New Network Goods*

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Abstract

New horizontally-differentiated goods involving product-specific network effects are quite prevalent. Consumers' market-wide preference for each of these goods typically is initially unknown. Later, as sales data begin to accumulate, agents learn market-wide preferences, which thus become common knowledge. We study such a market, pinpointing the factors which determine whether the market-wide preferred firm reinforces its lead as time elapses, penetration and under-cost pricing prevail, and first- or last-mover effects in market-wide preferences occur.

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1 Introduction

NEW HORIZONTALLY-DIFFERENTIATED GOODS involving product-specific network effects often reach the market almost simultaneously without consumers and firms knowing which is favored by the majority of consumers. Recent examples are the consoles market where Nintendo, Sony and Microsoft compete, or the storage-media market were Imation and Iomega compete with the SuperDisk and Zip formats, respectively. In these markets, network effects are present since consumers' utility increases with the number of other users.[1] Quite frequently, such goods are incompatible, implying that network effects are product specific. Thus, consumers who bought the less well-liked good may find themselves stranded with the minority format.[2] Even though when such goods are introduced, neither con-

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[1]In the examples cited, due to game sharing (a direct network effect) and variety (an indirect network effect), and file swapping, respectively.

[2]A fact well known to Beta videotape-system patrons.

sumers nor firms know which most consumers will prefer, as sales data begin to accumulate, market-wide preferences become common knowledge.[3] Also, a particular good may be preferred by the majority of consumers because of differences intrinsic to the goods, in which case such an advantage lasts over time, or as a result of an initially more-successful advertising and marketing campaign, in which case it may change.

For the sake of realism, to all these issues one must add horizontal differentiation since consumers idiosyncratically differ in their valuation of the competing goods' characteristics.[4]

We consider a model where two firms compete in prices over two periods by selling horizontally-differentiated goods involving product-specific network effects. Either product may be preferred by the majority of consumers due to the non-observable realization of a random variable common to all consumers. This unobservable common term adds to the usual idiosyncratic horizontal-differentiation term to determine gross surplus which, added to the network benefit, yields willingness to pay. Thus, initial consumers who enjoy one good more than the other do not know if the majority of other consumers also show the same relative preference, or if this is instead an idiosyncratic trait. Afterwards, second-period consumers become aware of which product enjoys a market-wide preference upon observing first-period sales. We thus capture the idea that with time, market-wide preferences become common knowledge.

Considering two periods, as well as profit-maximizing and information-processing firms, allows us to capture strategic price decisions in this setup where market-wide preferences must be learned. Not surprisingly, penetration pricing prevails as long as network effects are felt, and under-cost pricing may occur depending on the relative strength of product differentiation vs. the network effect.

We find that the firm that obtains the larger market share in the first period reinforces its lead in the following period if and only if the network effect is significant enough compared to the degree of product differentiation. This finding contrasts sharply with that of Arthur and Ruszczynski (1992), who show that a firm's increase in market share, when it finds itself

[3]Imation discontinued the production of its SuperDisk drive perhaps as a consequence of learning that most consumers preferred the Zip storage format.

[4]Thus, we *explicitly* capture in a dynamic setting the tension between horizontal differences that tend to split the market among firms, and network effects that induce the opposite tendency.

with a bigger installed base, depends on the discount rate.

By considering a variant of the model with two independent realizations of the non-observable random variable, each affecting consumers buying in one period, we are able to compare the case where a good's preference by the majority of consumers is unchanging in time to the case where such preference may vary. The former depicts an advantage inherent to the good (e.g., its design or performance) whereas the latter describes an extrinsic and possibly fleeting advantage (for instance, resulting from a more-successful introductory campaign). Surprisingly, this has a striking impact on the previous paragraph's conclusion. We find that in the latter case, when a firm obtains the same market-wide preference in both periods, it *always* reinforces its lead.

This variant of the model also allows one firm to be preferred by the majority of consumers in one period, while the other firm benefits from the very same advantage in the following period. In this case, we show that a first-mover advantage prevails, a result that contrasts with that of Katz and Shapiro (1986).[5] Insofar as advertising budgets for promoting new network goods aim at affecting market-wide preferences, this result provides a rationale for the often-observed concentration of spending before and during the launching of the new product, as opposed to later.

By considering yet another variant of the model where the realization of the common term is known from the outset (i.e., which good is preferred by the majority of consumers and by how much is common knowledge resulting, for instance, from advanced testing reported by the media), we show that the parameters' range for which the firm with a larger installed base after the first period increases its dominance in the second period is smaller. Thus, one concludes that firms and initial consumers ignoring which product enjoys a market-wide preference enlarges the set of circumstances under which one firm continually increases its market share as time elapses.

Related issues have been discussed by a plethora of authors.[6] However, almost all studies assume within-period consumer preferences' homogeneity, implying that the same firm captures the whole market in each period while excluding horizontal-differentiation issues. Instead, we assume within-period consumers' heterogeneity while also departing from the lit-

[5] See Liebowitz and Margolis (1994, p. 143) who criticize this type of result.

[6] See Farrell and Klemperer (forthcoming) for a survey of the extant literature.

erature in assuming that consumers and firms initially do not know (other) consumers' valuations with certainty, a realistic feature of paramount importance when network effects are present.

The paper is organized as follows. In Section 2, we describe the model. In Section 3, we solve it. In Section 4, we present the main results. Finally, Section 5 briefly concludes. All material not needed for a quick understanding of the model, its solution and main results is found in several appendices.

2 The Model

We consider a model with two periods. In each period, N consumers reach the market and must decide which good to buy. All have unitary demand. Regardless of when they reach the market, all consumers begin using the good after the second period.[7] Two firms, A and B, sell two differentiated goods endowed with product-specific network effects, i.e., incompatible, which are also denoted A and B, respectively. We assume that firms compete in prices, which they set in each period. Without loss of generality, let the cost of serving an additional consumer be zero.

The final size of network A is given by $N(x_1 + x_2)$, where $x_i \in [0,1]$ is the proportion of consumers who choose network A in period $i = 1, 2$. Each consumer enjoys a surplus resulting from the network effect, S, which increases linearly at rate $E > 0$ with the final size of the network, i.e., $S = E \times N(x_1 + x_2)$.[8] Thus, E is a constant that measures the intensity of the network effect.

In each period, consumers choose the good that offers the greater expected net surplus. To determine it, consumers must consider (i) the gross surplus excluding the network effect, (ii) the expected network benefit which depends on the expected network size and (iii) the price.

For each consumer, the *difference* between the gross surplus yielded by network A and that yielded by network B is given by random variable $v(\cdot)$. A consumer with a positive value of $v(\cdot)$ obtains a larger gross surplus by choosing network A rather than B. Otherwise, it obtains a larger gross surplus by choosing network B.

[7] This straightforwardly models situations where the two buying-periods' time lengths are insignificant when compared to the overall lifetime of the goods. We thus exclude the durable goods' issue, not juxtaposing it to the coordination issue at the root of network goods' markets. This modeling option is widespread in the literature.

[8] Thus, we adhere to Metcalfe's law.

Random variable $v(\cdot)$ equals the sum of two components, random variable z, common to all consumers, and random variable $a(\cdot)$, specific to each consumer, i.e., idiosyncratic:

$$v\left(x_i, z\right) = a\left(x_i\right) + z.$$

The value of z measures how much, on average, *all* consumers prefer network A to B. We assume it to have uniform distribution with support $[-w, w]$:

$$z \rightsquigarrow U\left(-w, w\right).$$

Random variable $a(\cdot)$ measures how much a particular consumer idiosyncratically prefers network A to B. It is built as follows. Assume that consumers are uniformly distributed along the segment $[0, 1]$ with A located at $x = 0$ and B located at $x = 1$. Let t measure the degree of product differentiation between the two goods. A consumer located at $x = 0$, *ceteris paribus*, prefers network A to B by an amount t, while a consumer located at $x = 1$ prefers network B to A by the same amount. Therefore, $a(x_i)$ is uniformly distributed with support $[-t, t]$:

$$a\left(x_i\right) = t - 2t x_i, \qquad i = 1, 2$$
$$x_i \rightsquigarrow U\left(0, 1\right) \Rightarrow a \rightsquigarrow U\left(-t, t\right).$$

We assume that each consumer knows the density functions of x_i, a, z and $v(\cdot)$, but can only observe the value taken by $v(x_i, z)$ in its particular case.[9] If a consumer observes $v(\cdot)$ taking a positive value, it knows that its gross surplus of choosing A exceeds that of choosing B by the amount $v(\cdot)$. However, it does not know if this is caused by a high realization of z, in which case most consumers also prefer network A to B, or a low realization of x_i, in which case it is she or he that idiosyncratically enjoys network A more than B.[10]

After defining $e \equiv E \times N$, the net surplus of period-i consumers from buying good A is given by

$$C + v\left(x_i, z\right) + e \times \left(\tilde{x}_1 + \tilde{x}_2\right) - p_i^A,$$

while the net surplus of buying good B is given by

$$C + e \times \left(2 - \left(\tilde{x}_1 + \tilde{x}_2\right)\right) - p_i^B,$$

[9] In plain words, each consumer knows how much it prefers one particular network over the other, all else equal.

