.DAGEM - A DYNAMIC APPLIED GENERAL EQUILIBRIUM
MODEL FOR TAX POLICY EVALUATION

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Alfredo Marvão Pereira

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Abstract

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ABSTRACT

This paper develops a sequential dynamic general equilibrium model of the U.S. economy - DAGEM. Economic behavior of every agent in this economy is derived from an intertemporal specification of the agent's objectives and constraints. Firms maximize the present value of the net cash flow in a technology with adjustment costs to determine endogenously optimal supplies and optimal demands for the different production inputs. In particular, investment decisions are forward looking. Real investment is financed by retained earnings and issuance of new debt and equity according to exogenously defined rules. Government intertemporal behavior is obtained from the maximization of a social welfare function defined over the domain of a public good and subject to an intertemporal budget constraint. The government is allowed to run deficits which are financed by issuing bonds. Optimal household behavior follows a life-cycle type of model generating endogenous savings and labor-leisure decisions. Household asset portfolio decisions merely accommodate the composition of demand for funds. Equilibrium in this economy is conceived as a temporary Walrasian equilibrium. All the markets clear, hence the Walrasian

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nature of equilibrium. Also, equilibrium in the short run is such that market clearing prices are parametric on the expectation formation rules, hence the temporary nature of equilibrium.

The second part of this paper addresses problems of implementation and policy analysis with the DAGEM. In the context of applied general equilibrium analysis, policy evaluations are typically carried out by contrasting a base case reflecting the status quo and several counterfactual equilibria reflecting different scenarios generated by the policy change under consideration. First, it is necessary to specify the base case equilibrium. In particular, the data requirements are reviewed and sources provided. Secondly, the different equilibria are made comparable by the use of the concept of equal yield. The concept of equal yield is generalized to accommodate the existence of government deficits. Thirdly, the information contained in the different equilibria is synthesized by using a scalar indicator. This indicator is the dynamic generalization of the Hicksian compensation tests to a context in which expectations are not self-fulfilling, and no future markets exist. This chapter contains also a discussion of the computation strategy and, in particular, the computation algorithm.

This paper concludes with a critical assessment of the DAGEM in terms of modeling and implementation as well as suggestions for future research. The potential of the methodology developed in this paper is emphasized. In particular, the merits of DAGEM to address several public finance issues, like the possible re-introduction of investment tax credits or the effects of political measures tending to balance the government budget, are discussed.

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CHAPTER 1 - INTRODUCTION

There is a well established body of literature focusing on tax policy evaluation using computational general equilibrium (CGE) techniques (see Shoven-Whalley (1984) and Pereira-Shoven (1988) for detailed surveys of this literature).[1/] The CGE approach has been developed as an answer to the three-way trade-off among the traditional methodologies in economics in their quest for comparative statics results. Partial equilibrium allows for highly disaggregated analysis at the cost of not considering market interactions. Macroeconomics allows for market interactions in the context of highly aggregated models. In turn, analytic general equilibrium allows for both disaggregated analysis and full consideration of market interactions. However, unlike partial equilibrium and macroeconomic analysis, general equilibrium fails, in general, to produce clear-cut quantitative and qualitative analytic comparative statics results. This is due to the complexity and dimensionality added to afford full-market feedbacks in a disaggregated setting.

Computational general equilibrium keeps the desirable features of analytic general equilibrium. In particular, it allows the study of differential impacts across sectors of production and consumer groups taking into full consideration market interactions. However, CGE is based on the use of flexible numeric - as opposed to analytic - techniques to obtain clear unambiguous comparative statics results. Furthermore, by adopting the numeric approach, the modeler does not have to be confined to small changes in parameters as with an analytic approach. This is also an important feature because large changes in policy parameters are required by most policy alternatives.

This paper develops a dynamic applied general equilibrium model for tax policy evaluation -

DAGEM. The economy is characterized by an incomplete, sequential market structure in a finite horizon and in a discrete time frame. Agents face a dynamic environment. Economic behavior of every agent in this economy is derived from an intertemporal specification of the agent's objectives and constraints. Firms maximize the present value of the net cash flow in a technology with adjustment costs to determine endogenously optimal supplies and optimal demands for the different production inputs. In particular, investment decisions are forward looking. Real investment is financed by retained earnings and issuance of new debt and equity according to exogenously defined rules. Government intertemporal behavior is obtained from the maximization of a social welfare function defined over the domain of a public good and subject to an intertemporal budget constraint. The government is allowed to run deficits which are financed by issuing bonds. Optimal household behavior follows a life-cycle type of model generating endogenous savings and labor-leisure decisions. Household asset portfolio decisions merely accommodate to the composition of demand for funds.

Economic decisions are formulated in a context of uncertainty about future prices and interest rates. In each period, expectations are formed as point expectations according to given rules. The concept of Temporary Walrasian Equilibrium (TWE) is adopted to capture the incomplete and sequential aspects of real world trading and the limitations of foresight into the future. All the markets for the several consumption goods, investment goods, labor, and financial funds, clear, hence the Walrasian nature of equilibrium. Also, equilibrium in the short run is such that market clearing prices are parametric on the expectation formation rules, hence the temporary nature of equilibrium.

The model in this paper departs from most of the numerical GE literature for tax policy evaluation, in several fundamental directions directly relevant for policy oriented analysis.[2/] First, it provides a comprehensive modeling of dynamic economic behavior. In particular, government deficits are optimally determined and investment decisions are forward looking and are the result of optimizing behavior. Secondly, it encompasses an endogenous sequential equilibrium

structure founded on dynamic economic behavior with flexible expectations. Thirdly, the model provides a detailed consideration of financial assets, public and private.

In the context of applied general equilibrium analysis, policy evaluations are typically carried out by contrasting a base case reflecting the status quo and several counterfactual - revised case - equilibria reflecting different policy scenarios. That being the case, it is first necessary to specify the base-case equilibrium. In particular, the data requirements are reviewed and sources provided. Secondly, the different equilibria are made comparable by the use of the concept of equal yield generalized to accommodate the existence of government deficits. Government optimal behavior in the revised cases is consistent with the base case social utility levels. Tax policy changes may be financed by increased tax revenues holding government deficits constant, or by increased debt holding tax revenues constant, or any combination of the two. The pure effects of the tax policy change can thereby be separated from the effects induced by marginal financial crowding out. Thirdly, the information contained in the different equilibria is typically synthesized by using a scalar indicator which is the dynamic generalization of the Hicksian compensation tests. This measure was derived specifically to accommodate intertemporal comparisons when perfect foresight is not assumed and future markets are not open.

Due to the absence of the requirement of intertemporally consistent action plans, the empirical implementation of the DAGEM for a T-period horizon, implies that T-separate rounds of one-period equilibria have to be computed. The link between adjacent periods is endogenously provided by the recursive relationships of the stock variables in the economy. The DAGEM model is implemented with a stylized data set for the U.S. economy, using a nonlinear optimization algorithm - NPSOL.

This paper concludes with the critical assessment of the different contributions in terms of modeling and implementation. Some directions for future research are also discussed. Also, the potential of the methodology developed in this paper is emphasized. In particular, the merits of DAGEM to address several public finance issues, such as the reintroduction of investment tax credits, or the effects of political measures tending to balance the government budget are discussed.

CHAPTER 2 - A DYNAMIC GENERAL EQUILIBRIUM MODEL FOR TAX POLICY EVALUATION

This chapter opens with a brief overview of "non-dynamic" CGE models. The rest of the chapter develops a fully dynamic applied general equilibrium model -DAGEM - designed for the purpose of tax policy evaluation. Section 2.2 provides a general characterization of the economic environment. The optimal behavior of households, producers, and government is spelled out in sections 2.3-2.5. Section 2.6, establishes the temporary Walrasian equilibrium conditions and the optimal transitions for the different stock variables in the economy. Section 2.7, develops the different rules of formation of expectations to be considered in this work. Sections 2.3-2.7, also include brief overviews of relevant aspects of dynamic CGE modeling for tax policy evaluation.

2.1 BRIEF OVERVIEW OF "NON-DYNAMIC" GENERAL EQUILIBRIUM MODELS

The origin of most of the CGE models goes back to the work of Scarf (1967, 1973), who first developed a reliable algorithm to compute equilibrium prices for an Arrow-Debreu economy. His algorithm used simplicial subdivision techniques and can be shown to be the computational analog of the fixed point theorems previously used to prove the existence of equilibrium. His technique could solve a model with an arbitrary number of consumers and commodities, as long as all agents were price takers, consumers were subject to budget constraints, demands were continuous, and production did not display increasing returns to scale. The algorithm, while guaranteed to converge, was relatively slow for problems involving more than say, 20 dimensions. A major improvement in computational speed was offered by Merrill's algorithm (1972), which used the same fundamental ideas as Scarf's procedure.

Scarf's model (as is true for the standard Arrow-Debreu model) does not include a government sector - neither taxes nor public goods. As one of the most promising applications of the new computational technique was in the area of tax policy evaluation, Shoven and Whalley (1973)

extended the general equilibrium model and computational approach to include a wide array of taxes and a government spending plan. The original Shoven-Whalley model was static (although the different commodities could be considered similar goods available at- different dates, as in -Arrow-Debreu), and government was assumed to run a balanced budget. The data are arranged as a social accounting matrix. The model's specification and calibration are checked by solving it in the presence of the base set of taxes. The result should be exactly the initial social accounting matrix. After having passed this replication check, the model is solved for a counterfactual equilibrium in the presence of a new tax design. The result is once again a social accounting matrix. The two equilibria are compared in order to assess the impact of the new tax plan. The first uses of this model for tax policy evaluation were Shoven-Whalley (1972), Whalley (1975), and Shoven (1976).

Completely static models as Shoven-Whalley's are unsatisfactory for many tax reform issues. These include corporate tax integration, effects of investment tax credits, effects of accelerated depreciations, consumer or expenditure taxes, and importance of saving subsidies like IRA's, etc. These are essentially dynamic issues. They involve not only the allocation of capital across sectors, but perhaps, more importantly, the capital intensity of the economy. But, the capital accumulation and capital reallocation take time and may involve adjustment costs. Because of these issues, Ballard-Fullerton-Shoven-Whalley took the first steps towards developing a dynamic model.

In this context, the Shoven-Whalley model has been extended and implemented for the U.S. economy in Ballard-Fullerton-Shoven-Whalley (1985). While their book completely documents the model, it was used in several publications beginning in 1978. Their model consists of nineteen production sectors, twelve households, and fifteen consumer goods. It includes a very detailed set of taxes, including the federal and state personal income taxes, federal and state corporate income taxes, Social Security taxes, property taxes, unemployment insurance, excise taxes, sales taxes, etc. It has been calibrated to reproduce the 1973 U.S. economy. Recently, a version corresponding to the 1983 U.S. economy has been developed.

.The Ballard-Fullerton-Shoven-Whalley model is dynamic in the sense that consumers face a choice between current consumption and leisure versus future consumption (which can be . purchased via savings). The consumer classes act as if they were maximizing a nested CES utility function over the domain of a CES aggregate of contemporaneous consumer goods and leisure and future consumption, subject to their income constraint. The parameters of those functions determine the shares of income devoted to each commodity, to saving, and to the "purchase" of leisure. They also determine the two key elasticities in the models - the elasticity of labor supply with respect to the real after tax wage rate, and the elasticity of saving with respect to the real after-tax rate of return to capital.

In the model, consumers have myopic expectations regarding future prices and, in particular, regarding the future rate of return to capital. Ballard (1983) and Ballard-Goulder (1985) incorporate both perfect foresight and limited foresight into this model. Future consumption is "acquired" by buying a fixed composition portfolio of real investments that offer an infinite annuity of returns.

The production side of the model is completely static. The model incorporates a constant elasticity of substitution between primary inputs in production (capital and labor) and fixed coefficients for intermediate inputs. The model distinguishes between industrial outputs and consumer goods for the simple reason that the data are classified differently. Industrial sectors involve such categories as forestry and fisheries, metal mining, and publishing and printing, while consumers purchase furniture, automobiles, and books. This fact is recognized in the model by incorporating a second stage of production which converts industrial outputs into consumer goods. This technology is usually modeled as a fixed-coefficient conversion matrix.

The Ballard-Fullerton-Shoven-Whalley model assumes that the private sector finances marginal investment with the same composition of debt and equity as currently exists in each sector. This is the same assumption that Harberger originally used. Investors all hold debt and equity in the same proportion. Therefore, this ownership can be aggregated simply into capital

ownership. For tax purposes, however, the separate treatment of debt and equity is taken into account at both the corporate and personal levels. Similarly, the dividend policies on corporate equity are established exogenously. There are no government bonds in the model, since there are no government deficits.

The model is solved for a sequence (as many as 100) of temporary equilibria, with consumers allocating income between present and future consumption at each point in time. The path for the economy is a set of connected equilibria. The connection is provided by capital accumulation. Capital accumulation is endogenous and determined by saving. The model starts with a social accounting matrix. In the base case the economy is assumed to be in a steady-state growth path (along which all relative prices are constant). The model solves for both the new steady-state growth path and the transition to it after a policy intervention. The authors have frequently addressed the question of how long it takes to effectively settle into a new steady-state growth path.

The dynamics of the model are limited, however, in that future consumption is collapsed into a composite commodity. Also, the absence of government deficits and the lack of production dynamics limits the realism of the Ballard-Fullerton-Shoven-Whalley model for the analysis of dynamic policy issues (such as the adoption of a consumption tax or the elimination of the investment tax credit).

More recently, a whole new generation of models has been developed by Andersson (1987), Auerbach-Kotlikoff (1983, 1984, 1987), Ballard (1983), Ballard-Goulder (1985), Bovenberg (1984, 1985, 1986), Erlich-Ginsburgh-Heyden (1987), Feltenstein (1984, 1986), Goulder (1985), Goulder-Summers (1987), and Jorgenson-Yun (1984). The model in this paper - DAGEM - is pretty much in the spirit of this new generation. Accordingly, throughout this chapter the DAGEM model will be compared to and contrasted with these recent contributions.

2.2 THE ECONOMY

The economy is characterized by a sequential market structure in a finite horizon and discrete time frame, t=z,....,T.· At period t, all current markets are open. Spot markets are allowed to re-open every period. There are no future markets at any t. J+3 markets are open at every t: J output/consumption goods markets, the physical capital good market, the labor market, and the financial assets market.

In this economy there are three types of agents: consumer groups, industries, and government. Agents face a dynamic environment. The economic behavior of every agent is derived from an intertemporal specification of its objectives and constraints. Intertemporal transfers of wealth are allowed and economic decisions are forward looking.

To make their real and financial decisions at each t, the economic agents use several types of information. They observe current prices at t. However, economic decisions are formulated in a context of uncertainty about future prices and interest rates. Intertemporal consistency in the usual Strötz (1956) sense is not imposed in the model. Agents are allowed to commit mistakes due to incorrect expectations. By generally assuming imperfect foresight, decisions will be taken that could have been improved upon should the agents have accurately foreseen the future. Thus, plans about the future will, in general, be revised. A minimum consistency requirement on individual behavior is imposed. Given intertemporally defined objectives and constraints, economic decisions are intertemporally consistent in the broad sense that they are the best possible decisions at every moment based on the available information about the future as synthesized by the rules of formation of expectations.

The information set at period t reflects what is known about the economy at t. It consists of all the structural information of individual preferences and technologies, and all the past equilibrium/observed prices and quantities. Individual expectations at t for all t+1,...,T, are based on available information as specified in the information set. Expectations are formed as point

expectations according to specific rules. Expectations are updated when new information comes into the information set. For example, the expectations of prices at t+h formulated at t and t+1 will, in general, be different. In terms of the the information structure of the economy, it is assumed that all the agents agree on what the future prices will be: they have homogeneous expectations.[3/] Therefore, the possibility of informational asymmetries across agents is ruled out.

Atomistic competition in each and every market is assumed. Even though the number of agents on each side of the market is finite, it is assumed that enough agents are involved to render their actions negligible in terms of the overall equilibrium outcomes.[4/] The concept of Temporary Walrasian Equilibrium, (TWE), is adopted to capture the incomplete and sequential aspects of real world trading and the limitations of foresight into the future which we want to capture in this model (see Grandmont (1982) for a detailed discussion of the temporary equilibrium literature). All current markets are assumed to clear, hence the Walrasian nature of equilibrium. Also, equilibrium in the short run is parametric on the expectations of future prices held by the different agents as well as future taxation parameters, hence the temporary nature of equilibrium. Actions of the economic agents are based on expectations which may turn out to be incorrect, i. e. price expectations are not self-fulfilling.[5/] The link between adjacent periods is endogenously provided by the recursive transitions of the stock variables in the economy.

2.3 ECONOMIC BEHAVIOR: HOUSEHOLDS

Overview of the Literature

Early efforts to build dynamic features into the economic behavior of consumers are due to Ballard (1983) and Auerbach-Kotlikoff (1983, 1984). Their work has been closely followed by several subsequent authors. In fact, dynamic household behavior is the single most pervasive aspect among the models more recent CGE models.

These models incorporate some form of life-cycle behavior. Household behavior is determined

by the maximization of an additively separable, time invariant intertemporal utility function. The utility function is defined over the domain of the consumption goods in the economy. In most of the models, leisure is also an argument in utility so that labor supply is optimally determined.

Utility maximization is subject to a lifetime intertemporal budget constraint which equalizes the present value of consumers' income and expenditure. More recently in Andersson (1987) and Bovenberg (1985) the constraint is defined as a sequence of recursive equations of motion on wealth. This has the potential advantage of accommodating liquidity constraints. However, It should be recognized that in the absence of liquidity constraints (i.e. when consumers are free to completely borrow against future income), the two specifications of the household constraint are essentially equivalent. Furthermore, in both versions saving is optimally determined as a way of transferring wealth intertemporally.

Some models now have a sophisticated dynamic specification of household behavior. In Ballard (1983), Ballard-Goulder (1985), and Auerbach-Kotlikoff (1983, 1984, 1987), consumer's behavior is embedded in an intergenerational setting. Each of them has 55 age cohorts simultaneously alive. Some measure of intergenerational altruism is considered in the form of bequest motives. In Ballard (1983) and Ballard-Goulder (1985), consumers derive utility directly from bequeathing part of their wealth. Therefore, bequests assume the form of a scrap value of terminal wealth of a generation. In Auerbach-Kotlikoff (1983, 1984, 1987), each generation empathizes with the utility of future generations over bequeathed wealth. In addition, Ballard (1983) includes detailed demographic projections with the intent of incorporating in the policy analysis the effects of the Post World War II baby-boom.

In the real world, household decisions also include the optimal allocation of saving among alternative physical or financial assets. Theories of household portfolio behavior are relatively well-established. However, they involve uncertainty as a crucial element. Accordingly, they have not been incorporated in the context of the deterministic dynamic models. We will come back to this issue below.

The DAGEM Model

The information structure underlying the economic problem of consumer group i at period z can be summarized as follows. At the beginning of period z, the consumer observes current commodity and labor prices and current interest rates. · Accordingly, the value of the current asset portfolio is observed. Then, the consumer formulates expectations about future prices and anticipates the parameters of future taxation policy. These parameters are used to determine his intertemporal plans for consumption, labor supply, and asset portfolio holdings. Due to the temporary equilibrium structure of the model, only current plans are strictly enforced. The actual intertemporal sequence of plans is obtained from the contemporaneous period decisions associated with an intertemporal sequence of optimization problems.

Intertemporal preferences of consumer group i defined over current and future commodity consumption/labor supply plans are represented by a time separable felicity function of the form:

(1) $\sum_{z \leq t \leq T} (1+\partial_i)^{-(t-z)} U_i(L^*_i - L_{it}, y_{i1t}, \dots, y_{iJt})$

where ∂_i is the time-invariant, subjective rate of discount for class i, and $U_i(.)$ is a well behaved, time-invariant utility function defined over the space of the J output goods y_j, and leisure, H_{it}. Leisure is given by $L^*_i - L_{it}$, where L^*_i is consumer i's total available time.

The consumer's behavior is constrained by a recursive set of budget constraints relating the intertemporal patterns of income, spending and savings. At t, consumer group i receives labor income, $p_{Lt} L_{it}$, and lump-sum transfers from the government, Tr_{it}. Also, consumer i receives wealth generated income, which includes capital gains:

(2) $[(1-\sum_j e_{jt})r_t + \sum_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]W_{it} + \sum_j [p_{jEt} - p_{jEt-1}]E_{ijt}$

where e_{jt} is the share at t of equity j in total wealth of individual i, $e_{jt} = p_{jEt-1}E_{jit}/W_{it}$, and $1-\sum_j e_{jt}$, represents the fraction of public and private debt, $1-\sum_j e_{jt} = (\sum_j B_{ijt} + B_{igt})/W_{it}$.

Labor and wealth income are taxable according to a linear progressive income tax schedule. Lump-sum transfers from the government are considered tax-exempt. Capital gains are taxed at a different rate, CGT_{it}. Accordingly, disposable income at t is given by:

$$(3) \quad b_{it}+(1-T_{it})[p_{Lt}L_{it}+[(1-\Sigma_j e_{jt})r_t+\Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]W_{it}]+Tr_{it}$$

$$+(1-CGT_{it})\Sigma_j[p_{jEt}-p_{jEt-1}]E_{ijt}$$

where b_{it} is negative to reflect the fact that marginal tax rates exceed average tax rates, and T_{it} is the marginal income tax rate for household group i.

At each t, $p_{jt}y_{ijt}$ represents pre-tax expenditure of group i in commodity j. Purchase of good j is subject to an ad valorem sales tax. Therefore the total after-tax expenditure of the i-th group in consumption goods is $\Sigma_j(1+T_{jt})p_{jt}y_{ijt}$.

Given the information above, the recursive set of budget constraints - the equation of motion for wealth - can for every t be written as:

$$(4) \quad W_{it+1}-W_{it}=b_{it}+(1-T_{it})[p_{Lt}L_{it}+[(1-\Sigma_j e_{jt})r_t+\Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]W_{it}]+$$

$$+(1-CGT_{it})\Sigma_j[p_{jEt}-p_{jEt-1}]E_{ijt}+Tr_{it}-\Sigma_j(1+T_{jt})p_{jt}y_{ijt}$$

$$(5) \quad W_{iT+1}=0$$

The terminal constraint on wealth (5) implies that the total present value of current and future expected income receipts has to be equal the present value of current and future expected spending.

