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Sub-Optimality of the Friedman Rule with Distorting Taxes

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Abstract

We find that the Friedman rule is not optimal with government transfers and distortionary taxation. This result holds for heterogeneous agents, standard homogeneous preferences, and constant returns to scale production functions. The presence of transfers changes the standard optimal taxation result of uniform taxation. As transfers cannot be taxed, a positive nominal net interest rate is the indirect way to tax the additional income derived from transfers. The higher the transfers, the higher is the optimal inflation rate. We calibrate a model with transfers to the US economy and obtain optimal values for inflation substantially above the Friedman rule.

JEL Codes: E52, E62, E63.

Keywords: Friedman rule, fiscal policy, monetary policy, taxes, transfers, inflation.

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1 Introduction

Friedman (1968) shows that a policy rule of zero nominal interest rate maximizes welfare. This policy rule is known as the Friedman rule. A zero nominal interest rate corresponds to a zero inflation tax and to a negative rate of inflation. This policy implies setting the price of obtaining real balances equal to zero, which is approximately equal to its production cost. As the marginal cost of supplying money is negligible, and the marginal benefit should be equal to the marginal cost to maximize welfare, then the nominal interest rate should be set to zero.

Phelps (1973) challenged the relevance of the result in Friedman (1968). According to Phelps, money should be taxed as any other good, taking into account its relative elasticity, if government expenditures must be financed with distortionary taxes. Following Ramsey (1927), the optimum taxation problem is the one of financing a given level of government expenditures that implies the minimum decrease in welfare. Taxes should then be set so that the marginal distortion caused by one unit of revenue collected with one tax is equalized across the different taxes. The standard implication applied to a monetary economy is that the optimal inflation tax would imply a strictly positive nominal interest rate.

It turns out that, when all sources of income can be taxed, the generalized use of the Ramsey policy to justify the Phelps result does not hold (see, among others, Kimbrough 1986, Correia and Teles 1999, Cunha 2008, and Schmitt-Grohe and Uribe 2011). Atkinson and Stiglitz (1972) establish that it is optimal not to distort the relative prices of different consumption goods when preferences are separable in leisure and homothetic in consumption goods. These rules were applied to the cash-credit goods economies by Lucas and Stokey (1983) and Chari et al. (1996) to study the optimal inflation tax. In these models, the inflation tax turns into an effective tax on cash goods. Following the result on uniform taxation of Atkinson and Stiglitz (1972), the Friedman rule is optimal. Moreover, consumption and labor income taxes are perfect substitutes. Optimal taxation is obtained by having zero seigniorage and taxing either labor income or consumption.

However, the Friedman rule might not be optimal when the tax system is not complete, according to the definition of Chari and Kehoe (1999). When the government is unable to tax all sources of income, positive inflation may be a desirable instrument to tax the part of income that cannot be taxed. As all types of private income are devoted to consumption at

some point, and because inflation acts as a tax on consumption, a positive nominal interest rate represents an indirect way to tax all sources of income. Schmitt-Grohe and Uribe (2004) consider the case when firms make profits that cannot be taxed. Nicolini (1998), Cavalcanti and Villamil (2001), and Arbex (2013), consider the presence of an informal sector where agents can evade taxes.

Schmitt-Grohe and Uribe (2011) calibrate three different models, each with a source of income that cannot be taxed to the US economy. They consider a model with decreasing returns to scale, another with monopolistic competition, and an underground economy in which firms can evade income taxes. They find that untaxed income alone cannot explain why the Federal Reserve and other monetary authorities follow an explicit or implicit inflation target of two percent per year, since the models with these frictions imply an optimal rate of inflation that is insignificantly above zero.

Here, we study the optimal monetary policy in the presence of transfers. Transfers are payments to economic agents that are not associated with any exchange of goods or services. Transfers can be payments such as social security, pensions, scholarships, financial aid, medicare, and subsidies. One of the main objectives of these transfers is to redistribute income from the richer to the poorer, and as such they are not taxed. They are substantial in all developed countries. In the US, government transfers payments increased from 4.6 percent of GDP in 1947 to 15 percent in 2016. Considering only federal transfers of social benefits to persons (the main component of transfers), the increase in the same period was from 3.2 percent of GDP to 10.8 percent.

In the presence of transfers, we show that the optimal tax policy changes significantly. Uniform taxation is not optimal and the efficient inflation tax is positive. In our calibration of the model to the US, we obtain optimal inflation rates that are significantly above zero. When transfers as a percentage of GDP are 10 percent, the optimal inflation rate is about 6 percent. Thus, a higher target for the inflation than the one followed by the generality of the central banks in the world, is warranted given the existence of transfers.

Moreover, we find that the equivalency between the tax instruments depends on the way in which transfers are introduced. We show this in a simple cash in advance economy with only a cash good. The tax on consumption and the tax on labor income are perfect substitutes if

the transfers, adjusted for the price gross of all taxes (including the nominal interest rate), are constant. Instead, if the path of transfers is constant, adjusted only for the price, then the optimal labor income tax is zero. In this case, the nominal interest rate and the consumption tax are perfect substitutes. The Friedman rule is optimal in this case, but there are other optimal policies that involve positive seigniorage.

The paper is organized as follows. Section 2 considers a model with heterogeneous households. Heterogeneous agents provides a justification for government transfers. Transfers are introduced as an instrument to reduce inequality. An economy with a cash good and a credit good is considered and we show that uniform taxation is not optimal. In section 3, we calibrate a simplified version of the model to the US and obtain an estimate of the optimal inflation rate. In section 4, we consider a version of the model with a cash good only. We show that the optimal tax policy changes with the particular way in which transfers are introduced. Section 5 states the main conclusions.

2 The Model

Consider an economy with heterogeneous households and two types of consumption goods: a cash good and a credit good. Each household makes decisions on consumption and labor, and cash is required to purchase the cash good. Households have measure 1, are uniformly distributed over $[0, 1]$, and are indexed by $i \in [0, 1]$. As in Correia (2010), households are different in two dimensions: the efficiency level e_i and the initial wealth \mathbb{W}_{i0} . The efficiency level affects the result of labor in the following way: l_{it} hours of work imply $e_i l_{it}$ units of efficiency.¹ Labor income depends on the efficiency units. The productivity, or real wage, for each unit of $e_i l_{it}$ is normalized to one. Time is discrete, $t = 0, 1, 2, \dots$. There is a constant returns to scale technology that transforms units of efficiency into output. The output can be used for private consumption of cash goods c_{1it} , credit goods c_{2it} , and public consumption g_t . The resource constraint is

$$\int_0^1 (c_{1it} + c_{2it}) di + g_t = \int_0^1 e_i l_{i,t} di. \quad (1)$$

¹Different levels of efficiency and of initial wealth are two ways of introducing inequality in the model. See Castaneda et al. (2003) and Diaz-Gimenez et al. (2011) for a discussion on inequality and the distribution of wealth, earnings, and income.

The utility function of household i is

$$\sum_{t=0}^{\infty} \beta^t U(c_{1it}, c_{2it}, 1 - l_{it}), \quad (2)$$

$0 < \beta < 1$, where the utility function U is strictly concave, satisfies the Inada conditions, is additively separable in leisure and homogeneous in consumption.

