PHYSICAL REVIEW D VOLUME 32, NUMBER 1 1 JULY 1985

Lorentz noninvariance and the Eötvös experiments

Ephraim Fischbach and Mark P. Haugan
Physics Department, Purdue University, West Lafayette, Indiana 47907

Dubravko Tadić
Zavod za Teorijsku Fiziku, Prirodoslovno Matematicki Fakultet, University of Zagreb, YU-41000 Zagreb, Croatia, Yugoslavia

Hai-Yang Cheng
Physics Department, Brandeis University, Waltham, Massachusetts 02254
(Received 17 December 1984)

We explore the consequences of Lorentz noninvariance (LNI) for the Eötvös experiments in several models. It is shown that a violation of Lorentz invariance leads to an anomalous difference in the gravitational acceleration of two test masses which depends on their composition. Using the present experimental limits from the Eötvös experiments, we then derive a limit on the magnitude of a possible violation of Lorentz invariance in the model of Nielsen and Picek (NP). In an Appendix we present a detailed discussion of the contribution to the nuclear binding energy due to the weak interactions, which are the presumed source of the LNI effects in the NP model.

I. INTRODUCTION

There has been renewed interest of late in the possibility that Lorentz invariance may not be an exact symmetry of nature.[1–11] On the experimental side, an analysis of high-energy data for the $K^0-\bar{K}^0$ system[1–4] has reported indications of an anomalous energy dependence of the fundamental parameters τ_S, Δm, and η_{+-} as determined in the proper frame of the kaons. On the theoretical side, various mechanisms have been proposed for breaking Lorentz invariance at appropriately high energies and/or short distances.[1–11] Manifestations of Lorentz noninvariance (LNI) that have been considered to date include an anomalous energy (or velocity) dependence of some physical parameter, an apparent violation of angular momentum conservation, and violations of parity conservation.

The object of this paper is to explore another, somewhat less obvious, manifestation of LNI, namely, a breakdown of the universality of free fall (UFF), i.e., of the so-called weak equivalence principle. The connection between LNI and UFF is of interest for several reasons: To start with, the great precision of the Eötvös-Dicke-Braginskii (EDB) experiments[12–14] places strong constraints on models of LNI, as we discuss below, and these constraints will become even tighter when current experiments with increased precision are completed. Conversely, any manifestation of LNI would imply a breakdown of the principle of the universality of free fall, and hence of the equivalence principle.

A general discussion of the interrelations among the equivalence principle, UFF, Lorentz invariance, and energy conservation has been given previously by Haugan.[15] For present purposes the main conclusions of that discussion can be summarized as follows: The weak equivalence principle (WEP), which is a variant of the UFF hypothesis, simply restates the null result of the EDB experiments, namely, the absence (to a great precision) of any difference in the gravitational acceleration of different test masses. However, conventional metric theories of gravity assume a stronger version of the equivalence principle in which one adds to the WEP the assumption that "the outcome of any *local test experiment* is independent of where and when in the universe it is performed, and independent of the velocity of the (freely falling) experimental apparatus."[16] Depending on whether the "local test experiment" excludes or includes gravitational forces, the resulting equivalence principles are termed, respectively, the Einstein equivalence principle (EEP) and the strong equivalence principle (SEP). The connection between the various forms of the equivalence principle and Lorentz invariance is embodied in a conjecture due to Schiff:[17] He noted that since the self-energies of dissimilar test bodies contain different relative contributions from strong, electromagnetic, and weak interactions, it would be difficult to understand why all bodies accelerate (gravitationally) at the same rate in the absence of a principle establishing that all forms of energy behaved kinematically in the same way. Schiff was thus led to suggest that the WEP (derived from the EDB experiments) actually implied Lorentz invariance as well, and hence that "any complete and self-consistent gravitation theory that obeys WEP must also, unavoidably, obey EEP."[16,17] It follows that a breakdown of Lorentz invariance also implies a breakdown of the WEP (or UFF), and hence leads to a non-null result in the EDB experiments. Haugan[15] has observed that a quantitative connection between LNI and the EDB experiments can be derived by using energy conservation: For a body falling in a gravitational field the decrease in potential energy is accompanied by a corresponding increase in its kinetic energy. However, in the absence of Lorentz invariance this increase in kinetic energy will not result in the expected increase in the body's velocity, and so the body will appear to have experienced an anomalous (composition-dependent) acceleration, which is what the

EDB experiments measure.

In the present paper we analyze the consequences of Lorentz noninvariance for the EDB experiments by elaborating on the LNI model of Nielsen and Picek[5] (NP). We have focused on this model because it leads to very specific quantitative predictions for a wide variety of processes, and at the same time possesses some general features which are likely to show up in any model of Lorentz noninvariance. In Sec. II we describe the NP model, and in Sec. III we apply this model to calculate the difference in the gravitational accelerations of two test masses due to a LNI contribution to the weak interactions. Section IV discusses the implications of our results, and in the Appendix we derive the expression for the weak-interaction contribution to the nuclear binding energy which is needed in Sec. III.

II. THE LNI MODEL OF NIELSEN AND PICEK

Nielsen and Picek[5] have suggested that LNI effects may arise in the weak interactions due to a modification of the Z^0 or W-boson propagator $D_{\mu\nu}(x-y)$. In the usual expression for $D_{\mu\nu}(x-y)$ appropriate for low energies,

$$D_{\mu\nu}(x-y)=\frac{-ig_{\mu\nu}}{m_{W,Z}^2}\delta^4(x-y)\ , \tag{2.1}$$

we make the replacement

$$g_{\mu\nu}\to g_{\mu\nu}+\chi_{\mu\nu}\ , \tag{2.2}$$

where $\chi_{\mu\nu}$ is a constant tensor. To specify the form of $\chi_{\mu\nu}$ NP impose the conditions (a) Hermiticity, $\chi_{\mu\nu}=\chi_{\nu\mu}^{\dagger}$, (b) tracelessness, $g^{\mu\nu}\chi_{\mu\nu}=0$, and (c) rotational invariance (in the preferred frame). Hermiticity of $\chi_{\mu\nu}$ is required to ensure that the overall Hamiltonian is Hermitian, and $\chi_{\mu\nu}$ can be chosen to be traceless by a suitable renormalization of the weak coupling constant. NP further assume that rotational invariance still holds in a frame at rest with respect to the 3°K cosmic radiation, and implement this with the assumption that $\chi_{\mu\nu}=\chi_{\nu\mu}$. Taken together these assumptions fix $\chi_{\mu\nu}$: In our conventions[18]

$$x^{\mu}=(\mathbf{x},x_4=icx^0)\ , \tag{2.3a}$$

$$g_{\mu\nu}=\delta_{\mu\nu} \tag{2.3b}$$

and hence,

$$\chi_{\mu\nu}=\alpha\begin{bmatrix}\frac{1}{3} & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{3} & \\ & & & -1\end{bmatrix}\ , \tag{2.4}$$

where α is an overall constant. The appeal of the NP model lies in part in its predictive power in that $\chi_{\mu\nu}$, which has been determined via these arguments up to an overall constant, can manifest itself in a variety of phenomena. For the sake of generality we can, however, allow α to be different for each interaction.

