On the Community Patent

Steffen H. Hoernig*
Universidade Nova de Lisboa
CEPR, London

December 2004

Abstract

The European Union will be introducing a Europe-wide patent, the so-called Community Patent. Its aim is to foster innovative activity, but strategic effects between firms competing in R&D have not been considered in the official discourse. We show that, even if these are taken into account, the Community Patent will increase innovative activity and welfare. On the other hand, if the decision of participating in R&D is considered, then this increased R&D will be concentrated into fewer firms. Furthermore, we show that existing asymmetries between countries and firms are bound to increase.

Keywords: Community patent, R&D race, Participation in R&D
JEL: L52, O34

*shoernig@fe.unl.pt. We would like to thank seminar participants at the 2003 SPiE and 2004 EARIE meetings, and at FEUNL, for their helpful comments. Financial support from FEDER and FCT under the research project POCTI/ECO/37925/2001 is acknowledged.

1 Introduction

After more than 20 years of discussion and negotiations the European Union is finally settling down on a proposal for the introduction of a "Community Patent". In August 2000, the European Commission put forward a proposal (COM(2000) 412 final) that seemed to finally pave the way for an agreement. The "Common political approach" of March 3rd 2003 of the European Council of Ministers on the "Proposal for a Council Regulation on the Community Patent", has subsequently been updated six times, and on March 8th 2004 a "Proposal for a Council Regulation on the Community Patent" has been published. Since then the issue has stalled, due to a struggle about translation requirements into the different EU languages. The Luxembourg presidency of January-June 2005 has put the relaunch of the process on its agenda.[1]

The stated aim of the Community Patent is to "boost innovation and competitiveness" by removing the need to ask for a new patent in each single country, thus lowering costs and speeding up the diffusion of new technologies (implying the assumption that inventions only are put on the market where they are protected by a patent). What is left out in the official statements are the *strategic effects* of the introduction of the Community Patent, including dynamic effects on the market structure in industries where R&D is important. These effects range from the simple observation that more firms become "visible" to each other because before their "regional" patents did not conflict, to the more subtle effects caused by the other firms' change in research strategies on their own R&D.

We consider a model of stochastic R&D races introduced by Lee and Wilde (1980), to which we add the new elements of participation decisions and the joining of two previously independent races.[2] Randomness seems essential in modelling patent races because no firm knows *ex ante* whether it will be the one who receives the patent; deterministic R&D models do not capture this aspect.

There has been an enormous amount of writing on patents. We just mention Scherer (1980) for an introduction into the economics of patents, and Denicoló (1996) for an exposition of the optimal choice between patent length and patent breadth. Though related to our research in the sense that the Community Patent increases the "breadth" of the patent in terms of countries covered, the latter does not touch upon the questions central to the Community Patent. In fact, we have not been able to spot any reference

[1] Situation as of December 2004. For latest news on the issue consult http://europa.eu.int/comm/internal_market/ en/indprop/patent/index.htm.

[2] See also Reinganum (1989), Beath *et al.* (1989) and Martin (2002).

to the Community Patent in the economic literature, nor any analysis of the union of hitherto separated patenting areas.

In most models of R&D firms are symmetric and the number of firms in the market is exogenous. This means that in these models certain questions cannot be addressed, such as: How much R&D is performed by different firms in the same market? How many firms want to participate in the first place? As for example Cefis (2003) stresses, there are persistent differences between firms' R&D investments, so that the symmetry assumption should sometimes be dropped to consider the consequences of this fact. Furthermore, as e.g. Scherer (1980) and Cohen and Klepper (1996) noted, there are significant fixed costs in research such as setting up laboratories, delegating personnel and attention etc. Therefore the question of *participation* in research, apart from how much to spend once the decision to participate is made, seems important.

Our results are as follows: With symmetric firms and for a given market structure, the Community Patent does increase research outlays at all firms if R&D investments are strategic complements, which may decrease firms' discounted expected profits as a result of the more intensive competition in R&D. With strategic substitutes the opposite is true. This leads to the insight that the equilibrium number of firms doing research will change once we let firms decide whether they want to participate in the first place. This would imply fewer or more firms, each doing more or less research, so the aggregate effect is ambiguous. Since consumer surplus increases if more research is done the total welfare effect is also ambiguous.

We endogenize market structure in two-stage game where firms first decide whether to participate, and then, given the number of participating firms, choose research outlays. In this case the Community Patent continues to increase research output as measured by a shorter expected time to invention on the market. It is possible that there will be concentration in research activities into fewer firms, at least for the market in question total welfare increases.

It has been argued that small and medium-sized firms (SME's) will not take up the Community Patent because they are individually better served by European Patents covering fewer countries. The reason advanced is that SME's do not have the organization capabilities, nor the leverage to enter successfully as many foreign markets as "big firms". Our theoretical results are as follows: If their R&D investment is a strategic complement, then SME's will do more research in equilibrium,but they will be subject to a negative externality imposed on them by the increase in research at big firms and the other SME's. That is, they suffer from the Community Patent without partaking in its advantages. On the other hand, if their R&D investment is a strategic substitute then they will do less R&D while firm value goes up.

We have also considered how the Community Patent affects firms with different research cost. Maybe surprisingly, we cannot say anything about the relative change in firm value. Once we take into account the participation decision, however, the picture becomes clearer. We find the stark result that under by chance will firms of different profitability or research cost will participate in the same R&D race. Any change to the conditions of the race, including the introduction of the Community Patent will likely upset this balance. If two regions of different profitability or research environment are merged into one R&D race then possibly only the firms of one region remain.

Section 2 contains a quick overview of the European Patent System and the proposed Community Patent, and Section 3 sets out the model. The case of symmetric firms without and with free entry is analyzed in Sections 4 and 5, respectively. Section 6 considers asymmetries, and Section 7 concludes. All proofs are in the appendix.

2 The European Patent System and the Community Patent

The "European Patent" under the European Patent System (EPS) and the Community Patent are at the same time quite different and intimately related. The EPS was established by an international treaty, the "Convention on the Grant of European Patents" in 1973 (Munich Convention).[3] Even though most member countries are members of the EU, it is not formally related with the EU. In fact, at the time of this writing (August 2003) all EU countries were members of the EPS, as well as a series of future accession and non-EU countries such as Bulgaria, Cyprus, Estonia, Liechtenstein, Monaco, Romania, Slovenia, Switzerland, Turkey. Likely future members are Albania, Hungary, Latvia, Lithuania, Slovakia. National patent offices (NPO's) forward applications to the European Patent Office (EPO) in Munich, or claims can be presented directly at the EPO. There are three official languages (English, French and German) in one of which the application must be presented. Successful applicants will be granted "European Patents", one for each country for which patent protection was requested; the average number of countries for which patent protection is requested is 8, and maintained for 10 years out of a maximum 20. That is, the main thrust of the European Patent does not stem from a wider coverage than a national patent, but from

[3]For more information consult www.epo.org or the website of the EU.

a standardized process of granting the patent. In fact, if a patent is granted, the whole patent, made up on average of 17 pages of description and three pages of claims, must be translated into the language of every country for which the European Patent was requested.

A further drawback of the EPS as it is now is that national courts are responsible for litigation over patents in every country, under national law. Thus there is no guarantee of consistent legislation, and decisions about cases may be contradictory. Because the resulting legal uncertainty is considered harmful, negotiations are under way for a unique court.

