Modelling a DS-CDMA Fading Channel with Bursty Traffic Arrivals

Jesse Landman

CS04-02-00
September 2003

Data Network Architecture
Department of Computer Science
University of Cape Town
Private Bag, RONDEBOSCH
7701 South Africa
e-mail: jlandman@cs.uct.ac.za

Abstract

We propose an analytic model for the performance of a faded DS-CDMA radio channel carrying bursty traffic. For the IP traffic model we propose a two-state Markov Modulated Poisson Process (MMPP) to model slow Poisson arrivals of packets interspersed by bursts of IP traffic with a much higher arrival rate. The channel model is a modified multi-state Gilbert-Elliot model which takes into account both the number of interfering users and whether the channel is in a faded or non-faded state. We analyse both Ricean and Rayleigh distributions for the user signal. For the final result we relied on a discrete event simulation of the same DS-CDMA radio channel data transmission process in order to parameterise the model. Results for metrics such as the slot success rate as a function of the number of interfering users and the packet throughput as a function of the IP traffic arrival rate are derived.

1 Introduction

With the increasing usage of wireless networks for IP traffic the performance of the radio link in the path from server to client in the fixed to wireless network path is receiving attention from several authors. This work concerns itself with a simple slotted direct sequence CDMA protocol where each user signal appears as low-level background interference to all other user signals in the network. The perspective of the work presented here is that of communication protocol modelling and analysis and, in particular, the modelling of the performance of mobile devices using TCP under different operational conditions.

This study also focuses on the phenomenon known as *fading*, which arises as a results of multiple versions of a signal existing due to scattering. Two kinds of fading environments are applicable: Non-Line-of-Sight (NLOS), where the fading signal is approximated by Rayleigh distributions, and Line-of-Sight (LOS), where the fading signal is approximated by Ricean distributions. The latter has a dominant specular component to its signal.

Several authors such as [2, 5, 6, 9] have taken up the challenge of modelling radio channels and the effect of Rayleigh or Ricean fading on the transmission accuracy. In most cases the models are for flat fading channels where the received signal is a function of the signal power and time-invariant interference noise. Judge and Takawira [5] extended Gilbert-Elliot models [7] to include the case where both the user signal power and the interference signal power are varying as is typically the case for DS-CDMA transmissions. In this paper we generalize their result for Rayleigh faded channels for Ricean fading, and introduce a somewhat more general traffic model.

The layout of the paper is the following. In the next section we describe a Markov Modulated Poisson Process (MMPP) traffic model for the arrival for transmission of messages on the DS-CDMA down link. In Section 3 we describe the performance metrics of interest in our analyzes. Next we find a general expression for the Ricean faded user signal and then derive an expression for the interference signal conditioned on the number of transmitting users in Section 6. The next section describes the Hidden Markov Model (HMM) and in Section 7 we present analytical numerical results and compare these to those obtained by simulation.

2 Traffic model

This analysis assumes that IP traffic arrives as *packets* of varying lengths for transmission to the end user. Each packet is segmented into *slots* which corresponds to a transport block (or MAC PDU) for transmission over the physical radio channel [1]. In the sequel we also refer to each such transport block as a *user*.

We assume that data packet lengths, that is the number of slots per packet, l, are geometrically distributed with parameter ν, the probability that a packet will terminate in a radio slot:

$$P_{pkt\_length}(l) = P(l) = (1-\nu)^{l-1}\nu \tag{1}$$

where the mean packet length is ν^{-1} slots.

The traffic model we use is a superposition of two Poisson arrival processes modulated by a two state continuous Markov chain. We denote the slot arrival rate of the MMPP process when the chain is in state j by $\lambda_j,\ \ j = 1, 2$ and the rate of change from state 1 to state 2 by r_{12}; from state 2 to state 1 by r_{21} respectively. State 1 corresponds to Poisson arrivals at a low rate while state 2 corresponds to bursts of high arrival rates.

