β decay of odd-A Cs isotopes in the interacting boson-fermion model

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The structure of odd-mass isotopes of Cs and Xe is described in the framework of the proton-neutron interacting boson-fermion model. The model provides a consistent description of β decay from Cs to Xe nuclei of mass number $A = 125$, 127, and 129.

I. INTRODUCTION

The interacting boson [1] and the interacting boson-fermion model [2] have been remarkably successful in the description of a variety of nuclear structure phenomena. As one of the possible applications, we consider here the description of β-decay rates in odd-mass nuclei. β-decay rates are very sensitive to details of wave functions and therefore can provide a fine test of the nuclear model. The application of the proton-neutron interacting boson-fermion model (IBFM2) [2–4] to the β decay [5,6] was already proposed for nuclei in the region $52 \leq Z \leq 58$ [7], for Ru and Tc nuclei [8,9] and for beta transitions from even-even to odd-odd nuclei [10]. For β transitions from spherical Rh to Pd odd-mass nuclei this approach was very successful [5]. In that case [near the SU(5) limit of the interacting boson model (IBM)] the wave functions are dominated by few big components and the calculation of β-decay rates tests those components. In this paper we analyze the β-decay from Cs to Xe isotopes of mass number $A = 125$, 127, 129. These isotopes can be considered to be around the O(6) limit of the IBM. The wave functions are very complex, some components can be more important in the β-decay, but contributions from many small components can add or cancel, and therefore one can have a more sensitive test of the model than in the spherical case. We first calculate the energy levels, then test the wave functions on the basis of electromagnetic matrix elements (transitions and static moments), and in the final step calculate the β-decay rates. This final step is parameter free and provides a unique test of wave functions.

II. THE IBFM2 MODEL

To describe an odd-A nucleus in the IBFM2, an odd nucleon is coupled to an even-even core of proton- and neutron-bosons. The Hamiltonian is written as

$$H = H^{\mathrm{B}} + H^{\mathrm{F}} + V^{\mathrm{BF}}, \tag{1}$$

where H^{B} is an IBM2 Hamiltonian [1]:

$$
\begin{aligned}
H^{\mathrm{B}} = {} & \epsilon_d (n_{d_\nu} + n_{d_\pi}) + \kappa (Q^{\mathrm{B}}_\nu \cdot Q^{\mathrm{B}}_\pi) \\
& + \frac{1}{2} \xi_2 [(d^\dagger_\nu s^\dagger_\pi - d^\dagger_\pi s^\dagger_\nu) \cdot (\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)] \\
& + \sum_{K=1,3} \xi_K ([d^\dagger_\nu d^\dagger_\pi]^{(K)} \cdot [\tilde{d}_\pi \tilde{d}_\nu]^{(K)}) \\
& + \frac{1}{2} \sum_{L=0,2,4} c^\nu_L ([d^\dagger_\nu d^\dagger_\nu]^{(k)} \cdot [\tilde{d}_\nu \tilde{d}_\nu]^{(k)}), \tag{2}
\end{aligned}
$$

where

$$Q^{\mathrm{B}}_\nu = d^\dagger_\nu s_\nu + s^\dagger_\nu \tilde{d}_\nu + \chi_\nu [d^\dagger_\nu \tilde{d}_\nu]^{(2)}, \tag{3}$$

$$Q^{\mathrm{B}}_\pi = d^\dagger_\pi s_\pi + s^\dagger_\pi \tilde{d}_\pi + \chi_\pi [d^\dagger_\pi \tilde{d}_\pi]^{(2)} \tag{4}$$

are the boson quadrupole operators. H^{F} is the Hamiltonian of the odd fermion,

$$H^{\mathrm{F}} = \sum_i \epsilon_i n_i, \tag{5}$$

where ϵ_i is the quasiparticle energy of the ith orbital while n_i is its number operator, and V^{BF} is the interaction between the bosons and the odd particle,

$$
\begin{aligned}
V^{\mathrm{BF}} = {} & \sum_{i,j} \Gamma_{ij} ([a^\dagger_i \tilde{a}_j]^{(2)} \cdot Q^{\mathrm{B}}_{\rho'}) + A \sum n_i n_{d_{\rho'}} \\
& + \sum_{i,j} \Lambda^j_{ki} \{ : [[d^\dagger_\rho \tilde{a}_j]^{(k)} a^\dagger_i s_\rho]^{(2)} : \cdot [s^\dagger_{\rho'} \tilde{d}_{\rho'}]^{(2)} + \mathrm{H.c.} \}. \tag{6}
\end{aligned}
$$

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TABLE I. IBM2 parameters taken from Ref. [12]. The unit is MeV except for the dimensionless χ_ν. The parameters $\chi_\pi = -0.80$ and $\xi_1 = \xi_2 = 0.24$ MeV, $\xi_3 = -0.18$ MeV are fixed.

