THE DIALECTIC RELATIONSHIP BETWEEN THEORY AND PRACTICE IN MATHEMATICS TEACHER EDUCATION

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Abstract

The study presented here is a part of ongoing research, in which 'situated knowledge' and 'cognitive apprenticeship' form the framework that allows us to deal with the knowledge and the learning process of pre-service elementary school teachers. Our aim is to determine how student teachers use conceptual tools provided in a mathematics methods course. The search context of this paper is the curricular analysis of the textbooks. Our research has shown the difficulty involved in using different conceptual tools to solve a proposed task. We have observed how, in situations in which the integration of conceptual tools has been achieved, the student teachers have found distinct features that lead to different decisions. This shows the professional relevance of such integration and points out the need of advancing in that research agenda.

Introduction

Efforts made in recent years to articulate mathematics teacher education programs have met with several problems. On the one hand, there is hardly any tradition in the co-ordination of educational innovation and teacher learning research. On the other hand, teacher-learning theories that can provide conceptual frameworks for our comprehension of the learning-to-teach process must be generated. In this sense, we have attempted to establish a dialectical relationship between educational innovation and teacher education research, between theory and practice. The information acquired from the analysis of practice provides the elements that allow us to start building up a theoretical framework. But, at the same time, theoretical references enable our comprehension of the variables in teacher learning to be extended, influencing task design. In a context of learning to teach mathematics, we should be aware that identifying domains of curricular content is as important as the way in which they are considered and incorporated into the methodological process.

The use of theoretical information from mathematics education research as conceptual knowledge becomes operative in solving tasks specially designed to articulate the process of learning to teach Mathematics. In particular, our overall aim is to achieve coherence with the theoretical perspective adopted, which considers 'situated knowledge' and 'cognitive apprenticeship' as the framework that allows us to deal with the knowledge and the learning process of the student teacher (Collins et al. 1989; García, 2000, in press; Llinares, 1994, 1999). The collaborative work between theory (researchers) and practice (mathematics teacher educators) has specific characteristics in our case. We are aware of assuming the two roles, which change depending on the particular aim of the research project.

An approach to the characterisation of learning to teach

Pre-service teacher development is a process during which knowledge and modes of reasoning similar to those of the expert should be acquired. Among the features that characterise this process, the following may be cited:

- It occurs through active participation in a context defined by "authentic activities" (understood as ordinary cultural practices (Brown et al, 1989)),
- Learning is based on participation in different activities, with the help of the teacher educator,
- The activity acquires full meaning from prior knowledge and beliefs,
- Participation in the different activities can increase or modify these concepts.

We also think that social practice is an integral and inseparable part of learning. According to Hanks' Foreword to Lave & Wenger (1991), 'This central concept [legitimate peripheral participation] denotes the particular mode of engagement of a learner who participated in the actual practice of an expert, but only to a limited degree and with limited responsibility for the ultimate product as a whole' (p. 14). This participation takes place in 'communities of practice' that portray a social group in which its members share a given activity. Although we are aware that pre-service student teachers of mathematics do not initially belong to the 'community of practice' of mathematics teachers, we do acknowledge that teacher education programs must provide the means to qualify them for becoming members of that community. These programs must therefore favour student participation in so-called 'apprentice communities'. We may describe these communities through 'learning environments' defined by the following elements: relevant tasks, active participation within the given context, group work, consideration of prior knowledge and beliefs and specification of reasoning processes (Llinares, 1999, 2002; García, 2000, in press).

This conception of situated learning causes several ideas related to the generation of the teacher's knowledge to emerge. Among these ideas, the integral nature of this knowledge, its continual development resulting from its use in new tasks, and the unending teacher training understood as continuous learning going beyond the initial education program, may be pointed out. The learning process of the future teacher may be seen as a "specific" reproductive cycle (Lave & Wenger, 1991) in which knowledge is integrated into the activity.

Some implications for primary school teacher education programs

Teacher education programs from the Didactics of Mathematics may be understood as the process of introducing pre-service elementary school teachers into the community of practice of mathematics teachers. Becoming an elementary school mathematics teacher means acquiring an understanding of the teaching of mathematics, learning to carry out teaching tasks. It also means learning to use and justify the tools involved in professional tasks like planning, assessment, task design, choice of textbook and curricular materials, and so on.

