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A Statistical Method of Screening Electrocardiograms

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A STATISTICAL METHOD

OF SCREENING ELECTROCARDIOGRAMS

BY

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RGB

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INTRODUCTION

Since the advent of the electrocardiogram, usually abbreviated ECG, the extent of its use in the medical field has broadened through research to find extensive applications in the diagnosis of heart ailments. One such application of the electrocardiogram is the diagnosis of the heart's operating condition and classification of its condition into normal and possibly abnormal categories.

A new approach to the use of the electrocardiogram to categorize the condition of the heart is described in this thesis. The purpose of this thesis is to present the new method of screening electrocardiograms into normal versus possibly abnormal categories detecting at least fifty percent of the normal electrocardiograms as normal with a maximum number of abnormal electrocardiograms detected abnormal. The results of a feasibility study to determine the effectiveness of the method are also discussed.

CHAPTER I

THE ELECTROCARDIOGRAM

Medical science is as old as civilization and each generation has added to the ever-increasing store of medical knowledge. New medicines for curing diseases and new, better methods of analyzing symptoms have been and continually will be proposed, studied and developed. One area of research is in the field of diagnosing heart malfunction by studying electrocardiograms. Time-varying electrical activity takes place in the muscular tissue of the heart and this electrical activity can be detected as a time-varying voltage on the surface of the skin. This time-varying voltage is recorded as an electrocardiogram (ECG).

The fact that there is electrical activity associated with heart beat has been known since 1856. In 1887, Augustus D. Waller demonstrated that differences of potential caused by electrical activity of the heart were measurable only if proper contact was made between electrometer leads and any two areas of the body which included the heart between them. The first usable clinical instrument to observe the ECG was the string galvanometer type electrocardiograph developed by Willem Enthoven in 1903.[1]

The voltage difference due to the heart's electrical activity is monitored by placing electrodes at various points on the

body. There are standard positions chosen for placement of these electrodes in order to receive and analyze repeatable electrical activity and thereby compare normal standard electrocardiograms with the electrocardiogram of any heart and determine its condition. For one set of leads, the electrodes are placed on the right and left arms as well as on both legs. The electrode placed on the right leg is used only as a machine ground to help eliminate alternating current. A particular lead is a specified arrangement of electrodes for monitoring the heart's electrical activity. The electrocardiograms obtained from the bipolar standard leads are measurements of the difference of potential caused by the heart's electrical activity between two different limb electrodes and are labeled by terminology such as Lead I or Lead II. Lead I measures the difference of potential detected between the electrode on the left arm and the electrode on the right arm. Lead II measures the difference of potential detected between the electrode on the left leg and the electrode on the right arm. Lead III measures the difference of potential detected between the electrode on the left leg and the electrode on the left arm.[2]

Bipolar chest leads monitor the electrical activity of the heart by an electrode located on a specific point on the chest, and by various limb electrodes. Lead CR measures the potential difference

between the chest electrode and the left arm electrode. Lead CF measures the potential difference between the chest electrode and the left leg electrode.[2]

Unipolar leads measure the heart's electrical activity as a voltage at one particular spot on the body with reference to a non-fluctuating point. Two electrodes are connected close to one another on the left arm, the right arm and the left leg. One electrode from each limb is connected in common to obtain the non-fluctuating reference point. The unipolar leads monitor the voltage between the second electrode on each limb and the common connection of the first electrode on each limb. The unipolar leads monitor the right arm, VR; the left arm, VL; and the left leg, VF.[2]

Augmented unipolar limb leads use a combination of three electrodes: one is connected to the left arm, AVC; the right arm, AVR; and the left leg, AVF. Unipolar chest leads, V_1, V_2, V_3, V_4, V_5, and V_6, are used to monitor the heart's electrical activity, each at a specific point on the chest. The electrical activity monitored is the difference of potential between an electrode on the chest and the common connection of the left arm, the right arm and the left leg.[2]

The Frank lead system measures the heart's electrical activity as a spacial vector. The heart's electrical activity can be pictured

as a vector rotating in three-dimensional space. The Frank lead system monitors the heart's electrical activity by electrodes placed at various points on the body as shown in Fig. 1. Electrodes A, C, E, I and M are all located on the same horizontal plane when the subject is standing upright. Electrodes A and I are placed in the left and right midauxiliary line, respectively. Electrodes E and M are applied at the midline anteriorly and posteriorly, respectively. Electrode C is applied at an angle of 45^{o} from and in the same plane as the left midauxiliary line. Electrode F is placed on the left leg, and electrode H is placed on the back of the neck.[3]

Three electrocardiograms represent the x, y and z components of the spacial vector in the Frank lead system. The x component of the spacial vector is obtained from the A, C and I electrodes. The y component of the spacial vector is obtained from the M, F and H electrodes. The z component of the spacial vector is obtained from the H, M and F electrodes. The connection between the electrodes and the necessary corrections for the three ECG outputs V_x, V_y, and V_z to represent the spacial vector of the heart's electrical activity are obtained by the standardization factors obtained by the resistor network shown in Fig. 2.[3] The Frank lead system is used to obtain the data used in this thesis.

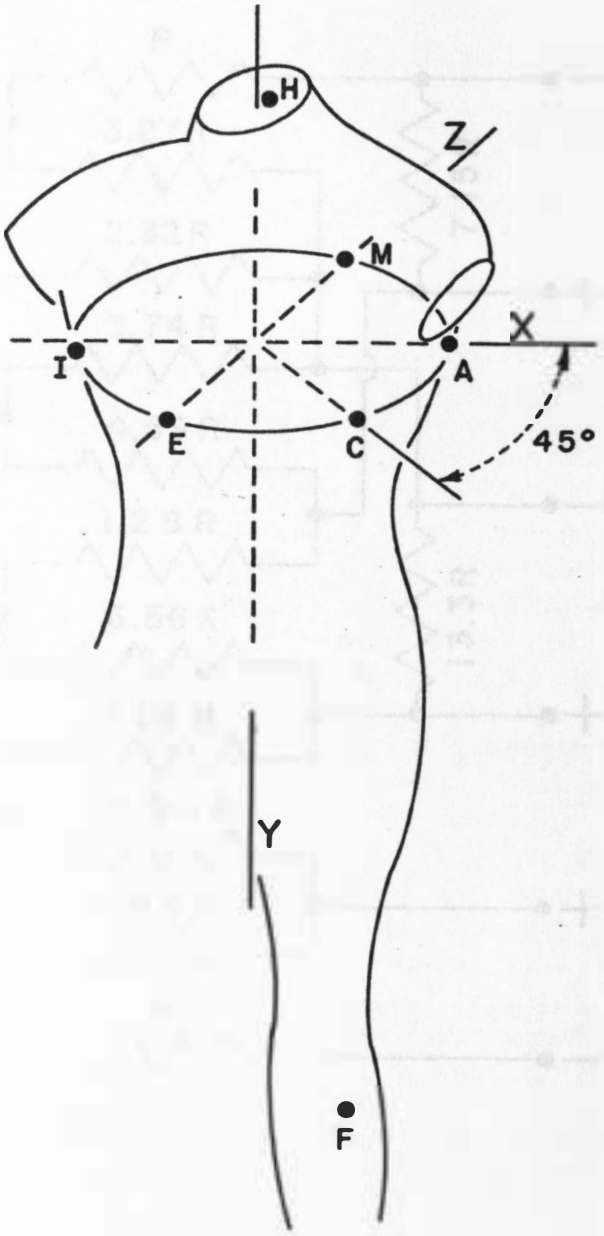


Figure 1

Electrode Arrangement of the Frank Lead System

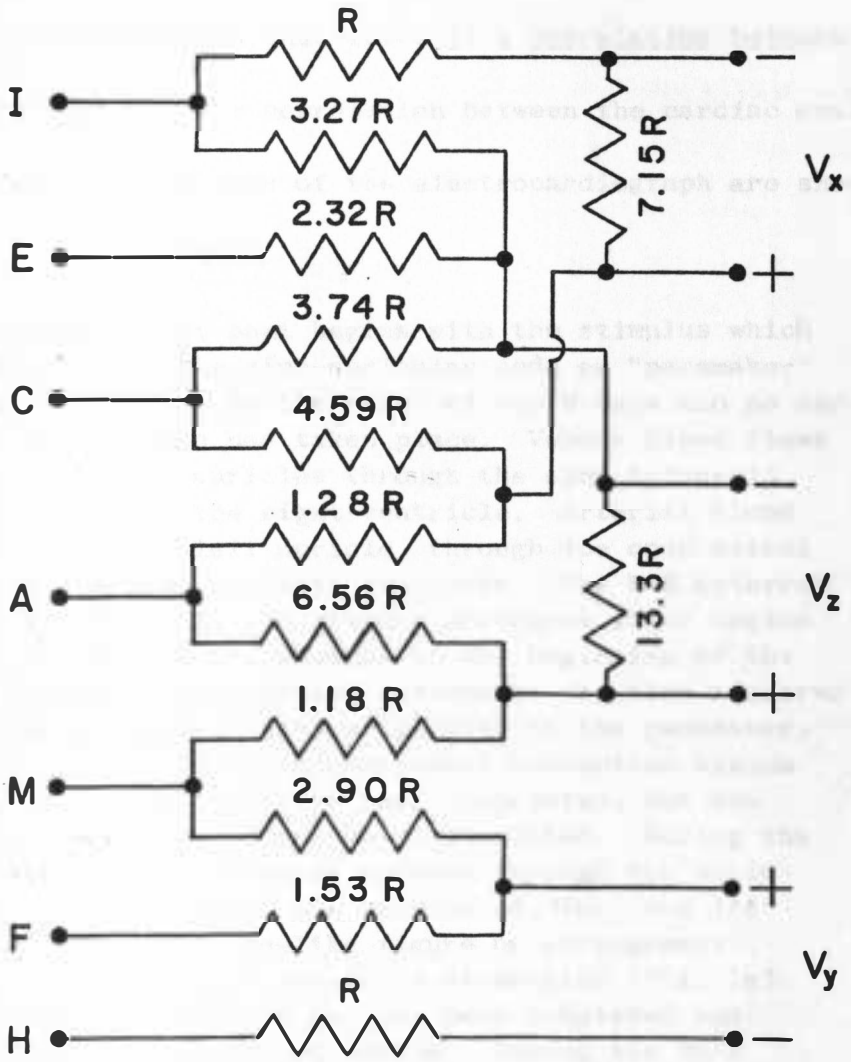


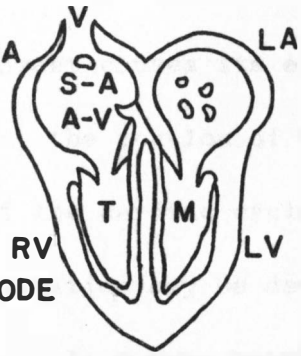
Figure 2

Standardization Factors of the Frank Lead System

Studies have shown that there is a correlation between ECG and the cardiac cycle. The correlation between the cardiac cycle and its corresponding portion of the electrocardiograph are shown in Fig. 3.

