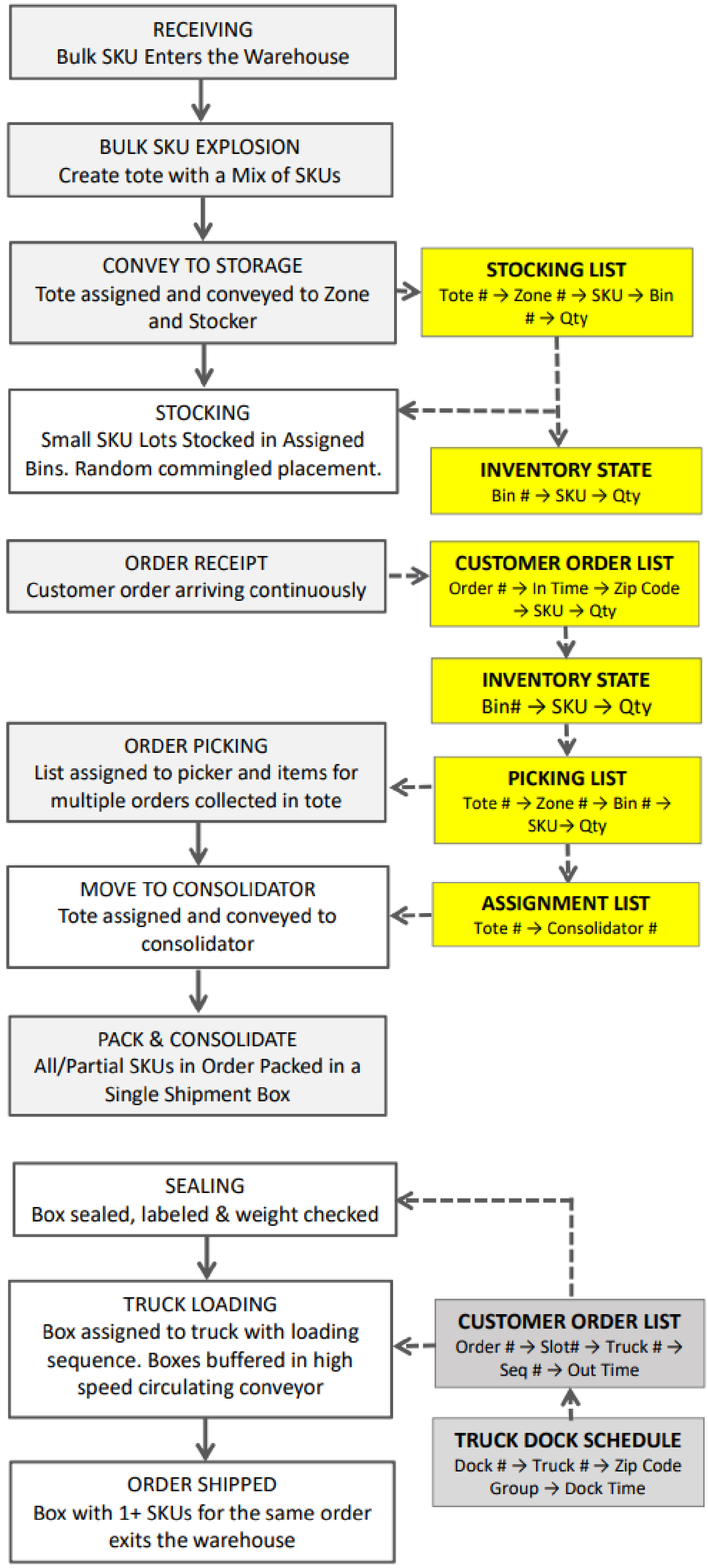

Figure 4.3 Amazon Fulfillment Warehouse process and data/decision flow diagram.

Since items in one picked tote are belonging to various customers, a brand-new process is demonstrated as supplement to picking strategy, as “Consolidation”. At consolidation station, an employee deals with 8 to 12 totes with picked items, scanning one by one and dividing them into different orders. These combined orders then are packed under box size and protection material suggestions and labeled to be assigned to truck. Conveyor takes all packed boxes to their corresponding delivery trucks.

Each process generates disparate decision problems. Following with data flows, optimal decision are made in separate phases; therefore a robust system with enhanced approaches would implement quick fulfillment warehouse.

A basic stocking and picking algorithm, where to assign receiving/customer orders to zone and create stocking/picking list, has been well-established and solved in our early research. The model describes the associated receiving and fulfillment product flows. Explosive storage of incoming bulk allows for much quicker fulfillment of incoming customer orders. Two decision algorithms for (i) generating a stocking list and (ii) creating an order picking list are formulated and presented.

A simulation model to evaluate the fulfillment time performance advantages of the explosive policy was built. Experimental runs were conducted on a problem with N=400, M=3240, bulk receipts $\sum_t R_t = 220$ and customer orders $\sum_t J_t = 22000$. The base case of $\chi_0 = 0.1$ was considered equivalent to traditional storage policy. The results show that increasing levels of explosions reduce the linear fulfillment time by as much as 16%, confirming that the IFW storage policy is beneficial.

In this chapter, the authors describe a stocking location assignment and tote composition problem and indicate a modified stocking algorithm that improve the warehouse inventory structure thereby fulfillment behavior.

4.2 Problem Formulation

Based on two visits to IFWs, the author observed that customer orders generally include single or very few items. Thus, the explosive storage and single unit pick require a high number of movements. Because even the smallest loss in time per order can be amplified in the big frame. In contrary to the traditional warehousing batching policies, in IFWs, clustering orders based on SKUs or customers would not be as efficient. Instead, orders split for each unique SKU and sorted by receiving time and fillable factor to minimize the effort to get these orders fulfilled. Therefore, as mentioned, storage process is considered to be an efficient aspect for picking improvement. A well-organized and tight inventory structure indicates the easiness to find diversity of items ordered around the same time.

For balanced picker utilization, the inventory dispersion must also consider customer order arrival behavior and demand correlations. The IFW stocking list problem is therefore different from traditional problems since multiple storage locations are selected for the same bulk, and the lots are stocked at different times. Minimize travel time is not a primary objective. IFW stocking objective is effective explosion of SKU to multiple stocking locations, reaching targeted distribution of SKU inventory through the warehouse for shorter fulfillment time. Decisions are:

- Assign bulk cases to an explosion station
- Assign SKUs and quantity to Tote #

- Assign Tote # to a Zone and further to a Bin#

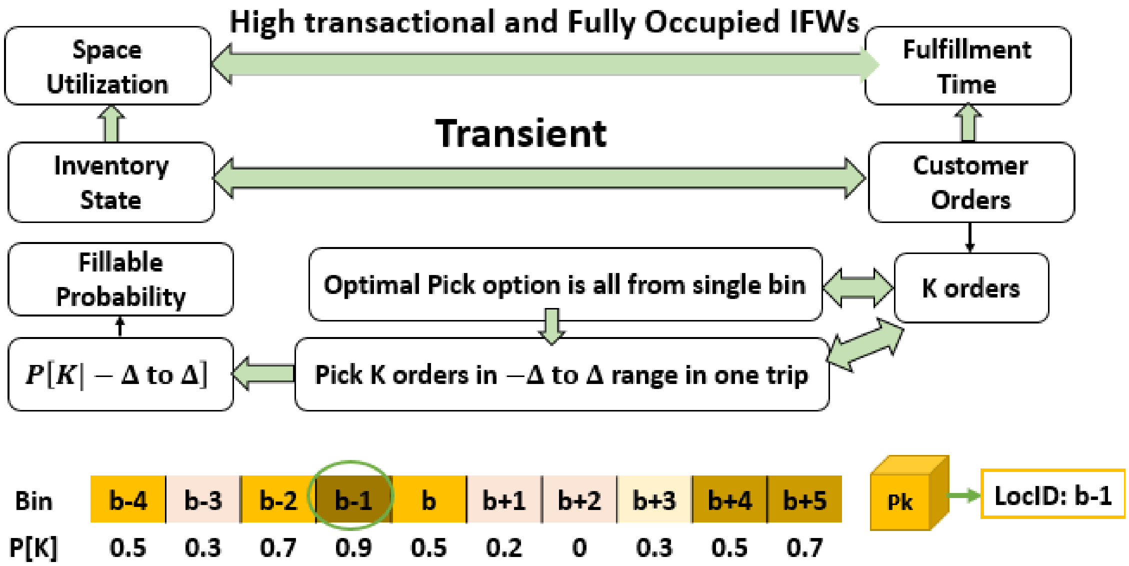

Figure 4.4 Stocking process and fulfillment objective in IFWs.

Figure 4.4 illustrates the transformation on objective in stocking phase in IFWs. For traditional warehouses, stocking process is developed to improve the space utilization and reduce the operational cost. In IFWs, storage location is almost fully occupied and under operation of millions of high transactional products. On basis of the new characteristics, fulfillment performance is more significant than to capture the small benefit from space utilization. Since the inventory state and customer orders are transient, a list of K orders is optimally picked in the same route with a reasonable inventory dispersion. An intermediate factor representing the probability that the above optimal case occurs is established as the objective in stocking strategy of IFWs.

4.2.1 Order-Oriented / Item-Related Stocking Policy

Items need to be stocked into warehouse locations / bins before they can be used to fulfill customer orders. Storage assignment problem is set up to be used to make location arrangements and sequentially stock them. There are lots of policies to implement storage assignments in traditional warehouses, as well as in IFWs. IFW aims to achieve immediate fulfillment, which requires a significant reduction in efforts to generate a complete picking list within an acceptable and reliable search band. Correlated with this research, some of the existing stocking policy are established as below.

Fragmented Warehouse was described in Ho and Sarma (2008) and Ho and Sarma (2009), as first considered the strategy of storing identical copies of an SKU in a fragmented manner, which creates a greater number of feasible pick list opportunities with greater choice, greater optimization follows. Fragmentation, defined as the "scattering" of identical stored items throughout the warehouse, break the traditional one-to-one mapping between SKU and storage location into multi-to-multi relationship. With multi-picking strategy, fragmented storage lead to additional choice when selecting which locations to visit to fulfill an order and increases the chance to optimization.

Another strategy is involving with order frequency. Distinct with turnover-based slotting strategies using COI to implement in practice, Ronald J. Mantel (2007) proposes a new and more logical way of slotting – order oriented slotting, which is based on multi-item orders instead of individual location visits to minimize total travelling time. In case of single-commands, such method is noted to be a modified dedicated storage policy with single order picking adopted and no order batching applied.

Frazelle (1989) first attempted to capture the correlation between two items and proposed a heuristic approach to cluster items into zones based on the joint probabilities that pairs of items occur in the same order, to reduce the pick time needed for more SKUs. Chuang, Lee, and Lai (2012) give a further extension on storage allocation problems by introducing between-item associations into family grouping to reduce picking efforts. The methodology can be described as the following procedures:

- Two phases
- 1st: to cluster items into group based on the correlation between items and to achieve the highest between-item-support
- 2nd: to assign items into storage locations
- Z-type picking method and one-block one-aisle warehouse layout as simple pilot experiments

In IFWs, explosive storage as a primary differentiator, is executed to modify current storage policies for fast-response, small-quantity and diversified-needs retailing with beehive commingled warehouse operations. To identify a more effective way to assign storage locations with exploded numerous packages, the authors establish a storage location assignment model combined order frequency with inventory dispersion to maximize the effect on picking process.

4.2.2 Storage Density

As mentioned in above sections, picking efficiency is limited by warehouse inventory structure. To represent fulfillment performance, picking process is occupying the most costly and beneficial procedures. The authors introduce the probability to complete a picking list in a narrow band of λ as the quantity measurement, to indicate how and how much storage structure can affect picking phase behavior. Depending on our early research, the number of successful picks located in a $\pm\lambda$ band away from a free picker follows Poisson Binomial distribution. For any receiving bulk $\{R_t\}$ opened at explosion station, $X_{i,t}$ packages are stored into different locations across the warehouse, where $X_{i,t} = 1 + int\left[\chi \cdot \frac{Q_{i,t}}{G_i}\right]$, if $Q_{i,t} - \left\lfloor\frac{Q_{i,t}}{X_{i,t}}\right\rfloor \cdot \left(X_{i,t} + 2\right) < 0$, else, $X_{i,t} = int[\chi \cdot \frac{Q_{i,t}}{G_i}]$. It is obvious that the larger explosion ratio - χ, the more scattering product would be stored. Thus, the probability of completing a pick list of k items in one trip where walking distance is less than $2\lambda + 1$ bins is:

$$\Pr(\mathrm{k}) = \sum_{A \in F_k} \prod_{h \in A} P_{h,t} \prod_{l \in A^c} Q_{l,t},$$

Where $P_{i,t} = [1 - \frac{X_{i,t}}{M}]^{2\lambda+1}$, $Q_{i,t} = 1 - [1 - \frac{X_{i,t}}{M}]^{2\lambda+1}$ and F_k is the set of all subsets of k integers that can be selected from $\{1, 2, 3, \dots, R\}$ as the set of receiving ID R in day t, A^c is the complement of A.

Bin\SKU	1	2	3	4	5	6	7	8	9	10	Fillable Factor
1	5	5	5	5	5	5	5	5	5	5	1
2	0	10	0	10	0	10	0	10	0	10	0.9922
3	0	0	15	0	0	15	0	0	15	0	0.951
4	20	0	0	0	20	0	0	0	20	0	0.8665
5	0	0	25	0	0	0	0	25	0	0	0.7903
6	0	0	0	33	0	0	0	0	0	34	0.6794
7	0	0	30	0	0	0	0	0	0	0	0.5217
8	0	0	0	50	0	0	0	0	0	0	0.5217
9	0	0	0	0	0	0	0	0	100	0	0.3017
10	15	0	0	0	10	0	2	0	22	0	0.972

Can fulfill five orders without any barriers among pickers

Can fulfill one order or need to deal with route problem

Figure 4.5 Storage structure and order fillable probability.

The simulation experiment established above shows that fulfillment time reduces according to the increase of explosion ratio. Here also indicates same conclusion. If raise the explosion ratio χ while keep all other factors constant, the number of storage slots of a specific item is larger which will escalate the probability to create a picking list with more items ordered from customer and located in a $\pm\lambda$ narrow band, therefore reduce the mean fulfill time. Figure 4.5 demonstrates the fillable factor, represented by the average quantity through M bins, under different inventory structures, showing a beneficial influence of explosive "scattering" storage strategy.

Bin\ SKU	1	2	3	4	5	6	7	8	9	10	Storage Density
1	1	1	1	1	1	1	1	1	1	1	1.00
2	0.6667	1	0.6667	1	0.6667	1	0.6667	1	0.6667	1	0.83
3	0.3333	0.6667	1	0.6667	0.6667	1	0.6667	0.6667	1	0.6667	0.77
4	1	0.6667	0.3333	0.6667	1	0.6667	0.3333	0.6667	1	0.6667	0.63
5	0.3333	0.6667	1	0.6667	0.3333	0.3333	0.6667	1	0.6667	0.3333	0.60
6	0	0.3333	0.6667	1	0.6667	0.3333	0	0.3333	0.6667	1	0.45
7	0.3333	0.6667	1	0.6667	0.3333	0	0	0	0	0	0.30
8	0	0.3333	0.6667	1	0.6667	0.3333	0	0	0	0	0.30
9	0	0	0	0	0	0	0.3333	0.6667	1	0.6667	0.15
10	1	0.6667	0.3333	0.6667	1	0.6667	0.4	0.2667	1	0.6667	0.73

Average density is 1

Average density is 0.6

Figure 4.6 Inventory structure and storage density.

Thus," Storage Density", individually for a specific item, is introduced in the following section, to manifest the average weighted inventory within all the bins in an IFW. It is not simply calculated by adding number of locations or inventory quantities from every slot, but weighted by the distance to selected center bin of defined searching band Δ, where it can be different with λ in creating picking list. It is showing that more explosive warehouse provides a higher average density, where the chance to pick up a required item from a free picker's right hand-side is much larger. For example, in Figure 4.6, a picker who is standing at slot #6 can easily raise his hand to pick up a item 1 and item 2 without moving while the other one has to walk through two bins to find a item 5 after fulfill one order for item 3.

These notations are used for storage density $W_{i,b}$. These results format a matrix as above Figure 4.6, which the authors named as Storage Dispersion Matrix.

C_i	The average order quantity of any item i, $i \in N$
δ	The distance index from current bin to center bin b, $-\Delta \leq \delta \leq \Delta$
$\hat{I}_{i,b}$	The initial inventory of item i stored in bin b, $i \in N,\ b \in M$
$I_{i,b}$	The current inventory of item i stored in bin b, $i \in N,\ b \in M$
$W_{i,b}$	The weighted inventory density of item i stored within $\pm\Delta$ of bin b, $i \in N,\ b \in M$

$$W_{i,b} = \sum_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot \min\left\{\frac{I_{b+\delta}}{C_i},\ 1\right\}$$

Then, $W_i = \sum_b W_{i,b} / M$

A pilot experiment have been completed for storage density within an aisle of 20 bins, where 10 types of inventory structures (shown partially in Figure 4.6) and eight scenarios with different weighted band are tested to demonstrate the effect on storage dispersion and pick-able probability. The average storage density for each experiment are shown as Table 4.1. Explosion provides opportunity to fulfill customer orders in multiple slots; also reduces the need to extend searching band to benefit from scattering or explosive storage.

Table 4.1 Storage Density vs. Item Fillable Probability Pilot Results

Delta \Case	3	4	5	6	7	8	9	10	Pick-able Probability
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.83	0.88	0.90	0.92	0.93	0.94	0.94	0.95	0.50
3	0.77	0.83	0.86	0.88	0.90	0.91	0.92	0.93	0.35
4	0.63	0.73	0.78	0.82	0.84	0.86	0.88	0.89	0.25
5	0.60	0.70	0.76	0.80	0.83	0.85	0.87	0.88	0.20
6	0.45	0.58	0.66	0.72	0.76	0.79	0.81	0.83	0.15
7	0.30	0.39	0.46	0.53	0.59	0.64	0.68	0.71	0.10
8	0.30	0.40	0.49	0.58	0.64	0.68	0.72	0.75	0.10
9	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.05
10	0.73	0.78	0.81	0.83	0.84	0.85	0.86	0.86	0.40

The performances among different searching band are shown in Figure 4.7, demonstrating a higher density behavior along with the extending of affective searching range.

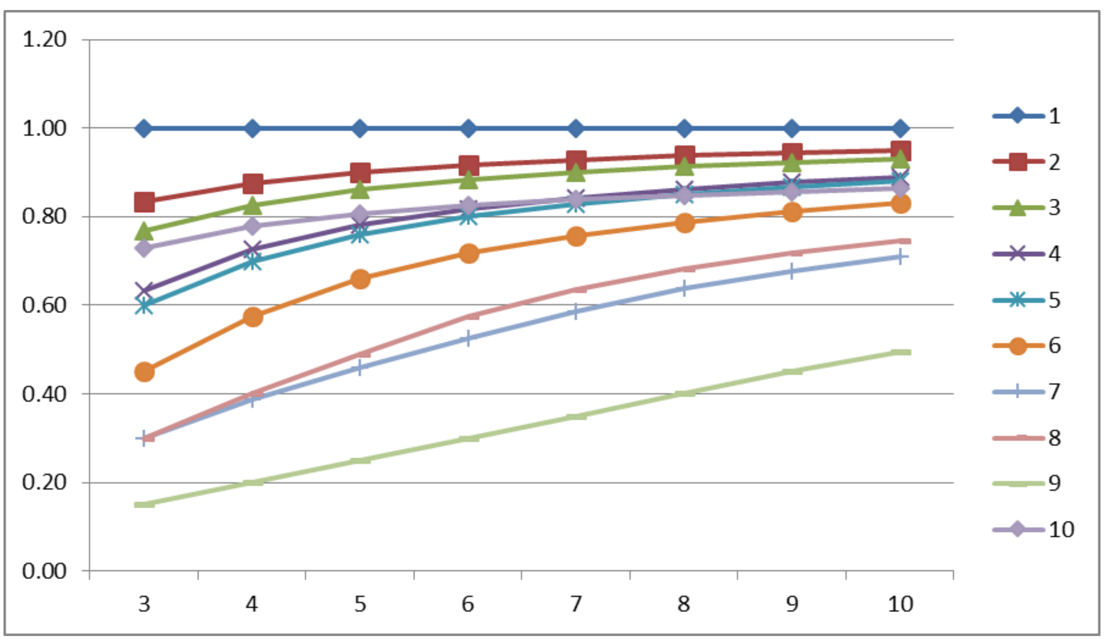

Figure 4.7 Storage density performance along with searching band.

