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# Framework for link reliability in inter-working multi-hop wireless networks

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#### ABSTRACT

With the increase in deployment of multi-hop wireless networks and the desire for seamless internet access through ubiquitous connectivity, the inter-working of heterogeneous multi-hop wireless networks will become prominent in the near future. To complement the quest for ubiquitous service access, multi-mode mobile terminals are now in existence. Inter-working heterogeneous multi-hop wireless networks can provide seamless connectivity for such multi-mode nodes but introduces a number of challenges due to its dynamic network topology. One of the challenges in ensuring seamless access to service through these terminals in an inter-working environment is the selection of reliable wireless point-to-point links by the multi-hop nodes. A wireless link is said to be reliable if its radio attribute satisfies the minimum requirements for successful communication. Successful communication is specified by metrics such as signal to interference and noise ratio (SINR), probability of bit error etc. However, the multi-hop wireless networks being inter-worked may operate with different link layer protocols. Therefore, how can the reliability of a wireless link be estimated irrespective of the link level technologies implemented in the networks being inter-worked so that optimal paths can be used for multi-hopping between nodes? In this paper, a generic framework which can estimate the reliability of a link in inter-working multi-hop wireless network is presented. The framework uses the relationship between inter-node interference, SINR and the probability of bit error to determine the reliability of a wireless link between two nodes. There is a threshold for the probability of bit error on a link for the link to be termed reliable. Using parameters such as the SINR threshold, nodes' transmission power, link distance and interfering node density, the framework can evaluate the reliability of a link in an interworking multi-hop network.

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#### 1. Introduction

There is a proliferation of wireless multi-hop networks due to their easy deployment. These networks are also being inter-worked in order to increase their coverage range and for the provisioning of seamless and ubiquitous wireless internet access [1]. Inter-working is termed as the seamless integration of two or more networks. A lot of benefits can be gained from the inter-working of multi-hop wireless networks. Inter-working multi-hop wireless networks allow users to access services such as multimedia, web browsing, video and news on demand, mobile office applications and stock market information anywhere, anytime in an uninterrupted and seamless manner. Inter-working multi-hop wireless networks can also allow efficient spatial re-use and it eases the provisioning of Internet access in areas with no initial wire-line network coverage e.g. rural communities [2]. In addition, if connectivity can be maintained between several inter-working multi-hop wireless

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networks, then mobile network users can make cheaper packetized voice calls on the move and seamlessly with the use of applicable mobile terminals.

Different multi-hop wireless access networks can be inter-worked. For example IEEE 802.11 mesh networks can be interworked with WiMAX mesh networks. However, to ensure seamless and uninterrupted communication, it is essential that the network traffic generated by the network user be transmitted through reliable links as the network user transits between the inter-worked networks.

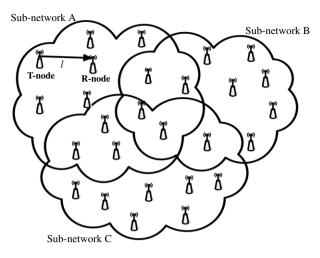
Consider an inter-working multi-hop wireless network which consists of at least three heterogeneous multi-hop wireless networks that are integrated by gateway nodes. The gateway nodes perform inter-network co-ordination and provide connectivity to the Internet. Nodes have the opportunity to remain connected and have access to their on-going service even if they are out of their parent network's gateway node coverage area. A node that does not have a direct link to its intended destination or gateway node will have its transmission relayed through other multi-hop nodes in the inter-working network. However, because of the instability of wireless links, it is difficult to guarantee that all the links on a multi-hop route will enable connectivity throughout the transmission duration. Even though there is an advantage of route diversity in the network, it is pertinent to route traffic through reliable links in order to maintain end-to-end and optimal connectivity for any node. Therefore, how can we estimate the reliability of a link in inter-working multi-hop wireless networks considering the heterogeneity of MAC/link level technologies that exists in such networks?

A wireless link is said to be reliable if its radio attributes satisfy the requirement for successful communication. A major aspect of reliability is the description of the quality under which a transmission is received by the intended receiver [2]. At the receiver node, the quality of the transmission is described by parameters which include interfering node density, inter-node interference level, signal to interference and noise ratio (SINR), bit error probability. Threshold values of these parameters are used to define successful communication in any network. The probability that successful reception can be guaranteed based on the threshold values for the parameters is termed reliability. In inter-working multi-hop wireless networks, it is necessary to be able to estimate the reliability of a link so that wasteful transmission on unreliable links can be avoided. Reliability is an important metric that can be used to make routing decisions in order to ensure optimal quality of service (QoS) provisioning and resource dimensioning. In this paper, link reliability is determined by using the interfering node density, inter-node interference level and SINR to evaluate the probability of bit error on a link. Then using the probability of bit error, a link's reliability can be evaluated. A link is reliable if the probability of bit error is below the threshold required for quality transmission in the network.

