



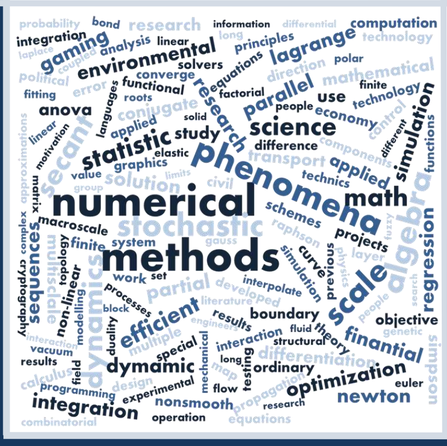



# Universidade da Beira Interior, Portugal
## Departamento de Matemática

## VI Workshop
## *on*
# COMPUTATIONAL DATA ANALYSIS AND NUMERICAL METHODS

# BOOK OF ABSTRACTS

**Joint organization:**

UNIVERSIDADE BEIRA INTERIOR

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# BOOK OF ABSTRACTS

UNIVERSIDADE DA BEIRA INTERIOR
DEPARTAMENTO DE MATEMÁTICA
June 27–29, 2019
Covilhã – PORTUGAL

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**Title:**

VI Workshop on Computational Data Analysis and Numerical Methods: Book of Abstracts

**Web page:**

http://www.wcdanm-ubi19.uevora.pt/

**Editor:**

UBI - Universidade da Beira Interior - Tipografia
Rua Marquês D´Ávila e Bolama
6201-001 Covilhã, Portugal

**Workshop place:**

Departamento de Matemática, Universidade da Beira Interior
27-29 June 2019, Covilhã, Portugal

**Authors:**

Alberto Simões (UBI), Célia Nunes (UBI), Ilda Inácio (UBI), Luís M. Grilo (IPT) and Fernando Carapau (UÉ)

**Published and printed by:**

UBI-Universidade da Beira Interior



**ISBN:** 978-989-654-545-1

# Welcome to the VI WCDANM|2019

## Welcome to the VI WCDANM|2019

Dear Participants, Colleagues and Friends,

it is a great honour and a privilege to give you all a warmest welcome to the sixth Workshop on Computational Data Analysis and Numerical Methods (VI WCDANM).

This Workshop takes place at the University of Beira Interior, located in the beautiful city of Covilhã, Portugal. The host institution, as well as the Polytechnic Institute of Tomar and the University of Évora have committed themselves to this challenge, hoping that the final result may exceed the expectations of the participants, sponsors and organizers. The important contributions of plenary speakers, the high scientific level of oral and poster presentations and an active audience will certainly contribute to the success of the meeting. A special thanks to all, since this event could not be possible without any of these essential and complementary parts.

An acknowledgment is also due to the Members of the Executive, Scientific and Organizing Committees, specially to Alberto Simões (Local Chair), Célia Nunes and Ilda Inácio (hosts from the University of Beira Interior) and Fernando Carapau (University of Évora), who have been relentless in the search for a balanced, broad and interesting program, having achieved an excellent result.

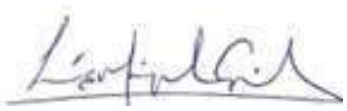
The submitted papers to the VI WCDANM, coming from Portugal and foreign countries, have been increasing over time, being this edition no exception, where the (theoretical and applied) papers in different research fields involve big data, data mining, data science and machine learning. For the second consecutive year, the Journal of Applied Statistics (Taylor & Francis) and Neural Computing and Applications (Springer) are also associated to the event.

It is a pleasure to join you all in Covilhã, in a familiar atmosphere, hoping that the Workshop can be intellectually stimulating and an opportunity

for the scientific community to work together, providing all unforgettable moments!

Covilhã, June 27 – 29, 2019

Chairman of the Executive Committee of VI WCDANM,



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# Program

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VI Workshop on Computational Data Analysis and Numerical Methods
VI WCDANM
Universidade da Beira Interior
June 27–29, 2019, Covilhã, Portugal

Event Place

Departamento de Matemática, Universidade da Beira Interior

**Thursday (June $27^{th}$)**

13:30-14:00 | Registration

14:00-14:15 | Open Ceremony of the VI WCDANM

14:15-15:00 | Plenary Session: Delfim Torres, University of Aveiro, Portugal

15:00-15:45 | Plenary Session: Ivette Gomes, University of Lisboa, Portugal

15:45-16:00 | Coffee Break and Poster Session

16:00-17:00 | Parallel Sessions

17:30 | Welcome Reception at the City Hall and walk through the historic center to see Urban Art

**Friday (June $28^{th}$)**

08:30-09:00 | Registration

09:00-10:00 | Parallel Sessions

10:00-10:45 | Plenary Session: Padmanabhan Seshaiyer, George Mason University, USA

10:45-11:00 | Coffee Break and Poster Session

11:00-11:30 | Round table about special issues in scientific journals: Milan Stehlík (Associate Editor)

11:30-12:45 | Parallel Sessions

12:45-14:00 | Lunch

14:00-18:00 | Social Program - Visit to the Belmonte Museums. Tasting Kosher products

20:30 | Conference Dinner - Hotel Puralã

**Saturday (June 29$^{th}$)**

09:00-09:30 | Registration

09:30-10:30 | Parallel Sessions

10:30-11:15 | Plenary Session: Leonid Hanin, Idaho State University, USA

11:15-11:30 | Coffee Break and Poster Session

11:30-12:45 | Parallel Sessions

12:45-14:00 | Lunch

14:00-15:00 | Parallel Sessions

15:00-15:15 | Coffee Break and Poster Session

15:15 | Closing Ceremony

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# Invited Speakers

# Numerical Methods for Fractional Optimal Control Problems

Delfim F.M. Torres[1]

[1]CIDMA, Department of Mathematics, University of Aveiro, Portugal

*Corresponding/Presenting author: delfim@ua.pt*

## Abstract

Optimal control theory allows us to choose control functions in a dynamical system to achieve a certain goal. The theory generalizes that of the classical calculus of variations and has found many applications in engineering, physics, economics, and life sciences. In 1996/97, motivated by nonconservative physical processes in mechanics, the subject was extended to the case in which derivatives and integrals are understood as fractional operators of arbitrary order. In this talk, we begin with a brief survey of main results and techniques of the fractional variational calculus and fractional optimal control. We consider variational problems containing Caputo derivatives and review both indirect and direct methods. In particular, we provide necessary optimality conditions for the fundamental, higher-order, and isoperimetric problems, and compute approximated solutions based on truncated Grünwald–Letnikov approximations of Caputo derivatives [3,2]. Although indirect methods are also very useful in practical terms [6], we then proceed restricting ourselves to direct methods, which allow us to consider more complex problems, where numerical methods are indispensable. We begin by presenting three direct methods, based on Grünwald–Letnikov, trapezoidal, and Simpson fractional integral formulas to solve fractional optimal control problems (FOCPs). At first, the fractional integral form of the FOCP is considered, then the fractional integral is approximated by Grünwald–Letnikov, trapezoidal, and Simpson formulas, in a matrix approach. Thereafter, the performance index is approximated either by trapezoidal or Simpson quadrature. As a result, FOCPs are reduced to nonlinear programming problems, which can be solved by well-developed algorithms. To improve the efficiency of the presented methods, the gradient of the objective function and the Jacobian of constraints are prepared in closed forms. The efficiency and reliability of the developed numerical methods are assessed by numerical tests involving a free final time FOCP with a path constraint, a bang-bang FOCP, and an optimal control problem of a fractional-order HIV-immune system [7]. We also present a Method Of Lines (MOL), which is based on the spectral

collocation method, to solve space-fractional advection-diffusion equations (SFADEs) on a finite domain with variable coefficients. We focus on the cases in which the SFADEs consist of both left- and right-sided fractional derivatives. To do so, we begin by introducing a new set of basis functions with some interesting features. The MOL, together with the spectral collocation method based on the new basis functions, are successfully applied to the SFADEs. Several numerical examples, including benchmark problems and a problem with discontinuous advection and diffusion coefficients, are provided to illustrate the efficiency and exponentially accuracy of the proposed method [4]. Such method can be extended to develop a direct numerical method for the solution of optimal control problems governed by two-side space-fractional diffusion equations. A direct numerical method containing two main steps is developed. In the first step, the space variable is discretized by using the Jacobi–Gauss pseudospectral discretization and, in this way, the original problem is transformed into a classical integer-order optimal control problem. The main challenge, which we faced in this step, is to derive the left and right fractional differentiation matrices. In this respect, novel techniques for derivation of these matrices are presented. In the second step, the Legendre–Gauss–Radau pseudospectral method is employed. With these two steps, the original problem is converted into a convex quadratic optimization problem, which can be solved efficiently by available numerical methods. Our approach can be easily implemented and extended to cover fractional optimal control problems with state constraints. Examples are provided to demonstrate the efficiency and validity of the presented method. The results show that our method reaches the solutions with good accuracy and a low CPU time [1]. We end our talk mentioning other efficient and reliable numerical methods for fractional optimal control problems, which have recently been developed on the basis of modified hat functions [5].

**Keywords:** optimal control, fractional calculus, direct methods, numerical methods.

**Acknowledgements**

This work was partially supported by *Fundação para a Ciência e a Tecnologia* (the Portuguese Foundation for Science and Technology) through *Centro de Investigação e Desenvolvimento em Matemática e Aplicações* (CIDMA), project UID-MAT-04106-2019.

## References

[1]M. S. Ali, M. Shamsi, H. Khosravian-Arab, D. F. M. Torres and F. Bozorgnia, A space-time pseudospectral discretization method for solving diffusion optimal control problems with two-sided fractional derivatives, J. Vib. Control **25** (2019), no. 5, 1080–1095.

[2]R. Almeida, D. Tavares and D. F. M. Torres, *The variable-order fractional calculus of variations*, SpringerBriefs in Applied Sciences and Technology, Springer, Cham, 2019.

[3]R. Almeida and D. F. M. Torres, A survey on fractional variational calculus. In A. Kochubei, Y. Luchko (Eds.), "Handbook of Fractional Calculus with Applications. Vol 1: Basic Theory", De Gruyter, Berlin, Boston, 2019, 347–360.

[4]M. K. Almoaeet, M. Shamsi, H. Khosravian-Arab and D. F. M. Torres, A collocation method of lines for two-sided space-fractional advection-diffusion equations with variable coefficients, Math. Methods Appl. Sci. 42 (2019), no. 10, 3465–3480.

[5]S. Nemati, P. M. Lima and D. F. M. Torres, A numerical approach for solving fractional optimal control problems using modified hat functions, Commun. Nonlinear Sci. Numer. Simul. **78** (2019), 104849, 14 pp.

[6]S. Rosa and D. F. M. Torres, Optimal control of a fractional order epidemic model with application to human respiratory syncytial virus infection, Chaos Solitons Fractals **117** (2018), 142–149.

[7]A. B. Salati, M. Shamsi and D. F. M. Torres, Direct transcription methods based on fractional integral approximation formulas for solving nonlinear fractional optimal control problems, Commun. Nonlinear Sci. Numer. Simul. **67** (2019), 334–350.

# Approximations for Extremes and Bounds for the Reliability of Large Systems

M. Ivette Gomes[1,2], Luísa Canto e Castro[2], Sandra Dias[3,4] and Paula Reis[5,2]

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## Abstract

The rate of convergence of the sequence of linearly normalized extremes to the corresponding non-degenerate *extreme value* (EV) limiting distribution is a relevant problem in the field of EV *theory* (EVT). In their seminal 1928 paper, Fisher and Tippett observed that, for normal underlying parents, if we approximate the distribution of the suitably linearly normalized sequence of maxima not by the so-called Gumbel limiting distribution, associated with an EV *index* (EVI) $\xi = 0$ but by an adequate sequence of other EV distributions with an EVI $\xi_n = o(1) < 0$, the approximation can be improved. Such approximations are often called penultimate approximations and have been theoretically studied from different perspectives. The modern theory of rates of convergence in EVT began in the Ph.D. thesis of Clive Anderson (1971), University of London, and M. Ivette Gomes (1978), University of Sheffield, and in the book by Janos Galambos (1978), on *The Asymptotic Theory of Extreme Order Statistics*. For papers on the subject prior to 1992, we refer the review in [1]. Developments have followed different directions that can be found in the review paper [2]. Recently, this same topic has been revisited in the field of reliability, where the exact *reliability function* (RF) of any complex technological system, $\mathcal{S}$, with lifetime $T$, can be difficult to obtain. It was shown in [3] that any coherent system can be represented as either a *series-parallel* (SP) or a *parallel-series* (PS) system. Its lifetime can thus be written as the *minimum of maxima* or the *maximum of minima*. Our strategy is now similar to the one in [4] and [5], among others: Assuming that the number $n$ of components of $\mathcal{S}$ goes to infinity, asymptotic EV or ultimate models can often provide an accurate estimation of the RF of $\mathcal{S}$, or at least of lower and upper bounds for such an RF. Considering that $n$ is fixed despite of large, pre-asymptotic or penultimate models provide an

improvement of the convergence rate and a better approximation. Ultimate and penultimate approximations for the RFs of regular and homogeneous PS and SP systems are discussed. A broader class of penultimate approximations is also put forward and a few conclusions are drawn.

**Keywords:** extreme value theory, Monte-Carlo simulation, penultimate and ultimate approximations, system reliability.

### Acknowledgements

This research was partially supported by Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00006-2019 (CEA/UL).

## References

[1]Gomes, M.I., Penultimate behaviour of the extremes. In J. Galambos, J. Lechner and E. Simiu (eds.), *Extreme Value Theory and Applications*, Kluwer Academic Publishers, 1994, pp. 403–418.

[2]Gomes, M.I. and Guillou, A., Extreme value theory and statistics of univariate extremes: A review. *Internat. Statist. Review*, **83**(2), 2015, pp. 263–292.

[3]Barlow, R.E. and Proschan, F., *Statistical Theory of Reliability and Life Testing*. Holt Rinehart and Winston, Inc., New York, 1975.

[4]Reis, P. and Canto e Castro, L., Limit model for the reliability of a regular and homogeneous series-parallel system, *Revstat* **7**(3), 2009, pp. 227–243.

[5]Reis, P., Canto e Castro, L., Dias, S. and Gomes, M.I., Penultimate approximations in statistics of extremes and reliability of large coherent systems, *Method. & Comput. in Appl. Probab.* **17**(1), 2015, pp. 189–206.

# Computational modeling, analysis and numerical methods for biological, bio-inspired and engineering systems

Padmanabhan Seshaiyer[1]

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## Abstract

Research in computational mathematics, which comprises of modeling, analysis, simulation and computing is quickly becoming the foundation for solving most multi-physics problems in science and engineering. These real-world problems often involve complex dynamic interactions of multiple physical processes which presents a significant challenge, both in representing the physics involved and in handling the resulting coupled behavior. If the desire to control and design the system is added to the picture, then the complexity increases even further. Hence, to capture the complete nature of the solution to the problem, a coupled multi-physics approach is essential. Performing research and teaching in computational mathematics, therefore, needs an in-depth understanding of the underlying mathematics and the fundamental principles that govern a physical phenomenon. It is well known that many physical systems can be described by partial differential equations. Thus, understanding the behavior of the numerical solution to such equations is of paramount importance for elucidating the actual physical problem. Some examples of physical systems include flow-structure interactions to understand rupture of aneurysms to non-linear dynamics of micro-air vehicles as well as uncertainty quantification [1–3]. Through our research in this field, we have come to appreciate that analyzing any numerical technique requires a combined theoretical and computational approach. Theory is needed to guide the performance and interpretation of the numerical technique while computation is necessary to synthesize the results. The focus of the proposed talk will be to describe how one can systematically combine sophisticated computational techniques, more specifically non-conforming finite element methods with high performance computing applied to several computationally challenging multi-physics problems involving fluid-structure interaction. We will also describe how to mathematically and computationally investigate the stability, convergence and control of a variety of non-conforming techniques and use this information to develop a general problem solving methodology that

can subsequently be used for solving such challenging problems efficiently. With this solution methodology one can hope to obtain efficient solutions that will not only benefit various engineering fields such as aerospace, material science, mechanical, thermal, chemical and bioengineering, but also aid in the process of designing better products and processes from automobiles and aircraft to prosthetic implants. We will also present how the research focus can be integrated with an education plan where the primary goal is to teach students to apply these well-developed research concepts in engineering, computer science, and mathematics to fundamental applications arising in other areas. This will be accomplished by incorporating these concepts into new or existing inter-disciplinary computational mathematical curriculum and also by mentoring students at the graduate, undergraduate, and high school levels and teachers through workshops, seminars and other enrichment activities [4,5].

**Keywords:** fluid-structure interaction, computational mathematics, finite element methods, aneurysms.

## References

[1]Raissi, M., and Seshaiyer, P. Application of local improvements to reduced-order models to sampling methods for nonlinear Partial Differential Equations with noise. *International Journal of Computer Mathematics, 95(5), 870-880.s*, 80(3), 2018, pp. 519–539.

[2]Badgaish, M., Lin, J.E. and Seshaiyer, P. Mathematical analysis and simulation of a coupled non-linear fluid structure interaction model with applications to aneurysms, *Communications in Applied Analysis*, 22, No. 4 (2018), 637-661.

[3]Aulisa, E., Manservisi, S., Seshaiyer, P., and Idesman, A.. Distributed Computational Method for Coupled Fluid Structure Thermal Interaction Applications. *Journal of Algorithms and Computational Technology*, 4(3), 2010, pp. 291-309.

[4]Seshaiyer, P. Leading Undergraduate Research Projects in Mathematical Modeling. *PRIMUS*, 27(4-5), 2017, pp. 476-493.

[5]Seshaiyer, P.. Transforming practice through undergraduate researchers. *Council on Undergraduate Research Focus*, 33(1), 2012, pp. 8-13.

# Mathematical Discovery of Natural Laws in Biomedical Sciences: A New Methodology with Application to the Effects of Primary Tumor and Its Resection on Metastases

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## Abstract

Mechanistic mathematical modeling of systemic biomedical processes faces two principal challenges: great complexity of these processes and enormous variability of individual characteristics of biological systems. As a result, mathematical models, statistical methods and computational tools chronically fall short and lag behind the rapidly expanding body of biomedical knowledge. We propose a new, alternative methodology that enables mathematical models and statistical methods to give, in certain cases, definitive answers to important biomedical questions of systemic nature that evade resolution both by empirical means and through mechanistic modeling. We call such definitive qualitative or quantitative answers, perhaps for lack of a better term, *natural laws*. Our new methodology is based on two ideas: (1) employing mathematical models that are so general that they can account for any, or at least many, biological mechanisms, both known and unknown; and (2) finding those model parameters whose optimal values are independent of observations. Here optimality refers to maximization of likelihood or minimization of a certain distance between theoretical and empirical distributions of some observable quantity. Every such universal parameter value, if it exists, reveals a natural law. Clearly, a model allowing such universal parameter values *must not* satisfy regularity conditions required for consistency of the relevant estimator. We illustrate this approach with the discovery of a natural law governing cancer metastasis. Specifically, we found in [1] that under very minimal mathematical and biomedical assumptions the likelihood-maximizing scenario of metastatic cancer progression in an individual patient is invariably the same: *Complete suppression of metastatic growth before primary tumor resection followed by abrupt growth acceleration after surgery*. Importantly, this scenario is widely observed in clinical practice, considered a common knowledge among veterinarians, and supported by a wealth of experimental studies on animals published over the

last 110 years starting with the early pioneering work by Paul Ehrlich of 1906 [2]. Furthermore, several tentative biological mechanisms behind the above natural law have been identified, see [1], [3]. The above natural law results from the application of the method of maximum likelihood to an extremely general individual-patient mathematical model of metastatic cancer progression. This type of models was initially proposed in [4]. Although the model is rooted in contemporary understanding of cancer progression, it is essentially free of specific biological assumptions of mechanistic nature. The model accounts for primary tumor growth and resection, shedding of metastases off the primary tumor according to a Poisson process, and their selection, dormancy and growth in a given secondary site. However, functional parameters descriptive of these processes are essentially arbitrary. In spite of such generality, the model still allows for computing the distribution of the sizes of detectable metastases in a given secondary site in closed form. Assuming exponential growth of metastases before and after primary tumor resection, we showed in [1] that, regardless of other model parameters and for any set of site-specific sizes of detected metastases, the likelihood-maximizing pre-surgery rate of growth of metastases is always zero, thus representing a universal parameter value. By contrast, the optimal post-surgery rate of metastatic growth is a positive quantity that depends on observations.

**Keywords:** cancer, metastasis, method of maximum likelihood, Poisson process.

## References

[1]Hanin, L. and Rose, J., Suppression of metastasis by primary tumor and acceleration of metastasis following primary tumor resection: A natural law?, *Bull. Math. Biol.*, 80(3), 2018, pp. 519–539.

[2]Ehrlich, P., Experimentelle Karzinomstudien an Mäusen, *Arch. Königlichen Inst. Exp. Ther. Frankfurt am Main*, 1, 1906, pp. 65–103 (in German).

[3]Krall, J.A., Reinhardt, F., Mercury, O.A., Pattabiraman, D.R., Brooks, M.W., Dougan, M., Lambert, A.W., Bierie, B., Ploegh, H.L., Dougan, S.K. and Weinberg, R.A., The systemic response to surgery triggers the outgrowth of distant immune-controlled tumors in mouse models of dormancy, *Sci. Transl. Med.*, 10, 2018, eaan3464.

[4]Hanin, L.G., Rose, J. and Zaider, M., A stochastic model for the sizes of detectable metastases, *J. Theor. Biol.*, 243, 2006, pp. 407–417.

# Organized Sessions

## Organized Session

# Multidisciplinary

**Organizer: VI WCDANM**

# Complex movement: symbolic dynamics for interacting agents

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## Abstract

We develop a deterministic discrete dynamical system which is used to classify a variety of types of movements, either two dimensional or three dimensional. The dynamical system is defined by a one-parameter family of bimodal interval maps, through iteration. The characterization of the movements is obtained from the topological classification of the discrete dynamical system. Techniques from symbolic dynamics and topological Markov chains are applied. Here we present a method to analyze the behaviour of a set of agents moving according to the referred model and subject to certain interactions.

**Keywords:** Topological dynamics, Markov chains, symbolic dynamics, complex motion, coupling dynamics.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações).

## References

[1]Lampreia, J.P., Severino, R. and Sousa Ramos, J. *Irreducible complexity of iterated symmetric bimodal maps.* Discrete Dynamics in Nature and Society Issue 1, Volume 2005. 69-85.

# Control Strategies for an epidemiological model with vaccination

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## Abstract

The study of epidemiological models is still a current research subject, and one of the reasons is the attempt of adjusting the models to reality. This allows us, for instance, to obtain better estimates regarding the disease evolution. In a previous work, [1], we analyzed the conditions that led to extinction and permanence of the disease for a discrete SIRVS model. In this work, using the same model, we compare two control strategies, involving the vaccination rate. To illustrate our findings we will present some simulation results using real data.

**Keywords:** Control theory, epidemic models, quasi-sliding mode control, Mickens Numerical Method.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00212-2013 (Centro de Matemática e Aplicações).

## References

[1]Mateus, J., Silva, C.S. and Vaz, S., A Consistent Discrete Version of a Nonautonomous SIRVS Model, *Discrete Dynamics in Nature and Society*, 2018, Article ID 8152032, 18 pages, 2018. https://doi.org/10.1155/2018/8152032.

[2]Baya, A., Meza, Control of Nonlinear Dynamic Models of Predator-Prey type, *Oecologia Australis*, 16(1), March, 2012, pp. 81–98.

[3]Utkin,V.I., *Sliding Modes and their application in variable structure systems*, MIR Publishers, Moscow, 1978.

# Performance analysis of different kernels on SVM classification using different feature selection methods on Parkinsonian gait

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## Abstract

Support Vector Machine (SVM) classifiers have been used extensively in classification problems based on gait data (e.g.[1]). A good performance of a SVM classifier depends on the right choice of the kernel function. It is also very well known that the selection of a feature subset can improve the prediction accuracy of classifiers and save computation time [3]. The objective of this study is to systematically investigate the impact of different kernel functions (linear, polynomial, Gaussian Radial Basis Function (RBF), and Sigmoid) and feature selection methods (step-wise regression, Lasso regression, and Gini-index) on the performance of SVM algorithm in the classification of Parkinsonian gait. The study group comprised 30 patients with Parkinsonism, subdivided into 15 patients with Vascular Parkinsonism (VaP) and 15 patients with Idiopathic Parkinson's Disease (IPD), and 15 age-matched healthy controls. The gait features of each subject were acquired by wearable sensors from a continuous walking course (for more detail about the participants and the experiment see [4]). The three feature selection methods were applied on 26 gait features. The selected features were then used in SVM with different kernel functions. To evaluate the performance of the different methods, the accuracy was assessed for each combination (feature selection method plus kernel function). This analysis has been examined on two different pairs of groups: all patients with Parkinsonism vs. Control, and VaP vs. IPD. When assessing the ability of a SVM classifier to discriminate between patients with Parkinsonism and controls, the linear kernel presented the higher accuracies (96.67%-100%) for all three feature selection methods. An accuracy of 100% was found when the features (5 in total) were selected

by the Lasso regression method. The best performance of SVM when distinguishing the two group of patients, VaP vs. IPD, was of 85% accuracy. This performance was obtained when using the RBF kernel and the Lasso regression for feature selection. In this case, 6 features were selected. In both cases, the Lasso regression presented the best classification performance. However, the selected feature subsets were different. These results suggest that the choice of the kernel function and feature selection methods for SVM classifier is crucial when working with data-driven modeling. Based on our findings the Lasso regression can be considered as the method with the best performance for gait feature selection.

**Keywords:** Support Vector Machine (SVM), Kernel functions, Feature selection methods, Parkinsonian gait

**Acknowledgements**

This work was partially supported by the projects NORTE-01-0145-FEDER-000026 (DeM-Deus Ex Machina) financed by NORTE-2020 and FEDER, and FP7 NETT project.

## References

[1]Joshi, D., Khajuria, A., and Joshi, P., An automatic non-invasive method for Parkinson's disease classification. *Computer methods and programs in biomedicine*, 145, 2017, pp. 135-145.

[2]Sangeetha, R. and Kalpana, B., Identifying efficient kernel function in multiclass support vector machines. International Journal of Computer Applications, 28(8), 2011

[3]Guyon, I. and Elisseeff, A., An introduction to variable and feature selection, *Journal of machine learning research*, 3, 2003, pp. 1157-1182.

[4]Ferreira, F., Gago, M.F., Bicho, E., Carvalho, C., Mollaei, N., Rodrigues, L., Sousa, N., Rodrigues, P.P., Ferreira, C., and Gama, J., Gait stride-to-stride variability and foot clearance pattern analysis in Idiopathic Parkinson's Disease and Vascular Parkinsonism. *Journal of Biomechanics*, 2019.