4.3 Joint Order-Frequency and Density Oriented (JOFDO) Stocking Algorithm

In this section, the algorithm is proposed for the Joint Order-Frequency and Density Oriented (JOFDO) stocking strategy. To develop the JOFDO model, the strategy is parted into two phases: (i) to select SKU depending on order frequency and determine location assignments based on storage density; (ii) to group the assignments and convey to predefined stocker. The first phase is possible to be completed before or after explosion which can be individually set up as a stage and eliminated from location assignments decision-making stage. Thus, in Section 4.3.1 the authors introduce storage location assignment (SLA) problem and the extension to Single-item SLA strategy as a footstone for JOFDO strategy. Section 4.3.2 shows the assumptions to adjust the presented strategy to actual operation flow in warehouse. Section 4.3.3 proposes the independent storage allocation as the second phase of JOFDO strategy. A mixed integer linear programming model is built in Section 4.3.4, followed by the JOFDO stocking algorithm developed in Section 4.3.5. At last, Section 4.3.6 presents the experimental results as the evaluation.

4.3.1 Single-item Storage Location Assignment (SSLA) Strategy

In traditional warehouse, single SKU is assigned to be stored in rack locations which are typically with large space and used to store multiple bulks of the same assigned SKU, either by dedicated, random or class based storage strategy. Warehouses like IFWs, store items in unit quantities and in multiple locations where each of them occupies small bins. Storage Location Assignment (SLA) Policy assigns incoming bulks into storage locations with certain rules to achieve predefined objectives. With introducing explosive strategy in IFWs, the scale of location assignment problem becomes larger, while the size of solution pool is multiplied. Three common objectives in SLA problem are: 1) to improve the storage space utilization, 2) to reduce the operating costs and 3) to improve picking efficiency. In IFWs, because of the large amount of SKUs and related high transactions, picking efficiency has a higher priority, leading to a directed and target driven fulfillment strategy.

As the corresponding storage policy, in IFWs, stocking phase works for a better fulfillment circumstance. Received supplies are assigned to predefined locations then grouped to be a stocking list and completed by a free stocker. A well-performed fulfillment process requires a highly efficient picking phase, motivated by a reasonable product inventory structure. Locations are decided by assigning certain criteria. For improving picking efficiency, the most intuitive stocking policy is based on both the order frequency or cube-per-order index (COI), further on a throughput-to-storage ratio (Liu, 2004; Montulet, Langevin, and Riopel, 1998). These criteria help to build the stocking strategy considering both storage space utilization and inventory transaction. Involving explosive storage process, the existing stocking strategy can be revised by a combination of two or more above criteria.

Considering M-to-M storage structure after explosion, storage density is introduced instead of item bin inventory as a measurement for location assignment. Multiple inventory slots in a certain neighborhood conveys to an integrated "bin" with less attractiveness compared to an empty range. Based on the neighbor effect, a location with lower storage density is arranged as a potential assignment for a replenishment package. Furthermore, because of the explosion strategy, receiving supplies are separated to be small packages with single SKU. Replenished products are processed one by one, depending on certain rule, either arrival time or order frequency of that item. Thus, SLA problem is simplified to be a sequential single-SKU storage location assignment (SSLA) model.

4.3.2 Assumptions

The actual processes in IFWs are complicated and covered by digital control. Stocking phase is carried out by a predefined stocking policy, either dedicated or random, which is composed of a set of parameterizations, rules and decisions. These factors are generated from other related stages, supporting the decision-making assignments in order to improve the fulfillment performance in the integrated warehousing processes. In order to reduce the difficulty on modeling the established SSLA problem, these assumptions are set as follows:

(1) Incoming bulks are exploded into small packages; one package is assigned to one slot.

(2) Only exploded lots for single SKU are processed for bin assignments at any time t.

(3) Location assignment is defined independently from SKUs.

(4) Location assignments are grouped within the minimized neighborhood to generate the list before assigned to stocker.

(5) The number of items on a stocking list is limited by list size.

(6) Stocker never wait at the conveyor. As such, a stocking list with no more available pending packages would be released to free stocker with items less than list size.

Based on the above assumptions, SSLA problem is identified to two consecutive processes. The first is storage allocation process, including SKU priority list and the single SKU independent storage assignments, while the second is clustering and grouping process. Further, the independent storage assignment strategy is developed and established in the following section.

4.3.3 Independent Storage Allocation

Among thousands of customer orders, picking strategy generally assign the items from the same order to the same picker to reduce the difficulty of packaging and shipping processes.

In IFWs, as described before, warehouse is facing to individual customers with unit quantity but random combination of products. To improve fulfillment performance, orders with multiple SKUs are discomposed into several "orders" with single item. These small orders are isolated then grouped with orders from other customer to a pick list and fulfilled by different picker. Corresponding to picking process, inventory stocking stage is motivated by picking movement. Order similarity as a criteria is generally involved into storage location assignments process, by which a frequent combination of SKUs is likely to be stocked together as a family group. The advantage from order analysis gains more complexity along with order correlation considered, rather than benefit on picking efficiency. To develop a basis SSLA model as benchmark, storage location assignments for each SKU are determined and evaluated individually, without correlation from other orders or SKUs.

In IFWs, warehouse operations can be described into several different functional phases. Stocking phase, after introducing SSLA problem, is established as below:

(1) Supply bulks are received and exploded by Receiving Phase;

(2) Depending on the arrival time of each bulk, assign the exploded packages with the earliest arrival SKU to stock-waiting list; the corresponding SKU is selected single-SKU – target SKU;

(3) Assign each package location with predefined criteria until all packages for the target SKU are arranged to a specific location;

(4) Assigned location assignments are grouped by close-to-next-free-stocker principle and released to the corresponding stocker;

(5) Stocker works with a certain number of lots as a stocking list; assignments beyond the size of list goes to next free stocker.

4.3.3.1 Notations. Notations in the Table 4.2 are established to describe the algorithm.

Table 4.2 Notations in SSLA strategy and JOFDO Stocking Algorithm

Variable	Description
$i = 1\ to\ N$	Index of SKU
$b = 1\ to\ B$	Index of Bin location
$r = 1\ to\ R$	Index of Receiving supplies
$o = 1\ to\ OD$	Index of Customer orders
$\{R\}$	Receiving bulks from suppliers
$\{r,\ U_r,\ A_r,\ OD_r\}$	Order number, identified SKU, Arriving time, Quantity
O_i	The order frequency of item i
C_i	The average quantity of item i in a single pick stop
$\hat{I}_{i,b}$	The original inventory level of item i in bin b
$I_{i,b}$	The dynamic inventory level of item i in bin b
V_i	The volume of a unit of item i
H_r	The number of exploded packages for order r
$k = 1\ to\ H_r$	Index of Exploded lots
$E_{r,\,k}$	The quantity of items in k^{th} exploded package
B_b	The available volume of bin b
$X_{r,\,k,\,b}$	A set of binary decision variables; denoting the decision if store the k^{th} exploded package of receiving order r into bin b
$F_{i,\,r}$	A set of binary variable, denoting if receiving order r has item i
$\delta = -\Delta$ to Δ	Index of density calculation searching band
$Z_{i,b}$	The fillable factor of item i from bin b
$W_{i,b}$	The storage density of item i at bin b

4.3.3.2 Independent Weighted Storage Density. As a chaotic warehouse, the IFW has an ordinary design difference with traditional one, which is the number and locations of I/O ports. Unlike the traditional warehouse with only one I/O port, an efficiency chaotic warehouse could have multiple loading port to satisfy the high speed transaction. In this chapter, the authors assume that two I/O ports locate at the edge of each aisle representing the belts used for moving exploded lots across the warehouse and splitting, delivering well-picked yellow plastic baskets to packing and shipping station. Assume that Δ is as large as a half of the aisle size L and the probability to generate a complete list with P $_{max}$ items from order list is p. It indicates that the maximum walking distance to pick up a picking list is L with probability and the longest picking time on such list is $cL + p\ P_{max}$. Exploded packages stock into different slots with a reasonable distance, increasing the probability that a pick list fulfilled within limited steps contains item i and other items stocked in the neighbor locations of that slot of item i.

As introduced above, W $_{i,b}$ as the weighted locations/lots of item i stored within ±Δ of bin b, representing the density of item i in a specific range of locations. After any of one item i is stored in a location b which has no or less than average order number of item i, the probability to pick up an order with item i in such range is increased with previous storage process. A picking list is generated by the system which would select a number of items appearing in the order list, having enough inventory located within ±Δ range of a specific bin to reduce the picking time and free walking distance. The probability of successfully assigning a full list is given by the equation below, which is improving when it becomes easier to pick up any of the item in the warehouse. The larger W $_{i,b}$ is, the more uniformly inventory of item i distributes, then with the higher probability a picker is able to fulfill an order for item i walking by less than 2Δ+1bins.

For a target SKU i, the independent weighted storage density is established as following equation.

$$W_{i,b} \leq \sum_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot Z_{i,b+\delta}\,;\; W_{i,b} \leq 1 \tag{4.1}$$

$$Z_{i,b} \leq \frac{I_{i,b}}{C_i};\; Z_{i,b} \leq 1 \tag{4.2}$$

Equation (4.1) indicates a central amplifying effect from neighbor bins in a certain range, in which location sets with existing inventory will avoid incoming replenishment packages, ensuring that each package is assigned to a different location. Equation (4.2) restricts the upper limit of density factor in which all bins with inventory able to fulfill an average customer order of item i are traded as the same priority. In SSLA and the JOFDO stocking algorithm stated in the following sections, storage density as a predominant parameter provides the guidance to an intuitive inventory allocation decision, improving the inventory structure of the warehouse in order to efficiently fulfill customer orders.

4.3.3.3 Storage Uniformity. In actual size warehouses, lots of the same items is a certain number at any time instance. For low inventory transient SKU, storage locations and the corresponding inventory are less than those of popular SKUs. When searching for next available location to assign to replenishment packages, the probability of existing multiple alternatives with no difference on density priority is not ignorable and to a large extend affecting the assignment decisions.

To deal with equivalent alternatives, uniformity is introduced into the single SKU location assignment model, which is expressed by the difference between average location number of all existing inventory slots and middle bin of all aisles. To reduce, even eliminate this difference, a direct solution is to stock all inventory in or among the warehouse center bin, however, to achieve high inventory density, the preference would be to separate small packages away from bins with pick-able products at the moment. Storage location are assigned with these two parameters to achieve a high storage density with little penalty from uniformity.

4.3.4 Mixed Integer Linear Programming (MILP) Model

As indicated, JOFDO stocking strategy can be described in two stages – Order frequency Oriented SKU Preselection and Independent Storage Density Oriented Location Assignment, with a Single-item Storage Location Assignment strategy involved.

4.3.4.1 Independent Storage Density Oriented Location Assignment Model. At first, based on the assumptions and the leading criteria – storage density, a Mixed Integer Linear Programming (MILP) Model (4.1) is established as follows.

For each SKU $i \leq N$,

Max: $$\sum_b W_{i,b} \cdot O_i \tag{4.3}$$

s.t.

$$I_{i,\,b} = \sum_r \sum_k X_{r,k,\,b} \cdot F_{i,\,r} \cdot E_{r,\,k} + \hat{I}_{i,b} \tag{4.4}$$

$$\sum_k \sum_b X_{r,k,\,b} = H_r \tag{4.5}$$

$$\sum_{r}\sum_{k} X_{r,k,b} \cdot F_{i,r} \cdot E_{r,k} \cdot V_i \leq B_b \tag{4.6}$$

$$X_{r,k,b} \cdot F_{i,r} \leq X_{r',k',b} \cdot F_{j,r'} \ if \ O_i \leq O_j \tag{4.7}$$

$$X_{r,k,b},\ F_{i,r} \ as \ Binary$$

$$Z_{i,b} \leq \frac{I_{i,b}}{C_i}$$

$$Z_{i,b} \leq 1$$

$$W_{i,b} \leq \sum\nolimits_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot Z_{i,b+\delta}$$

$$W_{i,b} \leq 1$$

In above equations, Objective Function (4.3) indicates the objective value is to achieve the highest improvement on storage density from a set of exploded lots stocking into the warehouse. Constraint (4.3) ensures that all the receiving packages are stocked in any defined bin location. Constraint (4.4) shows the inventory flow and illustrate the inflow and outflow balance. Constraint (4.6) presents the space availability while SKU priority from order analysis is conveyed in Constraint (4.7). The results are a set of location assignments corresponding to each exploded lot, which await the grouping and stocker arrangement in next stage.

In Section 4.3.3, the authors illustrate that multiple sets of solutions would be reached from the above MILP model since these assignments can achieve the same benefit from a certain number of replenishment packages. Here storage uniformity is involved as the secondary factor and the second part of objective value to distinguish a better solution from equivalent alternatives in Model (4.1).

Notations U_i and M_i as the indicators to represent storage uniformity are introduced to develop the modified MILP model in Table 4.3.

Table 4.3 Additional notations in SSLA strategy and JOFDO Stocking Algorithm

Variable	Description
Π_i	Uniformity Reference – the summary of bin numbers if all inventory lots of item i are distributed uniformly in the warehouse
M_i	The number of total lots of item i if none of the new packages is assigned to a bin with target item
U_i	The penalty of storage uniformity from current inventory distribution for item i
$\hat{Z}_{i,b}$	The original fillable factor of item i from bin b

Based on the supplements of notations above, a revised MILP Model (4.2) is defined as below. For any SKU $i \leq N$,

Max: $$\left(\sum_b W_{i,b} - \frac{U_i}{M_i}\right) \cdot O_i \tag{4.8}$$

s.t.

$$I_{i,\,b} = \sum_r \sum_k X_{r,k,\,b} \cdot F_{i,\,r} \cdot E_{r,\,k} + \hat{I}_{i,b}$$

$$\sum_k \sum_b X_{r,k,\,b} = H_r$$

$$\sum_r \sum_k X_{r,k,\,b} \cdot F_{i,\,r} \cdot E_{r,\,k} \cdot V_i \leq B_b$$

$$X_{r,\,k,\,b} \cdot F_{i,\,r} \leq X_{r',\,k',\,b} \cdot F_{j,\,r'} \; if \; O_i \leq O_j$$

$$X_{r,k,\,b},\ F_{i,\,r}\ as\ Binary$$

$$Z_{i,b} \leq \frac{I_{i,b}}{C_i}$$

$$Z_{i,b} \leq 1$$

$$W_{i,b} \leq \sum\nolimits_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot Z_{i,b+\delta}$$

$$W_{i,b} \leq 1$$

$$U_i \geq \sum_{b} Z_{i,b} \cdot b - \Pi_\mathrm{i} \tag{4.9}$$

$$U_i \geq \Pi_\mathrm{i} - \sum_{b} Z_{i,b} \cdot b \tag{4.10}$$

$$\Pi_\mathrm{i} = \frac{1}{2} \cdot (1 + \mathrm{B}) \cdot M_i \tag{4.11}$$

$$M_i = \sum_{b} \hat{Z}_{i,b} + H_r \tag{4.12}$$

In MILP Model (4.2), Objective Function (4.8) includes two components. One is the total weighted storage density, which is same as Model (4.1). The other is storage uniformity, to be subtracted as a penalty from the difference between solved inventory distribution and uniformly allocation. Constraint (4.11) shows the calculation for uniformity reference number, in which total number of lots is presented in Constraint (4.12). Constraint sets (4.9) and (4.10) indicate the evaluation for uniformity of slotting, which is approaching zero while replenishment packages are stocking in such a way that all the bin ranges with target item are undifferentiated.

4.3.4.2 Problem Reduction. The proposed formulation can solve for optimal within a small-size warehouse, dealing with small amount bulks, but the difficulty to solve such problem is emphasized along with increase of the size of the formulation, which make it hard, if feasible, to solve. The difficulty derives from the number of integer decision variables and constraints. The established Model (4.2) has $\sum_r H_r B + NR$ binary variables, $(2B + 2)N$ other variables and $(7B + 5 + (I - 1)B)N$ constraints. For example, the total number of variables and constraints from a small size of the problem (2000 bins, 100 SKUs and 1000 orders) is ten million variables and constraints. Most of the existing optimization software or platforms takes days or even weeks to find an exact optimal solution if feasible or would fail before running out of memory. Thus, a systematic approach is provided to approximately solve this independent SSLA problem in an efficient way, while an acceptable tolerance is shown compared with optimal solutions obtained by optimization software.

The proposed approach is solution space reduction. By identifying the characteristic of parameters, the MILP model can be simplified by either predefining values for decision variable or releasing the constraints with adjustable assumptions.

As stated that SKU is processed individually in location assigning stage, without interaction from either sales orders or other items, notations are simplified to eliminate the subscript of index i. Meanwhile, in receiving and explosion phase, a bulk of large quantity of items is equally distributed across H_r lots with quantity of $E_{r,k}$ units (Onal et al., 2017). To simplify the calculation in MILP Model (4.2), instead of selecting one location for each lot sequentially, a set of K lots is solved and randomly assigned to each package. Thereupon, a reduced MILP Model (4.3) is proposed followed by the revised notations in Table 4.4.

Table 4.4 Notations in Reduced MILP Model

Variable	Description
C	The average quantity of target SKU in a single pick stop
$\hat{I}_b$	The original inventory level of target SKU in bin b
I_b	The dynamic inventory level of target SKU in bin b
V	The volume of a unit of target SKU
K	The number of exploded lots for target SKU
$k = 1\ to\ K$	Index of Exploded lots
E	The quantity of items in one exploded lot
X_b	A set of binary decision variables; denoting the decision if assign one exploded lot of target SKU into bin b
Π	Uniformity Reference – the summary of bin numbers if all inventory lots of target SKU are distributed uniformly in the warehouse
M	The number of total lots of target SKU if none of the new packages is assigned to a bin with target item
U	The penalty of storage uniformity from current inventory distribution for target SKU
$\hat{Z}_b$	The original fillable factor of target SKU from bin b
Z_b	The fillable factor of target SKU from bin b
W_b	The storage density of target SKU at bin b

The reduced MILP Model (4.3) executes after system decides next receiving order or target SKU to be exploded and wait for stocking, to capture the relationship between the decision variables and the performance objective to allocate incoming inventory.