The effective combined Poisson arrival rate λ is given by

$$\lambda = \frac{r_{21}\lambda_1 + r_{12}\lambda_2}{r_{12} + r_{21}}$$

Hence the probability that m packet slots will arrive for transmission in a radio slot is given by

$$f(m) = \frac{e^{\lambda}\lambda^m}{m!} \tag{2}$$

Where we assume that data slot retransmissions are absorbed into the arrival process.

3 Performance metrics

The measures of performance of interest are the average bit error and the average packet throughput. In order to compute these we need to know the probability that all slots belonging to the packet are received without error. The success of a slot depends on the state of the network during its attempted transmission. The following performance metrics are introduced from the work of [5].

We define state x as the number of transmitting users in a timeslot and $P(x = j)$ as the probability of being in state j. According to our Poisson assumption,

$$P\{x = j\} = \sum_{i=0}^{\infty} P\{x = i\}\pi_{ij}, \sum_{j=0}^{\infty} P\{x = j\} = 1 \tag{3}$$

where π_{ij} is the steady state probability of moving from state i to state j and is given by

$$\pi_{ij} = \begin{cases} \sum_{k=0}^{i} \binom{i}{k} \cdot \nu^k \cdot (1-\nu)^{i-k} \cdot f(j-i+k) & i <= j \\ \sum_{k=(i-j)}^{i} \binom{i}{k} \cdot \nu^k \cdot (1-\nu)^{i-k} \cdot f(j-i+k) & i > j \end{cases} \tag{4}$$

where $f(m)$ is the Poisson distribution and ν is the probability that a packet will terminate in a time slot.

In order to observe the error sequence process, we write α_{ij} as the joint conditional probability of the successful transmission of the n^{th} and the $(n-1)^{th}$ slots in the packet conditioned on there being j users present during the transmission of the n^{th} slot while there were i simultaneous users during the transmission of the $(n-1)^{th}$ slot.

To complement α_{ij}, we write α_{0j} as the probability of success of the first slot in the packet conditioned on there being j users in the channel at the time of transmission of this first slot. These two definitions are used in $R_n(j)$, which is the probability of success of all slots in a packet up to and including the n^{th} slot given that the n^{th} slot sees j other transmitting users. This probability is solved recursively using

$$R_1(j) = P\{x = j\}\alpha_{0j} \tag{5}$$

$$R_n(j) = \sum_{i=1}^{\infty} R_{n-i}(i)\pi_{ij}\alpha_{ij} \quad n > 1 \tag{6}$$

The probability of success of a packet containing L slots, R_L, and the overall average packet success probability P_S, are as follows:

$$R_L = \sum_{j=0}^{\infty} R_L(j) \tag{7}$$

$$P_S = \sum_{i=1}^{\infty} R_i P(i) \tag{8}$$

with $P(i)$ the probability that a packet will contain i slots given by (1). Finally, the packet throughput S is simply the fraction of the offered load λ that is successfully transmitted across the channel in a time slot:

$$S = \lambda P_S \tag{9}$$

4 Hidden Markov model (HMM)

The behaviour of the physical channel is a function of the state $\Omega \in \{Bad, Good\}$ of the channel and the number $j, \quad j = 1, \ldots$ of interfering users. This is an extension of the well-known Gilbert-Elliot Markov Chain (GEC) [7].

The GEC is described by two states, a *Good (non-fade)* state, and a *Bad (faded)* state [7]. Errors occur in the *Bad* state with probability *h* (large), and in the *Good* state with probability *k* where $0 \leq k \leq h \leq 1$.

