Uluslararası Alanya İşletme Fakültesi Dergisi
Yıl:2013, C:5, S:3, s. 87-93

International Journal of Alanya Faculty of Business
Year:2013, Vol:5, No:3, s. 87-93

Generalized Formulae for the Periodic Fixed and Geometric-Gradient Series Payment Models in a Skip Payment Loan with Rhythmic Skips

Bir Borcun Ritmik Atlamalı Sabit ve Geometrik Değişimli Taksitlerle Geri Ödenmesi Problemleri İçin Genel Formüller

Abdullah EROĞLU*
Prof. Dr., Süleyman Demirel University, Dept. of Business Administration, (abdullaheroglu@sdu.edu.tr)

Erdal AYDEMİR
Research Assistant, Süleyman Demirel University, Dept. of Industrial Engineering, (erdalaydemir@sdu.edu.tr)

Yusuf ŞAHİN
Msc. Industrial Engineer, Süleyman Demirel University, Dept. of Business Administration (yusufytu@gmail.com)

Nigar KARAGUL
Lecturer, Pamukkale University, Honaz MYO, (ntokat@pau.edu.tr)

Kenan KARAGUL
Lecturer, Pamukkale University, Honaz MYO, (kkaragul@pau.edu.tr)

Keywords:

Payments, Rhythmic skip, Arbitrary skip, Loan payment models

ABSTRACT

Nowadays, the periodic level payment model is the most widely used loan payment model by the banks. In addition, the periodic geometric and linear gradient payment models are available in the financial mathematics books. The arbitrary skip periodic level (or equal) loan payment model was firstly developed by Formato (1992). Formato's model was extended to the geometric gradient loan payment model by Moon (1994) and the linear gradient model by Eroglu and Karaoz (2002). This loan payment models that have periodic level and geometric gradient series payment with rhythmic skips instead of arbitrary skips have been discussed. General formulae have been derived for these models. Numerical examples are solved to show the practical application of the developed payment models on home financing.

Anahtar Kelimeler:

Anüite, Ritmik atlamalı, Rastgele atlamalı, Borç ödeme modelleri

ÖZ

Bankalar tarafından en çok kullanılan borç ödeme modeli, sabit taksitli modeldir. Bunun yanı sıra, finans matematiği kitaplarında geometrik ve aritmetik değişimli taksitlerle borç ödeme modelleri de mevcuttur. Rastgele atlamalı sabit taksitli borç ödeme modeli Formato (1992) tarafından geliştirildi. Formato'nun modeli; Moon (1994) tarafından geometrik değişimli taksitlerle ve Eroğlu ve Karaöz (2002) tarafından aritmetik değişimli taksitlerle borç ödeme modellerine genişletildi. Bu çalışmada, rastgele atlamalı sabit ve geometrik değişimli taksitler yerine, ritmik atlamalı sabit ve geometrik değişimli taksitler içeren borç ödeme modelleri ele alınıp modeller için genel formüller türetilmiştir. Geliştirilen ödeme modellerinin pratik uygulamasını göstermek için sayısal örnekler konut finansman modeli kurularak çözülmüştür.

1. INTRODUCTION

The term of annuity is used in finance theory to refer fixed payments over a specified period of time. Individuals who want to provide a certain accumulation and the investment-strapped companies can make debt payment or capital accumulation in financial corporations by determined amount of installments when they want to make savings (Baskaya, 1998).

In case of debt payment with equal installments, the amount of installments to be paid is obtained by annuity formulae. In long-term debt models, the installments/annuities paid include the repayments of the capital with its calculated interest rate. In this type of loan models, it's difficult to pay only interest amount of the related period during the loan date and to pay the capital at the end of period.

* Corresponding Author: abdullaheroglu@sdu.edu.tr , +90-246-2113192

In series composed of re-payments (installment series), the payment of a debt by equal, geometric-gradient and arithmetic-gradient installments is a frequent method used in the scientific literature.

However, as a result of special situations, these formulae are insufficient to satisfy the repayment models (Eroglu, 2000). The problem of loan repayment with installments is based on the fact that the present value of the debt and the present sum of the repayments are equal (Iscil, 1997). In case of variability of insolvency of indebted people and/or companies during the year, series of skipped payment loans have become important. Formato (1992) mentioned skipped installments in development of a new payment series, has addressed the problem of repayment of a debt in equal installments rounds without installments (back payments) in a certain time period at some randomly selected periods.

