

# The Macroeconomics of Uncertainty

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Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 25 September 2019

European University Institute **Department of Economics** 

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I confirm that chapter 1 draws upon an earlier article I published in the EUI-ECO Working Papers in June 2019.

I confirm that chapter 2 was jointly co-authored with Dario Bonciani and I contributed 50% of the work. The chapter was published in the Bank of England Staff Working Papers in June 2019.

I confirm that chapter 3 was jointly co-authored with Anna Rogantini Picco and I contributed 50% of the work.

Signature and Date: Joonseok Oh 06/09/2019

# Abstract

This thesis comprises three essays that analyze how uncertainty affects the macroeconomy. Each essay investigates a particular feature of uncertainty propagation. The first essay studies the effects of uncertainty shocks on economic activity, focusing on inflation. I consider standard New Keynesian models with Rotemberg-type and Calvo-type price rigidities. Despite the belief that the two schemes are equivalent, I show that they generate different dynamics in response to uncertainty shocks. In the Rotemberg model, uncertainty shocks decrease output and inflation, in line with the empirical results. By contrast, in the Calvo model, uncertainty shocks decrease output but raise inflation because of firms' precautionary pricing motive. The second essay, written with Dario Bonciani, shows that uncertainty shocks negatively affect economic activity not only in the short, but also in the long run. We build a New Keynesian model with endogenous growth and Epstein-Zin preferences. A decline in R&D by higher uncertainty determines a fall in productivity, which causes a long-term decrease in the macroeconomic aggregates. This long-term risk affects households' consumption process, which exacerbates the overall negative effects of uncertainty shocks. The third essay, prepared with Anna Rogantini Picco, illustrates how economic agents' heterogeneity is crucial for the propagation of uncertainty shocks. We build a heterogeneous agent New Keynesian model with search and matching frictions and Calvo pricing. Unemployment risk for imperfectly insured households amplifies their precautionary savings through increased uncertainty, thus further depressing consumption. Therefore, uncertainty shocks have considerably adverse effects and lead to a decrease in inflation.

"Uncertainty is the only certainty there is."

John Allen Paulos

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# Preface

The Great Recession has sparked a wide debate on how uncertainty affects economic activity. Many researchers have discussed that uncertainty is an important factor in determining business cycle fluctuations. In this thesis, I study the propagation of uncertainty shocks throughout the macroeconomy from various perspectives.

In Chapter 1, I study the effects of uncertainty shocks on economic activity, focusing in particular on inflation. By conducting a VAR analysis, I show that increased uncertainty has negative demand effects, reducing both GDP and prices. To explain this empirical evidence, I consider standard New Keynesian models that feature Rotemberg-type and Calvo-type price rigidities. Contrary to the belief that the two schemes are observationally equivalent, I show that they generate different dynamics in response to uncertainty shocks. In the Rotemberg model, uncertainty shocks reduce output and inflation. Since all firms are symmetric in this model, uncertainty shocks have only an aggregate demand effect. By contrast, in the Calvo specification, uncertainty shocks are stagflationary, as they decrease output and increase inflation. This pricing assumption generates heterogeneity in firms' prices. For this reason, uncertainty shocks have not only an aggregate demand effect but also a precautionary pricing effect that pushes inflation up. I conclude that the implications of the Rotemberg model are more consistent with the empirical results.

In Chapter 2, Dario Bonciani and I argue that shocks increasing macroeconomic uncertainty negatively affect economic activity not only in the short but also in the long run. In a sticky-price DSGE model with endogenous growth through investment in R&D, uncertainty shocks lead to a short-term fall in demand because of precautionary savings and rising markups. The decline in the utilized aggregate stock of R&D determines a fall in productivity, which causes a long-term decline in the main macroeconomic aggregates. When households feature Epstein-Zin preferences, they become averse to these long-term risks affecting their consumption process (long-run risk channel), which strongly exacerbates the precautionary savings motive and the overall negative effects of uncertainty shocks.

In Chapter 3, Anna Rogantini Picco and I show how economic agents' heterogeneity is crucial for the propagation of uncertainty shocks throughout the economy. First, using a SVAR model with aggregate data, we show that an identified uncertainty shock generates a drop of consumption and inflation, a response which conventional representative agent New Keynesian models have difficulty in qualitatively and quantitatively matching. Then, using the Consumer Expenditure Surveys, we show that the response of consumption is heterogeneous across households' income distribution: consumption decreases more for middle-income households. To rationalize our empirical findings, we build a heterogeneous agent New Keynesian model with search and matching frictions and Calvo pricing, and study the propagation of uncertainty shocks. Uncertainty shocks induce households' precautionary saving and firms' precautionary pricing behaviors, causing a fall in aggregate demand and aggregate supply respectively. When markets are incomplete and unemployment risk is countercyclical, these two precautionary behaviors increase unemployment risk for imperfectly insured households that strengthen their precautionary saving behavior, thus further depressing consumption and aggregate demand. When the feedback loop between unemployment risk and precautionary saving is strong enough, a rise in uncertainty leads to a decrease in inflation. This model is able to qualitatively and quantitatively match the empirical evidence on uncertainty shock propagation, in contrast to standard representative agent New Keynesian models.

### Chapter 1

# The Propagation of Uncertainty Shocks: Rotemberg vs. Calvo

#### 1.1 Introduction

Recently, uncertainty has received substantial attention in the wake of the Great Recession and the subsequent slow recovery. Many researchers have argued that uncertainty is an important factor in determining business cycle fluctuations. In a New Keynesian framework, increased uncertainty leads to a decrease in aggregate demand because of precautionary saving motives and time-varying markups. While the impact of uncertainty on aggregate demand is well understood, the effects on inflation have not been yet explored in the literature.

In this paper, I study how increased uncertainty affects economic activity, concentrating in particular on inflation. Firstly, I conduct a structural vector autoregression (VAR) analysis on quarterly U.S. macroeconomic data. I consider eight widely cited U.S. uncertainty measures from the literature. These eight measures can be categorized into four groups: (*i*) macroeconomic uncertainty, (*ii*) financial uncertainty, (*iii*) survey-based uncertainty, and (*iv*) policy uncertainty. The VAR analysis shows that an exogenous increase in any of these uncertainty indices results in significant falls in output and prices. In other words, uncertainty shocks act in the same way as aggregate demand shocks.

To explain these empirical findings, I compare two standard New Keynesian models with the most common sticky price assumptions: the Rotemberg (1982)-type quadratic price adjustment cost and the Calvo (1983)-type constant price adjustment probability. In the Rotemberg model, a firm can adjust its price whenever it wants after paying a quadratic adjustment cost. On the other hand, in the Calvo model, each firm may reset its price only with a constant probability each period, independent of the time elapsed since the last adjustment. Although the two assumptions have different economic intuitions, the predictions of the New Keynesian model are robust against the pricing assumption up to a first-order approximation around a zero-inflation steady state. For this reason, there is a widespread agreement in the literature that the pricing assumption is innocuous for the dynamics of the standard New Keynesian model. However, by employing a third-order perturbation, I show that the Rotemberg and Calvo models generate very different results in response to uncertainty shocks. In particular, I separately consider five different sources of uncertainty shocks in the models: (i) preference uncertainty, (ii) productivity uncertainty, (iii) markup uncertainty, (iv) government spending uncertainty, and (v) interest rate uncertainty. In all cases, increased uncertainty leads to a decrease in inflation in the Rotemberg model, and to an *increase* in inflation in the Calvo model, while still resulting in a decrease in output in both models. This result is important because inflation stabilization is one of the main goals of monetary policy. For this reason, it is important to understand which propagation mechanism holds in the data.

Uncertainty shocks have two effects on firms: an aggregate demand effect and a precautionary pricing effect, as pointed out by Fernández-Villaverde et al. (2015). Increased uncertainty induces risk-averse households to consume less. The fall in aggregate demand lowers the demand for labor and capital, which decreases firms' marginal costs. In the Rotemberg model, only the aggregate demand effect is at work for firms. To be specific, since their pricing decision is symmetric, all firms behave as a single representative firm. Thus, the firms are risk-neutral concerning their pricing decision: the firms' marginal profit curve, a function of the reset price, is constant. Therefore, the decrease in marginal costs induces firms to lower their prices. Consequently, inflation decreases in the Rotemberg model. On the other hand, in the Calvo model, both the precautionary pricing effect as well as the aggregate demand effect are operative when an uncertainty shock hits. The Calvo pricing assumption generates heterogeneity in firms' prices. This implies that firms are risk-averse regarding their pricing decision: the firms' marginal profit curve is strictly convex. Thus, higher uncertainty induces firms which are resetting their prices to increase them so as to self-insure against being stuck with low prices in the future. If firms lower their prices, they may sell more but at negative markups, thereby incurring losses. As a result, inflation increases in the Calvo model. Using a prior predictive analysis, I show that the predictions of the two models are robust against the exact model parametrization and the different sources of uncertainty. Therefore, the Rotemberg model is more consistent with the empirical evidence than the Calvo model.

**Related Literature** This paper is related to three main strands of literature. First of all, this paper contributes to the literature that studies the propagation of uncertainty shocks in New Keynesian models. This is the first paper which highlights the different responses to uncertainty shocks in the Rotemberg and Calvo models. The following papers which assume the Rotemberg pricing argue that uncertainty shocks reduce output and inflation in the same way as negative demand shocks: Bonciani and van Roye (2016), Leduc and Liu (2016), Basu and Bundick (2017), Cesa-Bianchi and Fernandez-Corugedo (2018), and Katayama and Kim (2018). On the contrary, Born and Pfeifer (2014) and Mumtaz and Theodoridis (2015), which adopt Calvo pricing, argue that uncertainty shocks result in a decrease in output but an increase in inflation, i.e., negative supply shocks. Exceptionally, Fernández-Villaverde et al. (2015) study an inflationary effect of uncertainty shocks in a Rotemberg-type New Keynesian model. However, this result is obtained because, in contrast to the abovementioned literature, their price adjustment cost directly affects firms' marginal costs. Basu and Bundick (2017) attribute this discrepancy to different sources of shocks and calibrations. However, I show that the primary reason for the different results found in the literature is the adopted assumption of price stickiness.

Second, this paper organizes the literature that looks at the empirical impact of uncertainty shocks on inflation. Caggiano et al. (2014), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), and Basu and Bundick (2017) argue that uncertainty shocks empirically induce a decrease in inflation. On the other hand, Mumtaz and Theodoridis

(2015) find an inflationary effect of uncertainty shocks, and Carriero et al. (2018b) and Katayama and Kim (2018) find an insignificant response of inflation to uncertainty shocks. However, they all use different uncertainty measures and time spans. Hence, I study eight widely cited U.S. uncertainty measures and, to avoid parameter instability, I start my sample only after the beginning of Paul Volcker's mandate as the Federal Reserve Chairman. I find that any kind of uncertainty has a negative effect on inflation.

Lastly, this paper adds to the literature that studies the difference between the Rotemberg and Calvo models. This is the first paper which compares the two models in terms of uncertainty shocks. Nisticó (2007) and Lombardo and Vestin (2008) compare the welfare implications of the two models. Ascari et al. (2011) and Ascari and Rossi (2012) investigate the differences between the two models under a positive trend inflation rate. Ascari and Rossi (2011) study the effect of a permanent disinflation in the Rotemberg and Calvo models. More recently, Boneva et al. (2016), Richter and Throckmorton (2016), Eggertsson and Singh (2018), and Miao and Ngo (2018) investigate the differences in the predictions of the Rotemberg and Calvo models with the zero lower bound for the nominal interest rate. Sims and Wolff (2017) study the state-dependent fiscal multipliers in the two models under a Taylor rule in addition to periods where monetary policy is passive. Moreover, Born and Pfeifer (2018) discuss the mapping between Rotemberg and Calvo wage rigidities.

The remainder of the paper is structured as follows. Section 1.2 provides the VARbased empirical evidence. Section 1.3 presents the two New Keynesian models. Section 1.4 explains the parametrization and the solution method. Section 1.5 compares the quantitative results. Section 1.6 investigates the robustness of the results. Finally, Section 1.7 concludes.

#### **1.2 Empirical Evidence**

In this section, I empirically investigate the impacts of uncertainty shocks on economic activity.



Figure 1.1: U.S. Uncertainty Indicies

Note: Each series is demeaned and standardized by its standard deviation. The sample period is 1985Q1 to 2017Q3. Shaded areas mark recessions as dated by the NBER.

#### 1.2.1 Measuring Uncertainty

Measuring uncertainty is inherently difficult. Ideally, one would like to know the subjective probability distributions over future events for economic agents. As this is almost impossible to quantify directly, there exists no agreed measure of uncertainty in the literature. For my analysis, I take eight widely cited U.S. uncertainty measures from the literature similarly to Born et al. (2018). Considering this wide range of uncertainty proxies has the advantage that I am able to capture different kinds of uncertainty, such as macroeconomic uncertainty, financial uncertainty, survey-based uncertainty, and economic policy uncertainty.

Specifically, the eight uncertainty measures are (i) the macro uncertainty proxy

Table 1.	1: Corre	lations
----------	----------	---------

	MU	TU	FU	VXO	CSU	FSU	EPU	MPU
Uncerta	ainty Indice	s						
MU	1	0.26***	0.66***	0.59***	0.27***	0.23***	0.34***	0.21**
TU	-	1	0.19**	0.09	$0.46^{***}$	-0.04	0.28***	-0.02
FU	-	-	1	0.86***	0.20**	0.23***	0.37***	0.36***
VXO	-	-	-	1	0.20**	0.29***	0.38***	0.50***
CSU	-	-	-	-	1	$-0.27^{***}$	0.70***	0.27***
FSU	-	-	-	-	-	1	-0.02	0.20**
EPU	-	-	-	-	-	-	1	$0.48^{***}$
MPU	-	-	-	-	-	-	-	1
Macro Variable								
$\Delta GDP$	$-0.56^{***}$	$-0.21^{**}$	$-0.31^{***}$	$-0.34^{***}$	$-0.39^{***}$	$-0.26^{***}$	$-0.37^{***}$	$-0.20^{**}$

Note: Numbers are pairwise unconditional time-series correlation coefficients. I test the hypothesis of no correlation against the alternative hypothesis of a nonzero correlation where \*\*\* denotes 1%, \*\* 5%, and \* 10% significance levels, respectively. Abbreviations: macro uncertainty (MU), TFP uncertainty (TU), financial uncertainty (FU), stock market volatility (VXO), consumers' survey-based uncertainty (CSU), firms' survey-based uncertainty (FSU), economic policy uncertainty (EPU), and monetary policy uncertainty (MPU).  $\Delta$ GDP is the quarterly growth rate of GDP. The sample period is 1985Q1 to 2017Q3.

measured by Jurado et al. (2015) and Ludvigson et al. (2019), (*ii*) the time-varying volatility of aggregate TFP innovations estimated by a stochastic volatility model (Born and Pfeifer, 2014; Fernald, 2014; Bloom et al., 2018), (*iii*) the financial uncertainty proxy estimated by Ludvigson et al. (2019), (*iv*) stock market volatility (VXO) studied by Bloom (2009) and Basu and Bundick (2017), (*v*) the consumers' perceived uncertainty proxy (concerning vehicle purchases) proposed by Leduc and Liu (2016), (*vi*) the firm-specific uncertainty proxy using the dispersion of firms' forecasts about the general business outlook constructed by Bachmann et al. (2013), (*vii*) the economic policy uncertainty index constructed by Baker et al. (2016), and (*viii*) the monetary policy uncertainty index constructed by Baker et al. (2016).

I present the evolution of the eight measures from 1985Q1 to 2017Q3 in Figure 1.1.<sup>1</sup> These eight measures can be categorized by four groups: (*i*) macroeconomic uncertainty, (*ii*) financial uncertainty, (*iii*) survey-based uncertainty, and (*iv*) policy uncertainty. Each category incorporates two indices respectively. For comparison, each series has been

<sup>&</sup>lt;sup>1</sup>The time span is determined by the availability of the monetary policy uncertainty index.

demeaned and standardized. The uncertainty indices are strongly countercyclical. Most of them increase noticeably before and during recessions while they are rather low during periods of stable economic expansion. Moreover, as shown in Table 1.1, there is generally a sizable degree of comovement between the uncertainty indices, consistent with Born et al. (2018).

#### 1.2.2 VAR Analysis

Following the existing literature of Bloom (2009), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), and Basu and Bundick (2017), I estimate a structural four-lag VAR model with a constant on quarterly U.S. macroeconomic data from 1985Q1 to 2017Q3:

$$\mathbf{A} \mathbb{Y}_t = c + \sum_{j=1}^{L} \mathbf{B}_j \mathbb{Y}_{t-j} + \epsilon_t, \qquad (1.1)$$

where  $\epsilon_t$  is a vector of unobservable zero mean white noise processes. The vector  $\mathbb{Y}_t$  comprises 7 variables: (*i*) the uncertainty measure, (*ii*) real GDP per capita, (*iii*) real consumption per capita, (*iv*) real investment per capita, (*v*) hours worked per capita, (*vi*) the GDP deflator, and (*vii*) the quarterly average of the effective federal funds rate.<sup>2</sup> Since the sample includes a period during which the federal funds rate hits the zero lower bound, I use the shadow federal funds rate constructed by Wu and Xia (2016) from 2009Q1 to 2015Q4, which is not bounded below by zero and is supposed to summarize the stance of monetary policy. With the exception of the federal funds rate and the shadow rate, all other variables enter the VAR in log levels. To identify uncertainty shocks, I use a Cholesky decomposition with the uncertainty measure ordered first. This ordering is based on the assumption that uncertainty is not affected on impact by the other endogenous variables in the VAR.<sup>3</sup> This assumption is supported by Angelini et

<sup>&</sup>lt;sup>2</sup>I use data on GDP, consumption, investment, hours worked, price, and the interest rate. My data set comes from the FRED database of St. Louis Fed. GDP is real GDP (GDPC1). Consumption is the sum of real consumptions on nondurable goods and services (PCNDGC96 and PCESVC96). Investment is the sum of real consumption on durable goods and real private fixed investment (PCDGCC96 and FPIC1). Hours worked are measured by hours of all persons in the business sector (HOABS). Price is based on the GDP deflator (GDPDEF). To convert them to per-capita terms, I use the quarterly average of the civilian non-institutional population (CNP16OV). The short-term interest rate corresponds to the quarterly average of the effective federal funds rate (FEDFUNDS) and the Wu and Xia (2016)'s shadow rate.

<sup>&</sup>lt;sup>3</sup>I also check a Cholesky decomposition with the uncertainty measure ordered last. The associated impulse response functions are consistent regardless of the ordering of the uncertainty measure. I display



Figure 1.2: Empirical Responses to Uncertainty Shocks: Macroeconomic and Financial Uncertainty Measures

Note: The solid lines represent median responses of the variables to a one-standard-deviation innovation to each uncertainty index. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. Abbreviations: macro uncertainty (MU), TFP uncertainty (TU), financial uncertainty (FU), and stock market volatility (VXO). The sample period is 1985Q1 to 2017Q3.

al. (2019). They argue that uncertainty is an exogenous source of decline of economic activity.

I display the impulse responses of GDP and prices to each uncertainty shock in Figure 1.2 and 1.3.<sup>4</sup> For each variable, the solid line denotes the median estimate of the impulse response and the shaded area represents the range of the one-standard-error bootstrapped confidence bands around the point estimates. Each uncertainty shock causes significant declines in GDP and prices. These results imply that uncertainty shocks act like aggregate demand shocks, consistently with Caggiano et al. (2014),

them in Appendix A.1.2.

<sup>&</sup>lt;sup>4</sup>I display the full sets of empirical impulse response functions in Appendix A.1.1. All kinds of uncertainty shocks have similar adverse demand effects on economics activity: GDP, consumption, investment, hours worked, prices, policy rate all decrease in response to uncertainty shocks.



Figure 1.3: Empirical Responses to Uncertainty Shocks: Survey-Based and Policy Uncertainty Measures

Note: The solid lines represent median responses of the variables to a one-standard-deviation innovation to each uncertainty index. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. Abbreviations: consumers' survey-based uncertainty (CSU), firms' survey-based uncertainty (FSU), economic policy uncertainty (EPU), and monetary policy uncertainty (MPU). The sample period is 1985Q1 to 2017Q3.

Fernández-Villaverde et al. (2015), Leduc and Liu (2016), and Basu and Bundick (2017).

#### 1.3 Models

In this section, I outline two standard New Keynesian models with different price setting assumptions.<sup>5</sup> Both economies are populated by identical infinitely-lived households. There are also a continuum of identical competitive final goods firms and a continuum of monopolistically competitive intermediate goods firms. Lastly, there are fiscal and monetary authorities.

<sup>&</sup>lt;sup>5</sup>In Appendix A.2, I report the equilibrium conditions in the two models.

#### 1.3.1 Households

The representative household maximizes the following lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t U\left(C_t, N_t\right), \qquad (1.2)$$

$$U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta},$$
(1.3)

where  $E_0$  is the conditional expectation operator,  $\beta$  is the subjective discount factor,  $C_t$  denotes consumption, and  $\gamma$  measures the degree of relative risk aversion.  $N_t$  denotes labor supply,  $\eta$  denotes the inverse elasticity of labor supply, and  $\chi$  indicates disutility from working.  $A_t$  is an exogenous preference shock which follows a stationary AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A, \qquad (1.4)$$

where  $0 \le \rho_A < 1$  and  $\varepsilon_t^A \sim N(0, 1)$ .

Every period, the household faces the following budget constraint:

$$P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} = B_t + W_t N_t + R_t^k K_t - P_t T_t + P_t \Pi_t,$$
(1.5)

where  $P_t$  is the price level,  $I_t$  is investment,  $B_t$  is one-period nominal bond holdings,  $R_t$  is the gross nominal interest rate,  $W_t$  is the nominal wage rate,  $R_t^k$  is the nominal rental rate of capital,  $K_t$  is capital stock,  $T_t$  is a lump-sum tax, and  $\Pi_t$  is profit income.

In addition, the capital stock evolves according to:

$$K_{t+1} = (1-\delta) K_t + \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t,$$
(1.6)

where  $\delta$  is the depreciation rate and  $\kappa$  controls the size of adjustment costs when the level of investment changes over time, as proposed by Christiano et al. (2005).

#### 1.3.2 Final Goods Firms

The final good  $Y_t$  is aggregated by the constant elasticity of substitution technology:

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}},\tag{1.7}$$

where  $Y_t(i)$  is the quantity of intermediate good *i* used as an input and  $\varepsilon$  is the elasticity of substitution for intermediate goods. The cost minimization problem for the final goods firm implies that the demand for intermediate good *i* is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t, \tag{1.8}$$

where  $P_t(i)$  is the price of intermediate good *i*. Finally, the zero-profit condition implies that the price index is expressed as:

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$
(1.9)

#### 1.3.3 Intermediate Goods Firms

There is a continuum of monopolistically competitive firms, indexed by  $i \in [0, 1]$ , which produce differentiated intermediate goods. Each intermediate goods firm produces its differentiated good *i* using the following Cobb-Douglas production function:

$$Y_t(i) = Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha} - \Phi,$$
(1.10)

where  $\alpha$  denotes capital income share and  $\Phi$  denotes the fixed cost of production.  $Z_t$  is an exogenous productivity shock which follows a stationary AR(1) process:

$$\log Z_t = \rho_Z \log Z_{t-1} + \sigma_t^Z \varepsilon_t^Z, \qquad (1.11)$$

where  $0 \le \rho_Z < 1$  and  $\varepsilon_t^Z \sim N(0, 1)$ .

Cost minimization implies that all intermediate goods firms have the same capital-tolabor ratio and the same marginal cost:

$$\frac{K_t(i)}{N_t(i)} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k},\tag{1.12}$$

$$MC_t = \frac{1}{Z_t} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t^k}{\alpha}\right)^{\alpha}.$$
 (1.13)

#### 1.3.4 Two Price Setting Mechanisms

To model price stickiness, I introduce Rotemberg (1982)'s and Calvo (1983)'s price setting mechanisms. Intermediate goods firms have market power and set prices to maximize

their discounted profits. They face frictions in adjusting prices and, thus, prices are sticky.