[10] As we will see, second-period consumers circumvent this informational problem by inferring the realization of z from first-period sales.

where $\tilde{x}_1$ and $\tilde{x}_2$ represent the expected first- and second-period market shares of network A, and C is a constant sufficiently large for all the market to be covered in equilibrium.

3 Solving the Model

This section contains the computations needed for solving the model and is thus of limited interest to readers interested only in results. For these, it is enough to know that equations (11) and (14) represent first- and second-period demand, whereas equations (15), (12) and (13) represent first- and second-period optimal prices when consumers and firms maximize utility and profits, respectively, while optimally learning the extent of market-wide preferences from sales data available at the end of the first period.

In order to choose a network, first-period consumers must compare the net surpluses yielded by networks A and B. This determines the first-period indifferent consumer, and thus first-period demand. First-period consumers indifferent between the two networks are such that:

$$\begin{aligned}
&C + v\left(a\left(x_1\right), z\right) + e\left(\tilde{x}_1 + \tilde{x}_2\right) - p_1^A = C + e\left(2 - \left(\tilde{x}_1 + \tilde{x}_2\right)\right) - p_1^B \Leftrightarrow \\
&t - 2tx_1 + z + e\left(\tilde{x}_1 + \tilde{x}_2\right) - p_1^A = e\left(2 - \left(\tilde{x}_1 + \tilde{x}_2\right)\right) - p_1^B \Leftrightarrow \\
&x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e\left(\tilde{x}_1 + \tilde{x}_2\right)}{2t}. \qquad (1)
\end{aligned}$$

First-period demand is a function of the expectation of the networks' market shares, i.e., $\tilde{x}_1$ and $\tilde{x}_2$, which we must thus compute.

In order to obtain the estimated demand for network A in the first period, $\tilde{x}_1$, we assume that all consumers are rational insofar as estimated demand equals expected demand:

$$\begin{aligned}
\tilde{x}_1 = E\left[x_1|v\right] &= \frac{p_1^B - p_1^A + E\left[z|v\right] + t - 2e + 2e\left(\tilde{x}_1 + \tilde{x}_2\right)}{2t} \Leftrightarrow \\
&= \frac{p_1^B - p_1^A + E\left[z|v\right] + t - 2e + 2e\tilde{x}_2}{2\left(t - e\right)}, \qquad (2)
\end{aligned}$$

where $E\left[z|v\right]$ is the expectation of z by a first-period consumer who has observed realization $v\left(x_i, z\right)$. Because the expected value of z is not the same for all consumers, they can have different expectations of the demand for network A in the first period.

This expected demand results in a unique and stable equilibrium when t exceeds e. If instead $e > t$, this expected demand is based on an non-unique and unstable equilibrium, in which case there are two other stable equilibria where *all* consumers choose one of the two networks. The reason is that when the network effect dominates the degree-of-product-differentiation effect, consumers may prefer to coordinate on all buying the same good rather than splitting and choosing the good which they prefer. In the end, the equilibrium turns out to be similar to one in which there is no product differentiation at all. Since we want to analyze the case where product differentiation also drives the results, we assume that $t > e$ for now. However, once we take into account the interaction between periods, this restriction will be strengthened.[11]

In order to determine first-period demand, we also need to compute the second-period expected demand, $\tilde{x}_2$. For that, one must model second-period consumers' behavior as well as firms' optimal second-period pricing.

Second-period consumers and firms, having observed the decisions of first-period consumers, namely actual first-period quantity demanded x_1^*, correctly infer the value of z.[12] Therefore, they determine exactly second-period demand.

In order to choose a network, second-period consumers compare the net benefit of adopting each network. A consumer indifferent between the two networks is such that:

$$C + v\left(a\left(x_2\right), z\right) + e\left(x_1^* + x_2\right) - p_2^A = C + e\left(2 - \left(x_1^* + x_2\right)\right) - p_2^B,$$

which yields

$$x_2 = \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2\left(t - e\right)}, \tag{3}$$

where x_1^* is the observed market share of network A at the end of the first period.

First-period consumers do not know the realization of z, x_1^* and second-period prices. Thus, they cannot determine the actual second-period demand, but only the expected demand:

$$E\left[x_2|v\right] = \tilde{x}_2 = \frac{E\left[p_2^B|v\right] - E\left[p_2^A|v\right] + E\left[z|v\right] + t - 2e + 2e\tilde{x}_1}{2\left(t - e\right)}. \tag{4}$$

First-period consumers determine expected second-period prices while assuming that these are chosen by profit-maximizing firms. To calculate expected second-period prices, $E\left[p_2^A|v\right]$ and $E\left[p_2^B|v\right]$, we consider firm A's profit-maximization problem in the second period, while bearing in mind

[11] See Appendix A for details.
[12] Appendix B explains this inference process in detail.

that firms, too, have inferred the realization of z at the end of the first period:

$$\underset{p_2^A}{\text{Max}} \qquad p_2^A x_2 N = p_2^A \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2(t-e)} N.$$

The f.o.c. equals

$$\begin{aligned} &\frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2(t-e)} N - p_2^A \frac{N}{2(t-e)} = 0 \Leftrightarrow \\ &p_2^B + z + t - 2e + 2ex_1^* = 2p_2^A, \end{aligned}$$

whereas the s.o.c. equals $-\frac{N}{t-e}$ and thus is strictly negative due to the assumption that $t > e$.

By symmetry we have for firm B:

$$p_2^A - z + t - 2ex_1^* = 2p_2^B.$$

By solving the system of equations formed by these first-order conditions, we obtain the prices charged in the second period:

$$\left\{ \begin{array}{lcl} p_2^A &=& \frac{1}{3}z + t + \frac{2}{3}ex_1^* - \frac{4}{3}e \\ p_2^B &=& -\frac{1}{3}z + t - \frac{2}{3}e - \frac{2}{3}ex_1^*. \end{array} \right. \tag{5}$$

First-period consumers compute expected second-period prices:

$$\begin{array}{lcl} E\left[p_2^A|v\right] &=& \frac{1}{3}E[z|v] + t + \frac{2}{3}e\tilde{x}_1 - \frac{4}{3}e \\ E\left[p_2^B|v\right] &=& -\frac{1}{3}E[z|v] + t - \frac{2}{3}e - \frac{2}{3}e\tilde{x}_1. \end{array} \tag{6}$$

By replacing these in (4), we obtain

$$\tilde{x}_2 = \frac{t - \frac{4}{3}e + \frac{2}{3}e\tilde{x}_1 + \frac{1}{3}E[z|v]}{2(t-e)}. \tag{7}$$

We now have two equations, (2) and (7), which together determine $\tilde{x}_1$ and $\tilde{x}_2$ as a function of all known parameters, first-period prices and $E[z|v]$. By solving this system of equations, we obtain

$$\tilde{x}_1 = \frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + E[z|v]\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}, \tag{8}$$

and

$$\tilde{x}_2 = \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + E[z|v]\,t}{3t^2 - 6te + 2e^2}. \tag{9}$$

Appendix A makes it plain that only for $t > 1.577e$ do we have a unique and stable intermediate equilibrium without consumer bunching on a network. Thus, we tighten the previously-made assumption $t > e$ to this more stringent inequality.

