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The Power of Pharmacist-to-Pharmacist Handoffs During Transitions of Care

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Time Symmetric Quantum Theory Without Retrocausality? A Reply to Tim Maudlin

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Abstract

In arXiv:1707.08641, Tim Maudlin claims to construct a counterexample to the result of Proc. Roy. Soc. A vol. 473, iss. 2202, 2017 (arXiv:1607.07871), in which it was shown that no realist model satisfying a certain notion of time-symmetry (in addition to three other assumptions) can reproduce the predictions of quantum theory without retrocausality (influences travelling backwards in time). In this comment, I explain why Maudlin's model is not a counterexample because it does not satisfy our time-symmetry assumption. I also explain why Maudlin's claim that one of the Lemmas we used in our proof is incorrect is wrong.

1 Introduction

In his book, Time's Arrow and Archimedes' Point [1], Huw Price argued that the "standard interpretation"[1] of quantum theory is time asymmetric, and that retrocausality is required to restore time symmetry. The argument runs as follows:

Consider a photon travelling between two crossed polarizers, let's call them the left polarizer and the right polarizer (see fig. 1(a)). As the photon exits the left polarizer, its polarization vector is aligned with the direction of that polarizer. It then encounters the right polarizer, still with its polarization vector aligned with the left polarizer, and either passes through or not with the probabilities ascribed by quantum theory. Suppose that, in this run of the experiment, it passes, and hence exits with a polarization vector aligned with the right polarizer.

If we run this description backwards in time (see fig. 1(b)), we find that the photon starts with its polarization aligned with the right polarizer, but as soon as it passes through the right polarizer its polarization immediately jumps to be aligned with that of the left polarizer. It then carries on towards the left polarizer and passes through it. If time symmetry holds then this ought to be a possible description of a forwards-in-time experiment, but that would involve retrocausality as the photon seems to anticipate the direction of the left polarizer, which could be chosen while the photon is still in flight between the two.

Of course, this is not at all what happens in the actual quantum description of the time reversed experiment (see fig. 1(c)). Instead, it is just like the description of the original experiment except with the roles of the left and right polarizers interchanged. In this description, the photon polarization is aligned with the right polarizer during its flight between the two polarizers, so there is no retrocausality, but we seem to have lost time symmetry, i.e. running the description of the first experiment backwards in time does not give us a valid description of a forwards-in-time experiment.

[1] A loaded term, because there is widespread disagreement about what the "standard interpretation" actually is, but here we mean something like the interpretation contained in the books of Dirac [2] and von Neumann [3], which endorses the collapse of the wavefunction and the eigenvalue-eigenstate link. This renders the quantum state an ontic property of the system that changes discontinuously upon measurement.

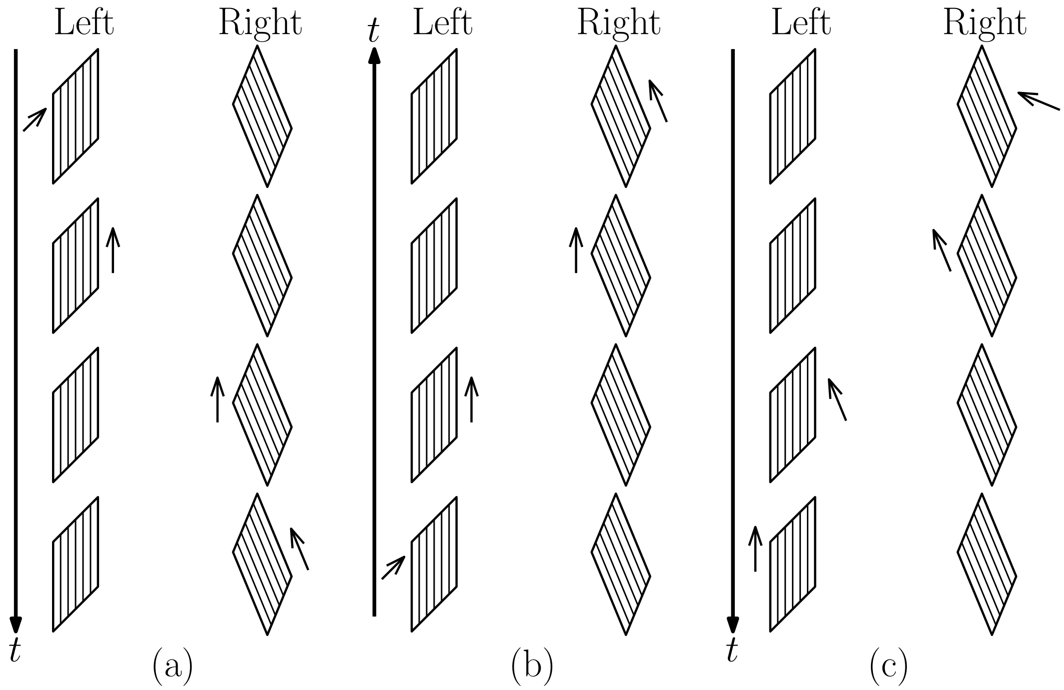

Figure 1: Time reversal in polarization experiments. (a) An experiment where the photon travels from the left to the right polaroid filter. Time runs from top to bottom. (b) The exact same experiment, but with the direction of time reversed to run bottom to top. (c) An experiment where the photon travels from the right polarizer to the left one, with time running top to bottom. The evolution of the polarization vector is not the same as in (b).

