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***Abstract*— This paper presents a cost effective design based on sensor reduction for backing-up control of a vehicle with triple trailers. To realize a cost effective design, we newly derive two linear matrix inequality (LMI) conditions for discrete Takagi-Sugeno fuzzy system. One is an optimal dynamic output feedback design that guarantees desired control performance. The other is an avoidance of jackknife phenomenon for the use of the optimal dynamic output feedback controller. Our results demonstrate that the proposed LMI-based design effectively achieves the backing-up control of the vehicle with triple trailers while avoiding the jackknife phenomenon. More importantly, we demonstrate that the designed optimal control can achieve the backing-up control without at least two potentiometers that were employed to measure the relative angles (of a vehicle with triple trailers) in our previous experiments. Since the relative angles directly relate to the jackknife phenomenon, the successful control results without two potentiometers are very interesting and important from the cost effective design points of view.**

***Index Terms*— fuzzy control, linear matrix inequality, sensor reduction, backing-up control, vehicle with triple trailers.**

I. Introduction

CONTROL theory mainly provides procedures for designing a controller to stabilize a system or to achieve desired control performance. However, in real control problems, we frequently need to consider a cost effective design in addition to achieving desired control performance. It is obviously more cost effective to be able to detect feedback information without the need of additional sensors. Thus, sensorless system [1] or at least reduction of the number of sensors (shortly, sensor reduction) is very useful for cost effective designs in real control problems. In this paper, we discuss sensor reduction for backing-up control of a vehicle with triple trailers.

Backing-up control problems for a vehicle with a single trailer or multiple trailers have been used as a testbed for a variety of control design methods, e.g., [2]-[10], etc. In order to successfully back up the trailer-truck, the so-called ”jackknife” phenomenon needs to be avoided. In the single trailer case, only two jackknife positions exist. On the other hand, eight jackknife positions exist in the triple trailer case. Thus, the triple trailer case is much more difficult than the single trailer case. The studies [2]-[7] have provided only simulation results for the single trailer case without discussing the stability issue of the designed control systems. In [11], the problem of asymptotic stabilization for backward motion has been addressed. However, the paper [11] has discussed only a single trailer case. We presented successful *experimental* results for the *triple* trailers case [12], [13] based on a linear matrix inequality (LMI) approach to stable fuzzy controller design. Recently, multiple trailer cases have been also presented in [14] and [15]. However, both the works [14] and [15] have provided no any theoretical guarantee for the stability of control systems. To the best of our knowledge, triple-trailers experimental results with guaranteeing the stability of the control system have been reported only in the literature [12], [13]. However, all the studied mentioned above were assumed that full sensor information is available in the backing-up control, that is, that full sensor information on relative angles was needed to realize the control purpose. Thus, cost effective designs for backing-up control have not been discussed in the literature. This paper attempts to reduce some of the important sensors to detect relative angles in the triple trailer case. Obviously, this is a challenging attempt in the backing-up control problem.

In this paper, we discuss a cost effective design in the triple trailers case. To realize a cost effective design, we newly derive two linear matrix inequality conditions for *discrete* Takagi-Sugeno fuzzy system. One is an *optimal* dynamic output feedback design that guarantees desired control performance (namely, guaranteed cost control design). The other is an avoidance of jackknife phenomenon for the use of the optimal dynamic output feedback controller. Over the last decade the design issues for Takagi-Sugeno fuzzy systems [16] have been considered extensively in nonlinear control frameworks, e.g., [17], [20]-[39]. The main advantage of such fuzzy model-based control methodology [17] is that it provides a natural, simple and effective design approach to complement other nonlinear control techniques (e.g., [18]) that require special and rather involved knowledge. Moreover, there is no loss of generality in adopting the Takagi-Sugeno fuzzy model based control design framework as it has been established that any smooth nonlinear control systems can be approximated by the Takagi-Sugeno fuzzy models (with linear rule consequence) [19]. Within the general framework of Takagi-Sugeno fuzzy model-based control systems, there has been, in particular, a flurry of research activities in the analysis and design of fuzzy

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control systems based on LMIs (e.g., [17]). This paper extends the framework to optimal dynamic output feedback designs while avoiding the jackknife phenomenon, where optimal control means the guaranteed cost control throughout this paper. To the best of our knowledge, optimal dynamic output feedback designs under the constrains on jackknife avoidance for discrete Takagi-Sugeno fuzzy systems have not been considered in the literature. Our results demonstrate that the proposed LMI-based design effectively achieves the backing-up control of the vehicle with triple trailers while avoiding the jackknife phenomenon. More importantly, we demonstrate that the designed optimal control can achieve the backing-up control without at least two potentiometers that were employed to measure the relative angles (of a vehicle with triple trailers) in our previous experiments [12], [13]. Since the relative angles directly relate to the jackknife phenomenon, the successful control results without two potentiometers are very interesting and important from the cost effective design points of view. Although potentiometers are inexpensive in general, the approach discussed here can be applied to more cost effective design problems for the systems with expensive sensors.

All the matrices and vectors in this paper are assumed to have appropriate dimensions.

II. Backing-up Control of a Vehicle with Triple Trailers

Figure 1 shows the vehicle model with three trailers and its coordinate system, where

$x_0(t)$: angle of vehicle,
$x_1(t)$: angle difference between vehicle and first trailer,
$x_2(t)$: angle of first trailer,
$x_3(t)$: angle difference between first trailer and second trailer,
$x_4(t)$: angle of second trailer,
$x_5(t)$: angle difference between second trailer and third trailer,
$x_6(t)$: angle of third trailer,
$x_7(t)$: vertical position of rear end of third trailer,
$x_8(t)$: horizontal position of rear end of third trailer,
$u(t)$: steering angle.

To design a fuzzy controller, we use the original model described in [13].

In this paper we have $l = 0.087[m]$, $L = 0.130[m]$, $\nu = -0.10[m/sec.]$, $\Delta t = 0.5[sec.]$, where l is the length of the vehicle, L is the length of the trailer, Δt is the sampling time, and ν is the constant speed of the backward movement.

For the relative angles $x_1(t), x_3(t)$ and $x_5(t)$, 90 [deg.] and -90 [deg.] correspond to eight jackknife positions. To successfully back up, the eight jackknife positions should be absolutely avoided. The control objective is to back the vehicle into the straight line ($x_7 = 0$) without any forward movement, that is, $x_1(t) \to 0,\ x_3(t) \to 0,\ x_5(t) \to 0,\ x_6(t) \to 0,\ x_7(t) \to 0$.

To employ our dynamic output feedback design method, we begin with the construction of a Takagi-Sugeno type of fuzzy model which represents the nonlinear dynamics of the vehicle with three trailers. We have the following Takagi-Sugeno fuzzy model [13] for the original model.

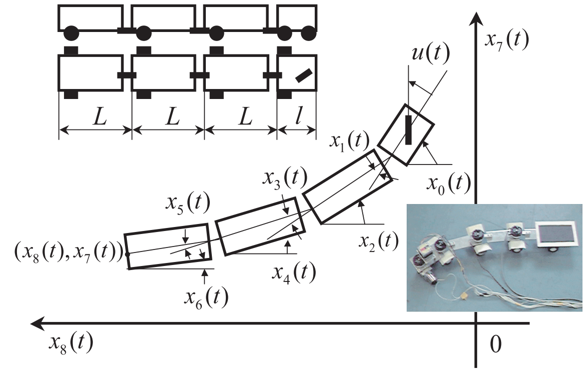

Fig. 1. Vehicle with Triple Trailers.

$$\begin{aligned}&\text{Model Rule 1 : If } z_1(t) \text{ is } \text{“about 0 [rad.]”},\\ &\qquad \text{then } \boldsymbol{x}(t+1) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_1 u(t),\end{aligned} \tag{1}$$

$$\begin{aligned}&\text{Model Rule 2 : If } z_1(t) \text{ is } \text{“about } \pi \text{ or } -\pi \text{ [rad.]”},\\ &\qquad \text{then } \boldsymbol{x}(t+1) = \boldsymbol{A}_2\boldsymbol{x}(t) + \boldsymbol{B}_2 u(t),\end{aligned}$$

where

$$\begin{aligned} z_1(t) &= x_6(t) + \frac{\nu \cdot \Delta t}{2L} x_5(t),\\ \boldsymbol{x}(t) &= [x_1(t)\ x_3(t)\ x_5(t)\ x_6(t)\ x_7(t)]^T. \end{aligned}$$

Figure 2 shows the membership functions “about 0 [rad.]” and “about π or $-\pi$ [rad.]”. For more details of the fuzzy model construction, see [13]. There is no loss of generality in adopting the Takagi-Sugeno fuzzy model based control design framework as it has been established that any smooth nonlinear control systems can be approximated by the Takagi-Sugeno fuzzy models (with liner rule consequence) [19]. The overall

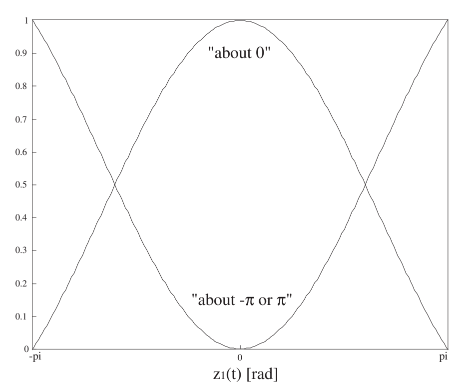

Fig. 2. Membership functions.

fuzzy model is inferred as

$$\boldsymbol{x}(t+1) = \sum_{i=1}^{2} h_i(z_1(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i u(t)\}, \tag{2}$$

where

$$h_i(z_1(t)) \;=\; \frac{w_i(z_1(t))}{\sum_{i=1}^{2} w_i(z_1(t))}.$$

$w_1(z_1(t))$ and $w_2(z_1(t))$ are the grades of membership of $z_1(t)$ in the membership functions "about 0 [rad.]" and "about π [rad.] or $-\pi$ [rad.]", respectively.