We have finally computed $\tilde{x}_1$ and $\tilde{x}_2$ and are now ready to obtain first-period demand. By replacing $\tilde{x}_1$ and $\tilde{x}_2$ in (1), one obtains

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{E[z|v]\,e\,(2t-e)}{t\,(3t^2 - 6te + 2e^2)}. \qquad (10)$$

At this point, one must tackle the inference problem encapsulated in $E[z|v]$, i.e, to compute the expectation of z by a consumer who observed a given realization of $v(x_i, z)$. Given the assumptions made, the support of v is $[-t-w, t+w]$. We now postulate that there are always some consumers who value network A more than B while others have the opposite valuation ordering when firms charge the same price and all consumers expect both networks to attain the same final size. This amounts to assuming that, whatever the realization of z, variable v can assume positive *and* negative values depending on the value of $a(x_i)$. This is tantamount to imposing $t > w$.[13]

We show in Appendix C how, given v, first-period consumers form their expectation of z. Also, Appendix C makes it plain that first-period demand is estimated by first-period consumers as follows:

(i) For consumers who observe a realization of $v \in [t-w, t+w]$:

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+w-t)\,e\,(2t-e)}{2t\,(3t^2 - 6te + 2e^2)}.$$

(ii) For consumers who observe a realization of $v \in [-t+w, t-w]$:

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \qquad (11)$$

(iii) For consumers who observe a realization of $v \in [-t-w, -t+w]$:

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+t-w)\,e\,(2t-e)}{2t\,(3t^2 - 6te + 2e^2)}.$$

Finally, though the first-period demand curve is estimated differently by different consumers depending on their observed realization of $v(\cdot)$, Appendix C makes it plain that (11) is the relevant demand curve in a symmetric equilibrium such that $p_1^A = p_1^B$. This has a very intuitive explanation. Begin by viewing the first case above as representing consumers who are quite "optimistic" about network A, the intermediate case as comprising

[13]Thus ensuring that the equilibrium values of x_1 and x_2 lie on $[0,1]$, as will be made clear shortly.

the "middle grounders," and the last one the "pessimists." Appendix C shows that "middle grounders" always determine market demand.[14]

To determine optimal first-period prices, firms have to take into account their effect on second-period demand and prices. The lower is a firm's first-period price, the greater will be its quantity demanded, and thus, due to the network effect, the greater will be its second-period demand and associated optimal price. For this reason, we must determine second-period demand and optimal prices as a function of first-period prices only.

By replacing (11) in (5), we obtain

$$p_2^A = \frac{1}{3}z + t - e + \frac{1}{3}\frac{ez}{t} + \frac{e\,(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}, \tag{12}$$

and

$$p_2^B = -\frac{1}{3}z + t - e - \frac{1}{3}\frac{ez}{t} - \frac{e\,(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{13}$$

By replacing (11), (12) and (13) in (3), we obtain

$$x_2 = \frac{1}{2} + \frac{\frac{1}{3}z + \frac{1}{3}\frac{ez}{t}}{2\,(t-e)} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{14}$$

The profit maximization problem of firm A is[15]

$$\underset{p_1^A}{\text{Max}} \quad \Pi^A = E\left[x_1\left(p_1^A, p_1^B\right) N p_1^A\right] + E\left[x_2\left(p_1^A, p_1^B\right) N p_2^A\right].$$

p_1^A is not a random variable, but p_2^A is because its value depends on the realization of z. Therefore, we can write

$$\underset{p_1^A}{\text{Max}} \quad \Pi^A = E\left[x_1\left(p_1^A, p_1^B\right)\right] N p_1^A + E\left[x_2\left(p_1^A, p_1^B\right) p_2^A\right] N.$$

We can now easily compute a symmetric equilibrium.[16] By replacing (11), (12) and (14) in the objective function, we obtain

$$\begin{aligned}\Pi^A = {} & E\left[\frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}\right] N p_1^A + E\left[\left(\frac{1}{2} + \frac{\frac{1}{3}z + \frac{1}{3}\frac{ez}{t}}{2\,(t-e)} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}\right) \times \right. \\ & \left. \times \left(\frac{1}{3}z + t - e + \frac{1}{3}\frac{ez}{t} + \frac{e\,(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}\right)\right] N.\end{aligned}$$

[14] Interestingly enough, even though "middle grounders" always determine actual demand—i.e., indifferent consumers are necessarily "middle grounders"—they may be wrong in their estimate of z. To see this, consider the case where the realization of z is extreme, namely w, in which case "optimists" are nearer to correctly estimating market-wide preferences (see Appendix C for details).

[15] The absence of discounting of second-period profits is in keeping with the remarks made previously in fn. 7.

[16] Note that firms are symmetric at the beginning of the game.

The first-order condition equals

$$\frac{1}{2}N+\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}N-\frac{3}{2}\frac{t-e}{3t^2-6te+2e^2}Np_1^A-\frac{1}{2}\frac{e\,(t-e)}{3t^2-6te+2e^2}N-$$
$$-\frac{1}{2}\frac{e\,(t-e)}{3t^2-6te+2e^2}N+\frac{1}{2}\frac{2e^2\,(t-e)\left(p_1^A-p_1^B\right)}{\left(3t^2-6te+2e^2\right)^2}N=0.$$

In a symmetric equilibrium we have $p_1^A=p_1^B$. Therefore,

$$\frac{1}{2}-\frac{3}{2}\frac{t-e}{3t^2-6te+2e^2}p_1^A-\frac{e(t-e)}{3t^2-6te+2e^2}=0.$$

Some more manipulation finally yields

$$p_1^A=t-\frac{5}{3}e-\frac{1}{3}\frac{e^2}{t-e}=p_1^B. \tag{15}$$

Equilibrium first-period prices depend positively on the degree of product differentiation and negatively on the extent of the network effect. A decrease in price increases expected network size. Thus, the stronger is the network effect, the greater is the impact of a decrease in price on each period's demand and so the lower is the first-period price which firms want to charge.

The second derivative of the problem at hand equals $\frac{1}{2}(t-e)\frac{-18t^2+36te-11e^2}{\left(3t^2-6te+2e^2\right)^2}$. This second derivative is negative if $t<0.376e$ or $t>1.623e$. Since we have already seen that only for $t>1.577e$ do we have a unique and stable equilibrium without full bunching on a network, we must retain $t>1.623e$ as the relevant constraint.

4 Results

4.1 Evolution of market shares

4.1.1 Initially-unknown time-invariant market-wide preferences

We want to check whether or not in a market with network effects and initial uncertainty concerning the market-wide preferences of consumers, the firm which obtains the larger market share in the first period tends to increase it in the next period.

Demand in both periods, given by (11) and (14), in a symmetric equilibrium collapses to

$$\begin{aligned} x_1 &= \frac{1}{2}+\frac{1}{2}\frac{z}{t} \\ x_2 &= \frac{1}{2}+\frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2\,(t-e)}. \end{aligned}$$

The first-period demand does not depend on the network effect because the expected size of both networks is the same.

Simple computations show that a firm increases its market share in the second period, $x_2 > x_1$, iff $t < 2e$. Recalling that we restrict our analysis to $t > 1.623e$, we conclude that the firm that obtains the larger market share in the first period will increase it in the second period iff $t \in (1.623e, 2e)$. In other words, market dominance is reinforced if and only if the network effect is strong enough *vis-à-vis* the degree of product differentiation.

The reason is that in the first period and when choosing its price, the preferred firm does not know that it has the greater demand since z is still unknown. So, it will charge a price lower than would have been optimal. In the second period, once firms infer the realization of z, the firm which realizes it has the higher demand will price accordingly.[17] This mispricing correction tends to reduce the firm's market share. Its impact on second-period equilibrium quantity demanded is proportional to the degree of product differentiation, t. To see it, note that if t is rather close to zero, the goods are very close substitutes, implying that any attempt by the preferred firm to significantly increase price would result in the loss of many sales.

Concurrently, consumers realize which firm is preferred by the majority of consumers, i.e., they learn the realization of z. This tends to increase that firm's second-period market share due to the network effect and, by the same token, reduce its opponent's. If the network effect is strong enough, the second effect dominates the first and the lead obtained by one firm in the first period is reinforced in the second.