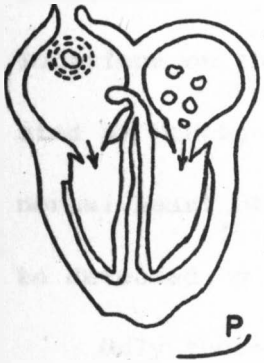
> The normal heart beat begins with the stimulus which originates in the sine-auricular node or "pacemaker" (Fig. 1b). This is the onset of the P wave and no cardiac contraction has taken place. Venous blood flows into the right auricles through the open tricuspid valve and into the right ventricle. Arterial blood flows into the left auricle, through the open mitral valve, and into the left ventricle. The P-R interval is the segment of the electrocardiogram which begins with the P wave and extends to the beginning of the QRS complex. This period represents the time required for the stimulus, which originated in the pacemaker, to spread to the intraventricular conduction system (Fig. 1d). The auricles have contracted, but the ventricles have not yet been stimulated. During the QRS complex the stimulus spreads through the auriculoventricular node, the bundles of His and its arborizations (a treelike figure or arrangement), and the ventricular muscle is stimulated (Fig. 1e). Auricular contraction has now been completed and ventricular contraction begins. During the RS-T segment and the beginning of the T wave the stimulus spreads throughout the ventricular musculature and the ventricles contract (Fig. 1f). The tricuspid and mitral valves become closed; the pulmonic and aortic valves are opened. The blood in the right and left ventricles is forced into the lungs and into the general circulation system. The end of the T wave represents the disappearance or recession of the stimulus--the beginning of the ventricular relaxation (Fig. 1g). The pulmonic and aortic valves close and prevent the return of blood. Venous blood again flows in from the periphery and oxygenated blood from the lungs. Thus, the normal heart cycle begins again.[4]

V. VENA CAVA
R.A. RIGHT AURICLE
R.V. RIGHT VENTRICLE
T TRICUSPID VALVE
S-A SINO-AURICULAR NODE

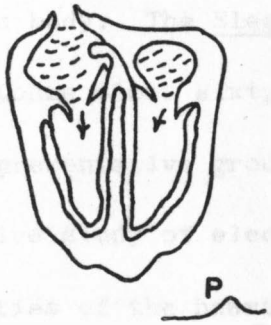


L.A. LEFT AURICLE
P PULMONIC VEINS
L.V. LEFT VENTRICLE
M MITRAL VALVE
A-V AURICULOVENTRICULAR NODE

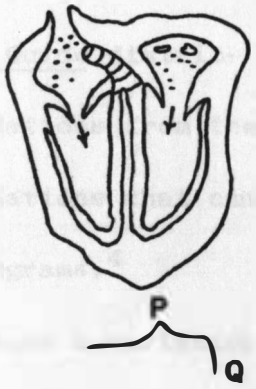
(a) THE HEART AT REST



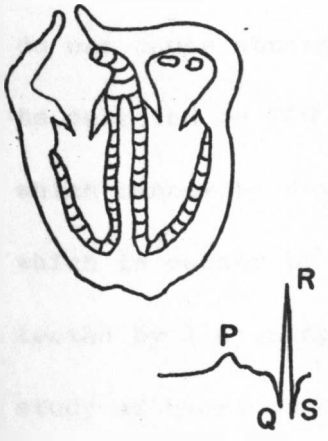
(b) P WAVE STARTS



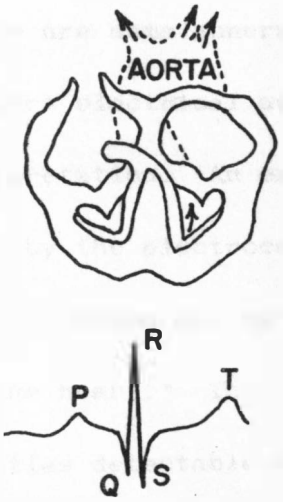
(c) P COMPLETED



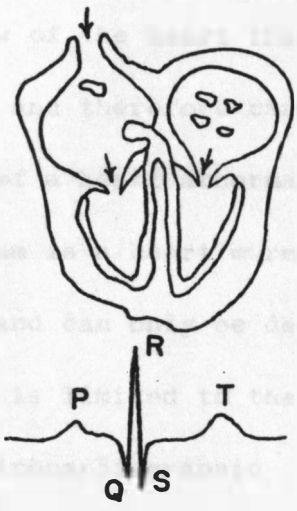
(d) P-R INTERVAL



(e) QRS COMPLEX



(f) RS-T SEGMENT



(g) T WAVE ENDS

Figure 3. The Normal Heart Beat[4]

When an abnormal heart causes its electrical activity to deviate from the normal pattern, the portion of the ECG which corresponds to the abnormal portion of the cardiac cycle differs from the normal ECG. The cause of the ECG deviation may be determined by pattern recognition of the ECG. To analyze many heart abnormalities, the cardiologist must examine several electrocardiograms obtained from monitoring the electrical activity of the heart at different locations on the patient's body. The Electrocardo Guide distributed by Merck, Sharp and Dohme shows sixty-one deviations from the normal heart beat as a representative group of deviations that can be detected by a comparative study of electrocardiograms.[4]

Only those abnormalities of the heart which cause a variation in the heart's electrical activity can be detected by the study of the patient's ECG. There are some abnormalities of the heart that do not cause abnormal heart electrical activity and therefore cannot be detected by ECG interpretation. An example of a heart abnormality which cannot be detected by the electrocardiogram is a heart murmur which is caused by a malformation of the heart and can only be detected by listening to the heart.[1] This thesis is limited to the study of heart abnormalities detectable by electrocardiographic diagnosis.

The science of electrocardiographic diagnosis of heart abnormalities is a science of pattern recognition. Diagnosis is based on the electrocardiogram's deviation from a normal pattern. The problem of pattern recognition is adaptable to the digital computer, but the effective use of the computer for evaluation of the electrocardiogram is dependent on the proper programming of the computer to detect the variations in the ECG waveform which are caused by heart abnormalities.

One purpose of this thesis is to propose a technique of screening normal electrocardiograms from possibly abnormal electrocardiograms. In the proposed technique, the ECG is changed from a time-varying plot to a statistical plot not involving time which is used for screening electrocardiograms into normal versus possibly abnormal heart categories.

How an existing method of computer diagnosis of electrocardiograms may be modified by the proposed technique of ECG screening and how the method may be used by clinics and hospitals to aid in rapid ECG diagnosis is explained in Chapter II. A brief review of probability theory and its application to the proposed method of ECG screening is covered in Chapter III. Chronological steps in the development of the proposed method of ECG screening are explained in Chapter IV. A brief review of the mechanization of the proposed

ECG screening method and the results and conclusions of this thesis are found in Chapter V.

CHAPTER II

APPLICATIONS OF THE PROPOSED ECG SCREENING METHOD

The method of ECG screening proposed in this thesis would quickly screen electrocardiograms into two categories: normal ECG or possibly abnormal ECG. The device which would mechanize the proposed method of ECG screening could be used by hospitals and clinics to rapidly eliminate those electrocardiograms detected as normal from any further analysis. The electrocardiograms detected as possibly abnormal would then be analyzed by the cardiologist, the computer or any method previously used by the hospital or clinic.

The actual hardware, which may be a small specialized digital computer, necessary for the mechanization of the proposed ECG screening method is not considered in this thesis which covers the theory and feasibility of the proposed ECG screening method. A computer simulation of the proposed method on an IBM 360/30 digital computer was used for the feasibility study.

The computer is also successfully being used as a tool for more complex screening of electrocardiograms into a normal category or into a specific heart malfunction category. The device which would mechanize the proposed method of ECG screening could be used to modify existing methods of computer analysis of electrocardiograms,

as explained in the above paragraph and thereby decrease analysis time for complex computer diagnosis.

A simplified block diagram of a method for computer analysis of electrocardiograms developed from a collaborative study between International Business Machines, Inc. and Mount Sinai Hospital, New York City, is shown in Fig. 4. A patient is connected to an ECG preamplifier by electrodes on his skin. At the body surface the detectable electrical activity of the heart is approximately one millivolt in magnitude. In order for the ECG signal to be used for diagnosis, it must be amplified to a convenient, usable level. The ECG preamplifier provides the amplification and impedance matching necessary for the electrical activity of the heart to be further conditioned and recorded as a useful electrocardiogram. The output signal of the ECG amplifier combination is the input signal to any device needed for monitoring or further transmission of the ECG, such as a strip chart recording or a data link to remote recorders. Data for patient identification is appropriately coded and placed with the ECG data.[5]

The ECG signal is recorded on an analog data recorder and is also used as input data for the computer. The analog data recorder records the ECG data to be used by the cardiologist for electrocardiographic diagnosis of heart abnormalities. The ECG signal can

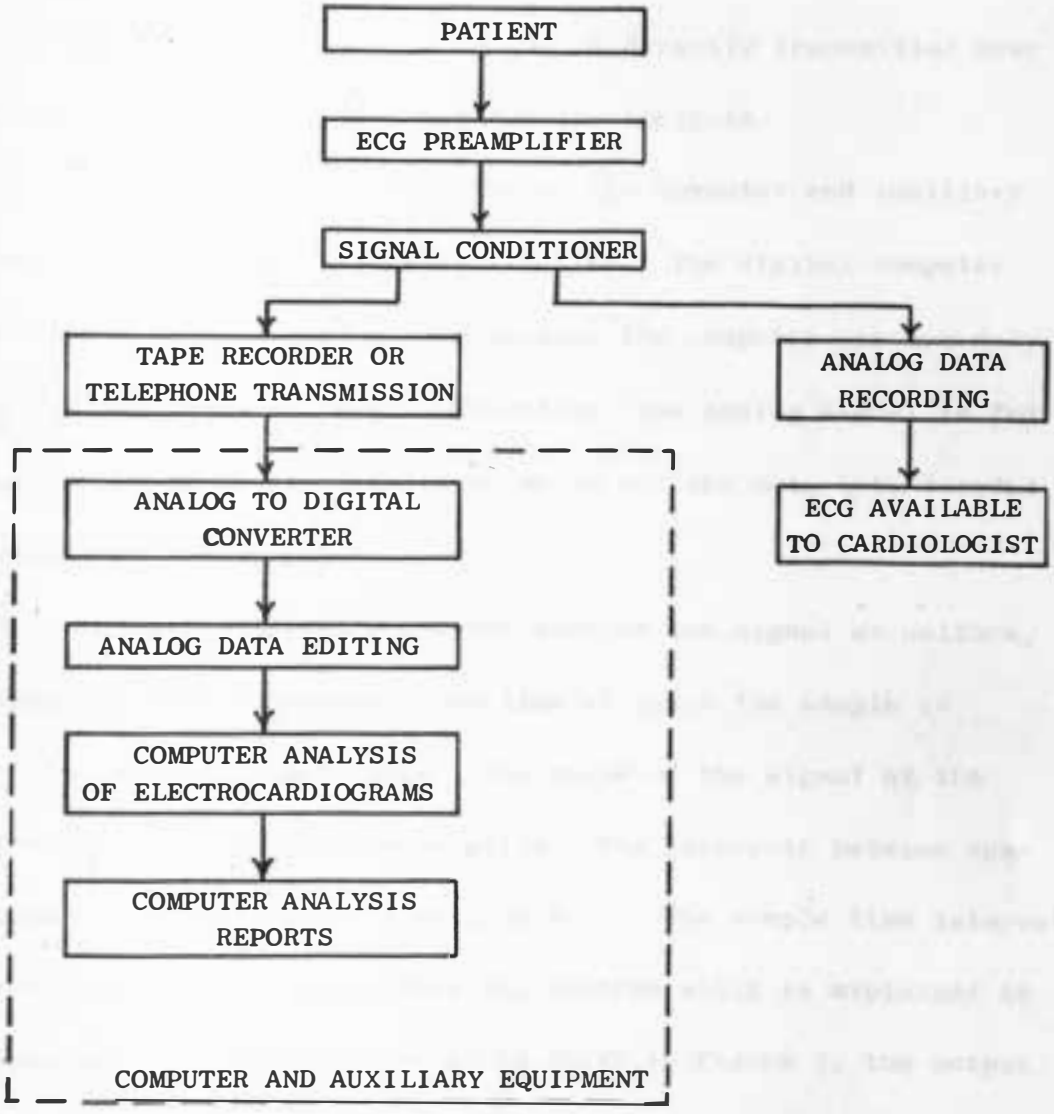


Figure 4

IBM - Mt. Sinai Hospital Computer System for ECG Analysis

be recorded on magnetic tape or it can be directly transmitted over an ordinary telephone to the computer for analysis.

The original ECG datum received by the computer and auxiliary equipment is a continuous function of time. The digital computer cannot accept data in analog form because the computer can use only digital number codes as input. Therefore, the analog signal is fed through an analog-to-digital converter to put the data into a coded form usable by the computer.

The analog-to-digital converter samples the signal at uniform, predetermined time intervals. The time at which the sample is taken is called the sample time. The value of the signal at the sample time is called the sample point. The intervals between sample times are called sample time intervals. The sample time interval is determined by the Nyquist Sampling Theorem which is explained in the Appendix. For the input waveform shown in Figure 5, the output of the analog-to-digital converter is represented by a series of discrete numbers. Each digital number is the value of the signal at a specific time as shown in Fig. 6.