The HMM developed here is also a function of the number of active users in a time slot. This means, the model has many states described by Ω_n where $\Omega_n = \{G_j, B_j\}$ i.e. G_j denotes being is a *Good* state with *j* transmitting users, $j = 1, \ldots$, at timeslot n, and B_j denotes being is a *Bad* state with *j* transmitting users, $j = 1, \ldots$, at timeslot n. $P(G_j)$ denotes the probability of being in this state and ω_{ij}^{CD} as the probability of moving from state C_i to state D_j with $\{C_i, D_j\} \in \Omega_n \forall i, j$. Then

$$\begin{aligned} P(G_j) &= \sum_{i=0}^{\infty} P(G_i)\omega_{ij}^{GG} + \sum_{i=0}^{\infty} P(B_i)\omega_{ij}^{BG} \\ P(B_j) &= \sum_{i=0}^{\infty} P(G_i)\omega_{ij}^{GB} + \sum_{i=0}^{\infty} P(B_i)\omega_{ij}^{BB} \end{aligned}$$

where clearly

$$\sum_{i=0}^{\infty}[P(G_i) + P(B_i)] = 1$$

Using Bayes' theorem we can write

$$P(C_j) = P\{\Omega_n = C|x = j\}.P\{x = j\}$$

and writing $P\{x_{n+1} = j|x_n = i\}$ as the steady state probability of moving from state *i* to *j* (given by (4)) we have

$$\omega_{ij}^{CD} = P\{\Omega_{n+1} = D|\Omega_n = C, x_{n+1} = j, x_n = i\}.P\{x_{n+1} = j|x_n = i\} \tag{10}$$

$P\{\Omega_{n+1} = D|\Omega_n = C, x_{n+1} = j, x_n = i\}$ is the steady state transition probability for being in state Ω_{n+1}.

With the above we can now return to (5) and write

$$\alpha_{0j} = P\{\Omega_n = G|x_n = j\}(1 - k) + P\{\Omega_n = B|x_n = j\}(1 - h) \tag{11}$$

and

$$\alpha_{ij} = P\{\Omega_{n+1} = G|\Omega_n = G, x_{n+1} = j, x_n = i\} \cdot (1 - k)^2 \tag{12}$$

these are our error processes i.e. they define, in section 3, the probabilities of a single slot being transmitted without error (equation (11)) and the probabillity that a slot will succeed given that the previous slot succeeded as well.

For the computation of the probabilities in (11) and (12), we need to know the conditional PDFs for the MAI and user signal amplitudes, given by $P_{MAI}(y|j)$ and $P_{sig}(u)$ respectively, since the probability of being in a faded channel depends on the relative amplitudes of the user signal and the MAI. To determine these, we define a parameter θ to be the MAI signal to user signal ratio and the threshold which determines the channel state:

$$y/u \leq \theta \Leftrightarrow \Omega_n = Good \ , \ \ y/u > \theta \Leftrightarrow \Omega_n = Bad$$

Using θ we can now write

$$P\{\Omega_n = G|x_n = j\} = \int_0^\infty \int_{\frac{y}{\theta}}^\infty P_{MAI}(y|j).P_{sig}(u)dudy \tag{13}$$

Note that $P\{\Omega_n = C|x_n = j\}$ is defined as the sum of all instantaneous amplitudes of the desired signal multiplied by the instantaneous amplitudes of the MAI signal (given in sections 5 and 6) over all possible amplitudes of MAI and over all desired user amplitudes greater than the threshold y/θ.

To compute the probabilities in (12), we note that

$$P\{\Omega_{n+1} = D|\Omega_n = C, x_{n+1} = j, x_n = i\}$$

can be written as:

$$\frac{P\{\Omega_{n+1} = D, \Omega_n = C|x_{n+1} = j, x_n = i\}}{P\{\Omega_n = C|x_n = j\}} \tag{14}$$

using Bayes' Theorem. $P\{\Omega_n = C|x_n = j\}$, assuming a stationary channel, is given by (13).