It is important for later purposes to understand more thoroughly the nature of the LNI effects that arise in such a model. Consider, for example, the behavior of the quantity $g_{\mu\nu}p^{\mu}p^{\nu}$:

$$-g_{\mu\nu}p^{\mu}p^{\nu}=m^2\to m^2-\chi_{\mu\nu}p^{\mu}p^{\nu}=m^2-\alpha(\tfrac{1}{3}\mathbf{p}^2+p_0^2)\ , \tag{2.5}$$

where m is the nucleon mass. For $\alpha\neq 0$ this is clearly not a Lorentz-invariant expression, and it has the property that its value in the proper frame of the nucleon varies with the velocity $\mathbf{v}$ of the nucleon with respect to the preferred frame. This is similar to the behavior that arises when the LNI effects are due to an external field whose sources are at rest in the laboratory (or perhaps in the frame of the galaxy, etc.). In this case the energy of a nucleon in its proper frame also varies with the velocity of that frame with respect to the field according to

$$m\to\gamma^J m\ , \tag{2.6}$$

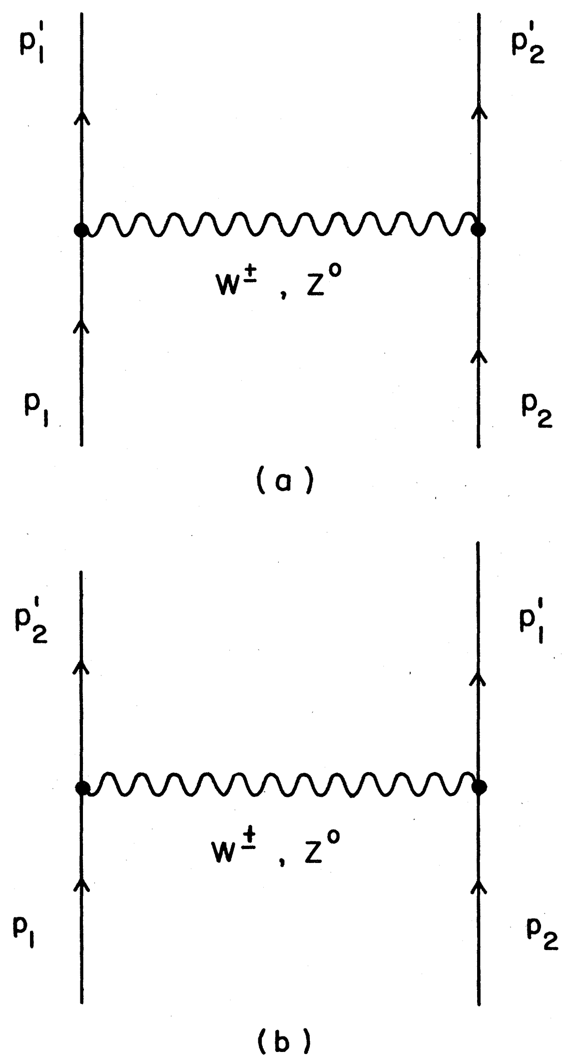

FIG. 1. Contributions to the parity-conserving amplitude for nucleon-nucleon scattering. (a) Direct diagram which gives $X(1,2)$ in Eq. (A5). (b) Exchange diagram which gives $Y(1,2)$. For purposes of evaluating M_w in Eq. (A4) we can set $p_1'=p_1$ and $p_2'=p_2$.

where $\gamma=(1-v^2)^{-1/2}$ is the usual relativistic factor, and J is the spin angular momentum of the quantum of the field ($J=1$ for photons, etc.).

Owing to the particular choice of $\chi_{\mu\nu}$ in the NP model, care must be taken in extracting the LNI effects in the EDB experiments. Consider the weak-interaction contributions arising from $W^{\pm}$ and Z^0 exchange shown in Fig. 1. As described in more detail below, we assume an effective low-energy current-current interaction at the nucleon level, which leads to a sufficiently accurate picture for present purposes. We then evaluate the *invariant* amplitude $\mathcal{M}$ arising from the diagrams in Fig. 1, where $\mathcal{M}$ is defined in terms of the S-matrix element S by[18]

$$S=1-i(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')\times\frac{m^2}{V^2}(p_{10}p_{20}p_{10}'p_{20}')^{-1/2}\mathcal{M}\ . \tag{2.7}$$

The purpose of focusing our attention on $\mathcal{M}$ is precisely because it is a Lorentz invariant in the absence of the contribution from $\chi_{\mu\nu}$. As we will see the presence of $\chi_{\mu\nu}$ leads to an explicit frame dependence of $\mathcal{M}$, in which the violation of Lorentz invariance manifests itself at low energies as an explicit violation of Galilean invariance. To see how this comes about let us consider as an illustration the calculation of the charged-current contribution in Fig. 1(b) to the weak self-energy of the deuteron. From Eqs. (2.1)–(2.4) the effective weak-interaction density is

$$\begin{aligned}\mathcal{H}_{\rm eff}(x)&=\frac{G_F}{\sqrt{2}}J_\mu^\dagger(x)(\delta_{\mu\nu}+\chi_{\mu\nu})J_\nu(x)+\text{H.c.}\ ,\\ J_\mu(x)&=i\bar{p}(x)\gamma_\mu(1+\gamma_5)n(x)\ ,\end{aligned} \tag{2.8}$$

where $G_F=1.166\,37(2)\times10^{-5}$ GeV^{-2} is the Fermi constant, and $p(x),n(x)$ are the field operators for the proton and neutron, respectively. Since we are interested in the weak contributions to the binding energy, we consider the spin-averaged parity-conserving amplitude $\langle\mathcal{M}\rangle$ which is given by

$$\begin{aligned}\langle\mathcal{M}\rangle&=\tfrac{1}{4}\sum_{\text{spins}}\mathcal{M}=\frac{G_F}{4\sqrt{2}}(\delta_{\mu\nu}+\chi_{\mu\nu})\text{Tr}\left[\gamma_\mu(1+\gamma_5)\frac{(-i\gamma\cdot p_1+m)}{2m}\gamma_\nu(1+\gamma_5)\frac{(-i\gamma\cdot p_2+m)}{2m}\right]T_{12}^{(+)}\\ &=\frac{G_F}{\sqrt{2}}\frac{1}{m^2}\left[p_1\cdot p_2-\alpha p_{10}p_{20}-\frac{\alpha}{3}\mathbf{p}_1\cdot\mathbf{p}_2\right]T_{12}^{(+)}\ ,\\ T_{12}^{(\pm)}&=\tau_1^+\tau_2^-\pm\tau_1^-\tau_2^+\ ,\end{aligned} \tag{2.9}$$

