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ASYMMETRIC INFORMATION AND ENDOGENOUS SIGNALLING THE CASE OF UNKNOWN INTERCEPT AND RANDOM OUTPUT

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Introduction

In economic models in which agents are asymmetrically informed about the structural parameters of the economy the acquisition or manipulation of information plays a crucial role. The incentive to affect the flow of information is especially important in models in which choice variables, and hence market variables, generate information used for future decisions. Examples of situations in which the flow of information is generated through endogenous (e.g., market) variables are numerous in economics. In these situations economic agents adjust their (myopically) optimal decisions in order to affect the flow of information. In this paper we study two different reasons why agents might change their (myopically) optimal decisions when they take account of the informational content of their decisions. The first is when informed agents manipulate the informational content of observed market variables through their own decisions in order to influence the learning of uninformed agents. The second is when uninformed agents "experiment" in order to influence the flow of information on which their own future decisions are based.

This paper presents a model in which both of these possibilities - the manipulation of information and experimentation - are present. We consider a duopoly model with asymmetrically informed agents. In order to study the simplest case a two period model is used so that information generated in the first period may be used in the second period. The market is characterized in each period by the same linear demand function. The intercept term of the demand curve is assumed to be stochastic. We use the Bayesian Nash equilibrium concept and study properties of a separating equilibrium.

The information is asymmetric in that each of the firms has different information about the stochastic intercept. In particular, one firm - the informed firm - knows the expected value of the intercept term while the other - the uninformed firm - knows only that the expected value of the intercept term can be one of two possible values. The uninformed firm has a subjective prior distribution over the pair of possible values.

In the first period each firm chooses a quantity which maximizes the expected sum of profits. Second period profitability for both firms depends upon the subjective beliefs of the uninformed firm in the second period, i.e., on the posterior distribution of the expected value of the intercept of the demand function. This posterior distribution, in turn, depends upon the quantity decisions of both firms in the first period as well as on the observation of the random price.

The essence of the problem is that the output decisions made by both firms in the first period imply a distribution - through the random intercept term - of prices. To the uninformed firm two possible price distributions are implied, one for each of the two possible values of the unknown parameter. Since the randomness of the intercept term obscures the true value of its mean, in a separating equilibrium, price observations do not reveal the true value of the unknown parameter. Hence an incentive is created for the informed firm to manipulate information through its quantity decision. This manipulation is designed to make the uninformed firm believe that the market warrants a small output (leaving more profit for the informed firm). The potential also exists for the uninformed firm to experiment since it too can alter its output to yield information about the unknown parameter. Of course, in a Nash equilibrium, both firms are aware of the strategies of their opponent so that no systematic deception is possible.

Unfortunately it is possible in this model that observing quantities (as well as prices) could reveal the true value of the unknown parameter to the uninformed firm. To see this, notice that in the first period game there are essentially two possible opponents for the uninformed firm, i.e., one opponent for each value of the unknown parameter. Each of these opponents can be thought of as having a strategy. These strategies yield, in equilibrium, a different output level for each of the (opponent) firms. Hence, viewing both the market price and the output decisions yields the true nature of the market to the uninformed firm. In order to remedy this problem we assume that the uninformed firm can not observe the output decision of the informed firm and that the output of the informed firm is random. The mean of the random output is the decision variable of the firm. Under this assumption the observation of the random output does not reveal the true output strategy of the informed firm. Hence, the true value of the expected intercept of the demand function remains obscured from the uninformed firm.

We thus have a model in which the decisions of both the informed and the uninformed firm play a crucial role in affecting the observed market variable and therefore the informational content of these variables. There are several currents in the economic literature which may be considered antecedents of this paper. The closest works are "Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand" by Matthews and Mirman [MM], "Experimental Consumption for a General Class of Disturbance Densities" by Fusselman and Mirman [FM] and "A Bayesian Approach to the Production of Information and Learning by Doing" by Grossman, Kihlstrom and Mirman [GKM].

The Matthews-Mirman paper can be viewed as the starting point for this paper since in MM a model of asymmetric information is presented in which the firm possessing the information (the incumbent) tries to manupulate the uninformed firm (i.e., the potential entrant) into believing that it is more likely that the market has a weak demand, thus reducing it's probability of entry.

Using the decision variable to manipulate the way information contained in the noisy observation of the market variable is interpreted by the firm, is a major theme of the paper. The approach of MM is employed in this paper with several differences. The first difference is that the value of the second period game in MM does not involve the use of the information gleaned by the uninformed firm. In MM the uninformed firm observes the relevant variable (i.e., the price) updates its information and decides - on the basis of this information and their prior beliefs - whether or not to enter. If entry occurs, the entrant becomes privy to the information held by the incumbent, in this case the value to the informed firm of the second period game does not depend on the subjective beliefs of the uninformed firm. In our model the value of the second period game depends on the information available to the uninformed firm through its posterior probability of the unknown parameter.

The second difference is that in our model the uninformed firm plays a role in the first period. This has two effects. The first is that the uninformed firm has an effect on the output and thus may have an influence on the informational content of the equilibrium. This yields the uninformed firm the opportunity to experiment. It also mitigates, through the Nash equilibrium concept, the ability of the informed firm to manipulate. The strategy of the uninformed firm must be accounted for in the strategy of the informed firm, limiting, to some extent, the effect of manipulation.

The phenomenon of experimentation is captured in GKM and FM. In these papers an uninformed agent changes his myopically optimal strategy in order to acquire information (experiment) for his own future use. The potential for the uninformed firm to experiment exists in this paper since the uninformed firm finds it useful to acquire information in order to make better informed decisions in the future as well as to counteract the effect of the manipulation by the informed firm. Thus the uninformed firm may give up profit in order to get information for the second period. However despite these incentives it is interesting that there is no experimentation. The reason is that it is the mean that is unknown. In this case all levels of output of the informed firm yields the same information. However, the output decision of the informed firm do have an effect on the uninformed firm even in this case.

The other consequence of the fact that the uninformed firm plays a role in the first period is that quantities rather than prices must be the decision variable in a Cournot-Nash equilibrium of the type we study. Since Matthews and Mirman use prices, an interesting question arises. How do our results compare to the MM results? It is shown in MM limit pricing is optimal, i.e., in MM the optimal policy for the informed firm is to reduce the price from its myopic level. The incumbent does not take account of the informational implementation of its decision in making myopic decisions. It is shown in this paper, under similar assumptions, that manipulation leads the informed firm to increase its output. On the surface these two results seem contradictory since price and quantity move in opposite directions along the demand curve. However the results are consistent since the response to a lowering of the intercept term in the demand function in the perfect information monopoly case reduces both the price and the quantity. Hence, it would seem that in the case of asymmetric information the key to how the decision is affected depends on the choice variable being used and the signal which is observed. Since each paper makes a different assumption it is not surprising that the results are different.

1. The Model and Preliminary Results

We consider a two period duopoly model with asymmetric information about a parameter of the stochastic linear demand function. One firm - the informed firm - knows the actual expected value of the intercept term while the other - the uninformed firm - does not know this value. However the uninformed firm has a subjective probability distribution over the possible values of the mean of the intercept term. Output is a random function of the choice of the informed firm. The uninformed firm uses the information contained in the observation of prices and quantities to update its beliefs about the unknown parameter in the demand function.

Assumptions

We make the following assumptions:

1) A duopolistic market is assumed. Each firm has a two period planning horizon.

2) The stochastic linear demand function in each period for this market is given by

$$\tilde{P} = \theta + \tilde{\epsilon} - bQ,$$

where θ is the mean of the unknown intercept parameter whose values belong to the set $\{\underline{\theta}, \overline{\theta}\}$, $\overline{\theta} > \underline{\theta}$. In order to insure interior solutions it is also assumed that $\overline{\theta} < 3\underline{\theta}$. The noise variable $\tilde{\epsilon}$ has support on the real line with distribution function $F(\epsilon)$, density function $f(\epsilon)$ and $E(\epsilon) = 0$. The noise terms are assumed to be independent and identically distributed in each period. The derivative of f exits and is continuous. All these assumptions on the distribution of ϵ are common knowledge

3) There is no possibility of entry or exit

4) The two firms produce the same homogeneous product in each of the two periods.