In order to compute $P\{\Omega_{n+1} = D, \Omega_n = C|x_{n+1} = j, x_n = i\}$, we need to sum all possible instantaneous amplitudes of the desired signal multiplied by the interfering signals for the current timeslot multiplied by the same for the next timeslot. We are only interested in the conditional probability corresponding to *[Good,Good]*, as is evident from (12). This is given by [5]:

$$P\{\Omega_{n+1} = G, \Omega_n = G|x_{n+1} = j, x_n = i\}$$

$$= \int_0^\infty \int_0^\infty P_{sig}(u_n).P_{MAI}(y_n < \theta u_n|i) \cdot \left[1 - \int_0^{\frac{y_{n+1}}{\theta}} P_{sig}(u_{n+1}|u_n)du_{n+1}\right] \cdot P_{MAI}(y_n|j)du_n dy_{n+1} \tag{15}$$

where

$$P_{MAI}(y < \theta|ji) = \int_0^\theta P_{MAI}(y|i)dy \tag{16}$$

The following 2 sections provide expressions for $P_{MAI}(u_n|j)$, $P_{sig}(u_n)$, and $P_{sig}(u_{n+1}|u_n)$.

5 User signal model

Before deriving an expressions for the user signal at the receiver, we first define the complex fading channel radio signal as done by Turin [9] to be

$$y(t) = c(t)x(t) + n(t)$$

where $x(t)$ is the transmitted signal, $y(t)$ the received signal, and $c(t)$ and $n(t)$ are complex random processes governing the fading and noise distortions respectively. The AWGN, $n(t)$, will be ignored for our purposes, because mobile radio channels are interference limited and not noise limited [8].

The autocorrelation function of $c(t)$, $R(\tau)$, is given by [7]:

$$R(\tau) = \sigma_u^2 J_0(2\pi f_D|\tau|)$$

Here, J_0 is the Bessel function of the first kind, order zero, σ_u^2 is the variance of $u(t)$, the envelope of $c(t)$. The parameter f_D is the channel Doppler bandwidth which is determined primarily by the speed and transmitting frequency of the mobile station. A value $f_D t_s = 0.02$, with t_s the timeslot duration, is considered to represent slow fading, while values of 0.1 and 0.3 represent medium and fast or uncorrelated fading respectively.

We follow the principles outlined in [9] to determine the relevant PDFs for our signal envelopes, with the exception that our envelope ($u_{\mathbf{k}}$) for Ricean fading is:

$$\boldsymbol{u_k} = (w_{u_1}, w_{u_2}, \cdots, w_{u_k})$$
$$w_{u_i} = \sqrt{A^2 + u_i^2 + 2u_i A\, cos(\theta_i)} \tag{17}$$

where A is signal power of the dominant component. This leads to the following expression (with $k = 1$ in [9]) for the Ricean PDF:

$$g(u_1) = \frac{u_1}{\mu} e^{-\frac{u_1^2 + A^2}{2\mu}} I_0\left(\frac{Au_1}{\mu}\right) \tag{18}$$

where $\mu = R(0)$ is the local mean scattered signal power [8].

The distribution in (18) can be expressed in terms of one unknown parameter by defining $K = \frac{A^2}{2\mu}$ where K is known as the *Rice Factor*, and is defined as the ratio between the Line of Sight (LOS) component of the signal and the scattered component [10]. If $K = 0$, the distribution becomes a Rayleigh faded distribution. K is adequate in order to completely specify the Ricean distribution [8, 10].

The general Ricean PDF is therefore given by

$$P_{sig}(u) = \frac{(1+K)}{\bar{p}} e^{-K} u e^{-\frac{1+K}{2\bar{p}}u^2} I_0\left(\sqrt{\frac{2K(1+K)}{\bar{p}}}u\right), \quad u >= 0 \tag{19}$$

$I_0(x)$ is the modified Bessel function of the first kind and order zero [4]. where $\bar{p}$ is the local mean power given by $\bar{p} = \frac{1}{2}A^2 + \mu$.