It is assumed from experimental points of view that the steering angle $u(t)$ has the saturation of $|u(t)| < \pi/3$ in the simulation. This means that the real steering angle to the vehicle is $\pm\pi/3$ if $u(t)$ exceeds $\pm\pi/3$.

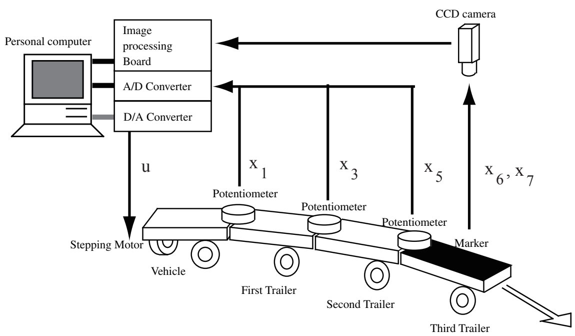

Fig. 3. Experimental setup.

Figure 3 shows our previous experimental setup in [13]. Three potentiometers are attached at the connecting parts between the vehicle and the first trailer, between the first trailer and the second trailer, and between the second trailer and the third trailer. The third trailer has a black rectangular marker for vision sensing using a charge coupled device (CCD) camera. The relative angles $x_1(t)$, $x_3(t)$ and $x_5(t)$ are observed (through an A/D converter) from three potentiometers. The control variables $x_6(t)$ and $x_7(t)$ are successively observed through an image processing board that receives vision information from the CCD camera. The steering angle $u(t)$ is controller by a stepping motor.

In [12], [13], a full-state feedback controller stabilizing the system was designed. The purpose of this paper is to discuss the possibility of reducing the number of potentiometers. The reduction of potentiometers directly relates to cost effectiveness for constructing control systems. To realize a cost effective design, we will newly derive two LMI conditions for discrete Takagi-Sugeno fuzzy systems in Section III.

III. Sensor Reduction via Dynamic output feedback control

Sensor reduction problems can be formulated as an output feedback design problem. To reduce the number of sensors, Section III provides dynamic output feedback designs based on LMIs. After discussing a stable dynamic output feedback design, we will newly derive an optimal dynamic output feedback design condition that guarantees desired control performance. By introducing some variable transformations, the optimal dynamic output feedback design condition derived in this section can be represented in terms of LMIs. We will see the fact in the proofs of Theorems 1 and 2.

Consider the following discrete Takagi-Sugeno fuzzy model.

Model Rule i:
if $z_1(t)\ is\ M_1^i\ and\ \cdots\ and\ z_p(t)\ is\ M_p^i$
then $\begin{cases} \boldsymbol{x}(t+1) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_i\boldsymbol{x}(t) \end{cases}$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$, $\boldsymbol{u}(t) \in \boldsymbol{R}^m$, $\boldsymbol{y}(t) \in \boldsymbol{R}^\ell$, and $i = 1, 2, \cdots, r$. r is the number of $Model\ Rules$. The membership function associated with the ith $Model\ Rule$ and jth premise variable component is denoted by M_j^i. $z_j(t)$ $(j = 1, 2, \cdots, p)$ is the premise variable. Each $z_j(t)$ is a measurable time-varying quantity that may be measurable states, measurable external variables and/or time. In other words, each $z_j(t)$ is needed to be independent of unmeasurable states. In Section III, we will discuss multiple inputs case although the vehicle with triple trailers has a single input. Hence the controller design discussed here can be applied also to multiple inputs case.

By using the center of gravity method for defuzzification, the Takagi-Sugeno fuzzy model is represented as

$$\boldsymbol{x}(t+1) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)), \tag{3}$$

$$\boldsymbol{y}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{C}_i\boldsymbol{x}(t), \tag{4}$$

where $\boldsymbol{z}(t) = [z_1(t),\ z_2(t),\ \cdots\ z_p(t)]$,

$$h_i(\boldsymbol{z}(t)) \;=\; \frac{w_i(\boldsymbol{z}(t))}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))},$$

$$w_i(\boldsymbol{z}(t)) \;=\; \prod_{j=1}^{p} M_j^i(z_j(t)).$$

Without loss of generality, we assume that $w_i(\boldsymbol{z}(t)) \geq 0$ and $\sum_{i=1}^{r} w_i(\boldsymbol{z}(t)) > 0$. The controller is constructed based on the dynamic parallel distributed compensation (DPDC).

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{C}_i^c\boldsymbol{x}^c(t), \tag{5}$$

$$\begin{aligned} \boldsymbol{x}^c(t+1) \;=\; & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \\ & \times(\boldsymbol{A}_{ij}^c\boldsymbol{x}^c(t) + \boldsymbol{B}_j^c\boldsymbol{y}(t)), \end{aligned} \tag{6}$$

where $\boldsymbol{x}^c(t) \in \boldsymbol{R}^n$. A key feature of the dynamic controller (5) and (6) is to have $\boldsymbol{A}_{ij}^c$ (instead of $\boldsymbol{A}_i^c$) in (6). The reason will be given later.

The control law (5) and (6) feedbacks only the outputs $\boldsymbol{y}(t)$ instead of the states $\boldsymbol{x}(t)$. In practical control, we generally select measurable states as the outputs $\boldsymbol{y}(t)$. In other words, the unmeasurable states caused by sensor reduction should be removed from the outputs. In our sensor reduction case, we will remove $x_1(t)$ and $x_3(t)$ from the outputs since these potentiometers concerning $x_1(t)$ and $x_3(t)$ are removed as will be seen in (46).

By substituting the dynamic output feedback controller (5) and (6) into the discrete fuzzy model (3) and (4), we have the closed-loop system,

$$\boldsymbol{x}(t+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \times(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{C}_j^c\boldsymbol{x}^c(t)), \tag{7}$$

$$\boldsymbol{x}^c(t+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \times(\boldsymbol{B}_j^c\boldsymbol{C}_i\boldsymbol{x}(t) + \boldsymbol{A}_{ij}^c\boldsymbol{x}^c(t)). \tag{8}$$

From (7) and (8), we arrive at the general form of the dynamic output feedback system.

$$\boldsymbol{x}^{cl}(t+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{A}_{ij}^{cl}\boldsymbol{x}^{cl}(t), \tag{9}$$

$$\hat{\boldsymbol{y}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{C}_{ij}^{*}\boldsymbol{x}^{cl}(t), \tag{10}$$

where

$$\boldsymbol{x}^{cl}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}^c(t) \end{bmatrix}, \quad \boldsymbol{A}_{ij}^{cl} = \begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{B}_i\boldsymbol{C}_j^c \\ \boldsymbol{B}_j^c\boldsymbol{C}_i & \boldsymbol{A}_{ij}^c \end{bmatrix},$$

$$\hat{\boldsymbol{y}}(t) = \begin{bmatrix} \boldsymbol{y}(t) \\ \boldsymbol{u}(t) \end{bmatrix}, \quad \boldsymbol{C}_{ij}^{*} = \begin{bmatrix} \boldsymbol{C}_i & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_j^c \end{bmatrix}.$$

We give stable and optimal dynamic output feedback designs for the discrete Takagi-Sugeno fuzzy system (3) and (4), respectively. As mentioned in Introduction, throughout this paper, the optimal control means guaranteed cost control.

A. Stable Controller Design

Theorem 1 shows stable controller design conditions. Since they are represented in terms of LMIs, Theorem 1 can be solved numerically.

Theorem 1: A dynamic output feedback controller (5) and (6) stabilizing (3) and (4) can be designed if there exist $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{G}_j$, $\boldsymbol{L}_j$ and $\boldsymbol{\Phi}_{ij}$ such that the following LMIs hold.

