Dark stationary matter waves via parity-selective filtering in a Tonks-Girardeau gas

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We propose a scheme for observing dark stationary waves in a Tonks-Girardeau (TG) gas. The scheme is based on parity-selective dynamical filtering of the gas via a time-dependent potential, which excites the gas from its ground state towards a desired specially-tailored many-body state. These excitations of the TG gas are analogous to linear partially coherent nondiffracting beams in optics, as evident from the mapping between the quantum dynamics of the TG gas and the propagation of incoherent light in one-dimensional linear photonic structures.

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I. INTRODUCTION

Trapped bose gases confined to one-dimensional (1D) geometry are highly attractive for studying quantum many-body dynamics because pertinent models yield exact solutions [1–4], these regimes are experimentally accessible [5–8], and quantum effects are enhanced [9–11]. Tonks-Girardeau (TG) gas is a system of 1D bosons with "impenetrable core" repulsive interactions [1]. In the fashion of the Pauli exclusion principle, impenetrable cores prevent bosons from occupying the same position in space, which causes TG gas to exhibit fermionic properties. This similarity is manifested in the mapping between 1D noninteracting fermions and TG bosons [1,4]. From the properties of atomic interactions in tight atomic waveguides [9] it follows that the TG regime can be reached at low temperatures, low linear densities, or stronger effective interactions [9–11]. The experimental realizations of the TG gas in 2004 [6,7] boosted the physical relevance of the model. A recent experiment has demonstrated that such gas does not relax to the thermodynamic equilibrium even after numerous collisions [8], due to the integrability of the underlying model. This work inspired a theoretical study of 1D impenetrable core bosons on a lattice, which suggested that the system can undergo irreversible relaxation to a steady state carrying more memory of the initial conditions than the usual thermodynamic equilibrium [12]. The process of equilibration of a 1D bose gas has also been recently studied within the two–particle irreducible (2PI) effective action approach [13]. Some aspects of the TG quantum dynamics have been studied theoretically within the context of so-called "dark solitons" [4,14], matter-wave interference [15], 1D expansion [16,17], irregular dynamics [18], and coherent states [19]. In this work, we study the dynamical tailoring of the TG gas via a time-dependent potential to produce dark stationary states, as well as point out the relation between TG dynamics and the propagation of incoherent light in linear photonic structures.

The Fermi-Bose mapping [1,4], applicable both in the static [1] and time-dependent case [4], prescribes the construction of the exact many-body wave function of the TG gas from single-particle (SP) wave functions, which obey a set of uncoupled linear SP Schrödinger equations. In Ref. [4] Girardeau and Wright discuss the dynamics of the TG gas within the context of dark solitons [20–23]. Dark solitons are fundamental nonlinear excitations, which have been studied theoretically within the nonlinear mean-field theories applicable for weakly interacting gases [20,22,23], and observed experimentally in these regimes [21]. For a strongly interacting TG gas on a ring, Girardeau and Wright [4] noticed that if the many-body wave function is constructed solely from the odd-parity SP eigenstates of the system, the SP density will have a dip at zero, similar in structure to dark solitons. However, such a specially structured many-body state is unlikely to occur without deliberate preparation, since even and odd parity SP states of that system are intermingled when ordered with respect to energy (see the discussion in Ref. [4]). In the study of Busch and Huyet [14], the collapses and reappearances of TG dark solitonlike structures in an harmonic trap are attributed to the mixture of the odd and the even components in the excitation. Generally, the SP eigenstates in parity-invariant 1D potentials can be chosen to be either even or odd, which makes them candidates for observing dark stationary structures in the TG gas. However, for their experimental realization under such confinement, it is essential to separate components of different parity.