5) Firm 1, the informed firm, knows the true value of θ. The choice of this firm is denoted by $q_1(\overline{\theta}) = \overline{q}_1$, if $\theta = \overline{\theta}$, $q_1(\underline{\theta}) = \underline{q}_1$, if $\theta = \underline{\theta}$ and $q_1(\theta)$ if θ is not specified. Firm 2, the uninformed firm, has an a priori belief function over $\{\underline{\theta}, \overline{\theta}\}$ i.e., ρ = Prob $\{\theta = \overline{\theta}\}$ and $1-\rho$ = Prob $\{\theta = \underline{\theta}\}$. The choice of firm 2 is denoted by q_2. Let $Q_1(\overline{\theta}) = \overline{Q}_1 = \overline{q}_1 + q_2$, $Q_1(\underline{\theta}) = \underline{Q}_1 = \underline{q}_1 + q_2$ and $Q(\theta) = q_1(\theta) + q_2$.

6) There are no costs.

7) Firms maximize expected profits.

8) The actual output of firm 1 is a random function of its choice i.e.,

$$\tilde{q}(\theta) = q_1(\theta) + \tilde{\mu}$$

For simplicity the output of firm two is assumed to be deterministic The noise term $\tilde{\mu}$ is distributed on the real line.[1] Let $H(\mu)$ be the distribution function of μ with $h(\mu)$ the corresponding density function. h is independent of Q, and has a continuous first derivative. Moreover $E(\mu) = 0$. All this information is common knowledge.

9) The density functions $f(P-\theta-bQ)$ and $h(Q-Q(\theta))$ have the monotone likelihood ratio property (MLRP) i.e., $\frac{f(P-\overline{\theta}+bQ)}{f(P-\underline{\theta}+bQ)}$ is nondecreasing in P whenever $\overline{\theta} > \underline{\theta}$ and $\frac{h(Q-\overline{Q})}{h(Q-\underline{Q})}$ is nondecreasing in Q whenever $\overline{Q} > \underline{Q}$.

10) The prior beliefs of Firm II are updated using Bayes Rule as information is acquired. The only additional information available to each firm after

[1]Under this assumption actual output may be negative. However, on average output is positive. A full discussion of this assumption appears in the conclusion.

the first stage of the game is both the market clearing price and output, i.e. there are two signals in the model P, the actual price and Q, the actual output.

<u>**Definition 1:**</u> A pure strategy in the first period for Firm I is a mapping $q_1: \{\overline{\theta}, \underline{\theta}\} \to R^+$. A pure strategy in the first period for Firm II is a mapping $q_2: [0,1] \to R^+$.

Note that since Firm II does not know the value of θ, its strategies, q_2, cannot depend on θ, i.e., given ρ, a strategy of Firm II is an output $q_2 \in R^+$.

In order to find properties of the first period strategies it is necessary to study the second period equilibrium, i.e., the equilibrium which incorporates the information from the first period. Thus we first consider the <u>second period</u> problem. Note that since the second period is the last period of the planning horizon only the static game need be considered.

Let ρ represent beliefs of Firm II in the second period game. Given an output q_2 the expected profits for Firm I, are

$$\Pi_1(q_1(\theta), q_2, \theta) = \int\int(\theta + \epsilon - b(q_1 + q_2 + \mu))(q_1 + \mu)\ f(\epsilon)h(\mu)d\epsilon d\mu \qquad (1)$$

and if the expected strategies of firm i are $\overline{q_1}$, $\underline{q_2}$, the expected profits for Firm II are,

$$\Pi_2(\overline{q_1}, \underline{q_1}, q_2) = \rho\int\int(\overline{\theta} + \epsilon - b(\overline{q_1} + q_2 + \mu))q_2 f(\epsilon)h(\mu)d\epsilon d\mu$$
$$+ (1-\rho)\int\int(\underline{\theta} + \epsilon - b(\underline{q_1} + q_2 + \mu))q_2 f(\epsilon)h(\mu)d\epsilon d\mu. \qquad (2)$$

Definition 2:

Given ρ, a second period equilibrium is a triple $(\overline{q}_1^*, \underline{q}_1^*, q_2^*)$ such that for each $\theta \in \{\overline{\theta}, \underline{\theta}\}$, and all $q_1, q_2 \in R^+$

$$\Pi_1(q_1^*(\theta), q_2^*, \theta) \geq \Pi_1(q_1, q_2^*, \theta),$$

and

$$\Pi_2(\overline{q}_1^*, \underline{q}_1^*, q_2^*) \geq \Pi_2(\overline{q}_1^*, \underline{q}_1^*, q_2)$$

Note that the dependence of these values on ρ is suppressed.

The assumption that the demand curves are linear implies, in this static game, that there is a unique equilibrium. Hence equilibrium expected profits of both firms are well defined and can be expressed as functions of ρ. Let the expected equilibrium profit function for Firm I be,

$$V_1(\rho, \theta) = \Pi_1(q_1^*(\theta), q_2^*, \theta) \quad , \tag{3}$$

and for Firm II,

$$V_2(\rho) = \Pi_2(\overline{q}_1^*, \underline{q}_1^*, q_2^*) \tag{4}$$

We shall now compute the value functions $V_1(\rho, \theta)$, $V_2(\rho)$. Let the <u>reaction function</u> for Firm I be denoted by $q_1(q_2, \theta) = \underset{q_1}{\text{Argmax}}\ \Pi_1(q_1, q_2, \theta)$. The <u>best response</u> for Firm II, if the expected strategies of Firm I are $\overline{q}_1$ and $\underline{q}_1$, is denoted by,

$$q_2(\overline{q}_1, \underline{q}_1; \rho) = \underset{q_2}{\text{Argmax}}\ \Pi_2(\overline{q}_1, \underline{q}_1, q_2).$$

Let $\hat{\theta} = \rho\overline{\theta} + (1-\rho)\ \underline{\theta}$.

Lemma 1. The reaction curves of Firm I are:

$$q_1(q_2;\overline{\theta}) = \frac{\overline{\theta}}{2b} - \frac{1}{2}q_2 \tag{5}$$

$$q_1(q_2;\underline{\theta}) = \frac{\underline{\theta}}{2b} - \frac{1}{2}q_2 \tag{6}$$

and the best response for Firm II is,

$$q_2 = q_2(\overline{q_1},\ \underline{q_1};\rho) = \frac{\hat{\theta}}{2b} - \frac{[\rho\ q_1(\overline{\theta}) + (1-\rho)q_1(\underline{\theta})]}{2}, \tag{7}$$

Proof.

The first order conditions for the maximization of $\Pi_1(\theta)$, $\theta = \overline{\theta}$, $\underline{\theta}$, respectively, yields (5) and (6). Similarly, the first order condition for the maximization of Π_2 yields (7). //

Given Firm I's reaction function and Firm II's best response function, the equilibrium may be viewed in terms of self-confirming conjectures. If Firm I conjectures that Firm II chooses q_2^*, then its best response, given θ, is to choose $q_1^*(\theta)$. In turn, if Firm II believes that Firm I chooses $q_1^*(\theta)$, $\theta = \overline{\theta},\underline{\theta}$, then its best response is to choose q_2^*. In equilibrium each firm's conjecture about the behavior of the other firm is correct.

Proposition 1. For each ρ, the unique Cournot-Nash equilibrium is given by the triple $q_1^*(\overline{\theta},\rho)$, $q_1^*(\underline{\theta},\rho)$, $q_2^*(\rho)$, such that

$$\bar{q}_1^* = q_1(\bar{\theta},\rho) = \frac{3\bar{\theta} - \hat{\theta}}{6b} \tag{8}$$

$$\underline{q}_1^* = q_1(\underline{\theta},\rho) = \frac{3\underline{\theta} - \hat{\theta}}{6b} \tag{9}$$

$$q_2^* = q_2(\rho) = \frac{\hat{\theta}}{3b} \ . \tag{10}$$

Proof. Since in equilibrium each firm's conjectures about the other firms behavior is correct, we solve the system of equations (5), (6) and (7) simultaneously. This yields (8), (9), and (10). //

Note that since $\bar{\theta} < 3\underline{\theta}$, $\bar{q}_1 > 0$, $\underline{q}_1 > 0$. The value functions are

$$\bar{V}_1(\rho) = V_1(\rho,\bar{\theta}) = \Pi_1(\bar{q}_1^*,q_2^*,\bar{\theta}) = \frac{1}{b}\left(\frac{3\bar{\theta} - \hat{\theta}}{6}\right)^2 \ , \tag{11}$$

$$\underline{V}_1(\rho) = V_1(\rho,\underline{\theta}) = \Pi_1(\underline{q}_1^*,\ q_2^*,\underline{\theta}) = \frac{1}{b}\left(\frac{3\underline{\theta} - \hat{\theta}}{6}\right)^2 \ , \tag{12}$$

and,

$$V_2(\rho) = \Pi_2(\bar{q}_1^*,\underline{q}_1^*,q_2^*) = \frac{1}{b}\left(\frac{\hat{\theta}}{3}\right)^2 \ , \tag{13}$$

Lemma 2.