The proposed Community Patent offers an alternative patent that covers all countries of the EU with one single patent. The main political difference is that the Community Patent is an instrument of the EU, and thus subject to European legislation. Applications are processed and patents are granted by the EPO, with costs to applicants lower as compared to a large number of EP's. Translation requirements still exist: First, the patent application must be presented to the EPO in one of the three official languages. Second, after the patent has been granted its claims (on average three pages) must within two years be translated into *all* languages of the EU. An integral part of the new Community Patent will be the Community Patent Court. This will be the unique court to pronounce itself on litigation involving Community Patents, and will do so based solely on European legislation, "increasing legal certainty".[4]

After the creation of the Community Patent the new and old systems will be coordinated and exist side-by-side. One possible definition of a Community Patent is an "European Patent that denotes all EU countries". Firms will be able to choose between the new Community Patent and any of the existing European Patent's. This is definitely true for firms who intend to cover a large number of countries, but an argument voiced in the discussion about the costs of the Community Patent, for example by the European Commission itself, or the "Union of Industrial and Employers' Confederations of Europe" (UNICE)[5], a Brussels pressure group, says that the Community Patent in its present form will be too expensive for SME's ("Small and Medium Enterprises"). The reasoning is straight-forward: Many of these companies do not have the organizational capability to serve many markets and make enough profits above patenting costs,[6] so they would choose an

[4]The latest snag introduced is the requirement that the translations of the claims in each country will attain legal value over the original claim submitted to the EPO in one of the EPO languages. That is, possible translation errors will increase legal uncertainty.

[5]http://www.unice.org

[6]Apart from the fixed cost of asking for a patent there are also yearly "maintenance"

European Patent with a small coverage. On the other hand, the Community Patent's additional benefits from covering many countries will be small, so that the Community Patent's higher cost make it unattractive. To this "accounting argument" the economist adds that strategic effects resulting from the non-adoption by SME's are likely to increase the gulf between SME's and big firms.

3 The Model

Assume that there are two "regions" or groups of countries. Before the introduction of the community patent firms in each region see themselves as independent of the firms in the other region because patents are only asked for one's own region, and there is no trade in the patented goods between these regions (since in each region a different firm holds a patent).

Apart from the legal aspects, the effects of the community patent are the following:

- Combines the product markets into one, raising the patentee's profits;
- lowers losers' profits (if they are not zero anyway) if the patentee is from the other region;
- decreases patenting costs;

and, which is the additional effect that we are interested in,

- exposes firms in any region to R&D competition from the other region, so that the total number of competing firms increases.

We will consider the R&D races that would take place with and without the introduction of the community patent. Generically, let W and L be the discounted payoffs of the winner and the losers[7] of the race, respectively, and let π be the flow payoff received at each moment during the race. It is natural to assume $W > \max\{\pi/r, L\}$, where r is the discount rate in continuous time.

Most of the analysis can be performed without defining these payoffs in more detail. Still, we will also illustrate certain results using the specific case

payments to the EPO.

[7] Since we let the number of participants vary, the assumption of a fixed L is restrictive. It is a useful simplification if the number of firms is limited, while the only value consistent with a large number of firms is $L = 0$.

of a drastic innovation: For the duration T of the patent the flow profits of the patent holder are $\pi^m > 0$, while the other firms earn zero profits. After this the patented technology becomes costlessly available to all firms, leading to discounted (up to the end of the race) profits of L. With patenting cost C, the winner's discounted payoffs are

$$W = \frac{1 - e^{-rT}}{r} \pi^m - C + L \equiv \Pi - C + L, \tag{1}$$

where we neglect the dependence on T because patent duration will not be changed. The two regions may differ in Π and the number of existing firms. Only the winner's payoff is changed by the introduction of the community patent.

In a first step the number of firms in each region is exogenously given and equal to n; we will consider the equilibrium number of firms in Section 5. Furthermore, for now all firms are assumed to be symmetric; the effects of asymmetries are considered in Section 6.

Time is continuous and the common discount rate is $r > 0$. Inventions occur according to independent Poisson processes with constant hazard rates. Each firm chooses its hazard rate $h_i \geq 0$, at a flow cost of $c(h_i)$ where $c(0) = 0$ and c is twice continuously differentiable, increasing and convex.[8] In the drastic innovation example we will use the special functional form $c(h_i) = kh_i^2/2$.

Consider firm i and let $H_i = \Sigma_{j \neq i} h_j$. It is well-known that firm i's value can be expressed as

$$V_i(H_i) = \max_{h_i} \frac{\pi - c(h_i) + h_i W + H_i L}{r + h_i + H_i} \tag{2}$$

This firm value consists of net flow profits $(\pi - c(h_i))$ during the expected duration of the race, plus expected profits of winning or losing. All of these are discounted by time and the probability of the race ending.

We now present a few useful preliminary results. All proofs can be found in the appendix.

Lemma 1 *In the patent race just described, for all participating firms,*

1. *a necessary and sufficient condition defining its optimal hazard rate is*

$$V_i(H_i) = W - c'(h_i), \tag{3}$$

which also implies that $V_i(H_i) < W$ for all H_i.

[8] Alternatively one could make the dual assumption that actions are research outlays x_i, with resulting hazard rates $h(x_i)$ where h is an increasing and concave function, but both games are equivalent. See e.g. Denicoló (1996).

2. *Hazard rates are strategic complements if and only if firm value decreases in rivals' R&D efforts. Both are equivalent to $V_i > L$. On the other hand, hazard rates are strategic substitutes if and only if $V_i < L$.*

3. *Best responses shift as expected: $\partial h_i/\partial W > 0$, $\partial h_i/\partial L < 0$ and $\partial h_i/\partial \pi < 0$.*

4. *The equilibrium is stable if and only if for all i*

$$D = (r + h_i + H_i)\, c''(h_i) - (n-1)(V_i - L) > 0.$$

The first result simplifies the exposition later on. As for the second one, the two cases of strategic substitutes or complements characterize two completely different scenarios. If a race becomes more intensive, with strategic complements the best response R&D investment increases but firm value decreases, while with strategic substitutes R&D investment decreases while firm value goes up. At first sight there seems to be a contradiction between the aims of increasing research spending, firm value and consumer surplus; this turns out to be the wrong questions once one remembers that the true aim is to increase total welfare where all these items are included.

Beath *et al.* (1989), established that R&D investments are strategic complements if the *profit effect* $(W - \pi/r)$ is small as compared to the *competitive effect* $(W - L)$, with the threshold being defined by $V_i = L$ or, letting $g = (c')^{-1}$,

$$(W - \pi/r) \leq (W - \pi/r)^* = (W - L)\left(1 + g(W - L)/r\right) - c\left(g(W - L)\right)/r, \tag{4}$$

and are strategic substitutes otherwise. This condition becomes less strict as W increases:

$$\begin{aligned} & \frac{d}{dW}\left[(W - L)\left(1 + g(W - L)/r\right) - c\left(g(W - L)\right)/r - (W - \pi/r)\right] \\ = \;& g(W - L)/r > 0. \end{aligned}$$

In other words, as W increases at some point competition will switch from strategic substitutes to complements. In any case, for large H_i the hazard rate h_i converges to $g(W - L)$. This means that individual investments under strategic substitutes (complements) are always larger (smaller) than this value.

With quadratic cost, investments are strategic complements if

$$(W - \pi/r) \leq (W - L)\left(1 + \frac{W - L}{2kr}\right), \tag{5}$$

which is true if either $L < \pi/r$ or $W \geq L + \sqrt{2kr\left(L - \pi/r\right)}$.

These results also provide the intuition for how firm value changes with rivals' investments. If the profit effect is large (strategic substitutes) then flow profits during the race π are small: it is better for a firm to have the race end rapidly, which is more likely if the others invest more. The opposite is true if the profit effect is small (large π), where each firm can endure a long race but will respond to an increase in rivals' investment.

As for the third result, best responses behave as expected. An increase in the patent holder's profits shifts them upwards, and an increase in the loser's profits or current flow profits shifts them downward. The fourth result provides an expression needed later for comparative statics.

4 Equilibrium R&D with a Fixed Number of Firms

We will follow two complementary approaches to evaluating the effect of the community patent. First, we will consider how marginal changes in parameters such as winner's and losers' payoff affect the properties of the equilibrium. If these effects have an unambiguous sign then they are also true after the discrete change that is the merging of two separate races into one. Second, we will pursue the case of the drastic innovation and compute explicit answers.