Next we will need an expression for the conditional PDF of the Ricean channel, i.e. the probability of fading of a signal amplitude u_n, conditioned on the amplitude u_{n-1} (the history of the channel). This can be found if $K = 2$ in [9], and the resulting integral is solved by Mondre [6] and is given by:

$$g(u_1, u_2) = \frac{u_1\, u_2}{\mu^2(1-\rho^2)} exp\left[-\frac{2K(1-\rho)}{1-\rho^2} - \frac{u_1^2 + u_2^2}{2\mu(1-\rho^2)}\right] \cdot \sum_{m=0}^{\infty} \epsilon_m\, I_m\left(\frac{\rho u_1 u_2}{\mu(1-\rho^2)}\right) I_m(\eta\, u_1) Im(\eta\, u_2) \tag{20}$$

where

$$\eta = \frac{\sqrt{2K(1+\rho^2)}}{\sqrt{\mu}(1-\rho^2)} \quad \epsilon_0 = 1. \quad \epsilon_m = 2, \quad m > 0. \tag{21}$$

and $I_m(x)$ is the modified Bessel function of the first kind and order m [4].

In order to compute the parameters of the HMM we need to compute $P_{sig}(u_{n+1}|u_n) = g(u_1|u_2)$ which, using Bayes theorem, is given by

$$P_{sig}(u_{n+1}|u_n) = g(u_2|u_1) = \frac{g(u_1, u_2)}{g(u_1)} \tag{22}$$

Hence

$$P_{sig}(u_{n+1}|u_n) = \frac{\frac{u_{n+1}\,u_n}{\mu^2(1-\rho^2)} exp\left[-\frac{2K(1-\rho)}{1-\rho^2} - \frac{u_{n+1}^2+u_n^2}{2\mu(1-\rho^2)}\right]}{\frac{(1+K)}{\bar{p}} e^{-K} u_n e^{-\frac{1+K}{2\bar{p}}u_n^2} I_0\left(\sqrt{\frac{2K(1+K)}{\bar{p}}} u_n\right)} . \sum_{m=0}^{\infty} \epsilon_m \, I_m\left(\frac{\rho u_{n+1} u_n}{\mu(1-\rho^2)}\right) I_m(\eta\, u_{n+1}) Im(\eta\, u_n) \tag{23}$$

6 Interfering signal model

Due to the imperfect orthogonality of the PN code sets used to multiplex the user signals in DS-CDMA, there is a level of interference that arises as more users access the system. This is called Multiple Access Interference, or MAI. If the MAI is too great, signals become distorted and errors occur. The channel is also a multi-path channel, and this creates yet another source of distortion due to signal reflection.

In order to obtain an expression for the MAI, one needs to take into account a summation of many Ricean random variables, and a closed form expression is not easy to obtain. However, [3] shows that the standard Gaussian approximation is an adequate approximation of the MAI, and can be given by [5]:

$$P_{MAI}(y|j) = \begin{cases} P_{sig}(u), & j = 1 \\ \frac{1}{\sqrt{2\pi\sigma_u^2}} exp\left(\frac{-(y-j\mu_u)^2}{2j\sigma_u^2}\right), & j > 1 \end{cases} \tag{24}$$

which is the PDF of the interference amplitude conditioned on the number of users present, j.

An expression for the conditional probability density function of y, $P\{y_n|y_{n-1}\}$ which would consider the correlation between successive samples of the MAI signal is analytically and computationally intractable. The reason is that the contribution from the individual interfering signals and their effect on the desired signal is not known. We therefore make the assumption that the MAI amplitude, y, is uncorrelated which would seems reasonable if the interfering users enter and leave the channel in a manner that the number of transmitting users is practically uncorrelated from timeslot to timeslot. This assumption is shown to be a valid one in [2].

7 Results

The results presented are for an analytic model with a validation of the main results using discrete simulation. The parameters used (where not indicated) are:

- Average number of slots per packet $\nu^{-1} = 4$.
- The Rice factor K (see Section 5). For Rayleigh fading $K = 0$; otherwise the fading is Ricean distributed (generally taken as $K = 4$).
- The product $F_D t_s$ of the Doppler bandwidth F_D (see Section 5) and the radio channel time slot duration t_s, where $0.02 \leq F_D t_s \leq 0.3$ [5].

