ESSAYS IN APPLIED ECONOMICS

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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

ESSAYS IN APPLIED ECONOMICS

presented by Rebecca Whitworth,

a candidate for the degree of doctor of philosophy,

and hereby certify that, in their opinion, it is worthy of acceptance.

Professor Joseph Haslag

Professor Christopher Otrok

Professor Chao Gu

Professor Dan French

…………………………To my daughter, may your determination carry you far.

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LIST OF ABBREVIATIONS

AIC	Akaike's Information Criterion
BIC	Schwarz Bayesian Information Criterion
BLUE	Best Linear Unbiased Estimator
BSM	Black Scholes Merton
CDF	Cumulative Distribution Function
CDS	Credit Default Swap
CES	Constant Elasticity of Substitution
CHIPs	Clearing House Interbank Payment Systems
IRT	Item Response Theory
ISDA	International Swaps and Derivatives Master Agreement
OLS	Ordinary Least Squares
OPM	Options Pricing Model
OTC	Over the Counter
TME	Test Measurement Error
USD	US Dollars
VAM	Value-Added Model

ABSTRACT

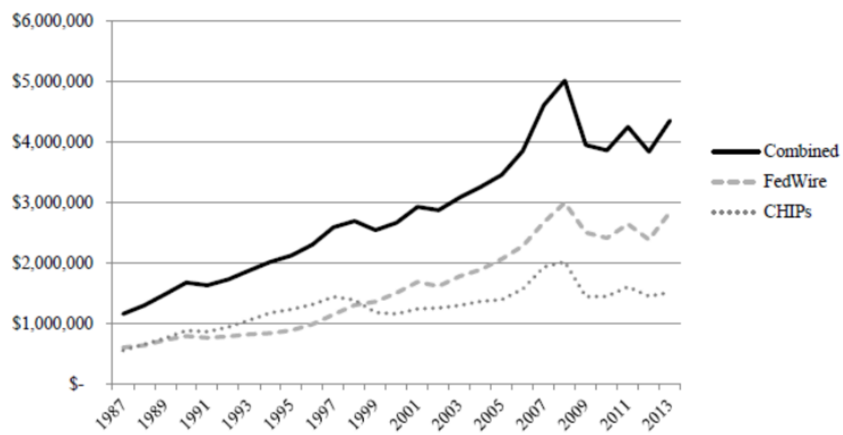
This dissertation examines several themes in applied economics. Specifically, Essay 1 examines the dynamics in an overlapping generations model with three-period lived agents, fiat money, and credit, Essay 2 reviews literature on value-added modeling and discusses a paper previously published, Essay 3 concludes by examining efficiency in the US bond market. While Essay 1 examines dynamics and 2 reviews tools used in estimating panel data, Essay 3 combines elements of both – empirically evaluating the efficiency of the bond market by looking at the movement of prices through time. That is, deriving the integral over *t* of the bond spread. While opportunities for more work exists, this paper suggests that the US Bond Market (the market for corporate debt) is informationally efficient, though it takes longer to converge than previously reported in the literature.

Chapter 1: Dynamics in Overlapping Generations Models with Settlement Risk

Introduction

Payment systems move the equivalent of US GDP in about one week. The average daily trading volume for FedWire and CHIPs combined is about $1.6 trillion USD as of January 2019. Figure 1 graphs the average trading volume of both services.

Figure 1. Average Daily Trading Volume for FedWire and CHIPs



Payments sent via FedWire and CHIPs are not without risk. Settlement risk is a subset of counter-party risk where one party fulfills its trade obligations but the other does not – generally because of timing differences, volatility in the payment systems, or a forced liquidation of the counterparty before the obligations come due. Differing from default risk – where one party is unable to fulfill its contractual obligation – settlement risk occurs when both parties could fulfill their obligations, but one is unable to transfer payment.

There is some evidence to suggest that settlement risk impacts the economy, notable examples include Herstatt Bank[1] or gridlock in payment systems. Jarrow and Yu (2001) examine the impact of general counter-party risk on the price of defaultable securities. They show that counter-party risk can have any number of impacts on the bond and derivative pricing schemes. Further, ignoring counter-party risk produces mis-pricings of derivatives and securities. Heider, Hoerova, and Holthausen (2009) show that the breakdown of the interbank market can change the equilibrium regime governing the economy under asymmetrical information. Kahn, McAndrew, and Roberds (2003) show that the type of settlement used in the payment system (gross verse net) can affect settlement risk – with net settlement being able to avoid gridlock events and real-time gross-settlement being susceptible.

This paper examines the impact of settlement risk on the dynamics within a model economy, specifically addressing the question "What happens if settlement risk exogenously increases?"[2] This situation is used to proxy an increase in settlement risk seen during recessions and financial crises. Using a three period lived agent in an overlapping generations model, I find that increasing settlement risk changes the equilibrium law of motion, increasing volatility along the hyper-inflationary path.

Dynamics are derived for the three-period lived agent model described by Gu and Haslag (2014). In this model, both fiat money and private debt are valued as a means of

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[1] The Germany based Herstatt Bank closure in 1974 illustrates settlement risk. Before the bank was forced into liquidation, several international banks released payment to Herstatt in the form of Dutsche Marks to be exchanged for US Dollars, deliverable to New York City. Herstatt was closed by regulators at the end of the day in local time. Because of the time-zone differences, Herstatt did not make payment in USD to New York before liquidation. To address counter-party risks between banks, the Basel Committee on Banking was formed and real-time gross settlement systems were implemented.

[2] There is also another consideration: is all counter-party risk created equal? Counter-party risk consists of default, replacement and settlement. I show in this paper that settlement risk can increase the volatility in the economy. Can default or replacement risk do the same?

payment. Each generation consists of two types of agents; "lenders" are endowed with κ units of capital when young, nothing when mid-age or old while "entrepreneurs" are endowed with 1 unit of labor effort when young and access to both the short- and long-term production technology. Both are named in anticipation of their equilibrium behavior. The entrepreneurs decision lies over whether to sell short- or long- term debt to the lender (and hence engage in short- or long- term production, respectively). The liquidity constraint is endogenized through the choice of production technology, participation frictions in an old-age markets, and the unverifability of inside money in the goods market. The settlement risk is tractable through an exogenous parameter controlling market participation.

Several features of this model correlate with observations from the 2007-2008 financial crisis. First, the liquidity constrain causes a misallocation of resources in production. Second, there is no correlation between base money and output. The distribution of liquidity is critical, but the aggregate level is not. Finally, a reduction is output is possible, even with well-functioning markets.

Debt is redeemed, either through fiat money which is used to purchase consumption goods, or the consumption good itself. The mechanism for settlement is equivalent to Freeman's Island Model (Freeman (1996)), except that debt can be repaid through both money and goods. The dynamics depend on the price effect in the agent's problem.[3] When the income effect dominates, dynamics are sensitive to the initial values of the system. That is, the dynamics are governed by chaos. When the substitution effect

[3] See Azariadis (1981), Benhabib (1992), Benhabib and Day (1982) and Barnett, Geweke, and Shell (1989) for a summary of work on the subject. De La Croix and Michel (2002) for a thorough derivation.

dominates, the monetary steady state is unstable and the economy follows a hyperinflation path or converges to the non-monetary steady state.

Within the constraints of this model, I find that an increase in settlement risk within the payment system alters the dynamics. Increasing settlement risk speeds convergence to autarky along the hyperinflationary path increasing price volatility within the economy.

The rest of the paper is arranged as follows. Section 2 summarizes the model described by Gu and Haslag. Section 3 summarize the equilibrium cases possible in this economy, partitioned through a discontinuity in the agent's problem and derives the feasible cases. Section 4 derives the dynamics for each feasible case. Section 5 addresses regime switching and the global dynamics of this system fixing the price effect in the entrepreneur's problem. Section 6 discusses relationship between settlement risk and the dynamics. Section 7 concludes.

Model

Overview

Environment

The environment consists of an infinite sequence of time periods, indexed $t = 1, 2, 3, \ldots$ There are two types of three-period lived agents, categorized by their young endowments. In each period, a continuum of measure 1 of each type is born. A continuum of measure 1 of the initial old and initial mid-age of each type is alive in the first period. All agents born at date $t = 1$ and later live for three periods. The three period life allows for agents to choose between issuing two lengths of debt contracts, which endogenizes debt and creates an environment with settlement risk. The commodity space consists of a single, perishable consumption good and physical capital.

Agents

In young life, agents make lending and investment decisions. In both mid-age and old, agents will meet in a Freeman Island type exchange in order to settle debt. The risk of meeting (or not) in the old-age settlement market introduces risk into the economy.

Lenders. The initial old and initial mid-age "lenders" own a fixed stock of fiat money of quantity M_0. Lenders born at date-1 and later are endowed with κ units of physical capital while young and nothing when mid-age or old. They are indifferent between mid-age and old consumption, seeking to maximize the sum. Their utility function is given as $v(q_{2t+1} + q_{3t+2})$. The subscript denotes the period of life followed by the date. Nature constrains $(1 - \pi)$ proportion of lenders to participating only in the mid-age debt settlement exchange. Differently, π is the exogenous proportion of lenders able to participate in all exchanges. Lenders are also free to choose to forego exchanges in which they are able to participate. Let $(1 - \alpha)$ be the endogenous proportion of lenders that choose to go to old-age settlement. Then, lender's participate in old-age IOU settlement (called late-leaving lenders) at rate $\Pi_t = \pi(1 - \alpha)$. The proportion of early-leaving lenders, those participating only in mid-life settlement, is given as $1 - \Pi_t$.

Entrepreneurs. The initial old and initial mid-age "entrepreneurs" are endowed with the consumption good. Entrepreneurs born at date-1 and later are endowed with one unit of time when young, to be divided between labor and leisure, and access to the production technology. Generation-t entrepreneurs can invest their young time into labor, which returns units of the consumption good in date-t at a one-for-one rate at cost $g(l_{1t})$. Also while young, entrepreneurs select either short- or long- term production, investing capital acquired from lenders to produce.

For any date-t investment, the short-term technology returns $f(k)$ units of the consumption good at date-*t+1*, while the long-term production technology returns $Af(k)$ units at date-*t+2*. $A > 1$ and $f(\cdot)$ is continuously differentiable, strictly increasing, and strictly concave.

Entrepreneurs are engaged in either short- or long-term production, with Λ_t being the measure choosing the short-term technology (termed short-term entrepreneurs) and $1 - \Lambda_t$ choosing the long-term technology (termed long-term entrepreneurs). Capital has no salvage value after production is complete. Entrepreneurs seek to maximize old-age consumption balancing it with the disutility they derive from working and the cost of securing capital. Their preferences are represented by $-g(l_{1t}) + u(x_{3t+2})$ where l_{1t} is the choice of labor and x_{3t+2} is the choice of consumption.

Both $u(\cdot)$ and $v(\cdot)$ are assumed to be strictly increasing and strictly concave. Assume $u(\cdot)$ is at least thrice differentiable with a negative third derivative, while $g(\cdot)$ is strictly increasing, strictly convex, thrice differentiable and satisfies $\lim_{l \to 0} g'(l) = 0$ and $\lim_{l \to 1} g'(l) = \infty$.

Markets

In the decentralized economy, four markets open sequentially; capital, settlement, secondary and consumption goods. With the incentives present, intragenerational trade occurs in the capital, settlement and secondary markets facilitated by IOUs and goods. Intergenerational trade occurs in the goods market and is facilitated by fiat money.

In the capital market, young entrepreneurs offer debt contracts to lenders, in exchange for their endowment of capital. Without loss of generality, all short-term IOUs are tied to short-term production and will be repaid next period at a gross nominal interest rate of 1. All long-term IOUs are tied to long-term production and will be repaid in two periods with a premium γ.

It is up to the lender to choose their portfolio optimally while they are certain about their own participation status in the old-age settlement market. Debt contracts can be repaid in fiat money or goods, both of which have the same value to lenders. Settlement is costlessly enforced. There is no default risk when accepting IOUs, but IOUs are not verifiable in the consumption goods market so they will not circulate as a means of payment.

After the capital market, lenders learn their participation status and make no further choices while young. Entrepreneurs allocate their unit of time between leisure and labor and invest the acquired capital into the chosen production technology before moving on to mid-age settlement.

In the settlement market, entrepreneurs repay lenders with the monetary proceeds from their young labor and goods from completed production. The participation rate among lenders is governed by their type. All mid-age lenders and Π_t proportion of old-age lenders meet with entrepreneurs to settle IOUs. Short-term entrepreneurs repay all IOUs in mid-age and move on to the secondary market where they invest any remaining proceeds. Long-term entrepreneurs cannot participate in the secondary market until all debt is repaid. When the return to IOUs purchased on the secondary market is "high enough" entrepreneurs will choose to forego the extra production from the long-term technology in favor of trading IOUs in the secondary market. Thus, even with well-functioning markets, output can be reduced.

In the secondary-market, early-leaving lenders (those excluded from old-age settlement) sell their remaining IOUs at price ρ_{t+1}. Late-leaving lenders and short-term entrepreneurs purchase these IOUs, holding them for one period until settling with the initial issuer.

Finally, in the consumption goods market, young entrepreneurs sell goods produced with their labor effort to generation- *t-1* and *t-2*. Old entrepreneurs buy goods with any remaining cash after settling debts on the settlement market. Old lenders buy goods with any remaining cash on hand. Finally, mid-age lenders may buy for consumption or sell for fiat money depending on their optimal choice in response to the price level.

Lenders

Lenders seek to maximize lifetime utility, $v(q_{2t+1}, q_{3t+2})$ subject to their budget constraint. While young, they trade capital in exchange for an IOUs. With non-satiation in utility, capital is inelastically lent to entrepreneurs. Lenders do not know their participation status, and choose their portfolio accordingly.

Denoting the proportion of short-term debt as h_t and long-term debt as h_t^* their portfolio allocation is

$$h_t + h_t^* = p_{kt}\kappa \qquad (1)$$

where p_{kt} is the price of capital at date-t. Variables that are associated with old-age participation are denoted with "*" throughout.

I categorize lenders based on their old-age participation status. Lenders that choose to forego old-age settlement, either because they are excluded or because it is their optimal choice, are called "early-leavers". Lenders that participate in old-age settlement are called "late-leavers". For early-leaving lenders, all long-term debt is sold on the secondary market in mid-age[4] while late-leaving lenders purchase debt.[5]

In mid-age, early-leaving lenders take debt proceeds (either goods or cash) and adjust their asset holdings in the goods market. If the price level is decreasing, early-leaving lenders want to consume as late as possible so they sell any consumption goods received to the previous generation. If the price level is increasing, they want to consume as early as possible and purchase consumption goods with their money holdings. When prices are constant, lenders are indifferent.

The consumption of an early-leaving lender born in date-t is:

$$h_t + \rho_{t+1}(1+\gamma_t)h_t^* = \min\{p_{xt+1},\ p_{xt+2}\}(q_{2t+1}, q_{3t+2}) \tag{2}$$

where ρ_{t+1} is the price of IOUs on the secondary market in period-*t+1*; γ_t is the net interest on long-term debt issued in period-*t* and redeemed in period-*t+2*; and p_x is the price of the consumption good in date-*t*. Debts that are still outstanding after mid-age settlement can be viewed as one-period discount bonds. The face value of these bonds is $(1+\gamma_t)$ and they are sold at a price ρ_{t+1}.

Late-leaving lenders eat when old, choosing to purchase IOUs on the secondary market with their cash/goods on hand. When mid-age, they use money and goods from their short-term debt settlement, h_t, to buy unsettled long-term IOUs on the secondary market with gross rate of return $\frac{1}{\rho_{t+1}}$. Late-leaving lenders consume

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[4] Early leaving lenders will never want to hold debt into old age – it cannot be used as a medium of exchange owing to verifiability concerns of debt in the goods market.
5 As a note, in the most flexible specification, they could, conceivably, forego participation in the secondary market and choose to eat in mid-age. Later, I show that the lower bound to ρ_{t+1} aligns with the late-leaving lender's incentives to participate in the secondary market. That is, they will never choose to forego participation in the secondary market in favor of consumption.

$$\frac{1}{\rho_{t+1}} h_t + (1+\gamma_t) h_t^* = p_{xt+2} q_{3t+2}^* \tag{3}$$

In equilibrium, the debt market must obey a non-arbitrage condition. This is formalized in Remark 1.

Remark 1: *In equilibrium, the interest rate satisfies* $(1+\gamma_t) = \frac{1}{\rho_{t+1}}$

Proof. Suppose $(1+\gamma_t) > \frac{1}{\rho_{t+1}}$, then the interest paid by long-term entrepreneurs is greater than the gross return. The optimal behavior for lenders is to purchase h_t^* and sell h_t with the optimal choice being to accumulate h_t^* to infinity violating the market clearing condition. The converse also holds. Suppose $(1+\gamma_t) < \frac{1}{\rho_{t+1}}$ then the optimal behavior is to accumulate h_t instead of h_t^* with the optimal choice being to accumulating h_t to infinity. The market still will not clear. The only situation where the debt market clears is $(1+\gamma_t) = \frac{1}{\rho_{t+1}}$ ■

To summarize the early-leaving lenders consumption, I can combine equation (2) with Remark 1 giving $h_t + h_t^* = \min\{p_{xt+1}, p_{xt+2}\}(q_{2t+1}, q_{3t+2})$. Substituting the budget constraint from (1) gives $p_{kt}\kappa = \min\{p_{xt+1}, p_{xt+2}\}(q_{2t+1}, q_{3t+2})$. Thus, early-leaving lenders consume;

$$\begin{cases} q_{2t+1} = \frac{p_{kt}\kappa}{p_{xt+1}}, q_{3t+2} = 0 & & p_{xt+1} < p_{xt+2} \\ q_{2t+1} = \left[0, \frac{p_{kt}\kappa}{p_{xt+1}}\right], q_{2t+1} + q_{3t+2} = \frac{p_{kt}\kappa}{p_{xt+1}} & if & p_{xt+1} = p_{xt+2} \\ q_{2t+1} = 0, q_{3t+2} = \frac{p_{kt}\kappa}{p_{xt+2}} & & p_{xt+1} > p_{xt+2} \end{cases} \tag{4}$$

Under Remark 1, late-leaving lenders consume

$$q_{2t+1}^* = 0, q_{3t+2}^* = \frac{1}{\rho_{t+1}} \frac{p_{kt}\kappa}{p_{xt+2}} \tag{5}$$

Consumption between early-leaving and late-leaving lenders differ by γ_t. The early-leaving lender consumes the nominal value of debt and the late-leaving lender gets extra return γ_t.

If a lender is not excluded from settlement when old, they choose optimally between early- and late- leaving behavior by comparing lifetime consumption. The probability a lender chooses to participate in old-age settlement is given as α. With symmetric strategies, the measure of late-leaving lenders in date-t is given by $\Pi_t = \pi(1-\alpha)$. When a lender strictly prefers long-term settlement, $\Pi_t = \pi$; $\Pi_t = 0$ when all lenders choose to forego old-age settlement with probability 1.

Entrepreneurs

An entrepreneur optimizes his life-time consumption by choosing his labor investment, level of capital, and production technology. I call entrepreneurs choosing the short-term production technology "short-term entrepreneurs" and the entrepreneurs choosing the long-term production technology "long-term entrepreneurs". Without loss of generality (shown below), I can tie the length of production to the length of the debt contract issued. That is, entrepreneurs choosing short-term production issue short-term debt; while entrepreneurs choosing long-term production issue long-term debt.[6] While they differ in the timing of income, all entrepreneurs derive a dis-utility from working when young and choose to consume in old-age. The following derives the short- and long- term entrepreneur's optimal choices.

[6] Lenders cannot use this information to optimize their portfolio as their exclusion status isn't learned until after the capital market closes.

Short-term Entrepreneurs

Short-term entrepreneurs maximize $-g(l_{1t}) + u(x_{3t+2})$ accounting for their lifetime budget constraint.

When young, entrepreneurs chooses the value of short-term debt, b_t which is linked to capital purchases through $b_t = p_{kt}k_t$. Young entrepreneurs also choose the amount of labor to invest in production, selling the goods produced to mid-age and old agents for units of fiat money. Generation-t's money holdings at the end of the first period of life is given by $m_t = p_{xt}l_{1t}$. In mid-age, production completes, and short-term entrepreneurs repay debt. After mid-age settlement, their net worth is $p_{xt+1}f(k_t) + m_t - b_t$. They use this wealth to purchase discounted long-term debt on the secondary market. When old, entrepreneurs realize return $\frac{1}{\rho_{t+1}}$ on any debt purchased. Old-age assets are then used to finance consumption. Their lifetime budget constraint is

$$p_{xt+2}x_{3t+2} = \frac{1}{\rho_{t+1}} p_{xt+1}f(k_t) + m_t - b_t)$$

which simplifies to

$$p_{xt+2}x_{3t+2} = \frac{1}{\rho_{t+1}}(p_{xt+1}f(k_t) + p_{xt}l_{1t} - p_{kt}k_t) \qquad (6)$$

after substituting for m_t and b_t.

Solving the maximization problem over k_t and l_{1t} yield first-order conditions:

$$\begin{gathered} f'(k_t) - \frac{p_{kt}}{p_{xt+1}} = 0 \\ \frac{p_{xt}}{\rho_t p_{xt+2}} u'(x_{3t+2}) - g'(l_{1t}) = 0 \end{gathered} \qquad (7)$$

Their consumption, x_{3t+2} is solved by substituting the implied values of l_{1t} and k_t into (6).

Long-term Entrepreneurs

Like short-term entrepreneurs, long-term entrepreneurs derive a dis-utility from work while young and seek to maximize old-age consumption. Their objective function is $-g(l_{1t}^*) + u(x_{3t+2}^*)$. Their lifetime budget constraint is derived as follows.

The nominal value of IOUs is given as $b_t^* = p_{kt}k_t^*$. Labor income is $m_t^* = p_{xt}l_{1t}^*$. Long-term entrepreneurs cannot participate in the secondary market until all debt is repaid. Mid-age, long-term entrepreneurs use their labor income, less debt repayment, to purchase discounted long-term debt with gross rate of return $\frac{1}{\rho_{t+1}}$. Old-age consumption is purchased with proceeds from completed production and the return to IOUs purchased on the secondary market. The long-term entrepreneur's budget constraint is

$$p_{xt+2}x_{3t+2}^* = p_{xt+2}Af(k_t^*) + \frac{1}{\rho_{t+1}}m_t^* - (1+\gamma_t)b_t^*$$

Recall, $(1+\gamma_t)$ is the premium paid on two period bonds. After substituting for m_t^* and b_t^*, the budget constraint simplifies to

$$p_{xt+2}x_{3t+2}^* = p_{xt+2}Af(k_t^*) + \frac{1}{\rho_{t+1}}(p_{xt}l_{1t}^* - p_{kt}k_t^*). \quad (8)$$

Solving the maximization problem yields the first-order conditions:

$$\begin{gathered} Af'(k_t^*) - \frac{p_{kt}}{\rho_{t+1}p_{xt+2}} = 0 \\ \frac{p_{xt}}{\rho_{t+1}p_{xt+2}}u'(x_{3t+2}^*) - g'(l_{1t}^*) = 0 \end{gathered} \quad (9)$$

Both l_{1t}^* and k_t^* are substituted into (8) to solve for x_{3t+2}^*.

Without loss of generality, short-term entrepreneurs issue short-term debt contracts. Long-term entrepreneurs issue long term debt contracts. This captures the trade-off in the decision problem faced by an entrepreneur. Short-term entrepreneurs earn higher profit by trading in discounted IOUs, but have less overall income from production. Long-term entrepreneurs enjoy more income, but cannot participate in the secondary market until all debt is repaid. This is formalized in Lemma 1.

Lemma 1: *Short-term entrepreneurs will only issue short-term debt and long-term entrepreneurs will only issue long-term debt. This formulation of the decision problem is equivalent to letting entrepreneurs optimize over the length of the debt contract in addition to the other variables.*

Proof. Suppose a short-term entrepreneur issues long-term debt. Denote the quantities chosen by the short-term entrepreneur with long-term debt with a prime. Their life-time budget constraint is

$$p_{xt+2}x'_{3t+2} = \frac{1}{\rho_{t+1}}(p_{xt+1}f(k'_t) + p_{xt}l'_{1t}) - (1+\gamma_t)p_{kt}k'_t$$

Under Remark 1, this is equivalent to the budget constraint for a short-term entrepreneur issuing short-term debt. Similarly, long-term entrepreneurs that issue short-term debt face the budget constraint

$$p_{xt+2}x^{*\prime}_{3t+2} = p_{xt+2}Af(k*'_t) + \frac{1}{\rho_{t+1}}(p_{xt}l^{*\prime}_{1t} - p_{kt}k^{*\prime}_t)$$

which is equivalent to the budget constraint for the long-term lender issuing long-term debt. ■

Let the measure of short-term entrepreneurs is denoted as λ. When short-term entrepreneurs have higher lifetime consumption, $\lambda = 1$. Alternatively, $\lambda = 0$ when all entrepreneurs choose long-term production. Otherwise, $\lambda \in [0,1]$. With symmetric strategies $\Lambda_t = \lambda$ by the law of large numbers.

When a positive measure of short- and long- term entrepreneurs exists in equilibrium they must consume the same amount, otherwise an agent would have an incentive to deviate. This implies that short-term and long-term entrepreneurs choose the same labor effort (that is, $l = l^*$) and quantity of capital ($k = k^*$), by curvature in $g(\cdot)$ and $u(\cdot)$.

Lemma 2. *In equilibrium,* $x_{3t+2} = x^*_{3t+2}$, $l_{1t} = l^*_{1t}$, *and* $k_t = k^*_t$.

Proof. Entrepreneurs are free to choose short- or long- term production in the first period of life. With perfect foresight, consumption is equal. Invoking the strict concavity of $u(\cdot)$ and convexity of $g(\cdot)$, the choice of l_{1t} must be the same by comparing (7) to (9). The budget constraint for the short- and long- term entrepreneur (respectively) is:

$$p_{xt+2}x_{3t+2} = \frac{p_{xt+1}f(k_t) + p_{xt}l_{1t} - p_{kt}k_t}{\rho_{t+1}}$$

$$p_{xt+2}x^*_{3t+2} = \frac{\rho_{t+1}p_{xt+2}Af(k^*_t) + p_{xt}l^*_{1t} - p_{kt}k^*_t}{\rho_{t+1}}$$

Equating the nominal value of consumption yields

$$p_{xt+1}f(k_t) + p_{xt}l_{1t} - p_{kt}k_t = \rho_{t+1}p_{xt+2}Af(k_t) + p_{xt}l_{1t} - p_{kt}k^*_t$$

With each type of entrepreneur choosing the same l_{1t},

$$p_{xt+1}f(k_t) - p_{kt}k_t = \rho_{t+1}p_{xt+2}Af(k_t) - p_{kt}k^*_t$$

The first order condition with respect to capital gives

$$\frac{f(k_t)}{f'(k_t)} - k_t = \frac{f(k^*_t)}{f'(k^*_t)} - k^*_t$$

By the concavity and monotonicity of $f(\cdot)$, this only holds at one point, $k_t = k^*_t$. Let the consumption, labor and capital choices of the long-term entrepreneur to denote the entrepreneur's choice. ■

Market Clearing

In the decentralized economy, there are six independent clearing conditions: the market for capital, goods, short- and long- term debt, money, and the secondary market. By Walras Law, I take the settlement (debt) market to be redundant.