Households trade money, bonds, and goods in markets that obey the Lucas and Stokey (1983) timing. At the beginning of period t they trade money and bonds in a centralized market. After this trading, the household splits into a shopper and a worker. The shopper uses money to buy the cash good and to purchase the credit good, the shopper issues nominal claims, which are settled in the assets markets at the beginning of period $t + 1$. The worker is paid in cash at the end of period t .

The budget constraint of each household for the asset market at the beginning of period 0 is given by

$$M_{i0} + B_{i0} \leq \mathbb{W}_{i0}, \quad (3)$$

and, at the beginning of period t , it is given by

$$\begin{aligned} M_{it} + B_{it} \leq & p_{t-1}(1 - \tau_{t-1})e_i l_{it-1} - p_{t-1}(1 + \tau_{1t-1})c_{1it-1} \\ & - p_{t-1}(1 + \tau_{2t-1})c_{2it-1} + R_{t-1}B_{it-1} + M_{it-1} + Z_{it-1}, \end{aligned} \quad (4)$$

$t \geq 1$, where M_{it} and B_{it} denote the stocks of money and bonds, p_t denotes the price level, R_t denotes the gross nominal interest rate, τ_{1t} and τ_{2t} denote the consumption tax rate on the cash good and on the credit good, respectively, τ_t denotes the tax rate on the labor income, and Z_{it} denotes the transfer.² The initial wealth \mathbb{W}_{i0} is exogenous. There is also a no-Ponzi condition

$$\lim_{t \rightarrow \infty} Q_t (M_{it} + B_{it}) \geq 0, \quad (5)$$

where $Q_t \equiv \prod_{k=0}^{t-1} R_k^{-1}$, $Q_0 \equiv 1$, is the price at 0 of a bond that pays 1 dollar at t .

²In this framework, where agents are heterogeneous, positive transfers from the government have the potential to reduce the inequality across households.

The cash-in-advance constraint is given by

$$p_t (1 + \tau_{1t}) c_{1it} \leq M_{it}, \quad (6)$$

for $t \geq 0$. Without loss of generality, we assume throughout that the cash in advance restriction holds with equality.³

The period t government budget constraint is

$$\begin{aligned} R_t \int_0^1 B_{it} di + \int_0^1 Z_{it} di + p_t g_t = p_t \tau_{1t} \int_0^1 c_{1it} di + p_t \tau_{2t} \int_0^1 c_{2it} di \\ + p_t \tau_t \int_0^1 e_i l_{it} di + \int_0^1 B_{it+1} di + \int_0^1 M_{it+1} di - \int_0^1 M_{it} di, \end{aligned} \quad (7)$$

for $t \geq 0$. The terms on the left hand side of (7) are outflows and the terms on the right hand side of (7) are inflows. The government must charge the same tax rate to each household, but it can discriminate the transfers across households. The government policy on taxes is given by $u_t \equiv (\tau_t, \tau_{1t}, \tau_{2t})$.

Let $x_{it} \equiv (c_{1it}, c_{2it}, l_{it}, M_{it}, B_{it})$ denote allocations for households $i \in [0, 1]$ and let $v_t \equiv (p_t, R_t)$ denote a price system for the economy. The problem of household i is to choose an allocation x_{it} that maximizes (2) given taxes u_t , prices v_t , transfers Z_{it} , and initial wealth \mathbb{W}_{i0} , and the constraints (3), (4) and (6). We define a competitive equilibrium an allocation $(x_{it})_{i=1}^\infty$, a price system v_t and a policy u_t such that: (i) each allocation x_{it} solves the problem of household i given the price system, the government policy and the transfers, $i \in [0, 1]$; and (ii) the resource constraint (1) is satisfied.

A Ramsey problem is defined as an allocation, a set of prices and policy variables such that welfare is maximized and the allocation can be decentralized as a competitive equilibrium with those prices and policy variables. In the context of the model, the Ramsey problem consists in choosing the paths of the nominal interest rate, the consumption tax rates, and the labor income tax rate, that implement the competitive equilibrium allocation and that yield the highest level of welfare to households. We say that the government follows an optimal policy if the policy solves the Ramsey problem.

We follow the primal approach to obtain the Ramsey allocation and policy variables.

³That will happen if $R > 1$.

According to this approach, the government maximizes welfare by choosing directly the allocations of households, taking into account the resource constraint of the economy, and the fact that households react to the tax rates. As in Lucas and Stokey (1983), we solve the problem in two steps. We first use the first order conditions of the maximization problem of the households to write the taxes rates as a function of the allocations. Then, we solve for the optimal allocations after replacing the tax rates by these functions.

Following this approach, we first solve the maximization problem of each household. The first order conditions of the maximization problem of household i imply

$$\frac{U_{i1}(t)}{U_{i2}(t)} = \frac{R_t(1 + \tau_{1t})}{1 + \tau_{2t}}, \quad (8)$$

$$\frac{U_{i2}(t)}{U_{i3}(t)} = \frac{1 + \tau_{2t}}{(1 - \tau_t)e_i}, \quad (9)$$

$$\frac{U_{i1}(t)}{p_t(1 + \tau_{1t})} = \beta R_t \frac{U_{i1}(t+1)}{p_{t+1}(1 + \tau_{1t+1})}. \quad (10)$$

The notation $U_{ij}(t)$, $j = 1, 2, 3$, denotes the first derivative of $U(c_{1it}, c_{2it}, 1 - l_{it})$, for household i at time t , with respect to the argument j . Equations (8)-(10) determine the tax rates as a function of the allocations.

In the second step, we maximize (2) subject to the resource constraints (1), the budget constraints and the cash-in-advance constraint of the household, substituting out the tax rates from (8)-(10). To do this, we write the budget constraint of household i in its present value form. This is obtained by multiplying condition (3) by Q_0 and condition (4) for time t by Q_t , $t = 1, 2, \dots$. Adding the resulting inequalities implies

$$\begin{aligned} & \sum_{t=0}^{T-1} Q_{t+1} p_t (1 + \tau_{1t}) c_{1it} + \sum_{t=0}^{T-1} Q_{t+1} p_t (1 + \tau_{2t}) c_{2it} + \sum_{t=0}^{T-1} Q_{t+1} (R_t - 1) M_{it} \\ & - \sum_{t=0}^{T-1} Q_{t+1} p_t (1 - \tau_t) e_i l_{it} - \sum_{t=0}^{T-1} Q_{t+1} Z_{it} + Q_t (M_{it} + B_{it}) \leq \mathbb{W}_{i0}, \end{aligned} \quad (11)$$

for $T > 1$. Using the no-Ponzi condition (5), this inequality implies

$$\begin{aligned} & \sum_{t=0}^{\infty} Q_{t+1} p_t (1 + \tau_{1t}) c_{1it} + \sum_{t=0}^{\infty} Q_{t+1} p_t (1 + \tau_{2t}) c_{2it} + \sum_{t=0}^{\infty} Q_{t+1} (R_t - 1) M_{it} \\ & - \sum_{t=0}^{\infty} Q_{t+1} p_t (1 - \tau_t) e_i l_{it} - \sum_{t=0}^{\infty} Q_{t+1} Z_{it} \leq \mathbb{W}_{i0}. \end{aligned} \quad (12)$$

