EXPLORATORY RESEARCH ON UNDERSTANDING PROOF BY COINCIDENCE: BUILDING AN ANALYTICAL FRAMEWORK BY COMPARING JAPANESE TEXTBOOKS

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INTRODUCTION

"Proof by coincidence" is a type of indirect proof defined as follows:

> Coincidence method consists in constructing a geometric figure which possesses all the properties in question and then showing that it coincides with the given geometric figure (Shute, Shirk, Porter, & Irwin, 1949, p. 68).

While there is an opinion that proof by coincidence is not an indirect proof (cf. Byham, 1969), it is usually treated as such in Japan (The Japan Society of Mathematical Education, 2013). The seven standard 9th-grade mathematics textbooks in Japan prove the Converse of the Pythagorean Theorem (CPT) by this method, although three textbooks apply it only in a single case. In general, research on indirect proof tends to focus only on proof by contradiction and proof by contrapositive (e.g., Antonini & Mariotti, 2008). Therefore, when researching indirect proofs, we should pay more attention to proof by coincidence.

The purpose of this paper is to build an analytical framework for empirical studies seeking to understand proof by coincidence. Due to a dearth of scientific literature regarding how to teach this topic, it is likely that textbooks have a high impact on its teaching practices, relative to other topics. To hypothesize the potential problems of teaching proof by coincidence, we will investigate the variety of ways in which the Pythagorean Theorem (PT) and the CPT are presented in Japanese textbooks.

METHOD AND RESULT

We examined all seven junior high school mathematics textbooks, focusing on the differences among the figures and the centered content. Table 1 shows a comparison of the figures for the PT and the CPT. Publishers *A*, *B*, and *C* use the same figure for both theorems, while the other four use different figures. The presence or absence of a right-angle mark is the key difference between Type I and Types II/III.

Table 2 shows the centered content used to present the CPT. When presenting the PT, all the textbooks center $a^2+b^2=c^2$, but they differ in their presentation of the CPT. Publishers *A*, *E*, *F*, and *G* center the antecedent $a^2+b^2=c^2$, which indicates that they consider the emphatic points of the PT and the CPT to be the same.

In Rezat, S., Fan, L., Hattermann, M., Schumacher, J., & Wuschke, H. (Eds.). (2019). *Proceedings of the Third International Conference on Mathematics Textbook Research and Development* (pp. 389–390). Paderborn: Universitätsbibliothek Paderborn.

Type	PT	CPT	Publisher
F1	Type I	Type I	*A*, *B*, *C*[*1]
F2	Type I	Type II	*D*, *F*, *G*
F3	Type I	Type III	*E*

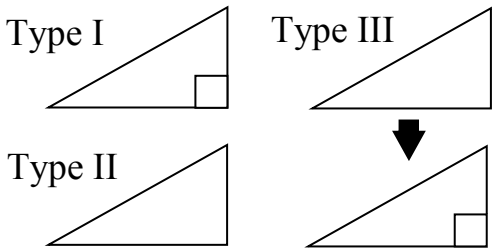

Table 1. Comparison between the PT and the CPT figures (*[1] Publisher *C* uses Type I for the CPT but with a different color.)

Type	Centered contents	Publisher
C1	$a^2+b^2=c^2$	*A*, *E*, *F*, *G*
C2	Nothing	*B*, *D*
C3	if $a^2+b^2=c^2$, then $\angle C = 90^\circ$	*C*

Table 2. Centered contents of the CPT

DISCUSSION

When we prove the CPT by coincidence, we must refer to the PT in the proof. As this can be very confusing for students, an effort should be made to resolve this confusion. However, as shown in Tables 1 and 2, the figures and centered contents in some textbooks do not differ for the PT and the CPT (Types F1 and C1). It is noteworthy that we are unsure whether the differences in the ways that textbooks present the PT and the CPT can contribute to resolving students' confusion. Therefore, empirical research on students' process of understanding proof by coincidence should be conducted from this point of view. Thus, we raise the following question: what influence does a teacher's way of summarizing the process of proving by coincidence have on students' understanding?

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References

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Shute, W. G., Shirk, W. W., Porter, G. F., & Irwin, M. E. (1949). *Plane geometry*. American Book Company.

The Japan Society of Mathematical Education (Ed.). (2013). Method of indirect proof. In *Japanese-English/English-Japanese Dictionary for Practical School Mathematics Usage* (p. 418). Tokyo: Toyokan Publishing.