where m is the nucleon mass, and the τ's are the usual Pauli spin operators $[\tau^{\pm}=\frac{1}{2}(\tau_x\pm i\tau_y)]$. We next introduce center-of-mass momentum $\mathbf{P}$ and the relative momentum $\boldsymbol{\pi}$ in the usual way:[19]

$$\begin{aligned}\mathbf{p}_j&=\boldsymbol{\pi}_j+\frac{m_j}{M_0}\mathbf{P}=\boldsymbol{\pi}_j+m_j\mathbf{v}\ ,\\ \sum_j\boldsymbol{\pi}_j&=0\ ,\end{aligned} \tag{2.10}$$

where $M_0=\sum_j m_j=2m$ is the deuteron mass in the absence of any interactions. In the nonrelativistic limit the weak-interaction terms arising from $\boldsymbol{\pi}_j$ contribute only to the *internal* energy and can be dropped for present purposes. We are thus finally left with

$$\langle\mathcal{M}\rangle=-\frac{G_F}{\sqrt{2}}\left[1+\alpha+\frac{4\alpha}{3}\mathbf{v}^2\right]T_{12}^{(+)}\ . \tag{2.11}$$

We see from (2.11) that for $\alpha=0$ $\langle\mathcal{M}\rangle$ is indeed Lorentz invariant, but for $\alpha\neq0$ Galilean invariance (and hence Lorentz invariance) is violated through the dependence of $\langle\mathcal{M}\rangle$ on the center-of-mass velocity $\mathbf{v}$.

It is evident from (2.11) that when the appropriate overall factors are included, the α-independent term will give the usual parity-conserving weak contributions to the mass of the nucleus, while the α-dependent terms will describe the LNI effects. The calculation described above can then be extended to an arbitrary nucleus containing N neutrons and Z protons interacting via any combination of charged and neutral currents. The generalization of Eq. (2.11) is then given by Eqs. (A17) and (3.1) below, where A_w and B_w are weak-interaction-model-dependent constants. We present the details of this calculation in the Appendix, and proceed in the next section to use these results to set a limit on α from the EDB experiments.

III. LIMITS ON LNI EFFECTS IMPLIED BY THE EÖTVÖS EXPERIMENTS

In this section we use the result in Eq. (A17) to derive a quantitative limit on α from the EDB experiments. Let $\overline{M}_0=\overline{M}_0(Z,N)$ denote the sum of all contributions to the inertial mass of a nucleus (with Z protons and N neutrons), *exclusive* of the weak-interaction contribution. It then follows from (A3) and (A17) that the *total* inertial mass $M=M(Z,N)$ is given by

$$M=\overline{M}_0+M_w=\overline{M}_0+A_w+\alpha B_w(1+\tfrac{4}{3}\mathbf{v}^2)\ . \tag{3.1}$$

Equation (3.1) can then be inserted into the expression for the total conserved energy E of a test mass located at a height z in a gravitational field,

$$E=M+\tfrac{1}{2}M\mathbf{v}^2+M'gz\ . \tag{3.2}$$

In Eq. (3.2) $g=|\mathbf{g}|$ is the local acceleration of gravity (due to either the Earth or the Sun), and M' is the passive gravitational mass. Combining Eqs. (3.1) and (3.2), and retaining only the leading contribution from $\mathbf{v}^2$, we find

$$E \cong M_0+\tfrac{1}{2}M_0\mathbf{v}^2+\alpha B_w(1+\tfrac{11}{6}\mathbf{v}^2)+M'gz \;, \qquad M_0=\overline{M}_0+A_w \;. \tag{3.3}$$

If the test mass is now released at z and falls to $z=0$, then its velocity $v=|\mathbf{v}|$ and acceleration $a=|\mathbf{a}|$ are given in terms of the elapsed time t by the usual nonrelativistic relations (v_0 is the initial velocity)

$$v=v_0+at \;, \quad z=v_0t+\tfrac{1}{2}at^2 \;. \tag{3.4}$$

When the resulting expressions for E at z and $z=0$ are equated we find immediately

$$a\left[1+\frac{11}{3}\frac{\alpha B_w}{M_0}\right]=\frac{M'}{M_0}g \;. \tag{3.5}$$

Since B_w/M_0 varies from material to material, it follows that for $\alpha\neq0$ test masses of different composition will experience different gravitational accelerations, even if $M'=M_0$. Thus for two dissimilar test masses 1 and 2 the fractional difference in their accelerations is given by

$$\frac{\Delta a}{g}=\frac{a_1-a_2}{g}\simeq-\alpha\frac{11}{3}\left[\frac{B_{w1}}{M_{01}}-\frac{B_{w2}}{M_{02}}\right] . \tag{3.6}$$

Equation (3.6) thus establishes one of the goals of this paper, namely, a quantitative relation between Lorentz noninvariance ($\alpha\neq0$), and a deviation from universal free fall ($\Delta a\neq0$).

It is of interest to contrast Eq. (3.6) to the result that would arise from a coupling of the test masses to a long-range hypercharge (or baryon-number) field.[2,4,20] Such a field would also produce apparent local violations of Lorentz invariance (such as the dependence of the proper mass of an object on its velocity in the laboratory[1–4]), but these would be fundamentally different in the EDB experiments from those due to $\chi_{\mu\nu}$, as we now discuss. Consider, for example, the force on object 1 due to the combined gravitational and hypercharge fields of the Earth:

$$F_1=m_1a_1=-G\frac{m_Em_1}{R^2}\left[1-f^2\frac{y_Ey_1}{Gm_Em_1}\right] . \tag{3.7}$$

Here f is the hypercharge analog of the electric charge e, G is the Newtonian gravitational constant, R is the radius of the Earth, and $m_E(y_E)$ is the Earth's mass (hypercharge), etc. From (3.7) it follows immediately that

$$\frac{\Delta a}{g}=-\frac{f^2}{Gm^2}\left[\frac{y_E}{\mu_E}\right]\left[\frac{y_1}{\mu_1}-\frac{y_2}{\mu_2}\right] , \tag{3.8}$$

where $\mu_j\equiv m_j/m$ is the mass of object j in units of the nucleon mass. We see from Eqs. (3.7) and (3.8) that even though a hypercharge field leads to an anomalous acceleration, $\Delta a\neq0$, it does so by making the effective gravitational coupling composition dependent,

$$m_i\to m_i\left[1-\frac{f^2y_E}{Gm_E}\left[\frac{y_i}{m_i}\right]\right] , \tag{3.9}$$

and not by introducing a LNI velocity dependence as in the NP model. The NP model and the hypercharge field are thus examples of the two fundamental classes of EEP violations, namely, preferred frame effects (NP) and preferred location effects (hypercharge). Further discussion of such LNI effects can be found in Refs. 4, 15, 16, and 21. It is interesting to note from Eqs. (3.6) and (3.8) that these two mechanisms leading to $\Delta a\neq0$ can be distinguished by their different dependences on the compositions of test bodies. It follows that if a nonzero result for Δa were actually observed, then either or both of these mechanisms as a source of this effect could be ruled out by an appropriate series of measurements on a collection of disparate test masses. It appears to be a rather general feature of LNI mechanisms that different sources of such effects do in fact lead to different expressions for the composition dependence of Δa.

