COMBINING CV AND RP DATA: A NOTE ON THE RELATIONSHIP BETWEEN CONSISTENCY AND RATIONALITY

by

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Abstract

In this paper, we show that, when combining revealed (RP) and stated (SP) data, for marginal changes in quality of environmental goods, rationality implies consistency, as the consistency conditions coincide with a subset of the conditions for rationality.

Keywords: combined (RP and SP) individual data; rationality; data consistency.

1. Introduction

In most empirical work, researchers assume data consistency when combining data of different origins, by developing methods to jointly utilize demand data revealed (RP) and stated (SP) by individuals. However, Huang, Haab and Whitehead (1997) found that RP and SP data should not be combined unless both observed and hypothetical decisions imply the same change in future decisions induced by a given change in quality of an environmental good, that is, should be consistent.

In Cunha-e-Sá and Ducla-Soares (1999), the conditions for the existence of an underlying rational preference structure based on a system of mixed demand functions are derived, which include symmetry of the diagonal blocks and skew symmetry of the off-diagonal blocks of the matrix $\overset{\wr}{S}_c$. [1] Besides, testable conditions for data consistency were also obtained for discrete changes in the quality of environmental goods. These involve the relationship between the direct compensated demand functions of the private goods and the indirect compensated demands of the public goods.

The purpose of this paper is to show that for marginal changes in quality: (i) a subset of the above referred consistency conditions coincide with the skew-symmetry of the off-diagonal blocks of the matrix $\overset{\wr}{S}_c$; (ii) the remaining consistency conditions are trivially satisfied. Therefore, in this context, consistency is necessary to rationality.

2. The relationship between consistency and rationality

Let us consider a system of mixed demand functions for the general case of *r* private goods and

[1] $\overset{\wr}{S}_c$ is the matrix of substitution effects in the case of mixed demand systems. See Cunha-e-Sá and Ducla-Soares (1999), pg.227.

n-r public (environmental) goods, and the corresponding Hicksian compensating variation function $s_{CV}^{j}(.)$ for a change in the quality of the environmental good $j \in \{r+1,\dots,n\}$, from $\overline{q}_j$ to $\overline{q}_j'$, respectively, the degraded and the improved levels of quality. If the reference level of utility is

$$u = v(p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j,\dots,\overline{q}_n,y),$$

then,

$$\begin{aligned} s_{CV}^{j}(.) &= y - \sum_{i=1}^{r} p_i q_i^m (p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j',\dots,\overline{q}_n, v(p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j,\dots,\overline{q}_n,y)) - \\ &- p_j^v (p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j',\dots,\overline{q}_n, v(p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j,\dots,\overline{q}_n,y))\,\overline{q}_j' - \\ &- \sum_{\substack{i=r+1 \\ i \neq j}}^{n} p_i^v (p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j',\dots,\overline{q}_n, v(p_1,\dots,p_r,\overline{q}_{r+1},\dots,\overline{q}_j,\dots,\overline{q}_n,y))\,\overline{q}_i \end{aligned}$$

where y is income, p_i and q_i^m, i=1,…,r, represent the price and compensated demand of good i, respectively, and $\overline{q}_i$ and p_i^n, i=r+1,…,n, are the fixed (nonnegative) quantities and virtual price of the public goods.[2]

Differentiating $s_{CV}^{j}(.)$ with respect to the price of the private good i, p_i, and income, y, and making use of well-known theoretical results, it follows that:[3]

$$q_i^{\mathrm{m}}(p_1,\dots,\mathrm{p_r},\overline{\mathrm{q}}_{\mathrm{r+1}},\dots,\overline{\mathrm{q}}_{\mathrm{j}}',\dots,\overline{\mathrm{q}}_{\mathrm{n}},u) + q\, q_i^{\mathrm{m}}(p_1,\dots,\mathrm{p_r},\overline{\mathrm{q}}_{\mathrm{r+1}},\dots,\overline{\mathrm{q}}_{\mathrm{j}},\dots,\overline{\mathrm{q}}_{\mathrm{n}},\mathrm{u}) = -b_i$$

for all i=1,…r, where $q = \dfrac{\partial s_{CV}^{j}}{\partial y} - 1$, and $b_i = \dfrac{\partial s_{CV}^{j}}{\partial p_i}$.