Odd nuclei	Core nucleus	ϵ_d	κ	χ_ν	c_0^ν	c_2^ν
^{125}Cs	^{124}Xe	0.70	-0.145	0.00	0.05	-0.10
^{125}Xe, ^{127}Cs	^{126}Xe	0.70	-0.155	0.20	0.10	-0.10
^{127}Xe, ^{129}Cs	^{128}Xe	0.70	-0.170	0.33	0.30	0.00
^{129}Xe	^{130}Xe	0.76	-0.190	0.50	0.30	0.10

The modified annihilation operator $\tilde{d}$ is defined by $\tilde{d}_m = (-1)^m d_{-m}$. The symbols ρ and ρ' denote π (ν) and ν (π) if the odd fermion is a proton (neutron). The creation operator of the odd particle is written as $a^\dagger_{jm}$, while the modified annihilation operator is defined as $\tilde{a}_{jm} = (-1)^{j-m} a_{j-m}$. The orbital dependence of the quadrupole and the exchange interactions are [11]

$$\Gamma_{i,j} = (u_i u_j - v_i v_j) Q_{i,j} \Gamma, \tag{7}$$

$$\Lambda^j_{k,i} = -\beta_{k,i} \beta_{j,k} \left(\frac{10}{N_\rho (2j_k+1)} \right)^{1/2} \Lambda, \tag{8}$$

where

$$\beta_{i,j} = (u_i v_j + v_i u_j) Q_{i,j}, \tag{9}$$

$$Q_{i,j} = \left\langle l_i, \frac{1}{2}, j_i \middle\| Y^{(2)} \middle\| l_j, \frac{1}{2}, j_j \right\rangle. \tag{10}$$

III. CALCULATIONS

A. Hamiltonian and energy levels

The Hamiltonian consists of the boson Hamiltonian, the Hamiltonian of the odd fermion, and the interaction between the bosons and the odd fermion. The IBM2 parameters for the even-even Xe isotopes are taken from Ref. [12]. To describe the odd-even isotopes of Cs (Xe), we couple a proton (neutron) to these cores.

In Table I we show the cores for the considered Cs and Xe isotopes and the IBM2 parameters taken from Ref. [12].

1. Cs isotopes

The odd-mass Cs isotopes are described by coupling an odd proton to the even-even Xe cores. The proton single-particle energies are taken from Ref. [13] (only the energy of $d_{5/2}$ has been reduced from the original value of 0.20 MeV to 0.05 MeV). The BCS equations are solved with the orbitals $g_{7/2}$, $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, $h_{11/2}$, $h_{9/2}$, and $f_{7/2}$ with $\Delta = 12/\sqrt{A}$ MeV. In the IBFM2 calculation for positive-parity states we include the first four orbitals. In the boson-fermion interaction, the quadrupole and the monopole interactions are included between the odd proton and the neutron bosons, in addition to the exchange interaction of the quadrupole type. We allow the interaction strengths to vary gradually depending on the mass number.

TABLE II. Single-particle energies (MeV) of the proton orbitals in Cs.

$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$d_{3/2}$	$h_{11/2}$	$h_{9/2}$	$f_{7/2}$
0.05	0.00	3.35	3.00	1.50	7.00	8.00

The adopted single-particle energies are shown in Table II. The quasiparticle energies and the u, v factors calculated with them have been used in H^F and V^{BF}.

In Table III we present the parameter values in V^{BF} interaction potential.

The results of the calculation are shown in Fig. 1 (the experimental data are taken from Refs. [14–16]). A generally reasonable agreement is seen. To see more detail, the calculated $3/2^+_1$ systematically comes lower than the experimental counterpart, while the calculated $5/2^+_1$ lies higher. This difference may be explained by the Coriolis effect. The locations of the yrast states with $I \geq 7/2$ are reasonably well reproduced. Nevertheless, we notice that the $\Delta I = 1$ structure consisting of $9/2^+$, $13/2^+$, $17/2^+$, $21/2^+$ yrast and $11/2^+$, $15/2^+$, $19/2^+$ yrare states, was proposed as intruder proton $g_{9/2}$ configuration in ^{125}Cs [17], as well as some levels in ^{127}Cs [18]. These levels are not presented in Fig. 1, but we notice that the excitation energies of these levels, calculated in the valence shell space, are very close to the $g_{9/2}$ experimental ones. This could be the reason why they have not been observed.