This argument is subject to two obvious objections. Firstly, it is only unitary time evolution according to the Schrödinger equation that is time symmetric in quantum theory. To get a time-symmetric description, we ought to treat the whole experiment, including the interaction with the polarizers, according to unitary quantum mechanics. This is easiest to do if we use polarizing beam-splitters rather than polaroid filters for the polarization measurements. Then, we see that, in the first experiment, the photon exits the right polarzing beam-splitter in a superposition of being in the two output ports. If we run this description backwards in time then the photon enters the right beam-splitter in a superposition of the two ports, and these interfere to yield a polarization direction that happens to be the same as the orientation of the left beam-spliiter. This is a valid description of a forwards-in-time experiment, so time symmetry is restored. There is also no retrocausality here as the photon polarization is determined by the initial amplitudes at the input ports of the right polarizer, and not by the orientation of the left polarizer, which can still be varied independently.

The error, according to this objection, is misapplication of the measurement postulates in our first attempt to describe time reversal. What happens during measurement is, of course, controversial, but in the conventional formalism the collapse of the quantum state at least looks like a time asymmetric process. It is only unitary evolution that is claimed to be time reversible in the standard interpretation of quantum theory.

However, this objection ignores the fact that there is a form of time symmetry that obtains even if we do apply the measurement postulates. Returning to our first attempt to describe time reversal, let $|\psi_l\rangle$ be the polarization state of a photon that passes the left polaroid filter with certainty and let $|\psi_r\rangle$ be the polarization state of a photon that passes the right polaroid filter with certainty. Then, the probability of passing the right polarizer given that the photon has passed the left polarizer is $|\langle\psi_r|\psi_l\rangle|^2$. In the time reversed experiment, the probability of passing the left polarizer given that the photon passed the right polarizer is $|\langle\psi_l|\psi_r\rangle|^2 = |\langle\psi_r|\psi_l\rangle|^2$, i.e. the exact same probability.

This time symmetry can be generalized. Consider n polaroid filters placed in sequence, with $|\psi_j\rangle$ being the state that passes the j^{th} filter with certainty. Then, the joint probability of passing filters $2, 3, \ldots, n$ given that the first filter was passed is $\prod_{j=1}^{n-1} |\langle\psi_{j+1}|\psi_j\rangle|^2$, and in the time reversed experiment the probability of passing filters $n-1, n-2, \ldots, 1$ given that the n^{th} filter was passed is also $\prod_{j=1}^{n-1} |\langle\psi_{j+1}|\psi_j\rangle|^2$.

Generalizing even further, we find that *any* sequence of measurements interspersed by dynamics can be described either in the conventional predictive formalism, in which states are evolved forwards in time, or in a retrodictive formalism, in which states are evolved backwards in time (see [4] and references therein). Because the predictive and retrodictive formalisms are mathematically identical, we conclude that for any forwards-in-time experiment consisting of measurements interspersed with dynamics there is another forwards-in-time experiment that predicts the same probabilities as the first experiment running backwards in time.

Therefore, it is possible that the apparent time asymmetry of quantum measurements is just an artifact of the way they are described in the conventional formalism. There exist other formalisms, such as the two state vector formalism of Aharonov et. al. [5], the consistent/decoherent histories formalism [6], and the path integral formalism [7], in which the symmetry just described is more explicit. These formalisms all either have the feature that the state at the present time depends on the future as well as the past, or that the formalism only refers to spacetime as a whole and does not assign states at a given instant of time. This seems to indicate some form of retrocausality, but one should be cautious because these are just mathematical formalisms, which can be interpreted in a variety of ways, rather than clean statements about the nature of reality.

In any case, what is certain is that there is a kind of time symmetry in quantum theory that includes measurements, and that is different from the conventional time reversal of unitary dynamics. On this notion, Price is correct in his description of time reversal, and his argument that time symmetry requires retrocausality holds more water. In his paper [8], Price presents a modified and more rigorous version of his argument, identifying more clearly the circumstances under which it holds. Our paper [9] follows on from this, and makes the argument even more general.

This brings us to the second objection, which applies if one takes the view that the quantum state ought to be thought of as an epistemic state (state or knowledge, information, or belief) akin to a probability distribution, as opposed to an ontic state (state of reality). In a time symmetric theory, we expect the ontic description of an experiment to be reversible in time, but there is no reason why an epistemic description should be. This is because the knowledge acquired by an observer also has a time direction to it, i.e. the observer assigns the photon a polarization state with the same orientation as a polarizer it has just passed through because the observer *knows* it has just passed through that polarizer. The apparent discontinuous jump of the photon polarization to the orientation of the left polarizer in the reversed description occurs because it is still the description assigned by an observer who knows that the photon will pass through left polarizer. If we also reverse the direction of the observer's information acquisition, so that they know the photon has passed the right polarizer in the reversed experiment but not whether it will pass the left one, then we get a different description, which is in fact just the usual quantum description of the time reversed experiment. In fact, all the qualitative features used in Price's argument occur within Spekkens' toy model [10], which is time symmetric and in which the analog of a quantum state is a probability distribution. This is discussed in detail in our paper.