# Sufficient conditions for heteroclinic solutions of $\phi$-Laplacian generalized differential equations

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## Abstract

In this paper we consider the second order discontinuous differential equation in the real line,

$$\left(a\left(t,u(t)\right)\ \phi\left(u'(t)\right)\right)' = f\left(t,u(t),u'(t)\right),\ a.e.t \in \mathbb{R},$$
$$u(-\infty) = \nu^-,\quad u(+\infty) = \nu^+,$$

with $\phi$ an increasing homeomorphism such that $\phi(0) = 0$ and $\phi(\mathbb{R}) = \mathbb{R}$, $a \in C(\mathbb{R}^2, \mathbb{R})$ with $a(t,x) > 0$ for $(t,x) \in \mathbb{R}^2$, $f : \mathbb{R}^3 \to \mathbb{R}$ a $L^1$-Carathéodory function and $\nu^-, \nu^+ \in \mathbb{R}$ such that $\nu^- < \nu^+$. The existence and localization of heteroclinic connections is obtained assuming a Nagumo-type condition on the real line, and without asymptotic conditions on the nonlinearities $\phi$ and $f$. To the best of our knowledge, this result is even new when $\phi(y) = y$, that is, for equation

$$\left(a\left(t,u(t)\right)u'(t)\right)' = f\left(t,u(t),u'(t)\right),\ a.e.t \in \mathbb{R}.$$

Moreover these results can be applied to classical and singular $\phi$-Laplacian equations and to the mean curvature operator.

**Keywords:** $\phi$-Laplacian operator, mean curvature operator, heteroclinic solutions, lower and upper solutions, Nagumo condition on the real line.

### Acknowledgements

This work has been partially supported by Center for Research in Mathematics and Applications (CIMA) related with the Differential Equations and Optimization (DEO) group, through the grant UID-MAT-04674-2019 of FCT-Fundação para a Ciência e a Tecnologia, Portugal.

## References

[1]Minhós, F., Heteroclinic solutions for classical and singular $\phi$-Laplacian non-autonomous differential equations, *Axioms*, 2019, 8 (1), 22; https://doi.org/10.3390/axioms8010022, https://www.mdpi.com/2075-1680/8/1/22.

# An optimal control problem of a non autonomous model for an outbreak of a Zombie infection

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## Abstract

In this work we revisit an icon of the American culture: A Zombie invasion. In the past several decades, several movies and TV series appeared concerning a Zombie invasion. A few years ago, several media (Tv and Journals) stated that the USA government had a plan againt a Zombie invasion. In 2009, see [1], the first mathematical investigation of the zombie community appears. Taking their cues from traditional zombie movies, in [1], studied the hypothesized the effect of a zombie attack and its impact on human civilization. According to their mathematical model, "*a zombie outbreak is likely to lead to the collapse of civilization, unless it is dealt with quickly*". While aggressive quarantine may contain the epidemic, or a cure may lead to coexistence of humans and zombies, the most effective way to contain the rise of the un dead is to hit hard and hit often. In this model we consider the case where there exists a self regulation from all the individuals of the susceptible population [1]. In addition, we assume the birth rate is a constant, $\Pi$. The model is as follows:

$$\begin{cases} \dot{S} = \Pi - \beta t S Z - \delta S \\ \dot{Z} = \beta t S Z + \zeta R - \alpha S Z \\ \dot{R} = \delta S + \alpha S Z - \zeta R \end{cases}$$

where $S$ represents the Susceptible individuals, $Z$ the Zombies and $R$, the Removed. We consider that the contact rate $\beta$ is not constant and reflects the behaviour of the susceptible elements of the population during the Zombies outbreak. The aim is to formulate an optimal control problem to minimize the number of infected elements (Zombies) of the population. The control is made by an aggressive behaviour to infected elements of the population. Namely, we consider the optimal control problem

$$\min_u \mathcal{J} Z, R = \min_u \int_0^T \kappa_1 Z + \kappa_2 u^2 dt$$

subjected to the conditions:

$$\begin{cases} \dot{S} = \Pi - \beta t S Z - \delta S \\ \dot{Z} = \beta t S Z + \zeta R - \alpha S Z - u Z \\ \dot{R} = \delta S + \alpha S Z - \zeta R \end{cases}$$

with initial conditions

$$S0 = S_0,\ Z0 = Z_0 \text{ and } R0 = R_0.$$

Basically, we are formulating an optimal control problem for a non autonomous SIR model of a disease propagation.

**Keywords:** zombies, SIR model, optimal control, non autonomous.

**Acknowledgements**

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00212-2019 (Centro de Matemática e Aplicações) and from Centro de Telecomunicações, UBI.

## References

[1]Munz, Philip, Hudea, Ioan, Imad, Joe. and Smith, Robert J., When zombies attack!: Mathematical modelling of an outbreak of zombie infection, *in Infectious Disease Modelling Research Progress*, pp. 133 – 150, 2009 Nova Science Publishers, Inc.

[2]Rebelo, Paulo, Rosa, Silvério and Silva, César M., *An optimal control problem for a non autonomous SIR model for the Ebola virus*, V Workshop on Computational Data Analysis and Numerical Methods, May 11 – 12, 2018, Instituto Politécnico do Porto & Escola Superior de Tecnologia e Gestão, Felgueiras

[3]Langtangen, H. P., Mardal, K. A. and Røtnes, P., *Escaping the zombie threat by mathematics*. In: Zombies in the Academy - Living Death in Higher Education, Chicago: University ofChicago Press, 2013.

[4]Rodrigues, Helena Sofia, *Application of SIR epidemiological model: new trends*, International Journal of Applied Mathematics and Informatics, 10: 92–97.

[5]Smith, Robert and Cartmel, Andrew, *Mathematical Modelling of Zombies*. University of Ottawa Press, 2014.

# State-space representation of MIMO 3-periodic behaviors

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## Abstract

The aim of this work is to provide an algorithm to obtain periodic state-space minimal representations for MIMO 3-periodic behavioral systems. The key tool for this procedure is based on a well-known technique which allows to associate a periodic behavior with a time-invariant behavior. Furthermore, this used approach differs from the classical method since we do not start from a transfer function description but rather from linear difference equations with periodically time-varying coefficients.

**Keywords:** mathematical systems theory, linear systems, discrete-time systems, behavioral systems.

### Acknowledgements

This work was supported by Project POCI-01-0145-FEDER-006933 - SYSTEC - Research Center for Systems and Technologies - funded by FEDER funds through COMPETE-2020 - Programa Operacional Competitividade e Internacionalização (POCI) - and by national funds through FCT - Fundação para a Ciência e a Tecnologia.

## References

[1]B. L. Ho and R. E. Kalman, "Effective construction of linear state-variable models from input/output functions." Regelungstechnik, vol. 14, pp. 545–548, 1966.
[2]I. Gohberg, M. A. Kaashoek, and L. Lerer, "Minimality and realization of discrete time-varying systems," in Time-variant systems and interpolation, ser. Oper. Theory Adv. Appl. Basel: Birkhäuser, 1992, vol. 56, pp. 261–296.
[3]E. Sánchez, V. Hernández, and R. Bru, "Minimal realizations for discrete-time linear periodic systems," Linear Algebra Appl., vol. 162-164, pp. 685–708, 1992.
[4]C. A. Lin and C. W. King, "Minimal periodic realizations of transfer matrices," IEEE Trans. Automat. Control, vol. 38, no. 3, pp. 462–466, 1993.

[5]C. Coll, R. Bru, E. Sánchez, and V. Hernández, "Discrete-time linear periodic realization in the frequency domain," Linear Algebra Appl., vol. 203-204, pp. 301–326, 1994.
[6]S. Bittanti, P. Bolzern, L. Piroddi, and G. De Nicolao, "Representation, prediction, and identification of cyclostationary processes. a statespace approach." Gardner, William A. (ed.), Cyclostationarity in communications and signal processing. New York, NY: IEEE. 267-294 (1994)., 1994.
[7]P. Colaneri and S. Longhi, "The realization problem for linear periodic systems," Automatica J. IFAC, vol. 31, no. 5, pp. 775–779, 1995.
[8]O. M. Grasselli, S. Longhi, and A. Tornamb'e, "System equivalence for periodic models and systems," SIAM Journal on Control and Optimization, vol. 33, no. 2, pp. 455–468, 1995.
[9]J. C. Willems, "Models for dynamics," in Dynamics Reported. Wiley, 1989, vol. 2, pp. 171–269.
[10]J. C. Willems, "Paradigms and puzzles in the theory of dynamical systems," IEEE Trans. Automat. Control, vol. 36, no. 3, pp. 259–294, 1991.
[11]M. Kuijper and J. C. Willems, "A behavioral framework for periodically time-varying systems," in Proceedings of the 36th IEEE Conference on Decision & Control - CDC'97, vol. 3, San Diego, California USA, 10-12 Dec. 1997, pp. 2013–2016.
[12]J. C. Aleixo, J. W. Polderman, and P. Rocha, "Representations and structural properties of periodic systems," Automatica J. IFAC, vol. 43, no. 11, pp. 1921–1931, 2007.
[13]José C. Aleixo, Paula Rocha and Jan C. Willems, "State space representation of SISO periodic behaviors," in Proceedings of the 50th IEEE Conference on Decision & Control and European Control Conference - CDC-ECC 2011, Orlando, Florida USA, 12-15 Dec. 2011, pp. 1545–1550.
[14]P. P. Khargonekar, K. Poolla, and A. Tannenbaum, "Robust control of linear time-invariant plants using periodic compensation," IEEE Trans. Automat. Control, vol. 30, no. 11, pp. 1088–1096, November 1985.
[15]V. Hernández and A. Urbano, "Pole-assignment problem for discrete-time linear periodic systems," Internat. J. Control, vol. 46, no. 2, pp. 687–697, 1987.
[16]J. C. Willems, "From time series to linear system. I. Finite-dimensional linear time invariant systems," Automatica J. IFAC, vol. 22, no. 5, pp. 561–580, 1986.
[17]M. Kuijper, "A periodically time-varying minimal partial realization algorithm based on twisting," Automatica J. IFAC, vol. 35, no. 9, pp. 1543–1548, 1999.
[18]J. C. Willems, "The behavioral approach to open and interconnected systems: modeling by tearing, zooming, and linking," IEEE Control Systems Magazine, vol. 27, no. 6, pp. 46–99, Dec. 2007.
[19]A. Urbano, "Estabilidad y control óptimo de sistemas lineales periódicos discretos," Ph.D. dissertation, University of Valencia, Spain, 1987.

# Dense numerical semigroups

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## Abstract

A numerical semigroup is a subset of the set of nonnegative integers (denoted by $\mathbb{N}$) closed under addition, containing zero element and with finite complement in $\mathbb{N}$. A numerical semigroup S is dense if for all $s \in S\backslash\{0\}$ we have that $\{s-1, s+1\} \cap S \neq \emptyset$. In this talk, we give algorithms to compute the whole set of dense numerical semigroups with fixed genus, Frobenius number and multiplicity. Furthermore, we solve the Frobenius problem for dense numerical semigroups with embedding dimension three.

**Keywords:** numerical semigroups, Dense Numerical semigroups, tree, Frobenius number, Multiplicity and genus.

### Acknowledgements

This work was partially supported by the research groups FQM-343 FQM-5849 (Junta de Andalucia-Feder) and by the project MTM-2014-55367-P (MINECO-FEDER, UE) and supported by the Fundação para a Ciência e Tecnologia (Portuguese Foundation for Science and Technology) through the project FCT PTDC-MAT-73544-2006.

## References

[1]R. Apéry Sur les branches superlinéaires de courbes algébriques. *C. R. Acad. Sci. Paris* **222** (1946), 1198-2000.
[2]V. Barucci, V. and D. E. Dobbs and M. Fontana *Maximality Properties in Numerical Semigroups and Applications to One-Dimensional Analytically Irreducible Local Domains*. Memoirs of the Amer. Math. Soc. **598** (1997).
[3]J. C. Rosales and P. A. García-Sánchez, *Numerical semigroups*. Developments in Mathematics, vol.**20**, Springer, New York, (2009).
[4]J. L. Ramirez Alfonsín, "The Diophantine Forbenius Problem", Oxford University Press, London (2005).
[5]J. J. Sylvester, Mathematical questions with their solutions, Educational Times, vol. 41, pg. 21, Francis Hodson, London (1884).
[6]E. S. Selmer, On linear diophantine problem of Frobenius, J. Reine Angew. Math., **293/294**, 1-17, (1977)

# Mixed models with random effects with known dispersion parameters

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## Abstract

There are some popular methods available for the estimation of variance components in linear mixed models, such as maximum likelihood, restricted maximum likelihood or analysis of variance based methods. The goal of this talk is to present a least squares estimation method, to estimate the variance components and the estimable vectors in linear mixed models. An advantage of this procedure is that it can be applied without requiring normality, unlike the ML, and REML methods. Besides this, the same procedure can be applied to balanced or unbalanced mixed models, contrary to the ANOVA.

**Keywords:** Least squares estimator method, linear mixed models, variance components.

### Acknowledgements

This work was partially supported by national founds of FCT-Foundation for Science and Technology under UID-MAT-00297-2019 and UID-MAT-00212-2019.

## References

[1]Henderson, C. (1953), Estimation of variance and covariance components, Biometrics **9**: 226–252.
[2]Jiang, J. (2007), Linear and Generalized Linear Mixed Models and Their Applications, Springer, New York.
[3]Khuri, A.I, Matthew, T. and Sinha, B.K. (1998), Statistical Tests for Mixed Linear Models, Jonh Wiley & Sons, New York.
[4]Radhakrishna R.C. (1972), Estimation of Variance and Covariance Components in Linear Models. Journal of the American Statistical Association **67**: 337, 112–115.
[5]Sahai, H. and Ojeda, M.M. (2000), Analysis of Variance for Random Models, Volume 2: Unbalanced Data: Theory, Methods, Applications, and Data Analysis, Birkhäuser, Cambridge.

# Tests for Multiple Additive Models

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## Abstract

In [3], [4] and [5] the authors considered a linear regression for each treatment of a base design. Then, they studied the action of the factors of this design on the vectors $\beta(l), l = 1, ..., d$. In this talk we will consider multiple additive models which are an extension of these models. We will show how to test hypothesis on fixed effects part. Then, we will use the structure of these models to ensure that the vectors of homologue components are approximately homoscedastic which enable us to apply ANOVA.

**Keywords:** Additive Models, ANOVA, Cumulants, Moments.

### Acknowledgements

This work was partially supported by national founds of FCT-Foundation for Science and Technology under UID-MAT-00212-2019 and UID-MAT-00297-2019.

## References

[1]Craig, C. C. (1931). On A Property of the Semi-Invariants of Thiele. Ann Math Statist, 2, 2, 154–164.
[2]Mexia, J. T. (1987). Multi-treatment regression designs. Research Studies N.1, Mathematics Department, Faculty of Sciences and Tecnology, Universidade Nova de Lisboa.
[3]Mexia, J. T. (1988). Standardized Orthogonal Matrices and the Decomposition of the sum of Squares for Treatments, Research Studies N.2, Mathematics Department, Faculty of Sciences and Tecnology, Universidade Nova de Lisboa.
[4]E. Moreira, A. Ribeiro, E. Mateus, J. Mexia, and L. Ottosen. (2005). Regressional modeling of electrodialytic removal of Cu, Cr and As from CCA treated timber waste: application to sawdust. Wood Science and Technology. 39(4): 291–305.
[5]Ribeiro, A. and Mexia, J. T. (1997), A dynamic model for the electrokinetic removal of copper from a polluted soil. Journal of Hazardous Materials, **56**: 257–271.

# Statistical analysis of reading results with Letrinhas software. A case study in primary schools in Portugal

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## Abstract

Letrinhas is an information system created by the Polytechnic Institute of Tomar, in partnership with the Grouping of Schools Artur Gonves (Portugal), to promote learning and reading development. This system makes use of mobile technologies, providing a more dynamic, interactive and meaningful learning. The creation of applications that allow for the diagnosis and interaction with students with reading difficulties is extremely important [1], which leads Letrinhas to occupy a prominent role in the Portuguese panorama, as it responds to this need and is an innovative system that takes advantage of the potential of mobile devices with touch based interfaces, while promoting the students' motivation and autonomy. In fact, studies show that ICT provides students with a more active role in building knowledge [2] [3]. Furthermore, Letrinhas distinguishes itself from similar tools for evaluating reading fluency because it agrees with the Curricular Goals established by the Portuguese Government and because it allows teachers to choose texts and exercises to work with the students according to the diagnosed needs [4]. Studies show that intervention in the context of reading difficulties should focus on three main aspects: 1) early identification; 2) prevention; 3) re-education. With this in mind, the implementation of reading programs should take place in the first two years of schooling [5]. In order to analyze the impact of Letrinhas in the promotion of reading, along with its implications in the organizational and educational dynamics of the schools, a study was carried out in several School Groups of the Center Region of Portugal. This was done with the collaboration of the Training Center The Templars which is running a Letrinhas

software training programme for teachers. A total of 37 teachers and 116 students with reading difficulties participated in this innovative learning process, but only the results of the regular students who attended the 2nd and 3rd years of schooling were considered (the total sample size is 65). Two groups of students were formed: one using the Letrinhas (experimental group, with 33 students) and another using traditional teaching methods (control group, with 32 students). In the students learning process, there are two essential reading components: fluency and precision. To compare statistically the data gathered for these two variables, in both groups (experimental and control), some (non)parametric tests were applied. The differences between the two teaching methods of reading, observed in graphs and in the values of some descriptive statistical measures, are statistically significant (also the values obtained for the effect sizes measures are considerable). The positive impact of the Letrinhas software on the fluency and precision of reading, empirically identified by teachers, was statistically confirmed.

**Keywords:** Educational technology, non(parametric) tests, reading difficulties, special education.

**Acknowledgements**

This work has the financial support of the Educational Technology Laboratory (University of Coimbra) and Centre for Technology, Restoration and Art Enhancement (Polytechnic Institute of Tomar).

## References

[1]Carvalho, A. O. D. C. and Pereira, M. A. M., O Rei: um teste para avaliação da fluência e precisão da leitura no 1 e 2 ciclos do ensino básico, Psychologica, 51, 2009, pp. 283-305.

[2]Li, X. and Atkins, M., Early Childhood Computer Experience and Cognitive and Motor Development, Pediatrics, 113 (6), 2004, pp. 1715-1722.

[3]Behrmann, M. M., Assistive technology for young children in special education, in ASCD Yearbook 1998: Learning with Technology, edited by. C. Dede (Association for Supervision and Curriculum Development, Alexandria), 1998, pp. 73-93.

[4]Marques, C. G., Manso, A., Ferreira, A. P., and Morgado, F., Using Mobile Technologies in Education: A New Pedagogical Approach to Promote Reading Literacy. International Journal of Technology and Human Interaction (IJTHI), 13(4), 2017, pp. 77-90.

[5]Foorman, B. R., Francis, D. J., Shaywitz, B. A. and Fletcher, J. M., The case for early reading intervention., in B. Blachman (Ed.), Foundations of reading acquisition and dyslexia (Erlbaum New Jersey), 1997, pp. 243-264.

# Confidence Ellipsoids for Additive Models

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## Abstract

In this talk we will apply Edgeworth Expansions to obtain (approximate) quantiles for the $U_h$, $h = 1, ..., n$ cumulants. Then we will adjust confidence ellipsoids to these quantiles reducing the quadratic form to an inner product. We will also show that the Edgeworth Expansions allows us to obtain estimated quantis and then confidence ellipsoids and hypothesis tests through duality.

**Keywords:** Edgeworth Expansions, Confidence Ellipsoids, Cumulants.

### Acknowledgements

This work was partially supported by national founds of FCT-Foundation for Science and Technology under UID-MAT-00297-2019.

## References

[1]Craig, C. C. (1931). On A Property of the Semi-Invariants of Thiele. Ann Math Statist, 2, 2, 154–164.

[2]Kendall, M. and Stuart, A. (1958). The Advanced Theory of Statistics. Charles Griffin & Company.

[3]Mexia, J. T. (1987). Multi-treatment regression designs. Research Studies N.1, Mathematics Department, Faculty of Sciences and Tecnology, Universidade Nova de Lisboa.

# Semigroup homomorphism generated by quasimorphism

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## Abstract

Erschler and Karlsson [3] constructed a homomorphism from a finitely generated group to the reals through a random walk approach. In our previous work, similar results were obtained for a semidirect product using the word length [1]. In [2], the word length was replaced by a quasimorphism. It turns out that, working with quasimorphisms made it possible to generalize further the construction to an infinite, finitely generated semigroup $S$, endowed with a probability measure. Specifically, using semigroup quasimorphisms, we construct a homomorphism from $S$ to $(\mathbb{R}, +)$.

**Keywords:** Random walks on semigroups, Semigroup homomorphism, Semigroup quasimorphism.

### Acknowledgements

This work was partially supported by FCT through CMA-UBI, project UID-MAT-00212-2019.

## References

[1]Bettencourt, G. and Mendes, S., Homomorphisms to $\mathbb{R}$ of semidirect products: a dynamical construction, *Appl. Math. Inf. Sci.*, 9.6, 2015, pp. 1–7.
[2]Bettencourt, G. and Mendes, S., Homomorphisms to $\mathbb{R}$ generated by quasimorphisms, *Mediterr. J. Math.*, 13, 2016, pp. 3205–3219.
[3]Erschler, A., and Karlsson, A., Homomorphisms to $\mathbb{R}$ constructed from random walks, *Annales de l'Institut Fourier*, 60.6, 2010, pp. 2095–2113.

# The role of supply, demand and external shocks in an open economy: The case of Mozambique

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## Abstract

This paper studies the dynamics of Mozambique real Gross DomesticProduct (GDP), uncovering the role of three aggregate shocks, supply, demand and external, taking as South Africa economy as key for measuring the latter shock. The data covers, on an annual basis, Mozambique real GDP –output variable–, Mozambique GDP deflator –aggregate prices– and South Africa GDP –external economic activity– over 1990 to 2012. Statistical analysis of the data reveals the existence of unit roots and cointegration, after resorting to standard ADF testing and applying Johansen cointegration analysis (see Johansen [5]). The results from the test led to consider all series as integrated and to the existence of cointegration. Then, we analyzed causality (see Granger [4]), using Dolado and Lütkepohl [3]), and proceeded to identify the Vector Error-Correction (VEC) employing a long run scheme in line with Blanchard and Quah [1]) and Breitung, et al. [2]). The identified VEC delivers impulses responses where permanent effects on GDP come, essentially, from aggregate supply shock, which is also the major source of output fluctuations.

**Keywords:** aggregate supply, demand and external shocks, structural vector error-correction, Mozambique.

### Acknowledgements

This paper was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013.

## References

[1]Blanchard, O. and Quah, D., The Dynamic Effects of Aggregate Demand and Supply Disturbances, *American Economic Review*, 79 (4), 1989, pp. 655-673.

[2]Breitung, J., Brüggemann, R., and Lütkepohl, Structural Vector Autoregressive Modeling and Impulse Responses, in Lutkepohl, H. and Krätzig, M. (Eds.), *Applied Time Series Econometrics*, 2004, Cambridge University Press.

[3]Dolado, J. and Lütkepohl, H., Making wald tests work for cointegrated VAR systems, *Econometric Reviews*, 15 (4), 1996, pp. 369-386.

[4]Granger, C., Investigating Causal Relations by Econometric Models and Cross-Spectral Methods, *Econometrica*, 37 (3), 1969, pp. 424-438.

[5]Johansen, S., Statistical analysis of cointegration vectors, *Journal of Economic Dynamics and Control*, 12 (2-3), 1988, pp. 231-254.

# Metric functionals: a new class of examples

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## Abstract

Inspired by Gromov's construction of horofunction compactification, Karlsson [1], [2], among others, started an entire program dedicated to the idea of transposing functional analysis constructions to metric spaces. Central in this realm of ideas is the concept of metric functional. In this work we consider the metric space of metrics, defined on a finite set [3], and obtain the entire description of metric functionals in the first nontrivial cases.

**Keywords:** Horofunction compactification, Metric space, Metric functional.

### Acknowledgements

This work was partially supported by FCT through CMA-UBI, project UID-MAT-00212-2019.

## References

[1]Karlsson, A., Elements of a metric spectral theory, *Preprint*
[2]Gouëzel, S., Karlsson, A., Subadditive and multilplicative ergodic theorems *To appear in J. Eur. Math. Soc.*
[3]Khare, A., The non-compact normed space of norms on a finite-dimensional Banach space, *arXiv:1810.06188*

## Organized Session

# Statistical Modeling and Applications

**Organizer: Swati DebRoy**

# The impact of psychosocial stressors on college students' wellbeing. A case study with application of PLS-SEM

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## Abstract

College students are exposed to psychosocial stressors during their studies (undergraduate|graduate) that can have a negative influence in their (psychological and physical) wellbeing. In fact, students experience multiple socio-economic, relational, and socio-professional outcome concerns that can lead them to the burnout psychological syndrome, usually associated with academic performance unbalance and school dropout [1]. To evaluate the academic burnout some questionnaires have been used despite some discussion in the scientific literature regarding their psychometric characteristics [3,4,6]. In this case study a questionnaire was developed by students of the course of Modelling and Simulation in Global Health and Neglected Tropical Diseases (School of Human Evolution and Social Change, Arizona State University, USA), where they tried to consider their own experience as students. The questions used correspond to ordinal variables measured on a 10-point scale and the data gathered have a character entirely observational (these ordinal observed/manifest variables operationalize latent variables|constructs such as cognitive stress, behavioral stress, quantitative demands, insecurity, somatic stress and distress). The Structural Equations Modeling (SEM) was applied to analyze the relationships between the manifest variables (directly measured) and the latent constructs (not directly observed), as well as among the latent constructs. Partial Least Squares (PLS), a variance-based estimator, was applied because it maximizes the explained variance of the available endogenous latent constructs and because it emphasizes prediction while it simultaneously relaxes the demands on data and the specification of those relationships [2,5]. Based on a priori perceptions about causal relations among

those constructs, and consistent with literature and authors' experience, a theoretical structural model was proposed. The PLS-SEM estimated model allows a better understanding of the variables that may be considered as a source of burnout in college students. Although PLS-SEM is basically an exploratory technique, the use of bootstrap resampling procedure allows limited but credible forms of inference leading to the conclusion that the main causal hypothesis expressed with the model were supported by data. The latent constructs distress and quantitative demands have a direct effect on the academic burnout, and the latent constructs behavioral stress and insecurity have an indirect effect on that endogenous variable. To compare the model obtained by gender (female and male) a multi-group analysis was developed, where the differences founded in the paths coefficients are not statistically significant. However, some of these differences might be considered interesting in the light of psychological theory.

**Keywords:** Bootstrap, Partial Least Squares, SmartPLS, Survey.

**Acknowledgements**

This research was partially supported by Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2019 (Centro de Matemática e Aplicações).

## References

[1]Dyrbye, L. N., Thomas, M. R., Power, D. V., Durning, S., Moutier, C., Massie, F. S., Harper, W., Eacker, A., Szydlo, D. W., Sloan, J. A. and Shanafelt, T. D., Burnout and serious thoughts of dropping out of medical school: a multi-institutional study. *Academic Medicine*, **85** (1), 2010, pp. 94–102.

[2]Hair, J. F., Hult, G. T. M., Ringle, C. M. and Sarstedt, M., *A Primer on Partial Least Squares Structural Equation Modeling (PLS-SEM).* 2nd Ed., Thousand Oakes, CA: Sage, 2017.

[3]Maroco, J. and Campos, J. A. D. B., Defining the Student Burnout construct: A structural analysis from three Burnout Inventories. *Psychological Reports: Human Resources & Marketing*, **111** (3), 2011, pp. 814–830.

[4]McCarthy, M. E., Pretty, G. M. and Catano, V., Psychological sense of community and student burnout. *Journal of College Student Development*, **31**(3), 1990, pp. 211–216.