Savings represent intertemporal transfers of wealth to finance future consumption. Accordingly, W_{it+1} represents the new total wealth at the end of period t to be transfered into period t+1 after all expenditures have been incurred. Additional wealth representing the total amount of new funds made available by group i to the rest of the economy is invested according to criteria detailed below in this section. Savings generated by group i at t are given by

(6) $\quad S_{it}=\Sigma_j[B_{ijt+1}-B_{ijt}]+[B_{igt+1}-B_{igt}]+\Sigma_j p_{jEt}[E_{ijt+1}-E_{ijt}].$

Formally, at each period z the economic problem of consumer group i can be stated as the maximization of the expected value at z of his felicity function subject to the recursive sequence of budget constraints, to terminal state constraints, and to a sequence of future price expectations. Such problem can be written as:

(7) $\quad \mathrm{Max}_{\{y,L\}}\Sigma_{z\leq t\leq T}(1+\partial_i)^{-(t-z)}U_i(L_{it},y_{i1t},...,y_{iJt})$

subject to:

i) <u>non-negativity constraint on controls</u> for all $z\leq t\leq T$

(8) $\quad y_{ijt}\geq 0$ for all $1\leq j\leq J$, $0\leq L_{it}\leq L^*_i$

ii) <u>equation of motion of wealth</u> for all $z\leq t\leq T$

(9) $\quad W_{it+1}-W_{it}= \Sigma_j[B_{ijt+1}-B_{ijt}]+[B_{igt+1}-B_{igt}]+\Sigma_j p_{jEt}[E_{ijt+1}-E_{ijt}]$

$= b_{it}+(1-T_{it})[p_{Lt}L_{it}+[(1-\Sigma_j e_{jt})r_t+\Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]W_{it}] +$

$+(1-CGT_{it})\Sigma_j[p_{jEt}-p_{jEt-1}]E_{ijt}+Tr_{it}-\Sigma_j(1+T_{jt})p_{jt}y_{ijt}$

iii) <u>state terminal conditions</u> for all $i\leq j\leq J+1$ (includes investment good industry)

(10) $\quad W_{iz}=W^*_i=\Sigma_j B^*_{ij}+\Sigma_j p_{jEz-1}E^*_{ij}+B^*_{ig}$

(11) $\quad W_{iT+1}=0; \quad B_{ijT+1}=0; \quad B_{igT+1}=0; \quad p_{jET}E_{ijT+1}=0.$

In what follows, the time invariant utility function will be characterized by a two-level nested structure. At the top level, consumer i will decide about his consumption/leisure allocation according to a linearly homogeneous Cobb-Douglas structure. At the lower level, consumers will decide about their pattern of disaggregated consumption/leisure according to a linearly homogeneous Cobb-Douglas structure. This is equivalent to the following specification:

(12) $\quad U_i(L_{it},...,y_{ijt},...)=\Sigma_j a_{ij}\ln(y_{ijt})+(1-\Sigma_j a_{ij})\ln(L^*_i-L_{it}).$

The problem (7)-(11) is solved using control theory techniques, in particular, Pontryagrin's

Maximum Principle to obtain the optimal demand functions. The relevant Hamiltonian at time t $H_t(L_i, y_i; W_i; q_i)$ and for $z \leq t \leq T$ is:

$$(13) \quad H_t(L_i, y_i; W_i; q_i) = (1 + \partial_i)^{-(t-z)}.$$

$$\{\Sigma_j a_{ij} \ln(y_{ijt}) + (1 - \Sigma_j a_{ij}) \ln(L_i^* - L_{it}) + (1 + \partial_i)^{-1} q_{t+1} \cdot$$

$$[W_{it} + b_{it} + (1 - T_{it})(p_{Lt} L_{it} + [(1 - \Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1} E_{jt})]W_{it})$$

$$+ (1 - CGT_{it})\Sigma_j[p_{jEt} - p_{jEt-1}]E_{ijt} + Tr_{it} - \Sigma_j(1 + T_{jt})p_{jt}y_{ijt}]\}$$

The Hamiltonian function represents the present value at z of the sum of the utility at t derived from contemporary consumption and leisure plus the implicit utility value of wealth transferred to next period, discounted back to the moment when decisions are being made, t. The dynamic shadow price associated with the equation of motion for wealth at t $(1+\partial_i)^{-1}q_{it+1}$ is to be interpreted as the marginal utility of wealth at t.

The <u>necessary conditions</u> for optimality at z are:

    i) <u>equation of motion for the state variable</u> for all $z \leq t \leq T$,

$$(14) \quad W_{it+1} = W_{it} + b_{it} + (1 - T_{it})[p_{Lt} L_{it} + [(1 - \Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1} E_{jt})]W_{it}]$$

$$+ (1 - CGT_{it})\Sigma_j[p_{jEt} - p_{jEt-1}]E_{ijt} + Tr_{it} - \Sigma_j(1 + T_{jt})p_{jt}y_{ijt}$$

    ii) <u>state terminal conditions</u>

$$(15) \quad W_{iz} = W_i^* \quad \text{(initial state condition)}$$

$$(16) \quad W_{iT+1} = 0 \quad \text{(final state condition)}$$

    iii) <u>adjoint equation for the marginal utility of wealth</u> for all $z \leq t \leq T$

$$(17) \quad q_{it} = (1 + \partial_i)^{-1}\{1 + (1 - T_{it})[(1 - \Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1} E_{jt})]\}q_{it+1}$$

    iv)<u>Hamiltonian variational conditions for control variables</u> H_{it} and y_{ijt} for all $z \leq t \leq T$ —

$$(18) \quad (1 - \Sigma_j a_{ij}) / (L_i^* - L_{it}^S) = (1 + \partial_i)^{-1}q_{it+1}(1 - T_{it})p_{Lt}$$

(19) $\quad a_{ij}/y_{ijt}=(1+\partial_i)^{-1}q_{it+1}(1+T_{jt})p_{jt}$

Closed form solutions to this problem are obtained in Appendix II. It is enough here to say that

both the demands for different consumption goods and the supply of labor depends on the

contemporaneous interest rate and on the respective price. Also, through the marginal utility of

wealth, both the demands for different consumption goods and the supply of labor depend on

expectation about future labor prices and interest rates, but not future commodity prices. In turn,

savings behavior depends on all contemporaneous prices and interest rate, as well as on future

labor prices and interest rates, but not on future commodity prices.

2.4 ECONOMIC BEHAVIOR: PRODUCERS

Overview of the Literature

The efforts to build dynamic features into the economic behavior of producers are more recent

and less widely adopted. The first attempts are due to Bovenberg (1984, 1985) and Summers

(1985). Dynamic behavior has been more fully incorporated in the recent models of Andersson

(1987), Auerbach-Kotlikoff (1987), and Goulder-Summers (1987).

Part of the reason why production side dynamics has been more slowly adopted is the weak

supply of accepted theories referring to the dynamic behavior of the firms. In the models referred

to above, dynamic production and investment behavior are induced by the existence of capital

adjustment costs and linked to Tobin's q theory. Adjustment costs are designed to capture both the

incomplete mobility of capital across industries and installation costs (i.e. the costs of adjusting

capital towards its optimal level).

Adjustment costs can be conceived as internal to the firm and measured in terms of foregone

output along the lines of Lucas (1967). This is the way the above model approaches the concept.

Alternatively, adjustment costs can be viewed as actual external costs incurred together with the

purchase costs, along the lines of Gould (1969).

The firms in the economy maximize their market value as the present discounted value of the future stream of dividends. In Andersson (1987), firms are seen as maximizing the present discounted value of net cash flow. Maximization is constrained by the adjustment cost technology and an equation of motion describing the evolution of the capital stock. It should be noted that under exogenous dividend/retention rules the two problems are equivalent.

The models with static formulation of producers' behavior are characterized by passive investment behavior. Investment merely accommodates to saving in the economy. With a dynamic formulation induced by adjustment costs, real investment decisions are forward looking. Investment is endogenously and optimally determined by the firms. A fundamental difference between the short run in which capital stock is given, and long run in which the level of capital is allowed to be optimally determined is emphasized.

This extra richness of production dynamics is not without costs. A careful look at the summary tables will clearly show an inverse relation between the adoption of dynamic features in production and the level of disaggregation of the production side of the economy. In fact, with production dynamics the dimension of the problems is immensely increased. Let us be more specific.

In a static framework with constant returns to scale production technology, the output level is indeterminate. The optimal allocation of inputs can be obtained by cost minimization with any feasible output level generating zero profits. Such zero profit conditions are used to solve for the output prices in terms of the factor prices and hence to reduce the dimensionality of the problem from the number of commodities and factors to the number of factors. Thus, even if the model deals with 30 production sectors, the computation of an economic equilibrium can take place using only the dimensionality of the primary inputs in the economy.

Now, under certain regularity conditions on the production and adjustment costs technologies, leading to enough concavity of the optimality objective, the intertemporal output path for the firm is endogenously, optimally, and uniquely determined even with constant returns to scale

technologies (see Pereira (1986,1987)). Accordingly, with adjustment costs in the model optimal profits will in general be non-zero. This result has crucial implications for the computation of equilibrium in the models with production dynamics. With adjustment costs, no reduction of dimensionality is possible. The curse of dimensionality returns as a binding constraint.

The introduction of adjustment costs adds another significant complication to the model. With capital being less than perfectly mobile in the economy, different rates of return on capital will exist in different sectors. This is a difficult problem to tackle conceptually in the absence of uncertainty. In the real world, producer maximization choices also include choice of financial ratios (debt/equity) and payout rates (dividend/retained earnings). These financial subjects are difficult, and much of this behavior is still taken as exogenous by the modelers. We will come back to these issues below.

The DAGEM Model

The information structure underlying the economic problem of production sector j at period z can be summarized as follows. At the beginning of period z, the producer observes current commodity and labor prices and interest rates. The producer then formulates expectations about future prices and anticipates the parameters of future taxation policy. This information is used to determine the intertemporal plans for output supply, demand for labor, investment and intermediate inputs, and demand for funds in the form of outstanding bonds and equity. Due to the temporary equilibrium structure of the model, only current plans are strictly enforced. The actual intertemporal sequence of plans is obtained from the contemporaneous period decisions associated with an intertemporal sequence of optimization problems.

Production technology at each t is represented by a time-invariant Leontieff structure of the form:

(20) $y_{jt} = Min\{F_j(L_{jt}, K_{jt}), y_{j1t}/a_{j1}, ..., y_{jJt}/a_{jJ}\}$.

The value-added production function, $VA_{jt} = F_j(L_{jt}, K_{jt})$, is twice continuously differentiable, strictly increasing in every input, and concave.

We further assume that adjusting capital stock towards its optimal level is not costless. This idea is captured by sector-specific cost functions "à la Gould" (1968), defined over gross capital stock accumulation. The adjustment cost functions can be interpreted to include both acquisition and internal, non-market adjustment costs. The twice continuously differentiable investment cost function for sector j is:

(21) $TC_j(I_{jt}) = p_{It}[I_{jt} + C_j(I_{jt})]$.

The adjustment cost function has the following properties:[6/]

(22) $C_j(0) = 0$, and $C_j(I_{jt}) > 0$ for $I_{jt} \neq 0$

(23) $\partial C_j(I_{jt})/\partial I_{jt} > 0$ for $I_{jt} > 0$, and $\partial C_j(I_{jt})/\partial I_{jt} < 0$ for $I_{jt} < 0$

(24) $\partial^2 C_j(I_{jt})/\partial I_{jt}^2 > 0$.

The evolution of capital stock through time - reflecting actual investment - is given by the equation of motion:

(25) $I_{jt} = K_{jt+1} - (1 - \emptyset_{jt}) K_{jt}$

where $\emptyset_{jt}$ is the depreciation rate of capital stock installed in sector j at period t. Depreciation rates are sector-specific.

The equation of motion of capital reflects the idea that, in the short run, capital stock is fixed, i.e., the capital stock in existence at t is not a decision variable at t, but it is determined by optimal decisions in previous periods. However, at t investment decisions will be made determining the capital stock at t+1. In the long run, capital stock is variable.

Each sector of production j faces ad valorem taxes on the use of labor services, which represent

the employer's portion of Social Security taxes. Therefore, if T_{Lt} is the tax rate, assumed constant across sectors of production, the cost for sector j of one unit of labor is given by $(1+T_{Lt})p_{Lt}$.

As a consequence of its decisions at period t, the sector realizes gross profits Π_{jt} - payment of capital services plus economic profits, i.e., sales revenues minus non-investment expenditures:

(26) $\cdot$ $\Pi_{jt}=[p_{jt}-\Sigma_{1\leq f\leq J}(a_{jf}p_{ft})]y_{jt}-(1+T_{Lt})p_{Lt}L_{jt}$.

Each sector j is subject to an ad valorem corporate tax on Π_{jt}. The after-tax gross profits are $(1-T_{cjt})\Pi_{jt}$, where T_{cjt} is the sector-specific corporate tax rate at t.

On the other hand, investment expenditures benefit from an investment tax credit which is an ad valorem subsidy. Actual investment expenditures are $(1-ITC_{jt})p_{lt}[I_{jt}+C_j(I_{jt})]$.

Interest payments are deductible from the corporate tax base so that the net interest paid on outstanding bonds is $(1-T_{cjt})r_tB_{jt}$.

Also, depreciation allowances DA_{jt} are to be deducted from the corporate tax base. Let $\emptyset'_{jt}$ and K'_{jt} be the depreciation rates for tax purposes and capital stock for tax purposes, respectively. The after-tax gross profits are increased by $T_{cjt}\emptyset'_{jt}K'_{jt}$.

Industry j's net cash flow at t NCF_{jt} can be written as:

(27) $NCF_{jt}=$

$=(1-T_{cjt})\{[p_{jt}-\Sigma_f(a_{jf}p_{ft})]F_j(K_{jt},L_{jt})-(1+T_{Lt})p_{Lt}L_{jt}\}-(1-ITC_{jt})[p_{lt}I_{jt}+C_j(I_{jt})]$.

The discounted value at t of the intertemporal sequence of net cash flows is obtained from the sequence of current and future expected market rates of return r_t's.

The producers' dynamic behavior with respect to real economic variables is determined by the maximization of the present value of net discounted cash flows at each moment z subject to strictly

convex adjustment costs, the equation of motion for the capital stock, and future price expectations.

Formally, this is for $z \leq t \leq T$, $z \leq s \leq t$,

(28) $\text{Max}_{\{y,L,I;K\}} \Sigma_t [\Pi_s (1+r_s)^{-1}] NCF_{jt} = \Sigma_t [\Pi_s (1+r_s)^{-1}] \cdot$

$\{(1-T_{cjt})[[p_{jt} - \Sigma_f (a_{jf} p_{ft})] F_j (K_{jt}, L_{jt}) - (1+T_{Lt}) p_{Lt} L_{jt}] - (1-ITC_{jt})[p_{It} I_{jt} + C_j (I_{jt})]\}$

subject to:

i) <u>non-negativity constraints</u> for all $z \leq t \leq T$ and $1 \leq f \leq J$,

(29) $y_{jft} \geq 0$, $L_{jt} \geq 0$, $K_{jt} \geq 0$;

ii) <u>equation of motion of capital stock</u> for all $z \leq t \leq T$

(30) $I_{jt} = K_{jt+1} - (1-\emptyset_{jt}) K_{jt}$;

iii) <u>state end conditions</u>

(31) $K_{jz} = K_j^*$

(32) scrap value of capital at $T+1$ is zero.

In what follows, the time invariant value added production function will be characterized by a linearly homogeneous Cobb-Douglas structure:

(33) $F_j (L_{jt}, K_{jt}) = (L_{jt})^{a_j} (K_{jt})^{1-a_j}$

Also, the adjustment cost function is assumed to be quadratic:

(34) $C_j (I_{jt}) = .5 \ b_j I_{jt}^2$

This problem is solved using control theory techniques, in particular, Pontryagrin's Maximum Principle, to obtain the optimal factor demand and supply functions. With this choice of functional forms the <u>Hamiltonian</u> associated at t with our problem becomes:

(34) $H_t (L_j, I_j; K_j; q_j) = [\Pi_{z \leq s \leq t} (1+r_s)^{-1}] \cdot$

$\{(1-T_{cjt})[[p_{jt} - \Sigma_f (a_{jf} p_{ft})][(L_{jt})^{a_j} (K_{jt})^{1-a_j}] - (1+T_{Lt}) p_{Lt} L_{jt}]$

$-(1-ITC_{jt}) p_{It} [I_{jt} + .5 b_j I_{jt}^2] + (1+r_{t+1})^{-1} q_{jt+1} [I_{jt} + (1-\emptyset_j) K_{jt}]\}$

The Hamiltonian function at t represents the present value at z of the sum of the net cash flow at t derived from contemporary production activities plus the imputed (shadow) discounted net cash flow value of capital to be installed next period. The dynamic shadow price associated with the equation of motion for capital at t is $(1+r_{t+1})^{-1}q_{jt+1}$. This is to be interpreted as the marginal value at t of capital to be installed at t+1.

The <u>necessary conditions</u> for optimality are:

    i) <u>equation of motion for the state variable</u> for all $z \leq t \leq T$

(35) $I_{jt} = K_{jt+1} - (1-\emptyset_{jt})K_{jt};$

    ii) <u>state terminal conditions</u>

(36) $K_{j0} = K_j^*$ (initial state condition)

(37) $q_{jT+1} = z$ (final co-state condition)

    iii) <u>adjoint equation for the co-state variable</u>   for all $z \leq t \leq T$

(38) $q_{jt} = (1-\emptyset_{jt})(1+r_{t+1})^{-1}q_{jt+1} + (1-T_{cjt})(p_{jt} - \Sigma_f(a_{jf}p_{ft}))[(1-a_j)(L_{jt}/K_{jt})^{a_j}]$

    iv) <u>Hamiltonian variational conditions for control variables</u> I_{jt} and L_{jt} for all $z \leq t \leq T$

(39) $[p_{jt} - \Sigma_f(a_{jf}p_{ft})][a_j(L_{jt}/K_{jt})^{(a_j-1)}] = (1+T_{Lt})p_{Lt}$

(40) $(1-ITC_{jt})p_{It}[1+b_jI_{jt}] = (1+r_{t+1})^{-1}q_{jt+1}$

Closed form solutions to this problem are obtained in Appendix III. Demand for labor and intermediate inputs as well as output supply depend on all current prices and on the rate of interest. No future prices are relevant. However, investment demand is forward looking in the sense that it not only depends on the current price of the investment good but also on next period's interest rate and all the other future prices via the shadow price of capital. In turn, demand for new funds depends on all the current prices and on the rate of interest as well as on all the future prices and rates of interest via investment demand.

2.5 ECONOMIC BEHAVIOR: GOVERNMENT

Overview of the Literature

Two issues dominate the modeling of government behavior. First, government behavior typically has been seen in the CGE literature for tax policy evaluation as constrained by yearly balanced budgets (see, for example, Ballard-Fullerton-Shoven-Whalley (1985)). The analysis of government deficits and public debt in a CGE context requires a dynamic setting. Second, the level of government expenditures is either exogenouly given as in Auerbach-Kotlikoff (1984, 1987), Bovenberg (1984, 1985), Feltenstein (1984, 1986), and Jorgenson-Yun (1984), or endogenously (but not optimally) determined by the balanced budget conditions as in Ballard-Fullerton-Shoven-Whalley (1985), Andersson (1987), Erlich-Ginsburgh-Heyden (1987), Goulder (1985), and Goulder-Summers (1987). In the second case, the composition of public expenditures is often optimally determined. However, the level of government expenditures can only be endogenously and optimally determined if the government is seen as an intertemporally optimizing agent and is allowed to run deficits.

The first attempts to deal with government deficits in CGE tax models are due to Auerbach-Kotlikoff (1984), Feltenstein (1984), and Goulder (1985). In Auerbach-Kotlikoff (1984), government expenditures are exogenous. They grow at the rate of growth of population. However, given the tax structure and tax revenues, yearly deficits and surpluses are allowed, subject to an intertemporal constraint that the present value of future tax revenues equals the present value of future expenditures. In Feltenstein (1984, 1986), government expenditures are also exogenously given. He also allows the government to run deficits to finance expenditures in excess to tax revenues. However, surpluses are returned to consumers in the form of transfers. Accordingly, government is not subject to any constraint regarding the future repayment of public debt. In Goulder (1985), the government maximizes a static social welfare function subject to a balanced

budget constraint. This follows the optimal allocation of government expenditures along the lines of Ballard-Fullerton-Shoven-Whalley (1985). The model is generalized to allow for exogenous changes in the time path of government expenditures. Two financing alternatives are considered and contrasted: additional tax revenues and bond issuance.

. The model in this paper attempts to address both the incorporation of deficits and the determination of government expenditures. The path of government expenditures and the path of deficits/surpluses (and therefore the path for debt) are endogenously and optimally determined. The government is seen as maximizing an intertemporal social welfare function given the tax structure. Optimization is subject to a sequence of recursive equations of motion reflecting the evolution of the public debt, allowing for government budget imbalances. At each moment government's decisions are based on the consideration of its current and future interests and constraints. Its evaluation of the future depends crucially on the expectations about the future state of the economy, in particular its expectations about future prices. This extra richness in the treatment of government behavior allows the examination of government debt policies in a truly dynamic setting. Also, financial crowding out of private investment induced by government deficits can be analyzed in a more meaningful way than in Auerbach-Kottlikoff (1987) and Feltenstein (1986) in which deficits are endogenous but not optimally determined.

The DAGEM Model

The information structure underlying the economic problem of government at period z can be summarized as follows. At the beginning of period z, the producer observes current commodity and labor prices and interest rates. Also, at the beginning of period z, the government formulates expectations about future prices and the parameters of future taxation policy. The government then determines its intertemporal plans for demand for consumption goods, labor, and investment as well as demand for funds in the form of new outstanding bond. Due to the temporary equilibrium structure of the model, only current plans are to be strictly enforced. The actual intertemporal

sequence of plans is obtained from the contemporaneous period decisions associated with an intertemporal sequence of optimization problems.

The government engages in four economic activities. First, it collects taxes according to an exogenously given tax regime. Second, it transfers discretionary lump-sum amounts to the private sector. Third, it purchases consumption goods, capital, and labor to accomplish general government activities through the production of a public good. Finally, since general government activities are constrained by a recursive set of budget constraints, and it is allowed to run yearly deficits, the government is also allowed to engage in the sale of public bonds to finance such imbalances.

The government raises revenue by levying taxes on the private sector. It is assumed that the government knows exactly how to compute the tax revenue it is going to collect at t. It is as if the government knows the closed form net demands of all the agents in the economy and therefore the tax base. The government can also infer future tax revenues which are relevant for current decisions from future price expectations.

The tax system and tax policies are institutionally given as the outcome of a process not captured by the model.[7] Six classes of taxes are considered in this model as described in the preceding sections. The total revenues they generate at t are accumulated as follows:

1. ad valorem labor tax on labor services used by the different industries ($j=i,...,J;I$) and government, representing Social Security taxes, unemployment insurance, and workmen's compensation and which generates revenue LT_t:

(41) $LT_t = (\Sigma_j T_{Lt} P_{Lt} L^D_{jt}) + T_{Lt} P_{Lt} L^D_{It} + T_{gLt} P_{Lt} L^D_{gt}$.

It should be noted that government is seen as paying taxes to itself on the use of labor. Consequently, the income effects of such a tax cancel out. However, the price effects measure the opportunity cost to government of hiring labor. Notice also that marginal labor tax rates in the private and public sectors are different, reflecting better pension plans for government employees.

2. ad valorem corporate income tax on industry $j=1,...,J,I$ generates revenue CT_t net of

interest deductibility and depreciation allowances:

(42) $CT_t = \Sigma_j T_{cjt}[((p_{jt} - \Sigma_f a_{ft} p_{ft}) y^S_{jt} - (1 + T_{Lt}) p_{Lt} L^D_{jt} - r w_t B_{jt} - \varnothing'_{jt} K_{jt}]$.

  3. ad valorem <u>investment tax credits</u> ITC_t, on industry $j = 1, ..., J; I$,

(43) $ITC_t = \Sigma_j ITC_{jt} p_{It}[I_{jt} + C_j(I_{jt})]$.

  4. ad valorem <u>sales tax</u> generates revenue ST_t:

(44) $ST_t = \Sigma_i [\Sigma_j T_{jt} p_{jt} y^D_{ijt}]$;

  5. a progressive <u>personal income tax</u> represented by a linear function for each i generates revenue IT_t:

(45) $IT_t = \Sigma_i \{-b_i + T_{it}[p_{Lt} L^D_{ti} + [(1 - \Sigma_j e_{jt}) r_t + \Sigma_j e_{jt}(Div_{jt}/E_{jt})] W_{it}]$

  6. <u>capital gains tax</u>:

(46) $CGT_t = \Sigma_i CGT_{it}(p_{jEt} - p_{jEt-1}) E_{ijt} + \Sigma_j CGT_{jt}(p_{jEt} - p_{jEt-1}) E_{jt}$.

Accordingly, <u>total taxes collected</u> at time t are TT_t:

(47) $TT_t = LT_t + CT_t + ST_t + IT_t - ITC_t + CGT_t$.

Total lump-sum redistributive <u>transfer payments</u>, i.e., transfers to households at t (Social Security, food stamps, AFDC, etc.) are exogenously given and represented by $Tr_t = \Sigma_i Tr_{it}$.