The main contribution of our argument is to show that, in order to reproduce the predictions of quantum mechanics, you need retrocausality to have the kind of time symmetry discussed above, even if the quantum state is epistemic.

The reason why I have described this history and motivation in detail is that Maudlin's purported counter example to our theorem [11] *is* just what I have been calling the standard interpretation of quantum theory. His state of reality is just the usual forwards evolving quantum state vector, and so his model is already subject to Price's arguments, without even having to consider ours. It is not time symmetric in the sense that includes measurements, just for the reasons given above.

For those still not convinced, in the remainder of this paper, I review our assumptions, show that

Maudlin's model fails to satisfy them, and show that Maudlin's criticism of Lemma VIII.2 in our paper is incorrect.

2 Our Assumptions

2.1 Operational Framework

In [9], we discussed a class of experiments, which are described operationally as a preparation procedure P, followed by a transformation T, followed by a measurement M. Maudlin correctly notes that the transformation T does not play any role in our main argument. We included it in our paper to get a cleaner relationship between quantum experiments that violate our assumptions and states that violate Bell inequalities in a spacelike Bell test, as discussed in §VIII.C of [9]. For present purposes, we can eliminate T and just discus prepare-and-measure experiments, as depicted in fig. 2.

The preparation procedure has an input X and an output A, and similarly the measurement M has an input Y and an output B. The outputs A and B are random variables that take values in (finite) sets Ω_A and Ω_B respectively[2], and the inputs are variables[3] X and Y that take values in sets Ω_X and Ω_Y respectively. A pair (P, M) is called an experiment if P and M can be performed sequentially in time on the same system. For each experiment (P, M), an operational theory provides predictions of the probabilities $p_{PM}(A = a, B = b|X = x, Y = y)$ for any $a \in \Omega_A$, $b \in \Omega_B$, $x \in \Omega_X$ and $y \in \Omega_Y$.

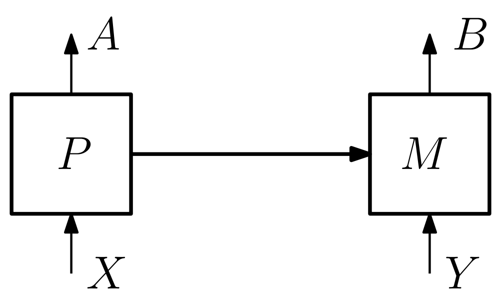

Figure 2: Schematic of an experiment.

In quantum theory, a preparation P is described by a set $\{\rho_{a|x}\}$ of unnormalized density operators, such that $\sum_{a\in\Omega_A} \rho_{a|x}$ is a normalized density operator for every $x \in \Omega_X$, and a measurement M by a set of Positive Operator Valued Measures (POVMs) $\{E_{b|y}\}$, i.e. for each $y \in \Omega_Y$, $\{E_{b|y}\}_{b\in\Omega_B}$ is a set of positive operators such that $\sum_{b\in\Omega_B} E_{b|y} = I$, where I is the identity operator. The probabilities predicted for an experiment are then

$$p_{PM}(A = a, B = b|X = x, Y = y) = \mathrm{Tr}\left(E_{b|y}\rho_{a|x}\right).$$

2.2 Ontological Framework

So far, we have just described the aspects of experiments that can be directly manipulated or observed in the lab, but we are interested in investigating realist accounts in which the system has some objective physical properties that are probed by the experiment.

We made four assumptions about such models in our paper: **(Single World) Realism**, **Free Choice**, **$\boldsymbol{\lambda}$-mediation**, and **No Retrocausality**. These are equivalent to assuming that the operational theory has an *ontological model*, which is a standard framework for discussing realist models of quantum theory [12]. The reason we broke this down into four assumptions is because we were interested in investigating the role of **No Retrocausality**, which is baked into the assumptions of an ontological model. Nonetheless, it is

[2] I am being more explicit about the fact that the outputs are random variables than in the original paper to make the meaning of our Time Symmetry assumption clearer.

[3] These are not random variables as they are chosen by the experimenter and always appear to the right of the conditional.

equivalent to just assume that our operational theory has an ontological model, so that is what we will do here.