[5]Sarstedt, M., Hair, J. F., Ringle, C. M., Thiele, K. O. and Gudergan, S. P, Estimation issues with PLS and CBSEM: Where the bias lies!, Journal of Business Research, 69, 2016, pp. 3998-4010.

[6]Schaufeli, W., Martínez, I. M., Pinto, A. M., Salanova, M. and Bakker, A. B., Burnout and engagement in university students: a cross-national study. Journal of Cross-cultural Psychology, **33**(5), 2002, pp. 464–461.

# Latin Text Authorship using Dynamical Measurements of Complex Networks

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## Abstract

The study of complex systems through has proved to be very useful, especially in the analysis of social networks. Using clustering techniques, communities are detected in networks of friendship or shared interests [1,3]. In this work we study a network of Latin text to obtain authorship verification [4]. The Latin texts are from Historia Augusta, a collection of biographies of Roman emperors stretching from Hadrian(117-138) to Carus (282-83) and his son Carinus (283-285) and Numerian (AD 283–284), and our goal is to verify the hypotesis that the authorship atribution of the texts is correct (six authors) or that all texts are written by the one author. We apply a spectral clustering tool that we have developed, based on the second eigenvector of the Laplacian matrix of the graph to obtain the topological chacterization of the networks: Degrees, Accessibility, Betweenness, Assortativity, Clustering coefficient e Average shortest path length. This techniques allows to avoid the high cost of combinatorial algorithms using numerical methods of linear algebra, well established in scientific computation, see [2]. In the case under study, the detection of communities identifies trends that allows to suspect of text clusters by the same author. To evaluate our results we study the Latin text authorship using other authors texts such as Res Gestae a Fine Corneli Taciti by Ammianus Marcelinus (4 AC) an author contemporary of Historia Augusta authors; and we use different computationaly methods to verify texts authorship such as Kmeans. To improve our authorship verification we use a list of Latin stop words, latin words lemmatization as well as some syntatic structures detection [5].

**Keywords:** Latin text Authorship, network graphs, clustering, weighted graphs, spectral clustering.

**Acknowledgements**

This work has been partially supported by Centro de Investigação em Matemática e Aplicações (CIMA) through the grant UID-MAT-04674-2013, by Lab- oratório de Informática, Sistemas e Paralelismo (LISP) through the grant UID-CEC-4668-2016, both research centers are supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal) and, also, by Departamento de Matemáticas, y Escuela Politécnica de Cáceres, de la Universidad de Ex- tremadura, Spain.

## References

[1]TEIXEIRA, Cláudia; RODRIGUES, Irene. Deciphering Latin sentences using traditional linguistic resources. Digital Scholarship in the Humanities, 2018.

[2]Stover, J. A., Kestemont, M. (2016). THE AUTHORSHIP OF THE HISTORIA AUGUSTA: TWO NEW COMPUTATIONAL STUDIES. Bulletin of the Institute of Classical Studies, 59(2), 140-157.

[3]Tohalino, J. V., Amancio, D. R. (2017, October). Extractive Multi-document Summarization Using Dynamical Measurements of Complex Networks. In 2017 Brazilian Conference on Intelligent Systems (BRACIS) (pp. 366-371). IEEE.

[4]Newman, M., Barabasi, A.L., and Watts, D.J. The structure and dy- namics of networks. Princeton University Press, 2011.

[5]Kannan, R., Vempala, S., Vetta, A. (2004). On Clusterings: Good. Bad and Spectral. Journal of the ACM, v.51, pp. 497–515.

# Mathematical Modeling Applied to Childhood Obesity

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## Abstract

School-based interventions provide the best obesity prevention and reduction strategy in children. Evaluating the isolated effectiveness of an intervention in light of changing height and body composition is challenging. Additionally, physiological and psychosocial responses to an intervention are dependent on baseline body mass index (BMI), sex, socioeconomic status, and age. We propose a non-linear ordinary differential equation model to quantify expected school specific BMI trends and determine how and if the BMI trajectory was altered due to the intervention. This model is modified from the adult obesity model formulated by Thomas, et al. [1]. The talk will also discuss how data gathered by our team through a middle school intervention has helped in direct parameter estimation for the model.

**Keywords:** ordinary differential equation, parameter estimation, childhood obesity.

### Acknowledgements

This work was partly supported by ASPIRE-I grant by University of South Carolina, a grant by National Institute on Minority Health and Disparity (USA) pass-through by TCC Morehouse, Morehouse School of Medicine, Atlanta, USA. Also supported by Sea Island Institute at University of South Carolina Beaufort, USA.

## References

[1]Thomas DM, Weedermann M, Fuemmeler BF, et al. Dynamic model predicting overweight, obesity, and extreme obesity prevalence trends. Obesity (Silver Spring). 2014; 22(2):590–597. doi:10.1002/oby.20520

## Organized Session

# Optimal Control Theory and Applications

**Organizer: Cristiana Silva**

# Numerical solution of variable-order fractional optimal control-affine problems using Bernoulli polynomials

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## Abstract

Optimal control theory lets us to choose control functions in a dynamical system to reach a specific goal and has found many applications in many life sciences [1,2]. When the dynamical system in an optimal control problem includes fractional differential equations, the problem is called a fractional optimal control problem. A recent generalization of the theory of fractional calculus is to allow the fractional order of the derivatives to be dependent on time [3]. In this work, we focus on the following variable-order fractional optimal control-affine problem

$$\min \ J = \int_0^1 \phi(t, x(t), u(t))dt,$$

s.t. the control-affine dynamical system

$${}_0^C D_t^{\alpha(t)} x(t) = \varphi\left(t, x(t), {}_0^C D_t^{\alpha_1(t)} x(t), \ldots, {}_0^C D_t^{\alpha_s(t)} x(t)\right) + b(t)u(t),$$

and the initial conditions

$$x^{(i)}(0) = x_0^i, \quad i = 0, 1, \ldots, n,$$

where $\phi$, $\varphi$ and $b \neq 0$ are smooth functions of their arguments, $n$ is a positive integer number such that for all $t \in [0,1]$: $0 < \alpha_1(t) < \alpha_2(t) < \ldots < \alpha_s(t) < \alpha(t) \leq n$, and ${}_0^C D_t^{\alpha(t)}$ is the (left) fractional derivative of the variable-order defined in the Caputo sense. Our aim is to find an approximation of the optimal value of the performance index $J$ using the properties of the Caputo derivative and Riemann–Liouville integral operators of variable-order. To this aim, we introduce an accurate operational matrix of variable-order fractional integration for the Bernoulli polynomials [4] and use this

matrix to give approximations for the state function $x(t)$ and its derivatives. Then, an approximation of the unknown control function $u(t)$ is given using the dynamical system. By employing Gauss–Legendre quadrature rule and using the optimality condition, the main problem is reduced to the solution of a system of nonlinear algebraic equations. Finally, in order to show the efficiency and accuracy of the method, we apply this new technique to a test problem.

**Keywords:** variable-order fractional optimal control problem, Bernoulli polynomials, operational matrix.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-04106-2019 (CIDMA).

## References

[1]Bertsekas, D.P., *Dynamic programming and optimal control*, Vol. I, fourth edition, Athena Scientific, Belmont, MA, 2017.

[2]Leong, A.S., Quevedo, D.E. and Dey, S., *Optimal control of energy resources for state estimation over wireless channels*, SpringerBriefs in Electrical and Computer Engineering, Springer, Cham, 2018.

[3]Almeida, R., Tavares, D. and Torres, D.F.M., *The variable-order fractional calculus of variations*, Springer, 2019.

[4]Costabile, F., Dellaccio, F. and Gualtieri, M.I., A new approach to Bernoulli polynomials, *Rendiconti di Matematica, Serie VII*, 26, (2006), pp. 1–12.

# Curtail the spread of cholera outbreaks through optimal control theory

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## Abstract

We propose and study several mathematical models for the transmission dynamics of some strains of the bacterium *Vibrio cholerae*, responsible for the cholera disease in humans. Control functions are added with the purpose to obtain different optimal control problems. Its study allow us to determine the best way to curtail the spread of the disease. Such models are applied to the cholera's outbreak of Haiti (2010-2011) and Yemen (2017-2018) (see [1,2]).

**Keywords:** disease-free and endemic equilibria, stability, quarantine, vaccination.

### Acknowledgements

This work was supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology – FCT) within projects UID-MAT-04106-2019 (CIDMA) and PTDC-EEI-AUT-2933-2014 (TOCCATA), funded by Project 3599 – Promover a Produção Científica e Desenvolvimento Tecnológico e a Constituição de Redes Temáticas and FEDER funds through COMPETE 2020, Programa Operacional Competitividade e Internacionalização (POCI). Lemos-Paião was also supported by the Ph.D. fellowship PD-BD-114184-2016; Silva by national funds (OE), through FCT, I.P., in the scope of the framework contract foreseen in the numbers 4, 5 and 6 of the article 23, of the Decree-Law 57-2016, of August 29, changed by Law 57-2017, of July 19.

## References

[1]Lemos-Paião, A. P. and Silva, C. J. and Torres, D. F. M., An epidemic model for cholera with optimal control treatment, *J. Comput. Appl. Math.*, 318, 2017, pp. 168–180.

[2]Lemos-Paião, A. P. and Silva, C. J. and Torres, D. F. M., A cholera mathematical model with vaccination and the biggest outbreak of world's history, *AIMS Mathematics*, 3(4), 2018, pp. 448–463.

# Ebola Modeling and Optimal Control with Vaccination Constraints

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## Abstract

The Ebola virus disease is a severe viral haemorrhagic fever syndrome caused by Ebola virus. This disease is transmitted by direct contact with the body fluids of an infected person and objects contaminated with virus or infected animals, with a death rate close to 90% in humans. Recently, some mathematical models have been presented to analyse the spread of the 2014 Ebola outbreak in West Africa. In this paper, we introduce vaccination of the susceptible population with the aim of controlling the spread of the disease and analyse two optimal control problems related with the transmission of Ebola disease with vaccination. Firstly, we consider the case where the total number of available vaccines in a fixed period of time is limited. Secondly, we analyse the situation where there is a limited supply of vaccines at each instant of time for a fixed interval of time. The optimal control problems have been solved analytically. Finally, we have performed a number of numerical simulations in order to compare the models with vaccination and the model without vaccination, which has recently been shown to fit the real data. Three vaccination scenarios have been considered for our numerical simulations, namely: unlimited supply of vaccines; limited total number of vaccines; and limited supply of vaccines at each instant of time. This talk is based on the recent paper [1].

**Keywords:** transmission of Ebola, control of the spread of the Ebola disease, optimal control with vaccination constraints, vaccination scenarios.

### Acknowledgements

Work partially supported by FCT through the project UID-MAT-04106-2019 (CIDMA) and the PhD fellowship PD-BD-150273-2019 (Ndaïrou).

## References

[1]I. Area, F. Ndaïrou, J. J. Nieto, C. J. Silva, D. F. M. Torres (2018). *Ebola model and optimal control with vaccination constraints.* J. Ind. Manag. Optim. 14 , no. 2, 427–446.

# Generating trajectories in fluid environments

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## Abstract

*Drag* is a mechanical force that acts in the opposite direction to the relative motion of a solid moving in a fluid environment. Its magnitude is typically proportional to the cube of the solid's speed and therefore the power needed to overcome the drag is proportional to the cube of the speed [1]. Motivated by trajectory planning problems of Autonomous Underwater Vehicles (AUV), we propose an optimization problem responsible for driving a vehicle from an initial position to a final target while minimizing both acceleration and drag [3]. The corresponding Euler–Lagrange equations will be derived. The absence of the drag term corresponds to the problem of finding cubic polynomials prescribing initial and final velocities. Even for this particular case, finding explicit solutions for the Euler–Lagrange equations is a major issue that remains unsolved and motivated several authors to look for alternative approaches [2]. Needless to say that the presence of the drag term will increase substantially the difficulty of the problem [5]. In order to overcome these difficulties, a numerical optimization procedure based on the discretization of the given functional will be proposed in order to obtain approximate solutions for the problem [4]. Several numerical illustrations of the solutions will be provided for some of the most important curved spaces used in robotics applications.

**Keywords:** drag, calculus of variations, Riemannian geometry, numerical optimization.

### Acknowledgements

This work was partially supported by OE - national funds of FCT-MCTES (PIDDAC) under project UID-EEA-00048-2019.

## References

[1]Cengel, Y., Cimbala, J., *Fluid Mechanics Fundamentals and Applications*, McGraw-Hill Science/Engineering/Math, 3rd edition, 2013.

[2]Crouch, P., Silva Leite, F., The Dynamic Interpolation Problem: on Riemannian Manifolds, Lie Groups and Symmetric Spaces, *Journal of Dynamical and Control Systems*, 1(2), 1995, pp. 177–202.

[3]Kruger, D., Stolkin, R., Blum, A., Briganti, J., Optimal AUV path planning for extended missions in complex, fast-flowing estuarine environments, *2007 IEEE International Conference on Robotics and Automation (ICRA)*, 2007, pp. 4265–4270.

[4]Machado, L., Silva Leite, F., Monteiro, M. T. T., Path planning trajectories in fluid environments, *2018 13th APCA International Conference on Automatic Control and Soft Computing (CONTROLO)*, 2018, pp. 219–223.

[5]Yudin, I., Silva Leite, F., Algebraic integrability for minimum energy curves, *Kybernetika*, 51(2), 2015, pp. 321–334.

# Can malware propagation be softened via optimal control theory and mathematical epidemiology?

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## Abstract

Purposing to lessen malware propagation, this work proposes optimal control measures using epidemiological modeling ([1] and [2]). By taking advantage of real-world data related to the number of reported cybercrimes in Japan from 2012 to 2017, an optimal control problem is formulated in order to minimize the number of infected devices in a cost-effective way ([3]). Overall, numerical simulations show the usefulness of the proposed control strategies in reducing the spread of malware infections.

**Keywords:** optimal control, optimal control application, malware propagation, SCIRS epidemiological model.

### Acknowledgements

This research has been supported by FCT - Fundação para a Ciência e Tecnologia within the project scopes: UID-MAT-04106-2019, through CIDMA - Center for Research and Development in Mathematics and Applications, and UID-CEC-00319-2019, through ALGORITMI R&D Center.

## References

[1]Piqueira, J. and Araujo, V., A modified epidemiological model for computer viruses. Applied Mathematics and Computation, 213(2), 2009, pp. 355–360.

[2]Khan, M.S., A computer virus propagation model using delay differential equations with probabilistic contagion and immunity. International Journal of Computer Networks & Communications, 6(5), 2014, pp. 111-128.

[3]Fleming, W. and Rishel, R., Deterministic and Stochastic Optimal Control. *Springer–Verlag, New–York* 1975.

# Optimality conditions for variational problems containing a derivative with a non-singular kernel

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## Abstract

The aim of this talk is to show sufficient and necessary optimality conditions [1] of certain problems of calculus of variations with a Lagrangian depending on a fractional derivative that have a non-singular kernel. Some special cases of fractional variational problems will be presented.

**Keywords:** Fractional calculus, calculus of variations, Caputo-Fabrizio fractional operator.

### Acknowledgements

This work was supported by the Portuguese Foundation for Science and Technology (Fundação para a Ciência e a Tecnologia), through CIDMA – Center for Research and Development in Mathematics and Applications, within project UID-MAT-04106-2019.

## References

[1]N.R.O. Bastos, Calculus of variations involving Caputo-Fabrizio fractional differentiation. Stat. Optim.Inform.Comput. (6), 2018, pp. 12–21.

# Shortest Paths for the Optimal Reorganization of a Nonholonomic Multi–Vehicle Formation

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## Abstract

Often, a group of mobile robots is used to perform a certain task as a team. On the one hand, they are coupled but, on the other hand, the movement of each one does not directly affects the others, which means they can be considered dynamically decoupled. Reported work in the areas of cooperative control and multiple vehicles systems can be found, for example, in [1,2,6]. When mobile robots perform as a team, there are some situations where we need to change the layout of the multi–vehicle formation. In many applications, this change is due to the adaptation to new tasks or to changes in the trajectory of the robots. A typical example involves the reshape of the formation to pass through a narrow passage or when there is an obstacle in the path. In these cases, the multi–vehicle formation must be reconfigured to a new geometry. In this work we address the problem of reconfiguring the layout of a formation of undistinguishable nonholonomic mobile robots, where each vehicle moves from its current position to a target oriented position using the shortest path [3]. We combine results from previous work on optimal formation switching of robots that are holonomic [2] with results on the structure of the shortest path for nonholonomic vehicles [1]. To solve the optimal control problem inherent to this application, we use direct methods [4]. A new feature that distinguishes this work from previous approaches is the fact that the shortest path between two oriented points in the plane for a nonholonomic vehicle has to be computed, which is done through Dubin's result [4].

**Keywords:** nonholonomic systems, multi–vehicle formation, formation reconfiguration, optimal control, direct methods, numerical methods, adaptive mesh refinement.

**Acknowledgements**

In this research, we acknowledge the support of FEDER-COMPETE-2020-NORTE-2020-POCI-PIDDAC-MCTES-FCT funds through grants SFRH-BPD-126683-2016, NORTE-01-0145-FEDER-000033-Stride, UID-IEEA-00147-006933-SYSTEC, POCI-01-0145-FEDER-031821-FAST, AAC2-SAICT-2017-31411-Harmony, PTDC-EEI-AUT-2933-2014, 16858-TOCCATA and POCI-01-0145-FEDER-031447-UPWIND.

## References

[1]A.C.D. Caldeira, L.T. Paiva, D.B.M.M. Fontes, F.A.C.C. Fontes. Optimal Reorganization of a Formation of Nonholonomic Agents Using Shortest Paths, *in Proceedings of CONTROLO'18 – 13th APCA International Conference on Control and Soft Computing*, pp. 177–182, Ponta Delgada, Azores, Portugal, June 2018.

[2]F.A.C.C. Fontes, D.B.M.M. Fontes, and A.C.D. Caldeira. Model predictive control of vehicle formations, *in Optimization and Cooperative Control Strategies*, ser. Lecture Notes in Control and Information Sciences, 381, M. Hirsch, C. Commander, P. Pardalos, and R. Murphey, Eds. Berlin: Springer Verlag, 2009.

[3]H.J. Sussmann, G. Tang. *Shortest paths for the reeds-shepp car: a worked out example of the use of geometric techniques in nonlinear optimal control*, Rutgers Center for Systems and Control, Tech. Rep. SYCON-91-10, September 1991.

[4]L.E. Dubins. On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents, *American Journal of mathematics*, 79(3), pp. 497–516, 1957.

[5]L.T. Paiva, F.A.C.C. Fontes. Adaptive Time–Mesh Refinement in Optimal Control Problems With State Constraints, *Discrete and Continuous Dynamical Systems*, 35(9), pp. 4553–4572, September 2015.

[6]R.M. Murray. Recent research in cooperative control of multi–vehicle systems, *Journal of Dynamic Systems, Measurement and Control*, 129, pp. 57–583, 2007.

# Replanning with fine meshes in irrigation systems

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## Abstract

Drinking water is an essential supply that is increasingly scarce, not only due to climate change but also due to the pollution caused by humans. It is extremely important to know how to use the water in a sustainable way. Agriculture uses the largest portion, about 70% of freshwater. More and more farmers are adopting water management strategies to improve the use of water in an efficient way, such as innovative irrigations systems. Using Optimal control [5], the authors have developed a model for the irrigation systems, [1]. This model has the advantage of planning the use of water for 10 days, instead of an ON-OFF irrigation system. The ON-OFF systems trigger the irrigation cycle when a minimum critical value of soil moisture is detected and suspend it when a defined maximum is reached (sometimes close to saturation). The excess of water in the soil, that arise frequently when applying of these techniques, is responsible for a significant water waste. Having a planning irrigation systems not only allows to safe water, but also allows to be adapted to different growth stages of the crop. In [2], predictive control techniques are used to improve the model described in [1], to correct the forecast imprecisions of the weather data. Using the humidity soil data, the software determines whether the water spent was lower or greater in comparison with the real needs, and the replanning is activated when the estimated humidity of the soil is not on a small vicinity of the real humidity of the soil. Based on the work develop in [4], we use a fine mesh and create several scenarios to understand the behaviour of the optimal irrigation systems, in [3]. The inclusion of hourly data in the model allows to consider scenarios where, for instance, rainfall is uniform or not uniform, the rainfall is concentrated in a period of time. The use of smaller time steps produce a smoother solution, with less water consumption. In this work, we propose to use predictive control to improve model [3] with the techniques described

in [2]. Thus, we obtain the possibility to replan the irrigation, using a finer mesh.

**Keywords:** optimal control, predictive control, mesh refinement, replan, irrigation systems.

### Acknowledgements

We acknowledge FEDER-COMPETE-NORTE-2020-POCI-FCT funds through grants UID-EEA-00147-2013, UID-IEEA-00147-006933-SYSTEC, PT-DC-EEI-AUT-2933-2014116858-TOCCATA, project, to CHAIR-POCI-01-0145-FEDER-028247 and UPWIND-POCI-FEDER-FCT-31447. Financial support from the Portuguese Foundation for Science and Technology (FCT) in the framework of the Strategic Financing UIDIFIS-04650-2013 and UID-MAT-00013-2013 is also acknowleged.

## References

[1]S.O. Lopes, F. A. C. C. Fontes, R. M. S. Pereira, M.D.R. de Pinho and A.M. Gonçalves. Optimal control applied to an irrigation planning problem . *Mathematical Problems in Engineering* **2016**, *2016*, 10 pages, doi:10.1155/2016/5076879.

[2]S.O. Lopes, M.F.P. Costa, R.M.P.S. Pereira, and F.A.C.C. Fontes, Replanning the irrigations systems, *SYMCOMP 2019 - 4th International Conference on Numerical and Symbolic Computation Developments and Applications*, accepted.

[3]S.O. Lopes, M.F.P. Costa, R.M.P.S. Pereira, M.T. Malheiro and F.A.C.C. Fontes. Irrigation planning with fine meshes, *Eurogen 2019 - Evolutionary and Deterministic Methods for Design, Optimization and Control with Applications to Industrial and Societal Problems*, submitted.

[4]L.T. Paiva, F.A.C.C. Fontes. Adaptive Time–Mesh Refinement in Optimal Control Problems With State Constraints, *Discrete and Continuous Dynamical Systems*, 35(9), pp. 4553–4572, September 2015.

[5]R. Vinter. Optimal control, *Birkhause*, Boston, 2000.

## Organized Session

# Dynamics and Games

**Organizer: Alberto Pinto**

# Firm Competition on a Hotelling Network

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## Abstract

We develop a theoretical framework to study the location-price competition under uncertainty of firms' production costs. Firms compete in a two-stage Hotelling- type network game, with linear transportation costs. We show the existence of a Bayesian-Nash equilibrium price if, and only if, some explicit conditions on the ex- pected production costs and on the network structure hold. Furthermore, we prove that the local optimal location of the firms are at the nodes of the network.

**Keywords:** Hotelling network mode, game theory, price competition, oligopoly models.

### Acknowledgements

The authors thank the financial support of LIAAD–INESC TEC and FCT–Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project Modelling, Dynamics and Games, with reference PTDC-MAT-APL-31753-2017.

## References

[1]Hotelling, H. (1929). Stability in Competition. *The Economic Journal*, 39(153), 41-57. doi:10.2307/2224214.

[2]Pinto, A.A., Almeida, J.P. and Parreira, T. (2016). Local market structure in a Hotelling town,J*ournal of Dynamics & Games*, 3(1), 75–100. doi: 10.3934/jdg.2016004

# A numerical characterization of the reinfection threshold

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## Abstract

The reinfection SIRI model describes the spreading of an epidemics in a population of susceptible (S), infected (I), and recovered (R) individuals, where after an initial infection the recovered individuals only have partial immunity against a possible reinfection. Grassberger, Chaté and Rousseau [1] considered similar models with partial immunization, and observed transitions between phases of no-growth, annular growth and compact growth. The transition between annular growth and compact growth corresponds to the reinfection threshold in epidemiology, introduced by Gomes et al. in 2004 [2]. In this work, we propose a new concept of reinfection threshold based on the global maximum of the positive curvature of the endemic stationary state. This new maximum curvature reinfection threshold forms a smooth curve with respect to the temporary immunity transition rate, and has the property of coinciding with the previous notion of the reinfection threshold at the limit of vanishing the temporary immunity transition rate.

**Keywords:** reinfection threshold, SIRI model, curvature.

### Acknowledgements

The authors thank the financial support of LIAAD-INESC TEC and FCT-Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project Modelling, Dynamics and Games, with reference PTDC-MAT-APL-31753-2017.

## References

[1]P. Grassberger, H. Chaté, and G. Rousseau (1997). *Spreading in media with long-time memory*, Phys. Rev. E 55, 2488-2495.

[2]M.G.M. Gomes, L.G. White and G.F. Medley (2004). *Infection, reinfection and vaccination under suboptimal protection: epidemiological perspective*, J. Theor. Biology, 228, 539-549.

# Pure price duopoly equilibria for discrete preferences

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## Abstract

We show that in a duopoly with homogeneous consumers, if these are negatively influenceable by each other behavior (e.g. through a congestion snob Veblen effect), a pure price equilibrium with positive profits for both firms exists. Furthermore, even in the case products are undifferentiated, an equilibrium where firms charge different (positive) prices and have different profits exists. In particular, such an equilibrium exists for atomic distributions of consumer preferences. We further show that in the case products are differentiated, social differentiation overcomes the effect of standard differentiation in creating price asymmetries. Furthermore, this provides a way to discretize the Hotelling model while guaranteeing pure price equilibria.

**Keywords:** Social influence, Bertrand duopoly, Bertrand paradox, Bertrand competition, consumption externalities, product differentiation, pure price equilibrium.

### Acknowledgements

This work is financed by National Funds through the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia within project Modelling, Dynamics and Games - MDG - with the reference PTDC-MAT-PL-31753-2017).

## References

[1] R. Soeiro, A. Mousa, and A. A. Pinto Externality effects in the formation of societies, *Journal of Dynamics and Games*, **2** (2015), 303–320.

[2] R. Soeiro, A. Mousa, T. R. Oliveira and A. A. Pinto Dynamics of human decisions, *Journal of Dynamics and Games*, **1** (2014), 121–151.

# Learning strategies in the Ignorant - Believer - Unbeliever rumor spreading model

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## Abstract

Based on the classical epidemiological SIR model, we propose a similar model to analyze the spreading of a false rumor in a homogeneous mixing population [1]. Regarding the rumor, individuals can be ignorants ($I$), believers ($B$) or unbelievers ($U$). Hence, we propose the $IBU$ spreading model. Assuming that the rumor is false, we study its spreading depending on the level of the real information attained by the individuals. Since the search for information can have costs but can also be very advantageous to an individual, we will introduce the expected learning payoff and we will compute the Nash and the evolutionary stable information search strategies [2].

**Keywords:** SIR model, Rumors.

### Acknowledgements

The authors thank the financial support of LIAAD-INESC TEC and FCT-Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project Modelling, dynamics and games, with reference PTDC-MAT-APL-31753-2017.

## References

[1]Daley D.J. and Kendall D.G., Epidemics and rumours, *Nature* 204, 1964, 1118.
[2]Hofbauer J. and Sigmund K., *Evolutionary Games and Population Dynamics*, Cambridge University Press, 1998.