Returning to Eq. (3.6) we insert the appropriate values of B_w/M_0 from Table I, which are obtained by using the specific numerical values of the various parameters given in the Appendix. We find

$$|\alpha|\lesssim4.1\times10^{-2} \;, \quad 95\%\ \text{C.L.} \quad (\text{Ref. 13 using Al and Au}) \;, \tag{3.10a}$$

$$|\alpha|\lesssim1.5\times10^{-3} \;, \quad 95\%\ \text{C.L.} \quad (\text{Ref. 14 using Al and Pt}) \;. \tag{3.10b}$$

We note from Table I and Fig. 2 that B_w/M_0 is a very slowly varying function across the periodic table, which then leads to a large cancellation between the two terms in parentheses in Eq. (3.6). The sensitivity of the EDB experiments to the weak interaction can be improved somewhat by comparing elements which are as far apart as is practicable in the periodic table. Thus if carbon and uranium were used, the sensitivity would increase by approximately 40% compared to Al and Au, other things being equal. For the sake of completeness we also quote the limit on f^2 obtained from the EDB experiments in Ref. 4,

$$\frac{f^2}{Gm^2}<6\times10^{-8} \;, \quad 95\%\ \text{C.L.} \tag{3.11a}$$

TABLE I. Values of B_w/M_0 from Eq. (A29) for some selected elements.

Element	Z	N	A	$10^9B_w/M_0$
C	6	6	12	4.468
Al	13	14	27	4.447
Fe	26	30	56	4.417
Cu	29	35	64	4.394
Ag	47	61	108	4.347
Sm	62	88	150	4.275
Pt	78	117	195	4.223
Au	79	118	197	4.227
Pb	82	125	207	4.207
U	92	146	238	4.165

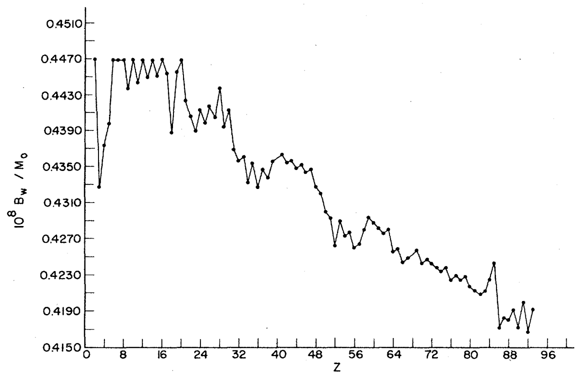

FIG. 2. Variation of B_w/M_0 as a function of Z. The explicit expression for B_w/M_0 is given in Eq. (A29) in the approximation of retaining only the vector-meson-exchange contributions to M_w.

using the Al-Au results of Ref. 13. The corresponding limit from the Al-Pt data of Ref. 14 is

$$\frac{f^2}{Gm^2} < 2\times 10^{-9} \ , \quad 95\% \text{ C.L.} \tag{3.11b}$$

It is of interest to compare the limits on α and f^2 implied by the EDB experiments to those obtained from the K^0-$\bar{K}^0$ system.[1–4] Nielsen and Picek[5] have shown that the effect of $\alpha\neq 0$ on the K_L-K_S mass difference $\Delta m=(m_L \ -m_S)$ is to impart to Δm a γ dependence which can be expressed in the form

$$\Delta m(\alpha)=\Delta m[1+\tfrac{4}{3}\alpha(\gamma^2-\tfrac{1}{4})] \ , \tag{3.12}$$

where Δm is the usual value that would obtain in a Lorentz-invariant world. For $\gamma^2 \gg 1$ this corresponds in the notation of Refs. 1–4 to a slope parameter $b_\Delta^{(2)}$ given by

$$b_\Delta^{(2)}=\tfrac{4}{3}\alpha \ . \tag{3.13}$$

Using the internal-fit results of Refs. 1–4 we find that at the 3σ level,

$$| b_\Delta^{(2)} | < 1.7\times 10^{-5} \ , \tag{3.14}$$

and hence

$$|\alpha| < 1.3\times 10^{-5} \ , \quad 99.7\% \text{ C.L.} \tag{3.15}$$

As shown in Ref. 4, we can also obtain from the K^0-$\bar{K}^0$ data the limit

$$f^2/Gm^2 < 1\times 10^{-14} \ , \quad 99.7\% \text{ C.L.} \tag{3.16}$$

IV. DISCUSSION AND SUMMARY

As noted in the Introduction, our objective has been to establish a quantitative connection between the violation of Lorentz invariance and the acceleration anomaly $\Delta a/g$ in the EDB experiments. Our results are given in Eqs. (3.6) and (3.10) for the specific model of Nielsen and Picek, in which the weak interactions are the source of the LNI effects. We see from Eqs. (3.10) and (3.15) that if the LNI mechanism were indeed universal, the limit implied by the K^0-$\bar{K}^0$ data would be stronger than that arising from EDB experiments, unless the pion contribution is substantially larger than the rough estimate given in the Appendix. However, on purely phenomenological grounds one could well imagine models in which the LNI mechanism manifested itself only in $\Delta S=0$ weak interactions, in which case an effect would be seen in the EDB experiments but not in the K^0-$\bar{K}^0$ system.

One can elaborate upon the phenomenological approach by asking for the limits on α that would follow from the EDB experiments if the source of the LNI effects were the strong or electromagnetic interactions. Up to an overall numerical factor, we would obtain in each case the analog of (3.6) with B_w replaced by B_{st} or B_{EM} for the strong and electromagnetic interactions, respectively. Since these constitute a larger fraction of M_0 than does B_w, the limits on α would be correspondingly stronger. Consider, for example, the Coulomb contribution B_{Coul} to the nuclear binding energy. Using the semiempirical mass formula[22,23] this is given by

$$B_{\text{Coul}}=\frac{3}{5}\frac{Z(Z-1)}{A^{1/3}}\frac{e^2}{R_0}\cong 0.81\frac{Z(Z-1)}{A^{1/3}}\text{MeV} \ , \tag{4.1}$$

where $R_0=1.07$ fm, and e is the electric charge. For the experiment of Ref. 14 comparing Al and Pt we have

$$\left|\frac{\Delta a}{g}\right| \approx |\alpha| \left|\frac{B_{\text{Coul}}(\text{Al})}{M_0(\text{Al})}-\frac{B_{\text{Coul}}(\text{Pt})}{M_0(\text{Pt})}\right| = 3\times 10^{-3}|\alpha| \ , \tag{4.2}$$

from which it follows that

$$|\alpha| \lesssim 4\times 10^{-10} \ , \quad 95\% \text{ C.L.} \tag{4.3}$$

We see that the limit on a presumed violation of Lorentz invariance in the nuclear Coulomb interactions is quite stringent, and those on the various components of the strong interactions would be better still.