(i) The functions $\bar{V}_1(\rho)$, $\underline{V}_1(\rho)$ are decreasing convex functions of ρ.

(ii) $\bar{V}_1(\rho) > \underline{V}_1(\rho)$. Also $\bar{V}_1' < \underline{V}_1'$ and $\bar{V}_1' < -\bar{V}_1''$.

(iii) The function $V_2(\rho)$ is an increasing convex function.

Since $\bar{V}'$ and $\underline{V}'$ are both negative, Firm I's interest is that ρ be as small as possible. Since the signals P and Q are endogenous or dependent on an endogenous variable, Firm I would like to manipulate them to its own advantage by the choice of outputs in the first stage of the game, i.e., it will try to control Firm II's learning process. Firm I chooses its quantities so that Firm II will interpret the market signals it receives to its own (Firm I's) advantage. To follow this behavior, however, is not costless. Firm I must take account of the tradeoff between present and future profits. In other words, Firm I's problem is one of optimal disclosure of information, over time, consistent with profit maximization.

2. The First Period and the Flow of Information

After the first period P and Q are observed. Firm II uses these signals to update its prior beliefs. In this way a posterior expectation function is determined.

Definition 3. A belief or expectation function ρ is a mapping from $R \times R \to [0,1]$. Interpret $\rho(P,Q)$ as the Firm II's subjective probability that $\theta = \bar{\theta}$, when P and Q are observed.[2]

[2]Note that there is a slight abuse of notation. The value ρ is used to represent the subjective beliefs of Firm II in both the first and the second period. From this point on ρ represents the beliefs of Firm II in the first period while $\rho(P,Q)$ is the posterior belief function used in the second period.

Definition 4. A belief function $\rho(P,Q)$ is called consistent if for $\theta \in \{\overline{\theta}, \underline{\theta}\}$, and for any observed P and Q,[3]

$$\rho(P,Q) = P_\rho[\theta = \overline{\theta} | \tilde{P} = P,\ \tilde{Q} = Q] = \frac{P\ [\theta = \overline{\theta},\ \tilde{P} = P,\ \tilde{Q} = Q]}{P_\rho[\tilde{P} = P,\ \tilde{Q} = Q]}$$

$$= \frac{P_\rho[\theta = \overline{\theta},\ \overline{\theta} + \epsilon - bQ = P,\ \overline{Q} + \mu = Q]}{P_\rho[\theta = \overline{\theta},\ \overline{\theta} + \epsilon - bQ = P,\ \overline{Q} + \mu = Q] + P_\rho[\theta = \underline{\theta},\ \underline{\theta} + \epsilon + bQ = P,\ \underline{Q} + \mu = Q]}$$

$$= \frac{\rho f(P - \overline{\theta} + bQ)h(Q - \overline{Q})}{\rho f(P - \overline{\theta} + bQ)h(Q - \overline{Q}) + (1 - \rho)f(P - \underline{\theta} + bQ)h(Q - \underline{Q})}.\text{[4]} \qquad (14)$$

Consistent beliefs are generated by Bayesian updating. Let expected profits in the first period, for Firm I, be,

$$\Pi_1(q_1(\theta),\ q_2,\ \theta) = \iint[(\theta + \epsilon - b(q_1(\theta) + q_2 + \mu)(q_1(\theta) + \mu)]\ f(\epsilon)\ h(\mu)\ d\epsilon\ d\mu\ ,\text{[5]}$$

and for Firm II

$$\Pi_2(\overline{q}_1,\ \underline{q}_1,\ q_2) = \rho\iint[(\overline{\theta} + \epsilon - b(\overline{q}_1 + q_2 + \mu))q_2]\ f(\epsilon)\ h(\mu)\ d\epsilon\ d\mu$$
$$+ (1-\rho)\iint[(\underline{\theta} + \epsilon - b(\underline{q}_1 + q_2 + \mu))q_2]\ f(\epsilon)\ h(\mu)\ d\epsilon\ d\mu.$$

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[3]The notation P_ρ denotes the posterior probability for a firm whose prior is ρ, i.e., $P_\rho[\theta = \overline{\theta}|\tilde{P} = P,\ \tilde{Q} = Q]$ is the posterior probability $\theta = \overline{\theta}$ after P and Q are observed, if prior beliefs are given by ρ.

[4]Recall that $\overline{Q} = Q(\overline{\theta})$ and $\underline{Q} = Q(\underline{\theta})$.

[5]Again a slight abuse of notation is employed. Profits in both periods are denoted by the same symbol. No confusion should arise since from here on only the second period value function will be used.

The problem faced by Firm I is,

$$\max_{0 \le q_1(\theta)} \{\Pi_1(q_1(\theta), q_2, \theta) + EV_1[\rho(P,Q), \theta]\} \quad , \tag{15}$$

where

$$EV_1[\rho(P,Q),\theta] = \iint V_1[\rho(P,Q),\theta]\; f\,(P-\theta + bQ)\; h(Q-q_1(\theta)-q_2)\; dP\, dQ. \tag{16}$$

Similarly, the problem faced by Firm II is,

$$\max_{0 \le q_2} \{\Pi_2(\overline{q}_1, \underline{q}_1, q_2) + EV_2[\rho(P,Q)]\} \tag{17}$$

where

$$\begin{aligned} EV_2[\rho(P,Q)] = {} & \rho\iint V_2[\rho(P,\theta)]\; f\,(P-\overline{\theta} + bQ)\; h(Q-\overline{q}_1-q_2)\; dP\, dQ \\ & + (1-\rho)\iint V_2[\rho(P,Q)]\; f\,(P-\underline{\theta} + bQ)\; h(Q-\underline{q}_1 - q_2)\; dP\, dQ \end{aligned} \tag{18}$$

Definition 5. An equilibrium is a triple $(\overline{q}_1^*, \underline{q}_1^*, q_2^*)$ and a belief function $\rho(P,Q)$ such that

(i) For all $q_1(\theta)$, $q_2 \in R^+$, $\theta \in \{\overline{\theta}, \underline{\theta}\}$

$$\Pi_1(q_1^*(\theta), q_2^*, \theta) + E[V_1^*(\rho(P,Q),\theta)] \ge \Pi_1(q(\theta), q_2^*, \theta) + E\{V_1(\rho(P,Q),\theta)]$$

and

$$\Pi_2(\vec{q}_1^*, \underline{q}_1^*, q_2^*) + E[V_2^*(\rho(P,Q))] \geq \Pi_2(\vec{q}_1^*, \underline{q}_1^*, q_2) + E[V_2(\rho(P,Q))] \text{ ,}^{6}$$

(ii) $\rho(P,Q)$ is consistent.

Before solving the first period problem, let us consider the flow of information. Given the observation (P,Q) in the market space, the noise term ϵ in the demand function makes it impossible for Firm II to deduce the true value of θ. In particular, corresponding to the observation (P,Q), there exists $\overline{\epsilon}$ and $\underline{\epsilon}$ such that $P = \overline{\theta} + \overline{\epsilon} - bQ = \underline{\theta} + \underline{\epsilon} - bQ$. Hence the pair $(\overline{\epsilon}, \overline{\theta})$ and $(\underline{\epsilon}, \underline{\theta})$ cannot be distinguished. A similiar statement can be made about observations Q in the strategy space, i.e., for each Q there exists $\overline{\mu}$ and $\underline{\mu}$ such that $Q = \overline{Q} + \overline{\mu} = \underline{Q} + \underline{\mu}$. Hence the pair $(\overline{\mu}, \overline{Q})$ and $(\underline{\mu}, \underline{Q})$ cannot be distinguished.

Firm II updates its prior beliefs on the basis of the observation (P,Q) using Bayes rule. Hence the posterior probability that $\theta = \overline{\theta}$ is,

$$\rho(P,Q) = \frac{\rho f(P-\overline{\theta} + bQ)\ h(Q-\overline{Q})}{\rho f(P-\overline{\theta} + bQ)\ h(Q-\overline{Q}) + (1-\rho)\ f\ (P-\underline{\theta} + bQ)\ h(Q-\underline{Q})} \ . \tag{19}$$

It has been assumed that both families of densities $f(P|\theta,Q)$ and $h(Q|Q(\theta))$ have the monotone likelihood ratio property (MLRP), that means that the ratio $\frac{f(P-\overline{\theta} + bQ)}{f(P-\underline{\theta} + bQ)}$ is a nondecreasing function of P, since for all Q, $\overline{\theta} - bQ > \underline{\theta} - bQ$.