4.1.2 Unknown time-variable market-wide preferences

In Appendix D, we modify our model by replacing variable z by variables z_1 and z_2 in the first and second periods, respectively. We further assume that z_1 and z_2 are independent. This allows us to consider the case where the market-wide advantage initially enjoyed by one firm may be non-permanent. Interestingly, if one firm happens to enjoy the same advantage in both periods, i.e. $z_1 = z_2 = z$, then this firm's market share *always* increases. In this case, neither the mispricing correction described previously, nor the network effect associated with second-period consumers

[17] Of course, the same applies *mutatis mutandis* to the other firm after it realizes that it is less favored by consumers.

learning the true value of z takes place. Thus, firms compete for second-period consumers *exactly* as they did for first-period ones, except that the initially-preferred firm now starts with the advantage of a larger installed base. Thus, it obtains an even larger percentage of second-period consumers than it did of first-period ones.

4.1.3 Known time-invariant market-wide preferences

In Appendix E, we develop another variant of our model where the realization of z is common knowledge, and thus immediately observable by first-period consumers. This accounts for the possibility that market-wide preferences may be apparent from the outset, due, for example, to technical reviews of the new products appearing in the press. We use it to show that z not being initially observable increases the range of circumstances under which the firm that gains a larger installed base gets to increase its market share subsequently.

In fact, when z is common knowledge from the outset, the interval of parameters for which the firm that obtains the greater market share in the first period increases it in the following period is reduced from $t < 2e$ to $t < 1.694e$. In this case, rational expectations ensure that the final size of each network is known in advance by all consumers, and is thus the same for first- and second-period consumers. Therefore, there is no reason for the firm that obtains the greater market share in the first period to increase it in the final period due to the network effect. The reason why we still have a positive trend in market share is firms' strategic pricing over time. To see this, suppose that we also impose that prices should be time invariant. Then, prices, as well as expected network size, are the same in both periods, and so consumers will split between networks in the same manner in both periods. Therefore, each firm will have the same market share in both periods. In this case and despite the network effect, the firm that obtains the larger market share in the first period will not increase it in the following period.

4.2 Strategic Prices

4.2.1 Penetration pricing

In a market with network effects, firms may want to act strategically by charging different prices in different periods even when costs and the demand they face are the same. To examine this issue, we compare the prices charged in each period. From (12), the second-period price charged by firm A in a symmetric equilibrium equals

$$p_2^A = t - e + \frac{1}{3}z + \frac{1}{3}\frac{ez}{t},$$

and thus, the average price charged in the second period equals

$$E\left[p_2^A\right] = t - e.$$

From (15), firm A's first-period price equals

$$p_1^A = t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t - e}.$$

The first-period price is smaller than the second-period average price.

4.2.2 Under-cost pricing

It is easy to show that under-cost pricing may occur. Simple computations show that $p_1^A = t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e} > 0$ iff $t > 2e$. Recall that cost is nil. Thus, if the degree of product differentiation is significant *vis-à-vis* the network effect's strength, firms will optimally price above cost in order to exploit "their" idiosyncratic clients at the cost of a reduced first-period installed base. Otherwise, if the network effect's strength is significant *vis-à-vis* the degree of product differentiation, firms will follow an under-cost pricing strategy.

4.3 First-mover Advantage

We now determine whether in a model where market-wide preferences may change over time, there is a first- or last-mover advantage in market-wide preference. To do so, in Appendix D we consider a particular realization profile of z_1 and z_2 such that, in the first period, firm A is the preferred one ($z_1 = K > 0$) while in the second period the symmetric situation occurs ($z_2 = -K < 0$). We conclude that if there are network effects—i.e., $e > 0$—then the firm which is preferred by consumers in the first period always obtains a larger total market share at the end of the two periods than the firm preferred in the second period. Therefore, in this model there is always a "first-mover advantage," in contrast to the "last-mover advantage" instances found in Katz and Shapiro (1986).

The reason for this is that in a market such as ours, in which individuals cannot anticipate the future evolution of the network, a firm that obtains a large market share in the first period benefits from the network effect in the following period because agents observe that advantage. On the other hand, the firm that gains advantage in the second period does not benefit from the network effect in the first period because that advantage was not anticipated by consumers.

5 Conclusion

We developed a two-period, differentiated-goods model of a market with network effects and consumer and firm uncertainty concerning consumers' overall valuation of the goods. We show that the firm that obtains the larger market share in the first period increases its market share in the last period if and only if the network effect is significant enough compared to the degree of product differentiation as long as market-wide preferences are time invariant. Strikingly, if market-wide preferences can vary over time, then the firm with a larger installed base will always reinforce its lead if it keeps enjoying the same market-wide preference.

The idea that, in a market with network effects, the firm that obtains a larger market share in the initial period tends to subsequently increase it is expressed by Varian and Shapiro (1999, pp. 174): "The new information economy is driven by the economics of networks (...) positive feedback makes the strong get stronger and the weak grow weaker." We qualify this observation in two respects. Most of the literature assumes homogeneous consumers and no uncertainty concerning market-wide preferences. Thus, the firm that wins in the first period immediately obtains 100% market share. There are no gradual dynamics in which success begets success and failure breeds failure, a feature that our model shows.

More importantly, we show that this is not always the case, depending on the relative strength of the network effect *vis-à-vis* product differentiation, as well as whether market-wide advantages are irreversible or fleeting. Also, we show that uncertainty over market-wide preferences increases the set of circumstances under which leaders amplify their market-share advantage.

Moreover, we show that in this context, firms do engage in penetration pricing and may engage in under-cost pricing. Finally, we find that the model shows a first-mover advantage concerning the market-wide preference term, z. This provides a rationale for allocating advertising budgets preferentially to pre-introduction promotion activities rather than to post-introduction advertising, insofar as these are mechanisms affecting, and possibly reversing, market-wide preferences.

Appendix A

In this appendix we show that a unique and stable equilibrium without bunching of all consumers on a network exists if and only if $t > 1.577e$, i.e., iff the degree of product differentiation is large enough compared to the intensity of the network effect.

For expositional clarity, we begin by showing that in a model with only one period, a unique and stable equilibrium without full bunching exists if and only if $t > e$.[18] The result for the two-period model in the main text then follows easily by analogy.

In a one-period model, the indifferent consumer is given by

$$C - tx_1 + z + e\tilde{x}_1 - p^A = C - t\,(1 - x_1) + e\,(1 - \tilde{x}_1) - p^B,$$

from which we obtain the following demand function

$$x_1 = \frac{p^B - p^A + z + t - e}{2t} + \frac{e}{t}\tilde{x}_1. \qquad \text{(A.1)}$$

A consumer's estimate of x_1 is then given by:

$$\begin{aligned} \tilde{x}_1 &= \frac{p^B - p^A + E\,[z|v] + t - e}{2t} + \frac{e}{t}\tilde{x}_1 \\ &= \frac{1}{2} + \frac{p^B - p^A + E\,[z|v]}{2\,(t - e)}. \qquad \text{(A.2)} \end{aligned}$$

If $t < e$, the intermediate expectation of x_1 given by equation (A.2), namely $0 < \tilde{x}_1 < 1$, is not the only one possible. Two other extreme expectations concerning x_1, namely $\tilde{x}_1 = 0$ and $\tilde{x}_1 = 1$, can consistently be entertained by consumers as part of an equilibrium. This is so because $t < e$ implies that all consumers—including those located at the far-off end of the horizontal-differentiation line—attach a higher value to belonging to the network to which all other consumers belong than to staying with their preferred network. In this case, equilibria involving complete bunching may occur.

Moreover, the intermediate equilibrium is unstable when $t < e$. If consumers hold an expectation slightly different from that given by (A.2), they will all migrate towards one network. Equation (A.1) makes this clear if one notes that $t < e \Rightarrow \frac{e}{t} > 1$—the latter being the coefficient affecting $\tilde{x}_1$ on the r.h.s. of (A.1)—implies $\frac{\partial x_1}{\partial \tilde{x}_1} > 1$.

[18] This is also the relevant interval in a model with two periods in which first-period consumers do not take into account the impact of their decisions on second-period consumers.

The extreme cases—in which all consumers are driven by the network effect to coordinate on consuming the same good—are tantamount to having no product differentiation at all.

We now consider the two-period model treated in the main text. Here, first-period consumers know the impact of their decisions on their second-period counterparts. The condition for a unique and stable intermediate equilibrium is now more demanding since an increase in the expected value of x_1 leads to an increase in the expected value of x_2 due to the network effect. This, in turn, leads to an increase of the expected value of x_1. Thus, the incentives for all consumers to choose the same network are stronger, and so the condition for a unique and stable intermediate equilibrium is more demanding.