Previous research works on reliability such as [3-5] focused mainly on single multi-hop networks such as ad-hoc or mesh networks. The analysis, models and techniques developed in the mentioned research studies will not be applicable in interworking multi-hop wireless networks for the following reasons. First, their scope is limited to single multi-hop networks operating with a single networking techniques where all nodes use the same mode of operation, but an interworking multi-hop wireless network may consist of multiple networks operating with different networking techniques. These networking techniques include the physical layer and MAC layer. Their results cannot be adapted to an interworking multi-hop wireless network due to its heterogeneity. Therefore, it is desirable to develop a generic reliability framework which is oblivious to underlying network technologies. Second, because these studies are based on single multihop networks, the effects of inter-network interactions were not considered. The effect of inter-network interactions such as interference has to be considered in achieving reliable end to end transmission in inter-working multi-hop wireless networks. For example, even if an end-to-end route exists for some time between two nodes, channel impairment due to inter-node interference may affect a link's reliability. Thus, it is desirable to be able to estimate the reliability on links in order to ensure an optimal route selection for seamless end-to-end connectivity for network users. Another plausible limitation of these research works is that a deterministic model (e.g. graph model) has been used to model a wireless network. They have represented the network as graph consisting of vertices and edges. However, since a wireless environment is stochastic, it cannot be adequately represented as a graph of vertices and edges. In order to be able to represent a wireless network properly, the randomness of node's location and the wireless medium must be modelled. Lastly, in the mentioned research works, the mobile terminals in the networks are homogenous where as in an inter-working multi-hop wireless network, heterogeneous mobile terminals may co-exist.

Very little work if at all exists on the subject of link reliability for inter-working multi-hop wireless networks. With the increasing interest in ubiquitous internet connectivity, research works on inter-working multi-hop wireless networks are up-coming. The frameworks presented in the previous research works will not be applicable to inter-working multi-hop wireless networks mainly because of the heterogeneity of technologies that may be present in such networks.

The major contribution of this paper is to answer the question: Given a network in which several multi-hop wireless networks are being integrated, how can the reliability of a link be estimated irrespective of the different link level technologies present? To answer the question, this paper presents a generic framework that can be used for the estimation of the reliability of a link in inter-working multi-hop wireless networks. A network model in which nodes are distributed according to a Poisson point process is considered. The probability of successful transmission is a function of the interfering nodes' density. By taking into consideration the probability of interference in the network, the density of interfering nodes can be estimated using Poisson splitting. The probability of interference for the receiver node (*R*-node) on a link of interest is taken as a function of (1) the distance between this *R*-node and other nodes concurrently transmitting with the transmitting node (*T*-node) on a link of interest and (2) the transmitting power of these other nodes. Several scenarios can be evaluated by placing the link of interest at different locations within the inter-working multi-hop wireless network.



**Fig. 1.** Network Ω showing a link (*l*) between a *T*-node and an *R*-node in an inter-working multi-hop wireless network comprising of three sub-networks (A, B and C).

The results obtained in this paper are relevant for the understanding of the relationship between the parameters that determine the reliability of a link in inter-working multi-hop wireless networks. In the framework, an interference region is defined for the *R*-node on a particular link of interest in the inter-working network. Then, the likelihood that the link will provide a successful transmission based on the number of nodes in the interference region is determined. Basically, any node in the interference region will affect the probability of bit error on a link of interest if it is transmitting at the same time as the *T*-node on the link of interest. With the target SINR for the link, the maximum number of interfering nodes that will give the upper bound for the acceptable probability of bit error on a link can be determined. If the probability of bit error evaluated for a link in the network is higher than the acceptable value, then the link is termed unreliable. In this way, unreliable links can be identified and optimized traffic routing decisions can be made in inter-working multi-hop wireless networks irrespective of the link/MAC layer technologies of the inter-working networks. The results reflected how the bit error probability at the receiver node is affected. The content of this paper is organized as follows: Section 2 describes the network model used in the analysis of the framework. Section 3 discusses the theory behind the SINR model implemented in this paper while Section 4 presents the probability of bit error model, the reliability framework and discusses the results obtained. A conclusion of the paper is provided in Section 5.

