Anisotropic cosmological solutions to the $Y(R)F^2$ gravity

Özcan Sert[a*] and Muzaffer Adak[b†]

[a]Department of Mathematics, Faculty of Arts and Sciences, Pamukkale University
20017 Denizli, Turkey

[b]Department of Physics, Faculty of Arts and Sciences, Pamukkale University
20017 Denizli, Turkey

August 1, 2019

Abstract

We investigate anisotropic cosmological solutions of the theory with non-minimal couplings between electromagnetic fields and gravity in $Y(R)F^2$ form. After we derive the field equations by the variational principle, we look for spatially flat cosmological solutions with magnetic fields or electric fields. Then we give exact anisotropic solutions by assuming the hyperbolic expansion functions. We observe that the solutions approach to the isotropic case in late-times.

PACS numbers: 04.50.Kd, 98.80.Jk

Keywords: Acceleration of universe, modified gravity, magnetic field

*osert@pau.edu.tr

†madak@pau.edu.tr

1 Introduction

The recent observations on the accelerated expansion of the universe [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] lead to re-questioning of Einstein's General Relativity and searching of new theories of gravitation. Firstly, it was considered that the acceleration may be caused by a constant energy arising from empty space and then Einstein's field equations were modified by adding a cosmological constant [12]. This modification also solved another problem of General Relativity which called dark matter [13, 14] by the model ΛCDM. But the observed value of the cosmological constant in this model is not consistent with the large value of the zero-point energy which calculated by quantum field theory. In addition, there are some observational discordance to overcome [15, 16, 17, 18, 19, 20, 21, 22] in ΛCDM model. Moreover it revealed some conceptual problems, such as what is the cosmological constant fundamentally?

To overcome these challenges and answer this question theoretically, the following two scenarios were mostly considered. The first is that it may be caused by a particular phase of a dark energy field (quintessence) which is a varying unknown energy source or a scalar field fills the universe [23, 24, 25]. Secondly the cosmological constant effects can be obtained by modifying gravity and then gravity behaves very differently than Einstein's theory of gravitation on the extremely large scales. Among the modified theories, mostly $f(R)$ theory was investigated in literature (for reviews see [26, 27, 28]).

On the other hand, gravitating systems such as galaxies, galaxy clusters, stars and planets have also intrinsic magnetic fields [29, 30, 31, 32, 33, 34]. In order to take into account the effects of electromagnetic fields to gravity, we have to consider the Einstein-Maxwell theory for obtaining an exact description. Since the Einstein's theory is modified to explain the dark matter and dark energy, then the Einstein-Maxwell theory can also be modified in the non-minimal form, especially, in the presence of the extreme situations with very strong fields such as the beginning of the universe.

Such a non-minimal modification with RF^2-type was firstly considered by Prasanna [35] to understand the complex nature between curvature and electromagnetic fields. Then the charge conservation was generalized to the such terms [36]. The non-minimal couplings also can appear from dimensional reduction of Gauss-Bonnet gravity [37] and R^2 gravity [38, 39]. It is remarkable that the non-minimal modifications with RF^2 form can arise from the calculation of QED one-loop vacuum polarization on a curved background [40]. Furthermore, the non-minimal RF^2 couplings may generate the primordial magnetic fields by quantum fluctuations at the inflation [41, 42], and the more general R^nF^2 couplings can increase the amplitude to sufficiently large seed fields which lead to the present galactic magnetic fields [43, 44, 45]. In other words, the couplings can lead to quantum fluctuations of electromagnetic fields at the inflationary stage by breaking the conformal invariance [41, 42, 43, 44, 45, 46]. Because of the inflation, the scale of the fluctuations can be stretched towards outside the Hubble horizon. Thus, they can be the reason of the large scale magnetic fields observed in galaxies.

Since the electromagnetic fields lead to anisotropic energy-momentum tensor and pressure without the averaged field assumption, the inflation may be explained by using the anisotropic Bianchi-I space-times in the non-minimal $Y(R)F^2$ theory [47]. The more general modifications have the spherically symmetric static solutions to explain the dark matter effects [48, 49, 50, 51], the regular black hole solutions to avoid the central singularity [52] and the pp-wave solutions [53]. It is interesting to note that the stability of the anisotropic Bianchi-I solutions were investigated for the extended $I(\phi, R, X)F^2$ theory recently in [54]. Furthermore, dark energy models with the non-minimally massive vector field couplings to gravity have also been investigated in [55].

Therefore the non-minimal cosmological models need to more investigation, in particular, to explain the late-time acceleration together with the origin of cosmic magnetic fields and its role in the evolution of the universe. Therefore, in this paper, we look for new anisotropic cosmological solutions with the hyperbolic expansion functions to the $Y(R)F^2$ gravity and determine the corresponding models.

2 The theory of $Y(R)F^2$ gravity

We obtain the field equations by varying an action $I = \int_M L$ where L denotes a Lagrange 4-form, and M a four-dimensional differentiable and orientable manifold endowed with a metric $g = \eta_{ab} e^a \otimes e^b$, $\eta_{ab} = \mathrm{diag}(-+++)$. We set the orientation by the Hodge star $*1 = e^0 \wedge e^1 \wedge e^2 \wedge e^3$. Here e^a figures the orthonormal basis 1-form. The Cartan-Maurer structure equations

$$T^a = de^a + \omega^a{}_b \wedge e^b \,, \tag{1}$$

$$R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{2}$$

define the torsion 2-form, and the curvature 2-form, respectively, where $\omega_{ab} = -\omega_{ba}$ is the metric compatible connection 1-form. Since we will force to vanish the torsion, ω_{ab} will be the Levi-Civita connection 1-form.