Next we present the results of various experiments with the analytic model.

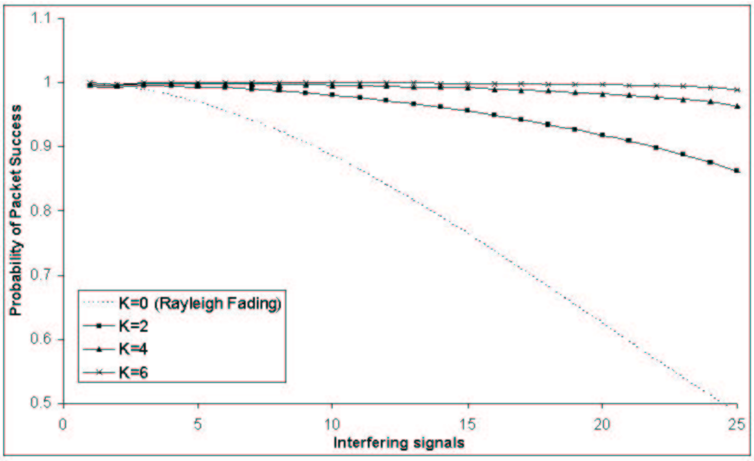

Figure 1: Slot success probability P_S as a function of the number of users j for $\theta = 25$dB for both Rayleigh ($K = 0$) and Ricean fading ($K \geq 1$).

7.1 Slot success probability

As mentioned in the introduction, the analysis of the physical channel performance presented here, has as in mind the modelling of the server to mobile device link for the purpose of protocol performance evaluation. The most important metric of interest is therefore the slot success rate, given certain channel conditions. The results confirm what one would expect: the higher the signal to noise ratio the higher the probability of packet success while Ricean fading environments are more stable.

Figure 1 is incomplete as it excludes the results of a discrete event simulator developed to validate the analytic model. In order to properly parameterise the model, we plotted the simulator results against HMM results for various values of θ, in order to find the best possible θ. The simulator uses the Standard Gaussian approximation to find the Bit Error Rate and is given by [5]:

$$P_{bit-error} = Q\left[\left(\frac{1}{3N}\sum_{k=2}^{\varphi}\frac{y_k^2}{u^2}\right)^{-1/2}\right] \quad (25)$$

where φ is the number of interfering users, N is the Spreading factor (number of chips per bit), y_k is the amplitude of user signal k, and u is the amplitude of the desired signal. $Q(r)$ is the Marcum-Q function.

Figure 2 shows that the best values of θ are roughly 18dB for Ricean Fading and 20dB for Rayleigh fading. It is also interesting to see that the model is more accurate for Ricean Fading (with a mean difference of 0.5 packets in throughput) than Rayleigh Fading (with a difference of 2 packets in throughput). At high levels of the interfering signal the system fails for all practical purposes beyond 25 users.

Using these parameters values for θ, we could proceed to study the packet throughput as a function of packet arrival rate.

7.2 Packet throughput

In the next experiment we plotted the packet throughput S (equation (9)) as a function of the average arrival rate λ when the MMPP is in burst mode for two different durations ($r_{12} = 0.1|0.5$). In all cases we used $F_D t_s = 0.3$ corresponding to a fast fading channel. As expected, with frequent bursts ($r_{12} = 0.5$) of prolonged high arrival rates the system gets overloaded fairly quickly and the throughput drops. This type of load situation is most likely to occur in 3G mobile environments where a sudden download of HTML pages after a period of browsing is a typical scenario.