The capital market clearing condition is

$$\kappa = \Lambda_t k_t + (1 - \Lambda_t) k_t^*$$

Under Lemma 2, $k_t = k_t^*$

The goods market clearing condition is

$$l_{1t}^* + \Lambda_{t-1} f(\kappa) + (1 - \Lambda_{t-2}) A f(\kappa) = x_{3t}^* + \Pi_{t-2} q_{3t}^* + (1 - \Pi_{t-1}) q_{2t} + (1 - \Pi_{t-2}) q_{3t}$$

The supply of goods comes from three sources: goods produced with generation-t's young labor, short-term production from generation-t-1, and long-term production from generation-t-2. Old entrepreneurs; mid-age, early-leaving lenders from generation-t-1, and old, early- and late- leaving lenders from generation-t-2 demand goods.

The debt market clearing conditions are

$$h_t = \Lambda_t b_t$$
$$h_t^* = (1 - \Lambda_t) b_t^*$$

These combine and simplify to

$$h_t + h_t^* = b_t^*$$

Let M_0 be the total quantity of fiat money held by the initial mid-age and old lenders. The money market clearing condition is

$$M_0 = \Lambda_t m_t + (1 - \Lambda_t) m_t^*$$

With Lemma 2, this simplifies to

$$M_0 = m_t^*$$

In the date-t secondary market, goods and fiat money available for settlement come from the short-term entrepreneur's production started last period, and labor production from both the short- and long- term entrepreneurs. The demand for liquidity comes from the mid-age, early-leaving lender's attempting to convert unsettled debt into fiat money and goods. The secondary market is written as

$$\Lambda_{t-1}f(k_t) + \frac{p_{xt-1}}{p_{xt}} l_{1t-1}^* \geq (1-\Pi_t)\frac{p_{xt+1}}{p_{xt}} \rho_t A f'(k_{t-1}^*)\kappa \tag{10}$$

With sufficient liquidity, IOUs sell at par.[7] Otherwise, IOUs are discounted and ρ is the market clearing price. Thus, (10) holds with equality if and only if $\rho < 1$.

Equilibrium

I define a competitive, rational expectations equilibrium as (1) entrepreneurs and lenders maximize expected lifetime utility, taking prices as given; (2) all markets clear; and (3) the subjective distribution of production types and settling types is equal to the objective distribution of production types and settling types.[8]

The previous equations are combined to get a recursive block for the system. Under Remark 1 and Lemma 1 and 2, the unique equilibrium allocation is given as:

$$\begin{cases} q_{2t+1} = \frac{p_{kt}\kappa}{p_{xt+1}}, q_{3t+2} = 0 & p_{xt+1} < p_{xt+2} \\ q_{2t+1} = \left[0, \frac{p_{kt}\kappa}{p_{xt+1}}\right], q_{2t+1} + q_{3t+2} = \frac{p_{kt}\kappa}{p_{xt+1}} \quad if & p_{xt+1} = p_{xt+2} \\ q_{2t+1} = 0, q_{3t+2} = \frac{p_{kt}\kappa}{p_{xt+2}} & p_{xt+1} > p_{xt+2} \end{cases} \tag{11}$$

$$q_{2t+1}^* = 0, q_{3t+2}^* = \frac{1}{\rho_{t+1}}\frac{p_{kt}\kappa}{p_{xt+2}} \tag{12}$$

$$f'(\kappa) = \frac{p_{kt}}{p_{xt+1}} \tag{13}$$

7 Note that $\frac{p_{xt+1}}{p_{xt}}\rho_t A f'(k_{t-1}^*)\kappa$ is the inflation adjusted capital income from long-term production.

8 Following Lucas (1972), Radnar (1979), and Grossman and Stiglitz (1976). For a discussion of the relationship between rational expectations and the dynamics of a system, see Azariadis and Guesnerie (1986).

$$Af'(\kappa) = \frac{p_{kt}}{\rho_{t+1}p_{xt+2}} \tag{14}$$

$$\frac{p_{xt}}{\rho_{t+1}p_{xt+2}} u'\left(A[f(\kappa) - f'(\kappa)] + \frac{p_{xt}}{\rho_{t+1}p_{xt+2}} l_{1t}^*\right) = g'(l_{1t}^*) \tag{15}$$

$$l_{1t}^* + \Lambda_{t-1}f(\kappa) + (1 - \Lambda_{t-2})Af(\kappa) =$$

$$x_{3t}^* + \Pi_{t-2}q_{3t}^* + (1 - \Pi_{t-1})q_{2t} + (1 - \Pi_{t-2})q_{3t} \tag{16}$$

$$M_0 = p_{xt}l_{1t}^* \tag{17}$$

$$\Lambda_t f(\kappa) + \frac{p_{xt}}{p_{xt+1}} l_{1t}^* \geq (1 - \Pi_t) \frac{p_{xt+2}}{p_{xt+1}} \rho_{t+1}Af'(\kappa)\kappa \tag{18}$$

By the concavity of $u(\cdot)$ and the convexity of $g(\cdot)$, the values of $(l_{1t}^*, k_t^*, x_{3t+2}^*)$ are unique. Under Lemma 2, this also gives the values of (l_{1t}, k_t, x_{3t+2}). Both Λ_t and Π_t are found by comparing the entrepreneur's and the lender's life-time consumption, respectively. The set of prices $(\rho_{t+1}, p_{xt}, p_{kt})$ are found by solving the respective market clearing conditions. The lender's optimal consumption is then determined. Since q_{2t+1} and q_{3t+2} are perfect substitutes, there are multiple equilibrium allocations satisfying the market clearing conditions.

To reduce the dimensionality of the system, use the entrepreneur's first-order condition for capital (14) to substitute for $\frac{p_{kt}}{\rho_{t+1}p_{xt+1}}$ in (11) and (12). These equations then find q^* and x^* in the goods market clearing condition equation (16). The money market clearing condition can be used to find the solution for p_x from equations (15) and (18). These substitutions given the following recursive block.

$$\begin{cases} q_{2t+1} = \frac{p_{kt}\kappa}{p_{xt+1}}, q_{3t+2} = 0 & & p_{xt+1} < p_{xt+2} \\ q_{2t+1} = [0, \rho_{t+1}Af'(\kappa)\kappa], q_{2t+1} + q_{3t+2} = \rho_{t+1}Af'(\kappa)\kappa & if & p_{xt+1} = p_{xt+2} \\ q_{2t+1} = 0, q_{3t+2} = \rho_{t+1}Af'(\kappa)\kappa & & p_{xt+1} > p_{xt+2} \end{cases} \tag{19}$$

$$q_{2t+1}^* = 0, q_{3t+2}^* = Af'(\kappa)\kappa \tag{20}$$

$$f'(\kappa) = \frac{p_{kt}}{p_{xt+1}} \tag{21}$$

$$\frac{l_{1t+2}^*}{\rho_{t+1}} u'\left(A[f(\kappa) - f'(\kappa)] + \frac{1}{\rho_{t+1}} l_{1t+2}^*\right) = g'(l_{1t}^*) l_{1t}^* \tag{22}$$

$$l_{1t}^* + \Lambda_{t-1} f(\kappa) + (1 - \Lambda_{t-2}) A f(\kappa)$$

$$= A[f(\kappa) - f'(\kappa)] + \frac{p_{xt-2}}{\rho_{t-1} p_{xt}} l_{1t-2}^* + \Pi_{t-2} A f'(\kappa)\kappa + (1 - \Pi_{t-1}) q_{2t}$$

$$+ (1 - \Pi_{t-2}) q_{3t} \tag{23}$$

$$\Lambda_t f(\kappa) + l_{1t+1}^* \geq (1 - \Pi_t) \frac{l_{1t+1}^*}{l_{1t+2}^*} \rho_{t+1} A f'(\kappa)\kappa \tag{24}$$

Equations (19)-(24) form the recursive block for this system. The equilibrium dynamics will depend on the specific starting value of the prices. These are considered piecewise. When addressing the individual case, the dimensionality of the system reduces even further. For example, lenders optimize their lifetime consumption by choosing between short- and long- term participation, when they aren't excluded from old-age settlement markets. Thus, the equilibrium space is partitioned by the trajectory of prices, and only one of the equations in (19) will be valid.

Equilibrium Cases

For both lenders and entrepreneurs the price (or rate of return) of IOUs purchased on the secondary market is driving the choice between short- and long- term participation. In order to derive the dynamics, I need to address the discontinuity in the agents' optimization problems.

The entrepreneur's choice is invariant to the behavior of prices but does depend on the return to IOUs. Entrepreneurs are only indifferent between short- and long- term production when the return to IOUs equals the additional output gained from choosing long-term production.

When lenders are not excluded from old-age settlement markets, they optimize over short-and long- term participation, choosing short-term participation when consuming in mid-age gives higher lifetime utility and choosing long-term participation when waiting until old age gives higher lifetime utility. This decision depends on the return to IOUs combined with the level of inflation, disinflation, or deflation in each period. To address this, I divide the equilibrium space based on the behavior of prices and the return to IOUs.

These discontinuities are formalized in in Lemmas 3 and 4.

Entrepreneur's Indifference Equation

Lemma 3. *Entrepreneurs are indifferent between short- and long- term production when*

$$\frac{1}{\rho_{t+1}} = \frac{A p_{xt+2}}{p_{xt+1}}$$

If the return to IOUs is less than the additional return for employing the long-term production technology, all entrepreneurs will choose long-term production. This would drive up the demand for liquidity on the secondary market, causing entrepreneur's to prefer the short-term production technology. Alternatively, when the return to secondary IOUs is greater, entrepreneurs choose short-term production. This drives up the demand for long term production on the goods market and is also not an equilibrium.

Lemma 4. *There is a positive measure of entrepreneur's selecting the long-term production technology. Differently,* $\Lambda_t < 1$.

Proof. When all entrepreneurs choose short-term production, all IOUs are redeemed in mid-age. Thus, no IOUs are supplied in the secondary market. The long-term production technology is return dominant, giving an incentive to deviate. Thus, all entrepreneurs choosing short-term production cannot be an equilibrium. ■

The equilibrium space is divided by the behavior of goods prices and the return to bonds on the secondary market. Equilibriums where IOUs return less than face value are excluded. Each possible case is considered individually. A case is called "feasible" if the market clearing conditions hold. For the feasible cases, I derive the equilibrium law of motion governing the system, what I term $H(\cdot)$.[9]

Increasing Prices

The inverse relationship between prices and labor comes from the money market clearing condition. When prices are increasing each period, the equilibrium quantity of labor is decreasing. This tightens the liquidity constraint on the secondary market. When prices are increasing, lenders choose between $q_{2t+1} = \frac{p_{kt}\kappa}{p_{xt+1}}$ (the lifetime consumption of early-leaving lenders) and $q^*_{3t+2} = \frac{1}{\rho_{t+1}}\frac{p_{kt}\kappa}{p_{xt+2}}$ (the lifetime consumption of late-leaving lenders). Lenders are indifferent when $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$. By assumption, this is greater than 1. The recursive block for inflationary prices is:

$$q_{2t+1} = \frac{p_{kt}\kappa}{p_{xt+1}}, q_{3t+2} = 0 \tag{25}$$

$$f'(\kappa) = \frac{p_{kt}}{p_{xt+1}} \tag{26}$$

$$\frac{l^*_{1t+2}}{\rho_{t+1}} u'\left(A[f(\kappa) - f'(\kappa)] + \frac{1}{\rho_{t+1}} l^*_{1t+2}\right) = g'(l^*_{1t})l^*_{1t} \tag{27}$$

$$\begin{aligned} l^*_{1t} + \Lambda_{t-1}f(\kappa) + (1-\Lambda_{t-2})Af(\kappa) \\ = A[f(\kappa) - f'(\kappa)] + \frac{p_{xt-2}}{\rho_{t-1}p_{xt}} l^*_{1t-2} + \Pi_{t-2}Af'(\kappa)\kappa + (1-\Pi_{t-1})q_{2t} \\ +(1-\Pi_{t-2})q_{3t} \end{aligned} \tag{28}$$

[9] Rather than derive this in the goods price, it is easiest to use the entrepreneur's labor choice.

$$\Lambda_t f(\kappa) + l_{1t+1}^* \geq (1 - \Pi_t) \frac{l_{1t+1}^*}{l_{1t+2}^*} \rho_{t+1} A f'(\kappa) \kappa \quad (29)$$

The inflationary equilibrium case has four subcase. The agent's choices are summarized in Table 1.

Table 1. Summary of Agents Choices with Inflation

Case	Condition on ρ_{t+1}	Preferred Choice of Agents in Equilibrium	Feasibility of Equilibrium
1a	$\frac{1}{\rho_{t+1}} = 1$	Lenders: participate only in mid-life exchange Entrepreneurs: long-term production	No
1b	$\frac{1}{\rho_{t+1}} \in \left(1, \frac{p_{xt+2}}{p_{xt+1}}\right)$	Lenders: participate only in mid-life exchange Entrepreneurs: long-term production	No
2	$\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$	Lenders: indifferent Entrepreneurs: long-term production	Yes
3	$\frac{1}{\rho_{t+1}} \in \left(\frac{p_{xt+2}}{p_{xt+1}}, \frac{Ap_{xt+2}}{p_{xt+1}}\right)$	Lenders: participate in all exchanges if able Entrepreneurs: long-term production	No
4	$\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$	Lenders: participate in all exchanges if able Entrepreneurs: indifferent	Yes

$\frac{Ap_{xt+2}}{p_{xt+1}}$ forms the upper bound for $\frac{1}{\rho_{t+1}}$. If $\frac{1}{\rho_{t+1}}$ were to increase beyond $\frac{Ap_{xt+2}}{p_{xt+1}}$, all entrepreneurs would opt for short-term production. Thus, all IOUs would be settled in mid-age and the secondary market would not operate.

The next four subsections investigates the feasibility of each case.

Case 1 – Early-Leaving Lenders and Long-Term Production

The case with only early-leaving lenders and long-term production is not an equilibrium. To see this, write the agent's choices in terms of l_1^* and substitute into the goods market clearing condition. For this to be sustainable, prices would need to be constant. This violates the initial assumption that prices are increasing.

Case 1(a). When $\frac{1}{\rho_{t+1}} = 1$, lenders choose to leave the settlement markets early ($\Pi_t = 0$) and entrepreneurs select the long-term production technology ($\Lambda_t = 0$). The goods market does not clear. Lenders are demanding repayment in mid-age for IOUs used to finance long-term production. The only source of liquidity for entrepreneurs comes from labor income. Entrepreneurs would need to choose a higher level of labor than the initial assumption of the case. After substituting for $\frac{1}{\rho_{t+1}}$ the secondary market clearing condition becomes

$$l_{1t+1}^* > \frac{p_{xt+2}}{p_{xt+1}} Af'(\kappa)\kappa$$

Combining with the money market clearing condition gives:

$$l_{1t+1}^* > \frac{l_{1t+1}^*}{l_{1t+2}^*} Af'(\kappa)\kappa$$

Simplifying yields

$$l_{1t+2}^* > Af'(\kappa)\kappa$$

which holds in every period.

The goods market clearing condition is

$$l_{1t}^* + Af(\kappa) = x_{3t}^* + q_{2t}$$

after substituting for $\frac{1}{\rho_{t+1}} = 1$. Substituting for *x* and q from the agent's optimal choices yields

$$l_{1t}^* + Af(\kappa) = Af(\kappa) - Af'(\kappa)\kappa + l_{1t}^* + \frac{p_{kt-1}\kappa}{p_{xt}}$$

which reduces to

$$Af'(\kappa)\kappa = \frac{p_{kt-1}\kappa}{p_{xt}}$$

Multiplying by $\frac{p_{xt+1}}{p_{xt+1}}$ gives

$$\frac{p_x}{p_{xt+1}} = \frac{p_{kt-1}\kappa}{p_{xt+1}} \frac{1}{Af'(\kappa)\kappa} \quad (30)$$

The entrepreneur's first-order condition with respect to capital gives $\frac{p_{kt}}{\rho_{t+1}p_{xt+2}} = Af'(\kappa)$. In this case, ρ is assumed to be 1. When combined with the price level from the money market clearing condition, (30) reduces to

$$\frac{l^*_{1t+1}}{l^*_{1t}} = \frac{Af'(\kappa)}{Af'(\kappa)}$$

By the initial assumption that prices are increasing, labor must be decreasing in each period. Thus $l^*_{1t+1} \neq l^*_{1t}$. When labor is decreasing in every period, $\rho \neq 1$.

Case 1(b). With $\frac{1}{\rho_{t+1}} \in \left(1, \frac{p_{xt+2}}{p_{xt+1}}\right)$, all lenders still choose to leave the debt market early. All entrepreneurs choose long-term production, as before. This case is also not sustainable in equilibrium for the same reasons. The secondary market clearing condition is

$$l^*_{1t+1} = \frac{l^*_{1t+1}}{l^*_{1t+2}} \rho_{t+1} Af'(\kappa)\kappa$$

after substituting for prices. Rearranging gives

$$\frac{1}{\rho_{t+1}} = \frac{Af'(\kappa)\kappa}{l^*_{1t+2}} \quad (31)$$

The goods market clearing condition from (28) becomes

$$l^*_{1t} + Af(\kappa) = x^*_{3t} + q_{2t}$$

Substituting for *x* and *q* from the agent's optimal choices, I have:

$$l^*_{1t} + Af(\kappa) = Af(\kappa) - Af'(\kappa)\kappa + Af'(\kappa)\kappa + \frac{p_{kt-1}\kappa}{p_{xt}}$$

which reduces to:

$$l_{1t}^{*} = \frac{p_{kt-1}\kappa}{p_{xt}}$$

To address the price of capital, we can multiply by $\frac{p_{xt+1}}{p_{xt+1}}$ giving

$$l_{1t+1}^{*} = Af'(\kappa)\kappa \tag{32}$$

Substituting (32) into (31) yields

$$\frac{1}{\rho_{t+1}} = \frac{Af'(\kappa)\kappa}{Af'(\kappa)\kappa} = 1$$

This case is only feasible when $\rho = 1$, which is outside the initial bounds of the case.

Case 2. With increasing prices and $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$, lenders are indifferent between early-leaving and participating in all exchanges in all periods of life. All entrepreneurs choose long-term production in equilibrium. Π is solved in the secondary market. This case is an equilibrium when a positive measure of lenders choose to participate in all exchanges. The equilibrium law of motion for labor is derived after confirming that the secondary market clears.

Starting with the secondary market clearing condition given in (29), substitute for $\frac{1}{\rho_{t+1}}$ from above giving

$$l_{1t+1}^{*} = (1 - \Pi_t)Af'(\kappa)\kappa$$

Rearranging

$$(1 - \Pi_t) = \frac{l_{1t+1}^{*}}{Af'(\kappa)\kappa}. \tag{33}$$

The goods market clearing condition is

$$l_{1t}^{*} + Af(\kappa) = x_{3t}^{*} + (1 - \Pi_{t-1})q_{2t} + \Pi_{t-2}q_{3t}^{*}$$

Substituting for $(1 - \Pi)$ from (33) and the agent's choice of *x* and *q* gives

$$l_{1t}^{*} + Af(\kappa) = Af(\kappa) - Af'(\kappa) + \frac{1}{\rho_{t-1}} l_{1t}^{*} + \frac{l_{1t}^{*}}{Af'(\kappa)\kappa} \frac{p_{kt-1}\kappa}{p_{xt}} + \left(1 - \frac{l_{1t-1}^{*}}{Af'(\kappa)\kappa}\right) Af'(\kappa)\kappa$$

Substituting for ρ

$$l_{1t}^{*} + Af(\kappa) = Af(\kappa) - Af'(\kappa) + l_{1t-1}^{*} + \frac{l_{1t}^{*}}{Af'(\kappa)\kappa} \frac{p_{kt-1}\kappa}{p_{xt}} + \left(1 - \frac{l_{1t-1}^{*}}{Af'(\kappa)\kappa}\right) Af'(\kappa)\kappa$$

Simplifying

$$l_{1t}^{*} = \frac{l_{1t}^{*}}{Af'(\kappa)\kappa} \frac{p_{kt-1}\kappa}{p_{xt}}$$

Multiplying by $\frac{p_{xt+1}}{p_{xt+1}}$ and using the entrepreneur's first order condition given in (27) gives

$$\frac{l_{1t+1}^{*}}{l_{1t}^{*}} Af'(\kappa)\kappa = \frac{l_{1t+1}^{*}}{l_{1t}^{*}} Af'(\kappa)\kappa$$

When $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$, the goods market clears

The entrepreneur's first order condition can be used to derive the law of motion for labor. Starting with equation (9)

$$\frac{p_{kt}}{\rho_{t+1} p_{xt+2}} u'(x_{3t+2}^{*}) = g'(l_{1t}^{*})$$

Substituting for x_{3t+2}^{*} from the budget constraint, p_{xt+2} from the partial derivatives of the agent's problem and ρ_{t+1} from (29) gives

$$l_{1t+1}^{*} u'(Af(\kappa) - Af'(\kappa) + l_{1t+1}^{*}) = g'(l_{1t}^{*}) l_{1t}^{*}$$

Which is a first-order, non-linear difference equation in l^{*}.

Case 3. By assumption prices are increasing and $\frac{1}{\rho_{t+1}} \in \left(\frac{p_{xt+2}}{p_{xt+1}}, \frac{Ap_{xt+2}}{p_{xt+1}}\right)$. Lenders choose to participate in all exchanges, if able, and entrepreneurs choose the long-term production technology. Because lenders strictly prefer to participate in all exchanges,

$\Pi_t = \pi(1-\alpha)$. The secondary market clearing condition (given in equation (29)) simplifies to

$$l_{1t+1}^* = (1-\pi(1-\alpha))\frac{l_{1t+1}^*}{l_{1t+2}^*}\rho_{t+1}Af'(\kappa)\kappa$$

Rearranging

$$\frac{1}{\rho_{t+1}} = \frac{(1-\pi(1-\alpha))Af'(\kappa)\kappa}{l_{1t+2}^*}$$

Equation (28) checks the goods market clearing condition. After substituting for the agents choices and prices, the goods market is characterized by

$$l_{1t}^* + Af(\kappa) = Af(\kappa) - Af'(\kappa) + \frac{(1-\pi(1-\alpha))Af'(\kappa)\kappa}{l_{1t}^*}l_{1t}^* + (1-\pi(1-\alpha))\frac{p_{kt-1}\kappa}{p_{xt}}$$
$$+ \pi(1-\alpha)Af'(\kappa)\kappa$$

Simplifying

$$l_{1t}^* = (1-\pi(1-\alpha))\frac{p_{kt-1}\kappa}{p_{xt}}$$

Addressing the ratio of capital to goods price from the entrepreneur's first order condition leaves

$$l_{1t}^* = (1-\pi(1-\alpha))Af'(\kappa)\kappa$$

The right hand side of this expression in constant and holds in each period. Thus $l_{1t}^* = l_{1t+1}^* = \big(1-\pi(1-\alpha)\big)Af'(\kappa)\kappa$. However, labor was assumed to be decreasing in each period. Thus, in this case the goods market clearing condition is not satisfied.

Case 4. The upper bound $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$ implies that lenders want to participate in all exchanges, if able, but entrepreneurs are indifferent between the short- and long- term production technology. As in Case 3, $\Pi_t = \pi(1-\alpha)$. With entrepreneurs being indifferent, Λ_t is solved in the secondary market. This gives

$$\Lambda_t f(\kappa) + l_{1t+1}^* = \big(1 - \pi(1-\alpha)\big) f'(\kappa)\kappa$$

Rearranging

$$\Lambda_t = \frac{\big(1 - \pi(1-\alpha)\big) f'(\kappa)\kappa - l_{1t+1}^*}{f(\kappa)}$$

Checking the goods market clearing condition

$$l_{1t}^* + \Lambda_{t-1} f(\kappa) + (1 - \Lambda_{t-2}) A f(\kappa) = x_{3t}^* + \pi(1-\alpha) q_{3t}^* + (1 - \pi(1-\alpha)) q_{2t}$$

Substituting for *x* and *q* from the agent's choices and simplifying yields

$$-A\big(1 - \pi(1-\alpha)\big) f'(\kappa)\kappa = -A\big(1 - \pi(1-\alpha)\big) f'(\kappa)\kappa$$

The supply does equal the demand for goods in equilibrium when prices are increasing and $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$.

The equilibrium law of motion is derived from the entrepreneur's optimal choice. Starting with (22) and substituting for $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$ gives

$$\frac{Ap_{xt}}{p_{xt+1}} \mathrm{u}'\left(Af(\kappa) - Af'(\kappa)\kappa + \frac{Ap_{xt}}{p_{xt+1}} l_{1t+1}^*\right) = g'(l_{1t}^*)$$

Substituting for p_x from the money market clearing condition yields

$$\mathrm{A} l_{1t+1}^* \mathrm{u}'\left(Af(\kappa) - Af'(\kappa)\kappa + A l_{1t+1}^*\right) = g'(l_{1t}^*) l_{1t}^*$$

Summary of Cases with Inflation

With inflation, the equilibrium quantity of labor is decreasing in each period. Cases 2 and 4 are feasible. That is, $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$ or $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$ in equilibrium.

Constant Prices

Equilibriums where IOUs return less than face value are excluded. Additionally, a positive measure of long-term production must exist in equilibrium. This gives the upper and lower bounds for the price of IOUs in the secondary market $\frac{1}{\rho_{t+1}} \in [1, A]$. With constant prices and labor, there are only three possible partitions for ρ; $\frac{1}{\rho_{t+1}} = 1$, $\frac{1}{\rho_{t+1}} \in (1, A)$, and $\frac{1}{\rho_{t+1}} = A$. Lenders that forego participation in the old-age settlement market consume $q_{2t+1} + q_{3t+2} = \frac{p_{kt}\kappa}{p_{xt+1}} = \frac{p_{kt}\kappa}{p_{xt+2}}$. Lenders that participate in all exchanges consume $q^*_{3t+2} = \frac{1}{\rho_{t+1}}\frac{p_{kt}\kappa}{p_{xt+2}}$. Comparing lifetime consumption of early- and late- leaving lenders, lenders strictly prefer to participate in all exchanges. Entrepreneurs are indifferent if $\frac{1}{\rho_{t+1}} = A$. Table 2 summarizes agent's equilibrium behavior when prices are constant.