The basic intertemporal consistency requirement imposed on government behavior is that its actions are constrained by an intertemporal balanced budget condition. The discounted sum of all the government expenditures on commodities, labor, and new capital investment cannot exceed the discounted sum of all its revenues, i.e. tax revenues net of transfers. The intertemporally recursive specification of the budget constraint can be written for each $z \leq t \leq T$ in the form:

(48) $LG_{t+1} = (1 + r_t) LG_t + [Tr_t + \Sigma_j p_{jt} y_{gjt} + (1 + T_{gLt}) p_{Lt} L_{gt} + p_{It} I_{gt}] - TT_t$,

with end conditions

(49) $LG_z = LG^*$

(50) $LG_{T+1} = 0.$

Optimal government spending is derived from the maximization of a social welfare function over the domain of an aggregate public good. Such public good is produced using capital, labor, and intermediate inputs according to a well behaved production function. This public good is not subject to market pricing. Accordingly, its production is financed by tax income and other sources of government income. This optimization objective is consistent with our modeling of consumers' behavior in which the public good does not enter the set of budget constraints and is not a decision variable. This is equivalent to having the public good enter additively in time t to the private utility functions. Thus the marginal rates of substitution between private goods do not depend on the level of availability of the public good. The government is then assumed to act empathetically with the private consumers according to a constrained social utility maximizing problem.[8/]

The social welfare function over the domain of the aggregate public good can be expressed indirectly in terms of a well-behaved, time invariant utility function defined at every t over the J commodities and labor and capital services:

(51) $U_g(K_{gt}, L_{gt}, y_{g1t}, \ldots, y_{gJt}).$

The intertemporal government preferences at z are characterized by an additively separable intertemporal felicity function of the form:

(52) $\sum_{z \leq t \leq T} (1 + \partial_g)^{-(t-z)} U_g(K_{gt}, L_{gt}, y_{g1t}, \ldots, y_{gJt})$

where ∂_g is the time invariant subjective rate of discount for the government.

The government's optimization problem at each period z can be formally stated as the maximization of the expected value of its felicity function subject to the recursive sequence of budget constraints as follows:

(53) $\text{Max}_{\{y,K,L\}}\{\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}U_g(K_{gt},L_{gt},y_{g1t},...,y_{gJt})\}$

subject to:

i) <u>non-negativity constraints</u> for all $z\leq t\leq T$,

(54) $y_{gjt}\geq 0$ for all $j=1,...,J$, $L_{gt}\geq 0$, $K_{gt}\geq 0$

ii) <u>equation of motion for government liabilities</u> for each $z\leq t\leq T$,

(55) $LG_{t+1}=(1+r_t)LG_t+[Tr_t+\Sigma_j p_{jt}y_{gjt}+(1+T_{gLt})p_{Lt}L_{gt}+p_{It}I_{gt}]-TT_t$

iii) <u>end conditions for government liabilities</u>

(56) $LG_z=LG^*$ (initial condition)

(57) $LG_{T+1}=0$ (terminal condition)

iv) <u>equation of motion for capital stock</u> for all $z\leq t\leq T$

(58) $K_{gt+1}=I_{gt}+(1-\emptyset_{gt})K_{gt}$

v) <u>end conditions for government capital stock</u>

(59) $K_{gz}=K^*_g$ (initial condition)

(60) scrap value of capital at $T+1$ is zero.

In what follows, government social welfare function is assumed to be a linearly homogeneous Cobb-Douglas function:

(61) $U_i(.)=a_{gk}\ln K_{gt}+a_{gL}\ln L_{gt}+\Sigma_j a_{gj}\ln y_{gjt}$ where,

(62) $a_{gk}+a_{gL}+\Sigma_j a_{gj}=1$

The interpretation of a function like (61) should be stressed again. Assume the public good G_t is produced using capital, labor, and consumption goods according to a CRTS Cobb-Douglas production function. Assume also that the public good G_t enters additively in the private utility functions in the form $a_{gi}\ln G_t$. Now government acts empathetically taking into consideration the

private utility derived from the public good. Then (61) can be interpreted as an utilitarian criterion of the form $\Sigma_i a_{gi} Ln G_t$, or given the nature of the public good $Ln G_t \Sigma_i a_{gi}$. The last step to recover (61) is to normalize $\Sigma_i a_{gi}=1$.

This problem is solved using control theory techniques, in particular, Pontryagrin's Maximum Principle, to obtain the optimal demand functions. With the above specification of the government social welfare function the <u>Hamiltonian</u> becomes

$$(63) \quad H_t(I_g, L_g, y_g; LG, K_g; q_g, q_{kg}) = (1+\partial_g)^{-(t-z)} \{ (a_{gk} ln K_{gt} + a_{gL} ln L_{gt} + \Sigma_j a_{gj} ln y_{gjt}) +$$

$$+ (1+\partial_g)^{-1} q_{gt+1}[(1+r_t)LG_t + Tr_t + \Sigma_j p_{jt} y_{gjt} + (1+T_{Lt})p_{Lt} L_{gt} + p_{It} I_{gt} - TT_t]$$

$$+ (1+\partial_g)^{-1} q_{kgt+1}[I_{gt} + (1-\emptyset_{gt})K_{gt}] \}$$

The Hamiltonian function represents the sum of the utility at t derived from contemporary consumption of the public good plus the implicit utility value of liabilities transfered to next period, plus the implicit discounted utility value of capital stock to be transfered to next period. The dynamic shadow price associated with the equation of motion for liabilities is $(1+\partial_g)^{-1} q_{gt+1}$. This is to be interpreted as the marginal social utility of government liabilities at t. Also, $(1+\partial_g)^{-1} q_{kgt+1}$ is the dynamic shadow price associated with the equation of motion for capital at t. This is to be interpreted as the marginal social utility of government capital at t.

The following are <u>necessary</u> conditions for optimality at t:

    i) <u>equation of motion for the state variable</u> LG_t for all $z \leq t \leq T$

$$(64) \quad LG_{t+1} = [(1+r_t)LG_t + Tr_t + \Sigma_j p_{jt} y_{gjt} + (1+T_{gLt})p_{Lt} L_{gt} + p_{It} I_{gt} - TT_t]$$

    ii) <u>state terminal conditions</u> for LG

$(65) \quad LG_z = LG^*$ (initial state condition)

$(66) \quad LG_{T+1} = 0$ (final state condition)

iii) <u>adjoint equation for the marginal utility of liabilities</u> for all $z \leq t \leq T$

(67) $\quad q_{gt} = (1 + \partial_g)^{-1} (1 + r_t) q_{gt+1}$

iv) <u>equation of motion for the state variable</u> K_g for all $z \leq t \leq T$

(68) $\quad K_{gt+1} = I_{gt} + (1 - \emptyset_{gt}) K_{gt}$

v) <u>state terminal conditions</u> for K_g

(69) $\quad K_{gz} = K_g^*$ (initial state condition)

(70) $\quad q_{kgT+1} = 0$ (final co-state condition)

vi) <u>adjoint equation for the marginal utility of capital</u> for all $z \leq t \leq T$

(71) $\quad q_{gkt} = (1 + \partial_g)^{-1} (1 - \emptyset_{gt}) q_{gkt+1} + a_{gk}(1/K_{gt})$

vii) <u>Hamiltonian variational conditions for control variables</u> L_{gt}, I_{gt}, y_{jgt}, for all $z \leq t \leq T$

(72) $\quad a_{gL}(1/L_{gt}) = (1 + \partial_g)^{-1} q_{gt+1} (1 + T_{gLt}) P_{Lt}$

(73) $\quad a_{gj}(1/y_{gjt}) = (1 + \partial_g)^{-1} q_{gt+1} P_{jt}$

(74) $\quad q_{gt+1} P_{It} = q_{kgt+1}$

Closed form solutions to this problems are obtained in Appendix IV. It is enough here to say that government demand for labor and the different consumption goods depend on current own prices - no cross price effects - and current interest rate, as well as on future interest rates, Tr's and TT's via the shadow prices. In turn, investment demand depends on current and next year's price of the investment good - no cross price effects - and on current and next year's interest rate, as well as on future interest rates, Tr's and TT's via the shadow prices. Finally, demand for new funds depends directly on all the current and future market prices.

2.6 FINANCIAL MARKETS AND FINANCIAL DECISIONS

Overview of the Literature

A dynamic economic structure not only provides the ideal environment to model many features of economic behavior, it also permits one to incorporate the financial side of the economy. If the government is allowed to run deficits, the question of deficit financing automatically follows. If investment is optimally determined and returns to capital are different across sectors, problems of investment financing arise. If there are several financial assets in the economy - government bonds, private bonds, and equity (or simply physical capital installed in different sectors), the problem of allocation of saving among assets with potentially different returns arises.

The different CGE models for tax policy evaluation vary greatly with respect to the extent of their attention to the financial side of the economy. At one extreme are the models in Ballard-Fullerton-Shoven-Whalley (1985), Andersson (1987), Ballard (1983), Ballard-Goulder (1985), Bovenberg (1985, 1986), and Erlich-Ginsburgh-Heyden (1987), which are devoted exclusively to the real side of the economy. In turn, Auerbach-Kotlikoff (1984, 1987), Feltenstein (1984, 1986), and Goulder (1985) allow for government debt. In these models saving finances changes in government debt and physical capital. Private and public assets are perceived by the households as perfect substitutes. The allocation of savings merely adjusts to the relative demands for funds.

Feltenstein (1984, 1986) is the only model surveyed which introduces money. Government deficits are financed by issuing money and bonds according to an exogenously given rule. Money is demanded by consumers for transaction motives and an exogenously given fraction as a store of value. On the other hand, government bonds and physical capital are the vehicles for the intertemporal transfer of wealth.

Summers (1985) and Goulder-Summers (1987) introduce firm specific equity capital. Different assets earn different rates of return. However, such rates are equal up to constant and

exogenous sector specific risk premia. Therefore, the introduction of constant exogenous risk premia, as helpful as it may be in the context of calibration, does not solve the main issue of the non-optimality of the allocation of saving. Also, talking about risk premia in a deterministic context is somewhat unsatisfactory.

The most advanced contribution in the modeling of savings allocation in a CGE setting is found in Slemrod (1980, 1983) in the context of a static one-period model. In his model, consumers act according to a two-stage, separable decision process. They first decide on how much to save. Then they decide on the allocation of saving according to an indirect utility function dependent on the rates of return and variances offered by the different assets in the economy. The source of riskiness in the economy comes from an uncertain marginal product of capital. On the other hand, aside from portfolio decisions, the rest of the economy is insulated from uncertainty.

The most complete contribution in terms of the treatment of the corporate financial rules is Fullerton-Gordon (1983). In a variant of the Ballard-Fullerton-Shoven-Whalley (1985) model, Fullerton and Gordon have capital intensity and optimal financial decisions jointly determined through a two-stage process. The cost of financing capital is minimized by trading off the tax advantages of debt against the expected real bankruptcy costs inherent with high debt/equity ratios. Given the optimal debt/equity ratio, the level of investment is chosen such that at the margin the return on equity equals the return on bonds plus an exogenous risk premium.

The Dagem Model

The DAGEM model considers a whole menu of financial assets, private and public bonds, and firm-specific equity. In terms of equilibrium analysis, consumers typically supply funds and production sectors, and government typically demands funds by issuing equity and private bonds and public bonds, respectively.

The current interest rate as well as the individual and market availability of funds are endogenously determined by the equilibrium conditions. On the other hand, individual asset

portfolio decisions are passive. Also, corporate financial rules and retention policies are either exogenous to the model or follow exogenous reaction rules parametric on the state of crucial variables in the economy.

The non-optimality of the allocation of saving and the absence or exogeneity of corporate financial rules reflect the way uncertainty is treated in the model. Uncertainty is solved by endowing the agents with point expectations about future prices. Under such circumstances, consumers either expect different rates of return (inclusive of risk premium) across assets, in which case they will buy only one asset (that with highest rate), or they expect equal rates of return, in which case they are indifferent about the asset composition of their portfolio. There is no way of trading off rates of return and risks to obtain an optimal interior solution to the problem of the allocation of saving. In the DAGEM all the assets are expected to yield the same after-tax rate of return (eventually corrected by exogenous risk premia), and therefore are perceived as perfect substitutes. Also, the endogenous determination of debt/equity policy parameters by trading off expected bankruptcy costs and the preferential tax treatment of bonds is difficult and problematic in the absence of uncertainty.

Despite the shortcomings of the analysis, the consideration of different financial assets is very important. First, it allow consideration of exogenous debt/equity and dividend/retention corporate financing rules and therefore several sources of corporate investment financing: bonds, equity, and retained earnings. Second, it allows the model to capture the fact that different assets are treated differently by the tax code both at the personal and corporate income levels.

Let us address now the financial decisions of consumers. Wealth W_{it+1}, representing the total amount of funds made available by consumer group i to the rest of the economy, is to be invested. There is a menu of assets in which savings F^S_{it} can be invested: private bonds, equity, and government bonds.

$$(75) \quad F^S_{it} = \Delta B_{igt} + \Sigma_j \Delta B_{igt} + P_{jEt}[E_{ijt+1} - E_{ijt}]$$

The 2J+3 financial assets are perceived by consumers as perfect substitutes, because all the assets are expected to yield the same after-tax rate of return. Accordingly, the asset composition of the portfolio is a matter of indifference for consumers. The only non-trivial financial decision is the amount of funds made available by the group to the rest of the economy:

The actual composition of the portfolio holdings is determined by the market equilibrium conditions. Furthermore, the portfolio composition will be the same for all consumer groups. Each group i will own, at t, a fraction of the market portfolio which corresponds to its share of the total wealth owned by consumers at t. Accordingly, group i's holdings at t of equity and private and public debt are given by:

$$(76) \quad W_{it} = \Sigma_j B_{ijt} + B_{igt} + \Sigma_j P_{jEt-1} E_{ijt}$$

$$(77) \quad B_{ijt} = s_{it} B_{jt}$$

$$(78) \quad B_{igt} = s_{it} B_{igt}$$

$$(79) \quad \Sigma_j P_{jEt-1} E_{ijt} = s_{it} \Sigma_j P_{jEt-1} E_{jt}$$

where $s_{it} = W_{it} / \Sigma_i W_{it}$.

To sum up, financial allocation of savings is exogenous to consumers but endogenous to the model. Also, the equilibrium conditions determine the equilibrium rate of return parametrically on corporate and government financing rules. However, due to the nature of the tax code, different consumer groups will have different after tax rates of return on their portfolios.

Financing its real investment, production sector j is constrained in the following way for all $z \leq t \leq T$:

$$(80) \quad F^D_{jt} = (1 - ITC_{jt}) P_{It} [I_{jt} + C_j(I_{jt})] + (1 - T_{cjt}) r_{Wt} B_{jt} +$$

$$(1 - CGT_{jt}) [P_{jEt} - P_{jEt-1}] E_{jt} - RE_{jt} - T_{cjt} \emptyset'_{jt} K'_{jt}$$

with terminal condition $FL_{jT+1} = 0$.

This means that real investment activities and the payment of interest on outstanding debt at t are financed through retained earnings, RE_{jt}, and external funds, $\bar{F}_{jt}$, which represent the increment in the financial liabilities of the sector FL_{jt}. Financial liabilities must be liquidated by the end of the model horizon.

Dividend-retention policies are exogenously given. Corporate dividend-retention policies are represented by parameter $\emptyset_{\Pi jt}$, the fraction of the after-tax gross profits generated at t which is retained by industry j. The remainder, $(1-\emptyset_{\Pi jt})$, represents the distributed portion of after-tax earnings. Total dividends at t, $(1-\emptyset_{\Pi jt})(1-T_{cjt})\Pi_{jt}$, are distributed among the t-th period shareholders. Notice that this criterion is consistent with the fact that the amount of capital in use at t by sector j is fixed so that gross profits reflect the existent capital stock and should be distributed among those who own it, the t-th period shareholders.

Corporate financing policies are exogenously given. External funds totalling F^D_{jt} are obtained by issuing additional equity and fixed price bonds:

(81) $F^D_{jt}=\Delta B_{jt}+p_{Ejt}(E_{jt+1}-E_{jt})$.

Issuance of new bonds and equity is governed by exogenous continuous corporate financing rules represented in this model by parameter $\emptyset_{Ejt}$. Such policy rules can be described as follows:

(82) $p_{Ejt}(E_{jt+1}-E_{jt})=\emptyset_{Ejt}F_{jt}$

(83) $\Delta B_{jt}=(1-\emptyset_{Ejt})F_{jt}$.

with end conditions,

(84) $p_{jEz-1}E_{jz}=p^*_{jEz-1}E^*_j$ and $p_{jET}E_{jT+1}=0$

(85) $B_{jz}=B^*_j$ and $B_{jT+1}=0$.

Perfect capital markets are assumed such that the price of equity at z p_{Ejz} is the present

discounted value of the future expected stream of dividends per share Div^e_{jt}/E_{jt} [9/]

$$(86) \quad p_{jEz} = \Sigma_t[\Pi_{s \le t}(1+r_s)^{-1}]Div^e_{jt}/E^e_{jt}, \quad \text{with} \quad z+1 \le t \le T.$$

Even though the real and financial decisions of firms are independently discussed, they are in a general equilibrium sense, interdependent. In fact, financial decisions will affect the current market prices and rate of interest, and they will directly and indirectly feedback into sector j's real side decisions. Consider the impact of changes in the dividend-retention rule. First, there is a direct general equilibrium effect in the sense that changes in $\varnothing_{\pi jt}$ change the amount of external funds to be demanded and therefore the equilibrium prices. Second, there is an indirect general equilibrium effect, via the change in disposable income - change in dividend income - of the different consumer groups. Now consider the impact of changes in the corporate finance rule. First, even though there is no direct contemporaneous general equilibrium effect, there will be subsequent effects induced by the changes in interest payments. Second, since the market interest rate and the effective rate of return on equity are different, a different debt/equity ratio affects disposable income of consumers in subsequent periods.

Government deficits and surpluses, which represent changes in government liabilities, are accommodated by open market operations in the bond market. These operations reflect the net demand for new funds by government:

$$(87) \quad LG^D_{t+1} - LG_t = \Delta B_{gt} = F^D_{gt}$$

$$(88) \quad F^D_{gt} = r_t LG_t + [Tr_t + \Sigma_j p_{jt}y^D_{gjt} + (1+T_{gLt})p_{Lt}L^D_{gt} + p_{It}I^D_{gt}] - TT_t.$$

The two different methods of government financing spending, taxation, and bond financing have different effects in the economy. This is a central issue in the model.

2.7 ECONOMIC EQUILIBRIUM: TEMPORARY WALRASIAN EQUILIBRIUM AND TRANSITIONS

Overview of the Literature

Virtually all of the available CGE models for tax policy evaluation are characterized by Walrasian market clearing assumptions. Also, all markets are perfectly-competitive. Atomistic competition among agents is assumed even though only a finite number of agents are considered. Virtually, no market disequilibria or price stickiness are considered. A recent exception is provided by Erlich-Ginsburgh-Heyden (1987). In their model of the Belgium economy, the wage rate is fixed in the short run. Therefore, in the short-run disequilibrium in the labor market will generate endogenous unemployment. However, in the long run all prices including the wage rate are flexible, and accordingly, all markets clear.

Given the dynamic nature of behavior in the economy, market-clearing prices in each period depend on expectations of future prices and on tax variables in the economy. There are essentially two ways of interpreting the economic equilibrium in such a dynamic context. If future prices are perfectly anticipated (i.e. expectations are self-fulfilling), a perfect foresight equilibrium prevails. Then, future actions are merely the implementation of current decisions for future periods. However, if price expectations are not perfect (i.e. agents make mistakes with respect to future prices), then a temporary or short-run equilibrium prevails. Markets clear, and clearing prices depend on future price expectations. Current plans for the future are typically not precisely implemented. They will be revised as more or better information becomes available to the economic agents.

The DAGEM Model

Atomistic competition in every market is assumed. Even though the number of agents on each side of the market is finite, it is assumed that enough agents are involved to render their actions negligible in terms of the overall equilibrium outcomes. The concept of Temporary Walrasian

Equilibrium is adopted to capture the incomplete and sequential aspects of real world trading and the limitations of foresight into the future which we want to capture in this model. All current markets are assumed to clear, hence the Walrasian nature of equilibrium. Also, equilibrium in the short run is parametric on the expectations of future prices held by the different agents as well as future taxation parameters, hence the temporary nature of equilibrium. Actions of the economic agents are based on expectations which may turn out to be incorrect, i. e. price expectations are not self-fulfilling. Therefore, the intertemporal equilibrium path in this economy is conceived as a sequence of short-run, temporary equilibria parametric on future price expectations.

The link between adjacent short-run equilibria is provided by the optimal transitions rules for the individual agents. In fact, given equilibrium prices, consumers decide not only how much to purchase of the several commodities available in the economy, but also how much to save, which is the change in the stock of privately owned wealth. The same is true about producers and government in terms of their decisions on the evolution of their capital stock and financial liabilities.

An equilibrium solution for our economy is a sequence of price vectors p_t and quantity vectors q_t defined over $\{1,....,t,....,T\}$ of the form $p_t=(p_{1t},....,p_{Jt};p_{Lt};p_{It};r_t)$ and $q_t=(y_{1t},....,y_{Jt};L_t;I_t;F)$. The vectors satisfy the following conditions:

i) For each and every $1\leq z\leq T$, and for $i=1,...,I$, given prices p_z, consumer group i maximizes the expected value of its intertemporal preferences subject to a recursive set of budget constraints and, point expectations about future prices $p^*_z=\{p_{z+1},....,p_T\}$;

ii) For each and every $1\leq z\leq T$, and for $j=1,...J+1$, given prices p_z, industry j (including the investment good sector) maximizes the expected present value of the net cash flows subject to the equation of motion of capital stock, strictly convex adjustment costs, and point expectations about future prices $p^*_z=\{p_{z+1},....,p_T\}$;

iii) For each and every $1 \le z \le T$, given prices p_z, the government maximizes the expected value of its intertemporal social preferences subject to a recursive set of budget constraints and point expectations about future prices $p^*_z = \{p_{z+1}, \ldots, p_T\}$;

iv) For each and every $1 \le z \le T$, given prices p_z, the $J+3$ markets in the economy clear based on common expectations about future prices p^*_z, individual preference parameters $¥_i$, technology parameters, corporate financing rules $¥_j$, and on social welfare parameters and current and expected tax policy rules $¥_g$. The market clearing equations are:

$$(89) \quad y^S_j(p_z; p^*_z; ¥_j) =$$

$$\Sigma_i y^D_{ij}(p_z; p^*_z; ¥_i) + y^D_{lj}(p_z; p^*_z; ¥_l) + y^D_{jgz}(p_z; p^*_z; ¥_g) + \Sigma_{1 \le f \le J} y^D_{fjz}(p_z; p^*_z; ¥_f)$$

$$(90) \quad \Sigma_j L^D_j(p_z; p^*_z; ¥_j) + L^D_l(p_z; p^*_z; ¥_l) + L^D_g(p_z; p^*_z; ¥_g) = \Sigma_i L^S_i(p_z; p^*_z; ¥_i)$$

$$(91) \quad \Sigma_j IT^D_j(p_z; p^*_z; ¥_j) + IT^D_l(p_z; p^*_z; ¥_l) + I^D_g(p_z; p^*_z; ¥_g) = I^S(p_z; p^*_z; ; ¥_l)$$

$$(92) \quad \Sigma_j F^D_j(p_z; p^*_z; ¥_j) + F^D_l(p_z; p^*_z; ¥_l) + F^D_g(p_z; p^*_z; ¥_g) = \Sigma_i F^S_i(p_z; p^*_z; ¥_i)$$

This economy satisfies Walras's Law for each and every $z \le T$ and for all current prices, i.e., the value of market excess demand is zero. This economy is characterized by a system of $J+3$ equations equating excess demands to zero in every market in $J+3$ unknown prices (J consumption goods, investment good, labor, and the interest rate). However, using Walras's Law, only $J+2$ equations are linearly independent, and therefore only relative prices can be obtained. Some sort of price normalization is necessary. In what follows, the prices are defined to be strictly positive and to sum up to one, i.e. they are defined in the unit simplex.