The first-period indifferent consumer is determined by

$$C - tx_1 + z + e\left(\tilde{x}_1 + \tilde{x}_2\right) - p_1^A = C - t\left(1 - x_1\right) + e\left(2 - \left(\tilde{x}_1 + \tilde{x}_2\right)\right) - p_1^B,$$

from which we obtain

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e\left(\tilde{x}_1 + \tilde{x}_2\right)}{2t},$$

and finally

$$\tilde{x}_1 = \frac{p_1^B - p_1^A + E\left[z|v\right] + t - 2e + 2e\tilde{x}_2}{2\left(t - e\right)}. \tag{A.3}$$

From (7) in the main text, we have

$$\tilde{x}_2 = \frac{t - \frac{4}{3}e + \frac{2}{3}e\tilde{x}_1 + \frac{1}{3}E\left[z|v\right]}{2\left(t - e\right)}.$$

After replacing $\tilde{x}_2$ in (A.3), we obtain

$$\tilde{x}_1 \quad = \quad \frac{p_1^B - p_1^A + E\left[z|v\right] + t - 2e + 2e\frac{t - \frac{4}{3}e + \frac{1}{3}E[z|v]}{2(t-e)}}{2\left(t - e\right)} + \frac{\frac{4}{3}e^2}{4\left(t - e\right)^2}\tilde{x}_1.$$

Now, the intermediate equilibrium is unique and stable iff the coefficient affecting $\tilde{x}_1$ on the r.h.s. of the previous equality is less than 1, i.e., $\frac{\frac{4}{3}e^2}{4(t-e)^2} < 1$. This is the case iff $t < 0.423e$ or $t > 1.577e$.

Appendix B

In this appendix we show that second-period consumers and firms deduce the realization of z upon observing x_1, provided a condition on accuracy of beliefs is met.

A first-period indifferent consumer is such that

$$C + a(x_1) + z + e(\tilde{x}_1 + \tilde{x}_2) - p_1^A = C + e(2 - (\tilde{x}_1 + \tilde{x}_2)) - p_1^B.$$

Thus, first-period demand equals

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e(\tilde{x}_1 + \tilde{x}_2)}{2t}. \tag{B.1}$$

The estimate of x_1 by first-period consumers equals

$$E[x_1|1,v] \equiv \tilde{x}_1 = \frac{p_1^B - p_1^A + E[z|1,v] + t - 2e + 2e(\tilde{x}_1 + \tilde{x}_2)}{2t},$$

where $E[x|1,v]$ denotes the estimate of random variable x by a first-period consumer who has observed realization $v\,(x_i, z)$. Thus,

$$\tilde{x}_1 = \frac{p_1^B - p_1^A + E[z|1,v] + t - 2e + 2e\tilde{x}_2}{2(t-e)}. \tag{B.2}$$

A second-period indifferent consumer is such that

$$C + a(x_2) + z + e(x_1^* + E[x_2|2,v]) - p_2^A = C + e(2 - (x_1^* + E[x_2|2,v])) - p_2^B,$$

where $E[x|2,v]$ denotes the estimate of random variable x by a second-period consumer who has observed realization $v(\cdot)$. Thus, the second-period demand curve equals

$$x_2 = \frac{p_2^B - p_2^A + z + 2eE[x_2|2,v] + t - 2e + 2ex_1^*}{2t}, \tag{B.3}$$

which yields

$$E[x_2|2,v] = \frac{p_2^B - p_2^A + E[z|2,v] + t - 2e + 2ex_1^*}{2(t-e)}. \tag{B.4}$$

Substituting (B.4) in (B.3), we obtain

$$x_2 = \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2(t-e)} + \frac{eE[z|2,v] - ez}{2t(t-e)}.$$

We need to compute the expected value of x_2 by first-period consumers:

$$\begin{aligned} \tilde{x}_2 = E[x_2|1,v] \quad &= \quad \frac{E\left[p_2^B|1,v\right] - E\left[p_2^A|1,v\right] + E[z|1,v] + t - 2e + 2e\tilde{x}_1}{2(t-e)} \\ &\quad + \frac{eE[E[z|2,v]|1,v] - eE[z|1,v]}{2t(t-e)}, \end{aligned} \tag{B.5}$$

where, from (6) in the main text, we have

$$E\left[p_2^A|1,v\right] = \frac{1}{3}E\left[z|1,v\right]+t+\frac{2}{3}e\tilde{x}_1-\frac{4}{3}e \tag{B.6}$$

$$E\left[p_2^B|1,v\right] = -\frac{1}{3}E\left[z|1,v\right]+t-\frac{2}{3}e-\frac{2}{3}e\tilde{x}_1. \tag{B.7}$$

By solving the equation system formed by equations (B.2), (B.5), (B.6) and (B.7), we conclude that

$$\tilde{x}_1\left(E\left[z|1,v\right],E\left[E\left[z|2,v\right]|1,v\right]\right)$$

and

$$\tilde{x}_2\left(E\left[z|1,v\right],E\left[E\left[z|2,v\right]|1,v\right]\right).$$

By replacing these expressions in (B.1), we obtain

$$\begin{aligned} x_1 &= \frac{p_1^B-p_1^A+z+t-2e+2e\tilde{x}_1\left(E\left[z|1,v\right],E\left[E\left[z|2,v\right]|1,v\right]\right)}{2t} \\ &\quad +\frac{2e\tilde{x}_2\left(E\left[z|1,v\right],E\left[E\left[z|2,v\right]|1,v\right]\right)}{2t}. \end{aligned}$$

Appendix C shows that $E\left[z|1,v\right]=E\left[z\right]=0$ for a first-period indifferent consumer in a symmetric equilibrium, a fact known to second-period consumers. Thus, we have

$$x_1=\frac{p_1^B-p_1^A+z+t-2e+2e\left\{\tilde{x}_1\left(E\left[E\left[z|2,v\right]|1\right]\right)+\tilde{x}_2\left(E\left[E\left[z|2,v\right]|1\right]\right)\right\}}{2t},$$

where we have simplified the notation by writing $E\left[E\left[z|2,v\right]|1,v\right]$ simply as $E\left[E\left[z|2,v\right]|1\right]$ for a first-period indifferent consumer who acts based on the posterior $E\left[z|1,v\right]=E\left[z\right]=0$.

The only variables determining x_1 whose values are unknown are z and $E\left[E\left[z|2,v\right]|1\right]$. We can thus write:

$$z\left(x_1,E\left[E\left[z|2,v\right]|1\right]\right). \tag{B.8}$$

In order to estimate the value of z after observing x_1^*, second-period consumers first need to estimate the value of $E\left[E\left[z|2,v\right]|1\right]$. In fact,

$$E\left[z|2,v\right]=E\left[z\left(x_1^*,E\left[E\left[z|2,v\right]|1\right]\right)|2,v\right]$$

becomes

$$E\left[z|2,v\right]=z\left(x_1^*,E\left[E\left[E\left[z|2,v\right]|1\right]|2,v\right]\right), \tag{B.9}$$

since x_1^* is observable by second-period consumers, they understand how $z(x_1, E[E[z|2,v]|1])$ is formed, and z is monotone in $E[E[z|2,v]|1,v]$, as inspection of (B.5) immediately shows.

We rule out equilibria based on $E[E[E[z|2,v]|1]|2,v] \neq E[E[z|2,v]|1]$. In plain words, we rule out equilibria based on first-period indifferent consumers forming beliefs on how second-period consumers estimate z which second-period consumers misunderstand even though they possess all the information which first-period indifferent consumers had at the time they formed their beliefs. Then, from (B.8) and (B.9), we have $E[z|2,v] = z$, i.e., second-period consumers deduce the true value of z after observing x_1^*. The same argument holds for firms in the second period.

Appendix C

Determination of $E[z|v]$

From

$$v = a + z$$
$$z \rightsquigarrow U(-w, w)$$
$$a \rightsquigarrow U(-t, t),$$

we have that v is itself a random variable with support $[-t - w, t + w]$. Moreover, it was also assumed that $t > w$.

Divide the support of v in three intervals.

(i) Intermediate values: $v \in [-t + w, t - w]$.