Table 2. Summary of Agents Choices with Constant Prices

Case	Condition on ρ_{t+1}	Preferred Choice of Agents in Equilibrium	Feasibility of Equilibrium
5	$\frac{1}{\rho_{t+1}} = 1$	Lenders: indifferent Entrepreneurs: long-term production	Yes
6	$\frac{1}{\rho_{t+1}} \in (1, A)$	Lenders: participate in all exchanges if able Entrepreneurs: long-term production	Yes
7	$\frac{1}{\rho_{t+1}} = A$	Lenders: participate in all exchanges if able Entrepreneurs: indifferent	Yes

The recursive block for this system is given as:

$$q_{2t+1} = [0, \rho_{t+1}Af'(\kappa)\kappa], q_{2t+1} + q_{3t+2} = \rho_{t+1}Af'(\kappa)\kappa \tag{34}$$

$$q^*_{2t+1} = 0, q^*_{3t+2} = Af'(\kappa)\kappa \tag{35}$$

$$f'(\kappa) = \frac{p_{kt}}{p_{xt+1}} \tag{36}$$

$$\frac{l^*_{1t+2}}{\rho_{t+1}} u'\left(A[f(\kappa) - f'(\kappa)] + \frac{1}{\rho_{t+1}} l^*_{1t+2}\right) = g'(l^*_{1t})l^*_{1t} \tag{37}$$

$$l^*_{1t} + \Lambda_{t-1} f(\kappa) + (1 - \Lambda_{t-2}) A f(\kappa)$$

$$= A[f(\kappa) - f'(\kappa)] + \frac{1}{\rho_{t-1}} l^*_{1t-2} + \Pi_{t-2} A f'(\kappa)\kappa + (1 - \Pi_{t-1}) q_{2t}$$

$$+ (1 - \Pi_{t-2}) q_{3t} \tag{38}$$

$$\Lambda_t f(\kappa) + l^*_{1t+1} \geq (1 - \Pi_t) \frac{l^*_{1t+1}}{l^*_{1t+2}} \rho_{t+1} A f'(\kappa)\kappa \tag{39}$$

Case 5. With $\frac{1}{\rho_{t+1}} = 1$, lenders are indifferent between participating in all exchanges or foregoing the old age settlement market. Π is solved in the secondary market. Entrepreneurs strictly prefer the long-term production technology, $\Lambda = 0$. The equation for the secondary market is

$$l^*_{1t+1} \geq (1 - \Pi_t) A f'(\kappa)\kappa$$

To check for feasibility, compare the supply and demand of goods. The goods market clearing condition from (38) simplifies to

$$(1 - \Pi_{t-2}) A f'(\kappa)\kappa = (1 - \Pi_{t-1}) A f'(\kappa)\kappa$$

The goods market clears if the proportion of lenders choosing to participate in both mid- and old- age exchanges remains constant.

Case 6. With $\frac{1}{\rho_{t+1}} \in (1, A)$, lenders strictly prefer long-term participation if able. Thus $\Pi = \pi(1 - \alpha)$. Entrepreneurs choose long-term production, so $\Lambda = 0$. The secondary market solves for ρ giving

$$\frac{1}{\rho_t} = \frac{\big(1 - \pi(1 - \alpha)\big) A f'(\kappa)\kappa}{l^*_{1t+1}}$$

The goods market clearing condition is rearranged to check for feasibility of the steady state.

$$l_{1t}^{*} + Af(\kappa) = Af(\kappa) - Af'(\kappa)\kappa + l_{1t}^{*}\frac{\big(1-\pi(1-\alpha)\big)Af'(\kappa)\kappa}{l_{1t}^{*}}\frac{l_{1t}^{*}}{l_{1t-2}^{*}}$$

$$+\pi(1-\alpha)Af'(\kappa)\kappa + \big(1-\pi(1-\alpha)\big)\frac{l_{1t}^{*}}{\big(1-\pi(1-\alpha)\big)}$$

Confirming the steady state, $l_{1t}^{*} = \ l_{1t-2}^{*}$

Case 7. By assumption, prices are constant and $\frac{1}{\rho_{t+1}} = A$. Lenders strictly prefer to participate in all market exchanges, if able, and entrepreneurs are indifferent between the short- and long- term production technology. Λ is solved in the secondary market. The clearing condition holds with equality.

$$\Lambda_t f(\kappa) + l_{1t+1}^{*} = \big(1-\pi(1-\alpha)\big)f'(\kappa)\kappa$$

Rearranging and lagging one period gives

$$\Lambda_{t-1} = \frac{\big(1-\pi(1-\alpha)\big)f'(\kappa)\kappa - l_{1t}^{*}}{f(\kappa)}$$

The goods market clearing condition is then:

$$l_{1t}^{*} + \big(1-\pi(1-\alpha)\big)f'(\kappa)\kappa - l_{1t}^{*} + Af(\kappa) - A\Big(\big(1-\pi(1-\alpha)\big)f'(\kappa)\kappa - l_{1t-1}^{*}\Big)$$

$$= A[f(\kappa) - f'(\kappa)] + Al_{1t}^{*} + \pi(1-\alpha)Af'(\kappa)\kappa + \big(1-\pi(1-\alpha)\big)f'(\kappa)\kappa$$

Cancelling confirms that

$$l_{1t-1}^{*} = l_{1t}^{*}$$

$\frac{1}{\rho_{t+1}} = A$ induces a steady state where prices and labor are constant in each period. The percent of entrepreneurs choosing the long-term production technology and the proportion of lenders opting to participate in all exchanges is constant through time.

Decreasing Prices

When prices are decreasing, $\frac{Ap_{xt+2}}{p_{xt+1}}$ can be less than one. This would imply that the price of a one period discount bond is above face value. No agent would ever purchase these bonds on the secondary market. For decreasing prices, I restrict $p_{xt+1} < Ap_{xt+2}$ to ensure an operating secondary market.

There are three possible cases of ρ. Lenders who are excluded from old age settlement or opt to forego the last settlement market consume $q_{3t+2} = \frac{p_{kt}\kappa}{p_{xt}}$. Lenders able to participate on the old-age settlement market earn an extra return of $\frac{1}{\rho_{t+1}}$ and consume $q_{3t+2}^* = \frac{1}{\rho_{t+1}}\frac{p_{kt}\kappa}{p_{xt}}$. Thus, lenders strictly prefer to participate in all settlement markets unless $\frac{1}{\rho_{t+1}} = 1$. Entrepreneurs are indifferent when $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$.

Table 3. Summary of Agents Choices with Decreasing Prices

Case	Condition on ρ_{t+1}	Preferred Choice of Agents in Equilibrium	Feasibility of Equilibrium
8	$\frac{1}{\rho_{t+1}} = 1$	Lenders: indifferent Entrepreneurs: long-term production	Yes
9	$\frac{1}{\rho_{t+1}} \in \left(1, \frac{Ap_{xt+2}}{p_{xt+1}}\right)$	Lenders: participate in all exchanges if able Entrepreneurs: long-term production	No
10	$\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$	Lenders: participate in all exchanges if able Entrepreneurs: indifferent	No

The recursive block with decreasing prices is

$$q_{2t+1} = 0, q_{3t+2} = \rho_{t+1}Af'(\kappa)\kappa \quad (40)$$

$$q_{2t+1}^* = 0, q_{3t+2}^* = Af'(\kappa)\kappa \quad (41)$$

$$f'(\kappa) = \frac{p_{kt}}{p_{xt+1}} \quad (42)$$

$$\frac{l_{1t+2}^*}{\rho_{t+1}} u'\left(A[f(\kappa) - f'(\kappa)] + \frac{1}{\rho_{t+1}} l_{1t+2}^*\right) = g'(l_{1t}^*) l_{1t}^* \quad (43)$$

$$l_{1t}^* + \Lambda_{t-1} f(\kappa) + (1 - \Lambda_{t-2}) A f(\kappa)$$

$$= A[f(\kappa) - f'(\kappa)] + \frac{p_{xt-2}}{\rho_{t-1} p_{xt}} l_{1t-2}^* + \Pi_{t-2} A f'(\kappa)\kappa + (1 - \Pi_{t-1}) q_{2t}$$

$$+ (1 - \Pi_{t-2}) q_{3t} \quad (44)$$

$$\Lambda_t f(\kappa) + l_{1t+1}^* \geq (1 - \Pi_t) \frac{l_{1t+1}^*}{l_{1t+2}^*} \rho_{t+1} A f'(\kappa)\kappa \quad (45)$$

A final case exists when $A = \frac{p_{xt+2}}{p_{xt+1}}$ causing both lenders and entrepreneurs to be indifferent simultaneously.

Case 8. With $\frac{1}{\rho_{t+1}} = 1$ by assumption, lenders are indifferent about participating in the old age settlement market and both early- and late- leaving lenders consume the same amount. Entrepreneurs strictly prefer the long-term production technology.

The goods market is clearing condition is

$$l_{1t}^* + A f(\kappa) = A f(\kappa) - A f'(\kappa)\kappa + l_{1t}^* + \Pi_{t-2} A f'(\kappa)\kappa + (1 - \Pi_{t-2}) A f'(\kappa)\kappa$$

which confirms that the supply for goods equals the demand for goods.

The difference equation comes from the entrepreneur's first-order condition with respect to labor. The second order difference equation is

$$l_{1t+2}^* \mathrm{u}'(A f(\kappa) - A f'(\kappa)\kappa + l_{1t+2}^*) = g'(l_{1t}^*) l_{1t}^*$$

Case 9. When $\frac{1}{\rho_{t+1}} \in \left(1, \frac{A p_{xt+2}}{p_{xt+1}}\right)$, lenders participate in all exchanges, if able. $\Pi = \pi(1 - \alpha)$. Entrepreneurs choose the long-term production technology, so $\Lambda_t = 0$. The secondary market clearing condition solves for $\frac{1}{\rho_{t+1}}$ giving

$$\frac{1}{\rho_t} = \frac{\big(1 - \pi(1 - \alpha)\big) A f'(\kappa)\kappa}{l_{1t+1}^*}$$

The goods market clearing condition is then

$$l_{1t}^{*} + Af(\kappa) = Af(\kappa) - Af'(\kappa)\kappa + \big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa + \pi(1-\alpha)Af'(\kappa)\kappa$$

$$+ \big(1 - \pi(1-\alpha)\big)\frac{l_{1t}^{*}}{\big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa}Af'(\kappa)\kappa$$

Cancelling and simplifying shows that the supply of goods equals the demand of goods in each period.

However, dynamics disappear with decreasing prices and $\frac{1}{\rho_{t+1}} \in \left(1, \frac{Ap_{xt+2}}{p_{xt+1}}\right)$. By substituting the solution for $\frac{1}{\rho_t}$ into the entrepreneur's first-order condition, I have

$$l_{1t+2}^{*}\frac{\big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa}{l_{1t+2}^{*}}u'\left(A[f(\kappa) - f'(\kappa)] + \frac{\big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa}{l_{1t+2}^{*}}l_{1t+2}^{*}\right)$$

$$= g'(l_{1t}^{*})l_{1t}^{*}$$

Simplifying $l_{1t+2}^{*}\frac{(1-\pi(1-\alpha))Af'(\kappa)\kappa}{l_{1t+2}^{*}}$ that is both inside the utility function and premultiplies the utility function gives

$$\big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa u'\Big(A[f(\kappa) - f'(\kappa)] + \big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa\Big) = g'(l_{1t}^{*})l_{1t}^{*} \quad (46)$$

Updating this equation one period gives

$$\big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa u'\Big(A[f(\kappa) - f'(\kappa)] + \big(1 - \pi(1-\alpha)\big)Af'(\kappa)\kappa\Big) = g'(l_{1t+1}^{*})l_{1t+1}^{*} \quad (47)$$

Taking the ratio of (47) to (46) implies

$$\frac{g'(l_{1t+1}^{*})}{g'(l_{1t}^{*})}\frac{l_{1t+1}^{*}}{l_{1t}^{*}} = 1$$

Deflation implies that the quantity of labor is increasing each period, $\frac{l_{1t+1}^*}{l_{1t}^*} > 1$. The cost function for the entrepreneur is strictly increasing and strictly convex so $\frac{g\prime(l_{1t+1}^*)}{g\prime(l_{1t}^*)} > 1$. This cannot be an equilibrium case.

Case 10. The upper bound on $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$ implies that all lenders choose to participate in all exchanges, if able. $\Pi = \pi(1 - \alpha)$. Entrepreneurs are indifferent between the short- and long- term production technology. Λ is found from the secondary market clearing condition, which holds with equality. Substituting for ρ and rearranging gives

$$\Lambda_{t-1} = \frac{\big(1 - \pi(1 - \alpha)\big)f'(\kappa)\kappa - l_{1t}^*}{f(\kappa)}$$

The goods market clearing condition is

$$l_{1t}^* + \big(1 - \pi(1 - \alpha)\big)f'(\kappa)\kappa - l_{1t}^* + Af(\kappa) - A\Big(\big(1 - \pi(1 - \alpha)\big)f'(\kappa)\kappa - l_{1t}^*\Big)$$

$$= Af(\kappa) - Af'(\kappa)\kappa + Al_{1t}^* + \pi(1 - \alpha)Af'(\kappa)\kappa$$

$$+ \big(1 - \pi(1 - \alpha)\big)\frac{l_{1t-1}^*}{l_{1t}^*}Af'(\kappa)\kappa$$

Simplifying yields

$$l_{1t}^* = l_{1t-1}^*$$

This is consistent with the assumption that prices are decreasing in each subsequent period. Case 10 is also not an equilibrium.

Lenders and Entrepreneurs are Simultaneously Indifferent. If the exogenous parameter *A* happens to equal $\frac{p_{xt+2}}{p_{xt+1}}$, both the lender and the entrepreneur are simultaneously indifferent. While this is possible with deflation, this is not supported as an equilibrium in the model. Consider the goods market

$$l_{1t}^{*} + f(\kappa)(\Lambda_{t-1} + (1 - \Lambda_{t-2})A)$$

$$= A[f(\kappa) - f'(\kappa)] + l_{1t}^{*} + \Pi_{t-2}Af'(\kappa)\kappa + (1 - \Pi_{t-2})Af'(\kappa)\kappa$$

Cancelling like terms and substituting for $A = \frac{p_{xt+2}}{p_{xt+1}}$ gives

$$\frac{l_{1t+1}^{*}}{l_{1t}^{*}} = \frac{\Lambda_{t-1} - \Lambda_{t-2}}{1 - \Lambda_{t-2}} \tag{48}$$

Deflation implies that $l_{1t+1}^{*} > l_{1t}^{*}$. However, Λ is the probability that an entrepreneur chooses the short term production technology and is strictly less than 1. $\Lambda_{t-1} - \Lambda_{t-2} < 1 - \Lambda_{t-2}$. Equation (48) does not hold with equality and the goods market does not clear in equilibrium.

Case 8, with $\frac{1}{\rho_{t+1}} = 1$ represents the only equilibrium with deflation.

Summary of Cases

Off steady state, the economy can only take certain values of $\frac{1}{\rho_{t+1}}$ in equilibrium. When prices are increasing, $\frac{1}{\rho_{t+1}}$ can take the value of $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$ (where lenders are indifferent, but entrepreneurs strictly prefer long term production) or $\frac{1}{\rho_{t+1}} = A\frac{p_{xt+2}}{p_{xt+1}}$ (where entrepreneurs are indifferent but lenders strictly prefer to participate in all periods). When prices are decreasing, $\frac{1}{\rho_{t+1}} = 1$ (lenders are indifferent, but entrepreneurs strictly prefer the long-term production technology). In steady state $\frac{1}{\rho_{t+1}} = A$ (entrepreneurs are indifferent, but lenders strictly prefer to participate in all market exchanges).

Table 4 summarizes the possible values of $\frac{1}{\rho_{t+1}}$ and the feasibility of each case.

Table 4. Summary of Cases

	$\frac{1}{\rho_{t+1}} = 1$	$\frac{1}{\rho_{t+1}} \in \left(1, \frac{p_{xt+2}}{p_{xt+1}}\right)$	$\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$	$\frac{1}{\rho_{t+1}} \in \left(\frac{p_{xt+2}}{p_{xt+1}}, \frac{Ap_{xt+2}}{p_{xt+1}}\right)$	$\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$
Inflation	No	No	Yes	No	Yes
Steady State	No			No	Yes
Deflation	Yes	No			No

"Yes" implies that the combination of prices (inflation, deflation, or steady state) with the particular value of $\frac{1}{\rho_{t+1}}$ represent an equilibrium for the economy. "No" implies the opposite – that the case is not feasible in equilibrium.

The law of motions depends on the value of $\frac{1}{\rho_{t+1}}$ and the price dynamics in the economy. Table 5 summarizes the law of motion for each feasible case – that is, each case where the combination of $\frac{1}{\rho_{t+1}}$ and the assumed movement in prices represents an equilibrium for the economy.

Table 5. Law of Motion for Equilibrium Cases Away from the Steady State

	Case	Value of $\frac{1}{\rho_{t+1}}$	Law of Motion for l_1^*
Inflation	2	$\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$	$l^*_{1t+1}u'(Af(\kappa) - Af'(\kappa) + l^*_{1t+1}) = g'(l^*_{1t})l^*_{1t}$
	4	$\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$	$Al^*_{1t+1}u'(Af(\kappa) - Af'(\kappa)\kappa + Al^*_{1t+1}) = g'(l^*_{1t})l^*_{1t}$
Deflation	8	$\frac{1}{\rho_{t+1}} = 1$	$l^*_{1t+2}u'(Af(\kappa) - Af'(\kappa)\kappa + l^*_{1t+2}) = g'(l^*_{1t})l^*_{1t}$

In cases 2 and 8, lenders are indifferent about participating in the old-age settlement market. Entrepreneurs strictly prefer the long-term production technology. In case 4, entrepreneurs are indifferent, however lenders strictly prefer to participate in all settlement markets.

Baseline Case-by-Case Dynamics

In this environment, dynamics are dependent on the behavior of prices. The discontinuity in the agent's problem carries over into the equilibrium law of motion. When prices are increasing in each period, the results are the same as in Azariadis (1981). The law of motion is a first-order, non-linear difference equation with slope dependent on the price effect derived from the entrepreneur's utility function. However, when prices are decreasing in each period, the law of motion is a second-order non-linear difference equation. I present two techniques for deriving the closed form solution to this problem. With first-order difference equations, the system has one positive steady state. With second-order difference equations, the system can have two positive steady states. However, the eigenvalues are repeated and rule out this case.

Inflationary Prices

When labor is decreasing in each period, there are two feasible equilibrium cases; Case 2 where $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$ and Case 4 where $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$. Both are first-order, non-linear difference equations that depend on the price effect in the entrepreneur's objective function.

Table 5 gives the equilibrium law of motion $H(l_{1t}^{*}, l_{1t+1}^{*})$ as derived from the entrepreneur's first-order condition with respect to labor, equation (15), and the money market clearing condition, equation (17). As in Azariadis (1981), the dynamics depend on the relative magnitude of the income and substitution effect in the entrepreneur's preferences. When the substitution effect dominates, the law of motion crosses the 45°-line from below. Starting below the positive steady state implies that the economy follows a hyperinflationary path. When the income effect dominates, the system exhibits chaotic dynamics. The movement of prices in equilibrium depends on the starting value of the system.

$H(l_{1t}^*, l_{1t+1}^*)$ depends on the value of $\frac{1}{\rho_{t+1}}$.

$$g'(l_{1t}^*)l_{1t}^* = \begin{cases} l_{1t+1}^* u'(Af(\kappa) - Af'(\kappa) + l_{1t+1}^*) & if \ \frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}} \\ \mathrm{A}l_{1t+1}^* \mathrm{u}'(Af(\kappa) - Af'(\kappa)\kappa + Al_{1t+1}^*) & \ \ \ \ \frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}} \end{cases} \quad (49)$$

For each case, the dynamics are found by totally differentiating the law of motion.

For Case 2, the total derivative is

$$\left(g''(l_{1t}^*)l_{1t}^* + g'(l_{1t}^*)\right)dl_{1t}^* = (u'(\cdot) + u''(\cdot)l_{1t+1}^*)dl_{1t+1}^*$$

Dynamics are given by

$$\frac{dl_{1t+1}^*}{dl_{1t}^*} = \frac{g''(l_{1t}^*)l_{1t}^* + g\prime(l_{1t}^*)}{u'(\cdot) + u\prime\prime(\cdot)l_{1t+1}^*} \quad (50)$$

The numerator is positive by the Indada Conditions imposed on $g(\cdot)$. The sign of (50) is determined by the curvature in the utility function: $\left(1 + \frac{u\prime\prime(\cdot)}{u'(\cdot)} l_{1t+1}^*\right)$. This is formalized in the Lemma below.

Lemma 5. *The slope of the law of motion* $H(l_{1t}^*, l_{1t+1}^*)$ *depends on the price effect in the entrepreneur's utility function.*

Proof. To derive the price effect, return to the first order condition for the entrepreneur (equation 15) and substitute $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$

$$\frac{p_{xt+2}}{p_{xt+1}} \frac{p_{xt}}{p_{xt+2}} u' \left(A[f(\kappa) - f'(\kappa)] + \frac{p_{xt+2}}{p_{xt+1}} \frac{p_{xt}}{p_{xt+2}} l_{1t}^*\right) = g'(l_{1t}^*)$$

Simplifying gives

$$\frac{p_{xt}}{p_{xt+1}} u' \left(A[f(\kappa) - f'(\kappa)] + \frac{p_{xt}}{p_{xt+1}} l_{1t}^*\right) = g'(l_{1t}^*) \quad (51)$$

The impact of a change in price on the amount of labor provided to the market, $\frac{dl_{1t}^*}{dp_{xt}}$, is found by differentiating (51) and setting $dp_{xt} = 0$.

$$\frac{dl_{1t}^*}{dp_{xt}} = -\frac{\left(\frac{1}{p_{xt+1}}u'(\cdot) + u''(\cdot)\frac{l_{1t}^*}{p_{xt+1}}\frac{p_{xt}}{p_{xt+1}}\right)}{\left(-g''(l_{1t}^*) + \left(\frac{p_{xt}}{p_{xt+1}}\right)^2 u''(\cdot)\right)}$$

The denominator is the entrepreneur's second-order condition with respect to labor – it is always negative. This cancels with the premultiplication of (-1). The price effect is always equivalent in sign to

$$\left(1 + \frac{u''(\cdot)}{u'(\cdot)}\frac{l_{1t}^* p_{xt}}{p_{xt+1}}\right)$$

Substituting for p from the money market clearing condition gives

$$\left(1 + \frac{u''(\cdot)}{u'(\cdot)}l_{1t+1}^*\right)$$

Which depends only on the curvature in $u(\cdot)$. The sign of both $\frac{dl_{1t}^*}{dp_{xt}}$ and $\frac{dl_{1t+1}^*}{dl_{1t}^*}$ depend on the sign of the expression $\left(1 + \frac{u''(\cdot)}{u'(\cdot)}l_{1t+1}^*\right)$. When the income effect dominates, the sign of $u''(\cdot)$ dominates and the price effect is negative. When the substitution effect dominates, $\left(1 + \frac{u''(\cdot)}{u'(\cdot)}l_{1t+1}^*\right) > 0$. ■

When the income effect dominates, an increase in price (wage) decreases labor effort. That is, $\frac{dl_{1t}^*}{dp_{xt}} < 0$. This depends directly on the amount of curvature in the utility function. That is $\left(1 + \frac{u''(\cdot)}{u'(\cdot)}l_{1t+1}^*\right) < 0$. This also determines the sign of the derivative of the law of motion. When the income effect dominates the entrepreneur's problem, $\frac{dl_{1t+1}^*}{dl_{1t}^*} < 0$. As shown in Azariadis, the dynamics of this system are chaotic. The law of motion $H(l_{1t}^{*}, l_{1t+1}^{*})$ crosses the 45°-line with a negative slope (from above). This is shown in Figure 2(a).

The differentiation is similar when $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$. For Case 4, the slope of the equilibrium law of motion is given by

$$\frac{dl_{1t+1}^{*}}{dl_{1t}^{*}} = \frac{g''(l_{1t}^{*})l_{1t}^{*} + g'(l_{1t}^{*})}{u'(\cdot) + u''(\cdot)A^2 l_{1t+1}^{*}}$$

With $A > 1$, the slope of $H(l_{1t}^{*}, l_{1t+1}^{*})$ is more shallow compared with the above case. The sign of $\frac{dl_{1t+1}^{*}}{dl_{1t}^{*}}$ is also dependent on the sign of the price effect. The first order condition is

$$A\frac{p_{xt}}{p_{xt+1}}u'\left(A[f(\kappa) - f'(\kappa)] + A\frac{p_{xt}}{p_{xt+1}}l_{1t}^{*}\right) = g'(l_{1t}^{*})$$

giving price effect of

$$\frac{dl_{1t}^{*}}{dp_{xt}} = -\frac{\left(\frac{A}{p_{xt+1}}u'(\cdot) + u''(\cdot)A^2\frac{l_{1t}^{*}}{p_{xt+1}}\frac{p_{xt}}{p_{xt+1}}\right)}{\left(-g''(l_{1t}^{*}) + \left(\frac{p_{xt}}{p_{xt+1}}\right)^2 u''(\cdot)\right)}$$

The sign of both the slope of the law of motion and the price effect depends on the sign of

$$\left(1 + \frac{u''(\cdot)}{u'(\cdot)}Al_{1t+1}^{*}\right) \qquad (52)$$

As before, when the income effect dominates, $\frac{dl_{1t}^{*}}{dp_{xt}} < 0$. This can only happen when $\left(1 + \frac{u''(\cdot)}{u'(\cdot)}Al_{1t+1}^{*}\right) < 0$. The sign of slope of the law of motion, $\frac{dl_{1t+1}^{*}}{dl_{1t}^{*}}$, also depends on the sign of (52). When the income effect dominates, $\frac{dl_{1t+1}^{*}}{dl_{1t}^{*}} < 0$. The law of motion again the crosses the 45°-line with a negative slope.

In both Case (2) , the law of motion is a first-order, non-linear difference equation.

Numerical Example

Figure 2 graphs the equilibrium law of motion when the income effect dominates the entrepreneur's preferences (panel (a)) and when the substitution effect dominates the entrepreneur's preferences (panel (b)).

To parameterize this system, let $u(x^*_{3t+2}) = \frac{x^{1-\sigma}}{1-\sigma}$, $g(l^*_{1t}) = -\phi(1-l^{\sigma}_{1t})^{\frac{1}{\sigma}}$, and $f(\kappa) = \kappa^{1/3}$. Further, assume $\kappa = 1$ and A=1.5. That is, long-run production returns 50% more output than short-run production. The derivatives for the functional form of the dynamical equation are: $u'(\cdot) = {x^*_{3t+2}}^{-\sigma}$ and $f'(\kappa) = \frac{1}{3}\kappa^{-2/3}$. The budget constraint for the entrepreneur is $p_{xt+2}x^*_{3t+2} = p_{xt+2}Af(\kappa) + \frac{1}{\rho_{t+1}}(p_{xt}l^*_{1t} - p_{kt}\kappa)$. To derive the equilibrium law of motion, the entrepreneur's problem is rewritten in terms of l^*. The budget constraint is rewritten as $x^*_{3t+2} = Af(\kappa) + \frac{1}{p_{xt+2}\rho_{t+1}}p_{xt}l^*_{1t} - \frac{p_{kt}}{p_{xt+2}\rho_{t+1}}\kappa$. Substituting for the optimal choice of capital and the return to IOUs on the secondary market $\frac{1}{\rho_{t+1}}$ gives $x^*_{3t+2} = Af(\kappa) + \frac{p_{xt}}{p_{xt+1}}l^*_{1t} - Af'(\kappa)\kappa$. Substituting for the money market clearing condition gives $x^*_{3t+2} = A(f(\kappa) - f'(\kappa)\kappa) + l^*_{1t+1}$. Substituting for $f(\kappa)$, $f'(\kappa)$, $\kappa = 1$, and $A = 1.5$ gives $x^*_{3t+2} = 1 + l^*_{1t+1}$. Returning to the entrepreneur's first order condition with respect to consumption, and substituting for x^*_{3t+2} gives $u'(1 + l^*_{1t+1}) = (1 + l^*_{1t+1})^{-\sigma}$. With the cost function above, the equilibrium law of motion in labor for the Constant Elasticity of Substitution (CES) utility function is

$$l^*_{1t+1}(1 + l^*_{1t+1})^{-\sigma} - \phi(1 - l^{\sigma}_{1t})^{\frac{1-\sigma}{\sigma}} l^{\sigma}_{1t} = 0$$

When the income effect dominates, $\sigma < 1$. In the neighborhood of the non-monetary steady state, the slope of $H(\cdot)$ is greater than 1.

