Technical report CS06-01-00 Department of Computer Science, University of Cape Town

Correct normal transformations for articulated models

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March 2006

Abstract

It is well-established that when a matrix is used to transform a rigid object, the normals should be transformed by the inverse transpose of that matrix. However, this is only valid where the transformation matrix is locally constant. This is not the case for models animated with skeletal subspace deformation (SSD), where the transformation matrix is computed for each vertex. We derive a formula for correctly transforming normals on SSD models.

1 Introduction

Skeletal subspace deformation (SSD) is a common real-time method for character animation in games and other virtual environments [LCF00, SRC01, MG03]. A model is created in a *rest* pose, and a skeleton is placed inside a model. The bones of the skeleton are controlled by the animator, either directly, or indirectly through inverse kinematics or physical simulation. Each bone defines a transformation from its rest pose. Vertices are attached to one or more bones, with weights to determine the influence of each bone on the vertex. The transformation matrices of the bones are combined linearly (according to the influence weights) to produce a blended matrix that is used to deform the vertex.

2 Preliminaries

We consider the behaviour at a single point on the surface — thus, there is no need to use indices for vertices, and all indices refer to bones. Vectors are written in bold (e.g., $\mathbf{v}$), and rest pose values are denoted with a prime (e.g., $\mathbf{v}'$). Vectors and matrices are generally homogeneous (4-vectors and 4×4 matrices); a bar over a vector indicates the first three components, while a bar over a matrix indicates the upper-left 3×3 sub-matrix.

For a single point on the surface, we label the rest pose and dynamic position by $\mathbf{v}'$ and $\mathbf{v}$, and the rest pose and dynamic normal by $\mathbf{n}'$ and $\mathbf{n}$. The influence of bone i on the vertex is w_i (which will be 0 for bones that do not influence the vertex), and M_i is the matrix that transforms bone i from its rest position to its dynamic position.

Skeletal subspace deformation defines the dynamic position as

$$\mathbf{v} = \sum_i w_i M_i \mathbf{v}'. \tag{1}$$

3 Normal transformations

We consider only models without creases, where a single normal is specified at each vertex. The polygonal mesh is treated as an approximation to a C^1 continuous surface. We extend this paradigm by treating the weights at the vertices to be an approximation to a C^1 continuous field defined over this smooth surface.

For a given point $\mathbf{v}'$, consider a differentiable parametrisation of the rest surface in terms of variables s and t, valid in a small neighbourhood around $\mathbf{v}'$. We assume that the parametrisation is differentiable. Since normals are translation-invariant, we make the assumption that $\mathbf{v}'$ is at the origin i.e., $\mathbf{v}' = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$. This leads to:

$$
\begin{aligned}
\frac{\partial \mathbf{v}}{\partial s} &= \frac{\partial}{\partial s}\left(\sum_i M_i \mathbf{v}' w_i\right) \\
&= \left(\sum_i M_i \frac{\partial \mathbf{v}'}{\partial s} w_i\right) + \left(\sum_i M_i \mathbf{v}' \frac{\partial w_i}{\partial s}\right) \\
&= \left(\sum_i w_i M_i\right) \frac{\partial \mathbf{v}'}{\partial s} + \left(\sum_i M_i \mathbf{v}' \frac{\partial w_i}{\partial \mathbf{v}'} \frac{\partial \mathbf{v}'}{\partial s}\right) \\
&= \left(\sum_i w_i M_i + M_i \mathbf{v}' \frac{\partial w_i}{\partial \mathbf{v}'}\right) \frac{\partial \mathbf{v}'}{\partial s}
\end{aligned}
$$

Let N be the matrix in parentheses. Since both $\frac{\partial \mathbf{v}}{\partial s}$ and $\frac{\partial \mathbf{v}'}{\partial s}$ must have homogeneous weight of zero, the last row and column of N play no role and hence

$$
\frac{\partial \overline{\mathbf{v}}}{\partial s} = \overline{N} \frac{\partial \overline{\mathbf{v}'}}{\partial s}, \tag{2}
$$

and similarly for t. The normal to the dynamic surface is parallel to

$$
\frac{\partial \overline{\mathbf{v}}}{\partial s} \times \frac{\partial \overline{\mathbf{v}}}{\partial t} = \left(\overline{N} \frac{\partial \overline{\mathbf{v}'}}{\partial s}\right) \times \left(\overline{N} \frac{\partial \overline{\mathbf{v}'}}{\partial t}\right) = \mathrm{co}(\overline{N})^T \left(\frac{\partial \overline{\mathbf{v}'}}{\partial s} \times \frac{\partial \overline{\mathbf{v}'}}{\partial t}\right), \tag{3}
$$

where $\mathrm{co}(\overline{N})$ is the cofactor matrix of $\overline{N}$. But $\frac{\partial \overline{\mathbf{v}'}}{\partial s}$ and $\frac{\partial \overline{\mathbf{v}'}}{\partial t}$ form a basis for the tangent plane to the rest surface at $\mathbf{v}'$, so their cross product is parallel to the rest normal $\mathbf{n}'$. Furthermore, the cofactor matrix is proportional to the inverse, so

$$
\mathbf{n} \parallel \overline{N}^{-T} \mathbf{n}'. \tag{4}
$$

References

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