Moreover, the term 'tool' not only denotes a physical object. It extends its meaning including the concepts and reasoning, etc., which enable and influence interaction within a community. In this context, such tools may be classified as either *technical or conceptual tools*. *Technical tools* are those tools entailed in the 'practice', such as teaching materials and software, techniques for managing discussion of procedures and answers to problems, etc. *Conceptual tools* are understood as those concepts and theoretical constructs that have been generated from research in mathematics teacher education leading to understanding and handling situations in which mathematics is taught and learned (Llinares, in press).

From this perspective, our program is articulated in tasks in which meanings related to the components of knowledge necessary to teach mathematics may be shared, discussed and negotiated (Sánchez, 1997). The activity becomes the core of a learning process, considered as a cognitive process and a means of participating, generated while attempting to carry out tasks (Llinares, 2002) that are similar to those of mathematics teachers, but without the responsibility of teachers (García, 2000, in press). Furthermore, considering the student teacher as a reflective individual leads us to assume that his/her knowledge can be built up from reflecting on what has been accomplished. In the relationship between thinking and learning, the student should consider conceptual knowledge acquired from research (i.e., information about mathematical concepts, learning processes, common mistakes regarding certain topics, and so on) as conceptual tools.

The study presented here is a part of ongoing research, which aims at determining how student teachers use conceptual tools provided in specially designed 'training itineraries' (García, 2000). We focus on the curricular analysis of the textbooks as the **search context** of this paper. Such an analysis is a typical task of elementary school mathematics teaching practice. The theoretical element object of our study is **'the use of conceptual tools in curricular analysis'**. This is understood to be 'the simultaneous setting in motion of the different tools, interaction and communication of the information coming from them leading to reasonable decisions'.

In particular, the curricular analysis centred on multiplicative structure arithmetic problems proposed in various elementary school textbooks in our country. Student teachers were provided with conceptual tools through articles, videos and information given them by the teacher educator. These tools included: different problem typologies of multiplicative structures and different perspectives of analysis (Vergnaud (1991), Nesher (1992)), learning difficulties associated with these problems and facilitating features, and support teaching/learning such as the relationship between the characteristics of comprehension of a situation and the use of mathematical symbols (translation process). We aimed to observe:

- The elements that appeared to be the grounds for their decisions,
- The characteristics of these elements, and
- Other aspects related to the difficulties that were considered.

In short, an attempt was made to see how these students used the conceptual tools to carry out the selected professional task (the analysis of textbook problems).

Methodology

Participants: The study included 130 primary school student teachers enrolled in two mathematics methods courses with similar characteristics. Following theircriteria, they were divided into 23 small groups (13 and 10 respectively). The groups were made up of 4 to 7 students.

Data collection procedures: Two different instruments were used as data sources in this study: a prepared list of additive and multiplicative classroom problems and a designed task that was one of the habitual tasks proposed in our course.

- *The set of problems.* Ten classroom problems of varying characteristics were selected. Five of them were additive structures and five multiplicative structures. The 10 word problems were presented in random order, and were given to each student for identification. They should include the justification of their answers. Our intention was for the students to analyse them individually, making use of the conceptual tools provided. Different structures were chosen because one of our aims was structure identification.

- *The task.* We chose two collections of classroom textbooks from different publishers that are very popular among primary teachers (labelled Publisher 1 (P1) and Publisher 2 (P2)). Although the texts chosen were not thoroughly analysed, some marked differences were considered:
- Inclusion or not of certain characteristics that match the traditional culture of primary school mathematics practice (subject revisions, recapitulations, etc.)
- Inclusion or not of supporting illustrations in the introduction of concepts and problems
- Integration of cross contents (other subjects or other mathematics topics).

The content was reviewed and all the pages relating to multiplicative structure problems were selected. These pages were used to elaborate two ‘abridged books’, one for each publisher. The first page described the professional task (a teacher must choose the textbook for his/her students) and posed several questions which considered the variables to be observed. Among these questions were: What were the assessment criteria that enabled you to make your decisions? Following these criteria, do you agree with the content? Do you agree with the organisation and presentation of that content? and so on. In other words, the first page was designed to summarize the characteristics of a situation that is habitual in teaching practice and initiate thought about that situation with the questions proposed. On the other hand, the overall task should allow to the student teachers to be able to situate both in a personal level and in a professional level. This enabled us to identify the various references employed in the answering process.