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} > \boldsymbol{0}, \tag{11}$$

$$\boldsymbol{W}_{ii} > \boldsymbol{0}, \quad \forall i \tag{12}$$

$$\boldsymbol{W}_{ij} + \boldsymbol{W}_{ji} \geq \boldsymbol{0}, \quad i < j, \tag{13}$$

where

$$\boldsymbol{W}_{ij} = \begin{bmatrix} \boldsymbol{X} & * & * & * \\ \boldsymbol{I} & \boldsymbol{Y} & * & * \\ \boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{G}_j & \boldsymbol{A}_i & \boldsymbol{X} & * \\ \boldsymbol{\Phi}_{ij} & \boldsymbol{Y}\boldsymbol{A}_i - \boldsymbol{L}_j\boldsymbol{C}_i & \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix}. \tag{14}$$

The symbol $*$ denotes the transposed element (matrix) for the symmetric position.

(proof) From Lyapunov stability theorem, we can have the following sufficient condition for ensuring the stability of (9) and (10).

$$\boldsymbol{A}_{ii}^{cl^T}\boldsymbol{P}^{cl}\boldsymbol{A}_{ii}^{cl} - \boldsymbol{P}^{cl} < \boldsymbol{0}, \quad \forall i \tag{15}$$

$$\boldsymbol{A}_{ij}^{cl^T}\boldsymbol{P}^{cl}\boldsymbol{A}_{ij}^{cl} + \boldsymbol{A}_{ji}^{cl^T}\boldsymbol{P}^{cl}\boldsymbol{A}_{ji}^{cl} - 2\boldsymbol{P}^{cl} \leq \boldsymbol{0}, \quad i < j \tag{16}$$

where $\boldsymbol{P}^{cl} > \boldsymbol{0}$.

Multiplying both sides of (15) by $\boldsymbol{P}^{cl^{-1}} = \boldsymbol{X}^{cl} > \boldsymbol{0}$ gives

$$\boldsymbol{X}^{cl}\boldsymbol{A}_{ii}^{cl^T}\boldsymbol{X}^{cl^{-1}}\boldsymbol{A}_{ii}^{cl}\boldsymbol{X}^{cl} - \boldsymbol{X}^{cl} < \boldsymbol{0}. \tag{17}$$

It easily follows that the above inequality can be transformed into (18) by Schur Complement.

$$\begin{bmatrix} \boldsymbol{X}^{cl} & \boldsymbol{X}^{cl}\boldsymbol{A}_{ii}^{cl^T} \\ \boldsymbol{A}_{ii}^{cl}\boldsymbol{X}^{cl} & \boldsymbol{X}^{cl} \end{bmatrix} > \boldsymbol{0} \tag{18}$$

Define that $\boldsymbol{X}^{cl} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{S} \\ \boldsymbol{S} & \boldsymbol{S} \end{bmatrix} > \boldsymbol{0}$. Then, we have

$$\begin{aligned} \boldsymbol{A}_{ii}^{cl}\boldsymbol{X}^{cl} &= \begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{B}_i\boldsymbol{C}_i^c \\ \boldsymbol{B}_i^c\boldsymbol{C}_i & \boldsymbol{A}_{ii}^c \end{bmatrix}\begin{bmatrix} \boldsymbol{X} & \boldsymbol{S} \\ \boldsymbol{S} & \boldsymbol{S} \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{C}_i^c\boldsymbol{S} & (\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{C}_i^c)\boldsymbol{S} \\ \boldsymbol{B}_i^c\boldsymbol{C}_i\boldsymbol{X} + \boldsymbol{A}_{ii}^c\boldsymbol{S} & (\boldsymbol{B}_i^c\boldsymbol{C}_i + \boldsymbol{A}_{ii}^c)\boldsymbol{S} \end{bmatrix}. \end{aligned} \tag{19}$$

Since $\begin{bmatrix} \boldsymbol{X} & \boldsymbol{S} \\ \boldsymbol{S} & \boldsymbol{S} \end{bmatrix} > \boldsymbol{0}$, note that $\boldsymbol{X} > \boldsymbol{S} > \boldsymbol{0}$. Assume that $\boldsymbol{Y} = (\boldsymbol{X} - \boldsymbol{S})^{-1}$. By using $\boldsymbol{T} \equiv \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{Y} & -\boldsymbol{Y} \end{bmatrix}$, we have

$$\boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} > \boldsymbol{0}. \tag{20}$$

By multiplying both sides of (18) by block-diag$[\boldsymbol{T}\ \boldsymbol{T}]$ and block-diag$[\boldsymbol{T}^T\ \boldsymbol{T}^T]$, respectively, (21) is obtained.

$$\begin{bmatrix} \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T & \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{A}_{ii}^{cl^T}\boldsymbol{T}^T \\ \boldsymbol{T}\boldsymbol{A}_{ii}^{cl}\boldsymbol{X}^{cl}\boldsymbol{T}^T & \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T \end{bmatrix} > \boldsymbol{0}, \tag{21}$$

where

$$\begin{aligned} &\boldsymbol{T}\boldsymbol{A}_{ii}^{cl}\boldsymbol{X}^{cl}\boldsymbol{T}^T = \begin{bmatrix} \boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{G}_i & \boldsymbol{A}_i \\ \boldsymbol{\Phi}_{ii} & \boldsymbol{Y}\boldsymbol{A}_i - \boldsymbol{L}_i\boldsymbol{C}_i \end{bmatrix}, \\ &\boldsymbol{G}_i = \boldsymbol{C}_i^c\boldsymbol{S}, \\ &\boldsymbol{L}_i = \boldsymbol{Y}\boldsymbol{B}_i^c, \\ &\boldsymbol{\Phi}_{ii} = \boldsymbol{Y}(\boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{C}_i^c\boldsymbol{S} - \boldsymbol{B}_i^c\boldsymbol{C}_i\boldsymbol{X} - \boldsymbol{A}_{ii}^c\boldsymbol{S}). \end{aligned}$$

Hence, the condition (21) becomes (12). The condition (13) is obtained from (16) in the same way as described above.

(Q.E.D.)

The LMI variables are $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{G}_j$, $\boldsymbol{L}_j$ and $\boldsymbol{\Phi}_{ij}$, where

$$\begin{aligned} &\boldsymbol{G}_j = \boldsymbol{C}_j^c\boldsymbol{S}, \\ &\boldsymbol{L}_j = \boldsymbol{Y}\boldsymbol{B}_j^c, \\ &\boldsymbol{\Phi}_{ij} = \boldsymbol{Y}(\boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{C}_j^c\boldsymbol{S} - \boldsymbol{B}_i^c\boldsymbol{C}_j\boldsymbol{X} - \boldsymbol{A}_{ij}^c\boldsymbol{S}), \\ &\boldsymbol{S} = \boldsymbol{X} - \boldsymbol{Y}^{-1}. \end{aligned}$$

$\boldsymbol{S}$ can be obtained as $\boldsymbol{S} = \boldsymbol{X} - \boldsymbol{Y}^{-1}$ from the solution $\boldsymbol{X}$ and $\boldsymbol{Y}$. $\boldsymbol{C}_j^c$ and $\boldsymbol{B}_i^c$ are obtained as $\boldsymbol{C}_j^c = \boldsymbol{G}_j\boldsymbol{S}^{-1}$ and $\boldsymbol{B}_j^c = \boldsymbol{Y}^{-1}\boldsymbol{L}_j$, respectively. $\boldsymbol{A}_{ij}^c$ are obtained as

$$\begin{aligned} \boldsymbol{A}_{ij}^c \ = \ \{&\boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{C}_j^c\boldsymbol{S} \\ &-\boldsymbol{B}_j^c\boldsymbol{C}_i\boldsymbol{X} - \boldsymbol{Y}^{-1}\phi_{ij}\}\boldsymbol{S}^{-1} \end{aligned} \tag{22}$$

from solutions $\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{S}, \boldsymbol{B}_j^c, \boldsymbol{C}_j^c$ and $\boldsymbol{\phi}_{ij}$.

The important point is to use $\boldsymbol{A}_{ij}^c$ instead of $\boldsymbol{A}_i^c$ as shown in (6). If $\boldsymbol{A}_i^c$ instead of $\boldsymbol{A}_{ij}^c$ is used, $\boldsymbol{A}_i^c$ can not be uniquely obtained by solving the following equation (instead of (22)) since the right-hand side of (23) has not only the subscript i but also j.

$$\begin{aligned}\boldsymbol{A}_i^c = \{&\boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{C}_j^c\boldsymbol{S} \\ &-\boldsymbol{B}_j^c\boldsymbol{C}_i\boldsymbol{X} - \boldsymbol{Y}^{-1}\boldsymbol{\phi}_{ij}\}\boldsymbol{S}^{-1}\end{aligned} \tag{23}$$

The premise variable $z_1(t)$ in (1) is dependent of $x_5(t)$ and $x_6(t)$ in the triple trailer case. A CCD camera is employed to detect both $x_6(t)$ and $x_7(t)$. Therefore, it is assumed in this design that $x_1(t)$ and $x_3(t)$ are unmeasurable. We will see in Section V that the designed controller can realize the control purpose without measuring the information on the relative angles $x_1(t)$ and $x_3(t)$. This means that any sensors are not employed to measure the relative angles $x_1(t)$ and $x_3(t)$. This is very interesting and important from the cost effective design points of view.