2.8 ECONOMIC EQUILIBRIUM: THE RULES OF FORMATION OF EXPECTATIONS

Overview of the Literature

Virtually all the dynamic CGE models for tax policy evaluation adopt the concept of perfect foresight equilibrium. An exception is Goulder (1985), who also assumes myopic expectations. In turn, Ballard-Goulder (1985) consider a flexible amount of foresight in terms of the number of years over which price movements are foreseen.

The model in this paper is flexible in a somewhat different way in that it can include any range of foresight from myopia to perfect foresight. The choice between the perfect foresight and temporary equilibrium is ultimately to be made on philosophic grounds. It can be argued that less than perfect expectations imply that agents are irrational in some way (see Auerbach-Kotlikoff (1987) p. 10). However, the reverse argument can be made: one can question whether agents are really rational and perfectly knowledgeable about future prices.

Recent evidence of Ballard (1987), Ballard-Goulder (1985), and Goulder (1985), confirmed by this work, suggests that the choice in modeling expectations is an important one. They show that the degree of foresight into the future (ranging from perfect foresight to myopic expectations) may have dramatic impacts on the policy conclusions of the model. Accordingly, the best research strategy may be to design models which are flexible enough to allow for different rules regarding the formation of expectations.

In terms of implementation, the two concepts of equilibrium - perfect foresight and temporary equilibrium - have different implications. The dimensionality of the equilibrium solution algorithm is involved. Suppose we have a model with ten markets to be run for a period of 50 years. Aside from normalization, a perfect foresight model implies computing prices in 500 dimensions. Due to the absence of the requirement of intertemporally consistent action plans, a temporary equilibrium model requires solving 50 sequential equilibria, each in ten dimensions. The link between adjacent periods is endogenously provided by the recursive transitions of the

stock variables in the economy, Given that computational speed often varies with the cube of the number of dimensions, the temporary equilibrium formulation is potentially and strikingly more feasible. However, Ballard (1987) and Goulder-Ballard (1985) have developed techniques to greatly speed the computation of a perfect foresight equilibrium.

The DAGEM Model

The information set at period t reflects what is known about the economy at t. It consists of all the structural information of individual preferences and technologies, and all the past equilibrium prices and quantities. Individual expectations at t, for all t+1...T, are based on information as specified in the information set. Price expectations are formed as Hicksian point expectations according to rules to be specified below. In each simulation of the intertemporal model, the agents maintain an intertemporally consistent rule of formation of point price expectations. Therefore, the possibility of the expectation formation, rules changing throughout time is ruled out. However, the price expectations are updated when new information comes into the information set. For example, the expectations of prices at t+h formulated at t and t+1 will, in general, be different. Finally, in terms of the the information structure of the economy it is assumed that all the agents have common price expectations. Therefore, the possibility of informational asymmetries across agents is ruled out.

The rules of formation of expectation are intended to capture the limitations of foresight into the future, and are therefore reasonably simple. In particular, the following three simplifying assumptions are made on the expectational price process.

Assumption 2.1: Bounded rationality - Price expectations depend only on past realized prices, not other variables in IS_t. This assumption can be interpreted as recognizing that information is costly to acquire and process, thus not all the information in IS_t is used.

(93) $p^e_{t+1} = P^e_t(p_t, p_{t-1}, ..., p_0)$ for all t.

This is a crucial assumption. The closed definition of the relevant information set excludes the possibility of the agents knowing, or at least using the knowledge of the model of the economy, hence the bounded rationality nature of the assumption. At a deeper level this assumption may be construed as revealing the source of uncertainty in this economy - if the agents knew the model of the economy they would be able to accurately forecast future prices.

Assumption 2.2: Markov Assumption - The price process $\{p_t\}$ is at most a second order Markov process, i.e., only p_t, p_{t-1} help to predict p_{t+1}. Thus for all t:

$$(94) \quad p^e_{t+1} = P^e_t(p_t, p_{t-1}, \dots, p_0) = P^e_t(p_t, p_{t-1})$$

Assumption 2.3: Stationarity - The parameters of the price process are time-invariant. For each simulation of the model to determine the equilibrium at each period, the agents will maintain expectations according to a stationary process. Thus for all t:

$$(95) \quad P^e_t(p_t, p_{t-1}) = P^e(p_t, p_{t-1})$$

Several rules of formation of point price expectations satisfying the above assumptions will be considered. Let current price (or interest rate) be p_z. Agents will form expectations at z of prices h periods into the future, $_z p^e_{z+h}$, (the first subscript will be dropped whenever it is not ambiguous) under several alternative rules. These rules are as follows:

i) _Static Expectations_: Current prices are expected to prevail into the future. For all $h \geq 0$:

$$(96) \quad p^e_{z+h} = p_z$$

ii) _Extrapolative Expectations_: Expectations about future prices reflect the expected changes in prices in previous periods. Extrapolative expectations are obtained according to the recursive rule for all h>0,

$$(97) \quad p^e_{z+h} = p^e_{z+h-1} + \emptyset(p^e_{z+h-1} - p^e_{z+h-2}).$$

Thus,

(98) $p^e_{z+1} = p_z + \emptyset(p_z - p_{z-1})$.

iii) <u>Constant Rate of Growth</u>: The expectations about future prices reflect the idea that the rate of change in current prices is expected to prevail. Forecasts are given according to the recursive rule for all h>0,

(99) $p^e_{z+h} = (p^e_{z+h-1}/p^e_{z+h-2})p^e_{z+h-1}$.

Thus,

(100) $p^e_{z+1} = (p_z/p_{z-1})\bar{p_z}$.

This recursive rule can be solved forward to yield $p^e_{z+h} = (p_z/p^e_{z-1})^h p_z$.

iv) <u>Adaptative Expectations</u>: Expectations of future prices reflect current prices and previous expectations, so that some adjustment is made for expectation errors and new information. The recursive rule is for all h>0,

(101) $p^e_{z+h} = p^e_{z+h-1} + \emptyset(p^e_{z+h-1} - z_{-1}p^e_{z+h-1})$,

(102) $p^e_{z+1} = p_z + \emptyset(p_z - z_{-1}p^e_z)$.

v) <u>Auto-Regressive Expectations of Order 2 - AR(2)</u>: Forecasts are given according to the recursive rule for all h>0,

(103) $p^e_{z+h} = b_{z+h} + a_1 p^e_{z+h-1} + a_2 p^e_{z+h-2}$,

(104) $p^e_{z+1} = b_{z+1} + a_1 p_z + a_2 p_{z-1}$.

Notice that if we set $b_{z+h}=0$ for all h, $a_1=1$, and $a_2=0$, (103)-(104) reduces to the static expectations as in equation (96). Also, if $b_{z+h}=0$ for all h, $a_1=1+\emptyset$, and $a_2=-\emptyset$, (103)-(104) reduces to the extrapolative expectations in equations (97)-(98). In turn, if we set $b_{z+h}=0$ for all h, $a_1=(p_z/p_{z-1})$, and $a_2=0$, (103)-(104) reduces to constant rate of growth expectations in

equations (99)-(100). Finally, if we set $b_{z+h} = -\emptyset_{z-1} p^e_{z+h-1}$, $a_1 = 1+\emptyset$, and $a_2 = 0$,

(103)-(104) reduces to adaptative expectations in equations. (101)-(102). Therefore static expectations, extrapolative expectations, constant rate of growth expectations, and adaptative expectations are special cases of the AR(2) expectation rule as in (103)-(104) above.

·To calculate the short-run equilibrium resulting from the different expectations rules above, it is sufficient to replace in the equilibrium expressions in section 2.7, the future price and interest rates according to the expectation rules outlined above.

CHAPTER 3 - MODEL IMPLEMENTATION AND POLICY EVALUATION

This chapter deals with issues related to the implementation of the dynamic general equilibrium model - DAGEM - described in Chapter 2. The implementation of this model involves the specification of a base case equilibrium, which is to be contrasted with the revised case equilibria resulting from different alternative policy scenarios. The link between the base case equilibrium and the revised case equilibria is provided by the concept of equal yield - the size of government is kept constant in a meaningful way. The ranking among different equilibria is provided by scalar welfare indicators defined as potential compensation tests. This chapter is organized as follows: Section 3.1 provides a complete description of the economy in terms of data on the stock variables and the behavioral parameter values; Section 3.2 generalizes of the concept of equal yield in the presence of government deficits; Section 3.3 generalizes the compensation tests to a context of no future markets and expectations which are not self-fulfilling. Finally, the details of the computer implementation are discussed in Section 3.4.

3.1 BASE CASE INTERTEMPORAL EQUILIBRIUM

Calibration

CGE models are typically parameterized by the use of a calibration procedure. Some parameters are exogenously given. However, some crucial parameters are determined in such a way that the model replicates the data for a given base year. See Mansur-Whalley (1984) for an extensive discussion of this issue.

Calibration in a dynamic context is often interpreted as requiring two consistency properties. First, as a static property replication of a base year data is required. Second, as an intertemporal requirement the model is parameterized to simulate a balanced growth path when the base policy is maintained. This is the approach followed by Ballard (1983), Ballard-Goulder (1985), Goulder (1985), and Goulder-Summers (1987). The two-requirement calibration strategy follows the practice of Ballard-Fullerton-Shoven-Whalley (1985). It is very much in the spirit of the traditional design of the comparison of alternative equilibria: comparison between a steady-state base case on one hand, and alternative paths including a transition period and a final steady-state on the other hand.

There are several potential problems this two-requirement calibration in the context of dynamic models. The assumption of a steady-state growth path in the base case can be questioned. First, while steady-state is a possibility, it certainly is not the only meaningful solution to dynamic models. Even in the case of a perfect foresight equilibrium, the model implies an equilibrium path which may or may not involve balanced growth. The model, not the modeler, should dictate the nature of the base case path. Second, in the context of a temporary equilibrium path, a steady-state solution is not a likely model outcome. In fact, unless expectations are static, short-run behavior consistent with a steady-state evolution will, in general, not be generated. On the other hand, if static expectations are self-fulfilling we have in fact a perfect foresight model. Third, even the base year replication requirement may cause problems in the context of temporary

equilibrium. Any calibration parameter would be conditional on expectation rules, which is probably an undesirable feature.

The arguments in favor of assuming a base case steady-state are based on the idea that the impact of policy changes can be observed most easily since all departures from the steady-state can be attributed to the alternative policy. On the other hand, the base case so defined as a steady-state is consistent with previous work in a "less dynamic" setting and therefore allows a common standard for comparing model results. At any rate, it seems fair to say that a balanced growth requirement is not necessary and may even be counterproductive, and the static requirement while desirable is not necessary either.

The alternative approach of qualitative calibration has been used by authors like Auerbach-Kotlikoff (1983, 1984, 1987) and Bovenberg (1984, 1985, 1986). The structural parameters are exogenously chosen so that the economy follows a reasonable path into the future.

The strategy of qualitative calibration is also adopted in this paper.[10/] First, qualitative calibration is ideal to exploit the recursive nature of the DAGEM. It minimizes the amount of information necessary to run the model. In fact, aside from the structural parameters, only initial stock values are needed. Given initial conditions on the stocks of private wealth, capital, and government debt, agents optimize thereby generating a set of net demands and short-run equilibrium conditions. In turn, short-run equilibrium prices determine the evolution of the stock variables into the next period. Second, it allows comparisons of different, not necessarily steady-state, equilibrium paths. As argued above, the model not the modeler should dictate the nature of the base case and revised case paths.

Data Requirements and Parameter Specification

The current data set and the parameter specification of the DAGEM is essentially consistent with the 1973 data set and parameter specification of the recent version of Shoven-Whalley's GEMTAP model as reported in Ballard-Fullerton-Shoven-Whalley (1985). The data set and

parameter specification was enlarged to cover aspects not considered in the GEMTAP. See Tables 1-3 for a detailed description of the data set and parameter specification in the DAGEM.

The implementation of the model in this paper requires the specification of a data set which consists of the initial values of the stock variables in the economy. The capital stocks for the different industries in the model are obtained from Ballard-Fullerton-Shoven-Whalley (1985) by applying an average after-tax rate of return to capital income. Industry specific debt and equity are obtained by applying the debt/capital ratios reported in Fullerton-Gordon (1983) to the capital stocks figures. The figure for government capital stock is based on the work of Boskin-Robinson-Roberts (1986) translated into 1973 numbers. Public debt is specified to reflect its current importance in the economy. The 1983 values of debt per capita and the proportion of debt to GNP are applied to the 1973 figures as reported in Ballard-Fullerton-Shoven-Whalley (1985). Since the formulation of the model assumes the existence of a market financial portfolio with individuals allocating savings by buying shares of the market portfolio, it is enough to determine the composition of ownership of global wealth in the economy by income class. This data is obtained from the Office of Tax Planning as reported in Galper-Lucke-Toder (1986). The number of households in each income class is as reported in Ballard-Fullerton-Shoven-Whalley (1985).

Running the model requires the specification of functional forms and parameter selection. For tractability, linear homogeneous Cobb-Douglas functional forms are chosen for all the utility and production functions in the current implementation of the model. Individual preference, government, and technology share parameter values, and the input/output structure are obtained from Ballard-Fullerton-Shoven-Whalley (1985) and correspond to 1973 values. Quadratic adjustment cost functions are postulated. The value of the adjustment cost parameters is consistent with the values reported in Summers (1981) and Goulder-Summers (1987). Finally, the private capital depreciation rates are from Fullerton-Gordon (1983), and public capital depreciation rate are from Boskin-Robinson-Roberts (1986).

Corporate financial rules are constant industry specific debt/equity ratios obtained from Fullerton-Gordon (1983) and the constant industry specific retention/dividend ratios are obtained from the Survey of Current Business (1983).

Tax parameters under the previous tax regime reflect sector specific labor taxes, corporate tax rates, investment tax-credits, and capital depreciation rates for tax purposes as reported by Fullerton-Gordon (1983). Marginal personal income taxes are those in Ballard-Fullerton-Shoven-Whalley (1985), and capital gains taxes are set at 5% as in Goulder-Summers (1987).

3.2 REVISED CASE INTERTEMPORAL EQUILIBRIUM

The link between the base case equilibrium and the revised case equilibrium obtained from different policy changes is provided by the concept of equal yield, government generated public utility is kept constant. The first subsection generalizes the concept of equal yield to a context in which the government is allowed to run deficits. The second subsection specifies government dual behavior according to the minimization of the discretionary expenditure necessary to finance a given path of intertemporal public utility. Finally, the third subsection describes the different methods of tax replacement used to make up for changes in government tax revenues induced by the policy changes under the different equal yield scenarios.

Equal Yield Alternatives and Government Deficits

The policy evaluation of tax changes is based on comparisons between a base case equilibrium which represents the status quo and a series of revised case equilibria which reflect the tax policy changes under discussion. The link between a base case and counterfactual simulations is usually provided by the concept of equal yield: to be comparable, base and revised equilibrium cases should be such that the size of government is kept constant in a meaningful way.

Shoven-Whalley (1977) provide a detailed discussion of the concept of equal yield in a general

equilibrium context. When government is confined to taxation and discretionary transfers, equal yield is interpreted to mean equal tax revenues. On the other hand, when government activities include purchases of private goods in addition to taxation and discretionary transfers, equal yield is interpreted to mean constant public utility. In this case, government base case utility is ·maintained in the counterfactual experiments.

In this paper equal yield is also assumed to mean equal government public utility in both the base and revised case equilibria. The intertemporal sequence of government cumulative utility $\{F_{gt}^b, \ldots, F_{gT}^b\}$ is retrieved from the base case. For the revised case, government purchases of commodities will be such that at revised equilibrium prices, the base case sequence of utilities is attained at minimum cost. Thus government behavior is consistent with compensated demand functions for base case utility levels.[11/]

·Running a revised case requires changes in the equilibrium conditions and the optimal transitions for the stock variables. First, the equilibrium conditions include government compensated demand functions as in the next subsection and Appendix VI, rather than the ordinary ·demand functions as in Section 2.5 and Appendix IV. Second, the government expenditure function in its budget constraints also reflects the compensated demands. Accordingly, the revised case transition for government liabilities can be written as

$$(105) \quad TT_t(IT) + \Delta B_g = P_{Lt} L_{gt}(F_{gt}^b) + \Sigma_j P_{jt} y_{gjt}(F_{gt}^b) + P_{It} I_{gt}(F_{gt}^b) + r_t B_{gt} + Tr_t$$

In Shoven-Whalley (1977), government is subject to a balanced budget constraint. With balanced budgets, the concept of equal yield is unambiguous. The new equilibrium prices and the balanced budget condition will determine the minimal expenditure and taxes needed to maintain base case public utility. Revised case tax revenues just match revised case minimum expenditures. Accordingly, in general, equal yield is inconsistent with equal nominal tax revenue. Some change in tax revenue is necessary. Different tax replacement schemes are considered to assure that enough tax revenue is collected.

In the DAGEM, because government is allowed to run deficits, the concept of equal yield tax replacement needs to be refined. If tax revenues are kept equal to new expenditures, the government supply of bonds is changed which introduces marginal financial crowding-out effects. Also, by keeping either revenues or debt constant, there is still one degree-of-freedom since additional expenditures plus discretionary transfers and interest payment on debt, may be financed via tax revenues, bond issuance, or both. Consequently there are several possible equal yield ways of computing a replacement tax rate in the revised case. The optimal level of expenditure for base case public utility can now be tax financed, bond financed or financed by a mix of bonds and taxation. Some measure of financial crowding-out effects of government deficits can be inferred from the comparison of the several equal yield alternatives.

The following three cases are considered.

1) tax financed policy change

In this alternative, equal yield is defined as the same utility levels and same deficits. The size of the deficits is kept as in the base case. A tax rate is endogenously changed such that tax revenues make up for the expenditure net of deficit financing. Formally, this adds to the model the following constraints:

(106) $\Delta B_{gt}^{r} = \Delta B_{gt}^{b}$ or $B_{gt}^{b} = B_{gt}^{r}$

(107) $TT_t(IT) + \Delta B_g^{b} = P_{Lt}L_{gt}(F_{gt}^{b}) + \Sigma_j P_{jt} y_{gjt}(F_{gt}^{b}) + P_{It}I_{gt}(F_{gt}^{b}) + r_t B_{gt}^{b} + Tr_t.$

2) bond financed policy change

This equal yield alternative implies the same government utility and same tax revenues. It should be emphasized that unlike Shoven-Whalley (1977) equal yield is now consistent with equal tax revenue. Tax revenues are kept constant at base case levels. For different equilibrium prices, the same tax revenue implies endogenous changes in tax rates. Adjustment to deficits, and therefore bond issuance, makes up the difference between tax revenue net of transfers, interest payment on the debt, and the minimizing expenditure to achieve base case utilities. Formally, this

adds to the model the following constraints:

$$(108) \quad \Delta B_g = P_{Lt} L_{gt}(F_{gt}^b) + \Sigma_j P_{jt} y_{gjt}(F_{gt}^b) + P_{It} I_{gt}(F_{gt}^b) + r_t B_{gt} + Tr_t - TT_t^b$$

$$(109) \quad TT_t^b = TT_t^r(IT).$$

3) same composition of expenditure financing

This equal yield aternative implies the same government utility and the same bond-tax revenue financing mix as in the base case. Formally:

$$(110) \quad \Delta B_{gt}^r = [TT_t^b / \Delta B_{gt}^b]^{-1} \, TT_t^r(IT)$$

$$(111) \quad \{1 + [TT_t^b / \Delta B_{gt}^b]^{-1}\} TT_t^r(IT) =$$

$$P_{Lt} L_{gt}(F_{gt}^b) + \Sigma_j P_{jt} y_{gjt}(F_{gt}^b) + P_{It} I_{gt}(F_{gt}^b) + r_t B_{gt} + Tr_t.$$

The comparison among these equal yield schemes is central to tax policy evaluation in the presence of government deficits. The three schemes differ in the marginal financial crowding out they generate. Tax financed change blocks marginal financial crowding out by keeping debt at base case levels. Bond financed equal yield maximize marginal financial crowding-out effects by keeping base case tax revenues constant and by allowing deficits to make up the necessary adjustments. Case 3. reflects an intermediate situation.

Minimization of Government Discretionary Expenditure

As discussed above, the concept of equal yield alternatives supposes that government in the revised cases maintains the same level of social utility as in the base case. Government acts as to minimize the discretionary expenditure necessary to finance a given path of intertemporal public utility.

Public utility at every t is given by a loglinear Cobb-Douglas function such that social felicity at z can be written as

$$(112) \quad F_{zg} = \Sigma_{z \leq t \leq T} (1 + \partial_g)^{-(t-z)} [\Sigma_j a_{gj} Ln(y_{gjt}) + a_{gL} Ln(L_{gt}) + a_{gK} Ln(K_{gt})]$$

The dual optimization problem for government can be written as:

(113) $\text{Min } \Sigma_{z\le t\le T}[\Pi_{z\le s\le t}(1+r_s)^{-1}][\Sigma_j p_{jt}y_{gjt}+(1+T_{gLt})p_{Lt}L_{gt}+p_{It}I_{gt}-TT_t+Tr_t+r_tLG_{gt}]$

subject to:

    i) <u>non-negativity constraints</u> on controls for all $z\le t\le T$

(114) $y_{it}\ge 0$ for all $1\le j\le J$, $L_{it}\ge 0$

    ii) <u>equation of motion for the state variable</u> capital stock for all $z\le t\le T$

(115) $K_{gt+1}=I_{gt}+(1-\varnothing_{gt})K_{gt}$

    iii) <u>end constraints</u> on the capital stock

(116) $K_{gz}=K^*_g$

    iv) <u>equation of motion of state variable</u> public felicity for all $z\le t\le T$

(117) $F_{gt}=\Sigma_j a_{gj}Ln(y_{gjt})+a_{gL}Ln(L_{gt})+a_{gK}Ln(K_{gt})+(1+\partial_g)^{-1}F_{gt+1}$

    v) <u>end conditions</u> for social felicity

(118) $F_{gz}=F^*_g$

(119) $F_{gT+1}=0$

The <u>Hamiltonian</u> associated to this problem is for all $z\le t\le T$

(120) $H_t(L_g,y_g,I_g;F_g,K_g;q_g,q_{kg})=[\Pi_{z\le s\le t}(1+r_s)^{-1}].$

$\{[\Sigma_j p_{jt}y_{gjt}+(1+T_{gLt})p_{Lt}L_{gt}+p_{It}I_{gt}]-TT_t+Tr_t+r_tLG_{gt}]+$

$(1+r_{t+1})^{-1}q_{gt+1}[(1+\partial_g)F_{gt}-(1+\partial_g)[\Sigma_j a_{gj}Ln(y_{gjt})+a_{gL}Ln(L_{gt})+a_{gK}Ln(K_{gt})]]+$

$+(1+r_{t+1})^{-1}q_{gkt+1}[I_{gt}+(1-\varnothing_{gt})K_{gt}]\}$

According to the Maximum Principle, the <u>necessary conditions</u> for optimality at t are:

    i) <u>equation of motion for the state variable</u> felicity for all $z\le t\le T-1$

(121) $F_{gt+1}=(1+\partial_g)F_{gt}-(1+\partial_g)[\Sigma_j a_{gj}Ln(y_{gjt})+a_{gL}Ln(L_{gt})+a_{gK}Ln(K_{gt})]$

ii) <u>felicity end conditions</u>

(122) $F_{gz}=F_i^*$

(123) $F_{gT+1}=0$

iii) <u>adjoint equation</u> for the marginal cost of utility for all $z \leq t \leq T$

(124) $q_{gt}=(1+\partial_g)(1+r_{t+1})^{-1}q_{gt+1}$

iv) <u>equation of motion</u> for capital for all $z \leq t \leq T$

(125) $K_{gt+1}=I_{gt}+(1-\varnothing_{gt})K_{gt}$

v) <u>capital stock initial condition</u>

(126) $K_{gz}=K_g^*$

vi) <u>adjoint equation</u> for the marginal cost of capital for all $z \leq t \leq T$

(127) $q_{kgt}=(1+r_{t+1})^{-1}[q_{gkt+1}(1-\varnothing_{gt})-q_{gt+1}(1+\partial_g)a_{gK}(1/K_{gt})]$

vii) <u>Hamiltonian variational conditions</u> for control variables for all $z \leq t \leq T$

(128) $(1+\partial_g)(1+r_{t+1})^{-1}q_{gt+1}a_{gj}/y_{jgt}=p_{jt}$

(129) $(1+\partial_g)(1+r_{t+1})^{-1}q_{gt+1}a_{gj}/L_{gt}=(1+T_{gLt})p_{Lt}$

(130) $p_{lt}+(1+r_{t+1})^{-1}q_{gkt+1}=0$

Closed form solutions are derived in Appendix VI. It suffices to say here that whenever revised case equilibria are to be computed, the government compensated demand functions should replace the government primal demand functions used in the computation of base case equilibria as discussed in Chapter 2.6 and described in Appendix V.