2. Xe isotopes

The odd-mass Xe isotopes are described by coupling an odd neutron hole to the neighboring even-even Xe cores. The single-particle energies are taken from Ref. [19], except small modifications for $g_{7/2}$ and $h_{11/2}$. For $g_{7/2}$ the values 0.3, 0.35, 0.4 MeV are taken for $A = 125$, 127, 129, respectively, while the single-particle energy of $h_{11/2}$ is set to 1.30 MeV for all isotopes (see Table IV).

The parameter values used in V^{BF} are shown in Table V. These values are almost identical to the approximate projection from the IBFM1 values in Ref. [19].

The results of the calculation are shown in Fig. 2. A reasonable agreement is seen. In ^{127}Xe, there are two different interpretations about the spin of the 510-keV level. Although Ref. [15] adopts $I = 3/2$, Refs. [19,20] insist on $I = 5/2$ because of very weak β-decay from $I = 1/2$ in ^{127}Cs and the level systematics in neighboring nuclei. We have chosen the latter on the basis of level systematics. However, the spin of 510 keV is still an open problem. The wave functions and organization of levels into bands in the present IBFM2 cal-

TABLE III. Parameters in the boson-fermion interaction (MeV) for Cs.

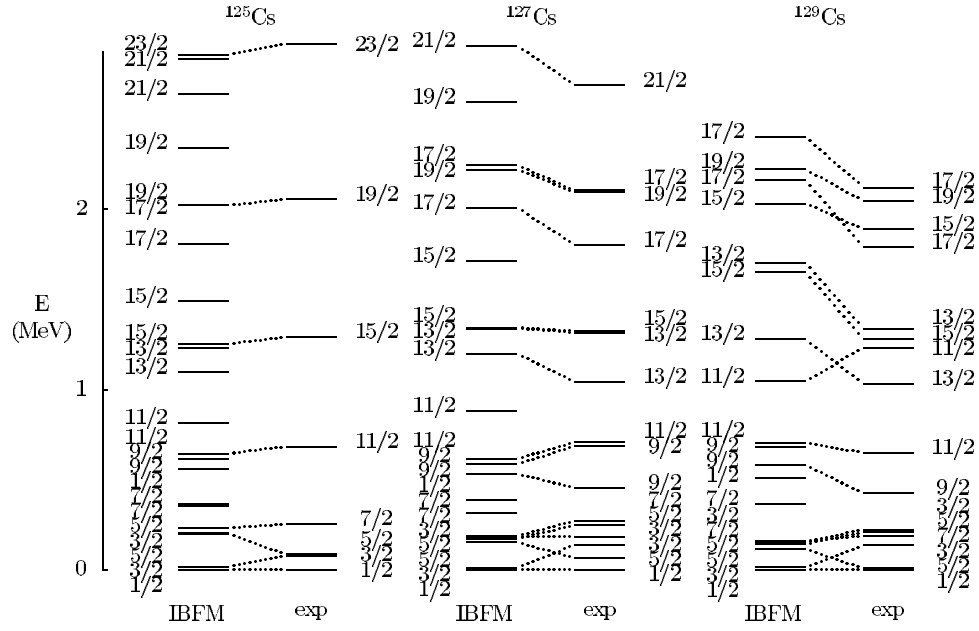| Isotope	Γ	A	Λ
^{125}Cs	0.90	-0.60	1.65
^{127}Cs	0.76	-0.66	2.30
^{129}Cs	0.74	-0.80	2.90

FIG. 1. Comparison between the calculated (IBFM) and the experimental (exp) energy levels of positive parity in ^{125,127,129}Cs. The experimental data are taken from Refs. [14–16].

culation are in very good agreement with the recent analysis in IBFM1 [19].

