Supplemental Analysis

Abstract

This document contains some supplemental material for the note "Restrictions in Spatial Competition: The Effects on Firms and Consumers." We introduce and discuss a particular model to substantiate the assertion made in the note that the novel cases can also emerge when restrictions are endogenously, not exogenously, determined. In the note we refer to this material in Section 4 in footnote 11.

A Model with Endogenous Choice of Restrictions

We consider here a simple variant of the standard market entry game. Positions X and consumers are as specified in the note. There are now two periods $t = 0, 1$. In period 0 a firm F1 is a monopolist in the market. In period $t = 1$ either F1 stays monopolist or a second firm F2 enters. The sequence of actions is as follows. (i) F1 chooses a set of feasible strategies S^1 and some initial position $s_0^1 \in S^1$. (ii) F2 chooses whether to enter the market and if it enters it chooses some position $s^2 \in X$.[1] (iii) F1 chooses a position $s^1 \in S^1$ to compete against F2. In a monopoly situation F1 serves all consumers, in a duopoly with $s^1 \neq s^2$ consumers are split as in the static model. If $s^1 = s^2$, we assume that all consumers stay at the incumbent and do not switch to the entrant.[2] One interpretation for this assumption is that consumers face small switching costs which cause inertia. Let $\pi : [0,1] \to \mathbb{R}_+$ be a continuously increasing function that assigns a profit to any mass of consumers. Moreover, let $0 < f^{entry} < \pi(\frac{1}{2})$ be the fixed costs of market entry. For F1, let $C : [0,1] \to \mathbb{R}_+$ be an increasing function that represents the costs of flexibility. We assume that the larger the range $[\underline{s}^1, \bar{s}^1]$, the higher these costs. Moreover, let $\delta \in (0,1]$ be F1's discount factor. The payoffs of F1 and F2 are then

$$\Pi^1, \Pi^2 = \begin{cases} \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1 - F(\hat{x})), \quad \pi(F(\hat{x})) - f^{entry}, & \text{if } s^2 < s^1 \\ \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(F(\hat{x})), \quad \pi(1 - F(\hat{x})) - f^{entry}, & \text{if } s^2 > s^1 \\ \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1), \quad \pi(0) - f^{entry}, & \text{if } s^2 = s^1 \\ \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1), \quad \pi(0), & \text{if no-entry.} \end{cases}$$

We now derive a subgame perfect Nash equilibrium (SPNE) by backward induction. Because of the open set issue this will be a "perfect epsilon-equilibrium" (Radner, 1980).

(iii) If F2 does not enter, then the choice $s^1 \in S^1$ is arbitrary. If F2 enters and $s^2 \in S^1$, then $s^1 = s^2$ is profit maximizing (because then F1 receives all consumers). If F2 enters and $s^2 \notin S^1$, then $s^1 = \underline{s}^1$ when $s^2 < \underline{s}^1$ and $s^1 = \bar{s}^1$ when $s^2 > \bar{s}^1$ is profit maximizing for F1.

[1] After observing F1's first move, there would be no incentive to build a strategy set that consists of more than one position.

[2] The result for the convention that the two firms split the market equally will be trivially that both firms choose the median. This observation stays true in the model variation, where firms first simultaneously choose feasible strategies and then simultaneously choose positions within their feasible set.

(ii) Given optimal behavior of F1 in decision (iii), we receive the following payoffs for different decisions of F2 at stage (ii):

$$\Pi^2 = \begin{cases} \pi(0) - f^{entry}, & \text{if } s^2 \in S^1 \\ \pi(F(\underline{s}^1 - \frac{\varepsilon}{2})) - f^{entry}, & \text{if } s^2 = \underline{s}^1 - \varepsilon \\ \pi(1 - F(\bar{s}^1 + \frac{\varepsilon}{2})) - f^{entry}, & \text{if } s^2 = \bar{s}^1 + \varepsilon \\ \pi(0), & \text{if F2 does not enter} \end{cases}$$

for $\varepsilon > 0$. Choosing $s^2 \in S^1$ is strictly dominated by not entering. In the two central cases, the payoff of F2 is decreasing in ε. Thus, we have an open set problem as in Case (IIb) of the short-term analysis. The supremum here is $F(\underline{s}^1)$ respectively $1 - F(\bar{s}^1)$ and it can be approached by letting ε shrink. Therefore F2 enters if

$$\pi(\max\{F(\underline{s}^1), 1 - F(\bar{s}^1)\}) > f^{entry} \tag{1}$$

and chooses a sufficiently small ε. Otherwise, i.e. if Condition (1) does not hold, F2 does not enter.

(i) To derive the optimal behavior of F1 in stage (i), we distinguish between the best entry deterring and the best entry admitting choice. Anticipating the behavior in stage (ii) and (iii) a strategy set S^1 is entry deterring if $\pi(F(\underline{s}^1)) \leq f^{entry}$ and $\pi(1 - F(\bar{s}^1)) \leq f^{entry}$. Let $\underline{y} := F^{-1}(\pi^{-1}(f^{entry}))$, i.e. the rightmost position that still does not allow for profitable entry to the left and, similarly, $\bar{y} := F^{-1}(1 - \pi^{-1}(f^{entry}))$.[3] Then the best entry deterring choice is $S^1 = [\underline{y}, \bar{y}]$. Note that $\underline{y}$ is increasing in f^{entry}, i.e. the larger the entry costs, the smaller the necessary flexibility to deter entry.