Max: $\sum_b W_b - \frac{U}{M}$

s.t. $X_b\ as\ Binary$

$$I_b = X_b \cdot E + \hat{I}_b$$

$$\sum_b X_b = K$$

$$\sum_k X_b \cdot E \cdot V_i \leq B_b$$

$$W_b \leq 1$$

$$W_b \leq \sum_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot Z_{b+\delta}$$

$$Z_b \leq \frac{I_b}{C}$$

$$Z_b \leq 1$$

$$U \geq \sum_b Z_b \cdot b - \Pi$$

$$U \geq \Pi - \sum_b Z_b \cdot b$$

$$\Pi = \frac{1}{2} \cdot (1 + \mathrm{B}) \cdot M$$

$$M = \sum_b \hat{Z}_b + K$$

With the reduction approach, the solution pool has been decreased into B decision variables, 2B+2 other variables and 6B+5 constraints for each SKU. For a small-scaled warehouse with 2000 bins and 100 SKUs, the reduced Model (4.3) has 6002 variables and

12005 constraints for a run with one target SKU. In optimization software, it is solved in a few minutes for a single SKU case and hours involved 100 SKUs.

4.3.4.3 Performance Analysis and Evaluation. To analyze the performance behavior of Model (4.3), a general-applied, powerful and free optimization software – OpenSolver (http://opensolver.org/) is used to solve several single-SKU cases. Based on the observed sensitivity of the performance, the experimental space is trimmed, with parameters defined in Table 4.5.

Table 4.5 Key Parameters for the Experimental Reduced SSLA Problem

$N = 1\ SKU$	$B = 1000\ Bins$	$\hat{B}_b \leq 3000\ in^3$
$V = 10 * 5 * 2 = 100\ in^3$	$C = 10$	$E = 25$
$\Delta = 20$	$\frac{\hat{I}_b}{M-K} = 50$	$\mathrm{K} = 20$
$M - K = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ - the number of set-up inventory lots		

Experimental results are inherently characterized by errors or variance, with specification from original setting of parameters. As a validation study, the replication number should be estimated to get more accurate experimental results. Since all problems are solved by OpenSolver, results for the same situation are static within several replications. Another factor is introduced into the experiments as the variance of bin allocation in inventory setup, which provides five different cases for each M-K. These five cases are generated by randomly select M-K bins as initial inventory lots, differentiated by randomized range size between each two locations. Thus, Cases 1 and 2 are selected from inventory allocation having a slight bias towards the front or back; Cases 4 and 5 have a heavy bias to either the start or the end bin, while Case 3 is approaching uniformly distributed.

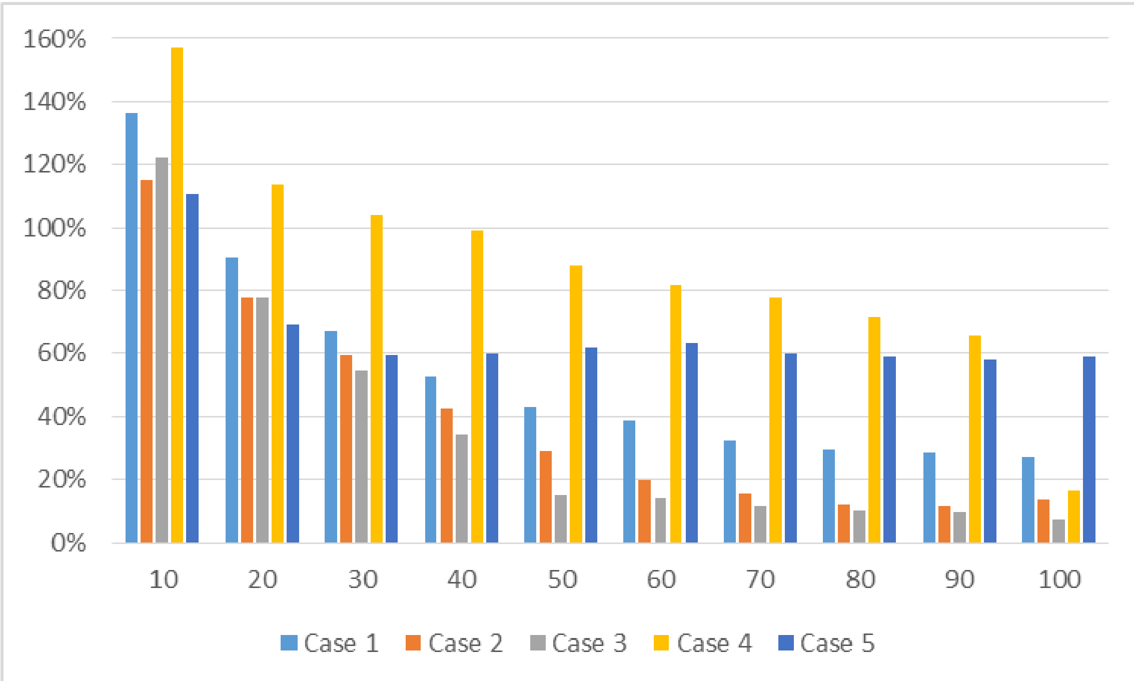

Figure 4.8 The objective value shows significant improvement within all parametric experiments differentiated by five cases with 10 types of M-K initial inventory allocation.

Figure 4.8 shows the experimental results, given improvement on objective value of Model (4.3) among five cases with each initial inventory setup. The primary objective is to increase the picking probability in order to obtain a quick customer order fulfillment performance. A comparison between the increase of picking probability of target SKU and average objective value improvement is proposed in Figure 4.9 as below.

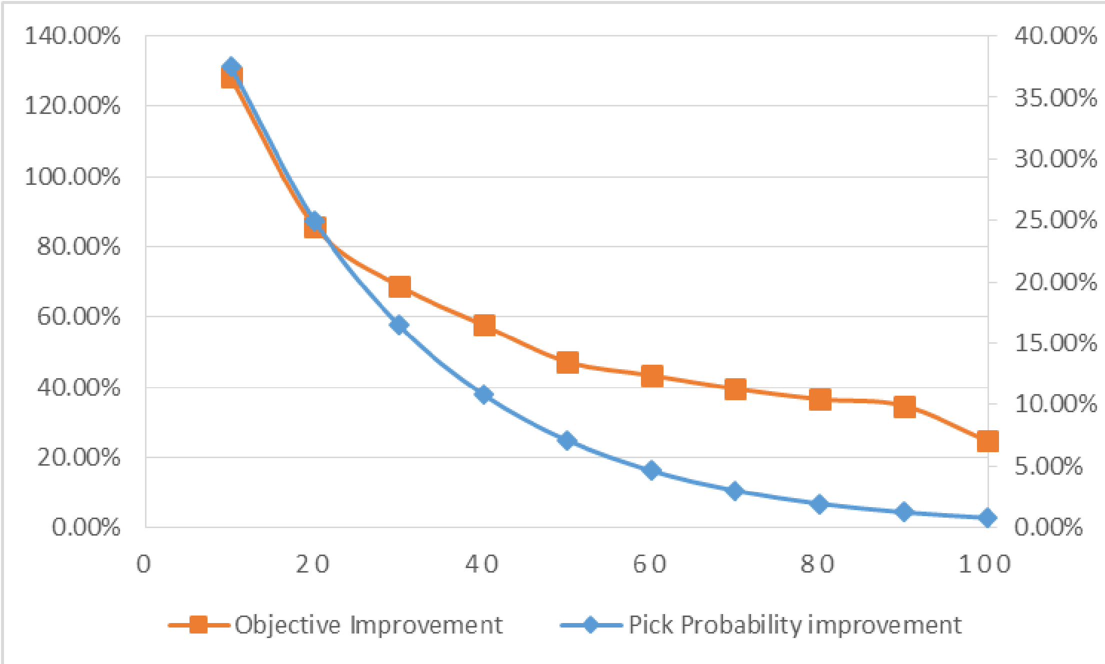

Figure 4.9 The pick-able probability of target SKU increases along with the average objective value improving in all parametric experiments.

The performance of the reduced SSLA model conveys a remarkable advancement of 25% to 130% compared with the original value, through the above experimental results. Note that the benefit margin is decreasing along with the additional initial inventory lots. Particularly, for Case 3, replenishment lots for an approximately uniform distributed storage structure indicate less enhancement with more initial lots and given searching band. This conclusion also works for other cases. Supplies to a warehouse with plenty of stocks would result in a higher holding cost instead of reducing fulfillment time since it is not necessary to stock four lots on the same aisle if two is the maximum picks on a pick list.

Figure 4.9 and Figure 4.10 shows a linear relationship between picking probability and the reduced SSLA model objective value, which presents that the solution obtained from the reduced SSLA strategy would convey to an improvement on picking efficiency, accordingly, the fulfillment performance.

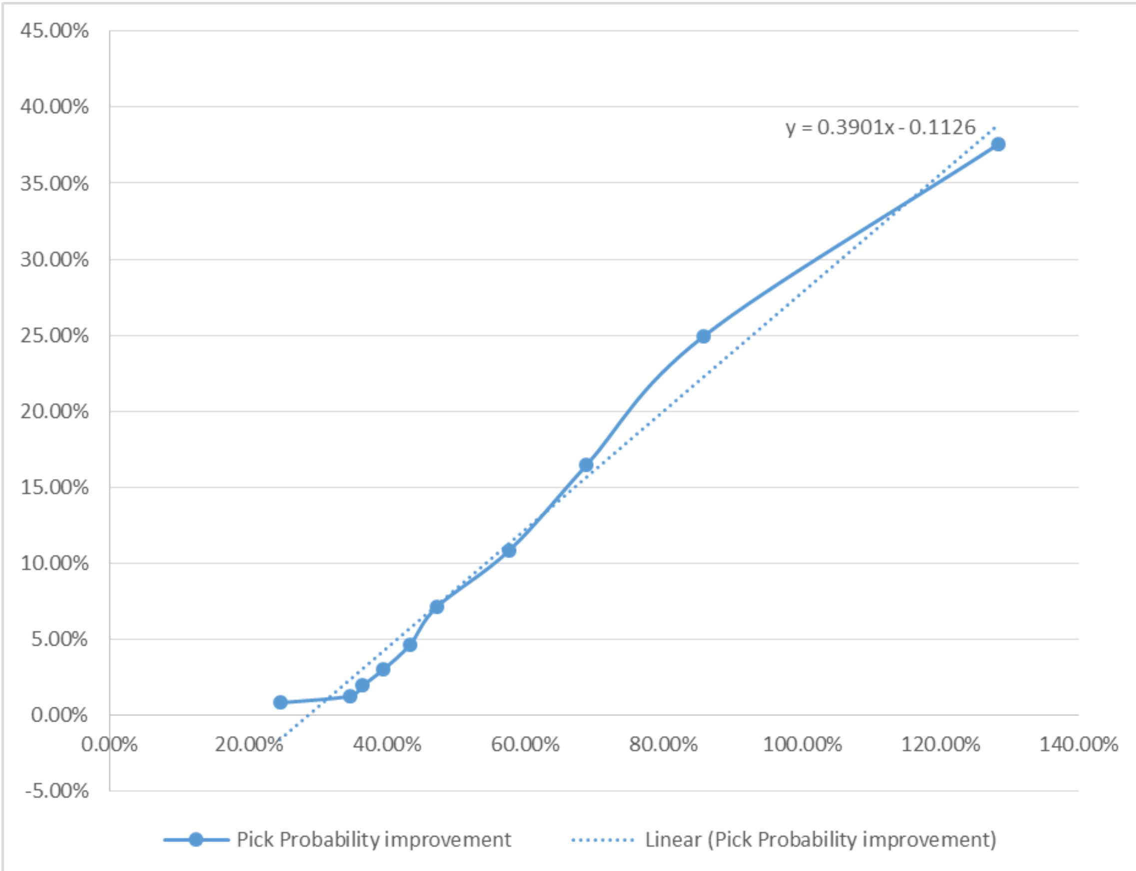

Figure 4.10 The pick-able probability of target SKU has a linear relationship with the average objective value improving in all parametric experiments.

4.3.5 JOFDO Stocking Algorithm

As introduced at the start of Section 4.3, SSLA strategy is presented to solve the location assignments problem after SKU selection and explosion. In Section 4.3.4, a reduced SSLA model is proposed to perform a likely efficient and intuitive design. To evaluate the strategy in a dynamic environment with a large problem size, the Joint Order Frequency and Density Oriented (JOFDO) stocking algorithm is established in the following sections, by combining the SSLA policy with order frequency to demonstrate a solution that assigning locations to selected pending packages and sequentially group them into stocking lists. This algorithm includes two phases: (i) to rank the SKU priority and solve the location assignment problem from reduced SSLA Model (4.3), (ii) to band pending packages into group to minimize the walking distance and stocking time.

The algorithm flows are as follows:

Phase I: SKU Priority and Single-SKU Storage Location Assignment

1. At time t, read Order Frequency Table from historical customer order database.
2. Read Order List $\{O\}$ for current pending orders.
3. Create the Receiving Replenishment List for current bulks not stored yet.
4. Do explosion and add exploded packages to Waiting List for stocking.
5. Call W_b Table – Weighted Storage Density Table.
6. If current inventory for SKU i couldn't satisfy the requirement from pending customer order list $\{O\}$, and this SKU is received and ready to be stocked, assign the SKU the highest rank of priority for location assignment as the target SKU.
7. Or not, depend on the order frequency table, give a priority rank for each SKU, which has replenishment at current time t. Select the SKU with highest order frequency as target SKU; If equivalent SKUs exist, depending on the arrival time of receiving bulks for each item, the earliest arrival SKU is first to be stocked.
8. Call the reduced SSLA Model (4.3) and solve for a set of location assignments.
9. Assign all exploded bulk of SKU i to the corresponding solution from SSLA strategy. Then update the Inventory Table and Weighted Storage Density Table.
10. Repeat step 1 to 9 until time shift to next period (t+1) or no more pending packages are waiting for stocking.

Phase II: Grouping and Stocker Arrangements

11. Read People Table for the free time of all stockers.
12. Select the earliest free stocker s and record the last location of stocker s. $l = 0$.
13. List all location-defined but not stocked lots for target SKU and Location ID to be the pending list at the moment t.

14. $l = l + 1$. Assign the closest pending location assignments to the last location of stocker s as the l^{th} item on the stocking list. Record this location assignment as the last location.

15. Redo step 14 until $l \geq S_{max}$ – Stocking list size, which indicates the maximum items a tote can carry, or the pending location assignments are completed.

16. Exit until time shift to next period (t+1) or no free stocker could be found in current time shift.

To perform JOFDO algorithm in a dynamic environment, MILP model which provides solutions by batch is not applicable. Thus, heuristic solving this problem within an acceptable tolerance compared with optimal solutions obtained by OpenSolver is established in the following section.

4.3.6 Heuristics

In this section, four Heuristics are developed to be combined with the 2nd phase in JOFDO stocking algorithm, in order to achieve the closeness to optimal inventory allocation solutions from MILP Model (4.3) presented in the Section 4.3.4. To approach to a heuristic with accuracy, optimal solution is used to do backward research. The objective function consists of two components, the weighted storage density and the uniformity penalty. A range of less inventory lots derives a higher enhancement on density but is possible to break the uniformity balance. Therefore, the strategy to develop heuristics is to optimize uniformity, then improve storage density within a predefined searching band.

4.3.6.1 Cut-off Heuristics. The first two heuristics come from straight-forward thinking to allocate new incoming inventory. Introduce that the range between two closest stocking lots of target SKU as Lots Gap. To uniformly stock the lots, long lots gap is cut-off into half, to obtain a higher density without affect the uniformity.

Heuristic #1 (H1): Cutoff longest gap and backward searching bin assignment.

- Read the current inventory of target SKU – I_b .
- Mark the range with longest gap $\max\{|(b-a)|\}$, where a and b indicates the start and end bin number correspondingly.
- Select the bin with highest improvement on objective value of Model (4.3) as next location assignment, when a backward comparison is executed from bin b to bin $b-2\Delta+1$.
- Set $b' = \left\{b-\delta \mid \max_{\delta}(Objective\ value\ after\ stocking\ in\ bin\ b - \delta)\right\}$.
- Redo the step above until no more pending exploded lots or current time shift is finished.

Heuristic #2 (H2): Cutoff longest gap and center bin assignment.

- Read the current inventory of target SKU – I_b .
- Mark the range with longest gap $\max\{|(b-a)|\}$, where a and b indicates the start and end bin number correspondingly.
- Select the center bin of current range as next location assignment, set $b' = \frac{1}{2}(b+a)$.
- Redo the step above until no more pending exploded lots or current time shift is finished.

Several tested experiments are executed to evaluate the results from these two heuristics. However, half of the tested problems have shown that after the first several assignments, the probability that cutoff heuristic is assigning the same location to the following packages, even available lots locates in the neighborhood of the result location, increases along with the number of exploded packages. The results are not indicative, which are not included in this document.

4.3.6.2 Uniform Seed Bin Heuristic. Another heuristic developed in this section consists of a two-phase decision approach: (1) seed bin locking on according to uniformity enhancement and (2) a band search to maximize the storage density, further the objective value.

To identify these two heuristics, a new factor – unbalanced difference (UBD_b) is introduced as a criteria to determine the characteristic of optimal model and solutions, where:

$$UBD_b = AVG\left(\sum_{a=1}^{b} W_a\right) - AVG(\sum_{a=b}^{B} W_a).$$

UBD_b is named as the calculation, aiming at the difference on the two side of bin b. A positive UBD_b represents the inventory bias in the range from start bin to bin b, while a negative UBD_b indicates the motivation to stock incoming lots into any bin with location number less than b, therefore to reduce uniformity penalty and improve storage density simultaneously.

Heuristic #3 (H3): Uniform seed bin and band searching location assignment.

- Read the current inventory of target SKU – I_b .
- Call the weighted density table – W_b.
- Calculate UBD_b based on the proposed equation above, $UBD_b = AVG\left(\sum_{a=1}^{b} W_a\right) - AVG(\sum_{a=b}^{B} W_a)$.
- Set uniform seed bin $b' = \min\left(abs\left(\frac{1}{2} \cdot (1 + B) \cdot \left(\sum_b \hat{Z}_b + 1\right) - \sum_b \hat{Z}_b \cdot b\right), B\right)$.
- Depending on the corresponding $UBD_{b'}$, determine the direction for band searching.

- If $UBD_{b'} \geq 0\ and\ b' \neq B$, from center bin b', towards to bin locations with location number larger than b', search for the closest range of $2\Delta - 1$ bins with zero or small inventory which shows unfillable to an average pick. Target range size is shrinking by reduce Δ to be $\Delta - 1$ along with a set of βM searching iterations. β is the restricted weight to avoid the number of failure iterations, which is default to be one.

- If $UBD_{b'} \geq 0\ and\ b' = B, or\ UBD_{b'} < 0$, perform the same procedure as above, from center bin b', towards to bin with location number less than b'.

- Select the center bin of target range as the location assignment.

- Redo the steps above until no more pending exploded lots or current time shift is finished.

With seed bin from uniform analysis involved, H3 presents a powerful strategy on location assignments. However, a patent defect is recognized in programming process. Unlike H1, H3 has no limitation on searching band. To keep on searching for target range, the total number of iteration could reach the number of total bins. Even a restricted weight factor α is used to control this procedure, a failure track is possible to have $\beta\text{M} \cdot (\Delta - 1)$ trials without finding the desired range and bin assignment.

Here Heuristic #4 is proposed as a revision of above heuristic, in which, searching strategy is replaced by a certain criteria based on UBD_b and the corresponding behavior.

Before presenting the modifications, the unbalance range is introduced as a referred parameter, to identify the trend of UBD_b.

- Set $w = 0$.

- If $UBD_1 \geq 0$, then assign unbalance range (UR_1) as 0; otherwise as 1, set $w = UR_1$.

- For any bin $b \geq 2$, if $UBD_b - UBD_{b-1} \geq 0$, then assign unbalance range (UR_b) as 0; else if $(UBD_{b-1} - UBD_{b-2}) < 0$, then assign UR_b as w; otherwise, assign UR_b as $w + 1$, and set $w = w + 1$.

- Exit until $b = B$.

