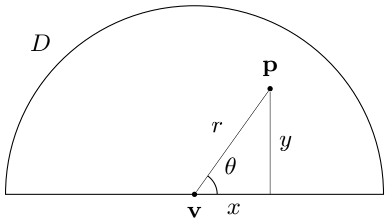

Fig. 1. Polar and cartesian parameterizations of the half-disc D.

Appendix
Derivation of the default γ value

Theorem 1: Let R be a positive real number. Let

$$w_{\mathbf{p}}(\mathbf{x}) = \left(1 - \min\left\{\frac{\|\mathbf{p}-\mathbf{x}\|}{R}, 1\right\}^2\right)^4,$$

for $\mathbf{p}, \mathbf{x} \in \mathbb{R}^3$, let H be a half-plane and $\mathbf{v}$ be a point on the boundary of H. Then

$$\frac{\|\int_{\mathbf{p}\in H} w_{\mathbf{p}}(\mathbf{v})(\mathbf{p}-\mathbf{v})\ dH\|}{\sqrt{(\int_{\mathbf{p}\in H} w_{\mathbf{p}}(\mathbf{v})\ dH)(\int_{\mathbf{p}\in H} H w_{\mathbf{p}}\|\mathbf{p}-\mathbf{v}\|^2\ dH)}} = \frac{512\sqrt{6}}{693\pi} \quad (1)$$

In the context of boundary detection, $\mathbf{v}$ is a point to be classified, while $\mathbf{p}$ is a splat with radius R.

Proof: Firstly note that if $\|\mathbf{p}-\mathbf{v}\| > R$ then $w_{\mathbf{p}}(\mathbf{v}) = 0$. Thus, if D is the subset of H within a distance R of $\mathbf{v}$, then we can replace H by D in the integrals in (1). We parameterize the half-disc D using polar coordinates (r,θ), with $r \in [0,R]$ and $\theta \in [0,\pi]$, as shown in Fig. 1. We can also Cartesian coordinates $(x,y) = (r\cos\theta, r\sin\theta)$.

$$\begin{aligned} w(r) &= \left(1 - \left(\frac{r}{R}\right)^2\right)^4 \\ &= 1 - \frac{4}{R^2}r^2 + \frac{6}{R^4}r^4 - \frac{4}{R^6}r^6 + \frac{1}{R^8}r^8. \end{aligned}$$

Let us now consider the integrals one at a time. The integral in the numerator is a vector integral, but from symmetry it is clear that the x component will vanish and we need only compute the y component

$$\begin{aligned} \int_H w_{\mathbf{p}}(\mathbf{v})(\mathbf{p}-\mathbf{v})_y\ dH &= \iint_{r,\theta} w(r) r\sin\theta \cdot r \cdot dr \cdot d\theta \\ &= \left(\int_0^R r^2 w(r)\ dr\right)\left(\int_0^\pi \sin\theta\ d\theta\right) \\ &= R^3\left(\tfrac{1}{3} - \tfrac{4}{5} + \tfrac{6}{7} - \tfrac{4}{9} + \tfrac{1}{11}\right)\cdot 2 \\ &= \frac{256}{3465}R^3. \end{aligned}$$

Next, the integrals in the denominator:

$$\begin{aligned} \int_H w_{\mathbf{p}}(\mathbf{v})\ dH &= \iint_{r,\theta} w(r)\cdot r \cdot dr \cdot d\theta \\ &= \left(\int_0^R w(r)\cdot r \cdot dr\right)\left(\int_0^\pi d\theta\right) \\ &= \left(\tfrac{1}{2} - \tfrac{4}{4} + \tfrac{6}{6} - \tfrac{4}{8} + \tfrac{1}{10}\right)R^2\pi \\ &= \frac{\pi}{10}R^2. \end{aligned}$$

$$\begin{aligned} \int_H w_{\mathbf{p}}(\mathbf{v})\|\mathbf{p}-\mathbf{v}\|^2 dH &= \iint_{r,\theta} w(r)\cdot r^3 \cdot dr \cdot d\theta \\ &= \left(\tfrac{1}{4} - \tfrac{4}{6} + \tfrac{6}{8} - \tfrac{4}{10} + \tfrac{1}{12}\right)R^4\pi \\ &= \frac{\pi}{60}R^4. \end{aligned}$$

The left hand side of (1) is thus

$$\frac{\frac{256}{3465}R^3}{\sqrt{\frac{\pi}{10}R^2 \cdot \frac{\pi}{60}R^4}} = \frac{256\sqrt{600}}{3465\pi} = \frac{512\sqrt{6}}{693\pi}.$$

□