#### **Rotemberg Model**

Rotemberg (1982) assumes that each intermediate goods firm *i* faces costs of adjusting price, which are assumed to be quadratic and zero at the steady state. Therefore, firm *i* sets its price  $P_t(i)$  to maximize profits given by:

$$\max_{P_{t+j}(i)} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( \left( \frac{P_{t+j}(i)}{P_{t+j}} - \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \right) Y_{t+j}(i) - \frac{\phi}{2} \left( \frac{P_{t+j}(i)}{P_{t+j-1}(i)} - 1 \right)^2 Y_{t+j} \right),$$
(1.14)

subject to its demand in Equation (1.8), where  $\Lambda_{t,t+j} \equiv \beta^j \frac{A_{t+j}}{A_t} \left(\frac{C_{t+j}}{C_t}\right)^{-\gamma}$  is the stochastic discount factor for real payoffs of the households, and  $\phi$  is the adjustment cost parameter which determines the degree of nominal price rigidity.  $M_t$  is an exogenous markup shock which follows a stationary AR(1) process:

$$\log M_t = \rho_M \log M_{t-1} + \sigma_t^M \varepsilon_t^M, \qquad (1.15)$$

where  $0 \le \rho_M < 1$  and  $\varepsilon_t^M \sim N(0, 1)$ .

The first-order condition associated with the optimal price is given by:

$$\left( \left(1-\varepsilon\right) \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} + \varepsilon \frac{MC_t}{P_t} M_t \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} - \phi \left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right) \frac{P_t(i)}{P_{t-1}(i)}\right) Y_t + \phi E_t \Lambda_{t,t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right) \frac{P_{t+1}(i)}{P_t(i)} Y_{t+1} = 0. \quad (1.16)$$

Since all intermediate goods firms face an identical profit maximization problem, they choose the same price  $P_t(i) = P_t$  and produce the same quantity  $Y_t(i) = Y_t$ . In a symmetric equilibrium, the optimal pricing rule implies:

$$\phi\left(\frac{P_t}{P_{t-1}} - 1\right)\frac{P_t}{P_{t-1}} = \phi E_t \Lambda_{t,t+1} \left(\frac{P_{t+1}}{P_t} - 1\right)\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{Y_t} + 1 - \varepsilon + \varepsilon \frac{MC_t}{P_t}M_t.$$
 (1.17)

#### Calvo Model

According to the stochastic time dependent rule proposed by Calvo (1983) and Yun (1996), in each period an intermediate goods firm *i* keeps its previous price with probability  $\theta$  and resets its price with probability  $1 - \theta$ . The firm that gets the chance to set its price,

chooses its price  $P_t^{\star}(i)$  to maximize:

$$\max_{P_{t}^{\star}(i)} E_{t} \sum_{j=0}^{\infty} \theta^{j} \Lambda_{t,t+j} \left( \frac{P_{t}^{\star}(i)}{P_{t+j}} - \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \right) Y_{t+j}(i),$$
(1.18)

subject to its demand in Equation (1.8).

The first-order condition with respect to the optimal price is given by:

$$E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( \left(1-\varepsilon\right) \left(\frac{P_t^{\star}(i)}{P_{t+j}}\right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left(\frac{P_t^{\star}(i)}{P_{t+j}}\right)^{-\varepsilon} \right) Y_{t+j} = 0.$$
(1.19)

The optimal reset price,  $P_t^* = P_t^*(i)$ , is the same for all firms resetting their prices in period *t* because they face the identical problem above. This implies that the optimal reset price is:

$$P_t^{\star} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} P_{t+j} \frac{\varepsilon M C_{t+j}}{P_{t+j}} M_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} P_{t+j} \frac{\varepsilon^{-1} Y_{t+j}}{Y_{t+j}}}.$$
(1.20)

Finally, I rewrite Equation (1.9) describing the dynamics for the aggregate price level:

$$P_t = \left( (1-\theta) P_t^{\star 1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$
(1.21)

#### 1.3.5 Fiscal and Monetary Authorities

The fiscal authority runs a balanced budget and raises lump-sum taxes to finance government spending  $G_t$ , which is given by:

$$G_t = T_t. (1.22)$$

The government spending  $G_t$  follows a stationary AR(1) process:

$$\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_t^G \varepsilon_t^G, \qquad (1.23)$$

where  $0 \le \rho_G < 1$  and  $\varepsilon_t^G \sim N(0, 1)$ . *G* is the deterministic steady-state government spending.

The monetary authority conducts monetary policy using the short-term nominal interest rate as the policy instrument. The gross nominal interest rate  $R_t$  follows a

conventional Taylor rule:

$$\log R_{t} = (1 - \rho_{R}) \log R + \rho_{R} \log R_{t-1} + (1 - \rho_{R}) (\phi_{\pi} (\log \pi_{t} - \log \pi) + \phi_{Y} (\log Y_{t} - \log Y)) + \sigma_{t}^{R} \varepsilon_{t}^{R}, \quad (1.24)$$

where  $0 \le \rho_R < 1$ ,  $\phi_\pi > 1$ ,  $\phi_Y \ge 0$ , and  $\varepsilon_t^R \sim N(0, 1)$ .  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate. *R*,  $\pi$ , and *Y* are the deterministic steady-state values of the corresponding variables.

#### 1.3.6 Market Clearing

In the Rotemberg model with the symmetric equilibrium, aggregate output satisfies:

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} - \Phi, \qquad (1.25)$$

and the equilibrium in the goods market requires:

$$Y_t = C_t + I_t + G_t + \frac{\phi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t.$$
 (1.26)

On the other hand, in the Calvo model where the equilibrium is not symmetric, aggregate output satisfies:

$$\Delta_t Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} - \Phi, \qquad (1.27)$$

where  $K_t = \int K_t(i) di$  and  $N_t = \int N_t(i) di$ .  $\Delta_t \equiv \int \left(\frac{P_t^*(i)}{P_t}\right)^{-\varepsilon} di$  is relative price dispersion and can be rewritten as the following recursive form:

$$\Delta_t = (1 - \theta) \left(\frac{P_t^{\star}}{P_t}\right)^{-\varepsilon} + \theta \left(\frac{P_t}{P_{t-1}}\right)^{\varepsilon} \Delta_{t-1}.$$
(1.28)

The equilibrium in the goods market for the Calvo model is given by:

$$Y_t = C_t + I_t + G_t. (1.29)$$

#### 1.3.7 Uncertainty Shock Processes

I consider the following uncertainty shock processes:

$$\log \sigma_t^X = (1 - \rho_{\sigma^X}) \log \sigma^X + \rho_{\sigma^X} \log \sigma_{t-1}^X + \sigma^{\sigma^X} \varepsilon_t^{\sigma^X}, \qquad (1.30)$$
where  $X \in \{A, Z, M, G, R\}$ ,  $0 \le \rho_{\sigma^X} < 1$ , and  $\varepsilon_t^{\sigma^X} \sim N(0, 1)$  is a second-moment uncertainty shock. An increase in the volatility of the shock process increases the uncertainty about the future time path of the stochastic process. All stochastic shocks are independent.

# 1.4 Parametrization and Solution Method

The two models are parameterized to a quarterly frequency. Table 1.2 provides a summary of the key parameters. To make sure that the differences in the Rotemberg and Calvo models hold independent of the parametrization, I conduct a prior predictive analysis as in Pappa (2009). This exercise formalizes, via Monte Carlo methods, standard sensitivity analysis. Firstly, I fix a zero inflation steady state ( $\pi = 1$ ) and a zero profit steady state ( $\Pi = 0$ ). I draw the values of the following 32 parameters uniformly: the discount factor ( $\beta$ ), the risk aversion ( $\gamma$ ), the inverse labor supply elasticity ( $\eta$ ), the steadystate hours worked (N), the capital depreciation rate ( $\delta$ ), the investment adjustment cost parameter ( $\kappa$ ), the elasticity of substitution between intermediate goods ( $\epsilon$ ), the capital income share ( $\alpha$ ), the Calvo price duration ( $\theta$ ), the steady-state government spending share  $(\frac{G}{Y})$ , the coefficients of the Taylor rule  $(\phi_{\pi} \text{ and } \phi_{Y})$ , and the coefficients of the shock processes ( $\rho_X$ ,  $\sigma^X$ ,  $\rho_{\sigma^X}$ , and  $\sigma^{\sigma^X}$ ). The parameters are allowed to vary over the ranges reported in Table 1.2. The ranges are based on theoretical and practical considerations. I impose the following 3 parameters to be fixed according steady state considerations and the first-order equivalence of the two models: the labor disutility parameter ( $\chi$ ), the production fixed cost ( $\Phi$ ), and the Rotemberg price adjustment cost parameter ( $\phi$ ).

I solve the two models using a third-order approximation to the equilibrium conditions around their respective deterministic steady states.<sup>6</sup> To solve the models, I use the Dynare software package developed by Adjemian et al. (2011) and the pruning algorithm designed by Andreasen et al. (2018). Then, I repeat this procedure 10,000 times. I construct the impulse response functions of the endogenous variables to uncertainty shocks

<sup>&</sup>lt;sup>6</sup>Fernández-Villaverde et al. (2011) explain that in the third-order approximation, in contrast to first and second-order approximations, the innovations to the stochastic volatility shocks enter independently the approximated policy functions.

Table 1.2:	Quarterly	Parametrization
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Parameter	Description	Range	Value/Target
β	Discount factor	[0.985, 0.995]	0.99
$\gamma$	Risk aversion	[1, 4]	2
η	Inverse labor supply elasticity	[0.25, 2]	1
χ	Labor disutility parameter	N=[0.2, 0.4]	$N = \frac{1}{3}$
δ	Capital depreciation rate	[0.01, 0.04]	0.025
κ	Investment adjustment cost parameter	[0, 6]	3
ε	Elasticity of substitution between goods	[6, 31]	11
α	Capital income share	[0.2 0.4]	0.33
$\Phi$	Production fixed costs	$\Pi = 0$	$\Pi = 0$
π	Steady-state inflation	1	1
$\theta$	Calvo probability of keeping price unchanged	[0.5, 0.9]	0.75
$\phi$	Rotemberg price adjustment cost parameter	$\frac{\theta(\varepsilon-1)}{(1-\theta)(1-\beta\theta)}$	$rac{ heta(arepsilon-1)}{(1- heta)(1-eta heta)}$
G	Steady-state government spending	$\frac{G}{Y} = [0.1 \ 0.3]$	$\frac{G}{Y} = 0.2$
$\phi_\pi$	Coefficient of inflation target in the Taylor rule	[1.1, 3]	1.5
$\phi_Y$	Coefficient of output target in the Taylor rule	[0, 0.5]	0.25
$ ho_X$	Persistence of level shocks	[0.5 <i>,</i> 0.99]	0.9 ( $\rho_R = 0.7$ )
$\sigma^X$	Volatility of level shocks	[0.005, 0.015]	0.01
$ ho_{\sigma^X}$	Persistence of uncertainty shocks	[0.5, 0.99]	0.7
$\sigma^{\sigma^X}$	Volatility of uncertainty shocks	[0.2, 0.8]	0.5

for each draw and rearrange them in ascending order.<sup>7</sup> Lastly, I generate pointwise 68% probability bands between the 84 and 16 percentiles in both models.

# 1.5 Quantitative Results

In this section, I quantitatively investigate the effects of uncertainty shocks on macroeconomic variables in the Rotemberg and Calvo models. I plot the pointwise 68% probability bands for the impulse response functions of output and inflation to each uncertainty shock in the Rotemberg (blue solid bands) and Calvo (red dashed bands) models in Figure 1.4. The figure shows that increased uncertainty has negative effects on output in both models. It increases inflation in the Calvo model. On the other hand, even though the bands of inflation slightly contain the zero line in the Rotemberg model, higher

<sup>&</sup>lt;sup>7</sup>As discussed by Fernández-Villaverde et al. (2011), a third-order approximation moves the ergodic means of the endogenous variables of the model away from their deterministic steady-state values. Hence, I compute the impulse responses in percent deviation from the stochastic steady state of each endogenous variable while keeping the level of corresponding standard shocks constant.



Figure 1.4: Pointwise 68% Probability Bands to Uncertainty Shocks in Rotemberg and Calvo Models

Note: The band of output is plotted in percent deviations from its stochastic steady state. The band of inflation is plotted in annualized percentage point deviations from its stochastic steady state.

inflation generally decreases in response to uncertainty shocks as compared to the Calvo model.<sup>8</sup> Hence, this exercise shows that the pricing assumptions are the main reason behind the different inflation responses and that the result is robust against different parameterization and sources of uncertainty. In the following subsections, I am going to explain why the effects of uncertainty shocks on inflation are different in the two models.

## 1.5.1 Households' Precautionary Decision: Rotemberg and Calvo

I display the pointwise 68% probability bands for the impulse response functions of the endogenous variables to a productivity uncertainty shock only in the Rotemberg (blue solid bands) and Calvo (red dashed bands) models in Figure 1.5. The effects of the other

<sup>&</sup>lt;sup>8</sup>Fasani and Rossi (2018) show that in the Rotemberg model, uncertainty shocks can have inflationary or deflationary effects depending on the monetary policy rule.



**Figure 1.5:** *Pointwise* 68% *Probability Bands to Productivity Uncertainty Shock in Rotemberg and Calvo Models* 

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.

uncertainty shocks are qualitatively similar and are displayed in Appendix A.3.1.

Increased uncertainty induces a precautionary saving effect on risk-averse households. This implies that when uncertainty increases, households want to consume less and save more. To save more, households would like to invest and work more. Since the fall in consumption implies a decline in aggregate demand, this decreases output. Lower output decreases the marginal products of capital and labor, thus leading to a fall in the demand for capital and labor. Consequently, this reduces the rental rates and wages, and thus decreases firms' marginal costs. To investigate the firms' pricing decision, I rewrite Equation (1.16) from recursive form to infinite sum form:

$$\begin{cases} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( (1-\varepsilon) \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon} \right) Y_{t+j} \\ - \phi \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{P_t(i)}{P_{t-1}(i)} Y_t = 0. \quad (1.31) \end{cases}$$

Since all intermediate goods firms solve an identical profit maximization problem, they choose the same price  $P_t(i) = P_t$ . In a symmetric equilibrium, the optimal pricing rule implies:

$$\left\{E_t\sum_{j=0}^{\infty}\Lambda_{t,t+j}\left(1-\varepsilon+\varepsilon\frac{MC_{t+j}}{P_{t+j}}M_{t+j}\right)Y_{t+j}\right\}-\phi\left(\frac{P_t}{P_{t-1}}-1\right)\frac{P_t}{P_{t-1}}Y_t=0.$$
(1.32)

Following Equation (1.32), when the marginal costs of the intermediate goods firms decrease, they lower their prices to stimulate the demand for output. This corresponds to a decrease in inflation. However, the prices do not decrease as much as the marginal costs due to the price adjustment costs. This implies an increase in price markups over marginal costs. Aggregate demand falls after all. Consequently, since the equilibrium is demand-determined, output, consumption, investment, and hours worked decrease. Under the Taylor rule, the monetary authority lowers the nominal interest rate to alleviate the adverse effects of uncertainty.

#### 1.5.2 Firms' Precautionary Decision: Calvo

Apart from the aggregate demand effect of uncertainty shocks discussed above, uncertainty shocks have an additional effect on firms' pricing decision in the Calvo model. Equation (1.19) can be rewritten as follows:

$$E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( \left(1-\varepsilon\right) \left(\frac{P_t^{\star}(i)}{P_{t+j}}\right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left(\frac{P_t^{\star}(i)}{P_{t+j}}\right)^{-\varepsilon} \right) Y_{t+j} = 0.$$
(1.33)



**Figure 1.6:** *Steady-State Expected Period Marginal Profits in Rotemberg and Calvo Models* Note: The period marginal profit is a function of the reset price.

The optimal reset price,  $P_t^{\star} = P_t^{\star}(i)$ , is the same for all firms resetting their prices in period *t*. This implies the following optimal pricing condition:

$$E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( (1-\varepsilon) \left( \frac{P_t^{\star}}{P_{t+j}} \right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left( \frac{P_t^{\star}}{P_{t+j}} \right)^{-\varepsilon} \right) Y_{t+j} = 0.$$
(1.34)

Therefore, the associated equilibrium is not symmetric.

Similarly to Fernández-Villaverde et al. (2015) and Born and Pfeifer (2019), without loss of generality, I explain firms' pricing decision in this model by using the steady-state period marginal profit function under the specific values of parametrization in Table 1.2. Under certainty, this function is as follows:

$$MP^{C} = \left( (1 - \varepsilon) \left( \frac{P_{certainty}^{\star}}{P} \right)^{1 - \varepsilon} + \varepsilon \frac{MC}{P} \left( \frac{P_{certainty}^{\star}}{P} \right)^{-\varepsilon} \right) Y.$$
(1.35)

I assume that the aggregate price P is equal to 1. Figure 1.6(b) displays that the  $MP^{C}$  is strictly convex in the reset price. This feature comes from the existence of the relative price dispersion. Economically, this implies that firms set their prices risk-aversely

like households discussed above. Under uncertainty, the steady-state expected period marginal profit function is as follows:

$$EMP^{C} = q \left( (1 - \varepsilon) \left( \frac{P_{uncertainty}^{\star}}{P^{l}} \right)^{1-\varepsilon} + \varepsilon \frac{MC}{P} \left( \frac{P_{uncertainty}^{\star}}{P^{l}} \right)^{-\varepsilon} \right) Y + (1 - q) \left( (1 - \varepsilon) \left( \frac{P_{uncertainty}^{\star}}{P^{h}} \right)^{1-\varepsilon} + \varepsilon \frac{MC}{P} \left( \frac{P_{uncertainty}^{\star}}{P^{h}} \right)^{-\varepsilon} \right) Y. \quad (1.36)$$

In this case, I assume that the aggregate price is either  $P^l = 0.95$  or  $P^h = 1.05$  with probability  $q = \frac{1}{2}$ . Figure 1.6(b) shows that to maximize their profits, the optimal price under uncertainty ( $P_{uncertainty}^{\star} = 1.02$ ) is higher than that under certainty ( $P_{certainty}^{\star} = 1$ ), applying Jensen's inequality. The firms which increase their prices will sell fewer goods but at higher price markups. In contrast, the firms which lower their prices may sell more but at negative markups, thereby incurring losses. Thus, when uncertainty increases, firms increase their prices to self-insure against being stuck with low prices in the future. Therefore, price markups increase by more. This precautionary pricing decision increases inflation and decreases output. Under the Taylor rule, the monetary authority increases the nominal interest rate to stabilize the increase in inflation.

On the other hand, those profit curves have zero curvature in the Rotemberg model as shown in Figure 1.6(a):

$$MP^{R} = EMP^{R} = \left(1 - \varepsilon + \varepsilon \frac{MC}{P}\right)Y = 0.$$
 (1.37)

Equation (1.37) implies that whatever the shocks realization is, all firms change their prices equally in the Rotemberg model.<sup>9</sup> This means that they do not face the trade-off present in the Calvo model where being an expensive firm is preferred to being a cheap one.

In sum, due to the precautionary pricing effect, inflation increases in the Calvo model, while it decreases in the Rotemberg model. Moreover, output, consumption, investment, and hours worked in the Calvo model decrease by more than those in the Rotemberg

<sup>&</sup>lt;sup>9</sup>One may argue that when capital is accumulated by the Rotemberg-type firms, this forward-looking behavior can induce a precautionary pricing behavior. However, due to a symmetric equilibrium, the capital accumulation by firms does not have any effects on their pricing behavior. In other words, those behaviors are independent from each other in the Rotemberg model. See Basu and Bundick (2017).



**Figure 1.7:** Steady-State Expected Period Marginal Profits with Different Elasticity of Substitution between Intermediate Goods in Rotemberg and Calvo Models

Note: The period marginal profit is a function of the reset price.

model. Thus, the Rotemberg model is qualitatively consistent with the empirical findings with respect to the transmission of uncertainty shocks. The opposite response of inflation to uncertainty shocks would prompt different monetary policy reactions. For this reason, understanding which propagation mechanism holds in the data becomes important.

# 1.6 Robustness Checks

To examine the robustness of my results, I conduct several robustness checks in this section.

#### 1.6.1 Elasticity of Substitution between Intermediate Goods

I show how important the elasticity of substitution between intermediate goods,  $\varepsilon$ , is for the responses of inflation to increased uncertainty. I display the steady-state expected period marginal profit functions for four levels of the elasticity of substitution ( $\varepsilon = 6$ , 11,



**Figure 1.8:** Impulse Responses of Inflation to Uncertainty Shocks with Different Elasticity of Substitution between Intermediate Goods in Rotemberg and Calvo Models

Note: The impulse response of inflation is plotted in annualized percentage point deviations from its stochastic steady state.

21, and 31) in the Rotemberg and Calvo models in Figure 1.7. These values imply a 20%, 10%, 5%, and 3.3% markup, respectively. As shown in Figure 1.7(a), the changes in  $\varepsilon$  do not have any effects on the marginal profits in the Rotemberg model. This confirms that unlike the Calvo model, uncertainty shocks do not have the precautionary pricing effects in the Rotemberg model. By contrast, as the elasticity becomes higher, the marginal profit curve becomes more convex in the Calvo model as shown in Figure 1.7(b). This means that firms become more risk-averse regarding their pricing decision. The more convex curve amplifies the precautionary pricing effect. Hence, the optimizing price increases for higher levels of  $\varepsilon$  in the Calvo model.

Furthermore, I conduct an impulse response function analysis. In this exercise, I set the specific values of parametrization in Table 1.2. Moreover, I fix  $\phi = 116.5$  in the Rotemberg model to evaluate the effects of the changes in  $\varepsilon$  only. Figure 1.8 displays the impulse responses of inflation to five different uncertainty shocks for four levels

of the elasticity of substitution ( $\varepsilon = 6$ , 11, 21, and 31) in the Rotemberg (blue line) and Calvo (red line) models. In the Rotemberg model, higher level of  $\varepsilon$  means less differentiation between the goods. As in differentiated Bertrand competition (Hotelling, 1929), less differentiation implies that firms lower their prices because they compete more vigorously. Therefore, inflation decreases by more in response to uncertainty shocks given the higher elasticity. However, in the Calvo model, the higher elasticity amplifies the precautionary pricing effect discussed above. Thus, the responses of inflation to uncertainty shocks are amplified for higher levels of  $\varepsilon$ . Exceptionally, inflation decreases in response to a government spending uncertainty shock in the Calvo model under the specific parametrization. This is because the drop in inflation triggered by the decrease in aggregate demand is not outweighted by the increase in inflation due to the precautionary pricing behavior of firms. Nevertheless, the feature of that higher elasticity amplifies the precautionary pricing behavior is preserved.

#### 1.6.2 Rotemberg Price Adjustment Costs

I show the importance of non-linearity when choosing two different types of price adjustment costs. One  $(AC_t^1)$  is scaled by aggregate output  $Y_t$  as in Bonciani and van Roye (2016), Leduc and Liu (2016), Basu and Bundick (2017), and Katayama and Kim (2018). The other  $(AC_t^2)$  is scaled by individual output  $Y_t(i)$  as in Fernández-Villaverde et al. (2015):

$$AC_t^1(i) = \frac{\phi}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t,$$
(1.38)

$$AC_t^2(i) = \frac{\phi}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t(i).$$
(1.39)

In a symmetric equilibrium ( $P_t(i) = P_t$  and  $Y_t(i) = Y_t$ ), the respective optimal pricing rules imply:

$$\phi\left(\frac{P_{t}}{P_{t-1}}-1\right)\frac{P_{t}}{P_{t-1}} = \phi E_{t}\Lambda_{t,t+1}\left(\frac{P_{t+1}}{P_{t}}-1\right)\frac{P_{t+1}}{P_{t}}\frac{Y_{t+1}}{Y_{t}} + 1 - \varepsilon + \varepsilon\frac{MC_{t}}{P_{t}}M_{t}, \quad (1.40)$$

$$\phi\left(\frac{P_{t}}{P_{t-1}}-1\right)\frac{P_{t}}{P_{t-1}} = \phi E_{t}\Lambda_{t,t+1}\left(\frac{P_{t+1}}{P_{t}}-1\right)\frac{P_{t+1}}{P_{t}}\frac{Y_{t+1}}{Y_{t}} + 1 - \varepsilon + \varepsilon\frac{MC_{t}}{P_{t}}M_{t} + \frac{\varepsilon\phi}{2}\left(\frac{P_{t}}{P_{t-1}}-1\right)^{2} \tag{1.41}$$



**Figure 1.9:** *Pointwise* 68% *Probability Bands to Uncertainty Shocks in Rotemberg* 1 *and Rotemberg* 2 *Models* 

Note: The band of output is plotted in percent deviations from its stochastic steady state. The band of inflation is plotted in annualized percentage point deviations from its stochastic steady state.