In this section we continue to assume that the number of firms in each region is exogenous, so that no entry or exit occur by definition. A symmetric equilibrium, generically, is given by the pair of equations

$$V^* = \frac{\pi - c\left(h^*\right) + h^* W + \left(n-1\right) h^* L}{r + nh^*}, \tag{6}$$

$$V^* = W - c'\left(h^*\right), \tag{7}$$

which joint define equilibrium firm value and hazard rate. The total effect of the community patent seems hard to judge because the three parameters W, L and n are all changed at the same time. In the following proposition we will disentangle their effects (remember that R&D investments are strategic complements (substitutes) if $V^* > (<) \, L$):

Proposition 2 *The symmetric equilibrium has the following properties:*

1. *Equilibrium hazard rates h^* are increasing in W and decreasing in L:*

$$\frac{\partial h^*}{\partial W} = \frac{r + \left(n-1\right) h^*}{D^*} > 0, \; \frac{\partial h^*}{\partial L} - \frac{\left(n-1\right) h^*}{D^*} < 0. \tag{8}$$

They are increasing (decreasing) in n if hazard rates are strategic complements (substitutes),

$$\frac{\partial h^*}{\partial n} = \frac{(V^* - L)\, h^*}{D^*}.$$

2. *Equilibrium firm value increases in L,*

$$\frac{\partial V^*}{\partial L} = c''(h^*) \frac{(n-1)\, h^*}{D^*} > 0, \tag{9}$$

and decreases (increases) in n if investments are strategic complements (substitutes):

$$\frac{\partial V^*}{\partial n} = -c''(h^*) \frac{(V^* - L)\, h^*}{D^*}. \tag{10}$$

The effect of the winner's payoff W is negative with strong strategic complements,

$$\frac{\partial V^*}{\partial W} = \frac{h^* c''(h^*) - (n-1)(V^* - L)}{D^*}, \tag{11}$$

but positive with strategic substitutes or weak strategic complements.

Since the community patent increases the winner's payoff W and the number competitors n, while decreasing the losers' payoff L, the total marginal effect on the equilibrium hazard rate h^* is unambiguously positive with strategic complements. In other words, if h_i^*, $i = 1, 2$, and h^* are the hazard rates before and after the introduction of the community patent then $h^* > h_i$. Since the total number of firms does not change, total hazard, the inverse of which is the expected time-to-innovation, is $(n_1 + n_2)\, h^* > n_1 h_1^* + n_2 h_2^*$, i.e. innovation is achieved faster than in any of the two regions alone. Total flow research outlays also increase, $(n_1 + n_2)\, c(h^*) > n_1 c(h_1^*) + n_2 c(h_2^*)$, while these are spent for a shorter period in time.[9] Thus the Community Patent clearly raises research efforts at an individual and aggregated level if one considers the number of participating firms as fixed. Nevertheless, as we will see below, one should take into account that firms must first decide whether they want to participate at all in the race. This becomes a relevant issue once we consider the effect of the community patent on the value of participating in the race. With strategic substitutes there are two opposing

[9] The correctly discounted total is included in the welfare calculations below.

effects: Higher $(W - L)$ increases R&D, but the larger number of effective competitors decreases it. *A priori* it is not clear which effect is stronger.

As expected, firm value decreases as the losers' payoff L decreases because losing the race becomes more damaging. It is also fairly obvious that V^* should decrease as more firms participate in the race for the same patent since the probability of winning decreases while research cost increase if investments are strategic complements. If they are strategic substitutes, though, firm value increases because the race ends earlier and because the firm invests less.

As concerns the effect of the increase in the winner's payoff, it is positive if investments are strategic substitutes or weak complements. With stronger complements the negative effect of a higher competitive intensity may even dominate and lower firm value. In this latter case firm value will certainly decrease since all three effects are negative . We will return to this question in our more specific model below.

Let us consider the welfare consequences of the equilibrium outcome. Let s_0 be the flow consumer surplus during the race, and S the discounted consumer surplus after the end of the race, taking into account the period during and after the validity of the patent. Total welfare is defined by the discounted sum of profits, research expenditures and consumer surplus:

$$\begin{aligned} B &= \frac{n\pi - nc(h^*) + s_0 + nh^*(W + (n-1)L + S)}{r + nh^*} \\ &= nV^* + \frac{s_0 + nh^* S}{r + nh^*} \end{aligned} \tag{12}$$

The consumer surplus term is increasing in nh^* if and only if $S > s_0/r$, that is, if and only if consumers gain from the race taking place. Let us assume that this is true. Then total welfare certainly increases if firm value and R&D investments increase, but may decrease if firm value or investments decrease strongly.

The preceding discussion is summed up in the following corollary:

Corollary 3 *With a fixed number of firms in both markets, the introduction of the community patent has the following effects:*

1. *If R&D investments are strategic complements, then*

    (a) *the Community Patent raises individual and total research investments $c(h^*)$ and $(n_1 + n_2)c(h^*)$, while decreasing expected time-to-innovation $1/(n_1 + n_2)h^*$.*

(b) *Firm value decreases if investments are strong strategic complements, otherwise increases.*

2. *If R&D investments are strategic substitutes, then*

    (a) *the Community Patent's effect on research investments is ambiguous; and*

    (b) *firm value increases.*

3. *Total welfare may increase or decrease. It increases with weak strategic complementarity, but otherwise the effect is ambiguous.*

We will now consider the question of whether firm value increases or decreases in our more specialized model, presuming that investments are strategic complements. With quadratic innovation cost, the conditions describing the equilibrium of the patent race in regions $i = 1, 2$ become

$$V_i^* = \frac{\pi - k\left(h_i^*\right)^2/2 + h_i^* W_i + (n_i - 1)\, h_i^* L_i}{r + n_i h_i^*} = W_i - k h_i^*, \tag{13}$$

with explicit solution

$$h_i^* = \frac{(n_i - 1)(W_i - L_i) - kr + R_i}{k(2n_i - 1)}, \tag{14}$$

$$D_i^* = R_i = \sqrt{\left[(n_i - 1)(W_i - L_i) - kr\right]^2 + 2kr(2n_i - 1)(W_i - \pi_i/r)} \tag{15}$$

Here we assume that firms from both regions have the same R&D cost function; this assumption will be relaxed later. The first observation is that firm value V_i^* decreases if and only if the following holds:

$$\frac{W - \pi/r}{W - L} < \frac{n-1}{n}\left[1 - \frac{1}{2kr}\frac{n-1}{n}(W - L)\right], \tag{16}$$

which is true if π/r is sufficiently close to W. In the terminology of Beath *et al.* (1989), the profit effect $(W - \pi/r)$ must be small compared to the competition effect $(W - L)$, i.e. investments must be strong strategic complements. On the other hand, if the profit effect is large then firm value increases; this is true for example with Bertrand price competition during and after the race, i.e. $L = 0$ and $\pi = 0$.

This condition is sufficient but not necessary, given that there are already two other effects lowering firm value. Let us consider the special case of two regions with n firms each and winner's payoff $W' = 2\Pi - C_{cp} + L$. Firm value decreases if

$$V' = W' - kh' \leq V = W - kh. \tag{17}$$

After some arduous computations we arrive at a result very similar to the one above:

Lemma 4 *If the Community Patent covers two previously independent equal-sized regions, and if the number of firms is fixed, then firms' value of participating in the race decreases if R&D investments are strong strategic complements.*

Since firm value decreases in this case it is questionable whether the assumption of a fixed number of firms before and after the introduction of the Community Patent continues to make sense. Indeed, if firms have to achieve at least a certain minimum value $\bar{V}$ then some firms may exit the race as the value becomes too low. This exit then raises equilibrium firm value but lowers equilibrium research outlays, so the total effect is not clear *a priori*. In a similar manner, if firm value increases then more firms will enter the race, which could at least partially compensate the decrease in research investments. We will deal with these questions in the following section.

5 Participation

5.1 The equilibrium number of firms

In the previous section we have taken the number of all firms to be constant, and considered how this given number of firms will compete under the Community Patent. That is, we have not touched on the issues of entry and exit, and how the introduction of the Community Patent may change the equilibrium number of firms participating in research. This is a relevant question because even if each remaining firm does more research, total research and welfare could decrease if there was a strong reduction in participation. As we have seen there could also be increased participation with less individual investment, and also in this case the total effect is unclear.