The ontological models framework says that a system has some objective physical properties, described by its *ontic state* L. This is again a random variable that takes values in a set Λ called the *ontic state space*. For simplicity, we shall assume that Λ is finite in our formal analysis, but the continuum and measure-theoretic generalizations are straightforward. The ontic state L is responsible for mediating any correlations between the preparation and measurement variables that we observe in the lab. For every experiment (P, M), an ontological model assigns probabilities $p_{PM}(A = a, B = b, L = \lambda | X = x, Y = y)$ of the form

$$\begin{aligned} p_{PM}(A = a, B = b, L = \lambda | X = x, Y = y) \\ = p_M(B = b | L = \lambda, Y = y) p_P(L = \lambda | A = a, X = x) p_P(A = a | X = x). \end{aligned} \quad (1)$$

These reproduce the operational predictions if

$$\sum_{\lambda \in \Lambda} p_{PM}(A = a, B = b, L = \lambda | X = x, Y = y) = p_{PM}(A = a, B = b | X = x, Y = y),$$

which we shall always assume.

Eq. (1) says that the observed probabilities are accounted for as follows. First, the experimenter chooses $X = x$ and the preparation device responds by outputting $A = a$ with probability $p_P(A = a | X = x)$ and outputting $L = \lambda$ with probability $p_P(L = \lambda | A = a, X = x)$. L is then transmitted to the measurement device, the experimenter picks $Y = y$, and the measurement device outputs $B = b$ with probability $p_M(B = b | L = \lambda, Y = y)$. Importantly, this does not depend on P, X or A as we want the properties of the system, i.e. the ontic state L, to mediate the correlations between preparation and measurement. The causal structure of this type of model is depicted in fig. 3.

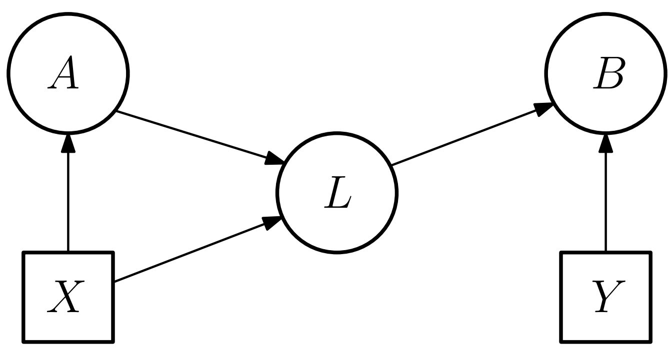

Figure 3: Influence diagram showing the causal structure of an ontological model of an experiment.

2.3 Time Symmetry

The most controversial assumption in our argument is our **Time Symmetry** assumption because it is intended to capture the time symmetry under sequential measurements discussed in the introduction, rather than the conventional notion of time symmetry under unitary evolution. We start by defining what we mean by this symmetry for a prepare-and-measure experiment.

An experiment (P, M) has an *operational time reverse* if there exists another experiment (P', M') (we use primes to denote the variables associated with (P', M')) such that $\Omega_{A'} = \Omega_B$, $\Omega_{B'} = \Omega_A$, $\Omega_{X'} = \Omega_Y$, $\Omega_{Y'} = \Omega_X$, and

$$p_{PM}(A = a, B = b | X = x, Y = y) = p_{P'M'}(A' = b, B' = a | X' = y, Y' = x). \quad (2)$$

If every experiment in an operational theory has an operational time reverse then the theory is *operationally time symmetric*.

Note that an experiment can be its own operational time reverse, in which case we have

$$p_{PM}(A = a, B = b|X = x, Y = y) = p_{PM}(A = b, B = a|X = y, Y = x).$$

We do not expect theories to be operationally time symmetric in general because of the possibility of sending signals into the future but not into the past. That is, we expect that for all experiments (P, M), $p_{PM}(B = b|X = x, Y = y) \neq p_{PM}(B = b|Y = y)$ but $p_{PM}(A = a|X = x, Y = y) = p_{PM}(A = a|X = x)$. However, for any operational theory, we can always consider the set of *no-signalling preparations*, i.e. the set of preparations P such that $p_{PM}(B = b|X = x, Y = y) = p_{PM}(B = b|Y = y)$ happens to be satisfied for all experiments (P, M) involving P. For some theories the set of no-signalling preparations might be empty, but it can always be constructed. The *no-signalling sector* of an operational theory is the operational theory that results from restricting the preparations to be no-signalling.

In [9], we showed that the no-signalling sector of quantum theory is operationally time symmetric. This also holds for several other theories, including classical probability theory and the Spekkens' toy theory [10]. Maudlin asserts that we proved that the no-signalling sector of any operational theory is operationally time-symmetric, but this is not the case. It can fail to hold for trivial reasons, e.g. if there is a preparation with exactly two possible outcomes but all the measurements in the theory have exactly three outcomes so no mapping from P to M' and M to P' exists. However, it can also fail for less trivial reasons. It is generically false for generalized probabilistic theories [13], in which the structure of the state space might be very different from the structure of the set of *effects* that describe measurement outcomes. Therefore, we claim that operational time symmetry of the no-signalling sector is a significant symmetry of quantum theory that is worth investigating. If Maudlin's claim were true then we would have no grounds for this.

We next follow the logic of Spekkens' noncontextuality [14] in asserting that this operational time symmetry would be best explained if there were an analogous time symmetry at the ontological level. This is a form of Leibniz's principle of the identity of indiscernibles: if two things look the same from the point of view of all observations we can make on them then we should think of them as actually being the same. This motivates the following definitions.