Recently, when we review of payment systems, there is only some paper in the literature. Mills (2006) explored alternative credit policies of central bank for liquidity provision by using a mechanism design approach as an agent system in a payments model. Martin et al. (2008) studied payment arrangements that resemble in some ways electronic payment networks for searching model of money. Kim and Lee (2010) developed a new model of debit card as a means of payments for overtime and the cross-sectional difference between cash and debit card payments. Galbiati and Soramäki (2011) offered and simulated an agent-based model of payment system on multi period model for real-time gross settlement (RTGS) payment system about liquidity stock between banks. Eroglu et al. (2012) presented the payment model about Islamic home financing under Musharakah and Mutanaqisah contracts. The literature of payment systems shows to us, there is no paper in loan payment models since Eroglu and Karaoz (2002) model so, we consider that this study is importance to fill in the gap of literature.

In this paper, instead of randomly choosing the time periods without installments, fixed and geometric-gradient series payment loan models have been examined and general formulae are derived to choose the time periods in terms of particular rules by supporting numerical examples.

2. GENERAL FORMULAE OF FIXED-GEOMETRIC AND ARİTHMETIC-GRADIENTS SERIES PAYMENT IN LOAN PAYMENT MODELS

Some known formulae are given as follows from Eroglu and Karaoz (2002). For Fixed series payment models the amount of payment d is defined as

$$d = \frac{pr}{1 - R^{-N}} \tag{1}$$

where p is the loan amount, r is the interest rate for period/term, $R = 1 + r$ and N is the payment/annuity number. For the geometric-gradients series payment models, the annuity values d_k at the end of the k^{th} period/term can be given in terms of d as

$$d_k = dG^{k-1}, \quad k = 1, \cdots, N \tag{2}$$

where G=1+g in which g is defined as relative gradients in annuity amount (geometric gradients). By defining i=GR^{-1}, the amount of payment is obtained different from Eq.(1) as

$$d = \begin{cases} \dfrac{p(r-g)}{1-i^N} & , \quad g \neq r \\ \dfrac{pR}{N} & , \quad g = r \end{cases} \tag{3}$$

For arithmetic-gradients series payment models

$$d_k = d + (k-1)v \quad , \quad k = 1, \cdots, N \tag{4}$$

$$d = \frac{pr^2R^N + v\left[1 + Nr - R^N\right]}{r\left(R^N - 1\right)} \tag{5}$$

where v is the amount gradients in annuity amount (arithmetic gradients).

In all of the above given formulae, the payments have been assumed to be paid at the end of each period. In general, the payment series models can be classified in four parts: equal amount, geometric-gradient, arithmetic-gradients and irregular payment series models. In some situations, a new model called skip payment loan with arbitrary skips is developed by Formato (1992) for customers that want to determine his payment periods, for example, holiday expenditure. Then, this model is extended to geometric-gradients skips by Moon (1994) and later to arithmetic geometric-gradient payment series model developed (Eroglu and Karaoz, 2002; Moon, 1994). For all of these three models, skips are randomly chosen for each model. So, Eroglu (2000) discussed four new models as periodic/irregular partials geometric-gradients and periodic/irregular partials arithmetic-gradient models ad give formulae that give the amount of instalment by using time value of money. By taking $M_0 = 1$ and $L_{s+1} = N$, the formulae of Formato (1992) and Moon (1994) are simplified by Eroglu (2001) . In addition, some new formulae are derived in Eroglu (2001) about the periodic and irregular piecewise of geometric-gradients and periodic and irregular piecewise of arithmetic-gradient models by supporting numerical examples in the same study.

3. DEVELOPED MODELS

3.1. The Periodic Geometric-Gradient Series Payment Model in A Skip Payment Loan with Rhythmic Skips

The repayments are made periodically, for instance monthly or quarterly etc., in the periodic geometric-gradient series payment (PGGSP) model. If a period has payments for a time interval with consecutive payment periods, it's called as *in-payment period* and if a period has no payments for a time interval with consecutive payment periods, it's also called as *non-payment period*. In this model, we mainly assume that the length of the in-payment period is equal to the length of the non-payment period with the same repayment numbers. So, it's called as *"rhythmic"* skips model. In previous studies, the length of payment periods was different. Thus, the annuity amount creates a geometric series as being another assumption for PGGSP model. The new notations are used for extended PGGSP model to Eroglu and Karaoz (2002). It is given as follows:

f *:* payment numbers/term numbers for a in-payment period.

h *:* non-payment numbers/term numbers for a non-payment period.

M_k *:* kth non-payment term after the first term number for in-payment period.