Tax Replacement Schemes

The equal yield alternatives discussed above involve endogenous replacement changes in the tax rates. Different tax replacement schemes are considered to assure that enough tax revenue is

collected. Tax replacements involve changes in the personal income tax rates. The personal income tax collected from individual i at time t is in the revised case:

(131) $b_{it} + [a\ IT_{it} + b](Taxable\ Income) + LST_{it}$

where a is a multiplicative change factor, b is an additive change factor, and LST_{it} is a lump sum tax levied on individual i at time t. This lump-sum tax corresponds to a fraction of the total endogenous tax revenue change equal to the i-th household share in total wealth.

The three replacement schemes are obtained as follows:

1) multiplicative replacement - set b=LST=0 and let a be endogenously determined;

2) additive replacement - set a=1, LST=0, and let b be endogenously determined;

3) lump sum replacement - set a=1, b=0, and let LST_{it} be endogenously determined.

In general, not all possible replacements schemes are feasible. The tax base that provides the additional revenues to match the tax revenues foregone by the policy changes has got to be important enough to generate the necessary revenues. Otherwise, counterfactual equilibrium may fail to exist (see Shoven-Whalley (1977) on this issue).

The three replacement schemes suggested here seem plausible on a priori grounds. Using the personal income tax as the base for the tax replacement in the context of corporate tax integration is conceptually appropriate in the light of the concept of "double taxation." Also, personal income tax as the base for the tax replacement in the context of corporate tax integration seems to minimize the likelihood of non-existence in that corporate tax revenues were about 8% of total tax revenues in 1985, while personal income tax revenues accounted for 46%.

3.3 POLICY EVALUATION INDICATORS

This section deals with the question of how to perform policy evaluations in the context of the DAGEM model. In the first subsection, the use of compensation tests is discussed. In the second

subsection, the concept of compensation test is generalized in a context of no future markets and of non-self-fulfilling expectations. Finally, the third subsection derives the policy evaluation indicators in the specific context of the DAGEM model.

On the Use of Compensation Tests

The ultimate goal of CGE analysis is to rank different policy alternatives according to some criterion reflecting desirability for society. There are several ways of associating a scalar welfare measure to the array of information which defines an economic equilibrium. The most general way is to follow an axiomatic approach. Several ethically desirable postulates on the space of social states would be established and a social welfare function over the domain of individual utilities derived accordingly. There are serious difficulties with such an approach. First, a general "impossibility theorem" for social welfare functions rules out the existence of such functions under a reasonable set of axioms (see Arrow (1963)). Second, even if we assume enough to guarantee the existence of a social welfare function, the specific functional form inevitably represents highly subjective values which will have an undue influence on the welfare conclusions to be drawn.

An alternative approach is to measure the actual benefits of policy changes by a Pareto superiority criterion. The most immediate drawback of this criterion is that it does not provide a complete ranking of the different social states of the economy. However, this problem can be overcome by the use of a potential benefit criteria, first suggested by Kaldor (1939). These criteria are based on the ability of winners to compensate the losers and still benefit from a certain policy change, even if such compensation does not actually take place.

Compensation tests in the form of aggregated Hicksian Equivalent Variations and aggregated Hicksian Compensated Variations (see Hicks 1940) are the most prevalent criteria to measure efficiency gains or losses in the CGE literature. The Compensated Variation (CV) measures at revised case prices the maximum amount of money the consumer is willing to pay or requires to receive to return to his base case utility. Positive CV's are to be interpreted as representing

welfare gains. The Equivalent Variation (EV) measures at base case prices the maximum amount of money the consumer is willing to receive or pay to attain the revised case utility level. Positive EV's are to be interpreted as representing welfare gains. These individual compensation EV's and CV's are constructed by using optimal cost functions. Therefore they are "objective" money metric indicators. Aggregation of compensation indicators across individuals does not pose any particular problem.

For empirical use, the concept of CV is useful to compare the status quo to one alternative policy. However, since the reference prices change with the alternative experiment, CV cannot be meaningfully used in multiple comparisons. That is exactly the comparative advantage of EV which always uses the status quo prices as reference.

Expectations, Optimal Intertemporal Expenditure, and Compensation Tests

The construction of compensation indicators from individual optimal cost functions in an intertemporal framework deserves some attention. Consider a consumer in an intertemporal framework. In general, the optimal intertemporal cost function associated with a certain path of utility depends on all present and future prices. If all future markets are open, or if future prices are perfectly anticipated - the case discussed by Pollak (1975) - there are no problems with the interpretation and use of the intertemporal cost function and "a fortiori" with the computation of intertemporal EV and CV indicators. Also, in a CGE context, when comparisons are confined to steady-state and/or perfect foresight is assumed that correct future prices are known and there are no difficulties associated with the use of the standard Hicksian indicators.

When some future markets are not open and/or future prices and interest rates are not perfectly anticipated, the concept of intertemporal cost function and associated policy evaluation indicators needs some refinements. Denton (1982) develops the notion of anticipated cost function to reflect expected long-run cost of utility. He also develops the idea of annuity costs associated with a constant flow of utility.

From the standpoint of meaningful empirical applications Denton's indicators are less than ideal. The true compensation indices must be based on ex-post, one-period, optimal cost function parametric on future price expectations, and not on an ex-ante anticipated cost function which is, in general, not self-fulfilling. Also, the true intertemporal compensation indices must be based on a consistent sequence of one-period, ex-post, optimal cost functions associated with a certain utility path. This will, in general, involve non-constant utility annuities. The basic concepts used in this subsection to build the true compensation indices are: anticipated cost function, and short-run and long-run realized cost functions. Let us be more formal.

Consider a sequential economy like in DAGEM, starting at z, lasting T periods, and evolving according to a temporary Walrasian structure. Consider also two different (T-z)-dimensional equilibrium trajectories for an economy associated with different policy specifications: a base case equilibrium with prices $\{...,p_t^b,...\}$ and associated price expectations $\{...,p_t^{eb},...\}$; and a revised case equilibrium with prices $\{...,p_t^r,...\}$ and associated price expectations $\{...,p_t^{er},...\}$. In both base and revised cases, the sequence of primal problems for the household induces a one-period utility sequence $\{U_z^*,...,U_t^*,...,U_T^*\}$. This utility sequence generates a decreasing sequence of felicity $\{F_t^*\}=\{F_z^*(.),...,F_t^*(p_z,p_{z+1},...,p_T;W^*),...,F_T^*(.)\}$ through a recursive dynamic programming algorithm, $F_t^*=U_t^*+(1+\partial_i)^{-1}F_{t+1}^*$. From the perspective of dual household behavior, there is a sequence of one-period cost functions $\{SRC_z^*(F_z^*),..., SRC_t^*(F_t^*)$ $=\Sigma_j p_{jt}H_{jt}^d,...,SRC_T^*(F_T^*)\}$, where H^d's is the one-period Hicksian demand functions, consistent with the primal felicity sequence. In turn, the one-period cost functions generates a sequence of cumulative future costs $\{C_z^*(F_z^*),...,C_t^*(p_z,p_{z+1},...,p_T;F_z^*), ...,C_T^*(F_T^*)\}$ through a recursive dynamic programming algorithm $C_t^*=SRCF_t+(1+r_t)^{-1}C_{t+1}^*$. Now, we want to compare from the point of view of the i-th household the two equilibrium sequences both in the short run and in the

long run.

If the i-th consumer correctly anticipates all future prices in both the base and revised cases, then all the plans into the future will be implemented without the need for revisions. In such a case, $F^*_z(.)$ is the actual intertemporal optimal felicity function at z. In turn, $C^*_z(.)$ is the actual intertemporal optimal cost function at z. The construction of intertemporal compensation tests is, in this context, a straightforward generalization of the static case.

However, If at z the i-th consumer cannot perfectly anticipate future prices, then $p_{z+1},...,p_T$ are to be interpreted as price expectations, $p^e_{z+1},...,p^e_T$ and not as actual prices. Furthermore, since expectations are not fulfilled, intertemporal primal and dual plans will be revised according to a sequence of optimization problems. At each z only current plans parametric on the expectation of future prices are actually implemented. Also, only current utility U^*_z and associated current optimal cost SRC^*_z are actually realized. Accordingly, the optimal functions $F^*_z(.)$ and $C^*_z(.)$ are to be interpreted as the long-run anticipated optimal felicity function and long-run anticipated optimal cost functions at z, respectively. Since $C^*_z(.)$ reflects current costs and future anticipated costs of obtaining a certain expected utility path level, as opposed to actual costs of financing an actual utility path, it should be rewritten as $C^e_z(.)$:

(132) $C^e_z(p_z, p^e_{z+1},...,p^e_T; F^*_z) = \Sigma_j p_{jz} H^d_{jz} + \Sigma_{z+1 \leq t \leq T} \Pi_{z+1 \leq s \leq t} (1+r_s)^{-1} [\Sigma_j p_{jt} H^{ed}_{jt}]$

Now, (132) may be used to generate indicators of the long-run effects anticipated at z of a certain policy change.

i) anticipated long run EV at z

(133) $EV^e_z = C^e_z(p^b_z, p^{eb}_{z+1},...,p^{eb}_T; F^b_z) - C^e_{iz}(p^b_z, p^{eb}_{z+1},...,p^{eb}_T; F^r_z)$

ii) anticipated long run CV at z

(134) $EV^e_z = C^e_z(p^r_z, p^{er}_{z+1},...,p^{er}_T; F^b_z) - C^e_{iz}(p^r_z, p^{er}_{z+1},...,p^{er}_T; ^r_z)$.

Inasmuch as consumers do not anticipate correctly future prices, the anticipated indicators are of no help to evaluate either the actual short-run effects at z or the actual long-run effects of the alternative policies. The relevant concept for short-run policy evaluation is the _realized short-run cost function_ at z, which gives current realized costs at z as a function of current prices as well as future price expectations and a given level of felicity:

(135) $\quad SRC_z(p_z, p^e_{z+1}, \ldots, p^e_T; F^*_z) = \Sigma_j p_{jz} H^d_{jz}(.)$

The following are the associated short-run evaluation indicators:

iv) _realized short-run EV at z_

(136) $\quad SREV_z = SRC_z(p^b_z, p^{eb}_{z+1}, \ldots, p^{eb}_T; F^b_z) - SRC_{iz}(p^b_z, p^{eb}_{z+1}, \ldots, p^{eb}_T; F^r_z)$

v) _realized short-run CV at z_

(137) $\quad SRCV_z = SRC_z(p^r_z, p^{er}_{z+1}, \ldots, p^{er}_T; F^b_z) - SRC_{iz}(p^r_z, p^{er}_{z+1}, \ldots, p^{er}_T; F^r_z)$

Let us now focus on the long-run evaluation indicators. The problem with obtaining such long-run indicators is to get a meaningful sequence of short-run indicators which are comparable and consistent for aggregation. We need first to construct a meaningful sequence of short-run optimal cost functions associated with an actual utility path, so that at each t the one-period costs reflect current utility and are consistent with future observed felicity.

The true _ex-post intertemporal cost function_ over the period z to T, C^*_z, corresponding to certain temporary equilibrium prices, future price expectations, and a given felicity sequence, is the present discounted value of the sequence of realized short-run cost functions:

(138) $\quad C^*_z(p_z, p_{z+1}, \ldots, p_T; \{F^*_t\}) =$

$$\Sigma_{z \leq t \leq T}[\Pi_{z \leq s \leq t}(1+r_s)^{-1}][\Sigma_j p_{jt} H^d_{jt}(p_t, p^e_{t+1}, \ldots, p^e_T; F^*_{it})]$$

where r_t and p_t are actual market prices and interest rates at t, and p^e_{t+h}'s are expected values at t of prices at $t+h$.

To obtain the _intertemporal evaluation indicators_, we just have to use the long-run optimal cost function as described above.

    vii) _Intertemporal EV at z_

(139) $LREV_z(p^b{}_z, p^b{}_{z+1}, ..., p^b{}_T; \{F^b{}_t\}, \{F^r{}_t\}) =$

$$C^*_z(p^b{}_z, p^b{}_{z+1}, ..., p^b{}_T; \{F^b{}_t\}) - C^*_z(p^b{}_z, p^b{}_{z+1}, ..., p^b{}_T; \{F^r{}_t\}) =$$

$$\Sigma_{z \leq t \leq T} [\Pi_{z \leq s \leq t}(1+r^b{}_s)^{-1}][SREV_t(p^b{}_t; p^{eb}{}_{t+1}, ..., p^{eb}{}_T; F^b{}_t, F^r{}_t)]$$

    viii) _Intertemporal CV at z_

(140) $LRCV_z(p^r{}_z, p^r{}_{z+1}, ..., p^r{}_T; \{F^b{}_t\}, \{F^r{}_t\}) =$

$$C^*_z(p^r{}_z, p^r{}_{z+1}, ..., p^r{}_T; \{F^b{}_t\}) - C^*_z(p^r{}_z, p^r{}_{z+1}, ..., p^r{}_T; \{F^r{}_t\}) =$$

$$\Sigma_{z \leq t \leq T} [\Pi_{z \leq s \leq t}(1+r^r{}_s)^{-1}][SRCV_t(p^r{}_t; p^{er}{}_{t+1}, ..., p^{er}{}_T; F^b{}_t, F^r{}_t)]$$

Intertemporal Compensation Indicators in the DAGEM

As discussed above, the construction of compensation tests requires individual short-run and long-run optimal cost functions. At z the dual problem for each consumer group i, the minimization of intertemporal net costs subject to a sequence of future felicity can be written as

(141) $Min_{\{y,L\}} \Sigma_{z \leq t \leq T} \Pi_{z \leq s \leq t}(1+r_s)^{-1} \{\Sigma_j(1+T_{jt})p_{jt}y_{ijt} - (1-T_{it}) \cdot$

$$[p_{Lt}L_{it} + [(1-\Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/E_{jt})]W_{it}] - b_{it} - Tr_{it}\}$$

subject to:

    i) _non-negativity constraints on controls_ for all $z \leq t \leq T$

(142) $y_{it} \geq 0$ for all $1 \leq j \leq J$, $0 \leq L_{it} \leq L^*{}_i$

    ii) _equation of motion for state variable_ felicity for all $z \leq t \leq T$

(143) $F_t = \Sigma_j a_{ij} Ln(y_{ijt}) + (1 - \Sigma_j a_{ij})Ln(L^*{}_i - L_{it}) + (1+\partial_i)^{-1} F_{t+1}$

iii) state end conditions

(144) $F_z = F^*_i$

(145) $F_{T+1} = 0$

The Hamiltonian associated to our problem is for $z \leq t \leq T$

(146) $H_t(L_i, y_i, W_i, q_i) = \Pi_{z \leq s \leq t}(1+r_s)^{-1}\{\Sigma_j(1+T_{jt})p_{jt}y_{ijt} \cdot (1-T_{it}) \cdot$

$[p_{Lt}L_{it} + [(1-\Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/E_{jt})]W_{it}] - b_{it} \cdot Tr_{it}\} +$

$(1+r_{t+1})^{-1}q_{t+1}[(1+\partial_i)F_t - (1+\partial_i)[\Sigma_j a_{ij}Ln(y_{ijt}) - (1-\Sigma_j a_{ij})Ln(L^*_i - L_{it})]]$

The necessary conditions for optimality at z derived from the Maximum Principle are:

i) equation of motion for the state variable felicity for all $z \leq t \leq T$

(147) $F_{t+1} = (1+\partial_i)F_t - (1+\partial_i)[\Sigma_j a_{ij}Ln(y_{ijt}) + (1-\Sigma_j a_{ij})Ln(L^*_i - L_{it})]$

ii) state end conditions

(148) $F_{iz} = F^*_i$

(149) $F_{iT+1} = 0$

iii) adjoint equation for the co-state variable marginal cost of utility for all $z \leq t \leq T$

(150) $q_{it} = (1+\partial_i)(1+r_{t+1})^{-1}q_{it+1}$

iv) Hamiltonian variational conditions for control variables for all $z \leq t \leq T$

(151) $(1+\partial_i)(1+r_{t+1})^{-1}q_{it+1}a_{ij}/y_{jit} = (1+T_{jt})p_{jt}$

(152) $(1+\partial_i)(1+r_{t+1})^{-1}q_{it+1}a_{ij}/(L^*_i - L_{it}) = -(1-T_{it})p_{Lt}$

This problem is solved in Appendix VII. It suffices to report here the optimal short-run cost function at z reflecting contemporaneous expenditure in consumption goods and leisure.

(153) $SRCF_{iz}(p_z, p^e_{z+1}, ..., p^e_T; F^*_z) = exp\{\{ F^*_i - \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}.$

$$[\Sigma_j a_{ij} Ln[a_{ij}/(1+T_{jt})p_{jt}]+(1-\Sigma_j a_{ij})Ln[(1-\Sigma_j a_{ij})/(1-T_{it})p_{Lt}]+$$

$$Ln[(1+\partial_i)^{-(t-z)}\Pi_{z+1\le s\le t}(1+r_s)]]\}/[\Sigma_{z\le t\le T}(1+\partial_i)^{-(t-z)}]\}$$

The intertemporal minimization problem is to be repeated each period to allow for revisions of optimal plans due to unfulfilled expectations. Then, the sequence of optimization problems induces a sequence of actual short-run cost functions $\{SRCF_{iz}, SRCF_{iz+1}, ..., SRCF_{iT}\}$ and cumulative intertemporal cost realizations $\{C^*_{iz}, C^*_{iz+1}, ..., C^*_{iT}\}$ consistent with the actual felicity sequence $\{F_{iz}, F_{iz+1}, ..., F_{iT}\}$. The aggregate short-run and long-run compensation indicators are reported in Appendix VIII.

3.4 COMPUTATION TECHNIQUES

Overview of the Literature

The development of dynamic models - in particular with adjustment costs and/or perfect foresight - has corresponded with the decline in the use of fixed point algorithms. In fact, given the relative large dimensions inevitably involved, such algorithms tend to be very inefficient at the best and often prohibitively slow. See Stone (1985) and Preckel (1985) for a comparative assessment of different computation techniques. Among the recent dynamic general equilibrium models for tax policy evaluation, only Ballard-Fullerton-Shoven-Whalley (1985) and Feltenstein (1985) use Merrill's variant of the fixed point algorithm technique (see Merrill (1972)).

Auerbach-Kotlikoff (1983, 1984, 1987) follow a three-stage procedure. They first compute a base case steady-state, then a revised case steady-state, and finally a transition path for the economy between these two steady-states. In all stages, a Gauss-Seidel iterative procedure is used.

Ballard (1982), Ballard-Goulder (1985), Goulder (1985), and Goulder-Summers (1987) use a method developed by Ballard and Goulder which is similar in many aspects to the Fair-Taylor

(1983) algorithm. Short-run equilibria are calculated (using Merrill's algorithm) parametrically on price expectations. The model is then iterated to generate self-fulfilling intertemporal expectations and the corresponding perfect foresight equilibrium. In a relatively similar approach, Andersson (1987) uses a simulation program, SIMNON, developed at the University of Lund. This program can handle two-point boundary problems in a fashion consistent with the multiple shooting algorithm (see Lipton-Poterba-Sachs-Summers (1982)).

Bovenberg's computational approach (1985, 1986) differs from other models in that he relies heavily on analytical techniques. Computations are done by using a dynamic version of Johanson's linearization method. Being essentially determined by the continuous-time nature of the model, this linearization model has the disadvantage of confining the analysis to infinitesimal changes around the base case equilibrium (see Bovenberg (1985) p. 53).

Erlich-Ginsburgh-Heyden (1987) follow a unique approach in that they use a variant of the optimization technique introduced by Negishi (1960). The economic equilibrium can be generated as a solution of a mathematical program, the objective function of which is a weighted sum of the utility functions of the various agents, while the constraints set consists of the market clearing conditions. Ginsburgh-Heyden (1985) have extended Negishi's result to the case of downward price rigidities.

Finally, the paper by Jorgenson-Yun (1984) is also unique in that it is, among the recent dynamic general equilibrium models for tax policy evaluation, the only econometrically estimated model. Different blocks for the consumption and production side of the model are separately estimated to provide the necessary structural parameters.

The diversity of computation techniques is yet another indicator of the exploratory nature of the dynamic CGE modeling for tax policy evaluation.

The DAGEM Model

Given the temporary equilibrium structure of the DAGEM, the computation of a t-dimensional

intertemporal equilibrium path involves the computation of a sequence of t, one-shot, short-run equilibria parametrically on price expectations. The model is typically run to produce a twenty-year equilibrium sequence in a decision time frame of one hundred years.[12/] The optimal transitions of the stock variables between adjacent short-run equilibria are determined endogenously given the equilibrium prices and net demands.

Each one-shot equilibrium is computed using NPSOL, an optimization algorithm developed by Gill-Murray-Saunders-Wright (1986). The equilibrium conditions are seen as nonlinear equality constraints in the minimization of an articifial objective function. The prices are normalized to the unit simplex by an additional linear equality constraint. The algorithm computes an equilibrium by finding a feasible point to this "bogus" minimization problem: by definition a feasible point satisfies the constraints of the problem, in this case the equilibrium conditions.

The DAGEM is implemented using an interactive FORTRAN program running on a IBM 4381. The computation program is composed of several complementary segments:

Segment 1

Prompts the user to state the number of equilibria to be computed, expectations rules, and the type of run, whether a base case or a revised case, and if the latter what type of equal yield revised case;

Segment 2

Reads from a common block all structural information: number of consumers, number of producers; reads from an input data file the initial values for stock variables and preference and technology parameters as well as tax parameters.

Segment 3

Describes the equilibrium conditions for both base case and revised cases, including the codes for the net demand functions derived from the specification of the production side, consumption side, and government behavior.

Segment 4

Provides the interface with the NPSOL subroutine. It solves for the one-shot, short-run equilibrium prices. The computation routine goes through T-z one-period loops, one for the computation of each temporary equilibria. The transition·between two consecutive loops includes information from the current equilibrium to generate new initial conditions for the computation of the next equilibrium.

Segment 5

The last step in the main computer program presents the equilibrium results and other final information: equilibrium prices and corresponding aggregate and individual net demands for each and every $t \leq T$; the transitions; individual utility and wealth; the net cash flow.and capital stock for each sector; the evaluation indicators and GNP, both for each and every $t \leq T$, and cumulatively.[13/]

As a closing remark it should be said that the NPSOL algorithm proved to be extremely efficient for the computation of the economic equilibrium for such a relatively complex model as the DAGEM. In fact it takes generally about one minute of terminal time on an IBM 4381 to compute a one-period equilibrium for an economy with eight markets and a time horizon of one-hundred periods. For the same specification of DAGEM, Merril's version of the fixed point algorithm takes about eight minutes.

CHAPTER 4 - SUMMARY AND CONCLUDING REMARKS

4.1 SUMMARY

This paper has developed a dynamic sequential general equilibrium model of the United States economy - DAGEM - with endogenous government deficits, forward looking investment decisions, and several financial assets. Given the model set up, the second step has been the discussion of

problems of implementation and policy analysis with such a model. The model has been implemented using a nonlinear optimization algorithm, NPSOL.

