Evaluation of Interference in Inter-working Multi-hop Wireless Networks

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Abstract

The consequence of simultaneous transmissions by nodes in inter-working multi-hop wireless networks is inter-node interference. Inter-node interference is a metric that is important for the performance evaluation of inter-working wireless networks. Several interference models exist in literature, however, these models are for specific wireless networks and MAC protocols. Due to the heterogeneity of link-level technologies in inter-working multi-hop wireless networks, it is desirable to have generic models for the evaluation of interference on links in such networks. A generic model can provide information about the interference level on a link irrespective of the MAC protocol in use. This paper presents a generic interference model. The model evaluates the probability of interference and uses the negative second moment of the distance between a receiver-node and interfering-nodes to evaluate the interference power on a link in an inter-working multi-hop wireless network. Numerical results of the performance of the model are presented.

Keywords: *Interference, Inter-working, Multi-hop, Wireless networks.*

1. Introduction

With the penetration of wireless access networks and an increasing interest in ubiquitous internet access, inter-working wireless networks are likely to be the next generation of wireless networks [1]. It is envisioned that the next generation of wireless networks will support a wide variety of services. Services such as multimedia web browsing, video and news on demand, mobile office system, and stock market information will be provided to mobile users anywhere, anytime in an uninterrupted and seamless way, using low-powered handsets [2]. Inter-working is a term which refers to the seamless integration of several networks. An advantage of inter-working wireless networks is that it allows ubiquitous internet access, most especially in areas with no initial wire-line network coverage e.g. rural areas [3]. Different wireless access networks can be inter-worked. However, a major challenge in integrating these networks is the issue of inter-node interference. Even though the integration of the networks can provide users with access choices, yet it can lead to mutual interference problems.

In inter-working multi-hop wireless networks, transmission flows are multi-hop and several communications can potentially take place simultaneously. Multi-hop means that data from a source node may travel through intermediate nodes to reach its destination. In addition the networks to be inter-worked may be similar or different networks. Hence, different user terminals (nodes) may co-exist in inter-working multi-hop wireless networks. As a result, node to node interference can either be internally or externally generated [4]. Internally

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generated interference is caused by nodes, which belong to the same wireless network while externally generated interference is caused by nodes in co-located wireless networks. Both internally and externally generated node-to-node interference hampers the reliability of the wireless channel (links) in terms of throughput and delay and thereby limits the performance gain of the network [5].

Research has identified that inter-node interference is one of the most important causes of performance degradation in wireless multi-hop networks. Hence, the research for analytical models for estimating the amount of inter-node interference in different wireless networks, has received a lot of attention over the past few years. The interest is expected to increase due to the advent of new architectures and communication technologies, such as wireless networks sharing the same frequency band (unlicensed), infrastructure-less wireless networks and ultra-wideband systems [6].

Hence, the modeling of inter-node interference in inter-working multi-hop wireless networks is an important step toward the analysis, design and deployment of such networks. Recent research papers such as [6], [7] [8], [9] [10], and [11] have developed models for interference in wireless networks. In [13], models for inter-node interference for aloha MAC protocol and the "Per-route" Carrier Sense Multiple Access (PR-CSMA) MAC protocols were derived. Also, in [14], the effect of interference in ALOHA ad-hoc network was investigated. In [15], a model for calculating interference in multi-hop wireless networks using CSMA for medium access control was proposed. In [7], Busson et al, proposed the use of Mat`ern point process and Simple Sequential Inhibition (SSI) point process, for the modeling of interference distribution in the case of CSMA/CA networks. In [12], the authors put forth a mathematical model for interference in cognitive radio networks, in wireless packet networks and in networks consisting of narrowband and ultra-wide band wireless nodes. [10] explained the analysis of interference in wireless ad-hoc networks with distributed MIMO. The research work in [11] presented a statistical model of interference in wireless networks, in which the power of the nearest interferer is used as a major performance indicator instead of the total interference power. The authors argued that at low outage region, the power of the nearest interferer and the total interference power have the same statistics so that the former is an accurate approximation of the latter. [8] provided the characterization of interference in cognitive radio networks.

These related research works have developed interference models for particular networks and they have assumed different network scenarios and different network topologies. For example, the model presented in [15] was specifically for ad-hoc networks in hexagonal network topology. Such deterministic placement of nodes (square, rectangular and hexagonal) may be applicable where the locations of nodes are known or they are constrained to a particular structure. However, the deterministic placement of nodes is not realistic for inter-working multi-hop wireless networks. Therefore, the interference models developed in research works based on deterministic location of nodes will not be applicable in inter-working multi-hop wireless networks where topology could be arbitrary.

Although, some of the research works mentioned have actually used stochastic models for nodes' locations, yet the interference models are inclined towards specific combinations of propagation models, spatial location of nodes, transmission technologies and multiple access schemes [12], These constraints make the results from the models not to be easily realizable in other wireless technologies where the parameters may differ.

The challenge associated with inter-working multi-hop wireless networks includes the variation in the operating standards of different wireless access networks. The standards are in terms of data rates, coverage areas, transmission technologies, MAC schemes and topology

etc. The variation in standards will make it difficult to adopt the interference models presented by the reviewed research works to inter-working multi-hop wireless networks. Therefore, it is desirable to characterize the inter-node interference on links in inter-working multi-hop wireless networks.