Once the task had been designed, it was analysed and some confusing points were identified and rectified or clarified. When the task was definitively configured, 46 abridged books, two (one for each Publisher) for each group of students, were prepared. Finally, the other elements in the 'training itinerary' related to the conceptual tools mentioned above were completed (García, 2000; Escudero et al, 2002; García and Sánchez, 2002). The abridged books were given to the different groups of students. Although the task was initially proposed in the classroom, the groups carried out the work by meeting as many times as they considered necessary by their own criteria, and taking all the time they required to discuss their choices. In coherence with the above-mentioned perspective, we insisted on the task being carried out in the groups. When the tasks were finished, each group made a report, in which all the answers were collected, including the arguments that had led them to their final decision.

Data analysis: The individual answers related to the identification of the set of problems were categorised on the basis of whether the student had identified (or not) the type of structure. When the identification was correct, we observed whether this identification had been made based on theoretical information and finally, whether it was based on the conceptual tools involved.

The reports were analysed by the following inductive process:

First, units of analysis were identified within the answers of the different groups, which were classified into the following sections:

- Criteria mentioned
- Elements considered basic (coherence with the criteria?)
- Presence of the theoretical information given in previous elements
- How difficulties (if any) are considered in the sequence set out in the introductions to the texts
- Establishment (or not) of relationships between the types of problems/difficulties in that sequence
- Relationships with cross contents.

Based on the above, the characteristics identified for each group (considered jointly) allowed four categories to be devised:

- Groups of students that based their answers on previous experience (from school, student teaching in the previous academic year).
- Groups of students that identified the tools provided, but did not incorporate these tools as decision-making elements.
- Groups of students that used and identified the tools provided.
- Groups of students that identified and used conceptual tools, incorporating the relationships among them in a more general framework.

Results

The analysis of the data showed that 76.7% of the students were able to individually identify the type of structure in all the sets of problems proposed. Within the students that identified the five multiplicative structure problems correctly, 38.9% classified them properly, but only 35.6% analysed the elements involved in the classification correctly. These results led us to delve into the cause of this low rate of identification and it was found that a problem of multiplicative comparison (see Table 1), which accumulated the greatest number of mistakes, was the cause. This shows the particularities of multiplicative-comparison problems, mentioned among others by Greer (1992), in Spanish-speaking student teachers.

Pedro has 8 marbles. Juan has 3 times as many marbles as Pedro. How many marbles does Juan have?

Table 1

On the other hand, the above-mentioned categories allowed different levels of use of the conceptual tools by the student teachers in the execution of the professional task to be identified. On a first level, the students are clearly situated at a personal stage, based on previous experiences. They do not identify the conceptual tools as useful in carrying out this task. A detailed analysis of the reports by these groups allowed us to appreciate that characteristics of the abridged books such as initial check/presence of drawings/final check, and criteria of what, how and what for in judging the contents, that is, aspects that they had found 'useful' in their student teaching experience, may have influenced their decisions. Six groups at this level selected P1 and one group chose P2. The comments quoted below illustrate these ideas:

- *'The elements that support our choice are the following:*
- *Problem structure. In general, problems are illustrated by drawings that facilitate visual comprehension*
- *The final revision is rather detailed and constructive*
- *The contents are developed at length and there is a brief summary at the beginning '* (Group 2)

On a second level, students were able to state that elements 'appeared' and detected when they 'did not appear', but they did not relate the presence/absence to any other aspect. The following Group 5 response is representative:

- '*In general, we think that some contents are missing in P1. All types of problems appear except for the following: quotative division, Cartesian multiplication and multiple proportions. At first, this led us to think that the other publisher (P2) was better, since we found all the types of multiplicative structure problems. However, the problem presentation (the drawings) and organisation (pages with too many problems) led us to choose the first Publisher (P1)'* (Group 5).

On this level, it may be seen how some groups used conceptual elements acquired from other subjects. This can be considered positive, since it represents the presence of transversal knowledge, which has been transferred from other subject matters. Nevertheless, just as in the case of the conceptual tools provided in our training itinerary, their use does not go beyond their identification. The most groups (eleven) were found to be on this level. Seven groups chose P1 and four P2.

