UNIVERSIDADE NOVA DE LISBOA
Faculdade de Economia

"ON TRADE IN A TWO COUNTRY WORLD"

Vasco d'Orey

Working Paper Nº 86/8

UNIVERSIDADE NOVA DE LISBOA
Faculdade de Economia
Travessa Estevão Pinto
1000 LISBOA

Junho, 1988

ON TRADE IN A TWO COUNTRY WORLD

by Vasco d'Orey*

* Universidade Nova de Lisboa, Faculdade de Economia, Travessa Estevão Pinto, 1000 Lisboa, Portugal. I am grateful to Antonieta da Cunha e Sá, Luis Cunha, Diogo Lucena and Mário Páscoa for their comments. Denis Cory must be acknowledged for a reading list relating to natural resources. Editorial assistance of John Huffstot is also acknowledged. Any error is my responsability.

This paper shows that, under the usual assumptions of the Hecksher-Ohlin-Samuelson model, extensivaly used in the pure theory of international trade, free trade can be seen as the solution to a Nash cooperative game. The question of free or fair trade is then related to different admissible solution concepts to a two person cooperative game.

INTRODUCTION

The standard approach to the pure theory of international trade assumes, in its utmost simplicity and beauty, that countries can be described by endowments of factors of production, usually assumed to be capital and labor, technology relating the use of factors and the output of two goods, and the preferences of nationals of each country, these being defined over different consumption bundles. Under autarchy each country produces efficiently, and any difference in factor endowments, technologies or tastes will result in different autarchy relative prices for the goods produced. Any such difference can then be exploited, and as the factors of production, capital and labor are assumed to be mobile within each country, but not between both countries, the exchange of goods produced (international trade), is the only avenue left in this endeavor. Once it is established, and a world relative price for the goods produced is found, one goes on to prove that, under the same conditions, both trading partners benefit from this situation. Trade is not usually caused by a strategic behavior, but rather, by the casual observation that there are differences in both partners that can be exploited for their mutual benefit. As a result, when two countries of equal dimension are considered, in the absence of tariffs and quotas, the outcome of this process is usually called a free trade solution.

In the traditional approach, the cooperative behavior that a free trade solution may entail is totally ignored. After all, it has been long known within the profession that it takes two to tango, and the autarchy point is always available for any trading partner. One can then ask if free trade can be arrived at after a strategic behavior is adopted by each country, a question that appears never to have been explicity spelled out in the literature. Our main point here is that, under the usual assumptions that are made in the standard Hecksher-Ohlin-Samuelson model, free trade can be seen as a solution to a cooperative game played by the two countries. Under this approach free trade may be seen as the consequence of some difference between the two countries, but is arrived at after some rules, (which must be obeyed by the solution to their cooperative approach to the joint welfare maximization), are agreed upon by the two players, (the two countries).

The remainder of the paper will proceed as follows. Section I will show that, under the usual assumptions of the Hecksher-Ohlin-Samuelson model, free trade can be seen as the solution to a Nash cooperative game[1] that is played by both countries. Section II will be devoted to the discussion of some other possible outcomes of a cooperative game played by the two countries, and relate them the ongoing arguments on free and fair trade. Section III will present a brief summary of the main conclusions presented here.

I. FREE TRADE AS THE OUTCOME OF A COOPERATIVE GAME.

We will assume in this section that the usual assumptions characterizing the Hecksher-Ohlin-Samuelson model are satisfied. There are two countries, A and B, two factors of production, capital (K) and labor (L) and two goods being produced in each country, good X and good Y, the technology of production being the same in each country. Consumers have preferences that are specified by an ordinal utility function, $U(C_x^A, C_y^A)$ and $V(C_x^B, C_y^B)$ being, respectively, the utility function of a A's and B's representative consumer, where C_i^j is the consumption of good i=X,Y in country j=A,B.