### 2. Network models

The network model in Fig. 1 is a collection of random nodes whose realization is called a spatial point pattern. A spatial point pattern is a set of location, irregularly distributed within a designated region and presumed to have been generated by some form of stochastic mechanism [6]. The nodes are contained in a Euclidean space of 2-dimensions ( $R^2$ ), and their positions in the network are independent of each other [1]. Since the positions of nodes are random, the spatial distribution of nodes is taken to be according to a homogeneous Poisson point process in two-dimensional plane. It follows that the probability that *p* points are inside region P, (Pr (*p* in P)) depends on the area of the region ( $A_p$ ) and not on the shape or location of the plane. Pr (*p* in P) is given by (1), where  $\mu$  is the spatial density of points.

$$\Pr(p \text{ in } P) = \frac{(\mu A_P)^p}{p!} e^{-\mu A p}, \quad p > 0.$$
(1)

The Poisson process has been chosen because the density function of the position of nodes is taken to be conditionally uniform over R [7]. Besides, the spatial Poisson distribution means that nodes are randomly and independently located. This is reasonable particularly in a network with indiscriminate node placement [8].

The inter-working network in Fig. 1 is represented as network  $\Omega$ , in which three multi-hop wireless networks are being integrated seamlessly. These three sub-networks are represented as A, B, and C. The total number of nodes in  $\Omega$  is denoted as  $N_{\Omega}$ , while the number of nodes in sub-networks A, B, C are  $N_a$ ,  $N_b$  and  $N_c$  respectively, where  $N_a + N_b + N_c = N_{\Omega}$ . The spatial density of each sub-network is given by  $\mu_A$ ,  $\mu_B$ ,  $\mu_C$  ( $\mu = N/s$ , N is the number of nodes in a sub-network, s is the sub-network's coverage area and  $\mu$  is given in nodes/unit square). The entire inter-working network is considered as a merging Poisson process with spatial density:  $\mu_A + \mu_B + \mu_C = \mu_{Net}$ .

In the network, node to node communication may be multi-hop and nodes transmit at a data rate of  $\Psi$  bps. In this paper, source-nodes are referred to as transmitter-nodes (*T*-nodes) while destination-nodes are referred to as receiver-nodes (*R*-nodes).

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#### 2.1. Link model

The links between nodes are represented by  $\{l : l = 1, 2, ..., n\} \subseteq L$ , where *L* is the set of all links in the entire network. The length of a communication link is represented by  $\beta_{T,R}$ , subscript *T* denotes the transmitter-node while subscript *R* denotes the receiver-node on the link.

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To account for path-loss, the channel attenuation for link *l* is denoted by  $A_l$ . where  $A_l = (\beta_{T,R})^{-\alpha}$ . It is typically given that the received power from a *T*-node at distance  $\beta_{T,R}$  from the *R*-node decays exponentially as  $(\beta_{T,R})^{-\alpha}$ . The path loss exponent is  $\alpha$ , which is a constant that can take on values<sup>1</sup> between 2 and 6, however, in this paper,  $\alpha$  is taken as = 2. Other link parameters are:

- *P*<sup>*t*</sup>: represents the transmitting power of the *T*-node on link *l*.
- $P_{a}$ : is the thermal noise power level at the *R*-node on link *l*. This is given by:  $P_{a} = FkT_{a}B$  where k is the Boltzman constant
- $(1.38 \times 10^{-23} \text{ J/}^{\circ}\text{K/Hz})$ .  $T_0$  is the ambient temperature, B is the transmission bandwidth and F is the noise figure [9].
- $P_l^r$  is the power received by the *R*-node on link *l*.

The free space propagation model is used to predict the received signal strength. For a packet transmitted by the *T*-node on link *I* and received by the *R*-node, the actual received power at the *R*-node can be expressed by the Friis equation given as:

$$P_l^r = cP_l^t A_l = cP_l^t \left(\beta_{T,R}\right)^{-\alpha} \quad \left[c = \frac{G_t G_r \lambda_g^2}{(4\pi)^2 L_f}\right]$$
(2)

where  $G_t$  and  $G_r$  are the transmitter and receiver gain respectively,  $\lambda_g$  is the wavelength,  $\lambda_g = g/f_g$ , g is the speed of light and  $f_g$  is the carrier frequency and  $L_f$  is the system's loss factor (with  $L_f \ge 1$ ).