At the optimum for household i , the intertemporal budget constraint holds with equality. Assume that real transfers are time invariant, that is, $Z_{it} = p_t (1 + \tau_{2t}) z_i$. Being time invariant is not important for our results. What is important for our results is that transfers are not taxed in the same way as labor income or purchases of consumption goods.⁴ With the cash-in-advance constraint holding with equality, the intertemporal budget constraint can then be rewritten as

$$\begin{aligned} & \sum_{t=0}^{\infty} q_{t+1} (1 + \tau_{1t}) R_t c_{1it} + \sum_{t=0}^{\infty} q_{t+1} (1 + \tau_{2t}) c_{2it} \\ & - \sum_{t=0}^{\infty} q_{t+1} (1 - \tau_t) e_i l_{it} - \sum_{t=0}^{\infty} q_{t+1} (1 + \tau_{2t}) z_i = \frac{\mathbb{W}_{i0}}{p_0}, \end{aligned} \quad (13)$$

where $q_t \equiv Q_t \frac{p_{t-1}}{p_0}$. This equation is known in the literature as the implementability condition. As is standard in the literature, we assume that the government is able to fully tax the initial wealth of the households \mathbb{W}_{i0} , but that this government revenue is not enough to pay for the present value of public expenditures.⁵ As a result, it is necessary to raise additional government revenues by resorting to distortionary taxation. Without loss of generality, therefore, we set $\frac{\mathbb{W}_{i0}}{p_0} = 0$.

Using the first order conditions, (8), (9) and (10), we can write (13) as

$$\sum_{t=0}^{\infty} \beta^t U_{i1}(t) c_{1it} + \sum_{t=0}^{\infty} \beta^t U_{i2}(t) c_{2it} = \sum_{t=0}^{\infty} \beta^t U_{i3}(t) e_i l_{it} + \sum_{t=0}^{\infty} \beta^t U_{i2}(t) z_i, \quad (14)$$

for $i \in [0, 1]$.

For a vector $(\tilde{x}_{it})_{i \in [0,1]} \equiv (c_{1it}, c_{2it}, l_{it})_{i \in [0,1]}$ that satisfies (14) and (1), it is always possible

⁴This condition is equivalent to $Z_{it} = p_t (1 + \tau_{1t}) R z_i$. Other alternatives for the evolution of the real transfers would have different implications. We discuss this issue in section 4, where we allow another alternative for real transfers.

⁵This can be done either with a tax over the initial wealth or by making the initial price level approaching infinity (when $\mathbb{W}_{i0} > 0$).

to find a policy u_t that satisfies conditions (9), (8) and a price system v_t that satisfies (10). This policy u_t and this price system v_t , together with the allocation $(x_{it})_{i \in [0,1]} \equiv (\tilde{x}_{it}, M_{it}, B_{it})_{i \in [0,1]}$, where M_{it} satisfies (6) with equality and B_{it} satisfies (4), is a competitive equilibrium.

The Ramsey allocation problem is the vector $(\tilde{x}_{it})_{i \in [0,1]}$ that maximizes

$$\int_0^1 \omega_i \sum_{t=0}^{\infty} \beta^t U(c_{1it}, c_{2it}, 1 - l_{it}) di, \quad (15)$$

for weights $\omega_i > 0$, and satisfies the restrictions (14), one for each household $i \in [0, 1]$, and the resource constraint (1). Let λ_i and $\beta^t \alpha_t$ be the Lagrange multipliers associated with the restrictions (14) and (1) respectively. The first order conditions of this problem are

$$U_{i1}(t) (\omega_i + \lambda_i) + \lambda_i U_{i11}(t) c_{it} + \lambda_i U_{i21}(t) q_{it} - \lambda_i U_{i21}(t) z_i + \alpha_t = 0, \quad (16)$$

$$U_{i2}(t) (\omega_i + \lambda) + \lambda_i U_{i12}(t) c_{it} + \lambda_i U_{i22}(t) q_{it} - \lambda_i U_{i22}(t) z_i + \alpha_t = 0, \quad (17)$$

$$- U_{i3}(t) (\omega_i + \lambda_i) + \lambda_i U_{i33}(t) l_{it} - e_i \alpha_t = 0. \quad (18)$$

We are now ready to prove the following proposition.

Proposition 1 *If the utility function is additively separable in leisure and homogeneous in consumption, and transfers are positive, then $\frac{U_{i1}(t)}{U_{i2}(t)} \neq 1$. As the optimal effective tax over the cash good, $R_t(1 + \tau_{1t})$, is different from the tax on the credit good, $(1 + \tau_{2t})$, then the optimal commodity taxation is not uniform.*

Proof. Since U is homogeneous in consumption, then

$$- \frac{U_{i11}(t) c_{1it} + U_{i21}(t) c_{2it}}{U_{i1}(t)} = - \frac{U_{i12}(t) c_{1it} + U_{i22}(t) c_{2i,t}}{U_{i2}(t)} \equiv \mu, \quad (19)$$

where $\mu (\neq 0)$ is a constant. From (16) and (17) we obtain

$$\frac{U_{i1}(t)}{U_{i2}(t)} = \frac{\omega_i + \lambda_i \left(1 - \mu - \frac{U_{i22}(t) z_i}{U_{i2}(t)}\right)}{\omega_i + \lambda_i \left(1 - \mu - \frac{U_{i21}(t) z_i}{U_{i1}(t)}\right)}, \quad (20)$$

where $\lambda_i \neq 0$. With $z_i > 0$, then $\frac{U_{i1}(t)}{U_{i2}(t)} \neq 1$ as in general $\frac{U_{i22}(t) z_i}{U_{i2}(t)} \neq \frac{U_{i21}(t) z_i}{U_{i1}(t)}$. ■

If $z = 0$, then the optimal allocation must be such that $\frac{U_{i1}(t)}{U_{i2}(t)} = 1$. This is the standard result. The optimal commodity taxation must be uniform. The effective consumption tax on the two goods must be the same. The policy that implements this allocation must satisfy condition (8), which implies $R_t(1 + \tau_{1t}) = 1 + \tau_{2t}$. There are many combinations of taxes and nominal interest rates that satisfy this condition. However, if the tax on the cash good cannot be different from the tax on the credit good, that is $\tau_{1t} = \tau_{2t}$, then the Friedman rule, $R_t = 1$, is the only efficient policy.

With $z > 0$, optimal taxes are not uniform anymore because we have an extra term, $U_{i22}(t)z_i$. This term appears in the first order conditions because, by assumption and as we usually observe in practice, transfers are not taxed. Transfers from the government are not taxed either for political reasons or because they are intended to decrease inequality across households.

3 Quantitative Results and Discussion

To calculate the quantitative implications of the model and discuss its results, we simplify some aspects of the general economy above. Consider an economy with homogeneous households, where $e_i = 1$, and $\mathbb{W}_{i0} = \mathbb{W}_0$. All households have standard preferences given by

$$U(c_{1t}, c_{2t}, 1 - l_t) = \frac{c_{1t}^{1-\theta}}{1-\theta} + \gamma \frac{c_{2t}^{1-\theta}}{1-\theta} + \eta \frac{(1 - l_t)^{1-\theta}}{1-\theta}, \quad (21)$$

where θ , γ and η are positive constants.⁶ We follow the same notation. That is, c_{1t} and c_{2t} denote the cash and credit goods at time t , respectively, and l_t denotes hours of work at time t . The parameter θ is the coefficient of relative risk aversion. The parameters γ and η determine the relative weight on credit goods and leisure $1 - l_t$, respectively.