Heuristic #4 is differentiated with above. After locking on uniform seed bin as start location, the algorithm identifies the unbalance range which the seed bin locates in. A bin in that range with the closest-to-zero $ABS(UBD_b)$ is picked as the location assignment for current exploded lot, if the seed bin has a positive UR_b value. Otherwise, according to the corresponding UBD, the range with positive unbalance range value closest to seed bin would be used as target unbalance range. Thereupon select the bin in that range with the closest-to-zero $ABS(UBD_b)$. Compared with the band searching algorithm, the UR oriented strategy reduces the number of iteration to be 1 or 2 for a single exploded package, which significantly improves the efficiency of location assignment phase.

Heuristic #4 (H4): Uniform seed bin and Unbalance Range oriented heuristic

- Read the current inventory of target SKU – I_b .
- Call the weighted density table – W_b.
- Calculate UBD_b based on the proposed equation above, $UBD_b = AVG\left(\sum_{a=1}^{b} W_a\right) - AVG(\sum_{a=b}^{B} W_a)$.
- Set uniform seed bin $b' = \min\left(abs\left(\frac{1}{2} \cdot (1+B) \cdot \left(\sum_b \hat{Z}_b + 1\right) - \sum_b \hat{Z}_b \cdot b\right), B\right)$, read the corresponding $UR_{b'}\ and\ UBD_{b'}$.
- If $UR_{b'} > 0$, then $b_k = \left\{b |\min_{\mathrm{b}}(abs(UBD_b)|UR_b = UR_{b'})\right\}$.
- Else, depending on the corresponding $UBD_{b'}$, determine the direction for next available unbalance range.
- If $UBD_{b'} \geq 0, b' \neq B\ or\ UBD_{b'} < 0, b' = 1$, from b', towards to bin locations with location number larger than b', search for the closest bin with a positive UR. Set this bin as b', then $b_k = \left\{b |\min_{\mathrm{b}}(abs(UBD_b)|UR_b = UR_{b'})\right\}$.
- If $UBD_{b'} \geq 0, b' = B, or\ UBD_{b'} < 0, b' \neq 1$, perform the same procedure as above, from b', towards to bin with location number less than b'.

- Redo the steps above until no more pending exploded lots or current time shift is finished.

4.3.6.3 Two-Phase Stocking Location Assignment Heuristic. The stocking location assignment heuristic is developed as JOFDO stocking algorithm with built in uniformity and unbalance range directed heuristic, consisting of two-phase solution, as stated above. In Phase I, location assignments are solved sequentially, with a preselection on SKU or receiving orders priority. Stocking list is generated and allocated to a specific stocker with pending lots grouping and stocker arrangement decisions with in Phase II, according to the slot solutions obtained in Phase I.

Phase I: SKU Priority and Location Assignment

1. Among all receiving supplies, select $\{RID\}_t = \{RID \mid ArrTime \le current\ time\ and\ ADay \le current\ day\}$;
2. Within the selections, calculate $order\ frequency$ for each SKU;
3. Select

$$rid_t = argmin(order\ frequency\ of\ rid \mid rid \in \{RID\}_t\ and\ order\ frequency\ of\ rid < 0)$$

   If $\{rid \mid rid \in \{RID\}_t\ and\ order\ frequency\ of\ rid < 0\} = \emptyset$, then select $rid_t = argmin\{ArrTime\ and\ ADay\ of\ rid \mid\ rid \in \{RID\}_t\}$;
4. $i = SKU_t = \{SKU \mid RID = rid_t\}$;
5. Do explosion, set k=1;
6. Let seed bin $b' = \min\left(abs\left(\frac{1}{2} \cdot (1 + B) \cdot \left(\sum_b \hat{Z}_b + 1\right) - \sum_b \hat{Z}_b \cdot b\right), B\right)$; Record $UR_{b'}\ and\ UBD_{b'}$;
7. If $UR_{b'} > 0$, then $b_k = \text{argmin}(abs(UBD_b) | UR_b = UR_{b'})$;
8. Else, from seed bin b', if $UBD_{b'} \ge 0, b' \ne B\ or\ UBD_{b'} < 0, b' = 1$, move to bin $\hat{b} = argmin(b - b' | UR_b > 0)$;

Set $b' = \hat{b}$ then $b_k = \text{argmin}(abs(UBD_b)|UR_b = UR_{b'})$;

9. If $B_{b_k} < V_i \cdot E_{i,k}$, set $b_k = b_k + 1$ and redo step 9 until $B_{b_k} \geq V_i \cdot E_{i,k}$;

10. If $UBD_{b'} \geq 0, b' = B, or\ UBD_{b'} < 0, b' \neq 1$, move to bin $\hat{b} = argmin(b' - b|UR_b > 0)$
Set $b' = \hat{b}$ then $b_k = \text{argmin}(abs(UBD_b)|UR_b = UR_{b'})$;

11. If $B_{b_k} < V_i \cdot E_{i,k}$, set $b_k = b_k - 1$ and redo step 11 until $B_{b_k} \geq V_i \cdot E_{i,k}$;

12. Update inventory of item $\boldsymbol{i}$ by adding quantity of $\boldsymbol{k^{th}}$ exploded package to location b_k; record as SID=SID+1, LID=b_k;

13. k=k+1;

14. Recalculate $Z_b\ W_b\ UBD_b\ and\ UR_b$;

15. Redo step **6** to **14** until $k = K$, then move to **Step 1** for next SKU;

16. Exit when no incoming packages or time shift is end and set SID=0.

Phase II: Group and Stocker Assignment

17. Select $eid_t = argmin\{Free\ time\}$ as next available stocker;

18. List $S_{max}(= the\ maximum\ number\ of\ a\ stocking\ list)\ packages$ where, $SID = argmin\{abs(LID - current\ location\ of\ eid_t)\}$ for 1st package, and $SID_n = argmin\{abs(LID - LID_{n-1})\}$ for the rest;

19. Exit when no pending packages or time shift is end.

Steps 1 to 4 initialize the current set of unassigned supplies and provide a priority list of available SKUs with current receiving bulks with predefined criteria – arriving time and order frequency. Step 5 calculate the number of pending lots of target SKU selected from the first four steps after explosion. Steps 6 to 12 perform a single-item storage assignment according to the proposed H4 with volume check. This single assignment is recorded in pending stocking lists with its SKU, quantity and assigned location ID. Steps 13 to 16 illustrate the continuous flow of location assignments within the same SKU and among SKUs. Phase II includes three steps, in which Step 17 determines next free stocker and Steps 18 to 19 split the pending stocking lists into a set of stocking lists completed by the corresponding stocker.

4.3.7 Experiments and Results

JOFDO stocking algorithm is approached by sequentially processing single SKU with discrete location decisions. Involving the established uniform seed bin heuristics (H3 and H4), pilot experiments are designed to evaluate the performance of the heuristics compared with optimal OpenSolver solution from Section 4.3.3.

The key parameters are stated in Table 4.6, in which, the majority of the setup follows the same setting in performance analysis of SSLA Model (4.3). Searching band and the number of replenishment packages are increased by three different situations each, which provide a sensitivity analysis simultaneously.

Table 4.6 Key Parameters for Valid Experiment on the JOFDO Algorithm with Heuristic

$N = 1\ SKU$	$B = 1000\ Bins$	$\hat{B}_b \leq 3000\ in^3$
$V = 10 * 5 * 2 = 100\ in^3$	$C = 10$	$E = 25$
$\Delta = 10, 20, 30, 40$	$\frac{\hat{I}_b}{M-K} = 50$	$\mathrm{K} = 5, 10, 20, 40$
$M - K = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ - the number of set-up inventory lots		

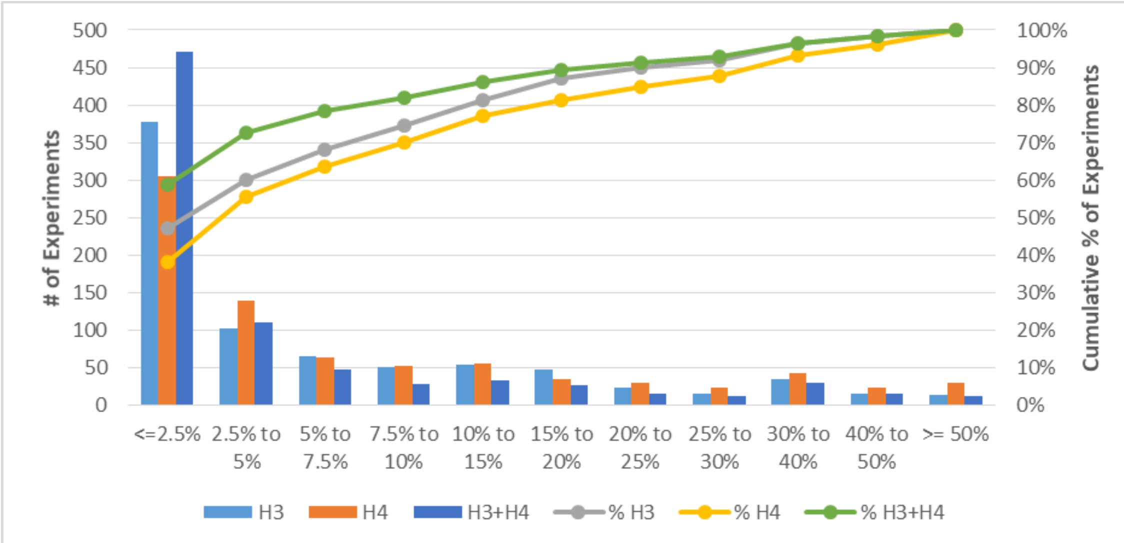

Figure 4.11 The overall performance analysis presents approximate results within an acceptable difference compared with optimal solution in MILP Model (4.3).

Figure 4.11 shows the results for the total of 800 experiments with different setting of Δ, K and M − K. Using optimal results as benchmark, around 80% among overall 800 experiments are presenting quick solutions without losing at most 15% accuracy by importing either H3 or H4 to replace the MILP Model (4.3). The number of worst cases under 50% accuracy occupy only 1.6% with H3 and 3.7% correspondingly with H4.

For the sake of better understanding, a paired two sample hypothesis test is conducted between each tested heuristic and the optimal.

Hypothesis is set up as follow:

$$\boldsymbol{H_0}: H3\ /\ H4\ is\ within\ 15\%\ difference\ of\ optimal\ MILP\ Model\ (4.3)$$

$$\boldsymbol{H_1}: H3\ /\ H4\ is\ \ out\ of\ 15\%\ difference\ of\ optimal\ MILP\ Model\ (4.3)$$

The results are illustrated in Table 4.7.

Table 4.7 t-Test: Paired Two Sample for Means between Optimal Solution and Heuristics

FACTOR	Optimal	H3	H4	H3+H4
Mean	1	0.918439	0.887594	0.935114
Variance	0	0.01466	0.0361	0.013615
Observations	800	800	800	800
Hypothesized Mean Difference		0.09	0.12	0.07
df		799	799	799
t Value		-1.97139	-1.1305	-1.23955
P(T<=t) two-tail		0.05	0.26	0.22

Based on the detailed behavior shown in above table, a paired t-Test provides clear evidence that H3 is acceptable within 9% difference compared to MILP solution while for H4, the difference is 12%, in which the null hypothesis is accepted that both two heuristics are within an identified difference of optimal result. The analyses are set at a significant level α of $\alpha = 0.05$. A straightforward method to improve the behavior is combining the two heuristics by using a higher results in between these two output, which improves the difference to be less than 7%. The corresponding behavior and paired t-Test are shown in Figure 4.11 and Table 4.7.

In Table 4.6, three key controllable key factors are proposed to represent the diversity in enviromental design. Consequently, the authors establish a set of sensitivity analysis to exploit insights into the heuristic against optimal SSLA method. Figures 4.12 to 4.14 illustrate the detailed behavior as below.

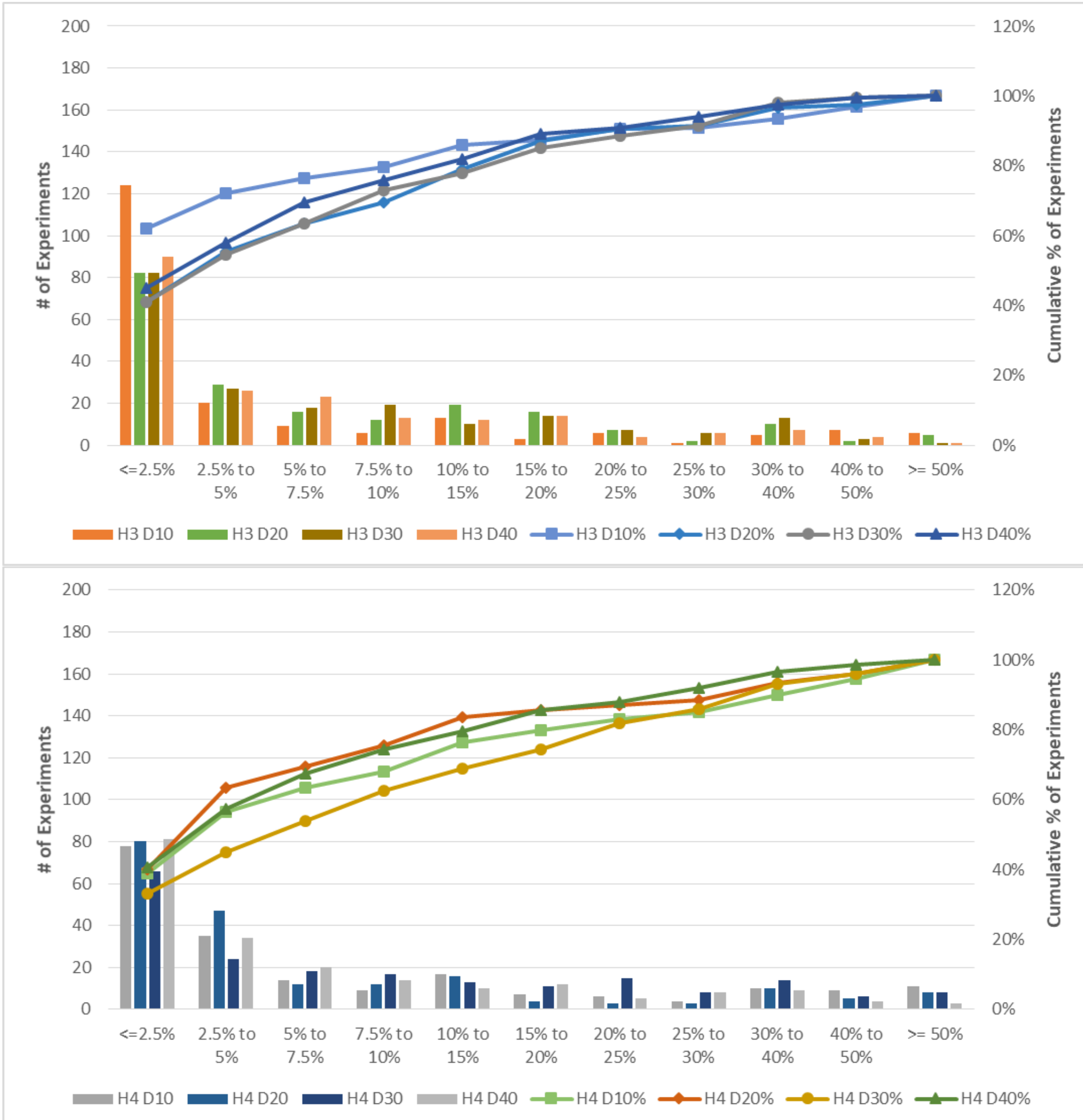

Figure 4.12 The performance analysis illustrates H3 is outperforming with a small searching band of 10 while H4 dominates on delta of 40 instead, compared with optimal solution in MILP Model (4.3).

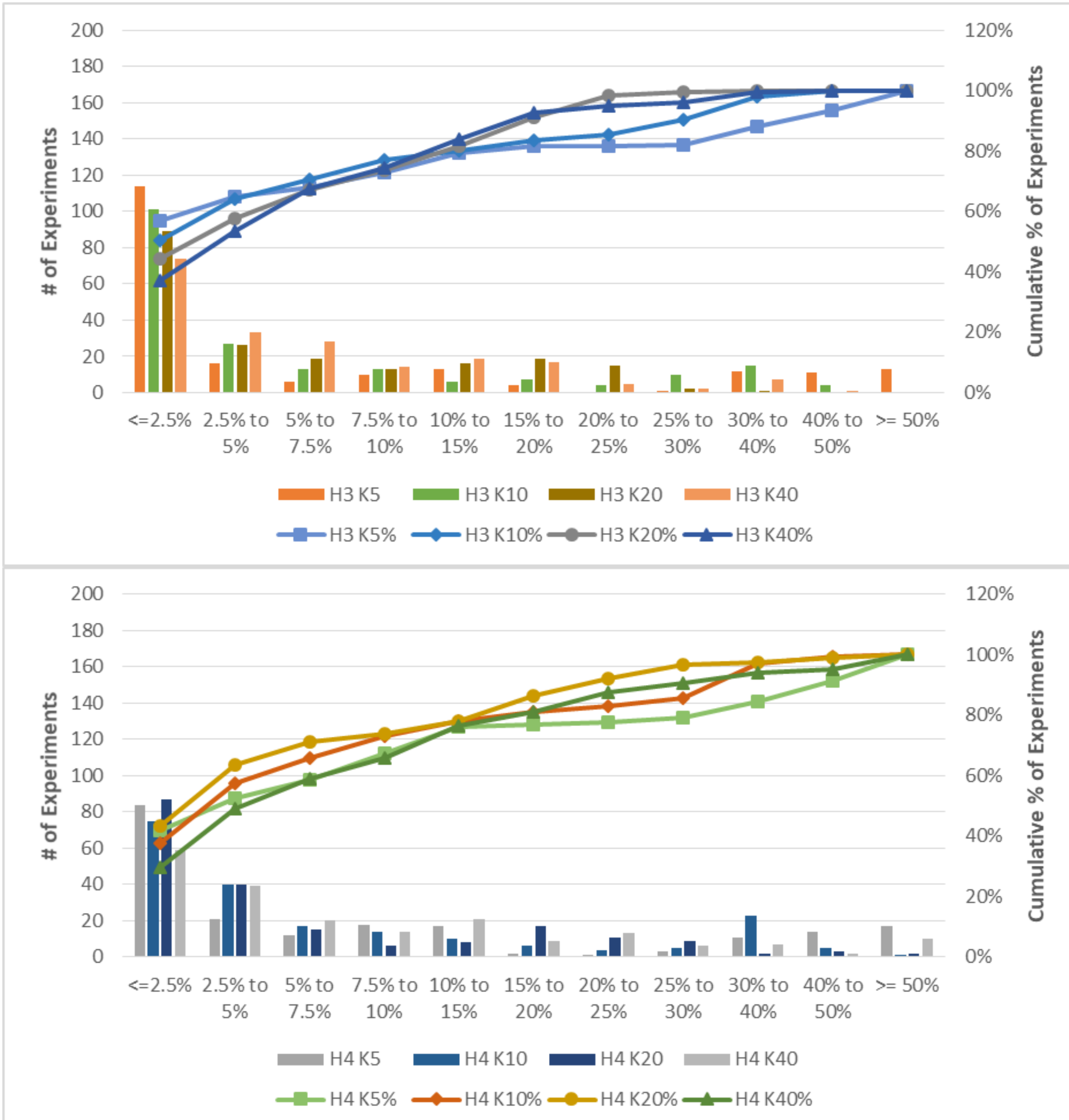

Figure 4.13 The performance analysis illustrates H3 presents better performance with SKU exploded into 5 lots while H4 dominates on K of 20 instead, compared with optimal solution in MILP Model (4.3).