An experiment (P, M) in an ontological model has an *ontological time reverse* if there exists another experiment (P', M') such that $\Omega_{A'} = \Omega_B$, $\Omega_{B'} = \Omega_A$, $\Omega_{X'} = \Omega_Y$, $\Omega_{Y'} = \Omega_X$, there exists a one-to-one map $f : \Lambda \to \Lambda'$, and

$$p_{PM}(A = a, B = b, L = \lambda|X = x, Y = y) = p_{P'M'}(A' = b, B' = a, L' = f[\lambda]|X' = y, Y' = x). \quad (3)$$

The reason for including the function f is that physical states often have to be transformed when taking the time reverse, usually by an involution. For example, in Hamiltonian mechanics, the momentum is reversed upon taking time reverses and, in quantum theory, the state vector $|\psi\rangle$ is mapped to its conjugate $|\psi^*\rangle$ with respect to some basis so we would need a nontrivial f in a model where the state vector is ontic.

Note that if (P, M) and (P', M') are ontological time reverses of each other then they must also be operational time reverses, because summing eq. (3) over λ yields eq. (2). However, the converse is not necessarily true.

Our **Time Symmetry** assumption is then the assertion that if an experiment has an operational time reverse then it should also have an ontological time reverse.

3 The Beltrametti-Bugajski Model

Before discussing Maudlin's model, it is useful to study a more general model, known as the Beltrametti-Bugajski model [15]. Here we will only need the special case of a spin-1/2 particle and orthonormal basis measurements. Maudlin's model is just this special case specialized further to only two possible preparation and measurement directions, with a relabelling of the inputs. The reason it fails the **Time Symmetry** assumption will be the same.

Consider the following preparation P of a spin-1/2 particle. A direction is chosen, which we represent by a unit vector $X = \vec{n}$. Then, with probability 1/2 the output $A = +1$ is generated and with probability 1/2 the output $A = -1$ is generated. If the output is +1 then the system is prepared in the state $|\vec{n}, +1\rangle$, i.e. spin-up in the direction $\vec{n}$ and if $A = -1$ the system is prepared in the state $|\vec{n}, -1\rangle$, i.e. spin-down in the direction $\vec{n}$. This is a no-signalling preparation, as the ensemble average state over A is always the maximally mixed state, regardless of the direction X.

The measurement M consists of choosing a direction $Y = \vec{m}$ and measuring the system in the $|\vec{m}, 1+\rangle, |\vec{m}, -1\rangle$ basis. We set $B = +1$ if the outcome is $|\vec{m}, +1\rangle$ and $B = -1$ if the outcome is $|\vec{m}, -1\rangle$.

For this experiment, quantum theory predicts the probabilities

$$p_{PM}(A = a, B = b|X = \vec{n}, Y = \vec{m}) = \frac{1}{4}\left(1 + (ab)\vec{n} \cdot \vec{m}\right).$$

This experiment is its own operational time reverse, i.e.

$$p_{PM}(A = a, B = b|X = \vec{n}, Y = \vec{m}) = p_{PM}(A = b, B = a|X = \vec{m}, Y = \vec{n}).$$

The Beltrametti-Bugajski model is really just a translation of the orthodox Dirac-von Neumann interpretation into the ontological models framework, i.e. the ontic state is identified with the quantum state and the measurement probabilities are those given by quantum mechanics.

One way of implementing this is to choose the ontic state space to be $\Lambda = S^2 \times \{+1, -1\}$, i.e. the Cartesian product of the unit sphere with the two possible values ± 1. The ontic states are of the form $\lambda = [\vec{\lambda}_1, \lambda_2]$ where $\vec{\lambda}_1 \in S^2$ and $\lambda_2 = \pm 1$. The probabilities of the ontological model are specified as follows:

$$\begin{aligned} p_P(A = a|X = \vec{n}) &= \frac{1}{2} \\ p_P(L = [\vec{\lambda}_1, \lambda_2]|X = \vec{n}, A = a) &= \delta(\vec{\lambda}_1 - \vec{n})\delta_{\lambda_2, a} \\ p_M(B = b|L = [\vec{\lambda}_1, \lambda_2], Y = \vec{m}) &= \frac{1}{2}\left(1 + (\lambda_2 b)\vec{\lambda}_1 \cdot \vec{m}\right), \end{aligned}$$

from which we can compute the probabilities

$$p_{PM}(A = a, B = b, L = [\vec{\lambda}_1, \lambda_2]|X = \vec{n}, Y = \vec{m}) = \frac{1}{4}\delta(\vec{\lambda}_1 - \vec{n})\delta_{\lambda_2, a}\left(1 + (\lambda_2 b)\vec{\lambda}_1 \cdot \vec{m}\right). \quad (4)$$

It is straightforward to see that this reproduces the quantum predictions.