The two optimal pricing conditions above are equivalent up to a first-order approximation.

By using these two Rotemberg models, I conduct a prior predictive analysis under the parametrization in Table 1.2. Then, I plot the pointwise 68% probability bands for the impulse response functions of output and inflation to each uncertainty shock in the Rotemberg 1 (blue solid line) and Rotemberg 2 (green dashed line) models in Figure 1.9.<sup>10</sup> The figure shows significant differences between the two models. Unlike in the Rotemberg 1 model, output decreases by more and inflation increases in the Rotemberg 2 model. Interestingly, the responses in the Rotemberg 2 model are similar to those in the Calvo model. However, the propagation mechanisms of uncertainty shocks in the

<sup>&</sup>lt;sup>10</sup>I display the full sets of model impulse response functions in Appendix A.3.2.

two models are totally different.

The different responses in the two Rotemberg models depend on how price adjustment costs are scaled. The Rotemberg 2 optimal pricing rule (1.41) has an additional quadratic price adjustment cost term  $\frac{e\phi}{2} \left(\frac{P_l}{P_{l-1}}-1\right)^2$  relative to the Rotemberg 1 optimal pricing rule (1.40). Therefore, the Rotemberg 2 model adds one further effect to the aggregate demand channel of uncertainty shocks already present in the Rotemberg 1 model as discussed in Section 1.5.1. To be specific, when marginal costs decrease due to the aggregate demand effect of uncertainty shocks, intermediate goods firms would like to lower their prices to stimulate the demand for output. However, the change in prices raises the quadratic price adjustment cost term. If the increase in the cost term dominates the decrease in marginal costs, firms would increase their prices. Consequently, inflation can increase in response to uncertainty shocks in the Rotemberg 2 model as in Fernández-Villaverde et al. (2015). This is a different channel of uncertainty propagation from the precautionary pricing channel of the Calvo model.

#### 1.6.3 Density of Inflation

An additional and simple way of confirming the firms' precautionary behavior mechanism in the Calvo model is to show the density of inflation. Using the specific values of parametrization in Table 1.2 and the policy functions, the Rotemberg and Calvo models are simulated separately for 20,000 periods in response to each uncertainty shock considered above. I then identify periods of increased uncertainty and finally plot the histograms of inflation to each uncertainty shock for the Rotemberg 1 (blue bar), Rotemberg 2 (green bar), and Calvo (red bar) models in Figure 1.10. I confirm important differences in the shape of the distributions.<sup>11</sup> Consistently, in the Rotemberg 1 model, the densities of inflation are left-skewed. On the other hand, in the Rotemberg 2 and Calvo models, the densities of inflation are right-skewed. Moreover, the densities of inflation are more right-skewed in the Calvo model than in the Rotemberg 2 model. These results are consistent with the impulse response function analysis discussed above.

<sup>&</sup>lt;sup>11</sup>In Appendix A.4, I show that there are little differences in the distributions of inflation to level shocks in the three models.



Figure 1.10: Histograms of Inflation to Uncertainty Shocks in Rotemberg and Calvo Models

Note: Sturges' rule is used to determine the number and width of the bins. The response of inflation is plotted in annualized percentage point deviation from its stochastic steady state.

# 1.7 Conclusion

This paper contributes to our understanding of the role of different sticky price assumptions in the propagation of uncertainty shocks. An important contribution of this paper is to show that in contrast to the Calvo model, the Rotemberg model does not generate a precautionary pricing effect of uncertainty shocks. For this reason, the response of inflation to uncertainty shocks is opposite in the Rotemberg and Calvo models. This result has important implications for monetary policy. Depending on the model adopted, the implied policy responses to higher uncertainty are qualitatively different. The implications of the Rotemberg model are qualitatively more consistent with the empirical findings than those of the Calvo model. However, from a quantitative perspective, in both models uncertainty shocks have much smaller effects on macro aggregates than those shown by the empirical evidence. To bring the theoretical models closer to the data, future research should focus on understanding the amplification channels of uncertainty shocks.

# Chapter 2

# The Long-Run Effects of Uncertainty Shocks

Co-authored with Dario Bonciani

# 2.1 Introduction

Heightened uncertainty is considered by policymakers and economists as one of the main factors behind the depth of the Great Recession and the subdued recovery (e.g. see Stock and Watson, 2012). Understanding the channels through which uncertainty propagates to the real economy is therefore relevant both from a research and a policy perspective. In this paper, we study how shocks to uncertainty can have a negative impact on economic activity in the short as well as in the long term.

To motivate that uncertainty may negatively affect economic activity in the long run, in Figure 2.1, we show how macroeconomic uncertainty is a strong predictor of future low-frequency movements in Total Factor Productivity (TFP). In particular, we compare the backward-looking moving average of macroeconomic uncertainty over the previous 20 quarters and the forward-looking moving average of the TFP growth rate over the next 20 quarters. The uncertainty measure considered is the one proposed by Jurado et al. (2015) and updated by Ludvigson et al. (2019).<sup>1</sup> The measure of TFP growth is taken

<sup>&</sup>lt;sup>1</sup>In Jurado et al. (2015), uncertainty is defined as the common time-varying volatility in the unforecastable component of a large set of macroeconomic time series.



Figure 2.1: Macro Uncertainty and TFP Growth, U.S.

Note: The solid blue line represents the 5-year backward-looking moving average of the macro uncertainty measure from Jurado et al. (2015) updated by Ludvigson et al. (2019). We use the annual average of their monthly series with h = 3 (i.e., 3-month-ahead uncertainty). The dashed red line represents the 5-year forward-looking moving average of the annualised TFP growth rate from Fernald (2014).

from Fernald (2014), which is adjusted for capacity utilization.<sup>2</sup> The left-hand-side and right-hand-side axes relate respectively to uncertainty and TFP growth. Evidently, there is a strong negative correlation between the two series (-53.91%).<sup>3</sup>

This result is consistent with the analysis conducted in the seminal study by Ramey and Ramey (1995), who find that countries with higher volatility have lower mean growth. The evidence provided in Figure 2.1, while suggestive, does not imply any causality in one direction or the other, nor it excludes the possibility that a third factor is driving both measures. To provide empirical evidence that uncertainty shocks cause a long-run downturn in economic activity, in section 2.2, we conduct an SVAR analysis for the US. We find that shocks increasing macroeconomic uncertainty induce significant reductions in the main macroeconomic aggregates and in TFP that persist over 40 quarters.

<sup>&</sup>lt;sup>2</sup>In particular, Fernald (2014) proposes a measure of TFP constructed as a Solow residual, cleansing for variations in factor utilization, which is an important source of non-technological cyclicality.

<sup>&</sup>lt;sup>3</sup>Section B.1 in the appendix shows how the correlation between *uncertainty* and tfp varies as we change the window over which we average the two measures.

We rationalize these results by estimating a dynamic stochastic general equilibrium (DSGE) model augmented with an endogenous growth mechanism of vertical innovation in the spirit of Grossman and Helpman (1991) and Aghion and Howitt (1992). Productivity has an endogenous component that depends on the aggregate level of R&D services. Spillovers stemming from the accumulation of R&D allow business cycle shocks to affect long-run growth. In this framework, rises in TFP uncertainty cause a fall in output, consumption, and investment in physical capital and R&D. The decrease in the aggregate level of R&D leads to a fall in productivity, and the decline in economic activity becomes therefore permanent. Moreover, we show that when households have recursive preferences and take risks about future long-term growth into account, the precautionary savings motive of households is strongly amplified and the overall effects of uncertainty shocks become quantitatively significant. To highlight the relevance of this "long-run risk" channel, we compare our baseline DSGE model featuring endogenous growth and EZ preferences with alternative model specifications that do not feature endogenous growth or EZ preferences (or both) and show how the combination of the two elements is necessary to obtain sizable effects of uncertainty shocks.

**Related Literature** This work is related to the growing literature on uncertainty shocks, which started with the seminal contribution by Bloom (2009). Numerous papers (e.g. Bachmann et al., 2013; Born and Pfeifer, 2014; Backus et al., 2015; Fernández-Villaverde et al., 2015; Leduc and Liu, 2016; Basu and Bundick, 2017; Katayama and Kim, 2018; Oh, 2019) have investigated how uncertainty shocks could generate business cycle fluctuations both with empirical and theoretical frameworks. From an empirical perspective, the literature has found that rises in uncertainty can cause a significant fall in economic activity. This result has been found using various measures of uncertainty such as financial volatility indexes (Bloom, 2009), macroeconomic uncertainty measures (Jurado et al., 2015; Rossi and Sekhposyan, 2015; Kozeniauskas et al., 2018) or political uncertainty news-based indexes (Baker et al., 2016; Caldara and Iacoviello, 2018).

The theoretical literature has concentrated on disentangling the potential transmission channels through which uncertainty can affect macroeconomic variables and on quantifying the effects within DSGE models. The main transmission channels that have been discussed in the literature are: (i) the precautionary savings channel, that leads risk-averse agents to reduce consumption and increase labor supply (Leland, 1968 and Kimball, 1990); (ii) the real options channel, which causes firms to postpone irreversible investments (Bernanke, 1983; Pindyck, 1991; Bertola and Caballero, 1994); (iii) the precautionary investment channel, for which a higher uncertainty in productivity raises investment, hours, and output if the optimal choices of capital and labor are convex in productivity (Oi, 1961; Hartman, 1976; Abel, 1983); (iv) the cost of financing channel, for which rises in uncertainty lead to increases in risk premia that in turn make borrowing more costly and therefore reduce investment (Christiano et al., 2014; Gilchrist et al., 2014; Arellano et al., 2016). While in partial equilibrium these transmission channels have clear-cut effects, they may offset each other in a general equilibrium framework. Basu and Bundick (2017) show that in a model with sticky prices and time-varying markups uncertainty shocks can generate business cycle fluctuations, i.e. co-movement between output, consumption, and investment.

The literature has provided mixed evidence on the quantitative relevance of uncertainty shocks. With standard business cycle models, the effects of uncertainty shocks tend to be economically insignificant (e.g. Bachmann and Bayer, 2013; Born and Pfeifer, 2014). The reason for the small effects found in the literature is that the shocks are small and the standard business cycle models are too linear to obtain a significant amplification. Accounting for nonlinearities such as the zero lower bound has been found to be an important source of amplification (Fernández-Villaverde et al., 2015; Basu and Bundick, 2017). A recent paper by Bianchi et al. (2018b) finds significant effects of uncertainty on both the business cycle and term premia dynamics in an estimated medium-scale Markov-Switching DSGE model. Another strand of the literature has also shown that uncertainty could be amplified in the presence of frictions in the financial sector (Christiano et al., 2014; Bonciani and van Roye, 2016) or in the labor market (Leduc and Liu, 2016). In this paper, we consider an additional source of nonlinearity deriving from the aversion of households to long-term risks to their consumption process, in the spirit of the finance literature on long-run risk (Bansal and Yaron, 2004; Kung and Schmid, 2015; Kung, 2015). This literature has shown how the equity premium puzzle could be solved in models featuring Epstein-Zin preferences and shocks to long-run

future consumption growth. Some papers in the literature on uncertainty shocks such as de Groot et al. (2018) also considered New Keynesian models with EZ preferences, but failed to find significant effects of uncertainty shocks, as they abstracted from the long-run risk channel.

By analysing how uncertainty affects economic activity in the long-run, we depart from the previous literature which only focused on the business cycle effects of uncertainty. Hence this work bridges the literature on uncertainty shocks with another relatively recent strand of the literature that analyses the long-run growth impact of business cycle shocks (e.g. Anzoategui et al., 2017; Bianchi et al., 2018a).

The rest of this paper is organized as follows. Section 2.2 presents the empirical evidence. In Section 2.3, we lay out the DSGE model, while in Section 2.4, we describe the model estimation. In Section 2.5, we present our results. Last, in Section 2.6, we provide some concluding remarks.

# 2.2 SVAR Analysis

In this section, we estimate a Vector Autoregressive (VAR) model for the US economy and analyse the impulse responses (IRFs). In our IRFs analysis, we look at a longer horizon than usually considered in the standard business cycle literature (40 quarters, i.e. 120 months). We identify the shocks with a recursive scheme (i.e. Cholesky identification). The baseline VAR contains 9 variables, entering in the following order: (i) the Standard and Poors 500 index, which is commonly included in the literature to control for movements in the stock market (*S&P500*); (ii) the measure of macroeconomic uncertainty estimated by Jurado et al. (2015) and updated by Ludvigson et al. (2019); (iii) GDP as a measure of aggregate macroeconomic activity (*Output*); (iv) personal consumption in nondurables and services (*Consumption*); (v) durable consumption and private fixed investment excluding R&D investment (*Capital Investment*); (vi) private fixed investment in R&D (*R&D Investment*); (vii) the GDP deflator, as a measure of the price level (*Price*); (viii) the shadow interest rate by Wu and Xia (2016), as a measure of the US monetary policy stance (Interest Rate); (ix) utilization-adjusted TFP as measured by Fernald (2014) (*TFP*). We take logs of the S&P 500 index and the uncertainty measure, to interpret the IRFs in percentage terms. Output, consumption, capital investment, and R&D investment are expressed in logs, real per capita terms. The ordering described above implies that uncertainty is contemporaneously affected by shocks to the S&P500 index, but not by the other macroeconomic variables. In subsequent periods, however, uncertainty responds to all shocks through its relation with the lags of the variables included in the VAR model. This identification strategy is in line with that in Bloom (2009), Leduc and Liu (2016), and Basu and Bundick (2017). The focus on macroeconomic uncertainty is supported by two recent empirical papers by Carriero et al. (2018a) and Angelini et al. (2019) that show that macroeconomic uncertainty can be considered an exogenous source of business cycle fluctuations.

In the baseline framework, data are at a quarterly frequency, spanning the period 1960Q3-2018Q2, and all variables that are available at a higher frequency are averaged over the quarter. We estimate the reduced-form VAR by ordinary least squares:

$$X_t = c + \sum_{k=1}^{L} A_k X_{t-k} + e_t$$
(2.1)

where  $X_t$  is the vector of endogenous variables,  $A_k$  is the coefficient matrix for the *k*-th lag of  $X_t$  and  $e_t$  is the vector of reduced form innovations, which have zero mean and variance  $\Sigma$ . We include two lags in our VAR, as suggested by the Akaike Information Criterion. All variables in the VAR enter in levels, since differencing or filtering the data discards information about the long-run properties of the data (Canova, 2007; Lütkepohl, 2013).

Figure 2.2 displays the impulse responses obtained from the VAR. The solid lines are the median responses of the endogenous variables to a one standard deviation uncertainty shock, while the shaded areas represent 68 (dark grey) and 95 (light grey) percent confidence intervals. Output (real GDP) declines by about 0.4 percent, while consumption and capital investment fall by 0.3 and 1.5 percent after 10 quarters. Rises in uncertainty also lead to an initial increase in prices and in the interest rate. Furthermore, the impulse responses show that uncertainty shocks significantly dampen R&D investment and TFP, which fall by approximately 0.6 and 0.2 percent. Last but not least, all real variables fall in a very persistent manner and do not revert to their trend within



Figure 2.2: Impulse Responses to a Macro Uncertainty Shock (Baseline VAR)

Note: Variables are in percent change except for the interest rate, which is in annualized percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

40 quarters.

**Robustness** In the appendix, Section B.2.2, we test the robustness of our baseline results to a variety of changes: (i) we change the ordering of the variables in our model and place uncertainty last in our VAR (Figure B.1); (ii) we include the inverse of the labor

share as a measure of markups, in line with Fernández-Villaverde et al. (2015) (Figure B.2); (iii) we consider alternative measures of macroeconomic uncertainty from Rossi and Sekhposyan (2015) (Figures B.3 and B.4); (iv) we increase the number of lags included in the VAR (Figure B.5); (v) we estimate an informationally rich monthly Factor-Augmented VAR (Figures B.6 and B.7); (vi) we estimate both the quarterly and monthly models with data from January 1985 until June 2018 (Figures B.8 and B.9), in order to take into account the regime shift in monetary policy induced by the *Volcker disinflation* (see e.g. Bianchi and Ilut, 2017). In all the robustness exercises, we find the baseline results to be confirmed. Macroeconomic uncertainty shocks lead to very persistent declines in the main macroeconomic aggregates and in total factor productivity. The responses of consumption and TFP tend to be the most persistent and the decline is in most instances significant (68% confidence) for over 40 quarters. For the sake of conciseness, we leave the details of the robustness checks to the appendix.

# 2.3 The Model

This section studies the transmission channels of uncertainty shocks in a New-Keynesian DSGE model with endogenous growth through R&D investment. Households have recursive preferences à la Epstein and Zin (1989) (EZ) to separately calibrate the parameters governing relative risk aversion and the elasticity of intertemporal substitution. Moreover, these preferences make households averse to long-term risk about their consumption process (Bansal and Yaron, 2004). The model features an endogenous growth mechanism of vertical innovation in the spirit of Grossman and Helpman (1991) and Aghion and Howitt (1992), which is introduced as in Kung (2015) and Bianchi et al. (2018a). Uncertainty shocks are modelled assuming that the exogenous component of TFP follows an AR(1) process with stochastic volatility as in Fernández-Villaverde et al. (2011).

#### 2.3.1 Households

The representative household maximizes its lifetime utility choosing consumption  $C_t$ , hours worked  $L_t$ , investment in physical capital  $I_t$  and in R&D  $S_t$ , the rates of utilization

of physical capital  $x_{K,t}$  and R&D  $x_{N,t}$  and next period bond holdings  $b_{t+1}$ . The aggregate stocks of physical capital and R&D are predetermined and denoted by  $K_t$  and  $N_t$ . The parameters  $\psi$  and  $\gamma$  govern the household's elasticity of intertemporal substitution and relative risk aversion. If  $\psi = \frac{1}{\gamma}$  the utility function reduces to the standard power utility. In our case instead, under the assumption  $\gamma \ge \frac{1}{\psi}$ , this type of utility function implies a preference for the early resolution of uncertainty, i.e. households dislike uncertainty over future utility. The problem of the household is formalized as follows:

$$V_{t} = \max\left[ (1-\beta) u_{t}^{1-\frac{1}{\psi}} + \beta \left( E_{t} V_{t+1}^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}},$$
(2.2)

where  $E_t$  is the conditional expectation operator and  $\beta$  is the subjective discount factor of the households. The term  $u_t$  aggregates consumption and leisure,  $\overline{L} - L_t$  (where  $\overline{L}$ represents the household's total time endowment), in a Cobb-Douglas fashion:

$$u_t = C_t \left( \bar{L} - L_t \right)^{\chi}. \tag{2.3}$$

The maximization problem is subject to the following budget constraint:

$$C_t + I_t + S_t + \frac{\pi_{t+1}}{R_t} b_{t+1} = w_t L_t + r_{K,t} x_{K,t} K_t + r_{N,t} x_{N,t} N_t + b_t + \Pi_t,$$
(2.4)

where  $R_t$  is the nominal return on the risk-free bonds, and  $\pi_t$  is today inflation. Variables  $r_{l,t}$  ( $l = \{K, N\}$ ) are the return on capital (either physical capital or R&D). The aggregate stocks of physical capital and R&D evolve according to the following laws of motion:

$$K_{t+1} = \left(1 - \delta_K \left(x_{K,t}\right)^{\xi_K}\right) K_t + \Lambda_K \left(\frac{I_t}{K_t}\right) K_t, \qquad (2.5)$$

$$N_{t+1} = \left(1 - \delta_N \left(x_{N,t}\right)^{\xi_N}\right) N_t + \Lambda_N \left(\frac{S_t}{N_t}\right) N_t, \tag{2.6}$$

where  $\delta_l$  ( $l = \{K, N\}$ ) is the depreciation rate. Utilisation  $x_{l,t}$  is introduced similarly as in Neiss and Pappa (2005) and enters the laws of motion (2.5) and (2.6) nonlinearly with parameter  $\xi_l$ . The function  $\Lambda_l$  ( $\cdot$ ) represents positive, concave adjustment cost functions, defined as in Jermann (1998):

$$\Lambda_K\left(\frac{I_t}{K_t}\right) = a_{K,1} + \frac{a_{K,2}}{1 - \frac{1}{\tau_K}} \left(\frac{I_t}{K_t}\right)^{1 - \frac{1}{\tau_K}},\tag{2.7}$$

$$\Lambda_N\left(\frac{S_t}{N_t}\right) = a_{N,1} + \frac{a_{N,2}}{1 - \frac{1}{\tau_N}} \left(\frac{S_t}{N_t}\right)^{1 - \frac{1}{\tau_N}}.$$
(2.8)

These adjustment costs capture the idea that changing the stocks of capital and R&D rapidly is more costly than changing them slowly. The presence of adjustment costs also implies that the shadow prices of  $K_t$  and  $N_t$  will not be constant. The household's stochastic discount factor derived under the EZ preferences is given by the following condition:

$$M_{t,t+1} = \beta \left(\frac{u_{t+1}}{u_t}\right)^{1-\frac{1}{\psi}} \left(\frac{C_t}{C_{t+1}}\right) \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$
(2.9)

#### 2.3.2 Final Goods Firms

The final good  $Y_t$  is produced by aggregating intermediate inputs  $Y_t(i)$  by a constant elasticity of substitution technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}},$$
(2.10)

where  $\varepsilon$  is the elasticity of substitution of intermediate goods. The cost-minimization problem for the final good firm implies that the demand for the intermediate good *i* is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t, \qquad (2.11)$$

where  $P_t(i)$  is the price of the intermediate input. Finally, the zero-profit condition implies that the price index is expressed as:

$$P_t = \left(\int_0^1 Y_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$
(2.12)

### 2.3.3 Intermediate Goods Firms

There exists a continuum of intermediate-goods producing firms indexed by  $i \in (0, 1)$ that rent labor  $L_t(i)$  and services of physical capital  $x_{K,t}(i)K_t(i)$  and R&D  $x_{N,t}(i)N_t(i)$ from the households at the respective prices  $w_t$  (real wage),  $r_t^K$  (rental rate of physical capital), and  $r_t^N$  (rental rate of R&D). These firms act in a monopolistically competitive environment and set their price  $P_t(i)$  facing quadratic adjustment costs à la Rotemberg (1982). Since firms are owned by the households, they discount future profits  $\Pi_{t+i}(i)$  by the stochastic discount factor  $M_{t,t+j}$  defined in Equation (2.9) and solve the following optimization problem:

$$\max E_t \sum_{j=0}^{\infty} M_{t,t+j} \Pi_{t+j}(i),$$
 (2.13)

$$\Pi_{t}(i) = \frac{P_{t}(i)}{P_{t}} Y_{t}(i) - w_{t} L_{t}(i) - r_{K,t} x_{K,t}(i) K_{t}(i) - r_{N,t} x_{N,t}(i) N_{t}(i) - \frac{\phi_{P}}{2} \left(\frac{P_{t}(i)}{\pi P_{t-1}(i)} - 1\right)^{2} Y_{t,t}$$
(2.14)

$$Y_t(i) = (x_{K,t}(i)K_t(i))^{\alpha} (Z_t(i)L_t(i))^{1-\alpha},$$
(2.15)

$$Z_t(i) = A_t \left( x_{N,t}(i) N_t(i) \right)^{\eta} \left( x_{N,t} N_t \right)^{1-\eta},$$
(2.16)

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t.$$
(2.17)

Equation (2.13) and (2.14) represent the stream of lifetime profits and  $\pi$  is the (nonstochastic) steady state level of inflation. Intermediate-good firm *i* produces product  $Y_t(i)$ using a Cobb-Douglas technology as defined in Equation (2.15). Firm *i*'s productivity  $Z_t(i)$  is given by the product of an exogenous component  $A_t$  and an endogenous part that depends both on the amount of R&D services rented by the individual firm  $x_{N,t}(i)N_t(i)$ and on the aggregate level of R&D services  $x_{N,t}N_t$ . The fact that productivity depends on the utilised stock of R&D represents the presence of technological spillovers and captures the idea that accumulated knowledge facilitates the creation of new knowledge. Finally, the parameter  $1 - \eta \in (0, 1)$  governs the degree of technological spillovers over the utilized stock of R&D.