B. Electromagnetic properties

The electromagnetic transition operators are

$$T^{(E2)} = e_\pi^B Q_\pi^B + e_\nu^B Q_\nu^B + \sum_{i,j} e'_{i,j} [a_i^\dagger \tilde{a}_j]^{(2)}, \quad (11)$$

where

$$e'_{i,j} = -\frac{1}{\sqrt{5}} (u_i u_j - v_i v_j) \langle i || r^2 Y^{(2)} || j \rangle. \quad (12)$$

For $M1$,

$$T^{(M1)} = \sqrt{\frac{3}{4\pi}} \left(g_\pi^B L_\pi^B + g_\nu^B L_\nu^B + \sum_{i,j} e_{i,j}^{(1)} [a_i^\dagger \tilde{a}_j]^{(1)} \right), \quad (13)$$

where

$$e_{i,j}^{(1)} = -\frac{1}{\sqrt{3}} (u_i u_j + v_i v_j) \langle i || g_l \mathbf{l} + g_s \mathbf{s} || j \rangle. \quad (14)$$

TABLE IV. Single-particle energies (MeV) of the neutron orbitals in Xe, taken from Ref. [19]. The energy of $g_{7/2}$ has been changed.

	$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$d_{3/2}$	$h_{11/2}$
^{125}Xe	0.00	0.30	1.55	2.00	1.30
^{127}Xe	0.00	0.35	1.55	2.00	1.30
^{129}Xe	0.00	0.40	1.60	2.00	1.30

We use the boson effective charge $e^B = 0.150e$ b in order to explain the $B(E2)$ values and the quadrupole moments in these odd-mass nuclei. The value used in our calculations is larger for some isotopes than the one determined from the corresponding even-even cores ($\approx 0.108e$ b) [21]. This difference may be due to a polarization effect caused by the odd fermion. For the odd proton in Cs $e_\pi^F = 1.5e$, while for the odd neutron in Xe $e_\nu^F = 0.5e$. For the magnetic dipole operator, the boson g factors for all the isotopes are $g_\nu^B = 0$, $g_\pi^B = 0.8\mu_N$. For the odd proton in Cs, the spin g factor is reduced by the factor of 0.85, while for the odd neutron in Xe, the spin g-factor is reduced by the factor of 0.5. The calculated electromagnetic transitions and static moments are in reasonable agreement with experimental data. In Fig. 3 some of the results of the calculations are shown. On the basis of calculated excitation energies and electromagnetic properties we may conclude that the IBFM2 description of the analyzed Cs and Xe isotopes is realistic. However, the β-decay rates, where both the parent and the daughter wave functions are involved, can give a more conclusive answer.

C. β decay

The Fermi $\Sigma_k t^\pm (k)$ and the Gamow-Teller $\Sigma_k t^\pm (k) \sigma(k)$ transition operators [22] can be expressed in the framework of IBFM2. They can be constructed by the transfer operators [2,7,11,22]

TABLE V. Parameters in the boson-fermion interaction (MeV) for Xe.

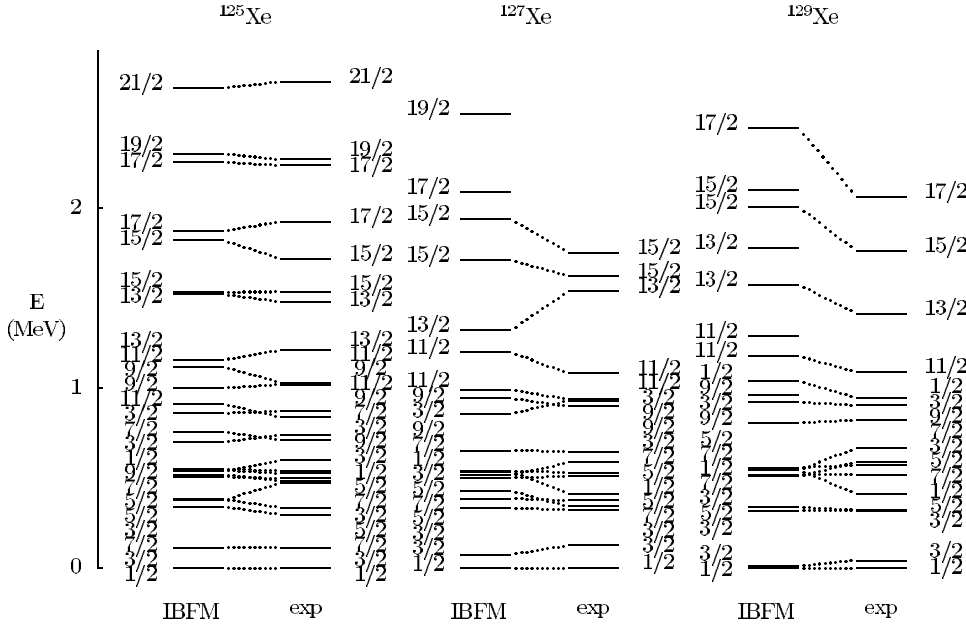| Isotope	Γ	A	Λ
^{125}Xe	0.39	-0.42	0.40
^{127}Xe	0.44	-0.42	0.40
^{129}Xe	0.50	-0.42	0.40