It is also easy to check that this model *does not* satisfy the **Time Symmetry** assumption. Since this experiment is its own operational time reverse, **Time Symmetry** says that we should have

$$p_{PM}(A = a, B = b, L = [\vec{\lambda}_1, \lambda_2]|X = \vec{n}, Y = \vec{m}) = p_{PM}(A = b, B = a, L = f[\vec{\lambda}_1, \lambda_2]|X = \vec{m}, Y = \vec{n}). \quad (5)$$

Define the function $g(a, b, \vec{\lambda}_1, \lambda_2, \vec{n}, \vec{m}) = p_{PM}(A = a, B = b, L = [\vec{\lambda}_1, \lambda_2]|X = \vec{n}, Y = \vec{m})$ and let $g(\vec{\lambda}_1, \vec{n}, \vec{m}) = \sum_{a,b,\lambda_2} g(a, b, \vec{\lambda}_1, \lambda_2, \vec{n}, \vec{m})$. By direct computation from eq. (4), we have

$$g(\vec{\lambda}_1, \vec{n}, \vec{m}) = \delta(\vec{\lambda}_1 - \vec{n}). \quad (6)$$

To apply eq. (5), we write $f[\vec{\lambda}_1, \lambda_2] = [\vec{f}_1(\vec{\lambda}_1, \lambda_2), f_2(\vec{\lambda}_1, \lambda_2)]$, where $\vec{f}_1(\vec{\lambda}_1, \lambda_2) \in S^2$ and $f_2(\vec{\lambda}_1, \lambda_2) \in \{+1, -1\}$. Then, combining eqs. (5) and (4) gives

$$g(\vec{\lambda}_1, \vec{n}, \vec{m}) = \delta(\vec{f}_1(\vec{\lambda}_1, \lambda_2) - \vec{m}). \quad (7)$$

However, eqs. (6) and (7) cannot possibly be the same function because the first depends on $\vec{n}$ and not $\vec{m}$ but the second depends on $\vec{m}$ and not $\vec{n}$. Therefore, eq. (5) cannot hold, so **Time Symmetry** is violated.

It is worth pausing for a moment to understand why this model violates **Time Symmetry** from a conceptual point of view. Because the experiment is its own operational time-reverse, when we exchange the preparation and measurement variables the quantum predictions can still be explained by the same ontological model. This is a sort of time symmetry and is perhaps what has lead Maudlin astray. However, the **Time Symmetry** assumption requires much more than this. It says that the ontic description of the time reversed experiment should be the same as ontic description of the original experiment running backwards in time. This means that the way in which the ontic state is correlated with the preparation in the original experiment has to be the same as the way in which the ontic state is correlated with the measurement in the time reversed experiment. Because the ontic state contains full information about preparation in the original experiment, it must contain full information about the measurement in the time reversed experiment. This means that the ontic state must be correlated with the measurement input. However, this would require retrocausality, which is not allowed in the ontological models framework. Hence, we get a contradiction. This is essentially the content of Price's argument for retrocausality.

4 Maudlin's Model

Maudlin's model is just the Beltrametti-Bugajski model specialized to only two possible preparation and measurement directions, with a relabelling of the inputs and outputs.

Let the preparation input value $X = 0$ correspond to the direction $\vec{n} = (0, 0, 1)$ and the value $X = 1$ to the direction $\vec{n} = (1/2, 0, \sqrt{3}/2)$, i.e. at $30°$ in the z-x plane. Similarly, let the measurement input value $Y = 0$ correspond to the direction $\vec{m} = (0, 0, 1)$ and the value $Y = 1$ to the direction $(-1/2, 0, \sqrt{3}/2)$, i.e. at $-30°$ in the z-x plane. Note the relabelling here as we have chosen $X = 1$ and $Y = 1$ to correspond to different directions, whereas X and Y just are the directions themselves in the Beltrametti-Bugajski model. Nevertheless, this is allowed as X, Y, A and B are just supposed to be abstract labels in our formalism, which can be mapped to physical degrees of freedom in any way you like.

With these definitions, the operational predictions are

$$p_{PM}(A = a, B = b|X = 0, Y = 0) = \frac{1}{4}(1 + ab)$$
$$p_{PM}(A = a, B = b|X = 0, Y = 1) = \frac{1}{4}\left(1 + \frac{\sqrt{3}}{2}ab\right)$$
$$p_{PM}(A = a, B = b|X = 1, Y = 0) = \frac{1}{4}\left(1 + \frac{\sqrt{3}}{2}ab\right)$$
$$p_{PM}(A = a, B = b|X = 1, Y = 1) = \frac{1}{4}\left(1 + \frac{1}{2}ab\right).$$

This experiment is its own operational time reverse, i.e. $p_{PM}(A = a, B = b|X = x, Y = y) = p_{PM}(A = b, B = a|X = y, Y = x)$.

The reason why Maudlin chose these directions is that these are the settings used by Bell to show that quantum theory violates local causality in a spacelike setting, and our main theorem shows that operationally time reversible experiments that satisfy our assumptions must satisfy local causality. Thus, if these predictions could be reproduced by a model that satisfies our assumptions then that would be a counter-example to our theorem.