#### 3. Link signal to interference and noise ratio model

A link's signal to interference and noise ratio is evaluated using the SINR model [10] in Eq. (3). A transmitted signal (packet) at a data rate  $\Psi$  bps can only be correctly decoded by the *R*-node on link *l* if  $\theta^{(l)}$  is not less than an appropriate threshold  $\theta^{(th)}$  throughout the duration of packet transmission as stated in [11,12].

$$\theta^{(l)} = \frac{P_l^r}{P_{\text{int}}} = \frac{P_l^r}{P_{\text{int}} + P_o} = \frac{cP_l^t(\beta_{T,R})^{-2}}{\sum\limits_{k=1}^{S} cP^{t(k)}(\beta_{k,R})^{-2} + P_o}.$$
(3)

In Eq. (3),  $P_{int}$  is the total interference power experienced by the *R*-node at the end of link *l*. It is the sum of the thermal noise power ( $P_o$ ) and the inter-node interference power ( $P_{ini}$ ). For link *l*,  $P_{ini}$  in Eq. (3) is the cumulative of the interfering power that the *R*-node experiences from nodes concurrently transmitting with *T*-node. All *k*-nodes (for  $k = 1, 2, 3...\infty$ ) are potential interfering nodes while S is the total number of simultaneously transmitting nodes that contributes to the effective interference power.  $P^{t(k)}$  is the transmitting power of a *k*-node and  $(\beta_{k,R})^{-\alpha}$  is the distance between a *k*-node and the *R*-node. Among other parameters, the probability of interference, the interfering node density, the distance between any potential interfering node (*k*-node) and the *R*-node on the link of interest, and the *k*-nodes' transmission power are important in the evaluation of  $P_{ini}$ .

For a particular link, an interference region ( $\delta r$ ) is defined for the *R*-node on the link. As shown in Fig. 2, a node will only interfere with the reception of the *R*-node on link *l* if the distance of the node to the *R*-node fulfills the constraint in Eq. (4) as follows.

$$r < |\beta_{k,R}| \le r + \delta r. \tag{4}$$

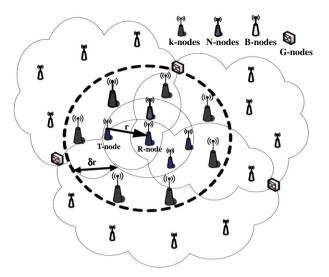
In Eq. (4), r represents the *T*-node's transmission range. According to [13] such nodes effectively contribute to the value of  $P_{ini}$  irrespective of the network topology or multiple-access technique. Normally, whenever a link is established between a *T*-node and an *R*-node, the MAC technique will prohibit nearby nodes in the network from simultaneous transmission. The portion of the network occupied by these nearby nodes is directly related to the size of *r* around the *R*-node [14].

Using the splitting property of the Poisson process in Theorem 1 [15], let all nodes in the inter-working network, with spatial density  $\mu_{Net}$  be sorted independently into 3 types, *k*-nodes, *N*-nodes, and *B*-nodes. If the probability of a node being an *k*-node, *N*-node or a *B*-node is  $P_I$ ,  $P_N$ , or  $P_B$  respectively such that  $P_I + P_N + P_B = 1$ , then these 3 types of nodes are mutually independent Poisson processes with spatial densities: :  $\mu_I = P_I \mu_{Net}$ ,  $\mu_N = P_N \mu_{Net}$ ,  $\mu_B = P_B \mu_{Net}$ , where  $\mu_{Net} = \mu_I + \mu_N + \mu_B$ .

<sup>&</sup>lt;sup>1</sup> The model in this paper can make use of other path-loss exponent values.

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**Fig. 2.** Representation of the transmission from a *T*-node to a *R*-node on link *l*, with interfering nodes (*k*-nodes), non-interfering nodes (*N*-nodes), nodes beyond δ*r* (*B*-nodes) and gateway nodes (*G*-nodes).

$2\pi r$
δι

Fig. 3. An approximation of the ring created by the interference cluster.