The following is a summary of the results and contributions brought forth by this paper in terms of dynamic general equilibrium modeling, model implementation and policy evaluation.

Dynamic General Equilibrium Modeling

This paper develops a stylized dynamic general equilibrium model of the U.S. economy - DAGEM. Economic behavior of every agent in this economy is derived from an intertemporal specification of the agent's objectives and constraints. Firms maximize the present value of the net cash flow in an adjustment costs technology in order to determine endogenously optimal supplies and optimal demands for the different production inputs. In particular, investment decisions are forward looking. Real investment is financed by retained earnings and issuance of new debt and equity according to exogenously defined rules. Government intertemporal behavior is obtained from the maximization of a social welfare function defined over the domain of a public good and subject to an intertemporal budget constraint. The government is allowed to run deficits which are financed by issuing bonds. Optimal household behavior follows a life-cycle type of model generating endogenous savings and labor-leisure decisions. Household asset portfolio decisions merely accommodate the composition of demand for funds. Equilibrium in this economy is conceived as a temporary Walrasian equilibrium. All the markets, for several consumption goods, investment good, labor and financial funds, clear, hence the Walrasian nature of equilibrium. Also, equilibrium in the short run is such that market clearing prices are parametric on the expectation formation rules, hence the temporary nature of equilibrium.

The model developed in this paper - DAGEM - is very much in the frontier of the computational general equilibrium modeling for tax policy evaluation. I would like to claim that with minor exceptions DAGEM provides an enlarged envelope of all the other dynamic CGE models available (see page 19 for a complete reference list and Pereira-Shoven (1987) for details).

Comparable models were simultaneously developed by Auerbach-Kotlikoff (1976) and Goulder-Summers (1.987). In terms of private individual behavior, only minimal differences can be found among DAGEM, Auerbach-Kotlikoff (AK), and Goulder-Summers (GS). In the three models, dynamic production behavior is induced by the existence of adjustment costs. In the three models, dynamic household behavior is characterized by a life-cycle type of structure. AK go further to consider a number of overlapping generations, which however are collapsed into a single consumer. DAGEM with six markets and three household income groups is more disaggregated than AK, which has three markets and one aggregate consumer, and at is closer to the level of disaggregation of GS which has seven markets and one aggregate consumer.

Marked differences among the three models are to be found, first, in the specification of government behavior. In GS, yearly balanced budgets are required. Expenditures are endogenous but not optimal in that they merely accommodate to the level of tax revenues. In AK, the government is subject to an intertemporal budget constraint. Therefore, government is allowed to run yearly deficits. However, the expenditure path is exogenously defined. Accordingly, deficits are endogenously but not optimally determined as a residual given tax revenues. In the DAGEM model both government expenditures and government deficits are endogenously and optimally determined through the maximization of a public utility function and subject to a sequence of budget constraints which are equivalent to an intertemporal budget constraint as in AK.

Differences are also to be found in the specification of financial markets. AK does not include any private financial asset. In turn, DAGEM and GS share similar specifications of the financial markets with a whole array of private financial assets. Both introduce firm-specific equity capital. In GS, different assets earn different rates of return. However, such rates are equal up to constant and exogenous sector-specific risk premia. DAGEM considers also firm-specific bonds and is the only to consider exogenous debt-equity ratios and the possibility of investment being financed by either new bonds or new equity in addition to retained earnings.

A third substantial difference is in the concept of equilibrium. Both AK and GS follow a perfect

foresight approach. In turn, DAGEM follows a temporary equilibrium approach with flexible expectation rules ranging from static expectations to any degree of foresight into the future. That is important because evidence fostered by Ballard (1987) and Ballard-Goulder (1985) suggests that empirical results tend to depend on the degree of foresight.

To summarize, the DAGEM model remains unique among the CGE literature in the extent of dynamic behavior, in the detailed consideration of financial assets, in that government deficits are optimally determined, and on the flexibility of expectation formation rules.

Model Implementation and Policy Evaluation

In the context of applied general equilibrium analysis, policy evaluations are typically carried out by contrasting a base case reflecting the status quo and several counterfactual equilibria reflecting different scenarios generated by the policy changes under consideration. The different equilibria are made comparable by confining the comparison to equal yield alternatives. Finally, the information contained in the different equilibria is typically synthesized in a scalar indicator.

In this paper the concept of equal yield is generalized to accommodate the existence of government deficits. In the counterfactual equilibria, government is seen as minimizing intertemporal expenditure associated with base case intertemporal "public utility." Equal yield is made consistent with tax financing changes in which bond issuance is kept at the base case level, or with bond financed changes in which tax revenues are kept at base case level, or any combination of the two. A measure of marginal financial crowding out induced by the policy changes may be inferred from comparing the different equal yield alternatives.

The dynamic generalization of the Hicksian compensating variation indicators is made necessary by the absence of future markets and perfect foresight in the DAGEM. The generalization is obtained by aggregating the present discounted value of a sequence of actual short-run individual cost functions. Alternative long-run anticipated, not actual, cost indicators were also developed.

The DAGEM is implemented with a nonlinear optimization algorithm, NPSOL. The equilibrium

conditions are interpreted as nonlinear constraints of an artificial optimization problem. While alternative approaches were possible, this computation strategy has two major advantages associated with the use of the NPSOL. First, NPSOL is an extremely efficient algorithm. I estimate that for the size of the model in this work NPSOL is at least eight- to ten-times faster than Merrill's fixed point algorithm. Second, this optimization approach has more flexibility than the standard techniques to solve systems of nonlinear equations in that it allows the treatment of market disequilibrium and fixed price situations without further complications.

4.2 CRITICAL ASSESSMENT OF THE CURRENT FORMULATION OF THE DAGEM

At this point it is only fair to illustrate the principle that "what you get from a model is directly related to what you put into the model." There are two groups of factors that should be taken into consideration when evaluating whatever policies are under consideration. First, "minor" technical problems will be discussed. Second, two more fundamental conceptual problems will be addressed.

Minor Technical Problems

i) The Assumption of Additive Time-Separability of Preferences

Household preferences are assumed to be time separable in a strongly additive form. This is a standard assumption in the context of dynamic CGE analysis as well as in many other areas of applied economics. Time separability is introduced mainly for the sake of analytic simplicity. However, the assumption of additive time separability creates excessive intertemporal substitutability.

ii) Restrictive Functional Forms

All the preferences and technologies are represented in the DAGEM by Cobb-Douglas type of functions. The specific empirical implications of this functional form are well known: fixed

expenditure shares; zero cross price elasticities; unit income elasticities; unit elasticities of substitution, etc. The specific net demand functions derived from the intertemporal optimization problems are reported in Appendices II-IV. Other studies have used either Cobb-Douglas or CES specifications.

While it is, at this stage, conceptually straightforward to generalize the DAGEM to include more flexible functional forms, such a task is extremely time consuming. Accordingly, this route has not been pursued thus far.

iii) Lack of Adequate Estimates of the Adjustment Cost Parameters

Several implementation of the DAGEM suggest that it takes time for the efficiency gains of specific policies to appear. This is in good part due to the adjustment cost specification. It reflects the adjustment lag in the interindustry investment decisions because of mobility and installation costs of adjustment. Therefore, the parametrization of the adjustment cost functions assumes crucial relevance. There aren't many estimates of adjustment costs parameters available (see Abel (1980), Summers (1981), and Pindyck-Rotemberg (1983a, 1983b) for the few exceptions). I am not aware of any estimates with the functional forms and disaggregation as in this paper. (Recall that the parameters used in this version of DAGEM are derived from the estimates in Summers (1981) which are obtained from different functional forms and disaggregation.) Accordingly, some effort should be developed to obtain more adequate estimates.[14/]

Conceptual Problems

i) Financial Behavior and Uncertainty

In the DAGEM, real private investment is financed by retained earnings and issuance of new debt and equity according to exogenously defined corporate financing and dividend-retention rules. Government finances deficits by issuing bonds. Household asset portfolio decisions merely accommodate to the composition of demand for funds - private assets and public bonds. However, there are policies which should be expected to affect both the optimal portfolio decisions and the

optimal corporate financial rules and dividend-retention policies.

Modeling uncertainty in a non-trivial way (recall that in the DAGEM uncertainty is "eliminated" by using point expectations) seems to be the only promising approach to meaningful treatment of both optimal financial decisions and optimal household portfolio decisions. In a stochastic context, optimal portfolio decisions may be addressed within a capital-asset-pricing-model framework (see Merton (1970, 1973) for seminal work along these lines). Also, corporate financial rules may be addressed by trading off the preference of debt against the potential bankruptcy costs of equity financing as in Fullerton-Gordon (1983).

Despite the advantages of stochastic analysis, CGE modeling and implementation in a stochastic setting is a very complex enterprise. It is not surprising that the first steps in this direction are still to be taken.

ii) The Closed-Economy Assumption

Most of the open economy CGE models follow the assumptions of balanced trade with import and export net demands characterized by constant elasticities along the lines of Ballard-Fullerton-Shoven-Whalley (1985). Such is the case of Ballard (1983), Ballard-Goulder (1985), and Goulder-Summers (1987).

A few models have a different treatment of the foreign sector. Feltenstein (1985) treats the rest of the world as an additional consumer group. Bovenberg (1986) develops a model in which two economies are considered, each following intertemporal perfect foresight paths. These economies meet in the international forum. Their trade relationships are characterized by yearly balanced trade accounts.

None of the new generation of dynamic CGE tax models has yet fully incorporated the international capital flows as done in Goulder-Shoven-Whalley (1983) on the earlier Ballard-Fullerton-Shoven-Whalley (1985) model. The first attempts to incorporate the international capital flows are by Andersson (1987), and Erlich-Ginsburgh-Heyden (1987). Andersson (1987) in his model of the Swedish economy adopts a small-economy approach in which rates of

return in the domestic economy are largely determined by the international capital markets. Fixed interest rates induce international capital flows. The international capital flows determine and finance the international trade imbalances. In turn, in Erlich-Ginsburgh-Heyden (1987), foreign trade is generated according to an intertemporal trade welfare function with constant import and export elasticities. In the short run they allow international trade imbalances which generate capital flows to the domestic households. In the long run, however, yearly trade balance is assumed. It is fair to say that meaningful modeling of trade imbalances and international capital flows in the CGE literature is still in a preliminary stage.

In its current formulation the DAGEM is a closed-economy model - no foreign trade or capital flows are considered. This is an undesirable feature in terms of descriptive realism. Furthermore, Goulder-Shoven-Whalley (1984) provide an extensive discussion of the importance of the modeling of the foreign sector for tax policy evaluation. In particular, they conclude that in the context of the Shoven-Whalley model the specification of the foreign sector substantially affects the results of several policy measures.

The introduction of international capital flows in the DAGEM assumes special interest in that government deficits are modeled. First, since foreign capital flows represent an important additional source of funds, the financial crowding-out effects induced by government deficits may change dramatically in an open economy framework. Second, a substantial part of government deficits in the U.S. is financed by inflows of foreign capital. In turn, the inflow of foreign capital tends to keep the dollar overvalued. An overvalued dollar makes imports relatively cheap and exports relatively expensive which creates further problems in terms of the trade balance.

The introduction in the DAGEM of a foreign sector as in Ballard-Fullerton-Shoven-Whalley (1985) does not pose any conceptual problem. However, the interesting aspects that would represent a substantial improvement are the modeling of trade imbalances and international capital flows in a dynamic framework. Modeling trade imbalances and international capital-flows in a dynamic framework are areas wide open for research.

4.3 DIRECTIONS OF FUTURE RESEARCH

Modeling efforts

This paper should be seen as a first step in the process of setting up and implementing an advanced, yet realistic, dynamic CGE model of the U.S. economy. In this sense; Section 4.2 provides an agenda for future research. Efforts are currently being developed in the direction of integrating into the DAGEM a foreign sector with both commodity and capital international flows. In addition, the disaggregation of the investment industry and in particular the introdution of housing capital and services is being considered.

Policy Applications

The DAGEM is flexible enough to support varied modeling generalizations. That being the case the DAGEM model is equally flexible enough to support the analysis of different public finance issues.

The DAGEM was originaly used to study the integration of the corporate and personal income taxes in the U.S. (see Pereira (1988)). Simulation results suggest first that the net welfare gains from integration are at best very modest and frequently negative. Such a dramatic change in tax codes, such as the complete elimination of the corporate tax and its replacement by increased personal income tax rates, is simulated to yield long-run benefits which are never larger than .17% of the present value of future consumption and leisure. This is between four-times and twelve-times lower than comparable results available in the literature. Secondly, it takes time for the efficiency gains of integration to appear. In particular, the average long-run gains are more than three times as large as the average short-run gains. This new intertemporal pattern of efficiency effects is due to the existence of costs of adjustment, and reflects an adjustment lag in the interindustry investment decisions. Thirdly, partial integration, achieved by excluding dividends from the corporate tax base, systematically generates negative effects. This is a new second-best

effect suggesting that less than complete integration may have perverse efficiency effects. Fourthly, unlike results in previous studies, integration is shown not to be a Pareto improvement action. In terms of the value of current consumption and leisure, the lowest income groups are worse off after the policy implementation. However, all income classes show an increase in wealth accumulation and, therefore, the potential is there for welfare gains at some point in the future. Fifthly, under the Tax Reform Act of 1986, the effects of integration show the same patterns and characteristics as under the old tax regime. However, under the new tax law the efficiency gains of integration are much lower. This suggests that the change in tax regimes in itself improved efficiency. In particular, the efficiency gains from both the new tax treatment of capital gains and depreciation allowances and the elimination of the investment tax credits dominate the additional distortions generated by an increase in the effective corporate income tax rates.

The results of the simulation experiments confirm the crucial importance of the marginal distortions in the labor-leisure decisions induced by the tax replacement mechanisms: the higher the marginal increases in the personal income tax rates the lower the efficiency gains from integration. In addition, the importance of marginal financial crowding out is illustrated: higher government deficits are associated with lower integration benefits. The importance of the rules of formation of expectations should be stressed. Different rules for formation of expectations and even different parametrization of the same rule lead to clear changes in the effects of the policies considered. Finally, the simulation results are very robust to different specifications of the debt-equity and dividend-retention parameters as was expected given the deterministic context of the model.

There are several other policy issues which the DAGEM model is particularly adequate to address. The following are currently under investigation. The first issue deals with the efficiency effects of deficit reduction mechanisms. The imposition of an upper bound on government deficits and the eventual elimination of such deficits was consecrated in the Gramm-Rudman-Hollings Bill. For fixed tax revenues, a deficit reduction will depress government expenditures. On the other

hand, a tax-financed deficit reduction will potentially depress private expenditures.. However, both the resource and financial crowding-out effects upon the private sector of government expenditures will be alleviated by the process of deficit reduction. Thus, the net efficiency effect of a deficit reduction mechanism is a matter to be determined empirically.

A related problem is the time frame of the elimination of government deficits. What is the time frame that minimizes the negative effects or maximizes the positive effects of the deficit elimination? Also, the permanent accumulation of new debt is widely perceived as having undesirable effects on the economy by raising interest rates. However, yearly balanced budgets may bring forth a lack of flexibility which is equally undesirable. Are yearly balanced budgets really a good idea? What is the optimal planning horizon for the government to balance revenues and expenditures?

The second policy issue under study is the role of investment tax credits and in particular whether investment tax credits should be reintroduced. Not long after the Tax Reform Act of 1986 was approved, the idea of re-introducing an ITC was thrown into the political arena. The re-introduction of an investment tax credit has been suggested as a way of lowering the price of new investment goods, thereby promoting investment and economic growth. The analysis of current economic conditions in the U.S. gives some credibility to the idea. The global impact of the Tax Reform on the economic growth is to a large extent dependent on incentives to save. Less favorable treatment of Individual Retirement Accounts for example, may suggest lower savings and a slower economic growth. Now, in the absence of any deficit reduction mechanisms, relatively high interest rates and the financial crowding-out effects generated by large government deficits may further depress investment demand. On the other hand, deficit reduction mechanisms of the Gramm-Rudman-Hollings type may alleviate the upward pressure upon interest rates. However, they are also likely to ultimately induce a drastic reduction of government spending and/or increase in the tax rates, and thereby create further depressive effects in the economy.

Considering all factors, it is possible that at some point in the near future undesirable forces

against economic growth will be unleashed. If such a scenario becomes reality, it is only natural that the possibility of re-instating an investment tax credit should be raised. Wouldn't it be a good measure to counteract the potential regressive effects on investment and economic growth of the current tax law in the context of a high deficit economy? Furthermore, if it is a good idea to re-introduce an investment tax credit, what would be the sectorial investment tax credit rates that maximize efficiency gains and economic growth?

APPENDIX I

DAGEM - NOTATION

1. General Notation

Time

current time	z
terminal time	T (finite)
future time	$z \leq t \leq T$

Agents

consumers	group $i = 1, ..., C$
producers of consumption goods	industry $j = 1, ..., J$
producers of physical capital	industry $j = I$
government	g

Commodities

consumption of good j by i	y_{ijt}
labor supplied by i	L_{it}
leisure of i	H_{it}
total available time of i	L^*_i
consumption good j	y_{jt}
value added by j	VA_{jt}
total investment cost by j	TC_{jt}
adjustment costs	$C_j(.)$
use of input f by j	y_{jft}
capital stock in sector j	K_{jt}
investment by industry j	I_{jt}
total demand for investment by j	$I_{jt} + C_j(.)$
labor used by industry j	L_{jt}
investment good	I_t
use of good j by g	y_{jgt}
capital stock demanded by g	K_{gt}
investment by g	I_{gt}
labor demanded by g	L_{gt}

Financial Flows and Assets

wealth of i	W_{it}
savings of i	$S_{it} = F_{it}$
j-th industry bonds owned by i	B_{ijt}
j-th industry equity owned by i	E_{ijt}
dividends from j received by i	Div_{ijt}

government bonds owned by i	B_{igt}
i's share of the market portfolio	s_{it}
share of debt in i's portfolio	$1 - \Sigma_j e_{jt}$
share of equity j in i's portfolio	e_{jt}
j-th's net cash flow	NCF_{jt}
j-th's gross/quasi profits	Π_{jt}
j-th industry bonds	B_{jt}
j-th industry capital equity	E_{jt}
sector j liabilities	FL_{jt}
dividends distributed by j	Div_{jt}
retained earnings by j	RE_{jt}
new funds demanded by j	F_{jt}
government bonds	B_{gt}
government liabilities	LG_t
new funds demanded by g	F_{gt}
labor tax revenue	LT_t
corporate tax	CT_t
investment tax credit	ITC_t
income tax	IT_t
sales tax	ST_t
total taxes	TT_t
transfers	Tr_t

Prices

consumption good j	P_{jt}
vector of consumption goods	P_t
physical investment	P_{It}
labor	P_{Lt}
interest rate	r_t
price of equity j	P_{jEt}

Optimal demands are referred to by superscript D. Optimal supplies are referred to by superscript S. Predetermined stock variables at z are denoted by a superscript * and do not have time subscript. Future expected prices are referred to by superscript e.

2. Structural Parameters

Preference and Technology Parameters

group i's discount rate	∂_i
Cobb-Douglas j-th share	a_{ij}
Cobb-Douglas labor share	$1 - \Sigma_j a_{ij}$

Leontiff parameters	a_{jf}
Cobb-Douglas labor share	a_j
Cobb-Douglas capital share	$1 - a_j$
adjustment costs parameter	b_j
j-th's capital depreciation rate	$\emptyset_{jt}$
dividend/retention parameter	$\emptyset_{\pi jt}$
new debt/equity paramenter	$\emptyset_{Ejt}$
government discount rate	∂_g
Cobb-Douglas labor share	a_{gL}
Cobb-Douglas capital share	a_{gK}
Cobb-Douglas j-th good share	a_{gj}
g's capital depreciation rate	$\emptyset_{gt}$

Tax Parameters

income tax rate	T_{it}
income tax rate intercept	b_{it}
capital gains tax	CGT_{it}
transfers received by i	Tr_{it}
sales tax rate	T_{jt}
labor tax rate	T_{Lt}
j's corporate tax rate	T_{cjt}
j's investment tax credit	ITC_{jt}
j's depreciation allowances	DA_{jt}
j's depreciation for tax purposes	$\emptyset'_{jt}$

APPENDIX II

DAGEM - DERIVATION OF HOUSEHOLD BEHAVIORAL FUNCTIONS

The optimal dynamic problem for the i-th household is described in (7)-(11) in Section 2.3. In turn, first-order necessary conditions are reported in (14)-(19) of the same section. Closed -form solutions to the household problem can obtained in several steps as follows.

Step 1: Finds the solution to the adjoint difference equation for the marginal utility of wealth parametric on its initial value, q_{iz}.

(A.1) $\quad q_{it} = (1+\partial_i)^{t-z} q_{iz} / \Pi_{z \leq s \leq t-1}\{1+(1-T_{it})[(1-\Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]\}$

Step 2: Finds the solution to the equation of motion for wealth.

(A.2) $\quad W^*_i = -\Sigma_{z \leq t \leq T}\{[b_{it}+(1-T_{it})p_{Lt}L_{it}+(1-CGT_{it})\Sigma_j[p_{jEt}-p_{jEt-1}]E_{ijt}$

$+Tr_{it}-\Sigma_j(1+T_{jt})p_{jt}y_{ijt}]/\Pi_{z \leq s \leq t}[1+(1-T_{it})[(1-\Sigma_j e_{js})r_s+\Sigma_j e_{jt}(Div_{js}/p_{jEs-1}E_{js})]]\}$

Step 3: Gets quasi closed-form solutions from the variational conditions parametric on q_{iz} by using the solution to the adjoint equation as in Step 1.

(A.3) $\quad y_{ijt} = \{a_{ij}(1+\partial_i)^{-(t-z)}.$

$\Pi_{z \leq s \leq t}[1+(1-T_{is})[(1-\Sigma_j e_{js})r_s+\Sigma_j e_{js}(Div_{js}/p_{jEs-1}E_{js})]]\}/[q_{iz}(1+T_{jt})p_{jt}]$

(A.4) $\quad L^*_i - L^S_{iz} = \{(1-\Sigma_j a_{ij})(1+\partial_i)^{-(t-z)}.$

$\Pi_{z \leq s \leq t}[1+(1-T_{is})[(1-\Sigma_j e_{js})r_s+\Sigma_j e_{js}(Div_{js}/p_{jEs-1}E_{js})]]\}/[q_{iz}(1-T_{it})p_{Lt}]$

Step 4: _Obtains the initial shadow price_ by plugging the variational conditions as in Step 3 into the expression for the initial wealth level W^*_i as in Step 2.