When the substitution effect dominates, $\sigma > 1$. The slope of $H(\cdot)$ is less than 1 in the neighborhood of the non-monetary steady state.[10] Figure 2 plots the equilibrium law of motion when the income effect dominates (Figure 2(a)) and when the substitution effect dominates (Figure 2(b)).

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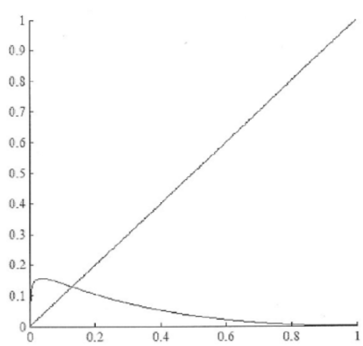
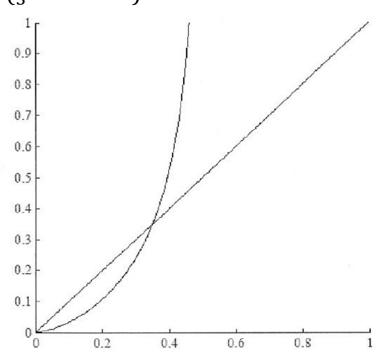
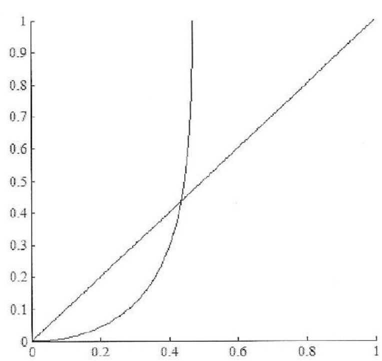
[10] The graphs of the equilibrium law of motion for $\sigma = \left\{\frac{1}{3}, 1.5, 2, 5\right\}$ are below:



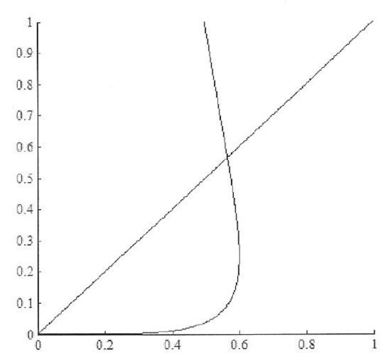
Equilibrium Law of Motion, $\sigma = 0.33$



Equilibrium Law of Motion, $\sigma = 1.5$



Equilibrium Law of Motion, $\sigma = 2$



Equilibrium Law of Motion, $\sigma = 5$

Figure 2(a). Equilibrium Law of Motion with Income Effect Dominating the Entrepreneur's Preferences

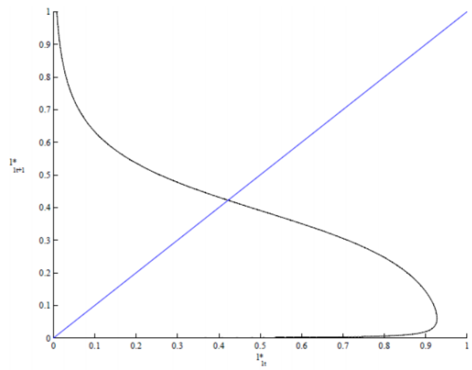
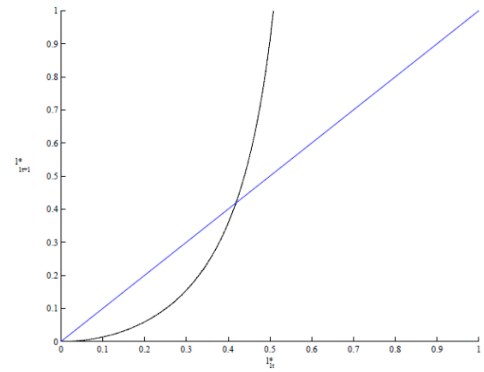


Figure 2(b). Equilibrium Law of Motion with Substitution Effect Dominating the Entrepreneur's Preferences



In Figure 2(a), starting above the 45°-line represents the situation where prices are decreasing each period. When the income effect dominates the entrepreneur's preferences, we observe an oscillatory convergence to the positive steady state. On the other hand, starting below the 45°-line represents a situation where prices are increasing in each period. This system diverges from the positive steady state. When the substitution effect dominates (figure 2(b)), starting below the positive steady state implies that the amount of labor falls each period – that is, prices are increasing exponentially. Starting above the positive steady state implies the amount of labor grows exponentially each period, diverging from the steady state and approaching autarky.

The behavior of $l_{1t+1}^{*}(l_{1t}^{*})$ is plotted through time in Figure 3. Panel (a) of Figure 3 shows the behavior of l^{*} for 14 periods when the income effect dominates. l^{*} converges to the steady state in 8 periods. Panel (b) shows the behavior of l^{*} when the substitution effect dominates the entrepreneur's problem. Within 6 periods, the system converges to the non-monetary steady state.

Figure 3(a). Behavior of Labor when the Income Effect Dominates the Entrepreneur's Problem

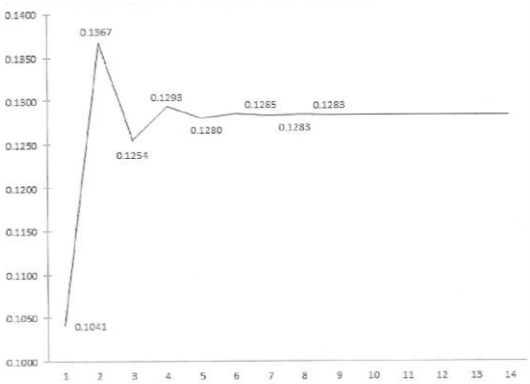
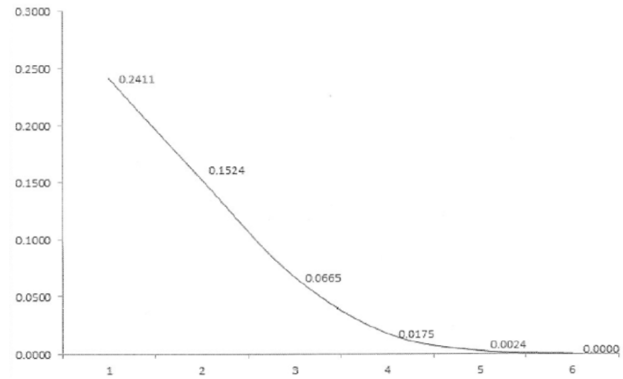


Figure 3(b). Behavior of Labor when the Substitution Effect Dominates the Entrepreneur's Problem



Deflationary Prices

When labor is increasing in each period, there is one equilibrium convergence path $\frac{1}{\rho_{t+1}} = 1$ (Case 8). The equilibrium law of motion is

$$H(l_{1t+2}^{*}, l_{1t}^{*}) = l_{1t+2}^{*}\mathrm{u}'\left(Af(\kappa) - Af'(\kappa)\kappa + l_{1t+2}^{*}\right) - g'(l_{1t}^{*})l_{1t}^{*} = 0$$

The dynamics of the second order, non-linear difference equation can be solved with Eigenvalues or numerically. Both are presented in turn below.

To solve for the dynamics with Eigenvalues, the system is linearized in the neighborhood of the positive steady state then written in matrix form. The Eigenvalues of a real matrix A are given by $|A - \lambda I| = 0$, where λ is the Eigenvalue and I is the identity matrix. The dynamics of the system are determined by the sign of λ.

Linearizing the system gives

$$0 = [u'(\cdot) + u''(\cdot)l_{1t+2}^{*}]dl_{1t+2}^{*} + 0dl_{1t+1}^{*} - \left[g''(l_{1t}^{*}) + g'(l_{1t}^{*})\right]dl_{1t}^{*}$$

In matrix form

$$\begin{pmatrix} l_{1t+2}^{*} \\ l_{1t+1}^{*} \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{g''(l_{1t}^{*}) + g'(l_{1t}^{*})}{u'(\cdot) + u''(\cdot)l_{1t+2}^{*}} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} l_{1t+1}^{*} \\ l_{1t}^{*} \end{pmatrix}$$

The Eigenvalues of this matrix are $\lambda_1, \lambda_2 = \pm\sqrt{\frac{g''(l_{1t}^*)+g'(l_{1t}^*)}{u'(\cdot)+u''(\cdot)l_{1t+2}^*}}$. Since both Eigenvalues have the same value, the system is not saddlepath stable.

As before, the magnitude of λ_1, λ_2 depend on the price effect in the entrepreneur's objective function. The price effect from the entrepreneur's preferences is

$$\frac{dl_{1t}^*}{dp_{xt}} = -\frac{\left(1 + \frac{u''(\cdot)}{u'(\cdot)} {p_{xt}}^2 \frac{l_{1t}^*}{p_{xt+2}}\right)}{\left(-g''(l_{1t}^*) + \left(\frac{p_{xt}}{p_{xt+1}}\right)^2 u''(\cdot)\right)}$$

after substituting for $\frac{1}{\rho_{t+1}} = 1$. The denominator is the second-order condition from the entrepreneur's maximization problem. This is negative, cancelling the leading negative in the expression. As before, the sign depends of $\frac{dl_{1t}^*}{dp_{xt}}$ depends on $\left(1 + \frac{u''(\cdot)}{u'(\cdot)} {p_{xt}}^2 \frac{l_{1t}^*}{p_{xt+2}}\right)$, which also determines the sign of $\frac{dl_{1t+2}^*}{dl_{1t}^*}$. Notice, when the economy is experiencing deflation, the dynamics oscillate in every other period.

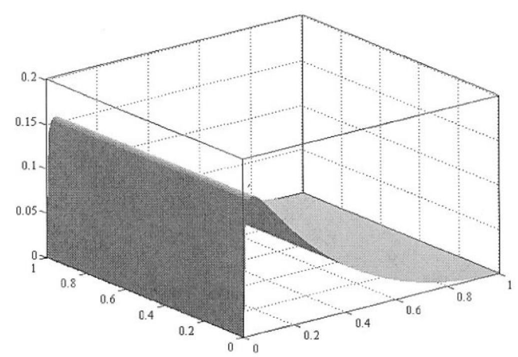
$$\frac{dl_{1t+2}^*}{dl_{1t}^*} = \frac{g''(l_{1t}^*)l_{1t}^* + g'(l_{1t}^*)}{u'(\cdot) + u''(\cdot)l_{1t+2}^*}$$

When the income effect dominates the entrepreneur's problem, $\frac{dl_{1t}^*}{dp_{xt}} < 0$. The sign of $\frac{dl_{1t}^*}{dp_{xt}}$ is determined by the expression $\left(1 + \frac{u''(\cdot)}{u'(\cdot)} {p_{xt}}^2 \frac{l_{1t}^*}{p_{xt+2}}\right)$. When this is negative, $\frac{dl_{1t+2}^*}{dl_{1t}^*}$ is also negative. The slope of the equilibrium law of motion is negative. The dynamics exhibit chaos. When the substitution effect dominates, $\frac{dl_{1t}^*}{dp_{xt}} > 0$ as $\left(1 + \frac{u''(\cdot)}{u'(\cdot)} {p_{xt}}^2 \frac{l_{1t}^*}{p_{xt+2}}\right) > 0$. In this case the slope of the equilibrium law of motion is also positive. $H(l_{1t+2}^*, l_{1t}^*)$ crossed the 45-degree line from below, meaning the monetary steady state is unstable.

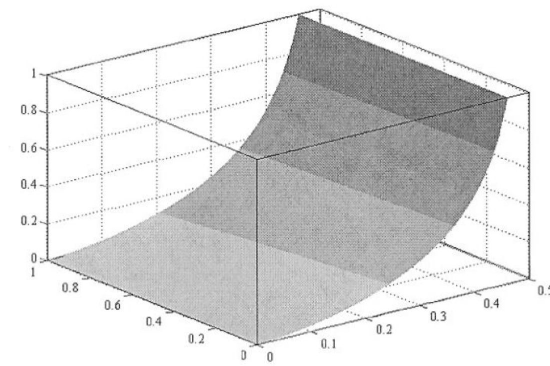
The intertemporal correlation with deflation is between l^*_{1t+2} and l^*_{1t}. The choice of labor, and subsequently the price level, has no inter-termporal connection between periods *t* and *(t+1)*.

Numerical Example

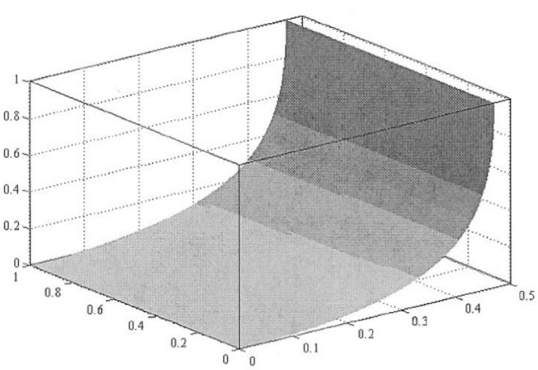
Using the same functional forms as above, the equilibrium law of motion can be graphed in the $l^*_{1t+2}(l^*_{1t})$ plane. Let $u(x^*_{3t+2}) = \frac{x^{1-\sigma}}{1-\sigma}$, $g(l^*_{1t}) = -\phi(1-l^{\sigma}_{1t})^{\frac{1}{\sigma}}$, and $f(\kappa) = \kappa^{1/3}$. Further, assume $\kappa = 1$ and A=1.5. The results are below, and mimic the shape of the first-order difference equation in the $l^*_{1t+1}(l^*_{1t})$ plane. The law of motion for for $\sigma = \left\{\frac{1}{3}, 1.5, 2, 5\right\}$ are below.



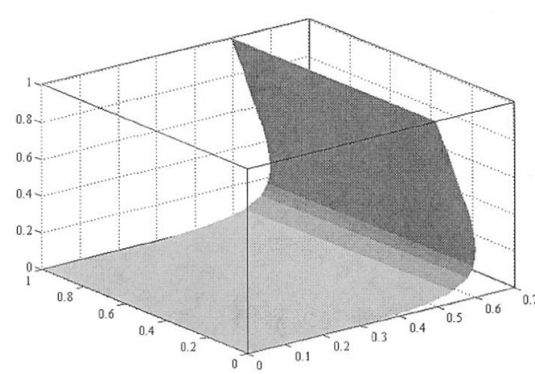
Equilibrium Law of Motion, $\sigma = 0.33$



Equilibrium Law of Motion, $\sigma = 1.5$



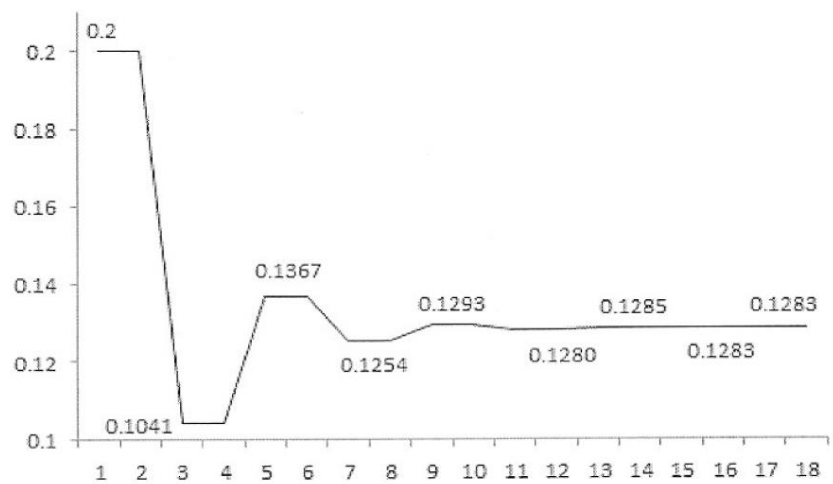
Equilibrium Law of Motion, $\sigma = 2$



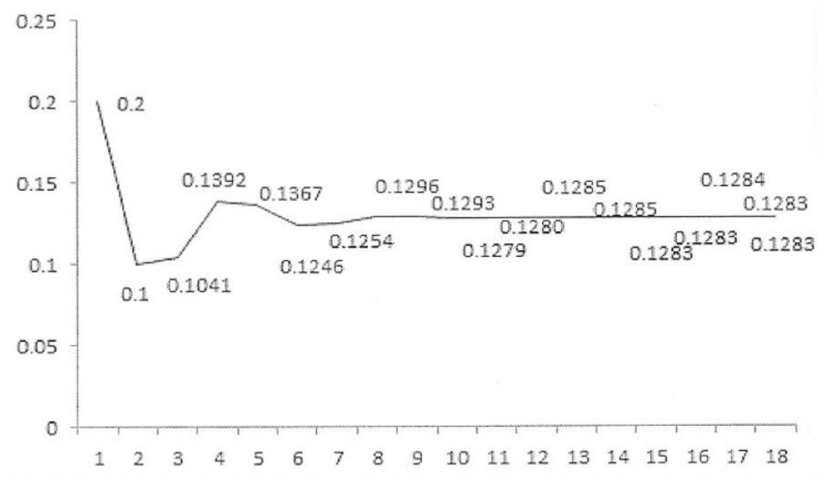
Equilibrium Law of Motion, $\sigma = 5$

In the following example, any initial value of l_{12}^* satisfies the equilibrium law of motion. This exercise shows that the system is sensitive to starting values and displays oscillatory convergence even when the substitution effect dominates. Using the same functional form as before, the initial values were chosen to be above the steady state value. The dynamics are then shown below.

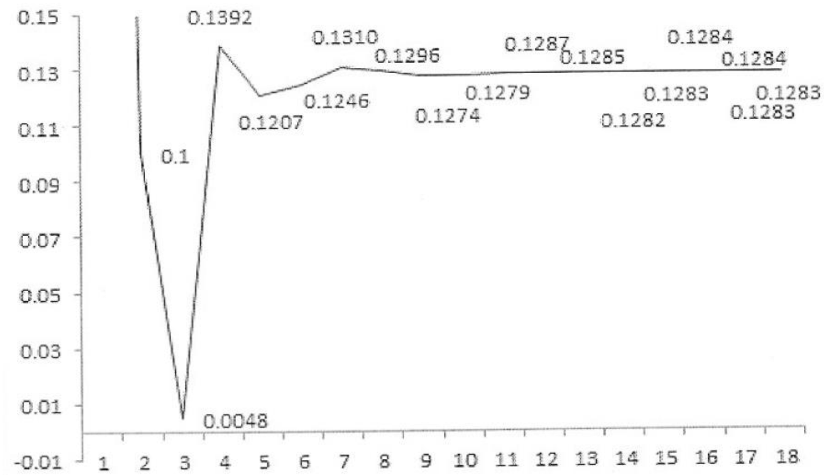
When $\sigma = 0.33$, the income effect dominates the planners problem. The law of motion for three different starting values of $l_{1\cdot}^*$ is shown below. If the initial values of the system are set to 0.2, $l_{1\cdot}^*$ follows the same path as before though the convergence is slower.



Starting with different values of $l_{1\cdot}^*$ (0.2 in period 1 and 0.1 in period 2) produces a faster cycle to the oscillation.

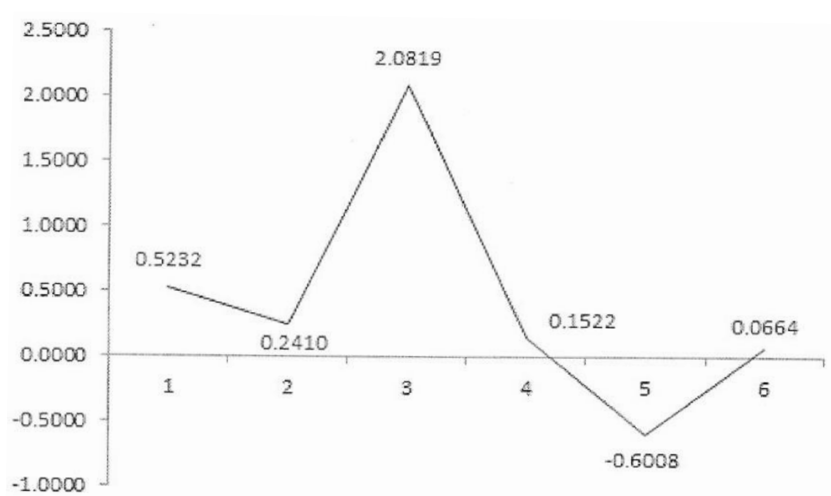

0.25
0.2
0.15
0.1
0.05
0
0.2
0.1
0.1041
0.1392
0.1367
0.1246
0.1254
0.1296
0.1293
0.1279
0.1280
0.1285
0.1285
0.1283
0.1283
0.1284
0.1283
0.1283
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Finally, if the initial values of $l_{1\cdot}^{*}$, $l_{11}^{*} = 0.8$ and $l_{12}^{*} = 0.1$ the system exhibits a large amplitude.



When the substitution effect dominates ($\sigma = 1.5$), the system can also display oscillatory behavior but converging to the non-monetary steady state. Consider the case when $l_{11}^{*} = 0.4$ (above the monetary stead state) and $l_{12}^{*} = 0.3$ (below the monetary steady state). The equilibrium law of motion then dictates:

Figure 4. Values of $l_{1\cdot}^{*}$ through time starting above the monetary steady state with the substitution effect dominating the entrepreneur's utility function



The amount of labor chosen in period 3 and 5 violates the boundary conditions on the amount of time given to each entrepreneur.

When the substitution effect dominates, the value of $l_{1\cdot}^{*}$ quickly reaches the boundary conditions on the amount of labor. This is not feasible. However, when the income effect dominates, the system displays oscillatory convergence to the positive steady state.

Summary

If the substitution effect dominates, the system will display divergence from the positive steady state. Starting above the steady state value of labor (below the steady value of prices) implies that prices decrease in each period moving the system towards the non-monetary steady state. Starting below the positive steady state in labor (above the steady state in price) moves the system along the explosive path towards hyper-inflation. If the income effect dominates the entrepreneur's problem, then the system displays oscillatory convergence to the positive (monetary) steady state. Additionally, when we start above the positive steady state, the system is governed with a second-order, non-linear difference equation. Starting below the positive steady state implies that the system is governed with a first-order, non-linear difference equation. This is summarized in Table 6.

Table 6. Summary of the Dynamical Behavior in Each Case Based on the Price Effect and Initial Conditions

	Labor Starting Below the Positive Steady State (Cases 2 and 4 of Table 5)	Labor Starting Above the Positive Steady State (Case 8 of Table 5)
Income Effect Dominates	Oscillatory Convergence to the monetary steady state $H(l_{1t}^{*}, l_{1t+1}^{*})$	Oscillatory Convergence to the monetary steady state $H(l_{1t}^{*}, l_{1t+2}^{*})$

Substitution Effect Dominates	Divergence along the hyperinflationary path $H(l_{1t}^*, l_{1t+1}^*)$	Convergence to the non-monetary steady state (autarky) $H(l_{1t}^*, l_{1t+2}^*)$

Global Dynamics

This analysis differs from before. Rather than check the feasibility of each equilibrium case, this exercise combines feasible cases by assuming a shape to the entrepreneur's utility function but allowing prices to fluctuate and lenders choose optimally

Several features suggest that the case specific dynamics derived above may not hold for the global system. Specifically, under deflationary prices (labor starting above the positive steady state), the system only pins the values of l_{1t+2}^* from l_{1t}^*. Nothing is required of the relationship between values in adjacent periods, for example the relationship between l_{1t+3}^* and l_{1t+2}^*. This presents a concern for global dynamics. While each case may be internally consistent, the oscillation found under the income effect may create dynamics that are globally inconsistent – that is, there may not be a value of l_{1t+1}^* that is able to induce a consistent value of l_{1t+2}^* and l_{1t+3}^* simultaneously.

Additionally, with deflationary prices (Case 8 from Table 5), the only equilibrium value of IOUs is par, that is $\frac{1}{\rho_{t+1}} = 1$. However, with inflationary prices (Cases 2 and 4 from Table 5), two different values of ρ are feasible. Namely, $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$ and $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$. Is it possible for ρ to switch between these values? That is, is there regime switching within this model? Global dynamics can be derived by establishing mutually exclusive cases and considering consistency between the possible equilibrium states.

To this end, equilibrium can be divided into disjoint cases. The choice of lenders and entrepreneurs depends, partly, on the trajectory of prices. Clearly, prices cannot be both increasing and decreasing in the same period. This gives a natural division between starting above the positive steady state[11] (thus following a second-order difference equation), and starting below the positive steady state (thus following a first-order difference equation). Additionally, entrepreneurs are constrained so that the utility function remains constant throughout the analysis. This gives three distinct global cases; (I) The substitution effect dominates the entrepreneur's utility function and the initial condition on labor is below the monetary steady state, (II) The substitution effect dominates the entrepreneur's utility function and the initial condition on labor is above the monetary steady state, (III) The income effect dominates the entrepreneur's utility function. The initial condition on labor is unimportant in this case as the behavior is always oscillatory.

In Case (I), the system displays hyperinflation in prices (explosive). Labor decreases in each period until entrepreneurs do not invest in labor in young life. Case (II) the system shows deflation in prices away from the positive steady state to autarky. Labor increases exponentially each period, implying the boundary conditions are quickly enforced. Of particular interest with this case, the value of labor is determined by a one-period lag. Thus, $l^*_{1t+2}(l^*_{1t})$ and $l^*_{1t+3}(l^*_{1t})$. The overall trajectory may display cyclicality when the substitution effect is dominating the entrepreneur's utility function. Case (III)

[11] In labor.

displays oscillation in prices towards the positive steady state though is inconsistent globally.[12] This is summarized in Table 7, which extends Table 6.

Table 7. Required Behavior of Prices with Under the Initial Value and Assumed Shape to the Entrepreneur's Utility Function

<table>
<tr><th>Initial Condition on Labor (Prices)</th><th>Required Behavior of Prices with the Income Effect</th><th>Required Behavior of Prices with the Substitution Effect</th></tr>
<tr><td>Above the monetary steady state (below the monetary steady state)</td><td rowspan="2">Oscillation implies that the starting value is irrelevant for convergence. The periodicity of $H(\cdot)$ rules out this case in equilibrium. (Case III)

Above the monetary steady state: $H(l_{1t}^*, l_{1t+2}^*)$
Below the monetary steady state: $H(l_{1t}^*, l_{1t+1}^*)$</td><td>Labor increases exponentially in each period implying prices are decreasing. The system is diverging from positive steady state to the non-monetary steady state. The divergence appears to oscillate though the substitution effect dominates the entrepreneur's problem. (Case II)

$H(l_{1t}^*, l_{1t+2}^*)$</td></tr>
<tr><td>Below the monetary steady state (above the monetary steady state)</td><td>Labor decreases in each period implying prices are increasing. The system is following the explosive path away from the positive steady state. (Case I)

$H(l_{1t}^*, l_{1t+1}^*)$</td></tr>
</table>

Regime switching can occur in Case I. When starting below the positive steady state with the entrepreneur's initial choice of labor, the system does display regime

[12] As a note, in the third case, the system is switching between a first-order and second-order difference equation. That is, the choice of labor is determined by the prior amount of labor in t+1, t+3, t+5…, but by a lag of the variable in t, t+2, t+4….

switching. The return on IOUs moves from the low return of $\frac{p_{xt+2}}{p_{xt+1}}$, to the high return of $A\frac{p_{xt+2}}{p_{xt+1}}$. This is formalized in the lemmas below.