This discussion illustrates the value of the EDB experiments in setting limits on LNI interactions, and other new interactions as well. Since almost any coupling will show up at some level as a contribution to the energy of a nucleus, the EDB experiments provide a general filter for a wide class of possible new interactions. As the limits on $\Delta a/g$ from the current generation of experiments improve, we will thus have available an increasingly more powerful tool for probing models of Lorentz noninvariance in a variety of interactions. Further discussion of the constraints implied by the EDB experiments for various interactions are given in Refs. 4, 21, 22, and 24.

We conclude with a brief discussion of a model of LNI mentioned in Ref. 3. There it was noted that a violation of Lorentz invariance could be introduced by taking lattice gauge theories seriously to the point of supposing that space-time really was a lattice. This would then lead to a modification of the usual relation between the energy $E(\mathbf{k})$ of a particle and its momentum $\mathbf{k}$,

$$E^2=\mathbf{k}^2+m^2 \rightarrow E^2=\mathbf{k}^2+m^2-\mathbf{k}^4/\Lambda^2 \ , \tag{4.4}$$

where Λ^{-1} is determined by the lattice spacing. Although the term proportional to $\mathbf{k}^4$ would, in principle, lead to an acceleration anomaly $\Delta a/g\neq 0$, the magnitude of this effect would be too small to show up in the present-day experiments. It thus appears that the best way to look for a LNI effect of the type suggested in (4.4) is in a high-energy experiment as discussed in Ref. 4.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy, under Grants Nos. DE-AC02-76ER01428 and DE-AC03-ER03230, the National Science Foundation, and the SiZ I of the S. R. Croatia under Grant No. YOR 82/051.

APPENDIX: CALCULATION OF THE WEAK-INTERACTION CONTRIBUTION TO THE NUCLEAR SELF-ENERGY

In this Appendix we calculate the weak-interaction contribution to the self-energy of an arbitrary nucleus in the presence of $\chi_{\mu\nu}$. Our discussion and results closely parallel those of Haugan and Will,[24] except that now some additional care must be taken to properly treat the LNI contributions. For our purposes it is sufficient to consider the effective low-energy interaction which arises from weak vector-meson exchange:[24,25]

$$\mathscr{H}_{\text{eff}}(x)=\frac{G_F}{\sqrt{2}}[J_\mu^{(c)}(x)^\dagger J_\nu^{(c)}(x)+J_\mu^{(n)}(x)^\dagger J_\nu^{(n)}(x)](\delta_{\mu\nu}+\chi_{\mu\nu})\ , \tag{A1}$$

$$J_\mu^{(c)}(x)=i\overline{\Psi}\gamma_\mu(1+\Delta\gamma_5)\tau^+\Psi\ ,$$
$$J_\mu^{(n)}(x)=A_p i\overline{\Psi}\gamma_\mu(1+a_p\gamma_5)\tau_p\Psi+A_n i\overline{\Psi}\gamma_\mu(1+a_n\gamma_5)\tau_n\Psi\ , \tag{A2}$$
$$\Psi=\begin{bmatrix}\psi_p\\ \psi_n\end{bmatrix},\quad \tau^\pm=\tfrac{1}{2}(\tau_x\pm i\tau_y)\ ,\quad \tau_{p,n}=\tfrac{1}{2}(I\pm\tau_z)\ .$$

Δ, A_p, A_n, a_p, and a_n are phenomenological parameters, whose values in some specific models will be discussed below. As noted previously, a description of $\mathscr{H}_{\text{eff}}$ in terms of fundamental quark fields (rather than in terms of phenomenological nucleon fields) would not change the results enough to be of interest at the present stage. We will return at the end of this Appendix to discuss this question in greater detail. The weak contribution M_w to the nucleon self-energy is given by

$$M_w=\int d^3x\langle 0|\mathscr{H}_{\text{eff}}|0\rangle\ , \tag{A3}$$

where $|0\rangle$ denotes the ground-state nuclear wave function. We can write M_w in the general form[26,27]

$$M_w=\frac{G_F}{2\sqrt{2}}\sum_{k,l}\int d^3x_1d^3x_2\phi_l^*(1)\phi_k^*(2)[X(1,2)\phi_l(1)\phi_k(2)-Y(1,2)\phi_l(2)\phi_k(1)]\delta^3(\mathbf{x}_1-\mathbf{x}_2)\ , \tag{A4}$$

where the factor $\frac{1}{2}$ corrects for double counting in $\sum_{k,l}$, and the presence of $\delta^3(\mathbf{x}_1-\mathbf{x}_2)$ reflects the short range of the vector-meson-exchange contribution to the weak interactions. $\phi_k(1)$ denotes the wave function for particle 1 in the nuclear state k, and $\sum_{k,l}$ accounts for the possibility that nucleons 1 and 2 can be in any of the nuclear states. The summation over k,l extends over all the quantum numbers of the nuclear levels including their isospins. However, the spin quantum number has already been summed over in obtaining $X(1,2)$ and $Y(1,2)$ which are given by

$$\begin{aligned}X(1,2)&=\tfrac{1}{4}(\delta_{\mu\nu}+\chi_{\mu\nu})\mathrm{Tr}\left[\gamma_\mu(1+\delta_1\gamma_5)\frac{(-i\gamma\cdot p_1+m)}{2m}\right]\mathrm{Tr}\left[\gamma_\nu(1+\delta_2\gamma_5)\frac{(-i\gamma\cdot p_2+m)}{2m}\right]\hat{\tau}\ ,\\ Y(1,2)&=\tfrac{1}{4}(\delta_{\mu\nu}+\chi_{\mu\nu})\mathrm{Tr}\left[\frac{(-i\gamma\cdot p_2+m)}{2m}\gamma_\mu(1+\delta_1\gamma_5)\frac{(-i\gamma\cdot p_1+m)}{2m}\gamma_\nu(1+\delta_2\gamma_5)\right]\hat{\tau}\ .\end{aligned} \tag{A5}$$

Here the parameters $\delta_{1,2}$ denote any of the constants Δ, a_p, or a_n in Eqs. (A2)–(A3), and $\hat{\tau}$ denotes one of the isospin factors

$$\hat{\tau}\sim T^{(+)}\ ,\ \tau_{1p}\tau_{2p}\ ,\ \tau_{1n}\tau_{2n}\ ,\ \tau_{1n}\tau_{2p}+\tau_{1p}\tau_{2n}\ . \tag{A6}$$