Here we propose a scheme for observing dark stationary waves in a TG gas. A time-dependent potential is used to selectively filter the even component of the many-body wave function, thereby creating a dark stationary wave. Such excitation of the strongly interacting TG gas is in fact an excited *many-body eigenstate* of the system, which distinguishes it from dark solitons of the nonlinear mean-field equations applicable for weak interactions [20–23]. Dark stationary waves are found in various types of parity-invariant potentials; we demonstrate these waves in a containerlike potential, and in a periodic (lattice) potential. We point out that such excited eigenstates of the TG gas are analogous to linear partially coherent nondiffracting beams in optics [24,25], as evident from the mapping between the quantum dynamics of the TG gas and the propagation of incoherent light in one-dimensional linear photonic structures, presented in this paper. This mapping adds to the analogies between optical and matter waves [26], and in particular to the similarity between nonlinear partially coherent optical waves and matter waves [27].

II. DARK STATIONARY WAVES VIA DYNAMICAL PARITY-SELECTIVE FILTERING

We consider N impenetrable bosons, confined within a 1D external potential $V_{ext}(x,t)$. The fully symmetrized many-body wave function describing the system, $\psi_B(x_1,\ldots,x_N,t)$, is constructed according to the Fermi-Bose mapping [1,4]. Let $\psi_m(\xi,\tau)$ denote a set of orthonormal SP wave functions obeying the set of uncoupled linear Schrödinger equations,

$$i\frac{\partial\psi_m}{\partial\tau}=\left[-\frac{\partial^2}{\partial\xi^2}+V(\xi,\tau)\right]\psi_m(\xi,\tau),\quad m=1,\ldots,N. \quad (1)$$

In order to unify notation and discuss the equivalence with optics, we find it convenient to use dimensionless units. The boson mass m and the (arbitrary) choice of spatial length-scale x_0 ($\xi=x/x_0$) determine the units of time $t_0=2mx_0^2/\hbar$ ($\tau=t/t_0$) and energy $\epsilon_0=\hbar^2/(2mx_0^2)$ [$V(\xi,\tau)=V_{ext}(x,t)/\epsilon_0$]. From the SP wave functions ψ_m one first constructs a fully antisymmetric (fermionic) wave function in the form of the Slater determinant, $\psi_F(x_1,\ldots,x_N,t)=\sqrt{x_0^{-N}/N!}\,\det[\psi_m(\xi_j,\tau)]$; ψ_F describes a system of spinless noninteracting fermions in the 1D potential $V_{ext}(x,t)$ [1,4]. The bosonic many-body solution $\psi_B(x_1,\ldots,x_N,t)$ is obtained after symmetrization of ψ_F,

$$\psi_B=A(x_1,\ldots,x_N)\sqrt{\frac{x_0^{-N}}{N!}}\det_{m,j=1}^{N}[\psi_m(\xi_j,\tau)], \quad (2)$$

where $A=\Pi_{1\le i<j\le N}\,\mathrm{sgn}(x_i-x_j)$ is a "unit antisymmetric function" [1]. Thus, the quantum dynamics of the TG gas is obtained from Eq. (2) after solving Eq. (1). For example, the evolution of the single-particle density $\rho_{SP}(x,t)=\int dx_2\ldots dx_N|\psi_B(x,x_2,\ldots,x_N,t)|^2$ corresponds to the evolution of $\rho(\xi,\tau)=\Sigma_m|\psi_m(\xi,\tau)|^2$ [4]. It should be noted that while some quantities of the corresponding fermionic system are identical (e.g., the SP density), some significantly differ (e.g., the momentum distribution [3,7]).