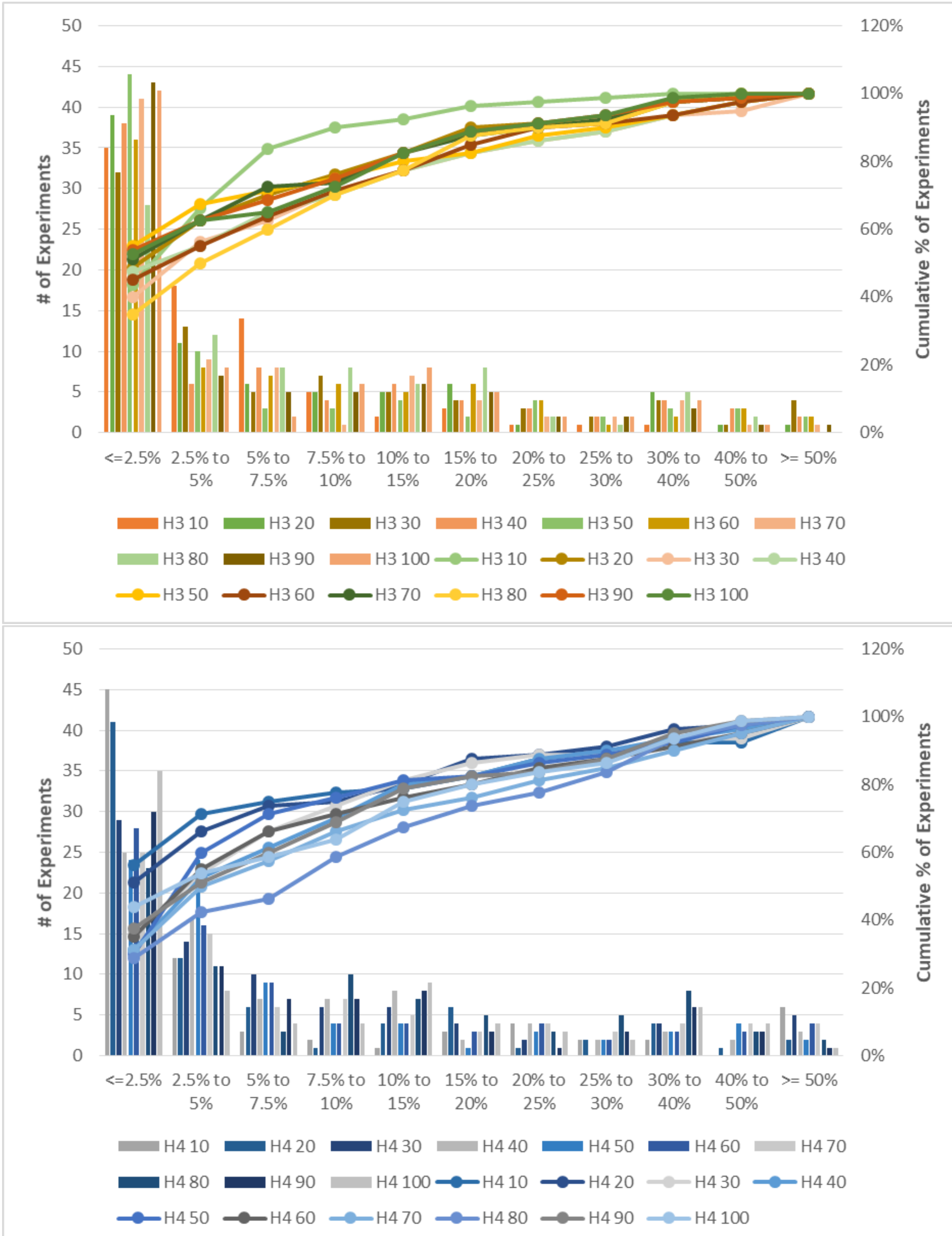

Figure 4.14 The performance analysis illustrates both of H3 and H4 show closer behavior within a warehouse with less initial inventory slots, when compared with optimal solution in MILP Model (4.3).

The inventory density and uniformity improvement sensitivity analysis exhibited above, as expected, across all factors, the presented two heuristics have reliable behavior and quick-solving methodology to improve the storage density performance without losing uniformity, in order to provide a picking circumstance that orders are easily fulfilled at a close location in a pick trip.

4.4 Joint Item-Correlation and Density Oriented (JICDO) Stocking Algorithm

Based on the observational visits to two IFWs, the product flow process and the associated data and decision flows are documented and presented in the above sections. Among them, the team collaboratively creates and exploits flow and decision models, sequentially updated in Onal et al. (2017) and the following two papers. In Section 4.3, the JOFDO stocking algorithm is established based on an independent SSLA strategy. With the updated stocking algorithm, the receiving bulk for a SKU figures out the weakest ranges with the original inventory allocation of that SKU and sends exploded lots to bins in the identified ranges.

Order picking efficiency can be improved by explosive storage policy and narrow-band group pick methodology. Since many pick combinations are possible when explosive storage is applied to perform a scattered lots distribution among the aisles, to narrow the searching band in group pick process is more value-added in this research. Storage density, as presented, provides a reliable approach to increase the pick-able probability in a location range with fixed number of bins, which is identified to be the searching band in pick phase. Another factor – item correlation is considered to influence inventory allocation, therefore the performance of picking performance.

In this section, a Joint Item-Correlation and Density Oriented stocking algorithm is proposed, as an advanced strategy compatible with the JOFDO model stated in Section 4.3. Involved with item correlation, the SLA problem is extended to be a Correlated-item SLA situation, which is described in Section 4.4.1. Followed by assumptions presented in Section 4.4.2 and the customer order analysis in Section 4.4.3, the storage density is enhanced with additional information from other SKUs, which would direct to sift out the available bins across the warehouse, in Section 4.4.4. Meanwhile, besides the storage uniformity in SSLA strategy, the author introduces proximity as a measurement of penalty from correlated SKU lots. Sections 4.4.5 and 4.4.6 propose the associated MILP models and a set of heuristics to solve the problem by Excel-VBA and OpenSolver. In Section 4.4.7, the valid environments are designed and results are presented to evaluate the heuristics approached.

4.4.1 Correlated-item Storage Location Assignment (CSLA) Strategy

As stated in Section 4.2, the existing correlated storage location assignment (CSLA) strategies consider the correlation among items to find more justified and economical solutions to enhance order picking performance. After Frazele and Sharp (1989) first provided the definition and calculation to measure SKU correlations., correlated slotting as a new storage policy besides the traditional dedicated, random or class-based strategies started to obtain adoption from researchers working in warehousing and operation management. CSLA is generally established to be a two-phase problem: (1) to cluster the correlated SKUs into groups and (2) to allocate locations to the clustered groups (Y. Zhang, 2016). For the sake of different research objectives, a diversity of models and algorithms are developed for the CSLA strategy. A CSLA algorithm combining clustering of SKU with sequencing of picking lists is presented by Liu (1999), in which a zero-one integer programming model is developed to optimally group items and customer orders. In Bindi, Manzini, Pareschi, and Regattieri (2009), a set of different storage allocation rules based on the similarity coefficients and clustering techniques are established and compared to demonstrate that the items often ordered together should be located near to each other.

Recently, the methodology of CSLA is further developed with involved other subjects. Ming-Huang Chiang, Lin, and Chen (2014) derives the modified class-base heuristic and the associated seed based heuristic with a proposed new measure, weighted support count (WSC) to facilitate efficient order picking from data mining studies. Wutthisirisart, Noble, and Alec Chang (2015) presents the adapted minimum delay algorithm with linear placement initially proposed in computer science for designing circuit boards. Different methods from other related subjects bring new approach to CSLA problem, which provides opportunities for researchers to expand or extend their theories.

On the basis of SSLA strategy, different from existing SLA models with correlations, a Correlated-item Storage Location Assignment stocking strategy is proposed in the following sections, in which the explosive storage policy is involved as the differentiators in IFWs.

4.4.2 Assumptions and Notations

Based on the defined processes in IFWs, stocking phase following with the receiving and explosion phase is dealing with pre-identified SKUs with full information from either item pool or inventory pool. To facilitate the modelling of location assignment model with correlation, the authors assume that:

(1) Incoming bulks are exploded into small lots; one lot is assigned to one location.

(2) Only exploded lots for single SKU are processed for bin assignments at any time t.

(3) Location assignment is defined by its inventory and correlations among SKUs.

(4) Lots with identified location assignments are grouped within the minimized bin range as a complete stocking list before assigned to stocker.

(5) Grouping process works on the pool including all location-identified but ungrouped lots for any SKU.

(6) The number of items on a stocking list is limited by list size.

(7) Stocker never wait at the conveyor. If no more available lots are pending to complete, a stocking list would be released to free stocker with items less than list size.

Compared with SSLA strategy, some restriction from assumptions in either location assignments or grouping processes are relaxed to generalize and adjust the setting of SLA problem to actual IFWs processes.

The notations in Table 4.8 are proposed to describe the CSLA model and algorithms in the following sections.

Table 4.8 Notations in CSLA strategy and JICDO Stocking Algorithm

Variable	Description
$i = 1\ to\ N$	Index of SKU
$b = 1\ to\ B$	Index of Bin location
O_i	The order frequency of item i
α_{ij}	The correlation of item j on item i
C_i	The average quantity of item i in a single pick stop

$\hat{I}_{i,b}$	The original inventory level of item i in bin b
$I_{i,b}$	The dynamic inventory level of item i in bin b
V_i	The volume of a unit of item i
B_b	The available volume of bin b
$\delta = -\Delta$ to Δ	Index of density calculation searching band
K_i	The number of exploded lots for item i
$k = 1\ to\ K_i$	Index of Exploded lots
$\hat{Z}_{i,b}$	The original fillable factor of item i from bin b
$Z_{i,b}$	The fillable factor of item i from bin b
$\widehat{D}_{i,b}$	The original correlated fillable factor of item i from bin b
$D_{i,b}$	The fillable correlated factor of item i from bin b
$W_{i,b}$	The storage density of item i at bin b
$\Pi_{i,U}$	Uniformity Reference – the summary of bin numbers if all inventory lots of item i are distributed uniformly in the warehouse
$M_{i,U}$	The number of total lots of item i if none of the new packages is assigned to a bin with target item
U_i	The penalty of storage uniformity from current inventory distribution for item i
$\Pi_{i,P}$	Uniformity Reference – the summary of bin numbers if all inventory lots of item i are distributed uniformly in the warehouse
$M_{i,P}$	The number of incoming and correlated lots of item i if none of the new packages is assigned to a bin with target item
P_i	The penalty of storage proximity from current inventory distribution for item i
$X_{i,b}$	A set of binary decision variables; denoting the decision if assign one exploded lot of item i into bin b

Some new notations are introduced to facilitate the description of CSLA model. Factors with subscript of P indicate the behavior of storage proximity, including $\Pi_{i,P}$ $M_{i,P}$ and P_i. As a significant differentiator with SSLA strategy, α_{ij} is derived to represent a single-direction correlation of item j on item i. In the following section, these parameters are described in detail.

4.4.3 Customer Order Analysis

With order splitting, customer who orders three different items may receive them separately since they can be picked up from at most three pick trips in IFW picking and consolidation processes. Thus, the general order correlation strategy, which analyses the order combination frequency, is not applicable in an IFW situation. Y. Zhang (2016) proposes a methodology to use picking frequency and correlation frequency since no order batching is executed in picking processes from the assumptions. It is an intuitive direction for this research, in which both correlation and order frequency are considered. In this research, the authors define item correlation as follow:

α_{ij} A static factor which is a two-decimal number between zero to one, indicating the likelihood an order for item j will arrive within a $\pm T$ hour window of any arriving order for the current SKU. The order j may or may not be placed by the same customer. Note it is a one-direction parameter.

To convey the behavior from customer orders, the similarity in a specific time slot is established to be the correlation among SKUs, instead of the order similarity used in general CSLA stocking algorithms. The order analysis is carried out in the following steps:

(1) Read historical sales data

(2) Calculate the order frequency of all the SKUs

(3) Split whole time horizon to be single hour time slot and count the number of orders placed in an associated time slot for all SKUs. Introduce $N_{i,t}$ to represent the counts of item i in time slot t.

(4) For an item i, lookup the time slots one by one. If item j has on less $N_{j,t}$ than $N_{i,t}$, set the number of correlated orders $N_{ij,t}$ as $N_{i,t}$. Otherwise, set it to be $N_{j,t}$.

(5) Sum all elements of $\{N_{ij,t}\}$ as the total number of correlated orders of item j to item i – $f_{ij,t}$; sum all elements of $\{N_{i,t}\}$ as the total number of original orders of item i – $f_{i,t}$.

(6) The item correlation α_{ij} is identified by the ratio of $f_{ij,t}$ and $f_{i,t}$.

(7) Redo Steps (4) to (6) for all $j \neq i$.

(8) Redo Steps (3) to (8) for all item i.

4.4.4 Correlated Storage Allocation

As proposed in Section 4.4.1, correlated stocking strategy is widely considered as an efficient policy to enhance an inventory environment, in order to reduce the picking effort. From the SSLA model, storage density and uniformity are two key controllable measurements, adjusting the current inventory allocation condition to a picking-efficiently structure.

Corresponding to the presented SSLA algorithm, a correlated weighted storage density is involved in the CSLA model as an advanced application of independent storage density. Meanwhile, storage uniformity is kept as the second measurement of the allocation behavior, with storage proximity established and derived from the item correlations presented above.

4.4.4.1 Correlated Weighted Storage Density. In IFW, the speed to fulfill customer orders is significantly indeed to be improved. To reduce routing time and walking distance, a picking list is assigned to a picker with multiple items from different customer order arriving at different times but stored at a narrow bin range in the warehouse. A well-structured stocking policy can increase the chance to generate such alternative group for efficient picking process.

In SSLA, the Independent Weighted Storage Density instead of simple inventory is used to represent the attractiveness of the bin to incoming replenishments. With picking range involved, an equivalent bin range with at least one fillable slot will be kicked out from the prior stocking location list. A similar approach works for multiple-SKU cases when the correlation among different items is considered into stocking decision. In CSLA, the weighted density is affected by current inventory lots of both target item and other correlated items. A bin slot with a large quantity of high correlated products is more likely to have the replenished lot since high correlation represents high probability that orders for both items come at the same time period and would be assigned to the same picking list which is picked in a single route within a given range.

For a target SKU i, the correlated weighted storage density is established as the following equation.

$$W_{i,b} \leq \sum_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot D_{i,b+\delta}\,;\ W_{i,b} \leq 1 \tag{4.13}$$

$$D_{i,b} = Z_{i,b} - \sum_{j} \hat{Z}_{j,b} \cdot \alpha_{ij} \tag{4.14}$$

$$Z_{i,b} \leq \frac{I_{i,b}}{C_i};\ Z_{i,b} \leq 1\ for\ target\ i\ or\ any\ item\ j \tag{4.15}$$

Equation (4.13) indicates a central amplifying effect from neighbor bins in a predefined range, by which the ranges without current inventory of target item is conspicuous in location assigning decision-making process. Equation (4.14) illustrates the correlated effect from other SKUs, to show the attractiveness from ranges with a high opportunity to generate a high-correlated-multi-item pick list in the following periods. Equation (4.15) bounds the density factor in which all bins with inventory able to fulfill an average customer order of item i are traded as the same priority. In CSLA and the JICDO stocking algorithm stated in the following sections, correlated storage density critically offers the approach to inventory allocation solutions, for the sake of efficient fulfillment to customer orders.

4.4.4.2 Storage Uniformity and Proximity. As indicated, the storage uniformity intuitively assists on decisions among alternatives providing equivalent improvement on inventory density. In SSLA model, uniformity is described by the difference between average location number of all existing inventory slots and middle bin of all aisles. The objective is to achieve high inventory density without losing uniformity. In CSLA algorithm, the formulation of uniformity follows the one in SSLA, expect for a justification on the share from objective value since correlated SKUs are taken into consideration equally.

Including the above two parameters, another state shows its significance on controlling storage assignments to better perform in picking phase, when item correlation is involved in SSLA problem, which is proximity. In multi-item storage process, the location to stock an item is determined by the inventory of target product and the correlated products. A searching band with high-correlated items attracts replenishment lots, which is selected as candidate since bins in this range state low density and high rank at current moment. Proximity is applied when two or more candidates establish equivalent situation, where, for example, two bins with same high-correlated items B for item A locate at range 10-20 and 20-30, correspondingly. Since both of two ranges have the same attractiveness to item A, the location assignment would be made to maximize the effect on both of the two ranges. Only one exploded lot pending in list will be assigned to bin 20, while two would fulfill different bins (bin 15 and 25) in the two ranges within the same stocking route.

4.4.5 Mixed Integer Linear Programming (MILP) Model

In multi-item storage problem, item correlation is considered into the justification of storage density, by which single SKU is exploded and processed at any moment t. Besides correlation weighted inventory density, storage location assignment problem is reduced to be a priority-ranking-and-grouping puzzle. Another request comes at the moment when two or more locations respond with same states, to make decision among these candidates. To deal with equivalent alternatives, uniformity and proximity are presented to be the measurements for different location assignments.

Table 4.9 Notations in SSLA strategy and JOFDO Stocking Algorithm

C	The average quantity of target SKU in a single pick stop
O	The order frequency of target SKU
α_j	The correlation of item j on target SKU
$\hat{I}_b$	The original inventory level of target SKU in bin b

I_b	The dynamic inventory level of target SKU in bin b
V	The volume of a unit of target SKU
K	The number of exploded lots for target SKU
$k = 1\ to\ K$	Index of Exploded lots
E	The quantity of items in one exploded lot
$\hat{Z}_b$	The original fillable factor of target SKU from bin b
Z_b	The fillable factor of target SKU from bin b
$\widehat{D}_b$	The original correlated fillable factor of target SKU from bin b
D_b	The fillable correlated factor of target SKU from bin b
W_b	The storage density of target SKU at bin b
Π_U	Uniformity Reference – the summary of bin numbers if all inventory lots of target SKU are distributed uniformly in the warehouse
M_U	The number of total lots of target SKU if none of the new packages is assigned to a bin with target item
U	The penalty of storage uniformity from current inventory distribution for target SKU
Π_P	Proximity Reference – the summary of bin numbers if all inventory lots of target SKU are distributed to correlated lots in the warehouse
M_P	The number of incoming and correlated lots of target SKU if none of the new packages is assigned to a bin with target item
P	The penalty of storage proximity from current inventory distribution for target SKU
X_b	A set of binary decision variables; denoting the decision if assign one exploded lot of target SKU into bin b

Based on the single SKU process, following the reduction methodology proposed with SSLA model, a SKU preselection is executed before location assignment process. Thus, notations can be simplified as shown in Table 4.9.

In respect to the performance of inventory allocation, the authors state three key measurements instead of two in SSLA algorithm. Correlated weighted storage density as the main factor is predominate in objective value. However, both uniformity and proximity are defined as penalty to the system, which will be subtracted from a calculated density value. As a group of reliable location assignments stocked, the optimal situation is to maximize the improvement on storage density, as well as minimize the penalty values to close to zero.

As presented, CSLA strategy has a two-phase solution, where the 1st phase generates location assignments to exploded packages and 2nd phase groups pending lots with predefined location ID into stocking list. The grouping and stocker arrangement phase is executed by a close-to-next-free-stocker algorithm, presented in SSLA heuristic. Thereupon, a mixed integer linear programming model to identify the 1st phase solution in CSLA is defined as below.