#### 2.3.4 Monetary Authority

The monetary authority sets the nominal rate  $R_t$  following a policy rule à la Taylor (1993). More specifically, we assume that the nominal policy rate depends on deviation of inflation from its non-stochastic steady state and on output growth. The monetary policy rule is formalized as follows:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\rho_{\pi}} \left(\frac{\Delta Y_t}{\Delta Y}\right)^{\rho_{Y}},\tag{2.18}$$

where *R* and  $\pi$  are the steady-state nominal interest rate and the steady-state inflation respectively and  $\rho_{\pi}$  and  $\rho_{Y}$  are the reaction coefficients to inflation and output growth.

#### 2.3.5 Closing the Model

The Rotemberg pricing assumption, as described by Equation (2.14), implies a symmetric equilibrium, such that all variables  $X_t(i) = X_t$ . Finally, the model closed by the usual resource constraint and assuming the risk-free bonds are in zero net supply ( $b_t = 0$ ):

$$Y_t = C_t + I_t + S_t + \frac{\phi_P}{2} \left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t,$$
(2.19)

which states that aggregate output  $Y_t$  is used for expenditure in consumption  $C_t$ , investment in physical capital  $I_t$ , investment in R&D  $S_t$ , and price adjustment costs.

#### 2.3.6 Exogenous Processes

The exogenous component of TFP follows a stationary AR(1) with stochastic volatility (see for example Fernández-Villaverde et al., 2011):

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A, \qquad (2.20)$$

where  $\rho_A$  is the parameter governing the persistence of the TFP shock  $\varepsilon_t^A$ , which is assumed to follow an iid standard normal stochastic process. Similarly, the time-varying standard deviation of the first moment shock,  $\sigma_t^A$ , follows itself a stationary AR(1) process:

$$\log \sigma_t^A = (1 - \rho_{\sigma^A}) \log \sigma^A + \rho_{\sigma^A} \log \sigma_{t-1}^A + \sigma^{\sigma^A} \varepsilon_t^{\sigma^A}.$$
(2.21)

The parameter  $\rho_{\sigma^A}$  measures the persistence of the uncertainty shock. The term  $\varepsilon_t^{\sigma^A}$  is the uncertainty shock, which follows an iid standard normal process.

# 2.4 Solution, Calibration, and Estimation

## 2.4.1 Solution Method

In order to induce stationarity, we divide all the trending variables ( $V_t$ ,  $u_t$ ,  $C_t$ ,  $I_t$ ,  $K_t$ ,  $S_t$ ,  $N_t$ ,  $Y_t$ ,  $w_t$ , and  $Z_t$ ) by the aggregate stock of R&D,  $N_t$ .<sup>4</sup> We then solve the model with perturbation methods, approximating the policy function to a third-order around

<sup>&</sup>lt;sup>4</sup>In appendix B.3, we report the detrended equilibrium conditions.

its non-stochastic steady state (Adjemian et al., 2011).<sup>5</sup> As emphasized in Fernández-Villaverde et al. (2011), the third-order approximation of the policy function is necessary to analyse the effects of uncertainty shocks independently of the first moment shocks. With lower orders of approximation, in fact, uncertainty shocks either do not matter at all (first-order approximation) or they enter as cross-products with the other state variables (second-order approximation). Furthermore, as discussed in Caldara et al. (2012), perturbation methods for DSGE models with stochastic volatility and recursive preferences are comparable, in terms of accuracy, to global solution methods such as Chebyshev polynomials and value function iteration, while being computationally more efficient.

#### 2.4.2 Calibrated Parameters

Table 2.1 reports the values of the parameters used for the simulations of the model. Some parameters are calibrated following the literature. In particular, the parameters relating to the household's preferences are specified in line with the long-run risk literature. The discount factor  $\beta$  is set equal to 0.997, while the coefficients of relative risk aversion  $\gamma$  and elasticity of intertemporal substitution  $\psi$  are set to 66 and 1.73, in line with the estimates by van Binsbergen et al. (2012). The risk-aversion parameter is lower than assumed in other works in the literature such as Rudebusch and Swanson (2012), Mumtaz and Theodoridis (2017), and Basu and Bundick (2017, 2018), who used values between 75 and 100. An intertemporal elasticity larger than 1 is also in line with Bansal and Yaron (2004). Similarly as in Neiss and Pappa (2005), the capital and R&D utilization parameters  $\xi_K$  and  $\xi_N$  are endogenously set to ensure steady-state values of utilization  $x_K$  and  $x_N$  of 1. The depreciation rate of physical capital is standard in the business cycle literature (0.02), used to match the average capital-investment ratio. The depreciation rate of R&D is set in line with Kung (2015) to 0.0375, which corresponds to an annualized depreciation rate of 15%, a standard value assumed by the Bureau of Labor Statistics in the R&D stock calculations. The share of capital in the production function  $\alpha$  is equal to 0.33 and the demand elasticity  $\varepsilon$  is equal to 6, implying a steady-state markup of

<sup>&</sup>lt;sup>5</sup>The model is solved using Dynare 4.4.3 (MATLAB R2018a). In order to obtain a non-explosive behavior of the simulations, Dynare relies on the pruning algorithm described in Andreasen et al. (2018).

Table 2.1:	Baseline	Quarterly	Parameters
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Parameter	Description	Value	Source/Target
Households	6		
β	Discount factor	0.997	van Binsbergen et al. (2012)
$\psi$	Elasticity of intertemporal substitution	1.73	van Binsbergen et al. (2012)
$\gamma$	Risk aversion	66	van Binsbergen et al. (2012)
$\delta_K$	Capital depreciation rate	0.02	Standard
$\delta_N$	R&D depreciation rate	0.0375	Kung (2015)
$ au_K$	Capital adjustment cost parameter	7.5036	Estimation
$ au_N$	R&D adjustment cost parameter	6.2454	Estimation
Firms			
α	Power on capital in production	0.33	Standard
ε	Elasticity of substitution between goods	6	20% markup
$\phi_P$	Price adjustment cost parameter	59.46	4Q stickiness
η	Technological spillovers	0.1	Kung (2015)
Monetary A	Authority	1.00	
$\pi$	Steady-state inflation	1.005	2% annualized inflation rate
$ ho_{\pi}$	Weight on inflation in policy rule	1.5	Standard
$ ho_Y$	Weight on output in policy rule	0.35	Standard
Exogenous	Processes		
A	Steady-state productivity	0.2375	1.64% annualized output growth
$\rho_A$	Persistence of productivity Shock	0.6586	Estimation
$\sigma^{A}$	Volatility of productivity shock	0.0115	Estimation
$\rho_{\sigma^A}$	Persistence of uncertainty Shock	0.8415	Estimation
$\sigma^{\sigma^A}$	Volatility of uncertainty Shock	0.3357	Estimation
-	· · · · · · · · · · · · · · · · · · ·		

20%. The Rotemberg price adjustment parameter  $\phi_P$  is set to 59.46, which to a first order approximation implies a Calvo parameter of 0.75 (i.e. firms, on average, update their price every 4 quarters). The parameter of technological spillovers  $\eta$  is set to 0.1, in order to match the R&D investment rate in the steady state (Kung, 2015; Kung and Schmid, 2015). The Taylor rule coefficients of inflation  $\rho_{\pi}$  and output growth  $\rho_Y$  are set respectively to 1.5 and 0.35, which are standard values in the New Keynesian literature. The steady state value of productivity *A* is calibrated to 0.2375 to match the mean growth rate of output (1.64% annualized).

# 2.4.3 Estimated Parameters

The parameters that appear in bold in Table 2.1 are estimated via indirect inference. The basic idea behind the estimation methodology is to find a vector of parameter estimates  $\hat{\lambda}$  that minimises both the distance between the impulse responses of our VAR ( $\hat{r}$ ) and those implied by the DSGE model (r), as well as the difference between some key empirical moments ( $\hat{m}$ ) from their counterparts obtained with simulations of our DSGE model (m). More formally, the estimation procedure involves solving the following minimization problem:

$$D = \min_{\lambda} \left[ \hat{r} - r(\lambda) \right]' W_r^{-1} \left[ \hat{r} - r(\lambda) \right] + \Omega \left[ \hat{m} - m(\lambda) \right]' W_m^{-1} \left[ \hat{m} - m(\lambda) \right], \quad (2.22)$$

where  $W_j^{-1}$  ( $j \in \{r, m\}$ ) is the inverse of the variance matrix of the moments. In line with Basu and Bundick (2017), the scalar  $\Omega$  is set to roughly equalize the weight on matching impulse responses and moments. The impulse responses we target to match are those of output ( $Y_t$ ), consumption ( $C_t$ ), capital investment ( $I_t$ ), and R&D investment ( $S_t$ ). Moreover, we target the unconditional standard deviations of the growth rates of the variables mentioned above.

Table 2.2 displays the results of our estimation procedure. As we will further discuss in Section 2.5.2, aside from our baseline framework, we also consider and estimate three alternative versions of our model. Model B is a version of the model with EZ preferences and no endogenous growth mechanism. In this case, we assume households can invest in physical capital and not in R&D. Model C features the endogenous growth mechanism but no EZ preferences. To this end, we set the RRA parameter  $\gamma$  equal to 2, as common in the business cycle literature, and the EIS equal to  $\frac{1}{\gamma}$ . Model D features neither the endogenous growth mechanism nor EZ preferences. In models B and D, productivity is purely exogenous and the steady-state level of TFP (*A*) is set equal to 1.

As for the baseline case, we estimate the parameters relating to the physical capital and R&D adjustment costs  $\tau_K$  and  $\tau_N$  to be equal to 7.5 and 6.2, in line with the calibrated values used in Kung (2015). We also estimate the parameters of the exogenous processes. For the persistence of the TFP level shock ( $\rho_A$ ), we find a value of 0.66, while for the steady-state level of TFP uncertainty ( $\sigma^A$ ) we obtain a value of 0.012. The autocorrelation

	Data	Baseline	Model B	Model C	Model D
Calibrated Parameter					
$\gamma$	-	66	66	2	2
$\psi$	-	1.73	1.73	0.5	0.5
A	-	0.2375	1	0.2990	1
Estimated Parameter					
$ au_K$	-	7.5036	15.3196	1.1159	0.9756
$ au_N$	-	6.2454	-	0.8076	-
$ ho_A$	-	0.6586	0.9016	0.4586	0.5187
$\sigma^A$	-	0.0115	0.0100	0.1030	0.0538
$ ho_{\sigma^A}$	-	0.8415	0.8781	0.9663	0.9714
$\sigma^{\sigma^A}$	-	0.3357	0.3029	0.1909	0.2367
Unconditional Volatility	Data	Baseline	Model B	Model C	Model D
ΔΥ	0.82	0.82	0.79	0.89	0.97
$\Delta C$	0.46	0.68	0.49	0.52	0.49
$\Delta I$	2.35	1.49	2.34	2.52	2.55
$\Delta S$	1.31	1.26	-	1.55	-

 Table 2.2: Empirical and Model-Implied Moments in Macroeconomic Aggregates

Note: The lower part of the table compares the empirical standard deviation of the growth rates (log first differences) with those from the models' simulations. Standard deviations are scaled by 100. The empirical sample period is 1960Q3-2018Q2. The baseline model features both EZ preferences and the endogenous growth mechanism. Model B features EZ preferences but no endogenous growth mechanism. Model C features non-recursive CRRA preferences and the endogenous growth mechanism. Last, model D features standard (non-EZ) preferences and no endogenous growth mechanism.

of TFP volatility  $\rho_{\sigma^A}$  is estimated to be equal to 0.84 and the standard deviation of the volatility shock  $\sigma^{\sigma^A}$  is 0.34. The relatively low persistence of the exogenous component of TFP can be explained by the presence of the endogenous growth mechanism that naturally introduces persistence in the aggregate TFP process. The other parameter estimates for the exogenous processes are broadly consistent with other papers in the literature (e.g. Born and Pfeifer, 2014; Leduc and Liu, 2016). For the alternative models, we find the parameter estimates to differ substantially from the baseline case. The capital adjustment costs parameter  $\tau_K$  is higher in model B (15.32), while much lower in models C and D (1.12 and 0.98). In order to match the empirical targets, we find that models C and D require a much larger steady-state volatility ( $\sigma^A$ ) with values of 0.1 and 0.05. Compared to the baseline model, in model B we find a much larger persistence of the TFP level shock (0.9) and lower steady-state standard deviation (0.0097), while the parameters of the uncertainty process are broadly similar.



Figure 2.3: Impulse Responses to Uncertainty Shocks (Estimation)

Note: Variables are in percent change. Grey shaded area represents 90 percent confidence bands. Variables are in percentage changes. The baseline model features both EZ preferences and the endogenous growth mechanism. Model B does not feature the endogenous growth mechanism. Model C does not feature EZ preferences, but standard CRRA utility. Model D does not feature either endogenous growth mechanism or EZ preferences.

In terms of fitting the data, the baseline model does a good job both at matching the empirical volatilities as well as the VAR-based IRFs. The model perfectly matches the volatility of output growth (0.82) and it implies volatilities of consumption (0.68) and R&D investment (1.26) that are close to their empirical counterparts (0.46 and 1.31). The standard deviation of investment in physical capital (1.49) is slightly lower than its empirical counterparts (2.35). In Figure 2.3, we can see how the baseline model is able to replicate the VAR-based IRFs both qualitatively and quantitatively.

In model B, the model-implied standard deviations of output growth (0.79), consumption (0.49), and investment (2.34) are close to those found empirically, yet at the cost of falling short with respect to the impulse responses, which are far smaller than in the VAR. In Models C and D, it is possible to obtain moments and IRFs that are close to their empirical counterparts, yet only with an unreasonably high steady-state TFP uncertainty. The model-implied volatility of output (0.89 in model C and 0.97 in model D), consumption (0.52 in model C and 0.49 in model D), investment in physical capital (2.52 in model C and 2.55 in model D), and investment in R&D (1.55 in model C) closely match the data. In both models C and D, the IRFs to an uncertainty shock are weaker than the median responses in the VAR, with the exception of consumption, which falls within the 90% confidence bands for most of the time horizon. The IRFs of all the different model specifications are discussed in detail in Section 2.5.

# 2.5 Impulse Response Analysis

We now analyze the effects of TFP uncertainty shocks on economic activity using our estimated model. First, we discuss the baseline results. Then, we describe the main transmission channels at play in our model and explain the importance of the long-run risk channel in amplifying the effects of uncertainty shocks.

As mentioned above, because of the endogenous growth mechanism, all real variables have to be detrended before solving the model. The impulse responses of output, consumption, investment in physical capital, and investment in R&D are obtained by adding back the trend. In particular, let  $\hat{x}_t$  be the detrended variable, i.e.  $\hat{x}_t \equiv \log(X_t) - \log(N_t)$ , and let  $\gamma_{N,t} \equiv \frac{N_t}{N_{t-1}}$  be the growth rate of the aggregate stock in R&D. Then the IRF of our variable of interest  $x_t = \log(X_t)$  is calculated as the sum of the IRF of  $\hat{x}_t$  and the cumulative sum of the IRF of  $\gamma_{N,t}$ .

#### 2.5.1 The Effects of TFP Uncertainty Shocks

Figure 2.4 displays the IRFs to a TFP uncertainty shock, i.e. an exogenous increase in the probability of large (either positive or negative) TFP shocks. As in the empirical section, an uncertainty shock causes a long-run decline in economic activity. In the short term, consumption falls by approximately 0.2, investment in physical capital by 0.5, and R&D Investment,  $S_t$ , by 0.45 percent. The fall in R&D investment leads to a decline in TFP of about 0.2 percent, which is quantitatively in line with the TFP response in the VAR. Output decreases by approximately 0.3 percent within the first 8 quarters. The



Figure 2.4: Impulse Responses to Uncertainty Shocks in Baseline Model (Estimation)

Note: Inflation and nominal interest rate are expressed in annualized percentage points. All other variables are in percent change.

fall in productivity causes an initial rise in inflation, analogously as in the empirical section. The negative effects of the uncertainty shock are partly offset by the reaction of the monetary authority that cuts the interest rate to counteract the strong fall in output growth. In the long term, TFP, output, consumption, capital investment, and R&D investment remain approximately 0.1 percent below trend, while the stationary variables (Hours, Inflation, Markup, and Interest Rate) revert back to their steady state within 40 quarters.



Figure 2.5: Precautionary Savings

Note: Variables are in percent change.

#### 2.5.2 Understanding the Transmission Channels

The responses of the endogenous variables described above are due to the interplay of precautionary savings, rising markups, endogenous growth, and long-run risk.

**Precautionary Savings** First, an uncertainty shock leads to a fall in consumption because risk-averse households desire to increase savings for precautionary reasons in order to be able to self-insure against possible negative events occurring in the future. The importance of this channel crucially depends on the degree of relative risk aversion of households. In Figure 2.5, we show the effect of varying the RRA parameter ( $\gamma$ ) on the transmission of uncertainty shocks. For conciseness, we focus on the effect on output and consumption. We display the effect on the other variables in the appendix (see Figure B.10). When we reduce the parameter from our baseline value (66) to 20, the agents' precautionary motive becomes more subdued and consumption falls less. Conversely, when we increase the parameter from 66 to 100, consumption drops by 0.1 percentage points more than in the baseline scenario.

**Time-Varying Markups** The precautionary motive of households leads to a fall in consumption as well as an increase in labor supply, which reduces nominal marginal



Figure 2.6: Time-Varying Markups

Note: Variables are in percent change.

costs and wages. When prices are fully flexible, real marginal costs are unaffected by the increase in labor supply and firms' markups remain constant. Since physical capital and R&D are predetermined, the increase in labor supply raises output and we cannot obtain the co-movement between consumption and output, which we find empirically. Under sticky prices instead, markups are time-varying and output is demand-driven in the short term. The fall in consumption for precautionary reasons leads firms to demand less labor, capital services and R&D services. Given that the aggregate stocks of physical capital and R&D are predetermined, we first have a drop in the rates of capital and R&D utilization and in capital and R&D investment. Hence, when prices are sticky, uncertainty shocks can be a source of business cycle fluctuations, as they cause a drop in all the main macroeconomic aggregates. Figure 2.6 displays the IRFs to an uncertainty shock when prices are flexible ( $\phi_p$  is set to 0) and in our baseline model ( $\phi_p$ equal to 59). Consistently with Basu and Bundick (2017), in a flexible-price model (red line), uncertainty shocks have expansionary effects on output, while in a sticky price model, we see an increase in markups, which causes a reduction in output. The effect of price stickiness on the other variables is left to the appendix (see Figure B.11).

**Endogenous Growth via R&D** The permanent effects of uncertainty shocks in this theoretical model are due to the endogenous growth mechanism. More specifically, the fall in R&D investment implies a decline in the aggregate stock of R&D, which reduces the accumulation of new ideas and has a negative impact on TFP and long-run growth. To highlight the role of technology spillovers, the top row of Figure 2.7 compares the transmission of an uncertainty shock under alternative calibration of the spillover parameter  $\eta$ . We find that the larger  $\eta$ , the larger are the effects of an uncertainty shock on R&D investment and hence on TFP. Intuitively, if we consider the extreme case of  $\eta = 0$ , then the endogenous component of TFP would be a pure externality. In other words, the larger  $\eta$ , the more the R&D choice is internalised by the firm. Hence, after an increase in uncertainty firm *i*'s demand for R&D will be more affected the larger  $\eta$ . In equilibrium, this leads to a stronger drop in aggregate R&D and therefore a more pronounced decline in TFP.

The degrees of capital and R&D adjustment costs can also affect the demand for R&D and hence influence the transmission of uncertainty shocks in the short and in the long run. The bottom two rows of Figure 2.7 display the effect of an uncertainty shock for different values of the adjustment cost parameters  $\tau_K$  and  $\tau_N$ . For larger values of the adjustment cost parameters adjustment costs) the model becomes more volatile as the drop in investment becomes more substantial. When we increase  $\tau_K$ , capital investment falls in a more pronounced way and, given input complementarity, this induces a stronger fall in the demand for R&D. Similarly, when we increase  $\tau_N$ , we see a sharper drop in R&D. As R&D falls more substantially, this translates into a larger decline in TFP and more severe effects in the long run on the overall economy. The effect of varying parameters  $\eta$ ,  $\tau_K$ , and  $\tau_N$  on the other variables is shown in the appendix in Figures B.12, B.13, and B.14.

**Long-Run Risk** The combination of Epstein-Zin preferences and the endogenous growth mechanism is the main source of amplification of uncertainty shocks in our model. Because of the EZ preferences, households are averse to risks to future long-term


Figure 2.7: Endogenous Growth via R&D

Note: Variables are in percent change.

growth<sup>6</sup> (Bansal and Yaron, 2004) and take therefore into account that shocks in this economy have permanent effects due to the endogenous growth mechanism described above. In other words, when shocks have effects in the long term, households become extremely risk-averse, which exacerbates their precautionary savings motive.

In order to highlight the amplification provided by the long-run risk channel, we

<sup>&</sup>lt;sup>6</sup>This is because, with EZ preferences, the continuation value does not enter linearly in the Bellman equation.

analyse the IRFs from the three alternative models previously described: model B that features EZ preferences but no endogenous growth mechanism; model C that features the endogenous growth mechanism but no EZ preferences; finally model D that does not feature either EZ preferences or endogenous growth. Figure 2.8 displays the IRFs of models B, C, and D. In order to make the responses comparable, we set the parameters of the uncertainty process in these alternative models equal to those in the baseline model.

First, we compare the results from our baseline model and those from the same model without R&D (model B). Given that model B does not feature the endogenous growth mechanism, shocks in this model specification will only be transitory. Comparing the IRFs from Figure 3.5 to those from model B highlights the importance of long-run risk. The long-run risk channel in the baseline model exacerbates the precautionary savings channel, causing a 200 times larger fall in consumption compared to that in model B. Markups rise approximately 200 times more in our baseline model, which leads to larger drops in investment (100 times more than in model B) and output (150 times more than in model B).

Second, we compare two alternative models with and without R&D in absence of EZ preferences (models C and D). In particular, we consider the standard case in which the EIS parameter  $\psi = \frac{1}{\gamma}$ , where gamma is the RRA parameter. The stochastic discount factor, in this case, writes as:

$$M_{t,t+1} = \beta \left(\frac{u_{t+1}}{u_t}\right)^{1-\gamma} \frac{C_t}{C_{t+1}}.$$
(2.23)

There are two key differences between the stochastic discount factor (SDF) in the baseline model with EZ preferences (Equation, 2.9) and the one with standard preferences (Equation, 2.23). First of all, in the standard SDF, one parameter governs both the degree of relative risk aversion and the elasticity of intertemporal substitution. Under EZ preferences instead, we can increase RRA without affecting the EIS.<sup>7</sup> Second, and most importantly, the SDF for non-recursive preferences does not depend on the continuation value  $V_{t+1}$ . With EZ preferences this is not the case, as  $V_{t+1}$  is not additive separable from

<sup>&</sup>lt;sup>7</sup>The EZ preferences boil down to the standard case when we set RRA = 1/EIS



Figure 2.8: Uncertainty Shock in Model B, C, and D (Baseline Calibration)

Note: Inflation and nominal interest rate are expressed in annualised percentage points. All other variables are in percent change. Model B does not feature R&D and the endogenous growth mechanism. Model C does not feature EZ preferences, but standard CRRA utility. Model D does not feature either endogenous growth mechanism or EZ preferences.

the instantaneous utility. The fact that  $V_{t+1}$  enters the Bellman equation in a non-linear way captures the idea that agents are averse to fluctuations in  $V_{t+1}$ , i.e. they fear long-run risk. With standard preferences instead, this fear is not accounted for.