Excited many-body eigenstates with solitonlike SP density are found in real, time-independent, and parity-invariant potentials, $V(\xi)=V(-\xi)$. Let $\phi_{\epsilon,\gamma}(\xi)$ denote the eigenstates, and let ϵ denote eigenvalues (energies) of the SP Hamiltonian $H=-d^2/d\xi^2+V(\xi)$ with appropriate boundary conditions. The extra index γ is used in case there are degenerate SP eigenstates. Since the Hamiltonian commutes with the parity operator, they can have a common complete set of eigenstates, in which case the eigenstates $\phi_{\epsilon,\gamma}(\xi)$ are either symmetric $\phi^+_{\epsilon,\gamma}(\xi)=\phi^+_{\epsilon,\gamma}(-\xi)$ (even parity) or antisymmetric $\phi^-_{\epsilon,\gamma}(\xi)=-\phi^-_{\epsilon,\gamma}(-\xi)$ (odd parity). Consider a bosonic many-body wave function, constructed according to Eq. (2), from odd-parity eigenmodes only, i.e., every ψ_m equals one of the eigenstates $\phi^-_{\epsilon,\gamma}(\xi)$ ($\psi_m\neq\psi_n$ for $m\neq n$). Such a wave function is an excited many-body eigenstate of the system. Its SP density $\rho^-(\xi)=\Sigma|\phi^-_{\epsilon,\gamma}(\xi)|^2$ is stationary, and $\rho^-(0)=0$ because $\phi^-_{\epsilon,\gamma}(0)=0$. The SP density of this excited many-body eigenstate thus has a dip at $\xi=0$, which resembles the structure of nonlinear dark solitons [20–23]. Therefore we will refer to these states as dark many-body eigenstates. If we construct waves ψ_m from symmetric eigenmodes $\phi^+_{\epsilon,\gamma}(\xi)$ only, the density $\rho^+(\xi)=\Sigma|\phi^+_{\epsilon,\gamma}(\xi)|^2$ is likely to have a pronounced peak at $\xi=0$, reminiscent of anti-dark solitons. Hence, they will be referred to as anti-dark many-body eigenstates.

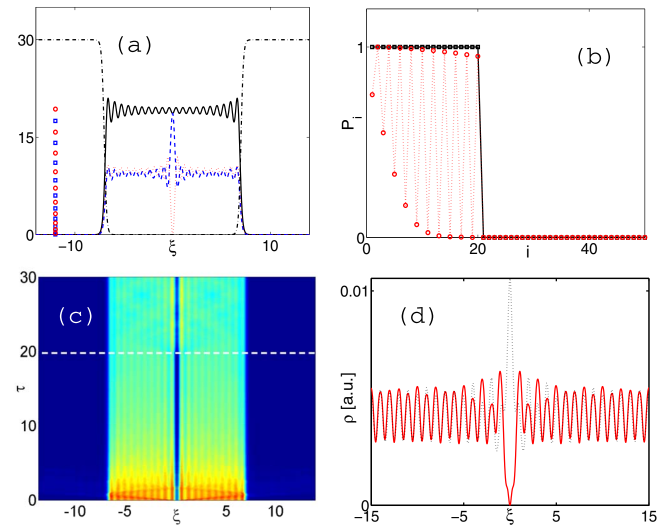

FIG. 1. (Color online) The dark and anti-dark many-body eigenstates: structure and excitation. (a) The container potential $V_c(\xi)$ (black dot-dashed line), the even ρ^+ (blue dashed line), odd ρ^- (red dotted line), and total SP density $\rho=\rho^++\rho^-$ (black solid line) of the ground state with 20 bosons; the first 20 eigenvalues are shown as blue squares (even SP eigenstates) and red circles (odd SP eigenstates). (b) The spectrum P_i of the excitation at $\tau=0$ (black squares) and $\tau=19.8$ (red circles), and (c) the dynamics of the SP density following the time-dependent perturbation. (d) The SP density $\rho(\xi)$ of an anti-dark (black dotted line) and dark (red solid line) many-body eigenstate in the periodic potential (see text for details).