**Theorem 1** ([15]). If each random point of a Poisson process in  $\mathbb{R}^d$  with density  $\lambda$  are of N different types and each point, independent of the others, is of type N with probability  $P_i$  for i = 1, 2, ..., N, such that,  $\sum_{i=1}^{N} P_i = 1$ , then the N point types are mutually independent Poisson processes with densities  $\lambda_i = P_i \lambda$  such that the  $\sum_{i=1}^{N} \lambda_i = \lambda$ .  $\mu_i$  represents the spatial density of k-nodes,  $\mu_N$  is the spatial density of the N-nodes and  $\mu_B$  is the spatial density of nodes

 $\mu_l$  represents the spatial density of k-nodes,  $\mu_N$  is the spatial density of the N-nodes and  $\mu_B$  is the spatial density of nodes beyond  $\delta r$ . From here, the effective density of k-nodes can be derived. If  $\beta_{x,R}$  represents the link distance between the R-node and an arbitrary node x in the network, then:

 $Pr(x \in N\text{-nodes}) = Pr(|\beta_{x,R}| \le r).$   $Pr(x \in k\text{-nodes}) = Pr(r < |\beta_{x,R}| \le r + \delta r).$  $Pr(x \in B\text{-nodes}) = Pr(|\beta_{x,R}| > r + \delta r).$ 

If node x is a *k*-node, the  $\beta_{x,R}$  becomes  $\beta_{k,R}$ .

#### 3.1. Distribution function of $\beta_{k,R}$

As shown in Fig. 2, nodes that can potentially interfere with the *R*-node's reception lie in the region outside the range *r*. However, nodes beyond the region  $(r + \delta r)$  cause negligible interference. The region within  $\delta r$  consists of the effective *k*-nodes. In order to find the probability that the distance between the *R*-node and all *k*-nodes fulfill the condition in (4), two events are defined:

 $\xi_1 = \{no \ k\text{-node exist within distance } r\}$ 

 $\xi_2 = \{ \text{at least one } k \text{-node exist within } \delta r \}.$ 

Similar to the nearest neighbor analysis in [16], the probability that concurrently transmitting nodes fulfill the condition in (4) is given by:

$$(\Pr[(\xi_1) \cap (\xi_2)]) = (\Pr(\xi_1))(\Pr(\xi_2))$$

$$\Pr(\xi_1) = e^{-\mu_1 \pi r^2}.$$
(6)

To evaluate  $Pr(\xi_2)$ , the interference cluster is laid as a strip with length  $2\pi r$  and width  $\delta r$  as shown in Fig. 3.

As  $\delta r$  approaches zero, the area of the annulus can be approximated by  $2\pi r \delta r$ . It follows from Poisson distribution that the probability of at least one node in the annulus is:

$$\Pr(\xi_2) = 1 - e^{-\mu_1 2\pi r \delta r}.$$
(7)

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From the first and second term of the Taylor's series [16],  $1 - e^{-\mu_l 2\pi r \delta r} = \mu_l 2\pi r \delta r$ . Therefore, the probability of having at least one *k*-node within  $\delta r$  is:

$$(\Pr(\xi_1))(\Pr(\xi_2)) = (2\mu_1 \pi r \delta r)(e^{-\mu_1 \pi r^2}).$$
(8)

This probability can be expressed as:

$$\Pr(r < |\beta_{k,R}| \le r + \delta r) = (2\mu_I \pi r \delta r)(e^{-\mu_I \pi r^2}) = f_{\beta_{K,R}}(r) \delta r.$$

Hence,

$$f_{\beta_{k,R}}(r) = 2\mu_{l}\pi r e^{-\mu_{l}\pi r^{2}}$$
(9)

The distribution of the distance between the *R*-node and *k*-nodes is  $f_{\beta k,R}(r)$  in (9),  $\mu_I = P_I \mu_{Net}$  and  $P_I$  is the probability of interference.

#### 3.2. Probability of interference

Actually, not all nodes within  $\delta r$  will transmit at the same time, with the *T*-node on link *l*, therefore *P*<sub>l</sub> is defined by two events:

 $\xi_3$  = at least a node exist within  $\delta r$  and

 $\xi_4$  = the node is transmitting.

In an inter-working multi-hop wireless network with spatial density  $\mu_{Net}$ ,  $\Pr(\xi_3)$  can be expressed as the probability that there exists at least a node within  $\delta r$  of the *R*-node and it is given by:  $1 - e^{-\mu_{Net}A_I}$  where  $A_I$  represents the area of  $\delta r$  for the *R*-node of interest.

$$\Pr(\xi_4) = \begin{cases} 1, & \text{if } P^{t(k)} > 0 \\ 0, & \text{if } P^{t(k)} = 0 \end{cases} \quad \forall P^{t(k)} \ge 0.$$

Thus:  $P_I = Pr(\xi_3)Pr(\xi_4) = 1 - e^{-\mu_{Net}A_I} \forall P^{t(k)} > 0.$ 

### 4. Probability of bit error and reliability model

The decoding performed at the receiving node on a link is a probabilistic process and it indicates whether a transmission is successful or not. Due to the potential interference from other transmitters, communication may not be totally error-free; hence success is specified in terms of an acceptable probability of bit error. Thus the probability of bit error is used to express the success or failure of a transmitted signal (packet) in terms of probability.