Maudlin's ontological model works in the same way as Beltrametti-Bugajski, i.e. the preparation input X and the output A are just transmitted to the measurement device. In this case, the ontic state space is just $\Lambda = \{0, 1\} \times \{+1, -1\}$, so ontic states are of the form $\lambda = [\lambda_1, \lambda_2]$ with $\lambda_1 \in \{0, 1\}$ and $\lambda_2 \in \{+1, -1\}$.

The ontological model probabilities are as follows

$$
\begin{aligned}
p_P(A = a|X = x) &= \frac{1}{2} \\
p_P(L = [\lambda_1, \lambda_2]|X = x, A = \vec{a}) &= \delta_{\lambda_1,x}\delta_{\lambda_2,a} \\
p_M(B = b|L = [0, \lambda_2], Y = 0) &= \frac{1}{2}\left(1 + \lambda_2 b\right) \\
p_M(B = b|L = [0, \lambda_2], Y = 1) &= \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\lambda_2 b\right) \\
p_M(B = b|L = [1, \lambda_2], Y = 0) &= \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\lambda_2 b\right) \\
p_M(B = b|L = [1, \lambda_2], Y = 1) &= \frac{1}{2}\left(1 + \frac{1}{2}\lambda_2 b\right),
\end{aligned}
$$

from which we can calculate

$$
\begin{aligned}
p_{PM}(A = a, B = b, L = [0, \lambda_2]|X = x, Y = 0) &= \frac{1}{4}\delta_{0,x}\delta_{\lambda_2,a}\left(1 + \lambda_2 b\right) \\
p_{PM}(A = a, B = b, L = [0, \lambda_2]|X = x, Y = 1) &= \frac{1}{4}\delta_{0,x}\delta_{\lambda_2,a}\left(1 + \frac{\sqrt{3}}{2}\lambda_2\right) \\
p_{PM}(A = a, B = b, L = [1, \lambda_2]|X = x, Y = 0) &= \frac{1}{4}\delta_{1,x}\delta_{\lambda_2,a}\left(1 + \frac{\sqrt{3}}{2}\lambda_2 b\right) \\
p_{PM}(A = a, B = b, L = [1, \lambda_2]|X = x, Y = 1) &= \frac{1}{4}\delta_{0,x}\delta_{\lambda_2,a}\left(1 + \frac{1}{2}\lambda_2 b\right).
\end{aligned}
$$

Since the experiment is its own operational time reverse, according to **Time Symmetry**, we should have

$$
p_{PM}(A = a, B = b, L = [\lambda_1, \lambda_2]|X = x, Y = y) = p_{PM}(A = b, B = b, L = f[\lambda_1, \lambda_2]|X = x, Y = y). \tag{8}
$$

If we define $f[\lambda_1, \lambda_2] = [f_1(\lambda_1), f_2(\lambda_2)]$, $g(a, b, \lambda_1, \lambda_2, x, y) = p_{PM}(A = a, B = b, L = [\lambda_1, \lambda_2]|X = x, Y = y)$, and $g(\lambda_1, x, y) = \sum_{a,b,\lambda_2} g(a, b, \lambda_1, \lambda_2, x, y)$ then, by the same argument we used for the Beltrametti-Bugajski model we get

$$
g(\lambda_1, x, y) = \delta_{\lambda_1,x} = \delta_{f_1(\lambda_1,\lambda_2),y},
$$

which is again a contradiction as the first expression depends on x but not y, and the second depends on y and not x.

5 Lemma VIII.2

Although we have shown that Maudlin has not provided a counterexample to our theorem, in his comment he asserts that our Lemma VIII.2 is incorrect, so we conclude by explaining why he is mistaken.

In our paper, we give the definition of an ontological time reverse as in eq. (3), but then immediately assert that if an ontological time reverse exists then there also exists a time symmetric ontological model in which f is trivial. Since Maudlin claims that our argument for this is difficult to follow, here it is again in more detail.

The reason we can always find a model in which f is trivial is because f is invertable. The condition $L' = f[\lambda]$ is exactly the same as the condition $f^{-1}[L'] = \lambda$. We can then define a new ontic state $L'' = f^{-1}[L]$ and we will have

$$
p_{PM}(A = a, B = b, L = \lambda|X = x, Y = y) = p_{P'M'}(A' = b, B' = a, L'' = \lambda|X' = y, Y' = x). \tag{9}
$$

The new ontic state L'' inherits all of the conditional independences that L had because the two are isomorphic. In particular, we have the defining equation for an ontological model

$$p_{P'M'}(A' = b, B' = a, L'' = \lambda | X' = y, Y' = x) \\ = p_{M'}(B' = a | L'' = \lambda, Y = x) p_{P'}(L'' = \lambda | A' = b, X' = y) p_{P'}(A' = b | X' = y). \quad (10)$$

This holds because the same equation holds if we replace $L'' = \lambda$ with $L' = f[\lambda]$, by definition of an ontological model, but $L'' = \lambda$ and $L' = f[\lambda]$ are the exact same condition.