To achieve the goal of inter-working, interference modelling is of great significance. The characterization of interference is necessary as it enables an understanding of the physical layer. This understanding will enable the design of strategies that will optimize network performance and resource allocation. Since the interfering nodes' (I-nodes') behavior (e.g. change in power levels, movement and distance relative to the receiving node (R-node)) influences network parameters such as throughput, delay and bit error rate, the modeling of interference is essential. An interference model helps in the design of power control strategies and traffic engineering strategies (e.g. routing, admission control and resource allocation). It is known that the higher the interference between nodes, the lower the effectiveness of any routing strategy in the network [12]. Consequently, the provisioning of quality of service (QoS) and resource dimensioning within the network are impacted. Hence, it is necessary to understand interference and its impact on network parameters so as to be able to design effect strategies that will optimize the network's capacity. Therefore, this paper presents a MAC protocol independent analytical model for inter-node interference (INI) in inter-working multi-hop wireless networks. Specifically, the statistical negative moment of distances between nodes and the probability of interference are used to evaluate the INI power on a link in a region within an inter-working multi-hop wireless network.

In general, interference has a stochastic nature, not only due to the randomness in the propagation channel (modeled by the free space propagation model), but also due to the random geographic dispersion of nodes (modeled by spatial Poisson process). As a result, INI power is characterized as a random variable in this paper.

In order to find the expected value of the INI power on a link in the network, firstly, the distribution of the distance (β_k, R) between the R node and the I nodes was determined. Then, the spatial density of interfering nodes was estimated using the probability of interference within the inter-working network. A region of interference is defined for each R-node and the interference from nodes beyond this region is said to negligible. An approximation for the negative second moment allowed a tractable mathematical analysis of the relationship between the INI power experienced on a link and other important parameters such as SINR, node transmit power and the spatial node density. The analysis also shows how a wireless link's performance in terms of SINR depends on these parameters. Such an understanding provides valuable insights to inter-working multi-hop wireless network designers.

The SINR model developed by Gupta and Kumar in [5] was used in the analysis because in reality, additive interference is the minimum level of detail that should be modeled in order to obtain accurate results and insights into interference in any wireless network. More so, the SINR model captures the decoding nature of most physical layer technologies [16]. Basically, the decoding of the signal meant for a receiver-node is done by treating the sum of all simultaneous transmission and disturbances as noise [17].

The numerical results presented validated the interference model by showing the influence of interference on the SINR on a link in inter-working multi-hop wireless network. The results provide some insights into the effect of the interfering node density on INI power. Since the power levels and the number of interfering nodes are the key players in determining interference power, the interference power was increased by increasing the power levels of the interfering nodes and the also the number of interfering nodes to see the influence of these parameters on the SINR on a link.

The scope of this paper is limited only to evaluating an inter-node interference model for Inter-working Multi-hop Wireless Networks. The design of a MAC or a routing protocol is outside the scope of this paper. The future work of this research includes applying this model in a simulation environment. The contents of this paper are as follows: section II discusses the network models used in this paper; these include the node distribution and inter-working models, which are presented in section 2.1 and 2.2. Section 2.3 explains the channel, propagation and mobility models; section 2.4 describes the node transmission and reception models, while the signal to interference and noise ratio model is given in section 2.5. Section 3 presents the analysis of the inter-node interference model, which includes determining the distribution function of $\beta k,R$, calculating the probability of interference and evaluating the interference power. Section 4 concludes the paper.

2. Network Models

So what do we need to characterize interference? A typical model of interference in any network requires:

1) A model, which provides the spatial location of nodes. 2) A channel propagation model, which explains the propagation characteristics of the network. These include the path loss, node mobility models etc. 3) A model for the transmission characteristics of nodes and a threshold-based receiver performance model.

2.1. Node Distribution Model

Since nodes' locations are completely unknown a priori in wireless networks, they can be treated as completely random. and hence the distribution of the nodes is random. The irregular location of nodes, which is influenced by factors like mobility or unplanned placement of the nodes may be considered as a realization of a spatial point pattern (or process) [3] [8].
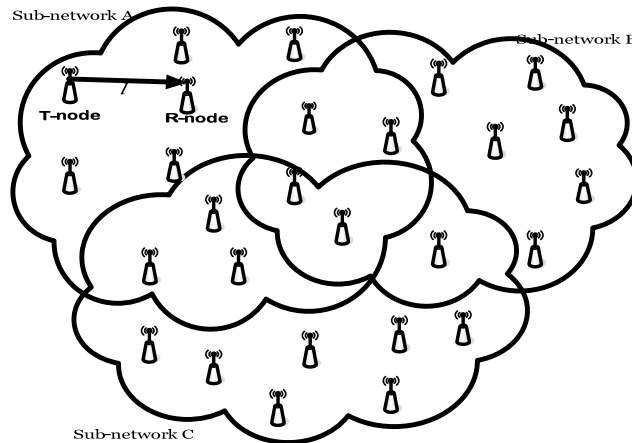

Fig. 1. Network Model

A spatial point pattern is a set of location, irregularly distributed within a designated region and presumed to have been generated by some form of stochastic mechanism. In most applications, the designation is essentially on planar Rd (e.g. d=2 for two-dimensional) Euclidean space [18]. The lack of independence between the points is called complete spatial

randomness (CSR) [3]. According to the theory of complete spatial randomness, for a spatial point pattern, the number of points inside a planar region P follows a Poisson distribution [18]. It follows that the probability of p points being inside region P, (Pr (p in P)) depends on the area of the region (Ap) and not on the shape or location of the plane.
Pr (p in P) is given by (1), where μ is the spatial density of points.

$$\Pr(\, p \text{ in } P) = \frac{(\mu A_{\mathrm{P}})^{p}}{p!} e^{-\mu Ap}, \, p > 0.$$

(1)

Poisson process' application to nodes' positions modeling in wireless environments was firstly done in [19] and then in [20]. In [9] it was proved that if the node positions are modeled with Poisson point process, then parameters such as the spatial distribution of nodes, transmission characteristics and the propagation characteristics of the wireless link can be easily accounted for. Therefore, in this paper, the distribution of nodes is according to homogenous Poisson point process. The nodes are randomly and independently located. This is a reasonable model particularly in a network with random node placement such as the inter-working multi-hop wireless networks. Moreover the most popular choice for the modelling of the nodes' spatial distribution is the Poisson point process [3] [6] [7] [8] [9] [11].