We consider homogeneous households and the preferences in (21) to facilitate the discussion of our results. Moreover, the preferences in (21) constitute an important case. We emphasize, however, that our results hold for heterogeneous households and for a general utility function $U(c_{1t}, c_{2t}, 1 - l_t)$ that satisfies the usual assumptions of concavity, separability, and homogeneity.

⁶Tiago Cavalcanti suggested this functional form for the utility function.

The first order conditions of the households' problem imply equations (9)-(10), which yield

$$R_t = \frac{1 + \tau_{2t}}{1 + \tau_{1t}} \frac{1}{\gamma} \left(\frac{c_{2t}}{c_{1t}} \right)^\theta, \quad (22)$$

$$\frac{1 + \tau_{2t}}{1 - \tau_t} = \frac{\gamma}{\eta} \left(\frac{1 - l_t}{c_{2t}} \right)^\theta, \quad (23)$$

$$1 + \pi_{t+1} = R_t \frac{1 + \tau_{1t}}{1 + \tau_{1t+1}} \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\theta}, \quad (24)$$

where $\pi_{t+1} \equiv p_{t+1}/p_t - 1$ is the inflation rate from period t to period $t + 1$.

The resource constraint is now given by

$$c_{1t} + c_{2t} + g = l_t, \quad (25)$$

where we let $g_t = g$ to focus on a stationary equilibrium.

As we did in the previous section, we now solve the second utility maximization problem, the Ramsey problem. The Ramsey problem is to obtain the allocation that maximizes (21) subject to the resource constraint (25) and the implementability condition below, which analogous to (14), with $e_i = 1$ and without the subscript i ,

$$\sum_{t=0}^{\infty} \beta^t U_1(t) c_{1t} + \sum_{t=0}^{\infty} \beta^t U_2(t) c_{2t} = \sum_{t=0}^{\infty} \beta^t U_3(t) l_t + \sum_{t=0}^{\infty} \beta^t U_2(t) z. \quad (26)$$

Let λ and $\beta^t \alpha_t$ be the Lagrange multipliers respectively associated with (26) and (25). As the objective function of this problem is concave, the Ramsey allocation must be stationary. That is, in equilibrium, $c_{1t} = c_1$, $c_{2t} = c_2$, $l_t = l$ and $\alpha_t = \alpha$.⁷ The first order conditions (16)-(18), (26) and (25) imply the system of equations

$$1 + \lambda(1 - \theta) = \alpha c_1^\theta, \quad (27)$$

$$1 + \lambda(1 - \theta) + \lambda \theta \frac{z}{c_2} = \frac{1}{\gamma} \alpha c_2^\theta, \quad (28)$$

$$1 + \lambda + \lambda \theta \frac{l}{1 - l} = \frac{1}{\eta} \alpha (1 - l)^\theta, \quad (29)$$

⁷This result is also a consequence of the fact that, in this formulation, $z_t = z$ and $g_t = g$ in the household budget constraint.

$$c_1^{1-\theta} + \gamma c_2^{1-\theta} = \eta(1-l)^{-\theta} l + \gamma c_2^{-\theta} z, \quad (30)$$

$$c_1 + c_2 + g = l. \quad (31)$$

The system (27)-(31) implies a set of five equations and five endogenous variables c_1 , c_2 , l , α and λ . The solution to this problem is the Ramsey allocation for this economy. We retrieve the optimal tax rates and interest rates using the first order conditions of the households' problem, constraints (22)-(24).

This problem is useful to understand that $z = 0$ implies uniform taxation and the Friedman rule, and that $z > 0$ implies a departure from the Friedman rule when it is not possible to set different consumption taxes on the cash and credit goods. To obtain analytical expressions, set $\gamma = 1$ and $\theta = 1$. From equations (27) and (28), we obtain

$$\frac{c_2}{c_1} = 1 + \lambda \frac{z}{c_2}. \quad (32)$$

Moreover, tax rates and the gross interest rate must satisfy

$$\frac{R(1 + \tau_1)}{1 + \tau_2} = \frac{c_2}{c_1}, \quad (33)$$

$$\frac{1 + \tau_2}{1 - \tau} = \frac{1}{\eta} \left(\frac{1-l}{c_2} \right), \quad (34)$$

$$1 + \pi = \beta R. \quad (35)$$

Without loss of generality, equations (33)-(35) focus on the case with constant interest rate and taxes.

Let $z = 0$. This is the case in which we have uniform taxation. From equation (32), we obtain $c_2 = c_1$. Moreover,

$$R = \frac{1 + \tau_2}{1 + \tau_1}, \quad (36)$$

$$\frac{1 + \tau_2}{1 - \tau} = \frac{1}{\eta} \left(\frac{1-l}{c_2} \right), \quad (37)$$

$$1 + \pi = \beta R. \quad (38)$$

The system (36)-(38) is indeterminate as there are five endogenous variables (R , τ_2 , τ_1 , τ and

π), and three equations. However, if the tax rate on the cash good cannot be different from the tax rate on the credit good, $\tau_1 = \tau_2$, then the Friedman rule, $R = 1$, is the unique solution to the system.

Let now $z > 0$. Equation (32) then implies $c_2/c_1 > 1$, as $\lambda > 0$. We still have an indeterminacy of the optimal taxes and interest rate from (33)-(35). However, from equation (33), optimality now requires non-uniform taxation, $R(1 + \tau_1) > 1 + \tau_2$. Uniform taxation is not optimal if transfers are positive.

Transfers are a pure rent and efficiency requires that they should be completely taxed, that is, they should have an 100 percent tax rate. Since transfers cannot be taxed directly, optimality requires that they should be taxed indirectly. This could be achieved with $R = 1$, and by taxing more the cash good than the credit good, $\tau_1 > \tau_2$.

The Ramsey planner has an incentive to inflate above the level implied by the Friedman rule as a way to levy an indirect tax on transfers. As Schmitt-Grohe and Uribe (2011) put it, if we add a source of income, then the government is likely to depart from the Friedman rule, if the instrument to tax that income is not available or if there is an upper limit on that instrument tax rate.⁸

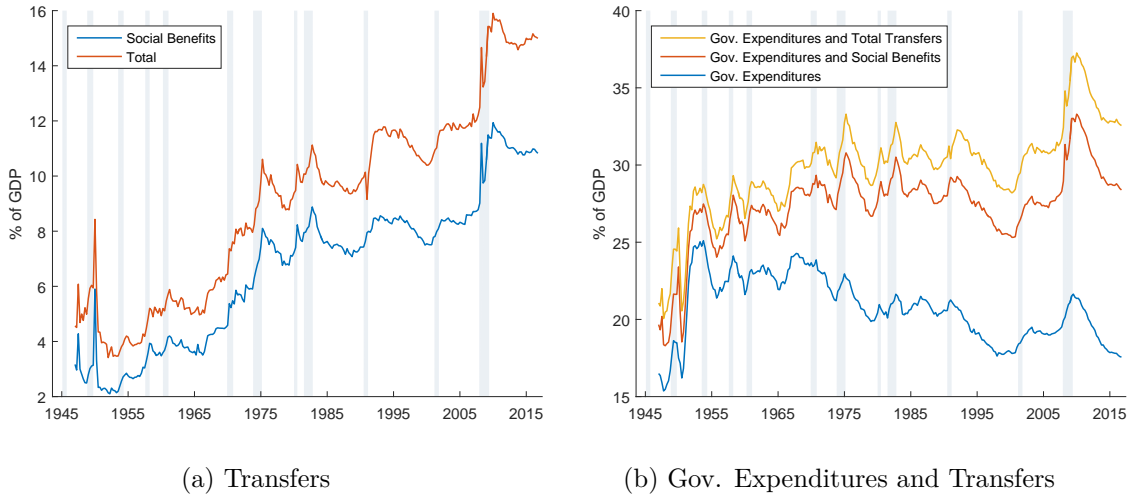


Figure 1: Transfers and government expenditures over time. Social benefits are total federal transfers of social benefits to persons (social security, medicare, veterans' benefits and other transfers). Total transfers include medicaid, state and local transfers, and transfers to the rest of the world. Shaded areas indicate NBER U.S. recessions. Source: Federal Reserve Bank of St. Louis.