A stable dynamic output feedback controller is designed by solving the LMIs in Theorem 1. Figures 4, 5 and 6 show the control results of the original model by the stable controller for the initial values $x_6(0) = \pi/6, \pi/2, \pi$, respectively, where $x_1(0) = x_3(0) = x_5(0) = 0$, $x_7(0) = 2$ and $\boldsymbol{x^c}(0) = 0$. The simulation stops (due to jackknife) when one of the relative angles $x_1(t)$, $x_3(t)$ or $x_5(t)$ at least exceed $\pm\pi/2$. When the vehicle-trailer system is at an "easy" initial position, even the stable controller works, i.e., the vehicle-trailer system approaches the desired straight line. On the other hand, the system starts from a "difficulty" initial position, the stable control occurs jackknife phenomenon as soon as control starts. To circumvent these problems, we invoke optimal dynamic output feedback design.

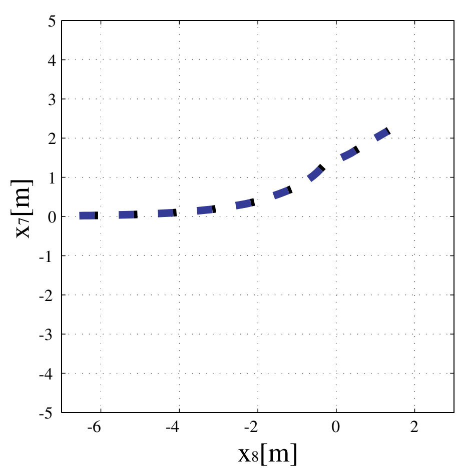

Fig. 4. Control result for stable controller ($x_6(0) = \pi/6$).

B. Optimal Controller Design

Section III-B presents the optimal controller design for the dynamic output feedback control. The design is achieved so

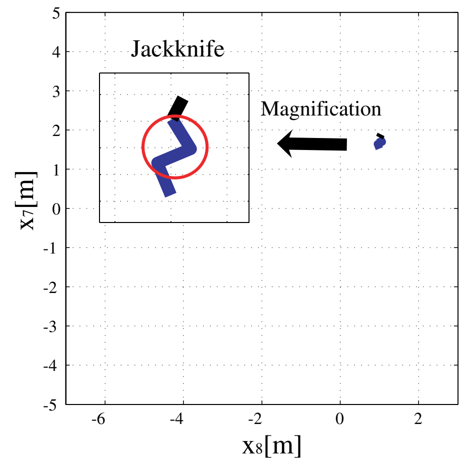

Fig. 5. Control result for stable controller ($x_6(0) = \pi/2$).

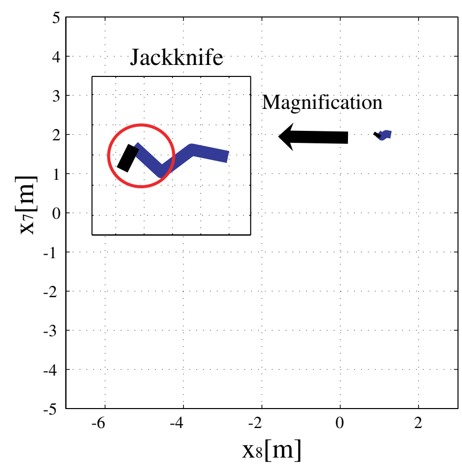

Fig. 6. Control result for stable controller ($x_6(0) = \pi$).

as to minimize the upper bound of a given performance index (24).

$$J = \sum_{k=0}^{\infty}\{\boldsymbol{y}^T(k)\boldsymbol{Q}\boldsymbol{y}(k) + \boldsymbol{u}^T(k)\boldsymbol{R}\boldsymbol{u}(k)\}, \tag{24}$$

where $\boldsymbol{Q} > \boldsymbol{0}$ and $\boldsymbol{R} > \boldsymbol{0}$.

Theorem 2 shows optimal controller design conditions. Since they are also represented in terms of LMIs, Theorem 2 can be solved numerically as well as in Theorem 1.

Theorem 2: Assume that $m = \ell$. By solving the following generalized eigenvalue minimization problem (GEVP), the optimal dynamic output feedback controller is designed. Then, the controller realizes $J < \lambda$.

$$\underset{\boldsymbol{X},\boldsymbol{Y},\boldsymbol{\phi}_{ij},\boldsymbol{L}_i,\boldsymbol{G}_j}{minimize} \quad \lambda$$

subject to

$$\begin{bmatrix}\boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y}\end{bmatrix} > \boldsymbol{0} \tag{25}$$

$$\begin{bmatrix} \lambda & \boldsymbol{x}_{k\ell}^T(0) & (\boldsymbol{x}_{k\ell}(0)-\boldsymbol{x}^c(0))^T\boldsymbol{Y} \\ \boldsymbol{x}_{k\ell}(0) & \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{Y}(\boldsymbol{x}_{k\ell}(0)-\boldsymbol{x}^c(0)) & \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} > \boldsymbol{0} \quad k,\ell = 1,2 \tag{26}$$

$$\boldsymbol{Z}_{ii} > \boldsymbol{0} \quad \forall i \tag{27}$$

$$\boldsymbol{Z}_{ij} + \boldsymbol{Z}_{ji} \geq \boldsymbol{0} \quad i<j \tag{28}$$

where

$$\boldsymbol{Z}_{ij} = \begin{bmatrix} \boldsymbol{X} & * & * & * & * & * \\ \boldsymbol{I} & \boldsymbol{Y} & * & * & * & * \\ -\boldsymbol{C}_i\boldsymbol{X} & -\boldsymbol{C}_i & \boldsymbol{Q}^{-1} & * & * & * \\ \boldsymbol{G}_j - \boldsymbol{C}_i\boldsymbol{X} & -\boldsymbol{C}_i & \boldsymbol{Q}^{-1} & \boldsymbol{Q}^{-1}+\boldsymbol{R}^{-1} & * & * \\ \boldsymbol{A}_i\boldsymbol{X}+\boldsymbol{B}_i\boldsymbol{G}_j & \boldsymbol{A}_i & 0 & 0 & \boldsymbol{X} & * \\ \boldsymbol{\Phi}_{ij} & \boldsymbol{Y}\boldsymbol{A}_i - \boldsymbol{L}_j\boldsymbol{C}_i & 0 & 0 & \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix},$$

$$\boldsymbol{x}_{k\ell}(0) = \begin{bmatrix} x_1(0)+\epsilon_1\cdot sgn(k-1.5) \\ x_3(0)+\epsilon_3\cdot sgn(\ell-1.5) \\ x_5(0) \\ x_6(0) \\ x_7(0) \end{bmatrix} \quad k,\ell=1,2.$$

ϵ_1 and ϵ_3 are positive scalars. $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{G}_j$, $\boldsymbol{L}_j$, $\boldsymbol{\Phi}_{ij}$ are LMI variables as well as in Theorem 1.

(proof)

Consider

$$J = \sum_{k=0}^{\infty} \hat{\boldsymbol{y}}^T(k)\boldsymbol{Q}^*\hat{\boldsymbol{y}}(k) \tag{29}$$

as a quadrtic performance function, where

$$\boldsymbol{Q}^* = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R} \end{bmatrix}, \boldsymbol{Q} > \boldsymbol{0}, \boldsymbol{R} > \boldsymbol{0}.$$

The proof begins with assuming that the following conditions hold.

$$\boldsymbol{P}^{cl} > \boldsymbol{0} \tag{30}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \times \begin{bmatrix} \boldsymbol{A}_{ij}^{cl^T}\boldsymbol{P}^{cl}\boldsymbol{A}_{ij}^{cl} - \boldsymbol{P}^{cl} & \boldsymbol{C}_{ij}^{*^T} \\ \boldsymbol{C}_{ij}^* & -\boldsymbol{Q}^{*^{-1}} \end{bmatrix} < \boldsymbol{0} \tag{31}$$

Then, it is clear that the augmented system (9) and (10) is asymptotically stable in the large. Note that $\boldsymbol{x}^{cl^T}(t)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t)$ is a quadratic Lyapunov function for the augmented system.