As previously mentioned, in models C and D, we fixed the RRA parameter  $\gamma$  to 2, a standard value in the business cycle literature (hence we are implicitly assuming an EIS of 0.5). From Figure 2.8 we first observe that in models C and D, uncertainty shocks have much smaller effects than in the baseline model. In model C (D), consumption falls approximately 50 (80) times less than in the baseline model, investment drops 30 (40) times less and output 35 (55). Second, unlike for models A and B, the effects of uncertainty shocks in the short term are not significantly different between models C and D. In the first 8 quarters, output falls by 0.007 percent in model C and 0.005 percent in model D, consumption by 0.0045 percent (model C) and 0.0025 (model D), and investment in physical capital drops by 0.016 (model C) and 0.012 (model D) percent.

As a bottom line, the comparison of the baseline model with model B shows how the presence of long-run risks in our model is crucial to amplify the precautionary savings and the overall effects of uncertainty shocks. Comparing model C and D with the baseline model highlights the importance of assuming that agents take long-run risks into account via EZ preferences. Finally, comparing model C with model D underscores that when households do not feature EZ preferences and do not take long-run risk into account, the presence of an endogenous growth mechanism does not significantly amplify the effects of uncertainty shocks. These three observations are evidence of the importance of the long-run risk channel. In all models in which long-run risk is not accounted for, the effects of an uncertainty shock become negligible.

## 2.6 Conclusion

In this paper, we argue that shocks to macroeconomic uncertainty have negative long-run effects on economic activity that persist well beyond the business cycle frequency. First, we conduct an SVAR analysis for the US and find that macroeconomic uncertainty shocks cause a significant decline in consumption, output, investment in physical capital and investment in R&D for over 40 quarters. Moreover, we find that these shocks lead to a persistent decline in total factor productivity. Second, we rationalize the empirical results through the lenses of a sticky-price DSGE model augmented with an endogenous growth mechanism of vertical innovations and recursive preferences à la Epstein-Zin. We find that this framework is able to provide a good fit to the data, both with respect to simple unconditional moments as well as with replicating the IRFs of the VAR. In this

model, uncertainty shocks reduce consumption for precautionary reasons and increase markups, which in turn leads to a fall in output and investment in both physical capital and in R&D. The decline in the aggregate stock of R&D induces a fall in productivity that makes the effects of uncertainty shocks permanent. The inclusion of EZ preferences allows us to capture households' aversion to both current and future uncertainty. When faced with permanent risks affecting their future consumption, agents become extremely risk-averse, which significantly exacerbates their precautionary savings motive and the overall negative effects of uncertainty shocks both in the short and in the long run. In particular, we show that this "long-run risk" channel amplifies the effects of uncertainty shocks on the main macroeconomic variables up to 2 orders of magnitude compared to models without either endogenous growth or EZ preferences. In light of our results, we believe future research should focus on further exploring alternative sources of nonlinearities within DSGE models that may be important to quantitatively account for the real effects of uncertainty.

# Chapter 3

# Macro Uncertainty and Unemployment Risk

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# 3.1 Introduction

The Great Recession has sparked a wide debate on the impact of uncertainty on the macroeconomy. After the seminal paper of Bloom (2009), close attention has been devoted to study the consequences of uncertainty shocks over the business cycle. An increase in uncertainty has been shown to cause a contraction of output and its subcomponents.<sup>1</sup>

While the existing literature has focused on the transmission of uncertainty shocks to the macroeconomy, it has not considered how households' heterogeneity affects their propagation. This paper illustrates how heterogeneity is key to the transmission of uncertainty to the macroeconomy and, in particular, to inflation. Empirical work has shown that an increase in uncertainty leads to a drop in output and its main components, as well as a drop in inflation, and an increase in unemployment. The theoretical literature, on the other hand, while being able to explain how a rise in uncertainty propagates to output, consumption, and unemployment, has not been successful in robustly explaining

<sup>&</sup>lt;sup>1</sup>Following the macro literature, we use the word 'uncertainty' to refer to 'objective uncertainty' or 'risk', in which the probabilities are well understood by all agents. There could be an alternative source of uncertainty, that is ambiguity, in which the probabilities are not well understood.

why inflation drops.<sup>2</sup> Our paper shows that households respond heterogeneously to increases in uncertainty and this heterogeneity is able to explain why inflation decreases following an uncertainty shock.

To corroborate the already existing empirical evidence on the propagation of macro uncertainty shocks, we start by estimating a vector autoregression (VAR) of macro variables and the macro uncertainty index of Jurado et al. (2015). We identify an uncertainty shock through sign restrictions. We show that a rise in macro uncertainty leads to a drop in consumption, inflation, and the policy rate. To gain a deeper understanding of the mechanism driving the macro dynamics, we estimate a VAR by using consumption and income micro data from the Consumer Expenditure Surveys (CEX). This allows us to study the heterogeneous response of consumption across different quintiles of households' income distribution. We show that the most responsive households to an increase in uncertainty are those belonging to the intermediate quintiles of the income distribution.

To rationalize these findings, our paper proposes a theoretical mechanism through which an increase in macro uncertainty results in a drop in inflation and generates responses of output, consumption and unemployment rate, which are quantitatively, as well as qualitatively in line with the empirical evidence. We develop a dynamic stochastic general equilibrium model with the following features: household heterogeneity induced by unemployment risk and imperfect risk sharing à la Challe et al. (2017), labor market search and matching (SaM) frictions à la Mortensen and Pissarides (1994), and Calvo (1983)-type price rigidities. We model uncertainty as a second moment shock to technology.

Within this framework, we study how a positive uncertainty shock propagates throughout the economy. In representative agent New Keynesian models (RANK) such as Born and Pfeifer (2014), Fernández-Villaverde et al. (2015), and Mumtaz and Theodoridis (2015), uncertainty shocks have two effects. The first effect is on aggregate demand and works through the precautionary saving behavior of risk-averse households.

<sup>&</sup>lt;sup>2</sup>While Leduc and Liu (2016) show that an uncertainty shock resembles an aggregate demand shock as it increases unemployment, while decreasing inflation, Fasani and Rossi (2018) argue that their result hinges on the Taylor rule specification and that this result could actually be flipped by using different Taylor rules.

Due to the convexity of the marginal rate of substitution between present and future consumption, higher uncertainty induces households to increase their savings. The second effect is on aggregate supply and works through the precautionary pricing behavior of firms. When uncertainty increases, firms which are allowed to reset their price, increase it to self-insure against the risk of being stuck with low prices in the future. Since the increase in prices induced by the precautionary pricing behavior of firms is stronger than the drop in prices induced by the precautionary saving behavior of risk averse households, inflation increases after a positive uncertainty shock. Enhancing this framework with households' heterogeneity adds an indirect channel of precautionary savings, which has powerful implications on the propagation of uncertainty shocks. This channel works as follows. The drop in aggregate demand and aggregate supply induces firms to lower their vacancy posting. This reduces households' job finding rate and increases unemployment risk. Since some households are borrowing constrained and subject to only partial risk sharing, an increase in unemployment risk pushes them to further strengthen their precautionary saving behavior. When the feedback loop between precautionary savings and unemployment risk sufficiently amplifies the negative demand effects of uncertainty shocks, the latter have deflationary effects. Moreover, this feedback effect is able to reinforce the responses of output, consumption, and unemployment rate so as to be quantitatively in line with the empirical evidence.

**Related Literature** Our paper belongs to the fast growing literature of heterogeneous agent New Keynesian (HANK) models, such as those developed by McKay and Reis (2016) and Kaplan et al. (2018). More specifically, it is related to the novel literature of HANK models with SaM frictions, which studies how labor market frictions interact with households' precautionary saving behavior. Within this literature, Gornemann et al. (2016) show how unemployment risk is endogenous to monetary policy, McKay and Reis (2017) investigate optimal social insurance against uninsurable risks to income and unemployment, Ravn and Sterk (2017) study how nominal and labor market rigidities along with household heterogeneity produce amplification and account for key features of the Great Recession, Ravn and Sterk (2018) revisit the qualitative results of the New Keynesian literature in light of the interaction between HANK and SaM, Cho (2018)

assesses the importance of unemployment risk for aggregate business cycle dynamics, and Dolado et al. (2018) analyze the distributional effects of monetary policy in the presence of SaM frictions and capital-skill complementarity. Closer to our paper, Challe et al. (2017) construct and estimate a tractable HANK model with SaM frictions, while Challe (2019) study optimal monetary policy in the presence of uninsured unemployment risk and nominal rigidities. To our knowledge, our paper is the first to study uncertainty shocks in the context of a HANK model with SaM frictions and highlight how these features are crucial to explain the propagation of uncertainty throughout the economy.

The second stream of literature this paper is related to is the one on uncertainty. Since the seminal work of Bloom (2009), many papers have studied how uncertainty affects economic activity. The literature has focused on different types of uncertainty: financial uncertainty (Ludvigson et al., 2019), stock market volatility (Bloom, 2009; Basu and Bundick, 2017), uncertainty as risk or ambiguity (Backus et al., 2015), consumers' perceived uncertainty (Leduc and Liu, 2016), firm-specific uncertainty (Bachmann et al., 2013), economic policy uncertainty (Baker et al., 2016), and fiscal policy uncertainty (Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015). This paper focuses specifically on macro uncertainty as estimated by Jurado et al. (2015) and updated by Ludvigson et al. (2019). The main contribution of this paper to the literature on uncertainty is to highlight the importance of the interaction between households' heterogeneity and labor market SaM frictions in the transmission of uncertainty shocks to the macroeconomy.

Our contribution is both empirical and theoretical. On the empirical side, this paper studies the propagation of macro uncertainty shocks across different levels of households' income by using CEX Surveys data. This data has been collected by Heathcote et al. (2010), and then used by Anderson et al. (2016) and Ma (2018) to study government spending shocks, by De Giorgi and Gambetti (2017) to analyze the interaction between business cycles and the consumption distribution, and by Wong (2019) to show the effects of demographic changes on the transmission of monetary policy to consumption. On the theoretical side, this paper adds to the literature on uncertainty shock propagation along two dimensions. First, it is able to match quantitatively the empirical responses of consumption and inflation to an identified uncertainty shock. Second, and most importantly, it is able to generate a decrease in inflation in response to an increase in

uncertainty and to uncover the underling mechanism explaining this decrease. Papers like Born and Pfeifer (2014) and Mumtaz and Theodoridis (2015) obtain an increase in prices as a consequence of higher uncertainty. This is due to price rigidities à la Calvo, which trigger a precautionary pricing behavior of firms. On the other hand, papers like Leduc and Liu (2016), Basu and Bundick (2017), and Cesa-Bianchi and Fernandez-Corugedo (2018) find that an increase in uncertainty leads to a decrease in prices. This is mainly due to their assumption of price rigidities à la Rotemberg (1982). There are multiple reasons why Calvo-type rigidities are preferable to Rotemberg-type rigidities, especially when solving a model at higher order approximation. First, it is quite difficult to attach a structural interpretation to the Rotemberg adjustment cost parameter, as there is no natural equivalent in the data. In contrast, for the Calvo approach various papers have computed average price durations, e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008). The literature on price rigidities has therefore regularly made use of the first-order equivalence of Rotemberg- and Calvo-type adjustment frictions by translating the Rotemberg adjustment costs to an implied Calvo price duration via the slope of the New Keynesian price Phillips Curve. Second, despite being equivalent to Calvo-type rigidities at first order approximation, Rotemberg-type rigidities generate opposite responses of prices to uncertainty shocks as shown in Oh (2019). In particular, Rotemberg-type rigidities lack the precautionary pricing channel, which has been shown to be at play by micro-founded menu cost models (Vavra, 2014; Bachmann et al., 2018). To the contrary, Calvo-type rigidities allow for this channel and are therefore preferable. Moreover, Fasani and Rossi (2018) show that the responses of inflation to uncertainty shocks in the presence of Rotemberg-type rigidities are very much dependant on the Taylor rule specification and could become positive once empirically plausible degree of interest rate smoothing is considered.

Another paper focusing on uncertainty and heterogeneity is Bayer et al. (2019). Our paper differs from it along several dimensions. While Bayer et al. (2019) study individual households' income volatility, we focus on the propagation of aggregate macro uncertainty. In addition, when solving for aggregate dynamics, Bayer et al. (2019) use a first-order perturbation. Instead, we solve the model at third order, which allows us to obtain a precautionary pricing motive for firms, which would not be present at a first order approximation. Third, we have a frictional labor market, which is necessary to explain the feedback effect between unemployment risk and precautionary saving, which is the one driving our main results.

Last but least, on the methodological side, we contribute to the literature by studying the propagation of uncertainty shocks in a heterogeneous agent framework that is tractable. Studying uncertainty shocks requires to solve the model to a third order approximation. This gets extremely complicated in fully fledged heterogeneous models, which are solved by Krusell and Smith (1998) projection method and Reiter (2009). However, Challe et al. (2017)'s assumptions on unemployment spells and binding borrowing constraints allow us to simplify the heterogeneity of households, thus being able to study uncertainty shocks in a tractable framework.

The rest of the paper is structured as follows. Section 3.2 shows empirical evidence on the responses of macroeconomic variables to an increase in uncertainty. Section 3.3 describes the HANK model. Section 3.4 illustrates the quantitative results. Section 3.5 compares our baseline results to a model where we substitute Rotemberg pricing to Calvo pricing. Section 3.6 concludes.

# 3.2 Empirical Evidence

#### 3.2.1 Inflation: Macro Data

Recent papers such as Carriero et al. (2018a) and Angelini et al. (2019) show that macroeconomic uncertainty can be considered exogenous when evaluating its effects on the US macro economy. To show how the US economy reacts to an exogenous increase in uncertainty, we estimate a quarterly frequency VAR with a constant and two lags.<sup>3</sup> The variables included in our VAR are: macroeconomic uncertainty, log of per capita real GDP, unemployment rate, log of per capita real consumption (including nondurable goods and services), inflation (first-differenced GDP deflator), and the policy rate.<sup>4</sup> To

<sup>&</sup>lt;sup>3</sup>We use the Hannan-Quinn information criterion to choose the number of lags.

<sup>&</sup>lt;sup>4</sup>We retrieve the following variables from the FRED of St. Louis Fed (FRED series IDs are in parentheses): Gross Domestic Product (GDP), Gross Domestic Product: Implicit Price Deflator (GDPDEF), Civilian Unemployment Rate (UNRATE), Personal Consumption Expenditures: Nondurable Goods (PCND), Personal consumption expenditures: Nondurable goods (chain-type price index) (DNDGRG3M086SBEA), Personal

measure macroeconomic uncertainty we use the macro uncertainty index estimated by Jurado et al. (2015) and then updated by Ludvigson et al. (2019).<sup>5</sup> As for the policy rate, we use the quarterly average of the effective Federal funds rate. However, since the sample includes a period during which the Federal funds rate hits the zero lower bound (ZLB), from 2009Q1 to 2015Q3 we use the shadow Federal funds rate constructed by Wu and Xia (2016).<sup>6</sup> This shadow rate is not bounded below by zero and better summarizes the stance of monetary policy.

We identify uncertainty shocks by using sign restrictions. In particular, we restrict macro uncertainty and the unemployment rate to be positive for four quarters and per capita real GDP to be negative for four quarters. On the other hand, we leave per capita real consumption, inflation, and the policy rate unrestricted. We use US quarterly data over the sample period 1982Q1-2015Q3. As it is common practice in this literature, to avoid parameter instability we start our sample only after the beginning of Paul Volcker's mandate as the Federal Reserve Chairman.<sup>7</sup>

Figure 3.1 shows the impulse responses to a one standard deviation shock in the macro uncertainty index. While we impose four quarter sign restrictions on uncertainty, GDP and unemployment, we leave consumption, inflation and the policy rate free to react. Consumption drops at its minimum of -0.2 percent after seven quarters, while the policy rate at roughly -0.3 percentage points after six quarters. Importantly, inflation falls after two quarters. The response of inflation is in line with what other papers studying uncertainty shocks find using different identification strategies - see Fernández-Villaverde et al. (2015), Bonciani and van Roye (2016), Leduc and Liu (2016), Basu and

Consumption Expenditures: Services (PCESV), Personal consumption expenditures: Services (chain-type price index) (DSERRG3M086SBEA), and Effective Federal Funds Rate (FEDFUNDS). Then, we obtain the quantity indices by deflating the expenditures. Per capita variables are divided by Civilian Noninstitutional Population (CNP16OV).

<sup>&</sup>lt;sup>5</sup>The updated version of the macro uncertainty series is obtained from the author's website, https://www.sydneyludvigson.com/data-and-appendixes. We use the quarterly average of their monthly series with h = 3 (i.e., 3-month-ahead uncertainty).

<sup>&</sup>lt;sup>6</sup>The shadow Federal funds rate is obtained from the author's website, https://sites.google. com/view/jingcynthiawu/shadow-rates.

<sup>&</sup>lt;sup>7</sup>Paul Volcker started his mandate on August 6, 1979.



Figure 3.1: Empirical Responses to One-Standard Deviation Macro Uncertainty Shocks

Note: Grey areas indicate 68 percent confidence bands.

#### Bundick (2017), and Oh (2019).<sup>8</sup>

To make sure that our results are robust to different sample periods and data series, Figure 3.2 shows several robustness checks. The first row reports the impulse responses when we exclude the ZLB period. The second row uses a different index of uncertainty, namely the VXO, which is one of the most commonly used indices in the uncertainty literature. The third row replaces the GDP deflator inflation with the CPI inflation to make sure that the inflation response does not depend on the specific price index used to compute inflation. In all cases, we get a decrease in consumption, inflation, and the policy rate following a positive uncertainty shock.

<sup>&</sup>lt;sup>8</sup>The few exceptions are Mumtaz and Theodoridis (2015), Katayama and Kim (2018), and Carriero et al. (2018b). The former finds an inflationary effects of uncertainty shocks, while the last two find a non-significant response of inflation to uncertainty shocks. However, they start their sample in 1975Q1, 1960Q3 and 1961M1 respectively, thus including the pre-Volcker period.



Figure 3.2: Robustness Checks for Empirical Responses

Note: Grey areas indicate 68 percent confidence bands.

Given this empirical evidence, Section 3.3 is going to build a model, which is able to replicate our empirical findings. In particular, our goal is to obtain a drop in inflation and a significant amplification in the response of macro variables following a positive uncertainty shock.

#### 3.2.2 Consumption: Micro Data

To gain a deeper understanding of the mechanism driving the macroeconomic dynamics, we carry out a similar VAR exercise to subsection 3.2.1, but we now use consumption micro data. This allows us to disentangle the responses of households' consumption across different quintiles of their income distribution and capture heterogeneous responses. We use the Consumer Expenditure Survey (CEX) data on consumption and income over the period 1982Q1-2015Q3. We follow Heathcote et al. (2010), Anderson et al. (2016), and Ma



**Figure 3.3:** Empirical Responses of Consumption across Income Quintiles to One-Standard Deviation Macro Uncertainty Shocks

Note: "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the highest income quintile. Grey areas indicate 68 percent confidence bands.

(2018) in defining nondurable consumption. This comprises food and beverages, tobacco, apparel and services, personal care, gasoline, public transportation, household operation, medical care, entertainment, reading material, and education. As in Ma (2018), income is defined as before-tax income, which is the sum of wages, salaries, business and farm income, financial income, and transfers. To get income and non-durable consumption for households in real per capita values, we divide them by family size (the number of family members), deflate by CPI-U series, and seasonally adjust by X-12-ARIMA.

Figure 3.3 exhibits the responses of average consumption to macro uncertainty shocks across income quintiles. The response of consumption is heterogeneous across income quintiles. In particular, the drop in aggregate consumption is mainly driven by the consumption response of households in the second and third quintiles. Instead, the



Figure 3.4: Robustness Checks for Empirical Responses of Consumption across Income Quintiles

Note: "1st" denotes the lowest income quintile, and "5th" denotes the highest income quintile. Grey areas indicate 68 percent confidence bands.

response of households in the fifth quintile is not significant, while the response of households in the first and fourth quintiles is less persistent. This suggests that the most responsive households to an increase in uncertainty are those belonging to the intermediate quintiles of the income distribution. On the other hand, households at the top and at the bottom of the distribution do not respond significantly or with the same persistence to an increase in uncertainty.

Figure 3.4 displays two robustness checks. The first row shows the responses of consumption across income quintiles when we exclude the ZLB period. In this case, households in the second quintile of the income distribution do respond very strongly to an increase in uncertainty lowering consumption by 0.4 percent. The second row reports responses to an uncertainty shock using the VXO. While the consumption response of households in the first and last quintile is small or insignificant, the drop in consumption

is driven by households in the three intermediate quintiles of the income distribution.

This micro data evidence suggests that households respond in a heterogeneous way across their income distribution. Therefore, households' heterogeneity is an important feature of the data that should not be overlooked when studying the propagation of uncertainty shocks. Hence, in Section 3.3 we are going to build a model with heterogeneous agents to study the propagation of uncertainty shocks throughout the economy.

# 3.3 The Model

To reproduce our empirical findings, we build a tractable heterogeneous agent New Keynesian model à la Challe et al. (2017) and Challe (2019), where we introduce a technology process with stochastic volatility. We then simulate a temporary increase in the stochastic volatility of technology and study how the economy reacts.

The model features *imperfect* insurance against idiosyncratic unemployment risk in a New Keynesian framework with labor market frictions à la Mortensen and Pissarides (1994). There are two types of households, an impatient and a patient one. Only patient households can own firms. Both impatient and patient households participate in the labor and bond market and are subject to idiosyncratic unemployment risk. However, while patient households fully share risk among each other, impatient households cannot fully insure themselves against unemployment risk and face a borrowing constraint. The two latter features generate precautionary saving motives for employed impatient households.

To simplify the introduction of both labor market frictions and nominal rigidities, the production side is made of four types of firms as in Gertler et al. (2008). First, labor market intermediaries hire labor from both patient and impatient households, subject to search and matching frictions, and transform it into labor services. Second, wholesale goods firms buy labor services in a competitive market to produce wholesale goods used by intermediate goods firms. Third, intermediate goods firms buy wholesale goods, differentiate it, and sell it monopolistically while facing price stickiness à la Calvo (1983). Fourth, a competitive final good sector aggregates the intermediate good into a final good used for consumption and vacancy posting costs. The nominal interest rate is set by a central bank which follows a standard Taylor rule.

To specify the timing of events within a period, every period can be divided into three sub-periods: a labor market transition stage, a production stage and a consumptionsaving stage. In the first stage, the exogenous state is revealed, workers are separated from firms, firms open vacancies and new matches are created. In the second stage, production takes place and the income components are paid out to the economy agents as wages, unemployment benefits, and profits. In the third stage, asset holding choices are made and the family heads redistribute assets across household members.

Challe et al. (2017)'s assumptions on imperfect risk sharing and a tight borrowing constraint faced by impatient households allow us to reduce the state space to a finite dimensional object. If in addition we assume that the borrowing constraint becomes binding after one period of unemployment spell, we can further reduce the heterogeneity of impatient households to three types. In Section 3.3.1 - 3.3.6, we are going to describe the model in detail by focusing on the specific case in which impatient households are reduced to three types.

For notation purposes, aggregate variables are in bold characters. In addition, variables corresponding to the beginning of the labor transition stage are denoted with a tilde.

#### 3.3.1 Households

There is a unit mass of households in the economy. Each household is endowed with one unit of labor. If at the beginning of the production stage the household is employed, she supplies her unit of labor inelastically. All households are subject to idiosyncratic changes to their employment status. A share  $f \in [0, 1]$  of the unemployed households at the beginning of the labor market transition stage finds a job by the beginning of the production stage, while a share  $s \in [0, 1]$  loses her job over the same period. There are two types of households: a measure  $\Omega \in [0, 1]$  of impatient ones and a measure  $1 - \Omega$  of patient ones. They all share the same period utility function  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ , but they have a different subjective discount factor. In particular, the discount factor  $\beta^{P}$  of patient

households is higher than the discount factor  $\beta^{I}$  of impatient ones.

#### **Impatient Households**

Impatient households face idiosyncratic shocks to their employment state and are subject to a borrowing limit that prevents them from borrowing beyond a given threshold  $\underline{a}$ .