The waveform can be reconstructed from the digital data as shown in Fig. 7. In this case, a first order reconstruction technique has been used because the reconstructed waveform assumes the

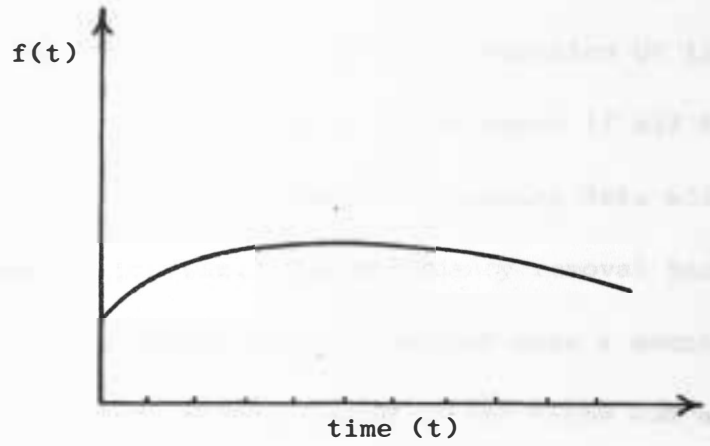


Figure 5. Input Waveform

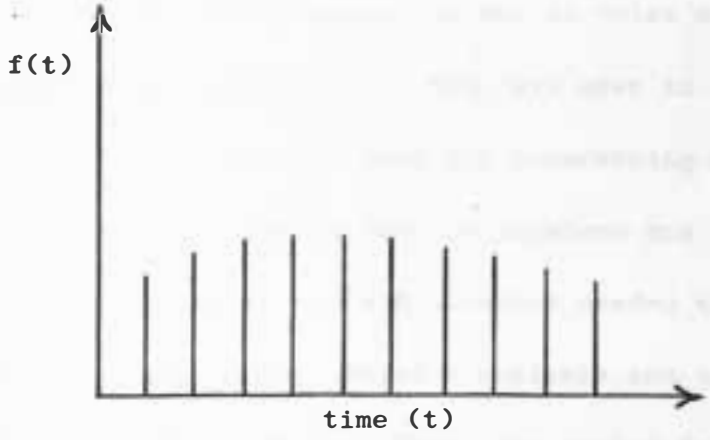


Figure 6. Digitized Waveform

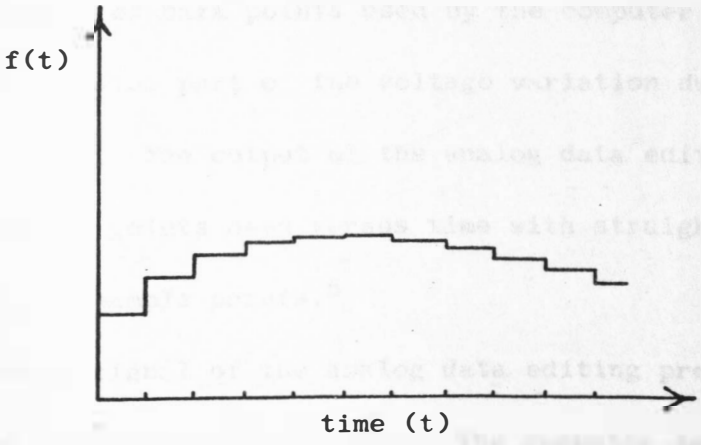


Figure 7. Reconstructed Waveform

value of the last sample point over the entire sample interval. The resulting waveform is not a continuous function of time.

Considerable computer time can be saved if all the sample points are not used by the computer. Analog data editing is a redundancy removal process. The redundancy removal technique applied in the IBM - Mount Sinai Hospital method uses a sample point only when there is a significant change in its slope and magnitude. The time relationship between sample points is preserved. There is a voltage variation of a small magnitude due to noise on the ECG waveform. The RMS noise voltage on the ECG data used in this thesis was 0.15 millivolts. The criterion used for determining when a sample point is taken are the slope of the ECG waveform and the magnitude of change in the voltage of the ECG waveform needed to preserve the waveform necessary for proper computer analysis and to eliminate the voltage variations due to noise. Thus, the analog data editing reduces the number of data points used by the computer and also automatically eliminates part of the voltage variation due to noise on the ECG waveform. The output of the analog data editing process is a plot of sample points used versus time with straight line segments joining adjacent sample points.[5]

The output signal of the analog data editing process is used as the input signal to the computer. The computer is programmed

for a sequence of operations on the digitized and edited ECG signal. The computer first locates the beginning of the QRS complex by detecting slope differences, then the computer analyzes the ECG data by a sequence of tests. The specific sequence used is determined at different points by the results of each previous test. The computer tests of the ECG use the slope, height and duration of the segments and combinations of segments of the electrocardiogram for analysis. The output of the computer is the results of the analyzed ECG, such as diagnosis and measurement data.[6]

The proposed device would quickly screen the electrocardiograms into normal or possibly abnormal categories. The proposed screening device could modify an existing method of computer analysis of electrocardiograms by screening normal electrocardiograms as normal, then extensively testing only those electrocardiograms detected by the proposed ECG screening device as possibly abnormal, as shown in Fig. 8. All electrocardiograms would then be indicated as normal or diagnosed as a specific heart abnormality, as in the previous system.

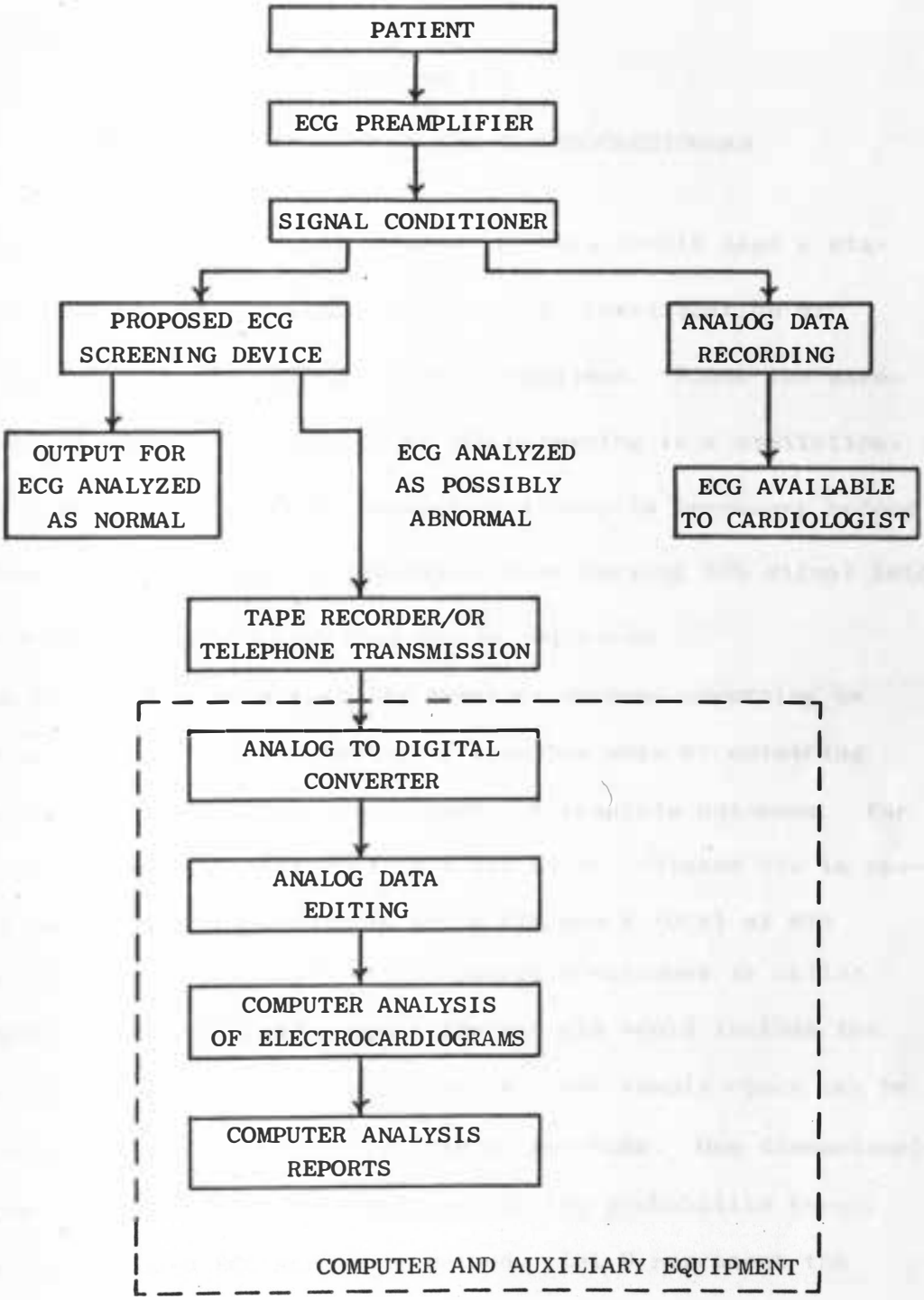


Figure 8

Modified IBM - Mt. Sinai Hospital Computer System for ECG Analysis

CHAPTER III

THE STATISTICAL PLOT OF THE ELECTROCARDIOGRAM

The ECG screening method proposed in this thesis uses a statistical plot of the ECG signal as data for classification of normal versus possibly abnormal heart conditions. Since the waveform used in the proposed method of ECG screening is a statistical plot, a brief review of some probability theory is necessary before the method of converting the continuous time varying ECG signal into a time-invariant statistical plot can be explained.

The probability of a specific event or outcome occurring is defined as the ratio of the number of possible ways of obtaining the specific outcome to the total number of possible outcomes. For example, the probability of rolling a six on an unbiased die is one-sixth because there is one way to get a six and a total of six possible outcomes. The total of all possible outcomes is called the sample space. The sample space for one die would include the possible outcomes of 1, 2, 3, 4, 5, and 6. The sample space can be more than one dimension as in the case of two dice. One dimensional sample space is adequate for understanding the probability theory used in the proposed ECG screening method. Let U represent the total number of possible outcomes in the sample space and R represent the number of ways the specific outcome can occur. The probability of the specific outcome x_i occurring is

$$P(x_i) = R/U \qquad (1)$$

and clearly

$$P(x_i) \leq 1 \qquad (2)$$

since R can never be greater than U.

If the outcome of two specific events cannot happen simultaneously in a given trial, the events are called mutually exclusive. Consider the roll of a die. Let event A be defined as an outcome of a 2, 3, or 4; event B be defined as an outcome of a 4, 5, or 6; and event C be defined as an outcome of a 1. Events A and B are not mutually exclusive because the outcomes of events A and B can happen simultaneously when the die is a four. Events A and C are mutually exclusive because there is no specific outcome of the die which satisfies both events A and C. Similarly, events B and C are mutually exclusive because there is no specific outcome of the die which satisfies both events B and C.