We consider the following Lagrangian for the non-minimal $Y(R)F^2$ gravity

$$L = \frac{1}{2\kappa^2} R * 1 - Y(R) F \wedge *F + \lambda_a \wedge T^a \,. \tag{3}$$

Here the form of the non-minimal function $Y(R)$ will be determined by solutions and κ is a gravitational constant, R is the Ricci curvature scalar, $F = dA$ is the electromagnetic field 2-form, λ_a is the Lagrange multiplier 2-form constraining torsion to zero ($T^a = 0$). We adhere the following shorthand notations throughout the paper: $e^a \wedge e^b \wedge \cdots = e^{ab\cdots}$, $\iota_a F = F_a$, $\iota_{ba} F =$

F_{ab}, $\iota_a R^a{}_b = R_b$, $\iota_{ba} R^{ab} = R$ where ι denotes the interior product such that $\iota_b e^a = \delta^a_b$ and $\iota_a \iota_b ... = \iota_{ab...}$. The independent variations of the Lagrangian with respect to e^a, $\omega^a{}_b$ and F yield (up to a closed form)

$$\begin{aligned} \delta L &= \frac{1}{2\kappa^2} \delta e^a \wedge R^{bc} \wedge *e_{abc} + \delta e^a \wedge Y(R)(\iota_a F \wedge *F - F \wedge \iota_a * F) + \delta e^a \wedge D\lambda_a \\ &+ \delta e^a \wedge 2Y_R(\iota_a R^b)\iota_b(F \wedge *F) + \frac{1}{2}\delta\omega_{ab} \wedge (e^b \wedge \lambda^a - e^a \wedge \lambda^b) \\ &+ \delta\omega_{ab} \wedge \Sigma^{ab} - \delta F \wedge 2Y(R) * F + \delta\lambda_a \wedge T^a \end{aligned} \tag{4}$$

where $Y_R = \frac{dY}{dR}$ and Σ^{ab} is the angular momentum 3-form

$$\Sigma^{ab} = D[\iota^{ab}(Y_R F \wedge *F)]. \tag{5}$$

We can solve λ_a from $\delta\omega_{ab}$-equation

$$\lambda^a = 2\iota_b \Sigma^{ab} + \frac{1}{2}(\iota_{bc}\Sigma^{bc}) \wedge e^a. \tag{6}$$

After the substitution of λ_a into δe^a-equation and some simplifications we arrive at the modified Einstein's equation

$$\begin{aligned} \frac{1}{2\kappa^2} R^{bc} \wedge *e_{abc} + Y(\iota_a F \wedge *F - F \wedge \iota_a * F) + 2Y_R(\iota_a R^b)\iota_b(F \wedge *F) \\ + D[\iota^b d(Y_R F_{mn} F^{mn})] \wedge *e_{ab} = 0, \end{aligned} \tag{7}$$

while the modified Maxwell equations read

$$dF = 0\,, \qquad d(Y * F) = 0\,. \tag{8}$$

where $d(Y_R F_{mn} F^{mn}) = D(Y_R F_{mn} F^{mn})$ and it can be shown that $2Y_R(\iota_a R^b)\iota_b(F \wedge *F) = Y_R F_{mn} F^{mn} * R^a$. In order to avoid the difficulties and instabilities of the last term in the gravitational field equation (7), we continue with the condition

$$Y_R F_{mn} F^{mn} = -\frac{1}{\kappa^2} \tag{9}$$

where the constant $-\frac{1}{\kappa^2}$ is determined by the trace of the gravitational field equation (see [47] for a detailed discussion). It is worthwhile to note that the condition (9) is not a new equation. Actually it corresponds to the conservation of energy-momentum tensor. If we take the exterior covariant derivative of the field equation (10) we obtain the condition again [56]. Furthermore, the condition (9) causes the Ricci scalar R to be dynamic and it relates the electromagnetic field with the derivative of the non-minimal function $Y(R)$. Here κ is the gravitational coupling constant and it determines the strength of the coupling between electromagnetic and gravitational fields. Then the gravitational field equation can be written as

$$-\frac{1}{2} R^{bc} \wedge *e_{abc} = \kappa^2 \tau_a\,, \tag{10}$$

where the effective energy momentum tensor $\tau_a = T_{ab} * e^b$ is

$$\tau_a = Y(\iota_a F \wedge *F - F \wedge \iota_a * F) - \frac{1}{\kappa^2} * R^a \ . \tag{11}$$

Thus the effective energy density and pressures are given by $\rho = T_{00}$, $p_x = T_{11}$, $p_y = T_{22}$, $p_z = T_{33}$.

3 Cosmological solutions

We look for cosmological solutions in the presence of electromagnetic fields to the modified fields equations (7) and (8) to describe the evolution of the universe. Since there is a preferential direction along the electromagnetic field, the energy-momentum tensor and the space-time metric become anisotropic unless assuming the averaged electromagnetic fields [57]. Then we consider the anisotropic, locally rotationally symmetric Bianchi-I metric

$$g = -dt^2 + a(t)^2 dx^2 + b(t)^2(dy^2 + dz^2) \tag{12}$$

where $a(t)$ is the expansion function in the x direction and $b(t)$ is the planar expansion function in the y and z directions. In order to obtain compatible solutions with the above geometry (12) with rotational symmetry around the x-axis, we choose the electromagnetic field along the x direction.

$$F = E(t)e^{01} + B(t)e^{23} \tag{13}$$

with the electric component $E(t)$ and magnetic component $B(t)$. Here $e^0 = dt$, $e^1 = a(t)dx$, $e^2 = b(t)dy$, and $e^3 = b(t)dz$ are the orthonormal basis 1-forms. Then we can set $B(t) = 0$ or $E(t) = 0$ to obtain solutions for the sub cases with only electric fields or magnetic fields. While the non-zero magnetic field is effective at later times, the non-zero electric field is more important at the beginning of the universe, in the charged plasma.