In most models of R&D races the number of participating firms is an exogenously given finite number, and this number is therefore independent of the equilibrium firm value. It has been observed that fixed costs play an important part in the decision whether to undertake research, as for example in Scherer (1980) or Cohen and Klepper (1996), therefore it is remarkable that it has been neglected in formal modelling of R&D races.

We will assume that there is a fixed cost $F > 0$ of participation in research, and that in a new first stage of the game firms decide whether to participate or not. A firm that does not participate in the race is assumed not to enter the market.[10] For simplicity we treat the number of firms as a continuous variable when solving the free-entry condition. Let $V^*(n)$ be the equilibrium firm value of the n participants in the race.

Let us first assume that $L < F < W$ (for larger fixed cost there is no entry). We will discuss the case $F \leq L$ below. If R&D investments are strategic substitutes then $V^*(n) < L$ for all n, therefore no firm enters the race. On the other hand, if they are strategic complements then $V^*(n)$ is larger than L; it is also decreasing in n, thus in equilibrium firms enter the race until

$$V^*(n) = F. \tag{18}$$

From this condition we can determine the equilibrium investments and the equilibrium number of firms (letting $g(.) = (c')^{-1}(.)$, as above):

Proposition 5 *If $F > L$ and R&D investments are strategic substitutes then there is no entry. If they are strategic complements then the equilibrium hazard rate is $h^* = g(W - F)$, and the equilibrium number of firms is*

$$n^* = \frac{\pi - c(h^*) - rF + h^*(W - L)}{h^*(F - L)}, \tag{19}$$

which is increasing in L, and decreasing in W if and only if $V^*(n)$ is decreasing in W :

$$\frac{dn^*}{dL} = \frac{n^* - 1}{F - L}, \; \frac{dn^*}{dW} = \frac{1}{F - L} - \frac{n^* - 1}{h^* c''(h^*)}.$$

Following Corollary 3 1b), the last result implies that the equilibrium number of firms may actually decrease in the winner's payoff. This happens if the strategic complementarity between R&D investments is very high. Nevertheless, this is not the most relevant question in this context: The question

[10] Here we suppose that firms are in a race for a new market or product; as a consequence the alternative is not to enter, with zero payoffs. If on the contrary an existing market is considered then the outside alternative is to stay and wait until the race is decided. In this latter case the decision to participate depends on the equilibrium time to innovation if there is one firm less in the market. This is significantly more difficult to analyse, but the qualitative conclusions are similar.

that we will now ask is whether the merging of the two races with n firms each will result in a new race with more or fewer firms than $2n$. More specifically, we will ask whether what are the "returns to scale" in the relationship between the size of the prize W and the equilibrium number of participating firms: If W is multiplied by the factor γ, will the number of firms increase by more or less than γ? The correct infinitesimal measure of this increase is the elasticity of n^* with respect to W, or

$$\varepsilon_{n^*W} = \frac{dn^*}{dW}\frac{W}{n^*}. \tag{20}$$

If ε_{n^*W} were always equal to 1 then the equilibrium number of firms would increase proportionally. In fact, we show that this number is smaller than 1 if the profit effect is small, i.e. if R&D investments are s

Proposition 6 *The equilibrium number of firms n^* increases less than proportionally in the winner's payoff W if R&D investments are strong strategic complements.*

This result means that if we join two equal races with n participants each then the new race needs more than to double its prize if it is to accommodate the $2n$ original participants, in the case the participants react sufficiently strongly to each others' investments. If on the other hand investment best responses are more or less constant (that is, there is little strategic effect) then more firms may join in the race. If we consider W as consisting of market profits minus patenting cost, and that the two markets' size does not change, then we obtain the following:

Corollary 7 *With the introduction of the Community Patent, if R&D investments are strong strategic substitutes then patenting cost must decrease sufficiently if the number of participating firms is not to decrease.*

In fact, the European Commission has argued that one of the strong points in favor of the Community Patent is the lower patenting costs for patents covering many countries. Unfortunately, contrary to the original Commission proposal successive alterations by the Councils of Ministers have resulted in increased patenting cost because of wider translation requirements. These and other alterations that increase patenting cost may therefore reduce the usefulness of the Community Patent not only from a legal but also economic point of view.

We will now consider the merger of the two markets directly in our model with quadratic investment cost. The equilibrium number of firms in each of the previous races is

$$n^*(W) = \frac{1}{2}\left(1 + \frac{W-L}{F-L}\right) + \frac{kr(\pi/r - F)}{(F-L)(W-F)}. \tag{21}$$

This number is positive if the profit effect is small enough,

$$(W - \pi/r) < (W-F)\left(1 + \frac{F - 2L + W}{2kr}\right), \tag{22}$$

which is condition that is stricter than (5), the condition defining strategic complements.

Assume that the winner's payoff is equal to $2W$, with equilibrium number of firms $n^*(2W)$. Therefore the number of firms in the market is maintained when the two races are merged if $n^*(2W) \geq 2n^*(W)$, or if

$$\left(W - \frac{\pi}{r}\right) > (W-F)\left(1 + \frac{(F-2L)(2W-F)}{2kr(3W-F)}\right). \tag{23}$$

This is the same type of condition as the one involved in proposition 6: R&D investments must be sufficiently weak strategic complements, otherwise the equilibrium number of firms in the merged market will be smaller than the previous number of firms in both markets.

Just for reference, with a quadratic cost function the expression dn^*/dW is negative, that is, the equilibrium number of firms decreases in W, if

$$\left(W - \frac{\pi}{r}\right) < (W-F)\left(1 - \frac{W-F}{2kr}\right). \tag{24}$$

As remarked above this implies that R&D investments are strong strategic complements.

The case $F < L$ turns out to be inconsistent with the assumptions of the model: With both strategic substitutes or complements, since as the number of firms increases their value converges to L, in equilibrium an infinite number of firms would enter. This is at odds with our simplifying assumption that the losers' payoff L is constant and maybe positive, because losers' future flow profits should converge to zero as n increases — the only constant L consistent with a large number of firms is $L = 0$. Alternatively, one could assume that L converges to 0 as the number of firms goes up. In either case, for large n at least we would in principle be back in the case of $F > L$ above.

5.2 Equilibrium R&D and welfare

As concerns the amount of R&D done, let us consider two measures: input (spending) and output (total hazard). We find

$$\frac{d}{dW}(n^* h^*) = \frac{1}{c''(h^*)} + \frac{h^*}{F-L} > 0, \tag{25}$$

$$\frac{d}{dW}(n^* c(h^*)) = \frac{dn^*}{dW} c(h^*) + n^* \frac{c'(h^*)}{c''(h^*)}. \tag{26}$$

Innovation unambiguously occurs faster as W increases since the expected time to innovation is $1/n^*h^*$. On the other hand, total spending increases if n^* increases in W. Only in the extreme case of very strong strategic complements will a decrease in the number of firms be enough to decrease total spending.

The welfare measure of a single race is given by the following expression, which now also takes account of the fixed cost of entry since the number of firms is variable:

$$B^* = n^*(V^*(n) - F) + \frac{s_0 + n^* h^* S}{r + n^* h^*} = \frac{s_0 + n^* h^* S}{r + n^* h^*}. \tag{27}$$

As we have just seen, n^*h^* increases, and therefore this welfare measure also increases in W since we have assumed that innovation raises consumer surplus. Because of our definition of welfare this result must be taken with care: Non-participating firms are not considered, in particular there is no notion of social costs of not participating in the race.

Still, the relevant question is not whether B^* is increasing in W as such, but whether merging the two races will lead to a higher joint welfare. That is, if N is the total hazard under the community patent, we would like to know whether

$$\frac{2s_0 + N(2S)}{r + N} > 2\frac{s_0 + n^* h^* S}{r + n^* h^*}. \tag{28}$$

Again this is definitely true since $N > n^*h^*$. The above discussion is summarized in the following corollary.

Corollary 8 *If the decision to participate in R&D is taken into account, the Community Patent*

1. *decreases time-to-innovation and, unless R&D investments are extremely strong strategic complements, increases research spending.*

2. increases welfare measure B^*.