L_{k+1} *:* kth non-payment term after the last term number for in-payment period.

d_{kj} *:* kth non-payment term after the payment amount at end of the jth term number for in-payment period.

g *:* Relative gradients in payment amount ($G = 1 + g$)

r *:* Interest rate for period/term ($R = 1 + r$)

s *:* number of non-payment terms

N : all term numbers for all loan repayments (in-payment and non-payment periods)

We know the length of the in-payment period is equal to the length of the non-payment period with the same repayment numbers so the equations are follows:

$$\begin{aligned} M_k &= k\left(f+h\right)+1, \quad k=0,\cdots,s \\ L_{k+1} &= k\left(f+h\right)+f, \quad k=0,\cdots,s \\ N &= L_{s+1} = s\left(f+h\right)+f \end{aligned} \tag{6}$$

Repayment amount can be calculated as

$$\begin{aligned} d_{k\,j} = dG^{j+k-1-Y_k} = dG^{j-1-kh}, \quad k=0,\cdots,s \\ , \quad j = M_k,\cdots,L_{k+1} = k\left(f+h\right)+1,\cdots,k\left(f+h\right)+f \end{aligned} \tag{7}$$

$$\text{Here } Y_k = \begin{cases} \sum_{t=1}^{k}\left(M_t - L_t\right) = k\left(h+1\right) & ,k \ge 1 \\ 0 & ,k < 1 \end{cases} . \tag{8}$$

And the same way, the present value of repayment amounts is equal to the sum of loan so Eq.9a and Eq.9b are derived as follows and detailed in appendix at end of the study:

$$p=\sum_{k=0}^{s}\sum_{j=M_k}^{L_{k+1}} d_{kj}R^{-j}=\begin{cases}\dfrac{d\left(i^f-1\right)\left[\left(i^f R^{-h}\right)^{s+1}-1\right]}{\left(g-r\right)\left(i^f R^{-h}-1\right)}, & g\neq r \quad (9a)\\[2ex] \dfrac{df\left(R^{-h(s+1)}-1\right)}{R\left(R^{-h}-1\right)}, & g=r \quad (9b)\end{cases}$$

And then, the payment amount (*d*) can be calculated by using Eq.9a and Eq.9b:

$$d=\begin{cases}\dfrac{p\left(g-r\right)\left(i^f R^{-h}-1\right)}{\left(i^f-1\right)\left[\left(i^f R^{-h}\right)^{s+1}-1\right]}, & g\neq r \quad (10a)\\[2ex] \dfrac{pR\left(R^{-h}-1\right)}{f\left(R^{-h(s+1)}-1\right)}, & g=r \quad (10b)\end{cases}$$

Here, $i=GR^{-1}$.

Consequently, PGGSP model has been inspired by Moon (1994) model; in fact it is a new rhythmic skips form for the Moon (1994) model.

3.2. The Periodic Fixed Series Payment Model in a Skip Payment Loan with Rhythmic Skips

Now, we have presented periodic fixed series payment (PFSP) model, therefore, relative gradients in payment amount $g=0$ and so, Eq.11 and Eq.12 are derived by using Eq.9a and Eq.9b.

$$p=\frac{d\left(1-R^{-f}\right)\left(1-R^{-(f+h)(s+1)}\right)}{r\left(1-R^{-(f+h)}\right)} \quad (11)$$

$$d=\frac{rp\left(1-R^{-(f+h)}\right)}{\left(1-R^{-f}\right)\left(1-R^{-(f+h)(s+1)}\right)} \quad (12)$$

Consequently, the PFSP model has been inspired by Formato (1992) model; in fact it is also a new rhythmic skips form for the Formato (1992) model.

4. NUMERICAL EXAMPLES

4.1. Numerical Example-1 (for $g\neq r$)

A residential home with a cash value of 100000 $ is planned to be bought with a PGGSP payment model (plan) that is 5 months in-payment and 1 month non-payment during 59 months and, the relative gradient 1.0% for payment amounts for each one. If the monthly interest rate is 0.5%, how much is the monthly payment amount?