In order to extract the momentum-dependence of M_w we have partially undone the usual nonrelativistic approximation for the wave function $\phi_k(1)$,

$$\phi_k(1)=R_k(\mathbf{r}_1)\chi(\tau_1)\chi(\sigma_1)\ . \tag{A7}$$

$R_k(\mathbf{r}_1)$ is the spatial wave function and $\chi(\sigma_1)$ and $\chi(\tau_1)$ are Pauli spinors for spin and isospin, respectively. Since $\chi(\sigma_1)$ arises from the use of the approximation

$$u(\mathbf{p}_1,\sigma_1)\xrightarrow{\mathbf{p}_1=0}\chi(\sigma_1) \tag{A8}$$

for the free-particle Dirac spinor $u(\mathbf{p}_1,\sigma_1)$, we have simply replaced $\chi(\sigma_1)$ by $u(\mathbf{p}_1,\sigma_1)$ in (A7) in order to obtain the leading momentum-dependent contributions in (A4). The single-particle momenta $\mathbf{p}_j$ can then be rewritten in terms of the center-of-mass momentum $\mathbf{P}$ and the relative momenta $\boldsymbol{\pi}_j$ as in Eq. (2.10). The effect of the substitution (A8) is then to boost the nucleus to the momentum $\mathbf{P}$ in such a way that each individual nucleon is boosted to a momentum $\mathbf{p}_j$ with $\sum\mathbf{p}_j=\mathbf{P}$. Note that since we are only interested in the leading contribution of $O(\mathbf{P}/M)$, we can safely use the nonrelativistic definitions of the center-of-mass and relative coordinates given in (2.10) in place of the more complex relativistic formulas.[28] The relative momenta π_j act on the internal wave functions R_k and lead to velocity-dependent corrections to the nuclear wave functions, which can be neglected for present purposes. With the approximation of neglecting π_j, the sum over spins in (A5) then allows us to express $X(1,2)$ and $Y(1,2)$ in terms of the center-of-mass momentum $\mathbf{P}$. The sum over isospins can be carried out in the following way. Consider, for example, the isospin average $\langle T^{(+)}\rangle$ of $T^{(+)}$,

$$\begin{aligned}\langle T^{(+)}\rangle &= \sum_{\tau,\tau'} \chi_\tau^\dagger(1)\chi_{\tau'}^\dagger(2) T^{(+)}[a\chi_\tau(1)\chi_{\tau'}(2) - b\chi_{\tau'}(1)\chi_\tau(2)] \\ &= \sum_{\tau,\tau'} \chi_\tau^\dagger(1)\chi_{\tau'}^\dagger(2)\tfrac{1}{2}(1-\tau_{1z}\tau_{2z})[a\chi_\tau(1)\chi_{\tau'}(2) - b\chi_{\tau'}(1)\chi_\tau(2)] ,\end{aligned} \tag{A9}$$

where a and b denote the spatial part of the matrix element in (A4). In obtaining (A9) we have used the identities

$$T^{(+)} = \tfrac{1}{2}(1-\tau_{1z}\tau_{2z})P_\tau , \quad P_\tau = \tfrac{1}{2}(1+\tau_1\cdot\tau_2) , \qquad P_\tau \chi_1\chi_2' = \chi_2\chi_1' , \tag{A10}$$

where χ and χ' are two different Pauli isospinors. For fixed τ the sum over τ' extends over all nucleons except for the last one (denoted by τ'). If the nucleon in question is a proton in a nucleus with Z protons and N neutrons, then the sum over τ' extends over $A-1$ nucleons and $Z-1$ protons. For our purposes we can approximate $A-1 \simeq A$ and $Z-1 \simeq Z$ which leads to a simplification of the final expression for M_w. Since the result of (A10) is to effectively replace $T^{(+)}$ by $\frac{1}{2}(1-\tau_{1z}\tau_{2z})$ which is a diagonal operator, we can use the normalization condition on the Pauli isospinors,

$$\chi_\tau^\dagger(1)\chi_{\tau'}(1) = \delta_{\tau,\tau'} , \tag{A11}$$

to write (A9) in the form

$$\langle T^{(+)}\rangle = \sum_{\tau,\tau'} \tfrac{1}{2}(1-\tau\tau')(a\delta_{\tau\tau'} - b) , \tag{A12}$$

where $\tau = \pm 1$ and $\tau' = \pm 1$ are the eigenvalues of τ_{1z} or τ_{2z}. It follows that the contribution proportional to a in (A12) vanishes, leaving only the exchange contribution proportional to b, which arises from Fig. 1(b). Since there are N contributions with $\tau' = -1$ and Z contributions with $\tau' = +1$ it follows that

$$\begin{aligned}\langle T^{(+)}\rangle &= -b\tfrac{1}{2}\sum_\tau [A + \tau(N-Z)] \\ &= -b\tfrac{1}{2}[A^2 - (N-Z)^2] = -2bNZ .\end{aligned} \tag{A13}$$

The averaging over the other isospin factors can be carried out in a similar fashion, and results are presented in Table II.

Having carried out the sum over spins and isospins, it remains to evaluate a and b in Eq. (A12) by carrying out the integrals over the nuclear wave functions. This can easily be done by making use of the δ function in (A4), and noting that by virtue of the normalization of the wave functions,

$$\int d^3x_1 \phi_k^*(\mathbf{x}_1)\phi_k(\mathbf{x}_1) = 1 . \tag{A14}$$

We can approximate $\phi_k^*\phi_k$ by $1/V$, where V is the nuclear volume. This gives[27]

$$\int d^3x_1 \phi_k^*(\mathbf{x}_1)\phi_l^*(\mathbf{x}_1)\phi_k(\mathbf{x}_1)\phi_l(\mathbf{x}_1) \simeq 1/V , \qquad V \cong A4\pi R_0{}^3/3 , \quad R_0 = 1.07 \text{ fm} . \tag{A15}$$

In principle, one could, of course, evaluate the integral in (A4) numerically but the approximations leading to (A15) are sufficiently good for present purposes. Combining all of the preceding results we find

$$\begin{aligned}M_w = 2^{-3/2}G_F V^{-1} \Bigg[&[2(\Delta^2-1) + \frac{1}{m^2}(1+\Delta^2)W_1]NZ + \left[-\frac{1}{m^2}W_2 + (a_p^2-1) + \frac{1}{2m^2}(1+a_p{}^2)W_1 \right] A_p{}^2Z^2 \\ &+ \left[-\frac{1}{m^2}W_2 + (a_n{}^2-1) + \frac{1}{2m^2}(1+a_n{}^2)W_1 \right] A_n{}^2N^2 - \frac{2}{m^2}W_2A_pA_nNZ \Bigg] ,\end{aligned} \tag{A16a}$$

$$\begin{aligned}W_1 &= m^2(1+\alpha+\tfrac{4}{3}\alpha\mathbf{v}^2) , \\ W_2 &= m^2(-1+\alpha+\tfrac{4}{3}\alpha\mathbf{v}^2) .\end{aligned} \tag{A16b}$$