In the paper, we use this construction to prove our results without having to include the function f in our equations. In case you do not buy the above argument, we will now prove the part of Lemma VIII.2 that Maudlin claims is wrong without eliminating f.

The relevant claim is that, if an experiment has an ontological time reverse, than an ontological model of it must satisfy

$$p_{PM}(L = \lambda | X = x, Y = y) = p_P(L = \lambda), \quad (11)$$

i.e. the ontic state is statistically independent of the measurement inputs.

To prove this, we start with the defining equation for an ontological model

$$p_{PM}(A = a, B = b, L = \lambda | X = x, Y = y) \\ = p_M(B = b | L = \lambda, Y = y) p_P(L = \lambda | X = x, A = a) p_P(A = a | X = x). \quad (12)$$

We can perform a Bayesian inversion of the second term to obtain

$$p_P(L = \lambda | X = x, A = a) p_P(A = a | X = x) = p_P(A = a | L = \lambda, X = x) p_P(L = \lambda | X = x),$$

which gives

$$p_{PM}(A = a, B = b, L = \lambda | X = x, Y = y) \\ = p_M(B = b | L = \lambda, Y = y) p_P(A = a | L = \lambda, X = x) p_P(L = \lambda | X = x). \quad (13)$$

Summing both sides over a and b then gives

$$p_{PM}(L = \lambda | X = x, Y = y) = p_P(L = \lambda | X = x), \quad (14)$$

which tells us that, for the experiment (P, M), L is conditionally independent of Y, given X.

We now do the same thing for the operational time reverse (P', M'), i.e. we start with

$$p_{P'M'}(A' = b, B' = a, L' = f[\lambda] | X' = y, Y' = x) \\ = p_{M'}(B' = a | L' = f[\lambda], Y' = x) p_{P'}(L' = \lambda | X' = y, A' = b) p_{P'}(A' = b | X' = y), \quad (15)$$

perform a Bayesian inversion to obtain

$$p_{P'}(L' = f[\lambda] | X' = y, A' = b) p_{P'}(A' = b | X' = y) = p_{P'}(A' = b | L' = f[\lambda], X' = y) p_{P'}(L' = f[\lambda] | X' = y),$$

which gives

$$p_{P'M'}(A' = b, B' = a, L = f[\lambda] | X' = y, Y' = x) \\ = p_{M'}(B' = a | L' = f[\lambda], Y' = x) p_{P'}(A' = b | L' = f[\lambda], X' = y) p_{P'}(L' = f[\lambda] | X' = y), \quad (16)$$

and then sum over a and b to obtain

$$p_{P'M'}(L' = f[\lambda] | X' = y, Y' = x) = p_{P'}(L' = f[\lambda] | X' = y). \quad (17)$$

Now, if (P, M) and (P', M') are operational time reverses then we have

$$p_{PM}(L = \lambda | X = x, Y = y) = p_{P'M'}(L' = f[\lambda] | X' = y, Y' = x), \tag{18}$$

which comes from summing eq. (3) over a and b. We already know that $p_{PM}(L = \lambda|X = x, Y = y) = p_P(L = \lambda|X = x)$ from eq. (14), so the only question is whether L is also independent of X. By definition, this is the case if, for every pair $x_1, x_2 \in \Omega_X$, $p_P(L = \lambda|X = x_1) = p_P(L = \lambda|X = x_2)$. However, applying eqs. (17) and (18) we have

$$p_P(L = \lambda|X = x) = p_{P'}(L' = f[\lambda]|X' = y).$$

Since the right hand side does not depend on x, the left hand side must also not vary with x, so we have the independence we need.

Maudlin complains that the equation $p_P(L = \lambda|X = x, Y = y) = p_P(L = \lambda)$ is implausible because it says that the ontic state is independent of both the preparation and measurement inputs, so it cannot convey any information at all between the two, and hence the preparation and measurement variables should be uncorrelated. This is not the case for two reasons:

1. $p_P(L = \lambda)$ can depend on P, the choice of preparation device. In quantum theory, for a no-signalling preparation, P corresponds to a fixed choice of ensemble average density operator, i.e. $\sum_a \rho_{a|x} = \rho$ is independent of x. However, different choices of P correspond to different density operators ρ, and $p_P(L = \lambda)$ can depend on this. Admittedly, this is somewhat obscured by the fact that we dropped the subscripts $_{PM}$ in the main argument of our paper.

2. Just because L is independent of X does not mean that it is independent of X and A, taken together, i.e. $p_P(L = \lambda|X = x, A = a) \neq p_P(L = \lambda)$. All we require is that $\sum_{a\in\Omega_A} p_P(L = \lambda|X = x, A = a)p_P(A = a|X = x) = p_P(\lambda)$. Thus, L can transmit information about A and X to the measurement device, provided it does so in such a way that there is no information about X left over when we average over A. If P is a no-signalling preparation, then this is exactly Spekkens' criterion of preparation noncontextuality [14], as we explained in our paper. The example of Spekkens' toy theory, also explained in our paper, shows how one can get nontrivial correlations between the preparation and measurement variables using this mechanism.

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