Then the modified Maxwell field equations (8) give

$$B = \frac{B_0}{b^2} \ , \qquad\qquad E = \frac{E_0}{Yb^2} \tag{14}$$

where B_0 and E_0 are integration constants. On the other hand, the modified Einstein field equation (10) yields

$$\frac{2\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} = \kappa^2 Y(E^2 + B^2) + 2\frac{\ddot{b}}{b} + \frac{\ddot{a}}{a} = \kappa^2 \rho \ , \tag{15}$$

$$\frac{2\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} = \kappa^2 Y(E^2 + B^2) + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{a}}{a} = -\kappa^2 p_x \ , \tag{16}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = -\kappa^2 Y(E^2 + B^2) + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} = -\kappa^2 p_y \tag{17}$$

where dot means the derivative with respect to cosmic time t and we have used the condition (9) as

$$Y_R(E^2 - B^2) = \frac{1}{2\kappa^2} \, . \tag{18}$$

By subtracting (15) from (16), we arrive

$$\dot{a}\dot{b} - a\ddot{b} = 0 \, . \tag{19}$$

Equation (19) gives the relation between the directional scale functions as

$$a = a_0 \dot{b} \tag{20}$$

where a_0 arbitrary constant. By summing equations (15) and (16), and using (19) we obtain (17) or its the following equivalent form

$$\frac{\ddot{a}}{a} - \frac{\dot{b}^2}{b^2} + \frac{\kappa^2}{b^4}\left(\frac{E_0^2}{Y} + YB_0^2\right) = 0 \, . \tag{21}$$

We noticed that the derivative of equation (21) gives the condition (18). In order to solve the differential equation (21) we can choose the non-minimal function $Y(R)$ which gives the expansion function $b(t)$, or alternatively we can choose the expansion function $b(t)$ which determines the non-minimal function $Y(R)$ in the Lagrangian, then $a(t)$ is determined by (20). In this study, we consider the second approach by taking the hyperbolic expansion.

$$b(t) = \sinh^k(\alpha t) \tag{22}$$

where α and k are positive real numbers. Most of the recent observations indicate the presence of the phase transition from deceleration to acceleration [4, 7]. The hyperbolic scale function (22) leads to a deceleration parameter which can change sign from positive to negative. Furthermore, it behaves $b(t) \propto t^k$ in the beginning of the universe and $b(t) \propto e^{\alpha k t}$ in late times. Here the constant α is in unit Gyr^{-1} and αk can be interpreted as the Hubble parameter in late times.

The scale function in the x direction is obtained via (20) as

$$a(t) = a_0 \alpha k \sinh^{k-1}(\alpha t)\cosh(\alpha t) \, . \tag{23}$$

When we look at the limit $\lim\limits_{t\to\infty} \frac{a(t)}{b(t)} = a_0 \alpha k$, we see that we should set $a_0 = \frac{1}{\alpha k}$, in order to obtain isotropic case in late times. Thus $a(t)$ becomes

$$a(t) = \sinh^{k-1}(\alpha t)\cosh(\alpha t) \, . \tag{24}$$

By using equation (21) we obtain the non-minimal function and the magnetic field as a solution of the model for $E = 0$

$$Y(t) = \frac{(3k-2)\alpha^2 \sinh^{4k-2}(\alpha t)}{\kappa^2 B_0^2} \, , \tag{25}$$

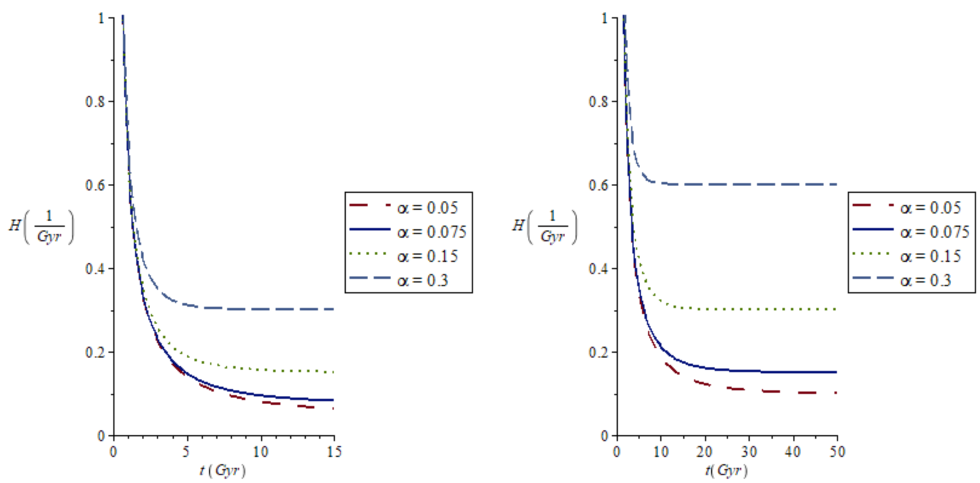
$$B(t) = \frac{B_0}{\sinh^{2k}(\alpha t)} \, . \tag{26}$$

The expansion functions (22) and(24) also give the following solution with the non-zero electric field and $B = 0$ which is another model

$$Y(t) = \frac{\kappa^2 E_0^2}{\alpha^2(3k-2)\sinh^{4k-2}(\alpha t)} \ , \tag{27}$$

$$E(t) = \frac{(3k-2)\alpha^2 \sinh^{2k-2}(\alpha t)}{E_0 \kappa^2} \ . \tag{28}$$

It is worthy to notice that the field equations (7) and (8) or the differential equations (14)-(21) have the duality transformation given by $(F, *F) \to (*YF, -YF)$ or $B \to -YE$, $B_0 \to -E_0$ and $Y \to \frac{1}{Y}$ [51]. Thus each model with magnetic field has a corresponding model with electric field and it can be found by taking the duality transformation.



(a) $k = 1$ (b) $k = 2$

Figure 1: The mean Hubble parameter $H(t)$ versus cosmic time t for various α values and $k = 1$, $k = 2$, respectively.