2.2. Inter-working Network Model

The network in fig 1 represents a set of inter-working multi-hop wireless networks. Each network is considered as a collection of random and independently positioned nodes. The nodes in the network in fig 1 are contained in a Euclidean space of 2-dimensions (R2). These set of multi-hop wireless networks are overlapping. They are inter-worked with a gateway i.e. there is inter-domain co-ordination between the networks. The gateways co-ordinate the handover issues within the inter-working networks.

The inter-working network in fig. 1 is represented as network Ω, which contain three subset networks (sub-networks) A, B, and C. The total number of nodes in Ω is denoted NΩ, while the number of nodes in sub-networks A, B, C are N_a, N_b and N_c respectively, where $N_a + N_b + N_c = N_\Omega$. The spatial density of each sub-network is given by μ_A, μ_B, μ_C (μ=**N/a**, **N** is the number of nodes in a sub-network, **a** is the sub-network's coverage area and μ is given in nodes /unit square). The entire inter-working network is considered as a merging Poisson process with spatial density: $\mu_A + \mu_B + \mu_C = \mu_{Net}$. In the network, node to node communication may be multi-hop and nodes transmit at a data rate of Ψ bps. In this paper, source-nodes are referred to as transmitter-nodes (T-nodes) while destination-nodes are referred to as receiver-nodes (R-nodes). $\{l : l = 1,2,3,\ldots\ldots n\} \in L$ represents the links between nodes, where L is the set of all links in the entire network. The length of a communication link is represented by $\beta_{T,R}$, subscript T denotes the transmitter-node while subscript R denotes the receiver-node on the link.

2.3. Channel, Propagation and Mobility Models

In fig.1, for a packet transmitted by the T-node on link l: l=1, 2, 3,….n and received by the R-node, the actual received power at the R-node can be expressed by the Friis equation given as:

$$P_l^{\,r} = cP_l^{\,t} A_l = cP_l^{\,t} (\beta_{T,R})^{-\alpha}.$$

(2)

$$\left[c = \frac{G_t G_r \lambda_c^{\,2}}{(4\pi)^2 L_f} \right]$$

The parameters in the network include:
- P_l^t: power transmitted by the transmitter- node on link l.
- P_l^r: power received by the receiver node on link l.
- G_t and G_r are the transmitter and receiver gain respectively.
- λ_c is the wavelength, $\lambda_c = g/f_c$ (g is the speed of light and f_c is the carrier frequency.
- L_f which is ≥ 1 is the system loss factor.

To account for path-loss, the channel attenuation for link l is denoted by A_l. Path-loss is an attenuation effect which results in the reduction of the transmitted signal power in proportion to the propagation distance between any T-node and corresponding R-node. It is typically given that the received power from a T-node at distance $\beta_{T,R}$ from the R-node decays exponentially as $(\beta_{T,R})^{-\alpha}$. α is the path loss exponent, which represents the exponential decay of the transmitted power. It depends on the environment and could be a constant between 2 and 6. In this paper[1], $\alpha=2$, so $A_l = (\beta_{T,R})^{-2}$. The exponential decay of power makes it possible to consider interference from nodes located at a far distance from the R-node as negligible.
In this paper, it is assumed that the randomness in the distance between nodes, irrespective of the topology of the network captures the movement of nodes. The movement of a node from one point to another changes its location and consequently its distance to a reference node[2]. Thus, the variation in the distances between any I-node and an R-node is highly coupled with the movement of the I-node.

2.4. Node Transmission Model

A case where transmitting nodes and interfering nodes use the same physical layer techniques (e.g. modulation techniques) is termed homogeneous. A heterogeneous case occurs when nodes use different physical layer techniques. Nodes are able to transmit signals at power levels, which are random and independent between nodes.

2.5. Link Signal to Interference and Noise Ratio Model.

The receiver performance model is based on the Signal to Interference and Noise (SINR) physical layer model. The (SINR) ratio is defined as the ratio of the power at which a transmitted signal was received to the total interference power experienced by the receiving node on a link. The total interference power is the sum of the inter-node interference power and the noise power level as in equation 3.

$$P_{\text{int}} = P_o + P_{ini} . \tag{3}$$

- P_o: thermal noise power level at the R-node on link l. $P_o = FkT_oB$ (k=1.38 × 10^{-23} J/^{o}K/Hz (Boltzman constant), T_o is the ambient temperature, B is the transmission bandwidth and F is the noise figure [21]).
- P_{int}: total interference power experienced by the receiver at the end of link l. It is the sum of the thermal noise power and the inter-node interference.
- P_{ini}: inter-node interference power given by equation 4.

$$P_{ini} = \sum_{k=1}^{S} c P'^{(k)} (\beta_{k,R})^{-\alpha} . \tag{4}$$

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[1] In free space α=2
[2] Reference node refers to the receiver node (R-node) on the link for which interference is being measured.

P_{ini} represents the total interference power from nodes simultaneously transmitting with the T-node on the reference link. For a T-node and an R-node on link l in fig. 1, P_{ini} is the cumulative of the interfering power that the R-node experiences from nodes concurrently transmitting with the T-node. I-nodes are the nodes that can transmit simultaneously with the T-node. S is the total number of I-nodes and k is a counter such that k=1, 2, 3…..S. $P^{t(k)}$ is the transmitting power of the kth I-node and $\beta_{k,R}$ is the distance between the kth I-node and the R-node. The value of P_{ini} depends mostly on the density of I-nodes. The density of I-nodes is determined by the number of nodes in the network and the distance between the R-node and the kth I-node.