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[6]The expected value functions in (i) are defined in (16). The left hand side is taken using $q_1^*(\theta)$, q_2^* while the right hand side is taken using $q_1(\theta)$, q_2^* in the argument of h in (16) for the first inequality and $q_1^*(\theta)$, q_2 for the second inequality.

i.e., since $\overline{\theta} > \underline{\theta}$. The MLRP implies that, given Q, observing higher prices increases the subjective probability that $\theta = \overline{\theta}$ for Firm II. The MLRP assumption also implies that the ratio $\frac{h(Q-\overline{Q})}{h(Q-\underline{Q})}$ is a nondecreasing (nonincreasing) function of Q, whenever $\overline{Q} > \underline{Q}$ $(\overline{Q} < \underline{Q})$ i.e., if $\overline{Q} > \underline{Q}$ higher values of Q also imply a higher probability that $\theta = \overline{\theta}$. Note that there is an important distinction between the MLRP for f and h. When applied to f the MLRP always holds since θ is an exogenous parameter and $\overline{\theta} > \underline{\theta}$. However for the MLRP to hold for h either $\overline{Q} > \underline{Q}$ or $\underline{Q} < \overline{Q}$ must hold. Since Q is an endogenous variable the relationship between $\overline{Q}$ and $\underline{Q}$ must be established. In the following proofs the inequality will be assumed. However, it will be established subsequently.

Consider the change in the expectation function $\rho(P,Q)$ when either P or Q changes. The following expressions will be needed when studying the maximization problem for each firm in the first stage of the game. Let $D = f(P-\overline{\theta} + bQ)\ h(Q-\overline{Q})\rho + f(P-\underline{\theta} + bQ)\ h(Q-\underline{Q})(1-\rho)$.

Lemma 3. If the densities $\{f(P|\theta,Q)\}$ have the MLRP, then the expectation function $\rho(P,Q)$ is a nondecreasing function of P.

Proof. This is clear from equation (19).

The next Lemma will be used in Proposition 2.2 and Lemma 5.

Lemma 4.

(i) $$\frac{\partial}{\partial Q}\rho(P,Q) = \frac{\rho(1-\rho)}{D^2} f(P-\overline{\theta} + bQ)\ f(P-\underline{\theta} + bQ)[-h'(Q-\overline{Q})\ h(Q-\underline{Q})], \qquad (20)$$

(ii) $$\frac{\partial}{\partial \underline{Q}}\rho(P,Q) = \frac{\rho(1-\rho)}{D^2} f(P-\overline{\theta} + bQ)\ f(P-\underline{\theta} + bQ)[h(Q-\overline{Q})\ h'(Q-\underline{Q})], \tag{21}$$

Proof. This is clear from equation (19).

Let $f(\theta) = f(P-\theta + bQ)$, $f'(\theta) = \frac{\partial f(\theta)}{\partial P}$; $h(\theta) = h(Q-Q(\theta))$ and $h'(\theta) = \frac{\partial h(\theta)}{\partial Q}$; and $f(\overline{\theta}) = f(\rho-\overline{\theta} + hQ)$, $h(\overline{\theta}) = h(Q-\overline{Q})$, etc.

Proposition 2. If the densities $\{f(P|\theta,Q)\}$ and $\{h(Q|Q(\theta)\}$ have the MLRP and if $\overline{Q} > \underline{Q}$ then $\rho(P,Q)$ is a nondecreasing function of Q. Moreover,

$$\frac{\partial \rho}{\partial Q} = b\ \frac{\partial \rho}{\partial P} - [\frac{\partial \rho}{\partial \overline{Q}} + \frac{\partial \rho}{\partial \underline{Q}}].$$

Proof. From equation (19),

$$\frac{\partial \rho}{\partial Q} = \{\rho[\frac{\partial f(\overline{\theta})}{\partial Q}h(\overline{\theta}) + f(\overline{\theta})\frac{\partial h(\overline{\theta})}{\partial Q}][f(\overline{\theta})h(\overline{\theta})\rho + f(\underline{\theta})h(\underline{\theta})(1-\rho)]$$
$$- f(\overline{\theta})h(\overline{\theta})\rho[\frac{\partial f(\overline{\theta})}{\partial Q}h(\overline{\theta})\rho + f(\overline{\theta})\frac{\partial h(\overline{\theta})}{\partial Q}\rho + \frac{\partial f(\underline{\theta})}{\partial Q}h(\underline{\theta})(1-\rho) + f(\underline{\theta})\frac{\partial h(\underline{\theta})}{\partial Q}(1-\rho)]\}\frac{1}{D^2}$$
$$= \{[\rho\ b\ f'(\overline{\theta})h(\overline{\theta}) + f(\overline{\theta})h'(\overline{\theta}))][f(\overline{\theta})h(\overline{\theta})\rho + f(\underline{\theta})h(\underline{\theta})(1-\rho)]$$
$$- f(\overline{\theta})h(\overline{\theta})\rho[bf'(\overline{\theta})h(\overline{\theta})\rho + f(\overline{\theta})h'(\overline{\theta})\rho + bf'(\underline{\theta})h(\underline{\theta})(1-\rho) + f(\underline{\theta})h'(\underline{\theta})(1-\rho)]\}\ \frac{1}{D^2}.$$

After cancelling terms,

$$\frac{\partial \rho}{\partial Q} = \frac{\rho(1-\rho)}{D^2}\{b[f'(\overline{\theta})f(\underline{\theta}) - f(\overline{\theta})f'(\underline{\theta})]\ h(\overline{\theta})h(\underline{\theta})$$
$$+ f(\overline{\theta})f(\underline{\theta})[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})]\}, \tag{22}$$

The sign of $\frac{\partial\rho}{\partial Q}$ depends on the sign of the terms $[f'(\overline{\theta})f(\underline{\theta}) - f(\overline{\theta})f'(\theta)]$ and $[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})]$. By the MLRP the first of the above terms is nonnegative. Similarly if $\overline{Q} > \underline{Q}$, then $\frac{\partial}{\partial Q}\frac{h(Q-\overline{Q})}{h(Q-\underline{Q})} = \frac{h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})}{(h(\underline{\theta}))^2} \geq 0$, i.e., $[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})] \geq 0$. Hence, $\frac{\partial\rho}{\partial Q} \geq 0$. Finally,

$$\begin{aligned}\frac{\partial\rho(P,Q)}{\partial Q} &= b\,\frac{\partial\rho}{\partial P} + f(\overline{\theta})f(\underline{\theta})[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})]\,\frac{\rho(1-\rho)}{D^2} \\ &= b\,\frac{\partial\rho}{\partial P} - \left[\frac{\partial\rho}{\partial\overline{Q}} + \frac{\partial\rho}{\partial\underline{Q}}\right] \end{aligned} \tag{23}$$

Lemma 5.

$$\frac{\partial\rho}{\partial q_2} = \frac{\partial\rho}{\partial\overline{Q}} + \frac{\partial\rho}{\partial\underline{Q}}$$

Proof. From Bayes rule,

$$\rho(P,Q) = \frac{f(P-\overline{\theta}+bQ)h(Q-\overline{q}_1-q_2)\rho}{f(P-\overline{\theta}+bQ)h(Q-\overline{q}_1-q_2)\rho+f(P-\underline{\theta}+bQ)h(Q-\underline{q}_1-q_2)(1-\rho)} .$$

Hence,

$$\begin{aligned}\frac{\partial\rho}{\partial q_2} &= \Big\{f(\overline{\theta})\,\frac{\partial h(\overline{\theta})}{\partial q_2}\rho[f(\overline{\theta})h(\overline{\theta})\rho+f(\underline{\theta})h(\underline{\theta})(1-\rho)] \\ &\quad - f(\overline{\theta})h(\overline{\theta})\rho[f(\overline{\theta})\frac{\partial h(\overline{\theta})}{\partial q_2}\rho+f(\underline{\theta})\frac{\partial h(\underline{\theta})}{\partial q_2}(1-\rho)]\Big\}\,\frac{1}{D^2} \\ &= \{-f(\overline{\theta})h'(\overline{\theta})\rho[f(\overline{\theta})h(\overline{\theta})\rho + f(\underline{\theta})h(\underline{\theta})(1-\rho)] \\ &\quad - f(\overline{\theta})h(\overline{\theta})\rho[-f(\overline{\theta})h'(\overline{\theta})\rho - f(\underline{\theta})h'(\underline{\theta})(1-\rho)]\}\,\frac{1}{D^2} .\end{aligned}$$

Cancelling and rearranging yields,

$$\frac{\partial \rho}{\partial q_2} = \frac{\rho(1-\rho)}{D^2} f(\overline{\theta}) f(\underline{\theta}) [h'(\overline{\theta}) h(\underline{\theta}) - h(\overline{\theta}) h'(\underline{\theta})] = \frac{\partial \rho_2}{\partial \overline{Q}_1} + \frac{\partial \rho_2}{\partial \underline{Q}_1} \quad , \tag{24}$$

by Lemma 4.