(A.5) $\quad q_{iz} = \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}/$

$$\{W^*_i + \Sigma_{z \leq t \leq T}\{[b_{it} + Tr_i + (1-T_{it})P_{Lt}L^*_i + (1-CGT_{it})\Sigma_j[P_{jEt} - P_{jEt-1}]E_{ijt}]/$$

$$/[\Pi_{z \leq s \leq t}[1+(1-T_{is})[(1-\Sigma_j e_{js})r_s + \Sigma_j e_{js}(Div_{js}/P_{jEs-1}E_{js})]]]\}\}$$

Step 5: _Obtains closed-form solutions_ for the consumption demands and labor supply by plugging this shadow price as in Step 4 back into the variational conditions as in Step 3. Only the net demands at z matter for the definition of equilibrium. Then, the _inputs from household behavior to the Temporary Walrasian Equilibrium_ are

Net Demands

-_Demand for Consumption Good_ $1 \leq j \leq J$:

(A.6) $\quad y^D_{ijz} = a_{ij}\{1+(1-T_{iz})[(1-\Sigma_j e_{jz})r_z + \Sigma_j e_{jz}(Div_{jz}/P_{jEz-1}E_{jz})]\}/\{q_{iz}(1+T_{jz})P_{jz}\}$

-_Labor Supply_:

(A.7) $\quad L^*_i - L^S_{iz} = (1-\Sigma_j a_{ij})\{(1+(1-T_{iz})[(1-\Sigma_j e_{jz})r_z + \Sigma_j e_{jz}(Div_{jz}/P_{jEz-1}E_{jz})]\}/$

$$/\{q_{iz}(1-T_{iz})P_{Lz}\}$$

in which the _initial marginal utility of wealth_ is given by

(A.8) $\quad q_{iz} = \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}/$

$$\{W^*_i + \Sigma_{z \leq t \leq T}\{[b_{it} + Tr_i + (1-T_{it})P_{Lt}L^*_i + (1-CGT_{it})\Sigma_j[P_{jEt} - P_{jEt-1}]E_{ijt}]/$$

$$/[\Pi_{z \leq s \leq t}[1+(1-T_{is})[(1-\Sigma_j e_{js})r_s + \Sigma_j e_{js}(Div_{js}/P_{jEs-1}E_{js})]]]\}\}$$

-Supply of New Funds:

(A. 9) $F^S_{iz} = b_{iz} + (1-T_{iz})[p_{Lz}L^S_{iz} + [(1-\Sigma_j e_{jz})r_z + \Sigma_j e_{jz}(Div_{jz}/p_{jEz-1}E_{jz})]W^*_i] +$

$+Tr_{iz} + (1-CGT_{iz})\Sigma_j[p_{jEz} - p_{jEz-1}]E_{iz} - \Sigma_j(1+T_{jz})p_{jz}y^D_{ijz}.$

Transitions

-Transition Equation for Wealth:

(A.10) $W^D_{iz+1} =$

$W^*_i + b_{iz} + (1-T_{iz})[p_{Lz}L^S_{iz} + [(1-\Sigma_j e_{jz})r_z + \Sigma_j e_{jz}(Div_{jz}/p_{jEz-1}E_{jz})]W^*_i]$

$+Tr_{iz} + (1-CGT_{it})\Sigma_j[p_{jEt} - p_{jEt-1}]E_{ijt} - \Sigma_j(1+T_{jz})p_{jz}y^D_{ijz}$

where both the initial value of wealth and the initial composition of wealth are predetermined and given by:

(A.11) $W^*_i = \Sigma_j B^*_{ij} + \Sigma_j p_{jEz-1}E^*_{ij} + B^*_{ig}$

-Transition for Wealth Composition:

(A.12) $W_{iz+1} = \Sigma_j B_{ijz+1} + B_{igz+1} + \Sigma_j p_{jEz}E_{ijz+1}$

(A.13) $B_{igz+1} = s_{iz+1}B_{igtz+1}$

(A.14) $B_{ijz+1} = s_{iz+1}B_{jz+1}$, for all j

(A.15) $p_{jEz}E_{ijz+1} = s_{iz+1}p_{jEz}E_{j1}$, for all j

(A.16) $s_{iz+1} = W_{iz+1}/\Sigma_i W_{i1}$

APPENDIX III

DAGEM - DERIVATION OF PRODUCERS BEHAVIORAL FUNCTIONS

The optimal dynamic problem for the j-th producer is described in (28)-(32) of Section 2.4 In turn, first-order necessary conditions are reported in (35)-(40) of the same section. Closed-form solutions to the producer problem can obtained in several steps as follows.

Step 1: Finds the solution to the state equation of motion for capital.

(A.1) $K_{jt} = I_{jt-1} + \Sigma_{z \leq s \leq t-2} I_{js} [\Pi_{s+1 \leq h \leq t-1} (1-\emptyset_{jh})] + K_{jz} \Pi_{z \leq s \leq t-1} (1-\emptyset_{js})$

Step 2: Incorporates the variational condition for labor into the adjoint difference equation

(A.2) $(L_{jt}/K_{jt}) = \{[(1+T_{Lt}) p_{Lt}] / a_j [p_{jt} - \Sigma_f (a_{jf} p_{ft})]\}^{1/(a_j-1)}$

(A.3) $q_{jt} = (1-\emptyset_{jt})(1+r_{t+1})^{-1} q_{jt+1} + (1-T_{cjt})(1-a_j)[p_{jt} - \Sigma_f (a_{jf} p_{ft})] \cdot$

$\{[(1+T_{Lt}) p_{Lt}] / a_j [p_{jt} - \Sigma_f (a_{jf} p_{ft})]\}^{a_j/(a_j-1)}$

Step 3: Solves the adjoint difference equation using the final adjoint condition.

(A.4) $q_{jt} = \Sigma_{t \leq s \leq T} [\Pi_{t<h<s-1} A_{jh}] C_{js}$, where

(A.5) $A_{jh} = (1-\emptyset_{jh})(1+r_{h+1})^{-1}$

(A.6) $C_{js} = (1-T_{cjs})[p_{js} - \Sigma_f (a_{jf} p_{fs})](1-a_j) \{[(1+T_{Ls}) p_{Ls}] / a_j [p_{js} - \Sigma_f (a_{jf} p_{fs})]\}^{a_j/(a_j-1)}$

Step 4: Solves the variational conditions for investment and labor

(A.7) $I^D_{jt} = [1/b_j]\{-1 + [(1+r_{t+1})^{-1} q_{jt+1} / (1-ITC_{jt}) p_{It}]\}$

where q_{jt+1} is given as in Step 3.

(A.8) $\quad L^D_{jt}=K_{jt}\{[(1+T_{Lt})p_{Lt}]/a_j[p_{jt}-\Sigma_f(a_{jf}p_{ft})]\}^{1/(a_j-1)}$

. where K_{jt} is given as in Step 1 and optimal investment is (A.7).

.Also, the optimal supply of good j at t can be obtained by using the value added production function and the optimal labor demand (A.8).

(A.9) $\quad y^S_{jt}=K_{jt}\{[(1+T_{Lt})p_{Lt}]/a_j[p_{jt}-\Sigma_f(a_{jf}p_{ft})]\}^{a_j/(a_j-1)}$

where K_{jt} is given as in Step 1 and optimal investment is (A.7).

Only the z-period net demands matter for the definition of equilibrium. Then the <u>inputs from producer's behavior to the Temporary Walrasian Equilibrium</u> are

<u>Net Demands</u>

  -<u>Total Investment Demand</u>:

(A.10) $IT^D_{jz}=I^D_{jz}+.5b_j(I^D_{jz})^2$, where

(A.11) $I^D_{jz}=[1/b_j]\{-1+(1+r_{z+1})^{-1}q_{jz+1}/(1-ITC_{jz})p_{Iz}]\}$

(A.12) $q_{jz+1}=\Sigma_{z+1\leq s\leq T}[\Pi_{z+1<h<s-1}A_{jh}]C_{js}$

(A.13) $A_{jh}=(1-\emptyset_{jh})(1+r_{h+1})^{-1}$

(A.14) $C_{js}=(1-T_{cjs})[p_{js}-\Sigma_f(a_{jf}p_{fs})](1-a_j)\{[(1+T_{Ls})p_{Ls}]/a_j[p_{js}-\Sigma_f(a_{jf}p_{fs})]\}^{a_j/(a_j-1)}$

  -<u>Labor Demand</u>:

(A.15) $L^D_{jz}=K^*_j\{[(1+T_{Lz})p_{Lz}]/a_j[p_{jz}-\Sigma_f(a_{jf}p_{fz})]\}^{1/(a_j-1)}$

  -<u>Supply of Good j</u>:

(A.16) $y^S_{jz}=K^*_j\{[(1+T_{Lz})p_{Lz}]/a_j[p_{jz}-\Sigma_f(a_{jf}p_{fz})]\}^{a_j/(a_j-1)}$

-<u>Demand for Intermediate Inputs</u> for all $f=1,...,J$:

(A.17) $y^D_{jft} = a_{jf}K^*_j\{[(1+T_{Lz})p_{Lz}]/a_j[p_{jz}-\Sigma_f(a_{jf}p_{fz})]\}^{a_j/(a_j-1)}$

-<u>Demand for External Funds</u>:

(A.18) $F^D_{jz} = (1-ITC_{jt})p_{Iz}[I_j+C_j(I_j)]+(1-T_{cjt})r_zB_{jz}+$

$(1-CGT_{jz})[p_{jEz}-p_{jEz-1}]E_{jz}-RE_{jz}-T_{cjz}\emptyset'_{jz}K'_{jz}$

Inter-agent transfers

-<u>Dividends</u> to be distributed to consumers:

(A.19) $Div_{jz} = (1-\emptyset_{\Pi jz})(1-T_{cjz})\{[p_{jz}-\Sigma_f(a_{jf}p_{fz})]y^S_{jz}-(1+T_{Lz})p_{Lz}L^D_{jz}\}$

Transitions

-<u>State Variable Transition</u>:

(A.20) $K_{jz+1} = I_{jz}+(1-\emptyset_{jz})K^*_j$

-<u>Liability Transitions</u>:

(A.21) $B^S_{jz+1} = B^*_j+(1-\emptyset_{Ejz})F^D_{jz}$

(A.22) $p_{jEz}E^S_{jz+1} = p_{jEz-1}E^*_j+\emptyset_{Ejz}F^D_{jz}$

(A.23) $FL^S_{jz+1} = B^S_{jz+1}+p_{jEz}E^S_{jz+1}$

In the above, FL^*_j, K^*_j, B^*_j and E^*_j are predetermined variables.

In turn, RE_{jz} is given by

(A.24) $RE_{jz} = \emptyset_{\Pi jz}(1-T_{cjz})\{[p_{jz}-\Sigma_f(a_{jf}p_{fz})]y^S_{jz}-(1+T_{Lz})p_{Lz}L^D_{jz}\}$

APPENDIX IV

DAGEM - DERIVATION OF GOVERNMENT BEHAVIORAL FUNCTIONS

The optimal dynamic problem for the government is described in (53)-(60) in Section 2.5. In turn, first-order necessary conditions are reported in (64)-(74) of the same section. Closed-form solutions to the government optimization problem can obtained in several steps as follows.

Step 1: Solves the adjoint difference equation for the marginal utility of public debt parametrically on q_{iz}.

(A.1) $q_{gt} = \Pi_{z \leq s \leq t-1}[(1+\partial_g)(1+r_s)^{-1}]q_{gz}$.

Step 2: Solves difference equation for state variable public debt.

(A.2) $LG^* = -[C_{gz}/A_{gz} + \ldots + (C_{gT-1}/\Pi_{z \leq t \leq T-1}A_{gt}) + (C_{gT}/\Pi_{z \leq t \leq T}A_{gt})]$, where

(A.3) $A_{gt} = [1+r_t]$

(A.4) $C_{gt} = [Tr_t + \Sigma_j p_{jt} y_{gjt} + (1+T_{gLt})P_{Lt}L_{gt} + P_{It}I_{gt} - TT_t]$

Step 3: Obtains quasi closed-form solutions for the demands for labor and commodities from the variational conditions and the marginal utility of public debt as in Step 1 parametrically on q_{iz}.

(A.5) $L_{gt} = [a_{gL}(1+r_t)]/[q_{gt}(1+T_{gLt})P_{Lt}]$

(A.6) $y_{gjt} = [a_{gj}(1+r_t)]/(q_{gt}p_{jt})$ for y_{jgt} $j=1\ldots J$.

Step 4: Obtains quasi closed-form solution for investment demand parametrically on q_{iz}, from

the variational condition for investment, the equation of motion for capital, and the adjoint equation

for the marginal utility of capital.

$$(A.7) \quad I^D_{gt} = [a_{gk}/q_{gt}]\{[(p_{It}-(1+r_{t+1})^{-1}(1-\varnothing_{gt+1})p_{It+1})(1+\partial_g)(1+r_t)^{-1}]^{-1}$$

$$-(1-\varnothing_{gt})[p_{It-1}-(1+r_t)^{-1}(1-\varnothing_{gt})p_{It}]^{-1}\}$$

Step 5: Obtains the initial shadow price of government liabilities q_{gz} by plugging the quasi

closed-form solutions for the several demands into the solution to difference equation for state

variable public debt as in Step 2.

$$(A.8) \quad q_{gz} = -\{(1-a_{kg})[1+1/(1+\partial_g)^{T-z}]+\Sigma_{z+1\leq t\leq T-1}(1+\partial_g)^{-(t-z)}$$

$$-[a_{kg}p_{IT-1}/(1+\partial_g)^{T-z}][(1+r_T)/[(1+r_T)p_{IT-1}-(1-\varnothing_{gT})p_{IT}]]\}/$$

$$\{LG^*_z+p_{Iz}K^*_g(1-\varnothing_{gz})/(1+r_z)+\Sigma_{z\leq t\leq T}[(Tr_t-TT_t)/\Pi_{z\leq s\leq t}(1+r_s)]\}$$

Step 6: Plug this shadow price q_{gz} into quasi closed-form solutions for the several demands as

in Steps 3 and 4 to get the desired closed-form solution for consumption, investment, and labor

demands. Only the net demands at z matter for the definition of equilibrium. Then, the inputs from

government behavior for the Temporary Walrasian Equilibrium are as follows.

Net Demands

-Demand for Labor:

$$(A.9) \quad L^D_{gz} = [a_{gL}(1+r_z)]/[q_{gz}(1+T_{gLz})p_{Lz}]$$

-<u>Demand for Consumption good j=1...J</u>:

(A.10) $y^D_{gjz} = [a_{gj}(1+r_z)]/[q_{gz}p_{jz}]$

-<u>Investment Demand</u>:

(A.11) $I^D_{gz} = a_{gk}/\{q_{gz}[p_{Iz} - (1+r_{z+1})^{-1}(1-\emptyset_{gz+t})p_{Iz+1}][(1+\partial_g)(1+r_z)^{-1}]\} -$

$-(1-\emptyset_{gz})K^*_z \cdot$

-<u>Demand for New Funds</u>:

(A.12) $F^D_{gz} = r_z LG^* + Tr_z + \Sigma_j p_{jz} y^D_{gjz} + (1+T_{gLz})p_{Lz}L^D_{gz} + p_{Iz}I^D_{gz} - TT_z$

Transitions

-<u>transition equation for capital stock</u>:

(A.13) $K^D_{gz+1} = I^D_{gz} + (1-\emptyset_{gz})K^*_g$

-<u>transition equation for liabilities</u>:

(A.14) $LG^D_{z+1} = (1+r_z)LG^* + Tr_z + \Sigma_j p_{jz} y^D_{gjz} + (1+T_{gLz})p_{Lz}L^D_{gz} + p_{Iz}I^D_{gz} - TT_z$

where LG^* and K^*_g are predetermined.

In turn, q_{gz} is given by

(A.15) $q_{gz} = \{(1-a_{kg})[1+1/(1+\partial_g)^{T-z}] + \Sigma_{z+1 \leq t \leq T-1}(1+\partial_g)^{-(t-z)} \cdot$

$\cdot[a_{kg}p_{IT-1}/(1+\partial_g)^{T-z}][(1+r_T)/[(1+r_T)p_{IT-1} - (1-\emptyset_{gT})p_{IT}]]\}/$

$/\{LG^*_z + p_{Iz}K^*_g(1-\emptyset_{gz})/(1+r_z) + \Sigma_{z \leq t \leq T}[(Tr_t - TT_t)/\Pi_{z \leq s \leq t}(1+r_s)]\}$

where TT_t is given by

(A.16) $TT_t = LT_t + CT_t + ST_t + IT_t + CGT_t - ITC_t \cdot$

(A.17) $LT_t = (\Sigma_j T_{Lt}p_{Lt}L^D_{jt}) + T_{gLt}p_{Lt}L^D_{gt}$

(A.18) $CT_t = \Sigma_j T_{cjt}[(p_{jt} - \Sigma_f a_{fjt} p_{ft}) y^S_{jt} - (1 + T_{Lt}) p_{Lt} L^D_{jt} - r_t B_{jt} - \varnothing'_{jt} K'_{jt}]$

(A.19) $ITC_t = \Sigma_j TC_{jt} p_{It}[I_{jt} + C_j(I_{jt})]$

(A.20) $ST_t = \Sigma_i[\Sigma_j T_{jt} p_{jt} y^D_{ijt}];$

(A.21) $IT_t = \Sigma_i\{b_i + T_{it}[p_{Lt} L^D_{ti} + [(1 - \Sigma_j e_{jt}) r_t + \Sigma_j e_{jt}(Div_{jt}/E_{jt})] W_{it}]\}.$

(A.22) $CGT_t = \Sigma_i CGT_{it}(p_{jEt} - p_{jEt-1}) E_{ijt} + \Sigma_j CGT_{jt}(p_{jEt} - p_{jEt-1}) E_{jt}.$

and Tr_t is given by

(A.23) $Tr_t = \Sigma_i Tr_{it}.$

APPENDIX V

DAGEM - EQUILIBRIUM CONDITIONS AND TRANSITIONS

1. **Equilibrium Conditions**

-**Consumption Goods Markets:** for $j=1,\ldots,J$:

(A.1) $\quad \Sigma_i y^D_{ijt}(p_t;p^*_t;\yen_i) + y^D_{ljt}(p_t;p^*_t;\yen_l) + y^D_{jgt}(p_t;p^*_t;\yen_g) +$

$\qquad + \Sigma_{1 \leq f \leq J} y^D_{fjt}(p_t;p^*_t;\yen_f) = y^S_{jt}(p_t;p^*_t;\yen_j)$

(A.2) $\quad y^S_{jt} = K^*_j \{[(1+T_{Lt})p_{Lt}]/a_j[p_{jt}-\Sigma_f(a_{jf}p_{ft})]\}^{a_j/(a_j-1)}$

(A.3) $\quad y^D_{jft} = a_{jf}K^*_j \{[(1+T_{Lt})p_{Lt}]/a_j[p_{jt}-\Sigma_f(a_{jf}p_{ft})]\}^{a_j/(a_j-1)}$

(A.4) $\quad y^D_{lft} = a_{lf}K^*_l \{[(1+T_{Lt})p_{Lt}]/a_l[p_{lt}-\Sigma_f(a_{lf}p_{ft})]\}^{a_l/(a_l-1)}$

(A.5) $\quad y^D_{gjt} = [a_{gj}(1+r_t)]/[q_{gt}p_{jt}]$

(A.6) $\quad y^D_{ijt} = a_{ij}\{1+(1-T_{it})[(1-\Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]\}/\{q_{it}(1+T_{jt})p_{jt}\}$

-**Labor Market:**

(A.7) $\quad \Sigma_j L^D_{jt}(p_t;p^*_t;\yen_j) + L^D_{lt}(p_t;p^*_t;\yen_l) + L^D_{gt}(p_t;p^*_t;\yen_g) = \Sigma_i L^S_{it}(p_t;p^*_t;\yen_i)$

(A.8) $\quad L^D_{jt} = K^*_j \{[(1+T_{Lt})p_{Lt}]/a_j[p_{jt}-\Sigma_f(a_{jf}p_{ft})]\}^{1/(a_j-1)}$

(A.9) $\quad L^D_{lt} = K^*_l \{[(1+T_{Lt})p_{Lt}]/a_l[p_{lt}-\Sigma_f(a_{lf}p_{ft})]\}^{1/(a_l-1)}$

(A.10) $\quad L^D_{gt} = [a_{gL}(1+r_t)]/[q_{gt}(1+T_{gLt})p_{Lt}]$

(A.11) $\quad L^*_i - L^S_{it} =$

$\qquad [(1-\Sigma_j a_{ij})(1+(1-T_{it})[(1-\Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]]/q_{it}(1-T_{it})p_{Lt}$

-Investment Good Market:

(A.12) $\Sigma_j IT^D_{jt}(p_t;p^*_t;\yen_j)+IT^D_{lt}(p_t;p^*_t;\yen_l)+I^D_{gt}(p_t;p^*_t;\yen_g)=I^S_t(p_t;p^*_t;\yen_i)$

(A.13) $IT^D_{jt}=I^D_{jt}+.5b_j(I^D_{jt})^2$

(A.14) $I^D_{jt}=[1/b_j]\{(1+r_{t+1})^{-1}q_{jt+1}-(1-ITC_{jt})p_{lt}]\}$

(A.15) $IT^D_{lt}=I^D_{lt}+.5b_l(I^D_{lt})$

(A.16) $I^D_{lt}=[1/b_l]\{(1+r_{t+1})^{-1}q_{lt+1}-(1-ITC_{lt})p_{lt}]\}$

(A.17) $I^D_{gt}=$

$a_{gk}/\{q_{gt}[p_{lt}-(1+r_{t+1})^{-1}(1-\varnothing_{gt+1})p_{lt+1}](1+\partial_g)(1+r_t)^{-1}\}-(1-\varnothing_{gt})K^*_t.$

(A.18) $I^S_t=K^*_l\{[[(1+T_{Lt})p_{Lt}]/a_l[p_{lt}-\Sigma_f(a_{fl}p_{ft})]\}^{a_l/(a_l-1)}$

-Financial Market:

(A.19) $\Sigma_j F^D_{jt}(p_t;p^*_t;\yen_j)+F^D_{lt}(p_t;p^*_t;\yen_l)+F^D_{gt}(p_t;p^*_t;\yen_g)=\Sigma_i F^S_{it}(p_t;p^*_t;\yen_i)$

(A.20) $F^S_{it}=b_{it}+(1-T_{it})[p_{Lt}L^S_{it}+[(1-\Sigma_j e_{jt})r_t+\Sigma_j e_{jt}(Div_{ji}/p_{jEt-1}E_{jt})]W^*_i]+$

$+Tr_{it}+(1-CGT_{it})\Sigma_j[p_{jEt}-p_{jEt-1}]E_{it}-\Sigma_j(1+T_{jt})p_{jt}y^D_{ijt}$

(A.21) $F^D_{jt}=(1-ITC_{jt})p_{lt}I^D_j+(1-T_{cjt})r_tB_{jt}+(1-CGT_{jt})[p_{jEt}-p_{jEt-1}]E_{jt}-RE_{jt}-\varnothing'_{jt}K''^*_j$

(A.22) $F^D_{lt}=(1-ITC_{lt})p_{lt}I^D_l+(1-T_{clt})r_tB_{lt}+(1-CGT_{jt})[p_{jEt}-p_{jEt-1}]E_{jt}-RE_{lt}-\varnothing'_{lt}K''^*_l$

(A.23) $F^D_{gt}=r_tLG^*+Tr_t+\Sigma_j p_{jt}y^D_{gjt}+(1+T_{gLt})p_{Lt}L^D_{gt}+p_{lt}I^D_{gt}-TT_t$

2. Transitions

-Capital Stock

(A.24) $K_{jt+1}=I^D_{jt}+(1-\varnothing_{jt})K^*_j.$

(A.25) $K_{It+1} = I^D_{It} + (1-\emptyset_{It})K^*_I$.