Employed households earn a wage w that gets taxed by a rate  $\tau$  to pay for the unemployment benefit  $b^u$  that unemployed households receive. Since the unemployment insurance scheme is balanced every period, the following equation has to hold:

$$\tau w \mathbf{n}^{\mathbf{I}} = b^{\mu} \left( 1 - \mathbf{n}^{\mathbf{I}} \right) , \tag{3.1}$$

where  $\mathbf{n}^{\mathbf{I}}$  is the impatient households' employment rate at the end of the labor market transition stage. Following the literature, we adopt the family structure according to which every impatient household belongs to a representative family, whose head makes consumption and saving decisions to maximize the family current and expected utility.

There are two crucial assumptions that Challe et al. (2017) make to keep the model tractable, while still preserving the heterogeneity across impatient households: *i*) the borrowing limit is tighter than the natural debt limit; *ii*) there is only partial risk sharing across members of the impatient households. In particular, only employed members can fully insure each other by transferring assets. Instead, no transfer is admitted between employed and unemployed members or across unemployed members.

Because of idiosyncratic shocks and imperfect risk sharing, there is heterogeneity across impatient households. This heterogeneity implies a distribution  $\mu(a^{I}, N)$  of impatient households over assets  $a^{I}$  and unemployment spells  $N \ge 0$ . Thanks to the two aforementioned assumptions, for every N the cross-sectional distribution  $\mu(a^{I}, N)$ of impatient households can be summarized by the unique mass point  $a^{I}(N)$  and the associated number of impatient households  $n^{I}(N)$ .

Given *X* the vector of aggregate states,<sup>9</sup> the head of a representative family of impatient households maximizes the family current and future utility with respect to

<sup>&</sup>lt;sup>9</sup>See Section 3.3.6 for the aggregate state definition.

assets a'(N) and consumption c(N):

$$V^{I}\left(a^{I}(N), n^{I}(N), X\right) = \max_{\{a^{I'}(N), c^{I}(N)\}_{N \in \mathbb{Z}_{+}}} \left\{ \sum_{N \ge 0} n^{I}(N) u\left(c^{I}(N)\right) + \beta^{I} \mathbb{E}_{\mu, X}\left[V^{I}\left(a^{I'}(N), n^{I'}(N), X'\right)\right] \right\}$$
(3.2)

subject to:

$$a^{l'}(N) \ge \underline{a},\tag{3.3}$$

$$a^{I'}(0) + c^{I}(0) = (1 - \tau) w + (1 + r) A, \qquad N = 0,$$
 (3.4)

$$a^{I'}(N) + c^{I}(N) = b^{u} + (1+r)a, \qquad N \ge 1.$$
 (3.5)

Equation (3.3) is the borrowing constraint, where  $\underline{a}$  is higher than the natural borrowing limit. Equation (3.4) is the budget constraint of an employed household (the unemployment spell N is zero). An employed household consumes  $c^{I}(0)$  and buys assets  $a^{I}(0)$ , while receiving after tax income  $(1 - \tau) w$  and return from previously held assets (1 + r) A. Equation (3.5) is the budget constraint of a household, who has been unemployed for N periods. This household consumes  $c^{I}(N)$ , buys assets  $a^{I}(N)$ , gets the unemployment benefit  $b^{u}$  and the return (1 + r) a from previously held assets (of course, if these are negative assets, i.e. debt, r is the interest paid on debt).

If N = 0, the value of assets and the employed households' law of motion are given by:

$$A' = \frac{1}{n^{I'}(0)} \left[ \left( 1 - s' \right) a^{I'}(0) + f' \sum_{N \ge 1} a^{I'}(N) n^{I}(N) \right],$$
(3.6)

$$n^{I'}(0) = (1 - s') n^{I}(0) + f'(1 - n^{I}(0)).$$
(3.7)

Equation (3.6) says that the next period value of assets that each employed impatient household gets is the total of assets that next period employed impatient households bring divided by the total number of employed impatient households  $n^{I'}(0)$ , who belong to the family. The total of assets that next period employed impatient households bring is given by the fraction of assets that households who *remain* employed bring to the family  $(1 - s') a^{I'}(0)$ , plus the fraction of assets that households, who *become* employed bring to the family  $f' \sum_{N \ge 1} a^{I'}(N) n^{I}(N)$ . Equation (3.7) says that next period employed impatient households are given by the fraction of this period employed

impatient households who *remain* employed  $(1 - s') n^{I}(0)$ , plus the fraction of this period unemployed impatient households who *become* employed  $f'(1 - n^{I}(0))$ .

If  $N \ge 1$ , the value of next period assets and next period unemployed households' law of motion are given by:

$$a^{I}(N) = a^{I'}(N-1),$$
 (3.8)

$$n^{I'}(1) = s'n^{I}(0) \text{ and } n^{I'}(N) = (1 - f')n^{I}(N - 1) \text{ if } N \ge 2.$$
 (3.9)

Equation (3.8) says that the value of next period assets of an impatient household, who has been unemployed for N - 1 periods is equal to the value of this period assets of an impatient household, who has been unemployed for N periods. Equation (3.9) says that next period unemployed people with one period unemployment spell are the fraction of this period employed households, who become unemployed, while next period unemployed with more than one period unemployment spell are the fraction of this period households, who stay unemployed.

Impatient households face a binding borrowing limit after  $\hat{N}$  consecutive periods of unemployment. This problem has a particularly easy solution for the case of  $\hat{N} = 1$ , which, following Challe et al. (2017), is supported by empirical evidence (liquid wealth is fully liquidated after one period). When  $\hat{N} = 1$ , in every period there are *three* types of impatient households: N = 0, N = 1, and  $N \ge 2$ . To these three types, there are the three following associated consumption levels  $c^{I}(0)$ ,  $c^{I}(1)$ , and  $c^{I}(2)$  for all  $N \ge 2$ , and the two following assets levels  $a^{I}(0)$ , and  $\underline{a}$ .  $a^{I}(0)$  is the asset level of employed households, while  $\underline{a}$  is the asset level of unemployed households. Since all unemployed households face a binding borrowing constraint, their asset level is the same regardless of their unemployment spell. These three types of impatient households are in number  $\Omega \mathbf{n}^{I}$ ,  $\Omega s \mathbf{\tilde{n}}^{I}$ , and  $\Omega (1 - \mathbf{n}^{I} - s \mathbf{\tilde{n}}^{I})$ . In equilibrium, for any  $N \ge 0$  the Euler condition for impatient households is:

$$\mathbb{E}_{\mu,X}\left[M^{I'}(N)\left(1+r'\right)\right] = 1 - \frac{\Gamma(N)}{u_c\left(c^{I}(N)\right)n\left(N\right)},$$
(3.10)

where  $M^{I}(N)$  is the intertemporal marginal rate of substitution (IMRS) and  $\Gamma(N)$  is the Lagrange multiplier associated to the borrowing limit. When the household is employed (N = 0), the borrowing limit is not binding. Therefore,  $\Gamma(N) = 0$  and the Euler condition

holds with equality:

$$\mathbb{E}_{\mu,X}\left[M^{I'}(0)\left(1+r'\right)\right] = 1.$$
(3.11)

Instead, when the household is unemployed ( $N \ge 1$ ), the borrowing limit is binding,  $\Gamma(N) > 0$  and  $\mathbb{E}_{\mu,X} \left[ M^{I'}(N) \left( 1 + r' \right) \right] < 1$ . The IMRS is the ratio of the next-period and the current period marginal utility:

$$M^{I'}(0) = \beta^{I} \frac{(1-s') u_{c}^{I'}(0) + s' u_{c}^{I'}(1)}{u_{c}^{I}(0)}, \qquad N = 0,$$
(3.12)

$$M^{I'}(N) = \beta^{I} \frac{(1-f') \, u_{c}^{I'}(N+1) + f' u_{c}^{I'}(0)}{u_{c}^{I}(N)}, \qquad N \ge 1.$$
(3.13)

Equation (3.12) is the IMRS of an employed household. The denominator is the current period marginal utility. The numerator is the next period marginal utility, which is a weighted average of the household's marginal utility if she remains employed  $u_c^{I'}(0)$  times the probability of remaining employed 1 - s', and her marginal utility if she becomes unemployed  $u_c^{I'}(1)$  times the probability of becoming unemployed s'. Similarly, Equation (3.13) is the IMRS of an unemployed household. In this case, the numerator is the weighted average of the household's marginal utility if she remains unemployed  $u_c^{I'}(N+1)$  times the probability of remaining unemployed while already being unemployed 1 - f', and her marginal utility if she becomes employed  $u_c^{I'}(0)$  times the probability of remaining unemployed  $u_c^{I'}(0)$  times the probability of remaining unemployed while already being unemployed 1 - f', and her marginal utility if she becomes employed  $u_c^{I'}(0)$  times the probability of becoming employed f'.

#### **Patient Households**

The fraction of employed members within every family of patient households before and after the labor-market transitions stage are denoted by  $\tilde{n}^{P}$  and  $n^{P}$ , respectively. We thus have:

$$n^{P'} = (1 - s') n^{P} + f' (1 - n^{P})$$
(3.14)

$$n^P = \tilde{n}^{P'}.\tag{3.15}$$

As before, these are family-level variables. The corresponding aggregate variables are denoted by  $\mathbf{\tilde{n}}^{P}$  and  $\mathbf{n}^{P}$ . Employed patient households earn after tax wage  $(1 - \tau)w^{P}$ , while unemployed patient households get unemployment benefit  $b^{u^{P}}$ . As for impatient households, also the unemployment insurance scheme of patient households is balanced

every period, thus the following equation holds:

$$\tau w^{P} \mathbf{n}^{\mathbf{P}} = b^{u^{P}} \left( 1 - \mathbf{n}^{\mathbf{P}} \right).$$
(3.16)

Besides having a higher discount factor, what differentiates patient households from impatient ones is that there is full risk sharing among their family members, regardless of their employment status. This implies that all family members are symmetric, consume  $c^{P}$  and save  $a^{P'}$ . The family head of patient households solves:

$$V^{P}\left(a^{P}, n^{P}, X\right) = \max_{a^{P'}, c^{P}} \left\{ u\left(c^{P}\right) + \beta^{P} \mathbb{E}_{n^{P}, X}\left[V^{P}\left(a^{P'}, n^{P'}, X'\right)\right] \right\},$$
(3.17)

subject to:

$$c^{P} + a^{P'} = w^{P} n^{P} + (1+r) a^{P} + \Pi, \qquad (3.18)$$

where  $w^{P}$  is the real wage that patient households get and  $\Pi$  is the profit from intermediate goods firms and labor intermediaries, which are owned by patient households.

Since all patient households are homogeneous, they have the same Euler equation:

$$\mathbb{E}_{X}\left[M^{p}\left(1+r'\right)\right] = 1, \qquad (3.19)$$

where the IMRS  $M^{P'}$  is given by:

$$M^{P'} = \beta^{P} \frac{u_{c}^{P'}}{u_{c}^{P}}.$$
(3.20)

#### 3.3.2 Firms

There are four types of firms in the economy. Labor intermediaries hire labor in a frictional labor market and sell labor services to wholesale goods firms. Wholesale goods firms buy labor to produce wholesale goods in a competitive market. Intermediate goods firms buy wholesale goods and sell them to the final goods firms while facing Calvo (1983) price rigidities. Final goods firms aggregate intermediate goods into a final good.

#### **Final Goods Firms**

A continuum of perfectly competitive final goods firms combine intermediate goods, which are uniformly distributed on the interval [0,1], according to the production

function:

$$y = \left(\int_0^1 y_i^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}},$$
(3.21)

where  $\varepsilon$  is the elasticity of substitution between two intermediate goods. Let  $p_i$  denote the real price of intermediate good variety *i* in terms of final good price. The final goods firm solves:

$$\max_{y} y - \int_{0}^{1} p_{i} y_{i} di, \qquad (3.22)$$

subject to Equation (3.21). The solution of the maximization gives the final firm's demand of intermediate good:

$$y_i(p_i) = p_i^{-\varepsilon} y, \tag{3.23}$$

while the zero-profit condition for final goods firms gives:

$$\left(\int_0^1 p_i^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} = 1.$$
(3.24)

#### **Intermediate Goods Firms**

Intermediate goods firm *i* produces  $x_i$  with a linear technology  $y_i = x_i - \Phi$ , where  $\Phi$  is a fixed cost of production. Firm *i*'s profit is then given by  $\Xi = (p_i - p_m)y_i - p_m\Phi$ , where  $p_m$  is the real price of intermediate goods in terms of final goods. Intermediate goods firms choose  $p_i$  to maximize the present discounted value of future profits subject to the demand curve (3.23). They face pricing frictions à la Calvo (1983). Therefore, every period only a share  $1 - \theta \in [0, 1]$  of firms is allowed to reoptimize over the price. The value of an intermediate goods firm  $V^R(X)$  that is allowed to reoptimize is:

$$V^{R}(X) = \max_{p_{i}} \left\{ \Xi + \theta \mathbb{E}_{X} \left[ M^{P'} V^{N}(p_{i}, X') \right] + (1 - \theta) \mathbb{E}_{X} \left[ M^{P'} V^{R}(X') \right] \right\}.$$
 (3.25)

The value of an intermediate goods firm  $V^N(p_{i,-1}, X)$  that is not allowed to reoptimize is:

$$V^{N}(p_{i,-1},X) = \Xi + \theta \mathbb{E}_{X} \left[ M^{P'} V^{N} \left( p_{i}, X' \right) \right] + (1-\theta) \mathbb{E}_{X} \left[ M^{P'} V^{R} \left( X' \right) \right].$$
(3.26)

Intermediate goods firms which do not reoptimize set their price by fully indexing it to steady state inflation  $\bar{\pi}$ :

$$p_i = \frac{1 + \bar{\pi}}{1 + \pi} p_{i,-1}.$$
(3.27)

Instead, optimizing firms set their price as:

$$p^{\star} = \frac{\varepsilon}{\varepsilon - 1} \frac{p^A}{p^B} \tag{3.28}$$

where

$$p^{A} = p_{m}y + \theta \mathbb{E}_{X} \left[ M^{P\prime} \left( \frac{1+\pi'}{1+\bar{\pi}} \right)^{\varepsilon} p^{A\prime} \right], \qquad (3.29)$$

$$p^{B} = y + \theta \mathbb{E}_{X} \left[ M^{P'} \left( \frac{1 + \pi'}{1 + \bar{\pi}} \right)^{\epsilon - 1} p^{B'} \right].$$
(3.30)

The inflation law of motion associated with the optimal price  $p^*$ , the indexation rule (3.27) and the zero profit condition (3.24) is

$$\pi = \frac{\theta(1+\bar{\pi})}{(1-(1-\theta)p^{\star 1-\varepsilon})^{\frac{1}{1-\varepsilon}}} - 1.$$
(3.31)

This pricing generates price dispersion. The price dispersion index  $\Delta = \int_0^1 p_i^{-\varepsilon} di$  evolves according to the following law of motion:

$$\Delta = (1 - \theta) p^{\star - \varepsilon} + \theta \left(\frac{1 + \pi}{1 + \bar{\pi}}\right)^{\varepsilon} \Delta_{-1}.$$
(3.32)

#### Wholesale Goods Firms

The wholesale good  $y_m$  is produced by a continuum of perfectly competitive identical firms, which use a linear technology in labor  $y_m = z\check{n}$ , where  $\check{n}$  is labor demand and z is technology. These firms solve:

$$\max_{n^d} \left\{ p_m z \check{n} - Q \check{n} \right\}. \tag{3.33}$$

The real unit price Q of labor services n is given by the first order condition:

$$Q = p_m z. \tag{3.34}$$

#### **Labor Intermediaries**

Labor intermediaries hire labor from both patient and impatient households in a frictional labor market and sell labor services to wholesale goods firms. Every period there is exogenous separation rate  $\rho$  between employers and workers. At the same time, labor intermediaries post vacancies at the unit cost  $\kappa$ . There is a skill premium for patient

households over impatient ones. In particular, while an employed impatient household provides one unit of labor services and earns a wage w, an employed patient household provides  $\psi > 1$  units of labor services and earns  $w^P = \psi w$ . Hence, the values for a labor intermediary of a match with impatient and patient households are:

$$J^{I} = Q - w + \mathbb{E}_{X} \left[ (1 - \rho') M^{I'} J^{I'} \right], \qquad (3.35)$$

$$J^{P} = \psi Q - \psi w + \mathbb{E}_{X} \left[ \left( 1 - \rho' \right) M^{P'} J^{P'} \right], \qquad (3.36)$$

which implies that  $J^{I} = \psi J^{P}$ . Moreover, given the vacancy filling rate  $\lambda$ , the free entry condition of labor intermediaries implies that the value of opening a vacancy has to equalize its cost:

$$\lambda \left(\Omega J^{I} + (1 - \Omega) J^{P}\right) = \kappa.$$
(3.37)

The aggregate employment rate at the beginning and at the end of the labor market transition stage are given respectively by

$$\tilde{\mathbf{n}} = \Omega \tilde{\mathbf{n}}^{\mathbf{I}} + (1 - \Omega) \psi \tilde{\mathbf{n}}^{\mathbf{P}}$$
(3.38)

$$\mathbf{n} = \Omega \mathbf{n}^{\mathbf{I}} + (1 - \Omega)\psi \mathbf{n}^{\mathbf{P}}$$
(3.39)

which implies that  $\mathbf{\tilde{n}}' = \mathbf{n}$ .

The aggregate unemployment rate **u** is given by the unemployed households  $1 - \tilde{n}$  at the beginning of the labor market transition stage plus the fraction  $\rho$  of employed households, who lose their job over the period:

$$\mathbf{u} = 1 - \tilde{\mathbf{n}} + \rho \tilde{\mathbf{n}}.\tag{3.40}$$

Firm-worker matches are created through the following matching technology

$$m = \mu \mathbf{u}^{\chi} v^{1-\chi}, \tag{3.41}$$

where v are the posted vacancies,  $\mu$  is the matching efficiency parameter, and  $\chi$  is the elasticity of matches with respect to unemployed households. The aggregate job finding and job filling rates are given by:

$$f = \frac{m}{\mathbf{u}},\tag{3.42}$$

$$\lambda = \frac{m}{v}.\tag{3.43}$$

Since the workers who lose their job at the beginning of the labor market transition period can be rematched within the same period, the period-to-period separation rate is:

$$s = \rho (1 - f).$$
 (3.44)

Given the job finding rate f and the job separation rate s, the law of motion of aggregate labor is:

$$\mathbf{n} = f\mathbf{\tilde{n}} + (1-s)\,\mathbf{\tilde{n}}.\tag{3.45}$$

As for wages, we assume that there are some rigidities à la Hall (2005). In particular, wages are set according to the following wage rule as in Challe et al. (2017):

$$w = w_{-1}^{\gamma_w} \left( \bar{w} \left( \frac{\mathbf{n}}{\bar{n}} \right)^{\phi_w} \right)^{1 - \gamma_w}, \qquad (3.46)$$

where  $\gamma_w$  indicates the indexation to previous period wage,  $\phi_w$  indicates the elasticity of wages to deviations of employment from its steady-state value  $\bar{n}$ , and  $\bar{w}$  is the steady state wage.

#### 3.3.3 Monetary Authority

The monetary authority follows a standard Taylor rule, where the nominal interest rate *R* reacts to inflation and output growth. The rule is:

$$\frac{1+R}{1+\bar{R}} = \left(\frac{1+R_{-1}}{1+\bar{R}}\right)^{\rho_R} \left(\left(\frac{1+\pi}{1+\bar{\pi}}\right)^{\phi_{\pi}} \left(\frac{y}{y_{-1}}\right)^{\phi_y}\right)^{1-\rho_R},$$
(3.47)

where  $\bar{R}$  is the steady-state nominal interest rate, and  $\phi_{\pi}$  and  $\phi_{y}$  are the reaction coefficients to inflation and output growth.

The real interest rate is determined as follows:

$$1 + r = \frac{1 + R_{-1}}{1 + \pi}.$$
(3.48)

#### 3.3.4 Exogenous Processes

The technology z used by wholesale goods firms is subject to first and second moment shocks according to the following stochastic processes:

$$\log z = \rho_z \log z_{-1} + \sigma^z \varepsilon^z \tag{3.49}$$

$$\log \sigma^{z} = (1 - \rho_{\sigma^{z}}) \log \bar{\sigma}^{z} + \rho_{\sigma^{z}} \log \sigma_{-1}^{z} + \sigma^{\sigma^{z}} \varepsilon^{\sigma^{z}}.$$
(3.50)

In particular,  $\varepsilon^z \sim N(0,1)$  is a first-moment shock capturing innovations to the level of technology, while  $\varepsilon^{\sigma^z} \sim N(0,1)$  is a second moment shock capturing innovations to the standard deviation  $\sigma^z$  of technology.  $\rho_z$  and  $\rho_{\sigma^z}$  indicate the persistence of the two processes and  $\sigma^{\sigma^z}$  is the standard deviation of  $\sigma^z$ . The second moment shock is how we introduce uncertainty into the model. We interpret a positive second moment shock as an increase in uncertainty in the economy.

### 3.3.5 Market Clearing

#### Labor Market

All households face the same job finding rate f and job separation rate s. Since we assume that employment is symmetric between patient and impatient households at the beginning of period zero, for the law of large numbers it remains symmetric at every point in time. Hence, the share of patient and impatient agents which is employed is the same, and family-level variables are equal to aggregate variables:

$$\tilde{n}^P = \tilde{n}^I = \tilde{\mathbf{n}}^P = \tilde{\mathbf{n}}^I = \tilde{\mathbf{n}},\tag{3.51}$$

$$n^P = n^I = \mathbf{n}^\mathbf{P} = \mathbf{n}^\mathbf{I} = \mathbf{n}.$$
(3.52)

Moreover, the aggregate labor supply is:

$$\Omega \mathbf{n}^{\mathbf{I}} + (1 - \Omega) \,\psi \mathbf{n}^{\mathbf{P}} = (\Omega + (1 - \Omega) \,\psi) \,\mathbf{n}, \tag{3.53}$$

and the labor market clearing condition is:

$$\left(\Omega + (1 - \Omega)\psi\right)\mathbf{n} = \check{n}.\tag{3.54}$$

#### **Assets Market**

All households participate in the assets market, which is in zero net supply:

$$\Omega\left(A + (1 - \mathbf{n})\underline{a}\right) + (1 - \Omega)a^{P} = 0.$$
(3.55)

There are  $\Omega$  impatient households and  $1 - \Omega$  patient households. Impatient households own either *A* if their budget constraint is not binding or <u>*a*</u> if it is binding.<sup>10</sup> Patient households own assets  $a^{P}$ .

#### **Goods Market**

The final good production *y* has to be equal to the final good aggregate consumption *c* plus the cost of posting vacancies:

$$c + \kappa v = y. \tag{3.56}$$

Aggregate consumption is the share  $\Omega$  of impatient households' consumption plus the share  $1 - \Omega$  of patient households' consumption  $c^P$ . The former is made of the consumption of impatient households who are employed  $n^I(0) c^I(0)$ , who have been unemployed for one period  $n^I(1) c^I(1)$ , and who have been unemployed for at least two periods  $n^I(2) c^I(2)$ :

$$c \equiv \Omega \left( n^{I}(0) c^{I}(0) + n^{I}(1) c^{I}(1) + n^{I}(2) c^{I}(2) \right) + (1 - \Omega) c^{P}.$$
(3.57)

Intermediate goods market is in equilibrium when the intermediate goods demand  $\Delta y$  is equal to its supply  $y_i - \Phi$ :

$$\Delta y = y_m - \Phi. \tag{3.58}$$

Finally, the market clearing condition for the wholesale goods is:

$$\int_{-0}^{1} x_{i} di = y_{m} = z\check{n}.$$
(3.59)

<sup>&</sup>lt;sup>10</sup>Since we have assumed that the borrowing constraint of unemployed impatient households becomes binding after one period of unemployment spell, the assets that they own is equal to the borrowing limit  $\underline{a}$  regardless of the length of their unemployment spell N. This would not be the case if the borrowing limit became biding after more than one period of unemployment spell.