$p=100000,\ f=5, h=1, s=9, n=59, g=0.01, r=0.005$ and $d=1817.29$

Table 1. Payment table for numerical example-1

Months	Payment amounts ($)		Remaining loan amounts ($)	
0				100000.00
1		1817.29	(100000.00*1.005) - 1817.29	98682.71
2	1817.29*1.01	1835.46	(98682.71*1.005) – 1835.46	97340.67
3	1835.46*1.01	1853.81	(97340.67*1.005) – 1853.81	95973.56
4	1853.81*1.01	1872.35	(95973.56*1.005) – 1872.35	94581.07
…	…	…	…	…
…	…	…	…	…
53	2787.68*1.01	2815.55	(14218.20*1.005) – 2815.55	14218.20
58	2900.87*1.01	2929.88	(5845.11*1.005) – 2929.88	2944.45
59	2929.88*1.01	2959.18	(2944.45*1.005) – 2959.18	0.00

4.2. Numerical Example-2 for $g = r$

A residential home with a cash value of 100000 $ is planned to be bought with a PGGSP payment model (plan) that is 11 months in-payment and 1 month non-payment during 59 months and, the relative gradient 1.0% for payment amounts for each one. If the monthly interest rate is 1.0%, how much is the monthly payment amount?

$p = 100000,\ f = 11, h = 1, s = 4, n = 59, g = 0.01, r = 0.01$ and $d = 1873.09$

Table 2. Payment table for numerical example-2

Months	Payment amounts ($)		Remaining loan amounts ($)	
0				100000.00
1		1873.09	(100000.00*1.01) - 1873.09	99126.91
2	1873.09*1.01	1891.82	(99126.91*1.01) – 1891.82	98226.36
3	1891.82*1.01	1910.74	(98226.36*1.01) – 1910.74	97297.89
4	1910.74*1.01	1929.85	(97297.89*1.01) – 1929.85	96341.02
5	1929.85*1.01	1949.14	(96341.02*1.01) – 1949.14	95355.28
6	1949.14*1.01	1968.64	(95355.28*1.01) – 1968.64	94340.20
7	1968.64*1.01	1988.32	(94340.20*1.01) – 1988.32	93295.28
8	1988.32*1.01	2008.21	(93295.28*1.01) – 2008.21	92220.03
9	2008.21*1.01	2028.29	(92220.03*1.01) – 2028.29	91113.94
10	2028.29*1.01	2048.57	(91113.94*1.01) – 2048.57	89976.51
11	2048.57*1.01	2069.06	(89976.51*1.01) – 2069.06	88807.22
12	0.00	0.00	(88807.22*1.01) – 0.00	89695.29
13	2069.06*1.01	2089.75	(89695.29*1.01) – 2007.42	88502.50
14	2089.75*1.01	2110.64	(88502.50*1.01) – 2110.64	87276.88
…	…	…	…	…
…	…	…	…	…
46	2816.66*1.01	2844.83	(36309.84*1.01) – 2652.38	33828.11
47	2844.83*1.01	2873.28	(33828.11*1.01) – 2873.28	31293.12
48	0.00	0.00	(31293.12*1.01) – 0.00	31606.05
49	2873.28*1.01	2902.01	(31606.05*1.01) – 2902.01	29020.10
50	2902.01*1.01	2931.03	(29020.10*1.01) – 2931.03	26379.27
51	2931.03*1.01	2960.34	(26379.27*1.01) – 2960.34	23682.72
52	2960.34*1.01	2989.94	(23682.72*1.01) – 2989.94	20929.61
53	2989.94*1.01	3019.84	(20929.61*1.01) – 3019.84	18119.06
54	3019.84*1.01	3050.04	(18119.06*1.01) – 3050.04	15250.21
55	3019.84*1.01	3080.54	(15250.21*1.01) – 3080.54	12322.17
56	3080.54*1.01	3111.35	(12322.17*1.01) – 3111.35	9334.04
57	3111.35*1.01	3142.46	(9334.04*1.01) – 3142.46	6284.92
58	3142.46*1.01	3173.89	(6284.92*1.01) – 3173.89	3173.89
59	3173.89*1.01	3205.62	(3173.89*1.01) – 3205.62	0.00

4.3. Numerical Example-3 (Fixed Payments)

A residential home with a cash value of 100000 $ is planned to be bought with a PFSP payment model (plan) that is 3 months in-payment and 1 month non-payment during 15 months. If the monthly interest rate is 1.0%, how much is the monthly fixed payment amount?