First, we derive (26) and the relation $J < \lambda$. We have the following relation.

$$\begin{aligned} &\boldsymbol{x}^{cl^T}(t+1)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t+1) - \boldsymbol{x}^{cl^T}(t)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r}\sum_{v=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))h_\ell(\boldsymbol{z}(t))h_v(\boldsymbol{z}(t)) \\ &\quad \times \boldsymbol{x}^{cl^T}(t)(\boldsymbol{A}_{ij}^{cl^T}\boldsymbol{P}^{cl}\boldsymbol{A}_{\ell v}^{cl} - \boldsymbol{P}^{cl})\boldsymbol{x}^{cl}(t) \\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \\ &\quad \times \boldsymbol{x}^{cl^T}(t)(\boldsymbol{A}_{ij}^{cl^T}\boldsymbol{P}^{cl}\boldsymbol{A}_{ij}^{cl} - \boldsymbol{P}^{cl})\boldsymbol{x}^{cl}(t) \\ &< -\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{x}^{cl^T}(t)\boldsymbol{C}_{ij}^{*^T}\boldsymbol{Q}^*\boldsymbol{C}_{ij}^*\boldsymbol{x}^{cl}(t) \\ &= -\hat{\boldsymbol{y}}^T(t)\boldsymbol{Q}^*\hat{\boldsymbol{y}}(t) \end{aligned} \tag{32}$$

Taking summation both side from 0 to ∞, we obtain

$$\begin{aligned} J &= \sum_{t=0}^{\infty} \hat{\boldsymbol{y}}^T(t)\boldsymbol{Q}^*\hat{\boldsymbol{y}}(t) \\ &< \sum_{t=0}^{\infty}\{\boldsymbol{x}^{cl^T}(t)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t) - \boldsymbol{x}^{cl^T}(t+1)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t+1)\} \\ &= \boldsymbol{x}^{cl^T}(0)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(0) - \boldsymbol{x}^{cl^T}(\infty)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(\infty). \end{aligned} \tag{33}$$

Since the augmented system is asymptotically stable in the large, $\boldsymbol{x}^{cl}(\infty) \to \boldsymbol{0}$. Hence, we have

$$\sum_{t=0}^{\infty} \hat{\boldsymbol{y}}^T(t)\boldsymbol{Q}^*\hat{\boldsymbol{y}}(t) < \boldsymbol{x}^{cl^T}(0)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(0). \tag{34}$$

Let us introduce λ such that

$$J = \sum_{t=0}^{\infty} \hat{\boldsymbol{y}}^T(t)\boldsymbol{Q}^*\hat{\boldsymbol{y}}(t) < \boldsymbol{x}^{cl^T}(0)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(0) < \lambda. \tag{35}$$

A part of the inequality (35) can be converted into

$$\begin{aligned} &\lambda - \boldsymbol{x}^{cl^T}(0)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(0) \\ &= \lambda - \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{x}(0) - \boldsymbol{Y}^{-1}\boldsymbol{Y}(\boldsymbol{x}(0)-\boldsymbol{x}^c(0)) \end{bmatrix}^T \boldsymbol{P}^{cl} \\ &\quad \times \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{x}(0) - \boldsymbol{Y}^{-1}\boldsymbol{Y}(\boldsymbol{x}(0)-\boldsymbol{x}^c(0)) \end{bmatrix} \\ &= \lambda - \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{Y}(\boldsymbol{x}(0)-\boldsymbol{x}^c(0)) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{I} & -\boldsymbol{Y}^{-1} \end{bmatrix}^T \boldsymbol{P}^{cl} \\ &\quad \times \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{I} & -\boldsymbol{Y}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{Y}(\boldsymbol{x}(0)-\boldsymbol{x}^c(0)) \end{bmatrix} > \boldsymbol{0}. \end{aligned} \tag{36}$$

From Schur Complement, the condition (36) is represented as

$$\begin{bmatrix} \lambda & \boldsymbol{x}^T(0) & \hat{\boldsymbol{x}}^T(0)\boldsymbol{Y} \\ \boldsymbol{x}(0) & \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{Y}\hat{\boldsymbol{x}}(0) & \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} > \boldsymbol{0}, \tag{37}$$

where $\hat{\boldsymbol{x}}(0) = \boldsymbol{x}(0) - \boldsymbol{x}^c(0)$.

As will be noted in Remark 1 later, if a polyhedron consisting of all its vertex points $\boldsymbol{x}_{k\ell}(0)$ can be selected so as to contain the unknown initial states $x_1(0)$ and $x_3(0)$, the initial state condition (37) can be replaced with the condition

(26), where ϵ_1 and ϵ_3 denotes the ranges of a polyhedron for $x_1(t)$ and $x_3(t)$, respectively.

Next, we convert (30) and (31) into the LMIs (25), (27) and (28). From (30), (25) is obtained in the same way as in Theorem 1. We derive (27) and (28) from (31). Multiplying the inequality (31) on the left and right by block-diag$[\boldsymbol{X}^{cl}\ \boldsymbol{I}]$, we rewrite the condition as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \begin{bmatrix} \boldsymbol{\Omega}_{ij} & \boldsymbol{X}^{cl}\boldsymbol{C}_{ij}^{*^T} \\ \boldsymbol{C}_{ij}^{*}\boldsymbol{X}^{cl} & -\boldsymbol{Q}^{*^{-1}} \end{bmatrix} < \boldsymbol{0}, \quad (38)$$

where

$$\boldsymbol{\Omega}_{ij} = \boldsymbol{X}^{cl}\boldsymbol{A}_{ij}^{cl^T}\boldsymbol{X}^{cl^{-1}}\boldsymbol{A}_{ij}^{cl}\boldsymbol{X}^{cl} - \boldsymbol{X}^{cl}. \quad (39)$$

Furthermore, multiplying the inequality (38) on the left and right by block-diag$[\boldsymbol{T}\ \ \boldsymbol{T}]$ and block-diag$[\boldsymbol{T}^T\ \boldsymbol{T}^T]$, respectively, we obtain

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \times \begin{bmatrix} \boldsymbol{T}\boldsymbol{\Omega}_{ij}\boldsymbol{T}^T & \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{C}_{ij}^{*^T}\boldsymbol{T}^T \\ \boldsymbol{T}\boldsymbol{C}_{ij}^{*}\boldsymbol{X}^{cl}\boldsymbol{T}^T & -\boldsymbol{T}\boldsymbol{Q}^{*^{-1}}\boldsymbol{T}^T \end{bmatrix}, \quad (40)$$

where we note that

$$\boldsymbol{T}\boldsymbol{C}_{ij}^{*}\boldsymbol{X}^{cl}\boldsymbol{T}^T = \begin{bmatrix} \boldsymbol{C}_i\boldsymbol{X} & \boldsymbol{C}_i \\ \boldsymbol{Y}\boldsymbol{C}_i\boldsymbol{X} - \boldsymbol{Y}\boldsymbol{G}_i & \boldsymbol{Y}\boldsymbol{C}_i \end{bmatrix},$$

$$\boldsymbol{T}\boldsymbol{Q}^{*^{-1}}\boldsymbol{T}^T = \begin{bmatrix} \boldsymbol{Q}^{-1} & \boldsymbol{Q}^{-1}\boldsymbol{Y} \\ \boldsymbol{Y}\boldsymbol{Q}^{-1} & \boldsymbol{Y}\boldsymbol{Q}^{-1}\boldsymbol{Y} + \boldsymbol{Y}\boldsymbol{R}^{-1}\boldsymbol{Y} \end{bmatrix}.$$

Multiplying the inequality (40) on the left and right by $\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Pi} \end{bmatrix}$, where $\boldsymbol{\Pi} =$ block-diag$[\boldsymbol{I}\quad \boldsymbol{Y}^{-1}]$, we arrive at

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \begin{bmatrix} \boldsymbol{T}\boldsymbol{\Omega}_{ij}\boldsymbol{T}^T & \boldsymbol{U}_i^T \\ \boldsymbol{U}_i & -\boldsymbol{V} \end{bmatrix} < \boldsymbol{0}, \quad (41)$$

where

$$\boldsymbol{U}_i = \begin{bmatrix} \boldsymbol{C}_i\boldsymbol{X} & \boldsymbol{C}_i \\ \boldsymbol{C}_i\boldsymbol{X} - \boldsymbol{G}_i & \boldsymbol{C}_i \end{bmatrix}, \quad \boldsymbol{V} = \begin{bmatrix} \boldsymbol{Q}^{-1} & \boldsymbol{Q}^{-1} \\ \boldsymbol{Q}^{-1} & \boldsymbol{Q}^{-1} + \boldsymbol{R}^{-1} \end{bmatrix}.$$

We rewrite the condition as follows.