If all specific events, x_i, contain the entire sample space and are mutually exclusive, then the sum of the probabilities of each specific outcome in the sample space equals one. For example, consider the outcome of a toss of a coin. All of the specific events, head and tail, contain the entire sample space, head or

tail, and are mutually exclusive. The probability of a head is 1/2 and the probability of a tail is 1/2.

$$P(\text{Head}) = 1/2 \tag{3}$$

$$P(\text{Tail}) = 1/2 \tag{4}$$

The total of all $P(x_i)$ is one

$$P(\text{Head}) + P(\text{Tail}) = 1 \tag{5}$$

The probability distribution function, $F_x(\alpha)$, is defined as the probability of x being less than or equal to some variable

$$F_x(\alpha) = P(x \leq \alpha) \tag{6}$$

In Fig. 9 all events x are mutually exclusive. The probability of each specific event x is

$$P(1) = 1/2 \tag{7}$$

$$P(2) = 1/4 \tag{8}$$

$$P(3) = 1/8 \tag{9}$$

$$P(4) = 1/8 \tag{10}$$

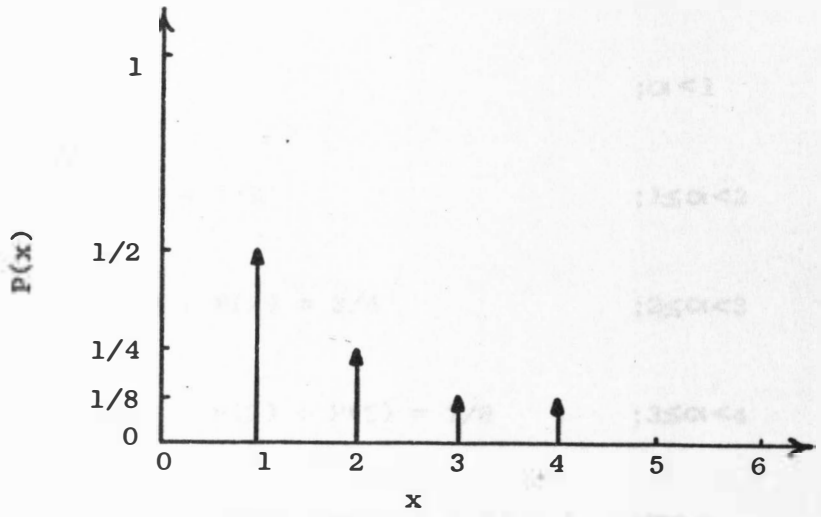


Figure 9. Probability Function

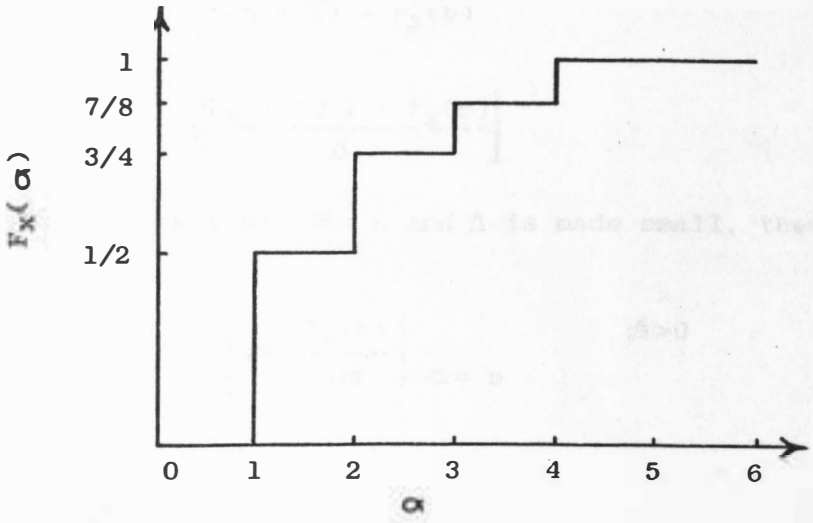


Figure 10. Probability Distribution Function

The probability distribution function $F_x(\alpha)$ is shown in Fig. 10.

By the definition $F_x(\alpha)$ is clearly

$$F_x(\alpha) = 0 \qquad ;\alpha<1 \qquad (11)$$

$$F_x(\alpha) = P(1) = 1/2 \qquad ;1\leq\alpha<2 \qquad (12)$$

$$F_x(\alpha) = P(1) + P(2) = 3/4 \qquad ;2\leq\alpha<3 \qquad (13)$$

$$F_x(\alpha) = P(1) + P(2) + P(3) = 7/8 \qquad ;3\leq\alpha<4 \qquad (14)$$

$$F_x(\alpha) = P(1) + P(2) + P(3) + P(4) = 1 \qquad ;\alpha\geq4 \qquad (15)$$

The probability that x lies in a small interval $(b, b + \Delta)$ is

$$P(b\leq x<b + \Delta) = F_x(b + \Delta) - F_x(b) \qquad (16)$$

$$= \Delta\left[\frac{F_x(b + \Delta) - F_x(b)}{\Delta}\right] \qquad (17)$$

If $F_x(\alpha)$ is differentiable at $\alpha = b$ and Δ is made small, then

$$\lim_{\Delta \to 0}\left[\frac{F_x(b + \Delta) - F_x(b)}{\Delta}\right] \approx \left.\frac{dF_x(b)}{d\alpha}\right|_{\alpha = b} \qquad ;\Delta>0 \qquad (18)$$

and

$$P(b\leq x<b + \Delta)\approx\Delta\left[\frac{dF_x(b)}{d\alpha}\right] \qquad (19)$$

If $F_x(\alpha)$ is everywhere differentiable and I is defined as a small interval, on α where I includes $N\Delta$; then

$$P(x \text{ in } I) = \lim_{\Delta \longrightarrow 0} \sum_{o}^{N} \left\{ \frac{F_x(b + \Delta) - F_x(b)}{\Delta} \right\} \Delta \tag{20}$$

$$= \int_I F'_x(\alpha)\, d\alpha \tag{21}$$

where $F'(\alpha)$ denotes the derivative of $F_x(\alpha)$, and P(x in I) means the probability that x lies in the region I.

The probability density function of x, p_x, is defined as

$$p_x(\alpha) = \frac{dF_x(\alpha)}{d\alpha} \tag{22}$$

therefore

$$P(x \text{ in } I) = \int_I p_x(\alpha) d\alpha \tag{23}$$

since $F_x(\alpha)$ is monotonic increasing and also,

$$F_x(+\infty) = 1 \tag{24}$$

from Eq. 23 it is evident that

$$p_x(\alpha) \geq 0; \text{ all } \alpha \tag{25}$$

and

$$\int_{-\infty}^{+\infty} p_x(\alpha) d\alpha = 1 \tag{26}$$

[7]

The method of obtaining the desired probability density function for the proposed ECG screening technique is explained in the following paragraphs. The ECG data is fed through an analog-to-digital converter, the operation of which is explained in Chapter II. The output of the analog-to-digital converter is a series of digitally coded numbers. Each coded number represents the digital value of the ECG signal at each specific sample time.

The range of the magnitude of the digitally coded ECG data is divided into a convenient number of equally spaced intervals. The range of an interval is the range of data between two consecutive levels. The range of the first interval would be the range of data greater than or equal to the most negative digitally coded data point but less than and not including the first level value. The range of the second interval would be the range of data greater than or equal to the first level number but less than and not including the second level number. The range of each consecutive interval would be determined in the same manner until the entire range of the expected ECG data has been accounted for as shown in Fig. 11. One hundred one levels were used to obtain the experimental results of this thesis.

The proposed screening method begins by feeding a given length of continuous ECG data through the analog-to-digital converter. Each

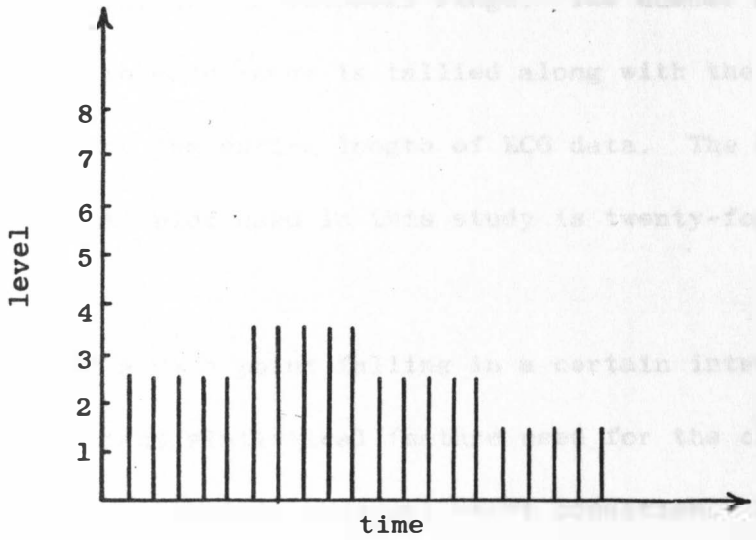


Figure 11. Digital Data

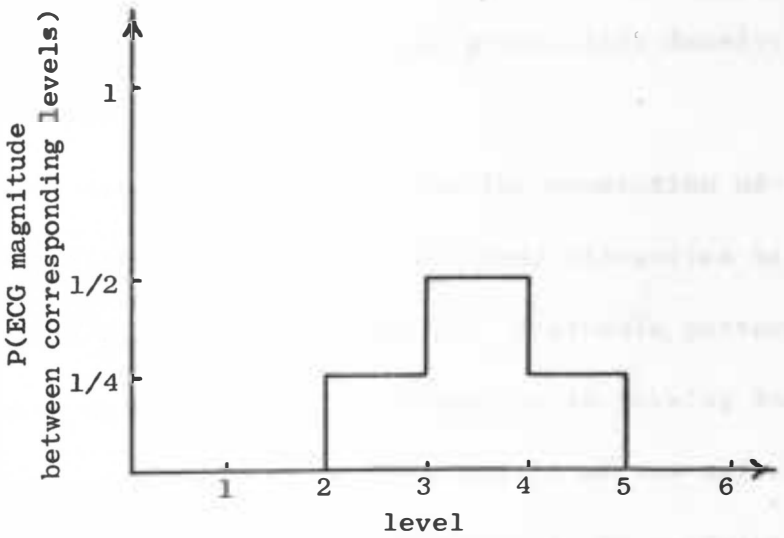


Figure 12. Statistical Plot

resulting data point falls in one interval range. The number of data points which fall in each range is tallied along with the total number of data points for the entire length of ECG data. The ECG data for each statistical plot used in this study is twenty-four seconds in length.

The probability of a data point falling in a certain interval range is the time-invariant statistical feature used for the classification of normal versus possibly abnormal heart condition. The probability density function is a plot of the ratio of the number of data points which fall into each level range to the total number of data points versus the corresponding level number as shown in Fig. 12. The method used to calculate the probability density function of Fig. 12 is shown in Table 1.

The problem of determining a technique for separation of the ECG statistical plot into normal versus abnormal categories is a problem of trainable pattern-classification. Trainable pattern-classification uses data rather than mathematics to develop the pattern classifier. The steps in the development of the data-classifying technique proposed in this thesis will be explained in Chapter IV.

Level No.	Number of Data Points in Level	Fraction of Data Points in Level
1	0	0.00
2	5	0.25
3	10	0.50
4	5	0.25
5	0	0.00
6	0	0.00
7	0	0.00
Total	20	1.00

Table 1. The Method of Calculating the Probability Density Function

CHAPTER IV

THE DEVELOPMENT OF THE PROPOSED METHOD FOR ECG SCREENING

The pattern classifying technique used to categorize the time-invariant statistical plot of the electrocardiographic waveform into normal versus possibly abnormal categories can be chosen from an infinite number of classifiers. A pattern classifier is a device that sorts patterns into categories. A block diagram of the pattern classifier is shown in Fig. 13. The probability density function described in Chapter III is the input to the pattern classifier. There is little theory to aid in selecting the type of measurement or input to the pattern classifier so the selection of such a measurement rests solely on the designer's ideas.[8] A number of ideas for measurement-selection-techniques that were tried for classifying the ECG statistical plot into normal versus possibly abnormal ECG are discussed in this chapter.

Once a measurement-selection-technique has been proposed, the optimum decision level for the technique can be found by a learning process. The measurement-selection-technique is applied to a finite set of data for which actual diagnosis is known. The finite set of data with a known diagnosis is called the training set. The optimum decision level can be found by repeatedly testing the training set of data at different levels and picking that level where the resulting categorization has the maximum correct results.[8]

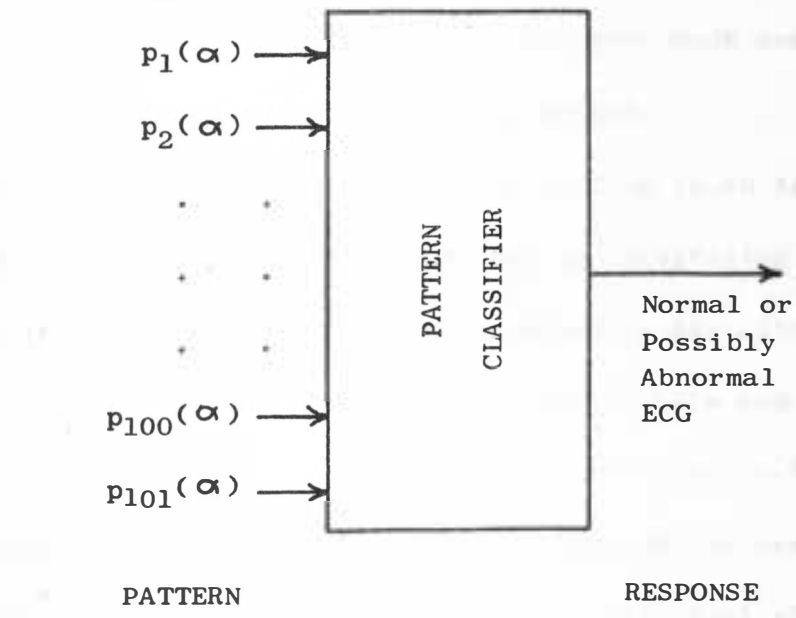


Figure 13. A Pattern Classifier[8]

A preliminary evaluation of the measurement-selection-technique can be made by checking the effectiveness of the classification of the training set of data. The technique is evaluated by testing a set of data which has not been used in the learning machine process. If the results of the test are satisfactory, the method may be considered as a possible method of ECG screening, but only after successful tests with large amounts of data have been made and evaluated can it be considered as a satisfactory method.

