

# The design of an $H_\infty$ /LPV active braking control to improve vehicle roll stability

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**Abstract:** The active braking control system is an active safety system designed to prevent accidents and to stabilize dynamic manoeuvres of a vehicle by generating an artificial yaw moment using differential braking forces. In this paper, the yaw-roll model of a single unit heavy vehicle is used for studying the active braking system by using the longitudinal braking force at each wheel. The grid-based LPV approach is used to synthesize the  $H_\infty$ /LPV controller by considering the parameter dependant weighting function for the lateral acceleration. The braking monitor designs are proposed to allow the active braking system to react when the normalized load transfer at the rear axle reaches the criteria of rollover  $\pm 1$ . The simulation results indicate that the active braking system satisfies the adaptation of vehicle rollover in an emergency situation, with low braking forces and improved handling performance of the vehicle.

*Keywords:* Vehicle dynamics, Active braking system, Rollover,  $H_\infty$  control, LPV system.

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## 1. INTRODUCTION

### 1.1 Context

Vehicle rollover accidents have been extremely hazardous to the occupants of the vehicle and identified as the most fatal vehicle crashes. Loss of roll stability is the main cause of rollover accidents involving heavy vehicles. According to the Japan Traffic Accidents Databases, rollover accidents were nearly 1/5 of all the single-vehicle accidents. The Federal National Highway Traffic Safety Administration has statistics that in the United States, there were 333,000 heavy vehicles involved in traffic crashes during 2012. There were 3,921 people killed in rollover crashes and 104,000 people injured. In 2013, more than 4,500 persons were killed in road traffic accidents involving heavy vehicles in the EU, constituting almost 18% of all road accident fatalities for that year (Evgenikos et al. (2016)).

Because of the high center of gravity, the disturbance effects such as gusts of wind, irregular road surfaces, and abrupt manoeuvres, heavy vehicles have a proclivity to rollover accidents. Thus, it is necessary to develop fast and safety control systems to detect and prevent vehicle rollover, so they will enhance vehicle stability. Rollover prevention and detection such as active steering, active braking, active suspension and active anti-roll bar systems have been studied extensively (Gaspar et al. (2004), Vu et al. (2017)). Among them, the active braking system is considered as the most effective way to improve vehicle stability in emergencies. With increasing emphasis on traffic safety in recent years, intensive efforts have been put towards improving the braking performance. Safety standards that specify performance requirements of various types of brake system have been introduced in many countries (Yakub and Mori (2015)).

In order to prevent a vehicle rollover situation, the rollover responsible pair of forces has to be reduced, which means the re-

duction of the lateral inertial force by means of speed reduction and the combined reduction of the lateral force component, by means of manipulating the tyre slip. These two effects together are enough to prevent rollover on both a combination and also a single unit vehicles.

### 1.2 Related works

Among the literature on active braking system in order to prevent the rollover situation of heavy vehicles, let us mention some few works below:

- In (Palkovics et al. (1999)), the authors discuss some of the problems of the commercial vehicle stability in general and heavy vehicles in particular, and offer a solution for detecting and avoiding rollover by using the existing sensors and actuators of the electronic braking system. An integrated control of yaw, roll and vertical dynamics based on a semi-active suspension and an active differential braking system was presented by (Soltani et al. (2017)). A coordinated control of the two systems is proposed, using a fuzzy controller and an adaptive sliding mode controller. With the active braking system, in the emergency situation the vehicle can avoid rollover by reducing the roll angle, the lateral acceleration and the lateral load transfer ratio.
- In (Jo et al. (2008)), in order to enhance vehicle roll stability, the reference yaw rate is designed and combined into a target yaw rate depending on the driving situation. A yaw rate controller is designed to track the target yaw rate based on sliding mode control theory. To generate the total yaw moment required from the proposed yaw rate controller, each brake pressure is properly distributed with effective control wheel decision.
- In (Gaspar et al. (2004)), the authors proposed a combined control structure between the active anti-roll bar system and the active braking system. The main objective of this

proposal is to allow the active anti-roll bar system to work in the normal driving situation and the active braking system is only activated when the vehicle comes close to a rollover situation.

### 1.3 Paper contribution

The active braking system is crucial in view of autonomous driving to accomplish the task of obstacle avoidance. With the aim of applying advanced control method to complete this system, hence the contributions of this paper are the following:

- A parameter dependant weighting function of the lateral acceleration is used to permit performance adaptation to the rollover risk of heavy vehicles, characterized by the normalized load transfers at the rear axle. The grid-based LPV approach is used to synthesize the  $H_\infty$ /LPV active braking controller by using the LPVTools<sup>TM</sup> toolbox.
- We propose two Braking Monitor Designs in order to satisfy simultaneously the improved vehicle performance and prevention of vehicle rollover in an emergency situation. To fit better with the real world application, the objective of the braking monitor designs is to allow the active braking system to react when the normalized load transfer at the rear axle reaches the criteria of rollover  $\pm 1$  by reducing the lateral acceleration.