FIG. 2. Comparison between the calculated (IBFM) and the experimental (exp) energy levels of positive parity in ^{125,127,129}Xe. The experimental data are taken from Refs. [20,14–16].

$$A_m^{\dagger(j)} = \zeta_j a_{jm}^{\dagger} + \sum_{j'} \zeta_{jj'} s^{\dagger} [\tilde{d} a_{j'}^{\dagger}]_m^{(j)} \quad (\Delta n_j = 1, \Delta N = 0), \tag{15}$$

$$B_m^{\dagger(j)} = \theta_j s^{\dagger} \tilde{a}_{jm} + \sum_{j'} \theta_{jj'} [d^{\dagger} \tilde{a}_{j'}]_m^{(j)} \quad (\Delta n_j = -1, \Delta N = 1). \tag{16}$$

The former creates a fermion, while the latter annihilates a fermion simultaneously creating a boson. Either operator increases the quantity $n_j + 2N$ by one unit. The conjugate operators are

$$\tilde{A}_m^{(j)} = (-1)^{j-m} \{A_{-m}^{\dagger(j)}\}^{\dagger} = \zeta_j^* \tilde{a}_{jm} + \sum_{j'} \zeta_{jj'}^* s [d^{\dagger} \tilde{a}_{j'}]_m^{(j)}$$
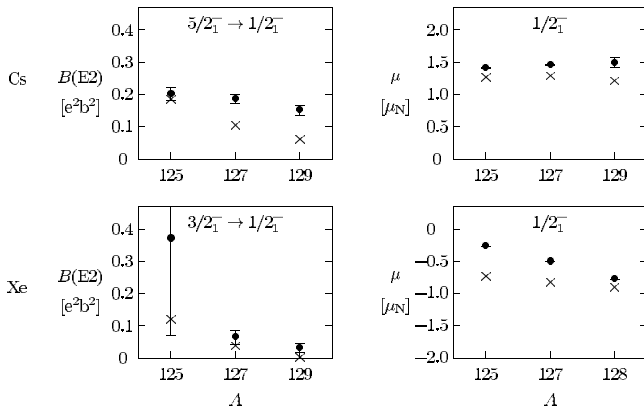
$$(\Delta n_j = -1, \quad \Delta N = 0), \tag{17}$$

FIG. 3. $B(E2)$ values and magnetic moments. The symbol ● with the error bar denotes the experimental data, while × shows the calculated values.

$$\tilde{B}_m^{(j)} = (-1)^{j-m} \{B_{-m}^{\dagger(j)}\}^{\dagger} = -\theta_j^* s a_{jm}^{\dagger} - \sum_{j'} \theta_{jj'}^* [\tilde{d} a_{j'}^{\dagger}]_m^{(j)}$$

$$(\Delta n_j = 1, \Delta N = -1), \tag{18}$$

where the asterisks mean complex conjugate. These decrease the quantity $n_j + 2N$ by one unit.

The IBFM image of the Fermi $\Sigma_k \mathbf{t}^{\pm}(k)$ and the Gamow-Teller transition operator $\Sigma_k t^{\pm}(k) \sigma(k)$ are written as

$$O^{\mathrm{F}} = \sum_j -\sqrt{2j+1} [P_\nu^{(j)} P_\pi^{(j)}]^{(0)}, \tag{19}$$

$$O^{\mathrm{GT}} = \sum_{j'j} \eta_{j'j} [P_\nu^{(j')} P_\pi^{(j)}]^{(1)}, \tag{20}$$

where

$$\eta_{j'j} = -\frac{1}{\sqrt{3}} \left\langle l' \frac{1}{2}; j' \left\| \boldsymbol{\sigma} \right\| l \frac{1}{2}; j \right\rangle$$

$$= -\delta_{l'l} \sqrt{2(2j'+1)(2j+1)} W\left(lj' \frac{1}{2}1; \frac{1}{2}j\right). \tag{21}$$