⁸This effect happens, for example, when the inflation tax is used to tax the underground economy (Nicolini 1998, Cavalcanti and Villamil 2003) or when the government has difficulties to enforce taxes (Arbex 2013).

Suppose, for example, that the government cannot set different taxes for cash and credit goods. In this case, $\tau_1 = \tau_2$.⁹ The government, for example, might not be able to distinguish cash and credit goods. One of the reasons for the difficulty to distinguish cash and credit goods is that the same good can be a cash good for some households and a credit good for others. Households are heterogeneous and this implies different consumption choices. Different consumption choices have implications with respect to the instruments used to make transactions. Avery et al. (1987), Kennickell et al. (1997), Mulligan and Sala-i-Martin (2000) and Attanasio et al. (2002), among others, point out that high-income households use a smaller fraction of cash on their transactions than low-income households. The poorest households do not own a checking account.¹⁰ The same good can be a cash good for a poor household and a credit good for a rich household.

If the government is constrained to set $\tau_1 = \tau_2$, equation (33) implies

$$R > 1. \tag{39}$$

The Friedman rule does not hold. As the government cannot set higher taxes for cash goods, as it would be implied by equations (32)-(33), then the government needs to set $R > 1$ to obtain the Ramsey policy. The cash good is taxed more than the credit good, but now this is done through the inflation tax. This policy reaches the cash good because this good is subject to the cash-in-advance constraint. From equation (38), we have $\pi > \beta - 1$. The inflation rate is higher than $\beta - 1$, which is the inflation rate implied by the Friedman rule. Although $R > 1$ when $\tau_1 = \tau_2$, the values of the consumption and labor taxes are still indeterminate. From equation (37), the labor tax is obtained once a value for the consumption tax is chosen.

To determine the quantitative implications of our findings, we parameterize the model based on US data and solve the system of equations (27)-(31) together with (33)-(35).

Figure 1 shows the evolution of transfers and government expenditures as a percentage of GDP over time. Transfers increased substantially from 1947 to 2016. As stated in the introduction, total government transfers payments during the period increased from 4.6

⁹It is common to assume the same tax rate for cash and credit goods. This is done, for example, in Cooley and Hansen (1992).

¹⁰Erosa and Ventura (2002) find that expected inflation acts as a regressive consumption tax, increasing inequality, as lower-income households tend to use more cash as a percentage of their total expenditures.

Table 1: Parameters

	Parameter	Value
Intertemporal discount factor	β	0.98
Coefficient of relative risk aversion (CRRA)	θ	1
Preference parameter on credit goods	γ	1
Preference parameter on leisure	η	4.213

The values of η , g and z are found simultaneously such that hours of work are equal to 0.3 when the transfers-to-GDP ratio z/y is equal to 8 percent and the government-to-GDP ratio g/y is equal to 20 percent. For $\theta = 0.5$ and 2, $\eta = 1.743$ and $\eta = 24.58$ respectively. $\theta = 1$ in figures 2 and 3. The value of g is maintained constant during the simulations while the value of z increases from $z = 0$ to a value such that $z/y = 15$ percent.

percent of GDP to 15 percent of GDP. Federal government transfers as social benefits to persons increased from 3.2 percent of GDP to 10.8 percent of GDP. In contrast, government expenditures have a more stable behavior. Government expenditures changed from 16.5 percent of GDP in 1947 to 17.6 percent to 2016; the average for the whole period is 20.7 percent of GDP. As we show below, this change in the composition of transfers and government expenditures has important consequences for optimal taxation.¹¹

Table 2: Interest rates, inflation and labor taxes for different values for transfers

z (% of GDP)	$\theta = 0.5$			$\theta = 1$			$\theta = 2$		
	R	π (% p.a.)	τ_l (%)	R	π (% p.a.)	τ_l (%)	R	π (% p.a.)	τ_l (%)
0	1.000	-2.00	11.6	1.000	-2.00	13.4	1.000	-2.00	14.1
5	1.027	0.61	17.6	1.029	0.84	18.3	1.030	0.93	18.6
10	1.084	6.27	23.2	1.083	6.15	22.5	1.083	6.15	22.2
15	1.199	17.5	28.6	1.172	14.9	25.6	1.167	14.38	24.6

Gross interest rate R , inflation π , and labor tax τ_l for different coefficients of relative risk aversion θ and different levels of transfers z . Inflation in percent per annum. To determine τ_l , the consumption tax is set to 6.5% ($\tau_1 = \tau_2$). Parameters in table 1. Results obtained from equations (27)-(31) and (33)-(35).

We need to set values for the preferences parameters, government expenditures g , and transfers z . We set $\beta = 0.98$ for the intertemporal discount, which implies a real interest rate of 2 percent per year. The value of the weight on leisure η is determined so that hours of work are equal to 0.3 when the ratio of government expenditures to GDP is equal to 20 percent

¹¹Federal transfers of social benefits to persons include social security, medicare, veterans' benefits, unemployment insurance, and other transfers. Social security and medicare are about 70 percent of social benefits since the mid 1960s. Veterans' benefits decreased from 70 percent of social benefits in 1947 to 5 percent in 2016. Unemployment insurance from 2000 to 2016 is on average 4 percent of social benefits. Total government transfers include federal social benefits, medicaid, state and local transfers, and transfers to the rest of the world. Medicare and medicaid together comprise about 40 percent of total government transfers. Government expenditures include consumption expenditures and gross investment. Data from the Federal Reserve Bank of St. Louis.

and the ratio of transfers to GDP is equal to 8 percent. The ratio of g over GDP reflects the mean of this variable since 1947. The value for transfers replicates the mean of the ratio of federal transfers in the form of social benefits to persons over GDP since 1970. We set $\gamma = 1$ so that cash and credit goods have an equal weight.¹² We follow the same procedure to obtain the parameters for $\theta = 0.5, 1,$ and 2 . The value $\theta = 1$ implies logarithmic utility. Once we set the parameters, we calculate the optimal allocations and taxes for different values of z . We change z so that the ratio of transfers to GDP to vary from zero to 15 percent. Table 1 shows the parameters used in the simulations.