## 2. THE LPV MODEL OF A SINGLE UNIT HEAVY VEHICLE USING AN ACTIVE BRAKING SYSTEM

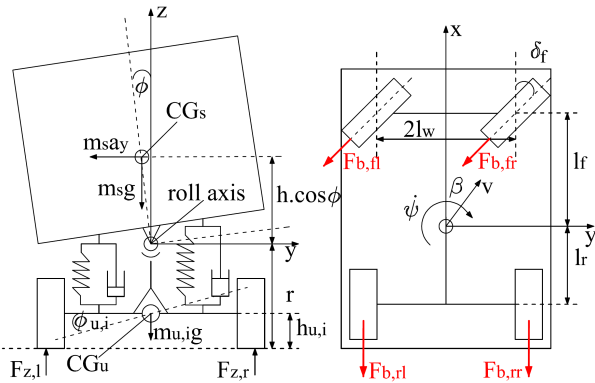


Fig. 1. Vehicle model (Gaspar et al. (2004)).

In this section, the yaw-roll model of a single unit heavy vehicle (two-axle vehicle) is used for studying the active braking system by using the four longitudinal braking forces at each wheel ( $F_{b,fl}$ ,  $F_{b,fr}$ ,  $F_{b,rl}$ ,  $F_{b,rr}$ ). The vehicle model is shown in Figure 1 with the parameters and values presented in (Gaspar et al. (2004)). The braking force  $F_{bij}$ , originating from the brake system and developed on the tyre-road interface is the primary retarding force. When the braking force is below the limit of tyre-road adhesion force, the braking force  $F_{bij}$ , is given by (J.Y.Wong (2001)):

$$F_{bij} = \frac{T_{bij} - \sum I\alpha}{r} \quad (1)$$

where  $T_{bij}$  is the applied brake torque,  $I$  the rotating inertia connected with the wheel being decelerated,  $\alpha$  the corresponding angular deceleration, and  $r$  the rolling radius of the tyre.

The maximum braking force that the tyre-road contact can support is determined by the normal load and the coefficient of road adhesion. With four-wheel brakes, the maximum braking

forces at the front axle are given by (assuming the maximum braking force of the vehicle  $F_{bimax} = \mu G_i$ ):

$$F_{b,flmax} = F_{b,frmax} = \frac{\mu G_f}{2} = \frac{\mu G[l_r + h(\mu + f_r)]}{2L} \quad (2)$$

at the rear axle:

$$F_{b,rlmax} = F_{b,rrmax} = \frac{\mu G_r}{2} = \frac{\mu G[l_f - h(\mu + f_r)]}{2L} \quad (3)$$

where  $\mu$  is the coefficient of road adhesion,  $G$  the vehicle weight,  $G_{f,r}$  the total axle loads,  $f_r$  the rolling resistance coefficient,  $h$  the height of CG from ground,  $L$  the wheelbase,  $l_{f,r}$  the distance from CG to the front and rear axes.

The yaw moment control generated by the active braking system, depends linearly on the difference between the left and right braking forces. In order to simplify building the control models, it is assumed that the braking forces are equal at the front and rear wheels on each side, so  $F_{b,fl} = F_{b,rl}$ ,  $F_{b,fr} = F_{b,rr}$ . Therefore the yaw moment control  $M_z$  is defined as:

$$\begin{cases} M_z = M_{z,rr} + M_{z,rl} \\ M_{z,rr} = F_{b,rr} \left(1 + \frac{\sqrt{l_f^2 + l_w^2}}{l_w} \sin(\arctan(\frac{l_w}{l_f}) - \delta_f)\right) \\ M_{z,rl} = F_{b,rl} \left(1 + \frac{\sqrt{l_f^2 + l_w^2}}{l_w} \sin(\arctan(\frac{l_w}{l_f}) + \delta_f)\right) \end{cases} \quad (4)$$

where  $l_w$  is the half of the vehicle's width,  $\delta_f$  the steering angle. Using the nonlinear functions in equation (4) will result in higher control efficiency, but it complicates the problem. Because the magnitude value of  $\delta_f$  is not too large compared to other parts, so to simplify the control design, we can use the approximate linear formula of  $M_{zj}$  (Gaspar et al. (2004)). The yaw moment (4) can be linearized as follows:

$$M_z = 2l_w(F_{b,rr} - F_{b,rl}) \quad (5)$$

The motion differential equations of the yaw-roll vehicle model using the active braking system are defined as follows:

$$\begin{cases} mv(\dot{\beta} + \dot{\psi}) - m_s h \ddot{\phi} = \mu C_f(-\beta + \delta_f - \frac{l_f \dot{\psi}}{v}) + \mu C_r(-\beta + \frac{l_r \dot{\psi}}{v}) \\ -I_{xz} \ddot{\phi} + I_{zz} \ddot{\psi} = \mu C_f(-\beta + \delta_f - \frac{l_f \dot{\psi}}{v}) l_f - \mu C_r(-\beta + \frac{l_r \dot{\psi}}{v}) l_r \\ + 2F_{b,rr} - 2F_{b,rl} \\ (I_{xx} + m_s h^2) \ddot{\phi} - I_{xz} \ddot{\psi} = m_s g h \phi + m_s v h (\dot{\beta} + \dot{\psi}) - k_f(\phi - \phi_{uf}) \\ - b_f(\dot{\phi} - \dot{\phi}_{uf}) + 4k_{AOf} \frac{t_{A^T B}}{c^2} \phi - 4k_{AOf} \frac{t_A^2}{c^2} \phi_{uf} - k_r(\phi - \phi_{ur}) \\ - b_r(\dot{\phi} - \dot{\phi}_{ur}) + 4k_{AO r} \frac{t_{A^T B}}{c^2} \phi - 4k_{AO r} \frac{t_A^2}{c^2} \phi_{ur} \\ - r \mu C_f(-\beta + \delta_f - \frac{l_f \dot{\psi}}{v}) = m_{uf} v (r - h_{uf}) (\dot{\beta} + \dot{\psi}) \\ + m_{uf} g h_{uf} \phi_{uf} - k_{uf} \phi_{uf} + k_f(\phi - \phi_{uf}) + b_f(\dot{\phi} - \dot{\phi}_{uf}) \\ + 4k_{AOf} \frac{t_{A^T B}}{c^2} \phi - 4k_{AOf} \frac{t_A^2}{c^2} \phi_{uf} \\ - r \mu C_r(-\beta + \frac{l_r \dot{\psi}}{v}) = m_{ur} v (r - h_{ur}) (\dot{\beta} + \dot{\psi}) - m_{ur} g h_{ur} \phi_{ur} \\ - k_{ur} \phi_{ur} + k_r(\phi - \phi_{ur}) + b_r(\dot{\phi} - \dot{\phi}_{ur}) \\ + 4k_{AO r} \frac{t_{A^T B}}{c^2} \phi - 4k_{AO r} \frac{t_A^2}{c^2} \phi_{ur} \end{cases} \quad (6)$$

where  $k_{AOf,r}$  are respectively the torsional stiffness of the anti-roll bar at the two axles,  $t_A$  half the distance of the two suspensions,  $t_B$  half the distance of the chassis and  $c$  the length of the anti-roll bar's arm (Vu et al. (2017)).

It is assumed that the driving throttle is constant during a lateral manoeuvre and the forward velocity depends only on the braking forces. The differential equation of the forward velocity is:

$$m\dot{v} = -2F_{b,rr} - 2F_{b,rl} \quad (7)$$

It is worth noting that the motion differential equation of the vehicle model depends on the forward velocity and its inverse, moreover, the forward velocity is a constantly changing parameter. Therefore in this case the forward velocity is chosen as a varying parameter. We also would like to emphasize here that, considering the forward velocity is a varying parameter, it is extremely important for vehicle brake system. Because during the braking process, this parameter changes very quickly, this cannot be met with the lower-level control methods. The motion differential equation can be written in the following LPV state-space representation:

$$\dot{x} = A(\rho_1).x + B_1(\rho_1).w + B_2(\rho_1).u \quad (8)$$

where  $\rho_1 = v$  is the varying parameter, the state vector  $x = [\beta \ \psi \ \phi \ \dot{\phi} \ \phi_{uf} \ \phi_{ur} \ v]^T$ , the exogenous disturbance  $w = [\delta_f]^T$ , and the control input  $u = [F_{b,rr} \ F_{b,rl}]^T$ .

### 3. THE PROPOSED $H_\infty$ /LPV ACTIVE BRAKING SYSTEM

The main objective of the active braking control system is to maximize the vehicle roll stability to prevent a rollover phenomenon in an emergency. The most important criteria is used to assess the vehicle roll stability being the normalized load transfer at the two axles  $R_{f,r}$ , which is defined as follows (Hsun-Hsuan et al., 2012):

$$R_f = \frac{k_{uf}\phi_{uf}}{G_f l_w}, \quad R_r = \Delta F_{zr} = \frac{k_{ur}\phi_{ur}}{G_r l_w} \quad (9)$$

where  $G_{f,r}$  are the total axle loads,  $k_{uf,r}$  the stiffness of the tyres,  $\phi_{uf,r}$  the roll angles of the unsprung masses at the two axles,  $l_w$  the half of the vehicle's width.