Lemma 6. *Starting with the lowest return to IOUs,* $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$, *the system can transition to a higher return to IOUs.*

Proof. When prices are increasing in each period, $\frac{1}{\rho_{t+1}} > 1$. Thus, the secondary market holds with equality. When $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$ entrepreneurs choose long-term participation, but lenders are indifferent. The secondary market clearing condition (equation 24) reduces to

$$l_{1t+1}^{*} = (1 - \Pi_t)\frac{l_{1t+1}^{*}}{l_{1t+2}^{*}}\rho_{t+1}A'(\kappa)\kappa$$

Solving for ρ_{t+1} gives

$$\rho_{t+1} = \frac{l_{1t+2}^{*}}{(1 - \Pi_t)A'(\kappa)\kappa}$$

Updating one period to ρ_{t+2} and taking the ratio of $\frac{\rho_{t+1}}{\rho_{t+2}}$ gives

$$\frac{\rho_{t+1}}{\rho_{t+2}} = \frac{l_{1t+2}^{*}(1 - \Pi_{t+1})}{l_{1t+3}^{*}(1 - \Pi_t)}$$

The goods market clearing condition (equation 23) can be used to find the magnitude of $\frac{(1-\Pi_{t+1})}{(1-\Pi_t)}$. After substituting for $\frac{1}{\rho_{t-1}} = \frac{l_{1t-1}^{*}}{l_{1t}^{*}}$ and $\frac{p_{kt-1}\kappa}{p_{x1}} = A'f(\kappa)\kappa$ from the money market clearing condition and the lender's arbitrage condition (respectively), the goods market clearing condition reduces to

$$l_{1t}^{*} - l_{1t-1}^{*} = (\Pi_{t-2} - \Pi_{t-1})A'f(\kappa)\kappa$$

With increasing prices, labor is decreasing in each period. So the left hand side of this is negative. This implies that $\Pi_{t-2} - \Pi_{t-1} < 0$. Thus, Π is decreasing in every period. That is, the proportion of lenders choosing to forego old-age market participation is shrinking in each period. Differently, the proportion of lenders participating in all exchanges grows. Thus $\frac{(1-\Pi_{t+1})}{(1-\Pi_t)} > 1$. This implies that $\frac{\rho_{t+1}}{\rho_{t+2}} > 1$, or $\frac{1}{\rho_{t+1}} < \frac{1}{\rho_{t+2}}$. The return to IOUs on the secondary market is growing faster than the equilibrium supply of labor. With long enough *t*, ρ will eventually switch to the higher return convergent path. ■

Lemma 7. *Starting with the high-return convergent path,* $\frac{1}{\rho_{t+1}} = A\frac{p_{xt+2}}{p_{xt+1}}$, *the return to IOUs will only grow. The system cannot transition back to the low return case.*

Proof. Let ρ follow the same form as above. Lenders strictly prefer to participate in all life exchanges, if able. With the above result ρ will only grow in every period. Thus, without an economic shock, ρ cannot transition to a lower return state. ■

If we start in a low return case, we can transition to the higher return case through normal growth in the return to IOUs on the secondary market. That is, the return to IOUs is growing faster than the labor supply. This puts upward pressure on the return to IOUs. As labor falls, the supply of IOUs increases, causing the price to be bid down. However, if we start in the high return case, we cannot transition back to the lower return case endogenously.

The following numerical example shows the equilibrium path of the system under the various assumptions.

Numerical Example

The dynamic movement of prices through time is considered using the same numerical example as above. Recall $u(x^*_{3t+2}) = \frac{x^{1-\sigma}}{1-\sigma}$, $g(l^*_{1t}) = -\phi(1 - l^{\sigma}_{1t})^{\frac{1}{\sigma}}$, and $f(\kappa) = \kappa^{1/3}$. Further, assume $\kappa = 1$ and A=1.5. To determine the price effect in the entrepreneur's problem, $\sigma = \frac{1}{3}$ (the income effect dominates) or $\sigma = 1.5$ (the substitution effect dominates). Results are below.

(I) The Substitution Effect Dominates Below the Monetary Steady State.

Starting below the monetary steady state in labor is equivalent to considering an inflationary path of prices (Cases 2 and 4 of Table 5). When the substitution effect dominates the entrepreneur's utility function the system can switch from a low return on IOUs to a high return on IOUs. The system cannot switch from a high return to a low return. Starting with $\frac{1}{\rho_{t+1}} = \frac{l^*_{1t+1}}{l^*_{1t+2}}$, the behavior of $l^*_{1t+1}(l^*_{1t})$ is examined. In each period, the value of $\frac{1}{\rho_{t+1}}$ is compared against the boundary conditions for the case. When the difference between $\frac{1}{\rho_{t+1}}$ and $\frac{1}{\rho_{t+2}}$ is greater than *A* (set at 1.5), the system has reached a tipping point and is now governed by the behavior for Case 4 in Table 5 where $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$, rather than Case 2 where $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$. With the parameters of the numerical example, the system reaches this tipping point in period 4. Figure 4 shows the trajectory of prices as parameterized. The solid vertical line represents the tipping point between Case 2 and Case 4. The dashed line represents the trajectory of $l^*_{1\cdot}$ had the regime switch not been considered. The solid line represents the trajectory of $l^*_{1\cdot}$ with the regime switching behavior. In short, switching from the lower return on IOUs to the higher return on IOUs slows the convergence to autarky as more lenders are opting to settle later in life (thus reducing the upward pressure on prices in the goods market).

Figure 5. Tipping Point with Inflationary Prices and Settlement Risk Dominating the Entrepreneur's Problem

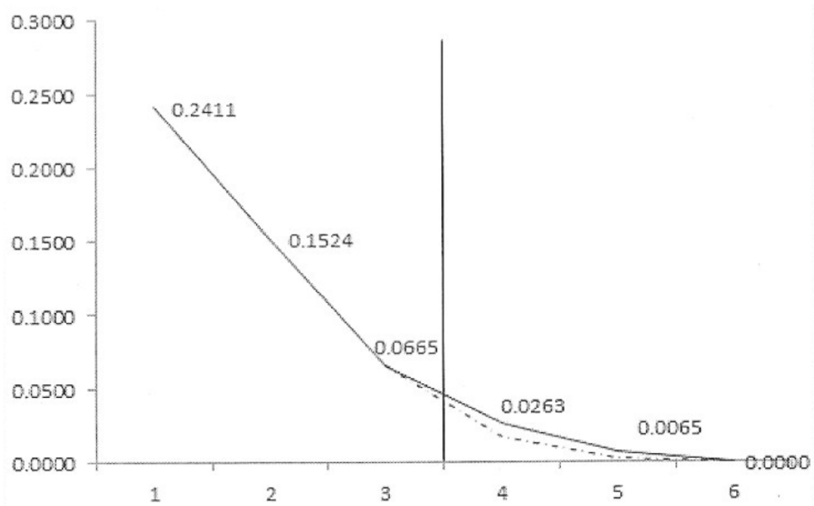
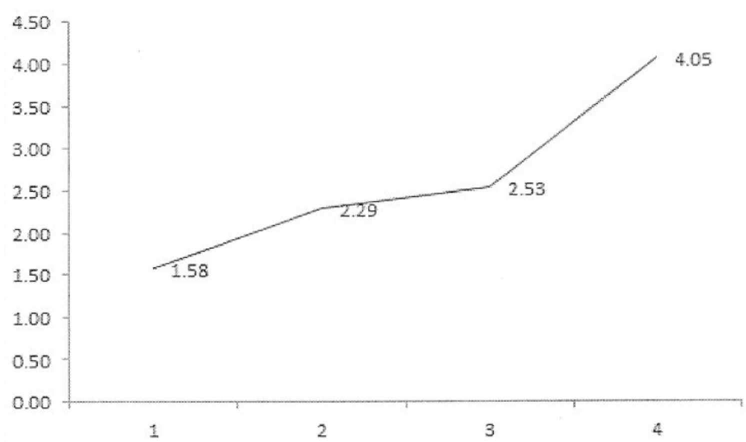


Figure 5 plots the value of ρ through time for this system.

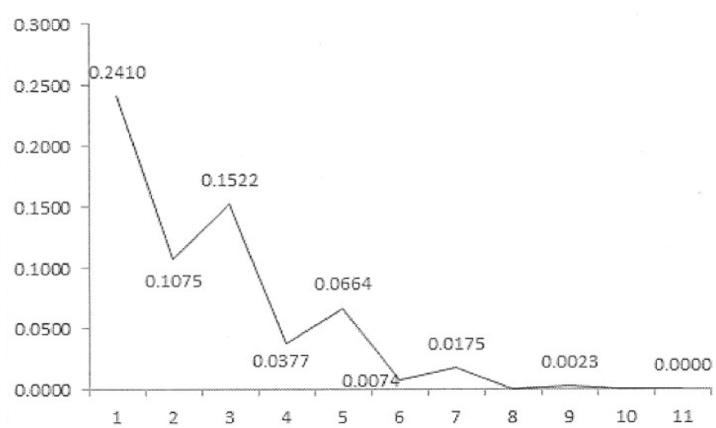
Figure 6. Value of ρ through time with regime switching



This plot confirms that the system has switched from the low return regime where $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$ to the high return regime where $\frac{1}{\rho_{t+1}} = \frac{Ap_{xt+2}}{p_{xt+1}}$. Lender's choices reduce the mid-age demand for goods, thus dampening the upward pressure on prices and slowing convergence to the non-monetary steady state.

(II) Substitution Effect Dominates Above the Monetary Steady State

Above the monetary steady state, the system is governed by a second-order, non-linear difference equation (Case 8 in Table 5). If both initial conditions are equal, for example let $l_{11}^{*} = l_{12}^{*} = 0.5$, then $l_{13}^{*} > 2$ from the difference equation. This violates the boundary conditions on amount of time entrepreneurs are able to invest in labor effort. On the other hand, if initial conditions are carefully chosen, the global dynamics can be feasible for a finite number of periods. The graph below shows the behavior of prices in equilibrium when l_{11}^{*} and l_{12}^{*} are in the neighborhood of the positive steady state.



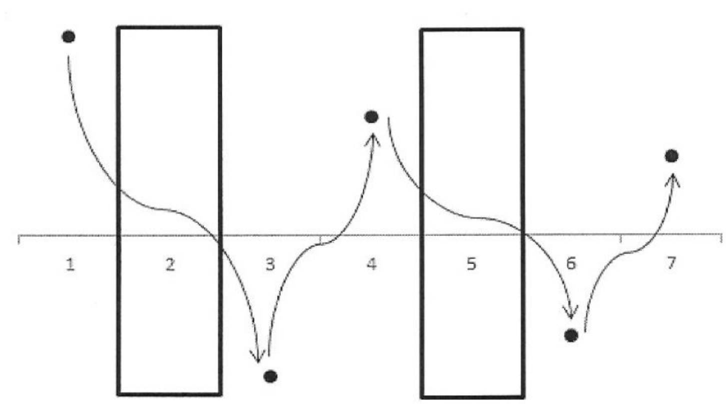
With the second-order non-linear difference equation, the system displays oscillatory convergence, regardless of the price effect in the entrepreneur's utility function. After 11 periods, the price level is near 0 and the amount of labor is near infinity.

(III) Income Effect Dominates the Entrepreneur's Utility Function

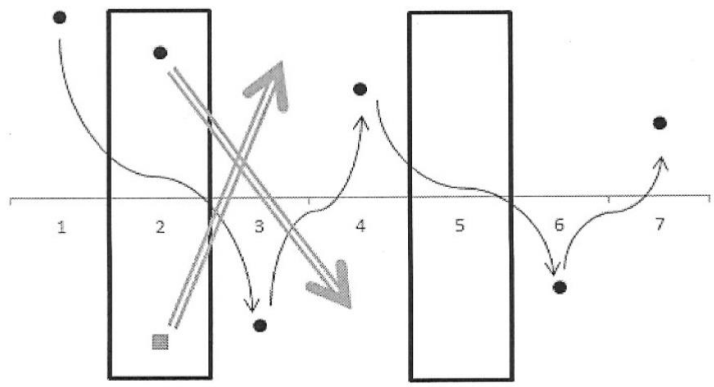
When the income effect dominates, the starting value of $l_{1\cdot}^{*}$ is irrelevant as the system oscillates. Starting above the monetary steady state means that the system is governed by a second-order, non-linear difference equation, $H(l_{1t}^{*}, l_{1t+2}^{*})$. Starting below the monetary steady state means that the system is governed by a first-order, non-linear difference equation $H(l_{1t}^*, l_{1t+1}^*)$. The difference in the periodicity of the system draws a contradiction in the behavior of prices. When the income effect dominates the entrepreneur's utility function, the system is internally inconsistent.

Lemma 8. *When the income effect dominates the entrepreneur's utility function, the dynamics in the system are internally inconsistent.*

Proof. All values above the positive steady state are governed by a second-order, non-linear difference equation. All value below the positive steady state are governed by a first-order, non-linear difference equation. Consider the case where l_{11}^* is initialized above the positive steady state. $l_{1\cdot}^*$ then oscillates with a two period cycle; l_{11}^* determines the value of l_{13}^*. The value of l_{13}^* is then below the positive steady state and governed with a first-order, non-linear difference equation. Thus, the value of l_{13}^* determines the value of l_{14}^*, as shown below:



There is no value of l_{12}^* that maintains consistent dynamics. If l_{12}^* is initialized below the positive steady state, l_{13}^* is determined simultaneously by l_{11}^* from a second-order difference equation (and should be below the steady state) and l_{12}^* from a first-order difference equation (and should be above the steady state). This draws an inconsistency. Alternatively, if l_{12}^* is initialized above the steady state, then the inconsistency is drawn in the fourth period. l_{14}^{*} is jointly determined by l_{12}^{*} to be below the positive steady state and l_{13}^{*} to be above the steady state. The inconsistency is shown below.



The black dot representing l_{12}^{*} shows the inconsistency in the fourth period. The gray square representing l_{12}^{*} shows the inconsistency in the third period. If l_{11}^{*} is initialized below the positive steady state, the same inconsistency occurs. ■

Summary

When the substitution effect dominates the entrepreneur's problem and the initial value of labor is below the monetary steady state, the system converges to autarky and displays a tipping point between the equilibriums (Case 2 and Case 4 from Table 5). When the initial value of the system is above the monetary steady state, the system can display oscillatory divergence with a careful choice of l_{11}^{*} and l_{12}^{*}. When the income effect dominates, the system is inconsistent with the value of either l_{11}^{*} or l_{12}^{*} drawing a contradiction in l_{13}^{*} or l_{14}^{*}.

Settlement Risk

The over-arching scope of this project is to examine the impact of settlement risk on the price-dynamics in the Overlapping Generations Model. Within this specification, two assets are valued – fiat money and IOUs (also described as bonds). The equilibrium dynamics are derived for trajectories away from the steady state where the Friedman Rule was optimal. The dynamics are stable when the substitution effect dominates the entrepreneur's utility function and the initial value of labor is below the monetary steady state (Cases 2 and 4 in Table 5, Case I in Table 7). In this situation, labor is decreasing in each period, implying that the system is diverging. If the system starts above the monetary steady state in labor (Case 8 in Table 5, Case II in Table 7), careful choices of the initial conditions can produce an oscillatory convergence to autarky. On the other hand, the dynamics are never stable when the income effect dominates the entrepreneur's problem.

Settlement risk is a feature of the model. The liquidity constraint is endogenized through the choice of production technology, participation frictions in the old-age settlement market, and the unverifiability of bonds in the goods market. Settlement risk becomes tractable through the exogenous parameter π, which controls market participation and is determined by nature. When $\pi = 1$, all lenders are able to participate in old-age settlement. There is no matching friction in this economy. When no lenders are able to participate in old-age settlement, $\pi = 0$. Any π between 0 and 1 represents a timing mismatch. Different lengths of debt contracts and a timing mismatch from the lenders ensures that some positive proportion of lenders will not meet with their entrepreneur in mid-age. With verifiability concerns in the goods market, lenders excluded from old-age settlement must sell on the secondary market at a discount in order to maximize consumption.

An exogenous change in settlement risk impacts dynamics through the demand for goods (from the young entrepreneur's labor supply) in the mid-age secondary market.

Changes in $(1-\pi)$ impact the market through prices. If more lenders are excluded from old-age settlement, the amount of liquidity demanded on the mid-age secondary market increases. This drives up the price of bonds on the secondary market and drives down the return. A change in the rate of return changes the proportion of entrepreneurs investing in the short-term production technology.

To investigate the impact of a change in settlement risk on the dynamics in the model economy, I calculate the comparative statics on the equilibrium law of motion for a change in $(1-\pi)$. To show this, use the secondary market clearing condition (equation 18) to write l_1^* in terms of Π. Recall that Π is the proportion of lenders that forego settlement market participation in old age. Part of these lenders are forced by nature. Additionally, lenders may choose to self-exclude, favoring early consumption. We can write $\Pi = \pi(1-\alpha)$ where π is the exogenous proportion of lenders that are able to participate in old age settlement and $(1-\alpha)$ is the portion of lenders that elect to continue to old-age settlement. When $(1-\alpha)$ is different from 0, π has a direct bearing on the demand for liquidity in the secondary market. I connect the equilibrium law of motion in labor to π through the secondary market.

For the remainder of this section, focus on deriving the impact in the case where $\frac{1}{\rho_{t+1}} = \frac{p_{xt+2}}{p_{xt+1}}$. This is easily extended to the case where $\frac{1}{\rho_{t+1}} = A\frac{p_{xt+2}}{p_{xt+1}}$. Settlement risk in the economy drives the rate of convergence along the hyper-inflationary path (Case I).

Derivation

Lemma 9. *An exogenous increase in settlement risk speeds divergence along the hyperinflationary path.*

Proof. Starting with (18),

$$l_{1t+1}^{*} = (1 - \pi(1 - \alpha))Af'(\kappa)\kappa$$

Emphasizing the relationship between π and l^{*} in equation (50) gives,

$$\frac{dl_{1t+1}^{*}}{dl_{1t}^{*}} = \frac{g''(l_{1t}^{*}(\pi))l_{1t}^{*}(\pi) + g'(l_{1t}^{*}(\pi))}{u'(\gamma + l_{1t+1}^{*}(\pi)) + u''(\gamma + l_{1t+1}^{*}(\pi))l_{1t+1}^{*}(\pi)}$$

where $\gamma = Af(\kappa) - Af'(\kappa)\kappa$ and has no impact on the derivation.

The impact of a change in π on the dynamics can be measured by $\frac{dH(l_{1t+1}^{*}, l_{1t}^{*})}{d\pi}$. The change in the slope of $H(l_{1t+1}^{*}, l_{1t}^{*})$ given an exogenous change in π. Let $\tau = g''(l_{1t}^{*}(\pi))l_{1t}^{*}(\pi) + g'(l_{1t}^{*}(\pi))$ and $\beta = u'(\gamma + l_{1t+1}^{*}(\pi)) + u''(\gamma + l_{1t+1}^{*}(\pi))l_{1t+1}^{*}(\pi)$. Then

$$\frac{dH(l_{1t+1}^{*}, l_{1t}^{*})}{d\pi} = \frac{\frac{d\tau}{d\pi}\beta - \frac{d\beta}{d\pi}\tau}{\beta^{2}}$$

Table 8 presents the derivatives of each expression and the signs of each derivative.

Table 8. Derivatives with respect to π

Derivative	Expression	Sign
$\frac{dl_{1\cdot}^{*}}{d\pi}$	$-(1 - \alpha)Af'(\kappa)\kappa$	< 0
$\frac{d\tau}{d\pi}$	$[g'''(\cdot)l_{1t}^{*}(\pi) + 2g''(\cdot)]\frac{dl_{1t}^{*}}{d\pi}$	< 0
$\frac{d\beta}{d\pi}$	$[u'''(\cdot)l_{1t+1}^{*} + 2u''(\cdot)]\frac{dl_{1t+1}^{*}}{d\pi}$	< 0

From the secondary market,

$$\frac{dl_{1\cdot}^{*}}{d\pi} = -(1 - \alpha)Af'(\kappa)\kappa < 0 \qquad (53)$$

As settlement risk increases, fewer lenders are participating in the old-age settlement markets, increasing the demand for liquidity in the mid-age settlement markets. Entrepreneurs need less labor effort to achieve the same income.

Differentiating τ with respect to π gives

$$\frac{d\tau}{d\pi} = g'''(\cdot)\frac{dl_{1t}^{*}}{d\pi}l_{1t}^{*}(\pi) + \frac{dl_{1t}^{*}}{d\pi}g''(\cdot) + g''(\cdot)\frac{dl_{1t}^{*}}{d\pi}$$

Assuming the cost function displays a positive third derivative – which is consistent with the Inada Conditions – the derivative is negative, taking the sign of (53).

Differentiating with respect to β gives

$$\frac{d\beta}{d\pi} = [u'''(\cdot)l_{1t+1}^{*} + 2u''(\cdot)]\frac{dl_{1t+1}^{*}}{d\pi}$$

Much like before, the sign depends on the curvature of $u(\cdot)$. The third derivative of the utility function $u'''(\cdot)$, is often termed to measure an individual's "prudence" – that is , how quickly do individuals increase precautionary savings in the presence of background risk.[13] For the constant elasticity of substitution function used in the simulations, the third derivative is ambiguous. If the second derivative of the utility function dominates in magnitude, then $\frac{d\beta}{d\pi} > 0$. If the third derivative of the utility function dominates in magnitude, then $\frac{d\beta}{d\pi} < 0$. Without clear theory and a need for precautionary savings in this model, assume that $u'''(\cdot) < 0$ giving $\frac{d\beta}{d\pi} < 0$. ■

[13] See Kimball (1990) for an origin of the term. Noussair, Trautmann, and Van de Kuilen (2013) provide a discussion. Huggett and Vidon (2002) show that a positive third derivative is not enough to induce precautionary savings.

The sign of $\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi}$ also depends on the whether the income or substitution effect dominates the entrepreneur's work preferences. Working with the substitution effect, gives $\beta > 0$. This implies that

$$\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi} < 0$$

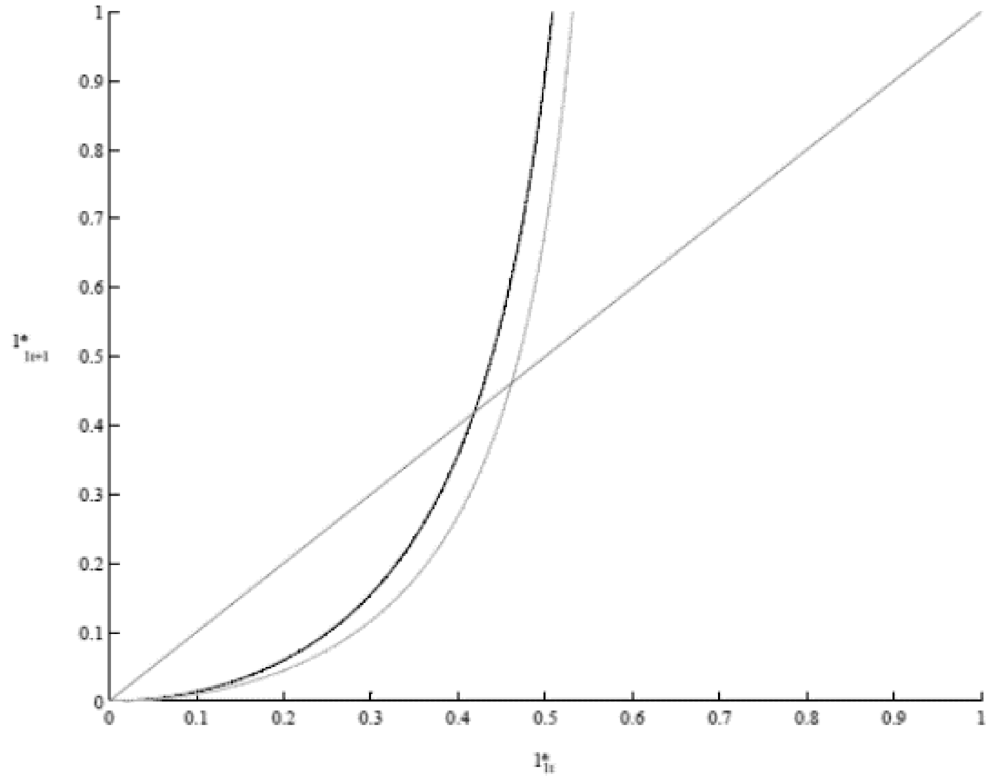
as formalized in Lemma 10.

Lemma 10. $\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi} < 0$ *when the substitution effect dominates the entrepreneur's choice of labor.*

Proof. Under the assumptions that $g'''(\cdot) \geq 0$ and $u'''(\cdot) \leq 0$, and the results that $\frac{d\tau}{d\pi} < 0$ and $\frac{d\beta}{d\pi} > 0$, it follows directly. ■

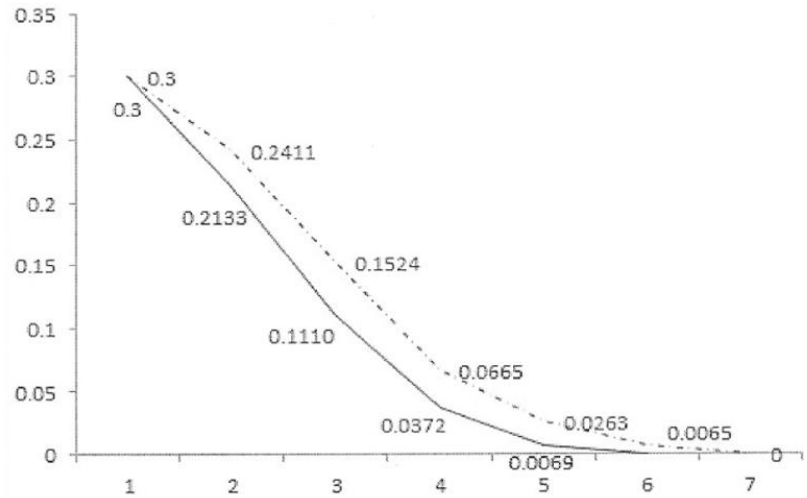
An exogenous increase in settlement risk (shown in the model as a decrease in π) increases the demand for bonds in mid-age secondary market. Workers respond by providing more work effort, which causes the slope of the equilibrium law of motion to flatten (become less positive). The impact of an exogenous increase in settlement risk is shown in Figure 7.