In other words, from an aggregate perspective the introduction of the Community Patent seems to be unambiguously positive, because welfare-increasing innovations will be made faster and therefore welfare increases. The perspective of our model is partial, however, since it does not take into account the economic and social effects of outside of the market in question. In particular, if the number of participating firms decreases then there may be negative effects which we do not consider explicitly.

Furthermore, R&D activity may become more concentrated in some regions even if we consider only symmetric firms.[11] As a consequence, if social benefits related to R&D are *local* then some regions would suffer while others would gain. We consider this aspect in more depth in the following section.

6 Asymmetries

In this section we will enlarge our analysis to incorporate various asymmetries that have an impact on R&D competition. Generically, if there are m_i firms of type $i = 1, 2$ with payoff parameters (π_i, W_i, L_i) and investment cost functions c_i, then the equilibrium with a fixed number of firms is described by

$$\begin{aligned} V_i^* &= \frac{\pi_i - c_i(h_i^*) + h_i^* W_i + \left((m_i - 1) h_i^* + m_j h_j^*\right) L_i}{r + m_i h_i^* + m_j h_j^*}, \\ V_i^* &= W_i - c_i'(h_i^*). \end{aligned} \tag{29}$$

We will consider some of these effects separately. In section 6.1 we consider firms that will not adopt the Community Patent while others do. Following this, in section 6.2 differences in the firms' cost of R&D are analyzed. Section 6.3 reconsiders the participation decision under asymmetry.

6.1 Non-adoption of the Community Patent

Once we take into account that firms differ in their capacity to exploit markets, especially if they are spread over several countries, there may be a group of firms that do not find it optimal to apply for the community patent. Specifically, we consider a group of firms who are already present in both regions and who will adopt the Community Patent (type $i = 1$), and a second group in each region who will not adopt the patent ($i = 2$). Assuming for

[11] For this effect see also Sutton (1996).

simplicity that both regions are of equal size, there are m_1 firms of type 1 in all, and in each region there are m_2 firms of type 2. We further assume that all firms of group 1 have high firm value ($V_1^* > L_1$) and therefore their R&D investments are strategic complements.

Given that firms of type 1 confront $2m_2$ firms of type 2, their value functions are

$$V_1 = \frac{\pi - c(h_1) + h_1 W_1 + ((m_1 - 1) h_1 + 2m_2 h_2) L_1}{r + m_1 h_1 + 2m_2 h_2}, \tag{30}$$

while the value functions of type 2 firms are as in (29).

First we consider how the adoption of the Community Patent by firms of type 1 affects the others of type 2; below we will discuss the adoption decision. The simplest way to model this is to consider an increase in W_1, due to the larger market covered, while W_2 does not change. As a result, by Lemma 1 only the best response functions of type 1 firms are shifted upwards. This means that these firms will increase their R&D investment in equilibrium. As for the firms who have not adopted the Community Patent, their R&D investment also increases if investments are strategic complements, or decreases if they are strategic substitutes.

As concerns the change in the value of the non-adopters:

Proposition 9 *The firm value of the non-adopters decreases (increases) if their R&D investments are strategic complements (substitutes).*

That is, there is a negative strategic effect on each firm that does not adopt the Community Patent. This effect is composed of the effect of more research at the adopters, and of the effect of more research at the other non-adopters. Therefore at this level of analysis we find some justification for the concern that a too expensive Community Patent would harm small firms. The possibility of choice between the European Patent and the Community Patent does not protect small firms from the choices made by the other firms.

Using the results in the proof of the last proposition, we find that the change in total hazard is

$$m_1 \frac{dh_1^*}{dW_1} + m_2 \frac{dh_2^*}{dW_1} \sim m_1 \left(c_2''(h_2^*) + \frac{V_2^* - L_2}{r + m_1 h_1^* + m_2 h_2^*} \right).$$

Total hazard definitely increases (and therefore time-to-innovation decreases) if the non-adopters' R&D investments are strategic complements, while it may decrease if they are strong enough strategic substitutes.

The firm value of a non-adopter who deliberates where to switch to the Community Patent, increasing his winner's payoff from W_2 to $\hat{W}_2$, is

$$\begin{aligned}\hat{V}_2 &= \frac{\pi_2 - c_2\left(\hat{h}_2^*\right) + \hat{h}_2^* \hat{W}_2 + \left(\left(m_2 - 1\right) h_2^* + m_1 h_1^*\right) L_2}{r + m_1 h_1^* + \left(m_2 - 1\right) h_i^* + \hat{h}_2^*} \\ &= \frac{\pi_2 - c_2\left(\hat{h}_2^*\right) + \hat{h}_2^* \hat{W}_2 + \hat{H}_2 L_2}{r + \hat{H}_2 + \hat{h}_2^*}, \qquad (31)\end{aligned}$$

where $\hat{h}_2^* = g\left(\hat{W}_2 - \hat{V}_2\right)$. The effect of increasing his W_2 to $\hat{W}_2$ is, using the envelope theorem,

$$\frac{d\hat{V}_2}{d\hat{W}_2} = \frac{\hat{h}_2^*}{r + \hat{H}_2 + \hat{h}_2^*} - \frac{\hat{V}_2 - L_1}{r + \hat{H}_2 + \hat{h}_2^*} \frac{\partial \hat{H}_2}{\partial \hat{W}_2}. \qquad (32)$$

The first term, the direct increase in expected profits, is positive. The second term, the strategic effect, is ambiguous. If R&D investments are strategic complements then it is negative: Competitors will increase investment which drives down value. On the other hand, if they are strategic substitutes for non-adopters, then the second term is positive if competitors' investment increases, and negative if it decreases. The latter may happen if there are only few type 1 firms, so that the investment reduction in type 2 firms dominates the increase in type 1 firms.

Since the second term may dominate the first one, it can be an equilibrium choice for type 2 firms to not switch to the Community Patent if their R&D investments are strategic complements, or if they are strategic substitutes and there are only few type 1 firms.

6.2 R&D Efficiency Differences

In this section we consider the situation where all firms involved compete over both regions already before the introduction of the Community Patent. What we are interested in is whether we can say anything how more or less efficient firms will fare once everybody switches to the Community Patent.

Let us assume that $c_1'(h) < c_2'(h)$, for all $h > 0$, while all the other parameters are equal, that is, firms of type 1 are more efficient in R&D. This section will check whether our model can say anything about how the gap between more and less efficient firms is changed by the introduction of the community patent if all firms adopt it. The equilibrium with m_i firms of type i is described by

$$V_i^* = \frac{\pi - c_i(h_i^*) + h_i^* W + \left((m_i - 1) h_i^* + m_j h_j^*\right) L}{r + m_i h_i^* + m_j h_j^*}, \qquad (33)$$

$$V_i^* = W - c_i'(h_i^*), \; i = 1, 2; \; j \neq i. \qquad (34)$$

We will first compare the equilibrium firm values and hazard rates:

Proposition 10 *For any given numbers m_1 and m_2 of firms of types 1 and 2, the value of the more efficient firms is higher: $V_1^* > V_2^*$. Still, without further assumptions the equilibrium hazard rates h_1^* and h_2^* cannot be compared.*

Even though the result on firm value is as one would expect, it is intuitive than one cannot compare firms' investment decisions at this level of generality: There are two opposing effects involved. The first effect arises because the more efficient firm finds it cheaper to do research, and has therefore higher incentives to do research; the second effect arises because the less efficient firm has more to gain from innovating: $W - V_2^*$ is larger than $W - V_1^*$.

The main aim of analysis in this section is to see how $(V_1^* - V_2^*)$ evolves with changes due to the introduction of the Community Patent. If the winner's payoff W increases for all firms, we find

$$\begin{aligned} \frac{d}{dW}(V_1^* - V_2^*) &= \frac{d}{dW}\left(c_2'(h_2^*) - c_1'(h_1^*)\right) \\ &= c_2''(h_2^*)\frac{dh_2^*}{dW} - c_1''(h_1^*)\frac{dh_1^*}{dW}. \end{aligned} \qquad (35)$$

At this level of generality this expression cannot be signed, unless the R&D investment of one group of firms is a strategic complement, and of the other group a strategic substitute. If they are both complements, or both substitutes, then the total effect is ambiguous (even when recurring to comparative statics as in Proposition 9).