The probability of the data point falling in an interval less than or equal to some level is obtained by integrating the probability density function and is called the probability distribution function. Typical examples of the Frank lead, x axis, y axis and z axis ECG probability distribution function curves are shown in Fig. 14, Fig. 15 and Fig. 16, respectively. The advantage of the probability distribution function of the time-invariant statistical plot is that each plot would have the same general shape regardless of the number of intervals used to obtain the plot. If fewer intervals are used, more of the total data points would fall in each level. The probability distribution, or the probability of a data point falling in an interval less than or equal to some level would have approximately the same shaped curve if the level axis is the same length.

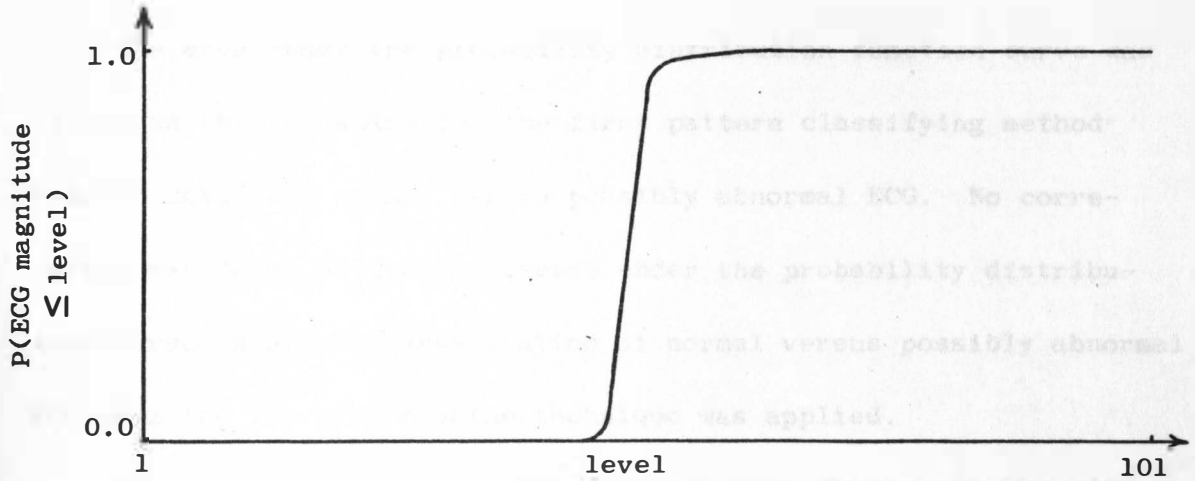


Figure 14. Typical x Axis Probability Distribution Function

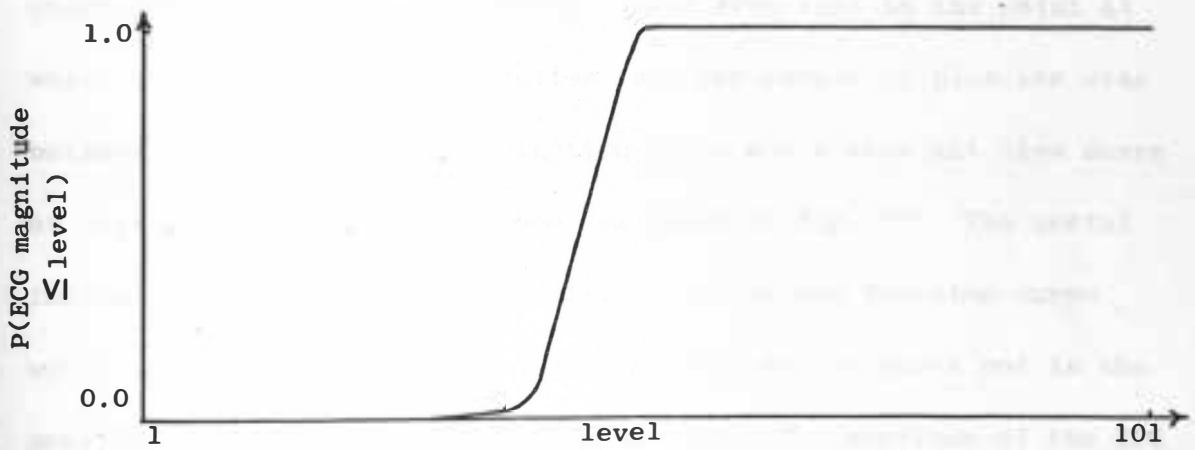


Figure 15. Typical y Axis Probability Distribution Function

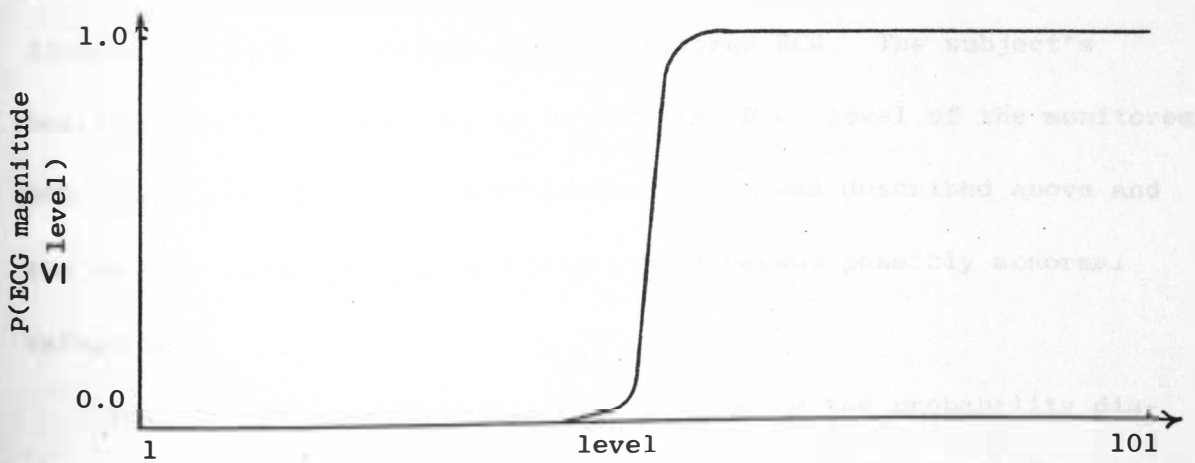


Figure 16. Typical z Axis Probability Distribution Function

The area under the probability distribution function curve was chosen as the mechanism for the first pattern classifying method used to determine normal versus possibly abnormal ECG. No correlation was found between the areas under the probability distribution curves and the classification of normal versus possibly abnormal ECG when the learning machine technique was applied.

The next mechanism for ECG categorization chosen was the area under the probability distribution curve from zero to the point at which the probability distribution function equals .5 plus the area between the probability distribution curve and a straight line curve of unit magnitude and zero slope, as shown in Fig. 17. The useful information from the ECG probability distribution function curve was considered to be contained in the shape of the curve not in the position of the curve along the level axis. The position of the ECG probability distribution function curve on the level axis is proportionate to the D.C. voltage of the monitored ECG. The subject's heart condition has nothing to do with the D.C. level of the monitored ECG. No correlation was found between the areas described above and the categorization of the ECG into normal versus possibly abnormal categories.

The measurement-selection-techniques using the probability distribution function were tried without success so the possibility of

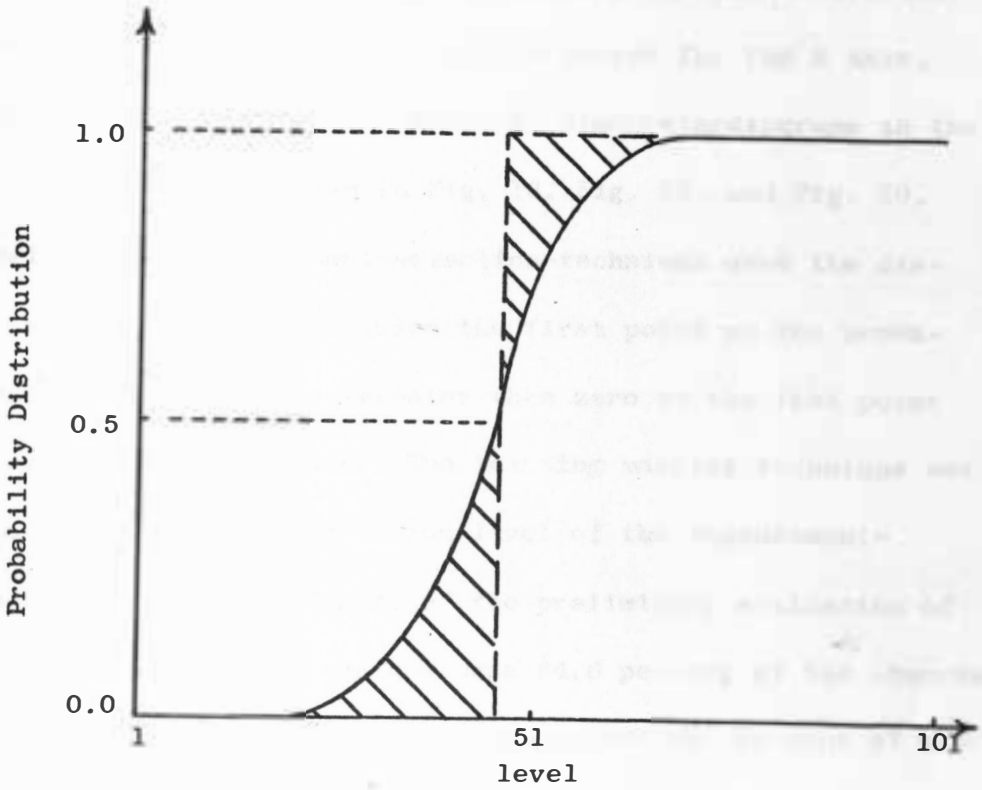


Figure 17. Decision Criterion for the Curve of the Probability Distribution Function

a technique using the probability of an ECG data point falling into a specific interval or the probability density function was tried. The over-all probability density function curves for the x axis, y axis, and z axis of normal and abnormal electrocardiograms in the training set of data, is shown in Fig. 18, Fig. 19, and Fig. 20, respectively. This measurement-selection-technique used the distance, in number of levels, between the first point on the probability density function curve greater than zero to the last point on the curve greater than zero. The learning machine technique was applied to find the optimum decision level of the measurement-selection-technique. The results of the preliminary evaluation of the measurement-selection-technique were 64.6 percent of the abnormal electrocardiograms detected abnormal and forty-eight percent of the normal electrocardiograms detected normal out of a test sample of forty-three electrocardiograms. An actual error occurs when an abnormal ECG is categorized as normal. Therefore, the error for the test sample of forty-three electrocardiograms was 14.3 percent.

The method described above used only the portion of the probability density function curve which was near zero. The above technique, expanded to use the whole curve with weighting functions as described below, is the proposed method of ECG screening. What the proposed method of ECG screening is and the preliminary evaluation of the method are discussed in the remainder of Chapter IV.