The normalized load transfer  $R_{f,r} = \pm 1$  value corresponds to the largest possible load transfers. The roll stability is achieved by limiting the normalized load transfers within the levels corresponding to wheel lift-off.

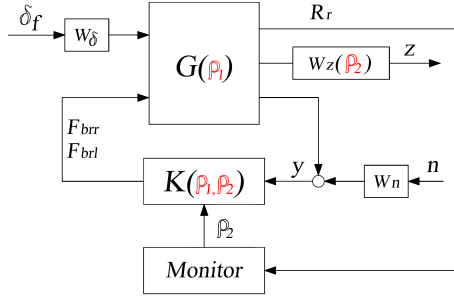


Fig. 2. The closed-loop interconnection structure.

According to the control objective, we propose a control scheme for the active braking system of heavy vehicles as shown in Figure 2. The  $H_\infty$ /LPV control structure includes the nominal model  $G(\rho_1)$ , the controller  $K(\rho_1, \rho_2)$ , the performance output  $z$ , the control input  $u$ , the measured output  $y$ , the measurement noise  $n$ . The steering angle  $\delta_f$  is the disturbance signal, set by the driver.  $W_\delta, W_z(\rho_2), W_n$  are the weighting functions for the steering angle, the performance output, the noises. The use of the two varying parameters ( $\rho_{1,2}$ ) has an important implication for adapting to vehicle's situation. The role of  $\rho_1$  is to help the vehicle model and the controller change according to the forward velocity, which is of great significance at high motion speeds because the dynamic vehicle characteristics are far different when moving at low motion speeds. Meanwhile we select the value of the varying parameter  $\rho_2$  depending on the

absolute value of the normalized load transfer at the rear axle. Because vehicle rollover is affected by the suspension stiffness to load ratio, so the wheels at the rear axle often lift off first. This aims to satisfy the two following goals: the first goal is to design the the parameter dependent weighting function to allow performance adaptation to the rollover risk of vehicle, the second goal is to design the monitor to allow the active braking system to be only activated when the vehicle reaches the critical rollover situation. This approach can ensure the active braking system works closest to the real situation of vehicles.

### 4. THE BRAKING MONITOR DESIGN

In the previous section, the proposed  $H_\infty$ /LPV active braking system aims to prevent vehicle rollover; the varying parameter  $\rho_2$  is changed according the absolute value of the normalized load transfer at the rear axle ( $\rho_2 = f(|R_r|)$ ). According to Figure 2, in order to satisfy simultaneously the improved vehicle performance and prevention of vehicle rollover in an emergency situation (critical state), in this section we propose the two following LPV Braking Monitor Designs.

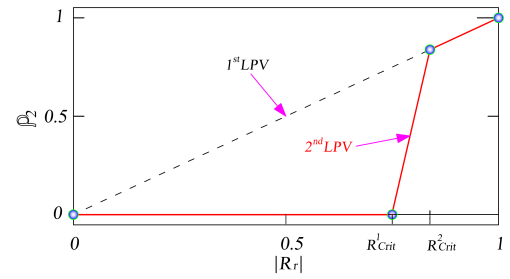


Fig. 3. The varying parameter  $\rho_2 = f(|R_r|)$ .

#### 4.1 The first braking monitor design (1<sup>st</sup> LPV):

The value of the varying parameter  $\rho_2$  is always equal to the absolute value of the normalized load transfer at the rear axle  $\rho_2^1 = |R_r|$ . It means that, the active braking system always operates even when the lateral acceleration is small. This is unrealistic, because when the active braking system is in use, it will reduce the various vehicle performance characteristics, which are highlighted by the increased energy consumption, the reduced longevity of the engine, the increased wear of the tyres and the brake actuators. The main purpose of this design is to evaluate the adaptability of the LPV active braking system when the varying parameter receives continuously the value from the roll stability criteria.

#### 4.2 The second braking monitor design (2<sup>nd</sup> LPV):

The previous research results have demonstrated that most of heavy vehicles are equipped with the passive anti-roll bar system to maintain roll stability. This passive system does not consume energy and its structure is quite simple. However, the main problem we need to consider is that when the vehicle is in a critical state (approaching to the threshold of rollover), the passive anti-roll bar system does not provide enough of the necessary restoring torque. This requires the addition of the anti-roll torque of proactive systems in critical states, not in normal states.