## Organized Session

# Integral and Differential Equations & Applications

**Organizer: M. Manuela Rodrigues**

# Time-fractional diffusion-wave equation

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## Abstract

In this talk, we study the multidimensional time-fractional diffusion-wave equation where the time fractional derivative is in the Caputo sense with order $\beta \in ]0, 2]$. Applying operational techniques via Fourier and Mellin transforms we obtain an integral representation of the fundamental solution (FS) of the time fractional diffusion-wave operator. Series representations of the FS are explicitly obtained for any dimension. From these we derive the FS for the time fractional parabolic Dirac operator in the form of integral and series representation. Fractional moments of arbitrary order $\gamma > 0$ are also presented. To illustrate our results we present and discuss some plots of the FS for some particular values of the dimension and of the fractional parameter. The results presented in talk can be found in [1].

**Keywords:** Time fractional diffusion-wave operator, Time fractional parabolic Dirac operator, Fundamental solutions, Caputo fractional derivative, Fractional moments.

### Acknowledgements

This work was supported by Portuguese funds through CIDMA-Center for Research and Development in Mathematics and Applications, and FCT–Fundação para a Ciência e a Tecnologia, within project UID-MAT-04106-2019. N. Vieira was also supported by FCT via the FCT Researcher Program 2014 (Ref: IF-00271-2014).

## References

[1]Ferreira, M. and Vieira, N., Fundamental solutions of the time fractional diffusion-wave and parabolic Dirac operators, *J. Math. Anal. Appl.*, 447(1), 2017, pp. 329–353.

# Time-fractional Borel-Pompeiu formula and hypercomplex fractional operator calculus

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## Abstract

In this talk we present a time-fractional operator calculus in fractional Clifford analysis. Initially we study the $L_p$-integrability of the fundamental solutions of the multidimensional time-fractional diffusion and parabolic Dirac operators. Then we introduce the time-fractional analogues of the Teodorescu and Cauchy-Bitsadze operators in a cylindrical domain, and we investigate their main mapping properties. As a main result, we prove a time-fractional version of the Borel-Pompeiu formula based on a time-fractional Stokes' formula. This allows us to present a Hodge-type decomposition for the $L^p$-sapce.. The obtained results exhibit an interesting duality relation between forward and backward parabolic Dirac operators and Caputo and Riemann-Liouville time-fractional derivatives. Some applications of the obtained results for solving time-fractional boundary value problems will be shown. The results presented in the talk can be found in [1].

**Keywords:** Fractional Clifford analysis, Fractional derivatives, Time-fractional parabolic Dirac operator, Fundamental solution, Borel–Pompeiu formula.

**Acknowledgements**

This work was supported by Portuguese funds through CIDMA-Center for Research and Development in Mathematics and Applications, and FCT–Fundação para a Ciência e a Tecnologia, within project UID-MAT-04106-2019, and also by the project New Function Theoretical Methods in Computational Electrodynamics Neue funktionentheoretische Methoden für instationäre PDE, funded by Programme for Cooperation in Science between Portugal and Germany (Programa de Ações Integradas Luso-Alemãs 2017 -

DAAD-CRUP - Acção No. A-15-17-DAAD-PPP Deutschland-Portugal, Ref: 57340281. N. Vieira was also supported by FCT via the FCT Researcher Program 2014 (Ref: IF-00271-2014).

## References

[1]Ferreira, M., Rodrigues, M.M. and Vieira, N., A Time-Fractional Borel–Pompeiu Formula and a Related Hypercomplex Operator Calculus, *Complex Anal. Oper. Theory*, 2019, pp. 1-32.

# Convolution theorems for oscillatory integral transforms on the positive half-line

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## Abstract

Motivated by the range of possible applications of convolutions in Mathematics and other sciences (e.g. [3,4]), the main purpose of this talk is to present three new convolutions for some oscillatory integral operators defined on the positive half-line and in the framework of $L^1$ Lebesgue spaces. For that purpose, we present some fundamental and operational properties of those integral transforms. Probably, the most important property of a convolution is to satisfy a factorization property which is typically associated with one or more than one integral operators (Convolution Theorem). In this sense, we prove that the convolutions introduced exhibit certain factorization identities when considering the operators under studied applied to all those functions. In most of the cases, such factorization property is fundamental to solve consequent integral equations which can be characterized by those convolutions (e.g. [1,2]).

**Keywords:** integral operators, operator properties, convolutions, factorization identities.

### Acknowledgements

This work was supported by Fundação para a Ciência e a Tecnologia (FCT), within project UID-MAT-04106-2019 (CIDMA) and by national funds (OE), through FCT, I.P., in the scope of the framework contract foreseen in the numbers 4, 5 and 6 of the article 23, of the Decree-Law 57-2016, of August 29, changed by Law 57-2017, of July 19.

## References

[1]Castro, L.P., Minh, L.T. and Tuan, N.M., New convolutions for quadratic-phase Fourier integral operators and their applications, *Mediterranean Journal of Mathematics* 15(13), 2018, 17pp.

[2]Anh, P.K., Castro, L.P., Thao T. and Tuan, N.M., Two new convolutions for the fractional Fourier transform, *Wireless Personal Communications*, 92(2), 2017, pp. 623–637

[3]Tuan, N.M. and Huyen, N.T., Applications of generalized convolutions associated with the Fourier and Hartley transforms, *Journal of Integral Equations and Applications* 24, 2012, pp. 111–130.

[4]Xiang, Q. and Qin, K., Convolution, correlation, and sampling theorems for the offset linear canonical transform, *Signal, Image and Video Processing* 8(3), 2014, pp. 433–442.

# New convolutions and their applicability to integral equations of Wiener-Hopf plus Hankel type

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## Abstract

We propose eight new convolutions exhibiting convenient factorization properties associated with two classes of finite integral transforms of Fourier-type. Properties of these new convolutions are derived and applied to the solvability of a class of the integral equations of Wiener-Hopf plus Hankel type (on finite intervals). Fourier-type series are used to produce the solution formula of such equations, and a Shannon-type sampling formula is also obtained.

**Keywords:** convolution, integral transform, integral equation, Wiener-Hopf plus Hankel equation, Shannon-type sampling formula.

### Acknowledgements

L. P. Castro and R. C. Guerra were supported in part by FCT– Portuguese Foundation for Science and Technology through the Center for Research and Development in Mathematics and Applications (CIDMA) of Universidade de Aveiro, within project UID-MAT-04106-2019. R. C. Guerra also acknowledges the direct support of FCT through the scholarship PD-BD-114187-2016. Tuan was supported by the Viet Nam National Foundation for Science and Technology Development (NAFOSTED).

## References

[1]Anderson, B. D. O. and Kailath, T., Fast Algorithms for the Integral Equations of the Inverse Scatting Problem, *Integral Equations and Operator Theory*, 1, 1978, pp. 132–136.

[2]Anh, P. K., Tuan, N. M. and Tuan, P. D., The finite Hartley new convolutions and solvability of the integral equations with Toeplitz plus Hankel kernels, *Journal of Mathematical Analysis and Applications*,397, 2013, pp. 537–549.

[3]Anh, P. K., Castro, L. P., Thao, P. T. and Tuan, N. M., Two New Convolutions for the Fractional Fourier Transform, *Wireless Personal Communications*, 92, 2017, pp. 623–637.

[4]Bogveradze, G. and Castro, L. P., Toeplitz plus Hankel operators with infinite index, *Integral Equations and Operator Theory*, 62(1), 2008, pp. 43–63.

[5]Chanda, K. and Sabatier, P. C., *Inverse Problems in Quantum Scattering Theory*, Springer-Verlag, New-York, 1977.

[6]Castro, L. P., Guerra, R. C. and Tuan, N. M., *Heisenberg uncertainty principles for an oscillatory integral operator*, American Institute of Physics, AIP Proceedings **1798**(1) (2017), 020037, 10pp. http://dx.doi.org/10.1063/1.4972629

[7]Castro, L. P., Guerra, R. C. and Tuan, N. M., On Wiener's Tauberian theorems for an Oscillatory Integral Operator, to appear, 24pp., DOI: 10.3906/mat-1801-90.

[8]Castro, L. P. and Rojas, E. M., Explicit solutions of Cauchy singular integral equations with weighted Carleman shift, *Journal of Mathematical Analysis and Applications*, 371, 2010, pp. 128–133.

[9]Castro, L. P., Rojas, E. M., Saitoh, S., Tuan, N. M. and Tuan, P. D., Solvability of singular integral equations with rotations and degenerate kernels in the vanishing coefficient case, *Analysis and Applications*, 13(1), 2015, pp. 1–21.

[10]Castro, L. P., Itou, H. and Saitoh, S., Numerical solutions of linear singular integral equations by means of Tikhonov regularization and reproducing kernels, *Houston Journal of Mathematics*, 38(4), 2012, pp. 1261–1276.

[11]Castro, L. P., Minh, L. T. and Tuan, N. M., New convolutions for quadratic-phase Fourier integral operators and their applications, *Mediterranean Journal of Mathematics*, 15(13), 2018, pp. 1–17.

[12]Debnath, L. and Bhatta, D., *Integral transforms and their applications, Second Edition*, Chapman & Hall/CRC, Boca Raton, 2007.

[13]Kailath, T., Levy, B., Ljung, L. and Morf, M., Fast time-invariant implementations of Gaussian signal detectors, *IEEE Transactions on Information Theory*, 24(4), 1978, pp. 469–477.

[14]Katznelson, Y., *An Introduction to Harmonic Analysis*, Cambridge University Press, Cambridge, 2004.

[15]Stein, E. M. and Shakarchi, R., *Fourier analysis: An introduction* (Princeton Lectures in Analysis, I), Princeton University Press, Princeton and Oxford, 2007.

[16]Tsitsiklis, J. N. and Levy, B. C., *Integral Equations and Resolvents of Toeplitz plus Hankel Kernels*. In: Technical Report LIDS-P-1170, Laboratory for Information and Decision Systems, M.I.T., 1981.

[17]Zygmund, A., *Trigonometric series*, Third edition: Volumes I & II combined, Cambridge University Press, Cambridge, 2002.

# Fundamental solutions of a time-fractional equation

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## Abstract

In this work we obtain the first and second fundamental solutions (FS) of the multidimensional time-fractional equation with Laplace or Dirac operators, where the two time-fractional derivatives of orders $\alpha \in ]0,1]$ and $\beta \in ]1,2]$ are in the Caputo sense. We obtain representations of the FS in terms of Hankel transform, double Mellin-Barnes integrals, and H-functions of two variables. As an application, the FS is used to solve Cauchy problems of Laplace.

**Keywords:** Laplace operator, fractional calculus, fundamental solution.

### Acknowledgements

The work of the authors was supported by Portuguese funds through the CIDMA - Center for Research and Development in Mathematics and Applications, and the Portuguese Foundation for Science and Technology (FCT–Fundação para a Ciência e a Tecnologia), within project UID-MAT-0416-2019.

## References

[1]Ferreira, M., Rodrigues, M. M. and Vieira, N., Fundamental solution of the multi-dimensional time fractional telegraph equation, *Fractional Calculus and Applied Analysis* 20(4), 2017, pp. 868–894.

[2]Ferreira, M., Rodrigues, M. M. and Vieira, N., Fundamental solution of the time-fractional telegraph Dirac operator, *Mathematical Methods in the Applied Sciences* 40(18), 2017, pp. 7033–7050.

## Organized Session

# Dental Research Applications of Statistical Methods

**Organizer: José A. Lobo Pereira**

# Characterization of a Plaque-Disclosing Test

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## Abstract

Dental plaque is a major biological determinant common to the development of dental caries and periodontal diseases [1]. Thus, elimination of bacterial plaque from tooth surfaces is essential for maintaining dental health [2]. Plaque disclosing agents (PDA) are preparations containing dye or other coloring agents used to identify bacterial plaque, providing a valuable visual aid, playing an important role in patient's motivation, control of dental hygiene efficacy and research. The Mira-2-Ton Tablets Hager & Werken® (M2T) is the PDA routinely used in the clinic of Faculdade de Medicina Dentária da Universidade do Porto (FMDUP) to estimate the plaque index (percentage of dental surfaces with plaque). However, until now, we do not have information on the power of M2T to separate plaque positive surfaces from plaque negative ones. Aiming to estimate the power of M2T test through accuracy, sensitivity, specificity, positive and negative predictive values, we compare M2T against the physical removing of plaque method, considered the gold standard. Twenty patients of FMDUP where randomly selected, and 4 surface per tooth screened for presence of plaque, firstly by the M2T followed by the physical removing technique. The result of the two diagnostic test were compared and the measures of M2T performance computed. The data were processed with $R$ software [3]. The performance of M2T varied among sextants with accuracy ranging from 0.54 to 0.85, sensitivity from 0.57 to 0.93, specificity from 0.12 to 0.96, positive predictive value from 0.47 to 0.98 and negative value from 0.12 to 0.80. The performance of M2T was satisfactory for motivational purposes but presents some limitations when applied to research.

**Keywords:** Dental plaque, Plaque disclosure, Test performance measures.

## References

[1]Sanz, M., et al. (2017) Role of microbial biofilms in the maintenance of oral health and in the development of dental caries and periodontal diseases. Consensus report of group 1 of the joint EFP(ORCA workshop on the boundaries between caries and periodontal diseases. Journal of Clinical Periodontology 44:S18, 5–11.

[2]Löe, H. et al. Experimental Gingivitis in Man. Journal of Periodontology, Chicago, v.36, p.177-87, 1965.

[3]Gustafson, K.E. and Rao, D.K.M., *Numerical range, the Field of Values of Linear Operators and Matrices*, Springer, New York, 1997.

[4]R. Core Team (2019). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

# Estimation of the Minimum Interval Between Orthopantomography to Detect Early Variations of Bone Level Through Logistic Regression Classifiers

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## Abstract

Periodontal disease (PD) is an infectious/inflammatory disease that affects the tooth supporting tissues and is characterized by alveolar bone loss. Being one of the most prevalent oral disorders, affecting about 20-50% of global population, [1] periodontitis, the destructive form of periodontal disease, is the leading cause of tooth loss among adults. The early diagnose of periodontitis and a good estimation of its progression rate allows an accurate diagnosis and prognosis and, consequently, an adequate treatment plan. The assessment of periodontal bone level changes is of paramount importance to estimate the rate of periodontitis progression, and therefore provide crucial information to achieve a more accurate prognosis. Panoramic radiographs (PR) are part of the patient care protocol at the FMDUP clinic and the first radiological medium for periodontal bone level screening. The interval between two PR needs to be small enough to detect early changes of bone level, however must minimize the amount of radiation received. Aiming to know the minimum interval between PR that allows detecting variations of bone level, we performed an observational study with 400 PR of 200 patients of the FMDUP clinic. Variation of interproximal bone level between two PR of the same patient was assessed through a percentile ruler graded at 5% and a binary classifier logistic regression was applied with R software. Binomial generalized linear models were fitted to data, with the predictive variables age, gender, and interval between PR. The results showed that the minimum interval between PR was 2.5 years with a sensitivity and specificity of 46.0% and 85.1%, respectively, and the area under the curve (AUC) was 69.6.

**Keywords:** Periodontal disease, Bone loss, Logistic Regression Classifiers.

## References

[1]Sanz M, D'Aiuto F, Deanfield J, Fernandez-Avilés F. (2010) European workshop in periodontal health and cardiovascular disease-scientific evidence on the association between periodontal and cardiovascular diseases: A review of the literature. Eur Heart J Suppl 2010;12 Suppl B:B3-12.

[2]R. Core Team (2019). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

# MANOVA Applied to a Randomized, Crossover, Double-Blind Study

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## Abstract

Halitosis, or bad breath, is used to describe an unpleasant and offensive odour in exhaled air. Although halitosis could result from systemic disorders [1], it is now widely accepted that the major source of bad breath is odorific compounds, particularly volatile sulphur compounds (VSC), produced by bacterial metabolic degradation of food debris, desquamated cells and saliva proteins [2] within the oral cavity [3]. The success of antihalitosis interventions appears to hinge on the reduction of VSC levels by mechanical or chemical methods. Mechanical interventions (i.e. toothbrushing, flossing and tongue scraping) aim to reduce the numbers of VSC-producing bacteria and bacterial substrate, while chemical interventions (toothpaste and mouthrinse use) have an antibacterial effect in addition to direct odour neutralization [4]. The aims of this study were to compare the volatile sulphur compounds (VSC)-reducing effect of two commercial mouthrinses using a morning bad breath model and to assess the role of mechanical plaque control (MPC) when performed previously to mouthrinse use. Eleven volunteers with good oral health were enrolled in a double-blind, randomized, six-step crossover design study with a 7-day washout period. Two commercial mouthrinses were tested using a saline solution (NaCl 0.9%) as a negative control: one mouthrinse contained 0.05% chlorhexidine, 0.05% cetylpyridinium chloride and 0.14% zinc lactate (CHX-CPC-Zn), while the other contained 0.05% chlorhexidine, 0.15% triclosan and 0.18% zinc pidolate (CHX-triclosan-Zn). A portable sulphide monitor (Halimeter®) was used for VSC quantification. Measurements were made at baseline, and 1, 3 and 5 h after rinsing. Significant differences were detected by multivariate analysis of variance (MANOVA). We were unable to demonstrate a significant influence of mechanical plaque control on the reduction of VSC levels when performed before mouthrinse use ($p = 0.631$). Both mouthrinses effectively lowered VSC levels in all test intervals ($p < 0.05$). No statistically significant differences were found between mouthrinses in any of the test intervals ($p = 0.629, 0.069$ and $0.598$ at $1, 3$ and $5$ h).

**Keywords:** halitosis, Volatile sulphur compounds, ANOVA.

## References

[1]Moshkowitz M, Horowitz N, Leshno M, Halpern Z. (2007) Halitosis and gastroesophageal reflux disease: a possible association. Oral Dis; 13: 581–585.

[2]Loesche WJ, Kazor C. (2002) Microbiology and treatment of halitosis. Periodontol 2000; 28: 256–279.

[3]Delanghe G, Ghyselen J, Bollen C, van Steenberghe D, Vandekerckhove BN, Feenstra L. (1999) An inventory of patients' response to treatment at a multidisciplinary breath odor clinic. Quintessence Int; 30: 307–310.

[4]Quirynen M, Zhao H, van Steenberghe D. Review of the treatment strategies for oral malodor. Clin Oral Invest 2002; 6: 1–10.

# Does Hyaluronan Application as Adjunct of Non-Surgical Periodontal Therapy Contribute to Periodontal Pocket Reduction? A Meta-Analysis

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## Abstract

Periodontal disease (PD) is one of the most prevalent oral disorders, affecting 20% - 50% of world population [1] and non-surgical periodontal therapy is the usual treatment approach. Hyaluronan (HY) is a naturally occurring high molecular weight glycosaminoglycan is present in all periodontal tissues and has been demonstrated to facilitate wound healing in a variety of organs and tissues, and hence has potential to be used in periodontal therapy. Individual clinical studies presented thus far provide evidence that HY associated with non-surgical periodontal therapy present no adverse effects, however the effect on periodontal pocket depth (PPD) reduction vary from null to substantial. This meta-analysis intends to find a point estimate of the effect of HY as adjunct of non-surgical periodontal treatment (PT) on PPD reduction. A total of 8 articles were selected through the search on the electronic databases (Medline (PubMed) and Web of Science). Although with similar experimental design, some potential sources of bias were detected. The standard mean difference effect size was computed from mean gain scores for each study. We applied the randon-effects model by summarizing the results of 8 studies, each of which has a sample size $n_k$. In each study, there is a true effect $\beta_k$ estimated by $\hat{\beta}_k$, with a true standard error $\sigma_k$ estimated by $\hat{\sigma}_k$, or, equivalently, a true variance $\beta_k^2$ estimated by $\hat{\sigma}_k^2$ and between-study variance $\tau^2$. The $\tau^2$ and $\beta$ were estimated by restricted maximum-likelihood estimator (REML) and the inference was attained by first-order statistics. The meta-analysis results were presented in a forest plot graph and publication bias was assessed by a funnel plot graph combined with the trim and fill method and tested for asymmetry by Egger's linear regression method. The statistical procedures were performed with *R* software [2]. The results suggest that HY associated with non-surgical periodontal therapy have a significant effect on PPD reduction for a confidence level of 95% (ES = 1; 95% CI = 0.46, 1.54) and a nonsignificant level of heterogeneity ($\tau^2 = 0.266$; $Q = 12.690$, $p - val = 0.080$).

**Keywords:** non-surgical periodontal therapy, Hyaluronan, Meta-analysis.

## References

[1]Sanz M, D'Aiuto F, Deanfield J, Fernandez-Avilés F. European workshop in periodontal health and cardiovascular disease-scientific evidence on the association between periodontal and cardiovascular diseases: A review of the literature. Eur Heart J Suppl. 2010;12(Suppl B):B3–12.

[2]R Core Team (2019). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

## Organized Session

# Mathematics, Education, Technology, Business and Society

**Organizer: Cristina Dias**

# Statistical Modelling of Portuguese Granites Response to Acid Attak: the case of RA and SPI Granites

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## Abstract

The development of methodologies to predicting the useful life of materials used as building materials or as ornamental stones is fundamental for the optimization of its choice, application criteria and maintenance in the short- and long-term period of time. Taking into account that the construction materials applied outside the buildings are currently exposed to aggressive atmospheres due to increased pollution, the analysis of their behaviour against these atmospheres, simulated laboratorially, allows to obtain data, that when analyzed with the proper techniques of statistical prevision modelling, provide us a precious information about their expected behaviour over time. In order to modelling the degradation of Portuguese granites used as material construction and ornamental stones exposed to three different acidified solutions (simulation of acid rain), we have applied a polynomial regression analysis to the laboratorial data obtained. For all the acidified solutions it was possible to verify that the pattern of the degradation curve is identical, in an $S$ form and with a very high coefficient of determination values, translating a very significant adjustment to this model.

**Keywords:** degradation Modeling, Polynomial Regression, Granite Decay.

## References

[1]Costa, J., M., *Modelos de Gestão da Degradação em Edifícios – Influência de Fatores de Degradação no Aparecimento de Manchas nas Fachadas*, Tese de Mestrado em Engenharia Militar, Instituto Superior Técnico, Academia Militar. Lisboa,2011, pp.91.

[2]Daniotti, B., Lupica Spagnolo, S., Service Life prediction for buildings' design to plan a sustainable building maintenance, *Portugal SB07 Sustainable construction, materials and practices, 12-14 Setembro, Lisboa*, 2007, pp.515-521.

[3]Gaspar, P., Brito, J., *Modelo de degradação de Rebocos* Revista de Engenharia Civil, Universidade do Minho,2005, nº 24; pp. 17 – 27

[4]Santos, M., I., and Porta Nova, A., M., Validation of nonlinear simulation metamodels, *In M. H. Hamza, editor, Proceedings of Applied Simulation and Modeling*, pp. 421–425, 2001.

[5]Scott, A.J., Knott, M., *A cluster analysis method for grouping means in the analyses of variance*, Biometrics, Washington, 1974, 30:507-512.

[6]Seber, G., A., F. and Wild, C., J., *Nonlinear Regression*, John Wiley and Sons, New York, NY, USA, 1989.
4

[7]Sen, Ashin and Srivastava, M., *Regression Analysis, Theory, Methods, and Applications*, Spring-Verlag, New York, 1990.

[8]Silva, A., de Brito, J. e Gaspar, P., *Service life prediction model applied to natural stone wall claddings (directly adhered to the substrate*, Construction and Building Materials, 25(9),2011, 3674-3684 (2011a).

[9]Welch, P., D., *The statistical analysis of simulation results*, In S. S. Lavenberg, editor, Computer Performance Modeling Handbook, páginas 268–328. Academic Press, New York, NY, USA, 1983.

# Problem posing tasks in the learning of probabilities

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## Abstract

Problem posing has long been under the shadow of problem solving in mathematics education, however, in recent years, problem posing has received incresing attention and has been described as beneficial to train students for a changing world and an effective stategy for teaching and learning Mathematics. The proper interpretation of probability problems involves, not only, the linguistic, semantic and schematic knowledge [9], necessary for the interpretation of any mathematical problem, but also the domain of probabilistic vocabulary, whose meaning is often discrepant from the common meaning [1]. The interpretation stage is, in fact, one of the critical points in the process of solving problems of probabilities, since an unsuccessful interpretation enhances the manifestation of misunderstandings and probabilistic fallacies and makes it impossible to reach the correct solution of the problem. There have been numerous warnings about the dire consequences of the study of probabilities based on routine and decontextualized tasks [5], particularly their inability to eliminate the misunderstandings and fallacies associated with the concepts of probabilities [7], [8]. For students to become aware of the complexity of probabilistic reasoning, it is essential to engage them actively in activities that provide knowledge building based on their effort, mistakes and interaction with peers. [4]. Problem-posing tasks are described in the literature as beneficial for problem-solving skills, contributing to deeper understanding of concepts, improved reasoning, motivation and creativity (eg, [3]). By requiring greater abstraction and proper use of natural and formal language [12], problem-posing tasks may be an ally in overcoming the difficulties associated with the interpretation of problems of probabilities.

**Keywords:** Problem posing, Problem solving, Fallacies, Misconceptions, Probabilities.

## References

[1]Ancker, J. (2006). The language of conditional probability. Journal of Stat. Educ., Vol 14 No 2.

[2]Bar-Hillel, M. e Falk, R. (1982) Some teasers concerning conditional probabilities. Cognition Vol 11, pp. 109–22.

[3]English, L. (1996). Children's problem posing and problem solving preferences, in J. Mulligan and M. Mitchelmore (Eds.), Research in Early Number Learning. Australian Association of Mathematics Teachers. English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. Educational Studies in Mathematics, Vol 34, 183- 217.

[4]Garfield, J. (1995). How students learn statistics. Intern. Statistical Review, Vol 63, pp. 25–34

[5]Garfield, J. e Ahlgren, A. (1988) Difficulties in Learning Basic Concepts in Probability and Statistics. Journal for Research in Mathem. Education, Vol 19 , No 1 , pp. 44-63

[6]Huerta, M.P. e Lonjedo, M.A. (2003) La resolución de problemas de probabilidad condicional: un estudio exploratorio con estudiantes de bachiller. VI Simposio SEIEM. Granada.

[7]Khazanov, L. (2005). An investigation of approaches and strategies for resolving students' misconceptions about probability in introductory college statistics. Unpublished doctoral dissertation, Teachers College, Columbia University.

[8]Konold, C. (1995). Issues in assessing conceptual understanding in probability and statistics. Journal of Statistics Education, Vol 3, No 1.

[9]Mayer, R. E. (1992) Thinking, problem solving, cognition. 2. ed. New York. Freeman and Co.

[10]Penalva, M. C., Posadas, J. A. e Roig, A. I. (2010). Resolución y planteamiento de problemas: contextos para el aprendizaje de la probabilidad. Educación Matemática, Vol 2, No 3, pp.23-54

[11]Polaki, M. V. (2005). Dealing with compound events. In G. A. Jones (Ed.), Exploring probability in school: challenges for teaching and learning (pp. 191-214). New York, NY: Springer

[12]Silver, E.A. (1994). On mathematical problem posing. For the learning of mathematics. Vol 14, No 1, pp.19-28.

[13]Stoyanova, E. e Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. Clarkson (Ed.), Technology in Mathematics Education pp.518–525. Melbourne: Mathematics Education Research Group of Australasia.