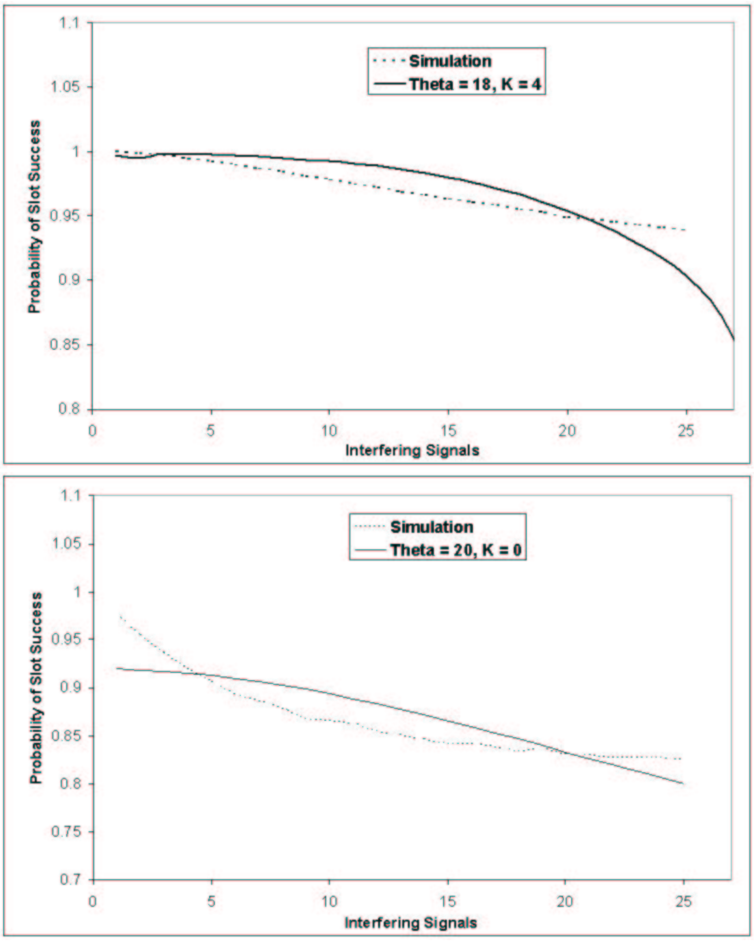

Figure 2: Slot success probability P_S as a function of the number of users j for Ricean (top) and Rayleigh (bottom) fading, plotted with their respective simulation results.

8 Conclusion

The model presented in this paper is a modification of the simple two-state Gilbert-Elliot Markov model in which the state of a DS-CDMA channel is a function of the number of interfering users present. We generalize past results to include Ricean fading which is the most likely scenario for stationary mobile users which use UMTS as a wireless interface. The Multiple Access Interference (MAI) amplitude is assumed to be Gaussian distributed and the sequence of MAI samples are uncorrelated for this study. We use a two-state MMPP Markov process to represent an IP packet arrival process which alternates between a low arrival rate such as would be the case for voice traffic and sudden bursts of HTML traffic.

The analytic and simulation results obtained correlate well enough to have sufficient confidence in the analytic model for analyzing trends in the packet error rates and packet throughput as a function of various parameters. Most important to extensions on this work was the accuracy of the Ricean model. Based upon this model the authors intend to extend it by including a model of user mobility, taking different physical channels into consideration as well as to include the all important scheduling and resultant queueing aspects of IP traffic over the mobile link.