Dark and anti-dark many-body eigenstates are specially tailored excitations of the TG gas, constructed solely from the odd or even SP eigenstates of the parity-invariant potential $V(\xi)$. Assuming the absence of degeneracy, the even and odd parity SP eigenstates alternate when ordered with respect to energy, meaning that such specially tailored states are unlikely to naturally occur. We illustrate this by studying $N=20$ TG bosons in the external containerlike potential $V_c(\xi)=V_c^0\{2+\Sigma_{i=1,2}(-)^{i+1}\tanh x_w[\xi+(-)^i x_c]\}$ ($V_c^0=15$, $x_w=4$, and $x_c=7$), shown in Fig. 1(a). The SP density ρ of the ground state is plotted as a solid black line, whereas the dotted red (dashed blue) line depicts the odd (even, respectively) component of the SP density: $\rho(\xi)=\rho^+(\xi)+\rho^-(\xi)$. The energies of the even (odd) parity SP eigenmodes are shown as blue squares (red circles, respectively). As expected odd and even-parity eigenstates alternate with increasing energy. Thus, half of the SP eigenstates comprising the ground state are even and half are odd. The even ρ^+ (odd ρ^-) component of the SP density has the structure of the anti-dark (dark, respectively) many-body eigenstates. In what follows we propose a method for the dynamical excitation of dark stationary waves.

Our scheme utilizes a time-dependent potential which tailors the many-body wave function in a specific desired fashion, and separates the odd from the even component. The system of $N=20$ TG bosons is initially in the ground state of the confining potential, see Fig. 1(a). In the spirit of Refs. [4,14], we perturb this system at its symmetry point $\xi=0$, with a spatially narrow repulsive potential, which may be obtained with a laser [4,14]. However, in contrast to [4,14], here we periodically switch this potential on and off in time. Such a time-dependent potential can be modeled as $V'(\xi,\tau)=V_0'\{\text{sgn}[\sin(2\pi t/\tau_p)]+1\}\exp[-(\xi/\sigma)^2]$, where $\tau_p=0.1$ is the periodicity of the laser signal, $V_0'=100$ corresponds to the peak intensity of the laser, and $\sigma=0.06$ to its spatial focusing width. As bosons are kicked by the time-dependent potential, they acquire energy which has a certain probability of being higher than the lip of the trap and are ejected away from it. However, because the laser is focused close to $\xi=0$, it strongly affects only the even-parity SP eigenstates, whereas the odd-parity eigenstates are left nearly unperturbed. Consequently, the many-body wave function within the container ($|x|<x_c$) takes on a specific structure: it is constructed via Eq. (2) mainly from the odd-parity SP eigenstates. This filtering process is depicted in Fig. 1(b), showing the spectrum of the SP wave functions $\psi_m(\xi,\tau)$ calculated according to $P_i(\tau)=\sum_m|\int_{-2x_c}^{2x^c}d\xi\psi_m(\xi,\tau)\phi_i^*(\xi)|^2$, where $\phi_i(\xi)$ is the ith eigenstate of the SP Hamiltonian. The spectrum at $\tau=0$ is flat (black squares) because the odd and the even eigenstates are equally present. However, the spectrum after $\tau=19.8$ (red circles) is mainly comprised from odd-parity eigenstates. Fig. 1(c) shows the evolution of the total SP density. The time-dependent potential acts within the interval $\tau=[0,19.8]$. After $\tau=19.8$ (marked by a horizontal line) it is turned off. The SP density nevertheless retains a dark notch at $\xi=0$ even after the time-dependent potential is turned off, clearly displaying a dark stationary wave evolution. The numerical evolution of Eq. (1) is performed with the split-step Fourier method.

The scheme proposed here should work for various types of containerlike potentials. It is also fairly robust. For lasers with larger intensity $\propto V_0'$, the filtering occurs on faster time scales (i.e., a smaller number of on-off switches is sufficient) and is more efficient. The focusing width of the laser σ limits the number of particles that can be efficiently filtered. This width should be sufficiently smaller than the period of the spatial oscillation (close to $\xi=0$) of the Nth SP eigenstate. It should be emphasized that even though we follow the spirit of Refs. [4,14], the proposed scheme separates the odd- and even-parity component in space, while the many-body wave function within the container assumes the particular structure of a dark stationary wave. Although different in nature, the parity-selective filtering of the TG gas bears some similarity to energy-selective removal of cold atoms from a tight optical trap by means of parametric excitation of the trap vibrational modes [28].