---

[1] Nash's solution of a two-person cooperative game is consistent with four axions: i) group rationality; ii) Von-Neuman-Morgenstern invariance of utility indicators; iii) Symmetry of players: iv) Independence of irrelevant alternatives. (See Nash, 1953, Friedman, 1986)

Given the endowments of capital and labor, each economy will be producing on its production possibility frontier, with the meaning that for any feasible production of good X the maximum feasible amount of good Y is being produced. Let

$$Q_x^A = \gamma(Q_y^A), \gamma' < 0, \gamma'' < 0 \tag{1}$$

$$Q_x^B = \psi(Q_y^B), \psi' < 0, \psi'' < 0 \tag{2}$$

be, respectively, the production possibility frontier of country A and country B, assuming that the technology of production and the factor endowments in each country are given, and where Q_x^j is the maximum amount of good X produced, given that the amount Q_y^j of good Y is being produced, where j=A,B.

The possibility of trade allows domestic consumption of any good to be different from domestic production of the good. Thus, when trade is allowed for, we have that

$$C_x^A = Q_x^A - M \tag{3}$$

$$C_y^A = Q_y^A + E \tag{4}$$

$$C_x^B = Q_x^B + M \tag{5}$$

$$C_y^B = Q_y^B - E \tag{6}$$

where M stands for the amount traded of good X and E stands for the amount traded of good Y. Equations (3), (4), (5) and (6) have the usual meaning that, once trade is allowed, consumption of any good in any country is equal to domestic production plus imports or minus exports of the same good. The requirement that, at the world relative price, trade must be balanced can be expressed by

$$E - p\,M = 0 \tag{7}$$

meaning that, at world relative prices, p, exports and imports balance.

In autarchy each country produces and consumes so that the domestic marginal rate of substitution in consumption equals the domestic marginal rate of substitution in production, the autarchy price level prevailing in country A and B being determined by this relation. As autarchy is always a possible regime, the utility levels associeted with the no trade situation for countries A and B are the natural threat points of the Nash cooperative game. Given that we are assuming cardinal utility functions, we take the utility levels associated with the autarchy situation as being zero.

It is widely know that under the Nash cooperative behavior the solution to the game can be found by the cooperative maximization of the product of the utility gains accrued to each player, these being measured from its threat point. Once that both players adopt the Nash behavior for the cooperative trade game, the solution can be found by:

$$\text{Max } U(C_x^A, C_y^A)\; V(C_x^B, C_y^B) \tag{8}$$

subject to the restrictions imposed by (1) to (7). As it is shown in the appendix, the solution to this problem can be characterized by the following conditions:

$$MRS^A = MRS^B = MRT^A = MRT^B \tag{9}$$

where MRSj and MRTj are, respectively, the marginal rate of substitution in consumption and the marginal rate of transformation in production in country j=A,B. What we have obtained in (9), taking into account that (7) is satisfied, are the conditions characterizing a free trade solution, meaning that the outcome of a cooperative Nash bargaining game between two countries, when balanced trade is allowed, is the free trade solution. This result can be most easily interpreted if we take into account the analysis developed by Roth (Roth, 1977). This author has shown that, under the conditions we have required for the bargaining game between the two countries, the result of the game will be Pareto optimal. The pure theory of international trade has been telling us that free trade is, from the world point of view, a particular Pareto optimal (Bahgwati and Srinivasan, 1986). Given the restrictions we are imposing on the game, the outcome will be a Pareto optimum, with the additional property that trade will be balanced. The result follows in an obvious way.