The best choice of S^1 given that F2 enters is the solution to the following maximization problem:

$$\max_{\underline{s}^1, \bar{s}^1} \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1 - \max\{F(\underline{s}^1), 1 - F(\bar{s}^1)\}). \tag{2}$$

Since any choice such that $F(\underline{s}^1) \neq 1 - F(\bar{s}^1)$ is a "waste" of flexibility costs, we have in equilibrium $F(\underline{s}^1) = 1 - F(\bar{s}^1)$. Thus, we can substitute $\underline{s}^1 = F^{-1}(1 - F(\bar{s}^1))$ to rewrite the maximization problem in dependence of one variable only:

$$\max_{\bar{s}^1 \in [q, \bar{y}]} \pi(1) - C(\bar{s}^1 - F^{-1}(1 - F(\bar{s}^1))) + \delta\pi(F(\bar{s}^1)) \tag{3}$$

A choice $\bar{s}^1 > \bar{y}$ is excluded by assumption because it deters entry and the last profit is the simplification of $\pi(1 - (1 - F(\bar{s}^1))$.

This maximization problem (3) incorporates the trade-off between leaving few consumers for a potential entrant (large $\bar{s}^1$) and saving flexibility costs (small $\bar{s}^1$). The solution to this problem depends on the specifications of the cost function C, of the entry costs f^{entry}, of the payoff function π, and of the distribution of consumers F, but it certainly exists because we maximize a continuous function over a compact set. Let $\bar{z}$ be a solution to this problem (3), be it an interior solution ($\bar{z} \in (q, \bar{y})$) or a boundary solution ($\bar{z} = q$ or $\bar{z} = \bar{y}$). Let $\underline{z} := F^{-1}(1 - F(\bar{z}))$. Then F1's profit maximizing behavior under entry and no-entry of F2 leads to the following payoffs:

$$\Pi^1 = \begin{cases} \pi(1) - C(\bar{y} - \underline{y}) + \delta\pi(1), & \text{if } S^1 = [\underline{y}, \bar{y}] \\ \pi(1) - C(\bar{z} - \underline{z}) + \delta\pi(F(\bar{z})), & \text{if } S^1 = [\underline{z}, \bar{z}] \end{cases}$$

[3] F and π are strictly increasing continuous functions such that they can be inverted.

The specific functional forms determine which choice leads to higher payoff and, hence, F1's choice in stage (i). Inspecting the two equilibrium payoffs above reveals that entry deterrence becomes relatively more attractive for lower costs of flexibility, for higher costs of entry, and for a larger discount factor. In Example 1 we illustrate how these model parameters determine the equilibrium path.

From the backward induction exercise we learn first of all that there always exists a subgame perfect epsilon-equilibrium. Moreover, there are two types of these equilibria, one entry admitting one entry deterring, which both satisfy the following two properties.

(a) $q \in S^1 \subseteq [\underline{y}, \bar{y}]$, i.e. F1 chooses a feasible set at the center of the market within certain boundaries and

(b) $F(\underline{s}^1) = 1 - F(\bar{s}^1)$, i.e. the 'niches' left for F2 at both sides of the center are of equal size.

In the entry deterring equilibrium, (i) F1 chooses $S^1 = [\underline{y}, \bar{y}]$, (ii) F2 does not enter, and (iii) F1's final position s^1 is arbitrary within S^1 because it acts as monopolist. An entry deterrent F1 gains $\pi(1) - C(\bar{y} - \underline{y}) + \delta\pi(1)$. Thus, it has the cost of flexibility $C(\bar{y} - \underline{y})$ to keep a threat to potential entrants. This is similar to a threat of a price war, but this threat is credible because after investments into flexibility have been made, a 'minimal differentiation war' is costless in our model. Welfare depends on the exact location of $s^1 \in S^1$ since the closer s^1 to the median, the smaller the total transportation costs. Thus, the size of the feasible set S^1 not only determines the cost of flexibility, but also provides an upper bound for the transportation costs. Since the size of F1's restriction is increasing in F2's costs of entry f^{entry}, entry barriers might even be considered as welfare enhancing.[4] Similarly, low marginal costs of flexibility increase the set of feasible positions S^1 and thus relax the upper bound of transportation costs. By property (a) this boundary for total transportation costs also applies to the entry admitting equilibrium.