In order for a transmission to be correctly decoded (successfully received), the value of  $\theta^{(l)}$ , which is the SINR experienced on the link must be greater than or equal to a SINR threshold value ( $\theta^{th}$ ). The threshold value ( $\theta^{th}$ ) is a pre-determined value that is used to ensure reliable transmission over a link. According to [17], the mean signal strength over the separation distance between a *T*-node and an *R*-node is appropriate for estimating the transmission strength of the link, whereas measures of signal variability are only appropriate in system design issues such as antenna diversity and signal coding. Hence, the mean value of  $\theta^{(l)}$  will be used for the estimation of the probability of bit error. In Eq. (10a),  $E(\theta^{(l)})$  is the expected value of  $\theta^{(l)}$ .

$$E(\theta^{(l)}) \ge \theta^{\text{th}}.$$
(10a)

Since  $\theta^{\text{th}}$  is the minimum SINR that is required for successful packet reception at the *R*-node, then a transmission error can be declared once there is a probability that  $E(\theta^{(l)})$  is below  $\theta^{\text{th}}$ . Let  $\Phi^{(l)}$  denote the probability of bit error on the *l*th link,  $\Phi^{(l)}$  is given by:

$$\Phi^{(l)} = P(E(\theta^{(l)}) < \theta^{\text{th}}) = 1 - P(E(\theta^{(l)}) \ge \theta^{\text{th}}).$$
(10b)

In evaluating  $E(\theta^{(l)})$ , the transmit power, received power and noise power level on link *l* have been kept constant.  $E(\theta^{(l)})$  is mostly influenced by the value of  $P_{ini}$  because as long as nodes operate in the same transmission frequency band, internode interference is bound to occur. However, the transmission from any node simultaneously transmitting with the *T*-node will not interfere with the reception at the *R*-node if the distance between the interfering nodes and the *R*-node is greater than the upper bound for  $\beta_{k,R}$  given by:

$$r + \delta r = \sqrt{\frac{SP^{t(k)}\theta^{\text{th}}}{P_l^t}}(\beta_{T,R}).$$
(11)

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In deriving (11) from (3), the SINR threshold  $\theta^{th}$  replaces  $\theta^{(l)}$  and the interfering nodes are assumed to have the same transmission power. Also, the thermal noise power has been considered to be negligible compared to the interference power ( $P_{ini}$ ) just as in [18]. Eq. (4) can now be expressed as:

$$r < \left|\beta_{k,R}\right| \le \sqrt{\frac{SP^{t(k)}\theta^{\text{th}}}{P_l^t}}(\beta_{T,R}).$$
(12)

However, at a certain time, not all of the nodes within  $\delta r$  may contribute to interference as some of them will not even transmit when the *T*-node is transmitting. Now, what is the threshold of the number of interfering nodes that can be in this interference region, beyond which, a transmission error can be declared? Note that in order to maintain a target SINR threshold, there is a maximum number of interfering users that a network can tolerate as stated in [19]. Let  $S_{th}$  represent the threshold of interfering nodes with the value of  $\beta_{k,R}$  of these nodes fulfilling Eq. (12). For analytical tractability, if we keep the interfering nodes' power constant, and since unsuccessful transmission is declared when  $E(\theta^{(l)}) < \theta^{th}$ , the probability of bit error at the receiver node occurs if:

$$S > S_{\rm th}$$
 (13)

where *S* is the number of interfering nodes. In order words,  $E(\theta^{(l)})$  becomes less that  $\theta^{\text{th}}$  as *S* increases beyond  $S_{th}$  and the probability of error occurs once  $E(\theta^{(l)}) < \theta^{\text{th}}$ . Therefore, the bit error probability can be approximated as:

$$\Phi^{(l)} \approx P(S > S_{\text{th}}) = 1 - P(S \le S_{\text{th}}). \tag{14}$$