III. DARK STATIONARY WAVES OF THE TONKS-GIRARDEAU GAS IN A PERIODIC LATTICE POTENTIAL

While the proposed method for exciting dark stationary states of the TG gas employs a containerlike potential, it should be emphasized that the notion of dark and anti-dark many-body eigenstates pertains to various parity invariant potentials. We illustrate this fact in a periodic potential $V(\xi)=V(\xi+D)$ (e.g., optically induced lattices). The SP eigenstates of this system are Bloch waves [29] of the form $\phi_{k,n}(\xi,\tau)=u_{k,n}(\xi)e^{ik\xi}e^{-i\epsilon_{k,n}\tau}$, where n denotes the band number, k is the Bloch wave vector, and $u_{k,n}(\xi)=u_{k,n}(\xi+D)$ describes the periodic spatial profile of the Bloch wave. Since $V(\xi)=V(-\xi)$, the Bloch waves $\phi_{k,n}$ and $\phi_{-k,n}$ are degenerate. By properly choosing the coefficients a_{k_m} and a_{-k_m} within the superposition $\psi_m(\xi,\tau)=a_{k_m}\phi_{k_m,n}+a_{-k_m}\phi_{-k_m,n}$, degenerate eigenstates $\phi_{\pm k_m,n}$ can be superimposed to obtain even $[\psi_m^+(\xi,\tau)=\psi_m^+(-\xi,\tau)]$ and odd-parity $[\psi_m^-(\xi,\tau)=-\psi_m^-(-\xi,\tau)]$ eigenstates. The many-body wave function comprised solely from $\psi_m^-(\xi,\tau)$ $[\psi_m^+(\xi,\tau)]$ via Eq. (2) is a dark (anti-dark) excited many-body eigenstate of the TG gas in the lattice. Figure 1(d) shows the (stationary) SP density of the dark and anti-dark many-body eigenstate in a periodic potential $V(\xi)=10\cos^2(\pi x)$, constructed by symmetrizing the lowest $N=21$ SP eigenmodes. The calculation is performed on the ring (periodic boundary conditions) of length $L=71$.

IV. RELATION BETWEEN TONKS-GIRARDEAU GAS AND INCOHERENT LIGHT IN 1D PHOTONIC STRUCTURES

Dark and anti-dark excited many-body eigenstates of the TG gas are analogous to partially coherent nondiffracting beams that were studied in the context of classical optics [24,25]. In order to clarify this point, we first demonstrate the mapping between the propagation of incoherent light in linear 1D photonic structures [30] and the TG gas dynamics. Consider a quasimonochromatic, linearly polarized, partially spatially incoherent light beam which propagates paraxially in the 1D photonic structure described by the spatially dependent index of refraction $n_{tot}^2=n_0^2+2n_0n(x,z)$; n_0 is constant, while $n(x,z)$ denotes spatial variation of the refraction index. The classical electromagnetic field $E(x,z,t)$ of the beam randomly fluctuates; z (x) denotes the propagation axis (spatial, respectively) coordinate. The state of the system is described by the mutual coherence function $B(x_1,x_2,z)=\langle E^*(x_2,z,t)E(x_1,z,t)\rangle$ [31], where brackets denote the time average, which equals the ensemble average assuming the light source is stationary and ergodic [31]. The equation of motion describing the dynamics of the intensity and coherence properties of the light along the propagation axis z (in the paraxial approximation) is

$$i\frac{\partial B}{\partial z}+\frac{1}{2k}\left(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2}\right)B+\frac{k}{n_0}[n(x_1,z)-n(x_2,z)]B=0, \tag{3}$$

where $k=n_0\omega/c$, and ω is the temporal frequency of the quasimonochromatic beam. The mutual coherence function B can be decomposed through an orthonormal set of coherent modes $\tilde{\psi}_m$ and their modal weights λ_m [31],

$$B(x_1,x_2,z)=\sum_m\lambda_m\tilde{\psi}_m^*(x_2,z)\tilde{\psi}_m(x_1,z). \tag{4}$$