As concerns the time-to-innovation, along the lines of the discussion above it is clear that it will decrease if R&D investments are strategic complements and will increase if they are substitutes. That is, we can make definite predictions about how consumer surplus changes exactly when we cannot say anything precise about how the relative position of firms changes.

6.3 Asymmetry and Participation

Above we have assumed that the number of participating firms remains fixed, an assumption which we relax in this section. Assume that firms' fixed cost of participation in R&D are $F_i > L_i$. In a generic market, the two types of firms are present in a free-entry equilibrium if $V_i^* (m_i^*) = F_i$, and equilibrium hazard rates are $h_i^* = g (W_i - F_i)$. Define the expression

$$\Gamma_i = \frac{\pi_i - rF_i - c_i (h_i^*) + h_i^* (W_i - L_i)}{F_i - L_i}, \tag{36}$$

then we find the following:

Proposition 11 *If firms decide whether to participate in the R&D race then*

1. *if* $\Gamma_i = \Gamma_j = \Gamma > 0$ *then both types of firm are present in equilibrium. The equilibrium number of firms of each type is indeterminate, while total hazard* $m_i^* h_i^* + m_j^* h_j^*$ *is equal to* Γ*;*

2. *if* $\Gamma_i > \Gamma_j$ *then no firms of type* j *enter the race. In this case the equilibrium number of firms is* $m_i^* = \Gamma_i / h_i^*$ *if* $\Gamma_i > 0$.

Since the value of Γ_i defines which firms will participate in the race, it is useful to do know its derivatives with respect to the various parameters at hand:

$$\begin{aligned} \frac{\partial \Gamma_i}{\partial W_i} &= \frac{1}{c_i'' (h_i^*)} + \frac{h_i^*}{F - L} > 0, \ \frac{\partial \Gamma_i}{\partial L_i} = \frac{\Gamma_i - h_i^*}{F - L} \\ \frac{\partial \Gamma_i}{\partial \pi_i} &= \frac{1}{F - L} > 0, \ \frac{\partial \Gamma_i}{\partial F_i} = -\frac{1}{c_i'' (h_i^*)} - \frac{r + \Gamma_i}{F - L} < 0. \end{aligned} \tag{37}$$

An increase in W or π_i, and decrease in F_i, increase Γ_i, while the second effect is ambiguous. Still, if firm i is present in equilibrium then $\Gamma_i \geq m_i h_i^* \geq h_i^*$, and the $\partial \Gamma_i / \partial L_i$ is positive. To see the effect of the cost of innovation, let $c_i (h) = \alpha_i c (h)$ and consider the effect of an increase in α_i, where now $g = (c')^{-1}$:

$$\frac{\partial \Gamma_i}{\partial \alpha_i} = -\frac{c (h_i)}{F_i - L_i} - \frac{W_i - F_i}{\alpha^2 c'' (h_i^*)} < 0. \tag{38}$$

That is, increases in payoffs and reductions in cost make firms more competitive. In our model, even slight differences in payoffs and costs have a large effect on the equilibrium: Any configuration where two types of firms are present is "unstable" in the sense that a small change in parameters destroys this equilibrium and leads to a new one where only one type of firm is present.

If we apply this reasoning to the introduction of the community patent, then our model predicts that any kind of asymmetry leads with probability 1 to an equilibrium outcome where only the more competitive firms remain in the market. These asymmetries can be, for example:

- Differences in the capacity to exploit larger markets; this applies in particular to SME's.
- Differences in the research cost function; this also applies to SME's.
- Differences in the research environment between regions, or even inside regions, leading to different fixed or variable cost of research, as considered above.
- Differences in the profitability of existing markets π_i.

The result that only one type of firms will exist in equilibrium is obviously too strong to be realistic. Still, it serves to indicate that as countries are opened up to competition even slight disadvantages can hamper firms significantly. Future research will be needed to determine which of the model's assumptions give rise to the extreme "bang-bang" outcome – this may go back as far as the assumption of the stochastic process itself (given that in deriving the participation condition we have not made any further assumptions about parameters or research cost functions).

7 Conclusions and Further Research

In this paper we analyze some strategic effects of the introduction of the Community Patent in the European Union, designed to be a complement to the European Patent System (EPS). These effects need to be clarified because the official justification for the introduction of the Community Patent considers only a direct positive effect when a firm adopts the Community Patent, without taking into account how market equilibrium and market structure are affected.

Analyzing a model of a stochastic R&D-race between symmetric firms we find that total investment in R&D indeed rises if market structure is exogenous, but that the expected discounted profits of participating firms may decrease. If there is free entry and exit in research activities then the total number of firms conducting research in equilibrium may decrease, that is, research activities will become more concentrated into fewer firms with higher research intensity. Total welfare related to the market under analysis, including consumer surplus, may increase or decrease because firm value and consumer surplus may change in opposite directions. With free entry, though, welfare does increase. Further research should address how a decrease in the number of firms performing R&D affects the whole of the economy.

Once we allow for asymmetries, additional effects emerge. First, the strategic effect of rivals' adoption of the Community Patent on non-adopters is negative (positive) if the non-adopters' R&D investments are strategic complements or substitutes, because adopters will increase R&D spending. Second, firms which are confronted with a higher cost of research may fall further behind or not — our model predicts nothing at the level of generality we maintain. Third, once we consider the participation decision, it becomes clear that any equilibrium where firms of different profitability or R&D capabilities are present is easily upset in favor of the more profitable or more efficient firms. This result is relevant for example if the Community Patent joins two regional markets, one of which is more offers a more profitable market or a better research environment.

In summary, the Community patent is likely to increase welfare, but at the possible cost of the concentration of research activities into fewer firms.

Finally, there are many open questions to still be answered in future research. We will mention here only the question of spillovers and research joint ventures: How does the presence of spillovers change the effects of the Community Patent, and how does the Community Patent change the incentives for and the social welfare effects of RJV's?

References

[1] Beath, John, Katsoulacos, Yannis, and Ulph, David (1989). "Strategic R&D Policy". *The Economic Journal*, 99 (Conference):74–83.

[2] Cefis, Elena (2003). "Is There Persistence in Innovative Activities?". *International Journal of Industrial Organization*, 21:489–515.

[3] Cohen, Wesley M. and Klepper, Steven (1996). "A Reprise of Size and R&D". *Economic Journal*, 106:925–951.

[4] Denicoló, Vincenzo (1996). "Patent Races and Optimal Patent Breadth and Length". *Journal of Industrial Economics*, 44(3):249–265.

[5] Lee, T. and Wilde, L. (1980). "Market Structure and Innovation: A Reformulation". *Quarterly Journal of Economics*, 94:429–436.

[6] Martin, Stephen (2002). "Spillovers, Appropriability, and R&D". *Journal of Economics*, 75(1):1–32.

[7] Reinganum, Jennifer F. (1989). "The Timing of Innovations". In Schmalensee, R. and Willig, R., editors, *Handbook of Industrial Organization*, chapter 14. Amsterdam: North-Holland.

[8] Scherer, Frederic M. (1980). *Industrial Market Structure and Economic Performance*. Chicago: Rand McNally.

[9] Seade, Jesus K. (1980a). "The Stability of Cournot revisited". *Journal of Economic Theory*, 23:15–27.

[10] Sutton, John (1996). "Technology and market structure". *European Economic Review*, 40:511–30.

8 Appendix

Proof of Lemma 1:
Proof. Taking the derivative on the right-hand side of (2) with respect to h_i, the necessary first-order condition for the maximization of profits is

$$\frac{(W - c'(h_i))(r + h_i + H_i) - (\pi - c(h_i) + h_i W + H_i L)}{(r + h_i + H_i)^2} = 0. \tag{39}$$

As pointed out in Martin (2002), this can be rewritten as stated in the text. The second derivative is negative at solutions to (3) because research costs are convex.