Using Eq. (1), the probability of bit error can be estimated as:

$$\Phi^{(l)} \approx 1 - \sum_{k=0}^{S_{\rm th}} \frac{(\mu_l A_l)^k}{k!} e^{-\mu_l A_{li}}$$
(15)

where  $\mu_l$  is the density of interfering nodes in the interfering region of area  $A_l$ . Eq. (15) approximates the probability of bit error as a function of the number of interfering nodes and  $\mu_l = P_l \mu_{Net}$  as in Section 3.1. A link's reliability, represented by  $\Phi^{(l)'}$  is estimated as the complement of Eq. (15). The contribution of this paper is in the evaluation of the reliability of a link in an inter-working multi-hop wireless network irrespective of the network's topology or the multiple access technique employed by the multi-hop nodes in the inter-working network. Inter-node interference is one of the major factors that degrades the reliability of a link in wireless networks, thus in the evaluation of a link's reliability, the effective density of interfering nodes is considered. This notion of considering the effective density of interfering nodes was adopted from [13] where the authors looked into the distribution of interference in cognitive radio networks and wireless packet switched network. The analysis leading to the evaluation of  $\Phi^{(l)'}$  allows an easy adoption of the probability of bit error and reliability as cost metrics in the design of routing techniques for inter-working multi-hop wireless networks. A routing technique that uses reliability as a metric can prevent wasteful transmissions over low quality links.

#### 4.1. Results and discussion

The network scenario is as shown in Fig. 2, where the link considered is at the centre of the inter-working multi-hop wireless network. Three wireless multi-hop networks with 10 nodes, 15 nodes and 25 nodes respectively have been interworked in a 1000 unit square area. The separation distance between the *T*-node and the *R*-node is 10 square units. All the nodes in the network transmit with the same power of 10 mW. The transmitter and receiver gains are represented by  $G_t$  and  $G_r$ , respectively and are assumed to be equal to 1,  $L_f = 1$  and  $\theta^{th}$  is taken as 6 dB. The interference region is 100 units square. The expression in (16) is used to calculate the threshold of interfering nodes for any value of  $\theta^{th}$ . The channel attenuation on links between the *k*-nodes and the *R*-node are considered as random variables (RV) and the expectation of these RVs is used in (16).

$$S_{\rm th} = \left\lceil \frac{1}{\varpi} \left( \frac{P_l^t(\beta_{T,R})^{-2}}{P^{t(k)}\theta_{\rm th}} - \frac{P_o}{cP^{t(k)}} \right) \right\rceil \tag{16}$$

where  $\varpi = E[(\beta_{k,R})^{-2}]^2$  is the solution to the expectation of the negative second moment of  $\beta_{k,R}$ , which can be found in [20].

Considering that noise power  $(P_o)$  is very weak relative to the interference power  $(P_{ini})$  the expression on the right side of (16) can be taken as negligible.

There is a minimum (threshold) probability of bit error that should be maintained for a link to be termed as reliable and consequently, for the transmission on the link to be successful. The maximum number of interfering nodes (nodes in

<sup>&</sup>lt;sup>2</sup> This can also be evaluated for any value of the path loss exponent, a.

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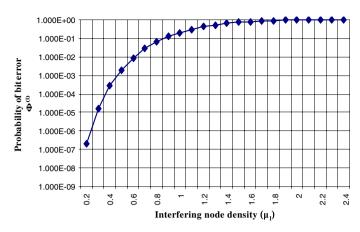


Fig. 4. Probability of bit error vs. interfering node density.

the  $\delta r$  region) that can be supported to maintain this probability of bit error is  $S_{th}$ . The value of  $S_{th}$  can be obtained from (16). Once the threshold of the probability of bit error is exceeded, unsuccessful transmission can be declared. For the interworking network, an increase in the probability of bit error beyond the threshold means that there is a high probability that the transmission on the link of interest is unsuccessful.

Fig. 4 shows the probability of bit error for the network scenario illustrated with the value of  $S_{th} = 12$ . The interfering node density ( $\mu_l$ ) was increased by increasing the probability of interference ( $P_l$ ) from the nodes in the interference region. In this network scenario, the value of  $\Phi^{(l)}$  increased continually as the density of interfering nodes increased beyond the threshold value at which a reliable transmission can be maintained on the link *l*. The link is reliable for all values of  $S \leq S_{th}$  and the threshold of the probability of bit error shown in Fig. 3 occurs for  $S = S_{th}(S_{th} = 1.15727E-06)$ . Thus, unsuccessful transmission is declared for values strictly greater than this threshold value. This gives an inclination that successful transmission on the link of interest in the inter-working network cannot be guaranteed any longer when  $\Phi^{(l)}$  becomes greater than the acceptable threshold probability. With a target probability of interference, the threshold of  $\Phi^{(l)}$  can be evaluated using  $S_{th}$  and the area of the interference region.