Max:

$$\sum_b W_b - \frac{U}{M_U} \cdot \frac{O_i}{(O_i + \sum_j O_j)} - \frac{P}{M_P} \cdot \frac{\sum_j \alpha_j}{J+1} \cdot \sum_j O_j \tag{4.16}$$

s.t.

$$X_b \ as \ Binary \tag{4.17}$$

$$W_b \leq 1 \tag{4.18}$$

$$W_b \leq \sum_{\delta=-\Delta}^{\Delta} \left(1 - \left|\frac{\delta}{\Delta}\right|\right) \cdot D_{b+\delta} \tag{4.19}$$

$$Z_b \leq \frac{I_b}{C} \tag{4.20}$$

$$Z_b \leq 1 \tag{4.21}$$

$$I_b = X_b \cdot E + \hat{I}_b \tag{4.22}$$

$$\hat{Z}_{j,b} \leq \frac{\hat{I}_{j,b}}{C} \text{ for } j \neq target\ sku \tag{4.23}$$

$$\hat{Z}_{j,b} \leq 1 \text{ for } j \neq target\ sku \tag{4.24}$$

$$D_b = Z_b - \sum_j \hat{Z}_{j,b} \cdot \alpha_j \tag{4.25}$$

$$\sum_b X_b = K \tag{4.26}$$

$$U \geq \sum_b \hat{Z}_b \cdot b - \Pi_\mathrm{U} \tag{4.27}$$

$$U \geq \Pi_\mathrm{U} - \sum_b \hat{Z}_b \cdot b \tag{4.28}$$

$$\Pi_\mathrm{U} = \frac{1}{2} \cdot (1 + \mathrm{B}) \cdot M_U \tag{4.29}$$

$$M_U = \sum_b \left[\frac{\hat{Z}_b}{abs(\hat{Z}_b)} \mid if\ \hat{Z}_b > 0 \right] + K \tag{4.30}$$

$$P = -\sum_b \left[\widehat{D}_b \cdot b \mid if\ \widehat{D}_b < 0 \right] \tag{4.31}$$

$$M_P = B + \sum_b \left[\frac{\widehat{D}_b}{abs(\widehat{D}_b)} \mid if\ \widehat{D}_b < 0 \right] + K \tag{4.32}$$

As established, the multi-item storage location assignment MILP Model (4.4) Objective Function (4.16) includes three components. First of all is the total correlation weighted storage density, which increases the number of pick-able slots therefore improves the picking efficiency. Second is uniformity, to be minimized to represent a uniformly distributed storages structure. The last part is proximity, similar to uniformity, to be reduced to state the closeness with correlated SKU stocked. Constraint (4.17) indicates that the decision variables are binary and non-negative. Constraint (4.26) ensures that all the exploded packages are assigned to a defined location. Constraint (4.22) represents the inventory flow balance after replenishment stocked. Constraint (4.25) establishes correlation effect on current inventory where a bin carrying correlated items but no target SKU would respond with a negative value showing the attractiveness from this location. Constraint sets (4.18) and (4.19) give the calculation and boundary to calculate density, by which bins are competing with involving the neighborhood effect in priority storage list. Constraint sets (4.20), (4.21), (4.23) and (4.24) ensure high inventory bin is equivalent with low inventory bin if both of them are fillable in order analysis. Constraint sets (4.27), (4.28), (4.29) and (4.30) provide the evaluation from uniformity of slotting, which is approaching zero while replenishment packages are stocking. Same as uniformity, Constraint sets (4.31) and (4.32) alleviate the penalty from correlated but unpaired inventory as proximity.

The proposed formulation takes a minute to find an optimal solution within a small-size warehouse. However, facing to a realistic situation within a million-square-feet warehouse, the complexity of the CSLA problem is amplified to be a large-scale system with thousands of data transaction in a second. It is hardly impossible to solve the problem with existing optimizer, before it run out of time or memory. For each SKU, this model has B binary variables, $2B + 2$ other variables and $7 + 7B + (N - 1)B$ constraints. A simple example is given that the total number of variables and constraints for a small size of the problem (2000 bins, 100 SKUs) is 6,002 variables and 212,007 constraints for each SKU. Thus, to develop an intuitive approach to solve this problem is indispensable within an acceptable tolerance compared with optimal solutions obtained by OpenSolver before involving the model into simulator.

4.4.6 JICDO Stocking Algorithm

In this section, item correlation is proposed to target the best solution from all bins, with a justification on storage density. The reduced CSLA model in above section performs an intuitively optimal but slow solution. With the consideration of SKU selection and grouping phases, the Joint Item-Correlation and Density Oriented (JICDO) stocking algorithm is presented as an extension of CSLA strategy, demonstrating the inventory allocation solution in a dynamic warehousing environment. Corresponding to JOFDO stocking policy, this algorithm consists of two sub-problem: (i) SKU priority and location assignment solution from CSLA Model (4.4) and (ii) pending lots group assignment and stocker arrangement.

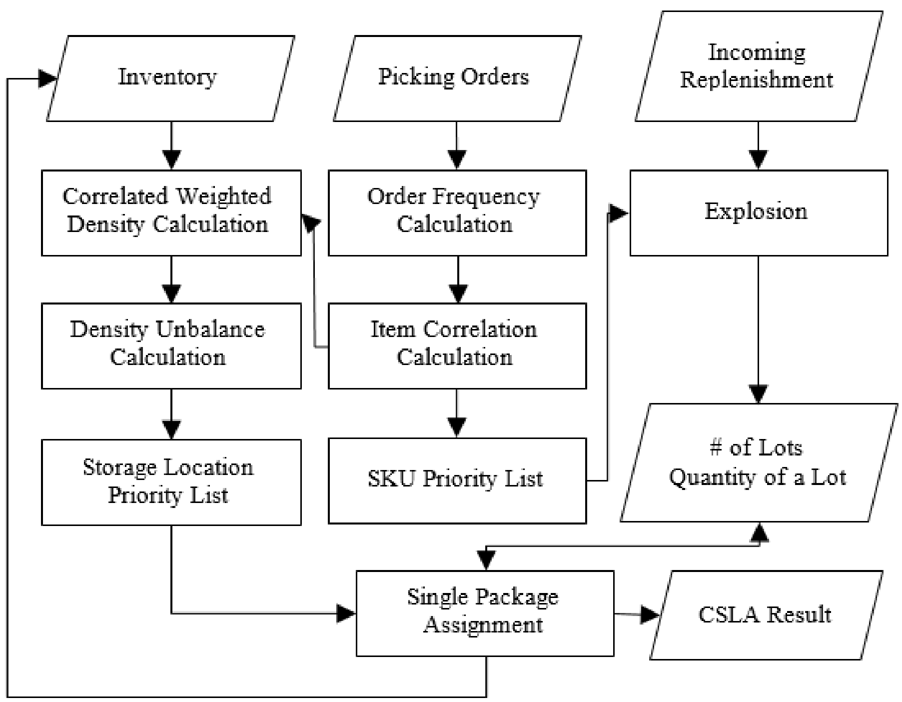

Figure 4.15 The work flow in SKU selection and location assignment phase illustrates decisions and information transaction in CSLA.

The 1st phase has two sub-steps, determining the next target SKU and allocating all replenishments of that SKU to certain locations. Figure 4.15 shows the algorithm flow.

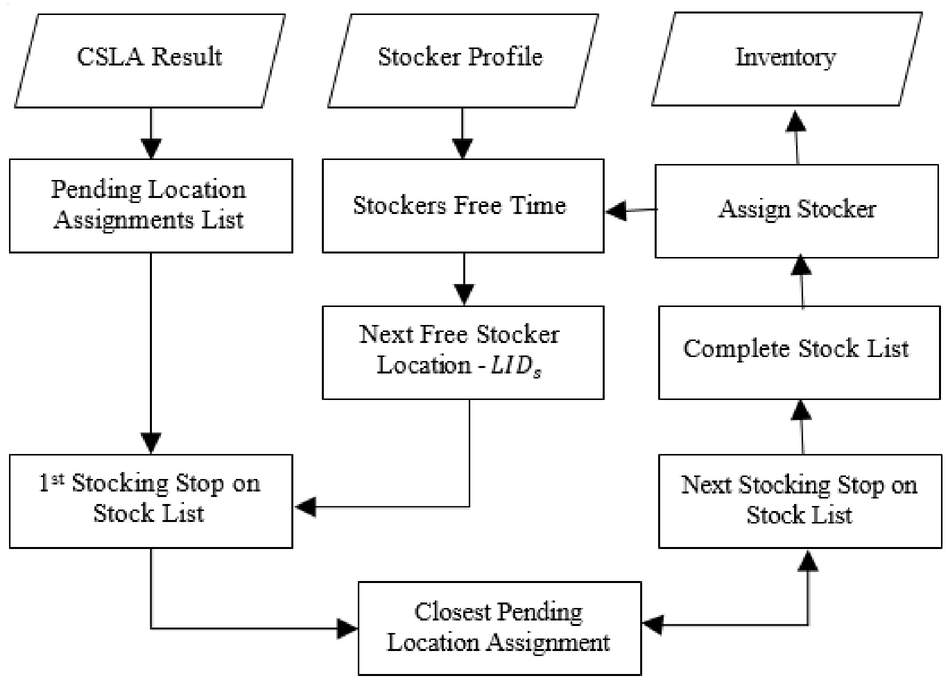

Figure 4.16 The work flow in grouping and stocker assignment phase illustrates decisions and information transaction in CSLA.

Phase I: SKU Priority and Correlated-Item Storage Location Assignment

1. At time t, read Order Frequency Table from historical customer order database.
2. Read Order List $\{O\}$ for current pending orders.
3. Create the Receiving Replenishment List for current bulks not stored yet.
4. Do explosion and add exploded packages to Waiting List for stocking.
5. Call Table α_j –Item Correlation Table.
6. Update W_b Table – Correlated Weighted Storage Density Table with correlated SKUs involved.

7. If current inventory for SKU i is less than orders from pending customer order list $\{O\}$, and this SKU is received and exploded, assign the SKU as the target SKU.

8. Otherwise, depend on the order frequency table, give a priority rank for each SKU, which has replenishment at current time t. Select the SKU with highest order frequency as target SKU; depending on the arrival time of receiving bulks, the earliest arrival SKU among equivalent alternatives is first to be stocked.

9. Call the CSLA Model (4.4) and solve for location assignment solutions.

10. Assign all exploded lots of SKU i to the corresponding bin solutions from CSLA strategy.

11. Update the Inventory Table and Correlated Weighted Storage Density Table.

12. Repeat step 1 to 11 until time shift to next period (t+1) or no more pending packages are waiting for stocking.

The 2nd phase is also identified with two sequential decisions – which stocker to be next assigned worker and which pending lots with predefined locations to be completed by this stocker. The associated algorithm flow is presented in Figure 4.16.

Phase II: Grouping and Stocker Arrangements

13. Read People Table for the free time of all stockers.

14. Select the earliest free stocker s and record the last location of stocker s. $l = 0$.

15. List and update all location-defined but not stocked lots with the associated SKU and Location ID to be the pending list at the moment t.

16. $l = l + 1$. Assign the closest pending location assignments to the last location of stocker s as the l^{th} item on the stocking list. Record this location assignment as the last location.

17. Redo step 15 to 16 until $l \geq S_{max}$, or all pending location assignments are completed.

18. Exit until time shift to next period (t+1) or no free stocker could be found in current time shift.

Considering the data requirements to track the exploded inventory, traditional discrete event simulation models could not be used. Instead, a data driven simulation model was built on the MS-Access/VBA platform (Onal et al., 2017). JICDO stocking algorithm with the reduced CSLA model built in solves the problem as a batch by using existing optimization software, which is not importable to MS-Access/VBA. Heuristics are established in following section as compatible alternatives for simulation analysis.

4.4.7 Heuristics

In order to develop the reliable heuristic to replace the CSLA Model (4.4) in JICDO stocking algorithm, the authors identify two characteristics as the potential breakthrough points to approach to an approximate solution.

The first approach is a benchmark method from JOFDO algorithm, which is the uniform seed bin heuristic.

Heuristic #1 (H1): Uniform seed bin and band searching location assignment.

- Read the current inventory of target SKU – I_b .
- Call the correlated weighted density table – W_b.
- Calculate UBD_b based on the proposed equation above, $UBD_b = AVG\left(\sum_{a=1}^{b} W_a\right) - AVG(\sum_{a=b}^{B} W_a)$
- Set uniform seed bin $b' = \min\left(abs\left(\frac{1}{2} \cdot (1+B) \cdot \left(\sum_b \hat{Z}_b + 1\right) - \sum_b \hat{Z}_b \cdot b\right), B\right)$
- Depending on the corresponding $UBD_{b'}$, determine the direction for band searching.
- If $UBD_{b'} \geq 0\ and\ b' \neq B$, from center bin b', towards to bin locations with location number larger than b', search for the closest range of $2\Delta - 1$ bins with zero or small inventory which shows unfillable to an average pick. Target range size is shrinking by reduce Δ to be $\Delta - 1$

along with a set of $\beta \cdot$ M searching iterations. β is the restricted weight to avoid the number of failure iterations, which is default to be 1.

- If $UBD_{b'} \geq 0$ and $b' = B, or$ $UBD_{b'} < 0$, perform the same procedure as above, expect the moving direction change towards to bin with location number less than b'.
- Select the center bin of target range as the location assignment.
- Redo the steps above until no more pending exploded lots or current time shift is finished.

The authors have executed a few tests to evaluate the results from the above heuristic. Similar to the cutoff heuristic in Section 4.3, the probability that H1 is assigning the same location to multiple packages increases along with the number of exploded packages, with available lots locating close to the result location. Furthermore, the solution has a strong bias to uniformity directed allocation, which would loss the beneficial storage density and reduction on proximity penalty if correlated items have plenty of inventory in the warehouse. The results not indicative are not included in this document.

Before establishing the following two heuristics, the storage location priority list is introduced as the second breakthrough, to represent the potency to bring an improvement by enriching the inventory of specific item in an identified bin. This priority is proposed to be a ranking score, which includes two different components -- correlation-weighted inventory density score as integer and inventory structural unbalance score as decimals. An example is a slot with density score of 100 out of B (which is the total bin number and the highest density score, e.g., 1000) and unbalance score of 234 out of B would have ranking score of 100 plus 234/B, which is 100.234. A lower score conveys to be a higher priority on the storage location list, representing more attractiveness to target SKU.

Heuristic #2 (H2): Density and unbalance difference priority oriented heuristic

- Read the current inventory of target SKU – I_b .
- Call the correlated weighted density table – W_b.
- Calculate UBD_b based on the proposed equation above, $UBD_b = AVG\left(\sum_{a=1}^{b} W_a\right) - AVG(\sum_{a=b}^{B} W_a)$
- Rank W_b and $abs(UBD_b)$ by ascending order, with the lowest W_b or $abs(UBD_b)$ assigned rank of 1.
- Calculate and record the storage location priority as $Rank(W_b) + Rank\left(abs(UBD_b)\right)/(\gamma B)$, where $\gamma = 1$ as default.
- $b_k = \left\{b \mid \min_{b}\left(Rank(W_b) + Rank\left(abs(UBD_b)\right)/(\gamma B)\right)\right\}$
- Redo the steps above until no more pending exploded lots or current time shift is finished.

From the description in H2, the sequential location assignment is solved by assigning the exploded lot to the bin with highest rank priority. Since it is a determined solution, the processing time for one iteration is limited within seconds. However, a distinct deficiency is the unavoidability of duplicate assignments because of the direct solution method.

An advanced heuristic is built on the basis of H2 to solve the duplicate assignment problem. The approach considered is to use the bin with highest priority as seed location, and perform a band-searching for the bin assignment with largest improvement among delta bins on one side of seed bin. Searching direction depends on the unbalance value of the seed location. Therefore, the 3rd heuristic is proposed as below.

Heuristic #3 (H3): W_b and UBD_b priority oriented band searching heuristic

- Read the current inventory of target SKU – I_b .
- Call the correlated weighted density table – W_b.

- Calculate UBD_b based on the proposed equation above, $UBD_b = AVG\left(\sum_{a=1}^{b} W_a\right) - AVG(\sum_{a=b}^{B} W_a)$

- Rank W_b and $abs(UBD_b)$ by ascending order, with the lowest W_b or $abs(UBD_b)$ assigned rank of 1.

- Calculate and record the storage location priority as $Rank(W_b) + Rank\left(abs(UBD_b)\right)/(\gamma B)$, where $\gamma = 1$ as default.

- $b' = \left\{b | \min_{\mathrm{b}}\left(Rank(W_b) + Rank\left(abs(UBD_b)\right)/(\gamma B)\right)\right\}$

- Depending on the corresponding $UBD_{b'}$, determine the direction for band searching.

- If $UBD_{b'} \geq 0, b' \neq B\ or\ UBD_{b'} < 0, b' = 1$, from b', towards to bin locations with location number larger than b', set $b_t = b' + \delta$ and $\delta = \delta + 1$, then update the corresponding bin inventory by adding a single exploded package to current inventory; recalculate all parameters and states, record current objective value as OF_δ and subtract the added inventory from current bin b_t.

- If $UBD_{b'} \geq 0, b' = B, or\ UBD_{b'} < 0, b' \neq 1$, perform the same procedure as above, expect the direction change from b'towards to bin with location number less than b' and set $b_t = b' + \delta$ and $\delta = \delta - 1$ instead.

- $b_k = \left\{b' + \delta | \max_{\delta}(OF_\delta)\right\}$

- Redo the steps above until no more pending exploded lots or current time shift is finished.

H3, compared with H2, provides a reliable solution approach with the band searching methodology involved. Instead of the instant decision, iterative trials are executed to select the most appropriate bin assignment. Contrast to the improvement, time consumption is a controversial aspect, which should be illustrated from a pilot test compared with optimal CSLA strategy.

4.4.8 Experiments and Results

JICDO stocking algorithm is composed of two phases, in which CSLA predominately determines the location assignments as the input of next phase – assignment grouping and stocker arrangement. Two compatible heuristics are presented in above. To evaluate and test the behavior along with the optimal solutions in CSLA Model (4.4).

Key parameters used in the experimental research are shown in Table 4.10.

Table 4.10 Key Parameters for Valid Experiment on the JICDO with Heuristic

$N = 1\ SKU$	$B = 1000\ Bins$	$\hat{B}_b \leq 3000\ in^3$
$V = 10 \cdot 5 \cdot 2 = 100\ in^3$	$C = 10$	$E = 25$
$\Delta = 5, 10, 20$	$Correlation\ type - 3\ cases$	$\mathrm{K} = 5, 10, 15$
$Inventory\ type - 3\ cases$ and $inventory\ of\ average\ lot = 50$		

Results from a total number of 243 experiments with three dimensional diversity of parameters setting are presented in Figure 4.17. Within 2% tolerance of optimal solution, over 90% of the tested cases proposed an approximate result. As expected, H4 provides a closer solution with a minute time window, while H3 states a quick solution with a little less accuracy but solving the problem in a few seconds. As evaluation results, both of these two heuristics are applicable, compatible and reliable solution methodology to be built in the simulator combining with the grouping phases and other processes in the warehouse in next section.