At the third level there are two sublevels, depending on how the role of difficulties in content organisation is considered. At the first sublevel, there are three groups. All of them related these difficulties to difficulties of types and characteristics of multiplicative structure problems.

• *'... we assessed the problems' quantity, quality and difficulty. Regarding quantity, we assessed positively that the textbook includes a greater number of problems. As for quality, we assessed positively appreciate that the textbook includes a wider variety of problems ... mapping rule, Cartesian multiplication, multiplicative comparison. With respect to difficulties, we judge the order of presentation of the different types of problems which takes into account these difficulties positive ...'* (Group 21)

In the other sublevel, there is only one group. This group related the difficulties to the conceptual level of hypothetical pupils, a wider idea that would include both the multiplicative and additive structures. The following is representative of the student teachers' explanations:

• *'.. in Publisher P2 ... it may be said that problems are introduced to make students transform addition into multiplication using direct modelling, since they are urged to use counters… '* (Group 12)

In the above-mentioned quotations, conceptual tools related to problem typology, learning difficulties and features facilitating the problem solving process are integrated. Practically all these groups chose P2.

Finally, there was only one group on the highest level, in which the conceptual tools were identified and used, incorporating the relationships among them in a more general framework.

'We did not choose Publisher 1 because:
- *All the problems mentioned by Nesher were not present ... order of difficulty of the problems is not considered given that the limited problems implying multiplicative comparison and Cartesian multiplication are treated superficially and no systematically. In other words, the Publisher considers them to be secondary problems and mere curiosities.*
- *The language used is complex and abstract.*
- *The exercises suggest an individual-oriented work methodology, obviating the advantages of students working in groups.*
- *Mental arithmetic is not sufficiently fostered.*' (Group 9)

Typologies of problems, learning difficulties, language, mental arithmetic, etc. were integrated by this group. The group chose Publisher P2.

It is important to underline that the choice of publisher made by the groups changed from P1 to P2 as the level progressed. This can show that greater integration and relating of conceptual tools allows other aspects that influence decision-making to be identified.

On a whole, the above-mentioned levels concerned the use that student teachers made of conceptual tools in carrying out a professional task, 16 of the 23 groups were able to identify the type of multiplicative problems in the abridged books and to some extent, considered the inclusion of different types of problems important. Nevertheless, only 5 of these groups established relationships among the different conceptual tools provided. We think these relationships to be very important in the teachers' instructional practices. Furthermore, it should be pointed out that the conceptual tool related to the different typologies and perspectives of analysis of multiplicative structure arithmetic problems was the theoretical instrument that the student teachers identified best.

Our research has shown the difficulty of establishing relationships among different conceptual tools, that is, the difficulty that implies their use in solving a task or, in other words, the difficulty of the translation process. This process, basic in the generation of meaningful learning, acquires essential relevance when a situated perspective is adopted, especially in pre-service elementary school teacher education programs. Program configuration through professional tasks allows student teachers to translate concepts, ideas and ways of reasoning into the process of solving those tasks. This implies the use of knowledge other than the propositional knowledge that is traditionally appraised in some teacher education programs. We think that one of the objectives for future research in mathematics teacher education should be the search for professional tasks that can be incorporated into mathematics teacher education programs and that foster the use of conceptual tools.

Conclusions: The dialectic relationship between theory and practice in teacher education

In conclusion, some ideas should be emphasized. Our research has shown that the learning environment generated in primary school teacher training enabled some of these student teachers to identify individually and collectively the conceptual tools provided. Nevertheless, integration of these instruments has been shown to be more complex. We have seen how, in situations in which such integration was achieved, the task was carried out differently, the students were aware of and evaluated distinct features that led to different decisions (e.g., to choose a different textbook). This shows the professional relevance of such integration and points out the need of advancing in this subject, extending this research agenda to both pre-service secondary mathematics teacher education and in-service programs at all levels.

Finally, we would like to get back to our practice. Research projects are not limited in time and scope. But the mathematics teacher educator practice is time-limited (an academic year), and has the main goal that student teachers become mathematics teachers. How many tasks are necessary to achieve this goal? What tasks? How about the diversity of student teachers? More feedback from the practice is required before returning to theory.

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