$\theta^{(l)}$ represents the SINR on the lth link in the network and it is expressed as:

$$\theta^{(l)} = \frac{P_l^r}{P_{int}} = \frac{cP_l^t(\beta_{T,R})^{-2}}{\sum_{k=1}^{S} cP^{t(k)}(\beta_{k,R})^{-2} + P_o}. \tag{5}$$

A transmitted signal (packet) at a data rate Ψbps can only be correctly decoded by the R-node if $\theta^{(l)}$ is not less than an appropriate threshold $\theta^{(th)}$ throughout the duration of packet transmission [4] [22]. This condition is given as:

$$\theta^{(l)} \geq \theta^{(th)}. \tag{6}$$

3. Inter-node Interference Model

From the denominator of equation 5, inter-node interference is a major metric that contributes to the SINR. Keeping metrics such as the transmit power and received power at fixed values and the noise power level constant, note that all $\beta_{k,R}$ are independent and identically distributed random variables (R.V.). These R.V. are distributed within the area of interference.
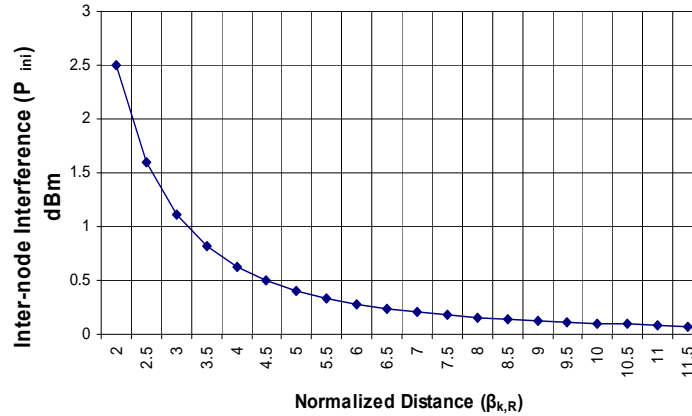

Fig.2. Inter-node Interference vs Distance.

Fig. 2 shows the effect of these R.V's on P_{ini}. With a constant number of I-nodes and varying ($\beta_{k,R}$), it can be observed that the higher the value of $\beta_{k,R}$, the smaller the P_{ini}. Fig. 3 confirms that as the number of I-nodes increases, intuitively, P_{ini} increases. From here, an interference constraint can be defined for the inter-working network. Fig. 4 illustrates the constraint, which is that nodes beyond a boundary (r+δr) contribute negligible interference due to the exponential decay of power caused by signal attenuation. With this constraint, an inter-node interference region bounded by equation 7 is defined.

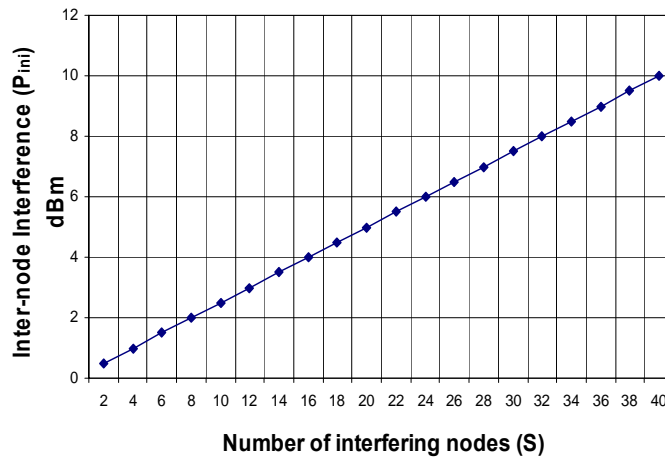$$r < \left| \beta_{k,R} \right| \le r + \delta r. \tag{7}$$

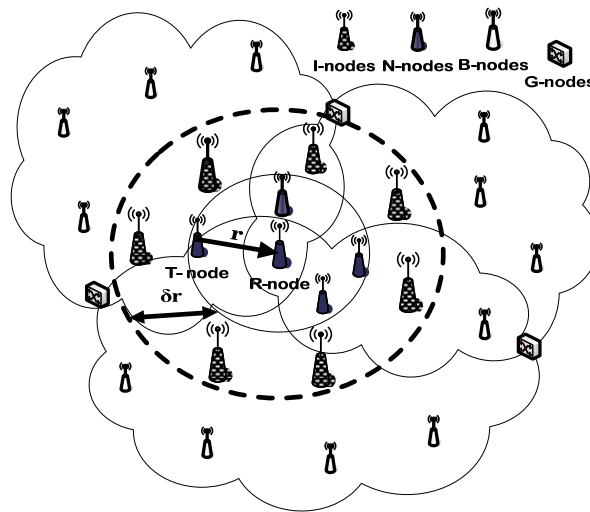Fig.3. Inter-node Interference vs number of interfering nodes.

Fig. 4. Representation of the transmission from a T-node to a R-node , with interfering nodes (I-nodes), non-interfering nodes (N-nodes), nodes beyond δr (B-nodes) and gateway nodes (G-nodes).

For all potential I-nodes, their separation distance to the reference R-node must fulfill equation 7. The bounded region (δr) is defined as the inter-node interference cluster. The interference cluster consists of nodes (the I-nodes) that can simultaneously transmitting within the frequency band of interest. According to [12] such nodes effectively contribute to the total inter-node interference and thus irrespective of the network topology or multiple-access technique, P_{ini} can be derived.