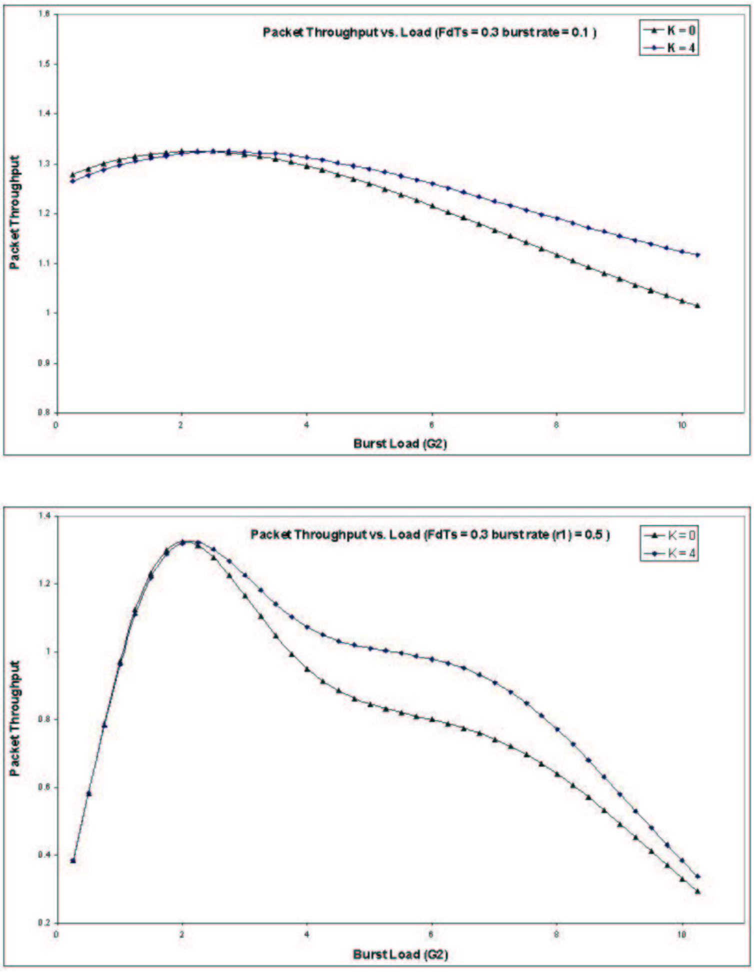

Figure 3: Packet throughput S as a function of the average burst load λ_2 for $r_{12} = 0.1$ and $= 0.5$ respectively for both Rayleigh ($K = 0$) and Ricean fading ($K = 4$).

9 Acknowledgement

Colette Consani and Heidi Proske developed the discrete event simulator used to validate the results of the analytic model. We express our sincere thanks for their collaboration.

References

[1] ETSI 3GPP TS 25.321. UMTS: MAC protocol specification. Available online: ftp://ftp.3gpp.org. Accessed: April 2003.

[2] J Cheng and N Beaulieu. Accurate DS-CDMA Bit-Error Probability Calculation in Rayleigh Fading. *IEEE Transactions on Communications*, January:3 – 15, 2002.

[3] E Geraniotis and M Pursley. Performance of Noncoherent Direct-Sequence Spread-Spectrum Communications over Specular Multipath Fading Channels. *IEEE Transactions on Communications*, March:219 – 226, 1986.

[4] Simon Haykin. *Communications Systems*. John Wiley and Sons, Inc., 2001.

[5] Garth Judge and Fambirai Takawira. A Simple Hidden Markov Model for a CDMA Channel with Correlated Rayleigh Fading. *The Transactions of the SAIEE*, March:17–26, 2002.

[6] Erwin Mondre. Complex and Envelope Covariance for Rician Fading Communication Channels. *IEEE Transactions on Communications*, February:80 – 84, 1971.

[7] Cecilio Pimentel and Ian F. Blake. Modelling Burst Channels Using Partitioned Fritchman's Markov Models. *IEEE Transactions on Vehicular Technology*, 47:885–899, 1998.

[8] T.S. Rappaport. *Wireless Communications: Principles & Practices*. Prentice Hall, 1996.

[9] William Turin and Robert van Nobelen. Hidden Markov Modeling of Flat Fading Channels. *IEEE Journal on Selected Areas in Communications*, 16:1809–1817, 1998.

[10] Michele Zorzi. Capture Probabilities in Random Access Mobile Communications in the Presence of Ricean Fading. *IEEE Transactions on Vehicular Technology*, Feb, 1997.