When $v \in [-t + w, t - w]$, for a given value of v, variable z can assume all values in the interval $[-w, w]$. Also, for a given value of v, to each value of z corresponds a unique value of a.[19] Since a and z are both uniformly distributed random variables, we conclude that for each value of v, variable z can assume all values in its support with the same probability. Therefore, the density function of z, given the realization of v, is

$$f[z|v] = \frac{1}{w - (-w)}, \qquad -w \leq z \leq w.$$

Thus, the posterior density function of z once a given value of v (x_i, z) has been observed, equals the prior density function of z:

$$E[z|v] = E[z] = 0.$$

For intermediate values of v, consumers cannot infer anything new about the expected value of z by observing their own relative valuation of the two networks as given by v.

In the extreme cases—high or low values of v—consumers can infer something about the expected value of z by observing their own relative valuation of the two networks. For instance, if a consumer observes a high value of v, it infers that this value cannot be associated with a low value of z and so the posterior expected value of z exceeds zero.

[19] To see this, consider the following example. If $v = 0$, then $z = w \Rightarrow a = -w$, and $z = 0 \Rightarrow a = 0$, and $z = -w \Rightarrow a = w$.

(ii) High values: $v \in [t - w, t + w]$.

If $v \in [t - w, t + w]$, then variable z cannot assume all values in $[-w, w]$. In particular, z cannot assume values towards the low end of its support, its posterior expected value no longer being zero but exceeding it, instead. For a given value of $v \in [t - w, t + w]$, z can assume values in the interval $[v - t, w]$. Thus, the density function of variable z, given the realization of v, is

$$f[z|v] = \frac{1}{w - (v - t)}, \qquad v - t \leq z \leq w.$$

Therefore, the posterior expected value of z equals

$$E[z|v] = \frac{w + (v - t)}{2}.$$

Therefore, $E[z|v]$ can assume values between 0 (when $v = t - w$) and w (when $v = t + w$).

(iii) Low values: $v \in [-t - w, -t + w]$.

Similar computations yield

$$f[z|v] = \frac{1}{v + t - (-w)}, \qquad -w \leq z \leq v + t,$$

and

$$E[z|v] = \frac{v + t + (-w)}{2}.$$

Therefore, $E[z|v]$ can assume values between $-w$ (for $v = -t - w$) and 0 (for $v = -t + w$).

First-period demand curve as a function of $E[z|v]$

For intermediate values of v, i.e., $v \in [-t + w, t - w]$, we have $E[z|v] = 0$. Then, (10) collapses to

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}.$$

For high values of v, i.e., $v \in [t - w, t + w]$, we have $E[z|v] = \frac{w+(v-t)}{2}$ which, replaced in (10), yields

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v + w - t)\, e\, (2t - e)}{2t\,(3t^2 - 6te + 2e^2)}.$$

For low values of v, i.e., $v \in [-t-w, -t+w]$, we have $E[z|v] = \frac{v+t+(-w)}{2}$ which, replaced in (10), yields

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+t-w)\,e\,(2t-e)}{2t\,(3t^2-6te+2e^2)}.$$

First-period demand curve

We now show that a first-period indifferent consumer has $E[z|v] = 0$ and thus $x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2-6te+2e^2}$ is the first-period demand function.

Take any realization of z, say, $\overline{z}$. By definition, $v = z + a$, $a \in [-t, t]$ and $z \in [-w, w]$. This, together with the assumption $t > w$, implies that $\exists \overline{x}_1, 0 < \overline{x}_1 < 1 : \overline{z} + a(\overline{x}_1) = 0$. Thus, for such a consumer located at $\overline{x}_1$, we have $v = 0 \in [-t+w, t-w]$. From the first subsection of this appendix, this implies $E[z|v] = E[z] = 0$.

Moreover, from (8) and (9), we have

$$\tilde{x}_1 = \frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + E[z|v]\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}$$
$$\tilde{x}_2 = \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + E[z|v]\,t}{3t^2 - 6te + 2e^2},$$

which, for a symmetric equilibrium and a consumer such that $E[z|v] = 0$, implies $\tilde{x}_1 = \tilde{x}_2 = \frac{1}{2}$. Thus, such a consumer fulfills the equality $C + v(a(\overline{x}_1), \overline{z}) + e(\tilde{x}_1 + \tilde{x}_2) - p_1^A = C + e(2 - (\tilde{x}_1 + \tilde{x}_2)) - p_1^B$. Consumers slightly to the right of $\overline{x}_1$ such that $x_1 > \overline{x}_1$ while $v \in [-t+w, t-w]$ strictly prefer network B because $v < 0$ and $\tilde{x}_1 = \tilde{x}_2 = \frac{1}{2}$. Consumers further to the right such that $x_1 > \overline{x}_1$ and $v \in [-t-w, -t+w]$ strictly prefer network B because $v < 0$ and $\tilde{x}_1 = \tilde{x}_2 < \frac{1}{2}$. A similar argument establishes that consumers to the left of $\overline{x}_1$ strictly prefer network A.

Appendix D

In this appendix we analyze a model similar in almost all respects to the one in the main text while assuming that the relative valuation of the two networks, z, can change over time. We thus now define variables $v_j(\cdot)$ as the sum of two random variables: $a(\cdot)$ and $z_j, j = 1, 2$. We assume that z_1 and z_2 are independent so that nothing can be inferred about z_2 after agents infer the realization of z_1. Summarizing,

$$\begin{aligned} v_j\left(x_i, z_j\right) &= a\left(x_i\right) + z_j \\ a\left(x_i\right) &= t - 2tx_i \\ x_i &\rightsquigarrow U\left(0, 1\right) \Rightarrow a \rightsquigarrow U\left(-t, t\right) \\ z_j &\rightsquigarrow U\left(-w, w\right) \qquad j = 1, 2. \end{aligned}$$

The first-period demand is similar to the one obtained in the main text:

$$x_1 = \frac{p_1^B - p_1^A + z_1 + t - 2e + 2e\left(\tilde{x}_1 + \tilde{x}_2\right)}{2t}. \tag{D.1}$$

The expected demand is obtained as in the main text:

$$\tilde{x}_1 = \frac{p_1^B - p_1^A + E\left[z_1|v_1\right] + t - 2e + 2e\tilde{x}_2}{2\left(t - e\right)}. \tag{D.2}$$

The second-period demand function is determined as in the main text, except that now the realization of z_2 is unknown at the beginning of the second period:

$$x_2 = \frac{p_2^B - p_2^A + z_2 + t - 2e + 2ex_1^* + 2eE\left[x_2|v_2\right]}{2t}. \tag{D.3}$$

The second-period demand expected by a second-period consumer with valuation v_2 equals

$$E\left[x_2|v_2\right] = \frac{p_2^B - p_2^A + E\left[z_2|v_2\right] + t - 2e + 2ex_1^* + 2eE\left[x_2|v_2\right]}{2t}.$$

Thus,

$$E\left[x_2|v_2\right] = \frac{p_2^B - p_2^A + E\left[z_2|v_2\right] + t - 2e + 2ex_1^*}{2\left(t - e\right)}. \tag{D.4}$$

From (D.3), the second-period demand expected by a first-period consumer with valuation v_1 is

$$\begin{aligned} \tilde{x}_2 = E\left[x_2|v_1\right] &= \frac{E\left[p_2^B|v_1\right] - E\left[p_2^A|v_1\right] + E\left[z_2|v_1\right]}{2t} + \\ &\quad + \frac{t - 2e + 2e\tilde{x}_1 + 2eE\left[E\left[x_2|v_2\right]|v_1\right]}{2t}, \end{aligned}$$

which simplifies to

$$\tilde{x}_2 = E[x_2|v_1] = \frac{E\left[p_2^B|v_1\right] - E\left[p_2^A|v_1\right] + t - 2e + 2e\tilde{x}_1}{2t} + \\ + \frac{2eE[E[x_2|v_2]|v_1]}{2t}, \tag{D.5}$$

because $E[z_2|v_1] = 0$ since z_1 and z_2 are independent. From (D.4), we have

$$E[E[x_2|v_2]|v_1] = \frac{E\left[p_2^B|v_1\right] - E\left[p_2^A|v_1\right] + E[E[z_2|v_2]|v_1] + t - 2e + 2e\tilde{x}_1}{2(t-e)}.$$