Then we calculate the Ricci scalar for the expansion functions (22) and (24)

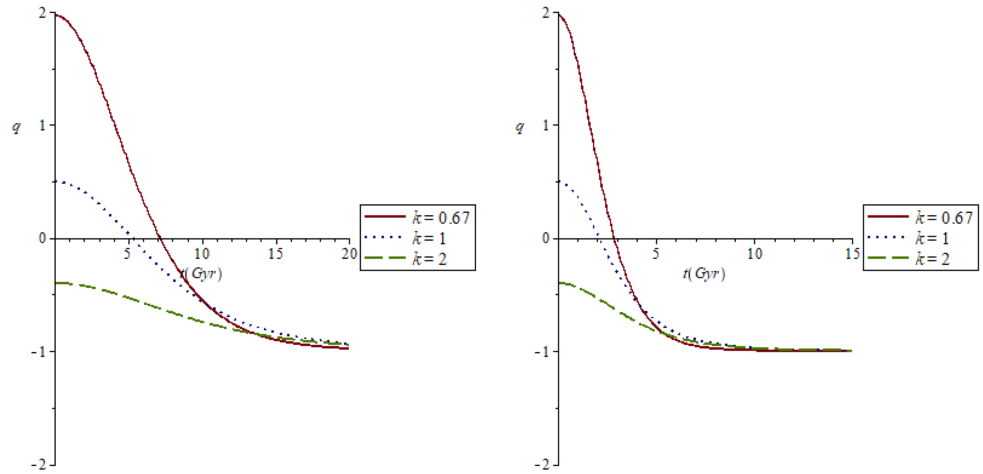
$$R(t) = \frac{2\alpha^2(6k^2 - 7k + 2)}{\sinh^2(\alpha t)} + 12\alpha^2 k^2 \ . \tag{29}$$

We see that the Ricci scalar is infinity at $t = 0$ and it approaches the constant value $R = 12\alpha^2 k^2$, as $t \to \infty$. When we take the inverse function of the Ricci scalar and substitute it in (25) and (27), we obtain the non-minimal function Y in terms of R. Then we can write our model

$$L = \frac{1}{2\kappa^2} R * 1 - \frac{(3k-2)\alpha^2}{\kappa^2 B_0^2} \left(\frac{2\alpha^2(6k^2 - 7k + 2)}{R - 12\alpha^2 k^2} \right)^{2k-1} F \wedge *F + \lambda_a \wedge T^a, \tag{30}$$

which admits the expansion functions (22), (24) and the magnetic field (26) as a solution to the field equations. Also, after the duality transformation [51] we obtain the corresponding model

$$L = \frac{1}{2\kappa^2} R * 1 - \frac{\kappa^2 E_0^2}{(3k-2)\alpha^2} \left(\frac{R - 12\alpha^2 k^2}{2\alpha^2(6k^2 - 7k + 2)} \right)^{2k-1} F \wedge *F + \lambda_a \wedge T^a, \tag{31}$$



(a) $\alpha = 0.08$ (b) $\alpha = 0.2$

Figure 2: The deceleration parameter q versus cosmic time t for two α values and the corresponding various k values.

admitting the same expansion functions (22), (24) and the electric field (28) as a solution. We note that the both models lead to the same scale factors and the cosmological parameters. Then we calculate the following parameters; the mean scale factor

$$v = (ab^2)^{\frac{1}{3}} = (\cosh(\alpha t)\sinh^{3k-1}(\alpha t))^{\frac{1}{3}} \ , \tag{32}$$

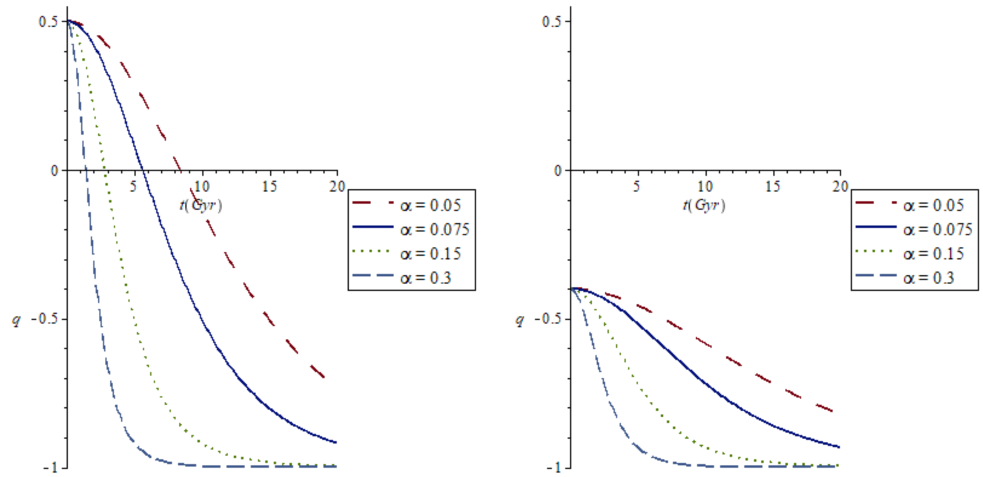
the mean Hubble parameter

$$H = \frac{\dot{v}}{v} = \frac{2\alpha(3k\cosh^2(\alpha t) - 1)}{3\sinh(2\alpha t)} \ , \tag{33}$$

and the mean deceleration parameter

$$q = -1 + \frac{d}{dt}(\frac{1}{H}) = \frac{3(3k\cosh^2(\alpha t) - 2\cosh^2(\alpha t) + 1)}{(3k\cosh^2(\alpha t) - 1)^2} - 1 \ . \tag{34}$$

We demonstrate the behaviors of the mean Hubble parameter in Figure 1 and the mean deceleration parameter q in Figure 2 and Figure 3 for various parameter values. As we see from Figure 1, the mean Hubble parameter (33) goes to infinity as $t \to 0$. Furthermore, it is a decreasing function of cosmic time and it approaches the constant value $H = k\alpha$ in late times. Additionally, Figure 2 shows that as α values increase, q approaches -1 faster. We see from (34) and the figures that the deceleration parameter q is a monotonically decreasing function of cosmic time. It starts from $q(0) = \frac{4-3k}{3k-1}$, at $t = 0$, and decreases to $q = -1$ as $t \to \infty$. Then the phase transition from deceleration to acceleration occurs only when the initial deceleration parameter positive, $q(0) = \frac{4-3k}{3k-1} > 0$. Therefore k must be in the interval $\frac{1}{3} < k < \frac{4}{3}$ for the



(a) $k=1$ (b) $k=2$

Figure 3: The deceleration parameter $q(t)$ for two k values and the corresponding various α values.

phase transition. In other cases the universe expands always in the accelerated phase without phase transition.