The presence of the velocity-dependent terms W_1 and W_2 in M_w is now the specific manifestation of Lorentz noninvariance and, as expected, these vanish when $\alpha = 0$. We can now write the final expression for M_w in the form

$$M_w = A_w + \alpha B_w + \tfrac{4}{3}\alpha B_w \mathbf{v}^2 , \tag{A17a}$$

$$A_w = 2^{-3/2}G_F V^{-1}\{NZ[(3\Delta^2-1) + 2A_nA_p] + \tfrac{1}{2}N^2A_n{}^2(1+3a_n{}^2) + \tfrac{1}{2}Z^2A_p{}^2(1+3a_p{}^2)\} , \tag{A17b}$$

$$B_w = G_F 2^{-3/2}V^{-1}\{NZ[(\Delta^2+1) - A_pA_n] + \tfrac{1}{2}N^2A_n{}^2(a_n{}^2-1) + \tfrac{1}{2}Z^2A_p{}^2(a_p{}^2-1)\} . \tag{A17c}$$

TABLE II. Results of isospin averaging for various operators.

Isospin operator $=f(\tau)$	$\langle f(\tau)\rangle$
$T^{(+)}$	$b(-2NZ)$
$\tau_{1p}\tau_{2p}$	$aZ^2+b(-Z^2)$
$\tau_{1n}\tau_{2n}$	$aN^2+b(-N^2)$
$\tau_{1p}\tau_{2n}+\tau_{1n}\tau_{2p}$	$a(2NZ)$

For $\alpha=0$, Eq. (A17) reduces to the result given in Eq. (10) of Haugan and Will.[24,29]

In the standard Glashow-Salam-Weinberg model of the weak interactions, the various parameters appearing in Eqs. (A2) and (A17) have the following values:[24]

$$\begin{aligned} a_p &= \frac{1}{1-4\sin^2\theta_W}\ , \quad a_n=1\ , \\ A_p &= \tfrac{1}{2}-2\sin^2\theta_W\ , \quad A_n=-\tfrac{1}{2}\ , \\ \Delta &= G_A/G_V=1.25\ . \end{aligned} \tag{A18}$$

These values of the parameters are appropriate to our simplified model in which the proton and neutron couple directly to $W^{\pm}$ and Z^0. In a more realistic model in which the fundamental fermion fields are quarks, the various parameters will be renormalized due to QCD effects. We can estimate the magnitude of these effects from the results already obtained for the parity-violating (PV) weak Hamiltonian.[30–32] Consider, for example, the PV operator $H^{(+)}$ proportional to $T^{(+)}$, which arises from weak $\rho^{\pm}$ exchange. Naively, we expect $H^{(+)}$ to be of order

$$H^{(+)}\simeq\frac{G_F}{\sqrt{2}}m_\rho{}^2\simeq\frac{10^{-5}}{\sqrt{2}}\left[\frac{m_\rho}{m}\right]^2\simeq 5\times10^{-6}\ . \tag{A19}$$

The effect of QCD corrections on the contribution from $T^{(+)}$, which is the sum of an isoscalar and isotensor,

$$\begin{aligned} T^{(+)} &= t_0-t_2\ , \\ t_0 &= \tfrac{1}{3}\tau_1\cdot\tau_2\ , \\ t_2 &= \tfrac{1}{2}(\tau_{1z}\tau_{2z}-\tfrac{1}{3}\tau_1\cdot\tau_2)\ , \end{aligned} \tag{A20}$$

is to renormalize each of these pieces differently:

$$\begin{aligned} & H^{(+)}T^{(+)}\to H^{(0)}t_0-H^{(2)}t_2\ , \\ & H^{(0)}=3g_\rho h_\rho^{(0)}\ , \quad H^{(2)}=-\left[\frac{3}{2}\right]^{1/2} g_\rho h_\rho^{(2)}\ . \end{aligned} \tag{A21}$$

Here g_ρ is the strong ρNN coupling constant ($g_\rho\simeq2.79$), and $h_\rho^{(j)}$ are the weak PV $NN\rho$ amplitudes in the isospin states $j=0,2$. These amplitudes are obtained by combining data on PV transitions with various theoretical arguments, but are only approximately known at present. The best estimates of $h_\rho^{(j)}$ are[32]

$$\begin{aligned} & -2.1\times10^{-6}\le h_\rho^{(0)}\le 0.89\times10^{-6}\ , \\ & -0.30\times10^{-6}\le h_\rho^{(2)}/\sqrt{6}\le -0.27\times10^{-6}\ . \end{aligned} \tag{A22}$$

Combined with (A19) and (A21) these lead to

$$\begin{aligned} & -3.5\le H^{(0)}/H^{(+)}\le 1.5\ , \\ & 0.45\le H^{(2)}/H^{(+)}\le 0.50\ . \end{aligned} \tag{A23}$$

We see from (A23) that the effect of the QCD corrections is to enhance the antisymmetric $I=0$ contribution and to suppress the symmetric $I=2$ contribution. The net result is that $H^{(+)}T^{(+)}$ can be enhanced by as much as a factor of 3.3. Actually the effects of QCD are somewhat more complicated: Since the summation over the nucleon isospins gives

$$\begin{aligned} \langle t_0\rangle &= a\tfrac{1}{3}(N-Z)^2+b(-2NZ)\ , \\ \langle t_2\rangle &= a\tfrac{1}{3}(N-Z)^2\ , \end{aligned} \tag{A24}$$

it follows that QCD corrections not only change the overall scale of the effects but also change the dependence of A_w and B_w on N and Z. We note that in contrast to the case for the charged currents, the parity-conserving (PC) neutral-current contributions cannot be related in a straightforward way to the analogous parity-violating ones. (There is, for example, no axial-vector analog of the electromagnetic current.) The extensive theoretical analysis which would be required for a complete treatment of these effects is not warranted at present, and in any case is unlikely to substantially modify A_w or B_w. Moreover, more significant uncertainties might arise from weak PC π exchange whose contributions we have not included in (A17). Here the PV and PC dynamics and kinematics differ in a way that might lead to a substantial relative enhancement in the PC case. The main contributions to the weak $NN\pi$ vertices (both PV and PC) come from the product of $+1$ and -1 helicity neutral currents (the so-called "penguin" contributions[33]), and the theoretical result is in reasonable agreement with the data.[31,32] Since the weak PV $NN\pi$ amplitude is S wave, whereas the PC amplitude is P wave, kinematic factors such as ($m_{u,d}$ are quark masses)

$$\frac{m_n-m_p}{m_d{}^2-m_u{}^2}$$

in the PV case might be replaced by much larger factors such as

$$\frac{2m}{(m_u+m_d)^2}$$

in the PC case.[33] We might thus expect a relative enhancement of the PC amplitude of order

$$\begin{aligned} A(NN\pi)_{\rm PC} &\simeq A(NN\pi)_{\rm PV}\frac{2m(m_u-m_d)}{(m_u+m_d)(m_p-m_n)} \\ &\simeq 10^3A(NN\pi)_{\rm PV}\equiv\kappa A(NN\pi)_{\rm PV}\ , \end{aligned} \tag{A25}$$

where we have used the current-quark masses. To actually estimate the magnitude of the PC π-exchange contribution to A_w and B_w we should replace G_F in (A1) roughly by

$$G_F \rightarrow g_{\pi NN} A(NN\pi)_{\rm PV} \kappa (m_\pi)^{-2} , \tag{A26}$$

where $g_{\pi NN} \simeq 13.5$ is the strong $NN\pi$ coupling constant. This leads to a net enhancement of the PC amplitude of order

$$\kappa g_{\pi NN} A(NN\pi)_{\rm PV} (G_F m_\pi{}^2)^{-1} \simeq 7 \times 10^{-3} \kappa \simeq 7 . \tag{A27}$$