3. First Order Conditions

For any quantity q_2 the best responses for Firm I in the first period is

$$q_1(q_2;\theta) \in \underset{q_1}{\text{Argmax}} \{ \Pi_1(q_1,q_2,\theta) + EV_1[\rho(\rho,Q),\theta] \} \tag{25}$$

The best response of Firm II in the first period is,

$$q_2(q_1(\overline{\theta}),q_1(\underline{\theta})) \in \underset{q_2}{\text{Argmax}} \{\Pi_2(q_1(\overline{\theta}),q_1(\underline{\theta}),q_2) + EV_2[\rho(P,Q)]\} \quad . \tag{26}$$

In theorem 1 it is shown that for every level of q_2, if

$$\overline{Q} = \overline{q}_1 + q_2 > \underline{q}_1 + q_2 = \underline{Q}$$

then the best responses (i.e., $\overline{q}_1$ and $\underline{q}_1$) for Firm I will occur where first period profits Π_1 (q_1, q_2, θ) have a nonnegative slope. The question of when $\overline{Q} > \underline{Q}$ will be postponed. Theorem 1 also shows that Firm II never experiments, i.e. it chooses that level of output which maximizes only first period expected profits.

Theorem 1. If $\overline{Q} > \underline{Q}$, then let $\overline{q_1} = q_1(q_2,\overline{\theta})$, $\underline{q_1} = q_1(q_2,\underline{\theta})$, and $q_2 = q_2(\overline{q_1},\underline{q_1})$ be the best reponses of Firm I and Firm II respectively, then

$$\text{(i)} \quad \frac{\partial \Pi_1(\overline{q_1},q_2,\overline{\theta})}{\partial \overline{q_1}} = \iint \overline{V_1}[\rho(P,Q)]f(P-\overline{\theta}+bQ)h'(Q-\overline{Q})dP\, dQ \geq 0 \tag{27}$$

$$\text{(ii)} \quad \frac{\partial \Pi_1(\underline{q_1},q_2,\underline{\theta})}{\partial \underline{q_1}} = \iint \underline{V_1}[\rho(P,Q)]f(P-\underline{\theta}+bQ)h'(Q-\underline{Q})\, dP\, dQ \geq 0 \quad , \tag{28}$$

where $\overline{Q} = \overline{q_1} + q_2$, $\underline{Q} = \underline{q_1} + q_2$. Also,

$$\text{(iii)} \quad \frac{\partial \Pi_2(\overline{q_1},\underline{q_1},q_2)}{\partial q_2} = 0 \quad . \tag{29}$$

Before proving theorem 1, the following results are needed.

Lemma 6. If $\overline{Q} > \underline{Q}$, then $\frac{\partial}{\partial Q}\{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\} \leq 0.$ (30)

Proof.

$$\frac{\partial}{\partial Q}\{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\}$$

$$= \int V_1'[\rho(P,Q),\theta]\,\frac{\partial \rho}{\partial Q}\,f(P-\theta+bQ)dP + \int V_1[\rho(P,Q),\theta]\,\frac{\partial}{\partial Q}\,f(P-\theta+bQ)dP. \tag{31}$$

Integrating the last term on the right hand side of (31) by parts yields,

$$\frac{\partial}{\partial Q}\{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\} = \int V_1'[\rho(P,Q),\theta][\frac{\partial\rho}{\partial Q} - b\frac{\partial\rho}{\partial P}]f(P-\theta+bQ)dP. \quad (32)$$

By equation (33) of proposition 2,

$$\frac{\partial\rho}{\partial Q} = b\frac{\partial\rho}{\partial P} + \frac{\rho(1-\rho)}{D^2}f(\overline{\theta})f(\underline{\theta})[h'(\overline{\theta})h(\underline{\theta}) - h(\theta)h'(\underline{\theta})].$$

Thus,

$$\frac{\partial}{\partial Q}\{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\}$$

$$= \frac{\rho(1-\rho)}{D^2}\int V_1'[\rho(P,Q),\theta]\ f(\overline{\theta})f(\underline{\theta})[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})]f(P-\theta+bQ)dP. \quad (33)$$

By Lemma 2, $V_1' < 0$, and by the assumption that $\overline{Q} > \underline{Q}$, and the MLRP of the densities $\{h(Q|Q(\theta))\}$, $[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})] \geq 0$. Thus $\frac{\partial}{\partial Q}\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP \leq 0$. //

<u>**Lemma 7.**</u> If $\overline{Q} > \underline{Q}$,

$$\int\{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\}h'(Q-Q(\theta))dQ \geq 0 \quad (34)$$

<u>**Proof.**</u> The MLRP implies that there exists $\hat{Q} \in (-\infty,\infty)$ such that $h'(Q) \geq 0$, $Q \leq \hat{Q}$ and $h'(Q) \leq 0$, $Q \geq \hat{Q}$. Moreover, from Lemma 6, $\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP$ is a nonincreasing, positive function of Q. Hence, since $h' > 0$ when $Q < \hat{Q}$,

$$
\begin{aligned}
&\int_{-\infty}^{\hat{Q}} \{\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)\,dP\} h'(Q-Q(\theta))\,dQ \\
&\ge \int_{-\infty}^{\hat{Q}} \{\int V_1[\rho(P,Q),\theta] f(P-\theta+b\hat{Q})\,dP\} h'(Q-Q(\theta))\,dQ \\
&= \{\int V_1[\rho(P,Q),\theta] f(P-\theta+b\hat{Q})\,dP\} \int_{-\infty}^{\hat{Q}} h(Q-Q(\theta))\,dQ \quad . \qquad (35)
\end{aligned}
$$

Also, since $h' < 0$ when $Q > \hat{Q}$,

$$
\begin{aligned}
&\int_{\hat{Q}}^{\infty} \{\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)\,dP\} h'(Q-Q(\theta))\,dQ \\
&\ge -\{\int V_1[\rho(P,Q),\theta] f(P-\theta+b\hat{Q})\,dP] \int_{\hat{Q}}^{\infty} h(Q-Q(\theta))\,dQ. \qquad (36)
\end{aligned}
$$

Adding equations (35) and (36) yields,

$$
\begin{aligned}
&\int \{\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)\,dP\} h'(Q-Q(\theta))\,dQ \\
&= \int_{-\infty}^{\hat{Q}} [\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)\,dP\} h'(Q-Q(\theta))\,dQ \; + \\
&+ \int_{\hat{Q}}^{\infty} \{\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)\,dP\} h'(Q-Q(\theta))\,dQ \\
&\ge \{\int V_1[\rho(P,Q),\theta] f(P-\theta+b\hat{Q})\,dP\} [\int_{-\infty}^{\hat{Q}} h'(Q-Q(\theta))\,dQ + \int_{\hat{Q}}^{\infty} h'(Q-Q(\theta))\,dQ] \\
&= \{\int V[\rho(P,Q),\theta] f(P-\theta+b\hat{Q})\,dP\} \int h'(Q-Q(\theta))\,dQ = 0,
\end{aligned}
$$

since $\int h'(Q-Q(\theta))\,dQ = 0$. //(37)

Next let $\overline{\Pi}_1 = \Pi(q_1, q_2, \overline{\theta})$ and $\underline{\Pi}_1 = \Pi_1(q_1, q_2, \underline{\theta})$. We return to the proof of theorem 1.