# Adaptation of higher education to the digital generation/AHEAD

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## Abstract

Learning through projects can be seen as a methodology that facilitates the acquisition of new knowledge and at the same time allows the development of new skills that can better prepare students and teachers for the job market.This project, reference number: 609551-EPP-1-2019-1-BG-EPPKA2-CBHE-JP, which we intend to develop in the Higher School of Technology and Management of the Polytechnic Institute of Portalegre, in partnership with otherinternational higher education institutions (Trakia University, Bulgária; State Pedagogical University Ion Creanga, Moldava; American University of Moldova, Moldava; Odessa National Academy of Food Technologies, Ukrain; National Technical University of Ukraine Igor Sikorsky Kyiv Polytechnic Institute, Ukraine, Bialystok University of Technology, Poland and Siauliai State College, Lithuania) targeting students and teachers through innovative training courses in the areas of Science, Technology, Engineering and Mathematics (STEM), and the creation of new courses in these areas. For this, it is foreseen the development of activities of project implementation - development, adaptation and test of new curricula and methodologies to improve the level of competences in the institutions of higher education. For the development of new courses, it is intended to integrate new ICT methods into the training practices in each of the partner institutions in the project. The objectives of the project are: (i) to provide training to target groups of students and teachers from different institutions; (ii) to develop innovative curricula and methodologies; and (iii) to create new courses with application of the practices developed in the STEM. The evaluation of the results achieved will be carried out through quantitative indicators to evaluate the quality of the project implementation by type of activity in accordance

with a work plan. The culmination of the project results in the dissemination of innovative practices developed, new curricula and methodologies as well as the improvement of the level of competences of the higher education institutions and the availability of the elaborated didactic materials and of free access. Dissemination of the results achieved may enable other institutions at regional and European level to adopt more appropriate curriculum methodologies and curricula in the areas of STEM.

**Keywords:** Methodologies, Pedagogical practices, Formation.

## References

[1]Mill, D. and Pimentel, N. (2010) Ensino, aprendizagem e inovação em Educação a Distância: desafios contemporâneos dos processos educacionais In Mill, D. e Pimentel, N. (org.) Educação a Distância. Desafios contemporâneos, São Carlos, EdUFSCar.

[2]Mishra, P. and Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers College Record, Vol. 108, N.° 6, 1017-1054. Columbia University.C. Dias, *Modelos e Famílias de Modelos para Matrizes Estocásticas Simétricas*, Ph.D. Thesis, Évora University, 2013

[3]Siemens, G. (2005). Connectivism: A learning theory for the digital age. Retrieved from http://www.elearnspace.org/Articles/connectivism.htm.

# Income, consumption and saving of Portuguese households in numbers – An update

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## Abstract

In this work the income, consumption and saving of Portuguese households during the last two decades (1995 to 2016) are analyzed. Through the use of PORDATA portal data different variables are used for each of the three aggregates. The results show that the majority of households live with a low income (below 10,000 euros per year), which justifies the great inequality in the distribution of income in Portuguese households (in 2016, the richest households earn six times more than the poor) and the high risk of poverty (almost 50%). This latter indicator is only attenuated to 20% due to the important role of Social Security and other social support institutions. It is also verified that more than 50% of households consumption is concentrated in three sectors: Food, beverages and tobacco; Housing, water, electricity, gas and fuels; and, Transport and communications Finally, the results show that the saving rate has been decreasing in the last two decades, standing at 5,3% of disposable income in 2016. Although this decrease in saving in relative terms, it shows an increase in absolute terms of the amount invested in term deposits from 2006. This is a reflection of a concern of the households to safeguard the future in a period of economic difficulties that the country faced.

**Keywords:** Portuguese households, Income, Consumption, Saving, Formation.

## References

[1]Attanasio, O. and G. Weber (2010). Consumption and saving: models of intertemporal allocation and their implications for public policy. Journal of Economic Literature, 48: 693-751.

[2]Bover, O., Schürz, M., Slacalek, J. and Teppa, F. (2016). Eurosystem household finance and consumption survey: Main results on assets, debt, and saving. International Journal of Central Banking, 12(2): 1-13.

[3]Cardoso, F., Farinha, L. and Lameira, R. (2008), "Household wealth in Portugal: revised series", Occasional Paper 1, Banco de Portugal.

[4]Carlos, G., Ribeiro, H., Alves, S., Veloso, C. and Pereira, J. (2017). The determinants of the evolution of family savings in the context of a high leveraged society. Proceedings of 23rd International Scientific Conference on Economic and Social Development, 78-99.

[5]Guerello, C. (2018). Conventional and unconventional monetary policy vs. households income distribution: An empirical analysis for the Euro Area. Journal of International Money and Finance, 12(2): 1-13.

# The use of information and communication technologies in the teaching-learning process in higher education: a case study

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## Abstract

The technological world provides new ways of learning and is part of the transformation that society has been suffering over the last century. Most schools have already incorporated the new technological transformations, since they provide many of the necessary pedagogical resources. This new context has broken with traditional teaching, school activities are no longer linked only to the classroom. The web has allowed the introduction of numerous tools that allow many of the school tasks and not only can be carried out, in the most varied places from the coffee, car, train, restaurants, libraries, etc. The dissemination of information of all kinds, can now be found online with the greatest of facilities, simply by having a computer connected in a network. The use of information technologies can contribute to help develop students' literacy either in terms of readings made or in terms of writing requirement. The objective of this work is to present a study carried out with students of a Polytechnic institution of higher education, which intends to ascertain the importance that these students give to the contents made available online by others and how they incorporate them in their learning. The results of this research indicate that although new technologies are present in the lives of students and teachers, and despite their importance in the learning process, the largest and key role continued to belong to the teacher. The content taught in the classroom, and the availability of the same in digital platforms, to which the students access and with which they interact, have importance recognized by the students. The search for information online, and its acceptance as true, seems to be consensual among the students interviewed. The study also allowed us to conclude that, students continue to privilege the contents made available by teachers in the classroom and

that, the use of internet is not the basis of their preparation, serving only as a supplement in case of questions on a subject, it has been less clear.

**Keywords:** Literacy, Information technologies, Higher education.

## References

[1]Barton, D., Hamilton, M. and Ivanic, R., *Situated literacies: reading and writing in context*, 2000, Routledge: London.

[2]Fisher, D., & Frey, N., *Implementing a Schoolwide Literacy Framework: Improving Achievement in an Urban Elementary School*, International literacy Association, 61(1),2011, pp.32–43, September, John Wiley & Sons.

[3]Christensen, R., Effects of technology integration education on the attitudes of teachers and students. *Journal of Research on Technology in Education*,2002, 34 (4), pp. 411-433.

[4]Fagan, M. H., Neill S. and Wooldridge, B. R., Exploring the intention to use computers: an empirical investigation of the role of intrinsic motivation, end perceived ease of use, *Journal of Computer Information Systems* 48 (3), 2008, pp.31-37.

[5]Ghiglione, R. and Matalon, B., *O inquérito: teoria e prática*. (4ª ed.). Oeiras, Celta Editora, 2001.

## Organized Session

# Quantitative Methods for Decision Making

**Organizer: Eliana Costa e Silva**

# State Space Modeling in Online Water Quality Monitoring

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## Abstract

In this study, the discussion focuses on the dynamic modeling procedure based on the state space approach (associated to the Kalman filter), in the context of surface water quality monitoring in a hydrological river basin, in order to analyze and evaluate the temporal evolution of the environmental water quality variables within a dynamic monitoring procedure ([1], [2]). This modeling approach allows to obtain pertinent findings concerning water surface quality interpretation and an online monitoring management. Issues such as trends, seasonality, temporal correlation and detection of change points are addressed ([3], [4]). From the environmental point of view, this research highlights the potential value of this proposed approach to identify unanticipated changes and behaviors that are important in the management process and for the assessment of water quality.

**Keywords:** water quality, state space modeling, Kalman filter, distribution-free estimation.

### Acknowledgements

This work is financed by FEDER funds through the Competitivity Factors Operational Programme - COMPETE, and by national funds through FCT (Fundação para a Ciência e a Tecnologia) within the framework of Project POCI-01-0145-FEDER-007136 and Project UID-MAT-00013-2013.

## References

[1]Gonçalves, A.M. and Costa, M., Predicting seasonal and hydro-meteorological impact in environmental variables modelling via Kalman filtering, *Stochastic Environmental Research and Risk Assessment*, 27(5), 2013, pp. 1021–1038.

[2]Costa, M. and Gonçalves, A.M., *Combining Statistical Methodologies in Water Quality Monitoring in a Hydrological Basin - Space and Time Approaches*, Chapter, pp. 121–142. Editors Kostas Voudouris and Dimitra Voutsa. Croacia: Intech, 2012.

[3]Costa, M. and Gonçalves, A.M., Clustering and forecasting of dissolved oxygen concentration on a river basin, *Stochastic Environmental Research and Risk Assessment*, 25(2), 2011, pp. 151–163.

[4]Gonçalves, A.M., Baturin, O. and Costa, M., *Time series analysis by state space models applied to a water quality data in Portugal*, in Proceedings of the International Conference on Numerical Analysis and Applied Mathematics 2017 (ICNAAM-2017), AIP Conference Proceedings, American Institute of Physics, 1978, 2018, pp. 470101-1–470101-4.

# Minimize the production of scrap in the extrusion process

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## Abstract

In the discussion on environmental policy, the notion of eco-efficiency is often used. The Eco-efficiency is defined by the delivery of products and services to competitive values, reducing the ecological impacts and the satisfying human needs [1]. This work presents an empirical study of a Portuguese company in the industrial sector. The problematic presented is based on the company's growing concern to reduce the amount of scrap produced, by rationalizing energy consumption and natural resources. The main objective is to model the aluminum extrusion process; in particular, minimize the production of scrap, taking into account the various variables involved in the process. In an environment of great competitiveness in which the companies live, the improvements in the productive process is one of the differentiating factors in relation to the competition. Thus, it is necessary to understand the main operations and dynamics of the company. From the bibliographic research, a strong relationship of dependence between the different variables was evident [2]. Using statistical techniques, in particular a multiple linear regression, allowed, additionally, to identify the level of significance of the variables under study for the scrap production.

**Keywords:** Aluminium Extrusion, Scrap, Extrusion variables, Multivariate Linear Regression.

### Acknowledgements

This work was partially supported by CIICESI-ESTG-P.PORTO.

## References

[1]Schmidheiny, S. and Timberlake, L., Changing course: A global business perspective on development and the environment. Vol. 1. MIT press, 1992.

[2]De Almeida, F., Correia, A. and Costa e Silva, E., Layered clays in PP polymer dispersion: the effect of the processing conditions. Vol. 45, 2017.

# Optimization of Aluminium Profiles Production Planning: a preliminary study

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## Abstract

In the current business world, the increasing competitiveness forces companies to adopt optimization strategies to ensure or improve their market position. It is crucial to take the best decisions from the production-planning point of view. The production-planning system aims at ensuring that the desired products are produced at the right time, at a minimal cost, in the exact quantities, while maintaining the established quality levels [1]. It is fundamental improve and understand all levels of planning, from strategic, to tactical and operational [2]. The tactic planning deals with the type of aggregate planning and the amount of production over a period of weeks up to six months. While, operational planning is responsible for decisions, such as, scheduling, i.e., the exact order in which operations are to be performed, and the sizing of production lots. At the operational level, one of the most important activities is production scheduling [3]. This is where the present work focuses on. More precisely, this work concerns the planning of the production of a portuguese company in aluminium production using direct extrusion. Extrusion is a process of mechanical conformation by plastic deformation of an aluminium billet, in which the material is subjected to high pressures applied by a punch, and forced to pass through the hole of a die, in order to reduce it and/or change the shape of its cross-section[5]. The production of aluminium profiles poses several challenges to the production manager. This paper address a real case of a company that operates in the aluminium market, whose core business is the development and production of aluminium profiles for application in several areas, such as, engineering, architecture and industry works in general. The aim is, on the one hand, to minimize production waste, commonly known as scrap, and on the other, to reduce product delivery times, maintaining quality and increasing productivity. This case study focuses on a scheduling flow shop problem involving

sequence-dependent setup times arising from the need to change the tools used in the process of aluminium extrusion. Each job has to be processed in each of the stages in the same order, i.e., each job has to be processed first in stage 1, then in stage 2, and so on [6,7]. A mixed integer programming model is being developed and implemented for answering the company's challenge. Several instances provided by the company will be solved and compared to the company's current procedure in terms of the fulfillment of the deadlines and also on the quantity of scrap that is generated during the production process. The quantity of scrap is modeled using multivariate regression models, that consider the dies geometry as well as other aspects of the dies and of the extrusion process. Furthermore, cluster analysis is used for selection of relevant test instances.

**Keywords:** Aluminium Extrusion, Optimization, Mixed Integer Models, Custer Analysis, Multivariate Regression Models.

**Acknowledgements**

This work was partially supported by CIICESI-ESTG-P.PORTO.

## References

[1], Singh, N., Systems approach to computer-integrated design and manufacturing, John Wiley & Sons, Inc., 1995.

[2]Olhager, J. and Wikner, J., Production planning and control tools, *Production Planning & Control*, Vol. 11 (3), 210–222, 2000.

[3]Fuchigami, H.Y. and Rangel, S., A survey of case studies in production scheduling: Analysis and perspectives, *Journal of Computational Science*, Vol. 25, 425–436, 2018.

[4]Liu, G. and Muller, D.B, Addressing sustainability in the aluminum industry: a critical review of life cycle assessments. *Journal of Cleaner Production*, 35, 108–117, 2012.

[5]Saha, P.K., *Aluminum extrusion technology*. ASM International, 2000.

[6]Allahverdi, A., Ng, C.T., Cheng, T.E. and Kovalyov, M.Y., A survey of scheduling problems with setup times or costs. *European Journal of Operational Research*, 187(3):985–1032, 2008.

[7]Gupta, J.N. and Stafford Jr., E.F., Flowshop scheduling research after five decades. *European Journal of Operational Research*, 169(3):699–711, 2006.

# Evaluating Zika literacy of emboarded staff from Portuguese Navy

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## Abstract

This article describes the knowledge, attitudes and preventive practices regarding the infection by the Zika virus (ZIKV) among the population embarked on Portuguese Navy ships. Once the surface ship fleet oj Portuguese Navy has some destinations that includes endemic Zika zones (see [1,2]), two distinct groups are considered in the present study: those who will navigate in endemic areas of Zika virus and navigators that have already traveled to endemic areas of ZIKV. A questionnaire based on [5] was implemented in the both groups revealing that the Knowledge, Attitudes and Practice (KAP) level about ZIKV reveals significant differences between the distinct questions and groups. A significant percentage of people under study did not know the existence of Zika virus. Between the individuals who knew the existence of ZIKA virus, the majority did not Known how could be infected. Also, between the individuals who knew the existence of ZIKA virus, the most part did not know how could make its prevention. A descriptive and detailed analysis of such data can be found in [3]. In [4] is done a more detailed statistical approach, where the questionnaire is validated, and some multivariate statistical techniques are applied. In this manuscript we present an Exploratory Factorial Analysis identifying some factors with a clear meaning. These factors can help to identify which are the themes that shall be focused in a disclosure issue.

**Keywords:** Literacy, questionnaire, statistical approach, Zika virus.

### Acknowledgements

M. Filomena Teodoro was supported by Portuguese funds through the *Center of Naval Research* (CINAV), Portuguese Naval Academy, Portugal and

*The Portuguese Foundation for Science and Technology* (FCT), through the *Center for Computational and Stochastic Mathematics* (CEMAT), University of Lisbon, Portugal, project UID-Multi-04621-2019.

## References

[1]Center for Disease Control and Prevention (CDC): `https://wwwnc.cdc.gov/travel/files/zika-areas-of-risk.pdf`. Accessed at 10/6/2019.

[2]ECDC. Rapid Risk Assessment. Zika virus disease epidemic. (2017) 10th update. Available from: `https://ecdc.europa.eu/en/publications-data/rapid-risk-assessment-zika-virus-disease-epidemic-10th-update-4-april-2017`. Accessed at 10/6/2019.

[3]Faria, J., Teodósio, R. and Teodoro, F., *Zika: literacy and behavior of individuals on board ships. A first approach.* In: Simos, T. and et al. International Conference on Computational Methods in Science and Engineering (ICCMSE 2019). AIP proceedings. (Submitted)

[4]Faria, J., Teodósio, R. and Teodoro, F., *Knowledge of the Boarded Population About Zika Virus.* In: Misra, S. and et al. International Conference on Computational Science and Applications (ICCSA 2019). Lecture Notes. (in Press)

[5]Rosalino, C. and et al., *Conhecimentos, atitudes e práticas sobre Zika de viajantes visitando o Rio de Janeiro, Brasil, durante os Jogos Olímpicos e Paraolímpicos de 2016.* Inquérito no Aeroporto Internacional António Carlos Jobim, Rio de Janeiro. 2016.

# On Different Time-Scales in Firm Financial Distress Probability Modelling

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## Abstract

In studies on firm bankruptcy or financial distress prediction, it is usual to adopt the survival analysis methodology to model the conditional probability that the event of interest occurs within a given time interval (e.g., [9], [10] [6] and [11]). The Cox Proportional Hazard model (CPHM)[4] is one of the mostly used as it is able to reflect the longitudinal factor of time and incorporate multiple covariates to test its effect on the risk of bankruptcy or financial distress. In the specific context of firm bankruptcy or financial distress, [7] make an interesting contribution to the choice of the most appropriate methodology, by empirically comparing discrete-time hazard models and continuous-time CPHM, focusing lightly on the problem in the choice of the time-scale on this type of models. Typically, two time-scales are used in practice: (i) time-on-study, the most frequently used, and (ii) chronological age time-scale. However, there is no general consensus about which time scale is the most appropriate. For example, [8] argue that if the cumulative baseline hazard is exponential or if the age-at-entry is independent of covariates, then time-on-study versus chronological age time-scale will give similar results. Alternatively, [3] show analytically and through a simulation study exactly the opposite. This problematic of time-scale choice in CPHM is addressed typically in health-related studies as in [8], [2], [12] and [3]. Thus, based on a bench-marking methodological approach, we aim to contribute to this problematic specifically in the context of small and medium-sized enterprises (SMEs) financial distress were age was shown as an important predictor (e.g. in the work of [1], [11] and [5]. Contributing to the growing literature on SMEs financial distress, by providing comparison of SMEs failure prediction using different time-scales. Using a dataset that includes financial information on 18 145 Portuguese SMEs, between 2007 and 2017, we test both time scales (testing also for non-linear of age effect). Comparing the different models by its estimates, predictive power (by means of the Harrell's C Index) and proportional hazard assumptions diagnostic, indicating also advantages and disadvantages between both procedures in this context.

**Keywords:** Survival Analysis, Cox Model, SMEs, financial distress.

## References

[1]Altman EI, Sabato G and Wilson N (2009). The value of nonfinancial information in SME risk management. Presented at Credit Scoring & Credit Control XI Conference, Edinburgh

[2]Canchola AJ, Stewart SL, Bernstein L, West DW, Ross RK, et al.. (2003) Cox Regression Using Different Time-Scales. Western Users of SAS Software. San Francisco, California.

[3]Chalise, P., Chicken, E., & McGee, D. (2010). Time Scales in Cox Model: Effect of Variability Among Entry Ages on Coefficient Estimates. Tallahassee: Florida State University.

[4]Cox, D. R. (1972). Regression models and life-tables. Journal of the Royal Statistical Society: Series B (Methodological), 34(2), 187-202.

[5]Cultrera, L., & Brédart, X. (2016). Bankruptcy prediction: The case of Belgian SMEs. Review of Accounting and Finance, 15(1), 101–119.

[6]Fotopoulos, G., & Louri, H. (2000). Determinants of hazard confronting new entry: Does financial structure matter? Review of Industrial Organization, 17(3), 285–300.

[7]Gupta, J., Gregoriou, A., & Ebrahimi, T. (2016). Empirical comparison of hazard models in predicting bankruptcy. Social Science Research Network.

[8]Kom, E. L., Graubard, B. I., & Midthune, D. (1997). Time-to-event analysis of longitudinal follow-up of a survey: choice of the time-scale. American journal of epidemiology, 145(1), 72-80.

[9]Mata, J., & Portugal, P. (1994). Life Duration of New Firms. The Journal of Industrial Economies, 42(3), 227–245.

[10]Mata, J., Portugal, P., & Guimarães, P. (1995). The survival of new plants: Start-up conditions and post-entry evolution. International Journal of Industrial Organization, 13(4), 459–481.

[11]Pérez, S. E., Llopis, A. S., & Llopis, J. A. S. (2004). The Determinants of Survival of Spanish Manufacturing Firms. Review of Industrial Organization, 25(3), 251–273.

[12]Thiebaut AC, Benichou J (2004) Choice of time-scale in Cox's model analysis of epidemiologic cohort data: a simulation study. Stat Med 23: 3803–3820.

# Optimization of the Grapes Reception

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## Abstract

Tâmega and Sousa is a region with a large concentration of wineries, which are essential to its economical development. This economical sector presents several challenges, most of them concerned with the harverest season. In fact, the process of grape reception in wineries during the short span of the yearly harvest days is a significant challenge [1,2]. The arrival of loaded trucks follows a non-uniform distribution across the day, which is difficult to plan ahead. Furthermore, each truck may be carrying different varieties of grapes and may have different characteristics (e.g. sifferent load capacities, and dump vs. non-dump). There are also restrictions on the side of the winery, which include the limited number of presses and grain-tanks, as well as the times for filling, discharging, pressing and cleaning these instruments. This represents a decision problem in which the manager must decide, at any given moment: 1) which presses should be working; 2) what grape variety or varieties should be assigned to them; 3) when a new truck arrives to which machine should it be allocated; and 4) in which position of the waiting queue for that machine should it be allocated. All this impacts the deterioration of the grapes, the quality of the wine produced[3] and the idle times of the winery's machines and operators. To address these challenges we propose the use of a Genetic Algorithm [4], to devise solutions with two main goals: to minimize the waiting time of each truck loaded with grapes and the idle time of the winery's machines. The developed tool, along with its graphical interfaces, constitutes a Decision-Support System that can be used with two main purposes: to simulate a given scenario beforehand and to solve a specific problem, in real-time. For instance, the press manager may use the system in real-time to, given the state of the winery at any given time (e.g. number of trucks in each queue, characteristics of trucks, number of machines available), decide on how to assign trucks and grapes to each machine. In its final version, the system will be constantly online, allowing a continuous optimization of the whole process in real-time, whenever the state of the problem changes.

**Keywords:** grapes reception process, genetic algorithms, optimization.

## References

[1]Chandes, J., Estampe, D., Berthomier, R., Courrié, L. A., Han, L., and Marquevielle, S., *Logistics performance of actors in the wine supply chain.* In Supply Chain Forum: An International Journal (Vol. 4, No. 1, pp. 12-27), 2003, Taylor & Francis.

[2]Michalewicz, M., Michalewicz, Z., and Spitty, R., *Optimising the wine supply chain.* In Proceedings of the Fourteen Australian Wine Industry Technical Conference (14 AWITC), Adelaide, Australia, 2010.

[3]Alturria, L., Solsona, J.E., Antoniolli, E.R., Winter, P., Ceresa. A.M., *Wine making: defects in the process that originate costs of non-quality.* Rev. FCA UNCuyo, (XL) (1), 2008, pp. 1–16.

[4]Fonseca, C. M., and Fleming, P. J., *Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization.* In Icga (Vol. 93, No. July, pp. 416-423), 1993.

# Entrepreneurship Conditions for Knowledge and Technology Transfer

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## Abstract

The main purpose of this study is to determine whether expert's perceptions about technology, science, and other knowledge are efficiently transferred from universities and public research centers to new and growing firms. This is a subject considered in National Expert Survey (NES), a survey of the Global Entrepreneurship Monitor (GEM). The GEM research project is an annual assessment of the national level of entrepreneurial activity in multiple, diverse countries, for twenty years. GEM conducted its inaugural survey of entrepreneurship in 10 developed economies in 1999. Since then, GEM has surveyed over 2.9 million adults in 112 economies. NES is part of the standard GEM methodology and it assesses various Entrepreneurial Framework Conditions (EFCs) as well as some other topics related to entrepreneurship. It is intended to obtain the views of additional experts capturing expert judgements to evaluate specific national conditions. In this work we used the 2014 NES dataset in orther to identify the entrepreneurship conditions favourable to this transfer. For this we used several multivariate analyzes, namely factor analysis and multiple linear regression. With factor analysis we find for factors in the variables suggested in literature: government subsidies / subsidies for companies; preparation provided by education to start a business; transfer of new technology and knowledge and entrepreneurship encouraged by primary and secondary education. Using multiple linear regression we find that several perceptions about entrepreneurship conditions suggested by literature are related with the perceptions about knowledge dissemination.

**Keywords:** Entrepreneurship, Knowledge and Technology Transfer, Factor Analysis, Multivariate Linear Regression.

### Acknowledgements

This work was partially supported by CIICESI-ESTG-P.PORTO.

## References

[1]Argote, L., & Ingram, P., Knowledge transfer: A basis for competitive advantage in firms. *Organizational behavior and human decision processes*, 82(1), 2000, pp. 150–169.

[2]Bekkers, R. and Freitas, I. M. B., Analysing knowledge transfer channels between universities and industry: To what degree do sectors also matter?. *Research policy*, 37(10), 2008, pp. 1837–1853.

[3]Bercovitz, J. and Feldman, M., Entpreprenerial universities and technology transfer: A conceptual framework for understanding knowledge-based economic development. *The Journal of Technology Transfer*, 31(1), 2006, pp. 175–188.

[4]Etzkowitz, H., The norms of entrepreneurial science: cognitive effects of the new university–industry linkages. *Research policy*, 27(8), 1998, pp. 823–833.

[5]Geuna, A. and Muscio, A., The governance of university knowledge transfer: A critical review of the literature. *Minerva*, 47(1), 2009, pp. 93-114.

[6]Lee, Y. S., 'Technology transfer'and the research university: a search for the boundaries of university-industry collaboration. Research policy, 25(6), 1996, pp. 843–863.

[7]Santoro, M. D. and Chakrabarti, A. K., Firm size and technology centrality in industry–university interactions. *Research policy*, 31(7), 2002 pp. 1163–1180.

[8]Santoro, M. D. and Gopalakrishnan, S., Relationship dynamics between university research centers and industrial firms: Their impact on technology transfer activities. *The Journal of Technology Transfer*, 26(1-2), 2001, pp. 163–171.

[9]Siegel, D. S. and Waldman, D. A. and Atwater, L. E. and Link, A. N., Toward a model of the effective transfer of scientific knowledge from academicians to practitioners: qualitative evidence from the commercialization of university technologies. *Journal of engineering and technology management*, 21(1-2), 2004, pp. 115–142.

[10]Siegel, D. S. and Waldman, D. A. and Atwater, L. E. and Link, A. N., Commercial knowledge transfers from universities to firms: improving the effectiveness of university–industry collaboration. *The Journal of High Technology Management Research*, 14(1), 2003, pp. 111–133.

[11]Szulanski, G., The process of knowledge transfer: A diachronic analysis of stickiness. *Organizational behavior and human decision processes*, 82(1), 2000, pp. 9–27.