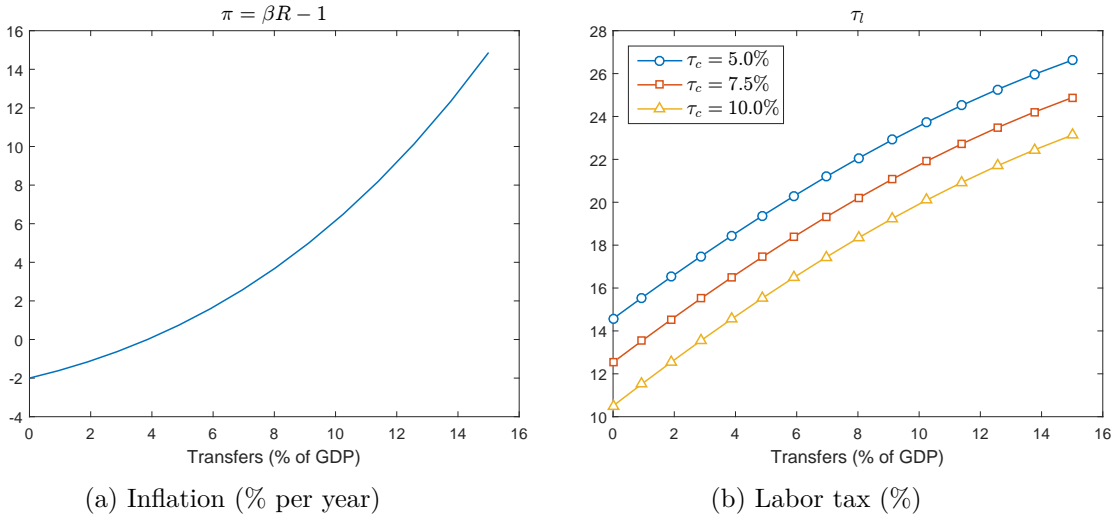


Figure 2: The Friedman rule holds for $R = 1$ or inflation equal to -2 percent per year. It holds when transfers are equal to zero. Results from simulations. See table 1 for parameters. Transfers z increase for a constant given value of government expenditures g .

Table 2 and figures 2 and 3 show the main results. Table 2 shows results for different values of θ . Figures 2 and 3 show the results for additional variables and $\theta = 1$. For $z = 0$, we see that the Ramsey policy implies the Friedman rule, with $R = 1$ and inflation of -2 percent per year. The consideration of positive transfers implies a substantial departure from the Friedman rule. As transfers increase to 5 percent of GDP, the optimal policy for $\theta = 1$ implies $R = 1.029$ and $\pi = 0.8$ percent per year. With transfers of 10 percent of GDP, the optimal policy implies inflation of 6.15 percent per year. The values are robust to changes in θ .

The increase in transfers requires higher inflation and higher labor taxes. As leisure is

¹²Cooley and Hansen (1991, 1992) use $\gamma = 1$ and smaller values such as $\gamma = 0.2$. We also used these values in the simulations and obtained results qualitatively similar.

not taxed, hours of work decrease with higher transfers. To determine the optimal value for the labor tax τ_l , we use equation (37) together with a value for the consumption tax, τ_c . To obtain a value for the consumption tax, we use estimates for the effective tax rates on consumption, as described by Mendoza et al. (1994).

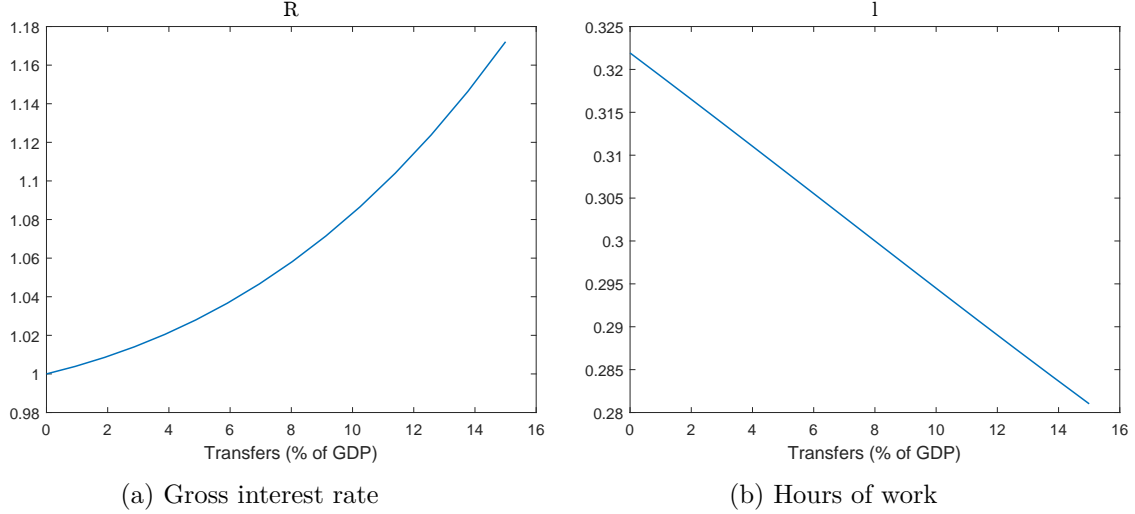


Figure 3: The Friedman rule ($R = 1$) holds when transfers are equal to zero. Total hours of work are normalized to 1. Results from simulations. See table 1 for the parameters.

The calculations of effective tax rates take into account aggregate tax revenues from consumption taxes and aggregate sales. In this way, a product with a high tax rate but low demand would not be over represented if the consumption tax were calculated as an average of the existing rates. Mendoza, Razin, and Tesar (1994) find τ_c between 6.4 and 5.1 percent for the US from 1965 to 1988, with smaller rates for the most recent periods. Silva (2008), using a similar procedure, finds values for τ_c for the US between 5 percent and 7.1 percent for 1970-2001. Carey and Rabesona (2002), with a revised methodology, find values between 6.4 percent and 6.7 percent. The values, therefore, are largely compatible across estimates.

For table 2, we set $\tau_c = 6.5$ percent and find $\tau_l = 22.5$ percent, for $\theta = 1$, when transfers are 10 percent of GDP. The values for the labor tax are similar for different values of θ . In figure 2, we calculate the optimal labor tax for $\tau_c = 5, 7.5$ and 10 percent. For higher consumption taxes, the required labor taxes are smaller. When transfers are 10 percent of GDP, the optimal labor tax varies between 19.9 percent and 23.6 percent.

The main result of this section is that transfers have a significant impact on the estimates

of optimal inflation in the standard cash-in-advance model with a credit good. The optimal inflation reacts strongly to changes in the level of transfers. When transfers are zero the optimal inflation rate is -2 percent, that is, equal to the negative of the real interest rate. When transfers are 10 percent of GDP, which according to the data is a conservative value, the optimal inflation is around 6 percent.

Our results were obtained assuming that the government maintains the real value of transfers every period, taking into account the full tax on the consumption goods. This assumption was made to simplify the analysis. In the next section, we investigate whether results are robust to changes in this assumption.

4 Alternative Paths for Real Transfers

The optimal tax policy depends on the way transfers are introduced. There are two interesting possibilities for the path of transfers, either real transfers adjusted for all taxes are time invariant, i.e., $Z_{it} = p_t (1 + \tau_2) z_i$, which is the assumption we have been using, or real transfers adjusted only for the price level are constant, i.e., $Z_{it} = p_t z_i$. We compare these two alternative assumptions. It turns out that the labor income tax and the consumption tax are equivalent instruments under the first assumption, but not under the second assumption. To study this issue, we consider a version of the economy in section 2 with one consumption good instead of two. Throughout, whenever possible, we keep the same notation.