The Non-interfering nodes (N-nodes) are found within range r. Normally, whenever a link is established between a T-node and an R-node, the MAC technique will prohibit nearby nodes in the network from simultaneous transmission. The portion of the network occupied

by these nearby nodes is directly related to the size of r around the R- node, which is a fixed value in case of no node power control [23].

   The interference cluster in fig.4 is defined with respect to an R-node. A particular R-node in the inter-working network is surrounded by both I-nodes and N-nodes. Since, interference could be internally or externally generated, there are different scenarios in which an R-node can find itself, based on a defined interference constraint as dictated by the MAC protocol. Some of the scenarios that can occur include:

1) The node could be surrounded by I-nodes and N-nodes from the same multi-hop wireless network.

2) The node could be surrounded by I-nodes and N-nodes from different multi-hop wireless networks.

As illustrated in fig. 4, the R-node is surrounded by nodes of other networks, which are its I-nodes, N-nodes and of course other nodes beyond δr (the B-nodes). Theorem 1, as stated below can be used to characterize these nodes within the inter-working multi-hop wireless network.

Theorem 1: If each random point of a Poisson process in $\mathbb{R}^d$ with density λ are of N different types and each point, independent of the others, is of type N with probability P_i for i = 1, 2, ··
·N, such that, $\sum_{i=1}^{N} P_i = 1$, then the N point types are mutually independent Poisson processes with densities $λ_i = P_iλ$ such that the $\sum_{i=1}^{N} λ_i = λ$ [24]

   Using the splitting property of the Poisson process in theorem 1, let all nodes in the inter-working network, which is characterized by a Poisson point process with spatial density $μ_{Net}$ be sorted independently into 3 types, I-nodes, N-nodes, and B-nodes. If the probability of a node being an I-node, N-node or a B-node is P_I, P_N, or P_B respectively such that $P_I+P_N+P_B=1$, then these 3 types of nodes are mutually independent Poisson processes with spatial densities:

$$\mu_I = P_I \mu_{Net}, \quad \mu_N = P_N \mu_{Net}, \quad \mu_B = P_B \mu_{Net}, \text{where } \mu_{Net} = \mu_I + \mu_N + \mu_B$$

$μ_I$ represents the spatial density of I-nodes, $μ_N$ is the spatial density of the N-nodes and $μ_B$ is the spatial density of nodes beyond δr. From here, the effective density of I-nodes can be derived. If $β_{x,R}$ represents the link distance between the R-node and an arbitrary node x in the network, then:

$$\Pr(x \in N - nodes) = \Pr(\left| \beta_{x,R} \right| \leq r).$$

$$\Pr(x \in I - nodes) = \Pr(r < \left| \beta_{x,R} \right| \leq r + \delta r).$$

$$\Pr(x \in B - nodes) = \Pr(\left| \beta_{x,R} \right| > r + \delta r).$$

   It was noted earlier that P_{ini} has a stochastic nature due to the random geographic dispersion of nodes, therefore, P_{ini} can be said to be a random variable. Since several nodes can simultaneously transmit in the δr region and they altogether influence the value of P_{ini}, then $θ^{(l)}$ (the SINR on a link) can be estimated using the expected value of P_{ini}, which is given as:

$$E[P_{ini}] = E\left[\sum_{k=1}^{S} cP^{t(k)} (\beta_{k,R})^{-\alpha} \right]. \tag{8}$$

   S is the total number of I-nodes and k is a counter such that k=1, 2, 3….S. For analytical plausibility and to avoid complexity, let all I-nodes transmission power ($P^{t(k)}$) be equal. Note that the T-node's transmission power and modulation technique are not necessarily the same as that of the I-nodes. Thus, the network in fig. 4 can be represented as a heterogeneous network, in which different multi-hop wireless networks are inter-working.

$$E[P_{ini}] = cP^{t(k)}E\left[\sum_{k=1}^{S}(\beta_{k,R})^{-2}\right]. \tag{9}$$

In order to solve equation 8, the distribution function of the distance between the R-node and the I-nodes ($\beta_{k,R}$), given by $f_{(\beta_{k,R})}(r)$, is of particular interest.

3.1 Distribution Function of $\beta_{k,R}$

From fig. 4, the total inter-node interference region is the area outside the range r. This region consists of nodes that can interfere with the R-node's reception. However, nodes beyond the bounded region (r+δr) cause negligible interference. The region within δr is the interference cluster, which consists of the effective number of I-nodes.

In order to find the probability that the distance between the R-node and all I-nodes fulfill the condition in (7), two events are defined.

$$\xi_1 = \{\text{no I-node exist within distance r}\}.$$
$$\xi_2 = \{\text{at least one I-node exist within δr}\}.$$

Similar to the nearest neighbor analysis in [25], the probability that concurrently transmitting nodes fulfill the condition in (7) is given by:

$$\left(\Pr[(\xi_1)\cap(\xi_2)]\right) = \left(\Pr(\xi_1)\right)\left(\Pr(\xi_2)\right). \tag{10}$$

$$\Pr(\xi_1) = e^{-\mu_I \pi r^2}. \tag{11}$$

To evaluate $\Pr(\xi_2)$, the interference cluster is laid as a strip with length 2πr and width δr as shown in fig. 5.
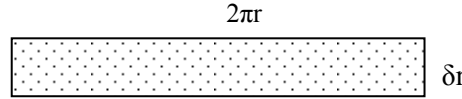
2πr

δr

Fig.5. An approximation of the ring created by the interference cluster.