One has $E[E[z_2|v_2]|v_1] = E[z_2|v_2]$ since z_1 and z_2 are independent. Moreover, as shown in Appendix C for first-period indifferent consumers, second-period indifferent consumers are such that $E[z_2|v_2] = E[z_2] = 0$. Thus, we have

$$E[E[x_2|v_2]|v_1] = \frac{E\left[p_2^B|v_1\right] - E\left[p_2^A|v_1\right] + t - 2e + 2e\tilde{x}_1}{2(t-e)},$$

which, replaced in (D.5), yields

$$\tilde{x}_2 = \frac{E\left[p_2^B|v_1\right] - E\left[p_2^A|v_1\right] + t - 2e + 2e\tilde{x}_1}{2(t-e)}. \tag{D.6}$$

Second-period firms do not know the realization of z_2 and act on the basis of its expected value, namely 0. Moreover, as said above, firms know that second-period demand is determined by consumers such that $E[z_2|v_2] = E[z_2] = 0$, i.e., by consumers whose posterior expectation of z_2 equals the prior and thus coincides with firms' expectation of this variable. Thus, by replacing (D.4) in (D.3) while bearing in mind that $E[z_2|v_2] = E[z_2] = 0$, we obtain the expected second-period demand faced by firm A, which we denote by $E[x_2]$:

$$E[x_2] = \frac{p_2^B - p_2^A + t - 2e + 2ex_1^*}{2(t-e)}. \tag{D.7}$$

The profit maximization problem of firm A in the second period is

$$\underset{p_2^A}{\text{Max}} \quad E\left[p_2^A x_2 N\right].$$

Since p_2^A is not a random variable, we can write

$$\underset{p_2^A}{\text{Max}} \quad p_2^A E[x_2] N = p_2^A \frac{p_2^B - p_2^A + t - 2e + 2ex_1^*}{2(t-e)} N.$$

The f.o.c. equals

$$\frac{p_2^B - p_2^A + t - 2e + 2ex_1^*}{2(t-e)} N - p_2^A \frac{1}{2(t-e)} N = 0 \Leftrightarrow$$
$$p_2^B + t - 2e + 2ex_1^* = 2p_2^A.$$

The s.o.c. equals

$$-\frac{1}{t-e} < 0.$$

By symmetry, we have for firm B

$$p_2^A + t - 2ex_1^* = 2p_2^B.$$

We can now solve the system of equations encompassing the first-order conditions, obtaining

$$\begin{cases} p_2^A &= t + \frac{2}{3}ex_1^* - \frac{4}{3}e \\ p_2^B &= t - \frac{2}{3}e - \frac{2}{3}ex_1^*. \end{cases} \tag{D.8}$$

First-period consumers must determine the expected value of these prices:

$$\begin{aligned} E\left[p_2^A|v_1\right] &= t + \frac{2}{3}e\tilde{x}_1 - \frac{4}{3}e \\ E\left[p_2^B|v_1\right] &= t - \frac{2}{3}e - \frac{2}{3}e\tilde{x}_1. \end{aligned}$$

By replacing them in (D.6), we obtain

$$\tilde{x}_2 = \frac{t - \frac{4}{3}e + \frac{2}{3}e\tilde{x}_1}{2(t-e)}. \tag{D.9}$$

By substituting (D.9) in (D.2), we obtain

$$\tilde{x}_1 = \frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + (t-e)E[z_1|v_1]}{3t^2 - 6te + 2e^2}. \tag{D.10}$$

By replacing (D.10) in (D.9), we obtain

$$\tilde{x}_2 = \frac{1}{2} + \frac{1}{2}\frac{eE[z_1|v_1] + e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{D.11}$$

By replacing (D.10) and (D.11) in (D.1), we obtain

$$x_1 = \frac{1}{2} + \frac{z_1}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{1}{2}\frac{e(3t-2e)E[z_1|v_1]}{t(3t^2 - 6te + 2e^2)}.$$

As shown in Appendix C, in a symmetric equilibrium, the indifferent consumers are such that $E[z_1|v_1] = E[z_1] = 0$. So, the previous expression collapses to

$$x_1 = \frac{1}{2} + \frac{z_1}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{D.12}$$

By replacing (D.12) in (D.8), we obtain

$$p_2^A = t - e + \frac{1}{3}\frac{ez_1}{t} + \frac{e(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}, \tag{D.13}$$

and

$$p_2^B = t - e - \frac{1}{3}\frac{ez_1}{t} - \frac{e(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{D.14}$$

By replacing (D.12), (D.13) and (D.14) in (D.4), we obtain

$$E[x_2|v_2] = \frac{1}{2} + \frac{\frac{1}{3}\frac{ez_1}{t} + E[z_2|v_2] + \frac{e(t-e)\left(p_1^B - p_1^A\right)}{3t^2-6te+2e^2}}{2(t-e)}. \tag{D.15}$$

By replacing (D.12), (D.13), (D.14) and (D.15) in (D.3), we obtain

$$x_2 = \frac{1}{2} + \frac{z_1 e}{6t(t-e)} + \frac{z_2}{2t} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{1}{2t}\frac{E[z_2|v_2]\,e}{t-e}.$$

Again, as Appendix C shows, in a symmetric equilibrium, demand is such that $E[z_2|v_2] = E[z_2] = 0$. Thus, the previous expression collapses to

$$x_2 = \frac{1}{2} + \frac{z_1 e}{6t(t-e)} + \frac{z_2}{2t} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{D.16}$$

The second-period demand depends on the random variable z_2 as well as the random variable z_1, due to the network effect.

The profit maximization problem of firm A is

$$\underset{p_1^A}{\text{Max}} \quad E\left[x_1\left(p_1^A, p_1^B\right)p_1^A + x_2\left(p_1^A, p_1^B\right)p_2^A\right]N,$$

or

$$\underset{p_1^A}{\text{Max}} \quad E\left[x_1\left(p_1^A, p_1^B\right)\right]Np_1^A + E\left[x_2\left(p_1^A, p_1^B\right)p_2^A\right]N.$$

Replacing (D.12), (D.13) and (D.16) in the profit maximization problem, we obtain

$$
\begin{aligned}
\operatorname*{Max}_{p_1^A} \quad & E\left[\frac{1}{2}+\frac{z_1}{2t}+\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right]Np_1^A+ \\
& +E\left[\left(\frac{1}{2}+\frac{z_1e}{6t(t-e)}+\frac{z_2}{2t}+\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right)\times\right. \\
& \left.\times\left(t-e+\frac{1}{3}\frac{ez_1}{t}+\frac{e(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right)\right]N= \\
= \quad & \left[\frac{1}{2}+\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right]Np_1^A+E\left[\frac{1}{2}\left(t-e+\frac{1}{3}\frac{ez_1}{t}\right)+\right. \\
& +\frac{1}{2}\frac{e(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}+\left(\frac{z_1e}{6t(t-e)}+\frac{z_2}{2t}\right)\left(t-e+\frac{1}{3}\frac{ez_1}{t}\right)+ \\
& +\left(\frac{z_1e}{6t(t-e)}+\frac{z_2}{2t}\right)\frac{e(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}+ \\
& +\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\left(t-e+\frac{1}{3}\frac{ez_1}{t}\right) \\
& \left.+\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\frac{e(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right)\right]N.
\end{aligned}
$$

The f.o.c. equals

$$
\begin{aligned}
& \frac{1}{2}N+\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}N-\frac{3}{2}\frac{(t-e)}{3t^2-6te+2e^2}p_1^AN-\frac{1}{2}\frac{e(t-e)}{3t^2-6te+2e^2}N- \\
& -\frac{1}{2}\frac{e}{3t^2-6te+2e^2}(t-e)N-\frac{1}{2}\frac{e^2(t-e)2\left(p_1^B-p_1^A\right)}{\left(3t^2-6te+2e^2\right)^2}N=0.
\end{aligned}
$$

By symmetry, we have

$$
\frac{1}{2}-\frac{3}{2}\frac{t-e}{3t^2-6te+2e^2}p_1^A-\frac{e(t-e)}{3t^2-6te+2e^2}=0,
$$

or

$$
p_1^A=t-\frac{5}{3}e-\frac{1}{3}\frac{e^2}{t-e}.
$$

Thus, equilibrium first-period prices are the same as in the main text. As to the s.o.c., we have

$$
(t-e)\frac{-3\left(3t^2-6te+2e^2\right)+e^2}{\left(3t^2-6te+2e^2\right)^2},
$$

which is negative if $t>\frac{5}{3}e$, a restriction we now retain.