II. TRADE - FREE OR FAIR?

The last section may shed some light upon a discussion that seems to be in the order of the day. Should trade be seen as free trade or as fair trade? Once we have established that, under some very mild and usual assumptions, free trade can be seen as the Nash solution to a cooperative game played by the two countries, why should the Nash solution be the one agreed upon by both trading partners? It is widely known that some other solutions have been proposed as a solution to a cooperative two person game. The Raiffa-Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975) and the minimal expectations solution (Friedman 1986) can be given as examples of different solution concepts that are perfectly plausible to generate outcomes for the cooperative game that the existence of trade implies. Any solution to the game that will result from the common adoption of playing rules that are different from the ones characterizing the Nash solution, may violate some of the conditions characterizing this one, meaning that some optimality condition related to a worldwide Pareto optimum situation, accompanied by balanced trade, may be violated. In this way both countries, by common agreement over a set of bargaining rules, may be disposing of the free trade solution, as the outcome of the trade game may be different from this situation. The question of free or fair trade is not passé (Krugman, 1987). Rather it can then be transposed to what kind of solution concept is adopted as the solution to the cooperative game which is played by the two trading partners. In this way fairness will not be a characteristic of the trading pattern that is arrived at, but rather, a property of the rules of the cooperative game which are agreed upon by both countries before the actual play has taken place, as these determine the outcome of the game.

III. CONCLUSIONS

In this paper it was shown that free trade can be seen as the result of a Nash cooperative game played by two countries, these being characterized as in the standard pure theory of international trade. The question of free or fair trade was also touched upon, and it was pointed out that, in the light of the previous result, fairness should not be seen as a property of the trading pattern, but rather, as a property of the rules which the solution of the cooperative game must obey

APPENDIX

The lagrangean corresponding to the non-linear optimization of (8) in the main text, subject to the restriction (1) to (7), can be written as:

$$\ell = U(C_x^A, C_y^A)\, V(C_x^B, C_y^B) + \lambda_1 (C_x^A - Q_x^A + E) + \lambda_2 (C_y^A - Q_y^A - M) +$$

$$+ \lambda_3 (C_x^B - Q_x^B - E) + \lambda_4 (C_y^B - Q_y^B + M) + \lambda_5 (Q_x^A - \gamma(Q_y^A)) +$$

$$+ \lambda_6 (Q_x^B - \psi(Q_y^b)) + \lambda_7 (E - p\,M). \qquad \text{(A1)}$$

Partial differentiation of (A1) leads to

$$\frac{\partial \ell}{\partial C_x^A} = V(C_x^B, C_y^B)\, \frac{\partial U}{\partial C_x^A} (C_x^A, C_y^B) + \lambda_1 = 0 \qquad \text{(A2)}$$

$$\frac{\partial \ell}{\partial C_y^A} = V(C_x^B, C_y^B)\, \frac{\partial U}{\partial C_y^A} (C_x^A, C_y^A) + \lambda_2 = 0 \qquad \text{(A3)}$$

$$\frac{\partial \ell}{\partial C_x^B} = U(C_x^A, C_y^A)\, \frac{\partial V}{\partial C_x^B} (C_x^B, C_y^B) + \lambda_3 = 0. \qquad \text{(A4)}$$

$$\frac{\partial \ell}{\partial C_y^B} = U(C_x^A, C_y^A)\, \frac{\partial V}{\partial C_y^B} (C_x^B, C_y^B) + \lambda_4 = 0. \qquad \text{(A5)}$$

$$\frac{\partial \ell}{\partial Q_x^A} = -\lambda_1 + \lambda_5 = 0. \qquad \text{(A6)}$$

$$\frac{\partial \ell}{\partial Q_x^A} = -\lambda_2 - \lambda_5 \gamma' = 0. \qquad \text{(A7)}$$

$$\frac{\partial \ell}{\partial Q_x^B} = -\lambda_3 + \lambda_6 = 0 \tag{A8}$$

$$\frac{\partial \ell}{\partial Q_y^B} = -\lambda_4 - \lambda_6 \psi' = 0. \tag{A9}$$

$$\frac{\partial \ell}{\partial E} = \lambda_1 - \lambda_3 + \lambda_7 = 0. \tag{A10}$$

$$\frac{\partial \ell}{\partial M} = -\lambda_2 + \lambda_4 - \lambda_7 p = 0. \tag{A11}$$

$$\frac{\partial \ell}{\partial p} = -\lambda_7 \bar{M} = 0 \tag{A12}$$

The partial derivatives of the lagrangean relative to the lagrange multipliers are just the restrictions. Assuming an interior solution, incomplete specialization and trade, M will be diffrent from zero so that λ_7=0, from (A12). It follows from (A10) and (A11).