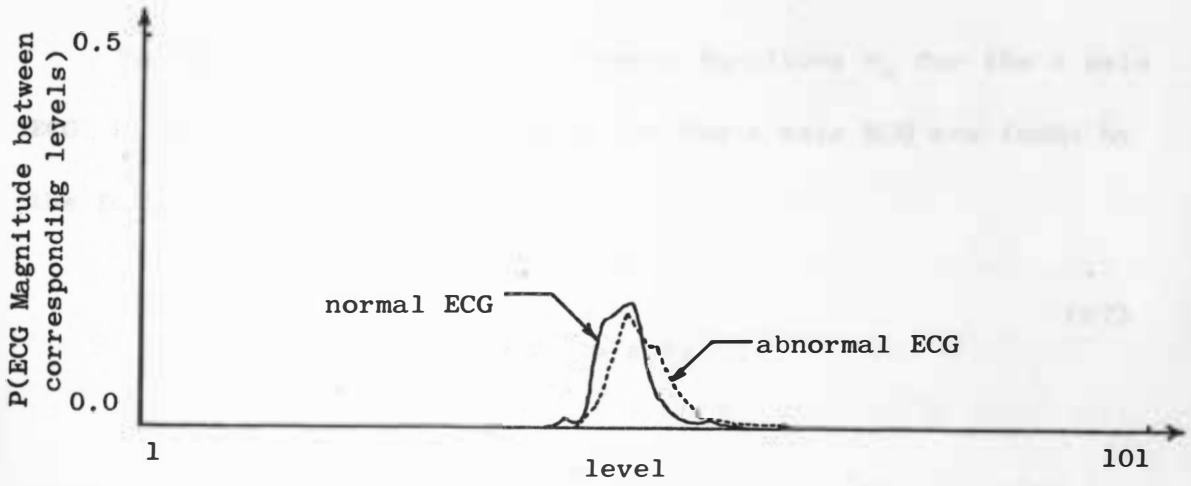


Figure 18. x Axis Probability Density Function

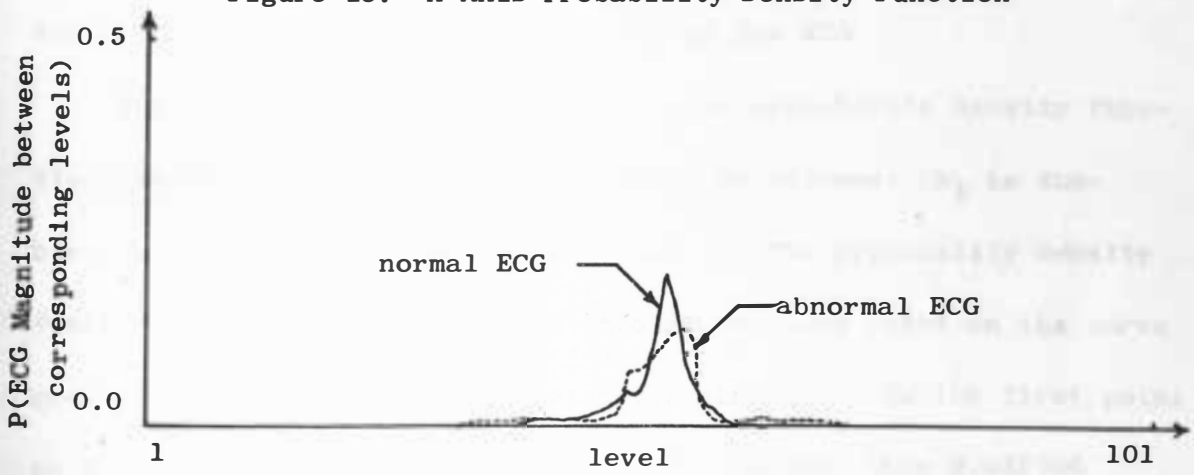


Figure 19. y Axis Probability Density Function

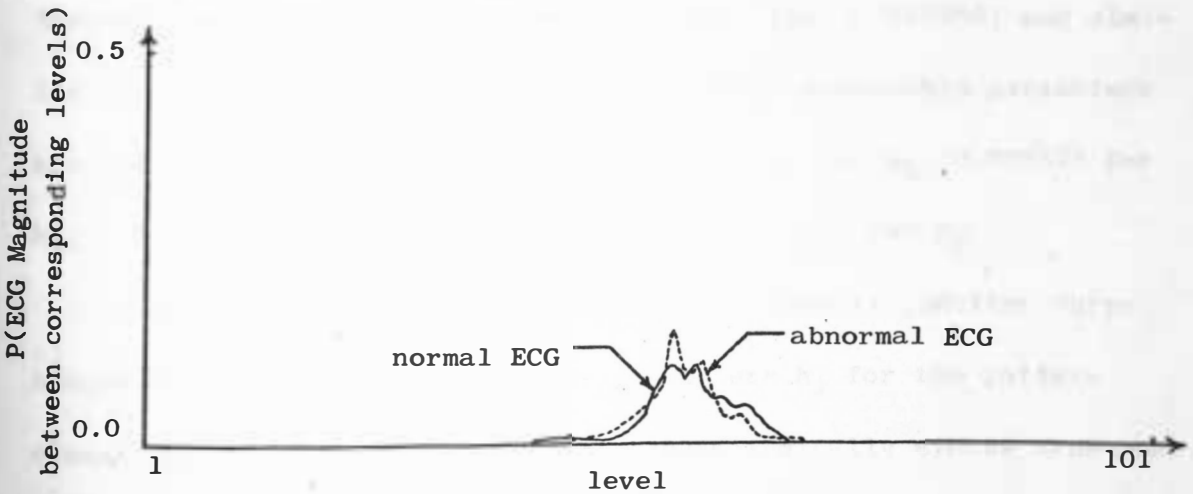


Figure 20. z Axis Probability Density Function

The pattern classifier measurement functions H_x for the x axis ECG, H_y for the y axis ECG, and H_z for the z axis ECG are found by the following equation:

$$H = \sum_{i=1}^{9} a_i h_i \tag{27}$$

where a_i is a weighting factor and h_i is a measurable parameter on the probability density function curve of the ECG.

The nine measurable parameters on the probability density function curve of the ECG signal are defined as follows: h_1 is the horizontal distance from the first point on the probability density function curve greater than zero through the last point on the curve greater than zero; h_2 is the horizontal distance from the first point on the probability density function curve greater than 0.012750 through the last point on the curve greater than 0.012750; and similarly, the critical numbers for the remaining measurable parameters are 0.024250 for h_3; 0.038750 for h_4; 0.055500 for h_5; 0.075375 for h_6; 0.099750 for h_7; 0.130750 for h_8 and 0.174750 for h_9.

The specific values of the probability density function curve chosen to determine the measurable parameters h_i for the pattern classifier measurement functions are logarithmically spaced from one-fourth to zero but not including the value of one-fourth as shown in

Fig. 21. The measurable parameters h_i are chosen in the above manner because the information received near the skirts of the probability density function is considered to be the most useful.

The weighting factors, a_i, as used in the Eq. 27 to evaluate the pattern classifier measurement function H, are calculated by using the average of the normal h_i's and the abnormal h_i's of the training set of ECG data. The averages of the normal h_i's and the abnormal h_i's seem to approach a value shown by Tables 2 and 3. The average h_i's of the normal electrocardiograms for a specific number of electrocardiograms is shown in Table 2. The average h_i's of the abnormal electrocardiograms are shown in Table 3. Biological data fluctuates from subject to subject because of the many uncontrolled variables present. The results of Tables 2 and 3 show that the averages of the h_i's from normal electrocardiograms and the averages of the h_i's from abnormal electrocardiograms may approach a limit as more data is used, but extensive data averaging must be done in order to establish this limit accurately.

The training set of data included forty-two ECG probability density functions; a_i was calculated using the total training set of data and the following formula:

$$a_i = \frac{(\text{average of abnormal } h_i) - (\text{average of normal } h_i)}{\text{average of normal } h_i} \qquad (28)$$

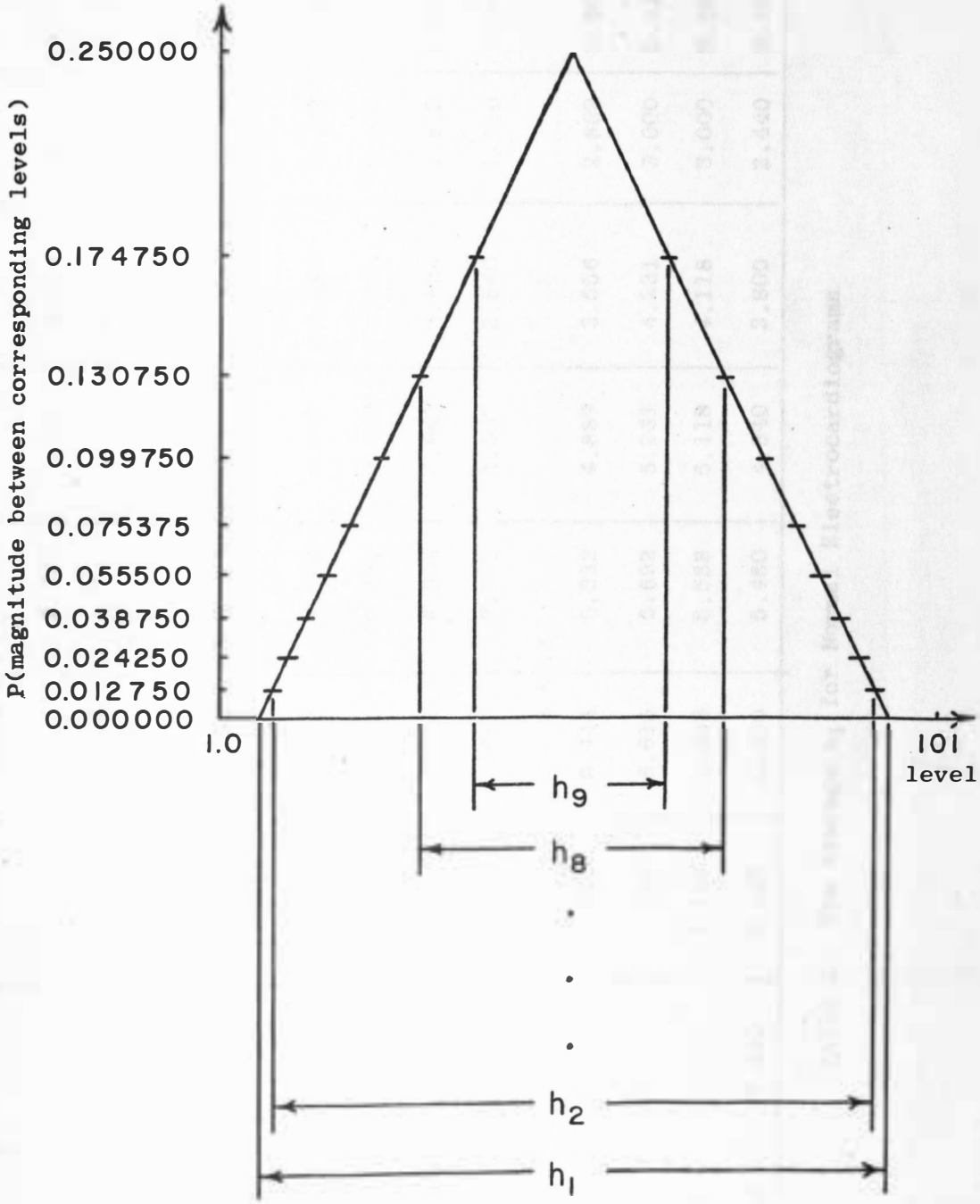


Figure 21. Measurable Parameters of the Probability Density Function Curve

Number Average	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9
				x Axis					
10	18.667	8.000	7.111	6.111	5.333	4.667	4.444	2.333	1.333
13	18.000	7.308	6.615	5.538	4.769	4.308	4.000	2.385	1.615
17	17.294	6.529	5.941	5.000	4.353	3.941	3.647	2.235	1.588
25	16.800	5.880	5.080	4.320	3.720	3.320	3.040	2.040	1.600
				y Axis					
10	22.111	13.000	12.667	12.111	12.000	9.667	2.889	2.111	0.889
13	19.462	11.615	11.077	10.615	10.231	7.923	2.923	2.077	0.923
17	18.118	10.471	9.824	9.353	8.824	6.941	2.824	1.882	0.941
25	15.920	9.000	8.440	7.880	7.120	5.520	2.640	1.920	1.080
				z Axis					
10	17.333	8.000	7.222	6.444	5.333	4.889	3.556	2.889	0.889
13	16.462	7.923	7.308	6.615	5.692	5.231	4.231	3.000	0.615
17	16.118	7.765	7.118	6.353	5.588	5.118	4.118	3.000	0.529
25	15.880	7.680	6.920	6.200	5.480	4.840	3.800	2.440	0.720

Table 2. The Average h_i for Normal Electrocardiograms

Number Average	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9
				x Axis					
10	20.100	7.300	6.400	5.300	4.800	4.400	2.700	1.900	1.000
14	19.615	6.846	6.154	5.154	4.692	4.231	2.692	1.923	1.000
18	19.889	7.111	6.111	5.333	4.778	4.222	3.000	2.000	0.833
				y Axis					
10	33.300	23.700	22.600	17.600	9.700	2.400	2.000	1.600	0.700
14	29.615	20.538	19.462	15.462	8.538	2.385	1.923	1.538	0.846
18	23.722	15.889	14.944	11.944	6.889	2.444	2.111	1.667	0.833
				z Axis					
10	22.500	11.200	10.200	5.100	4.300	2.800	2.200	2.000	1.000
14	20.000	9.923	8.846	4.769	4.154	3.000	2.308	2.154	1.154
18	18.444	9.056	8.222	5.111	4.500	3.611	2.833	2.333	1.056

Table 3. The Average h_i for Abnormal Electrocardiograms

for i = 1,2,...,8,9. This technique of evaluating a_i weights the data with a greater spread in the average h_i for normal and abnormal electrocardiograms, more than data which does not have as great a spread.