The slope of best responses is

$$\frac{dh_i}{dh_j} = \frac{V_i(H_i) - L}{(r + h_i + H_i) c''(h_i)}, \tag{40}$$

and on the other hand,

$$\frac{\partial V_i(H_i)}{\partial h_j} = -\frac{V_i(H_i) - L}{r + h_i + H_i}$$

by the envelope theorem or $V_i(H_i) = W_i - c'(h_i)$. As Beath *et al.* (1989) have shown best responses are either increasing, constant, or decreasing.

At a stable symmetric equilibrium with n firms the slope of the reaction function must be less than $1/(n-1)$ (see e.g. Seade 1980), so that we obtain the stability condition

$$D = (r + h_i + H_i) c''(h_i) - (n-1)(V_i - L) > 0. \tag{41}$$

Of interest are the shifts caused in best responses by changes in own flow profits: Using (3),

$$\frac{dh_i}{dW} = -\frac{\frac{d}{dW}(V_i - W + c'(h_i))}{\frac{d}{dh_i}(V_i - W + c'(h_i))} = \frac{r + H_i}{r + h_i + H_i} \frac{1}{c''(h_i)} > 0, \tag{42}$$

and similarly

$$\frac{dh_i}{dL} = -\frac{H_i}{r + h_i + H_i} \frac{1}{c''(h_i)} < 0, \quad \frac{dh_i}{d\pi} = -\frac{1}{r + h_i + H_i} \frac{1}{c''(h_i)} < 0. \tag{43}$$

■

Proof of Proposition 2:

Proof. The first observation is that the direct effect of an equal rise in research spending by all firms, no matter its cause, is to decrease equilibrium value:

$$\frac{\partial V^*}{\partial h^*} = -\frac{(n-1)(V^* - L)}{r + nh^*} < 0. \tag{44}$$

By the envelope theorem a small increase in own research does not change one's value, so what remains is the strategic effect through the increase in the other firms' research which is negative by assumption. Using this result we obtain

$$\frac{d}{dh^*}(V^* - W + c'(h^*)) = c''(h^*) - \frac{(n-1)(V-L)}{r + nh^*} > 0$$

by the stability condition

$$D^* = (r + nh^*)\, c''(h^*) - (n-1)(V^* - L) > 0.$$

The effects of W, L and n are, respectively,

$$\begin{aligned}
\frac{\partial h^*}{\partial W} &= -\frac{\frac{h^*}{r+nh^*} - 1}{c''(h^*) - \frac{(n-1)(V^*-L)}{r+nh^*}} = \frac{r + (n-1)h^*}{D^*} > 0, && (45)\\
\frac{\partial h^*}{\partial L} &= -\frac{\frac{(n-1)h^*}{r+nh^*}}{c''(h^*) - \frac{(n-1)(V^*-L)}{r+nh^*}} = -\frac{(n-1)h^*}{D^*} < 0, && (46)\\
\frac{\partial h^*}{\partial n} &= -\frac{-\frac{h^*L - Vh^*}{(r+nh^*)}}{c''(h^*) - \frac{(n-1)(V^*-L)}{r+nh^*}} = \frac{(V^* - L)h^*}{D^*} > 0. && (47)
\end{aligned}$$

As concerns firm value, consider the effect of an increase in W:

$$\begin{aligned}
\frac{\partial V^*}{\partial W} &= \frac{\partial}{\partial W}(W - c'(h^*)) = 1 - c''(h^*)\frac{r + (n-1)h^*}{D^*} && (48)\\
&= \frac{h^* c''(h^*) - (n-1)(V^* - L)}{D^*},
\end{aligned}$$

which is positive if $V^* \approx L$ and negative if $V^* \gg L$. On the other hand,

$$\begin{aligned}
\frac{\partial V^*}{\partial L} &= \frac{\partial}{\partial L}(W - c'(h^*)) = c''(h^*)\frac{(n-1)h^*}{D^*} > 0, && (49)\\
\frac{\partial V^*}{\partial n} &= \frac{\partial}{\partial n}(W - c'(h^*)) = -c''(h^*)\frac{(V^* - L)h^*}{D^*} < 0. && (50)
\end{aligned}$$

■

Proof of Lemma 4:

Proof. Denote by V', h' and W' the parameters and variables after the introduction of the community patent. Firm value decreases if

$$V' = W' - kh' \leq V = W - kh$$

Substituting h from above and

$$h' = \frac{(2n-1)(W'-L)-kr+\sqrt{(2(n-1)(W'-L)-kr)^2+2k(4n-1)(W'r-\pi)}}{(4n-1)},$$

after some transformations we arrive at

$$B_0\pi + 2kn\pi^2 \geq \frac{B_1B_2 - A_1^2}{8kn(2n-1)^2(4n-1)^2}$$

where

$$\begin{aligned}
B_0 &= \frac{(4n-1)(W'-W)((2n-1)(W'-L)-(n+1)(W-L))+n(W'-L)(W-L)+2nkr(W-L-2W')}{1} \\
B_1 &= 4(4n-1)^2\left(\left(4n^2-2n\right)y-\left(4n^2-n\right)w-2nkr\right)^2 \\
B_2 &= ((n-1)(W-L)-kr)^2 + 2k(2n-1)rW \\
A_1 &= (2n-1)^2((2n-1)(W'-L)-kr)^2 - (4n-1)^2((n-1)(W-L)-kr)^2 \\
&\quad +2(2n-1)^2k(4n-1)rW' - 2(4n-1)^2k(2n-1)rW \\
&\quad -\left(\left(4n^2-2n\right)(W'-L)-\left(4n^2-n\right)(W-L)-2nkr\right)^2
\end{aligned}$$

This condition means that flow profits π must be large enough. ■

Proof of Proposition 5:

Proof. From the first-order condition (3) we know that $c'(h^*) = W - V^*(n)$, and, denoting $g = (c')^{-1}$ as above, we find the equilibrium hazard rate as

$$h^* = (c')^{-1}(W-F) = g(W-F). \tag{51}$$

Individual hazard rates h^* and research spending $c(h^*)$ are increasing in W and decreasing in F, because g is an increasing function. After substituting h^* we can solve (18) for the equilibrium number of firms:

$$n^* = \frac{(\pi - rF) - c(h^*) + h^*(W-L)}{h^*(F-L)} \tag{52}$$

Taking derivatives, and using $g' = 1/c''$, we obtain

$$\begin{aligned}
\frac{dn^*}{dW} &= \frac{g(W-F)-(n^*-1)(F-L)g'(W-F)}{(F-L)g(W-F)} \\
&= \frac{h^*c''(h^*)-(n^*-1)(F-L)}{(F-L)h^*c''(h^*)},
\end{aligned} \tag{53}$$

which has the same sign as dV^*/dW in (11) taking into account that $F = V^*(n)$. Furthermore,

$$\frac{dn^*}{dL} = \frac{n^* - 1}{F - L} > 0. \quad \blacksquare$$

Proof of Proposition 6:

Proof. The elasticity ε_{n^*W} can be written as

$$\varepsilon_{n^*W} = \frac{h^* c''(h^*) - (n^* - 1)(F - L)}{h^* c''(h^*)(F - L)} \frac{W}{n^*}.$$

Substituting the equilibrium value of n^*, the inequality $\varepsilon_{n^*W} > 1$ can be shown to be equivalent to

$$\left(W - \frac{\pi}{r}\right) > P^* = \frac{(W - L)(r + h^*) - c(h^*)}{r} - \frac{(r + h^*)(F - L)W + c''(h^*)h^*(r(F - L) + h^* W)}{r(c''(h^*)h^* + W)}.$$

The threshold for R&D investments being strategic complements is

$$\left(W - \frac{\pi}{r}\right) < \left(W - \frac{\pi}{r}\right)^* = (W - L)(1 + g(W - L)/r) - c(g(W - L))/r.$$

The right-hand side is increasing in g since:

$$\begin{aligned} \frac{d}{dg}((W - L)(1 + g/r) - c(g)/r) &= \frac{W - L - c'(g)}{r} \\ &= \frac{V - L}{r} > 0. \end{aligned}$$