Moreover, vehicle rollover often occurs when the lateral acceleration reaches around 0.5g (Palkovics et al. (1999)). So to fit better with the real world application, the active braking system should be only activated when the vehicle reaches the critical rollover situation to reduce the lateral acceleration. The

objective of this braking monitor is to allow the active braking system to react when the normalized load transfer at the rear axle reaches the criteria of rollover  $\pm 1$ . Therefore, in this design the varying parameter  $\rho_2$  is chosen as follows:

$$\rho_2^2 = \begin{cases} 0 & \text{when } |R_r| \leq R_{Crit}^1 \\ |R_r| \frac{|R_r| - R_{Crit}^1}{R_{Crit}^2 - R_{Crit}^1} & \text{when } R_{Crit}^1 < |R_r| < R_{Crit}^2 \\ |R_r| & \text{when } |R_r| \geq R_{Crit}^2 \end{cases} \quad (10)$$

The values of  $R_{Crit}^1$  and  $R_{Crit}^2$  are chosen so that they respond to the emergency situation, satisfying the time delay of the braking system and limiting the influence of the switch point. Here, the authors propose  $R_{Crit}^1 = 0.75$  and  $R_{Crit}^2 = 0.8$ .

To better understand the value of the varying parameter  $\rho_2$  in the two cases (the 1<sup>st</sup> LPV and 2<sup>nd</sup> LPV braking monitor designs), let us look at the diagram in Figure 3. In the 1<sup>st</sup> LPV braking monitor design, the value of  $\rho_2$  is always equal to the absolute value of the normalized load transfer at the rear axle. Meanwhile the 2<sup>nd</sup> LPV braking monitor design, when the absolute value of the normalized load transfer at the rear axle is less than 0.75, the vehicle roll stability is determined by the passive anti-roll bar system, the active braking system is only activated when this value is greater than 0.75, meaning when the vehicle is close to the critical rollover situation.

## 5. THE $H_\infty$ /LPV CONTROL DESIGN

### 5.1 Performance specifications for the $H_\infty$ /LPV control design

The purpose of choosing weighting functions and performance output in Figure 2 is to enhance vehicle roll stability and avoid actuator saturation, over the desired frequency range up to 4rad/s, which represents the limited bandwidth of the driver (Gaspar et al. (2004), Sampson and Cebon (2003)). These weighting functions can be considered as penalty functions, that is, weights which should be large in the frequency range where small signals are desired and small where larger performance outputs can be tolerated. According to the considered performance objectives, the weighting functions  $W_z(\rho_2) = \text{diag}[W_{zFb,rr}, W_{zFb,rl}, W_{zay}(\rho_2)]$  are selected as follows:

$$\begin{cases} W_{zFb,rr} = \frac{1}{\zeta_{Fb,rr}}, & W_{zFb,rl} = \frac{1}{\zeta_{Fb,rl}}, \\ W_{zay}(\rho_2) = \rho_2 \frac{s\chi_{ay} + 2}{s\vartheta_{ay} + \tau_{ay}} \end{cases} \quad (11)$$

Here, the parameters in equations (11) are chosen as:  $\zeta_{Fb,rr} = \zeta_{Fb,rl} = 8270$ ;  $\chi_{ay} = 0.5$ ;  $\vartheta_{ay} = 0.2$ ;  $\tau_{ay} = 90$ .

The varying parameter is defined as  $\rho_2 = f(|R_r|)$ . We stress that the interest of parameter dependant weighting functions  $W_{zay}(\rho_2)$  is to allow performance adaptation to the rollover risk of heavy vehicles. For example, as far as the normalized load transfer at the rear axle is concerned, when the varying parameter  $\rho_2 \rightarrow 1$ , the gain of the weighting function  $W_{zay}$  is large, and therefore the lateral acceleration is penalized, this leads to vehicle roll stability improvement.

The weighting function for the steering angle is selected as  $W_\delta = \pi/180$  in order to manage the maximum expected command. The weighting function for sensor noise models in the control design  $W_n$  is selected as a diagonal matrix of  $0.01(m/s^2)$  for the lateral acceleration  $a_y$  and  $0.01(^{\circ}/sec)$  for the derivative of the roll angle  $\dot{\phi}$  (Gaspar et al., 2004).

### 5.2 The solution of the $H_\infty$ /LPV control problem

According to Figure 2, the concatenation of the nonlinear model (8) with the performance weighting functions has a partitioned

representation in the following form:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho) & B_1(\rho) & B_2(\rho) \\ C_1(\rho) & D_{11}(\rho) & D_{12}(\rho) \\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho) \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} \quad (12)$$

where the performance output vector  $z(t) = [a_y \ F_{b,rr} \ F_{b,rl}]^T$ , the exogenous input  $w(t) = [\delta_f \ n]$ , the control input  $u(t) = [F_{b,rr} \ F_{b,rl}]$ , the measured output vector  $y(t) = [a_y \ \dot{\phi}]^T$ .