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \left(\begin{bmatrix} \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T & -\boldsymbol{U}_i^T \\ -\boldsymbol{U}_i & \boldsymbol{V} \end{bmatrix} - \begin{bmatrix} \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{A}_{ij}^{cl^T} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{X}^{cl^{-1}} \begin{bmatrix} \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{A}_{ij}^{cl^T} \\ \boldsymbol{0} \end{bmatrix}^T \right) > \boldsymbol{0}. \quad (42)$$

From Schur Complement, the condition (42) can be converted into

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \times \begin{bmatrix} \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T & -\boldsymbol{U}_i^T & \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{A}_{ij}^{cl^T} \\ -\boldsymbol{U}_i & \boldsymbol{V} & \boldsymbol{0} \\ \boldsymbol{A}_{ij}^{cl}\boldsymbol{X}^{cl}\boldsymbol{T}^T & \boldsymbol{0} & \boldsymbol{X}^{cl} \end{bmatrix} > \boldsymbol{0}. \quad (43)$$

Finally, multiplying the inequality (43) on the left and right by block-diag$[\boldsymbol{I}\ \ \boldsymbol{I}\ \ \boldsymbol{T}]$and block-diag$[\boldsymbol{I}\ \ \boldsymbol{I}\ \ \boldsymbol{T}^T]$, respectively, we have

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \times \begin{bmatrix} \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T & -\boldsymbol{U}_i^T & \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{A}_{ij}^{cl^T}\boldsymbol{T}^T \\ -\boldsymbol{U}_i & \boldsymbol{V} & \boldsymbol{0} \\ \boldsymbol{T}\boldsymbol{A}_{ij}^{cl}\boldsymbol{X}^{cl}\boldsymbol{T}^T & \boldsymbol{0} & \boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T \end{bmatrix} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \\ &\times \begin{bmatrix} \boldsymbol{X} & * & * & * & * & * \\ \boldsymbol{I} & \boldsymbol{Y} & * & * & * & * \\ -\boldsymbol{C}_i\boldsymbol{X} & -\boldsymbol{C}_i & \boldsymbol{Q}^{-1} & * & * & * \\ \boldsymbol{G}_i - \boldsymbol{C}_i\boldsymbol{X} & -\boldsymbol{C}_i & \boldsymbol{Q}^{-1} & \boldsymbol{Q}^{-1} + \boldsymbol{R}^{-1} & * & * \\ \boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{G}_j & \boldsymbol{A}_i & 0 & 0 & \boldsymbol{X} & * \\ \boldsymbol{\Phi}_{ij} & \boldsymbol{Y}\boldsymbol{A}_i - \boldsymbol{L}_j\boldsymbol{C}_i & 0 & 0 & \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} > \boldsymbol{0}. \end{aligned} \quad (44)$$

By replacing the matrices in (44) with $\boldsymbol{Z}_{ij}$, (44) can be rewritten as

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{Z}_{ij} \\ =&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\boldsymbol{Z}_{ii} \\ &+ 2\sum_{i=1}^{r}\sum_{i<j}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t)) \left(\frac{\boldsymbol{Z}_{ij} + \boldsymbol{Z}_{ji}}{2} \right) > 0. \end{aligned} \quad (45)$$

Since $h_i(\boldsymbol{z}(t)) \geq 0$ for all i and $\sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1$, we arrive at (27) and (28) from (45).

(Q.E.D.)

[Remark 1]

As mentioned before, $x_1(t)$ and $x_3(t)$ are unmeasurable. However, if a polyhedron consisting of all its vertex points $\boldsymbol{x}_{k\ell}(0)$ can be selected so as to contain the unknown initial states $x_1(0)$ and $x_3(0)$, the condition (26) implies the initial state condition (37). The reason is that the condition (26) is convex with respect to the vertex points $\boldsymbol{x}_{k\ell}(0)$ containing the unknown initial states $x_1(0)$ and $x_3(0)$.

[Remark 2]

ϵ_1 and ϵ_3 denote the ranges of a polyhedron for $x_1(t)$ and $x_3(t)$, respectively. In the simulation, $\epsilon_1 = \epsilon_3 = 10 \times \frac{\pi}{180}$ [rad.]. This means that the margin for the initial unknown unmeasurable states are permitted to have 10 [deg.] error margins. For example, suppose that we want to set the initial relative angles to zero. Then, it is difficult to set the initial relative angles to exact zero without potentiometers. However, even in this situation, if it is possible to set the initial relative angles $x_1(0)$ and $x_3(0)$ to any values on the closed interval $[-10 \times \frac{\pi}{180}[rad.] \quad 10 \times \frac{\pi}{180}[rad.]]$, the stability is guaranteed for the initial states. It is possible to set up these initial relative angles within the margins (without potentiometers) in real experiments.

[Remark 3]

The design condition in Theorem 2 minimizes the upper bound of the quadratic performance function (24) in the worst case for all the values of $h_i(\boldsymbol{z}(t)) \in [0 \quad 1]$.

[Remark 4]

In the case of $m \neq \ell$, we can add dummy zero vectors to $\boldsymbol{B}_i$ and $\boldsymbol{C}_i$ so as to be matrices of appropriate dimensions. The details will be concretely shown later.

The optimal dynamic output feedback design requires that $m = \ell$. In this case, $m \neq \ell$. Therefore, we add dummy zero vectors to $\boldsymbol{B}_i$ and $\boldsymbol{C}_i$ so as to be matrices of appropriate dimensions. Hence, the output equation of the Takagi-Sugeno fuzzy system for the controlled object is as follows.

$$\boldsymbol{y}(t) = \frac{\sum_{i=1}^{2} w_i(z_1(t))\boldsymbol{C}_i\boldsymbol{x}(t)}{\sum_{i=1}^{2} w_i(z_1(t))}, \tag{46}$$

where

$$\boldsymbol{C}_1 = \boldsymbol{C}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

According to the above modification of $\boldsymbol{C}_i$, $\boldsymbol{B}_i$ is modified so as to be a matrix of appropriate dimension.

$$\boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} \frac{\nu \cdot \Delta t}{l} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The optimal dynamic output feedback controller is designed by solving the GEVP in Theorem 2. Figures 7, 8 and 9 show the control results of the original model by the optimal controller for the initial values $x_6(0) = \pi/6, \pi/2, \pi$, respectively, where $x_1(0) = x_3(0) = x_5(0) = 0$, $x_7(0) = 2$ and $\boldsymbol{x^c}(0) = 0$. In the optimal design, we use $Q = 0.001 \times \mathbf{I}$ and $R = 10 \times \mathbf{I}$. Table I shows the performance index values J calculated in the simulations. Table I also shows performance index values in the stable control result in Fig. 4. The performance index values of the optimal controller is better than those of the stable controller. The optimal controller successfully realizes the backing-up control for the second initial states, but still not for the third initial states, although the stable controller realizes the backing-up control only for the first initial states.

IV. LMI Condition for Avoiding Jackknife

To perfectly realize the backing-up control, we derive an LMI condition for avoiding the jackknife phenomenon. The condition of avoiding the jackknife is equivalent to

$$||x_1(t)|| \leq \beta, ||x_3(t)|| \leq \beta, ||x_5(t)|| \leq \beta,$$

where $\beta = \pi/2$. The derivation begins with representing $x_k(t)$ with a vector $\boldsymbol{d}_k$ and $\boldsymbol{x}^{cl}(t)$.

$$\begin{aligned} x_k(t) &= \begin{bmatrix} \boldsymbol{d}_k & \mathbf{0} \end{bmatrix} \boldsymbol{x}^{cl}(t) \\ &= \hat{\boldsymbol{d}}_k \boldsymbol{x}^{cl}(t) \quad k = 1, 3, 5. \end{aligned} \tag{47}$$

where

$$\begin{aligned} \boldsymbol{d}_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \boldsymbol{d}_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \\ \boldsymbol{d}_5 &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \end{aligned}$$

TABLE I

Performance index values J.

$x_6(0)$	Stable	Optimal
Initial state $\pi/6$	9.7	2.4
Initial state $\pi/2$	jackknife	1.6
Initial state π	jackknife	jackknife

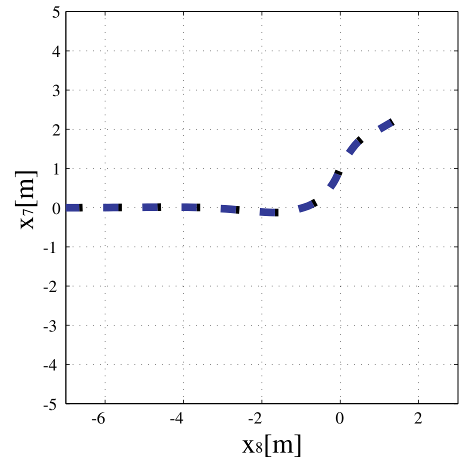

Fig. 7. Control result for optimal controller ($x_6(0) = \pi/6$).

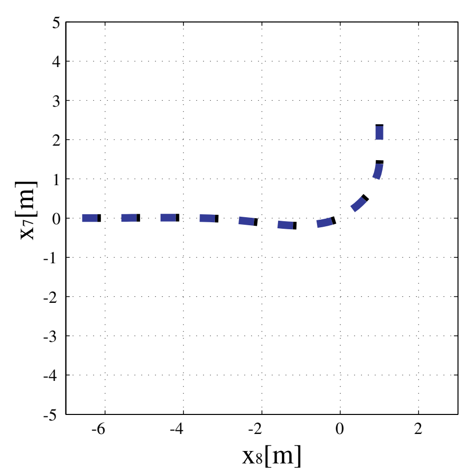

Fig. 8. Control result for optimal controller ($x_6(0) = \pi/2$).