The transfer operators $P_\rho^{(j)}$ are chosen from Eqs. (15)–(18) depending on the nuclei. In the present case,

$$P_\pi^{(j)} = \tilde{A}_\pi^{(j)}, \tag{22}$$

$$P_\nu^{(j)} = \tilde{B}_\nu^{(j)}. \tag{23}$$

The squares of the β-decay matrix elements are

$$\langle M_{\mathrm{F}} \rangle^2 = \frac{1}{2I_i+1} |\langle I_f \| O^{\mathrm{F}} \| I_i \rangle|^2, \tag{24}$$

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$$\langle M_{GT}\rangle^2 = \frac{1}{2I_i+1}|\langle I_f||O^{GT}||I_i\rangle|^2 \qquad (25)$$

from which the ft value is calculated by

$$ft = \frac{6163}{\langle M_F\rangle^2 + (G_A/G_V)^2\langle M_{GT}\rangle^2} \qquad (26)$$

in units of second where $(G_A/G_V)^2 = 1.59$.

Now we estimate the coefficients η_j, $\eta_{jj'}$, θ_j, $\theta_{jj'}$ appearing in Eqs. (15)–(18), following the formulation of Ref. [2]:

$$\zeta_j = u_j \frac{1}{K_j'}, \qquad (27)$$

$$\zeta_{jj'} = -v_j\beta_{j'j}\left(\frac{10}{N(2j+1)}\right)^{1/2}\frac{1}{KK_j'}, \qquad (28)$$

$$\theta_j = \frac{v_j}{\sqrt{N}}\frac{1}{K_j''}, \qquad (29)$$

$$\theta_{jj'} = u_j\beta_{j'j}\left(\frac{10}{2j+1}\right)^{1/2}\frac{1}{KK_j''}, \qquad (30)$$

where N is N_π or N_ν, depending on the transfer operator, and K, K_j', K_j'' are determined by

$$K = \left(\sum_{jj'}\beta_{jj'}^2\right)^{1/2}, \qquad (31)$$

and the conditions

$$\sum_{\alpha J}\langle\text{odd};\alpha J||A^{\dagger j}||\text{even};0_1^+\rangle^2 = (2j+1)u_j^2, \qquad (32)$$

$$\sum_{\alpha J}\langle\text{even};0_1^+||B^{\dagger j}||\text{odd};\alpha J\rangle^2 = (2j+1)v_j^2. \qquad (33)$$

Formulas (27)–(33) are valid when the odd nucleon is a particle. If the latter half of a major shell is partly occupied (e.g., $N=73$), we consider the fully closed shell as the vacuum (e.g., $N=82$ core to deal with $66<N<82$), according to the convention of IBM and IBFM. In this case, the microscopic derivations are done for the holes in the shell. For example, Eq. (15) is an IBFM image of a hole creation operator. Then the formulas corresponding to Eqs. (27)–(33) can be obtained by interchanging u_j and v_j.

The scheme is essentially the same as in Ref. [7]. Namely, the IBFM images of the real particle creation and annihilation operators are constructed [11,22], and then they are coupled to form the Fermi and the Gamow-Teller operators. Although some previous works introduced overall normalization factors to account for the absolute values of the beta-transition rates [7], we did not introduce any adjustable parameters in the β-decay operators.
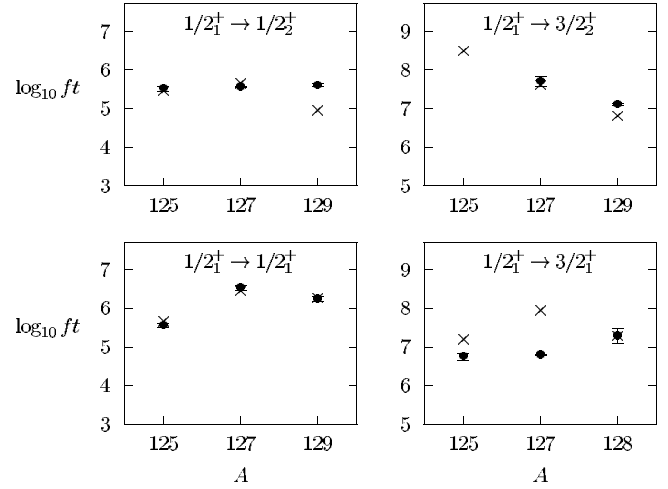

FIG. 4. The β-decay rates from ^{A}Cs to ^{A}Xe shown in terms of $\log_{10}ft$ values. The symbol ● with the error bar denotes experimental data, while × presents the calculated value.