The LPV model of the active braking system (12) uses the varying parameters  $\rho = [\rho_1, \rho_2]$ , which are known in real time. The parameter  $\rho_1 = v$  is measured directly, while the parameter  $\rho_2 = f(|R_r|)$  can be estimated by the unsprung mass at the rear axle  $\phi_{ur}$ .

The  $H_\infty$ /LPV controller in Figure 2 is defined in the form (13). The control goal is to minimize the induced  $\mathcal{L}_2$  norm of the closed-loop LPV system  $\Sigma_{CL} = LFT(G, K)$ , with zero initial conditions, which is shown in equation (14).

$$\begin{bmatrix} \dot{x}_K(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_K(\rho) & B_K(\rho) \\ C_K(\rho) & D_K(\rho) \end{bmatrix} \begin{bmatrix} x_K(t) \\ y(t) \end{bmatrix} \quad (13)$$

where  $A_K(\rho)$ ,  $B_K(\rho)$ ,  $C_K(\rho)$ ,  $D_K(\rho)$  are continuous bounded matrix functions.

$$\|\Sigma_{CL}(\rho)\|_{2 \rightarrow 2} = \sup_{\substack{\rho \in \mathcal{P} \\ \forall \rho \leq \nu}} \sup_{\substack{w \in \mathcal{L}_2 \\ \|w\|_2 \neq 0}} \frac{\|z\|_2}{\|w\|_2} \quad (14)$$

The existence of a controller that solves the parameter dependent LPV  $\gamma$ -performance problem can be expressed as the feasibility of a set of linear matrix inequalities (LMIs), which can be solved numerically.

In this study, the authors use the grid-based LPV approach and the LPVTools<sup>TM</sup> presented in (Hjartarson et al. (2015)) to synthesize the  $H_\infty$ /LPV active braking control system. It requires a gridded parameter space for the two varying parameters  $\rho = [\rho_1, \rho_2]$ . The  $H_\infty$  controllers are synthesized for 12 grid points of the forward velocity in the range  $\rho_1 = v = [20km/h, 130km/h]$  and 5 grid points of the normalized load transfer at the rear axle in a range  $\rho_2 = f(|R_r|) = [0, 1]$ .

## 6. PERFORMANCE ASSESSMENT USING SIMULATION

In this section, the simulation results of a single unit heavy vehicle using the active braking system are shown in the time domain. The proposed 2<sup>nd</sup> LPV braking monitor design (dash-dotted line) is compared with the passive anti-roll bar system (solid line) and the case of the 1<sup>st</sup> LPV braking monitor design (dashed line). The parameter values of the vehicle model are given in (Gaspar et al. (2004)). The double lane change manoeuvre to avoid an obstacle is used in this scenario, with the steering angle as shown in Figure 4a.

Figure 4 shows the time response of the vehicle. The initial forward velocity is 100 km/h as in Figure 4b. In the case of the passive anti-roll bar system, the forward velocity is kept constant at 100 km/h because there is no resistive force to reduce it. In the case of the 1<sup>st</sup> LPV braking monitor design, the forward velocity reduces continuously by 19 km/h from 0.5s to 6s. However, in the case of the 2<sup>nd</sup> LPV braking monitor design, the forward velocity only decreases when the normalized load transfer at the rear axle reaches its limitation with the reductions of 12 km/h.

Figures 4e, f show the normalized load transfer at the two axles  $R_{f,r}$ . In the case of the 2<sup>nd</sup> LPV braking monitor design, the maximum absolute value of the normalized load transfer at the two axles is less than 1, this means that with this design the vehicle can avoid rollover. Additionally, in the case of the