The directional components of the parameters are obtained as

$$H_x = \frac{\dot{a}}{a} = \frac{2\alpha(k\cosh^2(\alpha t)-1)}{\sinh(2\alpha t)} \,, \qquad H_{y,z} = \frac{\dot{b}}{b} = k\alpha\coth(\alpha t)\,, \tag{35}$$

$$q_x = -1 + \frac{d}{dt}\left(\frac{1}{H_x}\right) = \frac{(k-2)\cosh^2(\alpha t)+1}{(k\cosh^2(\alpha t)-1)^2} - 1\,, \quad q_{y,z} = -1 + \frac{d}{dt}\left(\frac{1}{H_y}\right) = \frac{1-k\cosh^2(\alpha t)}{k\cosh^2(\alpha t)}\,. \tag{36}$$

The anisotropy parameter Δ and the shear scalar σ^2 are obtained by

$$\Delta = \frac{1}{3}\sum_{i=x,y,z}\left(\frac{H_i - H}{H}\right)^2 = \frac{2}{(3k\cosh^2(\alpha t)-1)^2}, \tag{37}$$

$$\sigma^2 = \frac{1}{2}\sum_{i=x,y,z}(H_i - H)^2 = \frac{4\alpha^2}{3\sinh^2(2\alpha t)}\,. \tag{38}$$

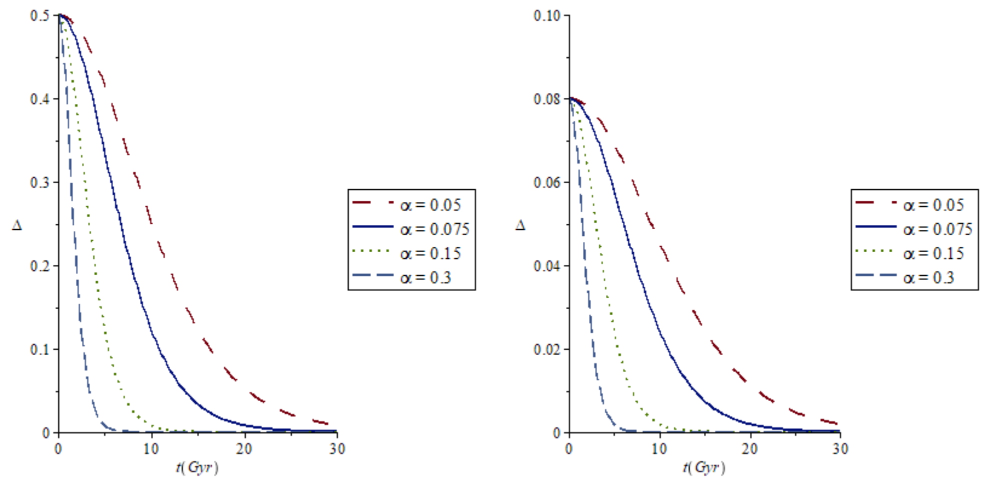
Then we calculate the effective energy density and pressures

$$\rho = \frac{k\alpha^2(3k\cosh^2(\alpha t)-2)}{\kappa^2\sinh^2(\alpha t)}\,, \qquad p_x = -\rho\,, \qquad p_y = p_z = \frac{\alpha^2(5k-2-3k^2\cosh^2(\alpha t))}{\kappa^2\sinh^2(\alpha t)} \tag{39}$$

which lead to the equation of state

$$w_x = \frac{p_x}{\rho} = -1\,, \qquad w_y = w_z = \frac{p_y}{\rho} = -\frac{3k^2\cosh^2(\alpha t)-5k+2}{k(3k\cosh^2(\alpha t)-2)}\,. \tag{40}$$

We note that the positive effective energy density condition of the model requires that $k > \frac{2}{3}$ from (39). We see that the model has the Big Bang singularity at the beginning of the universe



(a) $k = 1$ (b) $k = 2$

Figure 4: The anisotropy parameter $\Delta(t)$ for various α values and $k = 1$, $k = 2$, respectively.

for $k > 1$, since $\lim_{t\to 0} H_{x,y,z} = \infty$, $\lim_{t\to 0} \rho = \infty$ and $\lim_{t\to 0} a, b = 0$. Furthermore, we also see $\lim_{t\to\infty} \Delta = 0$ from (37) and $\lim_{t\to\infty} \frac{a(t)}{b(t)} = 1$ from (22) and (24) which means that the universe approaches isotropy and homogeneity at late-times for all positive k values, see Figure 4.

3.1 The Model with $k = 1$

In order to demonstrate features of the models, we focus on the simple case with $k = 1$ in which the expansion function (22) takes the form

$$b(t) = \sinh(\alpha t) \tag{41}$$

then, the equation (24) leads to

$$a(t) = \cosh(\alpha t)\ . \tag{42}$$

By using equation (21) we obtain the non-minimal function and the magnetic field as a solution with $E = 0$

$$Y(t) = \frac{\alpha^2 \sinh^2(\alpha t)}{\kappa^2 B_0^2}\ , \tag{43}$$

$$B(t) = \frac{B0}{\sinh^2(\alpha t)}\ . \tag{44}$$

After the duality transformation given by $(F, *F) \to (*YF, -YF)$ or $B \to -YE$, $B_0 \to -E_0$ and $Y \to \frac{1}{Y}$ [51], we can obtain the corresponding solution with non-zero electric field and $B = 0$

as

$$Y(t) = \frac{\kappa^2 E_0^2}{\alpha^2 \sinh^2(\alpha t)} \ , \tag{45}$$

$$E(t) = -\frac{\alpha^2}{E_0 \kappa^2} \ . \tag{46}$$

for the same expansion functions (41), (42). By using the expansion functions, we calculate the Ricci scalar as

$$R(t) = \frac{2\alpha^2}{\sinh^2(\alpha t)} + 12\alpha^2 \ . \tag{47}$$

When we take the inverse function of the Ricci scalar and substitute it in (43) and (45), we obtain the non-minimal function Y in terms of R. Then our model for the non-zero magnetic field becomes

$$L = \frac{1}{2\kappa^2} R * 1 - \frac{2\alpha^4}{\kappa^2 B_0^2 (R - 12\alpha^2)} F \wedge *F + \lambda_a \wedge T^a, \tag{48}$$

which gives the solution (41), (42) and (44). After the duality transformation the corresponding model for the non-zero electric field