Figure 7. Exogenous Increase in Settlement Risk when the Substitution Effect Dominates the Entrepreneur's Preferences



The black line shows the original equilibrium law of motion $H(l^*_{1t+1}, l^*_{1t})$ from Figure 2. The gray line shows $H(l^*_{1t+1}, l^*_{1t})$ after an exogenous decrease in π by 0.1 (10% more agents are excluded from old-age settlement by nature). Starting below the positive steady state (inflationary prices) speeding divergence away from the steady state. Both utility and cost channels respond in the same way to an increase in settlement risk – higher risk leads to a higher return in the mid-age secondary market. With the substitution effect dominating the entrepreneur's problem, agents provide more work effort (in order to increase the number of goods available with which to purchase IOUs).

The trajectory of $l^*_{1\cdot}$ through time is plotted below. The dashed line is the original trajectory of labor. The solid line represents the equilibrium choices of labor after an increase in settlement risk when the income effect dominates the entrepreneur's problem and the initial condition is below the steady state value.



An exogenous increase in settlement risk flattens the equilibrium law of motion, speeding divergence from the positive steady state along the hyperinflationary path.[14]

Conclusion

Using an Overlapping Generations model with 3-period lived agents, I examine the impact of an exogenous increase in settlement risk on the dynamics of both labor and

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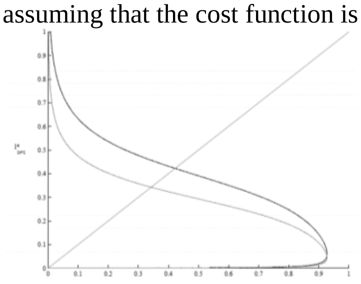
[14] **Income Effect.** When the income effect dominates, $\beta < 0$. The sign of $\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi}$ is indeterminant. If the cost function (and thus τ) is more responsive to changes in π, then $\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi} > 0$. On the other hand, if β is more responsive to changes in π, then $\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi} < 0$. Figure 4 graphs the impact on $H(l_{1t+1}^*, l_{1t}^*)$ assuming that the cost function is more responsive to changes in π.



With $\frac{dH(l_{1t+1}^*, l_{1t}^*)}{d\pi} > 0$, $H(l_{1t+1}^*, l_{1t}^*)$ flattens. The amplitude of the convergence path increases, increasing volatility in price along the equilibrium path. The amount of labor needed to sustain the positive steady state falls.

prices on off-steady state, equilibrium trajectories. I find that the impact of settlement risk depends on whether the income or substitution dominates the entrepreneur's preferences. When the substitution effect dominates, an exogenous increase in settlement risk speeds divergence away from the positive steady state. That is, as settlement risk increases, the system becomes more volatile.

Chapter 2: Literature About Value Added Modeling

Introduction

Improving public education has the attention of researchers and practitioners, with most studies focusing on improving student outcomes through teachers. Teachers, specifically teachers with seniority, represent the largest line-item expenditure for school districts (see Ballou and Podgursky (1997, 2002)). Additionally, intuition and experience suggest higher quality teaching makes the biggest difference to individual learners. However, results are mixed. While most studies agree that teachers account for the largest overall impact on students (see Koedel and Betts (2007)), few agree on what determines teacher quality – or even how to define a quality teacher.

Since the Coleman Report (Coleman, 1966), most work has centered on uncovering the educational production function. Under this umbrella, two separate research veins exist – assessing aggregate teacher quality through time and assessing the quality of a particular teacher on student outcomes. These approaches are interrelated. While it is appropriate to ask how well a particular teacher does at their job, we need to be ready with recommendations to improve teacher training, recruitment and retention. It is necessary to evaluate individual teachers and the system that creates teachers.

In this essay, I position Koedel, Leatherman, and Parsons (2012) within the existing literature. I start be reviewing the educational production function and discussing value-added modeling. After addressing some standing concerns with the modeling approach, I discuss the paper. Section 3 addresses an alternative approach to teacher quality – namely estimating aggregate quality over time.

Educational Production Function

To provide a basis for estimating teacher effects with any technique, researchers appeal to the "educational production function" put forth by Coleman. An educational production function maps inputs in the form of family influences, peer influences, community influences, and teacher and school inputs from birth through the students current grade into the student's currently level of achievement. As summarized by Hanushek and Rivkin (2006), the educational production function is generally written as

$$O_g = f(F^{(g)}, P^{(g)}, C^{(g)}, T^{(g)}, S^{(g)}, \alpha) \qquad (54)$$

The grade-g outcome for student-i is a function of Family, Peer, Community, Teacher and School inputs from birth through grade-g, conditioned on ability α. Several assumptions underlie this model. Namely, that the impact of prior experience is cumulative on the student's current achievement. This accumulation can decay, and often does. Additionally, it assumes that a student's achievement is conditioned on some underlying, unobservable latent ability.

Rather than focus efforts on each input, as would be the case with a structural estimation,[15] quantitative modeling conditions on some previous measure of achievement in an attempt to remove persistent student characteristics. Value-added models (VAM) condition on the achievement of student-i in grade-g-1 – which removes the impact of unobservable, fixed student characteristics.

The educational production function, as given in (54) is often expressed as

$$O_g = f'(O_{g-1} \middle| F^{(g)}, P^{(g)}, C^{(g)}, T^{(g)}, S^{(g)}, \alpha)$$

[15] See Hanushek (1979) for a discussion of the confounding factors in structural estimation.

to indicate conditioning the model on prior achievement, which is conditional on the cumulative impacts of the student's environment.

Value-Added Models

In practice, VAMs are fixed-effect models where the coefficients of each fixed effect is the estimation aim. Fixed-effects are used not to absorb variation in the model, but produce meaningful estimations of the teacher's average contribution to student learning in that year. They are estimated with least-squares using large student-by-teacher-by-school datasets. The contribution of a teacher to student learning is defined as the difference between the predicted score and the actual score on statewide standardized achievement exams, conditioning on observables. A typical value-added specification is

$$Y_{it} = \beta_0 + Y_{it-1}\beta_1 + X_{it}\beta_2 + T_{it}\tau + \varepsilon_{it}$$

where Y_{it} is the achievement of student-i at time-t, Y_{it-1} is the lagged score for student-i from the standardized exam taken 1 period before, X_{it} is a vector of student-level observables – such as family background, socioeconomic observables, school level information, etc. – and T_{it} contains dummy variables indicating the official teacher assignment for the student. A predicted score is estimated by conditioning for past achievement and other observables. The binary variable associated with the teacher absorbs any remaining variation in scores. The teacher effect estimate is the weighted average of this variation across all of their students.

The way in which prior year achievement enters the model depends intrinsically on the assumption the researcher makes on learning decay. When researchers assume no decay in learning, prior year test scores enter with a coefficient set at unity. These are often moved to the other side and estimated as a gains- or change- score model, as we did in Koedel, Leatherman, and Parsons. Other researchers allow the coefficient on prior achievement to vary, consider increased lag length, or some combination of above approaches. Todd and Wolpin (2003) examine different estimation techniques and show the assumptions necessary for each to be valid. All model specifications place assumptions on the evolution of student learning through time and impose restrictions on the underlying data-generating process.

Teacher-Effect Estimation Problems

Rothstein (2009) shows that fifth-grade teachers have a meaningful impact on forth-grade student gains-scores. With the exclusion restrictions violated, it's clear that accounting for persistent student characteristics still doesn't produce consistent estimates of teacher quality. One explanation for this result is the manner in which students are assigned to classrooms. Non-random assignment to classrooms biases the effect estimates, particularly when students are grouped on underlying. Receiving less attention, but just as problematic, is the non-random attrition and movement of teachers within schools and districts (see Greenberg and McCall (1974), Murnane (1981), Hanushek, Kain, and Rivkin (2004)). Both create simultaneous equations bias in the estimate of the quality of a particular teacher. An accurate teacher effect estimate depends on random matching between students and teachers. Systematic sorting will bias the teacher effect estimate, even after conditioning on past achievement.

Model specifications that address this issue are presented in Rothstein, Aaronson, Barrow, and Sander (2007) and Hanushek, Kain, O.Brien, and Rivkin (2005). In a more general form, school or community fixed effects are incorporated into the model reducing the teacher comparison group to within school peers. Rockoff (2004) addresses the issue by estimating both student- and teacher- level fixed effects to account for all unobserved student characteristics. Koedel and Betts (2011) show that a sufficiently rich VAM minimizes the bias in estimating teacher-effects. That is, enough covariates reduce the impact of non-random sorting. Using a random assignment experiment, Kane and Staiger (2008) show that controlling for prior year's achievement and mean classroom characteristics yield the highest accuracy in predicting impact on learning outcomes.

The reasonableness of any effect estimates intrinsically depends on the estimation techniques and the underlying exam construct. McCaffrey, Lockwood, Koretz, and Hamilton (2003) discuss sources of error in estimating teacher effects, mainly owing to confounding by included or omitted variables. Ballou (2009) addresses issues related to vertical scaling of the exam. He finds that the conditions necessary for vertical scaling in item response theory (IRT) exams – a class of exam-construction metric that account for the majority of standardized tests – are rarely met or often nonsensical. In the last section of his paper, he shows that VAM estimates are sensitive to the underlying scale of the exam. Mariano, McCaffrey, and Lockwood (2010) present a Baysenian estimation technique for teacher effects that can be used when exams are not vertically scaled. Boyd, Lankford, Loeb and Wyckoff (2013) provide a method to estimate measurement error from all sources using the test-retest framework. They account for more error than just what is induced by the construction of the achievement test. By using three or more consecutive tests in a single subject, they generalize the test-retest framework to account for growth or decay in skills, non-vertically scaled tests, and the degree of test-induced measurement error (TME) varying across exams. They show that Bayesian estimation of posterior means of achievement and achievement gains based on observed scores can increase the power of the estimate. Many of these methods to correct for TME rely on advanced estimation techniques.

Summary of Koedel, Leatherman, and Parsons (2012)

In Koedel, Leatherman, and Parsons, we incorporate error estimates given by the test-publisher for each possible score used into existing models as a weight. Students with low signal-to-noise ratio have scores down-weighted in the effect estimate. This low-cost approach can improve point estimates with the same amount of accuracy afforded to decreasing class size by 11- to 17- percent.

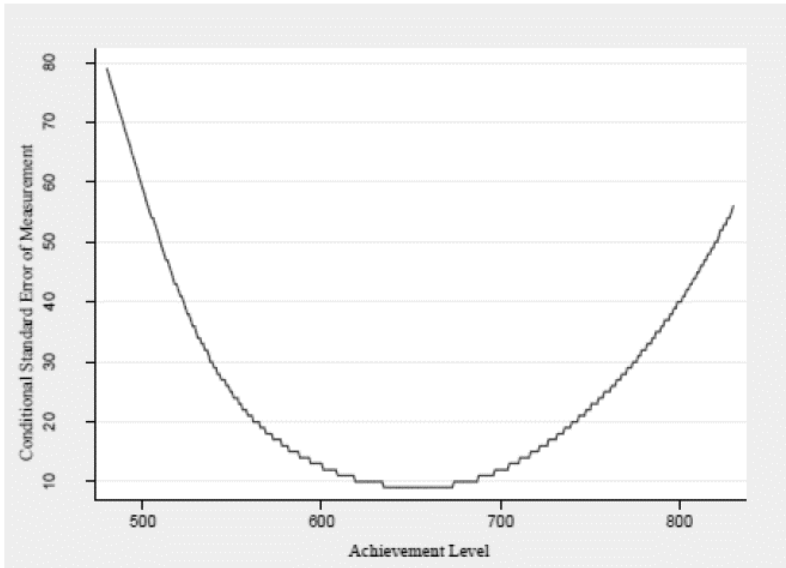
It is widely accepted that the score on a standardized test measures student learning with noise. We can divide this noise into two "types"; idiosyncratic noise owing to environmental and student factors (say the room is loud or the student has a headache) and noise induced by the testing instrument. Idiosyncratic noise in the independent variable still produces unbiased and consistent OLS estimates, at least asymptotically. However, systematic noise that is also correlated with observables (such as underlying student achievement, socioeconomic factors, and classroom selection) can produce inconsistent OLS estimates, particularly when students are non-randomly assigned to classrooms.[16] Test publishers have provided a measure or error induced by the exam.

Figure 1 in Koedel, Leatherman, and Parsons (Figure 9 below) shows the bimodal property of the TME. The correlation between the test-score outcome and error-in-dependent-variable is evident. Achievement on the exam is related to any other number of conditional variables. When students are not randomly sorted into classrooms across

[16] With non-random assignment, teacher effect estimates are based on students more tightly clustered on the test-distribution. If a teacher has a high proportion of students at either tail, their effect estimate are based more on error than peers.

ability, this implies that some teacher's effect estimates are reported based on noisier measures than counter-parts in the comparison group.

Figure 8. Distribution of Test Measurement Error in Standardized Achievement Exams



To account for the error, we propose using the estimate provided by the publisher. Through Monte Carlo simulations, we show that weighting by the inverse of the report error increases the correlation between the actual teacher contribution and the estimated teacher contribution. The positive impact decreases as the simulated class size increases. By using micro-level data from Missouri, we con.rm that accounting for the test-induced measurement error produces the same efficiency gains that would be seen from increasing the class size by 11 to 17 percent.

Alternative Approaches to Modeling Teacher Quality

Rather than estimate the impact of individual teachers on student outcomes, early work focused on estimating the dynamics of aggregate teacher quality. As Hanushek and Rivkin point out, teacher salaries have been falling since 1940. As the return to teaching declines, a basic labor supply model would suggest that the worker quality declines, too.

In an attempt to explain this trend, researchers have looked at many different aspects of teacher salary. For example, Lakdawalla (2001) posits that districts have not fully absorbed the cost of an increase in skilled labor – meaning that relative quality should decline. Studies, such as those by Flyer and Rosen (1997), Corcoran, Evans, and Schwab (2002), Bacolod (2007), and Hoxby and Leigh (2004) have examined the impact of increases in job opportunities for women on teacher quality.

While the decline in real earnings relative to other industries could impact quality, several veins of research have shown that this relationship isn't clear. For example, Ballou and Podgursky point out, the impact isn't obvious in the short-term because the decline in relative salary impacts the supply of new teachers and retention of employed teachers. Additionally, Scafidi, Sjoquist and Stinebrickner (2006) add that there are non-pecuniary returns to teaching (such as summer vacations) that may negate the impact of a lower wage. Hanushek, Kain, and Rivkin further this argument by providing evidence that non-pecuniary compensation provides a much stronger determinant of turnover than salary.

Teacher turnover is discussed by Lankford, Loeb, and Wyckoff (2002) and Boyd, et al. Podgursky, Monroe and Watson (2004) find that high ability teachers are more likely to leave teaching – but, these teachers do not leave for higher pay in other fields.[17] Chingos and West (2012) argue that the current compensation system does not account for increased wage opportunities in other industries, for high quality teachers – creating incentive for high quality teachers to exit the industry. A finding confirmed by Wiswall

[17] Scafidi, Sjoquist, and Stinebrickner (2006) show that non-pecuniary school characteristics are highly correlated with teacher turnover. Feng and Sass (2017) show that teachers who rank above their peers are more mobile. Increasing the share of more experienced, higher educated colleagues within the district decreases the intra-district mobility.

(2013), who abstracts from the wage argument, but finds that high quality teachers are more likely to leave teaching for other opportunities. Temin (2002) argues that teacher pay needs to increase to attract higher-quality applications.

In closely related worked, researchers have shown that the decline in quality matters disproportionally for urban/disadvantaged areas. Lankford, Loeb, and Wyckoff, using the New York public school database, discuss teacher sorting – arguing that the least qualified teachers select to urban, high-poverty and low-achieving schools. Clotfelter, Glennie, Ladd, and Vigdor (2008) suggest that increasing pay in urban and struggling schools increase retention, suggesting that school characteristics have some impact on teacher quality and retention. Sass, Hannaway, Xu, Figlio, and Feng (2012) show that effect estimates of teachers in schools with high poverty rates are less than their counterparts in low-poverty schools. Hanushek and Rivkin (2010), who suggest teachers that remain in their school out-perform those that move – at least in terms of student achievement on standardized exams. Boyd, Grossman, Lankford, Loeb and Wyckoff (2008) find that less effective teachers leave teaching, but more effective teachers transfer from low-performing schools to higher-performing schools.

While most agree that quality is declining, recent work has asked; "does it matter?" Differently, would hiring high quality teachers have a meaningful economic impact? Hanushek (2011) estimates the economic impact of replacing the bottom 6-8 percent of teachers with average teachers has a $100 trillion impact in future student earnings. This finding is corroborate by Chetty, Friedman, and Rockoff (2011), who find that a one standard-deviation increase in teacher quality (estimated by a VAM) increases earnings by 1% at age 28.

In an attempt to disentangle the relationship between pay and quality, researchers have focused on estimating the return to various inputs that "should" be correlated with quality, such as experience and education, performance on exams, and certification. Most findings indicate that teacher performance cannot be predicted by any measurable characteristic, even though measurable characteristics are directly related to compensation.[18] Harris and Sass (2011) find returns to experience for the first five years of teaching, but little evidence that "pre-service" training (certification programs) had any impact on student outcomes. Boyd, Grossman, Lankford, Loeb and Wyckoff support a similar finding – the most effective teacher training programs are those with a strong emphasis on practice, however other aspects were insignificant.

As Hanushek and Rivkin (2006) show the results are hardly consistent. Of 170 estimates linking teacher education to student outcomes, 86-percent were statistically insignificant. Nine studies showed a positive impact while 5 uncovered a negative relationship. The link between experience and student outcomes is only slightly clearer. Out of 206 studies (at the time of writing), 66-percent were statistically insignificant, with 29 showing a positive relationship and 5 showing a negative.[19] Kane, Rockoff, and Staiger (2008) show that certification does little to explain teacher quality on educational outcomes – teachers with the same experience and certification status have sometimes large, persistent differences in outcomes. However, using high-school data Goldhaber and Brewer (2000) show that across certification status math teachers with traditional

[18] Many summary papers exist (see Hanushek and Rivkin (2006), Wayne and Youngs (2003)).

[19] Murnane and Phillips (1981) attempts to reconcile the literature relating experience to student learning (which was already displaying divergent results) by breaking experience into three distinct categories; learning-by-doing, vintage, and selection. By using spline estimators and explicitly accounting for vintage, they show a stronger relationship between experience and student outcome. Because of potential non-linearities in the data-generating process, cross-sectional analysis can lead to inconsistent results.

certification perform better than private school certification, but those with emergency certification perform no worse than their traditionally certified counter-parts.

Chingos and Peterson (2011) show that inputs that "should" predict quality rarely matter for student outcomes – for example, university attended, educational major and master's degrees do not correlate with effectiveness, however quality does increase early in a career and may decline later. Clearly confounding results exist.

Rather than address quality through salary, other studies have look at the relationship between licensing exams and/or college entrance exams and quality or other school characteristics. For example, Podgursky, Monroe, and Watson find that college graduates with above-average ACT scores do not self-select into teaching. Hanushek and Pace (1995) point out that individuals who self-select into teaching do not differ in measurable cognitive ability from all other college entrants, but those who complete teacher training are low in the graduation distribution.

Table 9 summarizes potential input to teacher quality and the impact noted in studies.

Table 9. Inputs to Teacher Quality

Characteristic	Finding
Falling Teacher Salaries	Teacher salaries have been falling since the 1940's, but the overall impact is unclear. - Salaries are not the only compensation for teachers, - Some studies show non-pecuniary benefits are more important than salary in deciding to teach - Higher skilled teachers are more likely to leave teaching, but these individuals do not earn higher pay in other fields
Falling Quality Matters Disproportionately for Urban/Disadvantaged Areas	Least qualified teachers often sort into urban areas - Paying urban teachers would reduce turnover suggesting that school characteristics have some impact on teacher turnover
Teachers impact student earnings	A 1-standard deviation increase in teacher quality increases student earnings by 1% at age 28.
Inputs to teacher quality are difficult to estimate	Most studies indicate that teacher performance cannot be estimated by any measurable characteristic

Conclusion

Value-added modeling is used in high-stakes environments to assess the quality of an individual teacher on student outcomes. While it is obvious that quality has been declining, few agree about what "makes" a quality teacher and what can be done to train better teachers. Because of the complexity of the data-generating process, there are serious problems with model specification .sometimes stemming from the exam construct, sometimes from the underlying optimization problem of agents, etc.

In Koedel, Leatherman, and Parsons, we attempt to address one such issue; namely, how does test-induced measurement error impact VAM estimates. We find that accounting for TME error alone improves effect estimates. Following a more general approach, such as that presented by Boyd, Lankford, Loeb and Wyckoff (2013), can only further increase accuracy of the effect estimates.

Chapter 3: Informational Efficiency of the US Corporate Bond Market

Introduction

The US Bond Market represents an important source of liquidity for firms. On average, daily trading volume equals $31.3 billion in the secondary debt market. While the stock market is considered "efficient", legal differences between the stock and bond market[20] may imply that the bond market does not respond the same way to informational shocks.

Fama (Malkiel and Fama, 1970) articulated the link between prices and information for the equity markets: "A market in which prices always 'fully reflect' available information is called 'efficient'." His work created what is known as the efficient market hypothesis. With subsequent work, the efficient market hypothesis was divided into three subtypes: one stating that the market price includes all past publicly available information (weak efficient market hypothesis), a second stating that price responds to all current publicly available information (semi-strong efficient market hypothesis), and a third stating that the market price responds to all information available both public and private (strong efficient market hypothesis). The efficient market hypothesis has a strong intuitive appeal -- if asset prices failed to reflect the information available, an opportunity for arbitrage exists. But the act of arbitraging a security itself would cause the asset price to adjust so that the arbitrage was no longer profitable.

[20] For example: preferential treatment of bond holds in bankruptcy and default

Several theories and extensions imply that this should also hold for the bond market. Katz (1974) discusses an extension of the efficient market hypothesis to debt. Taking a different approach, Modigliani-Miller Theorem states that the specific finance portfolio of a firm is irrelevant as the overall cost must remain the same. That is, both the bond and the stock market must be simultaneously efficient and offering the firm the same interest charges. Were they not, an investor could earn differing returns on the same exposure.

This paper examines the informational efficiency of the US Corporate Bond Market by examining the dynamic response of the bond spread[21] to new information. This paper proposes quantitative answers to the question: "What is the impact of exogenous information regarding credit worthiness of a firm on the price of bond over time?" that is, "What is the integral over time for the impulse response function for the US bond market?" by using a robust panel data set and standard least squares regression techniques. Evaluating the dynamic impact furthers an existing body of literature that views a change in the information set as having a static impact on the credit spread. By expanding the time horizon, I show that the bond market does incorporate new information into the price of the debt on the secondary market and the convergence in price to the pre-shock level follows an oscillatory pattern.

The initial issue of the credit default swap (CDS) is used to proxy a change in the information set regarding the creditworthiness of a firm. A set of binary variables indicating the date the CDS first appeared for a specific firm is incorporated into an ad-hoc credit spread regression. Changes in the spread attributed to information conveyed by

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[21] The bond spread is measured as *Yield to Maturity – Term Matched Treasury Security*.

the CDS is measured by the parameter estimates and statistical significance of those parameters.[22]

Quantitatively evaluating the price dynamic both before and after the CDS issue provides a test of the efficient market hypothesis within the debt market. Additionally, I distinguish between strong, semi-strong, and weak efficient market hypotheses as it maps the price convergence through time after an informational shock that is first known to some, then to the market overall.

Credit Default Swap

At the end of June 2017, the Bank of International Settlements estimated the notational value of all derivatives be $542 trillion (USD) covering $13 trillion in assets.[23] CDSs comprise $8.3 trillion of this market. The average CDS contract exceeds $5 million.

The relationship between the credit default swap on the bond market has been studied, both theoretically and empirically. The credit default swap is little more than a vanilla insurance policy. In exchange for a series of premiums, the seller promises to pay the holder if a particular firm (termed the “reference entity”) defaults on their bonds. The bond need not be owned by the CDS holder. A CDS can be issued covered (when the CDS holder owns the bond) or naked (when the CDS holder does not own the bond). A covered CDS creates a risk-free security bundle, while the naked CDS is used to speculate against future credit worthiness of a firm. Investors purchasing a covered CDS create a risk-free security when held to maturity (Blanco, Brennan, and Marsh, 2005). On

[22] This approach was chosen for its prevalence in the literature, accuracy in explaining bond spreads, and simplicity of interpretation.

[23] This is down from At the end of 2015, the notional value of credit defaults swaps (CDSs) was estimated to be $553 trillion (USD) by the Bank of International Settlements covering $15.5 trillion in underlying assets (measured in market value).

the other hand, a naked CDS is equivalent to taking the short position on a bond (Stulz, 2010).[24]

Credit default swaps represent an important counterpart to the US debt market. CDS contracts require an International Swaps and Derivatives Master Agreement (ISDA Agreement) and are limited to large investment houses, banks, and commercial investment firms. In its own right, the CDS market is interesting. CDSs form a web of linkages between banks and other financial institutions amplifying the impact of financial shocks in the economy (Jorion and Zhang, 2009). The underlying network created by the swaps amplified the contagion effect between banks, contributing the 2007-2008 financial crisis (Stulz, 2010).

At its core, the CDS is a trade in risk. An entity owning a firm's bond or speculating against default is able to purchase an insurance policy that pays a set amount if default does occur. Risk is transferred to a party more able to bear the consequences of a default. Like other financial instruments, after the CDS is first issued they trade over the counter (OTC) where the end user of the CDS trades through a dealer who may or may not have their own stock. Buyers of CDSs tend to purchase proactively. Prior to default, the spread on the CDS widens, indicating that demand for a particular firm's CDS increases.

A CDS is issued only after it is requested, either for protection against or to speculate on default. The date the CDS is issued contains a signal about the creditworthiness of a firm. Initially, it is only known or believed by the person requesting the CDS. After the CDS is issued, it becomes public knowledge. By using the CDS as a

[24] The market impact is different: shorting bonds reduces market volatility by increasing liquidity in the market; trade in naked CDSs appears to increase volatility.

proxy for new information, I uncover the response of the bond market to new information about a firm.

Literature Review

This is an asset pricing paper that focuses on exploiting changes in the information set to determine efficiency within the market. Both the literature on asset pricing, as well as the link between the CDS and bond market and the studies on efficiency within the bond market play a key role in the background for this study. The first develops a test of the efficient market hypothesis in both the debt and equity markets. The second links the bond and CDS markets together. Taken together, they address the base questions considered in the paper; (1) the price of the bond responds to changes in relevant information known about that bond, and (2) the CDS issue represents relevant information to the bond market.

Current research into asset pricing utilizes structural models or factor models, though an increasing body of literature explores ad hoc[25] models as a reliable alternative for absorbing known variation in asset spreads.

Structural models, pioneered by Modigliani and Miller (1958), Black and Scholes (1973), and Merton (1974), explain default risk as a function of the firm's expected future cash flows. Assets are priced based on the probability of default and the recovery value at maturity. In this way, assets are viewed as European Call Options, however models may be extended to account for early sales. Yields are estimated with the value functions

[25] The bond pricing literature defines structural models as firm value based models of credit -- such as Merton (1974). Reduced form, or ad hoc models link prices directly to default probabilities and other observable factors -- for example, see Jarrow and Turnbull (1995). This project uses OLS to directly link yield to maturities to observable characteristics and follows the reduced form approach. The structural approach is presented in the appendix.

derived from Black-Scholes-Merton (BSM). Accuracy is based on the assumption made in the model, namely;

> BSM.1: All options are European and are exercised only at the time of expiration;
> BSM.2: No dividends are paid out;
> BSM.3: Market movements are unpredictable (markets are efficient);
> BSM.4: No transactions costs;
> BSM.5: The underlying volatility and risk-free rate are constant;
> BSM.6: Returns on underlying stocks are regularly distributed.