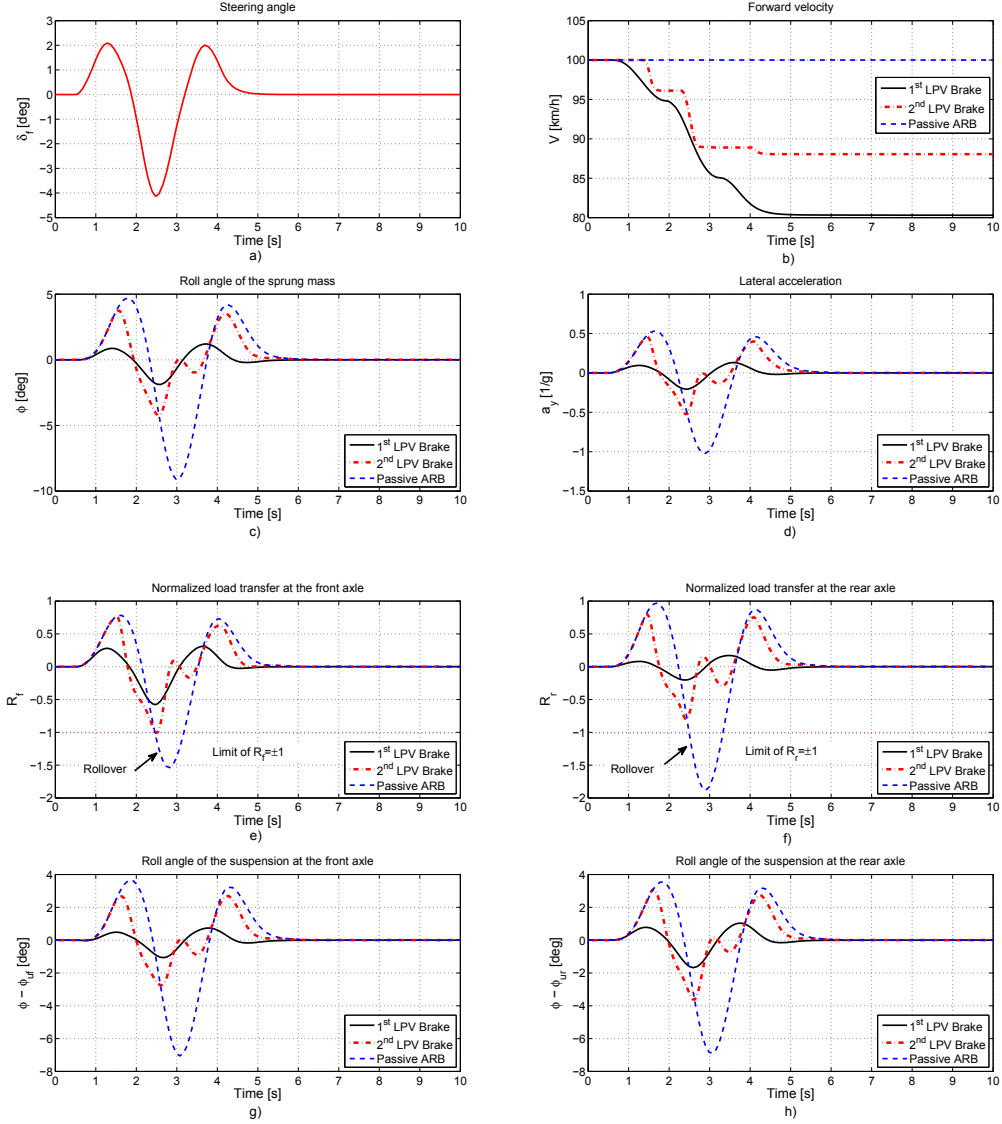


Fig. 4. Time response of the vehicle.

passive anti-roll bar system, the normalized load transfers at the two axles exceed the limitation of  $\pm 1$ . So the use of the passive anti-roll bar in this scenario is not enough to prevent vehicle rollover.

The simulation results indicate that, due to the braking monitor, the active braking system can be activated when the vehicle comes close to the rollover situation. In the normal situation, the vehicle can manoeuvre as in the case of using the passive anti-roll bar system. Therefore, the lateral acceleration  $a_y$  in the case of the  $2^{nd}$  LPV braking monitor design is kept to less than the limitation of  $0.5g$  (Palkovics et al. (1999)) as shown in Figure 4d. The maximum absolute value of the roll angle of sprung mass, lateral acceleration, suspension roll angles and normalized load transfers at both axles are listed in Table 1 for the two cases of the  $H_\infty/LPV$  designs and the passive anti-roll bar system. From Figure 4 and Table 1, we can see that the  $2^{nd}$  LPV braking monitor design behaves better than the passive anti-roll bar system in term of avoiding vehicle rollover in a emergency situation. And it does not affect the various vehicle performance characteristics as in the case of the  $1^{st}$  LPV braking

monitor design.

Table 1. Maximum absolute value of the signals.