There is a constant returns to scale technology that transforms units of efficiency into output. Output can be used for private consumption of cash goods and public consumption. The resource constraint is

$$\int_0^1 c_{it} di + g_t = \int_0^1 e_i l_{it} di. \quad (40)$$

The private consumption good must be bought with money according to the standard cash-in-advance constraint

$$p_t (1 + \tau_{ct}) c_{it} \leq M_{it}, \quad (41)$$

where τ_{ct} is the tax rate on the consumption good. The utility function of household i is given

by

$$\sum_{t=0}^{\infty} \beta^t U(c_{it}, 1 - l_{it}), \quad (42)$$

with $0 < \beta < 1$. As before, we normalize total time to one; $1 - l_{i,t}$ denotes leisure. Function U is strictly concave and satisfies the Inada conditions.

The budget constraint of each household for the asset market at the beginning of time t is given by

$$M_{it} + B_{it} \leq R_{t-1}B_{it-1} + M_{it-1} - p_{t-1}(1 + \tau_{ct-1})c_{it-1} + p_{t-1}(1 - \tau_{t-1})e_i l_{it-1} + Z_{it}. \quad (43)$$

Households are subject to the no-Ponzi condition (5).

The household i 's problem is to choose a vector $(M_{it}, B_{it}, c_{it}, l_{it})_{t=0}^{\infty}$ that maximizes (42) subject to (43), (41), the no-Ponzi condition, and the initial conditions \mathbb{W}_{i0} . The first-order conditions of household i 's problem include

$$\frac{U_{i1}(t)}{U_{i2}(t)} = \frac{R_t(1 + \tau_{ct})}{(1 - \tau_t)e_i}, \quad (44)$$

$$\frac{U_{i1}(t)}{p_t(1 + \tau_{ct})} = \beta R_t \frac{U_{i1}(t+1)}{p_{t+1}(1 + \tau_{ct+1})}. \quad (45)$$

The notation $U_j(t)$, $j = 1, 2$ denotes the first derivative of $U(c_{it}, 1 - l_{it})$ with respect to the argument j .

The intertemporal budget constraint for the household i is

$$\sum_{t=0}^{\infty} q_{t+1}(1 + \tau_c)Rc_{it} = \frac{\mathbb{W}_{i0}}{p_0} + \sum_{t=0}^{\infty} q_{t+1} \left(\frac{Z_{it}}{p_t} + (1 - \tau)e_i l_{it} \right). \quad (46)$$

An efficient way of raising government revenue is to tax the initial real wealth, as it is equivalent to a lump-sum tax. Therefore, we assume that the initial real nominal wealth of the household is fully taxed.¹³

As the utility function U is strictly concave by assumption and transfers and public consumption are time invariant, then consumption and leisure are also time invariant. It follows from (44) that time invariant consumption and leisure can be achieved with constant

¹³That can be done by making the initial price level extremely high or by having a 100 percent tax on the initial wealth.

tax rates and nominal interest rate. Thus, from now on, without loss of generality, we assume that τ_{ct}, τ_t, R_t are time invariant.¹⁴

Denote the path of transfers $Z_{it} = p_t(1 + \tau_c)Rz_i$ as path A, and the path $Z_{it} = p_tz_i$ as path B. The following lemma states that, in a cash in advance economy, the consumption tax, the labor income tax and the nominal interest rate are equivalent instruments when the path for the transfers follows path A.

Lemma 1 *Assume that transfers follow path A. The policy vector (τ_c^a, τ^a, R^a) is equivalent to the policy vector (τ_c^b, τ^b, R^b) where $\frac{(1+\tau_c^a)R^a}{(1-\tau^a)} = \frac{(1+\tau_c^b)R^b}{(1-\tau^b)}$.*

Proof. When transfers follow path A and initial wealth is fully taxed, the budget constraint for each household i can be written as

$$\sum_{t=0}^{\infty} q_{t+1} \frac{(1 + \tau_c) R}{(1 - \tau)} c_{it} = \sum_{t=0}^{\infty} q_{t+1} \left(\frac{(1 + \tau_c) R}{(1 - \tau)} z_i + e_i l_{it} \right). \quad (47)$$

This constraint for the vector (τ_c^a, τ^a, R^a) is identical to the one for the vector (τ_c^b, τ^b, R^b) . In the same way, the first order conditions are identical under the two alternative policies. Given a path for government consumption and transfers, the equilibrium prices gross of taxes are the same under the two policies. Moreover, aggregate and individual allocations are also the same under the two policies. Therefore, the two policy vectors are equivalent. ■

We prove below that, when transfers follow path B, there is no equivalency between the nominal interest rate and the labor tax rate, and that the optimal labor income tax is zero.¹⁵ First, it is convenient to write the budget constraint (46) of individual i with $Z_{it} = p_tz_i$ and replace q_{t+1} using (45). We obtain

$$\sum_{t=0}^{\infty} \beta^t (1 + \tau_c) R c_{it} = \Psi_i + \sum_{t=0}^{\infty} \beta^t (1 - \tau) e_i l_{it}, \quad (48)$$

where $\Psi_i = \sum_{t=0}^{\infty} \beta^t z_i$. The variable Ψ_i is exogenous wealth of household i . Consider fiscal policies of the type $f = (\tau_c, \tau, R, L)$, where L is a virtual levy on Ψ_i . The virtual levy L on the present value of transfers, defined as $(1 - L)\Psi_i$, is equivalent to a lump-sum tax. The next Lemma implies that the policy $f^a = (\tau_c^a, \tau^a, R^a, 0)$ is equivalent to the virtual policy

¹⁴To avoid confusion, we do not suppress the subscript t in c_{it} and l_{it} .

¹⁵In the appendix, we provide an alternative proof of this result.

$f^{av} = (\tau_c^{av}, \tau^{av}, R^{av}, L)$, where $(1 + \tau_c^{av}) R^{av} = (1 + \tau_c^a) R^a / \phi$, $(1 - \tau^{av}) = (1 - \tau^a) / \phi$, and $1 - L = 1/\phi$, with $\phi > 1$.

Lemma 2 *When transfers follow path B, the policy $f^a = (\tau_c^a, \tau^a, R^a, 0)$ is equivalent to the virtual policy $f^{av} = (\tau_c^{av}, \tau^{av}, R^{av}, L)$, where $R^{av} = R^a / \phi$, $(1 - \tau^{av}) = (1 - \tau^a) / \phi$, and $1 - L = 1/\phi$.*

Proof. When transfers follow path B the budget constraint (48) of household i , under policy f^a , is

$$\sum_{t=0}^{\infty} \beta^t (1 + \tau_c^a) R^a c_{it} = \Psi_i + \sum_{t=0}^{\infty} \beta^t (1 - \tau^a) e_i l_{it}. \quad (49)$$

Dividing by ϕ , the budget constraint of household i becomes

$$\sum_{t=0}^{\infty} \beta^t (1 + \tau_c^{av}) R^{av} c_{it} = (1 - L) \Psi_i + \sum_{t=0}^{\infty} \beta^t (1 - \tau^{av}) e_i h_{it}. \quad (50)$$

The individual first order conditions are identical under the two alternative policies. Given the same path for government consumption and the same paths for the consumption-leisure pairs of households, the resource constraint will be satisfied and, by Walras law, the government budget constraint will also be satisfied. As a result, the equilibrium prices gross of taxes are identical under f^a or f^{av} . Moreover, aggregate and individual allocations are the same under the two policies. ■

We now establish that the inflation tax is a better instrument to finance government transfers than the labor income tax. To simplify notation, let $\tau_c = 0$.