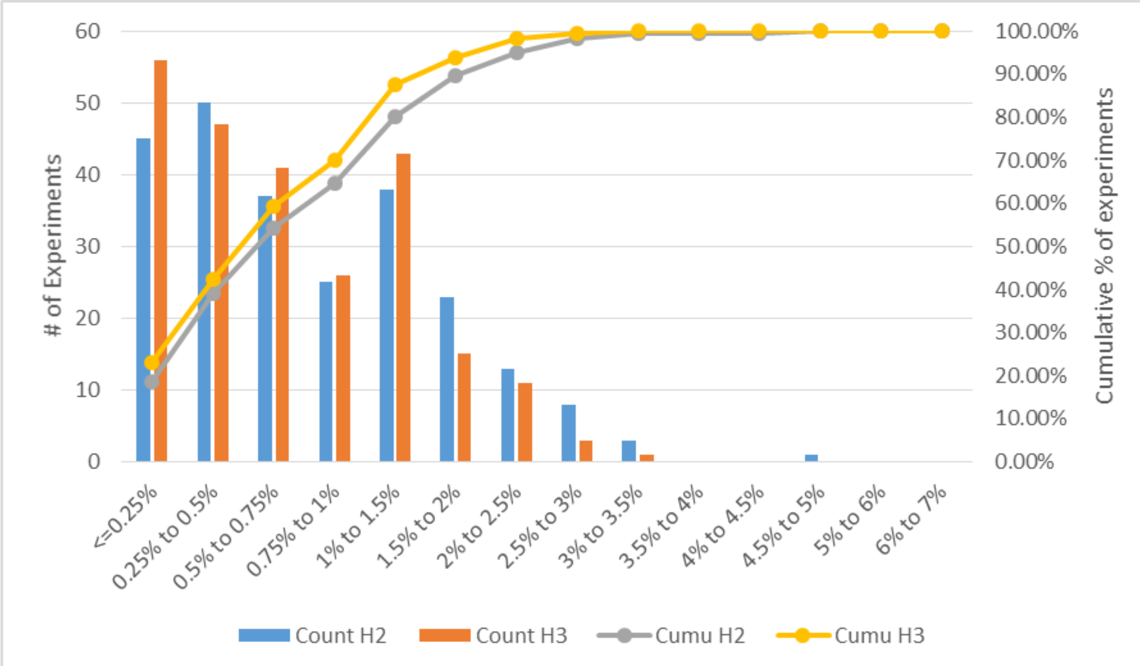

Figure 4.17 The performance analysis illustrates both of H2 and H3 provides a reliable quick solution in a warehouse with less initial inventory slots, when compared with optimal solution in CSLA Model (4.4).

4.5 Simulation Experiment and Evaluation

To evaluate the performance of the heuristic built-in JICDO algorithm, the authors applied the heuristic into a dynamic processing simulator-based warehouse. With the original random stocking algorithm, this simulator has been used to prove the enhancement obtained from the key differentiator – explosive storage. As a benchmark, the authors establish a set of pilot experiments with new JICDO algorithm, to illustrate the influence on the performance of the average order fulfillment time.

4.5.1 Simulation Design

A simulation model is used to analyze the linear fulfillment performance behavior of an IFW. Considering the big data required to track the exploded inventory and order information, a data driven simulation model is built with the MS-Access/VBA platform (Onal et al., 2017). Given the processing time limits, parameters to satisfy a feasible model are established in Table 4.11.

As proposed, the simulator is built with 3240 bins and working for five days, dealing with around 120 to 140 receiving bulks and 12,000 customer orders since it is designed to be nine-day task in the default setting.

Table 4.11 Key Parameters for the Experimental IFW with the JICDO Algorithm

$N = 400\ SKUs$	$B = 3240\ Bins$	$\hat{B}_b \leq 3000\ in^3$
$Z = 3\ (Equal\ size\ zone)$	$S_z = 6/zone$	$P_z = 6/zone$
$T = 5\ days$	$T_S = 8\ Hours$	$T_P = 8\ Hours$
$\sum_t R_t = 220\ for\ 9\ days$	$\sum_t O_t = 22{,}000\ for\ 9\ days$	$PL = 15/List$
$\Delta = 20, 30, 40$	$\chi = 0.5, 0.6, 0.7, 0.8$	

S_z and P_z representing the number of stokers and pickers are constant for everyday assignment, but which worker would works as what role is identified at the beginning of each day. PL as picking list size, demonstrates convexity with sensitivity analysis of a range of values from 10 to 20. Result shows that opportunity to add one or two more stops on a quick pick turn is considered and value-added, in contrast to the cumulative delay form a long pick cycle and waiting time, when PL is limited to around 13 under the current parametrical setting. Here, the authors use PL=15 instead of 13, to capture the optimism of picking list size and be regarded as a benchmark of searching band in stocking process, where location allocation decisions are worked out by this factor.

Another significant factor is explosion ratio χ. Given that a larger explosion ratio indicates more location assignments decisions based on the same receiving bulks. In this section, four explosion ratios are defined to demonstrate the diversity of warehouse operation structure, among which, 0.8 as the optimal explosion ratio within the established simulator in (Onal et al., 2017) .

4.5.2 Simulator with Heuristic Built-in

In simulator preparation, distinguished with the randomized storage policy, stocking algorithm is updated within the VBA platform. Meanwhile, new parameters are prepared on the database and data relation levels.

The proposed JICDO stocking algorithm with heuristic H4, which is better performed in the pilot experiment tests, is described into the following three phases, as a reference to update the simulator. Note that a minor modification made to reduce the data transactions from the recalculation steps is to use bin inventory check instead of optimizing the objective value with iteratively update inventory table, since that a bin with less inventory of target SKU than the average customer order quantity shows 50% probability of non-fillable in picking process. Thus, the criteria of min pack of target SKU, which is generally a multiplier of average order quantity, is introduced as a filter instead of repetitive calculations with in the warehouse.

Phase 0: Dataset Preparation

a. Add $\boldsymbol{C_i}$ column in $\boldsymbol{Table - Item}$ where $\boldsymbol{C_i}$ represents the average customer order quantity of item $\boldsymbol{i}$ in a time slot; add $\boldsymbol{O_i}$ column in $\boldsymbol{Table - Item}$ where $\boldsymbol{O_i}$ represents the order frequency of item $\boldsymbol{i}$ (=# of orders for item $\boldsymbol{i}$ / total # of customer orders);

b. Create $\boldsymbol{Table - \alpha_{ij}}$ represents one direction correlation for item $\boldsymbol{i}$ of item $\boldsymbol{j}$ from sales dataset, where 1st column represents solution SKU, 2nd column provides the correlated SKU and last column is corresponding correlation value. Or add columns $\boldsymbol{\alpha_{l,j}^{p}}$ ***and*** $\boldsymbol{\alpha_{i,l}}$ represent the $\boldsymbol{l^{th}}$ highest correlated SKU of item $\boldsymbol{i}$ and the corresponding correlation, where $1 \le l \le L,\ L = 5$;

c. Add $\boldsymbol{Z_{i,b}}$ column in ***Table – Inventory*** where $Z_{i,b} = \min\left\{\frac{I_{i,b}}{C_i}, 1\right\}$; Add $\boldsymbol{D_{i,b}}$ column in ***Table – Inventory*** where $D_{i,b} = Z_{i,b} - \sum_j \hat{Z}_{j,b} \cdot \alpha_{i,j}\ for\ all\ j \ne i\ and\ j\ is\ correlated\ to\ i$;

d. Calculate $\boldsymbol{Array - R_{ab}}$ represents the coefficient value for bin b from center bin a, where $R_{ab} = 1 - \frac{\max\{|b-a|, \Delta\}}{\Delta}\ for\ all\ b$;

Phase 1: SKU selection and priority ranking oriented bin assignment

Phase 1.1: SKU Priority

i. Among all receiving supplies, select $\{RID\}_t = \{RID \mid ArrTime \le current\ time\ and\ ADay \le current\ day\}$; $\boldsymbol{Table - Receiving}$

ii. Within the selections, calculate $inventory\,insufficiency = \#\,of\ inventory\ slots - average\ orders$ for each SKU; $\boldsymbol{Table - Inventory\ \ and\ Table - Item}$ $(\boldsymbol{O_i} \cdot \#\boldsymbol{of\ total\ sales\ orders/9})$

iii. Select $rid_t = argmin(inventory\ insufficiency\ of\ rid \mid rid \in \{RID\}_t\ and\ inventory\ insufficiency\ of\ rid < 0)$; if $\{rid \mid rid \in \{RID\}_t\ and\ inventory\ insufficiency\ of\ rid < 0\} = \emptyset$, then select $rid_t = argmin\{ArrTime\ and\ ADay\ of\ rid\}$, where $rid \in \{RID\}_t$;

iv. $i = SKU_t = \{SKU \mid RID = rid_t\}$

v. Create $\boldsymbol{Table - W_{i,b}}$ where $W_{i,b} = \min\left\{\sum_a R_{ab} \cdot D_{i,a}, 1\right\}$;

vi. Add column $\boldsymbol{UBD_{i,b}}$ in $\boldsymbol{Table - W_{i,b}}$; calculate unbalance value $UBD_{i,b} = average\left(\sum_{a=1}^{b} W_{i,a}\right) - average\left(\sum_{a=b}^{B} W_{i,a}\right)$;

vii. Add column $\boldsymbol{R_{w,i,b}}$ in $\boldsymbol{Table - W_{i,b}}$ where $\boldsymbol{R_{w,i,b}}$ represents the priority of bin b when ranking all bins by ascending order of $\boldsymbol{W_{i,b}}$ – a bin with lower density has higher probability to be selected; if there are tie-up bins, give same rank value to all of them.

viii. Add column $\boldsymbol{R_{u,i,b}}$ in $\boldsymbol{Table - W_{i,b}}$ where $\boldsymbol{R_{u,i,b}}$ represents the priority of bin b when ranking all bins by ascending order of $\boldsymbol{abs(UBD_{i,b})}$ – a bin with more balanced neighborhood has higher probability to be selected; if there are tie-up bins, give same rank value to all of them. The columns in step k to m can be reused for all SKUs since the algorithm is processing single SKU at a moment;

ix. Add column $\boldsymbol{R_{p,i,b}}$ in $\boldsymbol{Table - W_{i,b}}$ where $\boldsymbol{R_{p,i,b} = R_{w,b} + R_{u,b}/B}$ representing rank priority of each bin for item $\boldsymbol{i}$ – the smaller, the higher priority and attractiveness**.**

x. Calculate

$$M_{i,U} = \sum_{b}\left[\frac{\hat{Z}_{i,b}}{abs(\hat{Z}_{i,b})} \mid if\ \hat{Z}_{i,b} > 0\right] + K_i$$

where K_i represents the number of exploded slots of incoming replenishment for item $\boldsymbol{i}$; Here stocking assignment is done one by one, the $K_i = 1$;

xi. Calculate

$$M_{i,P} = \sum_{b}\left[-\frac{\widehat{D}_{i,b}}{abs(\widehat{D}_{i,b})} \mid if\ \widehat{D}_{i,b} < 0\right] + K_i$$

where K_i represents the number of exploded slots of incoming replenishment for item $\boldsymbol{i}$; Here stocking assignment is done one by one, the $K_i = 1$;

xii. Calculate $\Pi_{\mathrm{i,U}} = \frac{1}{2} \cdot (1 + \mathrm{B}) \cdot M_{i,U}$;

xiii. Calculate

$$U_i = abs\left(\sum_{b} \hat{Z}_{i,b} \cdot b - \Pi_{\mathrm{i,U}}\right)$$

$$and\ P_i = abs\left(\sum_{b}\left[\ \hat{Z}_{i,b} \mid if\ \hat{Z}_{i,b} < 0\right]\right)$$

xiv. Calculate $OF = \sum_b W_{i,b} - \frac{U_i}{M_{i,U}} \cdot \frac{O_i}{(O_i + \sum_j O_j)} - \frac{P_i}{M_{i,P}} \cdot \frac{\sum_j \alpha_{i,j}}{J+1} \cdot \sum_j O_j$;
J is the number of correlated SKUs of item i;

Phase 1.2: Bin Assignment

a. After explosion, set k=1;

b. Let seed bin $\boldsymbol{b'} = \boldsymbol{argmin}\left(R_{p,i,b}\right) \boldsymbol{for\ all\ b}$. If $\boldsymbol{UBD_{i,b'}} \geq \mathbf{0},$ go to step **c**, else, go to step **g**. $\boldsymbol{Table - W_{i,b}}$

c. For seed bin $\boldsymbol{b'}$, if $\boldsymbol{UBD_{i,b'}} \geq \mathbf{0}$, $\delta = 0$, Do while $\boldsymbol{\delta} \leq \mathbf{min}\{\boldsymbol{\Delta}, \boldsymbol{B} - \boldsymbol{b'} + \mathbf{1}\}$, If $\mathbf{I}_{\boldsymbol{i,b'+\delta}} > \boldsymbol{Minpack\ of\ item\ i},$ then $\boldsymbol{\delta} = \boldsymbol{\delta} + \mathbf{1}$, Else, Exit Do; $\boldsymbol{Table - Inventory}$

d. Set current location assignment as $SLA_{i,k} = \hat{b}$ where $\hat{b} = b' + \delta$; $\boldsymbol{Table - Inventory}$

e. If $V_{\hat{b}} < V_i \cdot E_{i,k}\ and\ \hat{b} \neq B$, set $b' = \hat{b} + 1$ and redo step **c to d** until $V_{\hat{b}} \geq V_i \cdot E_{i,k}$; else if $V_{\hat{b}} < V_i \cdot E_{i,k}\ and\ \hat{b} = B$, then go to step **g**; else, go to step **k**. $\boldsymbol{Table - Inventory\ and\ Table - Item}$

f. If none, set $\boldsymbol{R_{p,i,b'}} = \boldsymbol{R_{p,i,b'}} + \boldsymbol{B}$ and return to step **b**. $\boldsymbol{Table - W_{i,b}}$

g. Corresponding to step **c**, if $\boldsymbol{UBD_{i,b'}} < \mathbf{0},$ $\delta = 0$, Do while $\boldsymbol{\delta} \leq \mathbf{max}\{\boldsymbol{\Delta}, \boldsymbol{b'} - \mathbf{1}\}$, If $\mathbf{I}_{\boldsymbol{i,b'-\delta}} > \boldsymbol{Minpack\ of\ item\ i},$ then $\boldsymbol{\delta} = \boldsymbol{\delta} + \mathbf{1}$, Else, Exit Do;

h. Set current location assignment as $SLA_{i,k} = \hat{b}$ where $\boldsymbol{\hat{b}} = \boldsymbol{b'} - \boldsymbol{\delta}$;

i. If $V_{\hat{b}} < V_i \cdot E_{i,k}\ and\ \hat{b} \neq 1$, set $b' = \hat{b} - 1$ and redo step **g to h** until $V_{\hat{b}} \geq V_i \cdot E_{i,k}$; else if $V_{\hat{b}} < V_i \cdot E_{i,k}\ and\ \hat{b} = 1$, then go to step **c**; else, go to step **k**.

j. If none, set $\boldsymbol{R_{p,i,b'}} = \boldsymbol{R_{p,i,b'}} + \boldsymbol{B}$ and return to step **b**.

k. Update inventory of item $\boldsymbol{i}$ by adding quantity of $\boldsymbol{k^{th}}$ exploded package to location $\hat{b}$; record as $RID = rid_t, LocID = \hat{b}$. $\boldsymbol{Table - SList2}$

l. Clear all $\hat{b}, b', \delta = 0$; k=k+1;

m. Recalculate $Z_{i,b}\ D_{i,b}\ W_{i,b}\ M_{i,U}\ M_{i,P}\ \Pi_{\mathrm{i,U}}\ U_i\ P_i\ OF\ and\ UBD_{i,b}$;

n. Redo step **b** to **m** until $k = K_i$, and then move to **Step 1.a** for next SKU.

o. Exit when no incoming packages or time shift is end.

Phase 2: Group and Stocker Assignment

a. Select $eid_t = argmin\{Free\ time\}$ as next available stocker; $\boldsymbol{Table - People}$

b. List $S_{max}(= the\ maximum\ number\ of\ a\ stocking\ list, e.g. 20)$ bulks, where, $RID = argmin\{abs(LID - current\ location\ of\ eid_t)\}$ for 1st package, and $RID_n = argmin\{abs(LID - LID_{n-1})\}$ for the rest; Record $SID = SID + 1$ for the selected $eid_t\ and\ listed\ S_{max}\ Lots$. $\boldsymbol{Table - SList1\ and\ Table - SList2}$

c. Exit when no pending packages or time shift is end.

4.5.3 Performance Analysis Results

Figures 4.18 and 4.19 shows the simulation results for three different setting of Δ with increasing explosion ratios. The longest fulfillment time of 63 minutes is used to benchmark the results. For all three searching band factors Δ, the fulfillment time has a drop of 3 to 4 minutes.

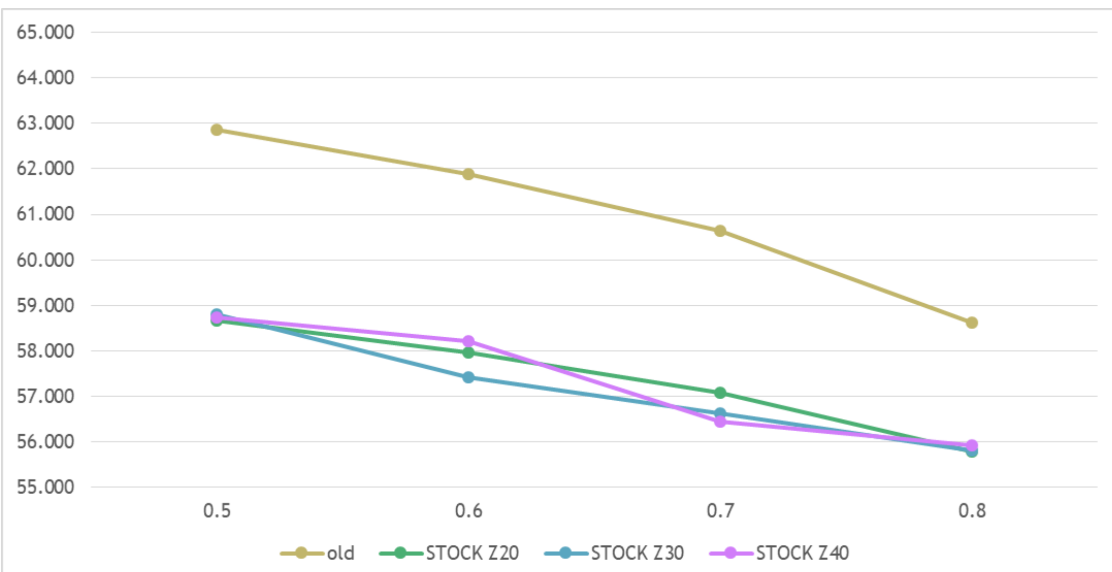

Figure 4.18 The results illustrate a consistent drop with the JICDO algorithm, when compared with random storage policy.

From Table 4.12, the decrease on order fulfillment is around 4% to 7%. The results ensure that the proposed JICDO algorithm with explosive storage strategy will reduce fulfillment time intuitively and reliably.