As δr approaches zero, the area of the annulus can be approximated by 2πrδr. It follows from Poisson distribution that the probability of at least one node in the annulus is:

$$\Pr(\xi_2) = 1 - e^{-\mu_I 2\pi r \delta r}. \tag{12}$$

From the first and second term of the Taylor's series [25], $1 - e^{-\mu_I 2\pi r \delta r} = \mu_I 2\pi r \delta r$. Therefore, the probability of having I-nodes within the cluster (annulus) is:

$$\left(\Pr(\xi_1)\right)\left(\Pr(\xi_2)\right) = \left(2\mu_I \pi r \delta r\right)\left(e^{-\mu_I \pi r^2}\right). \tag{13}$$

This probability can be expressed as:

$$\Pr(r < |\beta_{k,R}| \le r + \delta r) = \left(2\mu_I \pi r \delta r\right)\left(e^{-\mu_I \pi r^2}\right) = f_{\beta_{k,R}}(r)\delta r$$

$$\therefore f_{\beta_{k,R}}(r) = 2\mu_I \pi r e^{-\mu_I \pi r^2}. \tag{14}$$

The distribution of the distance between the R-node and I-nodes is $f_{\beta_{k,R}}(r)$ in (14). Now, it is clear that $\beta_{k,R}$ has a Poisson distribution. To evaluate the expected value of $\beta_{k,R}$ in equation 9, the summation of the negative-second moment of a Poisson random variable ($\beta_{k,R}$), must be solved.

$$E\left[\sum_{k=1}^{S}(\beta_{k,R})^{-2}\right] = \sum_{k=1}^{S}E[(\beta_{k,R})^{-2}]$$

$$= \sum_{k=1}^{S} \varpi \qquad \text{(15)}$$

Very few approximations for the solution of the negative moments of Poisson R.Vs exist in the literature. Two solutions that have been identified by the authors of this paper are the Tiku's estimators [26] and the approximations developed by C. Matthew Jones et al in [27]. However, in this paper, the Tiku's approximation has been adopted. It follows from [26] that:

$$\varpi \approx \frac{1}{(\mu_I - 1)(\mu_I - 2)........ ...(\mu_I - \tau)}. \qquad \text{(16)}$$

for the τth negative moment of $\beta_{k,R}$ (τ represents the positive value of the power of $\beta_{k,R}$), $\mu_I = P_I \mu_{Net}$ and P_I is the probability of interference.

3.2. Probability of Interference

In practice, not all nodes within δr will transmit at the same time,, therefore P_I can be defined by two events:

- ❖ ξ_3 -at least a node exist within δr and
- ❖ ξ_4 -the node is transmitting.

For inter-working multi-hop wireless network, with density μ_{Net}, $\Pr(\xi_3)$ is the probability that the distance between an arbitrary node and the R-node is > r and $\leq$ r+ δr, (r and δr are defined with reference to the R-node of interest). This probability can also be expressed as the probability that > 0 nodes exist within δr of the R-node and it is given by: $1 - e^{-\mu_{Net} A_I}$ where A_I is the area of δr for the R-node of interest.

$$\Pr(\xi_4) = \begin{cases} 1, & \text{if } P^{t(k)} > 0 \\ 0, & \text{if } P^{t(k)} = 0 \end{cases} \qquad \forall P^{t(k)} \geq 0.$$

Thus: $P_I = \Pr(\xi_3)\Pr(\xi_4) = 1 - e^{-\mu_{Net} A_I} \quad \forall P^{t(k)} > 0.$

3.3. Evaluation of the Interference Power (P_{inj})

Since μ_I can now be evaluated, from (8);

$$E[P_{ini}] \approx cP^{t(k)} \times S \times \varpi. \qquad \text{(17)}$$

$S \approx P_I \times N_\Omega$, N_Ω is the total number of nodes in the network. Equation 17 expresses the expected value of the effective P_{ini} experienced on a link. P_{ini} is dependent on the spatial density of the interfering nodes (μ_I) and the interfering nodes' transmitting power. In order to validate equation 17, P_{ini} has been used to estimate the value of $\theta^{(l)}$ and numerical results have been obtained as shown in fig. 6 and 7. Thus, $\theta^{(l)}$ can be approximated as:

$$\theta^{(l)} \approx \frac{cP_l^{t}(\beta_l)^{-2}}{cP^{t(k)} \times S \times \varpi + P_o}. \qquad \text{(18)}$$