$$L = \frac{1}{2\kappa^2} R * 1 - \frac{\kappa^2 E_0^2 (R - 12\alpha^2)}{2\alpha^4} F \wedge *F + \lambda_a \wedge T^a, \tag{49}$$

gives the solution (41), (42) and (46). Both models lead to the same scale factors and the cosmological parameters. Then we calculate the following mean scale factor

$$v = (ab^2)^{\frac{1}{3}} = (\cosh(\alpha t) \sinh^2(\alpha t))^{\frac{1}{3}} \ , \tag{50}$$

the mean Hubble parameter

$$H = \frac{\dot{v}}{v} = \frac{2\alpha(3\cosh^2(\alpha t) - 1)}{3 \sinh(2\alpha t)} \ , \tag{51}$$

and the mean deceleration parameter

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = \frac{3(\cosh^2(\alpha t) + 1)}{(3\cosh^2(\alpha t) - 1)^2} - 1 \ . \tag{52}$$

We see form (52) and Figure 3a that the phase transition from deceleration to acceleration occurs in the case with $k = 1$, because of $q(0) = \frac{1}{2} > 0$ and $\lim_{t \to \infty} q(t) \longrightarrow -1$. It is clearly seen from the figures that the mean deceleration parameter changes sign after a certain time from the beginning which depends on the parameter α and it approaches the value -1, monotonically.

In order to obtain the mean deceleration parameter in terms of the redshift $z = -1 + \frac{v_0}{v}$, we isolate $\cosh(\alpha t)$ from (50) as

$$\cosh(\alpha t) = \frac{X}{6} + \frac{2}{X} \tag{53}$$

where $X = (\ 108v^3 + 12\sqrt{81v^6 - 12} \ \ )^{1/3}$, $v = \frac{v_0}{1+z}$ and v_0 present value of the scale factor. After substituting (53) in (52), we obtain

$$q(z) = \frac{\frac{X^2}{12} + \frac{12}{X^2} + 5}{(\frac{X^2}{12} + \frac{12}{X^2} + 1)^2} - 1 \ . \tag{54}$$

The directional components of the parameters are obtained as

$$H_x = \frac{\dot{a}}{a} = \alpha \tanh(\alpha t) \ , \qquad H_{y,z} = \frac{\dot{b}}{b} = \alpha \coth(\alpha t) \ , \tag{55}$$

$$q_x = -1 + \frac{d}{dt}(\frac{1}{H_x}) = -\coth^2(\alpha t) \ , \qquad q_{y,z} = -1 + \frac{d}{dt}(\frac{1}{H_y}) = -\tanh^2(\alpha t) \ . \tag{56}$$

The anisotropy parameter Δ and the shear scalar σ^2 are obtained by

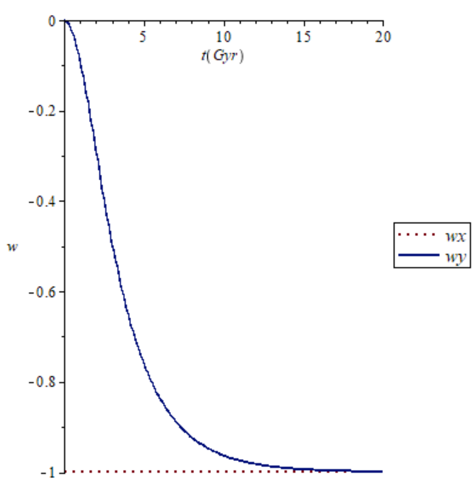
$$\Delta = \frac{2}{(3\cosh^2(\alpha t) - 1)^2}, \qquad \sigma^2 = \frac{4\alpha^2}{3\sinh^2(2\alpha t)} \ . \tag{57}$$

Then we calculate the effective energy density and pressures

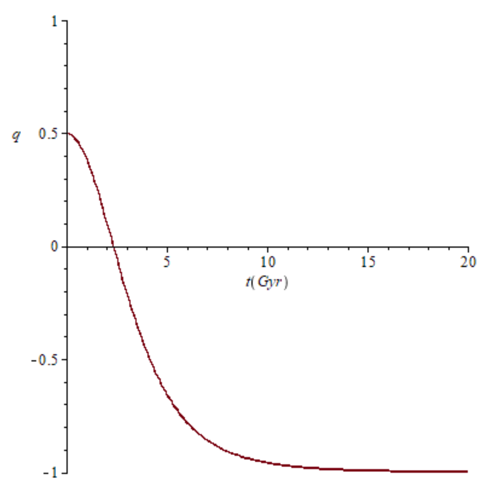
$$\rho = \frac{\alpha^2(3\cosh^2(\alpha t) - 2)}{\kappa^2 \sinh^2(\alpha t)} \ , \qquad p_x = -\rho \ , \qquad p_y = p_z = -3\alpha^2 \tag{58}$$

which leads to the equation of state

$$w_x = \frac{p_x}{\rho} = -1 \ , \qquad w_y = w_z = \frac{p_y}{\rho} = -\frac{3\sinh^2(\alpha t)}{3\cosh^2(\alpha t) - 2} \ . \tag{59}$$



(a) w_x and $w_y = w_z$



(b) q

Figure 5: The directional equation of state parameters w_x, $w_{y,z}$ and deceleration parameter q versus cosmic time t for $\alpha = 0.08$.

We see from equation (57) and Figure 4 that the anisotropy parameter Δ goes to zero, as t approaches to infinity, $\lim\limits_{t\to\infty} \Delta = 0$ and $\lim\limits_{t\to\infty} \frac{a(t)}{b(t)} = 1$ from (22) and (24). It means that the universe becomes isotropic in late-times. We also see that the model has a singularity at $t = 0$.

4 Conclusions

We have investigated anisotropic cosmological solutions of the non-minimally coupled gravity in $Y(R)F^2$ form. After casting our model by a Lagrangian 4-form we have obtained the variational field equations. Then we found solutions with only electric or magnetic fields under the assumption of a spatially flat anisotropic space-time. We note that the anisotropy parameter approaches to zero at late-times and $\lim_{t\to\infty} \frac{a(t)}{b(t)} = 1$. Therefore the universe becomes homogeneous and isotropic at late-times. We also found that the model has a singularity at $t = 0$, since $H \to \infty$, $\rho \to \infty$, $b \to 0$ and $v \to 0$, as $t \to 0$, for all α when $k > \frac{2}{3}$.