The entry admitting equilibrium path is as follows: (i) F1 chooses $S^1 = [\underline{z}, \bar{z}]$ such that $\underline{z}$ solves (3), i.e. it optimizes the trade-off between low costs of flexibility and a large market share; (ii) F2 enters and chooses an adjacent position to F1's restriction, i.e. $s^2 = \underline{s}^1 - \varepsilon$, respectively $s^2 = \bar{s}^1 + \varepsilon$; and (iii) F1 reacts with choosing its restriction adjacent to s^2, i.e. $s^1 = \underline{z}$ or $s^1 = \bar{z}$. Observe that the outcome of this dynamic model corresponds to Case (IIb) of the static analysis, where F1 is in the role of the more-central player. We discussed in Subsection 3.4 of the note that this is the case with potentially high inequality and low welfare. In given examples, the specific inequality and the total transportation costs are determined by the size of the interval S^1 such that we get the following comparative static effects. Both equality of firms' payoffs and welfare are increasing in F1's marginal costs of flexibility (called c in Example 1 below) and in F1's discount factor δ. In the worst situation, F1 values the second period highly ($\delta = 1$), while flexibility is relatively cheap. Then it chooses a large feasible set S^1 with only small niches left for F2 such that market shares are highly unequal, while consumers' transportation costs are large because two similar products away from the center of the market are offered. Of course, this can only be an entry admitting equilibrium if F2's costs of entry f^{entry} are sufficiently low.

[4] The intuition is that low costs of entry lead to costly investments into flexibility that allow the incumbent to offer products which are not close to the center of the market.

To study how costs of market entry and other model parameters determine which equilibrium is played and to illustrate further comparative static effects, we use a specific example for which an explicit solution can be easily obtained.

Example 1. *Consider the special case of uniform distribution of consumers, i.e.* $F(x) = x$*, quadratic costs of flexibility, i.e.* $C(r) = cr^2$ *with cost parameter* c*, and linear payoff function, i.e.* $\pi(a) = a$*. From* (1) *we get that F2 enters if* $\max\{\underline{s}^1, 1 - \bar{s}^1\} > f^{entry}$*. Moreover, let* $f^{entry} < \frac{3}{8}$*, which in this case (*π *is the identity function) can be interpreted as the market share that is necessary to make market entry profitable. F1 can optimally deter entry by choosing* $\underline{s}^1 = \underline{y} = f^{entry}$ *and* $\bar{s}^1 = \bar{y} = 1 - f^{entry}$*. The optimal choice of F1 given that F2 enters is the solution to the maximization problem (cf.* (2)*), which simplifies to*

$$\max_{\bar{s}^1 \in [\frac{1}{2}, 1 - f^{entry}]} 1 - c(2\bar{s}^1 - 1)^2 + \delta \bar{s}^1. \tag{4}$$

Analogous to Eq. (3)*, the main idea of the simplification is that best actions satisfy here* $\underline{s}^1 = 1 - \bar{s}^1$*. If* $f^{entry} > \frac{1}{2} - \frac{\delta}{8c}$*, then we have the boundary solution* $\bar{z} = \bar{y} = 1 - f^{entry}$ *and* $\underline{z} = \underline{y} = f^{entry}$*. In that case entry admission is never profitable and we have the entry deterring equilibrium. On the other hand, if* $f^{entry} \leq \frac{\delta}{8c}$*, then the unique solution to this maximization problem is* $\bar{z} = \frac{1}{2} + \frac{\delta}{8k}$*. In that case we have to compare the payoff of F1 under the optimal entry admitting choice* $S^1 = [\frac{1}{2} - \frac{\delta}{8k}, \frac{1}{2} + \frac{\delta}{8k}]$ *with the payoff of the optimal choice that deters entry* $S^1 = [f^{entry}, 1 - f^{entry}]$*. Low enough entry costs* f^{entry}*, high marginal costs of flexibility* c*, as well as low enough valuation of the future* δ*, make the entry admitting choice of restrictions more profitable than entry deterrence.*

For instance, for $c = 1$ *and* $\delta = 0.8$*, F1 prefers to admit entry of F2 if* $f^{entry} < \frac{1}{5}$*, i.e. if the required market share to make entry profitable is below* 20%*. In that case we get the following equilibrium path: (i) F1 chooses* $S^1 = [0.4, 0.6]$ *and* $s_0 \in S^1$ *arbitrary, e.g.* $s_0 = 0.5 = q$*. F2 enters with strategy* $s^2 = 0.4 - \varepsilon$ *(or with* $s^2 = 0.6 + \varepsilon$*) for some small* $\varepsilon > 0$*. F1 reacts with* $s^1 = 0.4$ *(respectively,* $s^1 = 0.6$*). The market share of F1 is approximately* 60%*, while F2 receives approximately* 40%*. The outcome is inefficient for two reasons. First costly investments into flexibility are not justified by some welfare benefit. Second, F1 locates at the position within* S^1 *that actually maximizes total transportation costs.*

There is an alternative interpretation for the model set-up of this section. Consider the incumbent's investment into flexibility as investment into patents that protect its initial product s_0^1. Specifically, the choice $S^1 = [\underline{s}^1, \bar{s}^1]$ can be interpreted as restricting the feasible strategies of a potential entrant, i.e. F2's strategy set is restricted to $X \setminus S^1 = [0, \underline{s}^1) \cup (\bar{s}^1, 1]$. The model results in an entry deterring or an entry admitting equilibrium as described above.

References

Radner, R. (1980). Collusive behavior in noncooperative epsilon-equilibria of oligopolies with long but finite lives. *Journal of Economic Theory*, **22** (2), 136–154.