For Xe, because the odd neutron is a hole in respect to the boson core, u_j and v_j are interchanged in Eqs. (27)–(33).

Figure 4 shows β-decay rates to the yrast and the yrare $1/2+$ and $3/2^+$ levels in terms of $\log_{10}ft$ values. The experimental values have been derived by electron capture and β^+ experiments in Refs. [14–16]. By taking the wave functions from our calculations, the decay rates from the ground states $(1/2_1^+)$ to the ground states $(1/2_1^+)$ are reproduced. In addition, we obtain reasonable agreement in decays to $1/2_2^+$, $3/2_1^+$, and $3/2_2^+$, except for the decay to $3/2_1^+$ in ^{127}Xe. Decay rates to higher excited levels are very sensitive to details in wave functions. In that sense, we notice a reasonable agreement in $\log_{10}ft$ values for decays to the $3/2_3^+$ levels in $A=125$, 127, and 129: theoretical values 6.402, 6.887, and 7.147, compared to the experimental data 6.360(70), 6.308(12), and 6.400(50), respectively.

We notice that once the wave functions are determined in IBFM2 calculations of energy levels, the $\log_{10}ft$ values are obtained in a parameter free calculation. In fact, in contrast to shell model calculations of $\log_{10}ft$ values, we do not use any additional normalization.

Looking at the wave functions, the ground states of ^{125,127,129}Cs are dominated by two orbitals $g_{7/2}$ and $d_{3/2}$ (30–40% each). The orbital $d_{5/2}$ has comparable amount of mixture, too. The mixture of the component $s_{1/2}$ (10–15%) is small. In the daughter nuclei ^{125,127,129}Xe, the dominant component of the ground state is $s_{1/2}$ (80–90%). The main contribution to the Gamow-Teller matrix elements comes from the term $[[\tilde{d}_\nu\nu s_{1/2}]^{(3/2)}\pi d_{3/2}]^{(1)}$. For the first excited states with $I=3/2$, the main component in the wave functions is $d_{3/2}$. The main contributions to the Gamow-Teller matrix elements come from the terms: $[[\tilde{d}_\nu\nu d_{3/2}]^{(3/2)}\pi d_{3/2}]^{(1)}$ and $[[\tilde{d}_\nu\nu d_{3/2}]^{(5/2)}\pi d_{3/2}]^{(1)}$, but cancellation occurs in these two contributions. That is one reason why $\log ft$ values to the $3/2_1^+$ states are larger (i.e., the matrix elements are smaller) than $\log ft$ values to the ground states $(1/2_1^+)$.

On the basis of calculated β-decay rates, the present calculation strongly supports the IBFM description of positive parity states in odd Xe isotopes, reported in Ref. [19]. On the other side, it does not confirm the structure of low-lying positive parity states in odd Cs isotopes, as described in the first calculation for these nuclei [4]. In fact, the main difference between that calculation and the present work is that in Ref. [4], due to a weak, almost negligible exchange interaction, the wave functions are not very mixed, while in the present calculation characterized by a strong exchange interaction, they show a strong configuration mixing. In the case of a weak exchange interaction for Cs isotopes, the wave functions of the parent nuclei give β-decay rates that are one to two order of magnitude wrong, even for the decays to the daughter ground states. Recent calculations for odd-mass [13] and odd-odd [23] Cs isotopes, with strong exchange interaction, in the light of the present calculation seem to be far more realistic.

IV. CONCLUSIONS

In a systematic calculation, we have performed an IBFM2 analysis of $A = 125$, 127, and 129 isotopes of Cs and Xe. The calculated energy level spectra and electromagnetic transition properties suggest that the choice of interaction parameters is realistic. In order to test how realistic the wave functions are, in the final step of our work we have calculated the β-decay rates from Cs to Xe nuclei. In our approach this type of calculation is a very sensitive test of wave functions, because it is parameter free, without any normalization of theoretical results. In addition to transition rates to ground states, it also gives transition rates to excited levels of daughter nuclei. The results of β-decay calculations are in very good agreement with observed data. The present analysis of β-decay in O(6) like nuclei, together with recent calculations for spherical nuclei, shows that the proton-neutron interacting boson-fermion model is appropriate for calculation of β-decay properties in odd nuclei.

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