Proposition 2 *When transfers follow path B, the inflation tax is a more efficient instrument than the labor income tax.*

Proof. Suppose that there are two policies $f^a = (R^a, \tau^a, 0)$ and $f^b = (R^b, \tau^b, 0)$ that generate the same fiscal revenue necessary to finance the exogenous transfers, with $R^b > R^a$ and $0 \leq \tau^b < \tau^a$. Using Lemma 2, the policy f^a is equivalent to the virtual policy $f^{av} = (1, \tau^{av}, L^{av})$ and f^b is equivalent to $f^{bv} = (1, \tau^{bv}, L^{bv})$. Since $R^b > R^a$, then $L^{bv} > L^{av}$. As the lump-sum tax is larger under the virtual policy b , then $\tau^{bv} < \tau^{av}$. Therefore, policy f^b is more efficient than policy f^a as the same path of government transfers is financed with a lower distortionary tax. ■

If transfers follow path B and there are no constraints on the consumption tax, then the labor income tax should be set to zero. The inflation tax and the consumption tax are indeterminate. If there are active constraints on the consumption tax, either for political reasons or because the inflation tax has lower administrative costs, then the Friedman rule is not optimal.

5 Conclusions

The Friedman rule is one of the most robust results in the literature. Departures from the Friedman rule in standard cash in advance models are associated with an incomplete set of tax instruments. Typically, if there are sources of income or transactions of goods and services that cannot be taxed, then the Friedman rule may cease to be optimal. For instance, if prices are sticky and consumption taxes are not available (Schmitt-Grohe and Uribe 2011), if there are positive firms' profits that cannot be taxed (Schmitt-Grohe and Uribe 2004), or if there is an underground economy where agents cannot be taxed (Nicolini 1998, Cavalcanti and Villamil 2003, and Arbex 2013). Although in all these examples optimal seigniorage is positive, they frequently imply insignificant levels of inflation. Schmitt-Grohe and Uribe (2011) calibrate economies with these frictions to the US and conclude that each friction by itself does not justify an inflation target above zero.

We investigate the implications of government transfers for the optimal rate of inflation. Surprisingly, we find that, unlike public consumption, the apparently innocuous introduction of government transfers changes the standard optimal taxation result of uniform taxation. As transfers cannot be taxed, a positive nominal net interest rate is the indirect way for the government to tax transfers. The higher the transfers, the higher is the optimal inflation rate. We calibrate a model with homogeneous households and transfers to the US economy. We obtain optimal values for inflation that are substantially higher than the ones obtained in the literature when other frictions are considered.

The model abstracts from the fact that households are asymmetric with respect to wealth and income and, as such, does not take into account the empirical distribution of transfers across households. A model and a calibration that allow for these asymmetries could imply different estimates, with possibly lower levels for the optimal inflation rate. Nevertheless,

the exercise shows that the presence of transfers has the potential to justify the targets for inflation of the order of magnitude of the targets followed by central banks.¹⁶

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¹⁶We focus here on the effect of transfers on optimal inflation. Of course, there are other reasons to keep seigniorage small. For instance, if there are costs of changing the composition of the portfolio of assets (Silva 2012, Adao and Silva 2016).

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A Appendix

We provide an alternative proof to proposition 2 of section 4.

Proposition 2 *When transfers follow path B, the inflation tax is a more efficient instrument than the labor income tax.*

Proof. Consider the simple economy of section 4 with the path of transfers following path B, $e_i = 1$, $g = 0$, and $\tau_c = 0$. Define $\Gamma_t \equiv \frac{1}{1-\tau_t}$ and $c_t \equiv f(\Gamma_t R_t)$ as the value of consumption that solves equations (40) and (44). Define the instantaneous indirect utility as $V(\Gamma_t R_t) \equiv U(f(\Gamma_t R_t), 1 - f(\Gamma_t R_t))$, using the fact that $c_t = l_t$. Since U is strictly concave, the optimal allocation is stationary, which implies that Γ_t and R_t should be stationary too. It is trivial to see that V is decreasing in ΓR . Therefore, the optimal tax policy solves the

problem $\min_{\Gamma, R} \Gamma R$ subject to the government budget constraint

$$\frac{\Gamma R - 1}{\Gamma} f(\Gamma R) = z. \quad (\text{A.1})$$

Suppose that $\tau > 0$, and so $\Gamma(\tau, R) \equiv \frac{1}{1-\tau} > 1$, and that Γ and R satisfy (A.1). We then show that it is always possible to decrease $\Gamma \geq 1$ and increase R so that ΓR decreases, but the constraint (A.1) is still satisfied. As a result, the solution of the problem cannot involve $\Gamma > 1$. First, a change in Γ together with a change in R so that $\frac{d\Gamma}{dR} = -\frac{\Gamma}{R}$ maintains the value of ΓR constant. Consider an increase in R , $dR > 0$. If Γ changes by $d\Gamma = -\frac{\Gamma}{R}dR - \varepsilon$, for $\varepsilon > 0$, then, as $d(\Gamma R) = \Gamma dR + R d\Gamma$, this change in Γ and R implies a change in ΓR equal to $-R\varepsilon < 0$. On the other hand, $d\Gamma$ and dR changes government revenues by $(\frac{1}{\Gamma^2}f(\Gamma R) + \frac{\Gamma R - 1}{\Gamma}f'(\Gamma R)R)d\Gamma + (f(\Gamma R) + (\Gamma R - 1)f'(\Gamma R))dR$. With $d\Gamma = -\frac{\Gamma}{R}dR - \varepsilon$, this change in government revenues is equal to

$$\left(1 - \frac{1}{\Gamma R}\right) f(\Gamma R) dR - \left(\frac{1}{\Gamma^2}f(\Gamma R) + \frac{\Gamma R - 1}{\Gamma}f'(\Gamma R)R\right) \varepsilon. \quad (\text{A.2})$$

As $\Gamma R > 1$ and $f(\Gamma R) > 0$, the coefficient on dR is strictly positive. Therefore, for any $dR > 0$, there is a sufficiently small $\varepsilon > 0$ such that the expression in (A.2) is positive. This means that ΓR decreases but government revenues increase. We then have that a pair (Γ, R) , with $\Gamma > 1$, cannot be the solution to the Ramsey problem as there is a decrease in the labor tax rate and an increase in the nominal interest rate such that the distortion ΓR decreases and government revenues do not decrease. It follows that the solution to the Ramsey problem requires $\Gamma^* = 1$. Moreover, setting $\Gamma^* = 1$ in (A.1) implies that R^* is equal to the smallest value that satisfies $(R - 1)f(R) = z$. As $\Gamma = \frac{1}{1-\tau}$ and $z > 0$, then $\tau = 0$ and $R > 1$. ■

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