#### 3.3.6 Aggregate State and Equilibrium

The aggregate state *X* is given by:

$$X = y_i = \left\{ \tilde{\mu}(.), a^p, a^I(0), R_{-1}, \Delta_{-1}, \tilde{\mathbf{n}}, z, \sigma^z \right\}.$$
 (3.60)

When  $\hat{N} = 1$ , i.e. when the borrowing constraint becomes binding after one period of unemployment spell, the heterogeneity of the impatient households can be reduced to three types: the employed type N = 0, the unemployed type for one period N = 1, and the unemployed type for more than one period  $N \ge 2$ . These types are in shares of respectively:  $\Omega \mathbf{n}$ ,  $\Omega s \mathbf{\tilde{n}}$ , and  $\Omega (1 - \mathbf{n} - s \mathbf{\tilde{n}})$ . In this specific case, a symmetric equilibrium is given by the following conditions:

- 1. the Euler condition (3.19) and the IMRS (3.20) for the patient households hold, and the Euler condition (3.11) and the IMRS (3.12) for the impatient households hold;
- 2. the budget constraint for the patient households (3.18) and the budget constraints for the three types of impatient households (3.4) and (3.5) with assets determined by (3.6) and (3.7);
- 3. the price set by optimizing firms, the inflation rate and the price dispersion are determined by (3.28) to (3.32), and the real unit price of labor services by (3.34);
- 4. the aggregate employment and unemployment rates are given by (3.38), (3.39), and (3.40), the job finding rate, the job filling rate, the period-to-period separation rate, and the matching function technology by (3.42), (3.43), (3.44) and (3.41), the aggregate labor law of motion by (3.45), the value of a match and the value of opening a vacancy are given by (3.35) to (3.37);
- 5. wages are determined according to (3.46), social contributions to (3.1) and (3.16), and nominal and real interest rates to (3.47) and (3.48);
- 6. the market clearing conditions (3.51) to (3.59) hold.

#### 3.3.7 Precautionary Savings

The model features precautionary savings induced by positive uncertainty shocks through two different channels, a direct and an indirect one. The direct channel is due to households' risk aversion. Since all households are risk averse, they behave in a precautionary manner when uncertainty increases. The indirect channel is due to uninsured unemployment risk. While both patient and impatient households bear unemployment risk, patient households fully share this risk, while impatient households face partial risk sharing. Partial insurance further strengthens the precautionary saving behavior of impatient households. We closely explain the two motives driving the precautionary saving behavior of patient and impatient households below.

#### **Direct Precautionary Savings: Household Risk Aversion**

Increased uncertainty directly triggers a precautionary saving behavior of risk-averse households. Let's assume that households have the following IMRS:

$$M = \beta \mathbb{E} \left(\frac{c'}{c}\right)^{-\sigma}.$$
(3.61)

Without loss of generality, we can shed light on the precautionary saving behavior by using the steady-state IMRS and our baseline parametrization of  $\sigma = 2$ . If we assume that under certainty, relative consumption is cc = 1,

$$\bar{M}^c = \beta c c^{-\sigma} = \beta. \tag{3.62}$$

If we assume that, under uncertainty, relative consumption can take either the low value of  $cc_l = 0.9$ , or the high value of  $cc_h = 1.1$ , both with probability  $q = \frac{1}{2}$ , then the IMRS is

$$\bar{M}^{u} = q \times \beta c c_{l}^{-\sigma} + (1 - q) \times \beta c c_{h}^{-\sigma} = 1.03 \times \beta, \qquad (3.63)$$

$$\bar{M}^c < \bar{M}^u. \tag{3.64}$$

Due to convexity, the IMRS under uncertainty is larger than that under certainty. A higher IMRS induces households to substitute out of consumption towards savings.

#### **Indirect Precautionary Savings: Uninsured Unemployment Risk**

Increased uncertainty further strengthens the precautionary behavior of impatient households through an indirect channel. In particular, higher uncertainty triggers a drop in aggregate demand. This, in turn, generates a fall in production and a decrease in posted vacancies. Less vacancies lead to a drop in the finding rate f, which increases the endogenous separation rate  $s = \rho(1 - f)$ . A lower finding rate and a higher separation rate increase the impatient households' propensity to save. This last implication can be derived from the IMRS of impatient households. In particular, if impatient households are employed (N = 0), their IMRS is as follows:

$$M^{I'}(0) = \beta^{I} \frac{(1-s') u_{c}^{I'}(0) + s' u_{c}^{I'}(1)}{u_{c}^{I}(0)}, \qquad N = 0.$$
(3.65)

Their marginal utility of consumption when becoming unemployed  $u_c^{l'}(1)$  is higher than their marginal utility of consumption when remaining employed  $u_c^{l'}(0)$ , as falling into unemployment generates a drop in consumption and marginal utility is decreasing in consumption. Therefore, whenever the separation rate s' rises, the IMRS increases, thus pushing impatient households to save more. A similar reasoning applies to the IMRS of impatient households who are unemployed ( $N \ge 1$ ):

$$M^{I'}(N) = \beta^{I} \frac{(1-f') \, u_{c}^{I'}(N+1) + f' u_{c}^{I'}(0)}{u_{c}^{I}(N)}, \qquad N \ge 1.$$
(3.66)

Whenever the finding rate f' drops, the IMRS increases as the marginal utility of consumption when remaining unemployed  $u_c^{I'}(N+1)$  is higher than the marginal utility of consumption when becoming employed.

Notice that since throughout the paper we assume that the borrowing limit becomes binding after one period of unemployment spell, only the Euler condition for N = 0 will hold with equality, while the Euler condition for N > 0 will be slack. This implies that the precautionary saving motive will only concern *employed* impatient households, who are the only type of impatient households allowed to save. To the contrary, *unemployed* impatient households will be at their borrowing limit, so their asset position will simply be <u>*a*</u>.

Parameter	Description	Value	Target/Source
	Households		
Ω	Share of impat. households	0.60	Challe et al. (2017)
<u>a</u>	Borrowing limit	0	Challe et al. (2017)
$\sigma_{\rm c}$	Risk aversion	2.00	Standard
$\beta^{I}_{-}$	Discount factor of impat. households	0.917	21% consumption loss
$\beta^P$	Discount factor of pat. households	0.993	3% annual real interest rate
$b^u$	Unemployment benefits	0.27	33% replacement rate
	Firms		
 E	Elasticity of substitution btw goods	6.00	20% markup
Φ	Production fixed cost	0.22	Zero steady-state profit
$\hat{\theta}$	Price stickiness	0.75	4-quarter stickiness
			1
	Labor Market		
μ	Matching efficiency	0.72	71% job filling rate
χ	Matching function elasticity	0.50	Standard
ρ	Job separation rate	0.23	73% job finding & 6.1% job loss rates
κ	Vacancy posting cost	0.037	1% of output
ψ	Skill premium	2.04	Bottom 60% consumption share (42%)
$\gamma_w$	Wage stickiness	0.75	Challe et al. (2017)
$\phi_w$	Wage elasticity wrt employment	1.50	Challe et al. (2017)
	Monetary Authority		
	Steady-state inflation	1.005	2% annual inflation rate
0 P	Interest rate inertia	0	Standard
ዮ አ	Taylor rule coefficient for inflation	1.50	Standard
$\varphi_{n}$	Taylor rule coefficient for output	0.20	Standard
Ψÿ	haytor fulle elementerin for output	0.20	
	Exogenous Processes		
$\rho_z$	Persistence of technology shock	0.95	Standard
$ar{\sigma}^z$	Volatility of technology shock	0.007	Standard
$ ho_{\sigma^z}$	Persistence of uncertainty shock	0.85	Katayama and Kim (2018)
$\sigma^{\sigma^z}$	Volatility of uncertainty shock	0.37	Katayama and Kim (2018)

# 3.4 Quantitative Results

# 3.4.1 Calibration and Solution Method

For our baseline calibration, we mainly follow Challe et al. (2017) and Cho (2018). Table 3.1 reports the parameter values for a quarterly calibration. The share of impatient

households  $\Omega$  is calibrated to 0.60. Risk aversion  $\sigma$  is set to the standard value of 2. The discount factor of patient households  $\beta^{p}$  is set to match an annual interest rate of 3%, while the discount factor of impatient households  $\beta^{I}$  is set to target a 21% consumption drop when falling into unemployment. The unemployment benefits are calibrated to target a replacement rate of 33%. As for parameters related to firms, we set the elasticity of substitution between goods to get a 20% markup. The fixed cost of production  $\Phi$  is set to have a zero steady-state profit, while the price stickiness  $\theta$  is calibrated to have a price resetting spell of four quarters. Moving to labor market parameters, the matching efficiency  $\mu$  is set to target a job filling rate of 71%, while the job separation rate  $\rho$  to target a job finding rate of 73%. The matching function elasticity  $\chi$  is set to the standard value of 0.5. The vacancy posting cost  $\kappa$  is calibrated to being 1% of output. The skill premium  $\psi$  is set to match the consumption share of the poorest 60% of the households to 42%. The wage stickiness  $\gamma_w$  and the wage elasticity with respect to employment  $\phi_w$  follow Challe et al. (2017). As far as monetary policy parameters are concerned, we set the steady-state inflation  $\pi$  to target a 2% annual inflation, the interest rate inertia  $\rho_R$  to zero, the interest rate responsiveness to inflation  $\phi_{\pi}$  to 1.5 and the interest rate responsiveness to output growth  $\phi_{y}$  to 0.2. Moving to the shock processes, we set the persistence  $\rho_z$  and the volatility  $\sigma^z$  of the technology shock to the standard values of 0.95 and 0.007. As for the uncertainty shock process, following Katayama and Kim (2018) we set the persistence  $\rho_{\sigma^z}$  and the volatility  $\sigma^{\sigma^z}$  to 0.85 and 0.37. These values are in line with Leduc and Liu (2016) as well.

To study the effects of uncertainty shocks, we solve the model using a third-order perturbation method, as suggested by Fernández-Villaverde et al. (2011). The third-order perturbation moves the ergodic means of the endogenous variables of the model away from their deterministic steady-state values. Hence, we compute the impulse responses in percent deviation from the stochastic steady state of each endogenous variable. For that, we use the Dynare software package developed by Adjemian et al. (2011) and the pruning algorithm designed by Andreasen et al. (2018).



Figure 3.5: Impulse Responses to One-Standard Deviation Technology Uncertainty Shocks

Note: Impulse responses of output, consumption, vacancy, and real wage are in percent deviation from their stochastic steady state, impulse responses of unemployment rate and job finding rate are in percentage point deviations from their stochastic steady state, while inflation and policy rate are in annualized percentage point deviations from their stochastic steady state.

#### 3.4.2 Baseline Results

Figure 3.5 shows the impulse responses of the variables of interest to a one standard deviation shock in technology uncertainty. The solid blue line shows the responses of the HANK model described in Section 3.3, while the dashed red line shows the responses
of the corresponding representative agent New Keynesian (RANK) model. This model is identical to the HANK model except that there are no impatient households, that is  $\Omega = 0$ . In this case, there is only one type of households, the patient ones, who fully share risk. As a benchmark, we first describe the responses of the RANK model, before illustrating the responses generated by the HANK model.

#### **Responses of the RANK Model**

In the RANK model, a positive uncertainty shock in technology has both an aggregate demand effect through households' saving decisions and an aggregate supply effect through firms' pricing decisions. On the one hand, higher uncertainty induces a negative wealth effect on risk-averse households, who increase savings and decrease consumption (see Fernández-Villaverde et al., 2015, Leduc and Liu, 2016, Basu and Bundick, 2017, and Oh, 2019 for this precautionary saving channel). This causes a drop in aggregate demand. The decrease in aggregate demand reduces the marginal cost that firms are facing and pushes them to lower prices to stimulate demand. On the other hand, an increase in uncertainty triggers a precautionary pricing behavior of firms, which are subject to Calvo pricing. When uncertainty increases, optimizing firms increase their prices to self-insure against the risk of being stuck with low prices in the future (see Born and Pfeifer, 2014, Fernández-Villaverde et al., 2015, and Oh, 2019 for this precautionary pricing behavior of firms is stronger than the drop in prices induced by the precautionary pricing behavior of households, inflation increases after a positive uncertainty shock.

#### **Responses of the HANK Model**

The HANK model adds a new channel of transmission and amplification of the uncertainty shock to the precautionary saving and pricing behavior described above for the RANK model.

As explained for the RANK model, an uncertainty shock causes a drop in aggregate demand triggered by the precautionary saving behavior of households. The drop in demand induces firms to lower their vacancy posting, thus reducing the job finding



Figure 3.6: Consumption Heterogeneity

Note: Impulse responses of consumption are in percent deviation from their stochastic steady state.

rate and increasing the unemployment rate. At this point the presence of impatient households becomes key to explain the dynamics of the model. Since impatient households cannot fully insure against unemployment as they are subject to *imperfect* risk sharing, a higher unemployment risk induces them to further increase savings and decrease consumption. The impatient households' precautionary saving behavior triggers a feedback loop, which reinforces the drop in aggregate demand. At the same time, firms precautionary pricing behavior generates a reduction in vacancy posting and an increase in unemployment. This further reinforces the precautionary saving behavior of impatient households and strengthen the feedback loop. Figure 3.6 illustrates the responses of consumption for both impatient (dashed line) and patient (dotted line) households. Because of the precautionary saving behavior that partial risk sharing induces on impatient households, their consumption response is much stronger than the one of patient households.

The presence of heterogeneous agents bears two consequences on the propagation



Figure 3.7: Different Degrees of Heterogeneity

Note: Impulse responses of output, consumption, vacancy, and wage are in percent deviation from their stochastic steady state, impulse responses of unemployment rate and job finding rate are in percentage point deviations from their stochastic steady state, while inflation and policy rate are in annualized percentage point deviations from their stochastic steady state.

mechanism of uncertainty shocks. First, the feedback loop triggered by the precautionary saving behavior of impatient households is strong enough to induce a drop in prices that outweighs the increase in prices due to the precautionary pricing behavior of optimizing firms. This is the reason why, after two quarters, inflation response becomes negative, which is in line with our empirical results as shown by Figure 3.1. Second, the feedback loop amplifies all the responses. The precautionary behavior of impatient households triggers a drop in aggregate demand, which is much stronger than in the RANK model. In parallel, the decrease in vacancy posting and the increase in unemployment rate are sharper.

It is worth noticing that our results hinge upon the interaction between the precautionary saving behavior of agents induced by imperfect risk sharing and the precautionary pricing behavior of firms induced by price rigidities à la Calvo (1983). It is the interaction between these two features that allows us to obtain a drop in inflation and an amplification of responses, which quantitatively match the empirical evidence. Absent these features, this would have been possible only by relying on unusual Taylor rules such as those in Fernández-Villaverde et al. (2015).

Since the presence of impatient households is crucial both to determine the response of inflation and to amplify the responses of the other variables, Figure 3.7 shows how the impulse responses vary when varying the share of impatient households. On impact, inflation increases regardless of the share of impatient households. As soon as the negative feedback loop on aggregate demand induced by the precautionary saving behavior of impatient households kicks in, inflation decreases. Indeed, the higher is the share of impatient households, the stronger the feedback effect becomes and the more inflation drops. Figure 3.7 also shows that a bigger share of impatient households amplifies the responses of the other variables. In particular, output, consumption, vacancies, job finding rate, and wages drop more, while unemployment rate increases more, the higher is the share of impatient households.

### 3.4.3 Sensitivity Analyses

This section illustrates sensitivity exercises on various parameters, which affect the strength of the precautionary saving motive for impatient households.

The first row of Figure 3.8 shows how consumption and inflation respond when we vary households' risk aversion  $\sigma$ . A higher risk aversion generates a stronger precautionary response of impatient households, who cannot fully insure against risk.



Figure 3.8: Sensitivity Analyses 1

Note: Impulse responses of consumption are in percent deviation from their stochastic steady state, while impulse responses of inflation are in annualized percentage point deviations from their stochastic steady state.

Hence, the more risk averse impatient households are, the bigger the shift of their response out of consumption and towards savings. At the same time, inflation, which increases on impact, drops faster the higher the risk aversion is. This is due to the feedback effect that the precautionary saving behavior of households has on aggregate demand.

The second row of Figure 3.8 shows sensitivity of consumption and inflation response to various consumption differences between employed and unemployed households. Indeed, the bigger the consumption differential is between the two employment states, the stronger the precautionary saving motive that leads employed impatient households to save more, thus triggering a sharper drop in consumption and inflation.

The third sensitivity exercise that we carry out is on impatient households' consumption share (C60/C). This share is important as it negatively affects the skill premium  $\psi$  of patient households over impatient ones (as shown in Table 3.1, we calibrate the skill premium by targeting the share of impatient households' consumption). The bigger the impatient households' consumption share, the more the precautionary saving behavior of impatient households affects aggregate consumption, thus amplifying the drop in consumption and inflation caused by an uncertainty shock.

The next sensitivity exercise is on the elasticity of substitution between two intermediate goods  $\epsilon$ . As shown in Oh (2019), a higher elasticity makes the marginal profit curve of intermediate firms more convex, thus strengthening the precautionary pricing behavior of firms. This is why, on impact, a higher elasticity causes a sharper increase in inflation. On the contrary, as soon as the higher prices set by intermediate firms trigger an increase in unemployment, the amplification effect of impatient households' precautionary saving behavior on aggregate demand kicks in, thus counteracting the price increase and leading to a sharper fall in inflation.

The first row of Figure 3.9 shows the sensitivity of consumption and inflation responses to different levels of wage rigidity. Wage stickiness affects unemployment risk. Namely, more rigid wages increase unemployment risk, thus strengthening the precautionary saving motive of impatient households and leading to a sharper drop in consumption. At the same time, wage stickiness also affects the pricing behavior of firms, leading to a higher price on impact and then to a sharper drop in inflation.

The next sensitivity exercises concern the parameters of the Taylor rule. The second row shows consumption and inflation responses when we vary the persistence  $\rho_R$  of the interest rate in the Taylor rule. The more persistent the interest rate is, the milder the precautionary saving motive of households, which makes consumption and inflation drop by less.



Figure 3.9: Sensitivity Analyses 2

Note: Impulse responses of consumption are in percent deviation from their stochastic steady state, while impulse responses of inflation are in annualized percentage point deviations from their stochastic steady state.

The third and fourth rows of Figure 3.9 show consumption and inflation responses to an uncertainty shock for different levels of monetary policy responsiveness. In particular, the more responsive monetary policy is to inflation (the higher  $\phi_{\pi}$ ), the smoother the real interest rate. A smoother real interest rate path reduces the inter-temporal substitution of impatient households, thus dampening the drop in consumption induced by an

uncertainty shock. Indeed, the more responsive monetary policy is to inflation, the less inflation responds to an uncertainty shock. Monetary policy responsiveness to output growth deviations from its steady state affects the impact response of consumption, but not of inflation. Consumption drops less on impact in response to higher uncertainty if monetary policy is more responsive. A more responsive monetary authority lowers the interest rate more, thus dampening the precautionary saving motive faced by impatient households.

## 3.5 Comparison to Rotemberg Pricing

To study how much of our results depends on the type of pricing friction that we use, this section compares the HANK model studied in the previous sections to an identical model where we substitute the Calvo (1983)-type price rigidity with the Rotemberg (1982)-type price rigidity.

As before, an intermediate good firm chooses price  $p_i$  to maximize the present discounted value of future profits subject to the demand curve (3.23). However, its value is now given by:

$$V^{Rotem}(p_{i,-1},X) = \max_{p_i} \left\{ \Xi - \frac{\eta}{2} \left( \frac{(1+\pi)p_i}{(1+\bar{\pi})p_{i,-1}} - 1 \right)^2 y + \mathbb{E}_X \left[ M^{P'} V^{Rotem}(p_i,X') \right] \right\},$$
(3.67)

where  $\frac{\eta}{2} \left( \frac{(1+\pi)p_i}{(1+\pi)p_{i-1}} - 1 \right)^2 y$  is a quadratic price adjustment cost. Imposing a symmetric equilibrium across firms implies  $p_i = 1$  and  $y_i = y$ . The optimal Calvo price equilibrium conditions (3.28), (3.29), and (3.30) are now replaced with the following equation:

$$\eta \left(\frac{1+\pi}{1+\bar{\pi}} - 1\right) \frac{1+\pi}{1+\bar{\pi}} = \eta \mathbb{E}_X M^{P'} \left(\frac{1+\pi'}{1+\bar{\pi}} - 1\right) \frac{1+\pi'}{1+\bar{\pi}} \frac{y'}{y} + 1 - \varepsilon + \varepsilon p_m.$$
(3.68)

Moreover, the intermediate goods market clearing condition (3.58) is replaced by

$$y = y_m - \Phi, \tag{3.69}$$

as Rotemberg-type frictions do not generate price dispersion. On the other hand, they generate price adjustment costs, which appear in the final good market clearing condition.

Hence, condition (3.56) is replaced with

$$c + \kappa v + \frac{\eta}{2} \left(\frac{1+\pi}{1+\pi} - 1\right)^2 y = y.$$
 (3.70)

Figure 3.10 compares the impulse responses to a positive uncertainty shock for the HANK ( $\Omega = 0.6$ ) and RANK ( $\Omega = 0$ ) models with both Calvo and Rotemberg pricing. Let's first focus on the RANK models. In both Rotemberg and Calvo pricing models, a positive uncertainty shock generates a negative wealth effect on risk-averse households, who decrease their consumption and increase their savings, thus lowering aggregate demand. The difference between the way pricing frictions are introduced in the two models becomes relevant when firms' behavior comes into play. As explained in Section 3.4.2, with Calvo-type frictions firms engage in a precautionary pricing behavior. This behavior leads them to increase prices to such an extent to overcompensate the downward pressure that the aggregate demand drop exerts on prices. That is why inflation response is positive on impact in the Calvo RANK model. On the contrary, the precautionary pricing motive is absent in the Rotemberg pricing model, where all firms are symmetric and are allowed to reset their price every period, even though subject to an adjustment cost - see Oh (2019) for a thorough comparison between the Calvo and Rotemberg pricing models in response to uncertainty shocks. The absence of the precautionary pricing motive results in a drop in the inflation response to an increase in uncertainty. In addition to the opposite response of inflation, a further difference between the two RANK models is that the Calvo pricing model generates more amplified responses. This difference is again induced by the precautionary pricing behavior of firms. Higher prices reduce consumption and push firms to cut on their vacancy posting, thus decreasing the job finding rate and increasing unemployment rate more than in the Rotemberg model.

To generate even more amplification and a response of inflation fully in line with the data, a HANK model with Calvo pricing is necessary. The blue and red solid lines in Figure 3.10 illustrate the responses of respectively Calvo and Rotemberg pricing HANK models. The heterogeneity of households enriches the dynamics of the RANK models with the precautionary saving behavior of impatient households. In particular, a HANK model with Calvo pricing generates more amplified responses than a HANK model with Rotemberg pricing. This is due to the precautionary pricing behavior of firms subject to



Figure 3.10: Comparison to Rotemberg Pricing

Note: Impulse responses of output, consumption, vacancy, and wage are in percent deviation from their stochastic steady state, impulse responses of unemployment rate and job finding rate are in percentage point deviations from their stochastic steady state, while inflation and policy rate are in annualized percentage point deviations from their stochastic steady state.

Calvo pricing. As in the RANK model, firms' precautionary pricing behavior triggers an initial increase in the prices set by optimizing firms. Higher prices induce firms to reduce their vacancy posting, lower their hiring rate and increase unemployment. Differently from the RANK model though, a higher unemployment risk further amplifies the precautionary saving behavior of impatient households, who reduce consumption more. This additional effect is not present in the Rotemberg model, where firms are symmetric.

### 3.6 Conclusion

This paper has shown how households' heterogeneity helps explaining the propagation of uncertainty shocks to the macroeconomy.

First, estimating a VAR of macro variables and the macro uncertainty index of Jurado et al. (2015) and using sign restrictions to identify uncertainty shocks, it has given empirical evidence that an increase in uncertainty generates a drop in consumption, inflation and the policy rate.

Second, estimating a VAR by using CEX Surveys data instead of aggregate consumption data, it has shown that households respond heterogeneously across the income distribution and that the most responsive households to a positive uncertainty shock are those belonging to the intermediate quintiles of the distribution.