The network scenario considered is a case of inter-working IEEE 802.11a/b/g mesh networks with 10 nodes, 15 nodes and 25 nodes respectively in a 1000 unit square area. G_t and G_r, the transmitter and receiver gains respectively are assumed to be equal to 1 and L_f=1. Nodes in the network transmit at 10mW. The area of the interference cluster is 200unit square. The evaluation of the interference power as shown in fig. 6-8 has been done with respect to an R-node on a link of interest (in fig. 4) in the inter-working multi-hop wireless network. The number of nodes in the inter-working network was increased as applicable.
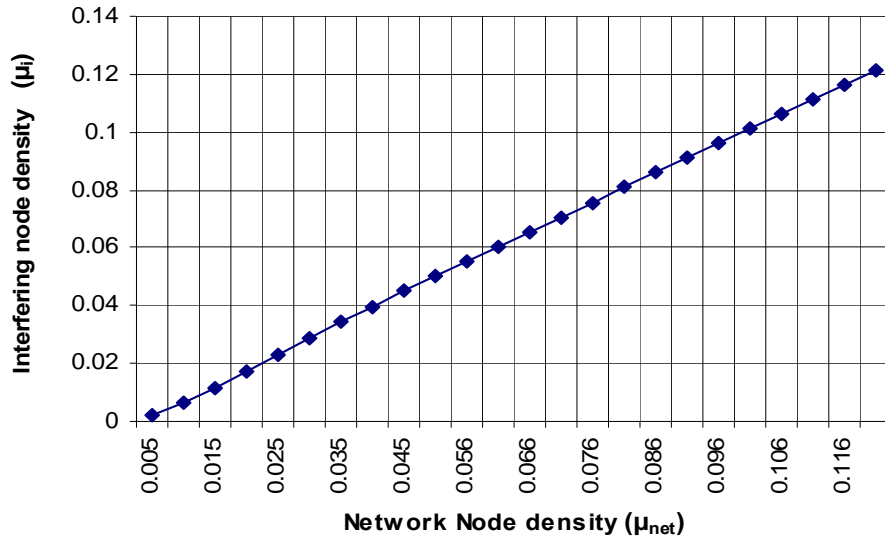
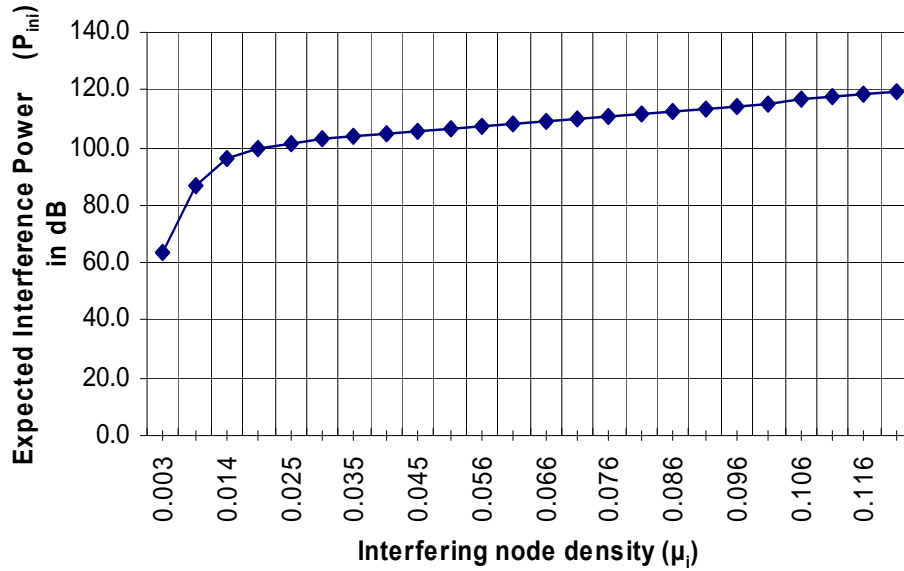Fig. 6. Expected SINR value vs interference power.

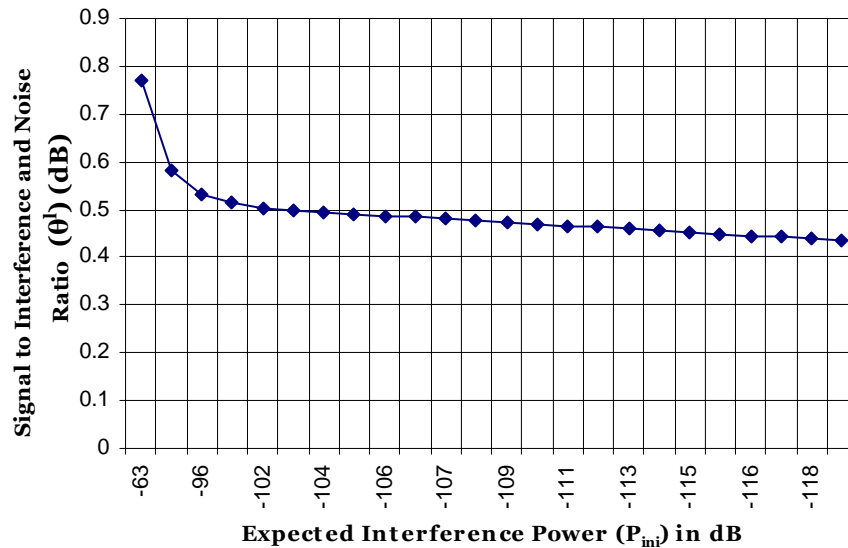Fig.7 Expected interference power vs interfering node density.

Fig. 8. Signal to Interference and Noise Ratio vs expected interference power

Fig. 6 shows plot of the network node density and the interfering node density. As more nodes are deployed in the inter-working network (i.e. the network becomes denser), the likelihood of having more nodes interfering with an R-node of interest increases. The increase in the density nodes increases the probability of interference and thus the density of the I-nodes. In fig. 7, a plot of the calculated values of equation 17 is given. By keeping the I-nodes' transmitting power at a fixed value, the expected value of the interference power (P_{ini}) rises as the density of the I-nodes is increased. To validate the model presented in this paper, the effect of the expected interference power on the link's Signal to Interference and Noise ratio ($\theta^{(l)}$) curve is as shown in fig. 8. It can be observed that the signal to interference and noise ratio on the link of interest decreases as interference power increases. In an inter-working multi-hop wireless network, nodes that can simultaneously transmit with a T-node on a link of interest effectively contribute to the total inter-node interference experienced by the R-node on the same link. Thus irrespective of the network topology or multiple-access technique, an approximation of the expected value of the inter-node interference power (P_{ini}) can be derived with the model presented in this paper.

4. Conclusion

The quality of a wireless link is a measure of how reliable it is. One of the physical layer metrics that can be used to measure a link's quality is the level of inter-node interference on the link. This paper presented a model for inter-node interference on a link in an inter-working multi-hop wireless network. The inter-node interference model incorporates the probability of interference in inter-working networks and uses the negative second moment of the distance between a receiver-node and nodes simultaneously transmitting with the transmitter-node to evaluate the expected value of the inter-node interference power on a link. Tiku's approximation for the negative moment of a random variable was adopted. The results

obtained confirm that the level of inter-node interference has a substantial effect on the expected quality of the signal received at the receiver node. Thus irrespective of the multiple-access technique, the expected value of the inter-node interference power (Pini) can be derived with the model presented in this paper, The future work of this research includes applying this model in a simulation environment.