Given the theoretical uncertainties involved in the preceding calculation, the best we can do at present is to estimate the π-exchange contribution as we have done. Our results suggest that pion exchange may in fact be more important than vector-meson exchange, which would then imply a correspondingly stronger limit on α from Eq. (3.6). Moreover, if the dependence of the pion-exchange contribution on Z, N, and A were such that B_w/M_0 in Eq. (3.6) varied more rapidly across the periodic table than is the case for vector-meson exchange, then the near cancellation that presently takes place in Eq. (3.6) would be prevented. This again would enhance the weak contribution to the binding energy, and would further tighten the limits on α.

Given the various uncertainties in the weak PC amplitude, the most reasonable approach to obtain a quantitative limit on α is to simply use the results in (A16)—(A18) with the parameters as specified. We have taken[34] $\sin^2\theta_W = 0.23$, and have expressed M_0 in the form

$$M_0 = (1 \text{ amu})A = 0.932A \text{ GeV}/c^2 . \tag{A28}$$

Combining (A28) and (A17) we can then write

$$\begin{aligned} \frac{B_w}{M_0} &= \left[1.705 \frac{NZ}{A^2} + 0.082 \frac{Z^2}{A^2} \right] \times 10^{-8} \\ &= \left[1.705 \frac{Z}{A} - 1.623 \frac{Z^2}{A^2} \right] \times 10^{-8} . \end{aligned} \tag{A29}$$

The values of B_w/M_0 are given in Table I for some selected elements, and the variation of B_w/M_0 across the periodic table is shown in Fig. 2.

[1]S. H. Aronson, G. J. Bock, H. Y. Cheng, and E. Fischbach, Phys. Rev. Lett. **48**, 1306 (1982).

[2]E. Fischbach, H. Y. Cheng, S. H. Aronson, and G. Bock, Phys. Lett. **116B**, 73 (1982).

[3]S. H. Aronson, G. J. Bock, H. Y. Cheng, and E. Fischbach, Phys. Rev. D **28**, 476 (1983); **28**, 495 (1983).

[4]E. Fischbach, in *Phenomenology of Unified Theories,* proceedings of the Topical Conference, Dubrovnik, Yugoslavia, 1983, edited by H. Galić, B. Guberina, and D. Tadić (World Scientific, Singapore, 1984), p. 156.

[5]H. B. Nielsen and I. Picek, Phys. Lett. **114B**, 141 (1982); Nucl. Phys. **B211**, 269 (1983).

[6]H. B. Nielsen and M. Ninomiya, Nucl. Phys. **B141**, 153 (1978); S. Chadha and H. B. Nielsen, *ibid.* **B217**, 125 (1983).

[7]R. Huerta and J. L. Lucio M., Phys. Lett. **131B**, 471 (1983); **124B**, 369 (1983).

[8]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B176**, 61 (1980); A. Zee, Phys. Rev. D **25**, 1864 (1982).

[9]P. R. Phillips, Phys. Rev. **139**, B49 (1965); **146**, 966 (1966); P. R. Phillips and D. Woolum, Nuovo Cimento **64B**, 28 (1969).

[10]L. B. Rédei, Phys. Rev. **145**, 999 (1966); **162**, 1299 (1967); L-E. Lundberg and L. B. Rédei, *ibid.* **169**, 1012 (1968).

[11]D. I. Blokhinstev, Phys. Lett. **12**, 272 (1964).

[12]R. v. Eötvös, D. Pekár, and E. Fekete, Ann. Phys. (Leipzig) **68**, 11 (1922).

[13]P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N.Y.) **26**, 442 (1964).

[14]V. B. Braginskii and V. I. Panov, Zh. Eksp. Teor. Fiz. **61**, 873 (1971) [Sov. Phys. JETP **34**, 463 (1972)].

[15]M. P. Haugan, Ann. Phys. (N.Y.) **118**, 156 (1979).

[16]K. S. Thorne, D. L. Lee, and A. P. Lightman, Phys. Rev. D **7**, 3563 (1973).

[17]L. I. Schiff, Am. J. Phys. **28**, 340 (1960).

[18]We adopt the conventions of J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1964), and set $c = \hbar = 1$.

[19]E. Fischbach, B. S. Freeman, and W. K. Cheng, Phys. Rev. D **23**, 2157 (1981).

[20]T. D. Lee and C. N. Yang, Phys. Rev. **98**, 1501 (1955).

[21]C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, 1981).

[22]M. G. Bowler, *Gravitation and Relativity* (Pergamon, New York, 1976), p. 29.

[23]S. de Benedetti, *Nuclear Interactions* (Wiley, New York, 1964), p. 97.

[24]M. P. Haugan and C. M. Will, Phys. Rev. Lett. **37**, 1 (1976).

[25]E. Fischbach and D. Tadić, Phys. Rep. **6C**, 123 (1973); As discussed in this reference, the weak $W^\pm$ and Z^0-exchange contributions can be equally well viewed as arising from weak ρ and ω exchange, assuming vector dominance of the weak form factors.

[26]F. C. Michel, Phys. Rev. **133**, B329 (1964); E. U. Condon and G. H. Shortly, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1951).

[27]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), p. 348.

[28]R. A. Krajcik and L. L. Foldy, Phys. Rev. D **10**, 1777 (1974).

[29]Equation (A17b) corrects a misprint in Eq. (10) of Haugan and Will (Ref. 24): In the term proportional to NZ the coefficient of $A_n A_p$ should be 2 rather than 4.

[30]H. Galić, B. Guberina, and D. Tadić, Z. Phys. **276A**, 65 (1976); Phys. Rev. D **14**, 2327 (1976).

[31]B. Desplanques, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (N.Y.) **124**, 449 (1980).

[32]B. Desplanques, Nucl. Phys. **A335**, 147 (1980).

[33]H. Galić, B. Guberina, and D. Tadić, Fortschr. Phys. **29**, 261 (1981).

[34]Particle Data Group (M. Aguilar-Benitez *et al.*), Rev. Mod. Phys. **56**, S1 (1984).