## Organized Session

# Linear Inference and Applications

**Organizer: Manuela Oliveira**

# On a priory estimation of random sequences predictability

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## Abstract

The problem of predicting the future values of time series and symbol sequences (and the problem of the sequence classification close to it) is one of the most important areas of modern theory and practice of big data processing. Considering that, on the one hand, different software tools based on various mathematical principles are used for forecasting, that can be rather expensive, and on the other hand, there may be high demands on the speed of forecasting (sometimes in real time), in order to choose a prediction method/tool requires the possibility of an a priory estimate of the fundamental possibility of making reliable predictions [1]. This leads to the problem of assessing predictability with respect to osed class of the prediction models. The accuracy of the prediction depends on the input information model used, on the prediction model (often determined by the data model), decision criteria (models). Among various means of ensuring the accuracy of predictions, such means as predictability, confidence, taking into account the specifics of prediction methods and the specifics of prediction objects (financial, telecommunication network systems) can be considered. (Note, that modern approaches to prediction can use not only single predictors but some sets of different predictors, with the subsequent formation of a forecast as a fusion of the results of individual predictors). Forecasting can be based either on probabilistic or on logical-ontological models. We consider models of the first type. The presentation will consider possible a priory estimates of predictability. The following aspects are considered: 1. Dependence on the used mathematical model of predicted data; 2. Dependence on the predictor model (which, of course, must be consistent with the data model); 3. Dependency on used Loss function. It is proposed to identify two classes of context models: context of individual sequence [2], and the model based on history of success rates predictions. As for latter, suppose we use a predictor $P$ to predict the value of the time series at time $t+1$, knowing the values at times $t$, $t-k$, where $k$ is the window size (training window). Let, besides,

for each $t-i, i=0,...k$, we know the success rate $SR = NP(i)/k$ of the predictions $t-i$ in the window $[t-i-k, t-i]$. We consider the sequence of the prediction outputs as binary (0- wrong prediction, 1- right). In this case, the predictability analysis is the answer is whether it possible to use the predictor properties of this sequence to assess the quality (confidence interval, for example ) of a prediction at $t+1$. The sequence of previous predictions can be considered either as a Bernoulli sequence, or Markov chain (state 0, 1), or k-order Markov chain. Some optimality property of such algorithm will be demonstrated. As possible approach to the Loss functions, the distance between accurate and predicted sequences can be considered. Levenshtein edit distance is one of interesting examples [3]. Say, there is an experience to use Levenshtein distance measurement technique as loss function for web session behavior prediction since it is applicable to strings of unequal size. However, in other cases, e.g. for prediction model of web caching, an ordering of web objects is an important aspect that is this distance metric is o not an appropriate way in this research [4]. Some modification of this metric will be considered. Another concept of predictability estimation deals with analysis of the inherent complexity of an empirical time series, then study the correlation of that complexity with the predictive accuracy. Theoretically, it can be expressed by the Kolmogorov-Sinai entropy (KSE) but this calculation is difficult. So called permutation entropy method of KSE approximation [5] (through frequency of that permutation occurring in various $l$-windows of time series $x_i$, $i=1,...,N$, $l<N$ is also rather computationally expensive. Therefore, in this presentation will be shown that permutation in the l-window for time series is similar to LSH for symbolic sequences. Then, it will be shown the correspondence between probability permutation (and Locality Sensitive Hashing(LSH) [6].

**Acknowledgements**

Research partially supported by the Russian Foundation for Basic Research under grant RFBR 18-07-00669.

## References

[1]Hassani,H.,Silva., E.S., Forecasting with Big Data: A Review, *Annals of Data Science*, 2, 1, 2015 pp 519.

[2]N. Merhav and M. J. Weinberger, On universal simulation of information sources using training data, *IEEE Trans. Inform. Theory* 50, 1, 2004, pp. 520.

[3]Li Yujian and Liu Bo, A Normalized Levenshtein Distance Metric, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 29, No. 6, 2007.

[4]Patel, D., Study of Distance Measurement Techniques in Context to Prediction Model of Web Caching and Web Prefetching, *International Journal on Soft Computing, Artificial Intelligence and Applications (IJSCAI)*, 5, 1, February 2016.

[5]Joshua, G, Ryan, G., Bradley,J., Model-free quantification of time-series predictability, *Physical Review E*, November 2014.

[6]Jure Leskovec, Anand Rajaraman, and Jeffrey David Ullman, *Mining of massive datasets*, Cambridge university press, 2014.

# Rank one symmetric stochastic matrices

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## Abstract

The models that we consider are based in the spectral decomposition of the mean matrices $\boldsymbol{\mu}$ . In this work we propose a new formulation introducing vec operators, which simplify for symmetric stochastic matrices the adjustment and validation. This validation being new, give us a theoretical support for the use of rank one symmetric stochastic matrices. These vectorial operators, besides presenting themselves as an important part in the new formulation for these models, also facilitate the presentation of these results. The models for symmetric stochastic matrices are the basis for inference for isolated matrices and for structured families of matrices. In these families the matrices, all of the same order, correspond to the treatments of base models. Since the matrices have all the same order, we are in the balanced case where we have the same number of degrees of freedom for the error for each treatment. The ANOVA and related techniques are, in the balanced case, robust techniques for heteroscedasticity and even more for non-normality.

**Keywords:** vec operators, adjustment and validation, ANOVA, base models.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações).

## References

[1]Areia, A., Oliveira, M. M. and Mexia, J. T., Modelling the Compromise Matrix in STATIS Methodology, 2011, 5, pp. 277-288.

[2]Dias, C., *Modelos e Famílias de Modelos para Matrizes Estocásticas Simétricas*, Ph.D. Thesis, Évora University, 2013

[3]Dias, C., Santos, S., Varadinov, M. and Mexia, J.T., ANOVA Like Analysis for Structured Families of Stochastic Matrices, *AIP Proceedings of the 12 International Conference of computational Methods in Sciences and Engineering (ICCMSE 2016)*, edited by American Institute of Physics, 2016, 1790, pp. 140006-1-140006-3

[4]Oliveira, M. M. and Mexia, J. T., AIDS in Portugal: endemic versus epidemic forecasting scenarios for mortality, *International Journal of Forecasting*, 2004, 20, pp.131–137.

[5]Scheffé, H., *The Analysis of Variance*, New York: John Wiley and Sons, 1959.

# Joint analysis of COBS

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## Abstract

Linear mixed models are useful in several disciplines,providing a flexible approach in situations of correlated data. When the variance-covariance matrix of the mixed model can be written as a linear combination of the matrices of its principal basis we obtain a more restrict class of mixed models, introduced by Nelder [3] [4], named models with orthogonal block structure (OBS). Imposing a commutativity condition between the orthogonal projection matrix on the space spanned by the mean vector and the variance-covariance matrix we get a particular class of OBS, that of models with commutative orthogonal block structure, COBS, introduced in Fonseca et al. [1]. This commutativity condition is a necessary and sufficient condition for the least square estimators, LSE, to be best linear unbiased estimators, BLUE, whatever the variance components, Zmyślony [7]. When dealing with COBS, we may be interested in performing the joint analysis of models obtained independently. For this, we can consider model crossing and model nesting, introduced in Mexia et al. [2], and model joining, introduced in Santos et al. [5]. Using these operations on COBS, in the resulting model the orthogonal projection matrix in the space spanned by the mean vector commutes with the variance-covariance matrix, thus the resulting model is COBS.

**Keywords:** Commutative orthogonal block structure, mixed models, model crossing, model nesting, model joining.

### Acknowledgements

This work was partially supported by national founds of FCT-Foundation for Science and Technology under UID-MAT-00297-2019 and UID-MAT-00212-2019.

## References

[1]Fonseca, M., Mexia, J.T., Zmyślony, R. (2006) Binary operations on Jordan algebras and orthogonal normal models. *Linear Algebra and its Applications*, 417(1):75—86.

[2]Mexia, J. T., Vaquinhas, R., Fonseca, M., Zmyślony R. (2010) COBS: segregation, matching, crossing and nesting. In Nikos E. Mastorakis, Vŭlko P. Mladenov and Z. S. Bojkovic, *Latest Trends and Applied Mathematics, Simulation, Modelling*, 4-th International Conference on Applied Mathematics, Simulation, Modelling (ASM'10), 249—255 WSEAS Press.

[3]Nelder, J. A. (1965a) The analysis of randomized experiments with orthogonal block structure I. Block structure and the null analysis of variance, *Proceedings of the Royal Society*, Series A 283, 147—162.

[4]Nelder, J. A. (1965b) The analysis of randomized experiments with orthogonal block structure II. Treatment structure and the general analysis of variance, *Proceedings of the Royal Society*, Series A 283, 163—178.

[5]Santos, C., Nunes, C., Dias, C., Mexia, J. T. (2017). Joining models with commutative orthogonal block structure. *Linear Algebra and its Applications*, 517: 235 – 245.

[6]Seely, J. (1970) Quadratic subspaces and completeness. *Ann. Math. Statist.*, 42, 710–721.

[7]Zmyślony, R. (1978) A characterization of best linear unbiased estimators in the general linear model, *Mathematical Statistics and Probability Theory* 2: 365-373.

# A Spatio-temporal model for burn severity data

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## Abstract

This research presents a spatio-temporal model for burn severity and ecosystem recovery. It takes as input satellite data acquired in the framework of the Monitoring Trends in Burn Severity (MTBS) project considering the period from 1984 to 2018. It develops a Hierarchical Bayesian model to analyze the burn severity and recovery trends in the Oregon and Washington wildfire perimeters. Results show that the Hierarchical Bayesian methods allow for multi-level parameter estimation using Bayesian procedures.

**Keywords:** hierarchical Bayesian model, burn severity, multi-level parameter estimation.

**Acknowledgements**

This research was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-04674-2019 (CIMA) (Centro de Matemática e Aplicações). This research was partially supported also by the Marie Skodowska-Curie Research and Innovation Staff Exchange (RISE) within the H2020-Work Programme (H2020-MSCA-RISE-2015), project Models and decision Support tools for integrated Forest policy development under global change and associated Risk and Uncertainty (SUFORUN) and by project LISBOA-01-0145-FEDER-030391 with the title 'Forest ecosystem management decision-making methods—an integrated bio-economic approach to sustainability' (BIOECOSYS), as well as from the Forest Research Centre, a research unit funded by Fundação para a Ciência e a Tecnologia I.P. (FCT), Portugal (UID-AGR-00239-2019).

## References

[1] An introduction to applied multivariate analysis with R, Everitt, Brian and Hothorn, Torsten, 2011, Springer Science & Business Media.
[2] Applied multivariate statistical analysis, Härdle, Wolfgang and Simar, Léopold, 2007, 2, Springer Science & Business Media
[3] Applied Spatial Data Analysis with R Bivand, Pebesma and Gómez-Rubio, 2013. Springer.
[4]Spatial and Spatio-temporal Bayesian Models with R-INLA Blangiardo and Cameletti, 2015 Wiley

## Organized Session

# Theory and Applications of Stochastic Differential Equations

**Organizer: Nuno Brites**

# Harvesting models in random environments with and without Allee effects. I. General models and their properties

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## Abstract

In a randomly varying environment, a harvesting model assumes the form of a stochastic differential equation (SDE)

$$dX(t) = f(X(t))X(t)dt + \sigma X(t)dW(t) - qE(t)X(t)dt, \quad X(0) = x,$$

where $X(t)$ is the harvested population size at time $t$, $f(X)$ is its mean (*per capita*) natural growth rate when its size is $X$ (assumed to be of class $C^1$ and such that $f(0^+)$ is finite $\neq 0$ and $f(+\infty) < 0$), $\sigma dW(t)/dt$ describes the effect of environmental fluctuations on the growth rate (with $W(t)$ a standard Wiener process and $\sigma > 0$ a noise intensity parameter), $E(t)$ is the harvesting effort at time $t$ and $q > 0$ the catchability. The yield per unit time is $H(t) = qE(t)X(t)$. The profit per unit time is $\Pi(t) = P(t) - C(t)$, where $P(t) = p_1 H(t) - p_2 H^2(t)$ ($p_1 > 0$, $p_2 \geq 0$) is the sale price and $C(t) = c_1 E(t) + c_2 E^2(t)$ ($c_1,\ c_2 > 0$) is the cost. We consider the case of no Allee effects ($f$ strictly decreasing with $f(0^+) > 0$) and the case of Allee effects (there are constants $0 < L < K$ such that $f(K) = 0$, $f$ increases strictly for $0 < X > L$ and strictly decreases for $X > L$). Allee effects ([1]) may occur at low population sizes ($0 < X < L$) when, for instance, the geographical dispersion makes if difficult for individuals to find mating partners or to mount an effective collective defence against predators. The case $E(t) \equiv 0$ (absence of fishing) was studied in [2] in terms of conditions for extinction and for existence of a stationary density (stochastic equilibrium). Here, based on [3], we do a similar study for sustainable harvesting policies with constant harvesting effort $E(t) \equiv E$. Following the ideas in [4] and [1], when a stationary density exists we also determine (see [3]) the effort

that maximizes $\mathbb{E}[\Pi(+\infty)]$ (expected profit per unit time at the stationary regimen) .

**Keywords:** stochastic differential equations, constant effort harvesting policies, profit optimization, stationary density.

**Acknowledgements**

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2019 (Centro de Investigação em Matemática e Aplicações).

## References

[1]Allee, W. C., Emerson, A. E., Park, O., Park, T. and Schmidt, K. P., *Principles of Animal Ecology*, Saunders, Philadelphia, 1949.

[2]Carlos, C. and Braumann, C. A., General population growth models with Allee effects in a random environment, *Ecological Complexity*, 30, 2017, pp. 26–33.

[3]Brites, N. M., Stochastic differential equation harvesting models: Sustainable policies and profit optimization, Ph. D. thesis, Universidade de Évora, Portugal, 2017.

[4]Braumann, C. A., Stochastic differential equation models of fisheries in an uncertain world: extinction probabilities, optimal fishing effort, and parameter estimation, in *Mathematics in Biology and Medicine*, Springer, 1985, pp. 201–206.

[5]Braumann, C. A., Variable effort fishing models in random environments, *Mathematical Biosciences*, 156, 1999, pp. 1–19.

# Harvesting models in random environments with and without Allee effects. II. Logistic type models and profit optimization

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## Abstract

In order to optimize profit, we model harvesting in a randomly varying environment by stochastic differential equation (SDE) versions of the most commonly used deterministic models with and without Allee effects, based on the logistic equation. They correspond to particular cases of the general SDE models of the form $dX(t) = f(X(t))X(t)dt + \sigma X(t)dW(t) - qE(t)X(t)dt$, where $X(t)$ is the harvested population size at time $t$, $f(X)$ is its mean (*per capita*) natural growth rate when its size is $X$, $\sigma dW(t)/dt$ describes the effect of environmental fluctuations on the growth rate (with $W(t)$ a standard Wiener process and $\sigma > 0$ a noise intensity parameter), $E(t)$ is the harvesting effort at time $t$ and $q > 0$ the catchability. The yield per unit time is $H(t) = qE(t)X(t)$. The logistic-like model with Allee effects corresponds to $f(X) = r\left(1 - \frac{X}{K}\right)\left(\frac{X-A}{K-A}\right)$ ($r > 0$, $K > 0$, $A$ is an Allee strength parameter such that $-K < A < 0$ gives weak Allee effects and $0 < A < K$ gives strong Allee effects); this is a re-parametrization (to facilitate comparisons) of a well-known model. The logistic model (without Allee effects) corresponds to $f(X) = r\left(1 - \frac{X}{K}\right)$ and can be obtained as the limiting case $A \to -\infty$. The profit per unit time is $\Pi(t) = P(t) - C(t)$, where $P(t) = p_1 H(t) - p_2 H^2(t)$ ($p_1 > 0$, $p_2 \geq 0$) is the sale price and $C(t) = c_1 E(t) + c_2 E^2(t)$ ($c_1,\ c_2 > 0$) is the cost. Here, following the ideas in [1] and [2], we present, based on the results of [3] and [4], optimal sustainable harvesting policies with constant harvesting effort ($E(t) \equiv E$), determining the conditions for non-extinction and existence of a stationary density and obtaining, under such conditions, the constant effort $E^{**}$ that maximizes the expected profit per unit time at the stationary regimen. We compare, for a particular case based on a real fishery data, the logistic model with the logistic-like models with Allee

effects, studying the effect of the Allee parameter $A$. Following [3] and [4] and using the same data, we present the application of stochastic optimal control theory and the Hamilton-Jacobian-Bellman equation to determine, using the numerical methods in [5], the optimal variable effort $E^*(t)$ policy that maximizes the expected discounted profit over a time horizon $T$. Again, we compare the logistic model with the logistic-like model and study the effect of the Allee parameter $A$. The implementation of this policy requires constant knowledge of the population size (an inaccurate, costly and difficult process) and leads to wildly varying efforts and heavy social implications, being inapplicable in practice. As we will see, the policy based on $E^{**}$ is less profitable, but it does not have these disadvantages and is very simple to apply; furthermore, if the Allee effects are absent or are mild, the difference in profit is very small.

**Keywords:** stochastic differential equations, profit optimization, Hamilton-Jacobi-Bellman equation, constant effort, optimal harvesting policies.

**Acknowledgements**

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2019 (Centro de Investigação em Matemática e Aplicações).

## References

[1]Braumann, C. A., Variable effort fishing models in random environments, *Mathematical Biosciences*, 156, 1999, pp. 1–19.

[2]Carlos, C. and Braumann, C. A., General population growth models with Allee effects in a random environment, *Ecological Complexity*, 30, 2017, pp. 26–33.

[3]Brites, N. M., Stochastic differential equation harvesting models: Sustainable policies and profit optimization, Ph. D. thesis, Universidade de Évora, Portugal, 2017.

[4]Brites, N. M. and Braumann, C.A., Fisheries management in random environments: Comparison of harvesting policies for the logistic model, *Fisheries Research*, 195, 2017, pp. 238–246.

[5]Suri, R. *Optimal harvesting strategies for fisheries: A differential equations approach*, Ph. D. thesis, Massey University, Albany, New Zealand, 2008.

# Parameter estimation of a piecewise linear stochastic differential system: the case of a bilinear oscillator subject to random loads

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## Abstract

Many mechanical and structural systems commonly used in engineering, in real life, are characterized by responding dynamically to random loads. The need to understand the responses of a system or a structure to actions of a stochastic nature, or to other types of uncertainties, involving its representation in the form of a mathematical model, and being able to understand the reasons for different behaviors (for example by identifying structural damage and failures), often lead to problems of statistical inference. In this work, we consider a simple harmonic oscillator with a degree of freedom modeled by a two-dimensional stochastic differential equation, assuming two behavioral regimes, subject to an external stochastic load. We assume that the oscillator changes from one regime to another at any given unknown time, as a result of damage or considerable deterioration of its operating conditions. The regimes differ in the value of the parameter called the stiffness coefficient, which characterizes the system. This formulation includes the scenario of a structure that suffers possibly from a loss of stiffness, of unknown value, after a certain unkown lifetime. The stochastic forces acting on the system are assumed to be modeled by a Wiener process. The system, written in state space form, describing displacements and velocities over time, is assumed to be completely observed and observations are assumed to be obtained in a sequence of discrete time instants. We describe how to calculate the maximum likelihood estimates of the parameters of the stochastic model from the observations as well as the natural frequencies of vibration of the oscillator in both regimes, i.e. before and after regime changes, using all data available over of the entire observation time interval. The method that we propose uses a model change–test that we designed, inspired by other tests existing in the literature that solve, in particular, problems in Finance. The novelty of this test lies in the fact that it applies to a class of stochastic differential equations of dimension larger than 1, see [4].

**Keywords:** piecewise linear stochastic model, maximum likelihood, Ornstein-Uhlenbeck process, stochastic harmonic oscillator.

**Acknowledgements**

This research was partially supported by CMUP (UID-MAT-450-00144-2013) and UID-Multi-04621-2013, funded by FCT (Portugal) with National (MEC) and European structural funds through the programs FEDER, under partnership agreement PT2020. The second author was also partially supported by project STR1-IDE ref. NORTE-01-0145-FEDER-000033, supported by NORTE-2020, through the European Regional Development Fund (ERDF).

## References

[1]Koncz, K., On the parameter estimation of diffusional type processes with constant coefficients (elementary Gaussian processes), *Journal of Anal.*, Math 13(1), 1987, pp. 75-91.

[2]Arato, M., *Linear Stochastic Systems with constant coefficients*, Springer-Verlag, Berlin, 1982.

[3]Bishwal, P., *Parameter estimation in stochastic differential equations*, Springer, Berlin, 2008.

[4]Brockwell, P., Hyndman, R. and Grunwald, G., Continuous time threshold autoregressive models, *Statistica Sinica*, 1, 1991, pp. 401-410.

# On a SDE coupled system for commodity pricing

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## Abstract

We will consider, similarly as in [1], a system of coupled SDE to describe the price evolution of both the processes of the spot prices $(S_t)_{t\geq 0}$ and the futures contract – expiring at date $T$ – price process $(F_t^T)_{t\geq 0}$, for commodities, given by:

$$\begin{cases} dS_t = k^S(\theta^S - \ln(F_t^T))S_t dt + \sigma^S S_t dB_t & S_0 \in \mathbb{R}^+ \\ dF_t^T = k^F(\theta^F - \ln(S_t))F_d^T t + \sigma^F F_t^T dB_t & F_0^T \in \mathbb{R}^+ \ . \end{cases}$$

This choice of modelling – a system of coupled equations of constant reverting type – was justified by the observation that $(y_t)_{t\geq 0}$ the *generalized convenience yield* defined to satisfy the following equation at all times $t \geq 0$,

$$F_t^T = S_t e^{(C-y_t)(T-t)} \text{ or } y_t = r_t + \frac{1}{T-t}\ln\left(\frac{S_t}{F_t^T}\right) \ ,$$

for $(r_t)_{t\geq 0}$ the spot interest rate process, had, in most of the commodities analyzed, a similar behaviour, to wit, $\ln(S_t/F_t^T)$ tended to quickly fade. In this presentation we will describe several properties related to this model.

**Keywords:** stochastic differential equations, commodities pricing models, spot and futures prices.

### Acknowledgements

This work was partially supported through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações) financed by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology).

## References

[1]Cabrera I., Esquível, M. L. Spot/Futures coupled model for commodity pricing, *Proceedings of the 6th St. Petersburg Workshop on Simulation I*, 2009, pp. 437–441.

# A note on the loss of the martingale property under the CEV process

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## Abstract

The discounted price process under the constant elasticity of variance (CEV) model is not a martingale (wrt the risk-neutral measure) for options markets with upward sloping implied volatility smiles. The loss of the martingale property implies the existence of (at least) two option prices for the call option: the price of Emanuel and MacBeth [1] for which the put-call parity holds and the (risk-neutral) price of Heston et al. [2] representing the lowest cost of replicating the call payoff. Based on the insights of Larguinho et al. [3], this article derives closed-form solutions for the Greeks of the risk-neutral call option pricing solution that are valid for any CEV process exhibiting forward skew volatility smile patterns. Using an extensive numerical analysis, we conclude that the differences between the call prices and Greeks of both solutions are substantial, which might yield significant errors of analysis for pricing and hedging purposes. See Dias et al. [4] for more details about this talk.

**Keywords:** bubbles, CEV model, Greeks, option pricing, put-call parity, local martingales.

### Acknowledgements

Financial support from FCT's grant number UID-GES-00315-2019 is gratefully acknowledged.

## References

[1]Emanuel, D.C. and MacBeth, J.D., Further Results on the Constant Elasticity of Variance Call Option Pricing Model, *Journal of Financial and Quantitative Analysis*, 17, 1982, pp. 533–554.

[2]Heston, S.L., Loewenstein, M. and Willard, G.A., Options and Bubbles, *Review of Financial Studies*, 20, 2007, pp. 359–390.

[3]Larguinho, M., Dias, J.C. and Braumann, C.A., On the Computation of Option Prices and Greeks under the CEV Model, *Quantitative Finance*, 13, 2013, pp. 907–917.

[4]Dias, J.C., Nunes, J.P. and Cruz, A., A Note on Options and Bubbles under the CEV Model: Implications for Pricing and Hedging, *Working Paper, ISCTE-IUL*, 2019.

## Organized Session

# Mathematical and Statistical Modeling

**Organizer: Milan Stehlík**

# Study of an Enhanced Total Least Squares Algorithm for Nonlinear Models

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## Abstract

The concept of optimization is used in many areas of engineering. It is devoted to the study of the minimum or maximum of a function with several variables on a certain domain of definition, particularly in the estimation of numerical model parameters from data [1]. This can be achieved using various methods depending on the applications. In many studies the optimization of a problem relies on the use of the least squares method [2], [3]. It is well known method that is usually used in order to quantify the errors of functions in order to compare the experimental data, which generally has measurement uncertainties, with the results of a mathematical model [3], [4]. This is used, in particular, if the application consists on minimizing a function describing a complicated problem, hard to estimate, and whose derivatives are not always available. To these difficulties are added the consideration of non-linear constraints [5]. The majority of studies that are based on this method choose to work with the easiest way to measure the distances between the measured data and the mathematical model which consists on calculating the vertical distances [2], [3], [6], due to its uncomplicated calculations, especially in the case of nonlinear functions. However, this approach does not consider the measurement uncertainties contained by the variable in X-axis which occurs for the real measured data. This work aims to establish a more practical method to solve the total lest squares problem in way that considers the measurement uncertainties contained in both variables. A comparative study done with numerical simulation is presented.

**Keywords:** Least Squares Algorithm, Nonlinear Models.

## References

[1]A. Antoniou and W.-S. Lu, *Practical optimization: algorithms and engineering applications*. Springer Science & Business Media, 2007.

[2]T. K. Moon and W. . C. Stirling, *Mathematical Methods and Algorithms for Signal Processing*, vol. 1. Prentice hall Upper Saddle River, NJ, 1999.

[3]S. Van Huffel and P. Lemmerling, *Total least squares and errors-in-variables modeling: analysis, algorithms and applications*. Springer Science & Business Media, 2013.

[4]S. Van Huffel and J. Vandewalle, *The total least squares problem: computational aspects and analysis*, vol. 9. Siam, 1991.

[5]E. Grafarend and J. L. Awange, *Applications of linear and nonlinear models: fixed effects, random effects, and total least squares*. Springer Science & Business Media, 2012.

[6]P. M. Ramos, F. M. Janeiro, M. Tlemçani, and A. C. Serra, Recent Developments on Impedance Measurements With DSP-Based Ellipse-Fitting Algorithms, *IEEE Trans. Instrum. Meas.*, vol. 58, no. 5, pp. 1680–1689, 2009.

# An alternative algorithm for the inverse numerical range problem

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## Abstract

The numerical range of a linear operator is the convex set in the complex plane comprising all Rayleigh quotients. The numerical range is a useful tool in the study of matrices and operators (see, e.g. [2] and references therein) which has applications in the stability analysis of dynamical systems and the convergence theory of matrix iterations, among others. For a given complex matrix, Uhlig [4] proposed the inverse numerical range problem: given a point $z$ inside the numerical range determine a unit vector $w_z$ for which this point is the corresponding Rayleigh quotient. Hence, this is an algebraic problem consisting of a system of two complex quadratic equations in the complex components of $w_z$. The inverse numerical range problem attracted the attention of several authors (e.g., Carden [1] and [1]), and different methods of solution have been proposed. In the present note we propose an alternative method of solution to those that have appeared in the literature. Our approach builds on the fact that the numerical range can be seen as a union of ellipses under a compression to the bidimensional case [3], in which case the problem has an exact solution. It requires a small number of eigenvector computations and compares well in execution time and in error with the existing ones in the literature.