10. References

1. Siddiqui F. and Zeadally S.: Mobility management across hybrid wireless networks: Trends and challenges. In: Computer Communications, Volume 29, Issue 9, Pgs 1363--1385, 31 (2006).
2. Li W. and Pan Y.: Resource Allocation in Next Generation Wireless Networks. In: Wireless Networks and Mobile Computing series, Volume 5, ISBN: 1-59454-583-9, 2005.
3. Salami O., Bagula A., and Chan H. A.: Analysis of Route Availability in Inter-working Multi-hop Wireless Networks. In: Proc. of the 4th International Conference on Broadband Communications, Information Technology and Biomedical Applications (BroadCom 2009), Wroclaw, Poland, 15th -19th (2009).
4. Blough D., Das S., Santi P.: Modeling and Mitigating Interference in Multi-Hop Wireless Networks. In: Tutorial presented at Mobicom 2008, San Francisco, Sept. 14 2008.
5. Gupta P. and Kumar P.R.: The Capacity of Wireless Networks. In: IEEE Transactions on Information Theory, Vol. 46, No.2, pp. 388--404, 2000.
6. Salbaroli E. and Zanella A.: Interference analysis in a Poisson field of nodes of finite area. In: IEEE Trans. on Vehicular Tech., vol. 58, no. 4, pp.1776–1783, May 2009.
7. Busson A., Chelius G. and Gorce J.: Interference Modeling in CSMA Multi-Hop Wireless Networks. INRIA research report, No 6624, (2009). ISSN: 0249-6399.
8. Babaei A.: Statistical Interference Modeling and Coexistence Strategies in Cognitive Wireless Networks. In : PhD thesis, George Mason University, (Spring semester 2009).
9. Pinto P.: Communication in a Poisson field of interferers. Master of Science Thesis, Massachusetts Institute of Technology, (February 2007).
10.Zanella A.: Connectivity properties and interference characterization in a Poisson field of nodes. WILAB (IEIIT/CNR) presentation, (Sept. 19, 2008).
11. Mordachev V. and Loyka S.: On Node Density – Outage Probability Tradeoff in Wireless Networks. In: IEEE Journal on Selected Areas in Communications, accepted for   publication. arXiv:0905.4926v1, (2009).
12. Win M., Pinto P. and Shepp L.: A Mathematical Theory of Network Interference and Its Applications. In: IEEE transaction, Vol 97, No 2, (2009). ISSN: 0018-9219
13.Ferrari G. and Tonguz O.K.: Performance of Ad-hoc Wireless Networks with Aloha and PR-CSMA MAC protocols. In: Proc of IEEE Globecom, San Francisco, pp 2824—2829 (2003).
14. Ganti R. and Haenggi M.: Spatial and Temporal Correlation of the Interference in ALOHA Ad Hoc Networks. In: IEEE Communications Letters, Vol. 13, No. 9, pg 631--633,   (2009).
15.Heckmat R. and Van Mieghem P.: Interference in Wireless Multi-hop Adhoc Networks and its effect on Network Capacity. In: Wireless Networks, Vol. 10, pp 389—399. Kluwer Academic Publishers (2004).
16. Iyer A., Rosenberg C. and Karnik A.: What is the right model for Wireless Channel interference? In: proc of the Third International Conference on Quality of Service in Heterogeneous wired/Wireless Networks. ACM International Conference Proceeding Series; Vol. 191, (2006).
17. Moscibroda T., Wattenhofer R., and Zollinger A.: Topology Control Meets SINR: The Scheduling Complexity of Arbitrary Topologies. In: proc of MobiHoc'06, Florence, Italy, (2006).
18. Diggle P. J.: Statistical Analysis of Spatial Point Patterns. A Hodder Arnold Publication, 2nd edition, 168pp, (2001). ISBN 0-340-74070-1.
19. Sousa E.: Performance of a spread spectrum packet radio network link is a Poisson field of interferes. In: IEEE Trans. Information Theory, vol 38, no 6, pp. 1743--1754, (1992).
20. Ilow J., Hatzinakos D, and Venetsanopoulos A.: Performance of FH SS radio networks with interference and modeled as a mixture of Gaussian and alpha-stable noise. In: IEEE Trans. Communication, vol. 46, no.4 pp.509--520, (1998).
21. Rappaport T. S.: Wireless Communications-Principles and Practice. Prentice Hall, (2002).
22. Avin C., Emek Y., Kantor E., Lotker Z., Peleg D., and Roditty L.: SINR Diagrams: Towards Algorithmically Usable SINR Models of Wireless Networks. In: proc. of the 28th ACM symposium on Principles of distributed computing, Pages 200--209 (2009). ISBN: 978-1-60558-396-9.
23. Hekmat R., and An X. : Relation between Interference and Neighbor Attachment Policies in Ad-hoc and Sensor Networks. In International Journal of Hybrid Information Technology, Vol. 1, No. 2, (2008).
24. Hohn N., Veitch D., Ye T.: Splitting and merging of packet traffic: measurement and modeling. In: Performance Evaluation, Volume 62, Issue 1-4, pp164--177, (2005). ISSN: 0166-5316.

25. Cherni S.: Nearest neighbor method, http://www.mcs.sdsmt.edu/rwjohnso/html/sofiya.pdf.
26. Tiku M. L.: A note on the negative moments of a truncated Poisson variate. In Journal of American Statistical Association vol.59, No 308, pp 1220--1224, (1964).
27. Matthew Jones C.: Approximating negative and harmonic mean moments for the Poisson distribution. In: Mathematical Communications vol. 8, pp157--172, (2003).

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