$p = 100000,\ f = 5, h = 1, s = 9, n = 59,\ r = 0.01$ and $d = 2060.57$

Table 3. Payment table for numerical example-3

Months	Payment amounts ($)	Remaining loan amounts ($)	
0			100000.00
1	2060.57	(100000.00*1.01) - 2060.57	98039.43
2	2060.57	(99126,91*1.01) – 2060.57	96076.90
3	2060.57	(98226,36*1.01) – 2060.57	94112.41
4	2060.57	(97297,89*1.01) – 2060.57	92145.95
5	2060.57	(96341,02*1.01) – 2060.57	90177.52
6	0.00	(95355,28*1.01) – 2060.57	90267.70
7	2060.57	(94340,20*1.01) – 2060.57	88297.40
8	2060.57	(93295,28*1.01) – 2060.57	86325.12
9	2060.57	(92220,03*1.01) – 2060.57	84350.88
10	2060.57	(91113,94*1.01) – 2060.57	82374.66
11	2060.57	(89976,51*1.01) – 2060.57	80396.46
12	0.00	(88807,22*1.01) – 0.00	80476.86
13	2060.57	(89695,29*1.01) – 2060.57	78496.77
14	2060.57	(88502,50*1.01) – 2060.57	76514.69
…	…	…	…
…	…	…	…
46	2060.57	(36309.84*1.01) – 2060.57	33828.11
47	2060.57	(33828.11*1.01) – 2060.57	31293.12
48	0.00	(31293.12*1.01) – 0.00	31606.05
49	2060.57	(31606.05*1.01) – 2060.57	29020.10
50	2060.57	(29020.10*1.01) – 2060.57	26379.27
51	2060.57	(26379.27*1.01) – 2060.57	23682.72
52	2060.57	(23682.72*1.01) – 2060.57	20929.61
53	2060.57	(20929.61*1.01) – 2060.57	18119.06
54	0.00	(18119.06*1.01) – 2060.57	15250.21
55	2060.57	(15250.21*1.01) – 2060.57	12322.17
56	2060.57	(12322.17*1.01) – 2060.57	9334.04
57	2060.57	(9334.04*1.01) – 2060.57	6284.92
58	2060.57	(6284.92*1.01) – 2060.57	3173.89
59	2060.57	(3173.89*1.01) – 2060.57	0.00

CONCLUSIONS

In the literature, there are a few studies about problems concerning loan repayment with skips. Firstly, Formato (1992) has presented a new generalized formula for periodic payment in a skip payment loan with arbitrary skips and later Moon (1994) has presented geometric-gradients and Eroglu and Karaoz (2002) has expanded formulae of arithmetic gradients by using Formato (1992) model. All studies have randomly chosen non-payment periods.

In this study, instead of randomly chosen periods/terms, for selecting by using a certain rule, fixed and geometric-gradient series payment loan models have been examined, formulae are derived and the proposed method is supported with numerical examples. Consequently, we believe that the models would be useful in recent economic and financial applications.

REFERENCES

BASKAYA, Z., (1998). "Business Mathematics". 1. Edition, In Turkish, Ekin Publishing, Bursa, Turkey, 83.

EROGLU, A., (2000). "Solution Approaches for Repayment Problems". Suleyman Demirel University Journal of Economics and Administrative Science Faculty, 5, (1), 87-102.

EROGLU, A., (2001), "Solution Approaches for in a Skip Repayment Problems with Piecewise Geometric-Gradient and Arithmetic-Gradient Payment Series". Dumlupinar University Journal of Social Sciences, 5, 297-307.

EROGLU, A., KARAOZ, M., (2002). "Generalized Formula for the Periodic Linear Gradient Series Payment in a Skip Payment Loan with Arbitrary Skips". The Engineering Economist, 47 (1), 75-83.

EROGLU, A., KALAYCI S., OZDEMIR, G., CETIN, A.C., USUL, H., (2012). "Generalized Formulae for Islamic Home Financing through the Musharakah Mutanaqisah Contracts". Afro Eurasian Studies, 1 (1), 126-133.

FORMATO, R.A., (1992). "Generalized Formula for Periodic Payment in a Skip Payment Loan with Arbitrary Skips". The Engineering Economist, 37 (4), 355-359.

GALBIATI, M., SORAMÄKI, K., (2011). "An agent-based model of payment systems". Journal of Economic Dynamics & Control, 35, 859-875.

ISCIL, N., (1997). "Business Arithmetics and Financial Algebra". Armagan Publishing, Ankara, Turkey, pp. 148.

KIM, Y. S., LEE, M., (2010). "A model of debit card as a means of payments". Journal of Economic Dynamics & Control, 34, 1359-1368.

MARTIN, A., ORLANDO, M. J., SKEIE, D., (2008). "Payment networks in a search model of money". Review of Economic Dynamics, 11, 104-132.

MILLS, JR. D. C., (2006). "Alternative central bank credit policies for liquidity provision in a model of payments". Journal of Monetary Economics, 53, 1593-1611.

MOON, I., (1994). "Generalized Formula for the Periodic Geometric-Gradient Series Payment in a Skip Payment Loan with Arbitrary Skips". The Engineering Economist, 39 (2), 177-185.