**Keywords:** field of values, inverse problem, generating vector, compression.

### Acknowledgements

This work was partially supported by the Centre for Mathematics of the University of Coimbra, UID-MAT-00324-2013, funded by the Portuguese Government through FCT-MEC and co-funded by the European Regional Development Fund through the Partnership Agreement PT2020.

## References

[1]Carden, R., A simple algorithm for the inverse field of values problem, *Inverse Problems*, 25, 2009, 115019.

[2]Crorianopoulos, C., Psarrakos, P. and Uhlig, F., A method for the inverse numerical range problem. *Electron. J. Linear Algebra*, 20, 2010, pp. 198–206.

[3]Gustafson, K.E. and Rao, D.K.M., *Numerical range, the Field of Values of Linear Operators and Matrices*, Springer, New York, 1997.

[4]Marcus, M. and Pesce, C., Computer generated numerical ranges and some resulting theorems, *Linear and Multilinear Algebra*, 20, 1987, pp. 121–157.

[5]Uhlig, F., An inverse field of values problem, *Inverse Problems*, 24, 2008, 055019.

# Some new aspects on the geometry of Krein spaces numerical ranges

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## Abstract

For $n \times n$ complex matrices $A$, $C$ and $H$, where $H$ is non-singular Hermitian, the Krein space $C$-numerical range of $A$ induced by $H$ is the subset of the complex plane given by $\left\{\mathrm{Tr}(CU^{[*]}AU) : \ U^{-1} = U^{[*]}\right\}$ with $U^{[*]} = H^{-1}U^*H$ the $H$-adjoint matrix of $U$. We revisit several results on the geometry of Krein space $C$-numerical range of $A$.

**Keywords:** $J$-Hermitian matrix, Krein space $C$-numerical range, indefininte inner product.

### Acknowledgements

The work of the first author is supported by the grant RFBR-16-01-00113. The work of the second author was supported by Portuguese funds through the Center for Research and Development in Mathematics and Applications (CIDMA) and the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), within the project UID-MAT-0416-2013. The work of the third author was financed by Portuguese Funds through FCT - Fundação para a Ciência e Tecnologia, within the Project UID-MAT-00013-2013..

## References

[1]N. Bebiano, R. Lemos, J. da Providência and G. Soares, On generalized numerical ranges of operators on an indefinite inner product space, *Linear Multilinear Algebra*, 52, 3–4, 2004, 203–233.

[2]N. Bebiano, R. Lemos, J. da Providência and G. Soares, Inequalities for $J-$Hermitian matrices, *Linear Algebra Appl.*, 407, 2005, pp. 125–139.

[3]N. Bebiano, H. Nakazato and J. da Providência, Convexity of the Krein space tracial range and Morse theory, *C. R. Math. Rep. Acad. Sci. Canada*, 32, 4, 2010, pp. 97–105.
[4]M. Goldberg, E.G. Straus, Elementary inclusion relations for generalized numerical ranges, *Linear Algebra Appl.*, 18, 1977, pp. 1–24.
[5]A. Guterman, R. Lemos and G. Soares, On the $C$-determinantal range for special classes of matrices, *Applied Mathematics and Computation*, 275, 2016, pp. 86-94.

# On modeling of asymmetric dependencies with ecological and medical applications

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## Abstract

Since symmetry is an idealized phenomenon and asymmetry is more typical for real problems, e.g. noncompatible bivariate conditioning, skewed errors, asymmetric KL divergences for non-normal distributions (see [3]), development of proper methods for medical applications can be of great interest both for theory and practice. I will illustrate this importance by several examples from Health and Ecological Sciences. We can take as examples data from bivariate relationships between cholesterol and blood pressures in cardiology risk preventive studies and studies on relationships between several quantitative measures for bone mineral density (see [1]). I will illustrate several paradoxes of statistical inference for the typical measures of dependence, like correlation coefficient prevalently used for measuring the association between cholesterols and blood pressures. This sever over-symmetrization prevents medical discoveries completely fundamental for proper treatment of e.g. hypertension for older patients. Thus, in general employing measures of linear association (e.g., correlation) may ignore the asymmetric and hierarchical levels of dependencies. Finally I will discuss on the importance of defining proper statistical invariants (see [2]), which can be defining abstract forms of statistical symmetries, not visible from a standard perspective, especially with ecological applications for Chilean mountain glaciers.

## References

[1]Filus, J., Filus,L. Arnold, B.C., Jordanova, P.K., Soza,L.N., Lu,Y. Stehlíková, S. and Stehlí k, M. (2019) On parameter dependence and related topics: The impact of Jerzy Filus from genesis to recent developments (with discussion), CRC press

[2]Francis, A., Stehlík, M. and Wynn, H. (2017) Building Exact Confidence Nets Bernoulli 23, No. 4B, 3145-3165.

[3]Stehlík, M., Economou, P., Kiselák, J. and Richter, W.-D. (2014) Kullback-Leibler life time testing, Applied Mathematics and Computation, 240 122-139

## Organized Session

# Numerical Methods and Application in Medicine

**Organizer: Telma Santos and Jorge Tiago**

# Optimal control applied to a HIV delayed model

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## Abstract

We consider an optimal control problem applied to an in-host HIV model with discrete time-delays in state and control variables. Direct methods to compute solutions of optimal control problems with distinct discrete time delays in state and control variables are analyzed. We apply a discretization method, proposed in [2], and formulate a large-scale nonlinear programming problem, using the Applied Modeling Programming Language AMPL [1] which can be linked to several powerful optimization solvers, for example, the Interior-Point optimization solver IPOPT developed by Wächter and Biegler [5]. Necessary optimality conditions are verified for the delayed optimal control problem, and for the undelayed case also sufficient optimality conditions are verified. Concrete delayed optimal control problems, applied to in host delayed HIV models, will be analyzed, see e.g. [4,3].

**Keywords:** optimal control, HIV delayed model, stability, optimal solutions, numerical simulations.

### Acknowledgements

This research was partially supported by the Portuguese Foundation for Science and Technology (FCT) within projects UID-MAT-04106-2019 (CIDMA) and PTDC-EEI-AUT-2933-2014 (TOCCATTA), co-funded by FEDER funds through COMPETE-2020 – Programa Operacional Competitividade e Internacionalização (POCI) and by national funds (FCT). Silva is also supported by national funds (OE), through FCT, I.P., in the scope of the framework contract foreseen in the numbers 4, 5 and 6 of the article 23, of the Decree-Law 57-2016, of August 29, changed by Law 57-2017, of July 19.

## References

[1]R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL: A Modeling Language for MathematicalProgramming*, The Scientific Press, South San Francisco, California, 1993.

[2]L. Göllmann and H. Maurer, *Theory and applications of optimal control problems with multiple time-delays*, J. Ind. Manag. Optim., **10** (2014), 413–441.

[3]F. Rodrigues, C. J. Silva, D. F. M. Torres and H. Maurer, *Optimal control of a delayed HIV model*, Discrete Contin. Dyn. Syst. Ser. B 23 (2018), no. 1, 443–458.

[4]C. J. Silva, H. Maurer, D. F. M. Torres, *Optimal control of a tuberculosis model with state and control delays*, Math. Biosci. Eng. 14 (2017), no. 1, 321–337

[5]A. Wächter and L. T. Biegler, *On the implementation of a primal-dual interior-point filter line-search algorithm for large-scale nonlinear programming*, Mathematical Programming, **106** (2006), 25–57.

# Three-dimensional velocity field for blood flow using the power-law viscosity function

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## Abstract

The three-dimensional model associated with blood flow in vessels with viscosity depending on shear-rate, e.g., power-law type, is a complex model to obtaining computational implementation, which in many important situations reveals impracticable. In order to simplify the three-dimensional model, and as an alternative to classic one-dimensional models, we will use the Cosserat theory related fluid dynamics to approximate the three-dimensional velocity field, and thus obtain a one-dimensional system. Therefore, this system consists on an ordinary or partial differential equation depending only on time and on a single spatial variable, the flow axis. From this reduce system, we obtain the unsteady equation for the mean pressure gradient depending on the volume flow rate, Womersley number and the flow index over a finite section of the tube geometry. Attention is focused on some numerical simulations for constant and non-constant mean pressure gradient using a Runge-Kutta method and on the analysis of perturbed flows. In particular, given a specific data we can get information about the volume flow rate and consequently we can illustrate the three-dimensional velocity field on the constant circular cross-section of the tube. Moreover, we compare the three-dimensional exact solution for steady volume flow rate with the corresponding one-dimensional solution obtained by the Cosserat theory.

**Keywords:** Cosserat theory, blood flow, shear-thinning fluid, one-dimensional model, power-law model, volume flow rate, mean pressure gradient.

### Acknowledgements

This work has been partially supported by Center for Research in Mathematics and Applications (CIMA) related with the Differential Equations and Optimization (DEO) group, through the grant UID-MAT-04674-2019 of FCT-Fundação para a Ciência e a Tecnologia, Portugal.

## References

[1] D.A. Caulk, and P.M. Naghdi, Axisymmetric motion of a viscous fluid inside a slender surface of revolution, *Journal of Applied Mechanics*, 54, pp. 190–196, 1987.

[2] F. Carapau and R. Conceição, Three-dimensional velocity field for blood flow using the power-law viscosity function, *WSEAS Transactions on Heat and Mass Transfer*, V.13, pp. 35-48, 2018.

[3] F. Carapau, and A. Sequeira, 1D Models for Blood Flow in Small Vessels Using the Cosserat Theory, *WSEAS Transactions on Mathematics*, 5(1), pp. 54–62, 2006.

[4] B.R. Bird, R.C. Armstrong, and O. Hassager, *Dynamics of Polymeric Liquids*, John Wiley & Sons Edition, 1(2), 1987.

# An alternative Viscoelastic Model for Blood Flow Simulation

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## Abstract

An alternative viscoelastic model is presented and applied to blood simulations. This model is validated using several 2D benchmark problems [2], designed to reproduce difficulties that arise in the simulation of blood in blood vessels or medical devices.

**Keywords:** Blood, Viscoelastic fluid, finite elements.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações).

## References

[1]Bodnar, T., Pires, M. and Janela, J. (2014). Blood Flow Simulation Using Traceless Variant of Johnson-Segalman Viscoelastic Model. , *Mathematical Modelling of Natural Phenomena* 9,(6), 117141.

[2]O. Winter, T. Bodnar (2017). Simulations of Viscoelastic Fluids Flows using a Modified Log-Conformation Reformulation. , *Topical Problems of Fluid Mechanics* (DOI: https://doi.org/10.14311/TPFM.2017.040)

# On the mathematical modeling of leukostasis

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## Abstract

Inflammation is a nonspecific response to injury that includes a variety of functional and molecular mediators, including recruitment and activation of leukocytes [1]. Leukocyte-endothelial interaction plays an important role in the early phase of the development of certain inflammation-associated diseases, such as the diabetic retinopathy (DR). Diabetic retinopathy is the most common microvascular complication of diabetes and remains one of the leading causes of blindness worldwide among adults aged 20 - 74 years. Leukostasis is the crucial step in the pathogenesis of DR and the major component of inflammatory processes, which increases significantly in the retinas of diabetic animals and may contribute to capillary nonperfusion in DR [2]. In this preliminary work, we will present the results obtained using models for an Oldroyd-B viscoelastic droplets [3] which are, qualitatively, in agreement with those observed by the experimentalists working in the field.

**Keywords:** inflammation, diabetic retinopathy, Oldroyd-B viscoelastic model.

### Acknowledgements

This work has been partially supported by Fundação para a Ciência e a Tecnologia - FCT (Portugal) through the Project UID-Multi-04621-2013 of CEMAT Center for Computational and Stochastic Mathematics (University of Lisbon).

## References

[1]Abu Al-Asrar, A.M., Role of inflammation in the pathogenesis of diabetic retinopathy, *Middle East African Journal of Ophthalmol*, 19(1), 2012, pp. 70–74.

[2]van der Wijk, A.E., Hughes, J.M., Klaassen, L., Van Noorden, C.J.F., and Schlingemann, R.O., Is leukostasis a crucial step or epiphenomenon in the pathogenesis of diabetic retinopathy?, *Journal of a leukocyte biology*, 102(4), 2017, pp. 993–1001.

[3]Boujena, S., Kafi, O. and Sequeira, A., Mathematical study of a single leukocyte in microchannel flow *Mathematical Modelling of Naturel Phenomena*, 13(5):43, 2018.

# Near-wall region analysis in intracranial aneurysms

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## Abstract

Cerebral aneurysms still represent a frightful threat. The mechanisms behind the natural history of the aneurysm development are not fully understood. However, hemodynamic factors, especially those related to the vessel wall, are considered to play a fundamental role in this pathophysiology. The effects of abnormal flow patterns and its mechanical phenomena, exerted by blood on the vessel wall favour degradation and deformations on the vessel wall. The near-wall region is where the blood flow interacts with the endothelium cell layer and, therefore, is extremely important to understand the physiological functions of the vascular system and how the mechanical forces act in the mechanotransduction. A crucial part of these functions is the mass transport process to/from the blood and the vessel wall. We present a CFD methodology to describe the near-wall flow field, by WSS and the WSS divergence, as proportional to the velocity components tangential to the wall or perpendicular to it, respectively. We present, also, the WSS critical points as local instantaneous streamlines, which describe locally the motion of the flow. We believe this analysis can provide a step further on a better understanding of the flow within the aneurysms and consequently the mechanisms which lead to a possible rupture[1,2].

**Keywords:** Intracranial aneurysms, Medical image segmentation, CFD, Near-wall transport, Critical Points.

### Acknowledgements

This project has been financially supported by Fundação para a Ciência e a Tecnologia (FCT - Portugal), namely through the scholarship SFRH-BD-90339-2012, by the projects UID-Multi-04621-2013 and UID-Multi-04621-2019 of CEMAT-IST-ID, and by the Department of Mathematics of the Instituto Superior Técnico, University of Lisbon.

## References

[1]V. Ardakani, X. Tu, A.M. Gambaruto, I. Velho, J. Tiago, R. Pereira and A. Sequeira, Near-wall flow in cerebral aneurysms, *Fluids* (2019).

[2]A.M. Gambaruto, D.J. Doorly and T. Yamaguchi, Wall shear stress and near-wall convective transport: Comparisons with vascular remodeling in a peripheral graft anastomosis, *Journal of Computational Physics* 229 (2010) 5339-5356.

## Organized Session

# Orthogonal Polynomials and Applications

**Organizer: Maria das Neves Rebocho and Edmundo Huertas Cejudo**

# Divided-difference operators from the geometric point of view

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## Abstract

The present communication is part of a Dissertation project that we have been developing since last year at the Faculty of Sciences of UBI in the Mathematics Course for Teachers in the area of Mathematical Analysis. Our communication has the purpose of presenting the relationship that exists between a conic and the operators. It should be noted that we consider the operators of differences $Df(x)$, $D_w f(x)$ and $D_q f(x)$ defined in the real function space to the real variable, with the property of that $\mathbb{D}$ transforms a polynomial of degree $n$ into another polynomial of degree $n-1$. For this study we give a classification of such operators, combined with a geometric interpretation.

**Keywords:** Operators of divided differences, Conic, geometric interpretation.

## References

[1]A.P. Magnus, Associated Askey-Wilson polynomials as Laguerre, Hahn orthogonal polynomials, Springer Lect. Notes in Math. 1329, Springer, Berlin, 1988, pp. 261-278.

# Relative asymptotics of Laguerre polynomials modified with an infinite number of discrete mass points

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## Abstract

In this talk we consider the sequences of polynomials $\{Q_n^{(\alpha)}\}_{n\geq 0}$, orthogonal with respect to the inner product

$$\langle f, g\rangle_\nu = \int_0^{+\infty} f(x)g(x)d\mu(x) + \sum_{j=1}^{m} a_j\, f(c_j)g(c_j),$$

where $d\mu(x) = x^\alpha e^{-x}$ is the Laguerre measure on the real line, $\alpha > -1$, $c_j < 0$, $a_j > 0$ and $f$, $g$ are polynomials with real coefficients. This polynomials were already considered in ([3]) and ([4]) concerning analytic and electrostatic properties. In this contribution we analyze two open problems considering the outer relative asymptotic behavior as $m \to \infty$, and the inner asymptotics of $\{Q_n^{(\alpha)}\}_{n\geq 0}$ when the discrete mass points are located in the support of the measure of the classical Laguerre polynomials.

**Keywords:** orthogonal polynomials, Krall polynomials, asymptotic behavior.

### Acknowledgements

The author is partially supported by the Departamento de Física y Matemáticas de la Universidad de Alcalá (UAH), Madrid, Spain.

## References

[1]E.J. Huertas, F. Marcellán and H. Pijeira, *An electrostatic model for zeros of perturbed Laguerre polynomials*, Proc. Amer. Math. Soc. **142** (5) (2014), 1733–1747.

[2]C. Hermoso, E.J. Huertas, and A. Lastra, *Determinantal form for ladder operators in a problem concerning a convex linear combination of discrete and continuous measures*, Springer Proceedings in Mathematics & Statistics (PROMS), **256** (2018), 263–274.

# q-analogs of Laplace transform. Towards different q-difference orthogonal polynomials

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## Abstract

The study of $q$-difference equations in the complex domain from the asymptotic point of view is of increasing interest. Different techniques such as $q-$analogs of Borel-Laplace procedure have been developed in the last decades. The main aim of the talk is to recall one of the definitions of $q-$Laplace transform, as described in [1,2] and its applications to find solutions of $q-$difference equations. More precisely, $q-$exponential growth of the functions in adequate domains allow to apply $q-$Laplace transformation and state summability results. We will also explain how orthogonal polynomials appear as solutions to certain equations of that type.

**Keywords:** q-Laplace transform, q-difference equation, orthogonal polynomials.

### Acknowledgements

The author is partially supported by the project MTM2016-77642-C2-1-P of Ministerio de Economía y Competitividad, Spain.

## References

[1]J-P. Ramis, C. Zhang, Développement asymptotique q-Gevrey et fonction thêta de Jacobi.C. R. Math. Acad. Sci. Paris 335 (2002), no. 11, 899–902.

[2]C. Zhang, Une sommation discréte pour des équations aux q-différences linéaires et á coefficients analytiques: théorie générale et exemples, (French) [Discrete summation for linear difference equations with analytic coefficients: general theory and examples] Differential equations and the Stokes phenomenon, 309–329, World Sci. Publ., River Edge, NJ, 2002.

# Analytic properties of discrete Sobolev–type ortogonal polynomials in non uniform (snul) and q-lattices

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## Abstract

In this contribution we analyze several ways to obtain sequences of polynomials $\{Q_n^\lambda\}_{n\geq 0}$ orthogonal with respect to the following inner product

$$\langle p,q\rangle_{M,N}=\int_I p\left(x\right)q\left(x\right)d\psi(x)+Mp(c)q(c)+N\left(\mathcal{D}p\right)\left(c\right)\left(\mathcal{D}q\right)\left(c\right),$$

$M,N\in\mathbb{R}_+$, involving the *first order divided difference operator at* $x$ (see [1], [2])

$$\left(\mathcal{D}f\right)\left(x\right)=\frac{f(\varphi_1(x))-f(\varphi_2(x))}{\varphi_1(x)-\varphi_2(x)}$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are the two roots in $y$ of the quadratic equation $Ay^2+2Bxy+Cx^2+2Dy+2Ex+F=0$. We explore possible orthogonality measures $\psi$ on special nonuniform lattices (and maybe in q-lattices), and the possible values of the discrete mass point $c\in\mathbb{R}$ such that $\langle p,q\rangle_{M,N}$ is positive definite.

**Keywords:** orthogonal polynomials, q-lattices, non uniform lattices (snul).

### Acknowledgements

The author is partially supported by the Departamento de Física y Matemáticas de la Universidad de Alcalá (UAH), Madrid, Spain.

## References

[1]A.P. Magnus, *Special nonuniform lattice (snul) orthogonal polynomials on discrete dense sets of points.* J. Comput. Appl. Maths., **65** (1-3), (1995), 253–265.

[2]G. Filipuk and M.N. Rebocho, Discrete semi-classical orthogonal polynomials of class one on quadratic lattices, J. Difference Equ. Appl. 25 (2019) 1–20.

# Contributed Posters

# Operators of vec type

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## Abstract

In this work we will show how to use vec operators in model validation. The operators vec are operators that when applied to a matrix, stack successively the columns of this matrix. Operators of type vec are operators that also match vectors to matrices, that is, they allow to reorganize some elements of an array in a column vector. The formulation presented here allows us to make inferences about the study series, since the results presented in this dissertation can be applied to the matrices of the Hilbert-Schmidt products, matrices that are very important in the first phase of the Statis methodology. Thus, the operators allowed us to present results that allow to make inference for models for symmetric stochastic matrices.

**Keywords:** symmetric stochastic matrices, Inference, Hilbert-Schmidt products.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações).

## References

[1]1. A. Areia, M. M. Oliveira and J. T. Mexia, Modelling the Compromise Matrix in STATIS, Methodology, 2011, 5, pp. 277-288.
[2]2. C. Dias, *Modelos e Famílias de Modelos para Matrizes Estocásticas Simétricas*, Ph.D. Thesis, Évora University, 2013.

[3]3. C. Dias, S. Santos, M. Varadinov and J.T. Mexia, "ANOVA Like Analysis for Structured Families of Stochastic Matrices, *AIP Proceedings of the 12 International Conference of computational Methods in Sciences and Engineering (ICCMSE 2016)*, edited by American Institute of Physics, 2016, 1790, pp. 140006-1-140006-3

[4]5. H. Scheffé, *The Analysis of Variance*, New York: John Wiley and Sons, 1959.

[5]M. M. Oliveira and J. T. Mexia, AIDS in Portugal: endemic versus epidemic forecasting scenarios for mortality, *International Journal of Forecasting*, 2004, 20, pp.131–137.

# The Brian Tracy Method Applied to Students of Higher Education

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## Abstract

Higher education has been gradually changing in the last 25 years in order to promote greater student autonomy, changing the traditional role of the teacher into a more coaching profile style, that is, a tutorial teaching. However, as teachers, we can easily realize that most students entering higher education system are systematically faced with extreme difficulty in managing their available time in order to maximize their use to achieve the best possible performance as students, without harming their personal quality of life and social relationship. This article intends to present to students of higher education, in a simple way, one of the several techniques available to carry out a planning and management of the scientific tasks that usually arise associated to their academic course, thus allowing a decrease in stress, better academic performance and also a better quality of life by avoiding getting overwhelmed.

**Keywords:** Higher education, Academic performance, Stress dealing .

## References

[1]Ackoff, R. L., *The Art and Science of Problem Solving*, Interfaces, Vol. 11, No. 1, 1981.
[2]Aladib, L., Rosli, M., Thamutharam, Y., *Task Management System (TMS) for University of Malaya Research Student*, Research Gate, 2015. DOI: 10.13140/RG.2.2.15613.36326
[3]Boland, J., Richard Jr. and Collopy, Fred (eds.), Managing as Designing, Stanford Business Books, *Stanford University Press*, California, USA,2004.
[4]Brian Tracy, *Eat That Frog*, ebook http://www.al-edu.com/wp-content/uploads/2014/05/Eat That Frog.pdf (acess 2019/03/10).
[5]
[6]Eiselt, H., A. and Laporte, G., Combinatorial Optimization with Soft and Hard Requirements, *Journal of Operations Research*, Vol 38, No 9, 1987, pp. 785-795. https://www.greycampus.com/blog/project-management/six-step-process-to-priroritize-project-tasks (acess 2019/03/13).

# Addressing risk to physical fitness with factor analysis

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## Abstract

Principal Components Analysis (PCA) is an appropriate tool for reduce low-dimension subspaces which capture most of the information of the whole data set. In this paper, using physical fitness tests (sit-and-reach, lower-limbs explosive power, sit-up's, and cardiorespiratory fitness), a PCA was conducted in 11th grade students. It was performed a descriptive and correlation analysis between variables. The adequacy of PCA was measured by KMO and chi-square tests. It was found that the proportion of variance of each variable, explained by principal components (communalities), was greater than 80%, which means that it is appropriate to describe the latent correlation structure between physical fitness tests. The eigenvalues and the proportion of variance explained by each component showed that the PCA followed by Varimax rotation gave the retention of two components; explaining about 87% of the total variability. Regarding component matrix, both without and with rotation of the components, it was recorded high and balanced factorial loads ($> 88\%$). The factorial loads were analyzed by component transformation matrix. The solution was presented after Varimax rotation, leading to a better definition of the components. With Kaiser normalization criterion we observe the scores of each variable in the components. All variables saturate in only one component, meaning that these tests can be explained in a single component. The component one had a close relationship with strength-resistance; the component two were more associated with exibility.

**Keywords** principal components analysis, physical fitness, secondary education.

### Acknowledgements

This research was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID- MAT-04674-2019 (CIMA) (Centro de Matemática e Aplicações).

## References

[1] An introduction to applied multivariate analysis with R, Everitt, Brian and Hothorn, Torsten, 2011, Springer Science & Business Media.
[2] Applied multivariate statistical analysis, Härdle, Wolfgang and Simar, Léopold, 2007, 2, Springer Science & Business Media
[3] Applied multivariate statistical analysis, Johnson, Richard Arnold and Wichern, Dean W and others, 4, 1999, Prentice-Hall New Jersey.
[4] Analysis of incomplete multivariate data, Schafer, Joseph L, 1997, CRC press

# A comparative study of exponential smoothing models for retail sales forecasting

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## Abstract

Forecasting future economic time series provides the best assessment of the business information available. Therefore, it is required an accurate forecasting system that plays a crucial role in the quality of the decision-making process. In the business operations of the retail sales segment, forecasting accuracy is even more important to the quality of the decision-making process because retailing is widely recognized as a competitive industry in both mature and developing markets [1]. The purpose of this study is to compare the accuracy of retail sales forecasting between two exponential smoothing models (additive and multiplicative Holt-Winters exponential smoothing) applied to a monthly retail sales time series in Portugal from 2000 to 2018. These methods are chosen because of their ability to model trend and seasonal fluctuations present in retail sales data [2]. The Holt-Winters Exponential Smoothing is an extension of the Holt method, and is applied whenever the data behaviour is trendy and seasonal [3]. This work aims to discuss and compare these two Holt-Winters exponential smoothing formulations based on the same economic dataset [4].

**Keywords:** retail sales, time series modeling, Holt-Winters exponential smoothing, forecast accuracy.

### Acknowledgements

This work is financed by FEDER funds through the Competitivity Factors Operational Programme - COMPETE, and by national funds through FCT (Fundação para a Ciência e a Tecnologia) within the framework of Project POCI-01-0145-FEDER-007136, and Project UID-MAT-00013-2013.

## References

[1]Scheider, M.J. and Gupta, S., Forecasting sales of new and existing products using consumer reviews: a random projections approach, *International Journal of Forecasting*, 32(2), 2016, pp. 243–256.

[2]Costa, M. and Goncalves, A.M., *Forecasting time series combining Holt-Winters and Bootstrap approaches*, in Proceedings of the International Conference on Numerical Analysis and Applied Mathematics 2014 (ICNAAM-2014), AIP Conference Proceedings, American Institute of Physics, 1648, 2015, pp. 110004-1–110004-4.