Third, to rationalize this empirical evidence, it has built a HANK model with SaM frictions and Calvo-type price rigidities. The interaction between the precautionary saving behavior of partially insured households, the labor market SaM frictions, and the precautionary pricing behavior of firms is able to generate in response to a positive uncertainty shock: i) a drop in inflation, and ii) responses of output, consumption, and the policy rate, which are quantitatively as well as qualitatively in line with the empirical evidence.

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## Appendix A

## **Appendix to Chapter 1**

### A.1 Full Sets of Empirical Impulse Response Functions

### A.1.1 First Cholesky Ordering

In Figure A.1 to A.8, I display the empirical impulse responses to each uncertainty shock under a Cholesky decomposition with the uncertainty measure ordered first.

### A.1.2 Last Cholesky Ordering

In Figure A.9 to A.16, I display the empirical impulse responses to each uncertainty shock under a Cholesky decomposition with the uncertainty measure ordered last.



Figure A.1: Empirical Responses to Uncertainty Shocks: Macro Uncertainty



Figure A.2: Empirical Responses to Uncertainty Shocks: TFP Uncertainty



Figure A.3: Empirical Responses to Uncertainty Shocks: Financial Uncertainty



Figure A.4: Empirical Responses to Uncertainty Shocks: Stock Market Volatility



Figure A.5: Empirical Responses to Uncertainty Shocks: Consumers' Survey-Based Uncertainty



Figure A.6: Empirical Responses to Uncertainty Shocks: Firms' Survey-Based Uncertainty



Figure A.7: Empirical Responses to Uncertainty Shocks: Economic Policy Uncertainty



Figure A.8: Empirical Responses to Uncertainty Shocks: Monetary Policy Uncertainty



Figure A.9: Last Cholesky Ordering: Macro Uncertainty



Figure A.10: Last Cholesky Ordering: TFP Uncertainty



Figure A.11: Last Cholesky Ordering: Financial Uncertainty



Figure A.12: Last Cholesky Ordering: Stock Market Volatility



Figure A.13: Last Cholesky Ordering: Consumers' Survey-Based Uncertainty



Figure A.14: Last Cholesky Ordering: Firms' Survey-Based Uncertainty



Figure A.15: Last Cholesky Ordering: Economic Policy Uncertainty


Figure A.16: Last Cholesky Ordering: Monetary Policy Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.

## A.2 Equilibrium Conditions in Two New Keynesian Models

In this section, I write the equilibrium conditions in the Rotemberg-type and Calvo-type New Keynsian models, respectively.

### A.2.1 Rotemberg Model

$$\phi(\pi_t - 1) \pi_t = \phi E_t \Lambda_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} + 1 - \varepsilon + \varepsilon m c_t M_t$$
(A.1)

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} - \Phi \tag{A.2}$$

$$Y_t = C_t + I_t + G_t + \frac{\phi}{2} (\pi_t - 1)^2 Y_t$$
 (A.3)

$$\Lambda_{t,t+1} = \beta E_t \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{A.4}$$

$$w_t = mc_t \left(1 - \alpha\right) \frac{Y_t + \Phi}{N_t} \tag{A.5}$$

$$r_t^k = mc_t \alpha \frac{Y_t + \Phi}{K_t} \tag{A.6}$$

$$\frac{\chi N_t{}^{\eta}}{C_t{}^{-\gamma}} = w_t \tag{A.7}$$

$$1 = q_t \left( 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) + E_t \Lambda_{t,t+1} q_{t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2$$
(A.8)

$$q_{t} = E_{t}\Lambda_{t,t+1} \left( r_{t+1}^{k} + q_{t+1} \left( 1 - \delta \right) \right)$$
(A.9)

$$K_{t+1} = (1-\delta) K_t + \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$
(A.10)

$$1 = E_t \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} \tag{A.11}$$

 $\log R_t = (1 - \rho_R) \log R + \rho_R \log R_{t-1}$ 

+ 
$$(1 - \rho_R) \left( \phi_\pi \left( \log \pi_t - \log \pi \right) + \phi_Y \left( \log Y_t - \log Y \right) \right) + \sigma_t^R \varepsilon_t^R$$
 (A.12)

$$\log A_t = \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A \tag{A.13}$$

$$\log Z_t = \rho_Z \log Z_{t-1} + \sigma_t^Z \varepsilon_t^Z$$
(A.14)

$$\log M_t = \rho_M \log M_{t-1} + \sigma_t^M \varepsilon_t^M \tag{A.15}$$

$$\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_t^G \varepsilon_t^G$$
(A.16)

$$\log \sigma_t^X = (1 - \rho_{\sigma^X}) \log \sigma^X + \rho_{\sigma^X} \log \sigma_{t-1}^X + \sigma^{\sigma^X} \varepsilon_t^{\sigma^X}, \quad X = A, Z, M, G, R$$
(A.17)

### A.2.2 Calvo Model

$$\left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{CC_t}{BB_t}$$
(A.18)

$$BB_t = Y_t + \theta E_t \Lambda_{t,t+1} \pi_{t+1}^{\varepsilon - 1} BB_{t+1}$$
(A.19)

$$CC_t = mc_t M_t Y_t + \theta E_t \Lambda_{t,t+1} \pi_{t+1} {}^{\varepsilon} CC_{t+1}$$
(A.20)

$$\Delta_t Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} - \Phi \tag{A.21}$$

$$\Delta_t = (1 - \theta) \left( \frac{1 - \theta \pi_t^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \theta \pi_t^{\varepsilon} \Delta_{t - 1}$$
(A.22)

$$Y_t = C_t + I_t + G_t \tag{A.23}$$

$$\Lambda_{t,t+1} = \beta E_t \frac{A_{t+1}}{A_t} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$
(A.24)

$$w_t = mc_t \left(1 - \alpha\right) \frac{\Delta_t Y_t + \Phi}{N_t} \tag{A.25}$$

$$r_t^k = mc_t \alpha \frac{\Delta_t Y_t + \Phi}{K_t} \tag{A.26}$$

$$\frac{\chi N_t{}^{\eta}}{C_t{}^{-\gamma}} = w_t \tag{A.27}$$

$$1 = q_t \left( 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) + E_t \Lambda_{t,t+1} q_{t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2$$
(A.28)

$$q_{t} = E_{t}\Lambda_{t,t+1} \left( r_{t+1}^{k} + q_{t+1} \left( 1 - \delta \right) \right)$$
(A.29)

$$K_{t+1} = (1-\delta) K_t + \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$
(A.30)

$$1 = E_t \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} \tag{A.31}$$

 $\log R_t = (1 - \rho_R) \log R + \rho_R \log R_{t-1}$ 

+ 
$$(1 - \rho_R) \left( \phi_\pi \left( \log \pi_t - \log \pi \right) + \phi_Y \left( \log Y_t - \log Y \right) \right) + \sigma_t^R \varepsilon_t^R$$
 (A.32)

$$\log A_t = \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A \tag{A.33}$$

$$\log Z_t = \rho_Z \log Z_{t-1} + \sigma_t^Z \varepsilon_t^Z \tag{A.34}$$

$$\log M_t = \rho_M \log M_{t-1} + \sigma_t^M \varepsilon_t^M \tag{A.35}$$

$$\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_t^G \varepsilon_t^G$$
(A.36)

$$\log \sigma_t^X = (1 - \rho_{\sigma^X}) \log \sigma^X + \rho_{\sigma^X} \log \sigma_{t-1}^X + \sigma^{\sigma^X} \varepsilon_t^{\sigma^X}, \quad X = A, Z, M, G, R$$
(A.37)

## A.3 Full Sets of Model Impulse Response Functions

### A.3.1 Rotemberg 1 vs. Calvo

I display the pointwise 68% probability bands for the impulse response functions of the endogenous variables to each uncertainty shock in the Rotemberg 1 (blue solid bands) and Calvo (red dashed bands) models in Figure A.17 to A.20.

#### A.3.2 Rotemberg 1 vs. Rotemberg 2

I display the pointwise 68% probability bands for the impulse response functions of the endogenous variables to each uncertainty shock in the Rotemberg 1 (blue solid bands) and Rotemberg 2 (green dashed bands) models in Figure A.21 to A.25.



**Figure A.17:** *Pointwise* 68% *Probability Bands to Preference Uncertainty Shocks in Rotemberg and Calvo Models* 



**Figure A.18:** *Pointwise* 68% *Probability Bands to Markup Uncertainty Shocks in Rotemberg and Calvo Models* 



**Figure A.19:** Pointwise 68% Probability Bands to Government Spending Uncertainty Shocks in Rotemberg and Calvo Models



**Figure A.20:** *Pointwise* 68% *Probability Bands to Interest Rate Uncertainty Shocks in Rotemberg and Calvo Models* 



**Figure A.21:** Pointwise 68% Probability Bands to Preference Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models



**Figure A.22:** Pointwise 68% Probability Bands to Productivity Uncertainty Shock in Rotemberg 1 and Rotemberg 2 Models



**Figure A.23:** Pointwise 68% Probability Bands to Markup Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models



**Figure A.24:** Pointwise 68% Probability Bands to Government Spending Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models



**Figure A.25:** Pointwise 68% Probability Bands to Interest Rate Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models

### A.4 Density of Inflation to Level Shocks

I plot the histograms of inflation to each level shock for the Rotemberg 1 (blue bar), Rotemberg 2 (green bar), and Calvo (red bar) models in Figure A.26. In contrast to the case of uncertainty shocks, there are little differences in the distributions of inflation to level shocks in the three models.



Figure A.26: Histograms of Inflation to Level Shocks in Rotemberg and Calvo Models

Note: Sturges' rule is used to determine the number and width of the bins. The response of inflation is plotted in annualized percentage point deviation from its stochastic steady state.

# Appendix **B**

# **Appendix to Chapter 2**

### **B.1** Correlation between Uncertainty and TFP

q/p	1	10	20	30	40
1	0.09 (0.27)	$\underset{(0.09)}{-0.20}$	$\underset{(0.01)}{-0.30}$	$\underset{(0.00)}{-0.40}$	$\underset{(0.00)}{-0.49}$
10	$\underset{(0.83)}{-0.01}$	$\underset{(0.29)}{-0.16}$	-0.27 (0.15)	$\underset{(0.02)}{-0.49}$	$\underset{(0.00)}{-0.60}$
20	$\underset{(0.54)}{-0.03}$	-0.19 (0.31)	-0.38 (0.11)	$\underset{(0.03)}{-0.51}$	-0.56 (0.00)
30	-0.07 (0.25)	-0.33 (0.07)	-0.46 (0.04)	-0.52 (0.01)	$\underset{(0.00)}{-0.48}$
40	$\underset{(0.05)}{-0.11}$	$\underset{(0.05)}{-0.35}$	$\underset{(0.04)}{-0.45}$	$\underset{(0.02)}{-0.44}$	-0.39 (0.00)

**Table B.1:** Correlation between Uncertainty and TFP

*Notes*: For each correlation (p,q) we show the estimate of  $\beta_1$  (upper value) and P-Values based on Newey-West standard errors (lower value).

In this subsection of the appendix we display the long-run correlations between p quarters backward-looking moving average of uncertainty and the q quarters forward-looking moving average of TFP growth. The correlations are calculated controlling for

past GDP growth. In practice, we run the following regression:

$$tf p_{t,t+q} = \beta_1 uncertainty_{t-p,t} + \beta_2 g dp_{t-p,t} + \varepsilon_t, \tag{B.1}$$

where *tfp*, *uncertainty*, and *gdp* are standardised moving averages, so that  $\beta_1$  can be interpreted as a correlation.

### B.2 VAR

In this subsection we describe the data sources and present the details and results of our robustness tests for the VAR analysis.

### **B.2.1** Data Sources

Name	Source	Ticker			
Peopline VAP					
Daseline VAR					
S&P 500 Index	Yahoo Finance	GSPC			
Macroeconomic Uncertainty	Sydney Ludvigson				
Gross Domestic Product	FRED (BEA)	GDP			
Services Consumption	FRED (BEA)	PCES			
Nondurables Consumption	FRED (BEA)	PCEND			
Services Consumption	FRED (BEA)	PCEDG			
Private Residential Fixed Investment	FRED (BEA)	PRFI			
Private Nonresidential Fixed Investment	FRED (BEA)	PNFI			
Private Fixed Investment R&D	FRED (BEA)	Y006RC1Q027SBEA			
GDP Implicit Price Deflator	FRED (BEA)	GDPDEF			
Labour Share	FRED	PRS85006173			
Shadow Interest Rate	FRBA				
Utilization-Adjusted TFP	FRBSF				
Robustness Exercises					
Alternative Macro Uncertainty	Rossi and Sekhposyan (2015)				
Downside Macro Uncertainty	Rossi and Sekhposyan (2015)				
Macroeconomic Dataset	FRED	FRED-MD			
Industrial Production	FRED	INDPRO			
Consumer Confidence	FRED (OECD)	CSCICP03USM665S			
Consumer Price Index	FRED	CPIAUCSL			
Spread Yields BAA - 10yr Treasury	FRED	BAA10Y			

**Table B.2:** Data Used in the VAR Analysis

### **B.2.2** Robustness Exercises

**Uncertainty Ordered Last** First, we change the Cholesky ordering assumed in the baseline setup and allow uncertainty to respond on impact to all the other variables in our model. The other variables instead, will respond only with a quarter lag to an

uncertainty shock. The results reported in Figure B.1 confirm those in the baseline VAR. We find a strong persistent decline in all the real macroeconomic variables. The response of prices and interest rate is insignificant throughout the 40 quarters.

**Including a measure of markup** To test the validity of the proposed short-run mechanism, i.e. uncertainty affecting the economy by raising price markups, we include the inverse of the labour share in our VAR. The markup proxy is placed below the macro uncertainty measure, implying that markup shocks do not affect uncertainty on impact. Similarly as in Fernández-Villaverde et al. (2015), in Figure B.2, we find the markup to initially fall, while it immediately rebounds and significantly rises by 0.1 per cent. The other responses are in line with the baseline results, although the response of capital investment and R&D investment become insignificant after approximately 20 quarters.

Alternative Measure of Uncertainty We also estimate the VAR above using the measure of macroeconomic uncertainty and downside macroeconomic uncertainty from Rossi and Sekhposyan (2015). They define uncertainty based on the percentile in the historical distribution of forecast errors associated with the realized error. Let  $e_{t+h}$  be the h- step ahead forecast error of  $y_{t+h}$  defined as  $y_{t+h} - E_t[y_{t+h}]$  and let f(e) be its forecast error distribution. Uncertainty is then defined as the cumulative distribution  $U_{t+h} = \int_{-\infty}^{e_{t+h}} f(e) de$ . Downside uncertainty is defined as  $U_{t+h}^- = \frac{1}{2} + max \{\frac{1}{2} - U_{t+h}, 0\}$ . As can be seen in figures B.3 and B.4, the median responses of output, consumption, R&D and TFP are extremely persistent and last well beyond the business cycle frequency, qualitatively and quantitative in line with our baseline results. However, for both alternative measures of macroeconomic uncertainty, the responses in the long-run are less significant than in the baseline case.

**Increase the Number of Lags** We increase the maximum number of lags included in our VAR to 2 to 5 to show that our baseline results are not due to the number of lags included in our VAR, as in Figure B.5.

**FAVARs** There are two potential issues with our baseline specification. The first one relates to the quarterly frequency of the data and the second to the potential insufficient

information contained in the model, which would not allow us to uncover the true effects of uncertainty shocks. One the one hand, the exact identification of uncertainty shocks could be undermined by the quarterly-data specification. Furthermore, by using quarterly data, the time-series dimension may not be sufficiently long considering the size of the VAR. In order to overcome these issues, we estimate a monthly-frequency Factor-Augmented VAR (FAVAR) model in the spirit of ?. The factors are extracted as principal components from a large monthly dataset for the US economy, FRED-MD (?), which includes 128 macroeconomic series. We include the first three factors in the VAR, which account for about 55% of the total variance of the data. The FAVAR contains the following variables  $X_t = [f^{(1)}; f^{(2)}; f^{(3)}; S\&P500; Confidence; Uncertainty; IP; C; CPI;$ FFR; Spread], where  $f^{(1)}$ ,  $f^{(2)}$ ,  $f^{(3)}$ , IP are respectively the three factors and industrial production. We include a measure of consumer confidence from ?, to avoid that the effects of uncertainty are confounded with the agents' perception of bad economic times. We also include the spread between the yield on BAA corporate bonds and the 10-year constant-maturity treasury bond. S&P500, Confidence, Uncertainty, IP, Consumption, CPI are in logs to interpret the IRFs in percentage changes terms. Figures B.6 and B.7 display the results of the FAVAR, assuming the ordering described above or placing uncertainty last. The responses confirm those found in the smaller quarterly VAR used in the baseline exercise. In particular, the responses in output and consumption fall significantly both in the short and in the long-run. The response of the nominal variables is less clear-cut, with both price and interest rate falling significantly on impact, but quickly becoming insignificant within the first year.

**Post-Volker Sample** Finally, we estimate the baseline quarterly VAR and the monthly FAVAR described above using the sample Jan-1985/Jun-2018 to account for the structural break in monetary policy induced by the *Volker disinflation*. Also in this case, as displayed in figures B.8 and B.9, the responses of output and consumption are extremely persistent and last well beyond the business cycle frequency. Prices significantly decline throughout the 40 quarters (120 months).



Figure B.1: Uncertainty Ordered Last



Figure B.2: VAR Including Markups



Figure B.3: VAR with Alternative Macro Uncertainty



Figure B.4: VAR with Macro Downside Uncertainty



Figure B.5: VAR with 5 lags



Figure B.6: Monthly FAVAR and Macro Uncertainty



Figure B.7: Monthly FAVAR and Macro Uncertainty Ordered Last



Figure B.8: Quarterly VAR: Time Span 1985Q1 - 2018Q2



Figure B.9: Monthly FAVAR: Time Span 1985Q1 - 2018Q2

## **B.3** Detrended Model

In order to solve the model in Dynare, we detrend the endogenous variables  $V_t$ ,  $u_t$ ,  $C_t$ ,  $I_t$ ,  $K_t$ ,  $S_t$ ,  $N_t$ ,  $Y_t$ ,  $w_t$ , and  $Z_t$  by  $N_t$ . We define the detrended variables and the growth rate of R&D as  $\hat{X}_t \equiv \frac{X_t}{N_t}$  and  $\gamma_{N,t} \equiv \frac{N_t}{N_{t-1}}$ . The detrended equilibrium conditions are provided below:

$$\hat{V}_{t} = \left[ (1-\beta) \, \hat{u}_{t}^{1-\frac{1}{\psi}} + \beta \left( E_{t} \left( \hat{V}_{t+1} \gamma_{N,t+1} \right)^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}} \tag{B.2}$$

$$\hat{u}_t = \hat{C}_t \, (\bar{L} - L_t)^{\chi} \tag{B.3}$$

$$\gamma_{N,t+1}\hat{K}_{t+1} = \left(1 - \delta_K \left(x_{K,t}\right)^{\xi_K}\right)\hat{K}_t + \Lambda_K \left(\frac{\hat{I}_t}{\hat{K}_t}\right)\hat{K}_t \tag{B.4}$$

$$\Lambda_K\left(\frac{\hat{I}_t}{\hat{K}_t}\right) = a_{K,1} + \frac{a_{K,2}}{1 - \frac{1}{\tau_K}} \left(\frac{\hat{I}_t}{\hat{K}_t}\right)^{1 - \frac{1}{\tau_K}}$$
(B.5)

$$\Lambda_{K,t}' = a_{K,2} \left(\frac{\hat{I}_t}{\hat{K}_t}\right)^{-\frac{1}{\tau_K}} \tag{B.6}$$

$$\gamma_{N,t+1} = \left(1 - \delta_N \left(x_{N,t}\right)^{\xi_N}\right) + \Lambda_N \left(\hat{S}_t\right)$$
(B.7)

$$\Lambda_N(\hat{S}_t) = a_{N,1} + \frac{a_{N,2}}{1 - \frac{1}{\tau_N}} \hat{S}_t^{1 - \frac{1}{\tau_N}}$$
(B.8)

$$\Lambda'_{N,t} = a_{N,2} \hat{S}_t^{-\frac{1}{\tau_N}}$$
(B.9)

$$M_{t,t+1} = \beta \gamma_{N,t+1}^{-\frac{1}{\psi}} \left(\frac{\hat{u}_{t+1}}{\hat{u}_t}\right)^{1-\frac{1}{\psi}} \frac{\hat{C}_t}{\hat{C}_{t+1}} \left(\frac{\hat{V}_{t+1}}{\left(E_t \hat{V}_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$$
(B.10)

$$\chi \frac{\hat{C}_t}{\bar{L} - L_t} = \hat{w}_t \tag{B.11}$$

$$1 = q_{K,t} \Lambda'_{K,t}$$
(B.12)

$$q_{K,t} = E_t M_{t,t+1} \left( r_{K,t+1} x_{K,t+1} + q_{K,t+1} \left( 1 - \delta_K \left( x_{K,t+1} \right)^{\xi_K} - \Lambda'_{K,t+1} \frac{\hat{I}_{t+1}}{\hat{K}_{t+1}} + \Lambda_{K,t+1} \right) \right)$$
(B.13)

$$r_{K,t} = q_{K,t} \delta_K \xi_K (x_{K,t})^{\xi_K - 1}$$
(B.14)

$$1 = q_{N,t} \Lambda'_{N,t} \tag{B.15}$$

$$q_{N,t} = E_t M_{t,t+1} \left( r_{N,t+1} x_{N,t+1} + q_{N,t+1} \left( 1 - \delta_N \left( x_{N,t+1} \right)^{\xi_N} - \Lambda'_{N,t+1} \hat{S}_{t+1} + \Lambda_{N,t+1} \right) \right)$$
(B.16)

$$r_{N,t} = q_{N,t} \delta_N \xi_N (x_{N,t})^{\xi_N - 1}$$
(B.17)

$$1 = E_t M_{t,t+1} \frac{R_t}{\pi_{t+1}}$$
(B.18)

$$\hat{w}_t = mc_t \left(1 - \alpha\right) \frac{\hat{Y}_t}{L_t} \tag{B.19}$$

$$r_{K,t} = mc_t \alpha \frac{\hat{Y}_t}{x_{K,t} \hat{K}_t}$$
(B.20)

$$r_{N,t} = mc_t \left(1 - \alpha\right) \eta \frac{\hat{Y}_t}{x_{N,t}}$$
(B.21)

$$\phi_P\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi} = \phi_P E_t M_{t,t+1}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}\frac{\hat{Y}_{t+1}}{\hat{Y}_t}\gamma_{N,t+1} + 1 - \varepsilon + \varepsilon mc_t \qquad (B.22)$$

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\rho_{\pi}} \left(\frac{\hat{Y}_t}{\hat{Y}_{t-1}}\frac{\gamma_{N,t}}{\gamma_N}\right)^{\rho_Y}$$
(B.23)

$$\hat{Y}_t = \left(x_{K,t}\hat{K}_t\right)^{\alpha} \left(\hat{Z}_t L_t\right)^{1-\alpha} \tag{B.24}$$

$$\hat{Z}_t = A_t x_{N,t} \tag{B.25}$$

$$\hat{Y}_{t} = \hat{C}_{t} + \hat{I}_{t} + \hat{S}_{t} + \frac{\phi_{P}}{2} \left(\frac{\pi_{t}}{\pi} - 1\right)^{2} \hat{Y}_{t}$$
(B.26)

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A$$
(B.27)

$$\log \sigma_t^A = (1 - \rho_{\sigma^A}) \log \sigma^A + \rho_{\sigma^A} \log \sigma_{t-1}^A + \sigma^{\sigma^A} \varepsilon_t^{\sigma^A}$$
(B.28)



Figure B.10: Uncertainty Shock with Different Levels of Risk Aversion



Figure B.11: Uncertainty Shock with Different Levels of Price Stickiness



Figure B.12: Uncertainty Shock with Different Levels of Technological Spillovers



Figure B.13: Uncertainty Shock with Different Levels of Capital Adjustment Costs



Figure B.14: Uncertainty Shock with Different Levels of R&D Adjustment Costs