Furthermore, as seen from the figures and equation (34), the deceleration parameter q starts from $q(0) = \frac{4-3k}{3k-1}$ and decreases to $q = -1$ as $t \to \infty$, monotonically. Additionally, as the parameter α increases it approaches -1 more faster. Then the phase transition from deceleration to acceleration occurs only for $\frac{1}{3} < k < \frac{4}{3}$. In other cases the universe expands continuously in the accelerated phase without phase transition, this can be seen easily in Figure 3b for some parameter values. The late-time acceleration can be realized by the constant curvature $R = 12\alpha^2k^2$ and the constant Hubble parameter $H = \alpha k$, as $t \to \infty$.

The recent observations indicate that the current value of the deceleration parameter is negative and it can take values in the interval $0 < q < -1$ [15, 22]. Additionally, the possible deviations from isotropy is predicted by the upper bound $\Delta \lesssim 10^{-4}$ [58] for type Ia supernovae through a model independent way.

By taking $t = 13.8$ Gyr in the anisotropy parameter (57), we find $\alpha \gtrsim 0.18$ for $k = 1$. Most of the recent models predict a phase transition from the early decelerated phase to the late time accelerated phase. In this case with $k = 1$, the phase transition is realized at $t \lesssim 2.3$ Gyr, see Figure 5b. Since after ~ 20 Gyr the anisotropy parameter becomes very small, it can be accepted that the universe almost reaches the isotropic phases about at that time. The deceleration parameter also reaches the value -1, which corresponds to de Sitter phase at the same time. For the case with $k > 1$, by using the same upper limit for the anisotropy parameter and (37), we can obtain smaller lower bound for α from $\alpha \simeq 0.18$. In all of these cases the deceleration parameter takes values in the interval $-1 < q < 0$ for late times. Thus, we have given some limits on the free parameters in order that our model would exhibit a behavior consistent with the current understanding of the universe.

Acknowledgement

This study was supported by the Scientific Research Coordination Unit (BAP) of Pamukkale University.

References

[1] D. N. Spergel *et al.*, First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, Astrophys. J. Suppl. 148 175 (2003)

[2] H. V. Peiris *et al* First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Inflation, Astrophys. J. Suppl. 148 213 (2003)

[3] D. N. Spergel *et al.*, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, Astrophys. J. Suppl. 170 377 (2007)

[4] S. Perlmutter et al., Measurements of Omega and Lambda from 42 High-Redshift Supernovae, Astrophys.J.517:565-586, 1999, arXiv:astro-ph/9812133

[5] R. A. Knop et al., New Constraints on Ω_M, Ω_Λ, and w from an Independent Set of Eleven High-Redshift Supernovae Observed with HST, Astrophys.J.598:102, 2003, arXiv:astro-ph/0309368.

[6] R. Amanullah et al., Spectra and Light Curves of Six Type Ia Supernovae at $0.511 < z < 1.12$ and the Union2 Compilation, Astrophys.J. 716:712-738, 2010, arXiv:1004.1711 [astro-ph.CO].

[7] A. G. Riess *et al.*, Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J. 116 1009 (1998)

[8] P. Astier *et al.*, The Supernova Legacy Survey: Measurement of Ω_M, Ω_Λ and w from the First Year Data Set, Astron. Astrophys. 447 31 (2006)

[9] A. G. Riess *et al.*, New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z > 1$: Narrowing Constraints on the Early Behavior of Dark Energy, Astrophys.J. 659 98 (2007)

[10] D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess, and E. Rozo, Observational Probes of Cosmic Acceleration, Physics Reports 530, 87 , 2013, arXiv:1201.2434 [astro-ph.CO].

[11] D. J. Schwarz, C. J. Copi, D. Huterer and G. D. Starkman, CMB Anomalies after Planck, Class. Quant. Grav. 33 184001, 2016, arXiv:1510.07929 [astro-ph.CO].

[12] S. Weinberg, The cosmological constant problem, Reviews of Modern Physics 61, 1 (1989).

[13] J. M. Overduin and P. S. Wesson, Dark Matter and Background Light, Physics Reports 402, 267 , 2004, arXiv:astro-ph/0407207.

[14] H. Baer, K.-Y. Choi, J. E. Kim, and L. Roszkowski, Dark matter production in the early Universe: beyond the thermal WIMP paradigm, Physics Reports 555, 1 , 2015, arXiv:1407.0017

[15] P. A. R. Ade et al., Planck 2015 results XIII. Cosmological parameters, Astrnomy Astrphysics, 594 ,A13 (2016).

[16] H. Hildebrandt, et al., Monthly Notices of the Royal Astronomical Society 465, 1454 (2016).

[17] J. T. de Jong, et al., Astronomy Astrophysics 582, A62 (2015).

[18] K. Kuijken, et al., Monthly Notices of the Royal Astronomical Society 454, 3500 (2015).

[19] I. Fenech Conti, R. Herbonnet, H. Hoekstra, J. Merten, L. Miller, and M. Viola, Monthly Notices of the Royal Astronomical Society 467, 1627 (2017).

[20] A. G. Riess, L. Macri, S. Casertano, H. Lampeitl, H. C. Ferguson, A. V. Filippenko, S. W. Jha, W. Li, and R. Chornock, The Astrophysical Journal 730, 119 (2011).

[21] A. G. Riess, L. M. Macri, et al., The Astrophysical Journal 826, 56 (2016).

[22] P. A. R. Ade et al., Planck 2013 results. XVI. Cosmological parameters, Astrnomy Astrphysics, 571, A16 (2014).

[23] P. J. E. Peebles and B. Ratra, The cosmological constant and dark energy, Reviews of Modern Physics 75, 559 (2003).

[24] T. Padmanabhan, Cosmological Constant-the Weight of the Vacuum, Physics Reports 380, 235 (2003).