Signals	Passive anti-roll bar	$1^{st}H_\infty/LPV$	$2^{nd}H_\infty/LPV$
$ \phi _{max}[deg]$	9.02	1.88	4.23
$ a_y _{max}[1/g]$	1.02	0.21	0.50
$ R_f _{max}$	<b>1.54</b>	0.57	0.99
$ R_r _{max}$	<b>1.88</b>	0.20	0.79
$ \phi - \phi_{ur} _{max}[deg]$	<b>7.06</b>	1.07	2.75
$ \phi - \phi_{ur} _{max}[deg]$	6.9	1.68	3.62

Figures 5a,b show the time response of the wheel braking forces as the control input. It shows that, the time to maintain the braking force in the  $1^{st}$  LPV is very large (during the entire period of vehicle manoeuvre - the lateral acceleration is different from zero). As mentioned above, this will affect the vehicle performance characteristics.

Besides the roll stability criteria, it is necessary to evaluate the vehicle handling performance by using the phase plane  $\beta - \dot{\beta}$  ( $|\dot{\beta} + k_{\beta\beta}\beta| < b$ ) and the stability index  $\lambda$  ( $\lambda = |2.39\dot{\beta} + 9.55\beta|$ ), where  $b = 24$  and the slope of the reference region

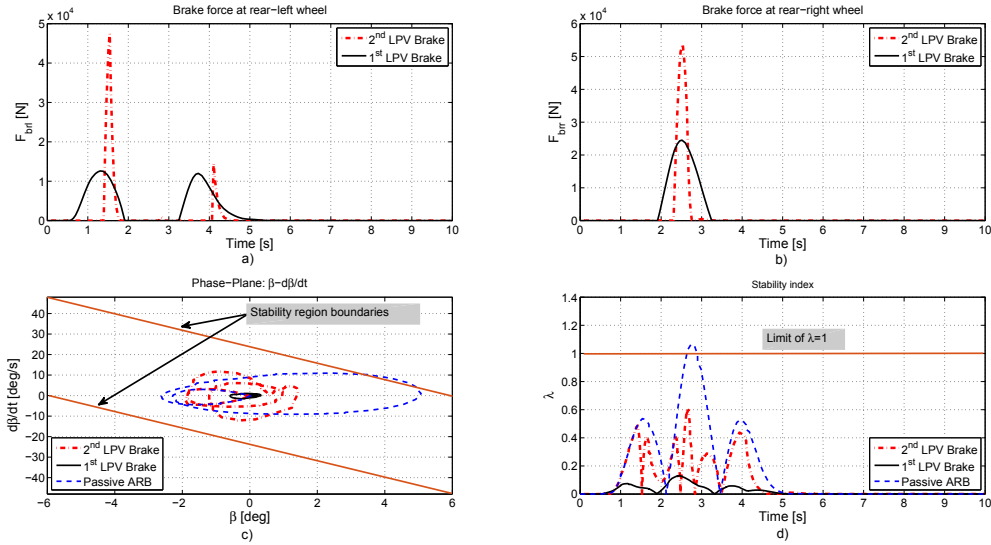


Fig. 5. Time responses of the (a,b) brake forces  $F_{br,l,r}$ , (c) phase - plane:  $\beta - \dot{\beta}$  and (d) stability index  $\lambda$ .

boundaries  $k_{\beta\dot{\beta}} = 4$  (Vu et al. (2017)). From Figure 5c,d, we can see that the handling performance is not satisfied for the passive anti-roll bar system. However, for the 2<sup>nd</sup> LPV braking monitor design, the trajectory of the phase plane  $\beta - \dot{\beta}$  is in the stability region boundaries and the stability index  $\lambda$  is still inside the limitation of 1. Therefore we can see that the vehicle handling performance is satisfied with the two proposed braking monitor designs. Therefore overall, the 2<sup>nd</sup> LPV can maintain the objective of preventing vehicle rollover situation and improving vehicle handling performance.

## 7. CONCLUSION

In this paper, the active braking system is studied with the four longitudinal braking forces at each wheel by using the yaw-roll model of a single unit heavy vehicle. The grid-based LPV approach is combined with the LPVTools<sup>TM</sup> toolbox to synthesize the  $H_{\infty}$ /LPV parameter dependant weighting function controller. The braking monitor designs are proposed to allow the active braking system to react when the normalized load transfer at the rear axle reaches the criteria of rollover  $\pm 1$ . The simulation results indicate that the 2<sup>nd</sup> LPV braking monitor design satisfies simultaneously the adaptation of the vehicle to rollover in an emergency situation, with low braking forces and improved handling performance of the vehicle. Therefore, in order to avoid vehicle rollover, the braking monitor design is an effective solution in studying the active braking system.

In the future, the characteristics of the brake actuators will be combined with the yaw-roll model. Studies on the nonlinear vehicle model and the comparison using nonlinear vehicle simulation packages (such as TruckSim) are also interesting topics. **Acknowledgements:** This work was supported by the project ‘‘Intelligent and autonomous vehicles for improving transport and road safety in Vietnam’’ from the French Embassy in Vietnam.

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