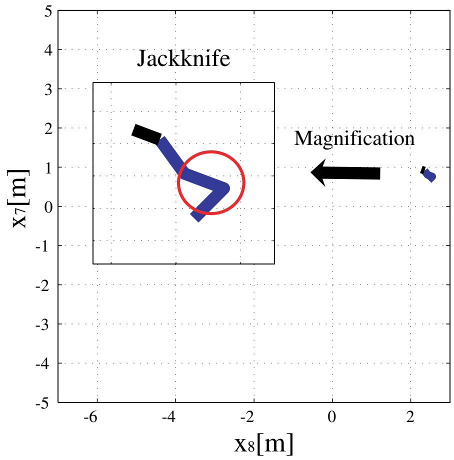

Fig. 9. Control result for optimal controller ($x_6(0) = \pi$).

Since

$$\begin{aligned} & x_k^T(t)x_k(t) \\ =& \boldsymbol{x}^{cl^T}(t)\hat{\boldsymbol{d}}_k^T\hat{\boldsymbol{d}}_k\boldsymbol{x}^{cl}(t) \le \beta^2, \end{aligned}$$

we have

$$\frac{1}{\beta^2}\boldsymbol{x}^{cl^T}(t)\hat{\boldsymbol{d}}_k^T\hat{\boldsymbol{d}}_k\boldsymbol{x}^{cl}(t) \le 1. \tag{48}$$

Next, without loss of generality, we assume that the upper bound of Lyapunov function is λ. Then, the Lyapunov function satisfies

$$\boldsymbol{x}^{cl^T}(t)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t) \le \boldsymbol{x}^{cl^T}(0)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(0) \le \lambda.$$

Therefore,

$$\frac{1}{\lambda}\boldsymbol{x}^{cl^T}(t)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t) \le 1. \tag{49}$$

The condition (48) is satisfied if

$$\frac{1}{\beta^2}\boldsymbol{x}^{cl^T}(t)\hat{\boldsymbol{d}}_k^T\hat{\boldsymbol{d}}_k\boldsymbol{x}^{cl}(t) \le \frac{1}{\lambda}\boldsymbol{x}^{cl^T}(t)\boldsymbol{P}^{cl}\boldsymbol{x}^{cl}(t). \tag{50}$$

Then, we have

$$\frac{1}{\lambda}\boldsymbol{P}^{cl} - \frac{1}{\beta^2}\hat{\boldsymbol{d}}_k^T\hat{\boldsymbol{d}}_k \ge \boldsymbol{0}. \tag{51}$$

Multiplying (51) on the left and right by $\boldsymbol{X}^{cl} = \boldsymbol{P}^{cl^{-1}}$ gives

$$\frac{1}{\lambda}\boldsymbol{X}^{cl} - \frac{1}{\beta^2}\boldsymbol{X}^{cl}\hat{\boldsymbol{d}}_k^T\hat{\boldsymbol{d}}_k\boldsymbol{X}^{cl} \ge \boldsymbol{0}. \tag{52}$$

Multiplying (52) on the left and right by $\boldsymbol{T}$ and $\boldsymbol{T}^T$, respectively, we obtain

$$\frac{1}{\lambda}\boldsymbol{T}\boldsymbol{X}^{cl}\boldsymbol{T}^T - \frac{1}{\beta^2}\boldsymbol{T}\boldsymbol{X}^{cl}\hat{\boldsymbol{d}}_k^T\hat{\boldsymbol{d}}_k\boldsymbol{X}^{cl}\boldsymbol{T}^T \ge \boldsymbol{0}.$$

By noting that

$$\hat{\boldsymbol{d}}_k\boldsymbol{X}^{cl}\boldsymbol{T}^T = \begin{bmatrix}\boldsymbol{d}_k\boldsymbol{X} & \boldsymbol{d}_k\end{bmatrix},$$

the inequality can be rewritten as

$$\frac{1}{\lambda}\begin{bmatrix}\boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y}\end{bmatrix} - \begin{bmatrix}\boldsymbol{X}\boldsymbol{d}_k^T \\ \boldsymbol{d}_k^T\end{bmatrix}\frac{1}{\beta^2}\boldsymbol{I}\begin{bmatrix}\boldsymbol{d}_k\boldsymbol{X} & \boldsymbol{d}_k\end{bmatrix} \ge \boldsymbol{0}.$$

By Schur Complement, we arrive at the following LMI condition.

$$\begin{bmatrix}\boldsymbol{X} & \boldsymbol{I} & \boldsymbol{X}\boldsymbol{d}_k^T \\ \boldsymbol{I} & \boldsymbol{Y} & \boldsymbol{d}_k^T \\ \boldsymbol{d}_k\boldsymbol{X} & \boldsymbol{d}_k & \frac{\beta^2}{\lambda}\boldsymbol{I}\end{bmatrix} \ge \boldsymbol{0} \quad k = 1, 3, 5. \tag{53}$$

The condition (49) is guaranteed if (26) holds. By solving (53) in addition to the LMIs in Theorem 2, we can design dynamic output feedback controller satisfying both optimality and avoidance of jackknife.

Figures 10, 11 and 12 show the control results of the original model by the optimal control considering avoidance of the jackknife for $x_6(0) = \pi/6, \pi/2, \pi$, respectively, where $x_1(0) = x_3(0) = x_5(0) = 0$, $x_7(0) = 2$ and $\boldsymbol{x}^c(0) = 0$. The optimal controller considering avoidance of jackknife successfully realizes the backing-up control for all three initial states.

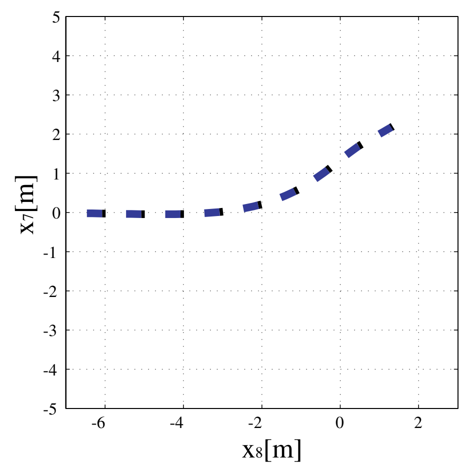

Fig. 10. Control result for optimal controller considering jackknife avoidance ($x_6(0) = \pi/6$).

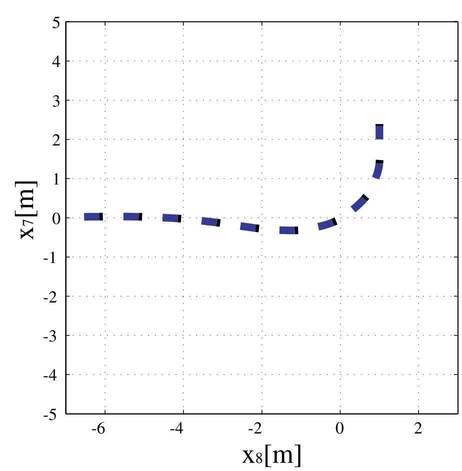

Fig. 11. Control result for optimal controller considering jackknife avoidance ($x_6(0) = \pi/2$).

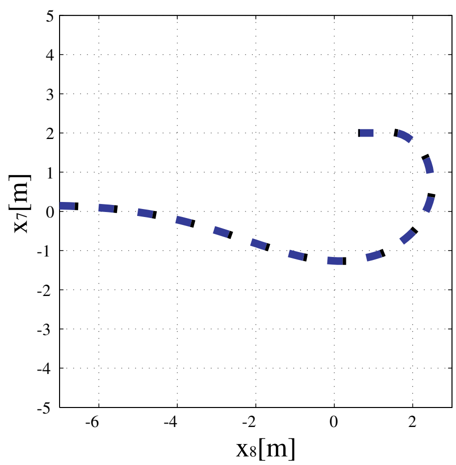

Fig. 12. Control result for optimal controller considering jackknife avoidance ($x_6(0) = \pi$).

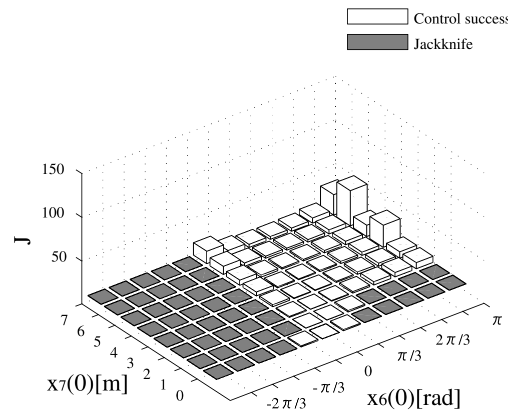

Fig. 14. Control performance for optimal controller.