While grounded in theory, structural models are known to present problems in estimation. When calibrated to historical rates, structural models tend to underpredict the credit spread (see Jones, Mason, and Rosenfeld (1984), Eom, Helwege, and Huang (2004), and Huang and Huang (2012)). Further, structural models may have little power in predicting credit spreads in general (Avramov, Jostova and Philipov (2007), and Schaefer and Strebulaev (2008)). Collin-Dufresn, Goldstein, and Martin (2001) show structural models do not perform well when explaining dynamic changes in bond spreads.

Fama (1970), Fama and MacBeth (1973), and Fama and French (1993), and Carhart (1997) pioneered factor modeling of debt prices. Building on the intuition learned in structural models, the spread is typically modeled as a function of the time to maturity of the debt (T-t), leverage of the firm (B), risk-free reference rate (r), value of the firm's assets (V_t), and volatility in the firm's assets ($\boldsymbol{\sigma}^2$). Factor models are ad-hoc, only attempting to explain the variance in spread and produce reliable estimates, rather than understand or provide insight into the variance itself. Some recent bond pricing factor models include as Bao, Pan, and Wang (2011); Elton, Gruber, Agrawal, and Mann (2001); and Houweling, Mentink, and Vorst (2005). Diebold, Piazzesi, and Rudebusch (2005) overview factor modeling, discussing common questions with implementing factor models.

Operationally, the accuracy of a factor model depends on the empirical modeling approach taken. Linear factor models can be estimated with Maximum Likelihood, Generalized Method of Moments, or Ordinary Least Squares (OLS). All the estimation restrictions apply. Estimating the model with OLS is common in literature. The entire set of Gauss-Markov and normality assumptions on the errors need to hold in order to recover reliable estimates of the factor loadings and use statistical inference.

Two different tests of market efficiency have been developed. The "event oriented" approach analyzes market efficiency by investigating the speed with which new information is incorporated into the market price. Numerous studies, including this project, utilize this method.[26] Alternatively, the "time-oriented" approach investigates the ability to use past price to predict the future and offers a test of the weak-form of the efficient market hypothesis.[27] Using the CDS issue as exogenous information to the bond market is evaluating market efficiency with the "event-oriented" approach.

Katz provides justification for extending the "event-oriented" approach to the debt market by analyzing the speed of price convergence in the debt market in response to an unexpected change in bond reclassification. Katz argued that, to date, stock market efficiency had been well studied both on the "time-horizon" and "event oriented" axes, but the bond market lacked the same depth of research. By using the bond spread in the one-year before and 5 months after a change in bond reclassification, he was able to show that (1) there was no anticipation in bond classification and (2) there appears to be a 6-10

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[26] See Katz (1974), Hettenhouse and Sartoris (1976), Grier and Katz (1976), Weinstein (1977), Davis, Boatsman, and Baskin (1978), Urich and Wachtel (1984), and Smirlock and Yawitz (1985) to name a few.
[27] For example, see Roll (1970), Cargill (1975), Fama (1975), Conroy and Rendleman (1983), Schiller (1979) and Schotman (1996).

week lag in incorporating new information into the price of the bond. His approach provides a test of market efficiency while the time lag provides evidence of a market slow to incorporate relevant information.

Kroon (1991) further develops this idea in both the debt and money markets by connecting empirically observed changes in price to the rational expectations hypothesis. Using evidence from the Dutch Bond Market between 1974 and 1990, he concludes that we cannot reject the weak form of the efficient market hypothesis using a “time-oriented” approach in the debt market.

Work by Hotchkiss and Ronen (2002) developed an event oriented test of the efficient market hypothesis by using a unique dataset of high-yield corporate bonds to estimate the speed of convergence in the price after an earnings announcement. Their results run contradictory to prior work -- suggesting that the debt market behaves like the stock market as new information is incorporated quickly even with short-return horizons. Their results suggest that the debt market may indeed follow a semi-strong form of market efficiency.

Studying bond market efficiency with an event-oriented approach -- one where the study focuses on the change in price caused by an exogenous change in information -- has been used many times for the equity market. In the debt market, Katz, Kroon, and Hotchkiss and Ronen develop event-oriented approaches to examining the informational efficiency in the bond market. This paper follows their approach, exploiting a link between the debt and CDS markets.

The link between the bond and CDS market is fundamental to the claim that the CDS market provides exogenous information to the bond market. The point in time link between the CDS and bond market has been investigated. At the highest level, the CDS adds to the set of Arrow-Debreu-McKenzie Securities allowing investors to trade in risk specifically (Fostel and Geanakoplos, 2012) increasing liquidity in the debt market. Stulz (2010) cites two ways the CDS impacts liquidity specifically -- (1) by transferring risk to parties more able to bear it, and (2) increasing efficient market clearing. Similarly, Oehmke and Zawadowski (2016) suggest that the CDS market provides an "alternative trading venue". While investors can typically make the same economic trade in the debt market, the CDS market is often used because the trading frictions are smaller. They note that the notional value of a reference entities CDS is larger when the entities bonds are fragmented into many separate issues, suggesting that the CDS market serves a liquidity and standardization role with respect to the bond market.

Blanco, Brennan, and Marsh (2005) establish that CDS trading onset informs the bond market about the risk characteristics of the underlying bond increasing information and market clearing efficiency. As would be expected, the message conveyed by the signal depends on the underlying characteristics of the firm and the type of CDS being purchased. Ashcraft and Santos (2009) find that CDSs reduce the cost of financing for "better" than average firms – as defined by information opacity and credit rating. If the firm is viewed as informationally transparent, investors are allayed when a CDS is issued and the price of the bond is bid up. The presence of a CDS increases the cost of financing for all others.[28] However, if the company is viewed as informationally opaque or lacking creditworthiness, a CDS issue is viewed negatively and the bond price is bid down. In

[28] Intuitively, if the firm is viewed as informationally transparent, investors are allayed when a CDS is issued and the price of the bond is bid up. However, if the company is viewed as informationally opaque or lacking creditworthiness, a CDS issue is viewed negatively and the bond price is bid down. In short, the CDS issue confirms investor's priors.

short, the CDS issue updates investor's priors. Darst and Refayet (2015) present a different signal conveyed by the CDS itself. If the CDS is used to hedge a bond against default, the presence of the CDS lower the cost of financing on the debt market.[29] On the other hand, if a CDS is purchased to speculate, the cost of financing increases -- investors believe the speculation to be a signal of declining creditworthiness.

Amato and Gyntelberg (2005) view the CDS as an insurance contract over the bond, connecting the CDS and bond markets through traunching of credit risk. While also increasing liquidity, this view presents some interesting counterpoints to the studies above. If CDSs were equivalent to traunching and securitization we would expect the underlying value of the asset to increase (Duffie and Garleanu (2001), for example). Empirically, however, Stulz (2010) finds that CDSs reduce the value of the underlying asset.

On the other hand, Subrahmanyam, Tang, and Wang (2014) ask "does the tail wag the dog?" and suggest that the CDS may not provide new information to investors -- its issue may be the consequence of information already available to the bond market and not information unique to a specific subset of investors. Following this logic, firms do not appear as a reference entity randomly, but chosen based on some unobservable, firm-level characteristic. If this is the case, the CDS issue would have no impact on the price of the bond. With the buyers of CDSs limited to institutional buyers, it is possible that they are responding to information not widely known.

The various theoretical relationships between the debt and CDS markets, plus differences between the bond and equity market leave opportunity for research in this

[29] Intuitively, the security is transferring risk to parties more able to bear it, driving down the risk premium paid by firms.

area. Noting differences between the legal structure of stocks and bonds, the length of time needed for the bond market to incorporate new information -- the integral over time -- is open for my study in both the quantitative and theoretical realms. This project uses an "event-oriented" approach to study the relationship between the bond and CDS market, thus commenting on the semi-strong form of the efficient market hypothesis within the bond market.

By combining these two bodies of literature, using a robust panel dataset of corporate bonds, I can use an information shock related to the risk characteristics of the bond (the issue of a CDS) to examine the speed with which the information is incorporated into the debt market by applying standard panel data modeling techniques to financial market data.[30]

Research Method

A panel dataset consisting of all bond trades between 2002 and 2010 was used for this project. Notable variables include individual bond trade information such as price paid and day of the trade, firm covariates such as earnings and market-to-book value, bond covariates such as face value and coupon rate, and the CDS trading onset date.

Consistent with the life cycle of the firm, debt issue comes from a specific subset of large, well established firms. These firms favor debt over other sources of financing (including equity issue and bank loans) usually owing to the total dollar value they need to raise.

[30] As with all "event oriented" studies, there is the potential for simultaneity or reverse causation. The CDS may not provide new information to investors but be the consequence of information already available, as discussed in Fostel and Geanakoplos (2012) and Subrahmanyam, Tang, and Wang (2012). Firms do not appear as a reference entity randomly, but chosen based on some unobservable, firm-level characteristic. By following the approach presented by Katz, I am able to investigate the potential for simultaneity or reverse causation inside the dataset.

The data was subset into the firms that just trade on the CDS market and contains 444698 trades for 270 bonds in the 8 years between 2002 Q3 and 2010 Q3. The average number of outstanding bond issues for these firms at any point in time in the dataset is 7. That is, each company that also appears as a reference entity in the CDS market also has 7 outstanding bond issues, on average. These companies are both larger than average with a higher than average number of outstanding bonds. The average company profile is below:

Company Profile (in thousands)	
Average Annual Revenue**	$48,089.77
Average Net Income**	$4,645.00
Average Outstanding Debt	$141,574.80
Average Assets	$186,824.00
Average Market to Book	2.52

That said, several large companies (notably Time-Warner and Verizon) have unusually high number of issues, skewing the right tail of the distribution. Throughout this time period, the average yield to maturity is 5.49%, yielding credit spreads of just over 1%. This is slightly lower than the all-time market average of 1.21%. Descriptive statistics for the bonds is presented in Table 10.

Table 10. Descriptive Statistics of Bonds

Descriptive	Proportion
Senior Notes	66%
Average Coupon Rate	5.75%
Investment Grade*	90%
Average Yield to Maturity	5.49%
Average Spread over Treasury	1.41%
Average Spread over Swap	1.01%
Average Time to Maturity	13.14 years

Median Time to Maturity	9.28 years

Bond trade information is from FINRA-TRACE. CDS data comes from a proprietary dataset consisting of all single entity trading onset dates for all CDSs between 2002 and 2010. The CDS trading onset date is the date the firm first appeared as the reference entity for the CDS contract.

Parameters affecting the bond spread will be estimated using a standard OLS fixed-effect regression. The entire set of Gauss-Markov and normality assumptions on the errors need to hold in order to recover unbiased and consistent estimates of the parameter estimates and use statistical inference. Specifically:

OLS.1: The model is linear in parameters
OLS.2: The observations are obtained by random sampling
OLS.3: $E(e_i|f_j) = E(e_i) = 0\ \forall i = 0 \dots n, j = 1 \dots k$ The mean of the error term is zero.
OLS.4: There is no multi-collinearity in the model.
OLS.5: Errors are spherical
OLS.6: Conditional on the regressors, errors follow an identical and independent normal distribution

Assumptions 1 through 4 ensure unbiased estimates of the intercept and parameters. OLS.5 ensures consistent estimates. Without OLS 5, estimates are consistent, but OLS is inefficient. When OLS.1-OLS.5 is supported by the data, the OLS estimate of the factor model is Best Linear Unbiased Estimator (BLUE) and statistical inference is valid. OLS.6 extends OLS.5 to pinpoint the exact variance and distribution of the errors and allows application of standard hypothesis testing using Student’s T-distribution.

Most high frequency finance data will invalidate some of these assumptions by displaying non-linear functional forms, autocorrelation and heteroscedasticity in the errors, high kurtosis and non-normality of the underlying distributions, and/or outliers with both leverage and influence. When OLS assumptions are not met by the data, the factor loading estimates are biased and inconsistent. However, Sakowski, Slepaczuk, and Wywial (2016) found that, even with the aforementioned issues, OLS estimates are still similar to those obtained through Maximum-Likelihood and Generalized Method of Moments.

Firm level variation based on unobservable characteristics also presents a problem in obtaining unbiased and consistent parameter estimates with OLS. Campbell and Taksler (2003) show that idiosyncratic firm-level variation explains the same proportion of bond spreads as credit rating. This observation suggests that data generates from a process that violates OLS.3 and must be modeled explicitly. Two types of models are known. A fixed-effect is one in which each unit of observation (for example firm, company, individual) displays a time invariant mean. Intuitively, each firm has the same factor loadings, but a different baseline spread that is constant through time. A random effect is one in which the firm-specific baseline spread is random; following a random walk for example.

Choosing an incorrect model for the idiosyncratic process can exacerbate the bias in the parameter estimates. A random-effects model is inconsistent if the firm-level effect is time-invariant. On the other hand, if the firm-specific mean is derived from a random data generating process the fixed-effects model remains consistent but loses efficiency. To guard against inconsistency, the fixed-effect model specification is appropriate, and often used, for this data structure.[31]

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[31] This is solidified in Data Assumption 1: Assume that an unobservable, firm-level factor is correlated with a latent, firm-specific, time-invariant mean. Under Data Assumption 1, a fixed-effect OLS model specification is BLUE.

Fixing ideas, the baseline estimation equation for this project relates the bond spread to a set of firm and bond specific observable characteristics, firm fixed effects to capture unmeasurable characteristics impact the spread, and time specific macroeconomic binary variables to absorb overall economic variation. The bond spread is calculated as the yield to maturity at time t minus the treasury security with the same time to maturity, though some evidence suggests that the repo rate is the relevant risk-free rate for investors. The baseline estimation equation is:

$$r_{i,t,j} = \alpha_0 + \beta_1 f_1 + \cdots + \beta_k f_k + \gamma_i X_i + \gamma_t T + e_{i,t,j} \tag{55}$$

where $r_{i,t,j}$ represent the spread of bond-*i*, at time-*t* issued by firm-*j*. The subscripts *i* indexes the bond, t index time, j indexes the firm. The set f represents characteristics specific to the bond or firm such as *LN(Sales)* to control for overall firm risk, *Profit Margin* to control for the firm's ability to generate a profit from sales (*Net Income/Sales*), *Leverage* (*debt/total assets*) controls for existing debt of the firm, *market-to-book* calculated as the firm's market cap divided by book value, *LN(time to maturity)* the log of the number of the years remaining until the bond is retired, and *Bond Grade* compares the relative spread of investment grade to high yield bonds, issued by the same firm. Because the regression also includes a fixed effect by firm (*X*) and time (*T*), all variation in the parameter estimates of β are comparing differences in bond issues within the same firm, not across firms. The parameter space consist of $\{\alpha_0, \beta_{1\ldots k}, \gamma_{1\ldots j}\}$ which are all taken to be constant through time. α_0 is the intercept representing the market average spread, $\beta_{1\ldots k}$ is the factor loading for the firm and bond specific characteristics (1 through k), and $\gamma_{1\ldots j}$ represents the fixed effect estimate for the firm and time specific binary variable. The regression is weighted at the firm-level by the inverse of the number of actively traded bonds. With firm-level fixed-effects, $r_{i,t,j}$ may be interpreted as the weighted average of credit spreads of bonds issued by firm-*i* at time-*t*.

LN(Sales) is expected to lower the bond spread -- a larger corporation should have a lower spread, ceteris paribus. Larger firms have better diversification in their income stream representing lower overall risk. *Profit Margin* is also expected to lower the bond spread. The larger the profit margin, the fewer dollars in sales it takes to earn a fixed amount of profit. Differently, firms with a larger profit margin are better equipped to generate profit. *Leverage* (*debt/total assets*) is expected to increase the bond spread. A firm with more leverage, all else equal, has a higher debt-to-asset ratio, representing an increase in risk. The impact of *market-to-book* on the bond spread is ambiguous. Although growth opportunities are vulnerable to financial distress, there could be a negative effect if it represents the liquidation value of the firm. *LN(time to maturity)* is expected to carry a positive parameter estimate. The longer time to maturity, the higher the spread. *Bond Grade* compares the relative spread of investment grade to high yield bonds issued by the same firm in the presence of firm-level fixed-effects. This number is expected to be positive; a bond deemed high yield should return a larger premium than those deemed investment grade, all else equal.

Estimation Issues

There are two related specification considerations for this model: (1) While the model is ad hoc, is there a theoretical basis for the covariates?, and (2) Which set of covariates fits the data best empirically?

Many of the covariates included in ad hoc models are grounded in the structural approach to modeling bond spreads. Work by Merton, and related papers, shows that the return to a bond depends on the time to maturity of the debt (entered as a natural log in the regression specification), leverage of the firm, risk-free reference rate (entered with the unitary coefficient when calculating the spread), value of the firm's assets V_t (controlled for in *ln(sales)*, *market-to-book*, *leverage,* and *net profit margin*), and volatility in the firm's assets σ^2 was highly collinear with the firm fixed-effect and excluded from the model.[32]

Estimating an OLS model with*out* group level fixed-effects, when the data does include a fixed-effect process, is well known to produce bias in the OLS estimates of $\hat{\beta}$. The intuition is straightforward. Consider the error components model where the error term of observational unit *i* (in this case firm-i) can be written as $e_{it} = X_i + \psi_{it}$. The first component of the error term X_i is common to firm-i and stationary through time. This is firm-level heteroskedasticity - also called the individual or fixed-effect. The second component $\psi_{it} \sim^{iid} N(0, \sigma^2)$ and is the standard time varying component of the error. $E\left[CDS, F_i, B_j | X_i\right] \neq 0$ by definition of the data generating process.[33] To correct for this, firm-level fixed-effects are included in the regression specification thus breaking the correlation between the parameters and the error term.

[32] R&D Expense and Credit Rating are not grounded in a structural model, but are commonly included in large factor regressions to absorb excess risk over and above market. These variables were excluded from this specification as they are relatively time invariant over the dataset and present as collinear with the firm-level fixed-effects.

[33] As a thought experiment, consider the linear regression model:

$$r_{i,t,j} = \overline{\overline{\alpha_0}} + \overline{\overline{\beta_F}} F_i + \overline{\overline{\beta_B}} B_J + (X_i + \psi_{it})$$

where the firm specific factor X_i is not explicitly included. Using the typical OLS estimation equation produces estimates of a factor model where the factors themselves are known to be correlated with the error term. The estimates of $\overline{\overline{\alpha_0}}$, $\overline{\overline{\beta_F}}$ and $\overline{\overline{\beta_B}}$ are biased, taking the form $\gamma \frac{s_{factor,effect}}{s^2_{factor}}$ where **s** is the sample standard error. Judson and Owen (1999) use Monte Carlo simulations to show that the estimates of long-t, narrow-k models with fixed-effects can still be biased, as the within group effect changes through time (a random effect model).

Model Selection

Empirically, to find a parsimonious model that explains the variation in the data I conduct a model selection exercise. Akaike (1974) showed that we can estimate the information lost when a particular model is used to estimate the true data generating process. Akaike's Information Criterion (AIC) is calculated as

$$AIC = 2k - 2\ln(\hat{L})$$

where $\hat{L}$ represents the maximized value of the log-likelihood function for the model and k represent the number of parameters in the model. The smaller AIC, the less informational loss owing to estimation. However, to accounting for the increase in explanatory power as the number of parameters increase, AIC punishes extraneous parameters at a rate of $2k$.

Schwarz (1978) developed an alternative information criterion that punishes extraneous parameters at the rate $\ln(n)$ tying the informational loss to the size of the dataset (with n observations). Schwarz Bayesian Information Criterion (SBC or BIC) is calculated as

$$BIC = \ln(n)\,k - 2\ln(\hat{L})$$

where $\hat{L}$ represents the maximized value of the log-likelihood function for the model and k represent the number of parameters in the model.

The AIC, BIC and adjusted-R^2 for the full set of covariates and the truncated fixed-effect model is included in Table 11. The regression specifications reported are: (1) the intercept only model, (2) covariate only, (3) covariate and firm fixed-effects, (4) covariate and bond fixed-effects, (5) covariate and year, (6) covariate and quarter, (7) covariate and time (year plus quarter together), (8) covariate, bond- and firm- fixed-effects, (9) covariate, firm fixed-effects and time dummies, (10) covariate, bond-effects and time dummies, and (11) covariate, bond and firm fixed-effects, and time dummies.

Table 11. Model Selection Criterion

Model	AIC	BIC	Log-Likelihood
1	1875.8	1875.8	-629.7
2	1791.4	1791.5	-895.7
3	1383.5	1384.2	-691.7
4	1259.4	1262.5	-629.4
5	1623.0	1623.2	-811.5
6	1786.7	1786.8	-893.3
7	1621.6	1621.8	-810.8
8	1259.4	1262.5	-625.2
9	1330.7	1331.6	-665.2
10	1213.5	1216.7	-606.5
11	1213.5	1216.7	-606.5

Including the bond-level fixed-effects absorbs excess variation but causes time-invariant parameters to be estimated with little power. Over-specifying the fixed-effects for the sake of absorbing variation in a reduced form regression can render relatively time-invariant parameters biased and statistically insignificant, when there is an economic justification for including them in the model. Heuristically, the fixed-effect absorbs most time-dependent variation in the credit spread, leaving little variation for relatively time-invariant parameters. Of the available covariates, *Coupon Rate* and *Coupon Type*, are perfectly collinear with the bond-level fixed-effect. Across the dataset, information on the CDS issue is also nearly collinear with the bond-level fixed-effect. Including these variables plus a firm-level fixed-effect increases the model variance exponentially. This leaves the option of including a truncated set of covariates plus fixed-effects (either bond or firm) or including all the covariates without the fixed-effects. Petersen (2009) and Skoulakis (2008) show, separately, that financial data is best modeled with a fixed-effect. Empirically, the goodness-of-fit, information criterion and log-likelihood indicate the fixed-effect specification outperforms the full set of covariates. I chose to include covariates plus the firm-level fixed-effects to recover accurate estimates of the impact of the credit default swap on the bond spread.

Parameter estimates for the spread over the treasury are below in Table 12. Specifications reference Table 11. Column I (Specification 2) includes all covariates, but no fixed-effects. Column II (Specification 7) adds time dummies for year and quarter, but no fixed-effects. Column III (Specification 9) adds firm-level fixed-effects to the covariates and time dummies. Column IV (Specification 10) adds bond level fixed-effects.

Table 12. Estimating the Credit Spread Over Term-Matched Treasury Securities Using Equation (1)

	(2)	(7)	(9)	(10)
ln(Sales)	-0.0221	-0.1066	-0.3325	-0.3410
	(0.1192)	(0.1040)	(0.4280)	(0.3957)
Profit Margin	-0.6795	-1.7293**	-0.1135	-0.1673
	(0.7392)	(0.6734)	(0.3068)	(0.3410)
Leverage	0.8848	2.8779***	3.7081	0.5987
	(1.2098)	(1.1051)	(3.3821)	(1.9248)
Market-to-Book	-0.2457***	-0.2602***	-0.0809	-0.0528
	(0.0814)	(0.0663)	(0.0863)	(0.0629)
ln(Time to Maturity)	0.2080	0.7892***	0.9265***	
	(0.2936)	(0.2012)	(0.2362)	
Investment Grade	0.3027	0.1309	0.1506	
	(0.2616)	(0.2002)	(0.2052)	
Time fixed-effects		Yes	Yes	
Company fixed-effects			Yes	
Bond fixed-effects				
Adjusted R-Squared	0.1887	0.437	0.7099	
F-Statistic	17240	20319	13770	
	(0.0001)	(0.0001)	(0.0001)	
Number of Companies		63		
Number of Bonds		270		
Number of Trades		444698		

Notes: Numbers in parentheses are standard errors.
"" is significant at the 10% level, "**" is significant at the 5% level, "***" is significant at the 1% level. Specification 9 was used to estimate this table.*

For most specifications many factors, except time to maturity, are statistically insignificant. The impact of time to maturity on credit spreads appears to be statistically significant (with a p-value consistently estimated below 0.0001). Quantitatively, a 1% reduction in the number of years to maturity reduces the spread by about 92 basis points.

These results are consistent with a large body of literature in this field (see Ashcraft and Santos (2009), Leland and Toft (1996), and Hull, Predescu and White (2004)). Specification 9 is used to investigate the impact of CDS trading on the bond spread.[34]

Incorporating the Credit Default Swap

A set of binary variable representing the CDS was added to equation (1). The model estimating the impact of the CDS on the bond yield is:

$$r_{i,t,j} = \alpha_0 + \eta CDS + \beta_1 f_1 + \cdots + \beta_k f_k + \gamma_i X_i + \gamma_t T + e_{i,t,j} \qquad (56)$$

This equation estimates the credit spread of bond-*i* at time-*t* from firm-*j* over the risk free reference rate. The set *f*, *X*, and *T* are as before.[35] *CDS* represents a set of binary variables spaced at 2 week distances starting 26 weeks before the CDS was issued and ending 26 weeks after the CDS was issued. This entire set of binary variables is estimated simultaneously.[36]

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[34] The reference rate needs attention. Historically, the treasury rate has functioned as the risk-free rate for studies. However, several empirical papers - see Houweling and Vorst (2005); Hull, Predescu, and White (2004); Longstaff, Mithal, and Neis (2005); and Blanco, Brennan, and Marsh (2005) - argue that the treasury rate is no longer the risk-free benchmark used by investors and put forth the swap (or repo rate) as an alternative. In all of their specifications, the swap rate performs better at option pricing than the treasury rate. Results are presented using the treasury rate as the risk-free benchmark. However, results using the swap are presented in the appendix. Little difference exists.

[35] $r_{i,t,j}$ represent the spread of the yield of bond-*i*, at time-*t* issued by firm-*j* over the risk free rate (treasury security). The set f represents characteristics specific to the bond or firm such as *LN(Sales)* to control for overall firm risk, *Profit Margin* to control for the firm's ability to generate a profit from sales (*Net Income/Sales*), *Leverage* (*debt/total assets*) controls for existing debt of the firm, *market-to-book* calculated as the firm's market cap divided by book value, *LN(time to maturity)* the log of the number of the years remaining until the bond is retired, and *Bond Grade* compares the relative spread of investment grade to high yield bonds, issued by the same firm.

[36] Because the regression also includes a fixed effect by firm (*X*) and time (*T*), all variation in the parameter estimates of β are comparing differences in bond issues within the same firm, not across firms. The parameter space consist of $\{\alpha_0, \beta_{1\ldots k}, \gamma_{i\ldots j}, \eta_t\}$ which are all taken to be constant through time. α_0 is the intercept representing the market average spread, $\beta_{1\ldots k}$ is the factor loading for the firm and bond specific characteristics (1 through k), $\gamma_{i,j}$ represents the fixed effect estimate for the firm and time specific binary variable, and η_t represents the impact of the CDS at time-t. The regression is weighted at the firm-level by the inverse of the number of actively traded bonds. With firm-level fixed-effects, $r_{i,t,j}$ may be interpreted as the weighted average of credit spreads of bonds issued by firm-*i* at time-*t*.