[3]Winters, P.R., Forecasting sales by exponential weighted moving averages, *Management Science*, 6(3), 1960, pp. 324–342.

[4]R Core Team. R: a language and environment for statistical computing. Version 3.5.0. Vienna, Austria: R Foundation for Statistical Computing, 2018, *URL https://www.R-project.org/*.

# Numerical simulations of a third-grade fluid flow on a tube through a contraction

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## Abstract

Based on a director theory approach related to fluid dynamics we reduce the nonlinear three-dimensional equations governing the axisymmetric unsteady motion of a non-Newtonian incompressible third-grade fluid to a one-dimensional system of ordinary differential equations depending on time and on a single spatial variable. From this new system we obtain the unsteady equation for the mean pressure gradient and the wall shear stress both depending on the volume flow rate, Womersley number and viscoelastic parameters over a finite section of a straight, rigid and impermeable tube with variable circular cross-section. We present some numerical simulations of unsteady flows regimes through a tube with a contraction using a nine-directors theory.

**Keywords:** third-grade fluid, one-dimensional model, unsteady flow, hierarchical theory.

### Acknowledgements

This work has been partially supported by Center for Research in Mathematics and Applications (CIMA) related with the Differential Equations and Optimization (DEO) group, through the grant UID-MAT-04674-2019 of FCT-Fundação para a Ciência e a Tecnologia, Portugal.

## References

[1]Caulk, D.A. and Naghdi, P.M., Axisymmetric motion of a viscous fluid inside a slender surface of revolution, *Journal of Applied Mechanics*, 54(1), 1987, pp. 190–196.

[2]Fosdick, R.L. and Rajagopal, K.R., Thermodynamics and stability of fluids of third grade, *Proc. R. Soc. Lond. A.*, 339, 1980, pp. 351–377.

[3]Carapau, F. and Correia, P., Numerical simulations of a third-grade fluid flow on a tube through a contraction, *European Journal of Mechanics B/Fluids*, 65, 2017, pp. 45–53.

# Results Related to Higher-Order Moments and Cumulants

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## Abstract

In this presentation we undertook a review of certain results on Cumulative Generation Functions and on Cumulants. This results will be useful to show how to obtain estimators for the cumulants of a class of mixed models, the additive models. As we shall explain, a simulation study is presented to show that these models are easy to implement and have many advantages.

**Keywords:** additive models, cumulants, mixed Models, moments.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2019 (Centro de Matemática e Aplicações) and UID-MAT-00212-2019.

## References

[1]Balakrishnan, N., Johnson, N. L. and Kotz, S. (1998), A note on relationships between moments, central moments and cumulants from multivariate distributions, Statistics & probability letters, **39**: 49–54, Elsevier.

[2]Dodge Y. and Valentin R. (1999), The Complications of the Fourth Central Moment, The American Statistician, **53** (3): 267–269.

[3]Craig, C. C. (1931). On A Property of the Semi-Invariants of Thiele. Ann Math Statist, **2** (2): 154–164.

[4]Harvey, J. R., (1972), On Expressing Moments in Terms of Cumulants and Vice Versa. The American Statistician, **26** (4): 38–39.

[5]Smith, P. J. (1995), A Recursive Formulation of the Old Problem of Obtaining Moments from Cumulants and Vice Versa. The American Statistician, **49** (2): 217–218.

# Discrimination Rules: An application to discrete variables

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## Abstract

In Discriminant Analysis (DA) the main task is to set rules that assigns an unknown subject to one of two or more groups on the basis of a multivariate observation. It is of much essence to consider the costs of assignment, the *priori* probabilities of belonging to one of the groups, and the number of groups involved [1,3]. The allocation rule is selected to optimize some function of the costs of making an error and the *priori* probabilities of belonging to one of the groups [2]. In this work, we consider discrete variables, using DA in connection with Statistical Decision Theory (SDT) to derive a rule, that minimizes the assignment costs. We present an application on a two-phase discrimination process on an HIV/AIDS dataset of 62676 subjects from Instituto de Higiene e Medicina Tropical, Universidade Nova de Lisboa.

**Keywords:** Discriminant Analysis, Discrimination rule, Two-Phase discrimination, Decision theory.

## References

[1]Philip J Brown, T Fearn, and MS Haque. Discrimination with many variables. Journal of the American Statistical Association, 94(448):1320–1329, 1999.

[2]Philip L Cooley. Bayesian and cost considerations for optimal classification with discriminant analysis. Journal of Risk and Insurance, pages 277–287, 1975.

[3]Peter A Lachenbruch and M Goldstein. Discriminant analysis. Biometrics, pages 69–85, 1979.

# Big Data and Ordinal Logistic Regression

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## Abstract

Logistic Regression, as well as all regressions, are used to predict a data value based on prior observations of a data set. Logistic Regression allows estimating the relation between several variables when the dependent variable is categorical. At the base of this technique is the logistic function that began to be used to describe the growth of a population, originally due to Pierre-François Verhulst in 1838, see [1]. The Logistic Regression is currently widely used in the most diverse areas of knowledge, see [2], [3] and [4]. This regression can be binomial, ordinal or multinomial, depending on the number of categories that the dependent variable assumes. In this study only the Ordinal Logistic Regression is considered.
In this work we intent to show the importance of this technique in rapidly expanding areas such as Big Data. According Wang et all (2016), [5], Big Data are data on a massive scale in terms of volume, intensity, and complexity that exceed the capacity of standard analytic tools (p.399). The role of statisticians in Big Data studies has been under-recognized. In our opinion it is important to change it. With this work we intent to contribute to this change, showing the important role that Logistic Regression already has in Big Data analysis. We believe that the research in Big Data present opportunities as well as challenges to statisticians.

**Keywords:** Logistic Regression, Logistic Regression Model, Ordinal Data, Big Data.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações).

## References

[1]Bacaër N. (2011) Verhulst and the logistic equation (1838). In: *A Short History of Mathematical Population Dynamics* (35–39), Springer, London.

[2]Jiang, Y.(2018) Using Logistic Regression Model to Predict the Sucess of Bank Telemarketing. *International Journal on Data Science and Technology* **4** (1): 35–41.

[3]Nie, G., Rowe, W., Zang, L., Tian, Y. and Shi, Y. (2011) Credit card churn forecasting by logistic regression and decision tree. *Expert Systems with Applications* **38**: 15273–15285.

[4]El aboudi, N. and Benhlima, L. (2018) Big Data Management for Healthcare Systems: Architecture, Requirements, and Implementation. *Advances in Bioinformatics*. Article ID 4059018, 10 pages.

[5]Wang, C., Chen, M., Schifano, E., Wu, J. and Yan, J. (2016) Statistical methods and computing for big data.*Stat Interface* **9(4)**: 399–414.

# Fitting mixtures of linear mixed models with the EM, CEM and SEM algorithms: a simulation study

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## Abstract

Finite mixture models are a well-known method for modelling data that arise from a heterogeneous population. In regression analysis, it has been a popular practice for modelling unobserved population heterogeneity through finite mixtures of regression models. Within the family of mixtures of regression models, finite mixtures of linear mixed models have also been applied in different areas of application since, besides taking into account the heterogeneity in the population, they also allow to take into account the correlation between observations from the same individual, which makes them particularly used in longitudinal data (see McLachlan and Peel [1]). One of the main issues in mixture models concerns the estimation of the parameters. The maximization of the log-likelihood function in mixture models is complex, producing in many cases infinite solutions whereby the maximum likelihood estimator may not exist, at least globally. In order to solve the problem, it is common to resort to iterative methods in the estimation of the parameters, in particular to the Expectation-Maximization (EM) algorithm (Dempster et al. [2]), an algorithm that consists of two steps, an E- and M-steps, which are used alternately until achieving convergence. In the attempt to overcome the slow convergence of the EM algorithm, several modified versions of this algorithm were developed over the years, the most usual being the Classification EM (CEM) and the Stochastic EM (SEM) (see Faria and Soromenho [3,4]). The CEM algorithm incorporates a classification step, C-step, between the E- and M-steps, which consists of assigning each observation to one of the components of the mixture model, the one that corresponds to the maximum posterior probability. On the other hand, the SEM algorithm incorporates a stochastic step, S-step, between the E- and M-steps, which consists of simulating a realization of the unobserved indicator for each individual, by choosing it randomly from its conditional distribution. In this work we compare the performance of the three algorithms in the estimation of the parameters of mixtures of linear mixed models through a simulation study. For

this, we analyse the computational effort of each algorithm by studying the mean number of iterations necessary to converge, we analyse two statistical properties of the estimators (the bias and the mean square error (MSE)) and we also analyse goodness of fit by computing the root mean-squared error of prediction (MRSEP) through K-fold cross-validation. Based on the mean number of iterations for convergence, we conclude that the CEM algorithm always converge in fewer iterations than the EM algorithm, whereas the slow convergence of the SEM algorithm can be a drawback to its use. On the other hand, the bias and MSE of the parameter estimates for the three algorithms show that three algorithms present approximately the same behaviour, with all the estimates having small bias and MSE. Finally, for studying goodness of fit we used 10-fold cross-validation to compute the MRSEP. The obtained values of the MRSEP show that, although once again the three algorithms perform similarly, the SEM algorithm performs generally better whenever the error variance increases and, on the other hand, it seems that both the EM and the CEM algorithms perform better for smaller error variances. In conclusion, in our simulation study it can be seen that the three algorithms provide similar maximum likelihood estimates for the parameters, both in the sense of lower bias and MSE and in the sense of goodness of fit. However, of the three algorithms, the CEM algorithm is the algorithm less computationally demanding, that is, it is always the one converging in fewer iterations, so we reccommend its use in every situation.

**Keywords:** mixture models, maximum likelihood estimation, goodness of fit, cross-validation.

**Acknowledgements**

This research was financed by FCT - Fundação para a Ciência e a Tecnologia, through the PhD scholarship with reference SFRH-BD-139121-2018.

## References

[1]McLachlan, G. and Peel, D., *Finite Mixture Models*, John Wiley & Sons, 2000.

[2]Dempster, A. P., Laird, N. M., and Rubin, D. B., Maximum likelihood from incomplete data via the EM algorithm, *Journal of the Royal Statistical Society: Series B (Methodological)*, 39(1), 1977, pp. 1–22.

[3]Faria, S. and Soromenho, G., Fitting mixtures of linear regressions, *Journal of Statistical Computation and Simulation*, 80(2), 2010, pp. 201–225.

[4]Faria, S. and Soromenho, G., Comparison of EM and SEM algorithms in Poisson regression models: a simulation study, *Communications in Statistics-Simulation and Computation*, 41(4), 2012, pp. 497–509.

# Stochastic diffusion process Based on generalized Goel-Okumoto curve

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## Abstract

In the present work we deal with a new stochastic diffusion process based on the generalized Goel-Okumoto curve ([1], [2]). We follow the methodology considered in the diffusion process associated with the generalized von Bertalanffy growth curve [3]. From the corresponding Itô's stochastic differential equation (SDE), First of all we establish the probabilistic characteristics of the studied process, such as the solution to the SDE, the probability transition density function and their distribution, the moments function in particular the conditional and non-conditional trend functions. Then, we treat the parameters estimation problem by using the maximum likelihood method in basis of the discrete sampling.

**Keywords:** Diffusion process, Genaralized Goel-Okumoto curve, Stochastic differential equation, maximum likelihood estimation.

## References

[1]Goel, A. L. and Okumoto, K., Time-dependent error-detection rate model for software reliability and other performance measures, *IEEE Transactions on reliability*, 28(3), 1979, pp. 206–211.

[2]Huang, C. Y. and Lyu, M. R., Estimation and analysis of some generalized multiple change-point software reliability models, *IEEE Transactions on reliability*, 60(2), 2011, pp. 498–514.

[3]Román-Román, P., Romero, D. and Torres-Ruiz, F., A diffusion process to model generalized von Bertalanffy growth patterns: Fitting to real data, *Journal of theoretical biology*, 263(1), 2010, pp. 59–69.

# Voronoi volumes by stochastic sampling: applications on lipid membrane studies

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## Abstract

Voronoi diagrams are geometrical constructs that divide the space into regions, named Voronoi cells, where each region comprises all the points that are closer to a particular point from a given set of central points (centroids) than to any of the other centroids. For a given set of $N$ distinct points $\{c_1, c_2, c_3, ..., c_N\}$ the Voronoi diagram is, thus, defined as a collection of Voronoi cells $\{V(c_1), V(c_2), V(c_3), ..., V(c_N)\}$ where each cell is defined as

$$V(c_i) = \{x \in \mathbb{R}^d : \|x - c_i\| \leq \|x - c_j\|, \forall j \neq i\}$$

The Voronoi cells thus formed are convex polyhedra and fill completely the space. They are also characterized by a set of geometric attributes like the edges number, vertices number, the face areas and the cell volume. The Voronoi tesselation construction has found many applications in a wide range of fields, particularly in the study of the structure of fluids [1], glasses [2], solids packing [3], proteins [4] and biological membranes [5] where the characterization of the spatial structure of the system is a challenging problem. For many of these applications, the relevant property to evaluate is the Voronoi polyhedra volumes and its distribution. This implies the determination of the Voronoi polyhedra for a set of configurations of particles, which requires considerable computational effort. In this work we apply a simple alternative procedure for the Voronoi cells volume determination by a Monte Carlo method and apply it to bilayer model membrane systems.

**Keywords:** Monte Carlo, area computation, volume computation, Voronoi tesselation.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through research project PTDC-DTP-FTO-2784-2014 and research center (Centro de Química de Évora) funding UID-QUI-0619-2016.

## References

[1]Idrissi, A., Vyalov, I., Kiselev, M., Fedorov, M. V. and Jedlovszky, P., Heterogeneity of the Local Structure in Sub- and Supercritical Ammonia: A Voronoi Polyhedra Analysis, *The Journal of Physical Chemistry B*, 115, 2011, pp. 9646-9652.

[2]Gedeon, O., Liska, M. and Machacek, J., Volume fluctuations in potassium silicate glass studied by molecular dynamics, *Journal of Non-Crystalline Solids*, 357, 2011, pp. 1574-1581.

[3]Yang, R. Y., Zou, R. P. and Yu, A. B., Voronoi tessellation of the packing of fine uniform spheres, *Physical Review E*, 65, 2002, pp. 041302.

[4]Poupon, A., Voronoi and Voronoi-related tessellations in studies of protein structure and interaction, *Current Opinion in Structural Biology*, 14, 2004, pp. 233–241.

[5]Prates Ramalho, J. P., Gkeka, P. and Sarkisov, L., Structure and Phase Transformations of DPPC Lipid Bilayers in the Presence of Nanoparticles: Insights from Coarse-Grained Molecular Dynamics Simulations, *Langmuir*, 27, 2011, pp. 3723–3730.

# Sum of a random number of independent and identically distributed random variables: An application to aggregate claims

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## Abstract

The distribution of the sum of a random number, $N$, of independent and identically distributed random variables, $X_1, X_2, \ldots, X_N$, is an important topic for example in risk management to model aggregate claims. However, in many cases the distribution of this random sum does not have a manageable form. Several techniques such as numerical algorithms or simulation based approaches have been used mainly for problems related with the distribution of aggregate claims [1–3]. Motivated by the results in [4,5] which combine the properties of mixtures with a matching moment technique to address the sum of a fixed number of random variables, we propose a new and simple computational technique to approximate the distribution of the sum of a random number of variables. In this first work we assume that $X_i$, $i = 1, \ldots, N$, follows a Loggamma distribution and that $N$ has a Poisson distribution. Numerical studies are conducted to assess the precision of the approximations developed, and an application related with aggregate claims is presented.distribution.

**Keywords:** Sum of random variables, random number of random variables, mixture distributions, aggregate claims.

### Acknowledgements

This work was partially supported by the Portuguese Foundation for Science and Technology through the projects UID-MAT-00212-2019 and UID-MAT-00297-2019.

## References

[1]Mohamed Amraja Mohamed, Ahmad Mahir Razali and Noriszura Ismail (2010). Approximation of Aggregate Losses Using Simulation. *Journal of Mathematics and Statistics*, 6 (3), 233-239.

[2]Philipe Heckman and Glening Meyers (1983). The calculation of aggregate loss distributions from claim severity and claim count distributions *Proceedings of the Casualty Actuarial Society*, 70, 22-61.

[3]Pavel V. Shevchenko (2010). Calculation of aggregate loss distributions. *The Journal of Operational Risk*, 5(2), 3-40.

[4]Marques, F.J., Coelho C.A. and de Carvalho, M. (2015). On the distribution of linear combinations of independent Gumbel random variables. *Statistics and Computing*, 25, 683-701.

[5]Marques, F.J. and Loingeville, F. (2016). Improved near-exact distributions for the product of independent Generalized Gamma random variables. *Computational Statistics & Data Analysis*, 102, 55-66.

# Dealing with overdispersed count data

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## Abstract

In many applications, the response variable of interest is a count, that is, takes on nonnegative integer values. Classical Poisson regression is the most well-known methods for modeling count data. However, its equidispersion constraint does not accurately represent real data. This equal mean-variance relationship rarely occurs in observational data and in most cases, the observed variance is larger than the mean. This excess variability is called overdispersion and has been widely considered in the literature (see Dupuy [1]). Various reasons make counts overdispersed, e.g. missing covariates or interactions, neglected or unobserved heterogeneity, violations in the distributional assumptions of the data, outliers in the response variable or correlation between responses, (see Hinde and Demetrio [2]). Two main problems are associated with overdispersion: a possible loss of efficiency in the estimations under different conditions and consequently incorrect inferences on the regression parameters (a variable may appear to be a significant predictor when it is in fact not significant) (see Quintero-Sarmiento, Cepeda-Cuervo, and Nunez-Anton [3]). To model overdispersion, many alternatives to Poisson regression models have been suggested in literature. Among them, we consider the negative binomial regression model (see Hilbe [4]) (which have been approached frequently to model overdispersion) and the generalized Poisson regression model introduced by Consul and Famoye [5]. In this work, we model the number of major derogatory reports recorded in the credit history of a sample of applicants for a type of credit card, applying Poisson regression models. However, the models developed show an overdispersion problem and the alternatives are negative binomial and generalized Poisson regression models.

**Keywords:** count data, generalized Poisson regression model, negative binomial regression model, overdispersion.

### Acknowledgements

This work was supported by the strategic programme UID-BIA-04050-2019 funded by national funds through the FCT.

## References

[1]Dupuy, J-F. *Statistical Methods for Overdispersed Count Data*, ISTE Press, Elsevier, 2018.

[2]Hinde, J., and Demetrio, C., Overdispersion: Models and estimation, *Computational Statistical Data Analysis*, 27(2), 1998, pp. 151–170.

[3]Hilbe, J., *Negative Binomial Regression*, Cambridge University Press, Cambridge, 2012.

[4]Quintero-Sarmiento, A., Cepeda-Cuervo, E. and Nunez-Anton, V., Estimating infant mortality in Colombia: Some overdispersion modelling approaches, *Journal of Applied Statistics*, 39(5), 2012, pp. 1011–1036.

[5]Consul, P.C. and Famoye, F., Generalized poisson regression model. *Communications in Statistics - Theory and Methods* , 21 (1), 1992, pp. 89–109.

# Portuguese student's performance in mathematics

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## Abstract

Several studies have been conducted to identify the different factors that influence students' school performance. International programmes of educational achievement, such as the OECD Programme for International Student Assessment (PISA), are being developed, which makes it easier to obtain information to carry out these studies (see OECD [1]). The purpose of this study was to explore the factors that affect the students' mathematics achievement in Portugal. Data on about 7325 Portuguese students and 246 schools who participated in PISA-2015 were used to accomplish our objectives. Studies based on PISA data to explain student performance are very common in literature (see Agasisti and Cordero-Ferrera [2] and Pereira an Reis [3]). The PISA data are hierarchically structured, in which students are nested within classrooms, classrooms, in turn, are nested within schools, schools are nested within regions and regions are nested within countries. Given the hierarchical structure of data, the models adopted for statistical analysis were multilevel regression models, which can take into account data variability within and among the hierarchical levels (see Goldstein [4]). A three-level modeling technique was used to model the variation in mathematics achievement as a function of student- (level 1), school- (level 2), and region-level (level 3) factors ( see Agasisti, Ieva, and Paganoni [5]). The study concluded that factors such as the economic, social and cultural index of the student, being a male student, starting the first year of schooling with 6 years, the total number of students in school and the proportion of girls in school positively influence the student's performance in mathematics. On the other hand, the grade repetition had a negative influence on the performance of the Portuguese student in Mathematics.

**Keywords:** mathematics achievement, multilevel regression models, Programme for International Student Assessment (PISA) 2015.

### Acknowledgements

This work was supported by the strategic programme UID-BIA-04050-2019 funded by national funds through the FCT.

## References

[1]OECD, PISA 2015 Results (Volume I) . Paris: Organisation for Economic Co-operation and Development, 2016.

[2]Agasisti, T., Cordero-Ferrera, J.M., Educational disparities across regions: A multilevel analysis for Italy and Spain. *Journal of Policy Modeling*, 35, 2013, pp. 1079–1102.

[3]Pereira, M. C., Reis, H., What accounts for Portuguese regional differences in students' performance? Evidence from OECD PISA. *Lisboa: Economic Bulletin*, Banco de Portugal, 2012.

[4]Goldstein, H., *Multilevel Statistical Models*, John Wiley and Sons, 2011.

[5]Agasisti, T., Ieva, F., Paganoni, A.M., Heterogeneity, school-effects and the North/South achievement gap in Italian secondary education: evidence from a three-level mixed model. *Statistical Methods and Applications*, 2017, 26 (1), pp. 157–180.

# Mixed effects models where the sample sizes are Binomial distributed

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## Abstract

This work presents an approach that considers orthogonal mixed models, under situations of stability, when the samples dimensions are not known in advance. In this case we assume that the samples sizes are realizations of independent random variables. This methodology is applied to the case where there is an upper bound for the sample dimensions, which may not be attained due the occurrence of failures. Accordingly, we assume that sample sizes are Binomial distributed. An application on the incidence of unemployed people in the European Union as well as a simulation study are conducted to illustrate the proposed methodology.

**Keywords:** mixed models, random sample sizes, stability situations, Binomial distribution, unemployment in European Union.

### Acknowledgements

This work was partially supported by the Portuguese Foundation for Science and Technology through the projects UID-MAT-00212-2019 and UID-MAT-00297-2019.

## References

[1]Mexia, J.T., Nunes, C., Ferreira, D., Ferreira, S.S. and Moreira, E. (2011). Orthogonal fixed effects ANOVA with random sample sizes, *Proceedings of the 5th International Conference on Applied Mathematics, Simulation, Modelling (ASM'11)*, Corfu, Greece, 84–90.

[2]Nunes, C., Ferreira, D., Ferreira S.S. and Mexia, J.T. (2012). $F$-tests with a rare pathology, *Journal of Applied Statistics*, 39(3): 551–561.

[3]Nunes, C., Ferreira, D., Ferreira, S.S. and Mexia, J.T. (2014). Fixed effects ANOVA: an extension to samples with random size, *Journal of Statistical Computation and Simulation* 84(11): 2316–2328.

[4]Nunes, C., Capistrano, G., Ferreira, D., Ferreira, S. S. and Mexia, J. T. (2019). Exact critical values for one-way fixed effect models with random sample sizes. *Journal of Computational and Applied Mathematics,* 354: 112-122.

[5]Nunes, C., Mário, A., Ferreira, D., Ferreira, S. S. and Mexia, J. T. (2019). Random sample sizes in orthogonal mixed models with stability. *Computational and Mathematical Methods*. Accepted.

# Statistical Challenges in Big Data and Data Science

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## Abstract

Big Data (BD) has the potential to help assess and improve the decision-making process, and in recent years has attracted strong interest from academics and practitioners. BD has become the new paradigm in the world of collecting and analyzing data and it is in general characterized by five dimensions: volume (number of observations, attributes and relations), variety (diversity of data sources, formats, media and content), velocity (production speed and data change), veracity (quality, origin and reliability of the data) and value. Big Data Analytics is increasingly becoming a trendsetter that many organizations adopt for the purpose of creating valuable BD information for decision-making. Areas such as Computer Science, Engineering and Statistics play a key role in analyzing BD, each with its own specificity, but all equally important. Data Science (DS) is an interdisciplinary area dedicated to the study and analysis of data, which also aims at extracting knowledge for possible decision making and combining the areas of BD and Machine Learning, as well as techniques from other interdisciplinary areas. DS can help to transform large amounts of raw data into business insights, and thereby aid decision-making to achieve better results. In this work we are going to present and discuss some of the challenges of Statistical Modeling in this new era of Big Data and Data Science.

**Keywords:** statistical modeling, big data, data science, e-learning.

**Acknowledgements**

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00006-2013 (CEAUL - Centro de Estatística e Aplicações, Universidade de Lisboa).

## References

[1]Agarwal R., Dhar V., Editorial – big data, data science, and analytics: the opportunity and challenge for is research. *Information Systems Research*, 25 (3), 2014, pp. 443–448.

[2]Ali, A., Qadir, J., urRasool, R., urRasool, R., Sathiaseelan, A., Zwitter, A., Crowcroft, J. Big data for development: applications and techniques, *Big Data Anal.*, 2016, 1,2.

[3]Brito, P., Data Science: Um desafio para os estatísticos? Boletim da Sociedade Portuguesa de Estatística, 2016, 40–42.

[4]Chen C.L.P., Zhang C.Y., Data-intensive applications, challenges, techniques and technologies: a survey on big data. *Information Sciences*, 275, 2014, pp. 314–347.

[5]Emani, C.K., Cullot, N., Nicolle, C., Understandable big data: a survey. *Comput. Sci. Rev.*, 17, pp. 70—81.

[6]Gandomi, A. and Haider, M., Beyond the hype: Big data concepts, methods, and analytics, *International Journal of Information Management*, vol. 35, no. 2, 2015, pp. 137—144.

[7]Jordan, J.M. and Lin, Dennis K.J., Statistics for Big Data: Are Statisticians Ready for Big Data? *Journal of the Chinese Statistical Association* Vol. 52, 2014 133–149.

[8]Kankanhalli, A., Hahn, J., Tan, S., Gao, G., Big data and analytics in healthcare: introduction to the special section. *Inf. Syst. Front.*, 18, 2016, pp. 233—235.

[9]Oliveira, T.A., Oliveira, A., Bonilla, A.P., Data Mining and Quality in Service Industry: Review and some applications. In J. Faulin, A. Juan, S. Grasman and M. Fry (eds.), Decision Making under Uncertainty in the Service Industry, 2013, 83–108, Boca Raton: *CRC/Taylor&Francis Book*, ISBN 9781439867341.

[10]Rajaraman, V., Big data analytics. *Resonance*, 21, 2016, pp. 695—716.

[11]Tukey, John W., The future of data analysis. The Annals of Mathematical Statistics, 1962, 33,1, 1–67.

[12]Ward, J. S. and A. Barker, A., Undefined by Data: A Survey of Big Data Definitions, 2013, arXiv:1309.5821 [cs.DB].

[13]Zicari R.V., Big Data: Challenges and Opportunities. Akerkar R. (Ed.), Big data computing, CRC Press, Taylor & Francis Group, Florida, USA, 2014, pp. 103–128

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978-989-654-545-1