[25] S. Tsujikawa, Quintessence: a review, Classical and Quantum Gravity 30, 214003 (2013)

[26] S. Nojiri and S. D. Odintsov, Unified cosmic history in modified gravity: from F(R) theory to Lorentz non-invariant models, to appear in Physics Reports, arXiv:1011.0544 (2011).

[27] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82, 451 (2010)

[28] S. Capozziello and V. Faraoni, Beyond Einstein Gravity, Springer, 2010

[29] K-T. Kim, P. P. Kronberg, P. E. Dewdney, T.L. Landecker, The halo and magnetic field of the Coma cluster of galaxies, Astrophys. J. 355 29 (1990)

[30] K-T. Kim, P. C. Tribble, P. P. Kronberg, Detection of excess rotation measure due to intracluster magnetic fields in clusters of galaxies, Astrophys. J. 379 80 (1991)

[31] T. E. Clarke, P. P. Kronberg, H. Boehringer, A New Radio - X-Ray Probe of Galaxy Cluster Magnetic Fields, Astrophys. J. 547 L111 (2001)

[32] M. Giovannini, The Magnetized Universe, Int. J. Mod. Phys. D 13 391 (2004)

[33] L.M. Widrow, Origin of galactic and extragalactic magnetic fields, Rev. Mod. Phys. 74 775 (2002)

[34] D. Grasso and H.R. Rubinstein, Magnetic Fields in the Early Universe, Phys. Rept. 348 163 (2001)

[35] A. R. Prasanna, A new invariant for electromagnetic fields in curved space-time, Phys. Lett. 37A, 331 (1971).

[36] G. W. Horndeski, Conservation of charge and the EinsteinMaxwell field equations, J. Math. Phys. 17, 1980 (1976).

[37] F. Mueller-Hoissen, Modification of Einstein Yang-Mills theory from dimensional reduction of the Gauss-Bonnet action, Class. Quant. Grav. 5 (1988) L35

[38] H.A. Buchdahl, On a Lagrangian for non-minimally coupled gravitational and electromagnetic fields, J. Phys. A 12, 1037 (1979)

[39] T. Dereli, G. Üçoluk, Kaluza-Klein reduction of generalised theories of gravity and non-minimal gauge couplings, Class. Q. Grav. **7**, 1109 (1990).

[40] I.T. Drummond, S.J. Hathrell, QED vacuum polarization in a background gravitational field and its effect on the velocity of photons, Phys. Rev. D 22, 343 (1980)

[41] M.S. Turner and L.M. Widrow, Inflation produced, large scale magnetic fields, Phys. Rev. D 37 (1988) 2743

[42] G. Lambiase, A. R. Prasanna, Gauge invariant wave equations in curved space-times and primordial magnetic fields, Phys. Rev. D 70 063502 (2004)

[43] F.D. Mazzitelli and F.M. Spedalieri, Scalar electrodynamics and primordial magnetic fields, Phys. Rev. D 52 (1995) 6694 [astro-ph/9505140]

[44] L. Campanelli, P. Cea, G.L. Fogli and L. Tedesco, Inflation-produced magnetic fields in RnF 2 and IF 2 models, Phys. Rev. D 77 (2008) 123002 [arXiv:0802.2630]

[45] G. Lambiase, S. Mohanty and G. Scarpetta, Magnetic field amplification in f(R) theories of gravity, JCAP 07 (2008) 019

[46] K. Bamba, S. Nojiri and S.D. Odintsov, Inflationary cosmology and the late-time accelerated expansion of the universe in non-minimal Yang-Mills-F (R) gravity and non-minimal vector-F(R) gravity, Phys. Rev. D 77 (2008) 123532 [arXiv:0803.3384]

[47] M. Adak, Ö. Akarsu, T. Dereli, Ö. Sert, Anisotropic inflation with a non-minimally coupled electromagnetic field to gravity, JCAP 11 026 , 2017, arXiv:1611.03393 [gr-qc].

[48] T. Dereli, Ö. Sert, Non-minimal $ln(R)F^2$ Couplings of Electromagnetic Fields to Gravity: Static, Spherically Symmetric Solutions, Eur.Phys.J.C71:1589, 2011, arXiv:1102.3863 [gr-qc].

[49] T. Dereli, Ö. Sert, Non-minimal $R^\beta F^2$ -Coupled Electromagnetic Fields to Gravity and Static, Spherically Symmetric Solutions Mod. Phys. Lett. A 26 1487 (2011)

[50] Ö. Sert, Gravity and Electromagnetism with Y(R)F2-type Coupling and Magnetic Monopole Solutions, Eur. Phys. J. Plus 127: 152 , 2012, arXiv:1203.0898 [gr-qc].

[51] Ö. Sert, Electromagnetic Duality and New Solutions of the Non-minimally Coupled Y(R)-Maxwell Gravity, Mod. Phys. Lett. A, 2013, arXiv:1303.2436 [gr-qc].

[52] Ö. Sert, Regular black hole solutions of the non-minimally coupled Y(R) F2 gravity, J. Math. Phys. 57, 032501 (2016), arXiv:1512.01172 [gr-qc]

[53] T. Dereli, Ö. Sert, Nonminimally coupled gravitational and electromagnetic fields: pp-wave solutions, Phys. Rev. D 83 065005 (2011)

[54] T.Q. Do, W.F. Kao, Anisotropic power-law inflation for a conformal-violating Maxwell model, Eur. Phys. J. C (2018) 78: 360. https://doi.org/10.1140/epjc/s10052-018-5846-5

[55] C. Böhmer, T. Harko, Dark energy as a massive vector field, Eur. Phys. J. C (2007) 50: 423. https://doi.org/10.1140/epjc/s10052-007-0210-1

[56] Ö. Sert, Radiation fluid stars in the non-minimally coupled $Y(R)F^2$ gravity, Eur. Phys. J. C 77, 97 (2017).

[57] R. Tolman and P. Ehrenfest, Temperature Equilibrium in a Static Gravitational Field, Phys. Rev. 36, 1791 (1930).

[58] L. Campanelli, P. Cea, G. L. Fogli and A. Marrone, Phys. Rev. D 83 (2011) 103503, arXiv:1012.5596 [astro-ph.CO].