In order to connect B to Eq. (1) we switch to dimensionless units: $\xi=x/x_0$, $\tau=z/(2kx_0^2)$ ("time" is the propagation length here). The potential V arises from the refractive index $V(\xi,\tau)=-2(kx_0)^2 n(x,z)/n_0$. From Eqs. (3) and (4) it follows that waves $\psi_m(\xi,\tau)=\sqrt{x_0}\tilde{\psi}_m(x,z)$ obey Eq. (1) (e.g., see Ref. [32]), affirming the mapping between the two systems. Note that each solution of Eq. (1) generates one bosonic many-body wave function via Eq. (2), and, due to the arbitrary choice of the modal weights λ_m, many correlation functions (4) corresponding to incoherent optical fields propagating in linear 1D photonic structures. If we were to consider only those functions $B(x_1,x_2,z)$ with the modal decomposition of the form $B=\Sigma_m \tilde{\psi}_m^*(x_2,z)\tilde{\psi}_m(x_1,z)$, i.e., where λ_m is either one or zero, the mapping would be one-to-one. Within this mapping, the single-particle density $\rho=\Sigma_m|\psi_m|^2$ of the TG gas corresponds to the time-averaged intensity $I=\Sigma_m\lambda_m|\tilde{\psi}_m|^2$ [32] of the light beam. However, the momentum distribution of the TG gas differs from the Fourier power spectrum of the incoherent beam [32], in the same fashion as it differs from the momentum distribution of the corresponding noninteracting fermionic system [3,7].

One particular example of a partially coherent nondiffracting optical beam propagating in vacuum, corresponds to the dark stationary TG wave on a ring studied by Girardeau and Wright [4]. An incoherent optical beam with the mutual coherence function $B(x_1,x_2)=\int dk_x G(k_x)\sin(k_x x_2)\sin(k_x x_1)$, or equivalently with the modal structure $\tilde{\psi}_{k_x}(x)=\sin(k_x x)/\sqrt{N(k_x)}$, and $\lambda_{k_x}=G(k_x)N(k_x)$, is propagation invariant; $N(k_x)$ serves to normalize $\tilde{\psi}_{k_x}(x)$. If its power spectrum $G(k_x)$ is rectangular, $G(k_x)=I_0/K$ for $|k_x|<K/2$, and zero otherwise, the intensity structure has the form $I_0/2[1-j_0(Kx)]$, which is exactly the form of the odd SP density component of the dark stationary wave on a ring in the thermodynamic limit [4]. If $B(x_1,x_2)=\int dk_x G(k_x)\cos(k_x x_2)\cos(k_x x_1)$, one obtains anti-dark optical propagation-invariant waves.

We have thus established a link between the dynamics of incoherent light in linear 1D photonic media and TG gas via Eq. (1). It should be kept in mind the former system is *classical*, while the latter is *quantum*, and the evolution of the quantities derived from the set of waves ψ_m (e.g., the density $\rho=\Sigma_m|\psi_m|^2$) should be properly interpreted. This mapping adds to the analogies between optical and matter waves [26], and in particular to the analogy between nonlinear partially coherent optical and matter waves [27]. We believe that the recently discovered wave phenomena [33,34] in the context of partially coherent optical-wave propagation in linear and nonlinear photonic lattices [32–34], have their counterpart in the context of matter waves.

Before closing, it should also be noted that the evolution equation (3) for the mutual coherence function $B(x_1,x_2,z)$ in linear photonic structures [32], is identical to the evolution of the reduced single-particle density matrix of noninteracting spinless fermions (in 1D, and 2D as well).

V. CONCLUSION

In conclusion, we have proposed a scheme for exciting dark stationary waves of the TG gas. Within our scheme, a time-dependent potential focused on the center of the trap, selectively filters a nondesirable part of the many-body wave function, thereby creating a dark stationary wave. The stationary waves of the TG gas are analogous to partially coherent nondiffracting beams in optics. This analogy is a consequence of the mapping between incoherent light in linear 1D photonic structures and the TG gas.

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