Since $h = g(W - F)$ is smaller than $\hat{h} = g(W - L)$ we find that

$$P^* - \left(W - \frac{\pi}{r}\right)^* < P^* - \frac{(W - L)(r + h^*) - c(h^*)}{r} < 0,$$

or $P^* < \left(W - \frac{\pi}{r}\right)^*$: For the elasticity ε_{n^*W} to be larger than 1, the profit effect must be between these two expressions. $\blacksquare$

Proof of Proposition 9:

Proof. . The effect of an increase in W_1 on the equilibrium value of firms of type 2 is

$$\frac{dV_2^*}{dW_1} = \frac{\partial V_2^*}{\partial h_1^*}\frac{dh_1^*}{dW_1} + \frac{\partial V_2^*}{\partial h_2^*}\frac{dh_2^*}{dW_1} = -\frac{V_2^* - L}{r + m_1 h_1^* + m_2 h_2^*}\left(m_1 \frac{dh_1^*}{dW_1} + (m_2 - 1)\frac{dh_2^*}{dW_1}\right). \quad (54)$$

Now we must calculate dh_i^*/dW_1. To compare how h_1^* and h_2^* change with any parameter x we will do comparative statics on the first-order conditions $c_i'(h_i^*) = W_i - V_i^*$, $i = 1, 2$:

$$\begin{bmatrix} \frac{dh_1^*}{dx} \\ \frac{dh_2^*}{dx} \end{bmatrix} = -\begin{bmatrix} \frac{\partial\left(V_1 - W_1 + c_1'(h_1)\right)}{\partial h_1} & \frac{\partial\left(V_1 - W_1 + c_1'(h_1)\right)}{\partial h_2} \\ \frac{\partial\left(V_2 - W_2 + c_2'(h_2)\right)}{\partial h_1} & \frac{\partial\left(V_2 - W_2 + c_2'(h_2)\right)}{\partial h_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial\left(V_1 - W_1 + c_1'(h_1)\right)}{\partial x} \\ \frac{\partial\left(V_2 - W_2 + c_2'(h_2)\right)}{\partial x} \end{bmatrix}$$

$$= -\begin{bmatrix} \frac{\partial V_1}{\partial h_1} + c_1''(h_1) & \frac{\partial V_1}{\partial h_2} \\ \frac{\partial V_2}{\partial h_1} & \frac{\partial V_2}{\partial h_2} + c_2''(h_2) \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial\left(V_1 - W_1 + c_1'(h_1)\right)}{\partial x} \\ \frac{\partial\left(V_2 - W_2 + c_2'(h_2)\right)}{\partial x} \end{bmatrix}$$

$$= -\begin{bmatrix} c_1''(h_1) - \frac{(m_1-1)(V_1-L_1)}{r+m_1h_1+2m_2h_2} & -\frac{2m_2(V_1-L_1)}{(r+m_1h_1+2m_2h_2)} \\ -\frac{m_1(V_2-L_2)}{r+m_1h_1+m_2h_2} & c_2''(h_2) - \frac{(m_2-1)(V_2-L_2)}{r+m_1h_1+m_2h_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial\left(V_1 - W_1 + c_1'(h_1)\right)}{\partial x} \\ \frac{\partial\left(V_2 - W_2 + c_2'(h_2)\right)}{\partial x} \end{bmatrix}$$

Assume that the equilibrium is stable, so that the determinant Δ of the above square matrix is positive. Therefore, we obtain with $x = W_1$:

$$= \frac{1}{\Delta}\begin{bmatrix} c_2''(h_2) - \frac{(m_2-1)(V_2-L_2)}{r+m_1h_1+m_2h_2} & \frac{2m_2(V_1-L_1)}{r+m_1h_1+m_2h_2} \\ \frac{m_1(V_2-L_2)}{r+m_1h_1+m_2h_2} & c_1''(h_1) - \frac{(m_1-1)(V_1-L_1)}{r+m_1h_1+2m_2h_2} \end{bmatrix} \begin{bmatrix} \frac{r+(m_1-1)h_1+2m_2h_2}{r+m_1h_1+2m_2h_2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{\Delta}\frac{r+(m_1-1)h_1+2m_2h_2}{r+m_1h_1+2m_2h_2}\begin{bmatrix} c_2''(h_2) - \frac{(m_2-1)(V_2-L_2)}{r+m_1h_1+m_2h_2} \\ \frac{m_1(V_2-L_2)}{r+m_1h_1+m_2h_2} \end{bmatrix}$$

Finally, we obtain

$$\frac{dV_2^*}{dW_1} \sim -(V_2^* - L)\, m_1 c_2''(h_2). \ \blacksquare$$

Proof of Proposition 10:

Proof. Assume that $V_2 \geq V_1$: from the first-order conditions

$$c_1'(h_1) = W - V_1 \geq W - V_2 = c_2'(h_2)$$

we obtain that $h_1 > h_2$. Equally from (33) we obtain that

$$V_2 - V_1 = \frac{[c_1(h_1) - c_2(h_2)] - (h_1 - h_2)(W - L)}{r + m_1h_1 + m_2h_2} \geq 0$$

by assumption, or

$$h_1 (W - L) - c_1 (h_1) \leq h_2 (W - L) - c_2 (h_2) .$$

On the other hand,

$$\begin{aligned} h_1 (W - L) - c_1 (h_1) &= h_1 (W - V_1) - c_1 (h_1) + h_1 (V_1 - L) \\ &> h_2 (W - V_1) - c_1 (h_2) + h_2 (V_1 - L) \end{aligned}$$

by the envelope theorem, and because $h_2 < h_1$. Furthermore since $c_1 (0) = c_2 (0)$ and $c_1' (h) < c_2' (h)$ it follows that $c_1 (h) < c_2 (h)$ for all $h > 0$, thus

$$\begin{aligned} ... &> h_2 (W - V_1) - c_2 (h_2) + h_2 (V_1 - L) \\ &= h_2 (W - L) - c_2 (h_2) , \end{aligned}$$

which contradicts $V_2 \geq V_1$. We can therefore conclude that $V_1 > V_2$.

As concerns the comparison between h_1 and h_2, $V_1 > V_2$ only implies that $c_1' (h_1) < c_2' (h_2)$, which does not tell us whether h_1 is larger or smaller than h_2. ■

Proof of Proposition 11:

Proof. . Solve the equilibrium value function

$$F_i = \frac{\pi_i - c_i (h_i^*) + h_i^* W_i + \left((m_i^* - 1) h_i^* + m_j^* h_j^* \right) L_i}{r + m_i^* h_i^* + m_j^* h_j^*}, \tag{55}$$

where $h_i^* = g (W_i - F_i)$, for the total hazard, to find

$$\left(m_i^* h_i + m_j^* h_j \right) = \frac{\pi_i - c_i (h_i^*) - rF_i + h_i^* (W_i - L_i)}{F_i - L_i} \equiv \Gamma_i, \tag{56}$$

This expression must be positive, or

$$\pi_i - c_i (h_i^*) + h_i^* (W_i - L_i) > rF_i.$$

The two expressions Γ_i and Γ_j must be equal for the firms of both types, and no further information about m_i^* can be found from these conditions.

If on the other hand $\Gamma_i > \Gamma_j$, then only one type of firms can exist in equilibrium. Assuming that it is firm j, we have $m_i^* = 0$ and $m_j^* = \Gamma_j / h_j^*$.We find

$$\begin{aligned} V_i &= \frac{\pi_i - c_i (h_i^*) + h_i^* (W_i - L_i) + \Gamma_j L_i}{r + \Gamma_j} \\ &> \frac{\pi_i - c_i (h_i^*) + h_i^* (W_i - L_i) + \Gamma_i L_i}{r + \Gamma_i} \\ &= F_i \end{aligned}$$

The strict inequality holds if $\pi_i - c_i(h_i^*) + h_i^*(W_i - L_i) > rL_i$, which is true since $F_i > L_i$. Now $V_i > F_i$ is not possible in equilibrium because more entry would follow. From this contradiction we can conclude that the firms of type i will be present in the market, and no firm of type j. ■