The data used to calculate the weighting functions a_i for the x, y and z axis pattern classifier measurement functions are shown in Tables 4, 5 and 6, respectively. The midpoint between the average H's calculated for the normal electrocardiograms and the abnormal electrocardiograms of the training set of data were the initial decision levels of H_X, H_y and H_Z used for the categorization of the ECG statistical plot into normal versus abnormal categories. The calculations of H_X, H_y and H_Z from the normal electrocardiograms and abnormal electrocardiograms are shown in Tables 7, 8 and 9, respectively.

The initial decision levels of the H's are calculated below, using the values obtained from Tables 7, 8 and 9:

$$H_x = (7.253 + 9.426)/2 = 8.340 \tag{29}$$

or the x axis ECG was considered normal if

$$H_x < 8.340 \tag{30}$$

and

$$H_y = (20.929 + 39.052)/2 = 29.99 \tag{31}$$

i	h_i's Abn. ECG Average	h_i's Norm ECG Average	(h_i's Abn. ECG Ave.)-(h_i's Norm. ECG Ave.)	a_i
1	19.889	16.800	3.089	0.184
2	7.111	5.880	1.231	0.209
3	6.111	5.080	1.031	0.203
4	5.111	4.320	0.791	0.183
5	4.778	3.720	1.058	0.284
6	4.222	3.320	0.902	0.272
7	3.000	3.040	-0.040	-0.013
8	2.000	2.040	-0.040	-0.020
9	0.833	1.600	-0.767	-0.479

Table 4. The Calculation of a_i for the x Axis

i	h_i's Abn. ECG Average	h_i's Norm ECG Average	(h_i's Abn. ECG Ave.)-(h_i's Norm. ECG Ave.)	a_i
1	23.722	15.920	7.802	0.490
2	15.889	9.000	6.889	0.765
3	14.944	8.440	6.504	0.771
4	11.944	7.880	4.064	0.516
5	6.889	7.120	-0.231	-0.032
6	2.444	5.520	-3.076	-0.557
7	2.111	2.640	-0.529	-0.200
8	1.667	1.920	-0.253	-0.131
9	0.833	1.080	-0.247	-0.229

Table 5. The Calculation of a_i for the y Axis

i	h_i's Abn. ECG Average	h_i's Norm ECG Average	(h_i's Abn. ECG Ave.)-(h_i's Norm. ECG Ave.)	a_i
1	18.444	15.880	2.564	0.161
2	9.056	7.680	1.376	0.179
3	8.222	6.920	1.302	0.188
4	5.111	6.200	-1.089	-0.176
5	4.500	5.480	-0.980	-0.179
6	3.611	4.840	-1.229	-0.254
7	2.833	3.800	-0.967	-0.254
8	2.333	2.440	-0.107	-0.044
9	1.056	0.720	0.336	0.467

Table 6. The Calculation of a_i for the z Axis

i	a_i	Normal h_i	Normal $a_i h_i$	Abn. h_i	Abn. $a_i h_i$
1	0.184	16.800	3.091	19.899	3.660
2	0.209	5.880	1.229	7.111	1.486
3	0.203	5.080	1.031	6.111	1.240
4	0.183	4.320	0.790	5.111	0.935
5	0.284	3.720	1.056	4.778	1.357
6	0.272	3.320	0.903	4.222	1.148
7	-0.013	3.040	-0.040	3.000	0.039
8	-0.020	2.040	-0.041	2.000	-0.040
9	-0.479	1.600	-0.766	0.833	-0.399
Hy	--	--	7.253	--	9.426

Table 7. The Calculation of H for the x Axis

i	a_i	Normal a_i	Normal $a_i h_i$	Abn. h_i	Abn. $a_i h_i$
1	0.490	15.920	7.800	23.722	11.624
2	0.765	9.000	6.885	15.889	12.155
3	0.771	8.440	6.507	14.944	11.522
4	0.516	7.880	4.066	11.944	6.163
5	-0.032	7.120	-0.228	6.889	-0.220
6	-0.557	5.520	-3.075	2.444	-1.361
7	-0.200	2.640	-0.528	2.111	-0.422
8	-0.131	1.920	-0.251	1.667	-0.218
9	-0.229	1.080	-0.247	0.833	-0.191
Hy	--	--	20.929	--	39.052

Table 8. The Calculation of H for the y Axis

i	a_i	Normal a_i	Normal $a_i h_i$	Abn. h_i	Abn. $a_i h_i$
1	0.161	15.880	2.557	18.444	2.97
2	0.179	7.680	1.375	9.056	1.621
3	0.188	6.920	1.301	8.222	1.546
4	-0.176	6.200	-1.091	5.111	-0.900
5	-0.179	5.480	-0.981	4.500	-0.806
6	-0.254	4.840	-1.229	3.611	-0.917
7	-0.254	3.800	-0.965	2.833	-0.720
8	-0.044	2.440	-0.107	2.333	-0.103
9	0.467	0.720	+0.336	1.056	+0.493
H_z	--	--	1.196	--	3.184

Table 9. The Calculation of H for the z Axis

or the y axis ECG was considered normal if

$$H_y < 29.99 \tag{32}$$

and

$$H_z = (1.196 + 3.184)/2 = 2.190 \tag{33}$$

or the z axis ECG was considered normal if

$$H_z < 2.190 \tag{34}$$

The initial test of the proposed pattern classifier indicated the ECG as abnormal if at least two out of three of the H_x, H_y, or H_z decisions were classified as abnormal, and normal if one or none of the three decisions were classified as abnormal. A summary of the results is as follows: twenty-two normal electrocardiograms detected normal; three normal electrocardiograms detected abnormal; five abnormal electrocardiograms detected abnormal; and thirteen abnormal electrocardiograms detected normal. The percentage of abnormal electrocardiograms detected as normal was thirty-six percent which is obviously unacceptable.

Levels for the H parameter decisions were not changed for the second test of the proposed pattern classification technique. The ECG being tested was considered abnormal if any of the three H parameters were classified as abnormal. A summary of the results is as follows:

fourteen normal electrocardiograms detected normal; eleven normal electrocardiograms detected abnormal; fifteen abnormal electrocardiograms detected as abnormal; and three abnormal electrocardiograms detected normal. If the abnormal electrocardiograms detected as normal were considered to be the only errors subject to the requirement that at least fifty percent of the normal were screened normal, there was a seven percent error.

The third criterion tested detected the electrocardiogram as abnormal if H_x indicated the electrocardiogram as abnormal or if both H_y and H_z indicated the ECG as abnormal. The classification levels for the H parameters were changed to the following:

the x axis ECG was considered normal if

$$H_x < 8.100; \tag{35}$$

the y axis ECG was considered normal if

$$H_y < 18.300; \tag{36}$$

the z axis ECG was considered normal if

$$H_z < 2.000. \tag{37}$$

The summary of the results is as follows: twenty-five normal electrocardiograms detected normal; five normal electrocardiograms

detected abnormal; eighteen abnormal electrocardiograms detected abnormal; and three abnormal electrocardiograms detected normal. The percent of normal electrocardiograms detected normal improved from fifty-six percent to seventy-two percent, although the percentage of error due to classifying abnormal electrocardiograms as normal remained at seven percent.

The final criterion detected the electrocardiogram being tested as abnormal if H_x indicated the electrocardiogram as abnormal or if both H_y and H_z indicated the electrocardiogram as abnormal. The decision levels for the H parameters were changed to the following: the x axis ECG was considered normal if

$$H_x < 7.000; \tag{38}$$

the y axis ECG was considered normal if

$$H_y < 16.000; \tag{39}$$

the z axis ECG was considered normal if

$$H_z < 2.000. \tag{40}$$

The summary of the results is as follows: fifteen normal electrocardiograms detected normal; ten normal electrocardiograms detected abnormal; seventeen abnormal electrocardiograms detected abnormal; one

abnormal electrocardiogram detected normal. The percentage of normal electrocardiograms detected normal dropped from seventy-two percent to sixty percent, but the percentage of error which occurs when an abnormal electrocardiogram was detected as normal decreased from seven percent to 2.3 percent.

The preliminary evaluation of the proposed method of ECG screening described above showed a detection of over fifty percent of the normal electrocardiograms as normal with a minimum of abnormal electrocardiograms detected as normal.

The results of the proposed method of ECG screening tested on some new data which was not in the training set of data is covered in Chapter V. A block diagram of a possible method of mechanizing the proposed method of ECG screening is also explained in Chapter V.

CHAPTER V

RESULTS AND CONCLUSIONS

The purpose of the proposed ECG screening method is to detect over fifty percent of normal electrocardiograms as normal and to detect a maximum percentage of abnormal electrocardiograms as abnormal. The ECG is obtained by the Frank lead system as explained in Chapter I. The Frank lead system includes an x axis, y axis and z axis signal, each of which is an input to the system, as shown in Fig. 22.

The subject's ECG is amplified to a usable level by the ECG preamplifier and amplifier. The analog ECG voltage signal is then converted to a series of digital numbers. The analog to digital converter samples analog data at two hundred fifty samples per second. The total number of points that fall in each interval of the digitalized ECG data is tallied and stored in the corresponding interval location, Li for i = 1,2,3,...,99, 100 and 101. The total number of data points per ECG sample is tallied and represented by the symbol T. The probability density function which is the probability of a datum point falling in a specific interval is computed after the complete ECG sample has been digitalized and tallied in the appropriated level and the total number of data points has been tallied. The probability density function value for a specific level i has