(A.26) $K^D_{gt+1} = I^D_{gt} + (1-\emptyset_{gt})K^*_g$

-<u>Liabilities</u>:

(A.27) $B^S_{jt+1} = B^*_j + (1-\emptyset_{Ejt})F^D_{jt}$

(A.28) $p_{IEt} E^S_{jt+1} = p_{IEt-1}E^*_j + \emptyset_{Ejt}F^D_{jt}$

(A.29) $FL^S_{jt+1} = B^S_{jt+1} + p_{jEt}E^S_{jt+1}$

(A.30) $B^S_{It+1} = B^*_I + (1-\emptyset_{EIt})F^D_{It}$

(A.31) $p_{IEt}E^S_{It+1} = p_{IEt-1}E^*_I + \emptyset_{EIt}F^D_{It}$

(A.32) $FL^S_{It+1} = B^S_{It+1} + p_{IEt}E^S_{It+1}$

(A.33) $LG^D_{t+1} = (1+r_t)LG^* + Tr_t + \Sigma_j p_{jt}y^D_{gjt} + (1+T_{gLt})p_{Lt}L^D_{gt} + p_{It}I^D_{gt} - TT_t$

-<u>Total Wealth and Composition</u>:

(A.34) $W^D_{it+1} =$

$= W^*_i + b_{it} + (1-T_{it})[p_{Lt}L^S_{it} + [(1-\Sigma_j e_{jt})r_t + \Sigma_j e_{jt}(Div_{jt}/p_{jEt-1}E_{jt})]W^*_i]$

$+ Tr_{it} + (1-CGT_{it})\Sigma_j[p_{jEt} - p_{jEt-1}]E_{ijt} - \Sigma_j(1+T_{jt})p_{jt}y^D_{jit}$

(A.35) $W_{it+1} = \Sigma_j B_{ijt+1} + B_{igt+1} + \Sigma_j p_{jEt}E_{ijt+1}$

(A.36) $\Sigma_j B_{ijt+1} = s_{it+1}\Sigma_j B_{jt+1}$

(A.37) $B_{igt+1} = s_{it+1}B_{igtt+1}$

(A.38) $p_{jEt}E_{ijt+1} = s_{it+1}p_{jEt}E_{jt+1}$ for all j

(A.39) $s_{it+1} = W_{it+1}/\Sigma_i W_{it+1}$

3. Specification of the Shadow Prices

$-q_{it}$ is given by

(A.40) $q_{it} = \Sigma_{t \leq s \leq T}(1+\partial_i)^{-(s-z)} /$

$$\{W^*_t + \Sigma_{t \leq s \leq T}\{[b_{is} + Tr_{is} + (1-T_{is})p_{Ls}L^*_i + (1-CGT_{is})\Sigma_j[p_{jEs}-p_{jEs-1}]E_{ijs}]/$$

$$/[\Pi_{t \leq h \leq s}[1+(1-T_{ih})[(1-\Sigma_j e_{jh})r_h + \Sigma_j e_{jh}(Div_{jh}/p_{jEh-1}E_{jh})]]]\}\}$$

$-q_{jt+1}$ is given by

(A.41) $q_{jt+1} = \Sigma_{t+1 \leq s \leq T}[\Pi_{t+1<h<s-1}A_{jh}]C_{js}$

(A.42) $A_{jh} = (1-\varnothing_{jh})(1+r_{h+1})^{-1}$

(A.43) $C_{js} = (1-T_{cjs})[p_{js}-\Sigma_f(a_{jf}p_{fs})](1-a_j)\{(1+T_{Ls})p_{Lc}/a_j[p_{js}-\Sigma_f(a_{jf}p_{fs})]\}^{a_j/(a_j-1)}$

$-q_{gt}$ is given by

(A.44) $q_{gt} = \{(1-a_{kg})(1+1/(1+\partial_g)^{T-t} + \Sigma_{t+1 \leq s \leq T-1}(1+\partial_g)^{-s} -$

$$-[a_{kg}p_{IT-1}/(1+\partial_g)^{T-t}][(1+r_T)/[(1+r_T)p_{IT-1}-(1-\varnothing_{jT})p_{IT}]]\}/$$

$$/\{LG^*_t + p_{It}K^*_g (1-\varnothing_{gt})/(1+r_t) + \Sigma_{t \leq s \leq T}[(Tr_s-TT_s)/\Pi_{t \leq h \leq s}(1+r_h)]\}$$

4. Inter-Agent Transfers

$-RE_{jt}$, Div_{jt} are given by

(A.45) $RE_{jt} = \varnothing_{\Pi jt}(1-T_{cjt})\{[p_{jt}-\Sigma_f(a_{jf}p_{ft})]y^S_{jt}-(1+T_{Lt})p_{Lt}L^D_{jt}\}$

(A.46) $Div_{jt} = (1-\varnothing_{\Pi jt})(1-T_{cjt})\{[p_{jt}-\Sigma_f(a_{jf}p_{ft})]y^S_{jt}-(1+T_{Lt})p_{Lt}L^D_{jt}\}$

$-TT_t$, Tr_t are given by

(A.47) $TT_t = LT_t + CT_t + ST_t + IT_t + CGT_t - ITC_t.$

$$(A.48) \quad LT_t = (\Sigma_j T_{Lt} p_{Lt} L^D_{jt}) + T_{gLt} p_{Lt} L^D_{gt}$$

$$(A.49) \quad CT_t = \Sigma_j T_{cjt} [(p_{jt} - \Sigma_f a_{fjt} p_{ft}) y^S_{jt} - (1 + T_{Lt}) p_{Lt} L^D_{jt} - r_t B_{jt}] +$$

$$+ T_{cIt} [(p_{It} - \Sigma_f a_{Ift} p_{ft}) I^S_t - (1 + T_{Lt}) p_{Lt} L^D_{It} - r_t B_{It}]$$

$$(A.50) \quad ITC_t = \Sigma ITC_{jt} p_{It} [I^D_{jt} + C_j(I^D_{jt})] + ITC_{It} p_{It} [I^D_{It} + C_1(I^D_{It})]$$

$$(A.51) \quad ST_t = \Sigma_i [\Sigma_j T_{jt} p_{jt} y^D_{ijt}] ;$$

$$(A.52) \quad IT_t = \Sigma_i \{ b_{it} + T_{it} [p_{Lt} L^D_{ti} + [(1 - \Sigma_j e_{jt}) r_t + \Sigma_j e_{jt} (Div_{jt} / p_{jEt-1} E_{jt})] W_{it}] \}$$

$$(A.53) \quad CGT_t = \Sigma_i CGT_{it} (p_{jEt} - p_{jEt-1}) E_{ijt} + \Sigma_j CGT_{jt} (p_{jEt} - p_{jEt-1}) E_{jt}.$$

$$(A.54) \quad Tr_t = \Sigma_i Tr_{it}.$$

APPENDIX VI

DAGEM - DERIVATION OF GOVERNMENT DUAL BEHAVIORAL FUNCTIONS

The problem of minimization of government discretionary expenditure is described in (113)-(119) of Section 3.2. In turn, first-order necessary conditions are reported in (121)- (130) of the same section. Closed-form solutions to this problem can be obtained in several steps as follows.

Step 1: Obtains the solution to the adjoint difference equation for the marginal cost of utility parametric on its initial value q_{gz}.

$$(A.1) \quad q_{gt} = [(1+\partial_g)^{-(t-z)}][\Pi_{z+1 \leq s \leq t}(1+r_s)]q_{gz}$$

Step 2: Obtains the solution to the equation of motion for the state variable cumulative utility:

$$(A.2) \quad F^*_z = \Sigma_{z \leq t \leq T}(1+\partial_g)^{-(t-z)}[\Sigma_j a_{gj} Ln(y_{gjt}) + a_{gL} Ln(L_{gt}) + a_{gK} Ln(K_{gt})]$$

Step 3: Gets quasi-closed form solutions from the variational conditions parametrically on q_{gz} by using the solution to the adjoint equation as in Step 1.

$$(A.3) \quad y_{gjt} = a_{gj}(1+\partial_g)^{-(t-z)}\Pi_{z+1 \leq s \leq t}(1+r_s)q_{gz}/p_{jt}$$

$$(A.4) \quad L_{gt} = a_{gL}(1+\partial_g)^{-(t-z)}\Pi_{z+1 \leq s \leq t}(1+r_s)q_{gz}/p_{Lt}$$

Step 4: Obtains a quasi-closed form solution for the optimal capital stock parametrically on the initial shadow price from the variational condition for investment stock owned by the government and the adjoint equation for the marginal cost of capital.

(A.5) $K_{gt}=a_{gK}[(1+\partial_g)^{-(t-z)}\Pi_{z+1\leq s\leq t}(1+r_s)]q_{gz}/[(1+r_t)p_{lt-1}-p_{lt}(1-\varnothing_{gt})]$

<u>Step 5</u>: <u>Obtains the initial shadow price of public cumulative utility</u> q_{gz} by plugging the variational conditions and K_{gt} into the expression for the solution of the equation of motion for cumulative utility as in Step 2.

(A.6) $q_{gz}=\exp\{\{F^*_z-a_{gk}LnK^*-\Sigma_j a_{gj}(Lna_{gj}-Lnp_{jz})-a_{gL}(Lna_{gL}-Lnp_{Lz})$

$-(1-a_{gk})-\Sigma_{z+1\leq t\leq T}(1+\partial_g)^{-(t-z)}$

$\cdot[Ln(\Pi_{z+1\leq s\leq t}(1+r^e_s)(1+\partial_g)^{-(t-z)})+\Sigma_j a_{gj}(Lna_{gj}-Lnp^e_{jt})+$

$a_{gL}(Lna_{gL}-Lnp^e_{Lt})+a_{gK}(Lna_{gK}-Ln((1+r^e_t)p^e_{lt-1}-p^e_{lt}(1-\varnothing_{gt})))]\}/$

$[\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}]\}$

<u>Step 6</u>: Plug the initial shadow price q_{gz} as in step 5 into the quasi closed-form solutions as in Steps 3 and 4 to get the desired <u>closed-form solution for the compensated demand functions</u>. In particular at z the demand functions are:

(A.7) $y^d_{gjz}=[a_{gj}/p_{jz}]\cdot\exp\{\{F^*_z-\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}\cdot$

$[Ln(\Pi_{z+1\leq s\leq t}(1+r^e_s)(1+\partial_g)^{-(t-z)})+\Sigma_j a_{gj}(Lna_{gj}-Lnp^e_{jt})+$

$a_{gL}(Lna_{gL}-Lnp^e_{Lt})+a_{gK}(Lna_{gK}-Ln((1+r^e_t)p^e_{lt-1}-p^e_{lt}(1-\varnothing_{gt})))]\}/$

$[\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}]\}$

(A.8) $L^d_{gz}=[a_{gj}/p_{Lz}]\cdot\exp\{\{F^*_z-\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}\cdot$

$[Ln(\Pi_{z+1\leq s\leq t}(1+r^e_s)(1+\partial_g)^{-(t-z)})+\Sigma_j a_{gj}(Lna_{gj}-Lnp^e_{jt})+$

$$a_{gL}(Lna_{gL}-Lnp^e{}_{Lt})+a_{gK}(Lna_{gK}-Ln((1+r^e{}_t)p^e{}_{lt-1}-p^e{}_{lt}(1-\varnothing_{gt})))]\}/$$

$$[\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}]\}$$

(A.9) $\quad i^D{}_{gz}=\{[a_{gK}(1+r_{z+1})(1+\partial_g)^{-1}]/[(1+r_{z+1})p_{lz}-p_{lz+1}(1-\varnothing_{gz+1})]\}$

$$exp\{\{F^*{}_z-\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}.$$

$$[Ln(\Pi_{z+1\leq s\leq t}(1+r^e{}_s)(1+\partial_g)^{-(t-z)})+\Sigma_j a_{gj}(Lna_{gj}-Lnp^e{}_{jt})+$$

$$a_{gL}(Lna_{gL}-Lnp^e{}_{Lt})+a_{gK}(Lna_{gK}-Ln((1+r^e{}_t)p^e{}_{lt-1}-p^e{}_{lt}(1-\varnothing_{gt})))]\}/$$

$$[\Sigma_{z\leq t\leq T}(1+\partial_g)^{-(t-z)}]\}-(1-\varnothing_{gz})K^*{}_g$$

APPENDIX VII

DAGEM - DERIVATION OF HOUSEHOLD DUAL BEHAVIORAL FUNCTIONS

The problem of minimization of household expenditure necessary to finance a certain utility path is described in (141)-(145) of Section 3.3. In turn, first-order necessary conditions are reported in (147)-(152) of the same section. Closed-form solution to this problem can be obtained in several steps as follows.

Step 1: Obtains the solution to the adjoint difference equation for the shadow price of utility parametric on the initial value q_{iz} :

(A.1) $\quad q_{it} = [(1+\partial_i)^{-(t-z)}][\Pi_{z+1\leq s\leq t}(1+r_s)]q_{iz}$

Step 2: Obtains the solution to equation of motion for the state variable utility:

(A.2) $\quad F^*_i = \Sigma_{z\leq t\leq T}(1+\partial_i)^{-(t-z)}[\Sigma_j a_{ij}Ln(y_{ijt})+(1-\Sigma_j a_{ij})Ln(L^*_i-L_{it})]$

Step 3: Gets quasi closed-form solutions from the variational conditions parametric on q_{iz} by using the solution to the adjoint equation as in Step 1.

(A.3) $\quad y_{ijt} = a_{ij}(1+\partial_i)^{-(t-z)}\Pi_{z+1\leq s\leq t}(1+r_s)q_{iz}/(1+T_{jt})p_{jt}$

(A.4) $\quad (L^*_i-L_{it}) = (1-\Sigma_j a_{ij})(1+\partial_i)^{-(t-z)}\Pi_{z+1\leq s\leq t}(1+r_s)q_{iz}/(1-T_{it})p_{Lt}$

Step 4: Obtains the initial shadow price of utility by plugging the variational conditions as in Step 3 into the expression for the initial felicity as in Step 2.

(A.5) $q_{iz} = \exp\{\{ F^*_i - \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)} \cdot$

$[\Sigma_j a_{ij} Ln[a_{ij}/(1+T_{jt})p_{jt}] + (1-\Sigma_j a_{ij}) Ln[(1-\Sigma_j a_{ij})/(1-T_{it})p_{Lt}] +$

$Ln[(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)]]\}/[\Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}]\}$

Step 5: Plug the initial shadow price as in Step 4 into the variational conditions as in Step 3 to get the desired closed-form solution for the compensated demand functions.

(A.6) $y^d_{ijt} = [a_{ij}(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)/(1+T_{jt})p_{jt}]$

$\exp\{\{F^*_i - \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)} \cdot$

$[\Sigma_j a_{ij} Ln[a_{ij}/(1+T_{jt})p_{jt}] + (1-\Sigma_j a_{ij}) Ln[(1-\Sigma_j a_{ij})/(1-T_{it})p_{Lt}] +$

$Ln[(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)]]\}/[\Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}]\}$

(A.7) $L^*_i - L^D_{it} = [a_{ij}(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)/(1-T_{it})p_{Lt}] \cdot$

$\exp\{\{ F^*_i - \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)} \cdot$

$[\Sigma_j a_{ij} Ln[a_{ij}/(1+T_{jt})p_{jt}] + (1-\Sigma_j a_{ij}) Ln[(1-\Sigma_j a_{ij})/(1-T_{it})p_{Lt}] +$

$Ln[(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)]]\}/[\Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}]\}$

and in particular for time z:

(A.8) $y^d_{ijz} = [a_{ij}/(1+T_{jz})p_{jz}] \exp\{\{ F^*_i - \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)} \cdot$

$[\Sigma_j a_{ij} Ln[a_{ij}/(1+T_{jt})p_{jt}] + (1-\Sigma_j a_{ij}) Ln[(1-\Sigma_j a_{ij})/(1-T_{it})p_{Lt}] +$

$Ln[(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)]]\}/[\Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}]\}$

(A.9) $L^*_i - L^D_{iz} = [(1-\Sigma_j a_{ij})/(1-T_{iz})p_{Lz}] \exp\{\{ F^*_i - \Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)} \cdot$

$$[\Sigma_j a_{ij} Ln[a_{ij}/(1+T_{jt})p_{jt}] + (1-\Sigma_j a_{ij}) Ln[(1-\Sigma_j a_{ij})/(1-T_{it})p_{Lt}] +$$

$$Ln[(1+\partial_i)^{-(t-z)} \Pi_{z+1 \leq s \leq t}(1+r_s)]]\}/[\Sigma_{z \leq t \leq T}(1+\partial_i)^{-(t-z)}]\}$$

APPENDIX VIII

DAGEM - POLICY EVALUATION INDICATORS

-Aggregate Short-Run Indicators:

(A.1) $\quad ASREV_t = \Sigma_i\{\exp[(F^r_{it} - F^b_{it})/\Sigma_{t \le s \le T}(1+\partial_i)^{-(s-t)}] - 1\}SRCF^b_{it}$

(A.2) $\quad ASRCV_t = \Sigma_i\{1 - \exp[F^r_{it} - F^b_{it})/\Sigma_{t \le s \le T}(1+\partial_i)^{-(s-t)}]\}SRCF^r_{it}$

-Aggregate Long-Run Indicators:

(A.3) $\quad ALREV_t = \Sigma_{t \le s \le T}[\Pi_{t+1 \le h \le s}(1+r^b_h)]ASREV_s$

(A.4) $\quad ALRCV_t = \Sigma_{t \le s \le T}[\Pi_{t+1 \le h \le s}(1+r^r_h)]ASRCV_s$

-Individual Short-Run Expenditure Functions:

(A.5) $\quad SRCF_{it}(p_t, r_t, p^e_{t+1}, r^e_{t+1} \ldots, p^e_T, r^e_T; F^*_t) =$

$= \exp\{\{F_{it} - \Sigma_{t \le s \le T}(1+\partial_i)^{-(s-t)}.$

$.[\Sigma_j a_{ij}Ln[a_{ij}/(1+T_{js})p_{js}] + (1 - \Sigma_j a_{ij})Ln[(1 - \Sigma_j a_{ij})/(1-T_{is})p_{Ls}] +$

$+ Ln[(1+\partial_i)^{-(s-t)}\Pi_{t+1 \le h \le s}(1+r_h)]]\}/[\Sigma_{t \le s \le T}(1+\partial_i)^{-(s-t)}]\}$

FOOTNOTES

1 /

CGE models have been extensively used in other areas of economics. See Shoven-Whalley (1984) for a survey of international trade applications, Decaluwé-Martens (1985) and Robinson (1986) for applications in the area of development, James (1985) for a survey of economic history applications, and A. Manne (1985) and Borges (1986) for all encompassing surveys.

2 /

Two models somewhat similar to DAGEM were developed simultaneously with this dissertation by Goulder-Summers (1987) and Auerbach-Kotlikoff (1987). While the treatment of government deficits, financial markets, and expectations remains unique to DAGEM, the modeling of consumer and producer behavior is essentially the same in the three models.

3 /

Common expectations are a convenient assumption. However, since expectations are in general not self-fulfilling, common expectations are not necessary for the existence of equilibrium (see Radner(1972)).

4 /

It has been pointed out in the literature on theoretical general equilibrium that such a hypothesis, even though widely assumed, is internally inconsistent. In fact, it is impossible to have a finite number of individually negligible agents. For individuals to be negligible we need a continuum of agents (see Aumann (1964) on this issue). A possible compromise not pursued here would be to consider a finite number of agent types, each including a continuum of agents.

5 /

It should be emphasized that equilibrium is not defined in terms of action plans, current prices, and price expectations such that all markets current and future are cleared, as would be the case with a rational expectations equilibrium concept. (See Radner (1983) for a survey of general.

equilibrium models under uncertainty.)

6 /

These assumptions are sufficient for existence and uniqueness of optimal intertemporal output plans even with constant returns to scale technologies (see Pereira (1986, 1987) on this issue).

7 / .

In this model, the government decides on the optimal intertemporal spending structure. However, the tax system is not a decision variable; it is institutionally given. This behavioral assumption is made for simplicity, and reflects the political rigidities associated with changing the tax structure as opposed to spending decisions.

8 /

The fact that government acts in a benevolent fashion is not welfare reducing. In fact, the government optimization process is confined to the space of a public good which only enters private utilities in a strongly separable way and is not subject to market pricing. (See Tesfatsion (1984) on the inconsistency of benevolent government behavior.)

9 /

It should be noted that a satisfactory rationale for the existence of dividends with the present tax code is missing in the profession (see Shoven-Simon (1987)).

1 0/

The base case with the DAGEM essentially reproduces the main relationships in the U.S. economy. In particular, household behavior implies an average savings elasticity with respect to interest rate of .20. This is well within the range of values in use in the CGE literature. On the other hand, household behavior implies an average labor supply elasticity with respect to wage rate of 1.11. This value is at the upper bounds of the set acceptable values (see Lucas-Rapping (1970)).

1 1 /

Unlike Shoven-Whalley's GEMTAP model, only the strong form of yearly constant felicity is available in the DAGEM (see Ballard-Fullerton-Shoeven-Whalley (1985) p. 152).

12/

The distinction between the decision time frame - decision horizon - and the number of equilibrium to be computed - equilibrium horizon - is very important. In fact, given the terminal conditions for economic behavior as in (11), (32), and (57), strong terminal effects should be expected were the model to be run for a number of equilibria equal to the decision time horizon. In turn, such terminal effects would severely bias the evaluation analysis. By introducing different decision horizon and equilibrium horizon, the terminal effects may be virtually avoided. Still, the economic decisions in the equilibrium horizon satisfy the terminal conditions for economic behavior as global consistency requirements. For example, while the government is not constrained to balance revenues and expenditures over the equilibrium horizon, the intertemporal balanced budget condition is imposed over the decision horizon. The government acts as though, given enough time, public debt will be repaid even if that is not necessarily the case within the equilibrium horizon.

13/

The simulation results are precise in the sense that they are robust to roundoff errors and are not affected by the degree of accuracy in the computation of economic equilibrium.

14/

The efficiency results generated by the DAGEM tend to be robust to small changes on the adjustment cost parameters around the original values. While this is a desirable sensitivity analysis result, the nature of the problem here is different. The "true" original values are not known with an acceptable degree of confidence. The parameter values used in the paper may be far off the true values.

Notice first, that to a large extent this problem is shared by any of the other dynamic CGE models with adjustment costs. Second, if it is likely that with the true parameter values the quantitative results in this model would be somewhat different, the qualitative results would not be likely to change.

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TABLE 1
CLASSIFICATION OF INDUSTRIES AND CONSUMER GROUPS IN THE DAGEM

CONSUMER GROUPS
(Households classified by $Thousands of 1973 gross income)

Group # 1	0 - 6
Group # 2	6 - 15
Group # 3	15 +

INDUSTRIES

Sector # 1	Agriculture, Mining, Energy
Sector # 2	Food, Textiles, Paper, Chemicals, Lumber, Metals
Sector # 3	Trade, Finance, Real Estate, Services
Sector # 4	Capital: Construction, Transportation, Machinery

TABLE 2
BASE CASE PARAMETER VALUES AND STOCKS FOR EACH INDUSTRY

	SECTOR 1 Agriculture	SECTOR 2 Manufacturing	SECTOR 3 Services	SECTOR 4 Capital
General Parameters				
Cobb-Douglas Labor Share	0.4	0.8	0.7	0.5
Adjustment Cost Parameter	0.035	0.02	0.025	0.016
Depreciation Rate	0.099	0.07	0.067	0.085
Equity/Capital	0.84	0.78	0.35	0.81
Retention/Earnings	0.63	0.3	0.28	0.25
Tax Parameters Old Tax Law				
Corporate Tax	0.01	0.46	0.3	0.45
Investment Tax Credit	0.038	0.04	0.013	0.039
Depreciation Rate (Tax)	0.203	0.13	0.124	0.128
Labor Tax	0.09	0.09	0.09	0.09
Sales Tax	0.024	0.02	0.01	0
Tax Parameters Current Tax Law				
Corporate Tax	0.01	0.3	0.2	0.3
Investment Tax Credit	-	-	-	-
Depreciation Rate (Tax)	0.15	0.9	0.8	0.9
Labor Tax	0.09	0.09	0.09	0.09
Sales Tax	0.024	0.02	0.01	0
Stock Values				
Capital	795	946	2509	110

TABLE 3
BASE CASE PARAMETER AND STOCKS FOR HOUSEHOLDS AND GOVERNMENT

	LOW INCOME	MED INCOME	HIGH INCOME	GOVERNMENT
General Parameters				
CD Share of Labor/Leisure	0.201	0.371	0.65	0.432
CD Share-Sector 1	0.011	0.008	0.008	0.004
CD Share-Sector 2	0.226	0.203	0.19	0.169
CD Share-Sector 3	0.562	0.418	0.452	0.062
Discount Rate	0.05	0.05	0.05	0.05
Population Distribution	0.38	0.41	0.21	-
Wealth Shares	0.06	0.25	0.69	-
Dep.Rate of Governt Capital	-	-	-	0.06
Tax Parameters **Old Tax Law**				
Personal Income Tax	0.1	0.2	0.3	
Capital Gains Tax	0.05	0.05	0.05	-
Tax Parameters **Current Tax Law**				
Personal Income Tax	0.08	0.15	0.25	-
Stock Values				
Government Capital Stock	-	-	-	2670
Public Debt	-	-	-	500

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