$$\lambda_1 = \lambda_3$$

$$\lambda_2 = \lambda_4 \tag{A13}$$

Dividing (A3) by (A2), (A5) by (A4), (A7) by (A6) and (A9) by (A8), it can be found that

$$\frac{\dfrac{\partial U}{\partial C_x^A}(C_x^A, C_y^A)}{\dfrac{\partial U}{\partial C_x^A}(C_x^A, C_y^A)} = \frac{\dfrac{\partial V}{\partial C_y^B}(C_x^B, C_y^B)}{\dfrac{\partial V}{\partial C_y^B}(C_x^B, C_y^B)} = -\gamma' = \Psi'$$

proving (9) in the main text.

REFERENCES

Bhagwati, Jagdish N. and Srinivasan, T.N., "Lectures on International Trade", Cambridge, Massachussetts; The M.I.T. Press, 1983.

Friedman, James W., "Game Theory with Applications to Economics", New York: Oxford University Press, 1986.

Kalai, Ehud and Smorodinsky, Meir, "Other Solutions to Nash's Bargaining Problem" Econometrica, 1982, 43, 513-18.

Krugman, Paul R., "Is Free Trade Passe?",The Journal of Economic Perspectives, Fall 1987, vol. 1, 2, 131-44.

Nash, J., "Two Person Cooperative Games", Econometrica, January 1953, 21, 128-40.

Roth, A. E., "Individual Rationality and Nash's Solution to the Bargaining Problem", Mathematics of Operations Research, 1977, 2, 64 - 66.

ÚLTIMOS WORKING PAPERS PUBLICADOS

nº 73 - ANTUNES, António Pais e GASPAR, Vitor : " Tributação, Incentivos e Investimentos: Análise Qualitativa". (Novembro, 1987).

nº 74 - BAROSA, José Pedro: "Optimal Wage Rigidity: A Suggested Methodology to Test the Theory and an Application". (Outubro, 1987).

nº 75 - CABRAL, L. M. B.: "Three Notes on Symmetric Games with Asymmetric Equilibria". (Novembro, 1987).

nº 76 - LUCENA, Diogo:"To Search or Not to Search".(Fevereiro,1987)

nº 77 - SÁ, Jorge Vasconcelos e: "A Theory of Synergy". (Fevereiro, 1987).

nº 78 - VILARES, Manuel José: "Os Bens Intermédios Importados Como Factor de Produção". (Julho, 1987).

nº 79 - SÁ, Jorge Vasconcelos e: "How To Compete And Communicate in Mature Industrial Products". (Maio, 1988).

nº 80 - ROB, Rafael: "Learning and Capacity Expansion IN A New Market Under Uncertainty". (Fevereiro, 1988).

nº 81 - PEREIRA, Alfredo Marvão: "Survey of Dynamic Computational General Equilibrium Models For Tax Policy Evaluation". (Outubro, 1987).

nº 82 - SÁ, Jorge Vasconcelos e: "Everything IS Important: But Some Things Are More Important Than Others". (Maio, 1988).

nº 83 - BÁRCIA, Paulo, GASPAR, Vitor e PEREIRA, Pedro T.: "Consumer Theory With NonConvex Consumption Sets". (Maio,1988).

nº 84 - CABRAL, Luís Martins Barata: "Política de Preços do Sector das Telecomunicações: Uma Análise de Equilíbrio Geral". (Junho de 1985; Revisto em Maio de 1988).

nº 85 - PEREIRA, Alfredo Marvão: "DAGEN- A Dynamic Applied General Equilibrium Model For Tax Policy Evaluation". (Abril, 1988).

nº 86 - D'OREY, Vasco: "On Trade in a Two Country World". (Junho, 1988).

Qualquer informação sobre os Working Papers já publicados será prestada pelo Secretariado de Apoio aos Docentes, podendo os mesmos ser adquiridos na Secção de Vendas da Faculdade de Economia, UNL, na Travessa Estevão Pinto, Campolide - 1000 LISBOA.