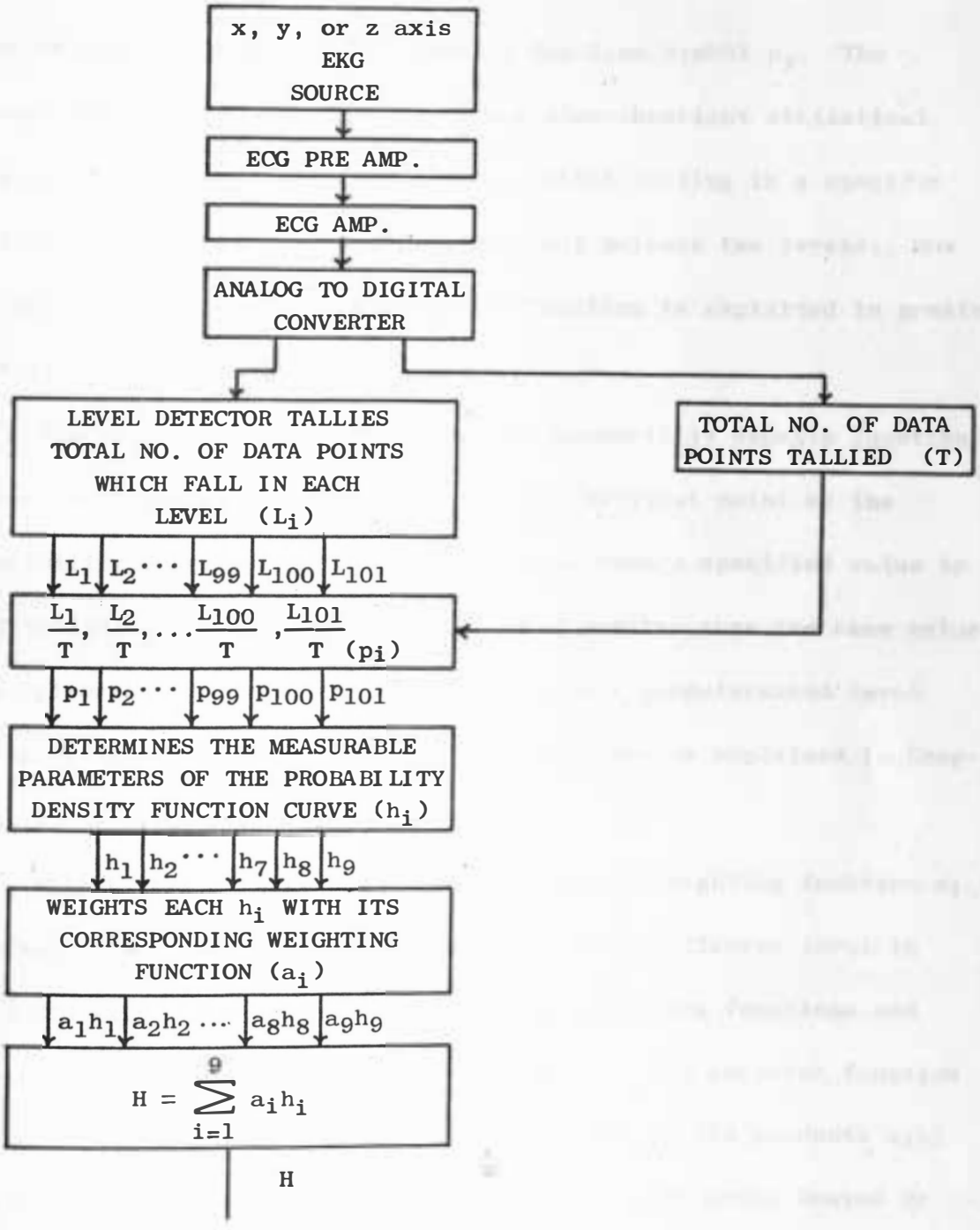


Figure 22. The Conversion of the ECG Signal to the H Function

the corresponding probability density function symbol p_i. The probability density function is then a time-invariant statistical plot of the probability of an ECG data point falling in a specific interval versus the corresponding interval between two levels. How to obtain the ECG probability density function is explained in greater detail in Chapter III.

The measurable parameter h_i of the probability density function curve is the horizontal distance between the first point on the probability density function curve greater than a specified value to and including the last point on the curve greater than the same value. The criterion for measurement of each h_i is a predetermined level value on the probability density function curve as explained in Chapter IV.

Each h_i has a corresponding predetermined weighting function a_i. The weighting functions are different for each different level in each ECG axis. The actual values for the weighting functions and their derivation are explained in Chapter IV. The decision function for each axis, H, is a number equal to the sum of the products a_ih_i for i = 1,2,3,...,8, and 9. The ECG from the axis being tested is considered normal if the H is less than a predetermined value or abnormal if the H is greater than or equal to the same predetermined value.

The heart's electrocardiogram is considered possibly abnormal if the x axis ECG waveform is detected as abnormal or if the y and z axis ECG waveforms are detected abnormal. If the heart's electrocardiogram is not considered as possibly abnormal, it is considered normal. The concept of normal versus possibly abnormal ECG is determined by the decision function H as shown in Fig. 23.

The actual numerical values of the H's used as the decision levels are 7.000 for H_x, 16.000 for H_y, and 2.000 for H_z. The proposed method of ECG screening using these values of the H's was applied to a set of data and the results are as follows: twenty-four of the twenty-nine abnormal electrocardiograms were detected abnormal, five of the ten normal electrocardiograms were detected normal. An error is considered to occur when an abnormal ECG is detected normal. Thus, there was a 12.8 percent error in the test of the above data.

Since the skirts of the probability density function curve are considered to contain most of the information necessary for proper categorization of the ECG into normal versus possibly abnormal categories, a modification to the above method was devised. This modification was to set the first negative and all following values of a_i for i increasing in value to zero. The weighting factors a_i for the decision function H as described by Eq. 27 are found by Eq. 28.

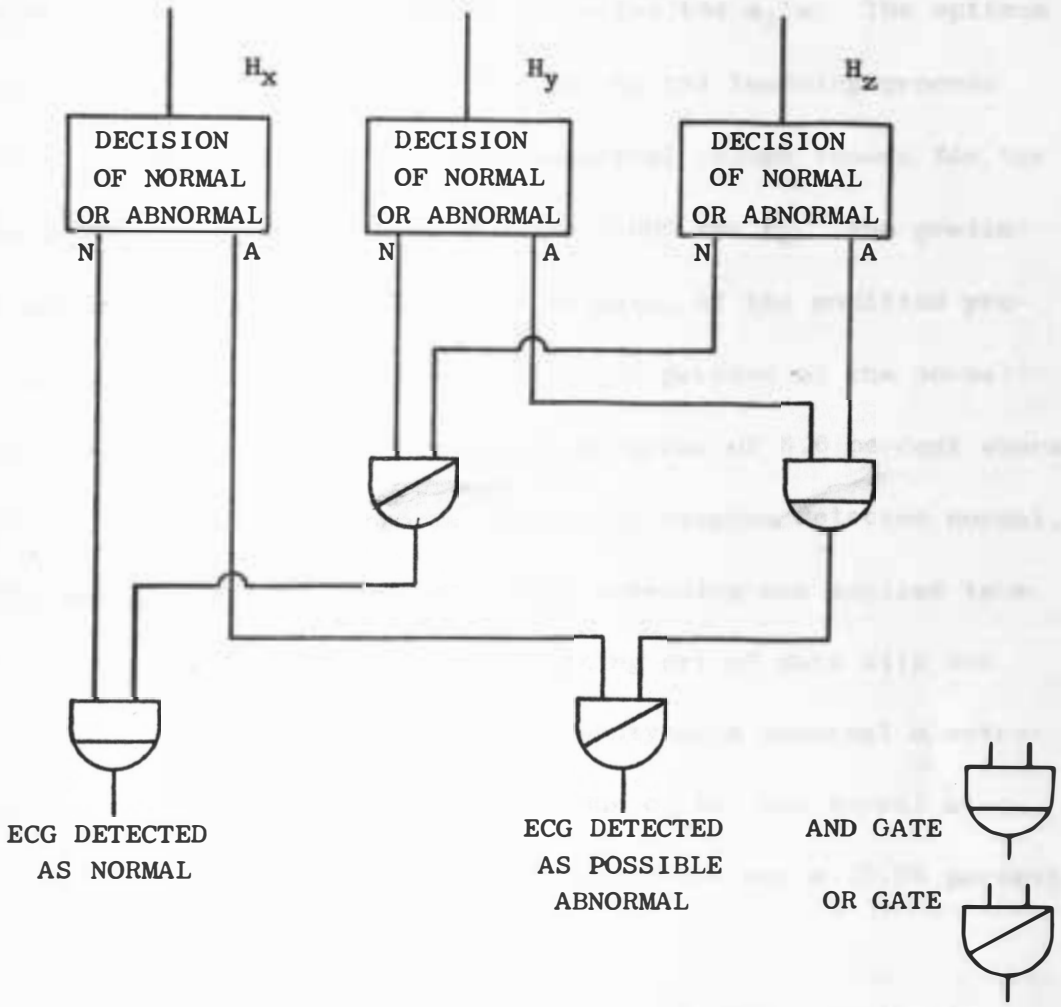


Figure 23. The Screening of the ECG into Normal versus Possibly Abnormal Categories by Using the H Functions

The training set of data was applied to the proposed method of ECG screening with the above modification for the a_i's. The optimum decision levels for the H's were obtained by the learning process described in Chapter IV. The actual numerical values chosen for the H's are 8.000 for H_x, 13.000 for H_y, and 6.000 for H_z. The preliminary results, using the training set of data, of the modified proposed method of ECG screening are sixty-four percent of the normal electrocardiograms detected normal with an error of 5.6 percent where an error is defined as an abnormal electrocardiogram detected normal.

The modified proposed method of ECG screening was applied to a new set of data not included in the training set of data with the following results: twenty-five of the twenty-nine abnormal electrocardiograms were detected abnormal and four of the ten normal electrocardiograms were detected normal. Thus, there was a 10.25 percent error in the test of the above data.

The test results of the proposed method of ECG screening do show a correlation between the ECG probability density function curve and the classification of the ECG into normal versus possibly abnormal categories. The learning process requires large amounts of data to properly set the values of the mechanization, but only limited amounts of data were available for the study. Therefore, the learning set of

data was adequate only for a study of the feasibility of the proposed method of ECG screening.

The computer method of ECG screening and diagnosis presently being used is successful, but a complex program must be used to detect specific heart abnormalities, the cost of which is not low enough to be within the budgets of most hospitals. The proposed method of ECG screening, which would categorize the electrocardiogram into normal or possibly abnormal categories, sacrifices the ability to detect specific heart abnormalities for simplicity and the probability of low cost of mechanization. A cardiologist's analysis would then be required for the final diagnosis of specific abnormalities.

The proposed method of ECG screening warrants further study and refinement to acquire more accurate weighting factors, a_i, and to find better decision levels for the H's. If the proposed method of ECG screening could be refined to a clinically acceptable method, its mechanization could be well within the budget limitations of most hospitals. The proposed method could be programmed for a small computer, or it could be mechanized with hardware. Tests of the proposed method of ECG screening show possibilities for its future in the science of detecting heart abnormalities by electrocardiograms.

APPENDIX

NYQUIST SAMPLING THEOREM

When sampling an analog signal, what sampling rate is necessary to insure that the samples contain all the information of the original signal? This question is answered by the Nyquist Sampling Theorem which states:

> If the Fourier transform X(f) (and therefore the power spectrum) of a time function x(t) is identically zero at all frequencies higher than W_{cps}, then x(t) is uniquely determined by specifying its ordinates at a series of points spaced 1/2 W seconds apart, the series extending throughout the time domain.[9]

The series representation of a function that is a uniformly convergent trigonometric series is a Fourier Series representable in the form:

$$f(x) = a_o/2 + \sum_{n=1}^{+\infty} a_n \cos(nx) + b_n \sin(nx) \qquad \text{(A-1)}$$

or

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{inx} \qquad \text{(A-2)}$$

where

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\cos(nx)\, dx \quad (n = 0,1,2,\ldots), \qquad \text{(A-3)}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\sin(nx)\, dx \quad (n = 1,2,\ldots) \qquad \text{(A-4)}$$

and

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx \quad (n = 0,\ 1,\ 2,\ldots) \qquad \text{(A-5)}$$

or in other words the function can be represented as a series of sine waves or cosine waves of various amplitudes depending upon the magnitudes of a_n, b_n or c_n.[10]

According to the Sampling Theorem, the function must be sampled at a rate which is twice the highest frequency of its Fourier Series representation in order for all the information to be contained in the sample. If n of the Fourier Series approaches infinity, the sampling rate would approach infinity, or the sampled waveform would be continuous.

In the practical application of the Sampling Theorem, the sampling rate is chosen to be more than twice the highest significant frequency of the function's frequency spectrum. Usually, the amplitudes of the higher frequency components greatly decrease as frequencies increase. Therefore, the function is considered band limited and the higher frequencies are eliminated by some predetermined criterion. A common criterion used is the approximation of the function by a sufficient number of terms so that the mean squared error is equal to or less than some specified value. The mean squared error is

$$E_n^2 = \left[\left(\frac{1}{T}\int f^2(x)dx - \sum_{-N}^{N} c_n c_n^*\right) / \frac{1}{T}\int f^2(x)dx \right] (100\%) \qquad \text{(A-6)}$$

The interval 0 to T is one period and c_n is defined by Eq. A-5. N is the number of terms included in the summation of c_n times the conjugate of c_n.[11] The sampling rate is usually chosen to be greater than twice the highest significant frequency in order to insure adequate information for reconstruction of the waveform from the sample values.

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