Table 4.12 Fulfillment Time Improvement with JICDO Algorithm

Fulfill Time	Z20	Z30	Z40
X	% Change		
0.5	6.67%	6.44%	6.55%
0.6	6.33%	7.22%	5.91%
0.7	5.88%	6.63%	6.92%
0.8	4.82%	4.79%	4.59%

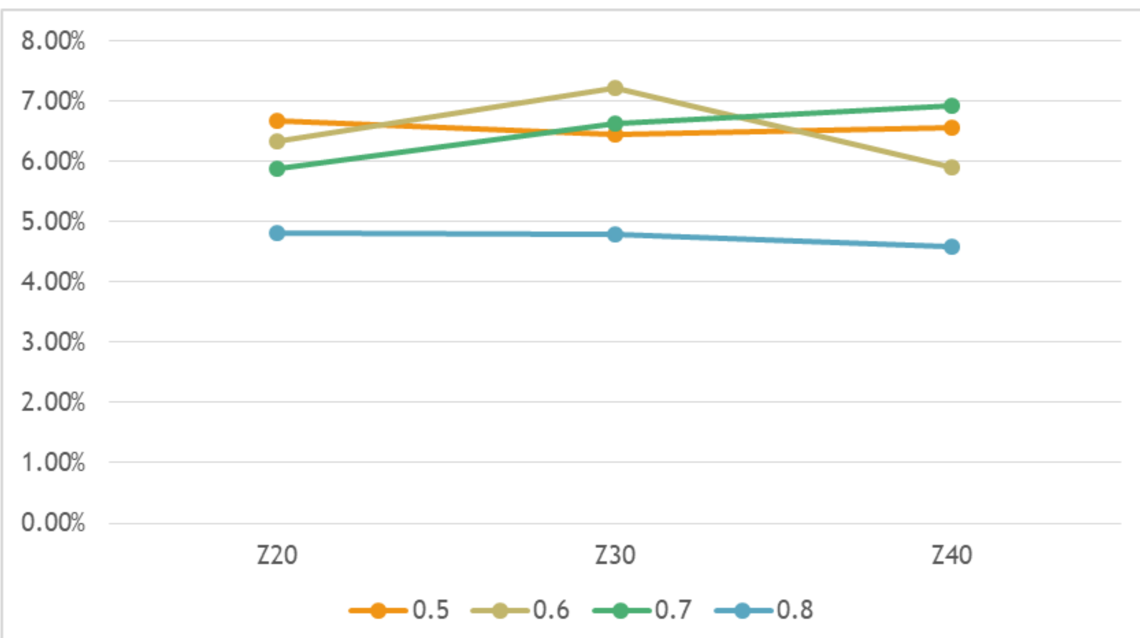

Figure 4.19 The performance analysis shows the percentage of improvement on fulfillment time in respect of the ones with random storage.

In Figure 4.18, the decreasing trend along with the increasing of explosion ratio is kept, while the improvements from Figure 4.19, shows little differences among the range of searching band. Clearly, the randomization in storage allocation is not compatible with explosion storage. With correlated storage assignments considered, the connection between SKU inventory allocation and picking strategy is strengthened. An advanced picking algorithm for a reasonable combination of pick lists will be an enhancement for both the efficiency of JICDO algorithm and the fulfillment performance.

CHAPTER 5

SUMMARY AND FUTURE RESEARCH

Ever since the change of the customer behavior with the popularity of online shopping, traditional retailers are required to provide quick and quality-controlled services for customers with diversity, intelligent thinking and full access to information. New requirements generate the new and adjusted decision problems, thereupon, new models are built to solve the corresponding problems to service new requirements. In this dissertation, two decision problems from different areas are figured out and presented as two typical examples, demonstrating the challenges and changes that industries are having, and the new features that Internet Impact has brought. Based on the quick response strategy, fashion industries has to enhance the design quality and reduce the deliver time, to lead customers' taste and to deal with unpredictable customer requirement uncertainty. Like Zara introduced fast fashion to the world, Amazon as the leading online shopping company, provides an incredibly quick service to customers with a chaotic warehouse system. To identify the insights from these two successful models, the following two topics have been presented in this document – decision model in Fast-Fashion Supply system and stocking problems in Internet Fulfillment Warehouses.

5.1 Summary

In this research, two distinguished problems are defined and formulated to be decision models. The channel switching decision model provides fast fashion retailers with an intuitive and effective approach to accomplish inventory management with operation control simultaneously. With real-time monitoring, fashion retailers are able to make immediate switching decisions between predefined discounting strategies. A multi-channel switching model is formulated to maximize horizontal revenue from a block inventory, where the Linear Moving Average Trend heuristic is established to make an instant decision on switching or not in the coming period.

The other topic is a continuous research on operation flows followed by an observational study and an empirical analysis related to Internet Fulfillment Warehouses. After stating that explosive stocking policy has a significantly improvement on fulfillment performance, stocking process, as a supportive stage to value-added picking process, is modelled as a two-phase problem with sequential decision-making procedures: (1) a SKU priority and location assignments phase with (i) SKU preselection decision (ii) single-SKU processing density oriented bin allocation decisions, and (2) a location assignments grouping and stocker arrangement phase with (i) group and stock list decisions and (ii) stocker assignment decision. Two algorithms – JOFDO and JICDO stocking algorithms are presented to approach to an optimal inventory allocation solution to maximize the improvement on picking efficiency after replenishment lots have been completely stocked in the assigned location. Simulation experiments proposed that with formulated JICDO algorithm, stocking decisions offer an enhancement on picking efficiency, thereupon to improve the fulfillment performance.

5.2 Future Research

With time and resource limitations, the research on both problems are still with plenty of future research opportunities.

As mentioned in Chapter 3, there are several controllable parameters in FFS problem. In this dissertation, the focus of the research is on the channel switching decision made with predefined prices series and block inventory. With relaxing the assumptions, this problem can be extended by including demand ratio diversity along with prices relationship, or a continuous production process with different reorder policy instead of initial constant supplies. Another extension is by solving a two dimensional problem in which $T_C\ and\ T_O$ are both independent decision variables. A new approach to this problem will be proved to proposing a new heuristic solution to deal with the complexity and capability of the optimal solution.

With the IFWs based problems, the team has indicated four decision models with the operation work flows in IFWs. As an extended study, a justification and modification on the combination of established stocking and picking algorithms is supposed to achieve a higher improvement than those with stocking or picking algorithm only. As assumed in Chapter 4, the current algorithm is processing SKUs sequentially in location assignment phase. Considering all exploded lots without preselection of SKU will increase the size of bin alternatives. Further, the identified consolidation assignment problem and truck load problem can describe a different viewpoint to improve customer order fulfillment.

REFERENCES

Bailey, J. P. (1998). Intermediation and Electronic Markets: Aggregation and Pricing in Internet Commerce. *Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA*. http://hdl.handle.net/1721.1/9835

Bindi, F., Manzini, R., Pareschi, A., and Regattieri, A. (2009). Similarity-Based Storage Allocation Rules in an Order Picking System: an Application to the Food Service Industry. *International Journal of Logistics Research and Applications*, *12*(4), 233-247. doi:10.1080/13675560903075943

Bozer, Y. A. (1985). Optimizing Throughput Performance in Designing Order Picking Systems. *Ph.D. Thesis, Georgia Institute of Technology, Atlanta, GA*. http://hdl.handle.net/1853/25588

Brynjolfsson, E., Hu, Y. J., and Rahman, M. S. (2013). Competing in the Age of Omnichannel Retailing. *MIT Sloan Management Review*, *54*(4), 23. http://sloanreview.mit.edu/article/competing-in-the-age-of-omnichannel-retailing/

Cachon, G. P., and Swinney, R. (2009). Purchasing, Pricing, and Quick Response in the Presence of Strategic Consumers. *Management Science, 55*(3), 497-511. doi:10.1287/mnsc.1080.0948

Cachon, G. P., and Swinney, R. (2011). The Value of Fast Fashion: Quick Response, Enhanced Design, and Strategic Consumer Behavior. *Management Science, 57*(4), 778-795. doi:10.1287/mnsc.1100.1303

Caro, F., and Gallien, J. (2010). Inventory Management of a Fast-Fashion Retail Network. *Operations Research, 58*(2), 257-273. doi:10.1287/opre.1090.0698

Caro, F., and Gallien, J. (2012). Clearance Pricing Optimization for a Fast-Fashion Retailer. *Operations Research, 60*(6), 1404-1422. doi:10.1287/opre.1120.1102

Caron, F., Marchet, G., and Perego, A. (2000). Layout Design in Manual Picking Systems: a Simulation Approach. *Integrated Manufacturing Systems, 11*(2), 94-104. doi:doi:10.1108/09576060010313946

Chakrabarti, R., and Scholnick, B. (2002). Exchange Rate Expectations and Foreign Direct Investment Flows. *Weltwirtschaftliches Archiv, 138*(1), 1-21. https://doi.org/10.1007/BF02707321

Chen, S., and Leteney, F. (2000). Get Real! Managing the Next Stage of Internet Retail. *European Management Journal, 18*(5), 519-528. doi:http://dx.doi.org/10.1016/S0263-2373(00)00041-4

Chiang, W.-y. K., Chhajed, Dilp, Hess, James. D. (2003). Direct Marketing, Indirect Profits a Strategic Analysis of Dual-Channel Supply-Chain Design. *Management Science, 49*(1), 1-20. doi:10.1287/mnsc.49.1.1.12749

Choi, T.-M. (2007). Pre-Season Stocking and Pricing Decisions for Fashion Retailers with Multiple Information Updating. *International Journal of Production Economics, 106*(1), 146-170. doi:10.1016/j.ijpe.2006.05.009

Chuang, Y.-F., Lee, H.-T., and Lai, Y.-C. (2012). Item-Associated Cluster Assignment Model on Storage Allocation Problems. *Computers and Industrial Engineering, 63*(4), 1171-1177. doi:http://dx.doi.org/10.1016/j.cie.2012.06.021

Commerce, D. o. (2017). Quarterly Retail E-Commerce Sales. *U.S. Census Bureau News*. doi:https://www.census.gov/retail/mrts/www/data/pdf/ec_current.pdf

Coughlan, A. T., and Soberman, D. A. (2005). Strategic Segmentation Using Outlet Malls. *International Journal of Research in Marketing, 22*(1), 61-86. doi:10.1016/j.ijresmar.2004.03.003

de Koster, R., Le-Duc, T., and Roodbergen, K. J. (2007). Design and Control of Warehouse Order Picking: A Literature Review. *European Journal of Operational Research, 182*(2), 481-501. doi:10.1016/j.ejor.2006.07.009

Ding, Q., Dong, C., and Pan, Z. (2016). A Hierarchical Pricing Decision Process on a Dual-Channel Problem with One Manufacturer and One Retailer. *International Journal of Production Economics, 175*, 197-212. doi:10.1016/j.ijpe.2016.02.014

Frazele, E. A., and Sharp, G. P. (1989). Correlated Assignment Strategy Can Improve Any Order-Picking Operation. *Industrial Engineering, 21*(4), 33-37

Frazelle, E. H. (1994). The Forward-Reserve Problem. *New York, NY: John Wiley and Sons, Inc*

Gallien, J., Mersereau, A. J., Garro, A., Mora, A. D., and Vidal, M. N. (2015). Initial Shipment Decisions for New Products at Zara. *Operations Research, 63*(2), 269-286. doi:10.1287/opre.2014.1343

Ghemawat, P., Nueno, Jose Luis. (2003). ZARA: Fast Fashion. *Harvard Business School Multimedia Case, 9-703-16*

Goetschalckx, M., and Ratliff, H. D. (1990). Shared Storage Policies Based on the Duration Stay of Unit Loads. *Management Science, 36*(9), 1120-1132. doi:10.1287/mnsc.36.9.1120

Grewal, D., Ailawadi, K. L., Gauri, D., Hall, K., Kopalle, P., and Robertson, J. R. (2011). Innovations in Retail Pricing and Promotions. *Journal of Retailing, 87*, S43-S52. doi:10.1016/j.jretai.2011.04.008

Gu, J., Goetschalckx, M., and McGinnis, L. F. (2007). Research on Warehouse Operation: A Comprehensive Review. *European Journal of Operational Research, 177*(1), 1-21. doi:10.1016/j.ejor.2006.02.025

Hackman, S. T., Rosenblatt, M. J., and Olin, J. M. (1990). Allocating Items to an Automated Storage and Retrieval System. *IIE Transactions, 22*(1), 7-14. doi:10.1080/07408179008964152

Ho, S. S., and Sarma, S. (2008). The Fragmented Warehouse Location Assignment for Unit-Load Picking. *2008 IEEE International Conference on Industrial Engineering and Engineering Management*, 1159-1163. doi:10.1109/IEEM.2008.4738052

Ho, S. S., and Sarma, S. (2009). The Fragmented Warehouse: Location Assignment for Multi-Item Picking. *2009 2nd International Symposium on Logistics and Industrial Informatics*, 1-6. doi:10.1109/lindi.2009.5258752

Huang, Y.-S., Chen, S.-H., and Ho, J.-W. (2013). A Study on Pricing and Delivery Strategy for E-Retailing Systems. *Transportation Research Part E: Logistics and Transportation Review, 59*, 71-84. doi:http://dx.doi.org/10.1016/j.tre.2013.09.002

Huang, Y.-S., Hsu, C.-S., and Ho, J.-W. (2014). Dynamic Pricing for Fashion Goods with Partial Backlogging. *International Journal of Production Research, 52*(14), 4299-4314. doi:10.1080/00207543.2014.881576

Iyer, A. V., Bergen, Mark E. (1997). Quick Response in Manufacturer-Retailer Channels. *Management Science, 43*(4), 559-570. doi:https://doi.org/10.1287/mnsc.43.4.559

Jarvis, J. M., and McDowell, E. D. (1991). Optimal Product Layout in an Order Picking Warehouse. *IIE Transactions, 23*(1), 93-102. doi:10.1080/07408179108963844

Karakul, M. (2008). Joint Pricing and Procurement of Fashion Products in the Existence of Clearance Markets. *International Journal of Production Economics, 114*(2), 487-506. doi:10.1016/j.ijpe.2007.03.026

Khouja, M., Park, S., and Cai, G. (2010). Channel Selection and Pricing in the Presence of Retail-Captive Consumers. *International Journal of Production Economics, 125*(1), 84-95. doi:10.1016/j.ijpe.2010.01.005

Kulturel, S., Ozdemirel, N. E., Sepil, C., and Bozkurt, Z. (1999). Experimental Investigation of Shared Storage Assignment Policies In Automated Storage/Retrieval Systems. *IIE Transactions, 31*(8), 739-749. doi:10.1080/07408179908969873

Lang, S. T., Logan; Zimmerman, Jarett; Harrison, Jeffrey S. (2012). Amazon.com- Offering Everything from A to Z. *Case Study. University of Richmond: Robins School of Business*. http://scholarship.richmond.edu/cgi/viewcontent.cgi?article=1001&context=robins-case-network

Laudon, K. C. T., Carol Guercio. (2007). E-Commerce: Business,Technology, Society, 4th Edition, *Upper Saddle River, NJ: Prentice Hall, Inc.*.

Liu, C.-M. (1999). Clustering Techniques for Stock Location and Order-Picking in a Distribution Center. *Computers and Operations Research, 26*(10), 989-1002. doi:http://dx.doi.org/10.1016/S0305-0548(99)00026-X

Liu, C.-M. (2004). Optimal Storage Layout and Order Picking for Warehousing. *International Journal of Operations Research, 1*(1), 37-46.

Malmborg, C. J., and Al-Tassan, K. (2000). An Integrated Performance Model for Orderpicking Systems with Randomized Storage. *Applied Mathematical Modelling, 24*(2), 95-111. doi:http://dx.doi.org/10.1016/S0307-904X(99)00009-8

Melis, K., Campo, K., Breugelmans, E., and Lamey, L. (2015). The Impact of the Multi-channel Retail Mix on Online Store Choice: Does Online Experience Matter? *Journal of Retailing, 91*(2), 272-288. doi:10.1016/j.jretai.2014.12.004

Ming-Huang Chiang, D., Lin, C.-P., and Chen, M.-C. (2014). Data Mining Based Storage Assignment Heuristics for Travel Distance Reduction. *Expert Systems, 31*(1), 81-90. doi:10.1111/exsy.12006

Montulet, P., Langevin, A., and Riopel, D. (1998). Minimizing the Peak Load: An Alternate Objective for Dedicated Storage Policies. *International Journal of Production Research, 36*(5), 1369-1385. doi:10.1080/002075498193372

Onal, S., Zhang, J., and Das, S. (2017). Modelling and Performance Evaluation of Explosive Storage Policies in Internet Fulfilment Warehouses. *International Journal of Production Research*, 1-14. doi:10.1080/00207543.2017.1304663

Petersen, C. G., and Schmenner, R. W. (1999). An Evaluation of Routing and Volume-Based Storage Policies in an Order Picking Operation. *Decision Sciences, 30*(2), 481-501. doi:10.1111/j.1540-5915.1999.tb01619.x

Pohl, L. M., Meller, R. D., and Gue, K. R. (2011). Turnover-Based Storage in Non-Traditional Unit-Load Warehouse Designs. *IIE Transactions, 43*(10), 703-720. doi:10.1080/0740817X.2010.549098

Ronald J. Mantel, P. C. S., Sunderesh S. Heragu. (2007). Order Oriented Slotting: A New Assignment Strategy for Warehouse. *European Journal of Industrial Engineering, 1*(3), 301-316. https://doi.org/10.1504/EJIE.2007.014689

Roodbergen, K.-J., Sharp, G., and Vis, I. (2008). Designing the Layout Structure of Manual Order-Picking Areas in Warehouses. *IIE Transactions, 40*(11), 1032-1045. doi:10.1080/07408170802167639

Ross, D. F. (2015). Statistical Inventory Management. *Distribution Planning and Control: Managing in the Era of Supply Chain Management*, 357-405. Boston, MA: Springer US

Sen, A. (2008). The US Fashion Industry: A Supply Chain Review. *International Journal of Production Economics, 114*(2), 571-593. doi:10.1016/j.ijpe.2007.05.022

Sierra, J. J., and Hyman, M. R. (2011). Outlet Mall Shoppers' Intentions to Purchase Apparel: A Dual-Process Perspective. *Journal of Retailing and Consumer Services, 18*(4), 341-347. doi:http://dx.doi.org/10.1016/j.jretconser.2011.03.002

Smith, S. A., McIntyre, S. H., and Achabal, D. D. (1994). A Two-Stage Sales Forecasting Procedure Using Discounted Least Squares. *Journal of Marketing Research, 31*(1), 44-56. doi:10.2307/3151945

Verhoef, P. C., Kannan, P. K., and Inman, J. J. (2015). From Multi-Channel Retailing to Omni-Channel Retailing. *Journal of Retailing, 91*(2), 174-181. doi:http://dx.doi.org/10.1016/j.jretai.2015.02.005

Wang, W., Li, G., and Cheng, T. C. E. (2016). Channel Selection in a Supply Chain with a Multi-Channel Retailer: The Role of Channel Operating Costs. *International Journal of Production Economics*, *173*, 54-65. doi:10.1016/j.ijpe.2015.12.004

Wutthisirisart, P., Noble, J. S., and Alec Chang, C. (2015). A Two-Phased Heuristic for Relation-Based Item Location. *Computers and Industrial Engineering, 82*, 94-102. doi:http://dx.doi.org/10.1016/j.cie.2015.01.020

Yan, W., Li, Y., and Sui, N. (2014). The Relationship between Recent Stressful Life Events, Personality Traits, Perceived Family Functioning and Internet Addiction among College Students. *Stress and Health, 30*(1), 3-11. doi:10.1002/smi.2490

Zhang, J., Onal, S., and Das, S., Price Differentiated Channel Switching in a Fixed Period Fast Fashion Supply Chain, *International Journal of Production Economics*, Vol 193, 31-39. doi:https://doi.org/10.1016/j.ijpe.2017.06.030

Zhang, J., Shou, B., and Chen, J. (2013). Postponed Product Differentiation with Demand Information Update. *International Journal of Production Economics, 141*(2), 529-540. doi:10.1016/j.ijpe.2012.09.007

Zhang, Y. (2016). Correlated Storage Assignment Strategy to Reduce Travel Distance in Order Picking**Zhang thanks the financial support of the china scholarship council (CSC). *IFAC-PapersOnLine, 49*(2), 30-35. doi:http://dx.doi.org/10.1016/j.ifacol.2016.03.006

Zhou, E., Zhang, J., Gou, Q., and Liang, L. (2015). A Two Period Pricing Model for New Fashion Style Launching Strategy. *International Journal of Production Economics, 160*, 144-156. doi:10.1016/j.ijpe.2014.10.008