V. Simulation Results and Discussions

To see the utility of the optimal dynamic output feedback design considering avoidance of jackknife, we investigate control performances for some combinations of $x_6(0)$ and $x_7(0)$, where $x_1(0) = x_3(0) = x_5(0) = 0$. Figures 13, 14 and 15 show control performances for each combination of $x_6(0)$ and $x_7(0)$ in the stable controller, the optimal controller and the optimal controller considering avoidance of jackknife, respectively. In these figures, the dark area and the white area denote jackknife and control success (namely, success of backing control), respectively. That is, the controls that start from any combinations of $x_6(0)$ and $x_7(0)$ in the dark area occurs jackknife. In the stable control (Figure 13), most of areas are dark (jackknife) areas. Note that J can not be calculated in the jackknife case. On the other hand, no dark areas exist in the optimal control considering avoidance of jackknife (Figure 15).

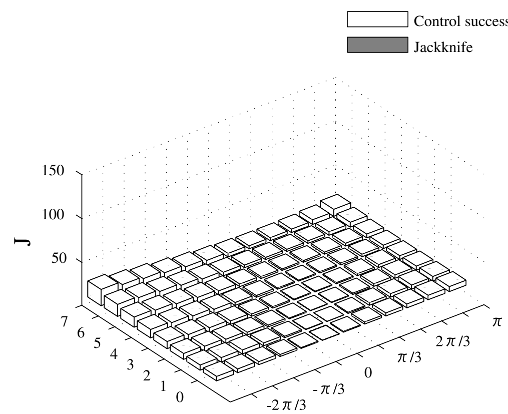

Fig. 15. Control performance for optimal controller considering jackknife avoidance.

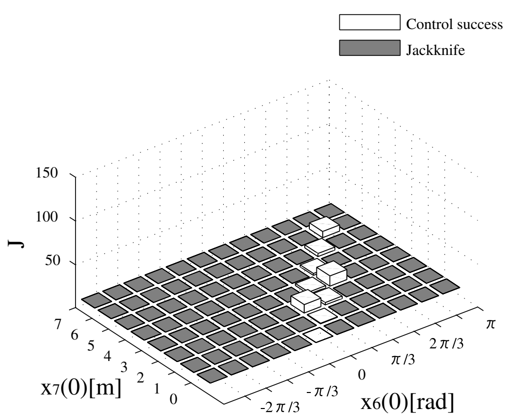

Fig. 13. Control performance for stable controller.

Next, in the case of considering the LMI condition (53) for avoiding the jackknife, we compare control performance difference between the optimal control and the stable control. Figures 16, 17 and 18 show the control results of the original model by the stable control considering the LMI condition (53) for avoiding the jackknife. Figures 19, 20 and 21 show the same control results as shown in Figures 16, 17 and 18, respectively. The dotted areas in Figures 19, 20 and 21 denote the same size as the overall areas shown in Figures 16, 17 and 18, respectively. It can be seen that the stable control considering the LMI condition for avoiding the jackknife realizes backing-up control. However, the control performance is much more poor than the optimal control. Clearly, a much wider experimental field is required for the stable control. Conversely, the optimal control (Figures 10, 11 and 12) realizes good speed of response in addition to stabilization.

Our results demonstrate that the optimal dynamic output feedback design considering avoidance of jackknife is effective for the backing-up control problem of a three-trailer truck. More importantly, our approach realizes the reduction of at least two potentiometers. Thus, our approach provides a cost effective design even for the difficult control problem.

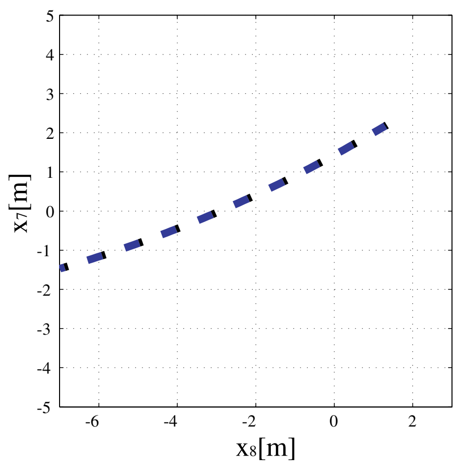

Fig. 16. Control result for stable controller considering jackknife avoidances ($x_6(0) = \pi/6$, Magnification).

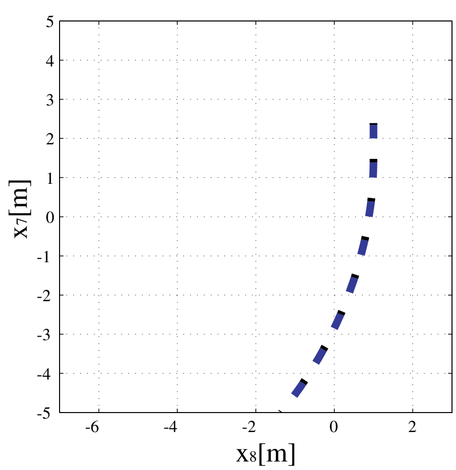

Fig. 17. Control result for stable controller considering jackknife avoidance ($x_6(0) = \pi/2$, Magnification).

VI. Conclusions

This paper has presented a cost effective design based on sensor reduction for backing-up control of a vehicle with triple trailers. To realize a cost effective design, we have newly derived two LMI conditions for discrete Takagi-Sugeno fuzzy system. One is an optimal dynamic output feedback design that guarantees desired control performance. The other is an avoidance of jackknife phenomenon for the use of the optimal dynamic output feedback controller. Our results have demonstrated that the proposed LMI-based design effectively

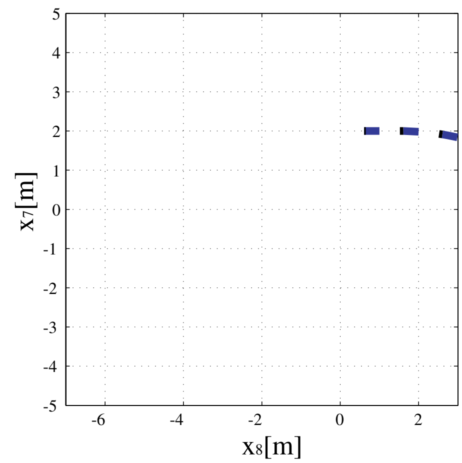

Fig. 18. Control result for stable controller considering jackknife avoidance ($x_6(0) = \pi$, Magnification).

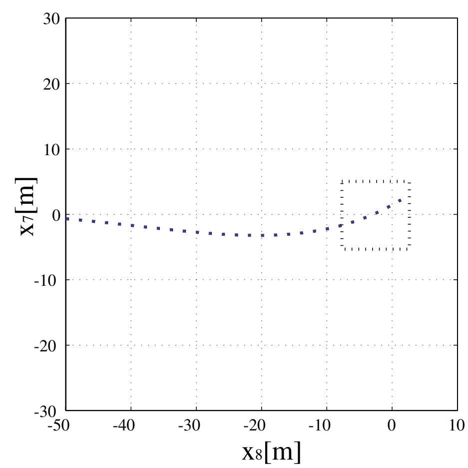

Fig. 19. Control result for stable controller considering jackknife avoidance ($x_6(0) = \pi/6$).

achieves the backing-up control of the vehicle with triple trailers while avoiding the jackknife phenomenon. More importantly, we have demonstrated that the designed optimal control can achieve the backing-up control without at least two potentiometers. The successful control results without two potentiometers are very interesting and important from the cost effective design points of view.

Our next subject is to demonstrate the utility of the cost effective design in experiments.

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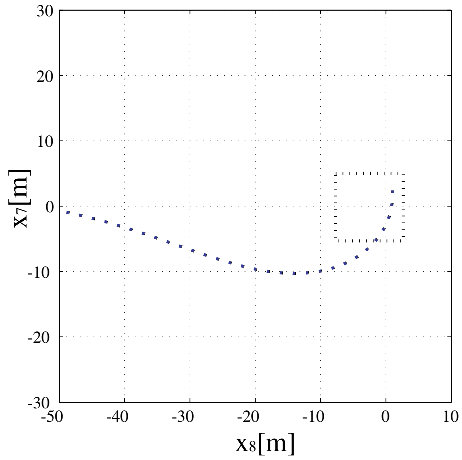

Fig. 20. Control result for stable controller considering jackknife avoidance ($x_6(0) = \pi/2$).

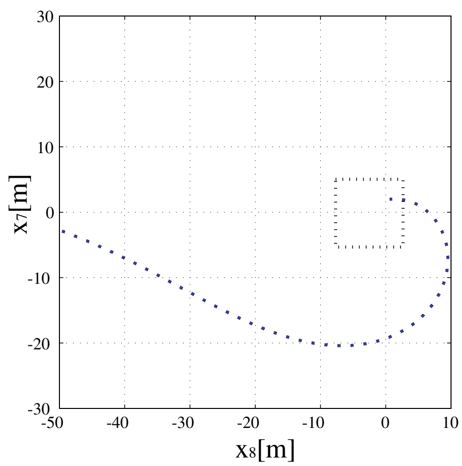

Fig. 21. Control result for stable controller considering jackknife avoidance ($x_6(0) = \pi$).

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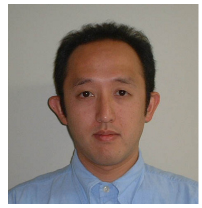

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