Proof of Theorem 1.

$$\text{(i)} \qquad 0 = \frac{\partial \Pi}{\partial q_1} + \frac{\partial}{\partial q_1}\{\int\int \overline{V}_1[\rho(P,Q)]f(P-\overline{\theta}-bQ)h(Q-q_1-q_2)dPdQ\}$$

$$= \frac{\partial \Pi}{\partial q_1} - \int\{\int \overline{V}_1[\rho(P,Q)]f(P-\overline{\theta}+bQ)dP\}h'(Q-q_1-q_2)dQ \qquad . \quad (38)$$

By Lemma 7, $\int\{\int \overline{V}_1[\rho(P,Q)]f(P-\overline{\theta}+bQ)dP\}h'(Q-\overline{Q})dQ \leq 0$. Thus,

$$\frac{\partial \Pi_1}{\partial q_1} = \int\{\int \overline{V}[\rho(P,Q)]f(P-\overline{\theta}+bQ)dP\}h'(Q-\overline{Q})dQ \geq 0 .$$

The first order condition for a maximum for Firm II for arbitrary $\overline{q_1}$ and $\underline{q_1}$ is,

$$\begin{aligned}
0 = {} & \frac{\partial \Pi_2(\overline{q}_1, \underline{q}_1, q_2)}{\partial q_2} \\
& + \frac{\partial}{\partial q_2}\{\rho\int\int V_2[\rho(P,Q)]f(P-\overline{\theta}+bQ)h(Q-\overline{q}_1-q_2)dP\ dQ \\
& + (1+\rho)\int\int V_2[\rho(P,Q)]f(P-\underline{\theta}+bQ)h(Q-\underline{q}_1-q_2)dP\ dQ\} \\
= {} & \frac{\partial \Pi_2}{\partial q_2} + \rho\int\int V_2' \frac{\partial \rho}{\partial q_2} f(P-\overline{\theta}+bQ)h(Q-\overline{q}_1-q_2)dP\ dQ \\
& + (1-\rho)\int\int V_2' \frac{\partial \rho}{\partial q_2} f(P-\underline{\theta}+bQ)h(Q-\underline{q}_1-q_2)dP\ dQ \\
& + \rho\int\int V_2 f(P-\overline{\theta}+bQ) \frac{\partial}{\partial q_2} h(Q_1-\overline{q}-q_2)dP\ dQ \\
& + (1-\rho)\int\int V_2 f(P-\underline{\theta}+bQ) \frac{\partial}{\partial q_2} h(Q-\underline{q}-q_2)dP\ dQ. \qquad (39)
\end{aligned}$$

Integration of the last two terms of the above expression by parts with $u = \int V_2 f(P-\theta+bQ)dP$, and $dv = \frac{\partial}{\partial q_2} h(Q-q_1(\theta)-q_2)dQ$ yields,

$$0 = \frac{\partial \Pi_2}{\partial q_2} + \rho \iint V_2' \frac{\partial \rho}{\partial q_2} f(P - \overline{\theta} + bQ) h(Q - \overline{q}_1 - q_2) dPdQ$$
$$+ (1-\rho) \iint V_2' \frac{\partial \rho}{2 q_2} f(P - \underline{\theta} + bQ) h(Q - \underline{q}_1 - q_2) dPdQ$$
$$- [\rho \int \{ \frac{\partial}{\partial Q} \int V_2 f(P - \overline{\theta} + bQ) dP \} h(Q - \overline{q}_1 - q_2) dQ + (1-\rho) \int \{ \frac{\partial}{\partial Q} \int V_2 f(P - \underline{\theta} + bQ) dP \} h(Q - \underline{q}_1 - q_2) dQ]$$
$$= \frac{\partial \Pi_2}{\partial q_2} + \rho \iint V_2' \frac{\partial \rho}{\partial q_2} f(P - \overline{\theta} + bQ) h(Q - \overline{Q}) dPdQ + (1-\rho) \iint V_2' \frac{\partial \rho}{\partial q_2} f(P - \underline{\theta} + bQ) h(Q - \underline{Q}) dPdQ$$
$$- \rho \int \{ \int V_2' \frac{\partial \rho}{\partial Q} f(P - \overline{\theta} + bQ) dP + \int V_2 b f'(P - \overline{\theta} + bQ) dP \} h(Q - \overline{q}_1 - q_2) dQ$$
$$- (1-\rho) \int \{ \int V_2' \frac{\partial \rho}{\partial Q} f(P - \underline{\theta} + bQ) dP$$
$$+ \int V_2 b f'(P - \underline{\theta} + bQ) dP \} h(Q - \underline{q}_1 - q_2) dQ.$$

Integration of $\int V_2 b f'(P - \theta + bQ) dP$ by parts, yields,

$$0 = \frac{\partial \Pi_2}{\partial q_2} + \rho \iint V_2' \frac{\partial \rho}{\partial q_2} f(P - \overline{\theta} + bQ) h(Q - \overline{Q}) dPdQ + (1-\rho) \iint V_2' \frac{\partial \rho}{\partial q_2} f(P - \underline{\theta} + bQ) h(Q - \underline{Q}) dP\, dQ$$
$$+ \rho \iint V_2' [\frac{\partial \rho}{\partial Q} - b \frac{\partial \rho}{\partial P}] f(P - \overline{\theta} + bQ) h(Q - \overline{Q}) dP\, dQ$$
$$+ (1-\rho) \iint V_2' [\frac{\partial \rho}{\partial Q} - b \frac{\partial \rho}{\partial P}] f(P - \underline{\theta} + bQ) h(Q - \underline{Q}) dP\, dQ.$$

Rearranging terms,

$$0 = \frac{\partial \Pi_2}{\partial q_2} + \rho \iint V_2' [\frac{\partial \rho}{\partial q_2} + \frac{\partial \rho}{\partial Q} - b \frac{\partial \rho}{\partial P}] f(P - \overline{\theta} + bQ) h(Q - \overline{Q}) dP\, dQ$$
$$+ (1-\rho) \iint V_2' [\frac{\partial \rho}{\partial q_2} + \frac{\partial \rho}{\partial Q} - b \frac{\partial \rho}{\partial P}] f(P - \underline{\theta} + bQ) h(Q - \underline{Q}) dP\, dQ. \tag{40}$$

By Lemma 5, $\frac{\partial \rho}{\partial q_2} = \frac{\partial \rho}{\partial \overline{Q}} + \frac{\partial \rho}{\partial \underline{Q}}$, and by Proposition 2, $\frac{\partial \rho}{\partial Q} = b \frac{\partial \rho}{\partial P} - [\frac{\partial \rho}{\partial \overline{Q}} + \frac{\partial \rho}{\partial \underline{Q}}]$. Thus, $[\frac{\partial \rho}{\partial q_2} + \frac{\partial \rho}{\partial Q} - b \frac{\partial \rho}{\partial P}] = \frac{\partial \rho}{\partial \overline{Q}} + \frac{\partial \rho}{\partial \underline{Q}} + b \frac{\partial \rho}{\partial P} - \frac{\partial \rho}{\partial \overline{Q}} - \frac{\partial \rho}{\partial \underline{Q}} - b \frac{\partial \rho}{\partial P} = 0$. Hence $\frac{\partial \Pi_2}{\partial q_2} = 0$. Note that this part of theorem 1 does not depend on the hypothesis $\overline{Q} > \underline{Q}$. //

Theorem 1 relies on the MLRP of both the conditional densities $\{f(P|\theta,Q)\}$ and $\{h(Q|Q(\theta)\}$ as well as on the condition $\overline{Q} > \underline{Q}$. Thus, in order to apply theorem 1 it is necessary to show that $\overline{Q} = \overline{q}_1 + q_2 > \underline{q}_1 + q_2 = \underline{Q}$. Alternatively it must be shown that, $\overline{q}_1 > \underline{q}_1$, for every q_2.

Lemma 8. Let $q_1(q_2,\overline{\theta}) = \overline{q}_1$ and $q_1(q_2,\underline{\theta}) = \underline{q}_1$, then, for all q_2,

$$\overline{q}_1 \neq \underline{q}_1 \tag{41}$$

Proof.

Suppose that $\overline{q}_1 = \underline{q}_1$, then $h(Q|\overline{Q}) = h(Q-\overline{q}_1-q_2) = h(Q-\underline{q}_1-q_2) = h(Q|\underline{Q})$, and

$$\rho(P,Q) = \frac{f(P-\overline{\theta}+bQ)h(Q-\overline{q}_1-q_2)\ \rho}{f(P-\overline{\theta}+bQ)h(Q-\overline{q}_1-q_2)\rho\ +\ f(P-\underline{\theta}+bQ)h(Q-\underline{q}-q_2)(1-\rho)}$$

$$= \frac{f(P-\overline{\theta}+bQ)\rho}{f(P-\overline{\theta}+bQ)\rho\ +\ f(P-\underline{\theta}+bQ)(1-\rho)}\ .$$

Hence, in this case, $\frac{\partial\rho}{\partial Q} = b\ \frac{\partial\rho}{\partial P}$ (c.f. Proposition 2).