These binary variables take the value 1 and stay 1, at 2 week intervals starting 26 weeks before through 26 weeks after CDS trading onset. The parameter estimate from these variables is identified on the yields measured in the 2-week period after the variable "turns on". So the 26-week binary variable measures the difference in the yield between all prior trades and the trades between 26 to 24 weeks before the CDS is actually issued. The 24-week binary variable measures the difference in the yield between 26 to 24 weeks and 24 to 22 weeks before the CDS is issued.

By controlling for covariates, macroeconomic indicators, and looking at the impact of the CDS through time, I estimate the biweekly average changes in the bond spread in the 6 months both before and after the CDS is issued. This approach provides an extension for existing literature by examining the dynamic impact of information on the bond market.[37] Statistical significance of this variable implies that the CDS is influencing price and this influence cannot be explained by another variable in the model, including both firm and time specific dummy variables.

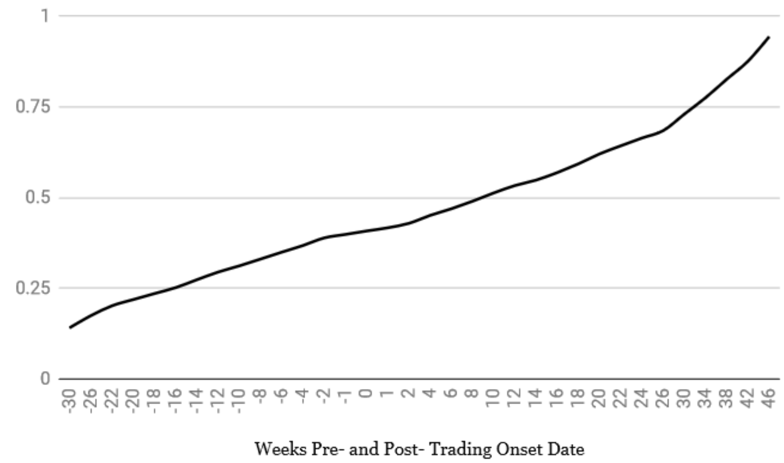
The accuracy of η relies on having sufficient data to estimate credit spread differences at two week intervals. There are enough trades to support this estimation.[38]

[37] This event-oriented approach provides a test of the efficient market hypothesis in the US corporate bond market depending on the timing and statistical significance of the estimation parameters. If the CDS is endogenous, having explanatory power before the CDS issue is public knowledge, then the data suggests that either (1) the US corporate bond market exhibits the strong form of the efficient market hypothesis or (2) the CDS issue is stemming from publicly available information that the market is already incorporating. If the CDS issue impacts the credit spread for US bonds after the CDS is publicly known, and the impact dissipates through time, the semi-strong form of the efficient market hypothesis is satisfied. Finally, if the impact of the CDS isn't significant until some time after the CDS is issued, and the impact dissipates, then the weak form of the efficient market hypothesis holds for the US bond market. There is always the case where the impact may not dissipate, or may not be measurable, implying that the bond market is not incorporating information or that investors do not care about the informational signal contained in the CDS.

[38] The distribution of trades in the 6 months before and after CDS trading onset is given in Table 8. With the dataset spanning 33 quarters, 3.03% of trades are expected to occur in each quarter. I do not find a smooth distribution. Namely, just over 40% of trades occur in the 21 quarters before the average trading onset date. I would expect 63% of trades to occur in this time frame. Additionally, about 5% of trades occur in the 18 months between Q1 2009 and Q3 2010. The majority of trades, specifically 80.1% occur within 2

The cumulative distribution of bond trades in the 7 months pre- and post- CDS issue is contained in Figure 10.

Figure 9. Cumulative Distribution Function (CDF) of Bond Trades in the 7 Months before CDS issue and 11.5 Months after CDS Issue



years of the trading onset date.

This distributional pattern has important implications. Any changes in the credit spread, owing to the credit default swap, occurring a year on either side of the onset date can be estimated with significant power because of the concentration in trades. However, the further away in time from the trading onset date, the lower the power of the estimate (it's being identified on fewer and fewer observations). This is seen in the standard errors of the regressions.

The cumulative distribution of trades is below:

Timing	Cumulative Distribution
6 weeks before CDS	35%
4 weeks before CDS	37%
2 weeks before	39%
1 week before	40%
On the CDS Trading Onset	41%
1 week after	42%
2 weeks after	43%
4 weeks after	45%
6 weeks after	47%

The interpretation of the CDS parameter estimate is key. Looking at the sign of the parameter: If $\eta<0$ and statistically significant, then a CDS lowers the spread of the bond's yield to maturity over the risk-free rate for the reference entity across all their outstanding bond issues on average. Lowering the spread means the yield to maturity must decrease -- investors are accepting a lower return for liquidity after the CDS is issued. If $\eta>0$ and statistically significant, then the CDS increases the bond spread over the risk-free rate for the reference entity. Investors are demanding a higher return for liquidity after the CDS is issued. The statistical significance of η also comments on the exogeneity of the CDS issue.[39] If η is significant before the CDS is issued, then the CDS is coming from some information already known to the bond market. The CDS and the bond market are co-moving. If η is statistically significant after the CDS is issued, then the CDS is adding new information to the bond market.

Results

Time independent parameter estimates are shown in Table 13 for the equation presented in (56). Time invariant parameter estimates allow us to observe the impact of accounting for the CDS as a proxy for information. The estimates in Table 13 are consistent with those presented for specification 9 in Table 12 in both sign and magnitude. Including the complete set of CDS dummies allowed several covariates to be estimated more precisely than before, increasing the significance of *ln(Sales), Leverage,* and *Market-to-Book.*

[39] For example, Rothstein (2009) used 5th grade teachers to predict 4th grade standardized achievement scores invalidating random sorting of students to classrooms.

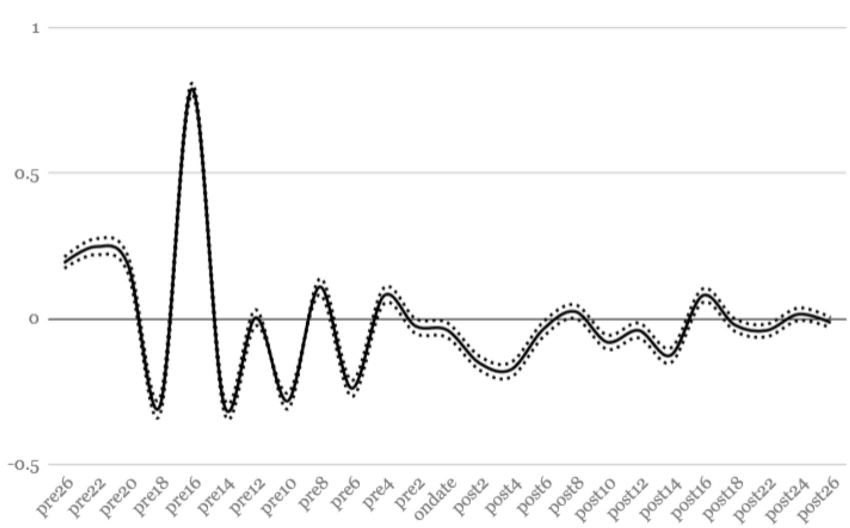
Table 13. Time Independent Parameter Estimates

	2-week Proxy Spacing	4 –week Proxy Spacing
ln(Sales)	-0.022	-0.033*
	(0.018)	(0.018)
Net Profit Margin	-0.086***	-0.078***
	(0.019)	(0.019)
Leverage	2.532***	2.571***
	(0.070)	(0.070)
Market-to-Book	-0.084***	-0.084***
	(0.002)	(0.002)
ln(Time to Maturity)	0.927***	0.927***
	(0.002)	(0.002)
Investment Grade	0.110***	0.111***
	(0.005)	(0.005)
Adjusted R-Squared	0.7316	0.7304
F-Statistic	12950	11550
	(0.0001)	(0.0001)

Notes: Numbers in parentheses are standard errors.
"" is significant at the 10% level, "**" is significant at the 5% level, "***" is significant at the 1% level. Specification 9 was used to estimate this table.*

The parameter estimates on the binary variable representing the CDS issue is graphed in Figure 11. Time is on the x-axis with the estimated impact on the yield to maturity on the y-axis. The 95% confidence interval is also plotted with a dashed line.

Figure 10. Impact of CDS on Bond Yields

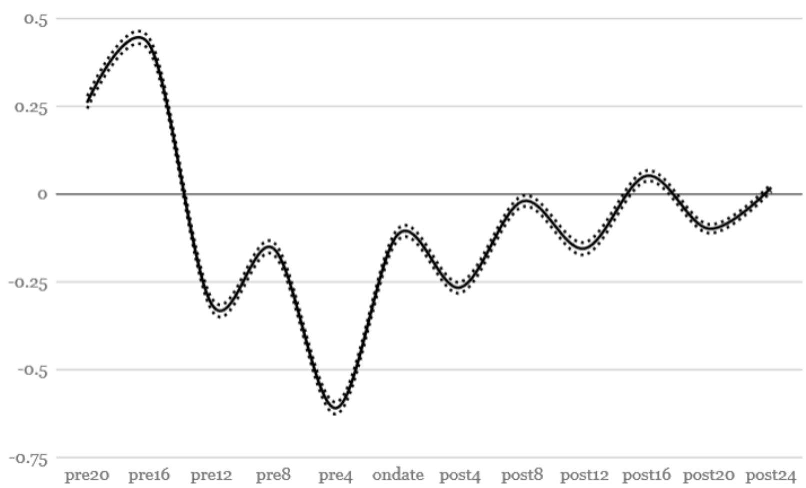


The solid black line represents the impact on the credit spread, while the dashed line provides the 95% confidence interval of the estimate. On average, the CDS has a 0.5 basis point impact on the spread of the bond over the risk free rate over the 6 months before and 6 months after the CDS is issued. 4 months before the CDS is issued, this spikes to a high of 79 basis points. This high spread quickly falls to 29 basis points below the estimated spread at 14 weeks when controlling for all firm and bond characteristics. This indicates that the CDS and bond spread are being jointly determined by some unobservable informational event. The issue of the CDS changes the dynamics of the bond spread, indicating that the CDS is providing the market with new information or context for the existing information. The market takes about 6.5 months to incorporate the information learned around the time the CDS was issued, controlling for other sources of variation. Any point in time estimate would capture an immediate negative impact on the spread, but fail to describe the entire path of price convergence. The OLS model shows that (1) the CDS is impacting credit spreads before the CDS is issued, indicating that the bond market and CDS market are responding to the same publicly known information; (2) information is incorporated into price slowly; and (3) price converges to the pre-shock level in an oscillatory pattern; and (4) the volatility in the oscillation decreases after the CDS is issued. Together suggests that, even though the CDS does not surprise the market, the information contained in the CDS is still incorporated slowly, rather than instantaneously.

In all, market is responding to public information, though the CDS and the bond market exhibit some form of spurious correlation – that is, both are likely responding to external information around the same time. The CDS was never more than a proxy for this information. Because the information is public, and incorporated into the price of the bond, this paper shows that the bond market is informationally efficient and exhibits semi-strong efficiency, at minimum. Future work isolating the root of the informational signal and testing the market for strong efficiency is warranted.

The model was also estimated by incorporating the CDS indicator at 4-week intervals and individually to perform a robustness check of the estimation. This is shown in Figure 12.

Figure 11. Impact of CDS on Bond Spread Estimated at 4-Week Intervals



Both the 2- and 4- week intervals follow the overall pattern given with the sequential regression. Estimating all parameters in one regression allows for a more precise estimation than the sequential model.[40] The overall negative estimate indicates that the CDS reduces the bond spread -- that is, investors view the investment as less of a

[40] The errors of the sequential model was adjusted for the re-estimation and reduction in the degrees of freedom leading to a much larger confidence interval as the firm-level and time-dependent fixed effects were re-estimated with each model. The single linear regression estimates the impact of the CDS with a lot more power as it uses less degrees of freedom.

risk with the CDS available. This is consistent with Stulz's result that the CDS actually increases the value of the underlying asset rather than reduces it.[41]

Under the semi-strong and strong forms of the efficient market hypothesis, the CDS and bond markets would react at the same time if being driven by the same publicly available information. The CDS would have statistical power to explain the bond spread at the date it was issued only through spurious correlation. The effect of the CDS on bond price would quickly dissipate as the information were incorporated. Put differently, as an informational lead to the bond market, savvy investors can use the emergent information to arbitrage the bonds of the reference entity. En masse, the arbitrage attempt should negate itself when markets are efficient. The information learned from the CDS issue is incorporated into the price of the bond, but much slower than previously predicted.

This project supports the view that the bond market follows the semi-strong efficient market hypothesis, that the bond market reacts to all publically available information, but suggests that the reaction is both oscillatory and slower than expected.

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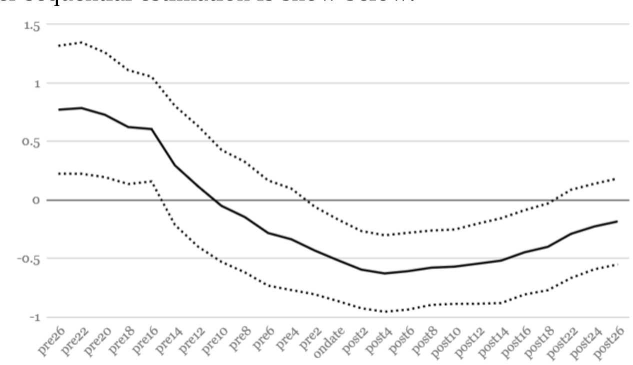
[41] The model can also be estimated sequentially as a nested regression. The parameter estimates for the impact of the CDS under sequential estimation is show below.



Sequential estimation is comparing the average yield on all prior trades to the average yield on all trades after the indicator variable takes the value 1. This is identifying the impact on a larger amount of data, but may not produce precise results. The trades 1 year pre- or post- the binary date may matter significantly less than the trades in the two weeks pre- and post- the binary date. Nevertheless, this regression displays the same overall pattern as the simultaneous estimation presented in Figures 11 and 12.

Transactions costs, informational asymmetries, or frictions in the market may be keeping the bond price from adjusting quickly.

This analysis was also repeated using Bayesian updating in the Black-Scholes-Merton Option Pricing Model (BSM-OPM) model, rather than an ad hoc least squares regression. The results are reported in the appendix.

Conclusion

This paper analyzes the informational efficiency between the bond and CDS markets, looking at the impact of a CDS on the price of a bond.

Principally a bond pricing paper, much was done in the debit pricing arena in order to extend factor models to account for both firm-level fixed-effects and a variable representing informational flow (the CDS instrument). Empirically, this exercise was carried out as a nested regression, attempting to characterize the impact of the credit default swap on the bond spread through time.

The dynamic impact of new information in the bond market was incorporated into price, but this wasn't linear nor a once and for all change. The integral over time of the impulse response function provides interesting insight in the debt market suggesting that transactions costs, informational asymmetries, or frictions in the market keep investors from responding immediately. However, investors do respond so the price does eventually incorporate this information.

Appendix: Bayesian Estimation of Bond Market Efficiency

Structural models, pioneered by Modigliani-Miller (1958), Black and Scholes (1973), and Merton (1974), explain default risk as a function of the firm's cash flows. Assets are priced based on the probability of default and the recovery value at maturity. In this way, assets are viewed as European Call Options, however models may be extended to account for early sales.

Yields are estimated with the value functions derived from Black-Scholes-Merton. Accuracy is based on the assumption made in the model, namely;

> *BSM.1:* All options are European and are exercised only at the time of expiration;
> *BSM.2:* No dividends are paid out;
> *BSM.3:* Market movements are unpredictable (markets are efficient);
> *BSM.4:* No transactions costs;
> *BSM.5:* The underlying volatility and risk-free rate are constant;
> *BSM.6:* Returns on underlying stocks are regularly distributed.

Further, assume that there are no bankruptcy charges, debt and equity are frictionless tradeable assets, and debt is taken to be a single outstanding bond with face value B that matures at time T. The value of the company at liquidation is the value of its assets. Let V_t represent the value of the company at date-t.

DEBT. The firm issues a single class of debt, a zero-coupon bond, with a face value B payable at T. Default occurs if $V_T < B$ at the maturity date. Bondholders exercise their right to liquidate the firm and receive liquidation value V_T . If, at maturity, $V_T > B$, bondholders are repaid B. Differently, bond holders own a zero coupon bond with par value B -- they are able to retrieve B if the company is worth more and V_T if $V_T < B$. The payoff to creditors at date T to retire the debt is given by the function

$$D(V_T, T) = min(V_T, B) = B - (B - V_T)$$

Creditors have a call option on the value of the firm's assets, exercised at date-T and capped at face value *B*. However, they can recover no more than *B* from the firm, leaving them without a put option for the residual value V_T - *B*, which is recovered by the owners (equity holders). The value of the firm's debt (*D*) at time-*t* is

$$D(V_t, t) = P(t, T) - Put_{BS}(V_t, B, r, T - t, \sigma)$$

Since the bonds are pure discount, creditors pay the discounted price $P(t, T)$, less the Black-Scholes put value over the residual V_T - *B*.

EQUITY. At t=T, the difference (V_t - *B*) is disbursed to equity holders, if $V_T > B$. If if $V_T < B$, equity holders get zero. The residual value of the firm can be written as

$$E(V_T, T) = max(V_T - B, 0)$$

Equity holders are the residual owners of the company. They (a) own the firm, (b) borrow amount *B* at t=0, and (c) own a put option on the assets of the firm at date-*T*, V_T, with strike price *B*. By the put-call parity relationship, equity is a call option on the company's assets.

The value of equity (*E*) at time-*t* can be expressed as:

$$E(V_t, t) = Call_{BS}(V_t, B, r, T - t, \sigma)$$

which is the value of the Black-Scholes Call Option.

Equations (1) and (2) provide insights into the relationship between bond prices and Black-Scholes options pricing. At time-*t* the bond price depends on the time to maturity of the debt (*T*-*t*), leverage of the firm (*B*), risk-free reference rate (*r*), value of the firm's assets (V_t), and volatility in the firm's assets ($\boldsymbol{\sigma}^2$).

The value of corporate debt depends on the value of the Black-Scholes put option, which needs to be calculated. Let $\tau = T - t$ and take N(·) as the standard Gaussian cdf function

$$N(x) \ = \ \frac{1}{\sqrt{2\pi}}\int_{\infty}^{x} e^{-y^2/2} dy$$

Then the value of corporate debt (evaluating the BS Put option) is

$$D(t,T) = V_t e^{-\delta\tau} N(-d_1) + \ BP(t,T)N(d_2) \text{ where}$$

$$d_1 = \ \frac{ln\left(\frac{V_t}{B}\right) + (r - \delta + 0.5\sigma^2)(\tau)}{\sigma\sqrt{\tau}} \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

where $N(-d_1)$ and $N(d_2)$ are the Black-Scholes probabilities. Nielsen (1992) shows that $N(d_1)$ is the probability by which the present value of the contingent (on exercise) receipt of the stock price exceeds the current value. $N(d_2)$ is the adjusted probability of exercise. Reinterpreting these probabilities into bond prices, the present value of the bond at time-*t* is the weighted average between the value of the firm at time T (weighted by the probability that the firm is worth more than *B* -- $N(-d_1)$) and the nominal value of the bond (weighted by the probability that the firm will default -- $N(d_2)$). Intuitively, the price investors are willing to pay for firm-*i*'s bonds at time-*t* is the weighted average between the face value of the bond and liquidation value of the firm.

Credit spreads, $R(\tau) - r$, are characterized as

$$R(\tau) \ - \ r \ = \ -\frac{1}{\tau} ln\left[V_t e^{-\delta\tau} N(-d_1) \ + \ BP(t,T)N(d_2)\ \right]$$

We can calculate the explicit probability of default $N(d_2) = \pi_Q$.[42]

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[42] Crouhy, Galai, and Mark (2000) derive the relationship between $N(d_2)$ and the risk-neutral rate

Options pricing models are implemented using Bayesian updating. The value of the firm's assets replaces *V(0)*, the total value of the firm's debt replaces the strike price *B*, the risk-free interest rate is used for *r*. With these parameters, the probability of default $N(d_2)$ and the credit spread are estimated. With the structural parameters recovered, these models are used to predict future yield curves given the current state.[43]

While grounded in theory, structural models are known to present problems in estimation. When calibrated to historical rates tend to underpredict the credit spread (Jones, Mason, and Rosenfeld (1984), Eom, Helwege, and Huang (2004), and Huang and Huang (2012)). Further, structural models may have little power in predicting credit spreads in general (Avramov, Jostova and Philipov (2007), and Schaefer and Strebulaev (2008)). Collin-Dufresn, Goldstein, and Martin (2001) show structural models do not perform well when explaining dynamic changes in bond spreads.

An estimate of the impact of the credit default swap on credit spreads can be obtained using the average[44] call price estimate in the Black-Schloes-Merton Option Pricing Model (BSM-OPM) both pre- and post- CDS instrument date.

The call price gives the estimated yield to maturity if the bond were purchased on the day the price is being estimated. That is, the expected price to purchase the instrument outright. To obtain valid estimates of the spread, the bond must be treated as a European Option (held to maturity). The average call price is estimated for a truncated dataset before the CDS instrument, then re-estimated for the remaining dataset after the CDS instrument. This creates a pre-CDS trading onset and post-CDS trading onset average

[43] Historical data can be used to calibrate the model, rather than calculate the statistical probability for each firm.
[44] Average over each trade in the dataset

spread that can be compared. Despite the noted problems, the two stage approach may yield more accurate results as options traders often reference the BSM-OPM call and strike prices when writing contracts.

Following the structural models, BSM-OPM is parameterized on the underlying asset price, the option strike price, the risk-free reference rate, underlying asset volatility, and time to maturity. The underlying price is taken to be the yield to maturity of the bond at the trade, the strike price is the face value, and volatility is the estimated iVolatility of the stock (following prior work by Ashcraft and Santos). The risk-free reference rate and time to maturity are obvious.

The average of the call price estimates (the yield to maturity over the treasury), as well as the differential induced by the CDS, is given in Table 14.

Table 14. CDS Trading Effect Estimates from BSM-OPM

	Instrument Date	Pre- Instrument Average Estimated Call Price	Post-Instruments Average Estimated Call Price	Difference
	30	5.8316	5.3704	-0.4612
	26	5.867	5.3443	-0.5227
	22	5.876	5.324	-0.552
	20	5.9044	5.3041	-0.6003
	18	5.9281	5.2838	-0.6443
	16	5.9324	5.2681	-0.6643
Pre-CDS Trading Onset	14	5.9129	5.2557	-0.6572
	12	5.9021	5.2405	-0.6616
	10	5.9022	5.224	-0.6782
	8	5.8923	5.2099	-0.6824
	6	5.8816	5.1957	-0.6859
	4	5.8727	5.181	-0.6917
	2	5.8752	5.1674	-0.7078

	1	5.8493	5.1615	-0.6878
CDS Trading Onset		5.839	5.1577	-0.6813
	1	5.8302	5.1536	-0.6766
	2	5.8175	5.1486	-0.6689
	4	5.7991	5.1371	-0.662
	6	5.7855	5.1263	-0.6592
	8	5.7672	5.117	-0.6502
	10	5.7536	5.101	-0.6526
	12	5.7362	5.0924	-0.6438
	14	5.7283	5.0805	-0.6478
	16	5.713	5.0696	-0.6434
Post-CDS Trading Onset	18	5.6919	5.0621	-0.6298
	20	5.6697	5.053	-0.6167
	22	5.653	5.0445	-0.6085
	24	5.6363	5.0387	-0.5976
	26	5.6237	5.0276	-0.5961
	30	5.5864	5.0278	-0.5586
	34	5.5581	5.0133	-0.5448
	38	5.5287	4.9933	-0.5354
	42	5.503	4.96	-0.543
	46	5.4683	4.8914	-0.5769
Average Estimated Yield To Maturity			5.43618	
Average Deviation from True Yield To Maturity			0.0001476	

Notice, the average yield over all trades is underpredicted by approximately 0.01%. This is consistent with prior work, starting as early as Cohen, Black, and Scholes (1972) and most recently highlighted by Sundaresan (2013).

Figure 12. Impact of CDS on Bond Spread Estimated with BSM-OPM

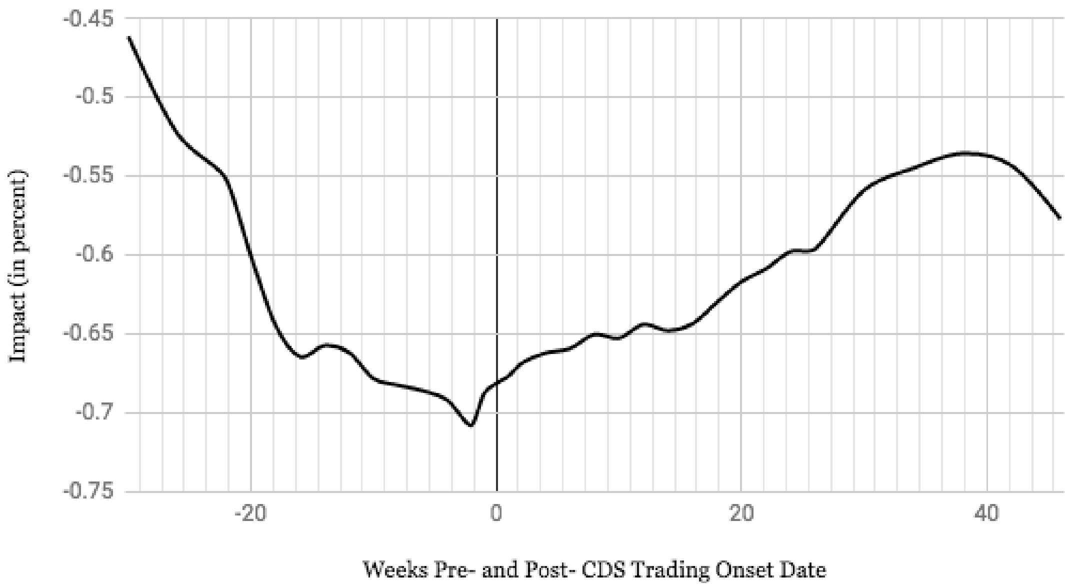

Figure 14 reinforces the patterns displayed in Figures 11 through 13. Overall, the CDS has a slightly negative impact on the bond spread. The impact is not smooth, but oscillates around a trend.

This observation is consistent with observing negative information in the week prior to CDS issue, which prompts investors to seek out insurance against debt.

To consider this approach, note that Darst and Refayet (working paper) show that CDSs can lower the cost of financing when used to hedge a bond against default. The observed impact of the CDS on credit spreads suggests that risk averse investors are seeking CDSs to hedge on bonds already owned, thus reducing the cost of financing for the firm overall. By combining these ideas with the specific pattern of impact of the CDS trading on the credit spread in the 6 months prior to and 1 year post trading onset, suggests that CDS are issued to otherwise “riskier” firms (those with higher than average credit spreads), as information about the firm’s change and investors perceive the firms as riskier. That is, the firm's enter a period of informational flux. This flux plus the presence of a CDS ultimately brings the firm's risk level to market risk (forming a risk-free security bundle, confirming the ideas assumed in Blanco, Brennan, and Marsh (2005)), but the CDS is needed or the average spread would again increase above the average market spread rather than the impact of the CDS return to zero as observed. This suggests that the CDS is issued to investors more risk averse than the average investor, as they are demanding additional insurance over and above what is incorporated into the market price.

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