In summary, for the *R*-node on a particular link of interest in the inter-working network, an interference region is defined. Using Eq. (15), we can find the likelihood that the link will provide a successful transmission based on the number of nodes in the interference region. Basically, any node in the interference region will affect the probability of bit error if it is transmitting at the same time as the *T*-node on the link of interest.

With the target  $\theta^{th}$ , the value of  $S_{th}$  that will give the upper bound for the acceptable probability of bit error on a link can be determined. If the probability of bit error evaluated for a link in the network is higher than the acceptable value, then the link is termed unreliable. In this way, unreliable links can be identified and optimized traffic routing decisions can be made in inter-working multi-hop wireless networks irrespective of the link/MAC layer technologies of the interworking networks. The results obtained in Fig. 4 are similar to the results provided in Appendix E, page 642 of [9] where the Gaussian Approximation, the Improved Gaussian Approximation and the Simplified Expression for the Improved Gaussian Approximation have been used to evaluate the probability of bit error as a function of the total number of users in a spread spectrum CDMA network.

Fig. 5 expresses the complement of the results of Fig. 4. The probability of successful transmission (link reliability) reduces as the density of interfering nodes in the interference region increases. Thus from the analysis, the reliability of a multi-hop route with  $l_{max}$  links/hops denoted by  $\Phi^{(r)'}$  is given by Eq. (17), where  $l_{max}$  is the maximum number of hops on that route

$$\Phi^{(r)'} \approx \prod_{l=1}^{\max} (1 - \Phi^{(l)}).$$
(17)

Figs. 6 and 7 further illustrates how  $P_{\text{ini}}$  in Eq. (3) affects Eqs. (10a) and (10b) from which the probability of bit error estimation in Eq. (15) was derived. As the density of interfering nodes increases, the value of  $P_{\text{ini}}$  increases and this causes  $\theta^{(l)}$  to decrease below  $\theta^{\text{th}}$  as shown in Fig. 7, hence probability of bit error increases on the link as observed in Fig. 4.

For example in Fig. 3, at  $\mu_I = 0.2$ , the probability of bit error is below the threshold value and  $P_{ini}$  is approximately 6.3 dB. At  $P_{ini}$  is 6.3,  $\theta^{(l)} = 7.79$  dB, which is higher than  $\theta^{th}$ . When  $\mu_I$  increases to 0.3,  $P_{ini}$  is approximately 8.7 dB, which causes  $\theta^{(l)}$  to decrease below the target  $\theta^{th}$  in the network scenario and thus the probability of bit error starts increasing.

Using Eq. (16), the effect of increasing interfering nodes' power  $(P_t^{(k)})$  on the value of  $S_{th}$  was obtained. Fig. 8 shows that an increase in  $(P_t^{(k)})$ , reduces the threshold of  $S_{th}$  Consequently, the acceptable probability of bit error for the link is increased. This means that an increase in  $(P_t^{(k)})$  aggravates reduces the reliability of a link.



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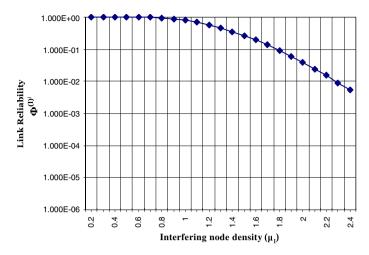
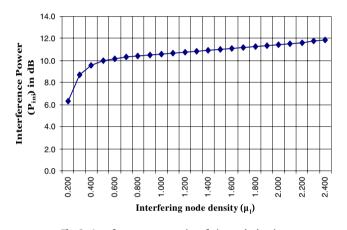


Fig. 5. Link reliability vs. number of interfering nodes.



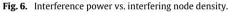




Fig. 7. Expected signal to interference and ratio noise vs. interference power.

#### 5. Conclusion

This paper presented a framework for the estimation of a link's reliability in inter-working multi-hop wireless networks. The results obtained shows that the reliability of a link in inter-working multi-hop wireless networks can be estimated irrespective of the under-lying link layer technologies present within the network. The results also give insights into the relationship between the parameters that determine the reliability of a link in inter-working multi-hop wireless networks.

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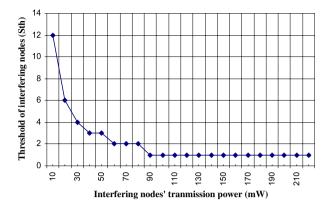


Fig. 8. Threshold of interfering nodes vs. interfering nodes transmission power.

The framework presented allows an easy adoption of the probability of bit error probability and reliability as metrics in the design of traffic engineering (routing and QoS) frameworks for inter-working multi-hop wireless networks.

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