Moreover,

$$0 = \frac{\partial\Pi_1(q_1,q_2,\theta)}{\partial q_1} - \int\ \{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\}h'(Q-q_1(\theta)-q_2)dQ. \tag{42}$$

Since $\frac{\partial \rho}{\partial Q} = b\,\frac{\partial \rho}{\partial P}$,

$$\frac{\partial}{\partial Q}\{\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)dP\} = \int V_1'[\rho(P,Q),\theta][\frac{\partial \rho}{\partial Q} - b\,\frac{\partial \rho}{\partial P}] f(P-\theta+bQ)dP = 0,$$

(c.f. **Lemma** 7, equation (32)). Thus, $\int V_1[\rho(P,Q),\theta] f(P-\theta+bQ)dP$ is constant for all Q. This implies that,

$$\int\{\int V[\rho(P,Q),\theta] f(P-\theta+bQ)dP\} h'(Q-q_1(\theta)-q_2)dQ$$
$$= \{\int V[\rho(P,Q),\theta] f(P-\theta+bQ)dP\}\int h'(Q-Q(\theta))dQ = 0$$

Therefore,

$$0 = \frac{\partial \Pi_1}{\partial q_1} .$$

Since $\Pi_1(q_1(\theta),q_2,\theta) = \int\int(\theta+\epsilon-b(q_1(\theta)+q_2+\mu))(q_1(\theta) + \mu)f(\epsilon)h(\mu)d\epsilon\; d\mu$ and $E(\epsilon) = E(\mu) = 0$,

$$\overline{q}_1 = \frac{\overline{\theta}}{2b} - \frac{1}{2}q_2$$
$$\underline{q}_1 = \frac{\underline{\theta}}{2b} - \frac{1}{2}q_2 \quad . \qquad (43)$$

Combining these values, $\overline{q}_1 - \underline{q} = \frac{\overline{\theta}-\underline{\theta}}{2b} > 0$, contradicting the assumption that $\overline{q}_1 = \underline{q}_1$. //

The next theorem shows that, $q_1(\theta)$ is in fact increasing.

Theorem 2. For each q_2, (again $q_1(q_2,\bar{\theta}) = \bar{q}_1$ and $q_1(q_2,\underline{\theta}) = \underline{q}_1$),

$$\bar{q}_1 > \underline{q}_1$$

Proof. Consider the pairs $(\bar{\theta},\bar{q}_1)$, $(\underline{\theta},\underline{q}_1)$ and suppose that $\bar{q}_1 < \underline{q}_1$ ($\bar{q}_1 \neq \underline{q}_1$ by lemma 8). Since $\bar{q}_1$, $\underline{q}_1$ are best responses, for the given q_2,

$$\bar{\Pi}_1(\bar{q}_1,q_2) + \int\int \bar{V}_1 f(P-\bar{\theta}+bQ)h(Q-\bar{Q})dP\, dQ \geq$$
$$\bar{\Pi}_1(\underline{q}_1,q_2) + \int\int \bar{V} f(P-\bar{\theta}+bQ)h(Q-\underline{Q})dP\, dQ,$$

and

$$\underline{\Pi}_1(\underline{q}_1,q_2) + \int\int \underline{V}_1 f(P-\underline{\theta}+bQ)h(Q-\underline{Q})dP\, dQ \geq \underline{\Pi}_1(\bar{q}_1,q_2) + \int\int \underline{V}_1 f(P-\underline{\theta}+bQ)h(Q-\bar{Q})dP\, dQ.$$

Hence,

$$\bar{\Pi}_1(\bar{q}_1,q_2)-\bar{\Pi}_1(\underline{q}_1,q_2) \geq \int\int \bar{V}_1 f(P-\bar{\theta}+bQ)[h(Q-\underline{Q}) - h(Q-\bar{Q})]dP\, dQ,$$

and

$$\int\int \underline{V}_1 f(P-\underline{\theta}+bQ)[h(Q-\underline{Q}) - h(Q-\bar{Q})]dPdQ \geq \underline{\Pi}_1(\bar{q}_1,q_2) - \underline{\Pi}_1(\underline{q}_1,q_2) \qquad (44)$$

Let $\bar{W}(Q) = \int \bar{V}_1 f(P-\bar{\theta}+bQ)dP$ and $\underline{W}(Q) = \int \underline{V}_1 f(P-\underline{\theta}+bQ)dP$ ($W(Q,\theta)$ is defined similarly).

Then, from (11) and (12), (recalling that $\hat{\theta} = \rho\ (\bar{\theta}-\underline{\theta}) + \underline{\theta})$),

$$\bar{W}(Q) - \underline{W}(Q) = \int \bar{V}_1 f(P-\bar{\theta}+bQ)dP - \int \underline{V}_1 f(P-\underline{\theta}+bQ)dP$$
$$= \frac{1}{36b}[\int (3\bar{\theta}-\hat{\theta})^2 f(P-\bar{\theta}+bQ)dP - \int (3\underline{\theta}-\hat{\theta})^2 f(P-\underline{\theta}+bQ)dP]. \qquad (45)$$

After algebraic manipulation, equation (45) becomes,

$$\overline{W}(Q) - \underline{W}(Q) = \frac{(\overline{\theta}-\underline{\theta})}{36b}\{9\overline{\theta} + 3\underline{\theta} + 2\underline{\theta}\int\rho f(P-\overline{\theta}+bQ)dP + 4\underline{\theta}\int\rho f(P-\underline{\theta}+bQ)dP$$
$$+ (\overline{\theta}-\underline{\theta})\int\rho^2[f(P-\overline{\theta}+bQ) - f(P-\underline{\theta}+bQ)]dP - 6\overline{\theta}\int\rho f(P-\overline{\theta}+bQ)dP\}. \quad (46)$$

Note that since the densities $\{f(\rho|\theta,Q)\}$ have the MLRP there exists a $\hat{Q}$ such that $[f(\rho-\overline{\theta}+bQ) - f(\rho-\underline{\theta}+bQ)] > 0$ (< 0) if $Q < \hat{Q}$ (if $Q > \hat{Q}$). The fact that $\frac{\partial\rho}{\partial P} > 0$ and $\int[f(P-\overline{\theta}+bQ) - f(P-\underline{\theta}+bQ)]d\rho = 0$ implies that

$$\int\rho^2[f(P-\overline{\theta}+bQ) - f(P-\underline{\theta}+bQ)]dP < 0.$$

Thus all the terms on the r.h.s. of equation (46) are nonnegative but the last one. However, since $\rho \in [0,1]$, $-6\overline{\theta}\int\rho f(P-\overline{\theta}+bQ) > -6\overline{\theta}$,

$$\overline{W}(Q) - \underline{W}(Q) > \frac{(\overline{\theta}-\underline{\theta})}{36b}\{3(\overline{\theta}+\underline{\theta}) + (\overline{\theta}-\underline{\theta})\int\rho^2[f(P-\overline{\theta}+bQ) - f(P-\underline{\theta}+bQ)]dP$$
$$+ 2\underline{\theta}\int\rho f(P-\overline{\theta}+bQ)dP + 4\underline{\theta}\int\rho f(P-\underline{\theta}+bQ)dP\} > 0 \quad . \quad (47)$$

Now suppose that, $\underline{q}_1 > \overline{q}_1$, then by Lemma 6, equation (33),

$$\frac{\partial}{\partial Q}W(Q,\theta) = \frac{\rho(1-\rho)}{D}\int V_1'[\rho(P,Q),\theta]f(\overline{\theta})f(\underline{\theta})[h'(\overline{\theta})h(\underline{\theta})-h(\overline{\theta})h'(\underline{\theta})]f(P-\theta+bQ)dP \geq 0.$$

The inequality follows since $V_1'(\cdot,\theta) < 0$, by Lemma 2, $\underline{Q} > \overline{Q}$ by assumption, and the densities $\{h(Q(Q(\theta))\}$ have the MLRP, i.e., $[h'(\overline{\theta})h(\underline{\theta}) - h(\overline{\theta})h'(\underline{\theta})] \leq 0$.

Thus the functions $\overline{W}(Q)$ and $\underline{W}(Q)$ are nondecreasing. Since $\overline{W}(Q) > \underline{W}(Q)$, the MLRP implies that,

$$\int \overline{W}(Q)\,[h(Q-\underline{Q}) - h(Q-\overline{Q})]\,dQ \geq \int \underline{W}(Q)\,[h(Q-\underline{Q}) - h(Q-\overline{Q})]\,dQ. \tag{48}$$

But, by equation (48), this means that

$$\overline{\Pi}_1(\overline{q}_1, q_2) - \overline{\Pi}_1(\underline{q}_1 - q_2) \geq \underline{\Pi}_1(\overline{q}_1, q_2) - \underline{\Pi}_1(\underline{q}_1, q_2), \tag{49}$$

i.e.,

$$(\overline{\theta} - b\overline{q}_1 - bq_2)\overline{q}_1 - (\overline{\theta} - b\underline{q}_1 - bq_2)\underline{q}_1 \geq (\underline{\theta} - b\overline{q}_1 - bq_2)\overline{q}_1 - (\underline{\theta} - b\underline{q}_1 - bq_2)\underline{q}_1$$

Hence

$$\overline{\theta}\,\overline{q}_1 - \overline{\theta}\,\underline{q}_1 \geq \underline{\theta}\,\overline{q}_1 - \underline{\theta}\,\underline{q}_1$$

i.e.,

$$\overline{\theta}(\overline{q}_1 - \underline{q}_1) \geq \underline{\theta}(\overline{q}_1 - \underline{q}_1)$$

Since $\overline{\theta} > \underline{\theta}$, $\overline{q}_1 - \underline{q}_1 \geq 0$, a contradiction. Hence $\overline{q}_1 > \underline{q}_1$. //

In Lemma 9 the best response functions for Firm I are compared to the myopic solutions, i.e. those solutions for which the future is not taken into account. These myopic solutions are denoted by a superscript m. Moreover the value $\overline{q}_1$ and $\overline{q}_1$ are functions of q_2 (i.e., points on the reaction curve) and q_2 depends on the conjectures $\overline{q}_1$, $\underline{q}_1$ of Firm II about the choice of Firm I.