[2] See Cunha-e-Sá and Ducla-Soares (1999), pg. 216, for the definition of virtual price.
[3] See Cunha-e-Sá and Ducla-Soares (1999) for all the derivations. In alternative, we could have used the expenditure function to obtain the same results.

Dividing both sides of the above equation by $\bar{q}_j' - \bar{q}_j$, we obtain:

$$\frac{q_i^{\mathrm{m}}(.,\bar{\mathrm{q}}_{\mathrm{j}}',.)+q\ q_i^{\mathrm{m}}(.,\bar{\mathrm{q}}_{\mathrm{j}},.)}{\bar{\mathrm{q}}_{\mathrm{j}}'-\bar{q}_j} = -\frac{b_i}{\bar{\mathrm{q}}_{\mathrm{j}}'-\bar{q}_j}. \qquad (1)$$

Since $$\lim_{\bar{q}_j \to \bar{q}_j'} \frac{q_i^{\mathrm{m}}(.,\bar{\mathrm{q}}_{\mathrm{j}}',.)+q\ q_i^{\mathrm{m}}(.,\bar{\mathrm{q}}_{\mathrm{j}},.)}{\bar{\mathrm{q}}_{\mathrm{j}}'-\bar{q}_j} = -\lim_{\bar{q}_j \to \bar{q}_j'} \frac{b_i}{\bar{\mathrm{q}}_{\mathrm{j}}'-\bar{q}_j}$$

and, given that $\lim_{\bar{q}_j \to \bar{q}_j'} q = -1$, conditions (1) collapse into the following marginal ones:

$$\frac{\P\ \mathrm{q}_{\mathrm{i}}^{\mathrm{m}}}{\P\ \bar{\mathrm{q}}_{\mathrm{j}}}(\bar{\mathrm{q}}_{\mathrm{j}}') = -\frac{\P\ \mathrm{p}_{\mathrm{j}}^{\mathrm{v}}}{\P\ \mathrm{p}_{\mathrm{i}}}(\bar{\mathrm{q}}_{\mathrm{j}}')$$

for all i=1,…,r and j=r+1,…,n. These conditions coincide with the skew-symmetry conditions derived in the rationality context, according to which individuals should be consistent when classifying the goods (private and public goods) as substitutes or complements in each data set.

Differentiating $s_{CV}^{j}(.)$ with respect to income, y, and the degraded quality level, $\bar{q}_j$,[4] and making use of well-known theoretical results, the *n-r* remaining conditions are obtained, as follows:

$$\frac{m}{q} = \mathrm{p}_{\mathrm{j}}^{\mathrm{v}}(p_1,..., p_r, \bar{q}_{r+1},..., \bar{q}_j,..., \bar{q}_n, u)$$

for j=r+1,…,n, where $m = \frac{\partial s_{CV}^{j}}{\partial \bar{q}_j}$. Taking the limits as $\bar{q}_j \to \bar{q}_j'$, the above conditions collapse into the following identities:

$$\mathrm{p}_{\mathrm{j}}^{\mathrm{v}}(p_1,..., p_r, \bar{q}_{r+1},..., \bar{q}_j',..., \bar{q}_n, u) \equiv \mathrm{p}_{\mathrm{j}}^{\mathrm{v}}(p_1,..., p_r, \bar{q}_{r+1},..., \bar{q}_j',..., \bar{q}_n, u).$$

3. Conclusions

The use of combined data has implications for empirical work. In this paper we show that, for marginal changes in quality, the skew-symmetry conditions included in the set of the rationality conditions for a mixed world, are actually the consistency conditions that evaluate the effect of a marginal change in the price of the private good on the variation function. Therefore, rationality implies consistency.

[4] All the derivations below can be identically obtained for a change in the improved quality level.

References

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