From (D.12) and (D.16), in a symmetric equilibrium, demand equals

$$
\begin{aligned}
x_1 &= \frac{1}{2}+\frac{z_1}{2t} \\
x_2 &= \frac{1}{2}+\frac{z_1e}{6t(t-e)}+\frac{z_2}{2t}.
\end{aligned}
$$

If $z_1 = z_2 > 0$, and since $t > e$, than $x_1 < x_2$. Thus, if the average relative valuation of the two networks is the same in both periods, i.e., if $z_1 = z_2$, the firm with the larger market share in the first period will always increase it in the following period.

We now consider another particular realization of the common terms such that in the first period, firm A is the preferred one, i.e. $z_1 = K > 0$, whereas in the second period the symmetric case occurs, $z_2 = -K$. In the first period, firms charge the same price, and so the market share of A equals

$$x_1 = \frac{1}{2} + \frac{1}{2}\frac{K}{t}.$$

In the second-period, $z_2 = -K < 0$, and A's market share equals

$$x_2 = \frac{1}{2} + \frac{Ke}{6t\,(t-e)} - \frac{1}{2}\frac{K}{t}.$$

When considering the two periods jointly, A always obtains a larger market share iff $e > 0$:

$$x_1 + x_2 > 1 \Leftrightarrow \frac{1}{2} + \frac{1}{2}\frac{K}{t} + \frac{1}{2} + \frac{Ke}{6t\,(t-e)} - \frac{1}{2}\frac{K}{t} > 1 \Leftrightarrow \frac{Ke}{6t\,(t-e)} > 0.$$

Thus, there is always a first-mover advantage in market-wide preferences whenever network effects are felt.

Appendix E

In this appendix we develop a model similar to the one in the main text but for the fact that random variable z is no longer unknown in the first period.

The first-period demand function is determined as in the main text. The only difference is that now the exact value of z is common knowledge:

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e\left(\tilde{x}_1 + \tilde{x}_2\right)}{2t}.$$

The expected value of x_1 is now equal to its actual value, i.e., $x_1 = \tilde{x}_1$:

$$\begin{aligned} x_1 &= \frac{p_1^B - p_1^A + z + t - 2e + 2e\left(x_1 + \tilde{x}_2\right)}{2t} \\ &= \frac{p_1^B - p_1^A + z + t - 2e + 2e\tilde{x}_2}{2\left(t - e\right)}. \end{aligned} \tag{E.1}$$

The second-period demand function and prices are determined as in the main text:

$$x_2 = \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1}{2\left(t - e\right)} \tag{E.2}$$

$$p_2^A = \frac{1}{3}z + t + \frac{2}{3}ex_1 - \frac{4}{3}e. \tag{E.3}$$

$$p_2^B = -\frac{1}{3}z + t - \frac{2}{3}e - \frac{2}{3}ex_1 \tag{E.4}$$

In contrast to the main text, since z is known from the outset, the expectations of x_2, p_2^B and p_2^A are equal to their actual value. By replacing (E.3) and (E.4) in (E.2), we obtain

$$x_2 = \frac{\frac{1}{3}z + t - \frac{4}{3}e + \frac{2}{3}ex_1}{2\left(t - e\right)}. \tag{E.5}$$

By substituting (E.5) in (E.1), bearing in mind that $\tilde{x}_2 = x_2$, we obtain:

$$x_1 = \frac{1}{2} + \frac{3}{2}\frac{\left(t - e\right)\left(p_1^B - p_1^A\right) + z\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}. \tag{E.6}$$

By substituting (E.6) in (E.5), we obtain:

$$x_2 = \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + zt}{3t^2 - 6te + 2e^2}. \tag{E.7}$$

By substituting (E.6) in (E.3) and (E.4), we obtain:

$$p_2^A = \frac{1}{3}z + t - e + \frac{e\left(t - e\right)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2},$$

and

$$p_2^B = -\frac{1}{3}z + t - e - \frac{e\left(t - e\right)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}.$$

The first-period profit-maximization problem of firm A is

$$\max_{p_1^A} \quad \left(x_1\left(p_1^A, p_1^B\right) p_1^A + x_2\left(p_1^A, p_1^B\right) p_2^A\right) N,$$

or

$$\begin{aligned}\max_{p_1^A} \quad & \left(\frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + z\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}\right) N p_1^A + \\ & + \left(\frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + zt}{3t^2 - 6te + 2e^2}\right) \times \\ & \times \left(\frac{1}{3}z + t - e + \frac{e(t-e)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}\right) N.\end{aligned}$$

The f.o.c. equals

$$\begin{aligned}&\frac{1}{2}\frac{54p_1^A t^2 e - 46p_1^A te^2 - 27p_1^B t^2 e + 22p_1^B te^2 - 26zt^2 e + 20zte^2 + 9t^4 + 8e^4}{\left(3t^2 - 6te + 2e^2\right)^2} + \\ &+\frac{1}{2}\frac{-18p_1^A t^3 + 10p_1^A e^3 - 42t^3 e + 66t^2 e^2 - 40te^3 + 9p_1^B t^3 - 4p_1^B e^3 + 9zt^3 - 4ze^3}{\left(3t^2 - 6te + 2e^2\right)^2} = 0.\end{aligned}$$

The second derivative equals

$$-\frac{-27t^2 e + 23te^2 + 9t^3 - 5e^3}{\left(3t^2 - 6te + 2e^2\right)^2} = \frac{-3t + e}{3t^2 - 6te + 2e^2} + \frac{e^2(e-t)}{\left(3t^2 - 6te + 2e^2\right)^2}.$$

As in the main text, one must have $t > 1.577e$ in order to have a unique and stable equilibrium without full bunching on one network. For $t > 1.577e$, the expression immediately above is negative, ensuring that the s.o.c. is verified.

The problem faced by firm B is

$$\max_{p_1^B} \quad \left(1 - x_1\left(p_1^A, p_1^B\right)\right) N p_1^B + \left(1 - x_2\left(p_1^A, p_1^B\right)\right) N p_2^B,$$

or

$$\begin{aligned}\max_{p_1^B} \quad & \left(\frac{1}{2} - \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + z\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}\right) N p_1^B + \\ & + \left(1 - \frac{1}{2} - \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + zt}{3t^2 - 6te + 2e^2}\right) \times \\ & \times \left(-\frac{1}{3}z + t - e - \frac{e(t-e)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}\right) N.\end{aligned}$$

The f.o.c. for firms B's problem equals

$$\begin{aligned}&-\frac{1}{2}\frac{27p_1^A t^2 e - 22p_1^A te^2 - 54p_1^B t^2 e + 46p_1^B te^2 - 26zt^2 e + 20zte^2 - 9t^4 - 8e^4}{\left(3t^2 - 6te + 2e^2\right)^2} - \\ &-\frac{1}{2}\frac{-9p_1^A t^3 + 4p_1^A e^3 + 42t^3 e - 66t^2 e^2 + 40te^3 + 18p_1^B t^3 - 10p_1^B e^3 + 9zt^3 - 4ze^3}{\left(3t^2 - 6te + 2e^2\right)^2} = 0.\end{aligned}$$

Solving the system of equations formed by the two first-order conditions, we obtain the optimal prices charged in the first period:

$$
\begin{aligned}
p_1^B &= -\frac{1}{3}\frac{56e^4 - 328te^3 + 12ze^3 - 60zte^2 + 582t^2e^2 - 378t^3e + 78zt^2e - 27zt^3 + 81t^4}{(e-t)\left(14e^2 - 54te + 27t^2\right)} \\
p_1^A &= -\frac{1}{3}\frac{56e^4 - 328te^3 - 12ze^3 + 60zte^2 + 582t^2e^2 - 378t^3e - 78zt^2e + 27zt^3 + 81t^4}{(e-t)\left(14e^2 - 54te + 27t^2\right)}.
\end{aligned}
$$

By replacing these in (E.6) and (E.7), we obtain

$$
\begin{aligned}
x_1 &= \frac{1}{2} + \frac{1}{2}\frac{9zt - 2ez}{14e^2 - 54te + 27t^2} \\
x_2 &= \frac{1}{2} + \frac{1}{2}\frac{-4e^2z + 15ezt - 9zt^2}{(e-t)\left(14e^2 - 54te + 27t^2\right)}.
\end{aligned}
$$

If $z > 0$, x_1 and x_2 exceed $\frac{1}{2}$ as was to be expected. Moreover, $x_2 > x_1$ if and only if $t \in (1,577e, 1.694e)$.

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