Lemma 9.

(i) $\bar{q}_1 \leq \bar{q}_1^{m}$, for all q_2 (50)

(ii) $\underline{q}_1 \leq \underline{q}_1^{m}$, for all q_2 (51)

(iii) $q_2 = q_2^{m}$, for all $\bar{q}_1$, $\underline{q}_1$. (52)

Proof.

(i) By theorem 1, $\dfrac{\partial \Pi_1(q_1, q_2)}{\partial q_1} \geq 0$. The myopic solution $\bar{q}_1^{m}$ satisfies the condition, $\dfrac{\partial \Pi_1(\bar{q}_1^{m}, q_2)}{\partial q_1} = 0$. Since Π_1 is concave, $\bar{q}_1 \leq \bar{q}_1^{m}$.

(ii) Follows similarly.

(iii) Again by Theorem 1, $0 = \dfrac{\partial \Pi_2}{\partial q_2}$ for any given $\bar{q}_1$, $\underline{q}_1$. The myopic solution satisfies $\dfrac{\partial \Pi_2}{\partial q_2} = 0$ as well. Hence $q_2^{m*} = q_2$.

Finally we study the <u>equilibrium</u> outputs and compare the Bayesian-Nash equilibrium outputs to the "myopic" equilibrium outputs. It should be noted that even though, from Lemma 9 (iii), $q_2^{m} = q_2$ for all possible conjectures, the output for Firm II is not the same in the Bayesian Nash case and myopic case since in these two cases the output of Firm I is different, and the equilibrium output of Firm II will reflect this difference.

The <u>myopic</u> equilibrium outputs are the equilibrium outputs in the one period game i.e.,

$$q_1^{m*}(\theta) = \underset{q_1}{\operatorname{argmax}}\ \Pi_1(q_1, q_2^{m*}, \theta) \ , \tag{53}$$

and,

$$q_2^{m*} = \underset{q_2}{\text{argmax}}\ \Pi_2(q_1^{m*}(\overline{\theta}),\ q_1^{m*}(\underline{\theta}),\ q_2) \tag{54}$$

Let $\overline{q}_1^* = q_1^*(\overline{\theta})$, $\underline{q}_1^* = q_1^*(\underline{\theta})$, q_2^*, be the equilibrium outputs for Firm I and Firm II, respectively, in the first period of the two period game and let $Q^*(\theta) = q_1^*(\theta) + q_2^*$. Similarly let $\overline{q}_1^{m*} = q_1^{m*}(\overline{\theta})$, $\underline{q}^{m*} = q_1^{m*}(\underline{\theta})$.

Theorem 3. If the densities $\{f(P|\theta,Q)\}$ and $\{h(Q|Q(\theta))\}$ both have the MLRP, then,

(i) $q_1^*(\theta) \le q_1^{m*}(\theta)$

(ii) $q_2^* \ge q_2^{m*}$

Proof. The first order conditions can be solved as follows,

$$q_1^*(\theta) = \frac{\theta}{2b} - \frac{1}{2}q_2^* - \frac{1}{2b}\int\{\int V_1[\rho(P,Q),\theta]f(P-\theta+bQ)dP\}\ h'(Q-Q^*(\theta))dQ$$

and

$$q_2^* = \frac{\hat{\theta}}{2b} - \frac{1}{2}[\rho\overline{q}_1^* + (1-\rho)\underline{q}_1^*]\ .$$

Also,

$$q_1^{m*}(\theta) = \frac{\theta}{2b} - \frac{1}{2}q_2^{m*}$$

and,

$$q_2^{m*} = \frac{\hat{\theta}}{2b} - \frac{1}{2}[\rho\bar{q}_1^{m*} + (1-\rho)\underline{q}_1^{m*}] \quad .$$

Combining these expressions yields,

$$q_1^*(\theta) - q_1^{m*} = -\frac{1}{2}[q_2^* - q_2^{m*}] - \frac{1}{2b}\int\{\int V_1[\rho(P,Q),\theta]f(\rho-\theta+bQ)dP\}\ h'(Q-Q^*(\theta))dQ \quad (55)$$

and

$$q_2^* - q_2^{m*} = -\frac{1}{2}[\rho\bar{q}_1^* + (1-\rho)\underline{q}_1^*] + \frac{1}{2}[\rho\bar{q}_1^{m*} + (1-\rho)\underline{q}_1^{m*}]$$
$$= \frac{1}{2}[\rho(\bar{q}_1^{m*} - \bar{q}_1^*) + (1-\rho)(\underline{q}_1^{m*} - \underline{q}_1^*)] \quad .$$

Hence,

$$q_2^* - q_2^{m*} = \frac{1}{3b}[\rho\int \{\int\bar{V}_1 f(P-\bar{\theta}+bQ)dP\}\ h'(Q-\bar{Q}^*)dQ$$
$$+ (1-\rho)\ \{\int\underline{V}_1 f(P-\underline{\theta}+bQ)dP\}\ h'(Q-\underline{Q}^*)dQ]$$

By Lemma 7, $\int\{\int V_1[\rho(P,Q),\theta]f(\rho-\theta+bQ)dP\}\ h'(Q-Q^*(\theta))dQ \geq 0$, thus $q_2^* \geq q_2^{m*}$. This implies, from (55) that $q_1^*(\theta) \leq q_1^{m*}(\theta)$. //

Conclusion

We have presented a model of a duopolistic market with asymmetric information. In this model there is one informed and one uninformed firm. Each firm maximizes the sum of two period profits. The second period profit function of both the informed and the uninformed firm depend upon the subjective (a posterior) beliefs of the uninformed firm. Hence the potential exists for the informed firm to manipulate its ouput and for the uninformed firm to experiment. Unfortunately it turns out that the uninformed firm does not gain from experimentation - and hence does not experiment. However this result of no experimentation depends on the fact that the intercept term, rather than, say, the slope, is the unknown parameter. In the unknown intercept case the informational content of the decision is independent of the output of the uninformed firm.

This conclusion leads immediately to the question of: when will the uninformed firm experiment? It is likely that if the slope is unknown and, as above, the output is a random variable whose mean is the choice variable of the informed firm, the uninformed firm will experiment. This question has not yet been studied. Another interesting question is; can the assumption that output is random be dispensed with? The answer to this question is considerably less clear and potentially more difficult. The reason that the random output assumption is made is that we found it necessary to separate the effect of the price signal and the quantity signal in order to invoke the MLRP. This separation can best be seen from equation (16). It is seen in (16) that the effect of the two variables which are unknown to the uninformed firm, the value of θ and the decision variable of the informed firm, can be separated. When these variables can be separated the MLRP must be invoked twice (once for h and once for f). Since $\overline{\theta} > \underline{\theta}$, the MLRP holds for f. However, in order for the MLRP to hold for h it must be shown that $Q(\theta)$ is monotonic in θ, i.e., $\overline{Q} > \underline{Q}$. Both this paper and MM need this monotonicity and it was in fact shown to hold in both papers.

The alternative assumption - that the uninformed firm does not have any information about the output of the informed firm - combines the two unknown variables. To be more precise, if in this paper Q were not observed then the probability of having seen P given that $\theta = \overline{\theta}$ and $Q = \overline{Q}$ is $f(P - \overline{\theta} + b\overline{Q})$. In order to invoke the MLRP it is necessary to show that $\theta - bQ(\theta)$ is monotonic in θ. Although we have not been able to prove that $\theta - bQ(\theta)$ is monotonic, it appears that it is not necessarily true. In other words, it might be optimal for the informed firm to increase as well as decrease its output to manipulate the uninformed firm. This remains an open question.

There are two questions that are important and interesting which has not been discussed. The first the effect of asymmetric information on total output. We have only studied the effect of asymmetric information on myopic decisions for each firm. The second is the question of existence. It appears that even stronger conditions than the MLRP is needed on the density functions to guarantee the existence of optimal solutions. Finally, examples are hard to find. It would be useful to illustrate these results with several simple examples.

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